Fundamental Symmetries

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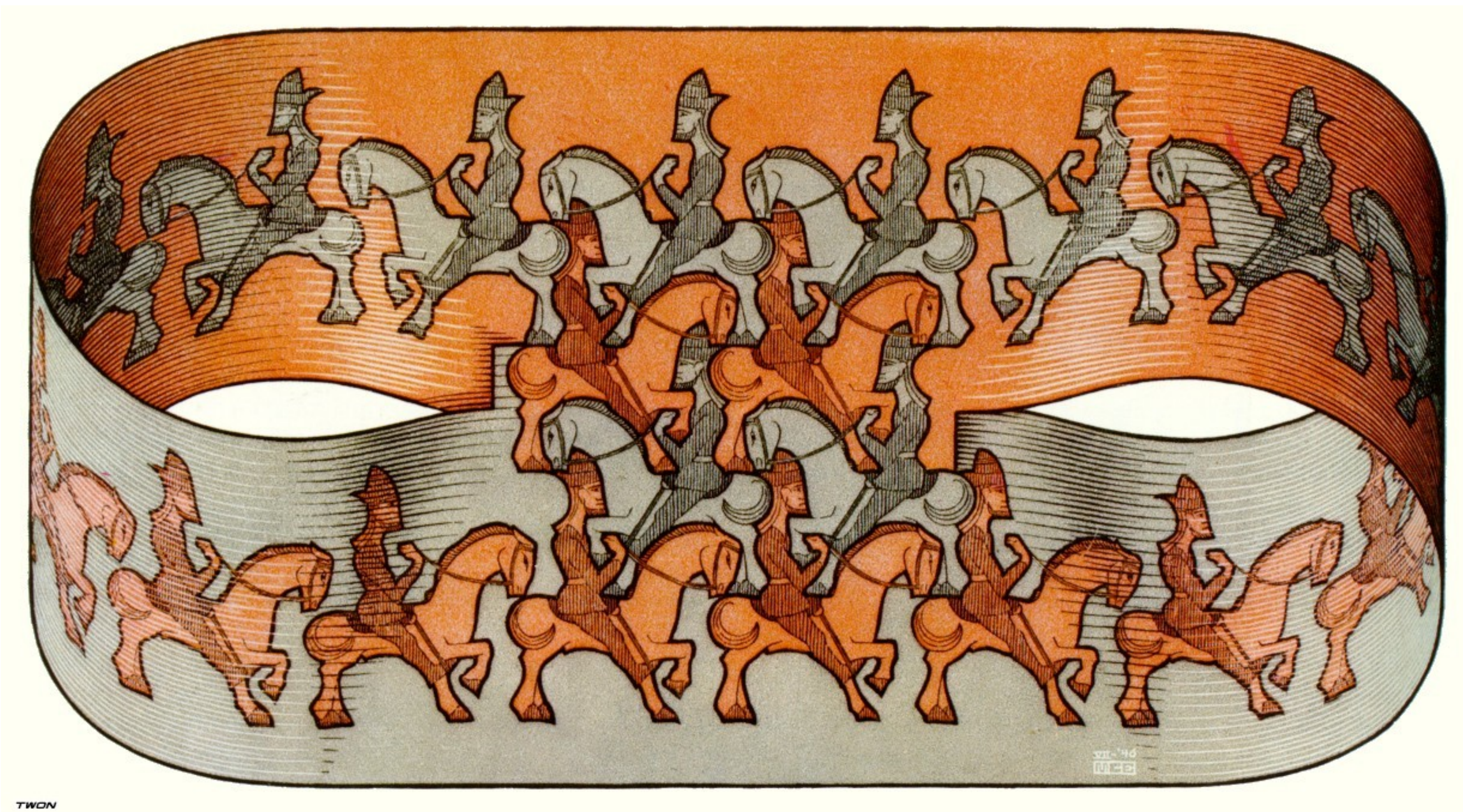


Fundamental Symmetries : Overview

- Standard Model : Inadequacies
- Experimental Tests of Standard Model and Symmetries
  - Baryon Number Violation : Proton Decay
  - Parity Violation : MOLLER at JLab
  - Charged Lepton Flavor Violation : $\mu N \rightarrow eN$
  - Electric Dipole Moment Searches : $e,\ \mu,\ n,\ p$, nuclei
  - Precision Test of the Standard Model : Muon g-2
- Summary and Outlook

- My experience : experimentalist, worked on polarized deep-inelastic scattering, muonium hyperfine structure (test of bound state QED), muon g-2, electron EDM searches in polar diatomic molecules, polarized proton-proton scattering with PHENIX collaboration at RHIC - to measure Δg and $\Delta \bar{u}$ and $\Delta \bar{d}$, new muon g-2

Standard Model : Inadequacies

- What is origin of the observed matter-antimatter asymmetry?
  - SM prediction off by $>$6 orders of magnitude
- SM doesn't explain $1/3$ relation between quark and lepton charges
- What is the origin of neutrino mass?
- What is dark matter? What is dark energy?
- Can we explain the extreme hierarchy of masses and strengths of forces?
- Why are there 3 families? Can the electroweak and strong forces be unified?

$\Rightarrow$ What about gravity ???

- Is Standard Model a low-energy limit of a more fundamental theory ??

- Noether : ∃ conserved quantity for every continuous symmetry of Lagrangian
- Baryon number : conserved by U(1)$_B$ symmetry in SM, but broken by non-perturbative weak effects ('t Hooft, PRL **37**, 8 (1976))

  ⇒ Proton can annihilate with neutron : $p + n \to e^+ + \bar{\nu}_\mu,\ p + n \to \mu^+ + \bar{\nu}_e$

  ⇒ SM proton decay rate contains pre-factor $e^{-4\pi \sin^2\theta_W/\alpha_{\rm QED}} \approx e^{-4\pi/0.0335..}$,
  so $\Gamma \ \propto\ 10^{-163}$ s^{-1} $\Leftrightarrow$ $\tau_{\rm proton} > 10^{150}$ years !

- But : baryon number violation *required for creation of matter in universe* (*i.e.* matter-antimatter asymmetry)
- Ultimate end of universe depends on proton stability

- Proton decay predicted in many Grand Unified Theories (GUTs)
- Scale at which forces unify, $M_G \approx 10^{16}$ GeV, well beyond EW scale $G_F^{-1/2} \approx 250$ GeV

⇒ Proton decay fantastic probe of profound physics, far beyond reach of accelerators

Why unify forces?

⇒ Standard model described by groups $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ with 3 distinct couplings - can this be simplified?

⇒ Even electroweak unification doesn't predict relative EM and weak couplings

⇒ Why are there 3 generations of fermions? Why large hierarchy of masses? $m_{\text{top}} > 10^5 m_e$

⇒ What is the origin of neutrino mass? Are neutrinos their own anti-particles?

⇒ What is the origin of the matter-antimatter asymmetry in the universe?

⇒ Quarks and lepton charged weak current doublets identical, $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} u \\ d' \end{pmatrix}_L$
Are they related at more fundamental level?

⇒ Why is charge quantized? Why is $Q(e) + Q(p) = 0$? Why is $Q(d) = Q(e)/3$? Why not $Q(d) = Q(e)/5$?

⇒ Higgs hierarchy problem : radiative corrections should push Higgs mass to $M_P \approx 10^{19}$ GeV. Explained by SUSY?

⇒ Gravity - not explained. Dark energy, dark matter, also unexplained, ...

⇒ Many of us will measure zero or consistency with SM for many years - but great new physics is almost certainly there, waiting to be discovered

$SU(N)$ Groups

- Elements of $SU(N)$ groups are $n \times n$ unitary matrices U with det 1 ($U^\dagger U = 1$, det(U)=1)
- Matrix elements are complex so nominally $2 \times n \times n$ elements; but $U^\dagger U = 1$ implies n constraints on diagonal elements, $n^2 - n$ constraints on off-diagonal, 1 constraint to make det(U)=1 $\Rightarrow$ $n^2 - 1$ independent parameters
- For $SU(2)$ there are three independent parameters : $\alpha,\ \beta,\ \gamma$; think of Euler angles

$$U(\alpha,\beta,\gamma) \;=\; \begin{pmatrix} e^{-i(\alpha+\gamma)/2}\cos\beta/2 & -e^{-i(\alpha-\gamma)/2}\sin\beta/2 \\ e^{i(\alpha-\gamma)/2}\sin\beta/2 & e^{i(\alpha+\gamma)/2}\cos\beta/2 \end{pmatrix}$$

- Can write $U = e^{iH}$ for H Hermitian ($H = H^\dagger$, $U^\dagger U = (e^{iH})^\dagger (e^{iH}) = e^{i(H-H^\dagger)} = 1$)
- Can pick $n^2 - 1$ Hermitian matrices G_i so any element U of $SU(N)$ can be written as :

$$U \;=\; \exp\left(\sum_{i=1}^{n^2-1} i\theta_i G_i\right),$$

- θ_i are real parameters, G_i are the generators of the group ($n^2 - 1$ of them)
- For $SU(2)$, can pick three Pauli matrices σ_i as generators
- Finally : $U = e^G$, det$(e^G) = e^{\mathrm{Tr}G}$, so det(U)=1 implies generators G_i traceless, Hermitian
- (See G. Kane, Modern Elementary Particle Physics or J.-Q. Chen, Group Representation Theory for Physicists)

- Georgi and Glashow, "Unity of All Elementary-Particle Forces", PRL **32**, 438 (1974) : propose a minimal $SU(5)$ as a possible GUT (minimal $\Leftrightarrow$ smallest Higgs sector)
- Fermions in $\bar{\mathbf{5}}$ and $\mathbf{10}$ representations (versus SM singlets, doublets, triplets)

$$\bar{\mathbf{5}} = \begin{pmatrix} \bar{d}_r \\ \bar{d}_g \\ \bar{d}_b \\ e^- \\ -\nu_e \end{pmatrix}_L , \quad \mathbf{10} = \left(\begin{array}{ccc|cc} 0 & \bar{u}_b & -\bar{u}_g & -u_r & -d_r \\ -\bar{u}_b & 0 & \bar{u}_r & -u_g & -d_g \\ \bar{u}_g & -\bar{u}_r & 0 & -u_b & -d_b \\ \hline u_r & u_g & u_b & 0 & e^+ \\ d_r & d_g & d_b & -e^+ & 0 \end{array}\right)_L$$

- $\mathbf{10}$ is antisymmetric, 15 particles total, $SU(5)$ gauge bosons enable transitions between multiplet members (like $SU(2)_L$ mixes doublet : $u + W^- \to d,\ e^- + W^+ \to \nu_e$)
- $SU(N)$ generators are traceless $\Leftrightarrow$ sum of eigenvalues is 0
- Electric charge Q is linear combination of generators from $SU(2)_L$ and $U(1)_Y$: $Q = T_3 + Y/2$

$\Rightarrow$ In $SU(5)$, Q is a (traceless) generator so sum of electric charges in a representation is zero

$\Rightarrow$ $Q(\nu_e) + Q(e^-) + 3Q(\bar{d}) \ = \ 0 \ \Rightarrow Q(\bar{d}) \ = \ \frac{1}{3}Q(e^-)$!

$\Rightarrow$ Electric charge of quarks is related to number of flavors, $Q(e^-) \equiv -Q(p)$ atoms neutral, charge quantized!

- Explain a remarkable amount, very appealing to think forces are unified

- What about $SU(N)$ gauge bosons?
- For $SU(5)$ should be $N^2 - 1 = 5^2 - 1 = 24$ bosons, versus $(3^2 - 1) + (2^2 - 1) + 1 = 12$ for SM
- Displayed in matrix form as (see G. Ross, Grand Unified Theories) :

$$V_{SU(5)} = \left(\begin{array}{ccc|cc} g_{r\bar{r}} - \frac{2}{\sqrt{30}}B & g_{r\bar{g}} & g_{r\bar{b}} & X_1 & Y_1 \\ g_{g\bar{r}} & g_{g\bar{g}} - \frac{2}{\sqrt{30}}B & g_{g\bar{b}} & X_2 & Y_2 \\ g_{b\bar{r}} & g_{b\bar{g}} & g_{b\bar{b}} - \frac{2}{\sqrt{30}}B & X_3 & Y_3 \\ \hline \bar{X}_1 & \bar{X}_2 & \bar{X}_3 & \frac{1}{\sqrt{2}}W^3 + \frac{3}{\sqrt{30}}B & W^+ \\ \bar{Y}_1 & \bar{Y}_2 & \bar{Y}_3 & W^- & -\frac{1}{\sqrt{2}}W^3 + \frac{3}{\sqrt{30}}B \end{array}\right)$$

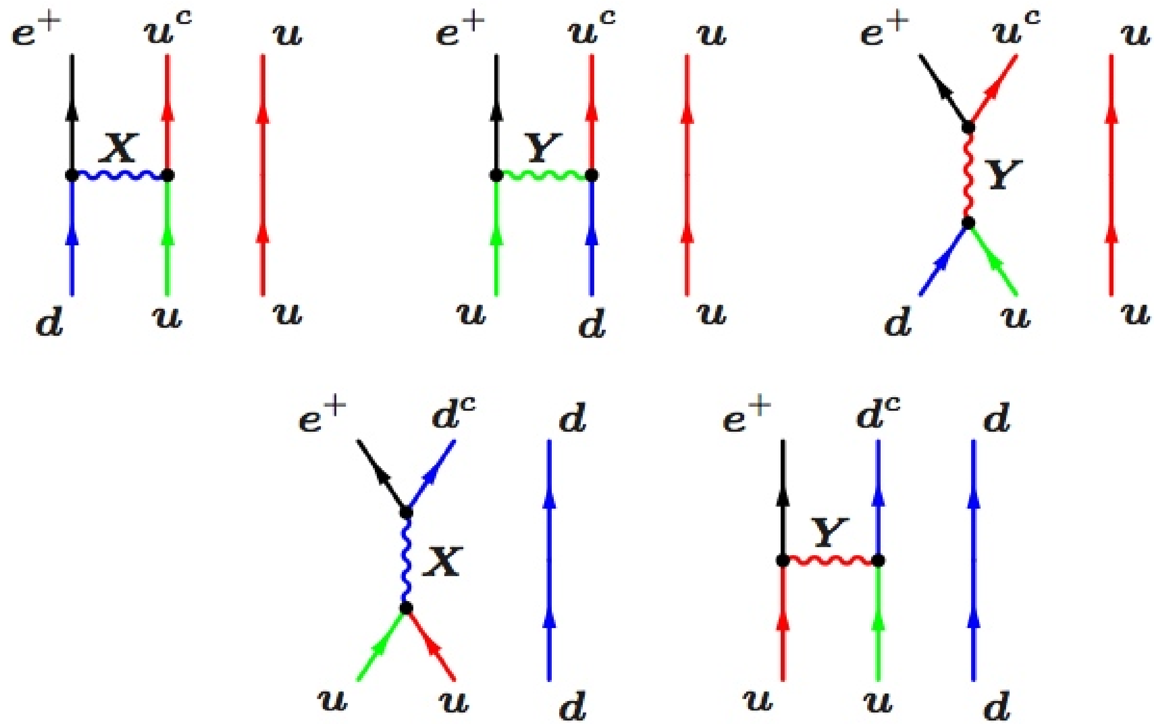
- Color group $SU(3)$ operates in first 3 rows and columns, $SU(2)$ on last two
- Twelve new gauge bosons $X_i,\ \bar{X}_i,\ Y_i,\ \bar{Y}_i,\ i = 1, 2, 3$
- New bosons mediate transitions between quarks and leptons

- Interaction part of $SU(5)$ Lagrangian (see C. Quigg) :

$$\begin{aligned}\mathcal{L}_{\rm int} \;=\; & -\frac{g_5}{2}G^a_\mu\left(\bar{u}\gamma^\mu\lambda^a u+\bar{d}\gamma^\mu\lambda^a d\right)-\frac{g_5}{2}W^i_\mu\left(\bar{L}_u\gamma^\mu\tau^i L_u+\bar{L}_e\gamma^\mu\tau^i L_e\right)\\ & -\frac{g_5}{2}\frac{3}{5}B_\mu\sum_{\rm fermions}\bar{f}\gamma^\mu Y f\\ & -\frac{g_5}{\sqrt{2}}\left[X^-_{\mu,\alpha}\left(\bar{d}^\alpha_R\gamma^\mu e^c_R+\bar{d}^\alpha_L\gamma^\mu e^c_L+\epsilon_{\alpha\beta\gamma}\bar{u}^{c\gamma}_L\gamma^\mu u^\beta_L\right)+H.C.\right]+\\ & +\frac{g_5}{\sqrt{2}}\left[Y^-_{\mu,\alpha}\left(d\bar{d}^\alpha_R\gamma^\mu\nu^c_R+\bar{u}^\alpha_L\gamma^\mu e^c_L+\epsilon_{\alpha\beta\gamma}\bar{u}^{c\beta}_L\gamma^\mu d^\gamma_L\right)+H.C.\right]\end{aligned}$$

- Doublets L given by $L_u = \begin{pmatrix} u \\ d' \end{pmatrix}_L$, $L_e = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$
- First three terms are from SM, though now with single coupling g_5
- Color $SU(3)$ $a = 1...8$, $SU(2)$ $i = 1, 2, 3$, $\alpha = r, g, b$, c indicates anti-particle
- X bosons (electric charge -4/3) and Y (electric charge -1/3) mediate quarks $\Leftrightarrow$ leptons
- X, Y boson exchange will allow baryon number violation $\Rightarrow$ proton decay

Proton decay in $SU(5)$



(Paul Langacker, Scholarpedia)

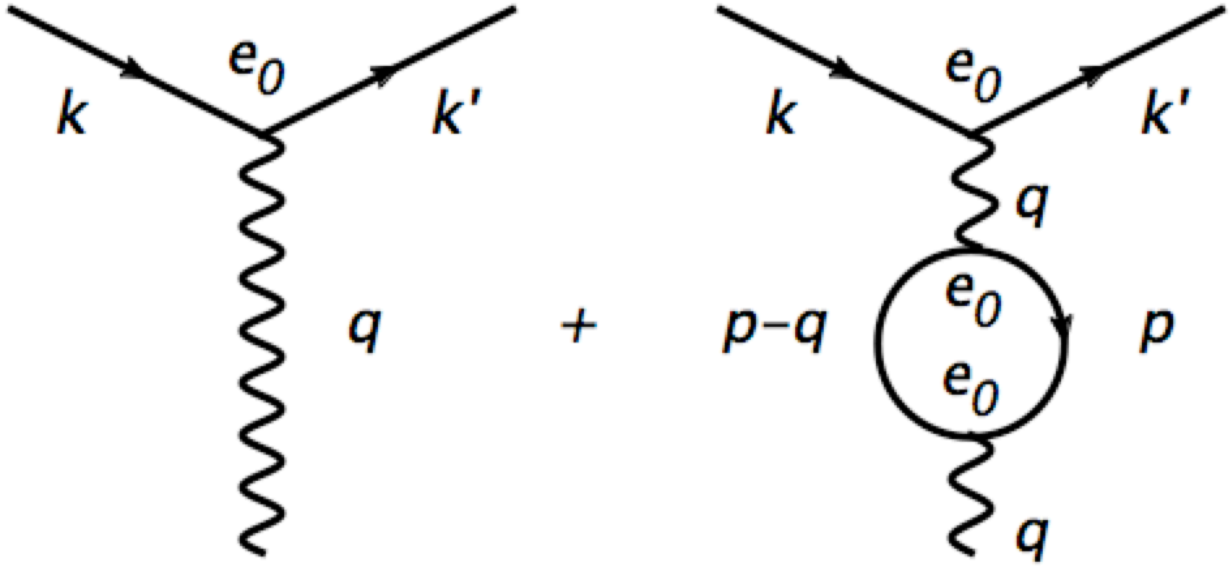
- See possible decay mode : $p \to e^+ + \pi^0$
- What about proton lifetime? Estimate similar to τ_μ

$$\tau_\mu = \left(\frac{M_W}{m_\mu g_w}\right)^4 \frac{12\hbar(8\pi)^3}{m_\mu c^2} \propto \frac{M_W^4}{m_\mu^5} \text{ so expect } \quad \tau_p \propto \frac{M_X^4}{m_p^5}$$

- What do we use for new gauge boson masses $M_X,\ M_Y$?

Unification of Forces

- Coupling strength depends on momentum transfer of virtual gauge bosons
- EM force increases at smaller length scale (α_1)
- Weak and strong force weaken at higher energy scales ($\alpha_2,\ \alpha_3$)
- Quickly review origin of this behavior



$$\mathcal{M} \propto e_0\bar{u}(k')\gamma^\mu u(k)\epsilon_\mu - \int \frac{d^4p}{(2\pi)^4}\left[e_0\bar{u}(k')\gamma^\mu u(k)\right] \times \frac{1}{q^2}\frac{\left[e_0\bar{u}(p)\gamma_\mu u(p-q)\right]\left[e_0\bar{u}(p-q)\gamma^\lambda u(p)\right]}{(p^2-M^2)\left[(p-q)^2-M^2\right]}\epsilon_\lambda$$

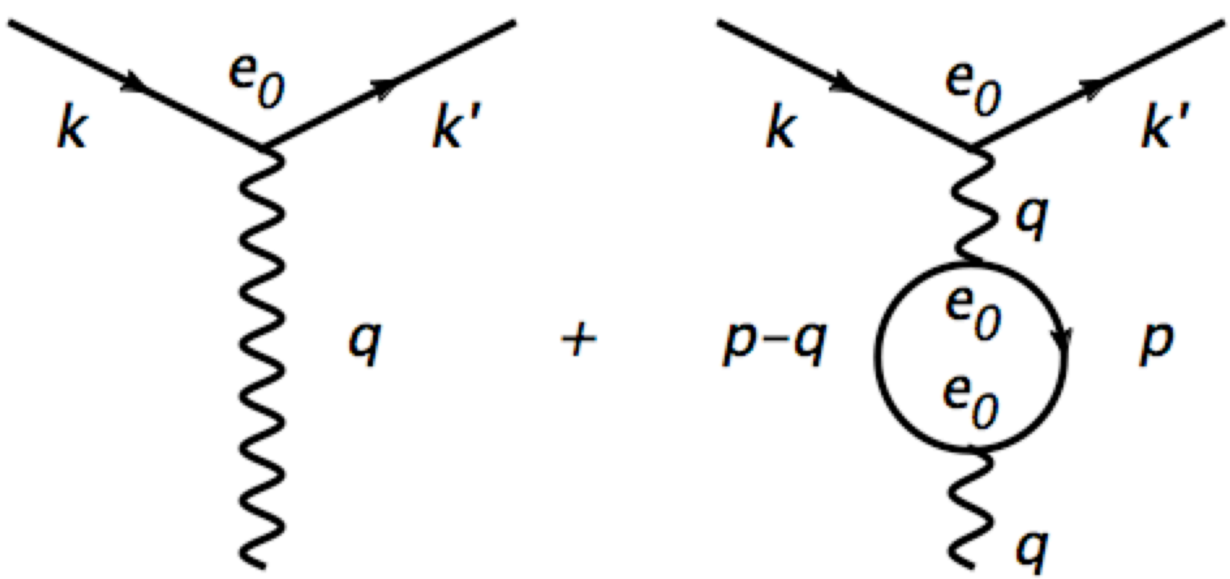
$$= e_0\bar{u}(k')\gamma^\mu u(k) \times \left[\epsilon_\mu - \frac{e_0^2\epsilon^\lambda}{q^2}\int \frac{d^4p}{(2\pi)^4}\frac{\left[\bar{u}(p)\gamma_\mu u(p-q)\right]\left[e\bar{u}(p-q)\gamma_\lambda u(p)\right]}{(p^2-M^2)\left[(p-q)^2-M^2\right]}\right]$$

$$= e_0\bar{u}(k')\gamma^\mu u(k) \times \left[\epsilon_\mu - \epsilon^\lambda T_{\mu\lambda}(q^2)\right], \quad T_{\mu\lambda} = g_{\mu\lambda}I(q^2) \ \text{ since } \ \epsilon_\mu q^\mu = 0$$

- What is $I(q^2)$? See C. Quigg or favorite QFT book

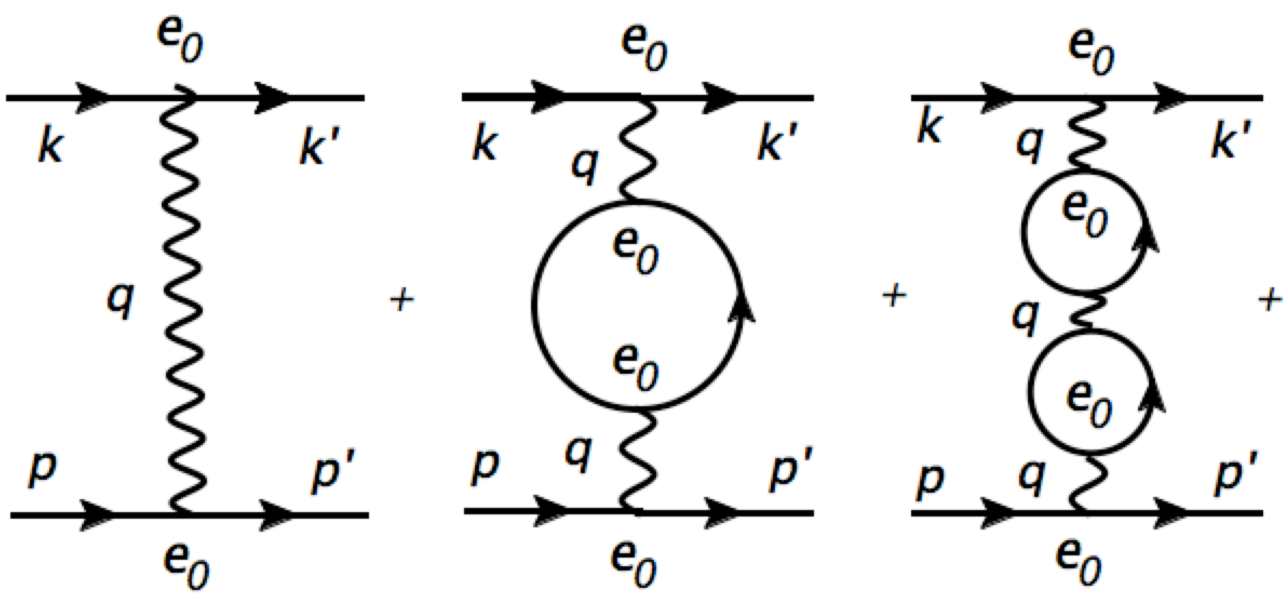
$$\begin{aligned} I(q^2) &= \frac{\alpha_0}{3\pi}\int_{M^2}^{\infty}\frac{dp^2}{p^2} - \frac{2\alpha_0}{\pi}\int_0^1 dx x(1-x)\ln\left[1-\frac{q^2x(1-x)}{M^2}\right] \\ &\approx \frac{\alpha_0}{3\pi}\ln\frac{\Lambda^2}{M^2} - \frac{\alpha_0}{3\pi}\ln\frac{-q^2}{M^2}; \quad \text{for large } \frac{q^2}{M^2}, \quad \text{cutoff } \Lambda, \quad \alpha_0 \equiv \frac{e_0^2}{4\pi} \\ &= \frac{\alpha_0}{3\pi}\frac{\Lambda^2}{(-q^2)} \end{aligned}$$

- So, amplitude describing diagram below is proportional to :



$$\mathcal{M} \propto e_0\left[1-\frac{\alpha_0}{3\pi}\ln\frac{\Lambda^2}{(-q^2)}\right]\left[\bar{u}(k')\gamma^\mu u(k)\right]\epsilon_\mu$$

- Can keep adding more loops



$$\mathcal{M} \approx e_0^2 \left[1 - \left(\frac{\alpha_0}{3\pi} \ln \frac{\Lambda^2}{(-q^2)} \right) + \left(\frac{\alpha_0}{3\pi} \ln \frac{\Lambda^2}{(-q^2)} \right)^2 + \ldots \right] \left([\bar{u}(k')\gamma^\mu u(k)] \, [\bar{u}(p')\gamma_\mu u(p)] \right)$$

$$\approx e_0^2 \left[1 - \epsilon_0 + \epsilon_0^2 - \epsilon_0^3 + \ldots \right] \left([\bar{u}(k')\gamma^\mu u(k)] \, [\bar{u}(p')\gamma_\mu u(p)] \right)$$

$$\approx \left[\frac{e_0^2}{1+\epsilon_0} \right] \; [\bar{u}(k')\gamma^\mu u(k)] \times [\bar{u}(p')\gamma_\mu u(p)] \,, \text{ where } \epsilon_0 = \frac{\alpha_0}{3\pi} \ln \frac{\Lambda^2}{(-q^2)}$$

$$\mathcal{M} \approx \left[\frac{e_0^2}{1 + \frac{\alpha_0}{3\pi} \ln \frac{\Lambda^2}{(-q^2)}} \right] \; [\bar{u}(k')\gamma^\mu u(k)] \times [\bar{u}(p')\gamma_\mu u(p)]$$

Unification of Forces (see Kane, Quigg, ...)

- Include higher order diagrams by replacing "bare" e_0 with q^2-dependent coupling :

$$e_0^2 \;\Rightarrow\; e^2(q^2) \;=\; \frac{e_0^2}{1 + \frac{\alpha_0}{3\pi} \ln \frac{\Lambda^2}{(-q^2)}}$$

- So coupling α *measured* at μ^2 includes all loops, given by :

$$\alpha(\mu^2) \;=\; \frac{\alpha_0}{1 + \frac{\alpha_0}{3\pi} \ln \frac{\Lambda^2}{\mu^2}}$$

- Use measurement of $\alpha(\mu^2)$ at μ^2 to determine α at any other momentum transfer q^2 :

$$\begin{aligned}
\alpha(q^2) \;&=\; \frac{\alpha_0}{1 + \frac{\alpha_0}{3\pi} \ln \left[\frac{\Lambda^2}{-q^2}\right]} \\
&=\; \frac{\alpha_0}{1 + \frac{\alpha_0}{3\pi} \ln \left[\frac{\Lambda^2}{\mu^2} \cdot \frac{\mu^2}{-q^2}\right]} \\
\Rightarrow\; \alpha(q^2) \;&=\; \frac{\alpha(\mu^2)}{1 + \frac{\alpha(\mu^2)}{3\pi} \ln \left[\frac{\mu^2}{-q^2}\right]}
\end{aligned}$$

- No more dependence on cut-off Λ or unmeasurable α_0, just depends on one finite, measured value $\alpha(\mu^2)$. Also see $\alpha(q^2)$ increases as momentum transfer increases

- Result above for $e^{\pm}$ in loops : need to include μ, τ, and quarks
- Should include contributions from all charged particles for which $|q^2| >> m^2$
- Multiply coefficient of correction by : $n_l + 3\left(\frac{4}{9}\right) n_u + 3\left(\frac{1}{9}\right) n_d$
- n_l is number of leptons, n_u is number of quarks with $Q = 2/3$, factor 3 is for three colors

⇒ Contribution depends on charge^2 since couple to γ on each side of loop

⇒ Each family contributes factor 8/3

⇒ Need to include loops with $W^{\pm}$ when $|q^2| >> M_W^2$

⇒ How much stronger is α at $q^2 = M_W^2$ versus $\alpha(4M_e)^2 \approx 1/137$?

⇒ Number particles in loops $n_l = n_d = 3$, $n_u = 2$ gives factor 20/3
($n_u = 2$ since $M_{\rm top} > M_W$, no contribution from top)

$$\frac{\alpha(M_W^2)}{\alpha(4M_e^2)} \approx \frac{1}{1 - \frac{20/3}{3\pi \times 137} \ln\left[\frac{M_W^2}{4M_e^2}\right]} \approx 1.066$$

$$\Rightarrow \alpha(M_W^2) \approx \frac{1}{128}$$

⇒ Running of coupling sensitive to particle content

- For QCD, similar effects but : no lepton contribution, quark color charges are the same, gluons self-couple
- For quark loops $\alpha(\mu^2)/3\pi \;\Rightarrow\; \alpha_3(\mu^2)/6\pi$ for each flavor
- Gluon loops lead to contribution with opposite sign, larger in magnitude
- Gluon loops lead to anti-screening, weakening with q^2, asymptotic freedon

$$\frac{\alpha(\mu^2)}{3\pi} \quad\Rightarrow\quad \frac{\alpha_3(\mu^2)}{4\pi}\left(\frac{2}{3}n_f - 11\right)$$

$$\alpha_3(q^2) \quad=\quad \frac{\alpha_3(\mu^2)}{1+\frac{\alpha_3(\mu^2)}{12\pi}\left(33-2n_f\right)\ln\left[\frac{-q^2}{\mu^2}\right]}$$

- Since $(33 - n_f) = (33 - 2\times 6) > 0$, QCD coupling decreases as momentum transfer increases $\Rightarrow$ asymptotic freedom
- At very large q^2, $\alpha_3(q^2)$ independent of $\alpha_3(\mu^2)$
- For small q^2, denominator approach zero as $q^2 \Rightarrow \Lambda_{\rm QCD}$

$$\Lambda_{\rm QCD} \;\approx\; \mu\exp\left(-\frac{6\pi}{(33-2n_f)\alpha_3(\mu^2)}\right) \;\approx\; 170\ \rm MeV$$

- Using $\mu \approx 10$ GeV, $\alpha_3(\mu^2) \approx\ 0.2,\ \ n_f = 5$
- Sets the approximate scale for bound states of strongly interacting particles

Unification of Forces (see Kane, Quigg, ...)

- For weak interaction : exchanged boson is Z, gauge bosons in loops ($W^{\pm},\ Z,\ H$) dominate over fermions since weak charge larger
- Running of weak coupling like strong coupling : gets weaker as momentum transfer increases
- Grand Unification : if 3 forces emerge from breaking symmetry of a simpler gauge group - reunification occurs at some high scale (for instance $SU(5) \supset SU(3)_c \times SU(2)_L \times U(1)_Y$)
- Strong and weak force, non-Abelian gauge groups, decrease in strength; EM has Abelian group, increases : could unite
- Can write EM and weak couplings reflecting normalization from EW unification :

$$\alpha_1 \equiv \frac{5}{3}\frac{g'^2}{4\pi} = \frac{5\alpha_{\rm QED}}{3\cos^2\theta_W}$$

$$\alpha_2 \equiv \frac{g^2}{4\pi} = \frac{\alpha_{\rm QED}}{\sin^2\theta_W}$$

$$\alpha_3 \equiv \frac{g_3^2}{4\pi}, \quad \text{so}$$

$$\frac{1}{\alpha_i(q^2)} = \frac{1}{\alpha_i(\mu^2)} + \frac{b_i}{4\pi}\ln\left[\frac{q^2}{\mu^2}\right] \quad \text{where } b_i = [-41/10,\ 19/6,\ 7]$$

(see A.V. Gladyshev and D.I. Kazakov, arXiv:1212.2548 [hep-ph])

Unification of Forces (see Kane : Modern Elementary Particle Physics)

- If forces unify, expect : $\alpha_5 = \alpha_1(M_G^2) = \alpha_2(M_G^2) = \alpha_3(M_G^2)$

$$\frac{1}{\alpha_2(\mu^2)} + \frac{b_2}{4\pi}\ln\left[\frac{M_G^2}{\mu^2}\right] = \frac{1}{\alpha_3(\mu^2)} + \frac{b_3}{4\pi}\ln\left[\frac{M_G^2}{\mu^2}\right]$$

$$\frac{1}{\alpha_2(\mu^2)} - \frac{1}{\alpha_3(\mu^2)} = 2\frac{b_3 - b_2}{4\pi}\ln\frac{M_G}{\mu}$$

$$\text{where } b_3 - b_2 = 11 - \frac{22}{3} = \frac{11}{3}, \text{ depends on gauge bosons only}$$

$$\ln\frac{M_G}{\mu} = \frac{6\pi}{11}\left(\frac{1}{\alpha_2(\mu^2)} - \frac{1}{\alpha_3(\mu^2)}\right)$$

$$\text{at } \mu = M_Z,\ \alpha_2(M_Z) \approx 0.034,\ \alpha_3(M_Z) \approx 0.118, \text{ so}$$

$$\ln\frac{M_G}{M_Z} \approx 35.8 \Rightarrow M_G \approx 10^{17}$$

- Result exponentially sensitive to measurement of couplings, affected by higher order corrections
- More exact treatment gives $M_G \approx 10^{15}$ GeV, $\tau_p \approx \frac{1}{\alpha_5^2}\frac{M_G^4}{m_p^5} \approx 10^{30\pm1.5}$ years
- Minimal $SU(5)$ ruled out by IMB experiment, $\tau_p > 5.5 \times 10^{32}$ years for $p \to e^+ + \pi^0$
- See P. Langacker, Phys. Rep. **72**, 185 (1981); C. McGrew *et al.*, Phys. Rev. D **59**, 052004 (1999).

Unification of Forces (see Kane : Modern Elementary Particle Physics)

- In SM, strength of EM and weak forces are independent, even though theory "unified"
- $\alpha_1 = e^2/4\pi, \; g_1 = e/\sin\theta_W, \; g_2 = e/\cos\theta_W, \; \sin^2\theta_W \approx 0.23$
- In GUTs, the mixing angle is predicted.
- In SM, $Q = T_3 - Y//2$, in $SU(5)$, expect $Q = T_3 + cT_1$, c depends on group
- Can write covariant derivative in $SU(5)$ in terms of $SU(5)$ gauge bosons V_a^μ and single coupling g_5 :

$$\begin{aligned}
\partial^\mu - ig_5 T_a V_a^\mu &= \partial^\mu - ig_5 \left(T_3 W_3^\mu + T_1 B^\mu + .. \right), \quad \text{now recall SM relation} \\
B^\mu &= A^\mu \cos\theta_W + Z^\mu \sin\theta_W, \\
W_3^\mu &= -A^\mu \sin\theta_W + Z^\mu \cos\theta_W \\
\Rightarrow \quad - g_5 T_3 \sin\theta_W + g_5 T_1 \cos\theta_W &= -g_5 \sin\theta_W \left(T_3 - \cot\theta_W T_1 \right) \\
&= eQ \text{ which is the coupling (charge) to photon } \; A^\mu
\end{aligned}$$

- So charge $e = g_5 \sin\theta_W$, $c = -\cot\theta_W$
- Try to solve for c : $Tr(Q^2) = Tr(T_3 + cT_1)^2 = TrT_3^3 + TrT_1^2$
- But $TrT_3^2 = TrT_1^2$ so $1 + c^2 = TrQ^2/TrT_3^2$
- From **5** multiplet : $TrQ^2 = 0 + 1 + 3(1/9) = 4/3 \; TrT_3^2 = \frac{1}{4} + \frac{1}{4} + 0 + 0 + 0 = 1/2$
- So $1 + c^2 = 8/3$, and $c^2 = 5/3$

- From this we *predict* :

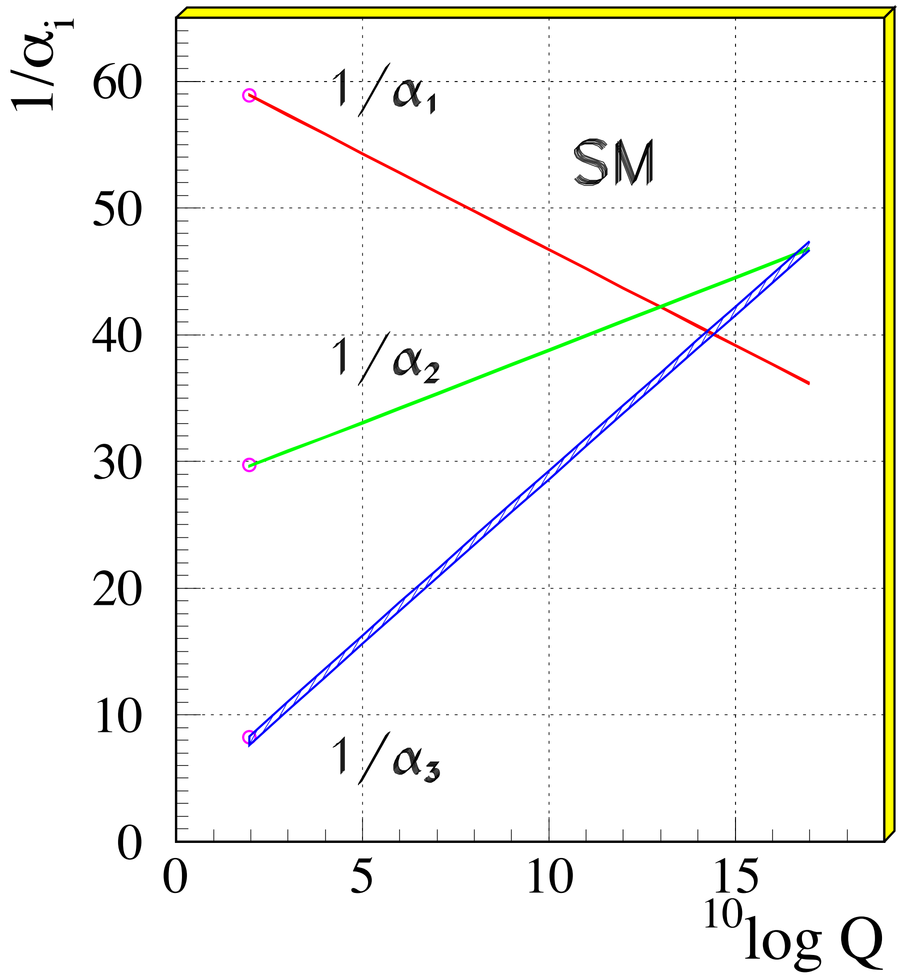
$$\sin^2\theta_W = \frac{g_1^2}{g_1^2+g_2^2} = \frac{1}{1+c^2} = \frac{3}{8} = 0.375 \text{ at unification scale}$$

- We can run couplings down to lower scale using $\alpha_5 = c^2\alpha_1,\ \alpha_2 = \alpha_5$:

$$\sin^2\theta_W = \frac{\alpha_1}{\alpha_1+\alpha_2} = \frac{1}{1+\alpha_2/\alpha_1}$$
$$= \frac{1}{1+\frac{0.033}{0.009}} \approx 0.21 \text{ at } M_W,\ \text{big change from } 3/8$$

- Was strong motivation to pursue these ideas

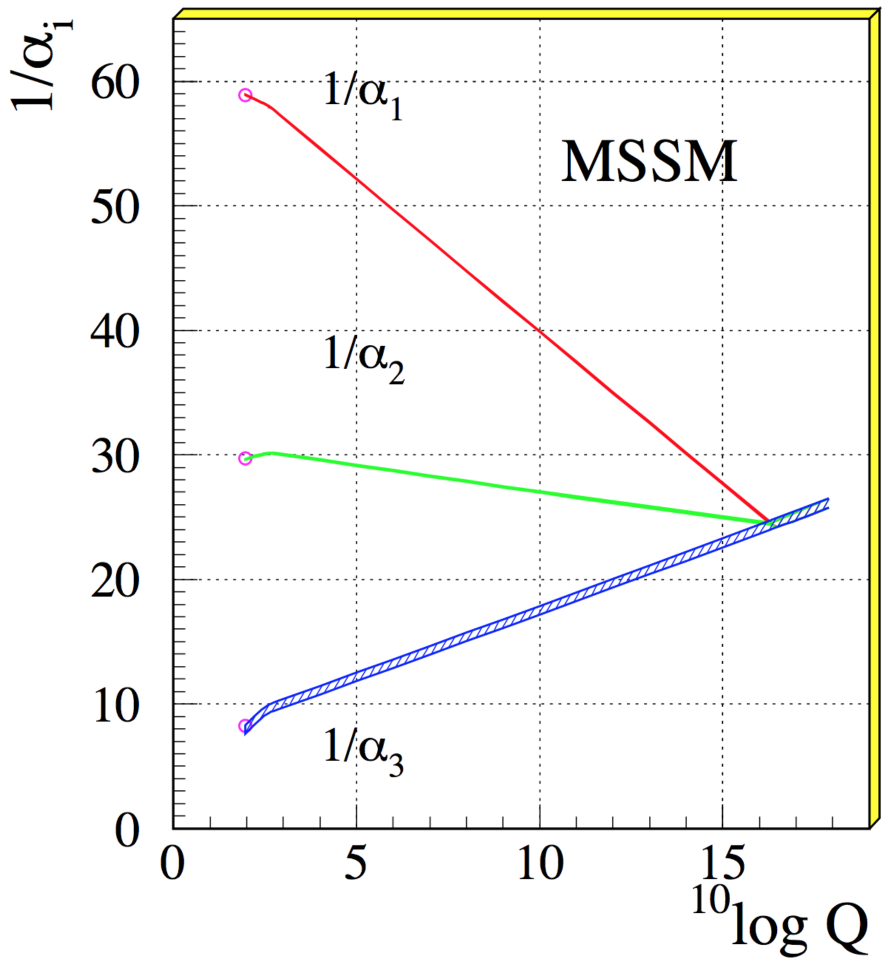
- Coupling strength depends on momentum transfer of virtual gauge bosons
- Familiar plot shows that in SM the couplings don't "unify"
- See for instance A.V. GLadyshev and D.I. Kazakov, arXiv:1212.2548v1 [hep-ph]



- Demonstrates importance of precision knowledge of couplings for extrapolation to higher scales
- For couplings to unify, slopes need to change - need new particle between 100 GeV scale and 10^{17} GeV

Unification of Forces

- For couplings to unify, slopes need to change - need new particle between 100 GeV scale and 10^{17} GeV : SUSY introduces many new gauge bosons
- Coefficients (slope parameters) $b_i = [-41/10, 19.6, 7] \rightarrow \ [-33/5, -1, 3]$

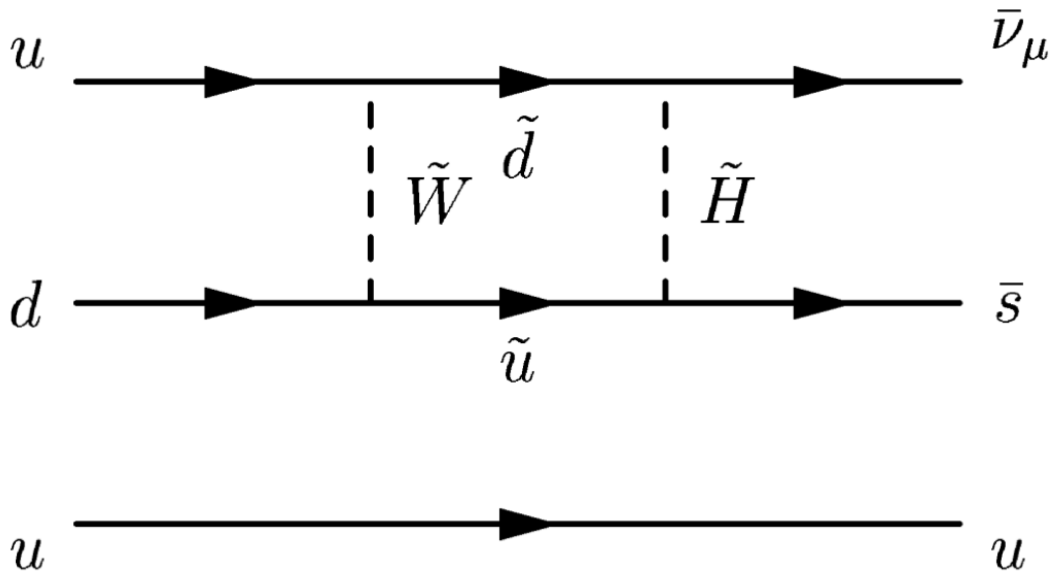


(A.V. GLadyshev and D.I. Kazakov, arXiv:1212.2548v1 [hep-ph])

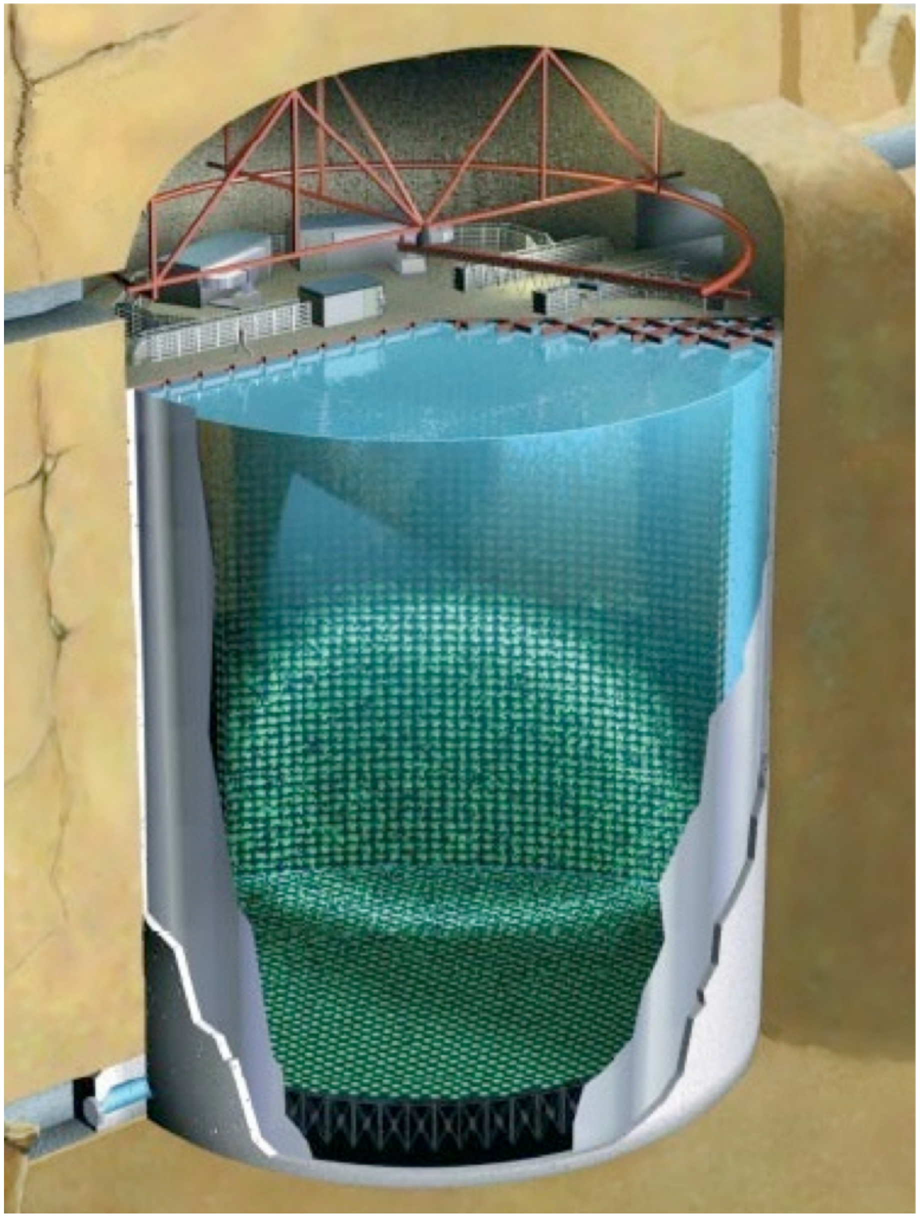
- Notice change of slope at thresholds for MSSM particles
- $M_{SUSY} \approx 10^{3.4 \pm 0.9 \pm 0.4}$ GeV
  $M_{GUT} \approx 10^{15.8 \pm 0.3 \pm 0.1}$ GeV
  $\alpha_{GUT}^{-1} \approx 26.3 \pm 1.9 \pm 1.0$
- Uncertainties from couplings, SUSY mass splittings
- SUSY GUTs solve Higgs hierarchy problem : ordinarily get contributions to Higgs mass of order $M_{X,Y}$
- In SUSY GUTs, superpartners contribute to M_H with same magnitude, opposite sign

Proton Decay in SUSY

- SUSY increases M_{GUT} by a rough factor of 10 compared to $SU(5)$, so τ_p increases by 104
- SUSY also predicts $\sin^2\theta_W = 0.233 \pm 0.003$, agrees with measurement $0.23116(12)$
- SUSY predicts new decay modes for proton - with Higgsino exchange, particles must be from different generations - so decay products must be 2nd or 3rd generations (see P. Nath and P.F. Perez, Phys. Rep. **441**, 191 (2007); arXiv:hep-ph/0601023)
- SUSY decay mode : $p \to \bar{\nu} K^+$

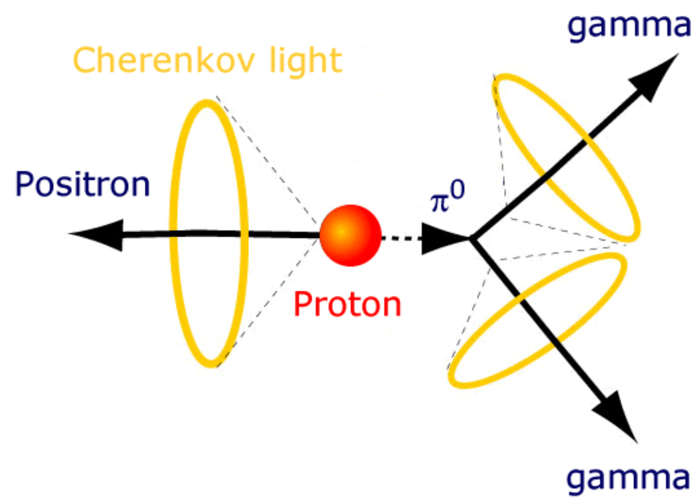


Proton Decay : Super-Kamiokande



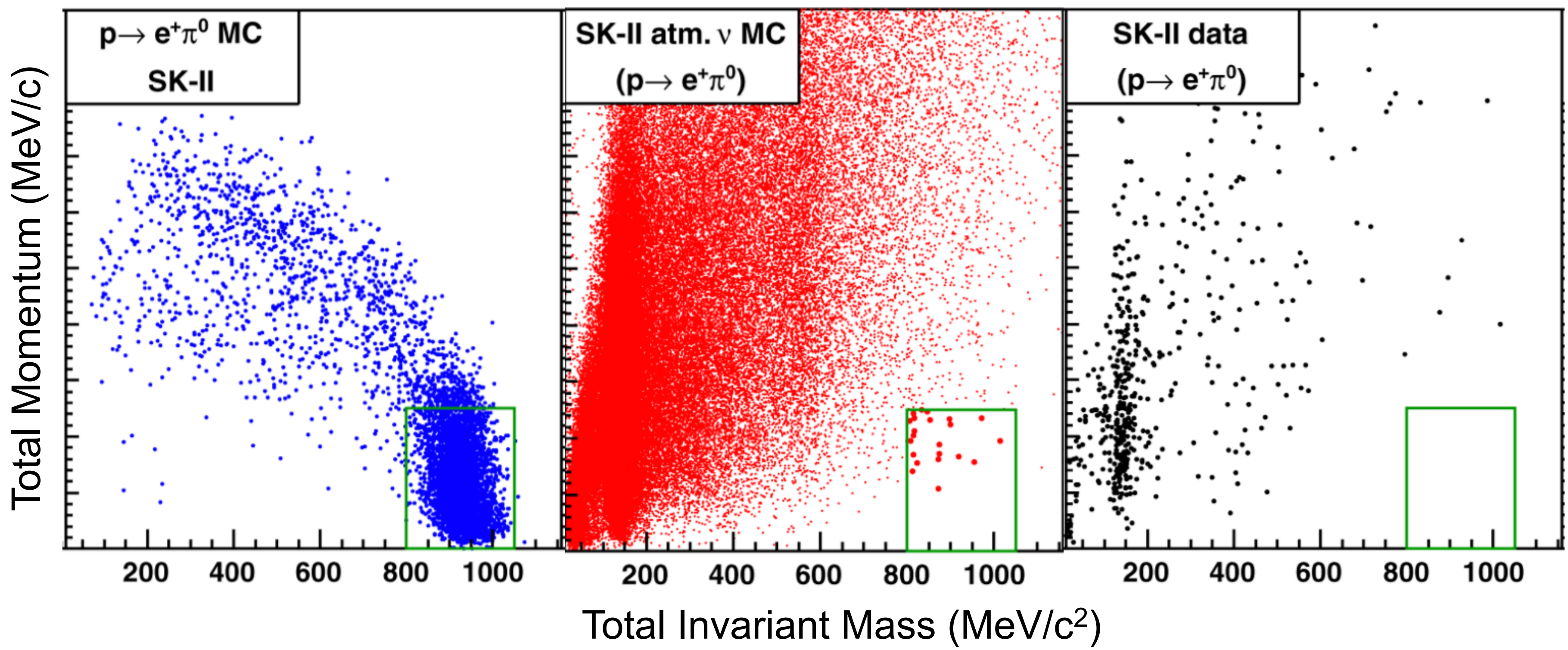
- 50 ktons water, 22.5 ktons fiducial volume, in Kamioka, Japan
- $7.5 \times 10^{33}\ p + 6 \times 10^{33}\ n$
- Stainless steel tanks, 39.3 m diameter, 41.4 m tall
- 1000 m rock overburden
- Inner detector : 20% coverage with 5182 20" PMTs
- Detect Cherenkov radiation from decay products, PID determines if e-like (e shower, multiple overlapping Cherenkov rings in diffuse cone) or μ-like (well defined circular ring)

Proton Decay : $p \to e^+ + \pi^0$ Detection in Super-K



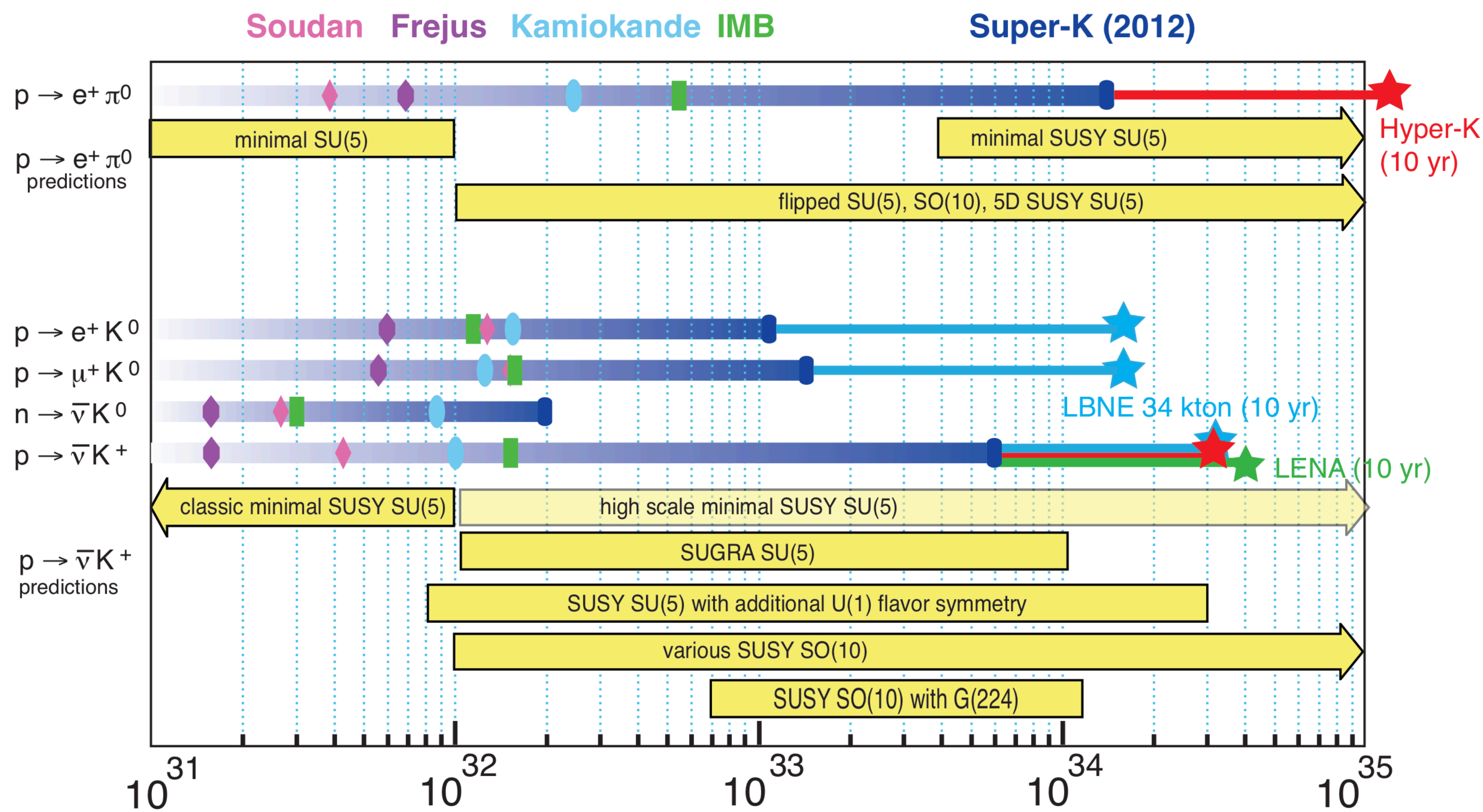
(from Super-K website)

- Good events : fully contained in fiducial volume, 2-3 rings consistent with EM shower
- Reconstructed π^0 mass of 85-185 MeV$/c^2$, no e from μ decay
- Total mass range 800-1050 MeV$/c^2$
- Net momentum < 250 MeV$/c$ (can have momentum from Fermi motion of nucleon in ^{16}O nucleus, meson-nucleon interactions (elastic scattering, charge exchange, absorption) : modeled carefully, include nuclear de-excitation with γ
- Efficiency $\approx 44\%$, mainly limited by π^0 absorption in ^{16}O nucleus
- Background from atmospheric neutrinos : $\bar{\nu}_e + p \to e^+ + \pi^0 + n$
- Invariant mass of backgrounds typically less than for p decay, momentum range larger



- H. Nishino *et al.*, Phys. Rev. D **85**, 112001 (2012)
- Set limits on nucleon decay to charged anti-lepton (e^+ or μ^+) and light mesons ($\pi^0,\ \pi^-,\ \eta,\ \rho^0, \rho^-, \omega$)
- No signals observed, backgrounds typically due to atmospheric neutrino interactions Limits from 3.6×10^{31} to 8.2×10^{33} years at 90% C.L. depending on mode
- Exposure 49.2 kiloton-years, for $p \to e^+ + \pi^0$, background 0.11 ± 0.02 events, no candidates, lifetime 8.2×10^{33} years at 90% C.L.

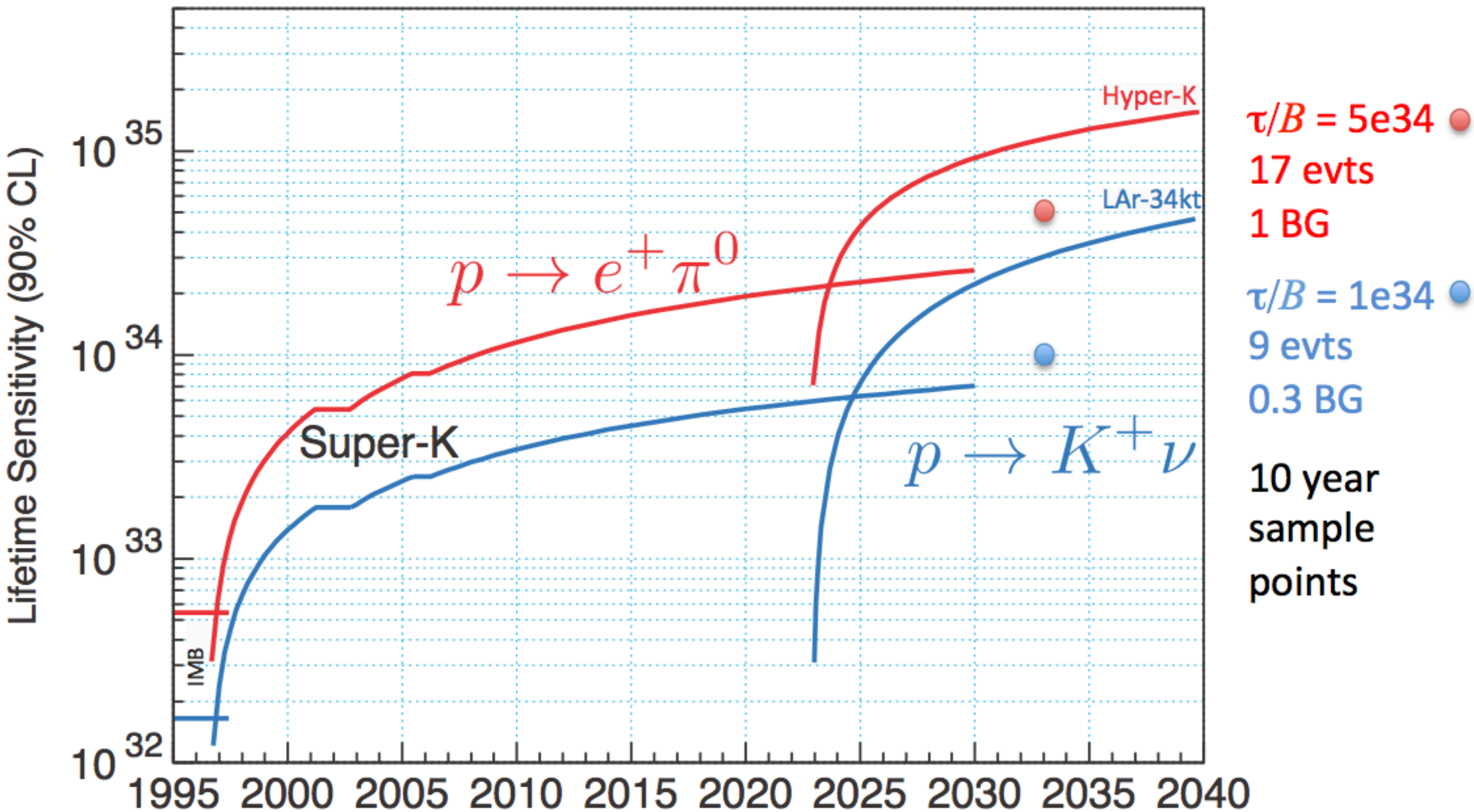
Proton Decay Limits versus Model Predictions



(from Ed Kearns, Boston University)

- Minimal $SU(5)$ ruled out from $p \to e^+ + \pi^0$
- Improving $p \to e^+ + K^0,\ p \to \mu^+ + K^0$ by order of magnitude would have big impact
- Plans to get to beyond $\tau_p > 10^{35}$ years

Proton Decay : Prospects



(from Ed Kearns, Boston University)

- Achieving another order of magnitude or more in τ very important
- Super Kamiokande will continue to run with improved analysis, searches in new channels
- Next generation detectors will be necessary

Proton Decay : Future Approaches

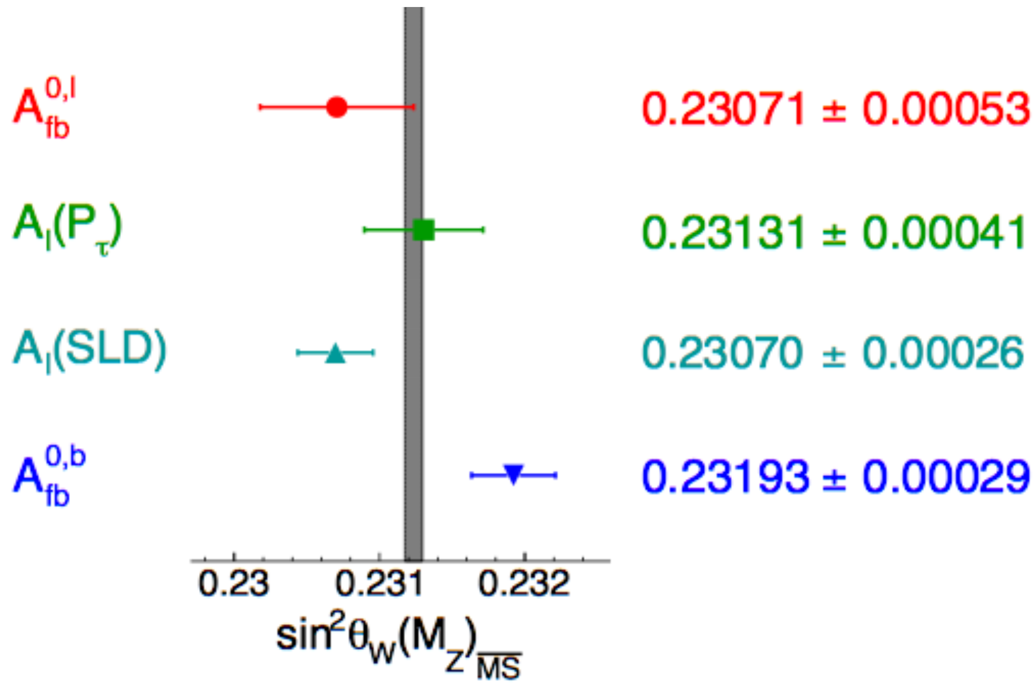
Technique	Examples	Comments
Water Cherenkov	22.5 kton Super-K 560 kton Hyper-Kamiokande	Best for $e^+\pi^0$ Good for all modes
Liquid Argon	34 kton LBNE LAr TPC 20 kton LBNO 2-phase TPC	Best for $K^+\nu$ Good for many other modes
Scintillator	50 kton LENA Next gen. reactor (DB2) ? Water-based LSc ?	Specific to $K^+\nu$

(from Ed Kearns, Boston University)

- Achieving another order of magnitude or more in τ very important
- Super Kamiokande will continue to run with improved analysis, searches in new channels
- Next generation detectors will be necessary

Precision Tests of Electroweak Physics

- Electroweak interactions tested extensively, consistency at 0.1% level
- No compelling discrepancies between electroweak observables and Standard Model
- $\sin^2\hat{\theta}(M_Z)(\bar{M}S) = 0.231\ 16(12)$, known at 5×10^{-4} level



⇒ Direct searches for new particles and new physics at LHC complemented by precision measurements

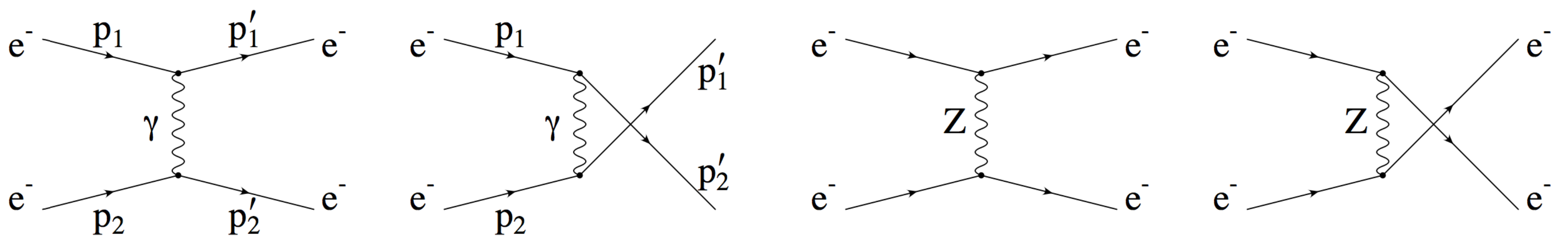
⇒ Look for deviations from Standard Model predictions at lower center of mass energies, through radiative corrections

⇒ Compelling theoretical arguments for new physics at TeV scale

- Proposes a measurement of parity-violating asymmetry A_{PV} in longitudinally polarized e^- off unpolarized e^-

$$A_{PV} \equiv \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma L}$$

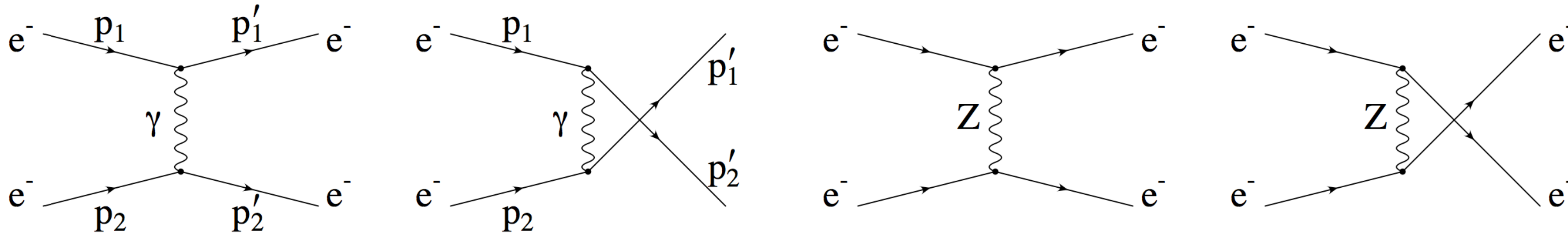
- σ_R (σ_L) is scattering cross-section for incident right (left) handed electrons
- $A_{PV} \neq 0$ violates parity
- At $Q^2 << M_Z^2$ parity nonconservation comes from interference between EM and weak amplitudes



- The unpolarized cross-section is dominated by photon exchange, given by :

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2m_e E}\frac{\left(3+\cos^2\theta\right)^2}{\sin^4\theta} = \frac{\alpha^2}{4m_e E}\frac{1+y^4+(1-y)^4}{y^2(1-y)^2},$$

- α is fine structure constant, E incident beam energy, θ scattering angle, $y \equiv 1 - E'/E$, E' energy of scattered e



- A_{PV} due to interference between photon and Z^0 exchange diagrams
- Remember - e coupling to Z^0 is different for left and right-handed e
- See E. Derman and W.J. Marciano, Annals Phys. **121**, 147 (1979)

$$A_{PV} \ = m_e E \frac{G_F}{\sqrt{2}\pi\alpha} \frac{4\sin^2\theta}{(3+\cos^2\theta)^2} Q_W^e m_e E \frac{G_F}{\sqrt{2}\pi\alpha} \frac{2y(1-y)}{1+y^4+(1-y)^4} Q_W^e$$

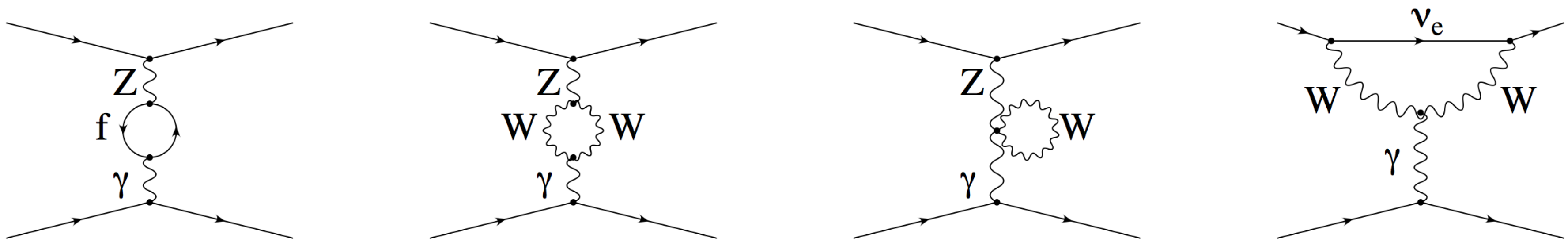
- Q_W^e proportional to product of electron vector and axial-vector coupling to Z^0
- Q_W^e weak charge of the electron
- At leading order $Q_W^e = 1 - 4\sin^2\theta_W$; modified at 1-loop and beyond $\rightarrow 1 - 4\sin^2\theta_W(Q^2)$
- At M_Z, $\sin^2\theta_W(M_Z) \approx 0.23116(12)$, $Q_W^e \approx 0.075$

$\Rightarrow$ At $Q^2 \approx 0.0056$ GeV^2 of MOLLER experiment, $\sin^2\theta_W$ ≈ 3% larger
$Q_W^2 \approx 0.0469 \pm 0.0006$, change of 40% compared to tree level value at M_Z!

- Very sensitive to running of $\sin^2\theta_W$

- $A_{PV} \approx$ 35 ppb, goal of MOLLER is measurement with statistical precision 0.73 ppb, 2.3% measurement of Q_W^e (Spokesperson Krishna Kumar, thanks for material)
- Determines $\delta(\sin^2\theta_W) \pm 0.00029$ (0.1%); comparable to single best measurements from LEP and SLC
- Would use 11 GeV polarized e^- beam in Hall A

- What is physics motivation for a precision measurement of $\sin^2\theta_W$?
- Electroweak theory provides precise predictions with negligible uncertainty - corrections at 1-loop level all known
- Comparison with precise experimental result ($\approx 10^{-3} \cdot G_F$) sensitive to new physics at TeV scale
- Uniquely sensitive to purely leptonic amplitudes at $Q^2 << M_Z^2$



- See A. Czarnecki and W. J. Marciano, Int. J. Mod. Phys. A **15**, 2365 (2000) [arXiv:hep-ph/0003049]

- Express amplitudes of new high energy dynamics as contact interaction between leptons :

$$\mathcal{L}_{e_1 e_2} = \sum_{i,j=L,R} \frac{g_{ij}^2}{2\Lambda^2} \bar{e}_i \gamma_\mu e_i \bar{e}_j \gamma^\mu e_j. \qquad (1)$$

- $e_{L/R} = \frac{1}{2}(1 \mp \gamma_5)Y_e$ chiral projections of electron spinor, Λ mass scale of new interaction, $g_{ij} = g_{ij}^*$ are new couplings, $g_{RL} = g_{LR}$
- For 0.023 measurement of Q_W^e, sensitivity to new interactions (like lepton compositeness):

$$\frac{\Lambda}{\sqrt{|g_{RR}^2 - g_{LL}^2|}} = \frac{1}{\sqrt{\sqrt{2} G_F |\Delta Q_W^e|}} \approx \frac{246 \text{ GeV}}{\sqrt{0.023 Q_W^e}} = 7.5 \text{ TeV} \qquad (2)$$

- For $\sqrt{|g_{RR}^2 - g_{LL}^2|} = 2\pi$, $\Lambda = 47$ TeV, electron structure probed at 4×10^{-21} m
- Best contact interaction limits on leptons from LEP, on quarks from Tevatron and LHC .
- But LEP only sensitive to g_{RL}^2 and $g_{RR}^2 + g_{LL}^2$ - insensitive to PV combination $g_{RR}^2 - g_{LL}^2$

- New Z' bosons, like Z_χ from $SO(10)$, predict PV couplings :

$$\sqrt{|g_{RR}^2 - g_{LL}^2|} = \sqrt{\frac{4\pi\alpha}{3\cos^2\theta_W}} \approx 0.2 \Rightarrow Z_\chi \approx 1.5 \text{ TeV}$$

- Get sensitivity up to $Z_{LR} \approx 1.8$ TeV from left-right symmetric models

MOLLER Experiment and Supersymmetry

- New particles in Minimal Supersymmetric Standard Model (MSSM) enter A_{PV} through radiative loops
- Effects from MSSM as large as $+8\%$ on Q_W^e, can be measured to significance of 3.5 σ
- If R-parity violated, Q_W^e can shift by -18%, an 8 σ effect
- MOLLER can help distinguish between R-parity conserving and violating SUSY; RPC lightest SUSY particle could be dark matter candidate (plot below from DOE proposal)

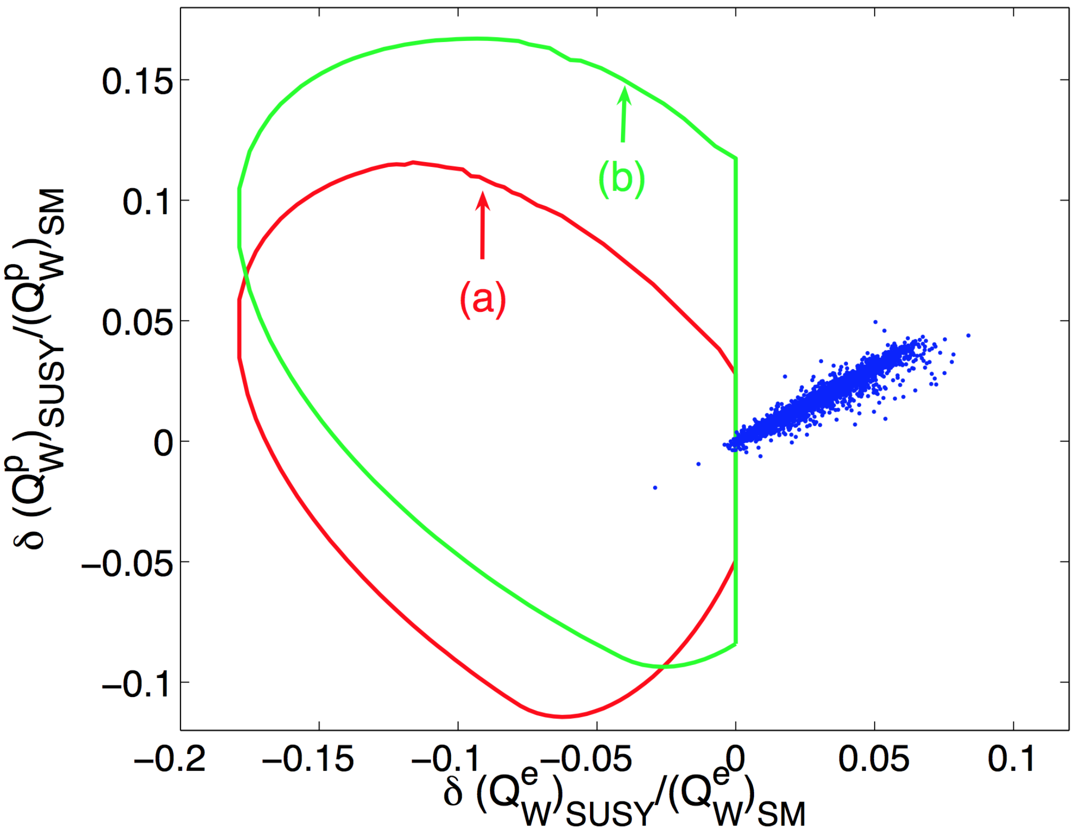
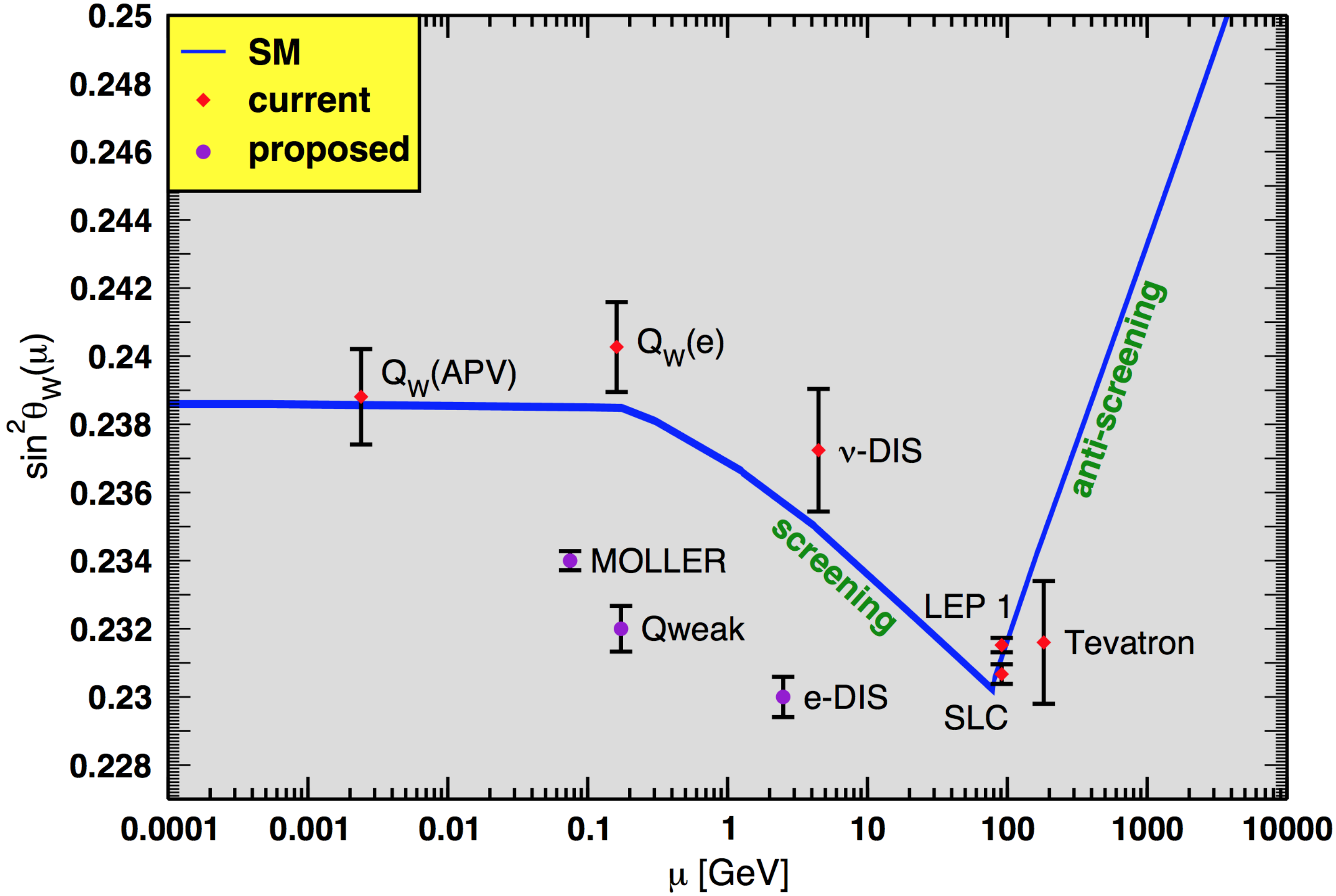
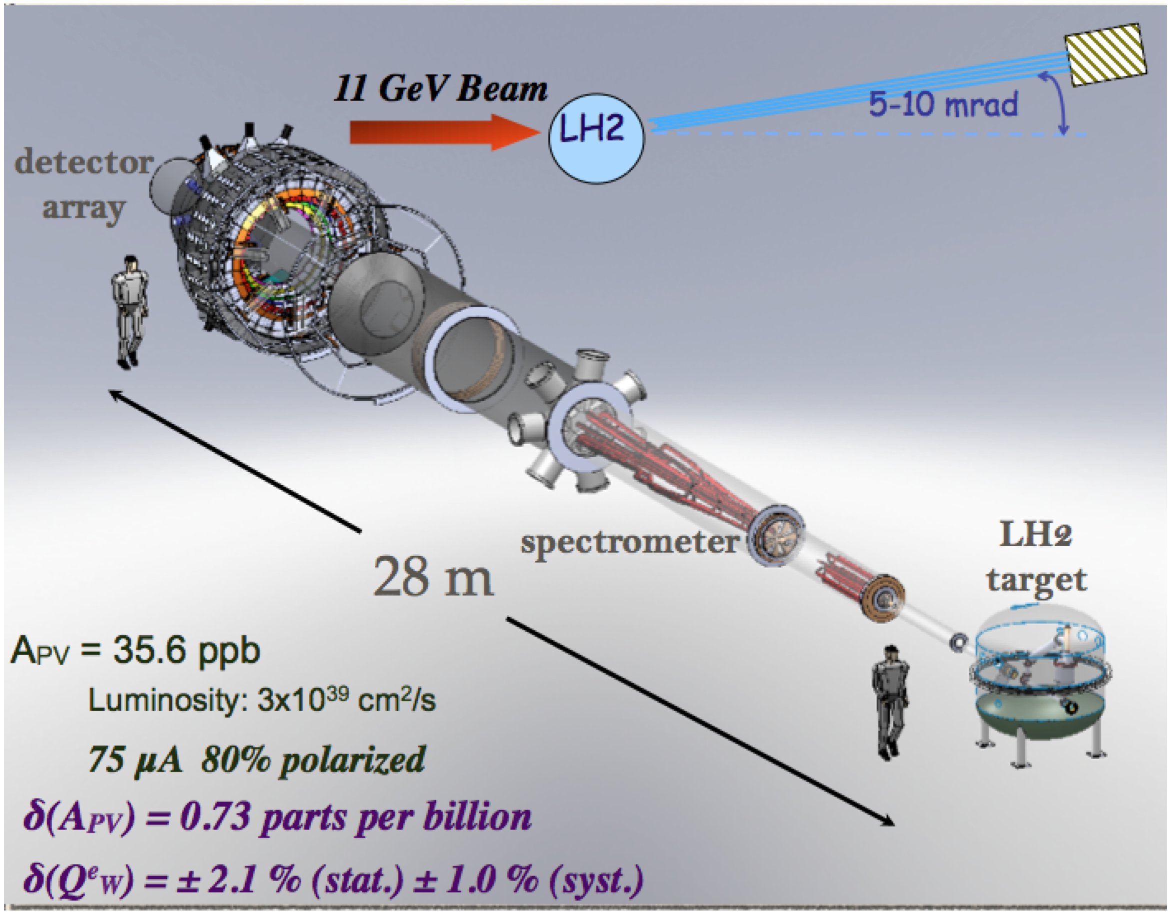


Figure 3: *Relative shifts in the electron and proton weak charges due to SUSY effects. Dots indicate the range of allowed MSSM-loop corrections. The interior of the truncated elliptical regions give possible shifts due to R-parity violating (RPV) SUSY interactions, where (a) and (b) correspond to different assumptions on limits derived from first row CKM unitarity constraints.*



- Plot from DOE proposal, shows 3 planned measurements with projected sensitivity, arbitrary central values
- Notice : some tension between left-right asymmetry in Z production at SLC $A_{LR}(had)$ vs forward backward asymmetry in Z decays to b-quarks $A_{FB}(b)$ at LEP
- MOLLER will achieve similar 0.1% accuracy, potentially influence world average

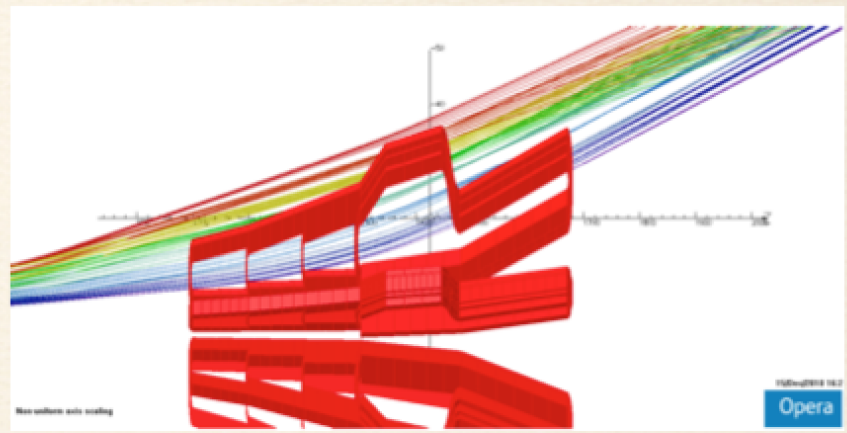


Parameter	Value
E [GeV]	≈ 11.0
E' [GeV]	1.8 - 8.8
θ_{cm}	46°-127°
θ_{lab}	0.23°-1.1°
$< Q^2 >$ [GeV^2]	0.0056
Maximum Current [μA]	85
Target Length (cm)	150
ρ_{tgt} [g/cm^3] (T= 20K, P = 35 psia)	0.0715
Max. Luminosity [cm^{-2} sec^{-1}]	$3.4 \cdot 10^{39}$
σ [μBarn]	≈ 40
Møller Rate [GHz]	≈ 135
Statistical Width(2 kHz flip) [ppm/pair]	≈ 83
Target Raster Size [mm]	5 x 5
ΔA_{raw} [ppb]	≈ 0.6
Background Fraction	≈ 0.08
P_{beam}	$\approx 85\%$
$< A_{pv} >$ [ppb]	≈ 35
$\Delta A_{stat} / < A_{expt} >$	2.1%
$\delta(\sin^2 \theta_W)_{stat}$	0.00026

Technical Challenges

- **~ 150 GHz scattered electron rate**
  - Design to flip Pockels cell ~ 2 kHz
  - 80 ppm pulse-to-pulse statistical fluctuations
- **1 nm control of beam centroid on target**
  - Improved methods of “slow helicity reversal”
- **> 10 gm/cm² liquid hydrogen target**
  - 1.5 m: ~ 5 kW @ 85 μA
- **Full Azimuthal acceptance with θ_{lab} ~ 5 mrad**
  - novel two-toroid spectrometer
  - radiation hard, highly segmented integrating detectors
- **Robust and Redundant 0.4% beam polarimetry**
  - Pursue both Compton and Atomic Hydrogen techniques

- **MOLLER Collaboration**
  - ~ 100 authors, ~ 30 institutions
  - Expertise from SAMPLE A4, HAPPEX, G0, PREX, Qweak, E158
  - 4th generation JLab parity experiment



- **20M$ proposal to DoE NP**
- **3-4 years construction**
- **2-3 years running**

- First data ≈ 2017