Non-perturbative QCD for Nuclear Physicists

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LECTURE I

Aim of lectures

To give an introduction to QCD intended for those interested in pursuing research in nuclear physics. Lecture are very selective...

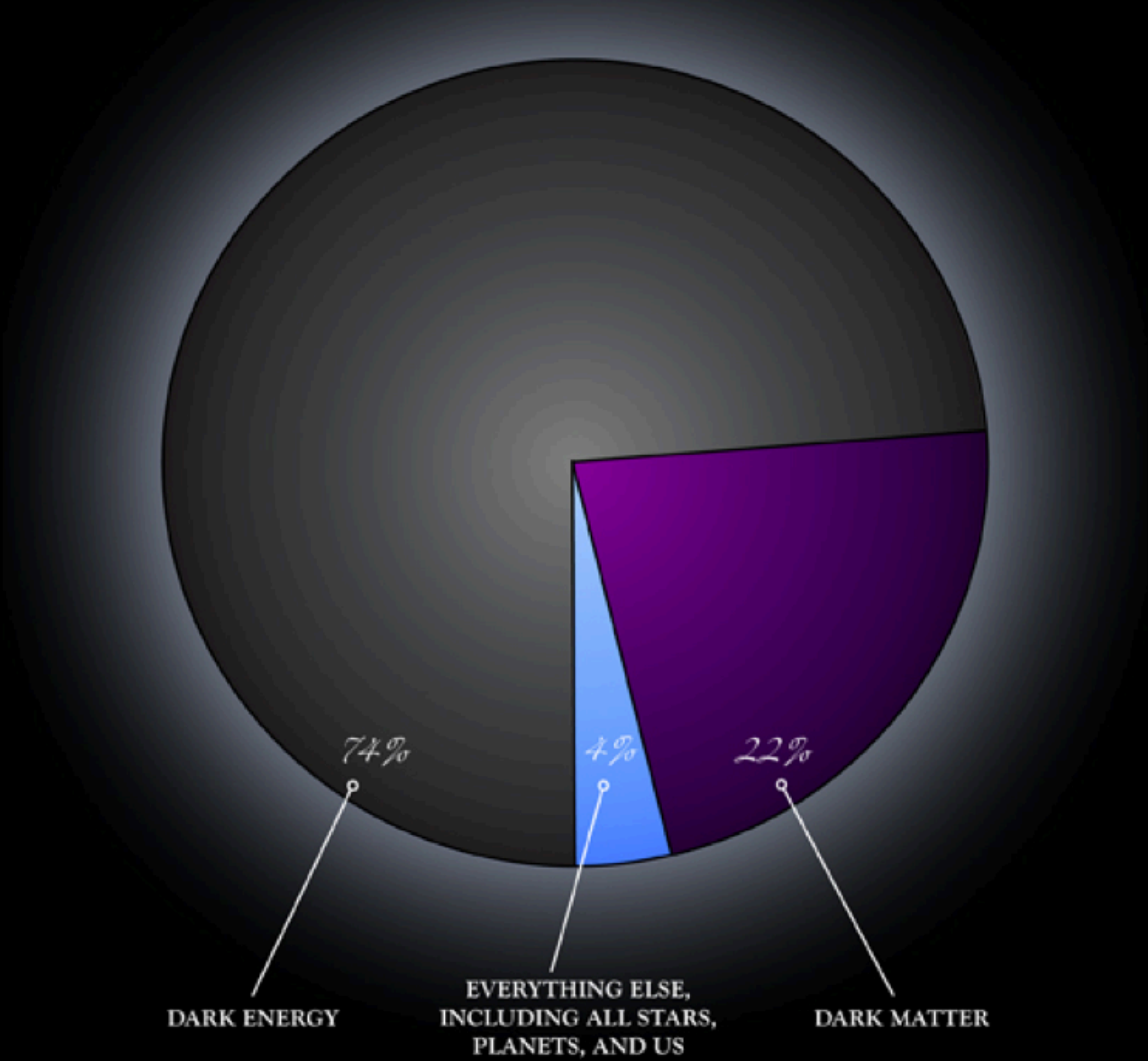
Organization

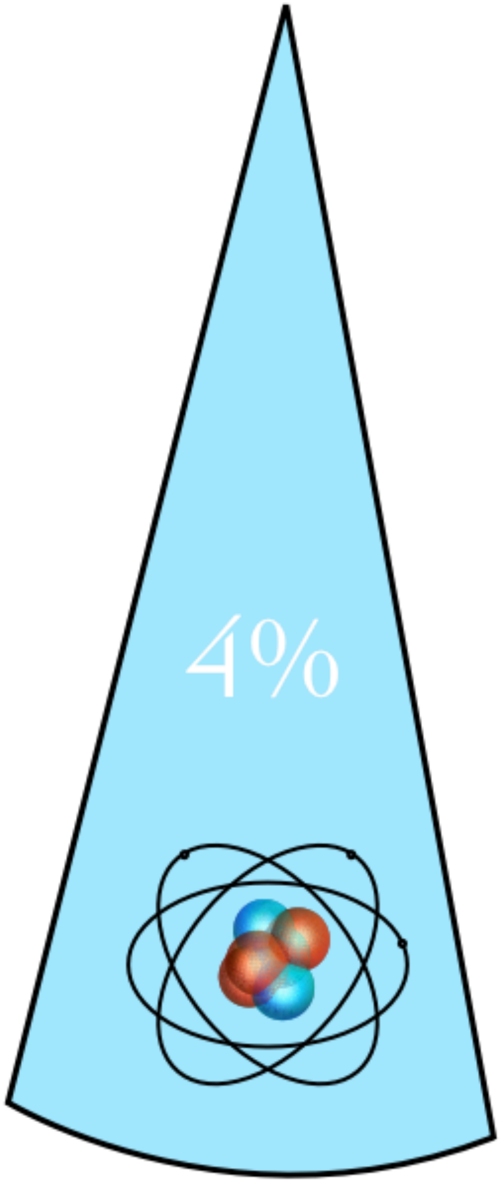
- Lecture I: Intro/EFT/ChiPT
- Lecture II: ChiPT(cont)/LQCD
- Lecture III: Nuclear Physics from QCD

Lecture I:

- Introduction
- Effective Field Theory
- ChiPT: a primer

Energy Budget of the Universe



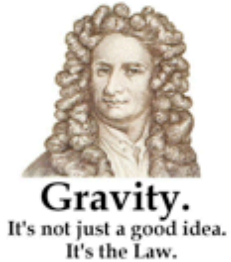


Nuclear physicists are interested in understanding this 4% quantitatively!

The Nature of the Visible Matter in the Universe

Dictated by the Standard Model:

gravity



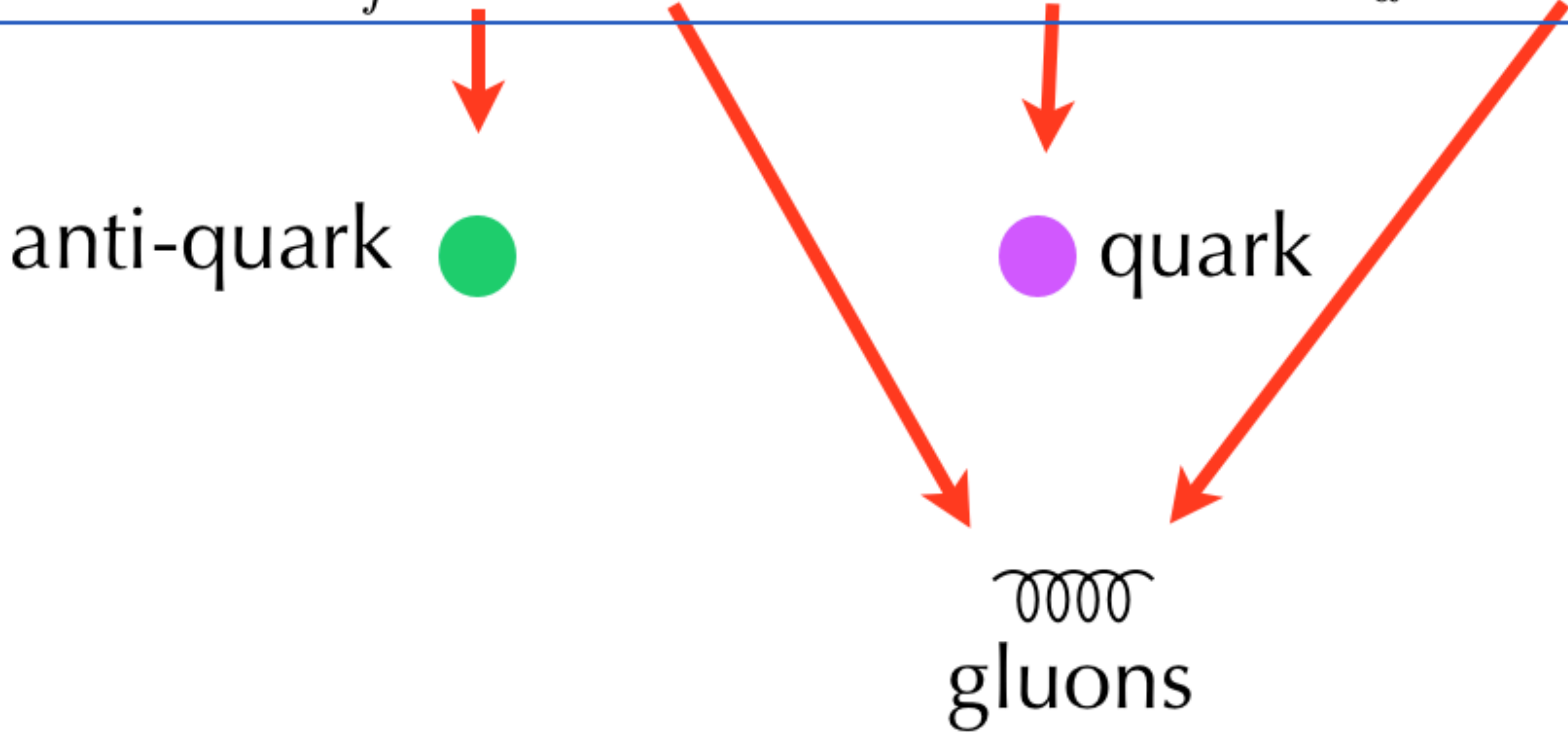
electromagnetism:QED

weak interaction

strong interaction: QCD

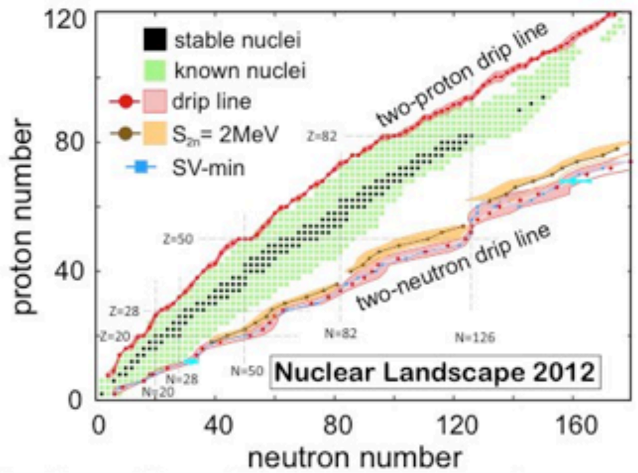
QCD looks simple!!

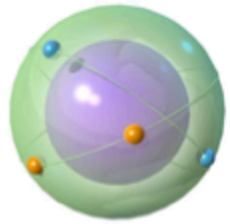
$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\psi}_f \left[D_\mu \gamma_\mu + m_f \right] \psi_f + \frac{1}{4} \sum_a G^a_{\mu\nu} G^{a\mu\nu}$$



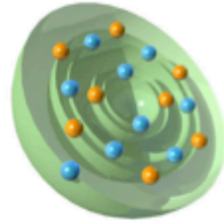
Complicated <u>many-body</u> problem

★ Together with E&M QCD must give:



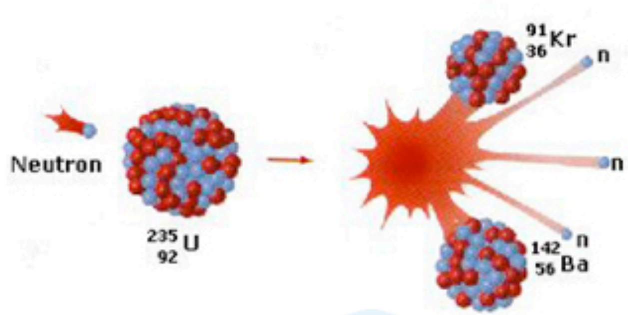
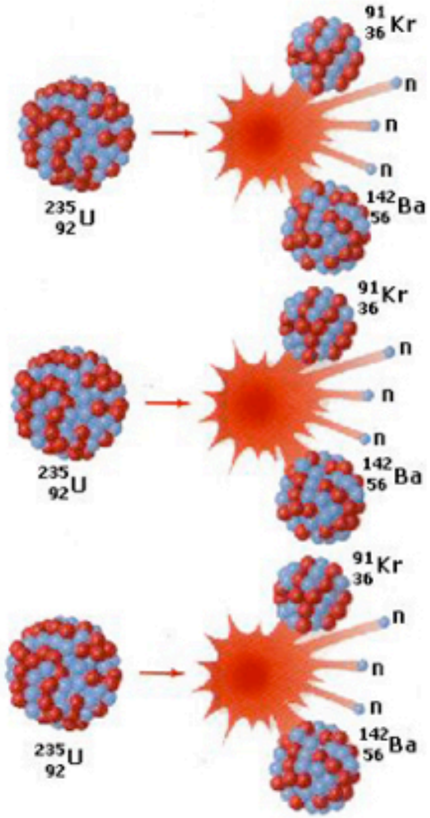


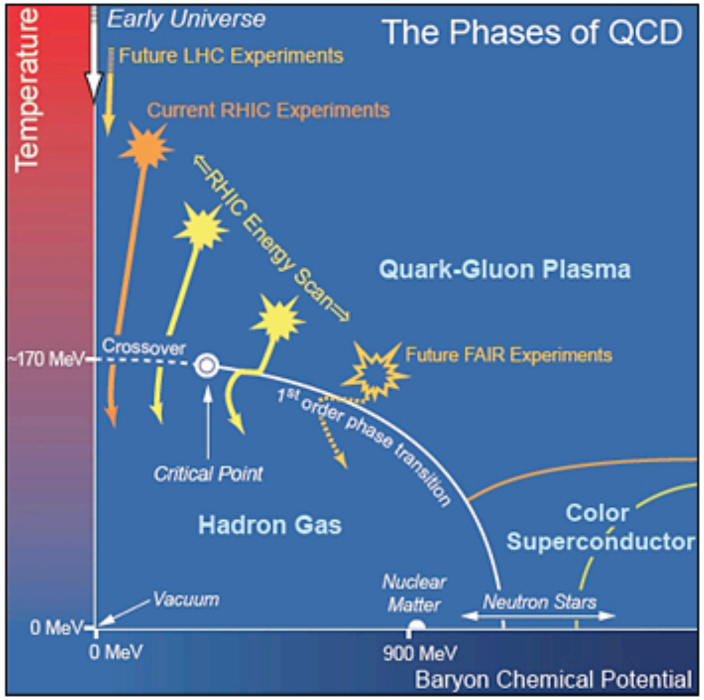
Spin-pairing

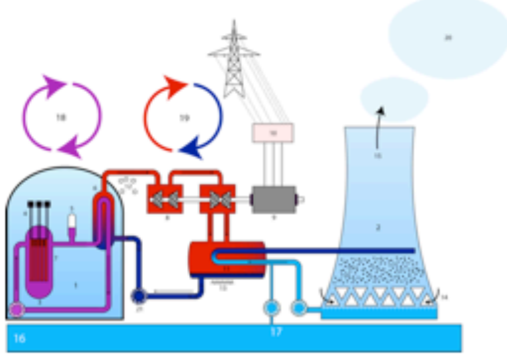


Shell-structure

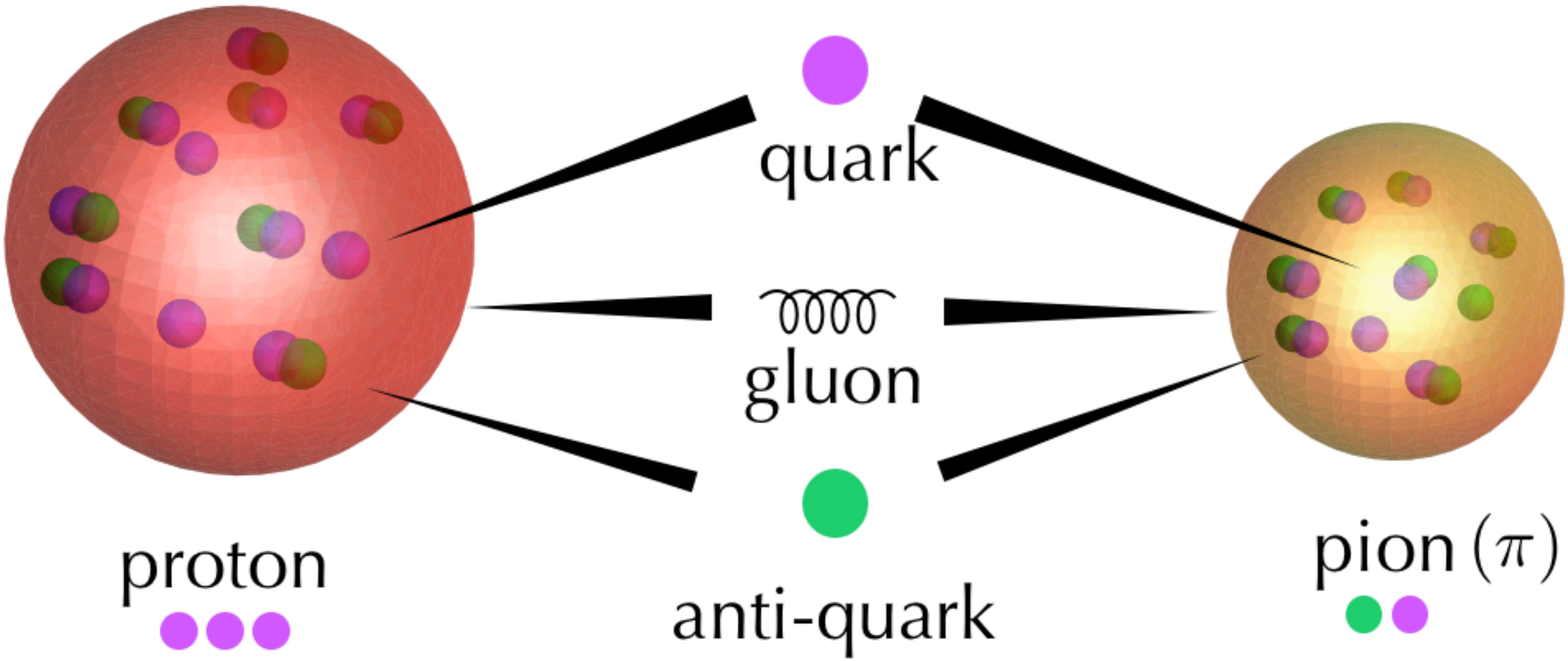
Vibrational and rotational excitations



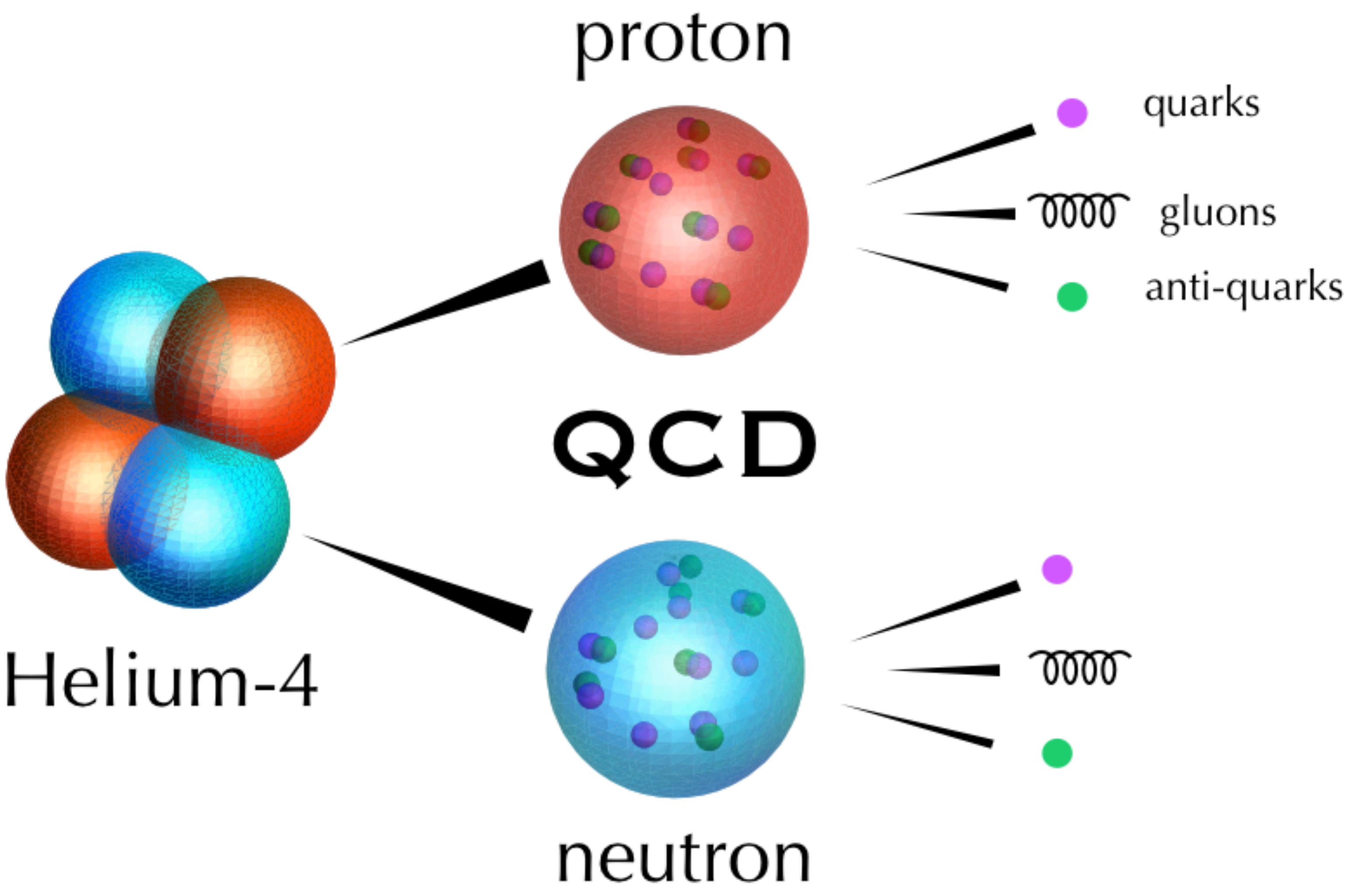


Even simplest systems are complex:



Requires high-performance computing!!

Nuclear Physics: two layers of complexity!!



✓ Nuclear physics from first principles is hard!

Current Lines of Attack

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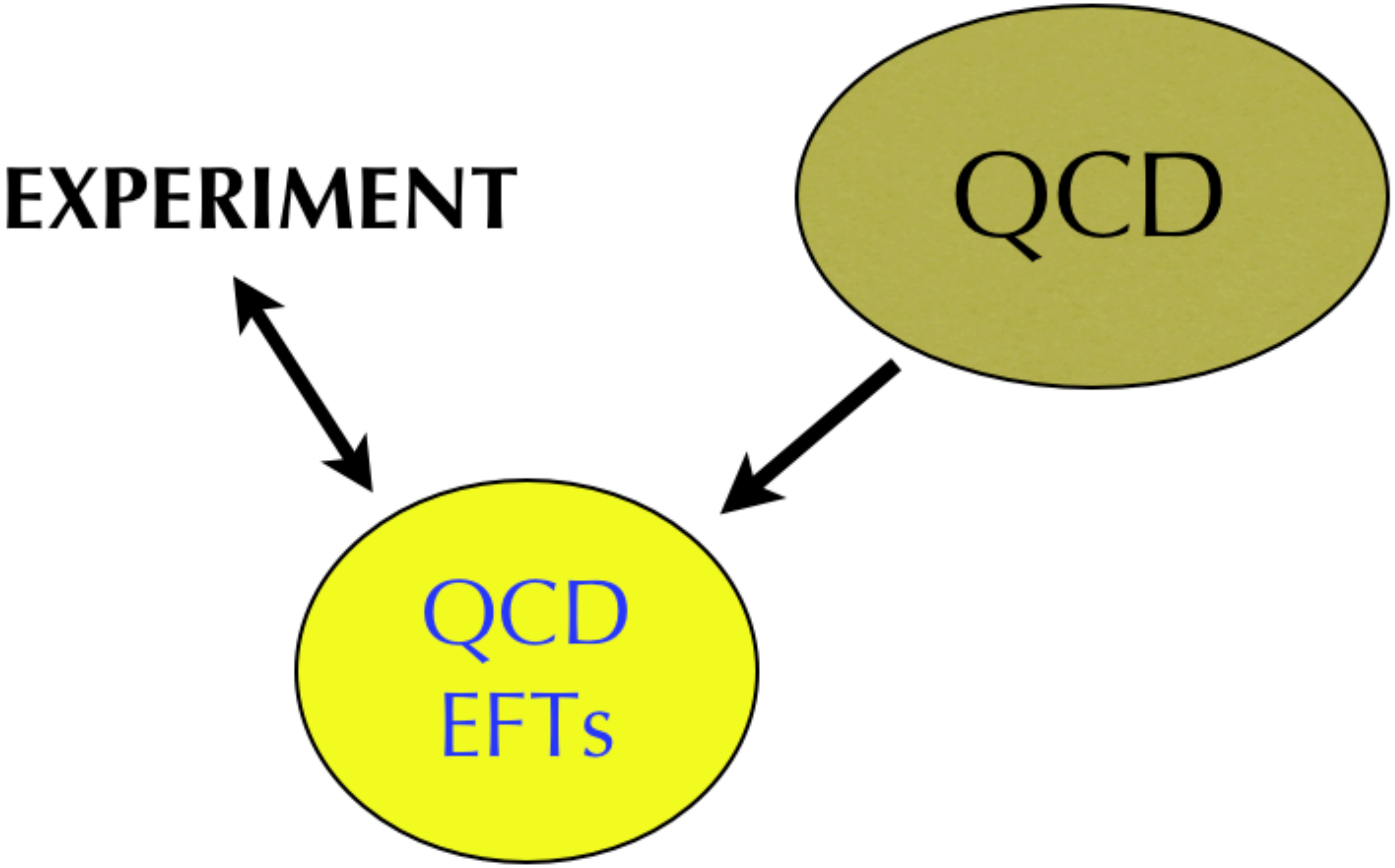
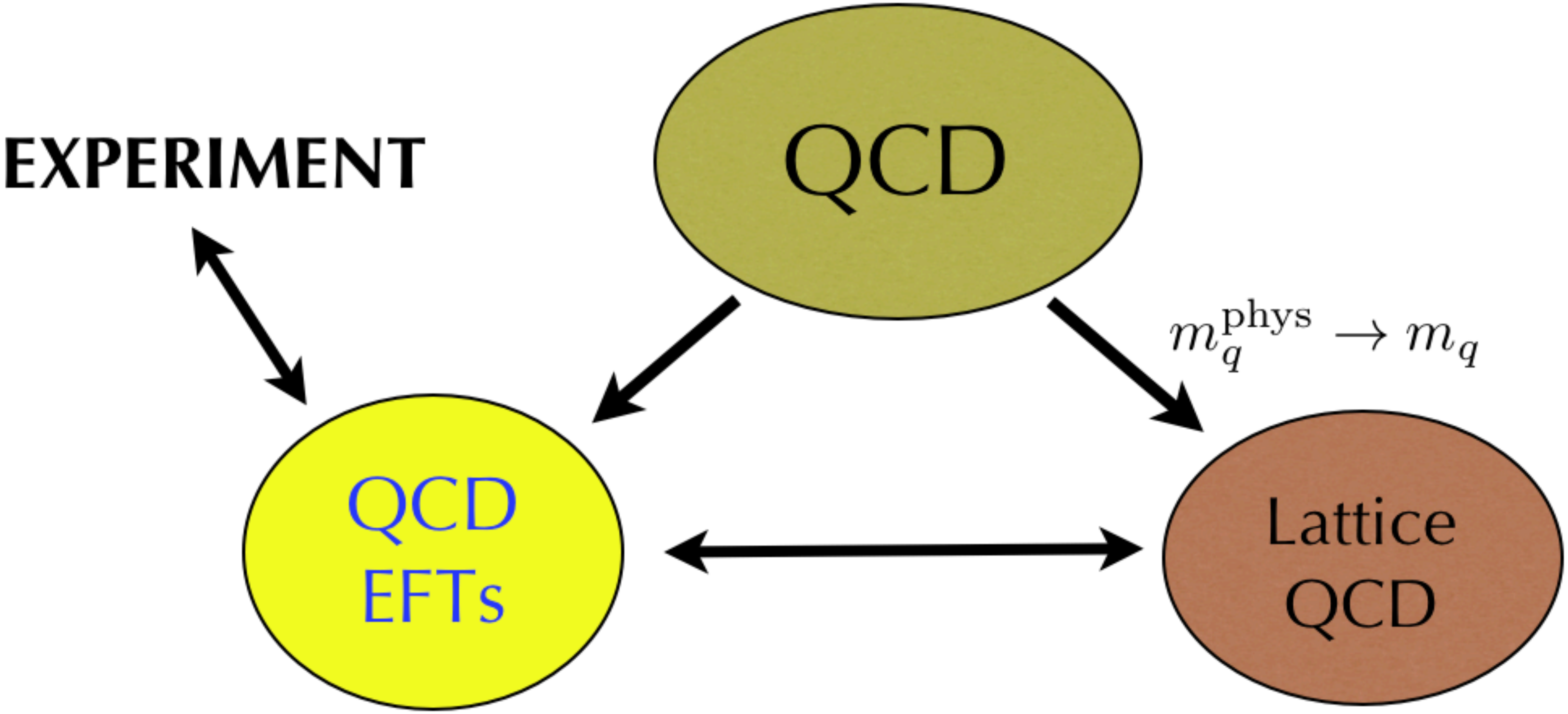
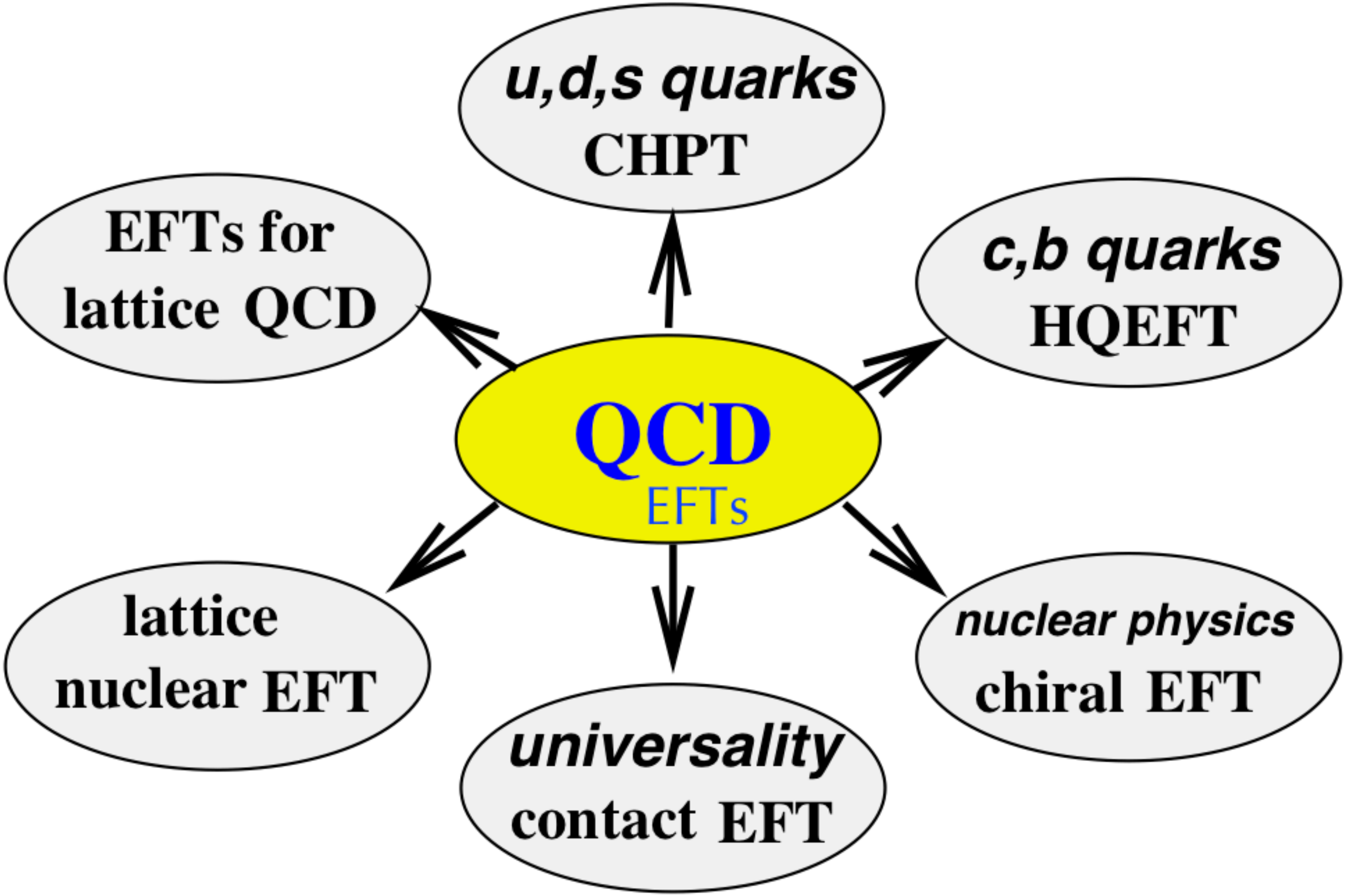


Current Lines of Attack




u,d,s quarks
CHPT
EFTs for
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HQEFT
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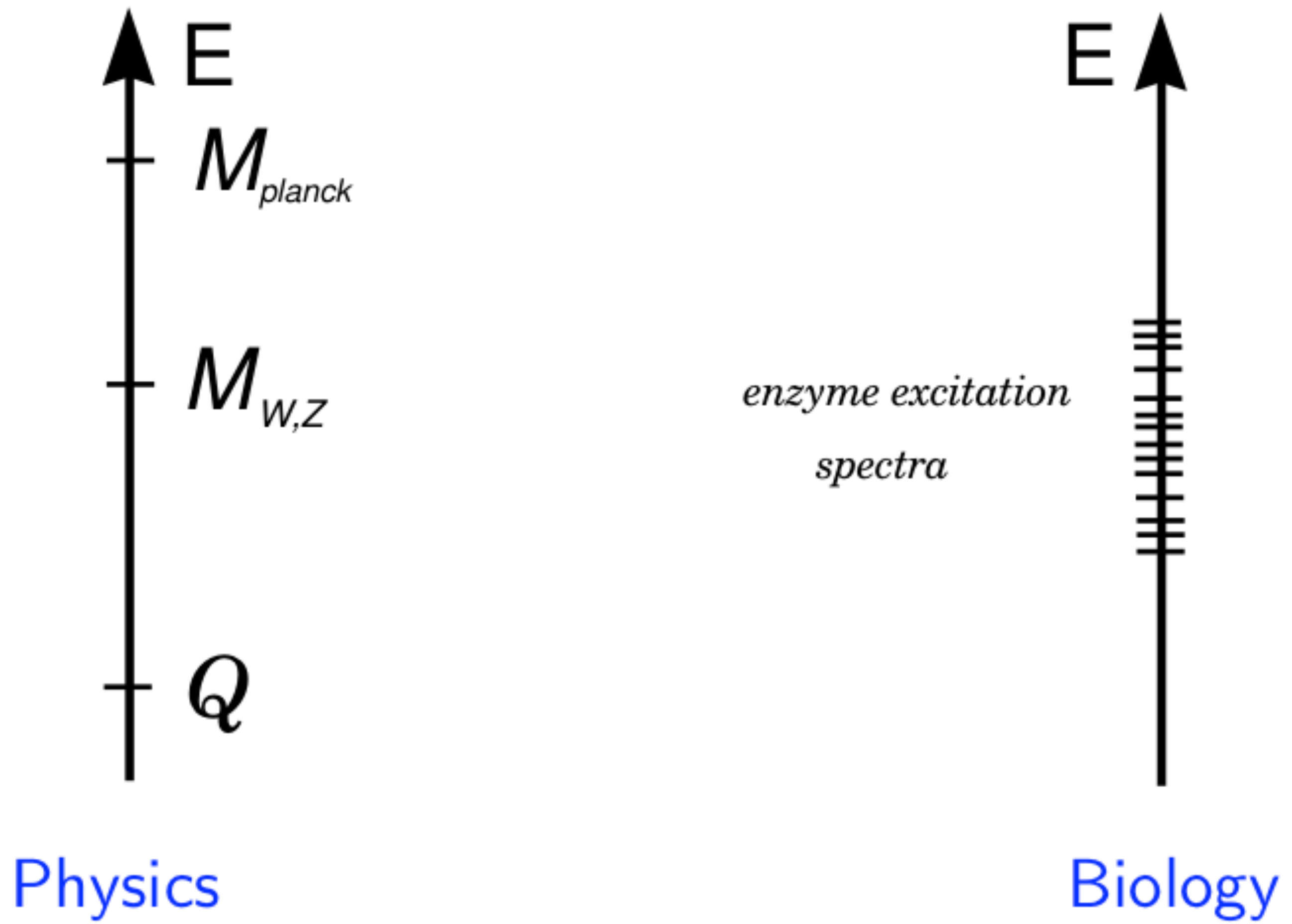
contact EFT
nuclear physics
chiral EFT

Physicists choose to study problems with widely separated scales



Consider a physical system with multiple scales

Arrange the various scales into two groups such that:

$$low - momentum \;\; scales \leq p$$
$$high - momentum \;\; scales \geq \Lambda$$

$p \ll \Lambda \quad \longrightarrow \quad$ <u>Effective Field Theory</u>

- ★ include low-momentum d.o.f.
- ★ omit high-momentum d.o.f.
- ★ systematically improve description in: $\left(\frac{p}{\Lambda}\right)^n$

Utility of EFT?

- Better understand problems with many length scales. (e.g. nuclear physics, atomic physics)

- Compute low-energy scattering without knowledge of short distance physics. (e.g. the Standard Model)

- Develop low-energy theory with non-perturbative full theory. (e.g. chiral perturbation theory)

Consider a system with N scalar fields

$$\phi, \Phi_1, \Phi_2, \ldots \Phi_{N-1}$$

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If $\quad m_{\Phi_i} \geq \Lambda$

$$\int D\phi\, D\Phi_1 \ldots D\Phi_{N-1} e^{-S} = \int D\phi\, e^{-S_{\rm EFT}}$$

non-local!

$$S_{\rm EFT} = \int d^4x\, \mathcal{L}_{\rm EFT}$$

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High-momentum d.o.f. are integrated out

Expand the non-local action:

$$\mathcal{L}_{\rm EFT} = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}c_{-2}\Lambda^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \sum_n \left(\frac{c_n}{\Lambda^{2n}}\phi^{4+2n} + \frac{d_n}{\Lambda^{2n}}(\partial_\mu\phi)^2\phi^{2+2n} + \ldots \right)$$

- ◇ Constrained by Lorentz invariance ... and $\phi \to -\phi$
- ◇ Assume $c_{-2},\, \lambda, c_n,\, d_n \ll 1$
- ◇ ∞ number of operators!

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Dimensional analysis:

$$\hbar = 1 \quad \longrightarrow \quad [\mathbf{x}] = -1 \quad [\mathbf{t}] = -1$$

$$[\int d^dx \mathcal{L}_{\rm EFT}] = 0 \quad \longrightarrow \quad [\phi] = d/2 - 1$$

Which operators are most important?

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$$\phi_\xi(x) \;=\; \phi(\xi x) \;\sim\; e^{ik\xi x}$$

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$$S_{\mathrm{EFT}}\left(\phi(\xi x); c_{-2}, \lambda, c_n, d_n, \ldots\right) \;=\; S_{\mathrm{EFT}}\left(\xi^{-1}\phi(x); \xi^{-2}c_{-2}, \lambda, \xi^{2n}c_n, \xi^{2n}d_n, \ldots\right)$$

$$\phi \to \xi^{-1}\phi \quad , \quad c_{-2} \to \xi^{-2}c_{-2} \quad , \quad \lambda \to \lambda \quad , \quad c_n \to \xi^{2n}c_n \quad , \quad d_n \to \xi^{2n}d_n$$

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Scaling to the infrared:

c_{-2}	*relevant*
λ	*marginal*
$c_n \;,\; d_n \;,\; \ldots$	*irrelevant*

In classical, relativistic EFT:

scaling dim = mass dim

$$[\phi] = 1 \quad , \quad [c_{-2}] = 2 \quad , \quad [\lambda] = 0 \quad , \quad [c_n] = [d_n] = -2n$$

Dominant effect from lowest dimensions!

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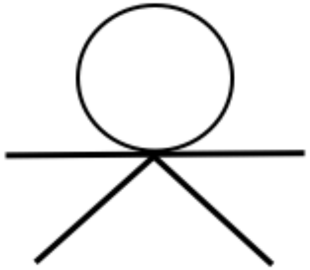
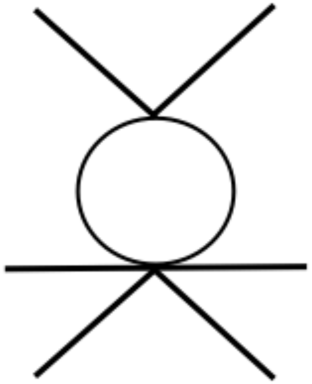
How do quantum effects alter scaling?

Operators *renormalize* each other via loops!

$\Delta\lambda \sim$  $\sim \frac{c_1}{\Lambda^2} \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_\phi^2} \sim \frac{c_1}{(4\pi)^2}$

$\Delta c_1 \sim$  $\sim c_1\,\lambda \int \frac{d^4q}{(2\pi)^4} \frac{1}{\left(q^2 - m_\phi^2\right)^2} \sim \frac{c_1\,\lambda}{(4\pi)^2} \log\Lambda$

These shifts are *perturbative* by assumption!

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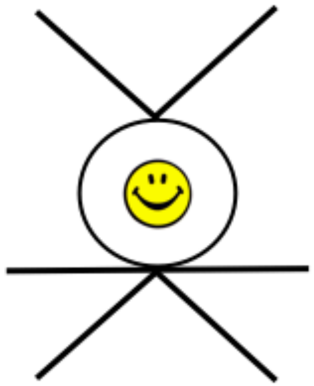

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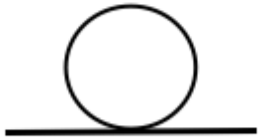
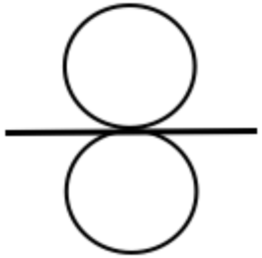
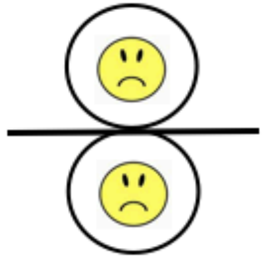
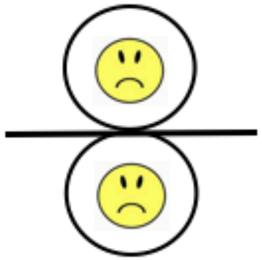
$\Delta c_{-2} \sim$  $+$  $+ \ldots \sim \left(\frac{\lambda}{(4\pi)^2} + \frac{c_1}{(4\pi)^4} + \ldots \right)$

$\Delta c_{-2} \sim$  $+$  $+ \dots \sim \left(\frac{\lambda}{(4\pi)^2} + \frac{c_1}{(4\pi)^4} + \dots \right)$

$m_\phi \ll \Lambda$ requires *fine tuning*!

Hierarchy/naturalness problem!

$\Delta c_{-2} \sim$  $+$  $+ \dots \sim \left(\frac{\lambda}{(4\pi)^2} + \frac{c_1}{(4\pi)^4} + \dots \right)$

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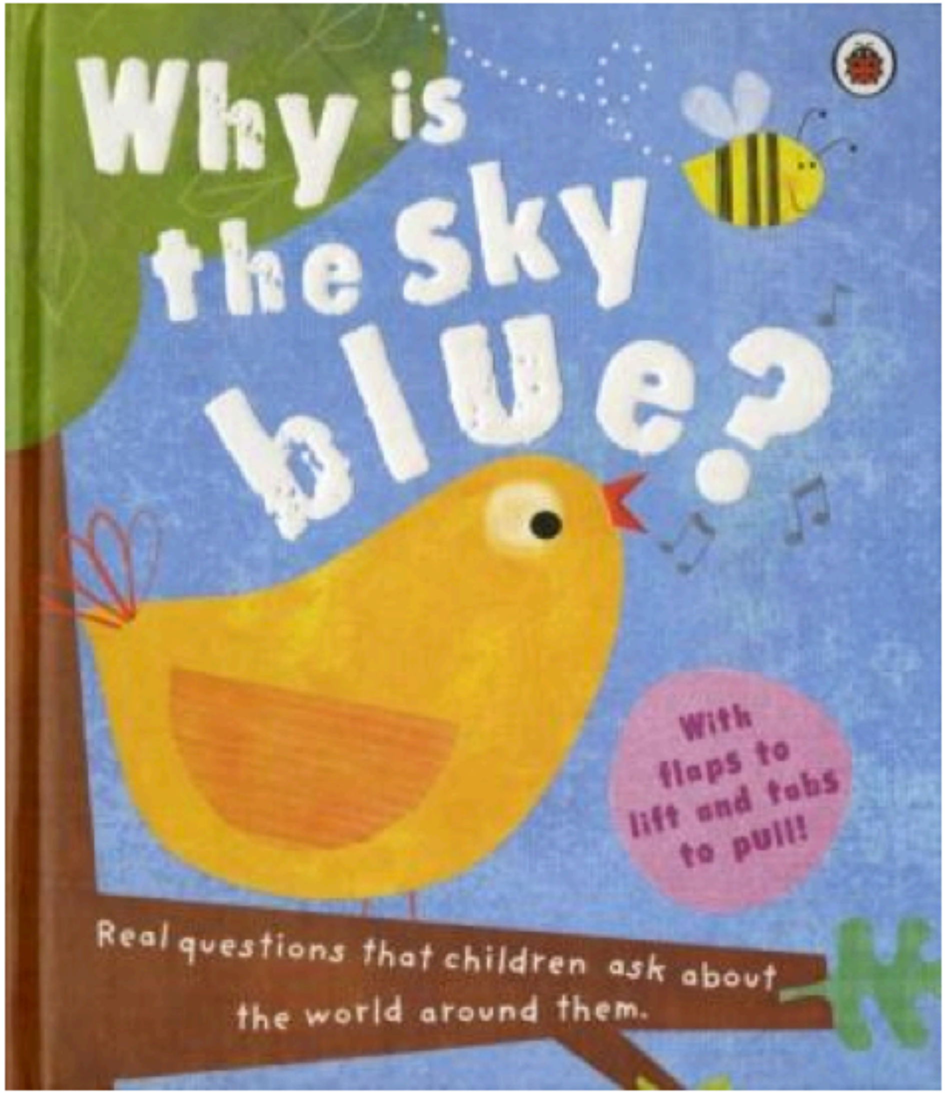
Hierarchy/naturalness problem!

Fermions do not have this problem!

Chiral symmetry

EFT Strategy

- Identify low-energy d.o.f
- Identify the symmetries
- Construct most general EFT
- Determine *power counting*
- Choose desired accuracy
- Determine parameters (*matching*)



Blue light scatters more strongly from atoms in the atmosphere than red light!

Consider interactions of photons with neutral atoms

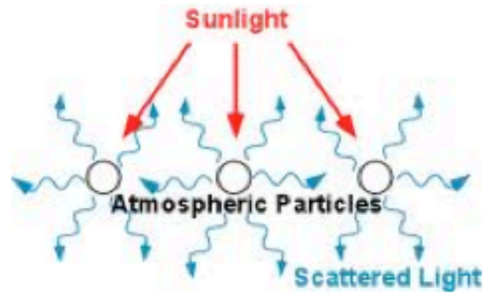
Physical scales

Photon energy:	ω
Atom mass:	$M_{\rm A}$
Atom size:	a_0^{-1}
Atom level spacing:	ΔE

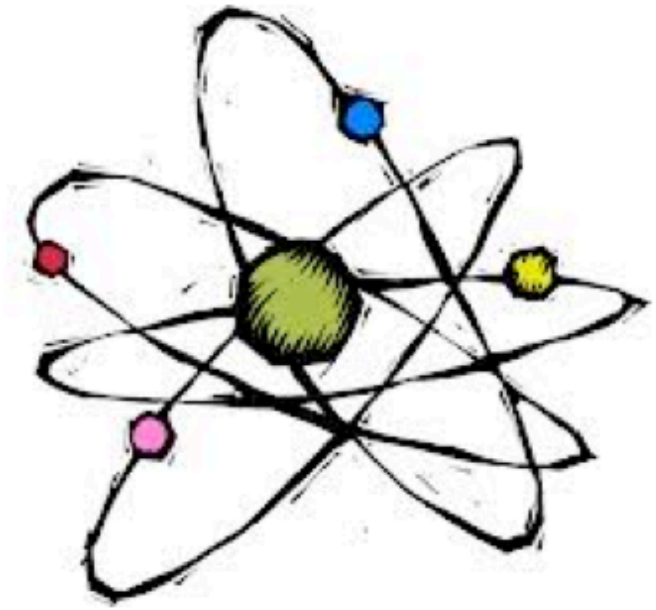
hierarchy of scales

$$\omega \ll \Delta E \ll a_0^{-1} \ll M_{\rm A}$$

$$M_A^{-1} \ll a_0 \ll \Delta E^{-1} \ll \omega^{-1}$$



Rayleigh Scattering



ω^{-1} | ΔE^{-1} | a_0 | M_A^{-1}

photons not probing these scales!

degrees of freedom?

A_μ creates and destroys photon

ϕ_v destroys atom with velocity $v_\mu = (1,0,0,0)$

$\phi_v^\dagger$ creates atom with velocity $v_\mu = (1,0,0,0)$

Constrained by Lorentz and gauge invariance

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Building blocks:

∂_μ	$\phi_v^\dagger \phi_v$	$F_{\mu\nu}$	v_μ

$$\mathcal{L}_0 = \phi_v^\dagger i v^\mu \partial_\mu \phi_v - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

kinetic terms

$$\mathcal{L}_0 \;=\; \phi_v^\dagger i v^\mu \partial_\mu \phi_v \;-\; \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

kinetic terms

Atom e.o.m: $\partial_t \, \phi_v \;=\; 0 \quad \Rightarrow \quad E \;=\; 0$

$$\mathcal{L}_{\rm EFT} = \; c_1 \phi_v^\dagger \phi_v F_{\mu\nu} F^{\mu\nu} + c_2 \phi_v^\dagger \phi_v v^\alpha F_{\alpha\mu} v_\beta F^{\beta\mu} + c_3 \phi_v^\dagger \phi_v (v^\alpha \partial_\alpha) F_{\mu\nu} F^{\mu\nu} + \ldots$$

∞ number of interaction operators ! need *power-counting*

$$[F_{\mu\nu}] \;=\; 2 \quad , \quad [\phi_v] \;=\; \frac{3}{2} \qquad \Longrightarrow \qquad [c_1] \;=\; [c_2] \;=\; -3 \quad , \quad [c_3] \;=\; -4$$

Dominant effect from lowest dimensions!

Dimensions must be made from high-energy scales!

$$\Delta E,\ a_0^{-1}, \ldots$$

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Scattering with $\omega \ll \Delta E\ ,\ a_0^{-1}$ ~ classical

$$\mathcal{L}_{\rm EFT} = a_0^3 \left(a_1 \phi_v^\dagger \phi_v F_{\mu\nu} F^{\mu\nu} + a_2 \phi_v^\dagger \phi_v v^\alpha F_{\alpha\mu} v_\beta F^{\beta\mu} \right) + \ldots$$

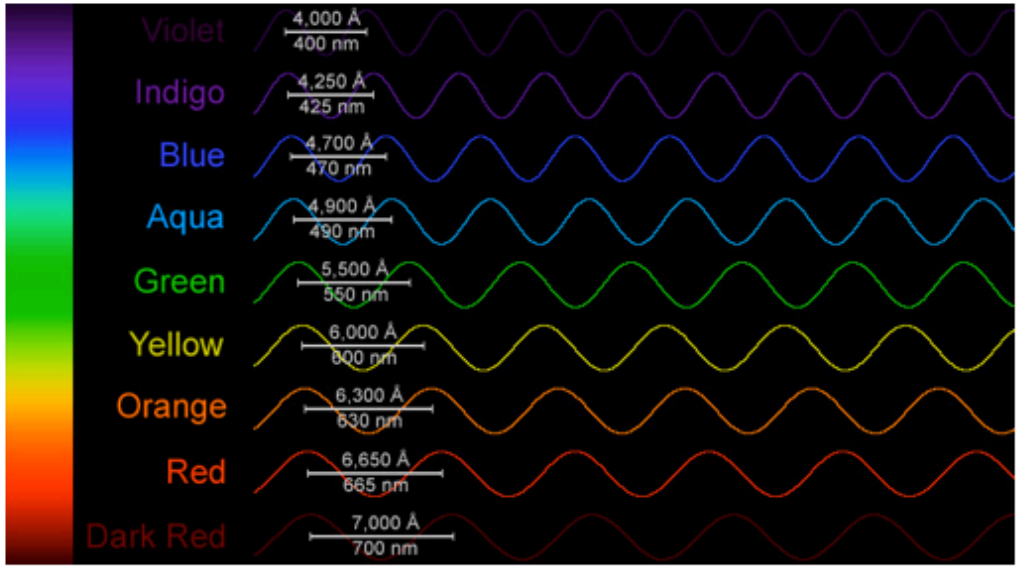
ω from derivative operators!

Scattering amplitude:	$\lvert\mathcal{A}\rvert^2 \sim a_0^6$
Cross-section:	$[\sigma] = -2$

$$\sigma(\omega) \propto \omega^4 a_0^6 \left(1 + \mathcal{O}\left(\frac{\omega}{\Delta E}\right)\right)$$

governed by "smallest" high-energy scale

Blue light scatters more strongly from atoms in the atmosphere than red light!



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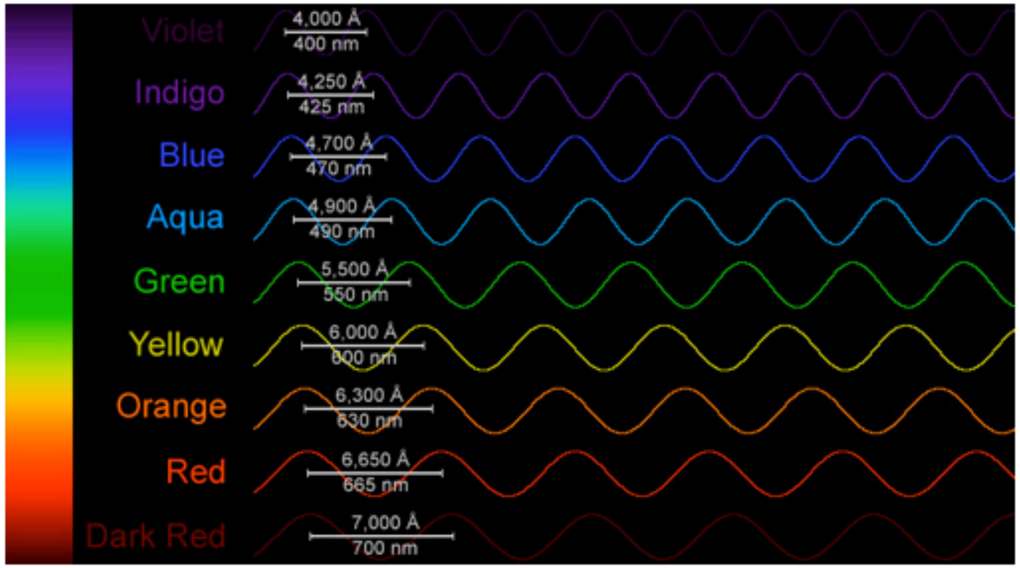
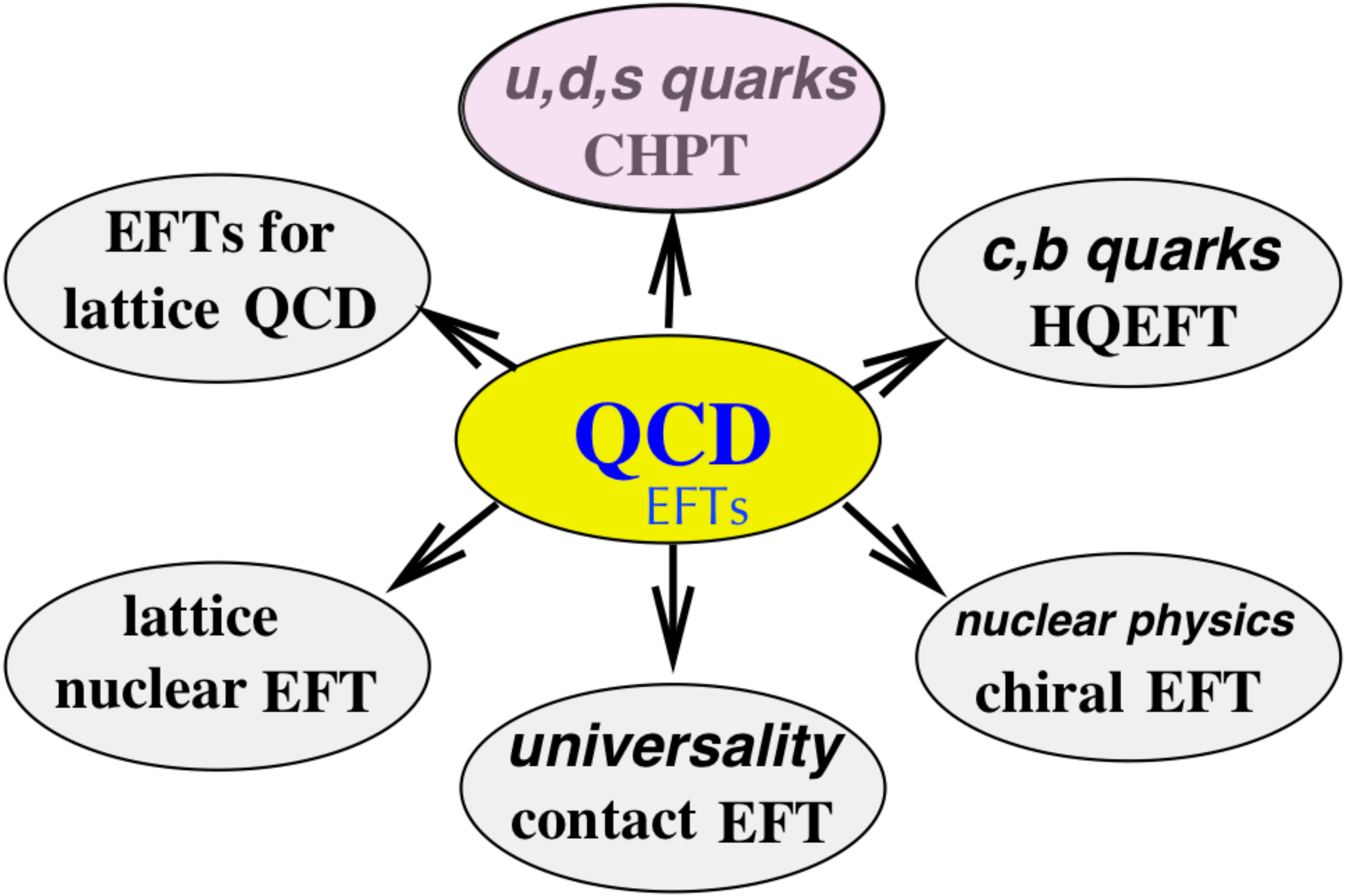


To calculate the coefficients must *match* the full theory to the EFT

(See Jackson, Classical E and M)



u,d,s quarks
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EFTs for
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contact EFT
nuclear physics
chiral EFT

$$\mathcal{L}_{\text{QCD}} = \sum_{i=1}^{3} \left(\bar{q}_i i \not{D} q_i - m_i \bar{q}_i q_i\right) - \frac{1}{2} \text{Tr} G_{\mu\nu} G^{\mu\nu}$$

u,d,s active flavors

$$D_\mu = \partial_\mu + igA_\mu \qquad A_\mu = A_\mu^a T_a$$

$$T_a \in SU(3)$$

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Chiral decomposition

$$\sum_i \bar{q}_i i \not{D} q_i = \sum_i \left(\bar{q}_{Li} i \not{D} q_{Li} + \bar{q}_{Ri} i \not{D} q_{Ri} \right)$$

$$q_{L,R} = \frac{1}{2}(1 \pm \gamma_5) q$$

$U(3)_L \times U(3)_R$ invariance

$U(1)_A$ anomalous

$$U(1)_V \times SU(3)_L \times SU(3)_R$$

Baryon number

Chiral symmetry: $q_{Li} \to L_{ij} q_{Lj} \qquad q_{Rj} \to R_{ij} q_{Rj}$

$$\boxed{\sum_i m_i \bar{q}_i q_i = \sum_{i,j} \bar{q}_{Ri} M_{ij} q_{Lj} + h.c.} \qquad M = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

If mass matrix were field with:

$$M \to R M L^\dagger$$

spurion

then mass term would be a chiral invariant

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NOTE

- $\underline{m_u\,,\, m_d \ll m_s}$ $SU(2)_L \times SU(2)_R$ better than $SU(3)_L \times SU(3)_R$
- $\underline{m_u = m_d = m_s \neq 0}$ $SU(3)_V$ $(L = R)$ is exact
- $\underline{m_u = m_d \neq 0}$ $SU(2)_V$ *Isospin* $(L = R)$ is exact
- $3 \times U(1) \leftrightarrow B + I_3 + Y$

Consequences of chiral symmetry?  

Assume ground state baryon octet of positive parity : $\mathcal{P}$

$$|B\rangle \sim |(1,8)\rangle + |(8,1)\rangle$$

$$\mathcal{P}|(L,R)\rangle = |(R,L)\rangle$$
$$\mathcal{P}|B\rangle = |B\rangle$$

Must also have:

$$|B^*\rangle \sim |(1,8)\rangle - |(8,1)\rangle$$

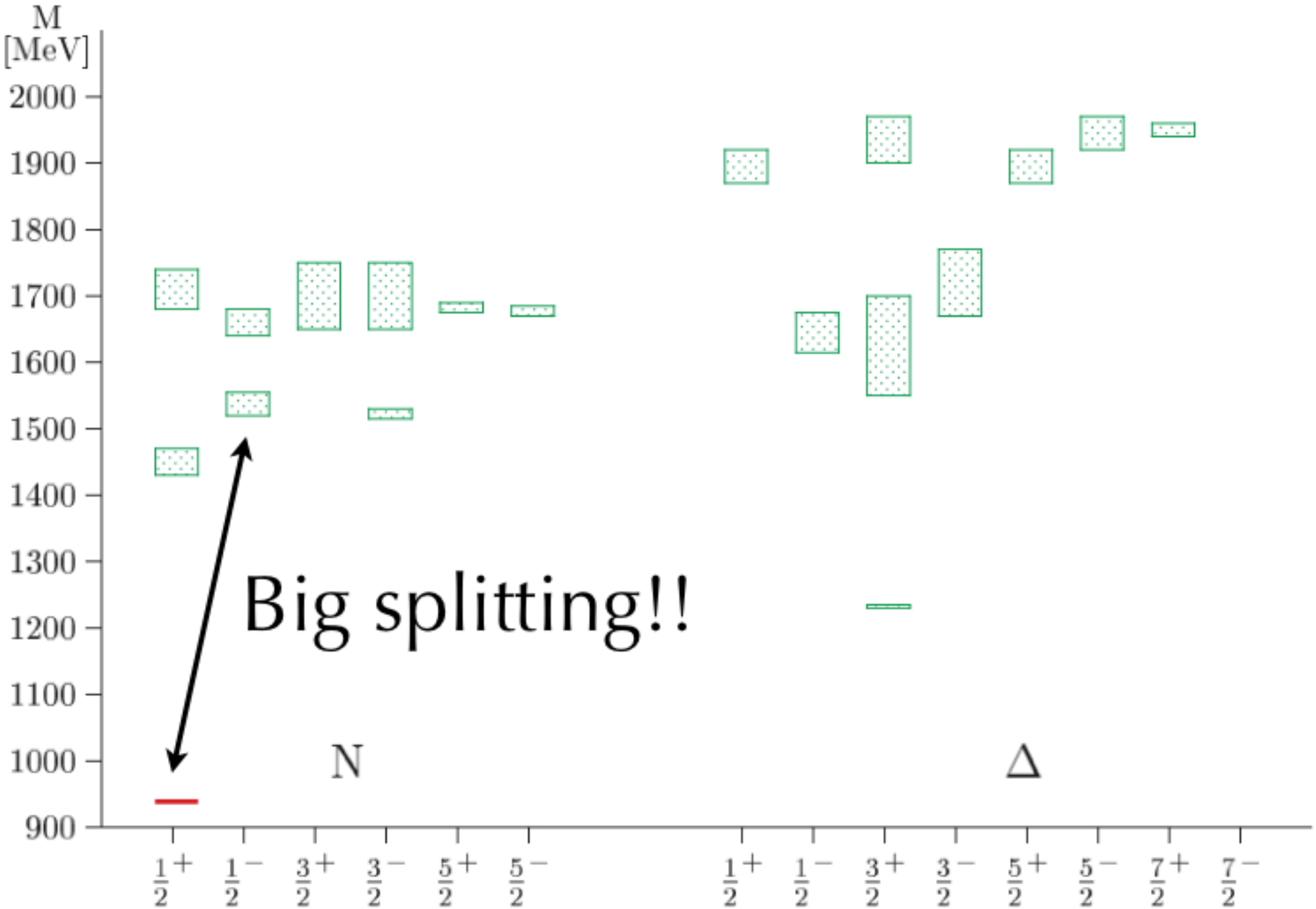
$$\mathcal{P}|B^*\rangle = -|B^*\rangle$$

$$\mathcal{H}_{QCD} \in (1,1)$$

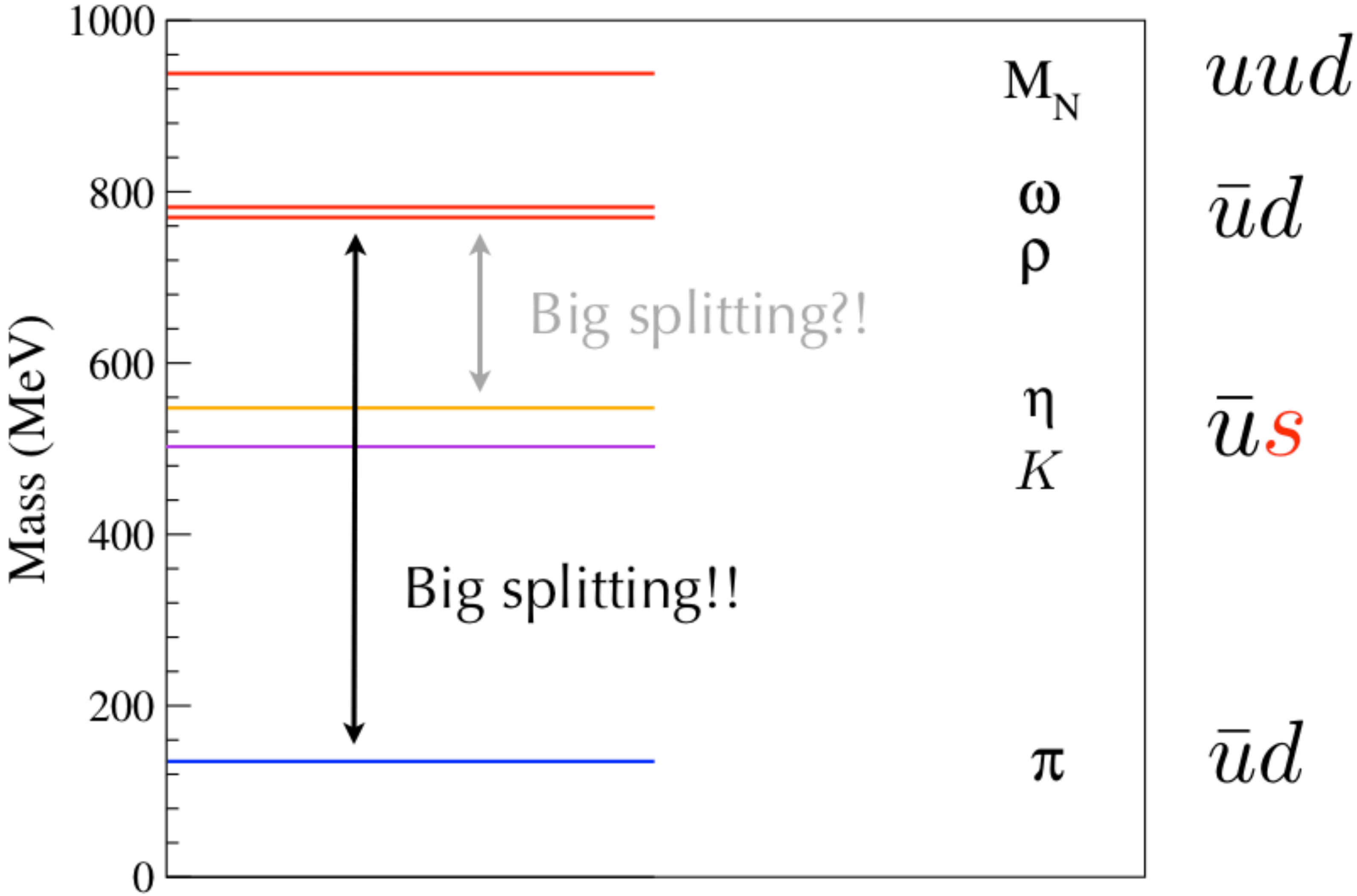
$$M_B = \langle B|\mathcal{H}_{QCD}|B\rangle = \langle B^*|\mathcal{H}_{QCD}|B^*\rangle = M_{B^*}$$

Parity doubling! (Wigner-Weyl)

EXPERIMENT: non-strange Baryons



EXPERIMENT: Mesons



$$G = SU(3)_L \times SU(3)_R$$

Wigner-Weyl realization of G
ground state is symmetric

$$\langle 0 | \bar{q}_R \, q_L \, | 0 \rangle = 0$$

ordinary symmetry
spectrum contains parity partners
degenerate multiplets of G

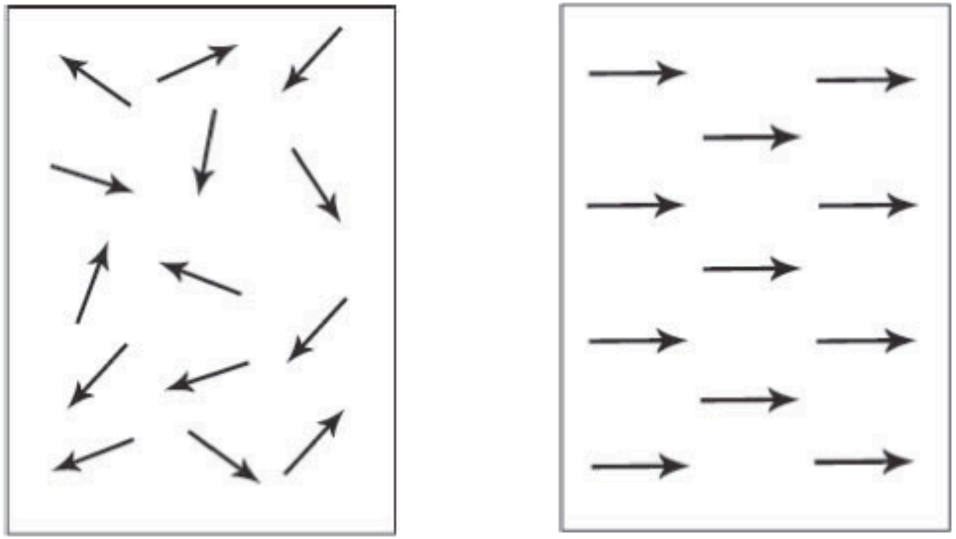
Nambu-Goldstone realization of G
ground state is asymmetric

$$\langle 0 | \bar{q}_R \, q_L \, | 0 \rangle \neq 0$$

“order parameter”
spontaneously broken symmetry
spectrum contains Goldstone bosons
degenerate multiplets of $SU(3)_V \subset G$

Analogy

Above T_c: Below T_c:



$\langle \mathbf{M} \rangle = 0$ $\langle \mathbf{M} \rangle \neq 0$

Ferromagnetism	**QCD**
Ground state $\lvert\text{magnet}\rangle$	QCD vacuum $\lvert 0\rangle$
$\langle\text{magnet}\lvert\mathbf{M}\rvert\text{magnet}\rangle$	$\langle 0\lvert\bar{q}q\rvert 0\rangle$
$O(3)$	$SU(2)_A$
Low temperature	Low energy, also T
Magnons	Pions