Introduction to the Physics of Saturation

Yuri Kovchegov
The Ohio State University

Outline

- General concepts
- Classical gluon fields, parton saturation
- Quantum (small-x) evolution
  – Linear BFKL evolution
  – Non-linear BK and JIMWLK evolution
- Phenomenology

General Concepts

Running of QCD Coupling Constant

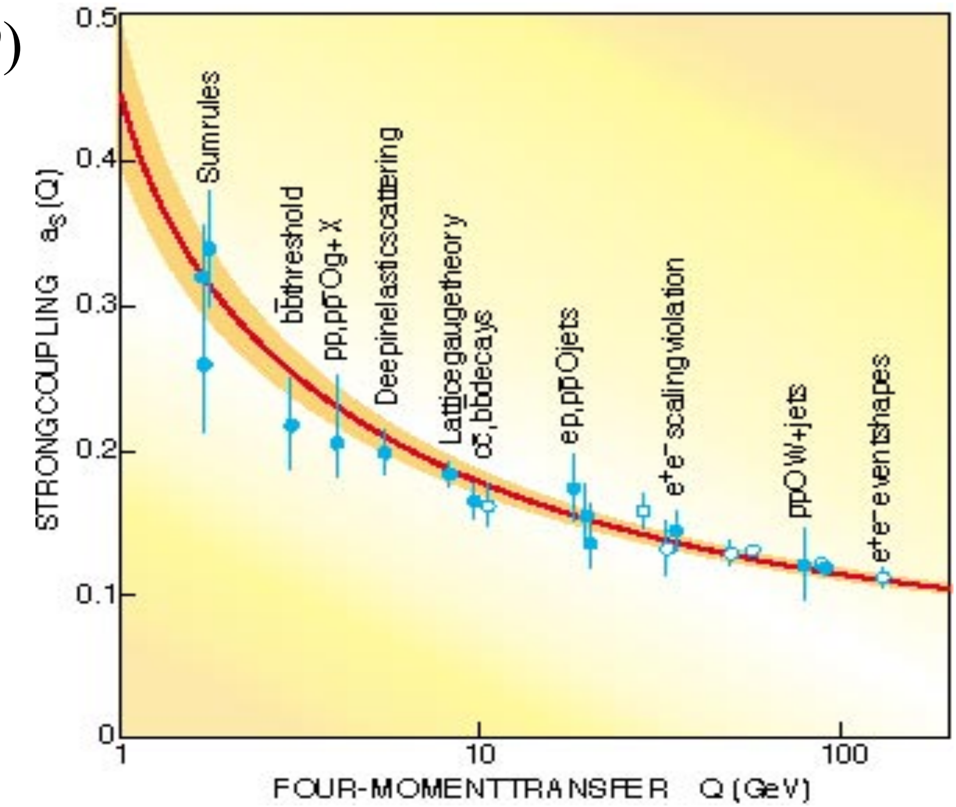
⇨ QCD coupling constant $\alpha_S = \dfrac{g^2}{4\pi}$ changes with the momentum scale involved in the interaction

$$\alpha_S = \alpha_S(Q)$$

Asymptotic Freedom!

Gross and Wilczek, Politzer, ca '73

Physics Nobel Prize 2004!



For short distances x < 0.2 fm, or, equivalently, large momenta k > 1 GeV the QCD coupling is small $\alpha_S << 1$ and interactions are weak.

A Question

- Can we understand, qualitatively or even quantitatively, the structure of hadrons and their interactions in High Energy Collisions?
  - What are the total cross sections?
  - What are the multiplicities and production cross sections?
  - Diffractive cross sections.
  - Particle correlations.

What sets the scale of running QCD coupling in high energy collisions?

- "String theorist": $\alpha_S = \alpha_S\left(\sqrt{s}\right) << 1$

  (actually wrong)

- Pessimist: $\alpha_S = \alpha_S\left(\Lambda_{QCD}\right) \sim 1$ we simply can not tackle high energy scattering in QCD.

- pQCD expert: only study high-p_T particles such that

$$\alpha_S = \alpha_S\left(p_T\right) << 1$$

  But: what about total cross section? bulk of particles?

What sets the scale of running QCD coupling in high energy collisions?

- Saturation physics is based on the existence of a large internal momentum scale Q_S which grows with both energy s and nuclear atomic number A

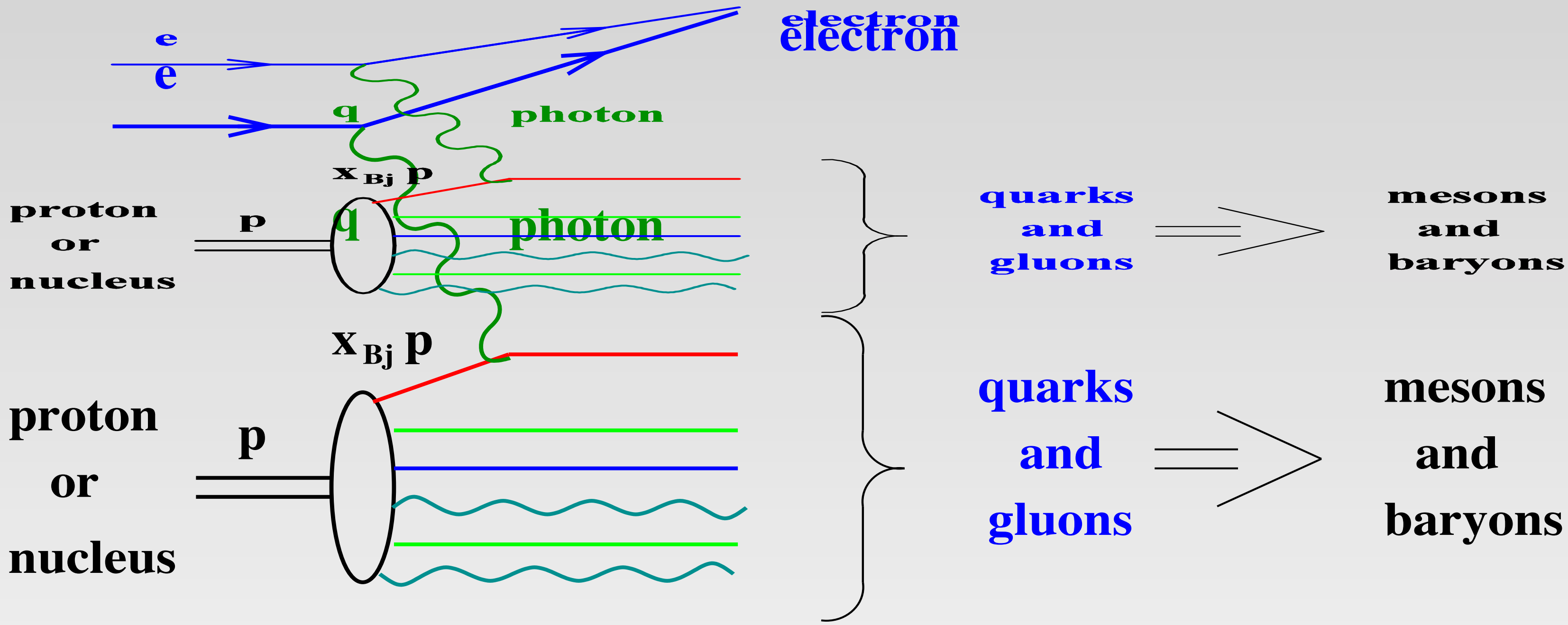
$$Q_s^2 \sim A^{1/3} x^{\lambda}$$

such that $\alpha_S = \alpha_S(Q_S) << 1$

and we can calculate total cross sections, particle spectra and multiplicities, etc, from first principles.

Classical Fields

Kinematics of DIS



- Photon carries 4-momentum q_μ , its virtuality is

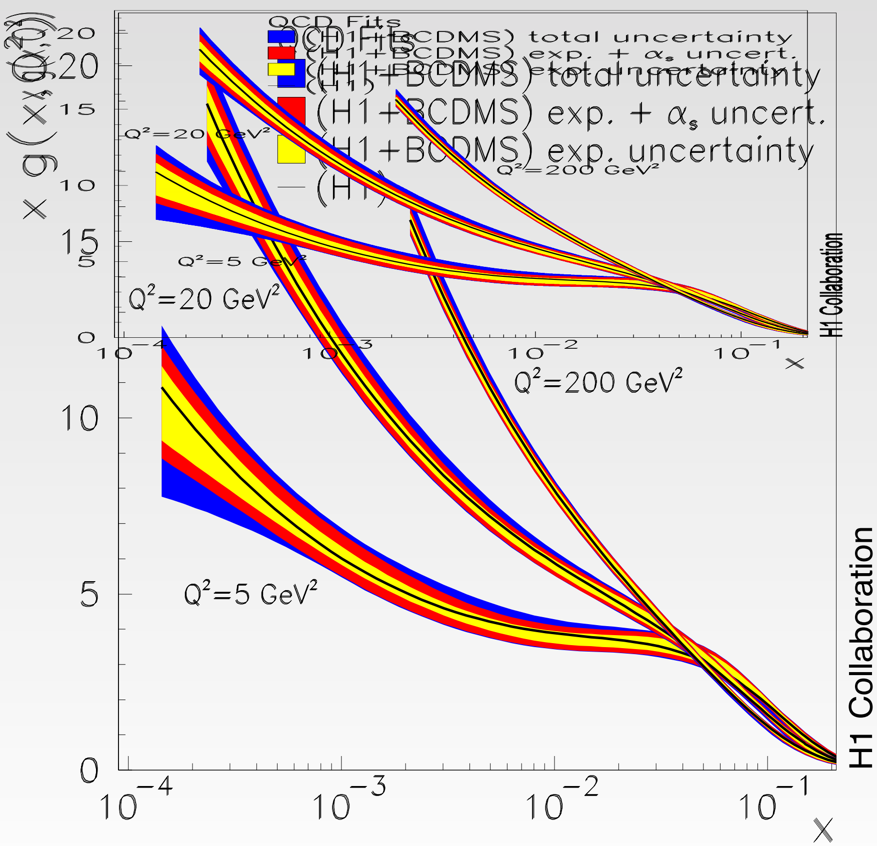
$$Q^2 = -q_\mu \, q^\mu$$

- Photon hits a quark in the proton carrying momentum $x_{Bj} p$ with p being the proton's momentum. Parameter x_{Bj} is called Bjorken x variable.

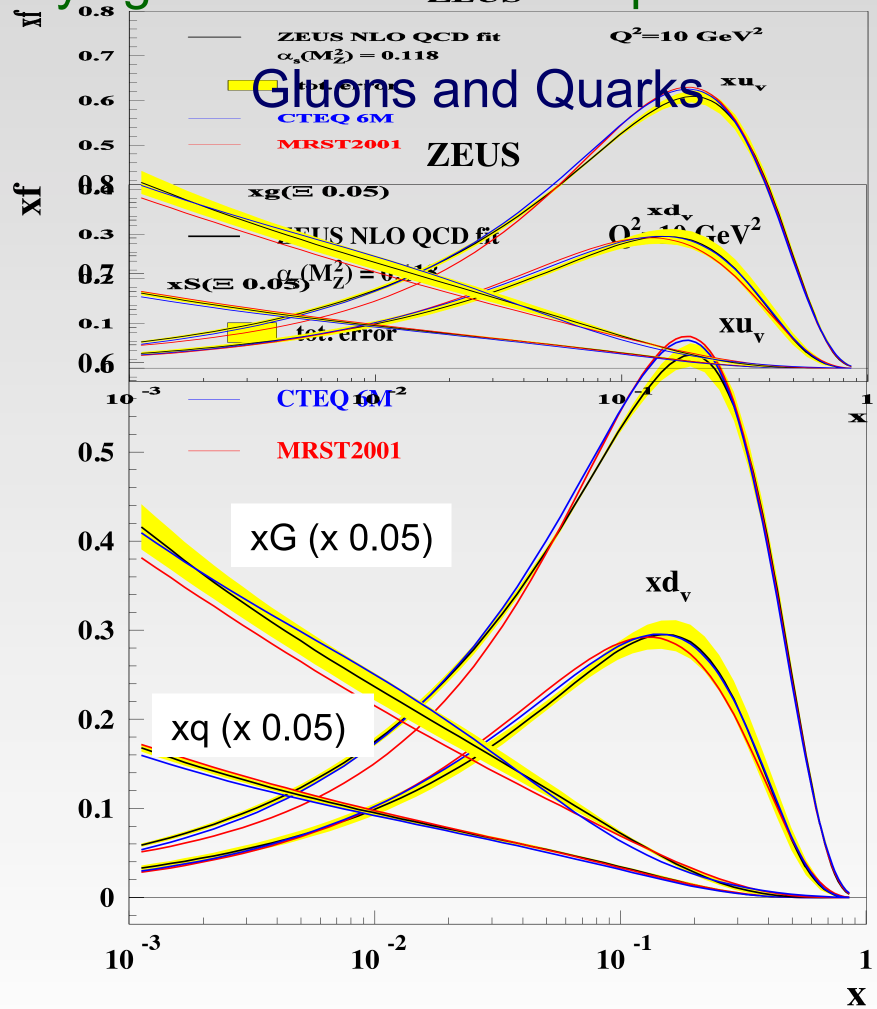
What have we learned at HERA?

Distribution functions $xq(x,Q^2)$ and $xG(x,Q^2)$ count the number of quarks and gluons with sizes ≥ 1/Q and carrying the fraction x of the proton's momentum.

Gluons only

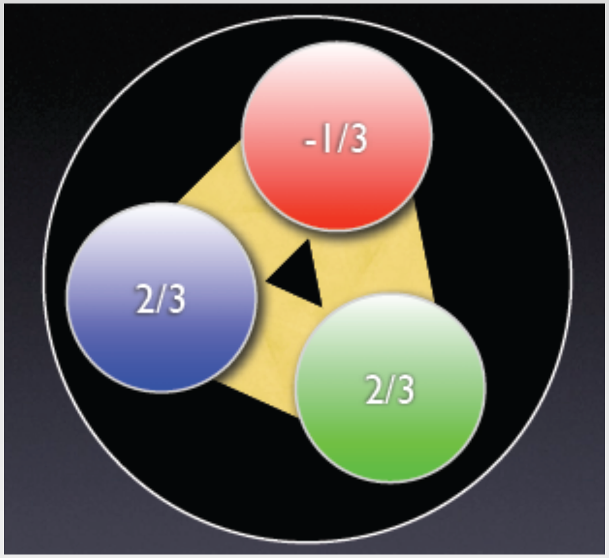


Gluons and Quarks



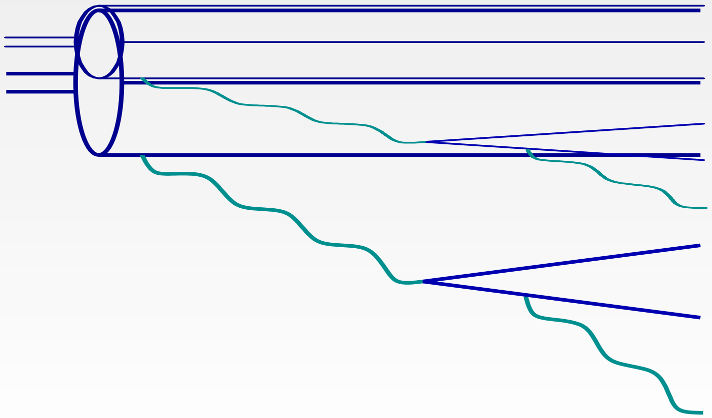
What have we learned at HERA?

⇨ There is a huge number of quarks, anti-quarks and gluons at small-x !



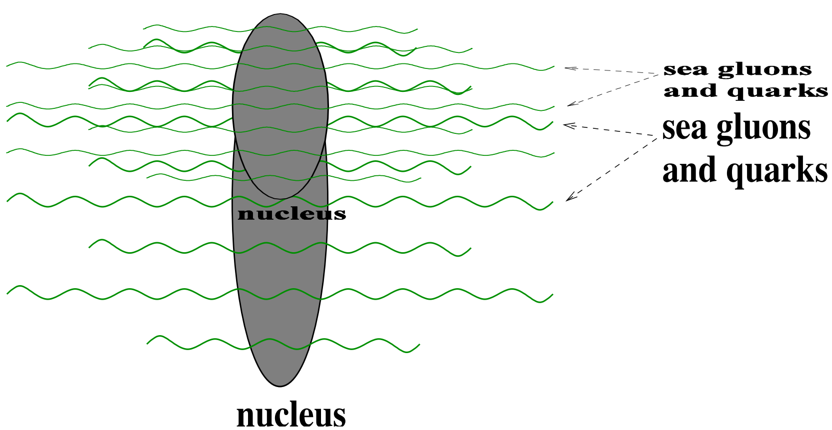
⇨ How do we reconcile this result with the picture of protons made up of three valence quarks?

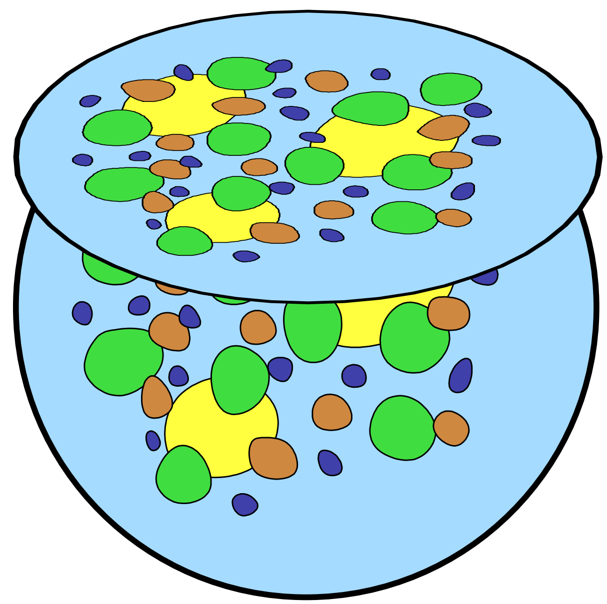
⇨ Qualitatively we understand that these extra quarks and gluons are emitted by the original three valence quarks in the proton.



McLerran-Venugopalan Model

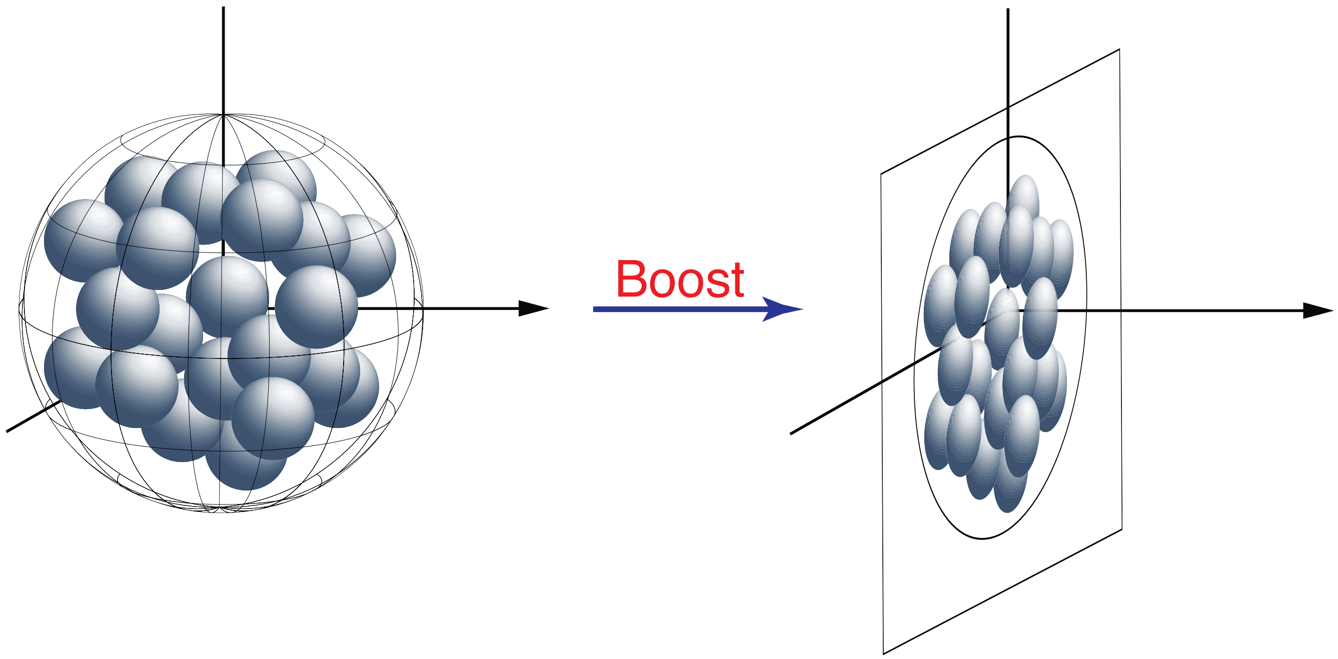
- The wave function of a single nucleus has many small-x quarks and gluons in it.
- In the transverse plane the nucleus is densely packed with gluons and quarks.





Large occupation number ⇨ Classical Field

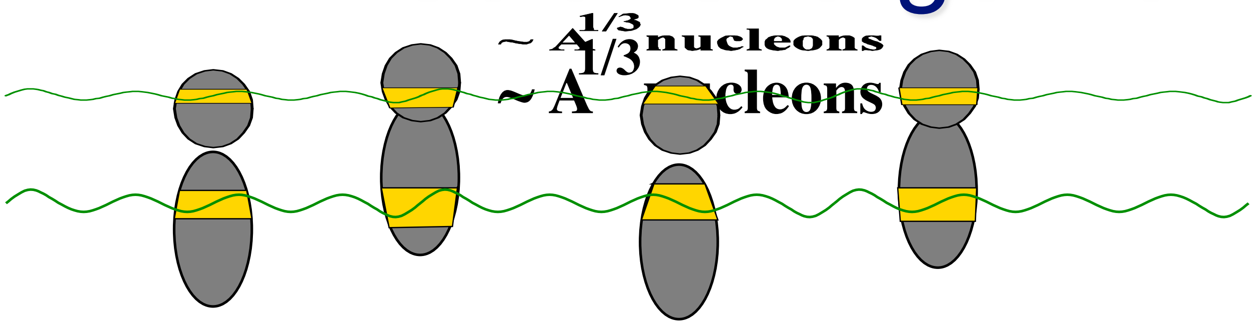
McLerran-Venugopalan Model

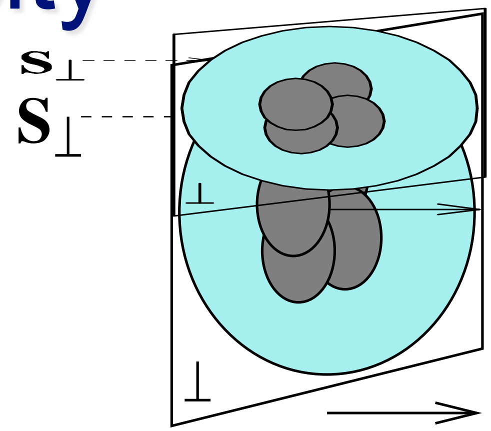


- Large parton density gives a large momentum scale Q_s (the saturation scale).
- For $Q_s >> \Lambda_{QCD}$, get a theory at weak coupling $\alpha_s(Q_s^2) \ll 1$ and the leading gluon field is classical.

McLerran, Venugopalan '93-'94

Color Charge Density





Small-x gluon "sees" the whole nucleus coherently in the longitudinal direction! It "sees" many color charges which form a net effective color charge Q = g (# charges)$^{1/2}$, such that Q^2 = g^2 #charges (random walk).

Define color charge density

$$\mu^2 = \frac{Q^2}{S_\perp} = \frac{g^2 \,\#\, charges}{S_\perp} \propto g^2 \frac{A}{S_\perp} \propto A^{1/3}$$

McLerran
Venugopalan
'93-'94

such that for a large nucleus (A>>1)

$$\mu^2 \propto \Lambda^2_{QCD} A^{1/3} \gg \Lambda^2_{QCD} \implies \alpha_s(\mu^2) \ll 1$$

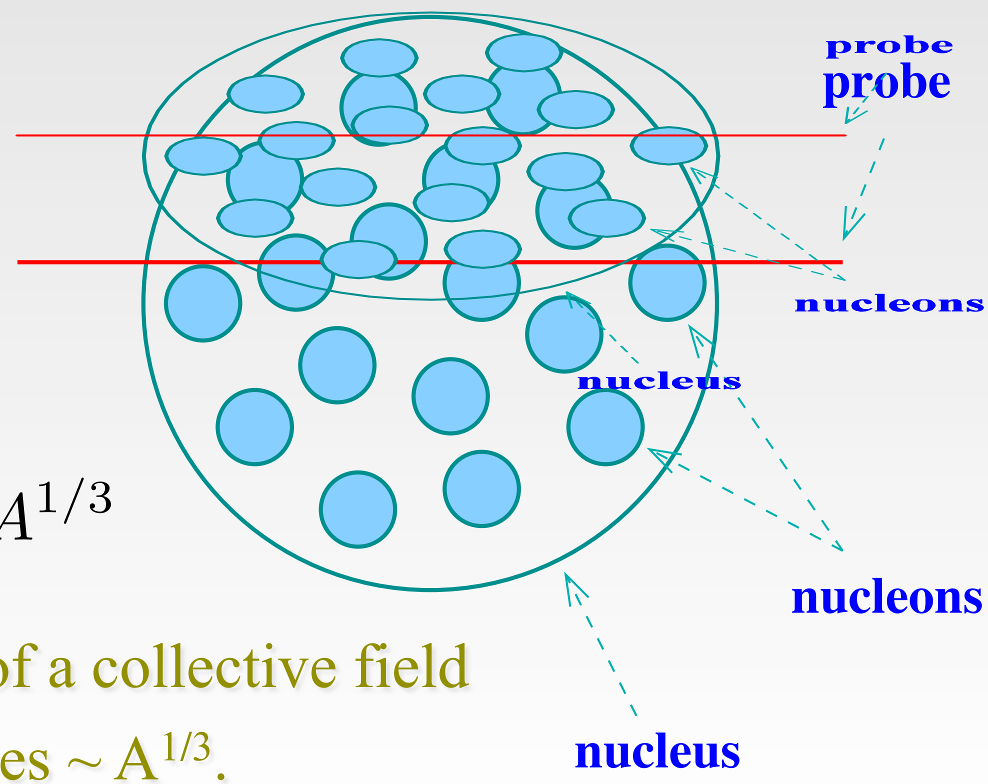
Nuclear small-x wave function is perturbative!!!

Saturation Scale

To argue that $Q_s^2 \sim A^{1/3}$ let us consider an example of a particle scattering on a nucleus. As it travels through the nucleus it bumps into nucleons. Along a straight line trajectory it encounters $\sim R \sim A^{1/3}$ nucleons, with R the nuclear radius and A the atomic number of the nucleus.

The particle receives $\sim A^{1/3}$ random kicks. Its momentum gets broadened by

$$\Delta k \sim \sqrt{A^{1/3}} \Rightarrow (\Delta k)^2 \sim A^{1/3}$$

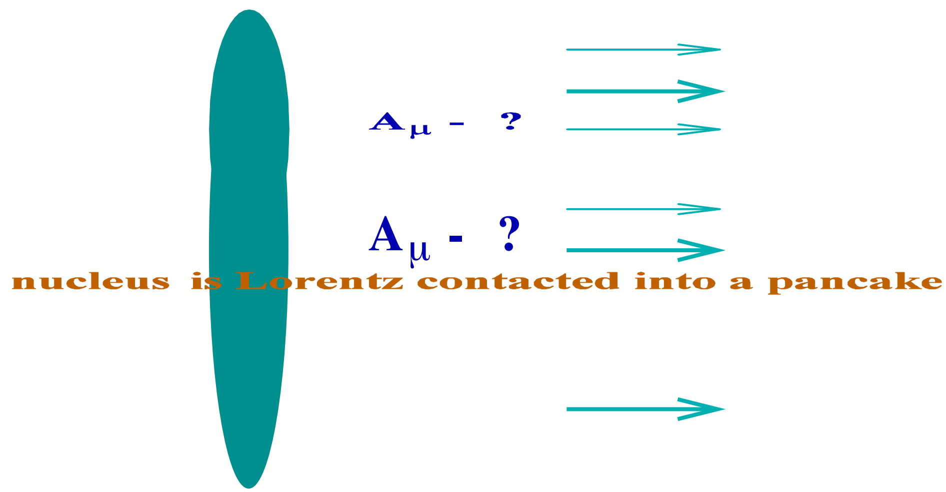


Saturation scale, as a feature of a collective field of the whole nucleus also scales $\sim A^{1/3}$.

McLerran-Venugopalan Model

- To find the classical gluon field A_μ of the nucleus one has to solve the non-linear analogue of Maxwell equations – the Yang-Mills equations, with the nucleus as a source of the color charge:

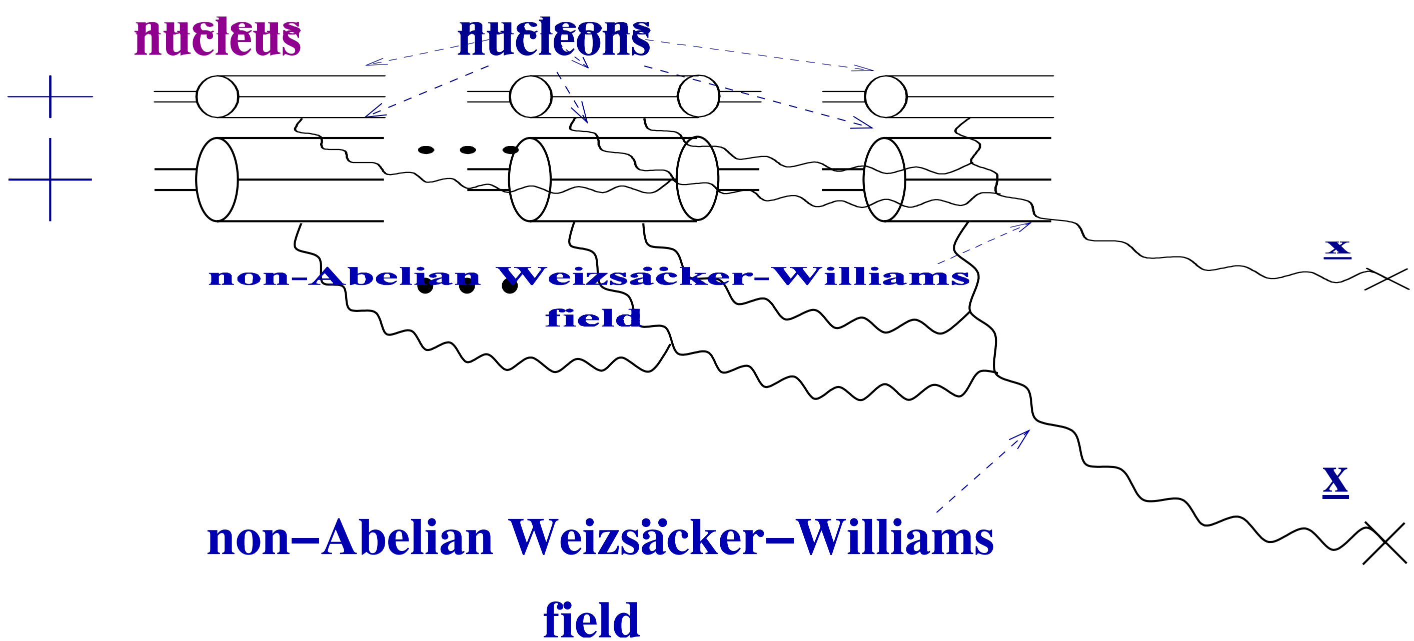
$$\mathcal{D}_\nu \, F^{\mu\nu} = J^\mu$$



nucleus is Lorentz contacted into a pancake

Yu. K. ’96; J. Jalilian-Marian et al, ‘96

Classical Field of a Nucleus

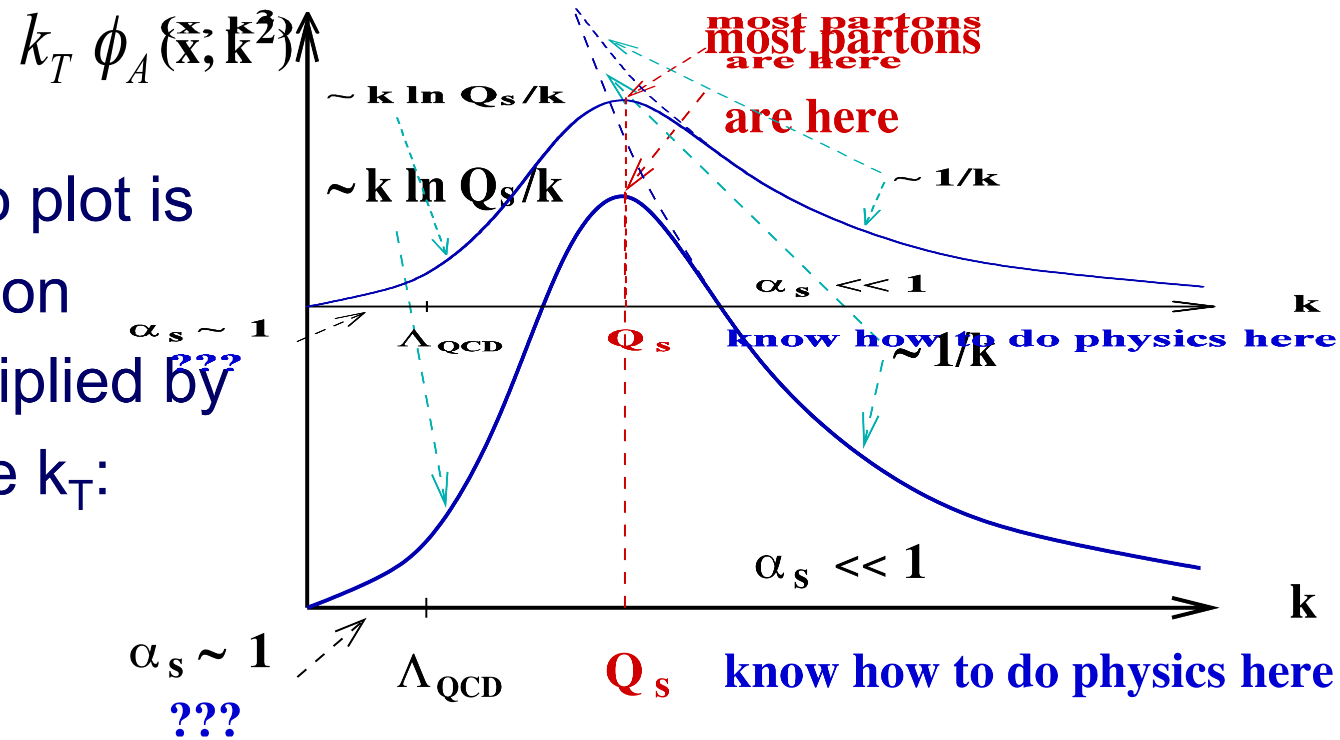


Here's one of the diagrams showing the non-Abelian gluon field of a large nucleus.

The resummation parameter is $\alpha_S^2 A^{1/3}$, corresponding to two gluons per nucleon approximation.

Classical Gluon Distribution

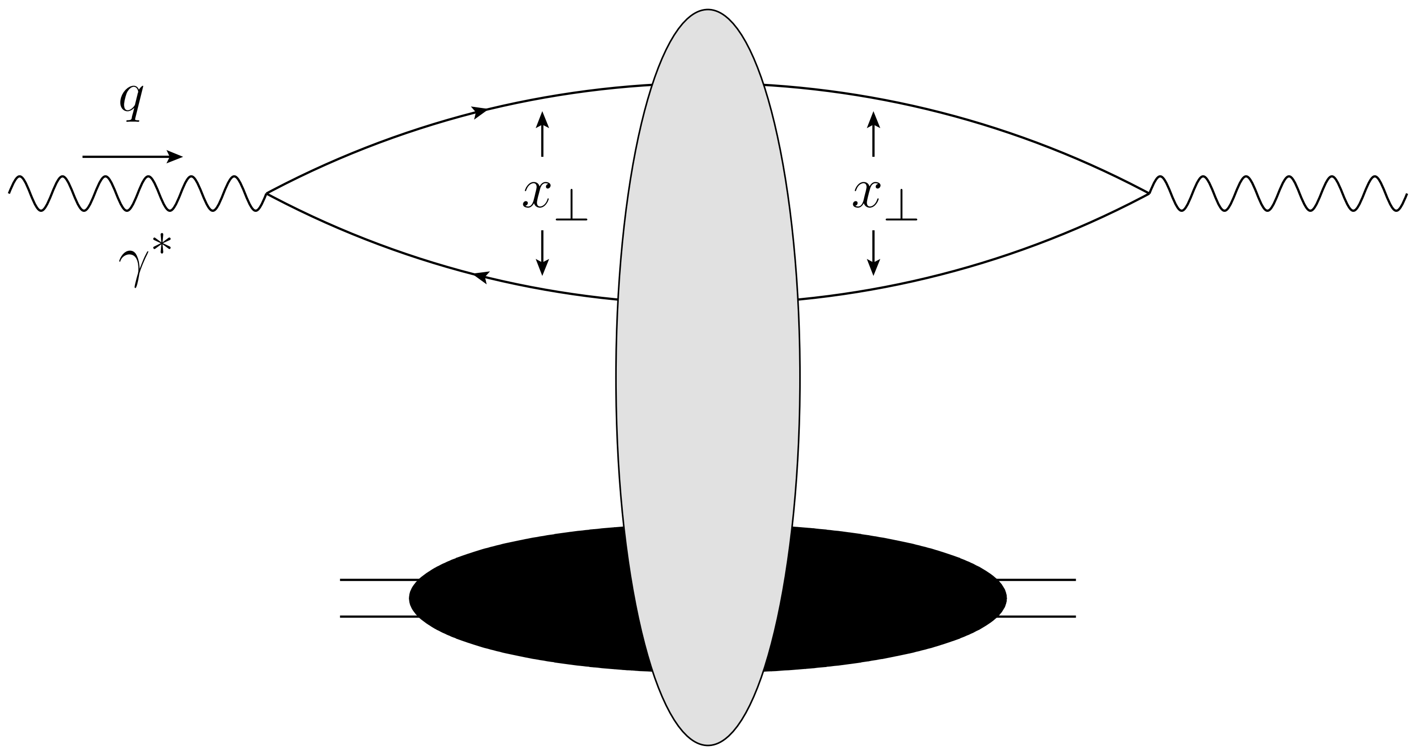
A good object to plot is the classical gluon distribution multiplied by the phase space k_T:



⇨ Most gluons in the nuclear wave function have transverse momentum of the order of $k_T \sim Q_S$ and $Q_S^2 \sim A^{1/3}$

⇨ We have a small coupling description of the whole wave function in the classical approximation.

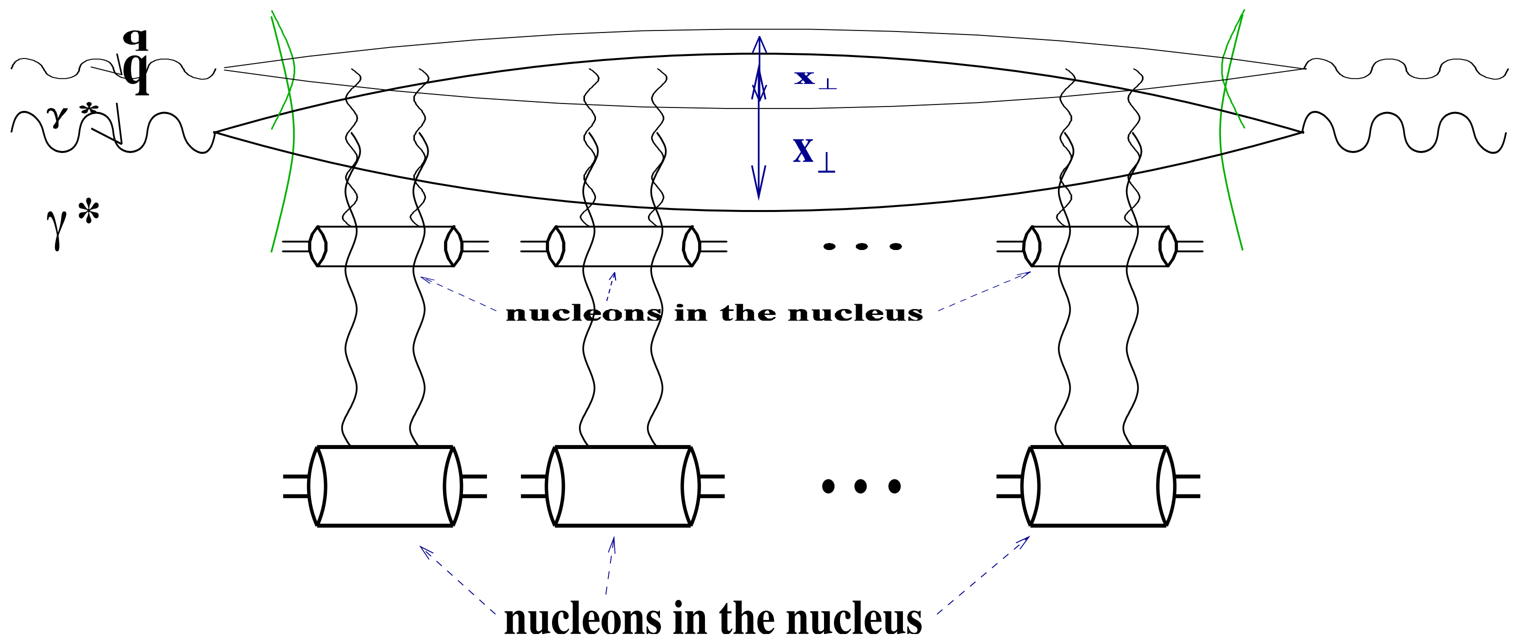
DIS in proton/nucleus rest frame



Dipole picture: the dipole interacts with the target.

DIS in the Classical Approximation

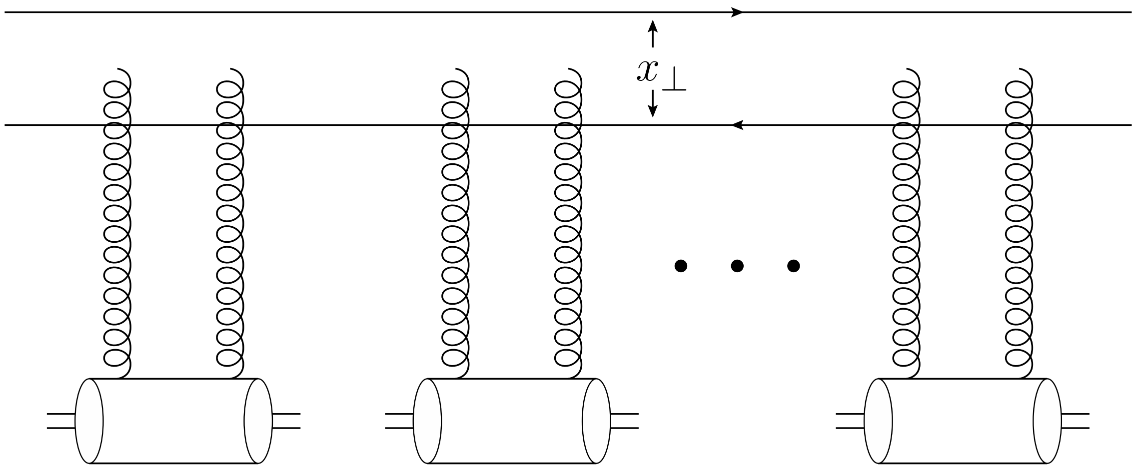
The DIS process in the rest frame of the target is shown below. It factorizes into



$$\sigma_{tot}^{\gamma * A}(x_{Bj}, Q^2) = |\Psi^{\gamma * \to q\bar{q}}|^2 \otimes N(x_\perp, Y = \ln 1/x_{Bj})$$

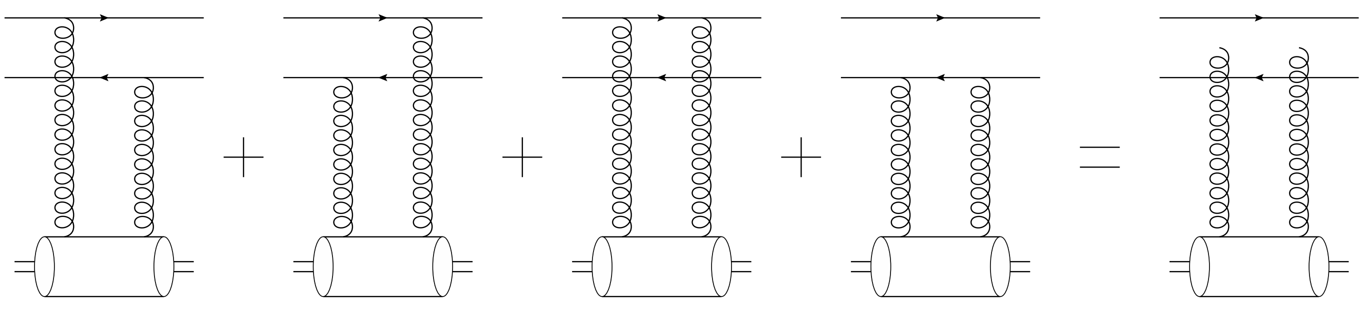
with rapidity Y=ln(1/x)

Quasi-classical dipole amplitude



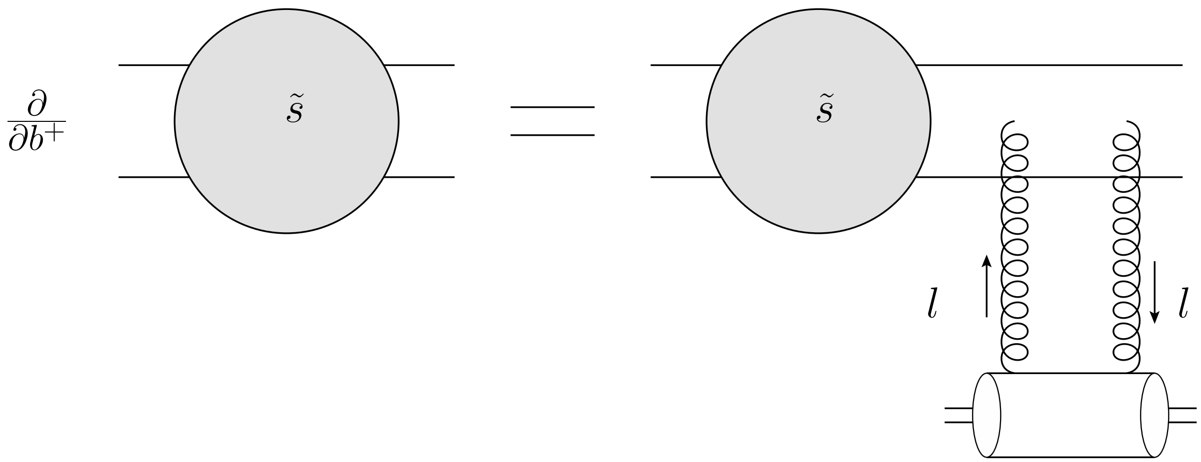
A.H. Mueller, ‘90

Lowest-order interaction with each nucleon – two gluon exchange – the same resummation parameter as in the MV model: $\alpha_s^2\, A^{1/3}$



Quasi-classical dipole amplitude

- To resum multiple rescatterings, note that the nucleons are independent of each other,and, hence, the scatterings are also independent.
- One then writes an equation (Mueller '90)



Each scattering!

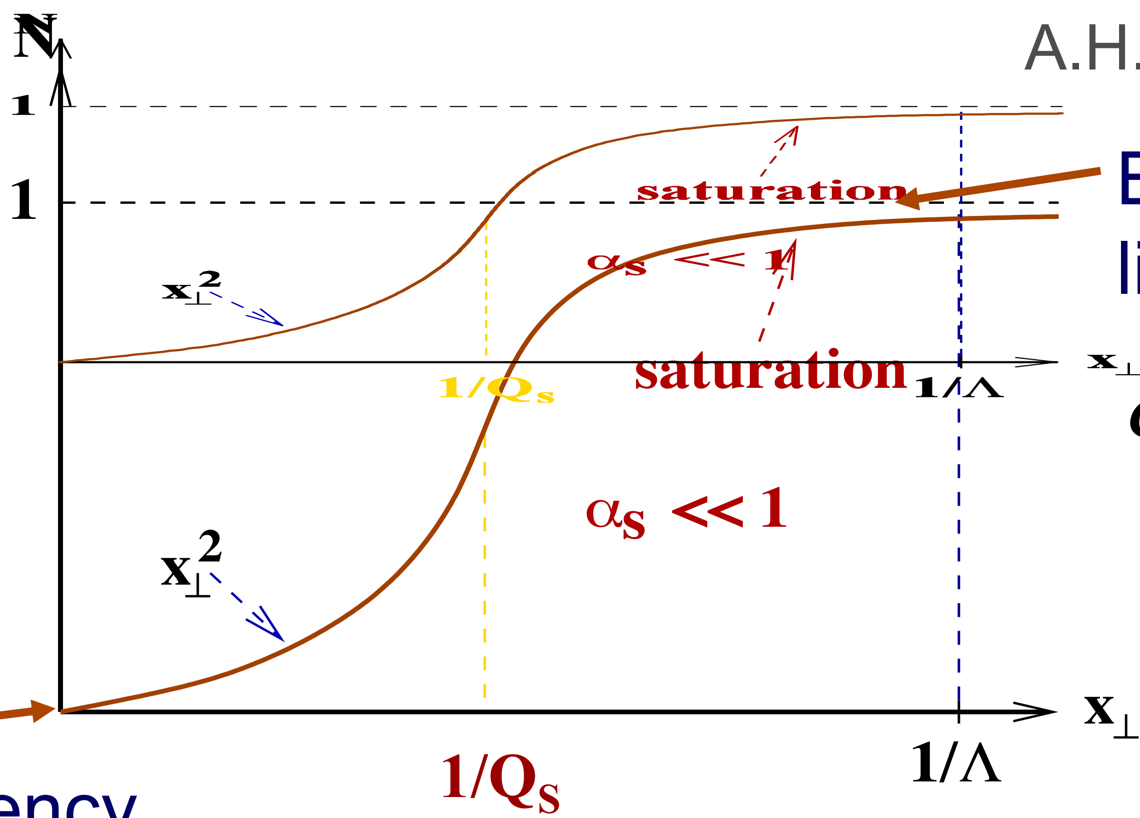
$$N(x_\perp, Y) = 1 - \exp\left[-\frac{x_\perp^2 \, Q_s^2}{4} \, \ln\frac{1}{x_\perp \, \Lambda}\right]$$

DIS in the Classical Approximation

The dipole-nucleus amplitude in the classical approximation is

$$N(x_\perp, Y) = 1 - \exp\left[-\frac{x_\perp^2\, Q_s^2}{4}\, \ln\frac{1}{x_\perp\, \Lambda}\right]$$

A.H. Mueller, ‘90



Black disk limit,

$$\sigma_{tot} < 2\pi R^2$$

Color transparency

Quantum Small-x Evolution

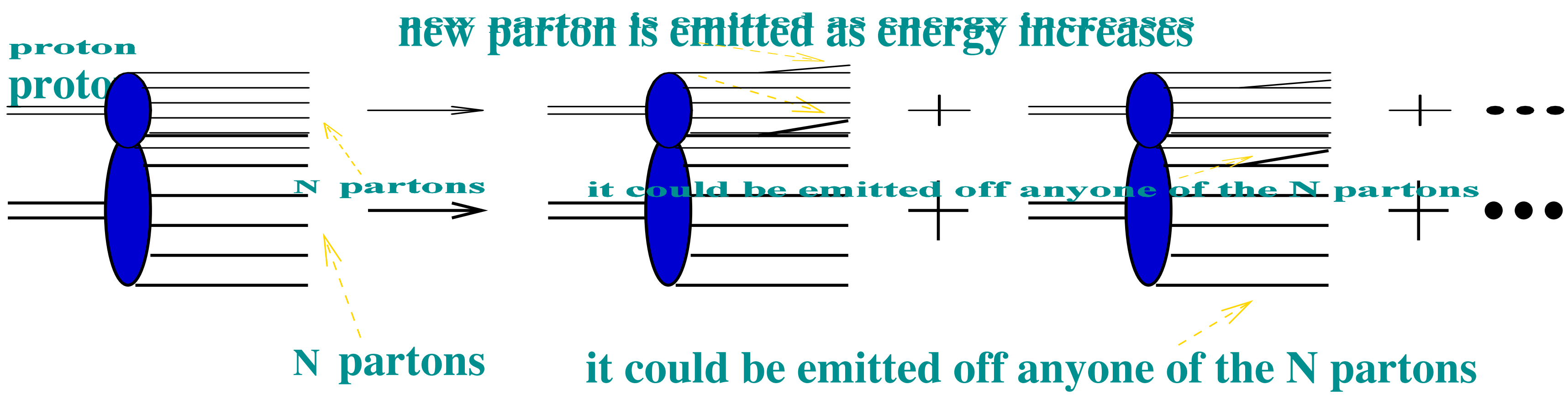
Why Evolve?

- No energy or rapidity dependence in classical field and resulting cross sections.
- Energy/rapidity-dependence comes in through quantum corrections.
- Quantum corrections are included through "evolution equations".

BFKL Equation

Balitsky, Fadin, Kuraev, Lipatov ‘78

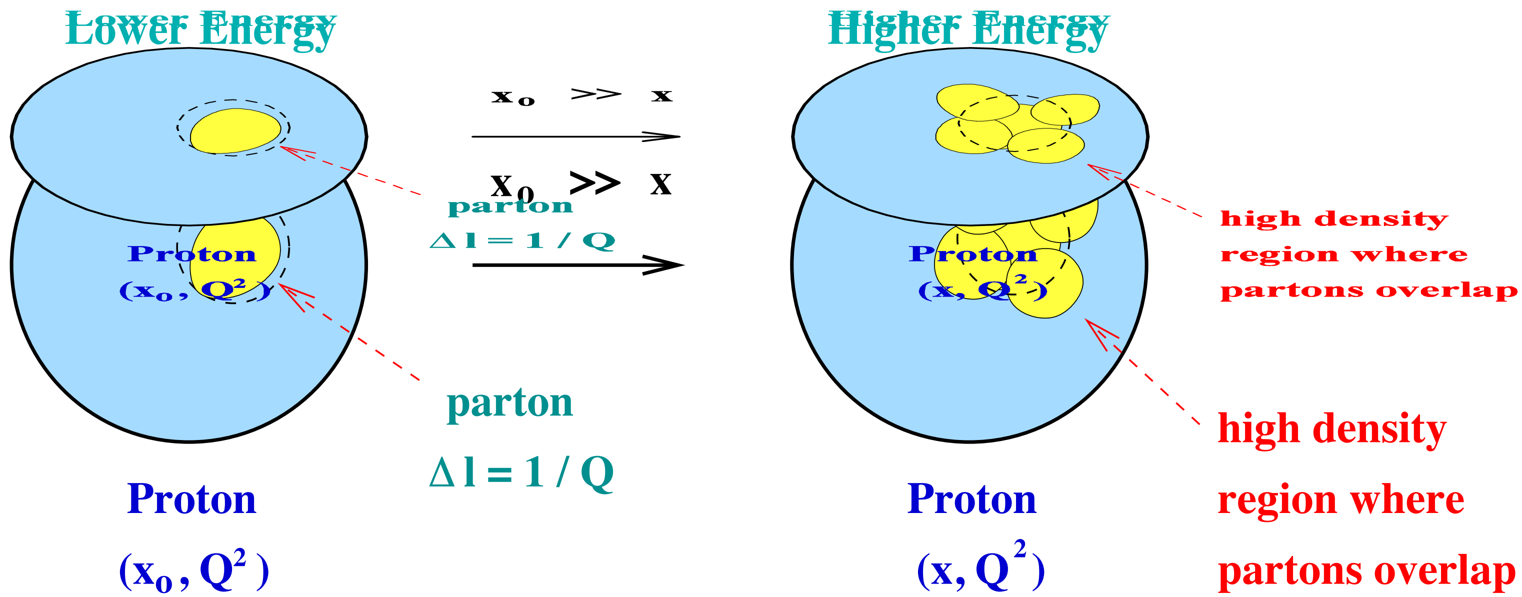
Start with N particles in the proton’s wave function. As we increase the energy a new particle can be emitted by either one of the N particles. The number of newly emitted particles is proportional to N.



The BFKL equation for the number of partons N reads:

$$\frac{\partial}{\partial \ln(1/x)} N(x,Q^2) = \alpha_s K_{BFKL} \otimes N(x,Q^2)$$

BFKL Equation as a High Density Machine



- As energy increases BFKL evolution produces more partons, roughly of the same size. The partons overlap each other creating areas of very high density.
- Number density of partons, along with corresponding cross sections grows as a power of energy

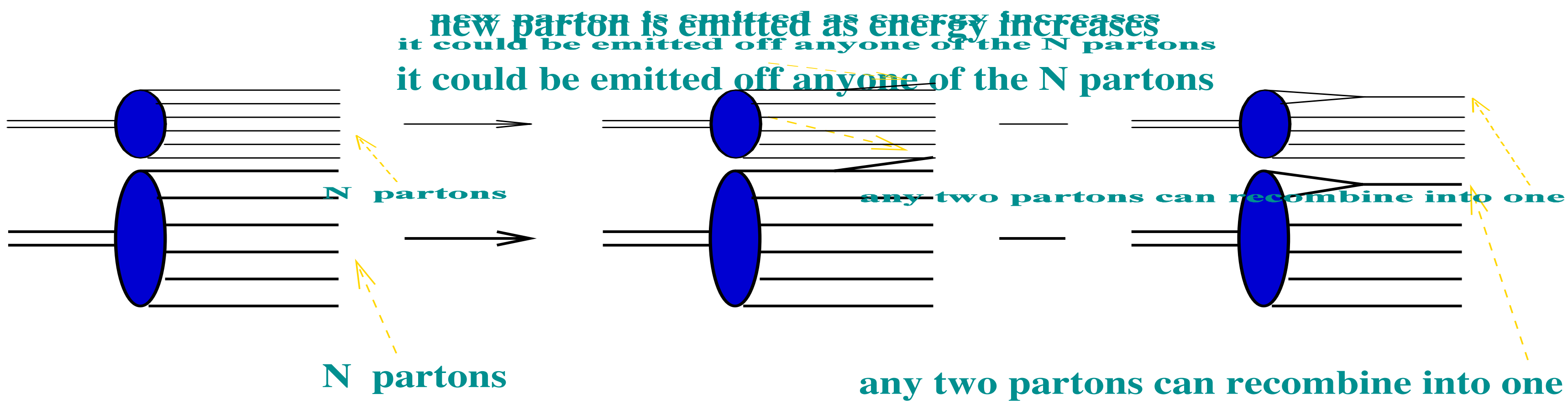
$$N \sim s^{\Delta}$$

- But can parton densities rise forever? Can gluon fields be infinitely strong? Can the cross sections rise forever?
- No! There exists a black disk limit for cross sections, which we know from Quantum Mechanics: for a scattering on a disk of radius R the total cross section is bounded by

$$\sigma_{tot} \leq 2 \pi R^2$$

Nonlinear Equation

At very high energy parton recombination becomes important. Partons not only split into more partons, but also recombine. Recombination reduces the number of partons in the wave function.

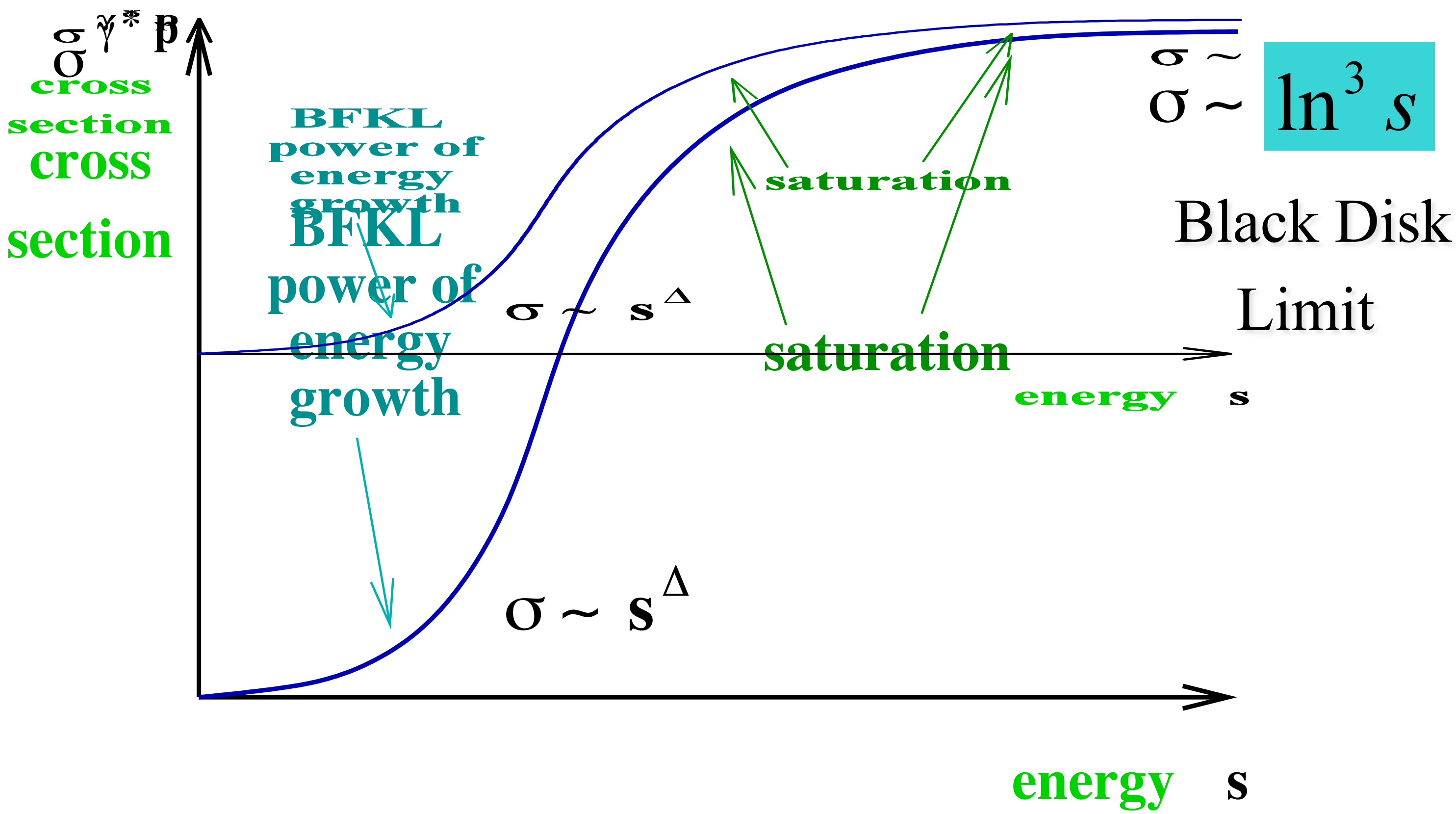


$$\frac{\partial}{\partial Y} N(x, k_T^2) = \alpha_s \, K_{BFKL} \otimes N(x, k_T^2) - \alpha_s \, [N(x, k_T^2)]^2$$

Number of parton pairs ~ N^2

I. Balitsky ’96 (effective Lagrangian)
Yu. K. ’99 (large N_C QCD)

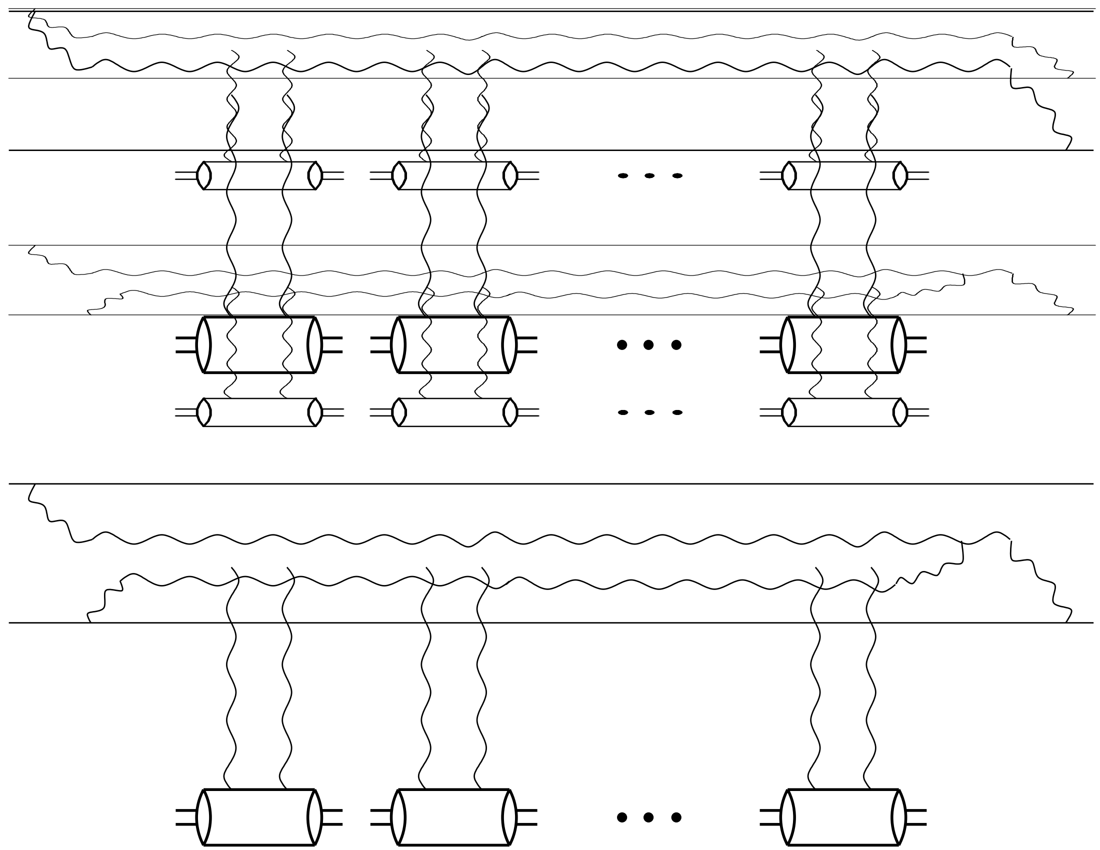
Nonlinear Equation: Saturation



Gluon recombination tries to reduce the number of gluons in the wave function. At very high energy recombination begins to compensate gluon splitting. Gluon density reaches a limit and does not grow anymore. So do total DIS cross sections. Unitarity is restored!

Quantum Evolution

As energy increases the higher Fock states including gluons on top of the quark-antiquark pair become important. They generate a cascade of gluons.

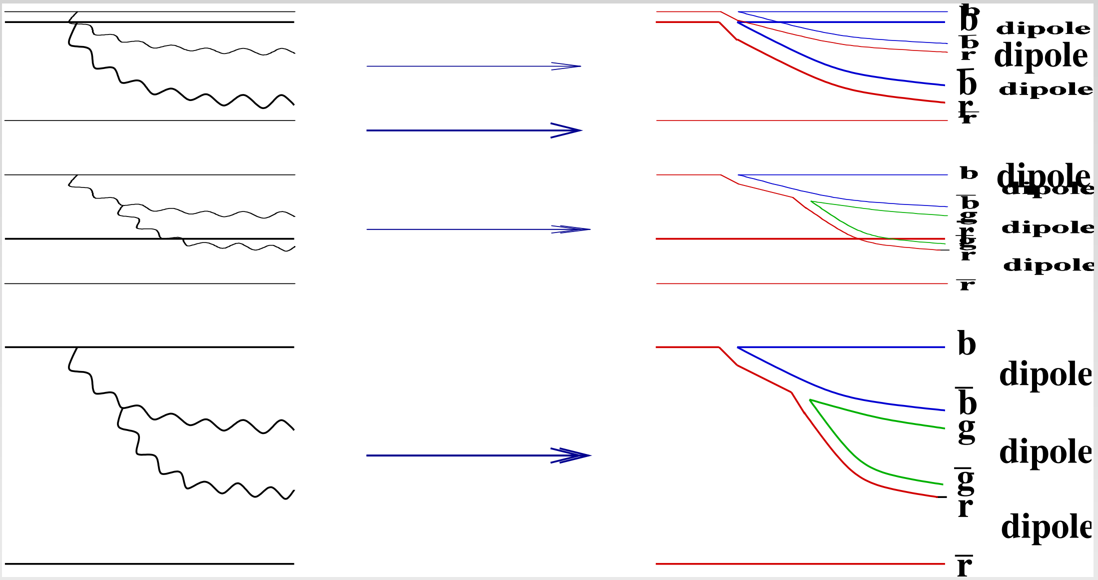


These extra gluons bring in powers of $\alpha_s \ln s$, such that when $\alpha_s << 1$ and $\ln s >> 1$ this parameter is $\alpha_s \ln s \sim 1$ (leading logarithmic approximation, LLA).

Resumming Gluonic Cascade

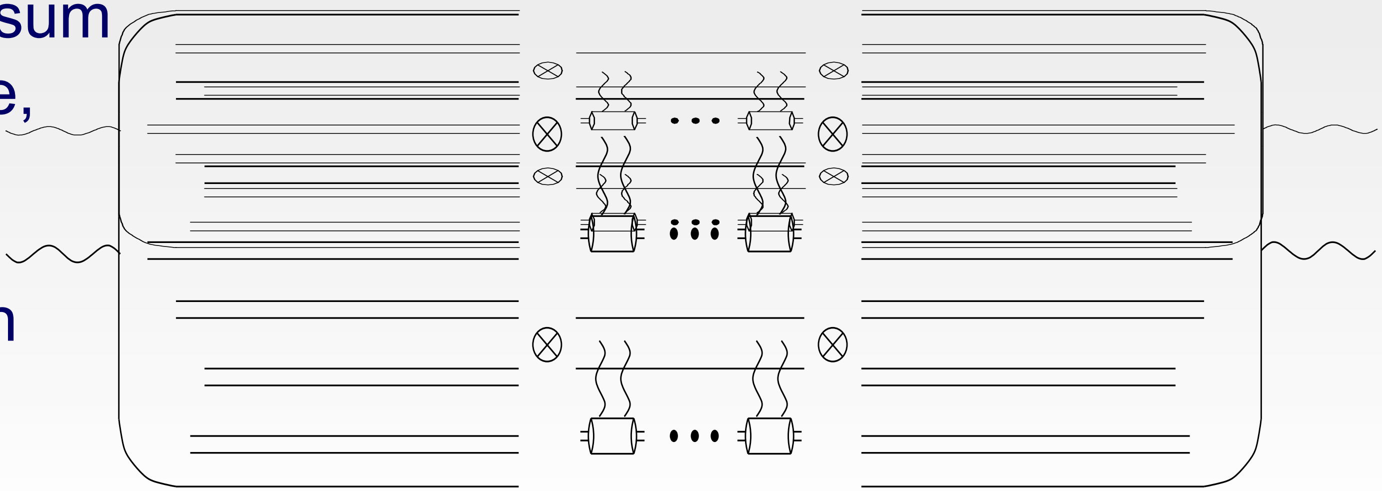
In the large-N_C limit of QCD the gluon corrections become color dipoles. Gluon cascade becomes a dipole cascade.

A. H. Mueller, ’93-’94

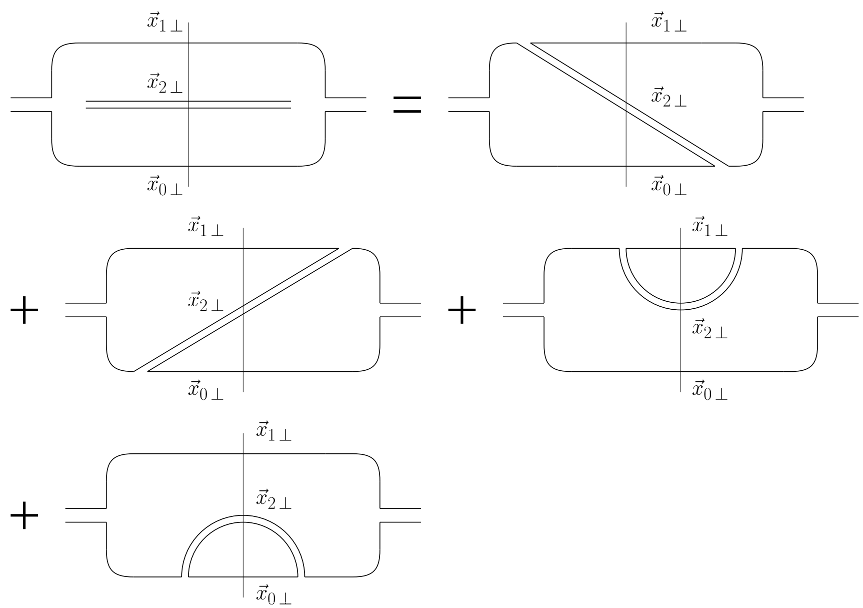


We need to resum dipole cascade, with each final state dipole interacting with the target.

Yu. K. ‘99

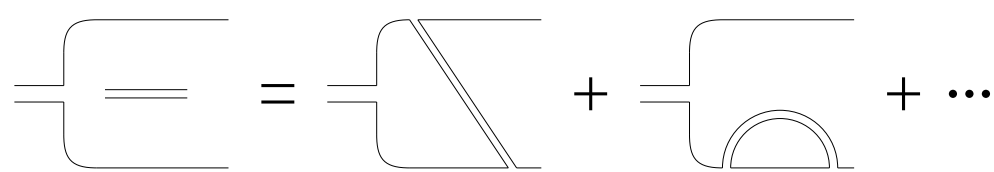


Notation

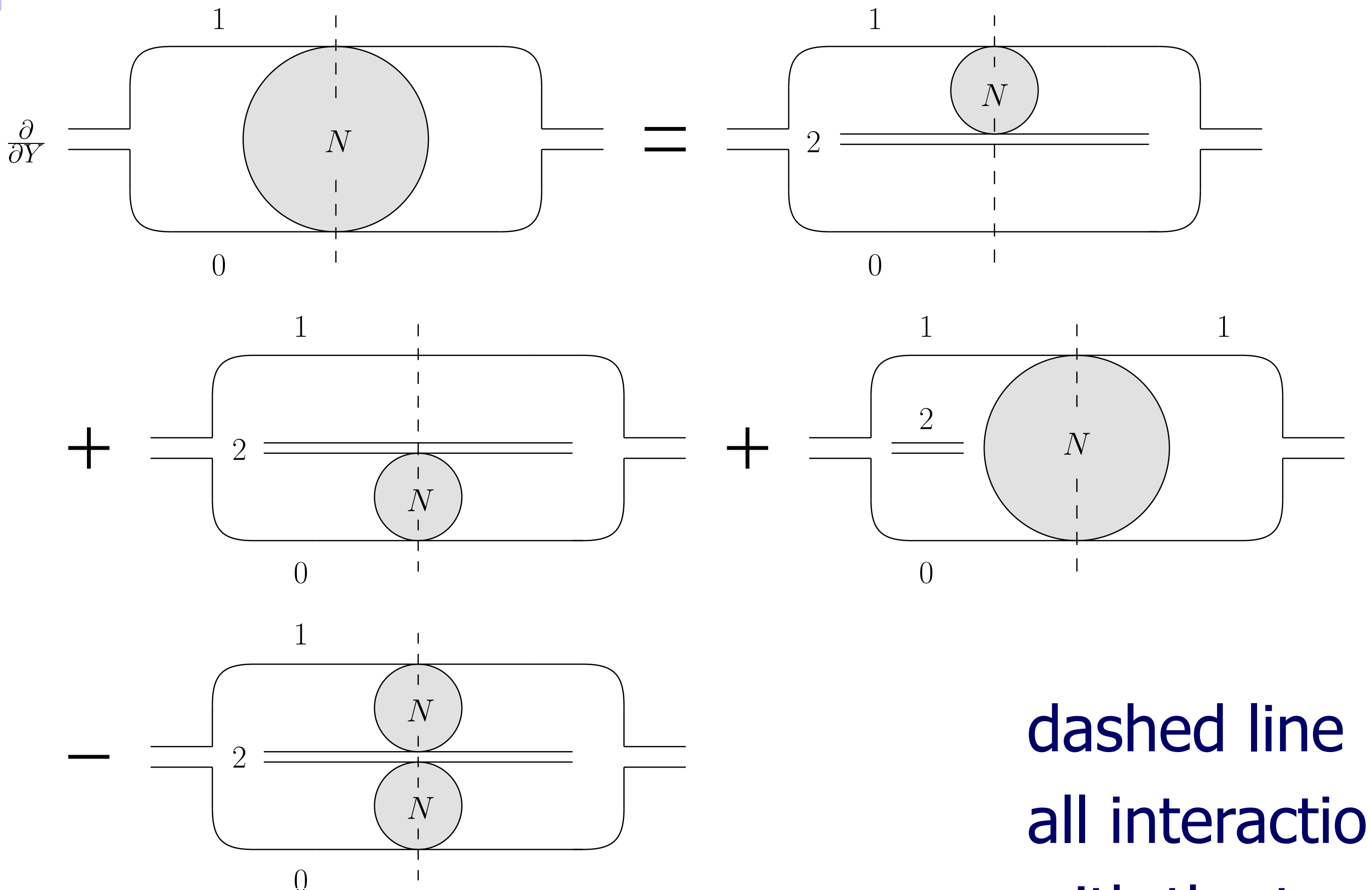


Real emissions in the amplitude squared

Virtual corrections in the amplitude
(wave function)

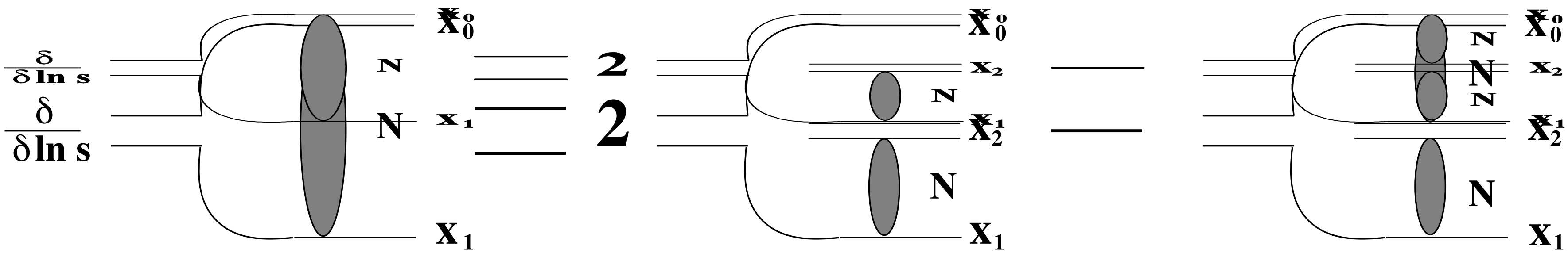


Nonlinear evolution



dashed line =
all interactions
with the target

Nonlinear Evolution Equation



We can resum the dipole cascade

$$\frac{\partial N(x_{01},Y)}{\partial Y} = \frac{\alpha_S N_C}{\pi^2} \int d^2 x_2 \left[\frac{x_{01}^2}{x_{02}^2 \, x_{12}^2} - 2\pi \, \delta^2(\underline{x}_{01} - \underline{x}_{02}) \ln\left(\frac{x_{01}}{\rho}\right) \right] N(x_{02},Y)$$

$$- \frac{\alpha_S N_C}{2\pi^2} \int d^2 x_2 \frac{x_{01}^2}{x_{02}^2 \, x_{12}^2} N(x_{02},Y) \, N(x_{12},Y)$$

I. Balitsky, ’96, HE effective lagrangian

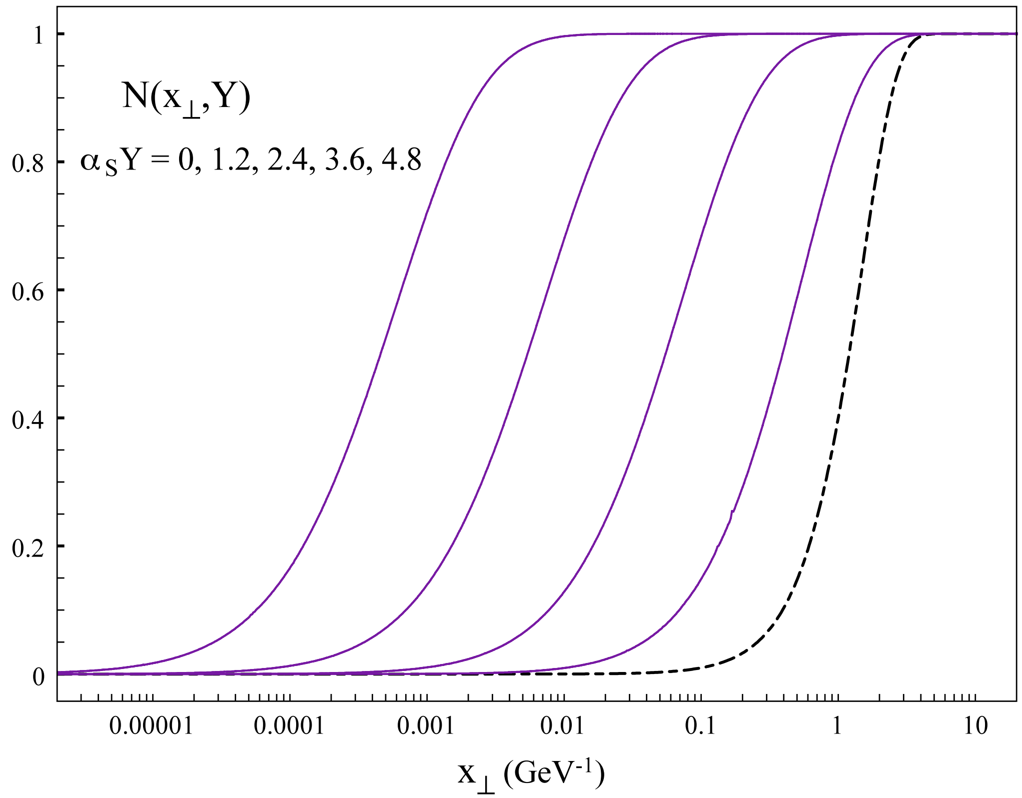
Yu. K., ’99, large N_C QCD

$$N(x_\perp, Y) = 1 - \exp\left[-\frac{x_\perp^2 \, Q_s^2}{4} \ln \frac{1}{x_\perp \Lambda} \right]$$

← initial condition

⇨ Linear part is BFKL, quadratic term brings in damping

Solution of BK equation

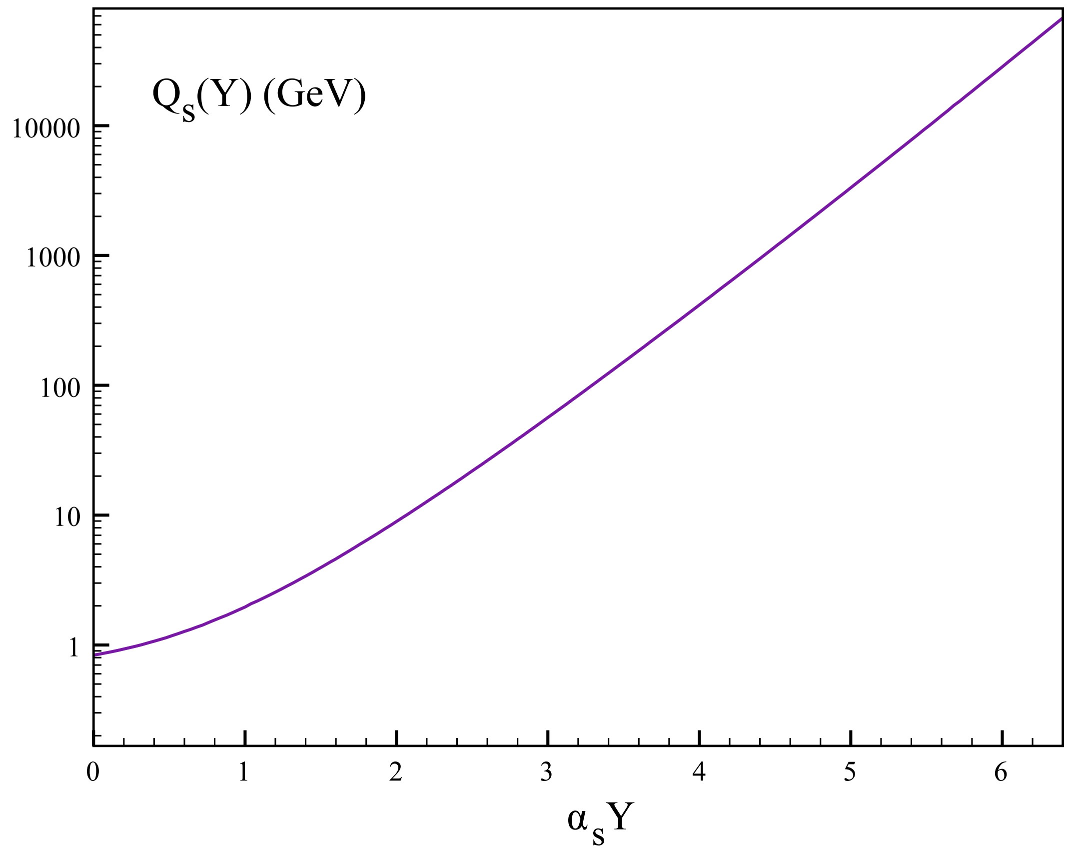


numerical solution by J. Albacete

BK solution preserves the black disk limit, N<1 always (unlike the linear BFKL equation)

$$\sigma^{q\bar{q}A} = 2 \int d^2b \, N(x_\perp, b_\perp, Y)$$

Saturation scale



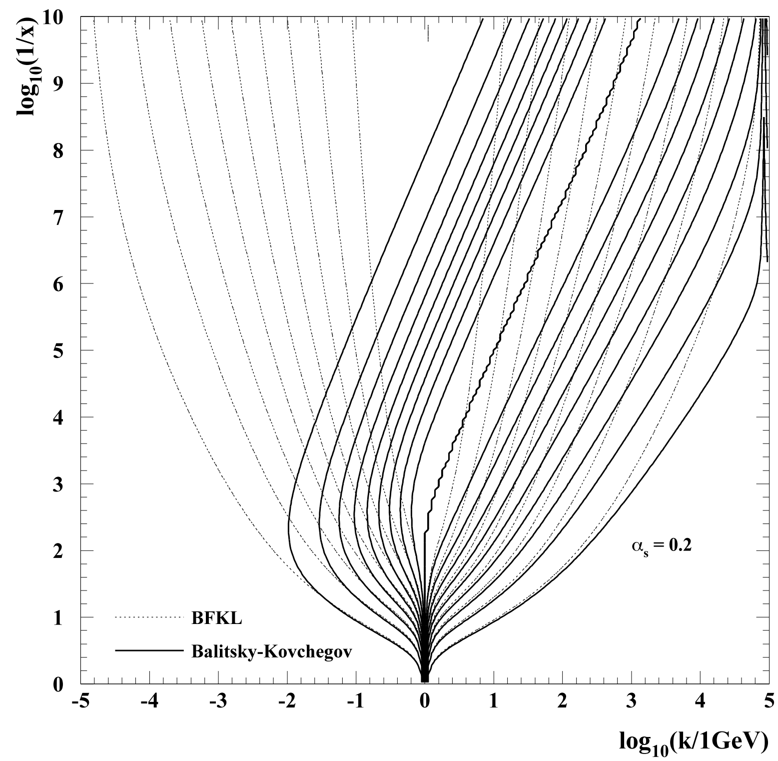
numerical solution by J. Albacete

BK Solution

- Preserves the black disk limit, N<1 always.

$$\sigma^{q\bar{q}A} = 2 \int d^2b\, N(x_\perp, b_\perp, Y)$$

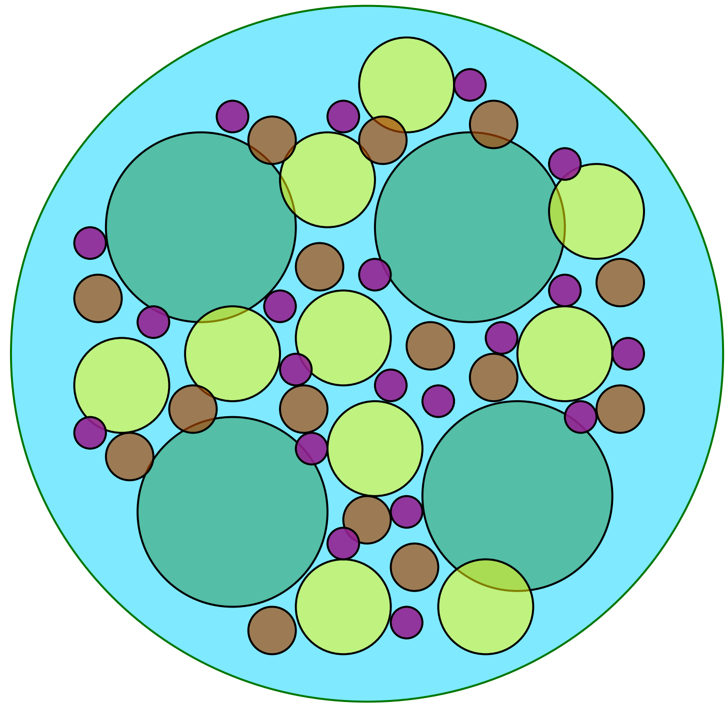
- Avoids the IR problem of BFKL evolution due to the saturation scale screening the IR:



Golec-Biernat, Motyka, Stasto ‘02

Nonlinear Evolution at Work

Proton

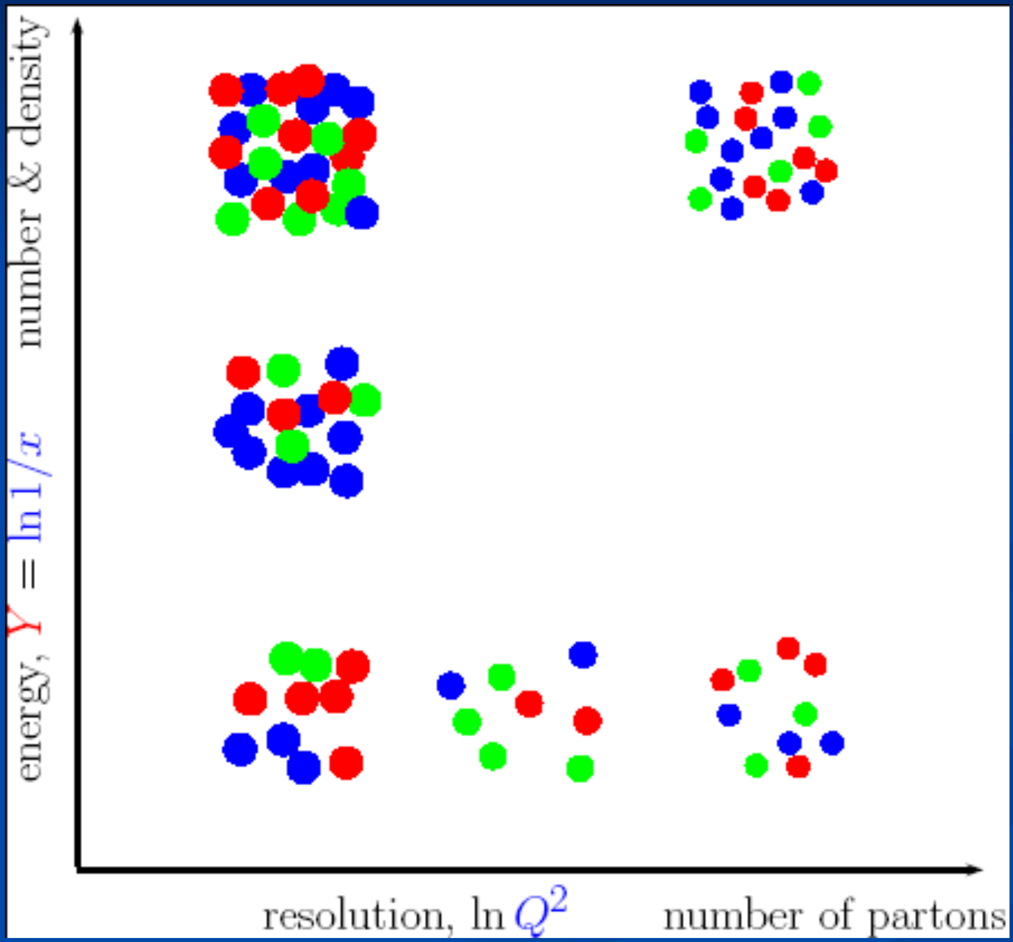


✓ First partons are produced overlapping each other, all of them about the same size.

✓ When some critical density is reached no more partons of given size can fit in the wave function. The proton starts producing smaller partons to fit them in.

Color Glass Condensate

Map of High Energy QCD

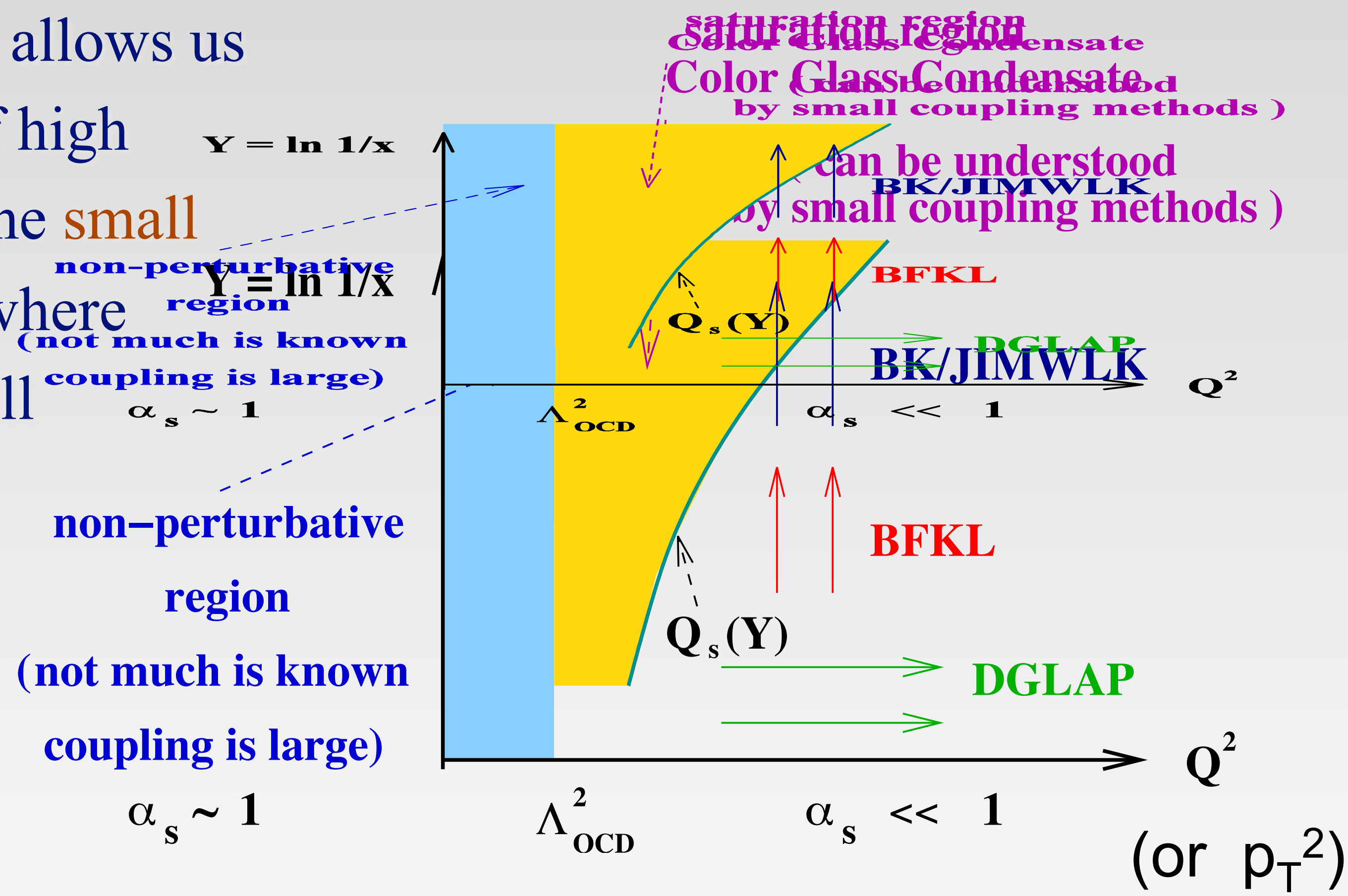


energy

size of gluons

Map of High Energy QCD

Saturation physics allows us to study regions of high parton density in the small coupling regime, where calculations are still under control!



Transition to saturation region is characterized by the saturation scale

$$Q_s^2 \sim A^{1/3}\left(\frac{1}{x}\right)^{\lambda}$$

Going Beyond Large N_C: JIMWLK

To do calculations beyond the large-N_C limit on has to use a functional integro-differential equation written by Iancu, Jalilian-Marian, Kovner, Leonidov, McLerran and Weigert (JIMWLK):

$$\frac{\partial Z}{\partial Y} = \alpha_s \left\{ \frac{1}{2} \frac{\delta^2}{\delta\rho(u)\,\delta\rho(v)} \left[Z\,\chi(u,v) \right] - \frac{\delta}{\delta\rho(u)} \left[Z\,\sigma(u) \right] \right\}$$

where the functional Z[ρ] can then be used for obtaining wave function-averaged observables (like Wilson loops for DIS):

$$\langle O \rangle = \int D\rho\, Z[\rho]\, O[\rho]$$

Going Beyond Large N_C: JIMWLK

- The JIMWLK equation has been solved on the lattice by Rummukainen and H. Weigert '04

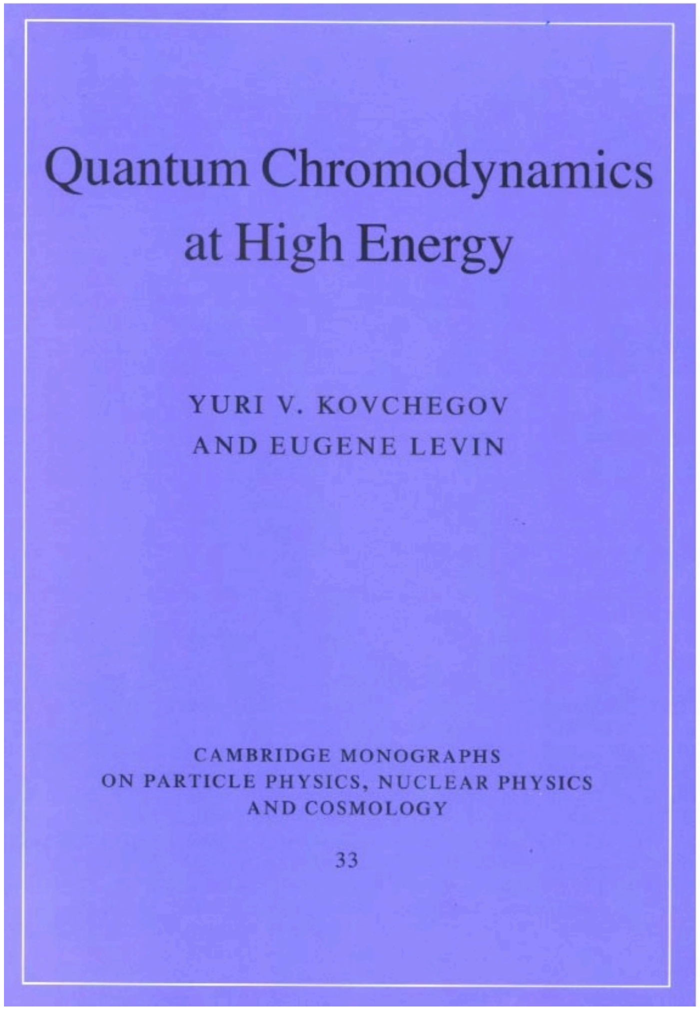
- For the dipole amplitude $N(x_0,x_1, Y)$, the **relative** corrections to the large-N_C limit BK equation are **< 0.001 !** Not the naïve $1/N_C^2 \sim 0.1$! (For realistic rapidities/ energies.)

- The reason for that is dynamical, and is largely due to saturation effects suppressing the bulk of the potential $1/N_C^2$ corrections (Yu.K., J. Kuokkanen, K. Rummukainen, H. Weigert, '08).

References

- E.Iancu, R.Venugopalan, hep-ph/0303204.
- H.Weigert, hep-ph/0501087
- J.Jalilian-Marian, Yu.K., hep-ph/0505052
- F. Gelis et al, arXiv:1002.0333 [hep-ph]
- and…

References



Published in Sept 2012
by Cambridge U Press

DIS Phenomenology

Geometric Scaling

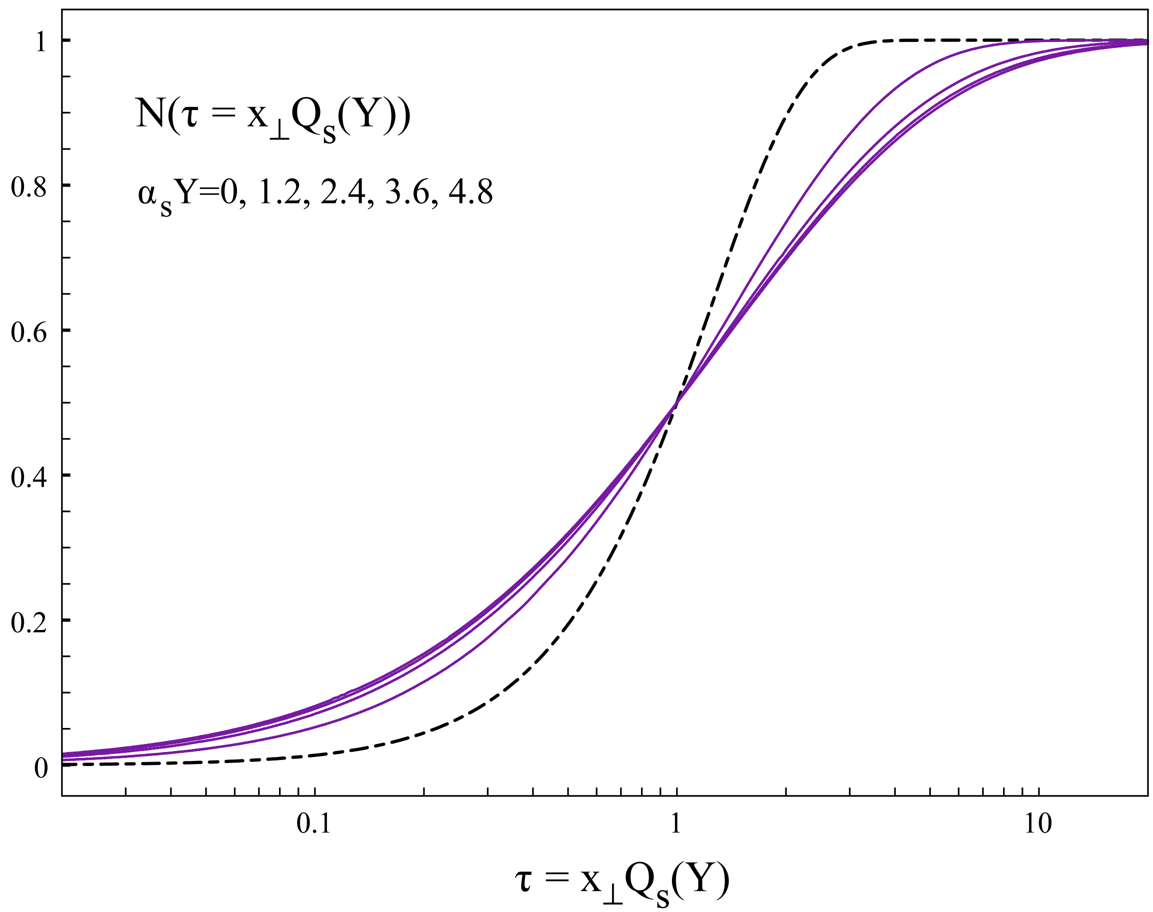
- One of the predictions of the BK/JIMWLK evolution equations is geometric scaling:

  DIS cross section at high energy (small-x) should be a function of one parameter:

$$\sigma_{DIS}(x, Q^2) = \sigma_{DIS}(Q^2 / Q_S^2(x))$$

(Levin, Tuchin ’99; Iancu, Itakura, McLerran ’02)

Geometric Scaling



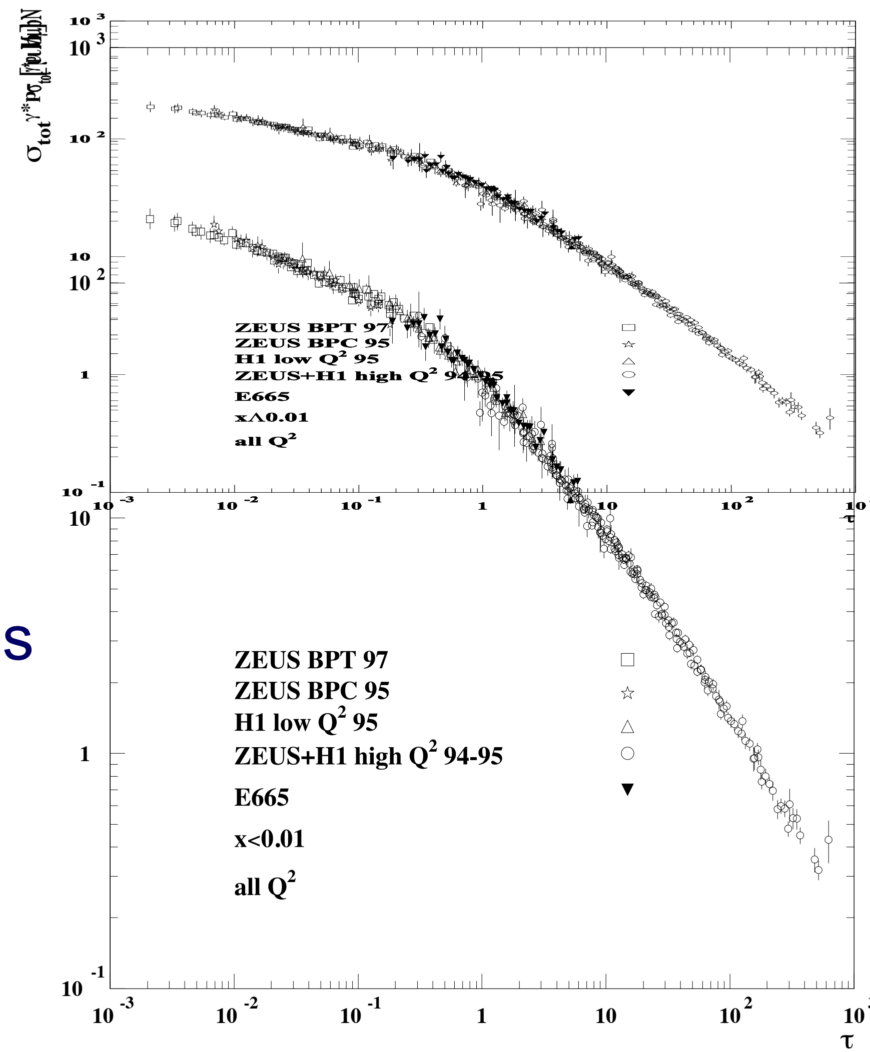
numerical solution by J. Albacete

Geometric Scaling in DIS

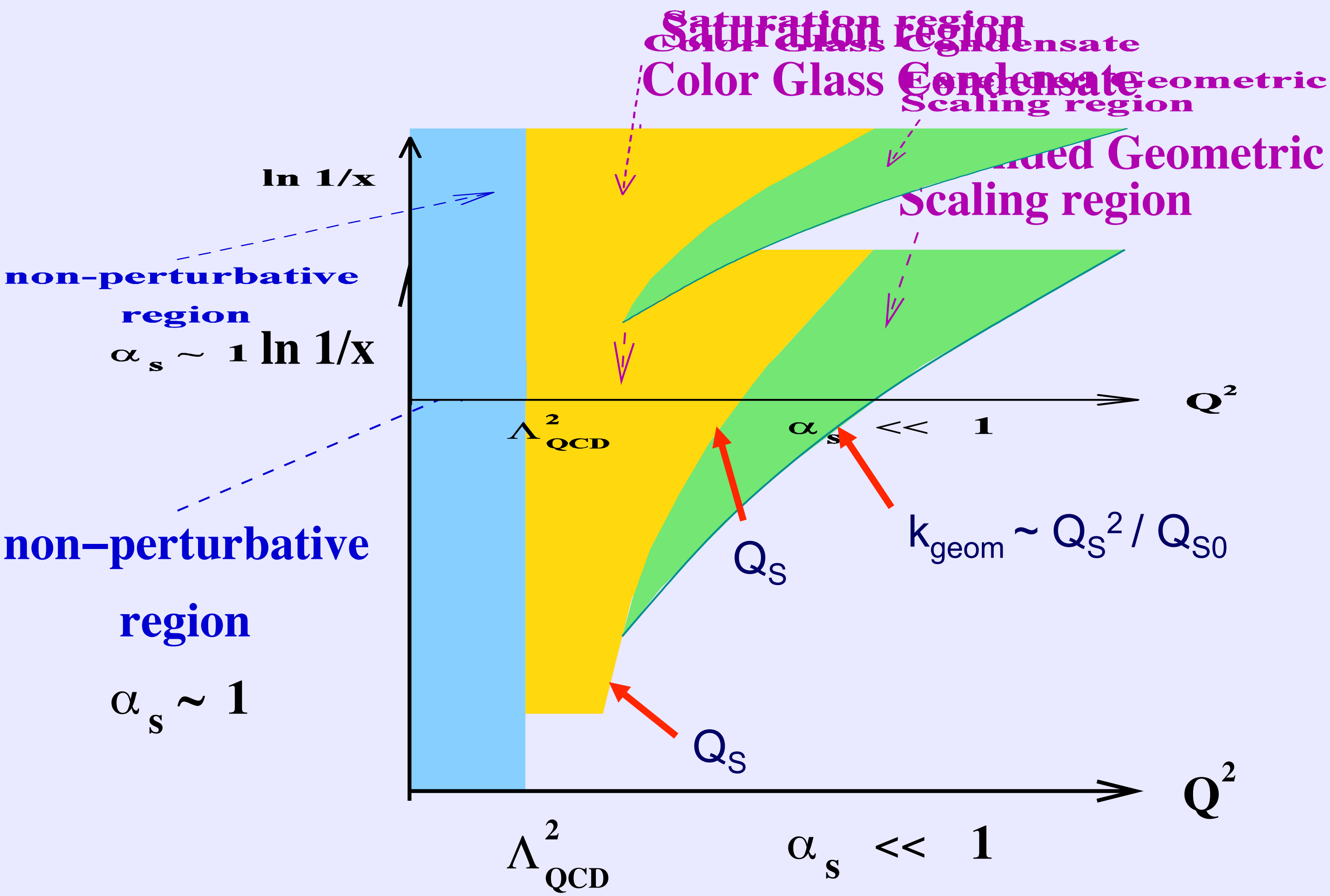
Geometric scaling has been observed in DIS data by Stasto, Golec-Biernat, Kwiecinski in ’00.

Here they plot the total DIS cross section, which is a function of 2 variables - Q^2 and x, as a function of just <u>one</u> variable:

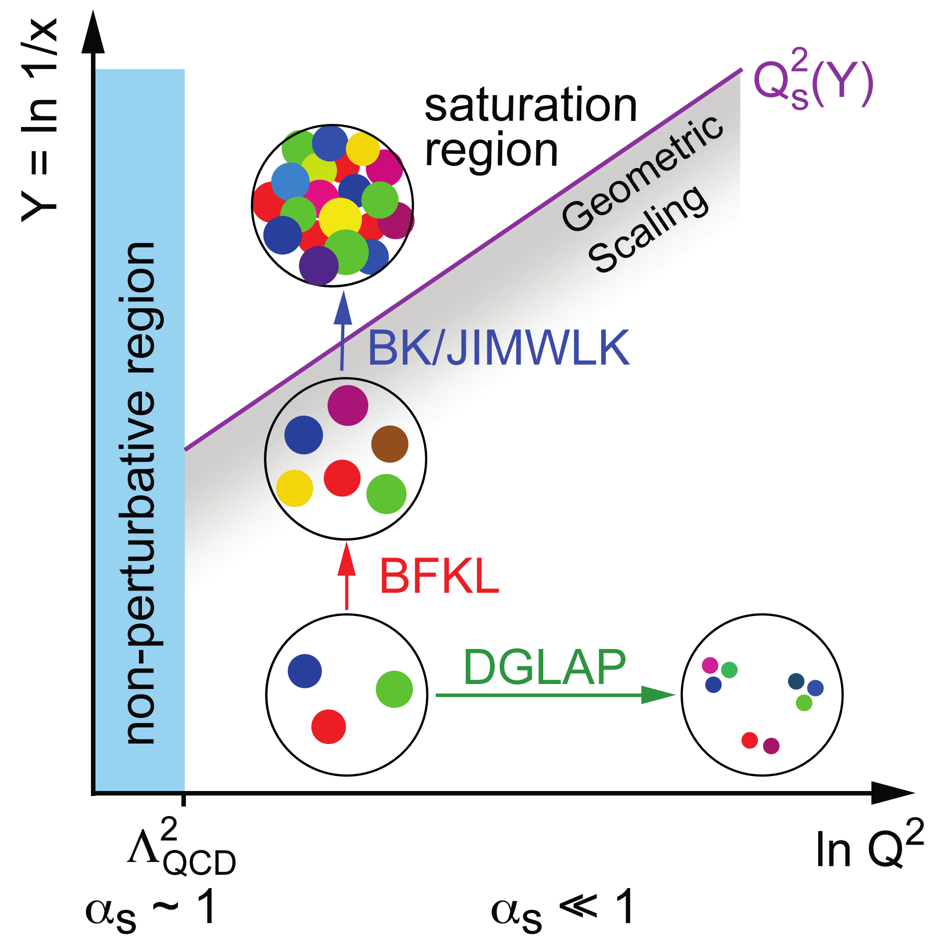
$$\tau = \frac{Q^2}{Q_s^2}$$



Map of High Energy QCD



Map of High Energy QCD

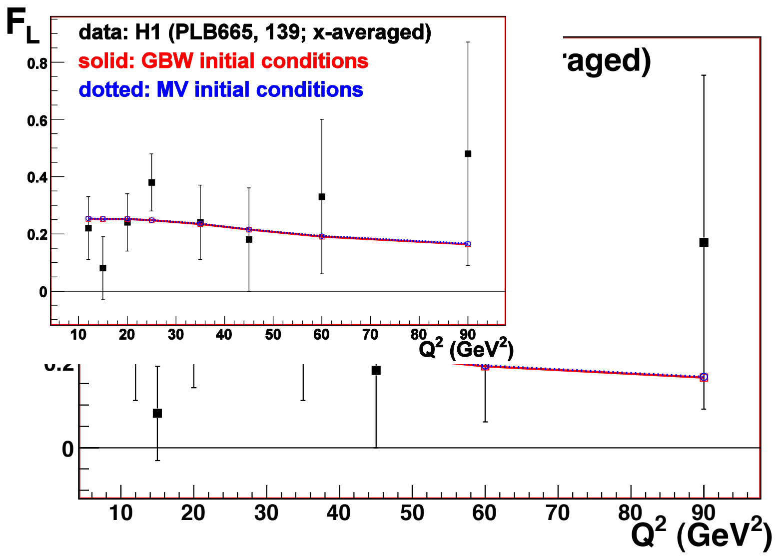


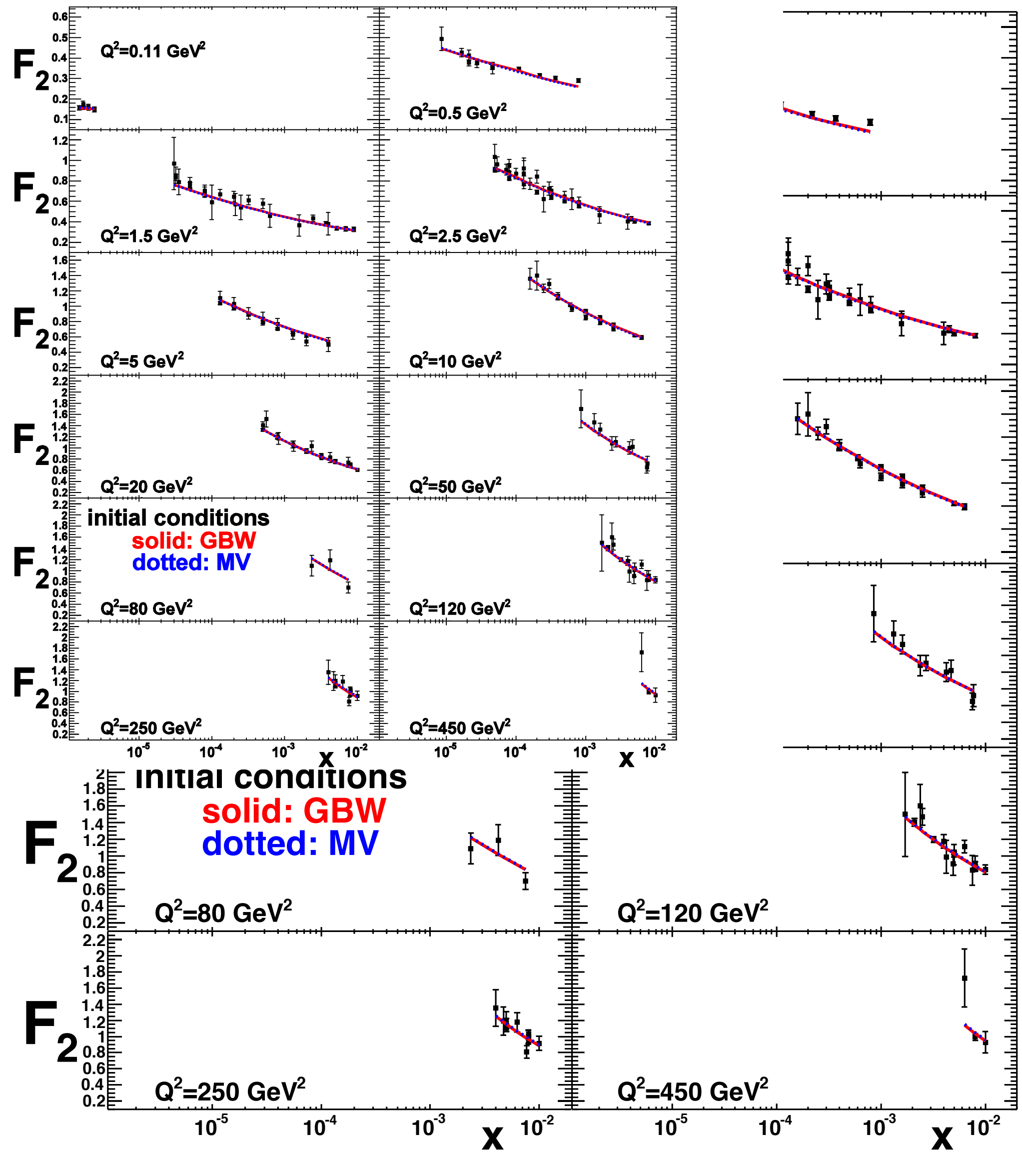
Comparison of rcBK with HERA F2 Data

DIS structure functions:

$$F_{2,\,L} = \frac{Q^2}{4\,\pi^2\,\alpha_{EM}}\,\sigma^{\gamma * p}_{tot,\,L}$$

from Albacete, Armesto, Milhano, Salgado '09





Conclusions

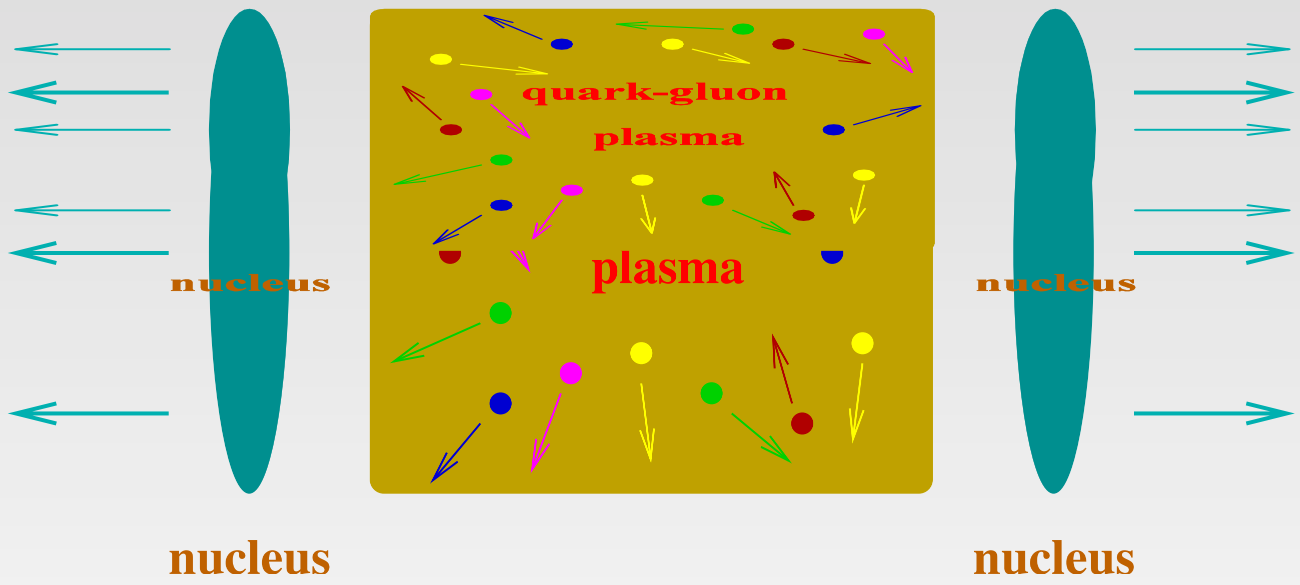
- We have constructed nuclear/hadronic wave function in the quasi-classical approximation (MV model), and studied DIS in the same approximation
- We included small-x evolution corrections into the DIS process, obtaining nonlinear BK/JIMWLK evolution equations
- We found the saturation scale justifying the whole procedure.

$$Q_S^2 \sim A^{1/3}\left(\frac{1}{x}\right)^{\lambda}$$

- Saturation/CGC physics predicts geometric scaling observed experimentally et HERA.

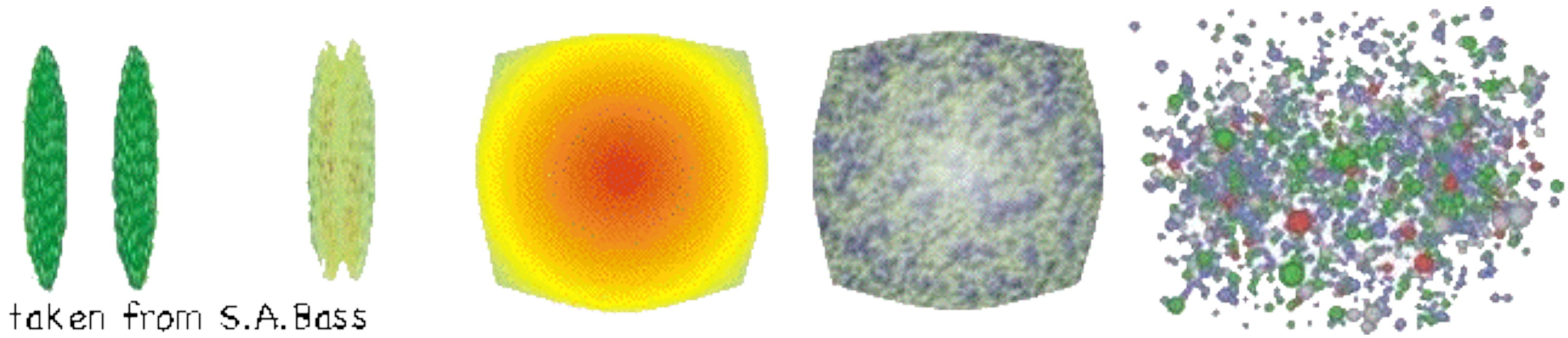
Heavy Ion Phenomenology

Heavy Ion Collisions



⇨ Quarks and gluons are confined inside hadrons. In heavy ion collisions people are trying to create a new state of matter called Quark-Gluon Plasma: a soup of de-confined quarks and gluons.

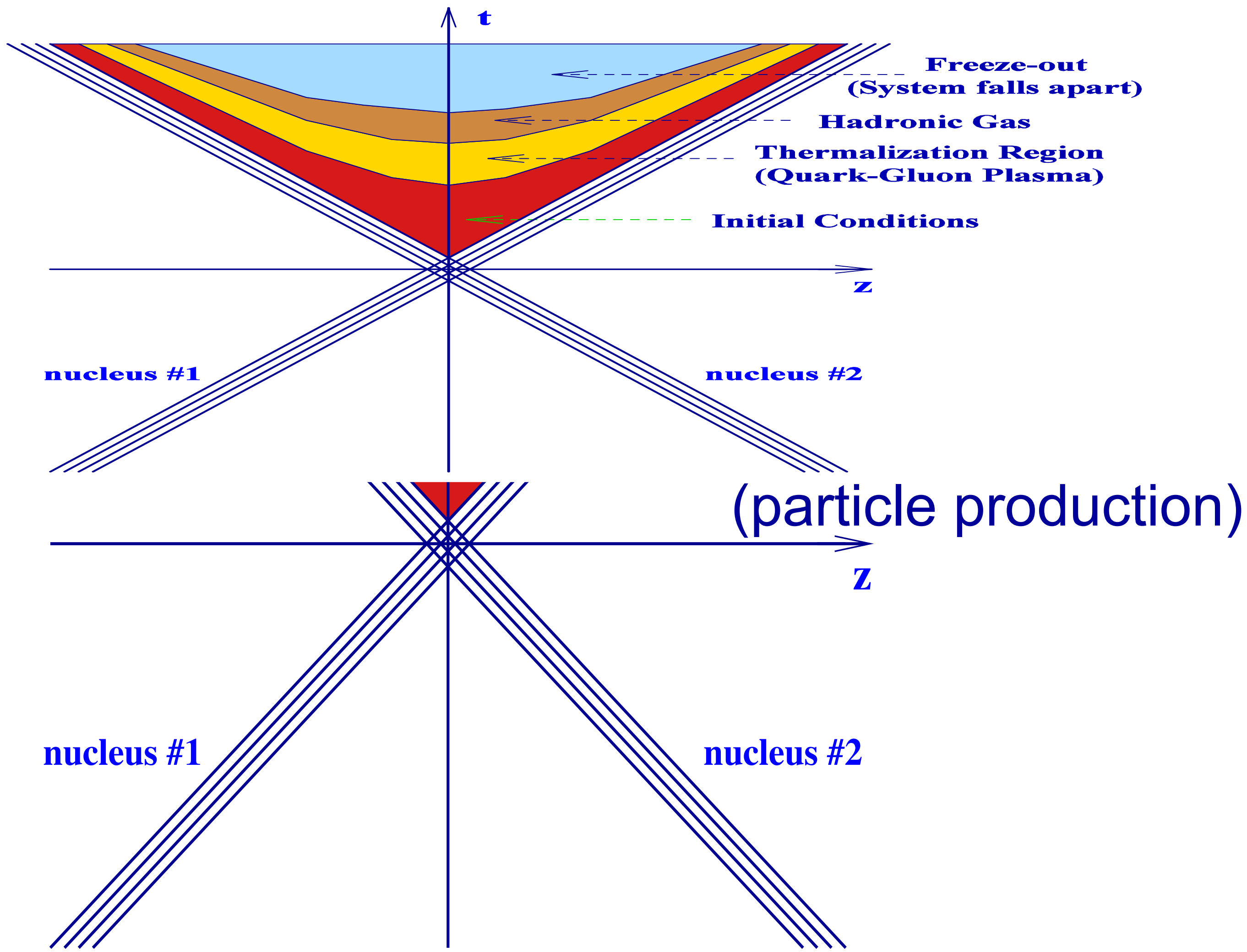
Heavy Ion Collisions



Time evolution of the collision:

- Initial collision and particle production
- Thermalization and formation of quark-gluon plasma (QGP)
- Hadronization: QGP becomes a hadron gas
- Decoupling followed by free-streaming

Timeline of a Heavy Ion Collision

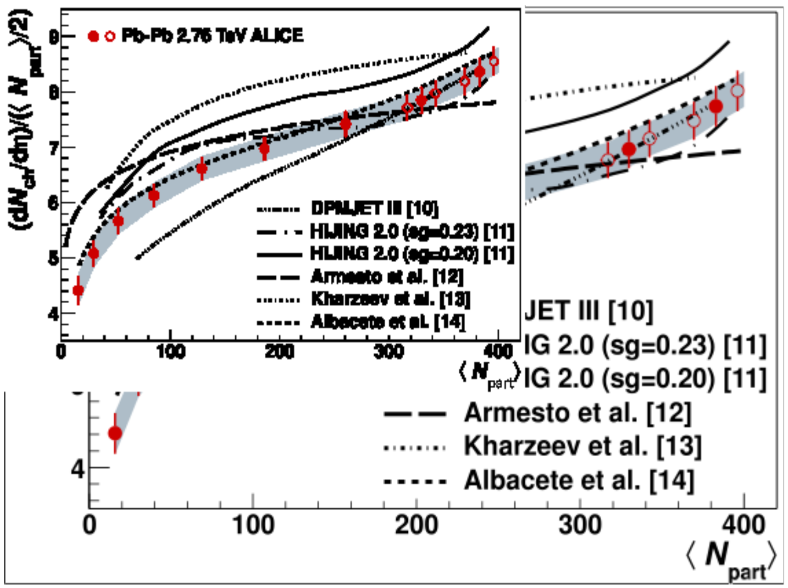


Three-step prescription

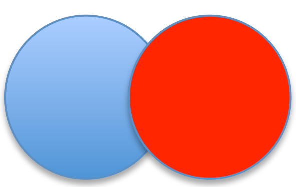
- Calculate the observable in the classical approximation.
- Include nonlinear small-x evolution corrections, introducing energy-dependence.
- To compare with experiment, need to fix the scale of the running coupling.

CGC Multiplicity Prediction

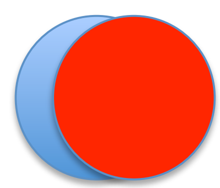
Number of hadrons
per nucleon-nucleon collision
at mid-rapidity.



CGC prediction by
Albacete and Dumitru '10
for LHC multiplicity
and its centrality
dependence was
quite successful.

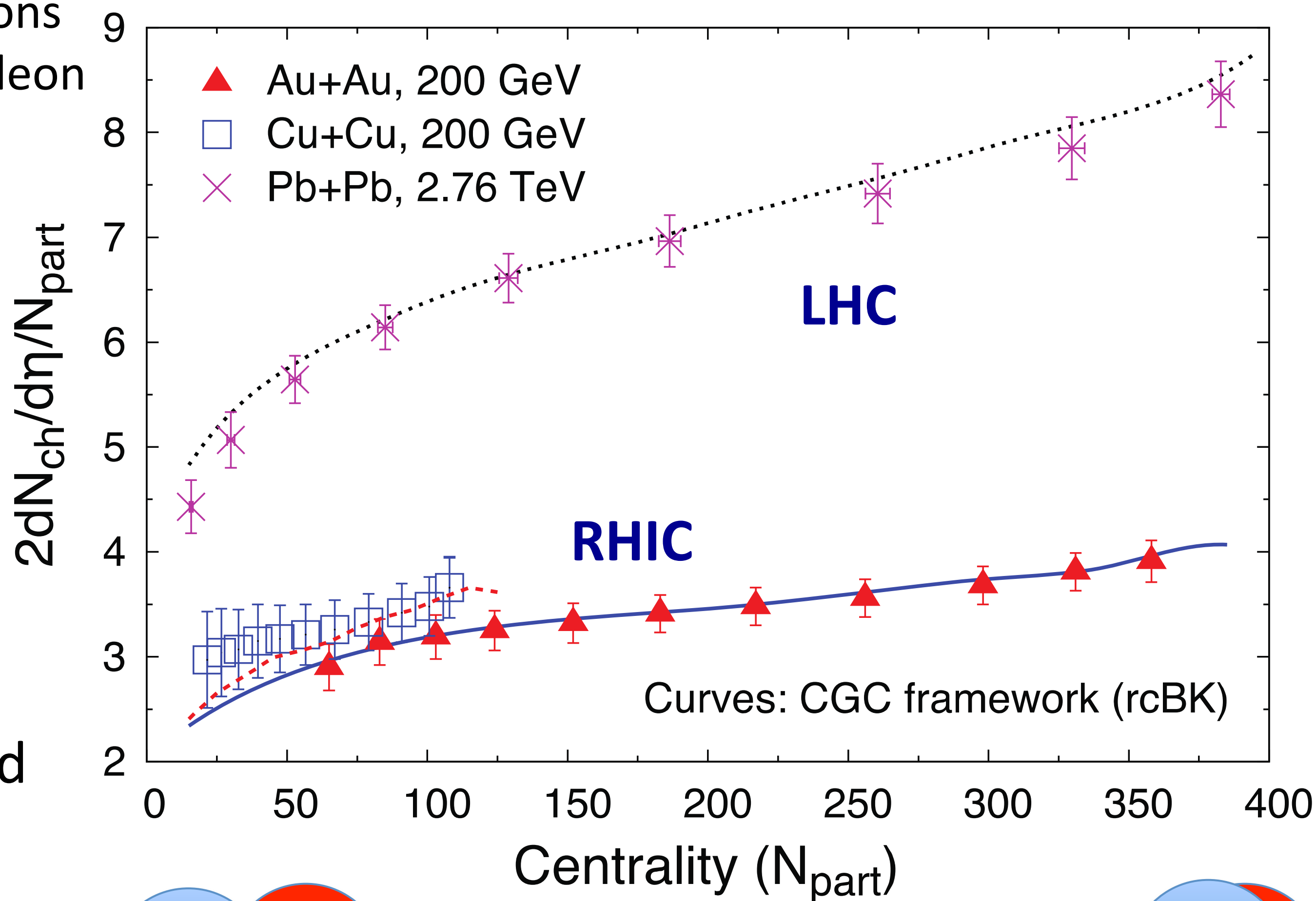


centrality

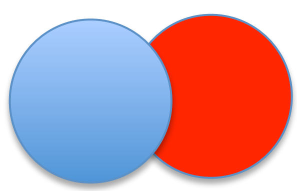


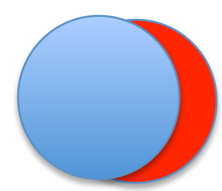
Multiplicity with centrality at RHIC and LHC

Number of hadrons per nucleon-nucleon collision at mid-rapidity.



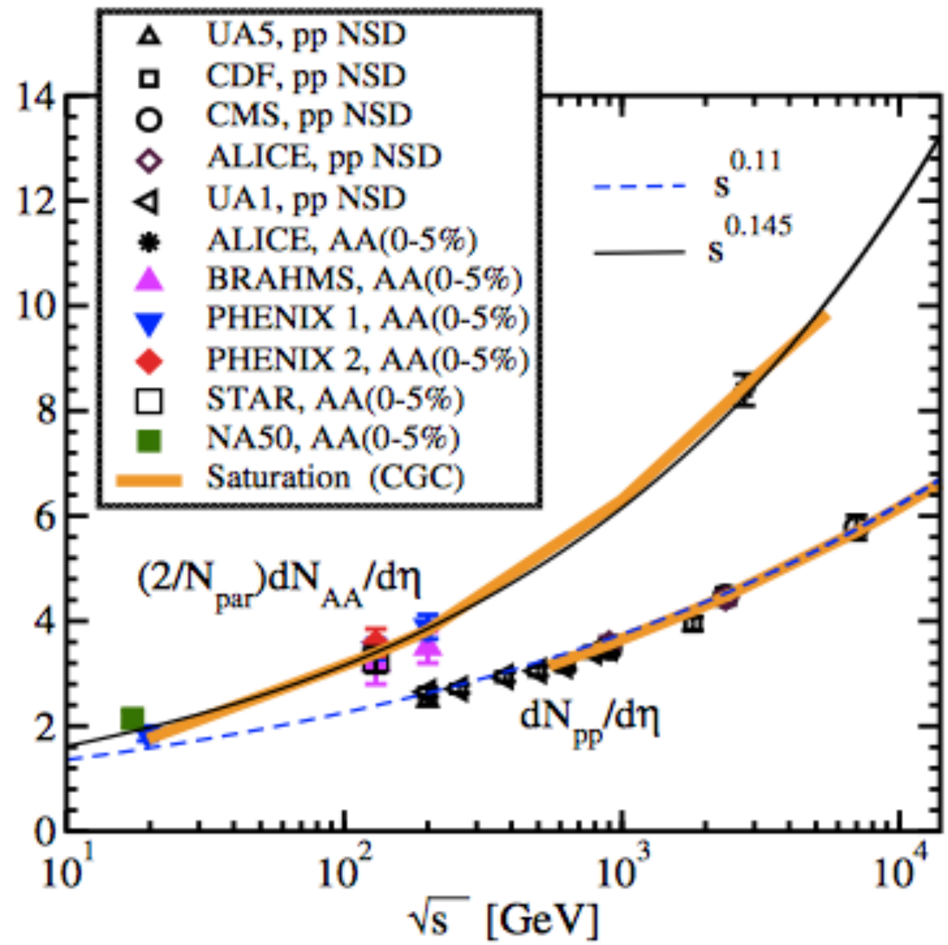
Albacete and Dumitru '10





Multiplicity vs. collision energy

Number of hadrons per unit rapidity at mid-rapidity

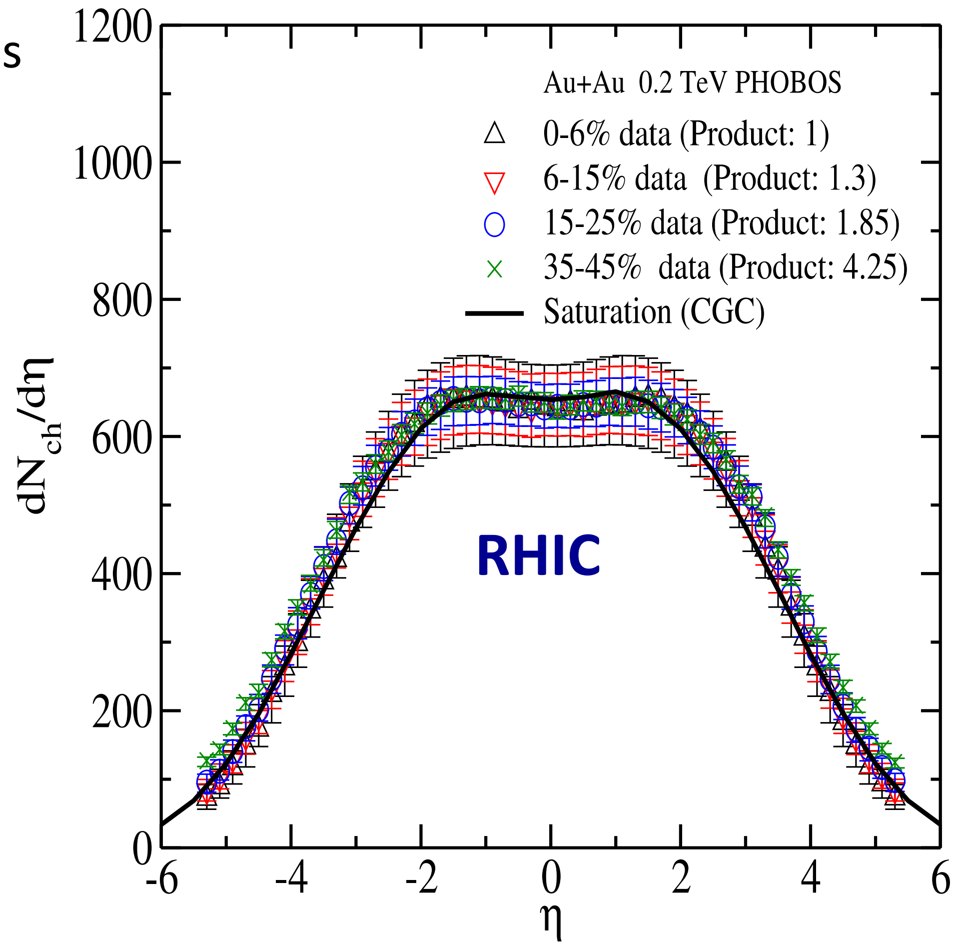


center-of-mass energy

Levin, Rezaeian '11

Multiplicity vs. rapidity

Number of hadrons per unit rapidity



rapidity

Levin and Rezaeian, '11

Conclusions

- In recent years we have made real conceptual progress in understanding QCD in high energy hadronic and nuclear collisions.
- High energy collisions probe a dense system of gluons (Color Glass Condensate), described by nonlinear evolution equations with highly non-trivial behavior.
- Progress on understanding higher order corrections led to an amazingly good agreement of saturation physics fits and predictions (!) with many DIS, p+A, and A+A experiments at HERA, RHIC, and LHC.

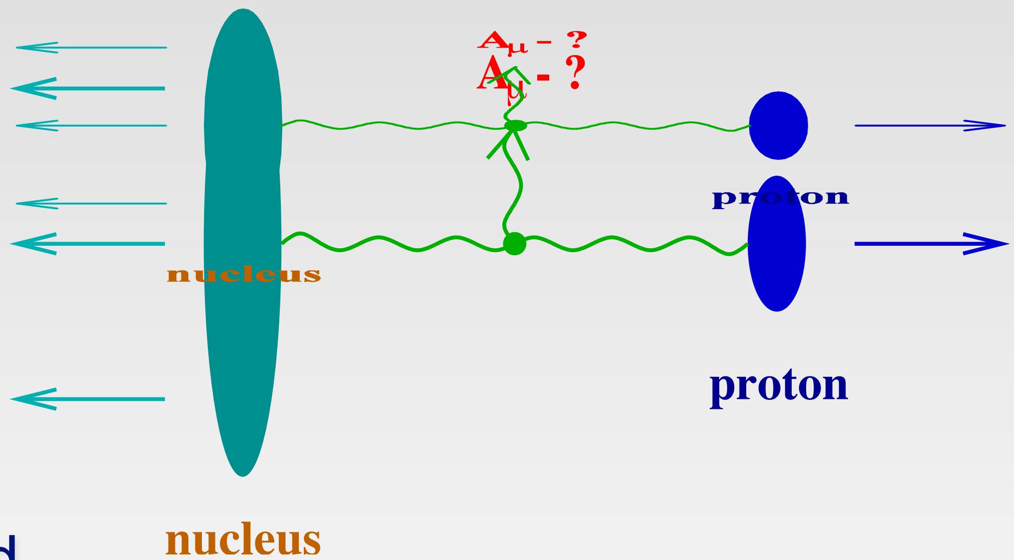
Backup Slides

Gluon Production in Proton-Nucleus Collisions (pA): Classical Field

To find the gluon production cross section in pA one has to solve the same classical Yang-Mills equations

$$D_\nu F^{\nu\mu} = J^\mu$$

for two sources – proton and nucleus.



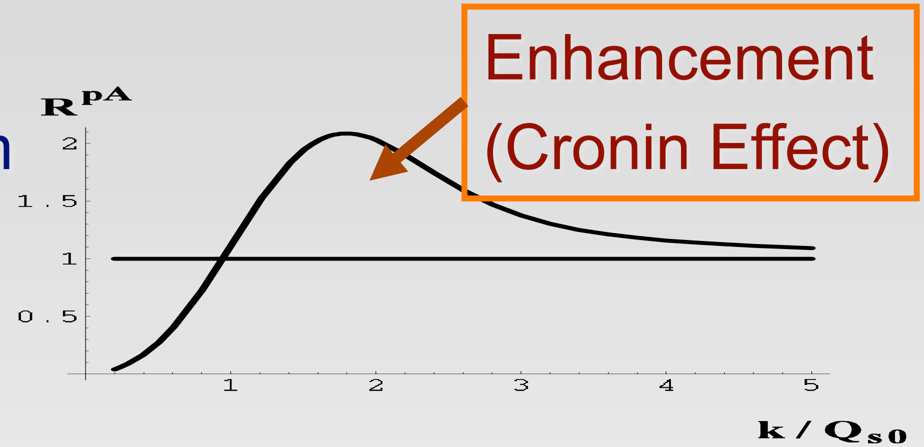
This classical field has been found by Yu. K., A.H. Mueller in '98

Gluon Production in pA: Classical Field

To understand how the gluon production in pA is different from independent superposition of *A* proton-proton (pp) collisions one constructs the quantity

$$R^{pA} = \frac{E \dfrac{d\sigma^{pA}}{d^3k}}{A \times E \dfrac{d\sigma^{pp}}{d^3k}}$$

which is = 1 for independent superposition of sub-collisions.

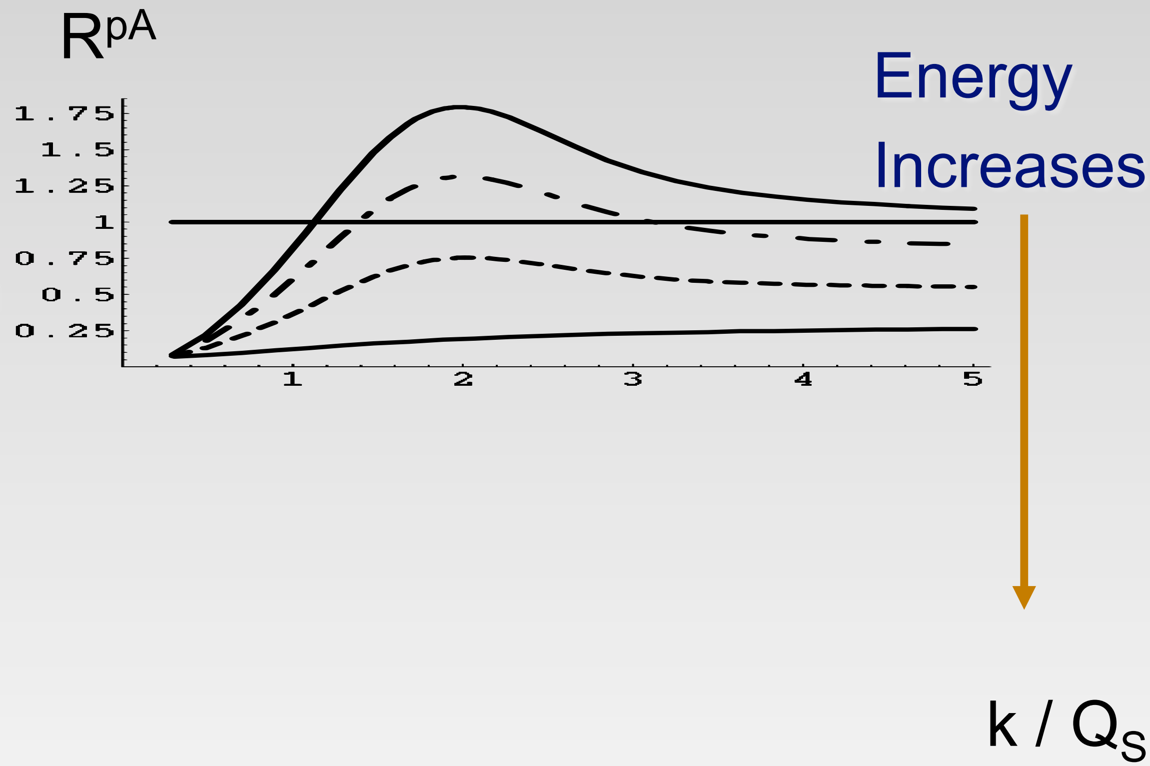


The quantity R^{pA} plotted for the classical solution found in Yu. K., A. Mueller, ‘98

Nucleus pushes gluons to higher transverse momentum!

Gluon Production in pA: BK Evolution

Including quantum corrections to gluon production cross section in pA using BK evolution equation introduces suppression in R^{pA} with increasing energy!



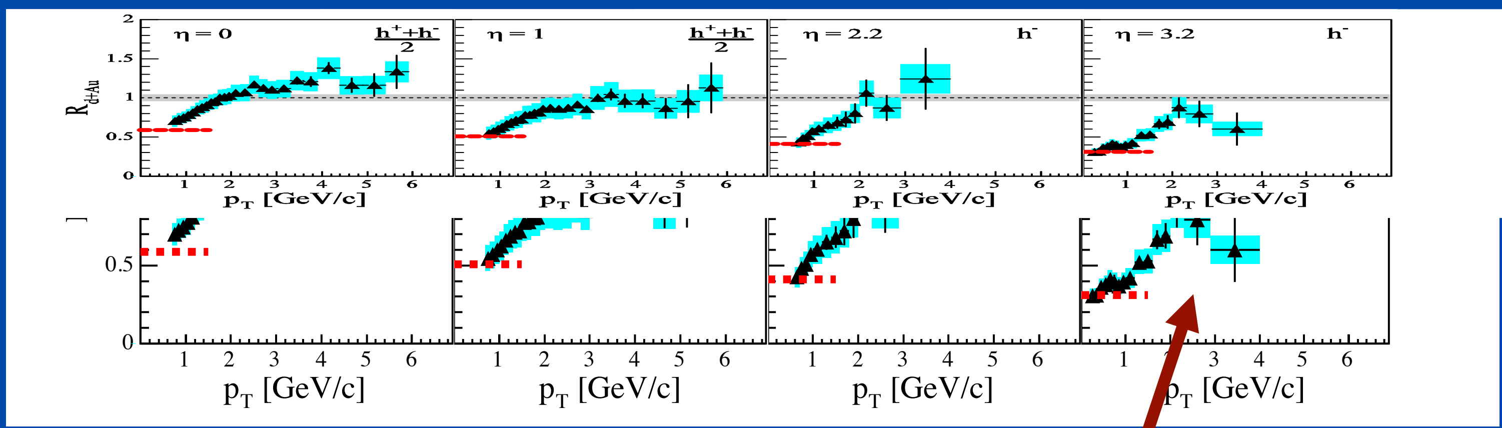
The plot is from D. Kharzeev, Yu. K., K. Tuchin '03
(see also Kharzeev, Levin, McLerran, '02, Albacete et al, '03)

Saturation/Color Glass Discovery

At RHIC they did deuteron-gold (dAu) collisions instead of pA (machine reasons).

BRAHMS collaboration data ‘04

R^{dAu}

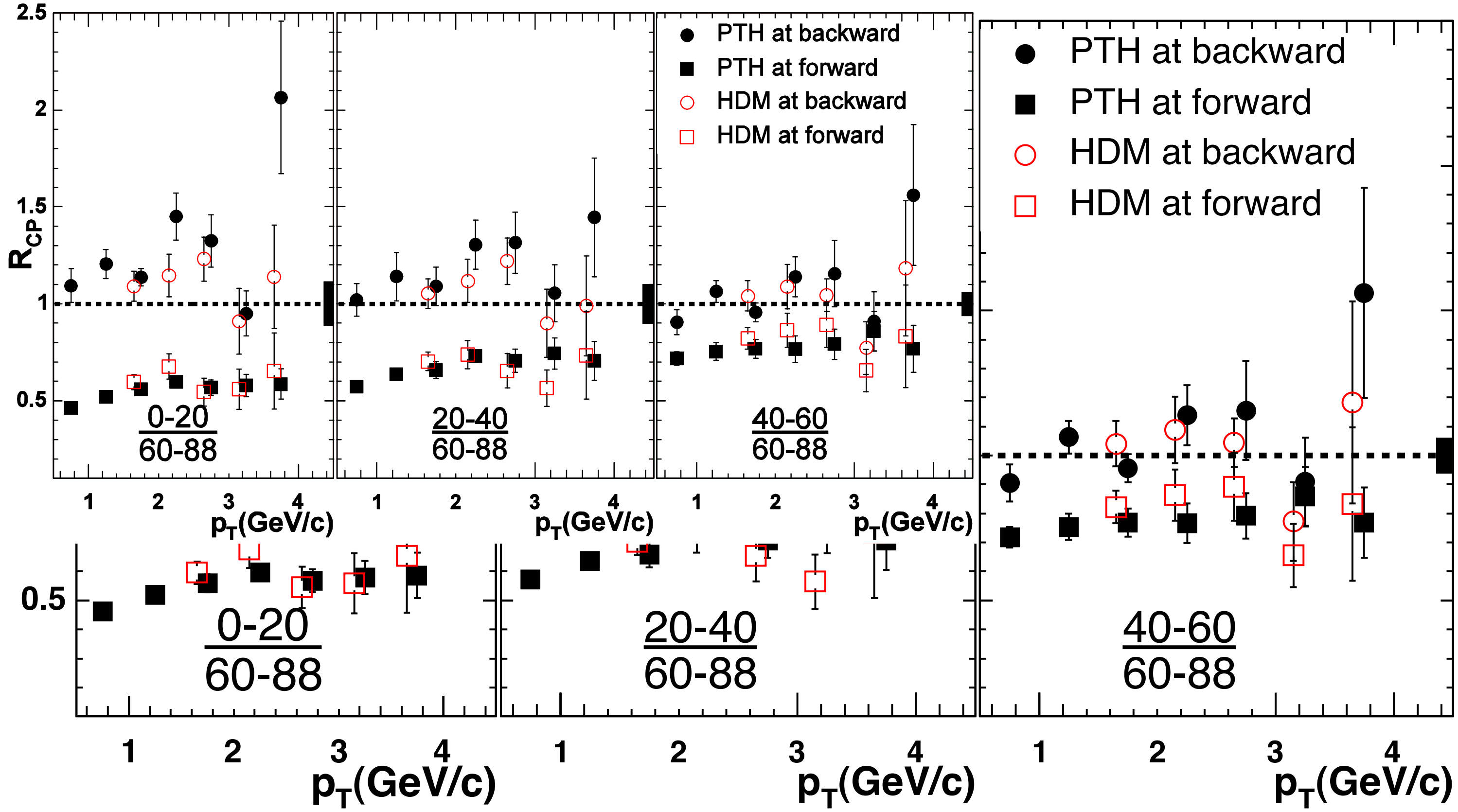


No other theory predicts this behavior!

Suppression!

⇨ <u>Our prediction based on BK evolution seems to work!</u>

R_{d+Au} at forward and backward rapidities

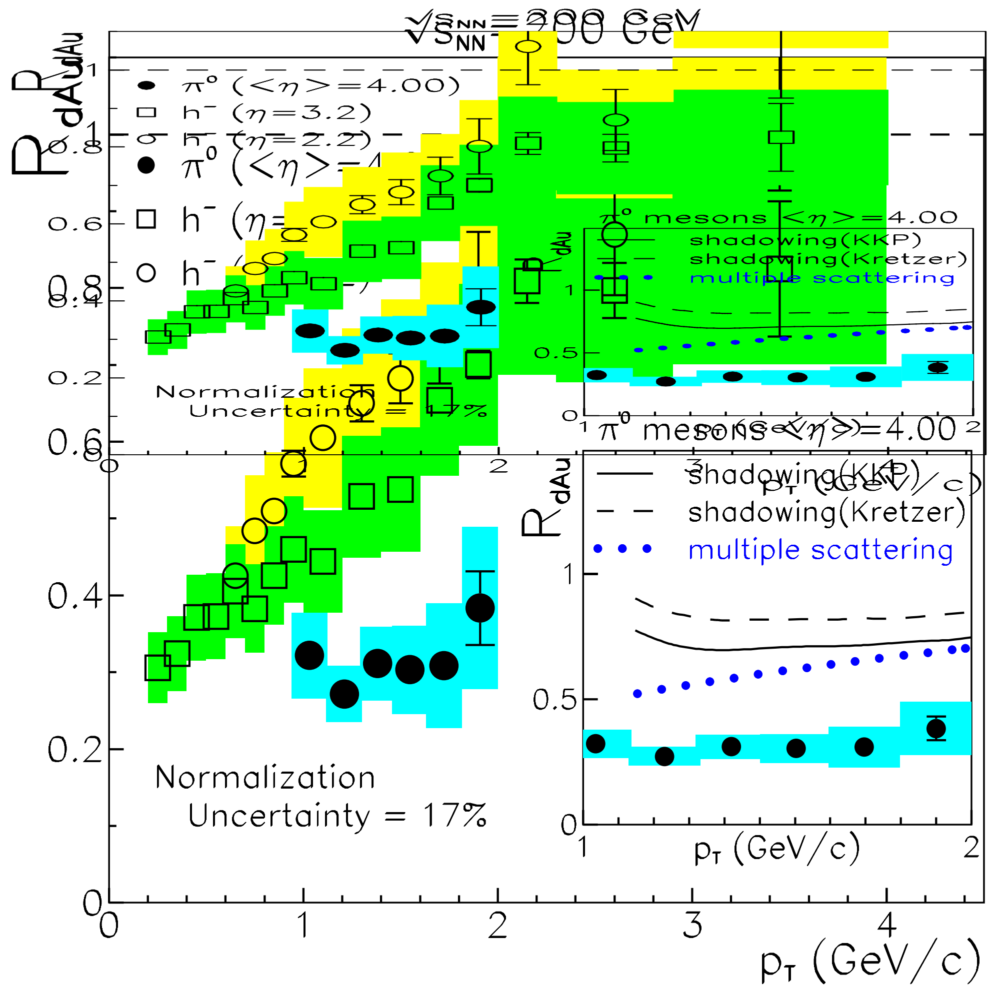


PHENIX data, nucl-ex/0411054

More Recent Data

STAR data
shows even stronger
suppression at
rapidity of 4.0,
strengthening the
case for CGC.

(figure from
nucl-ex/0602011)



More Recent Progress

A. Running Coupling

Non-linear evolution: fixed coupling

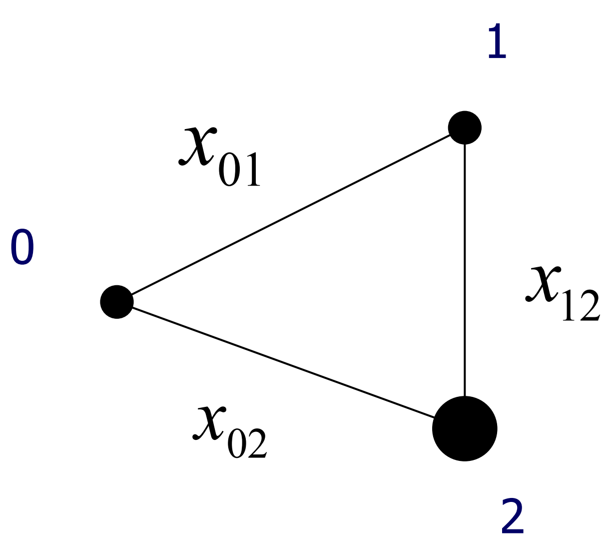
- Theoretically nothing is wrong with it: preserves unitarity (black disk limit), prevents the IR catastrophe.
- Phenomenologically there is a problem though: LO BFKL intercept is way too large (compared to 0.2-0.3 needed to describe experiment)

$$\alpha_P - 1 = 2.77 \frac{\alpha_s N_c}{\pi} \approx 0.79$$

- Full NLO calculation (order-α^2 kernel): tough, but done (see Balitsky and Chirilli ’07).
- First let’s try to determine the scale of the coupling.

What Sets the Scale for the Running Coupling?

$$\frac{\partial N(x_0, x_1, Y)}{\partial Y} = \frac{\alpha_S N_C}{2\pi^2} \int d^2 x_2 \frac{x_{01}^2}{x_{02}^2 x_{12}^2}$$
$$\times [N(x_0, x_2, Y) + N(x_2, x_1, Y) - N(x_0, x_1, Y) - N(x_0, x_2, Y) N(x_2, x_1, Y)]$$



transverse plane

What Sets the Scale for the Running Coupling?

$$\frac{\partial N(x_0, x_1, Y)}{\partial Y} = \frac{\alpha_S N_C}{2\pi^2} \int d^2 x_2 \frac{x_{01}^2}{x_{02}^2 x_{12}^2}$$
$$\times [N(x_0, x_2, Y) + N(x_2, x_1, Y) - N(x_0, x_1, Y) - N(x_0, x_2, Y) N(x_2, x_1, Y)]$$

$$\alpha_S(???)$$

In order to perform consistent calculations it is important to know the scale of the running coupling constant in the evolution equation.

There are three possible scales – the sizes of the "parent" dipole and "daughter" dipoles x_{01}, x_{21}, x_{20} . Which one is it?

Preview

- The answer is that the running coupling corrections come in as a **"triumvirate"** of couplings (H. Weigert, Yu. K. '06; I. Balitsky, '06):

$$\alpha_\mu \Rightarrow \frac{\alpha_s(\ldots)\,\alpha_s(\ldots)}{\alpha_s(\ldots)}$$

cf. Braun '94, Levin '94

- The scales of three couplings are somewhat involved.

Main Principle

To set the scale of the coupling constant we will first calculate the $\alpha_S N_f$ corrections to BK/JIMWLK evolution kernel to all orders.

We then would complete N_f to the QCD beta-function

$$\beta_2 = \frac{11 N_C - 2 N_f}{12\pi}$$

by replacing $N_f \to -6\pi\beta_2$ to obtain the scale of the running coupling:

$$\alpha_s(Q^2) = \frac{\alpha_\mu}{1 + \alpha_\mu \beta_2 \ln(Q^2/\mu^2)}$$

BLM prescription (Brodsky, Lepage, Mackenzie '83)

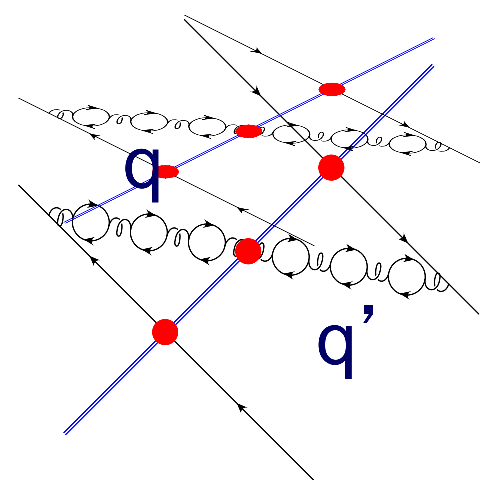
Results: Transverse Momentum Space

The resulting JIMWLK kernel with running coupling corrections is

$$\alpha_\mu K(\mathbf{x}_0,\mathbf{x}_1;\mathbf{z}) = 4\int \frac{d^2q\, d^2q'}{(2\pi)^4}\, e^{-i\mathbf{q}\cdot(\mathbf{z}-\mathbf{x}_0)+i\mathbf{q}'\cdot(\mathbf{z}-\mathbf{x}_1)}\, \frac{\mathbf{q}\cdot\mathbf{q}'}{\mathbf{q}^2\,\mathbf{q}'^2}\, \frac{\alpha_S(\mathbf{q}^2)\,\alpha_S(\mathbf{q}'^2)}{\alpha_S(Q^2)}$$

where $$\ln\frac{Q^2}{\mu^2} = \frac{\mathbf{q}^2\ln(\mathbf{q}^2/\mu^2) - \mathbf{q}'^2\ln(\mathbf{q}'^2/\mu^2)}{\mathbf{q}^2-\mathbf{q}'^2} - \frac{\mathbf{q}^2\,\mathbf{q}'^2}{\mathbf{q}\cdot\mathbf{q}'}\,\frac{\ln(\mathbf{q}^2/\mathbf{q}'^2)}{\mathbf{q}^2-\mathbf{q}'^2}$$

The BK kernel is obtained from the above by summing over all possible emissions of the gluon off the quark and anti-quark lines.



Running Coupling BK

Here's the BK equation with the running coupling corrections (H. Weigert, Yu. K. '06; I. Balitsky, '06):

$$\frac{\partial N(x_0, x_1, Y)}{\partial Y} = \frac{N_C}{2\pi^2} \int d^2 x_2$$

$$\times \left[\frac{\alpha_S(1/x_{02}^2)}{x_{02}^2} + \frac{\alpha_S(1/x_{12}^2)}{x_{12}^2} - 2\, \frac{\alpha_S(1/x_{02}^2)\,\alpha_S(1/x_{12}^2)}{\alpha_S(1/R^2)} \frac{\mathbf{x}_{20} \cdot \mathbf{x}_{21}}{x_{02}^2\, x_{12}^2} \right]$$

$$\times [N(x_0, x_2, Y) + N(x_2, x_1, Y) - N(x_0, x_1, Y) - N(x_0, x_2, Y)\, N(x_2, x_1, Y)]$$

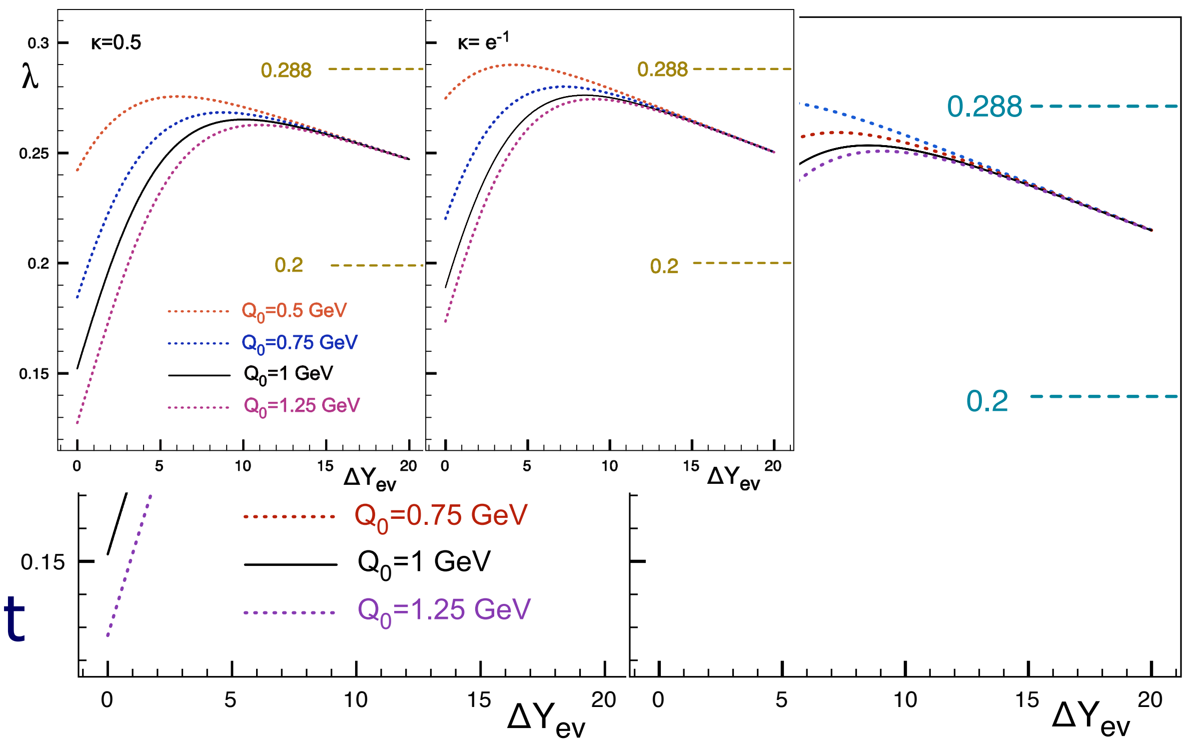
where

$$\ln R^2 \mu^2 = \frac{x_{20}^2 \ln(x_{21}^2\, \mu^2) - x_{21}^2 \ln(x_{20}^2\, \mu^2)}{x_{20}^2 - x_{21}^2} + \frac{x_{20}^2\, x_{21}^2}{\mathbf{x}_{20} \cdot \mathbf{x}_{21}} \frac{\ln(x_{20}^2 / x_{21}^2)}{x_{20}^2 - x_{21}^2}$$

What does the running coupling do?

- Slows down the evolution with energy / rapidity.

$$\lambda = \frac{d\ln Q_s^2(Y)}{dY}$$



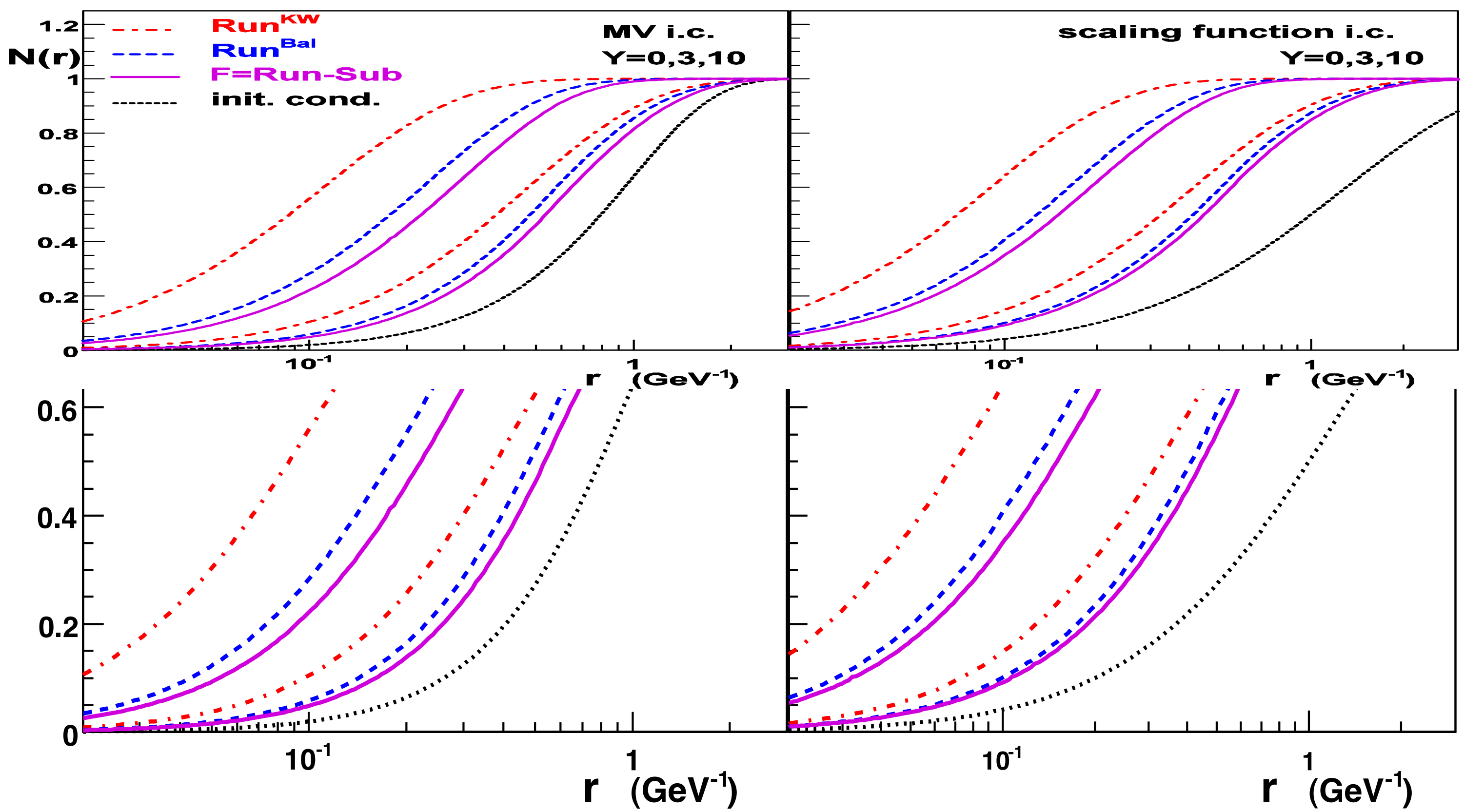
down from about

$\lambda \approx 0.7 \div 0.8$

at fixed coupling

Albacete '07

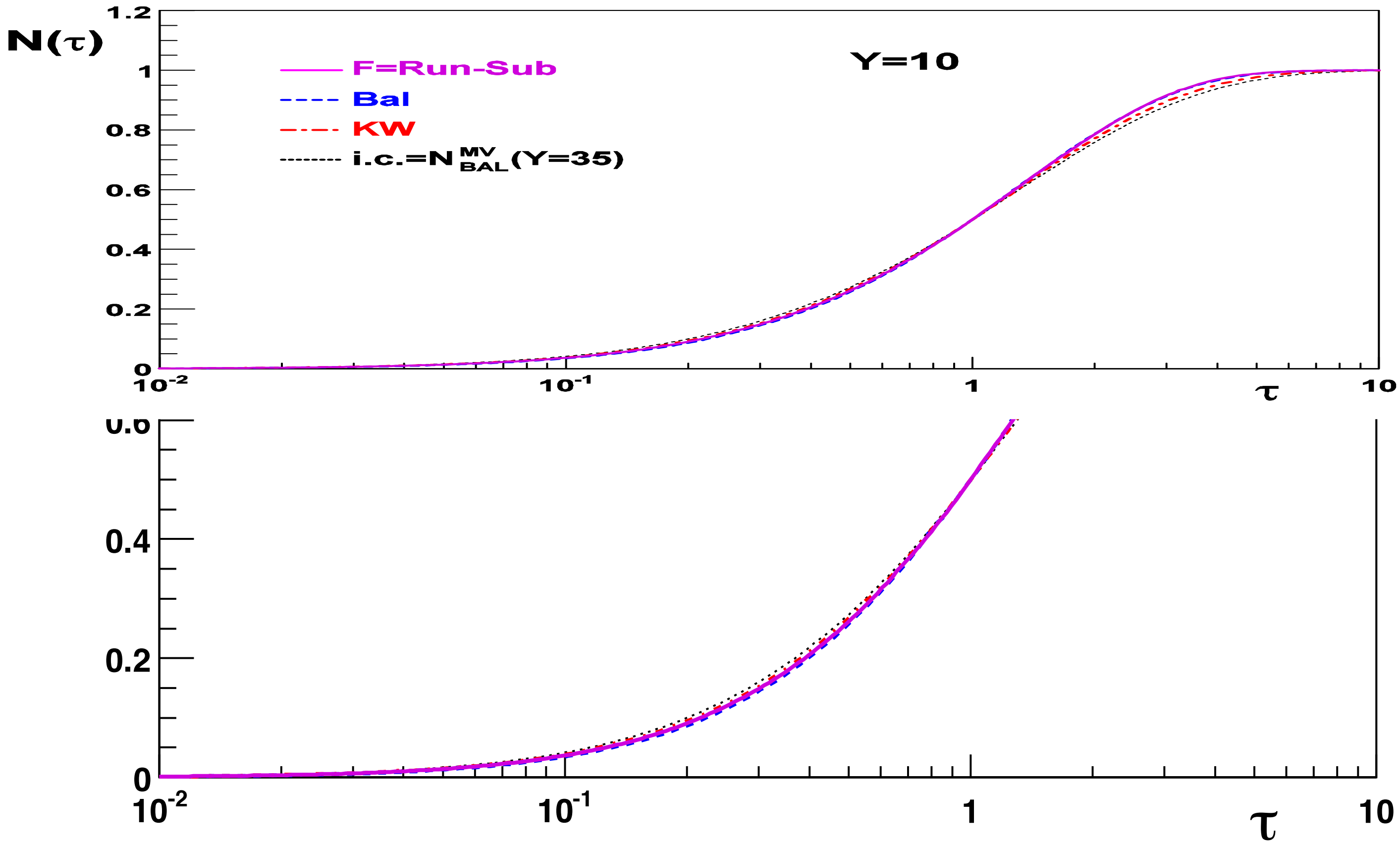
Solution of the Full Equation



Different curves – different ways of separating running coupling from NLO corrections. Solid curve includes all corrections.

J. Albacete, Yu.K. '07

Geometric Scaling



$$\tau = r\, Q_S(Y)$$

At high enough rapidity we recover geometric scaling, all solutions fall on the same curve. This has been known for fixed coupling: however, the shape of the scaling function is different in the running coupling case!

J. Albacete, Yu.K. ‘07

B. NLO BFKL/BK/JIMWLK

NLO BK/JIMWLK Evolution

- NLO BK/JIMWLK was calculated by Balitsky and Chrilli ’07
- The answer is simple:

$$
\begin{aligned}
&\frac{d}{d\eta}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\} \\
&= \frac{\alpha_s}{2\pi^2}\int d^2z\;\frac{(x-y)^2}{X^2Y^2}\Big\{1+\frac{\alpha_s}{4\pi}\Big[b\ln(x-y)^2\mu^2 - b\frac{X^2-Y^2}{(x-y)^2}\ln\frac{X^2}{Y^2}+\Big(\frac{67}{9}-\frac{\pi^2}{3}\Big)N_c-\frac{10}{9}n_f \\
&\qquad - 2N_c\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Big]\Big\}\;[\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_z\}\mathrm{Tr}\{\hat{U}_z\hat{U}^\dagger_y\}-N_c\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\}] \\
&+\frac{\alpha_s^2}{16\pi^4}\int d^2z\,d^2z'\Big[\Big(-\frac{4}{(z-z')^4}+\Big\{2\frac{X^2Y'^2+X'^2Y^2-4(x-y)^2(z-z')^2}{(z-z')^4[X^2Y'^2-X'^2Y^2]} \\
&+\frac{(x-y)^4}{X^2Y'^2-X'^2Y^2}\Big[\frac{1}{X^2Y'^2}+\frac{1}{Y^2X'^2}\Big]+\frac{(x-y)^2}{(z-z')^2}\Big[\frac{1}{X^2Y'^2}-\frac{1}{X'^2Y^2}\Big]\Big\}\ln\frac{X^2Y'^2}{X'^2Y^2}\Big) \\
&\qquad\times[\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_z\}\mathrm{Tr}\{\hat{U}_z\hat{U}^\dagger_{z'}\}\mathrm{Tr}\{\hat{U}_{z'}\hat{U}^\dagger_y\}-\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_z\hat{U}_{z'}U^\dagger_y\hat{U}_z\hat{U}^\dagger_{z'}\}-(z'\to z)] \\
&+\Big\{\frac{(x-y)^2}{(z-z')^2}\Big[\frac{1}{X^2Y'^2}+\frac{1}{Y^2X'^2}\Big]-\frac{(x-y)^4}{X^2Y'^2X'^2Y^2}\Big\}\ln\frac{X^2Y'^2}{X'^2Y^2}\;\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_z\}\mathrm{Tr}\{\hat{U}_z\hat{U}^\dagger_{z'}\}\mathrm{Tr}\{\hat{U}_{z'}\hat{U}^\dagger_y\} \\
&+4n_f\Big\{\frac{4}{(z-z')^4}-2\frac{X'^2Y^2+Y'^2X^2-(x-y)^2(z-z')^2}{(z-z')^4(X^2Y'^2-X'^2Y^2-)}\ln\frac{X^2Y'^2}{X'^2Y^2}\Big\}\mathrm{Tr}\{t^a\hat{U}_xt^b\hat{U}^\dagger_y\}[\mathrm{Tr}\{t^a\hat{U}_zt^b\hat{U}^\dagger_{z'}\}-(z'\to z)]\Big]
\end{aligned}
$$

NLO BK/JIMWLK

- It is known that NLO BFKL corrections are numerically large.
- Could it be that saturation effects make NLO BK/ JIMWLK corrections small? TBD

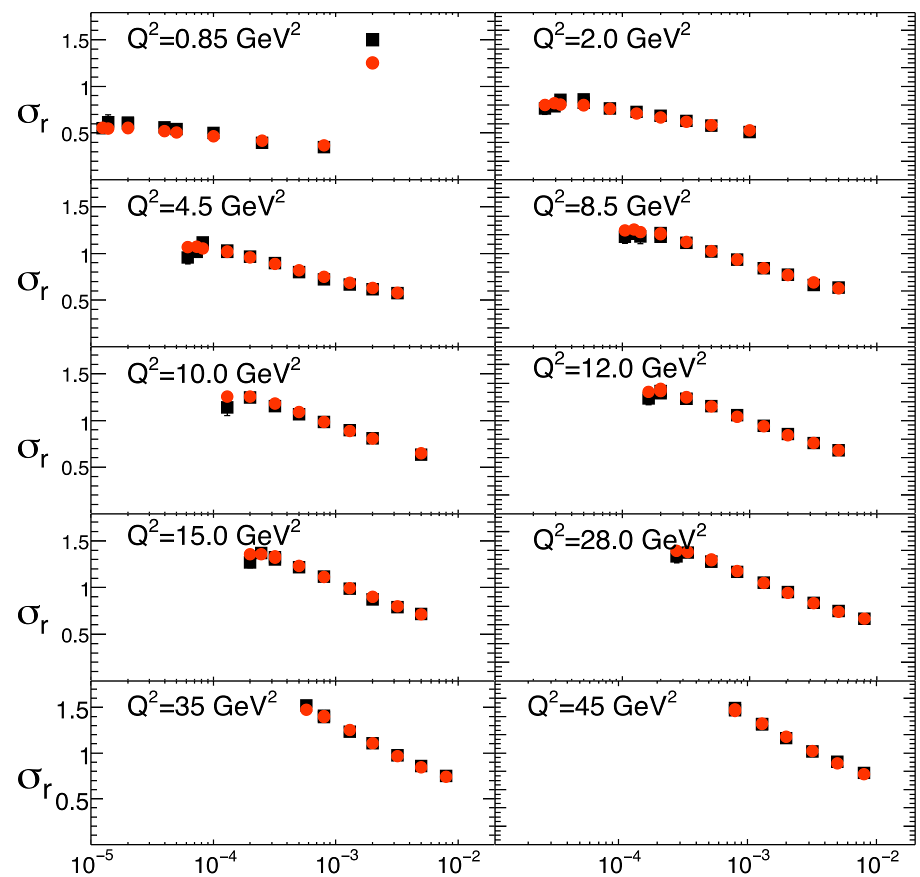
DIS Phenomenology

Comparison with the combined H1 and ZEUS data

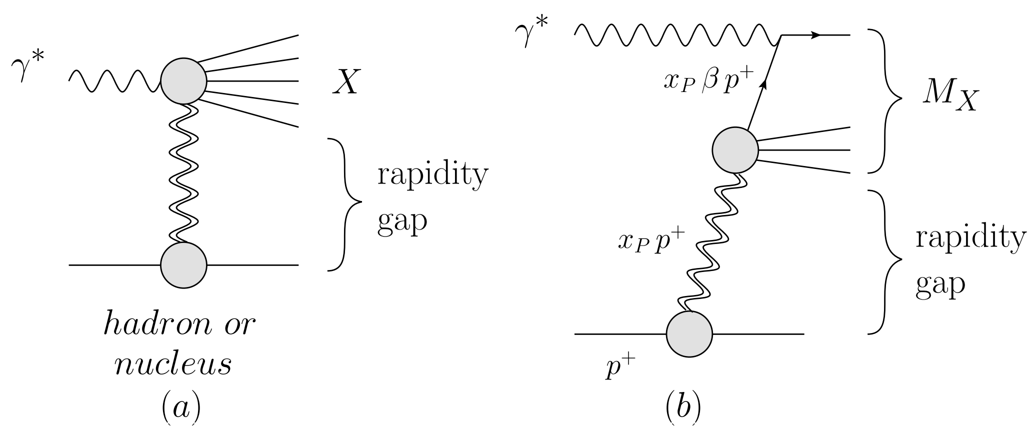
Albacete, Armesto, Milhano, Qiuroga Arias, and Salgado '11

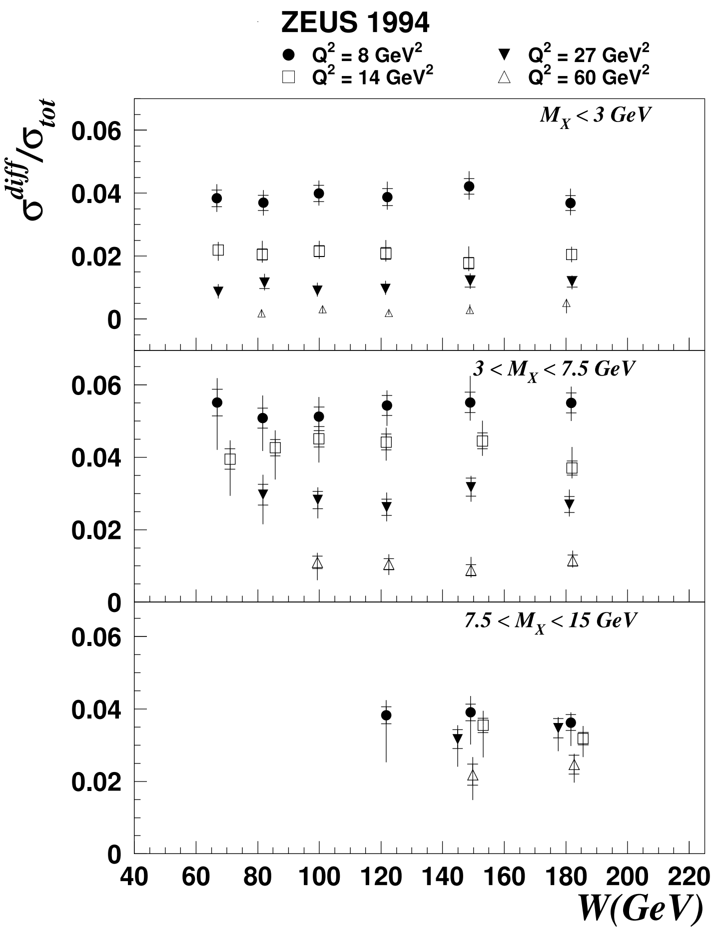
reduced cross section:

$$\sigma_r = F_2 - \frac{y^2}{1+(1-y)^2} F_L$$



Diffractive cross section





Also agrees with the saturation/CGC expectations.

Dipole universality

- So far a wide range of observables, from total DIS cross section and structure functions, to the hadronic p_T spectra in pA are described in terms of a single quantity – dipole scattering amplitude.
- This is a new universal degree of freedom. (Gelis, Jalilian-Marian '02; Goncalves, Kugeratsky, Machado, Navarro '06; AGBS '12, etc.)
- However, there are observables, like two-particle correlations, which are described in terms of the higher-order correlators, like quadrupoles, etc.

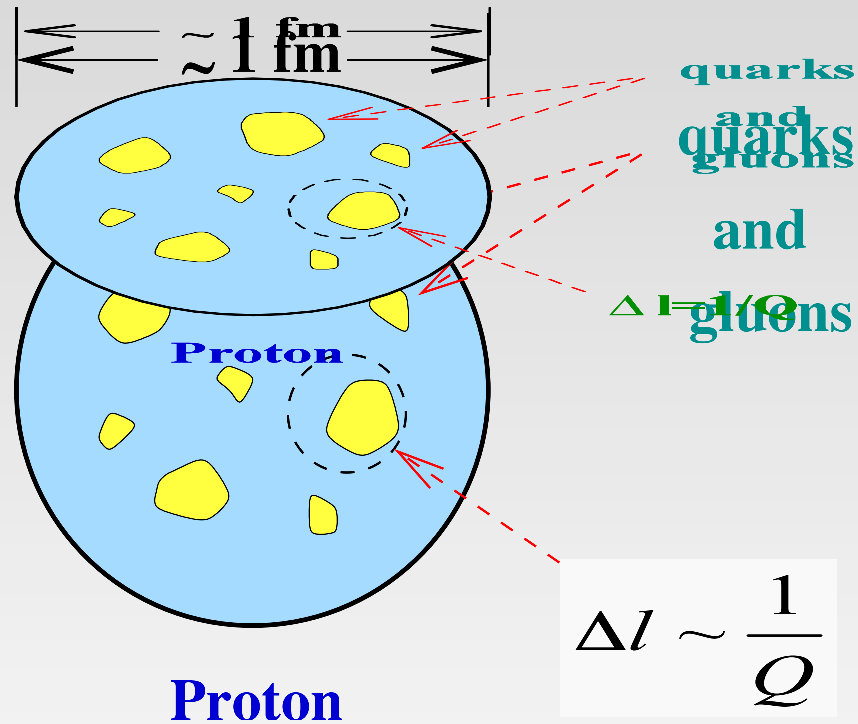
Physical Meaning of Q

Uncertainty principle teaches us that

$$\Delta p \Delta l \approx \hbar$$

which means that the photon probes the proton at the distances of the order (ħ=1)

$$\Delta l \sim \frac{1}{Q}$$



Large Momentum Q = Short Distances Probed

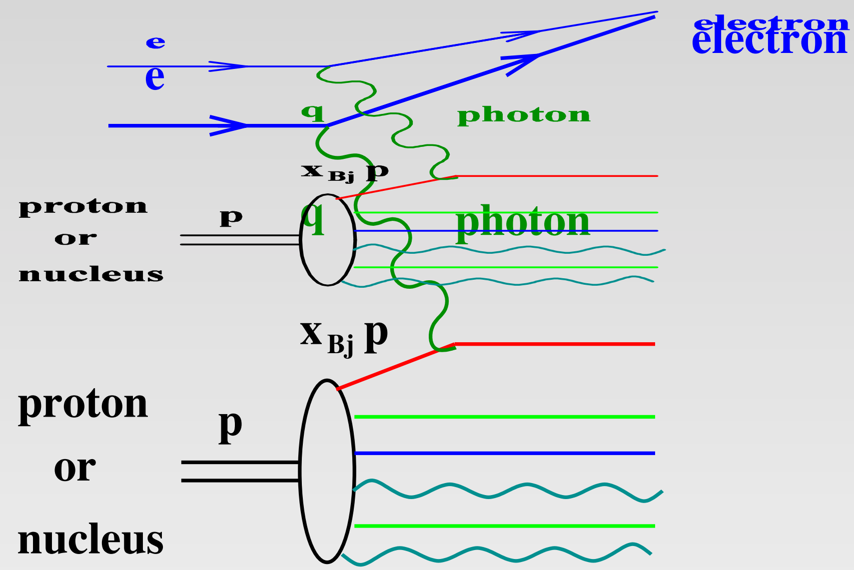
Physical Meaning of Bjorken x

In the rest frame of the electron the momentum of the struck quark is equal to some typical hadronic scale m:

$$x_{Bj}\, p \approx m$$

Then the energy of the collision

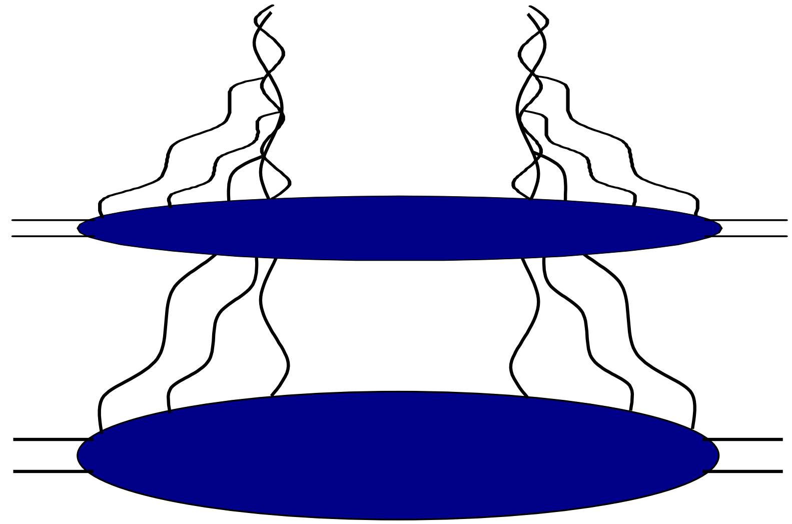
$$E \sim p \sim \frac{1}{x_{Bj}}$$



High Energy = Small x

Classical Gluon Field of a Nucleus

Using the obtained classical gluon field one can construct corresponding gluon distribution function $\phi_A(x,k^2) \sim \langle \underline{A}(-k) \cdot \underline{A}(k) \rangle$



with the field in the $A^+=0$ gauge

$$\phi_A(x, k_T^2) = \frac{C_F}{\alpha_s \pi} \int \frac{d^2 x_\perp}{x_\perp^2} e^{i \underline{k} \cdot \underline{x}} \left[1 - \exp\left(-\frac{x_\perp^2 Q_s^2}{4} \ln \frac{1}{x_\perp \Lambda} \right) \right]$$

J. Jalilian-Marian et al, ’97; Yu. K. and A. Mueller, ‘98

⇨ $Q_S=\mu$ is the saturation scale $Q_s^2 \sim A^{1/3}$

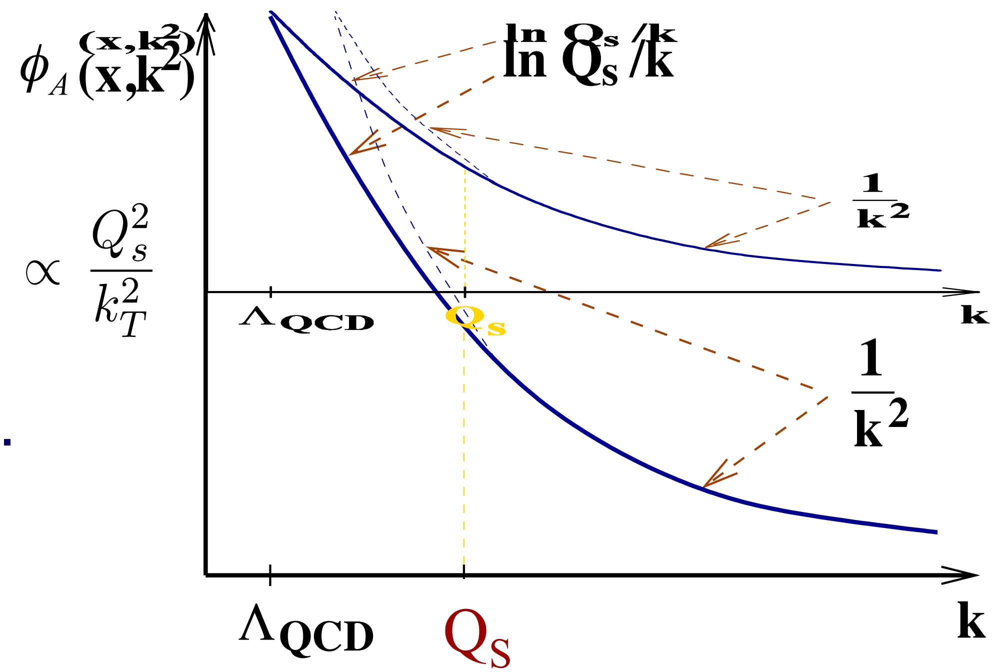
⇨ Note that $\phi \sim \langle A_\mu A_\mu \rangle \sim 1/\alpha$ such that $A_\mu \sim 1/g$, which is what one would expect for a classical field.

$$\phi_A(x,k_T^2) = \frac{C_F}{\alpha_s \pi} \int \frac{d^2x_\perp}{x_\perp^2} e^{i\,\underline{k}\cdot\underline{x}} \left[1 - \exp\left(-\frac{x_\perp^2 Q_s^2}{4} \ln\frac{1}{x_\perp \Lambda}\right)\right]$$

⇨ In the UV limit of k→∞, x_T is small and one obtains

$$\phi_A(x,k_T^2) \sim \int d^2x_\perp\, e^{i\,\underline{k}\cdot\underline{x}}\, Q_s^2 \ln\frac{1}{x_\perp \Lambda} \propto \frac{Q_s^2}{k_T^2}$$

which is the usual LO result.



⇨ In the IR limit of small k_T, x_T is large and we get

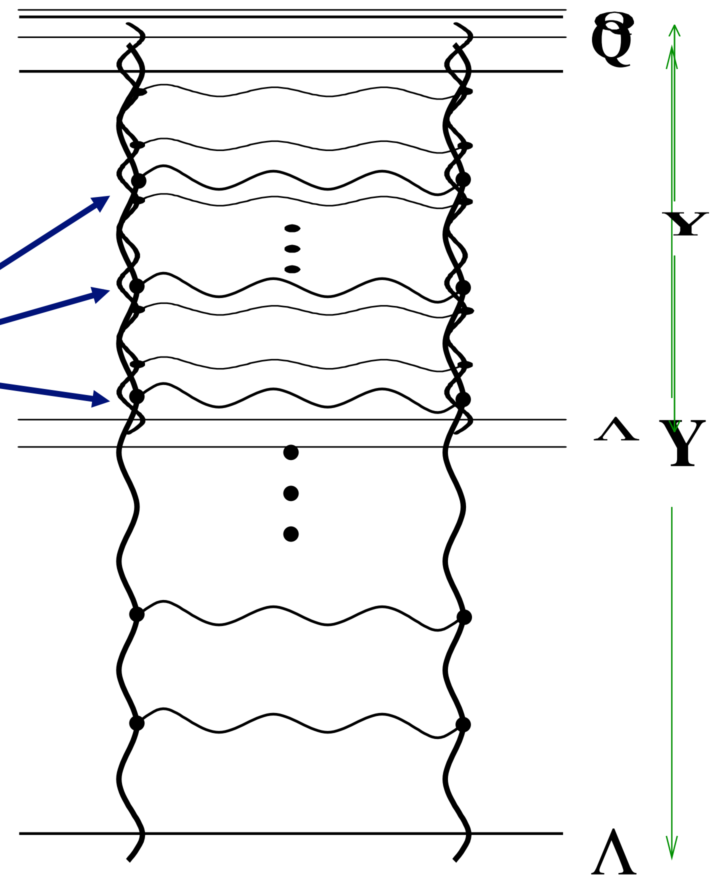
$$\phi_A(x,k_T^2) \approx \frac{C_F}{\alpha_s \pi} \int_{1/Q_s} \frac{d^2x_\perp}{x_\perp^2} e^{i\,\underline{k}\cdot\underline{x}} \propto \ln\frac{Q_s}{k_T}$$

SATURATION !

Divergence is regularized.

BFKL Equation

In the conventional Feynman diagram picture the BFKL equation can be represented by a ladder graph shown here. Each rung of the ladder brings in a power of α ln s.



The resulting dipole amplitude grows as a power of energy

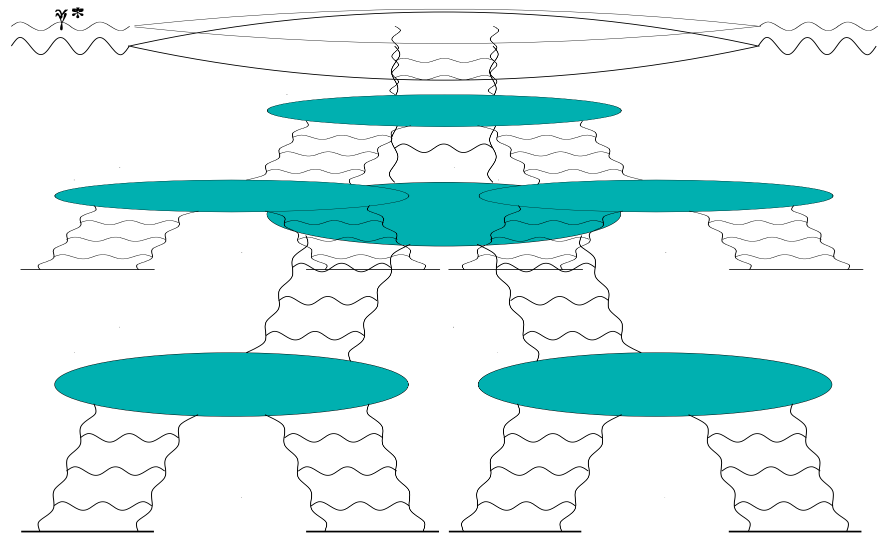
$$N \sim s^{\lambda}$$

violating Froissart unitarity bound

$$\sigma_{tot} \le const \ln^2 s$$

GLR-MQ Equation

Gribov, Levin and Ryskin ('81) proposed summing up "fan" diagrams:



Mueller and Qiu (' 85) summed "fan" diagrams for large Q^2.

The GLR-MQ equation reads:

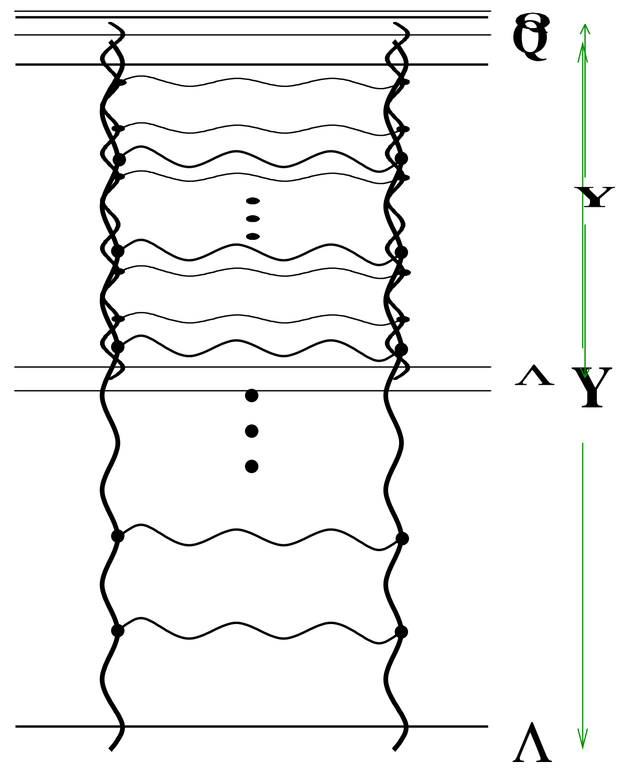
$$\frac{\partial}{\partial \ln 1/x} \phi(x, k_T^2) = \alpha_s \, K_{BFKL} \otimes \phi(x, k_T^2) - \alpha_s \left[\phi(x, k_T^2)\right]^2$$

GLR-MQ equation has the same principle of recombination as BK and JIMWLK. GLR-MQ equation was thought about as the first nonlinear correction to the linear BFKL evolution. An AGL (Ayala, Gay Ducati, Levin '96) equation was suggested to resum higher-order nonlinear corrections.

BK/JIMWLK derivation showed that for the dipole amplitude N (!) there are no more terms in the large-N_C limit and obtained the correct kernel for the non-linear term (compared to GLR suggestion).

Energy Dependence of the Saturation Scale

Single BFKL ladder gives scattering amplitude of the order $N \sim \frac{\Lambda}{k_T} s^{\Delta}$



Nonlinear saturation effects become important when N ~ N^2 ⇨ N ~ 1. This happens at

$$k_T = Q_s \sim \Lambda\, s^{\Delta}$$

Saturation scale grows with energy!

Typical partons in the wave function have $k_T \sim Q_S$, so that their characteristic size is of the order $r \sim 1/k_T \sim 1/Q_S$.

⇨ Typical parton size decreases with energy!