Hydrodynamics in Heavy Ion Collisions

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Hydrodynamics of QGP

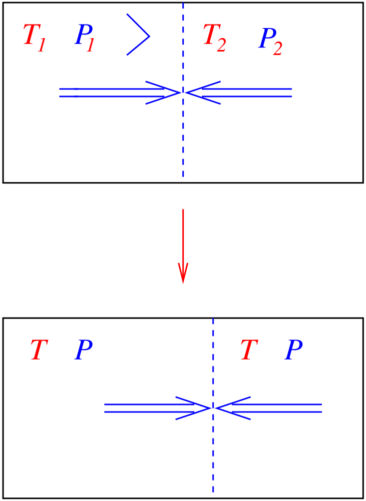
Equilibrium

- How does one describe a system of $N \gg 1$ bodies?
- Depends on how much “information” one wants.
- Worst case: $\psi(x_1, \cdots, x_N)$
- Best case: Minimum information content. I.e. maximum entropy. Need only a handful of numbers such as temperature and chemical potential.
- $n(p) = 1/(e^{E_p/T - \mu/T} \mp 1)$
- These quantities are the Langrange multipliers that constraints conserved quantities such as energy and charge.

Non-Equilibrium

- How does one describe a system of $N \gg 1$ bodies?
- Depends on how much "information" one wants.
- Worst case: $\psi(x_1, \cdots, x_N)$
- Best case: Minimum information content. I.e. maximum entropy. Need only a handful of numbers such as temperature, and chemical potential. But locally.
- $n(p, x) = 1/(e^{p_\mu u^\mu/T(x) - \mu(x)/T(x)} \mp 1)$
- You only need to know few functions: $T(t, \mathbf{x}), \mu(t, \mathbf{x})$ as well as the collective velocity $\mathbf{u}(t, \mathbf{x})$
- These quantities are the Langrange multipliers that constraints conserved quantities such as energy, momentum and charge.

Schematic idea of hydro evolution



- System is made up of "fluid cells".
- Each fluid cell feels a force according to the pressure difference (gradient) w.r.t. its neighbors
- System evolves by "flowing" towards lower pressure
- NR: $\mathbf{F} = m\mathbf{a}$

$$-\nabla P = n_m \partial_t \mathbf{u}$$

where n_m: mass density,
P: pressure,
$\mathbf{u}$: flow velocity

Physics from Hydro – What are we trying to learn?

- What is the nature of the initial condition?
- Do we reach local equilibrium in heavy ion collisions?
- How hot is it?
- How viscous is QGP?
- (Is there a phase transition? If so what kind?)

Physics from Hydrodynamics

- Information content of single particle spectra

$$\frac{dN_i}{dy\,d^2p_T} = \frac{dN_i}{2\pi p_T dp_T dy}\left(1 + \sum_{n=1}^{\infty} 2v_{i,n}(p_T, y)\cos(n\phi)\right)$$

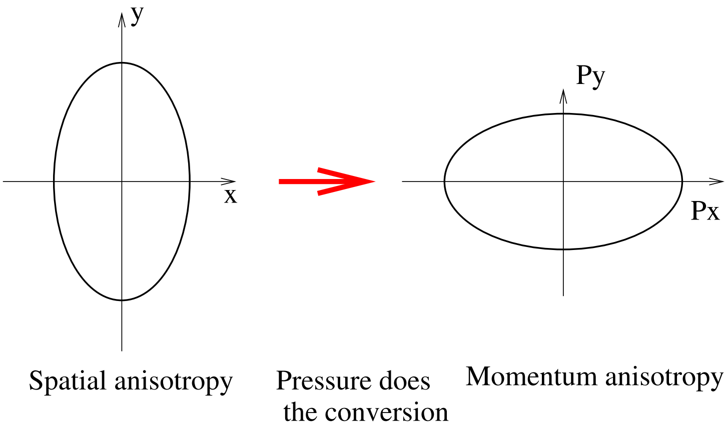
- "Flow": $v_{i,n}(p_T)$
- Came from

$$\varepsilon(\mathbf{x}_T, \eta) = \varepsilon(r_T, \eta)\left(1 + \sum_{n=1}^{\infty} 2\epsilon_n(r_T, \eta)\cos(n\phi)\right)$$

  - *Pressure* converts it into $v_{i,n}(p_T)$
  - History matters
- $\epsilon_n \to v_{i,n}$ conversion contains information on the medium and its evolution

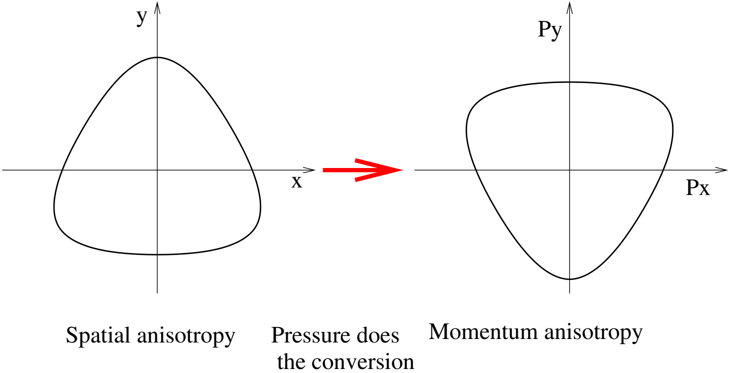
Physics from Hydrodynamics

- Elliptic Flow – $\cos(2\phi)$ component



Physics from Hydrodynamics

- Triangular Flow – $\cos(3\phi)$ component



[Alver and Roland, Phys.Rev.C81:054905, 2010]

Physics from Hydrodynamics

- Flows are about: Pressure converting
  Spatial morphology $\epsilon_n(r_T, \eta) \Longrightarrow$ Momentum space morphology $v_n(p_T, y)$
- This is sensitive to
  - Initial Conditions
  - Flow dynamics (η/s)
  - Equation of State (to a less extent)

Physics from Hydrodynamics

- Why is initial condition important?
  - Initial temperature (distribution) $T_0 > T_c$
  - Beginning time of hydro ($\sim$ thermalization time) τ_0
  - The size of the hot spots σ_0
  - What happens *before* the hydro stage?
- v_2 alone cannot determine all these $\Longrightarrow$ $v_3, v_4, \cdots$

Physics from Hydrodynamics

- Why is η/s important?
  - One of the central properties of QGP
  - Calculable in perturbative QCD $\eta/s \sim 1/g^4 \ln(1/g)$
  - Calculable in AdS/CFT $\eta/s = 1/4\pi$
  - If $\eta/s \sim 1/4\pi$, QGP must be sQGP

Formulating Hydrodynamics

- Equations for $T, \mathbf{u}$ – Conservation laws

$$\partial_\mu \langle T^{\mu\nu} \rangle = 0$$

- Stress-energy tensor $T^{\mu\nu}$ has only 10 d.o.f. Cons. laws provide 4 constraints $\Longrightarrow$ 6 d.o.f. left.
- No dynamical content yet.
- Energy density and flow vector

$$T^{\mu\nu} u_\nu = -\varepsilon u^\mu$$

- u^μ: Time-like eigenvector of $T^{\mu\nu}$. Normalized to $u^\mu u_\mu = -1$.
- ε: Local energy density
- This is always possible since $T^{\mu\nu}$ is real and symmetric.

Hydrodynamics

- So far:

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu + W^{\mu\nu}$$

with

$$W^{\mu\nu} u_\nu = 0$$

- This is just math. No physics input except that ε is the energy density and u^μ is the velocity of the energy flow.
- Physics - Small scale physics is thermal $\Longrightarrow$ Local equilibrium $\Longrightarrow$ Equation of state (i.e. $P = P(\varepsilon)$)
  - $W^{\mu\nu} = (g^{\mu\nu} + u^\mu u^\nu)P(\varepsilon) + \pi^{\mu\nu}[\varepsilon, u]$ with $\pi^{\mu\nu} u_\nu = 0$
  - Ideal Hydro: $\pi^{\mu\nu} = 0$ gives $\partial_t((\varepsilon + P)\mathbf{u}) = -\nabla P$ for small $\mathbf{u}$
  - Viscous Hydro:

$$\pi^{ij} = -\frac{\eta}{2}\left(\partial^i u^j + \partial^j u^i - g^{ij}(2/3)\nabla\cdot\mathbf{u}\right) - \zeta g^{ij}\nabla\cdot\mathbf{u}$$

Validity of Hydrodynamics

- $\partial_\mu T^{\mu\nu} = 0$: This is an operator statement. This is valid no matter what.
- $\partial_\mu \langle T^{\mu\nu} \rangle = 0$: This is a statement about average. This is valid no matter what.

  Ideal Hydro

- $T^{\mu\nu}_{\rm id.} = \epsilon u^\mu u^\nu + P(g^{\mu\nu} + u^\mu u^\nu)$
  This assumes that the the system has isotropized $\Longrightarrow$ Ideal Hydrodynamics is valid only after the system has isotropize. But this is not enough.
- $P(x) = P(\varepsilon(x))$: Equation of state. Valid only if local equilibrium is reached. Recent most complete characterization of QCD thermalization process: 1107.5050 by Moore and Kurkela. $t_{\rm eq} \sim \alpha^{-2} Q^{-1}$.

Validity of Hydrodynamics

Viscous Hydro

- $\pi_{ij} = -\eta\partial_{\langle i}u_{j\rangle}$ (tranceless, symmetric and transverse to u^μ)
- Gradient expansion must be valid $\Longrightarrow$ Higher derivatives are smaller.
- This means local equlibrium is established in the length scale much longer than the microscopic mean free path.
- In fact, $\pi_{ij} = -\eta\partial_{\langle i}u_{j\rangle}$ induces unphysical faster-than-light propagations.
  $\Longrightarrow$ Second order Israel-Stewart formalism: π_{ij} relaxes towards $-\eta\partial_{\langle i}u_{j\rangle}$

$$\frac{d}{d\tau}\pi_{ij} = -\frac{1}{\tau_r}\left(\pi_{ij} - (-\eta\partial_{\langle i}u_{j\rangle})\right)$$

Ideal Hydro

- Stress-energy tensor

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu + P(u^\mu u^\nu - g^{\mu\nu})$$

- Energy momentum conservation

$$0 = u^\mu \partial_\mu \varepsilon + (\varepsilon + P)(\partial_\mu u^\mu)$$

and

$$(\varepsilon + P) u^\mu \partial_\mu u_\alpha = \partial_\alpha P - u_\alpha u_\nu \partial^\nu P$$

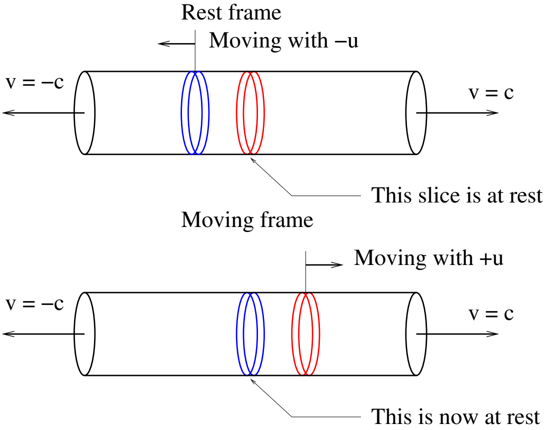
- One can easily show that entropy is conserved

$$\partial_\mu (s u^\mu) = 0$$

using $sT = \varepsilon + P$ and $TdS = dU + PdV \rightarrow Tds = d\varepsilon$

Solving Hydro – Need for τ, η

- Idealized physical picture: Two infinitely energetic ($v = c$) pancakes pulling away from each other



- Can't distinguish the two cases $\Longrightarrow$ Boost invariance if $E_{\text{beam}} = \pm\infty$.

Need for τ, η

- Dynamic rapidity y is defined by:

$$E = \sqrt{m^2 + p_T^2} \cosh(y)$$
$$p_z = \sqrt{m^2 + p_T^2} \cosh(y)$$

- Ends with $\pm c$ are at $y = \pm\infty$ $\Longrightarrow$ The system occupies the whole rapidity axis.
- With $\gamma = \cosh \Delta y$ and $\gamma v = \sinh \Delta y$, Lorentz boost is just a translation in the rapidity space

$$E' = \gamma E + \gamma v p_z = m_T \cosh(y + \Delta y)$$
$$p'_z = \gamma p_z + \gamma v E = m_T \sinh(y + \Delta y)$$

- The system must be homogeneous in y $\Longrightarrow$ Independent of y

Solving Hydro – Need for τ, η

- Space-time rapidity η defined by

$$t = \tau \cosh \eta$$
$$z = \tau \sinh \eta$$

- Lorentz boost is just a translation in the rapidity space

$$t' = \gamma t + \gamma v z = \tau \cosh(\eta + \Delta y)$$
$$z' = \gamma z + \gamma v t = \tau \sinh(\eta + \Delta y)$$

- A boost invariant system is independent of η as well.

Solving Hydro – 1+1 D (Bjorken)

- Simplify some more – No dependence on x, y. No dissipation.
- The only thing a boost can do: Lorentz transform the fluid velocity u^μ.
- Boost invariance: Fluid velocity can only be $u^\mu = (t/\tau, 0, 0, z/\tau) = (\cosh\eta, 0, 0, \sinh\eta)$.
- Let

$$\varepsilon = \varepsilon(\tau)$$
$$P = P(\tau)$$

- The energy-momentum conservation becomes

$$\frac{d\varepsilon}{d\tau} = -\frac{\varepsilon + P}{\tau}$$

- $\dfrac{d\varepsilon}{d\tau} = -\dfrac{\varepsilon + P}{\tau}$ can be rewritten as

$$\frac{ds}{d\tau} = -\frac{s}{\tau}$$

using $Ts = \varepsilon + P$ and $TdS = dU + PdV$.
Also,

$$\frac{d\varepsilon}{d\tau} = -\frac{(1 + v_s^2)\varepsilon}{\tau}$$

using $v_s^2 = \dfrac{\partial P}{\partial \varepsilon}$

- Solutions

$$s(\tau) = s_0 \left(\frac{\tau_0}{\tau}\right)$$

and

$$\varepsilon(\tau) = \varepsilon_0 \left(\frac{\tau_0}{\tau}\right)^{1+v_s^2}$$

Solving Hydro – Need for τ, η

- At RHIC: $y_{\max} \approx \pm 5.4$
- At LHC: $y_{\max} \approx \pm 8.0$
- Not $\pm\infty$, but big enough
- More technical reason: Hard to contain this system in $t - z$ as the boundary of the system linearly increases with time
- In τ, η, $\eta_{\max} > y_{\max}$ is enough.
- Price to pay: $\partial_\mu T^{\mu\nu} = 0$ becomes complicated.

Including Viscosity

- Generalized Israel-Stewart
- For example, shear viscosity: Baier, Romatschke, Son, Starinets, Stephanov (0712.2451)

$$\Delta^{\mu\alpha}\Delta^{\nu\beta}D\pi_{\alpha\beta} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\mu\rangle} + \frac{4}{3}\tau_\pi\pi^{\mu\nu}(\partial_\alpha u^\alpha)\right)$$

Physics Issue 1: Initial state

Reminder

- What we want to do:
  Study how initial state spatial anisotropy

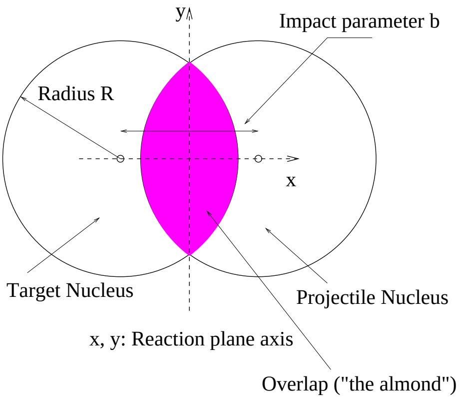
$$\varepsilon(\mathbf{x}_T, \eta) = \varepsilon(r_T, \eta) \left(1 + \sum_{n=1}^{\infty} 2\epsilon_n(r_T, \eta) \cos(n\phi)\right)$$

  turns into the final state momentum anisotropy

$$\frac{dN_i}{dy\, d^2p_T} = \frac{dN_i}{2\pi p_T dp_T dy} \left(1 + \sum_{n=1}^{\infty} 2v_{i,n}(p_T, y) \cos(n\phi)\right)$$

- Wants to get history of physical quantities $P, \mathbf{u}, \eta/s, \cdots$ from the flow coefficients $v_{i,n}(p_T)$ – Need many different measurements

Smooth Geometry



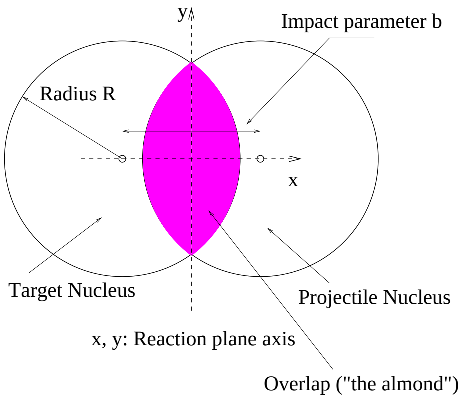
- Thickness function

$$T_A(\mathbf{s}) = \int dz\, \rho_A(\mathbf{s}, z)$$

- Overlap function:

$$T_{AB}(\mathbf{s}, \mathbf{b}) = T_A(\mathbf{s}) T_B(\mathbf{b} + \mathbf{s})$$

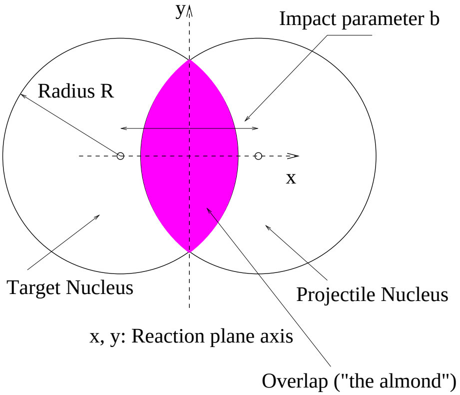
Smooth Geometry



- Participants: $N_{\rm part}(\mathbf{s}, \mathbf{b}) \propto T_A(\mathbf{s}) + T_B(\mathbf{b} + \mathbf{s})$
- Binary scatterings: $N_{\rm bin}(\mathbf{s}, \mathbf{b}) \propto T_{AB}(\mathbf{s}, \mathbf{b})$
- Initial energy density

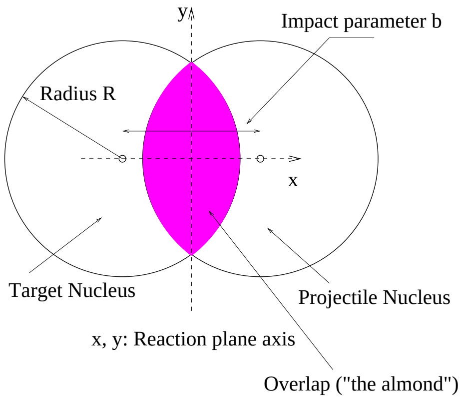
$$\varepsilon(\mathbf{s}, \mathbf{b}) = c_1 \left[T_A(\mathbf{s}) + T_B(\mathbf{b} + \mathbf{s})\right] + c_2 T_{AB}(\mathbf{s}, \mathbf{b})$$

Smooth Geometry



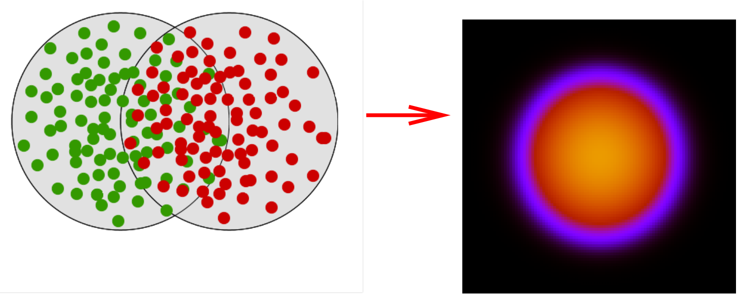
- Ultimately, initial geometry determines the initial conditions and the final flow pattern.
- Initial geometry also determines number of jets at **s** and the path conditions for those jets.

Smooth Geometry



- Smooth initial states have up-down, left-right symmetry: Initial states only has $\cos(2n\phi)$ components such as $v_2, v_4, v_6, \cdots$

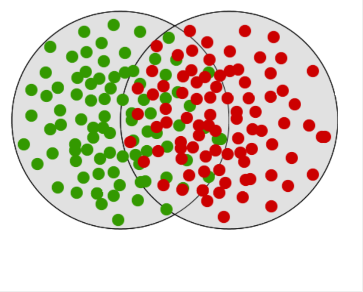
What determines the initial shape?

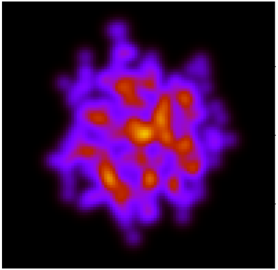


- Averaged smooth initial condition $\Longrightarrow$ Only $v_{\rm even}$'s survive.

Physics from Hydrodynamics

What determines the initial shape?



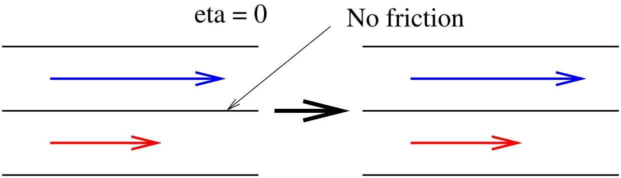


- Fluctuating initial condition $\Longrightarrow$ All v_n are non-zero.

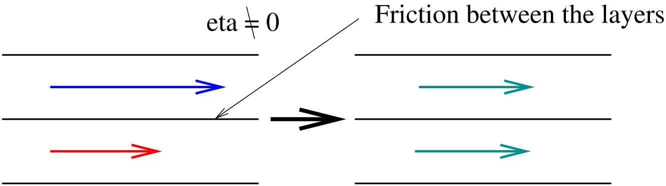
Why go beyond v_2?

- $v_{\rm odd} \neq 0$ due to fluctuations is obvious *once* you see it
  [Alver and Roland, Phys.Rev.C81:054905, 2010]
- v_2 and v_3 are sensitive to the different features of the initial condition
- Elliptic flow: Sensitive to the overall almond shape
- Triangular flow: Less so. More local in the sense that average initial condition gives zero v_3.
- Viscosity effect on different features is *different*
  - Viscosity smears out lumps.
  - Viscosity reduces differential flow - Triangle is “rounder” than ellipse

Effect of viscosity

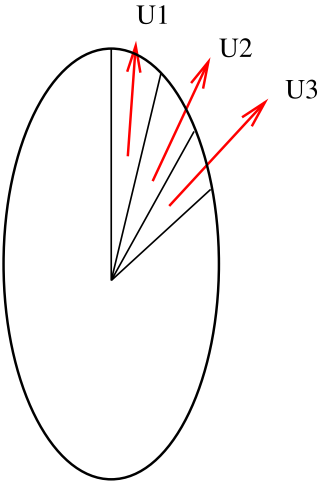


The relative velocity of the two layers does not change.

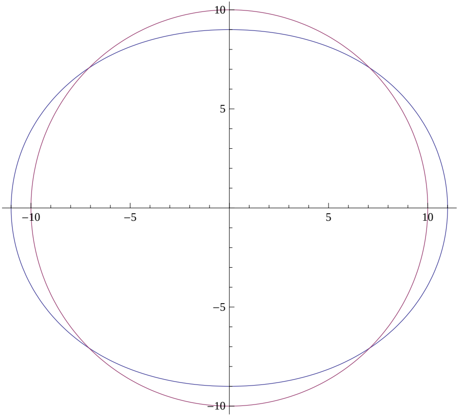


The velocities eventually become the same.

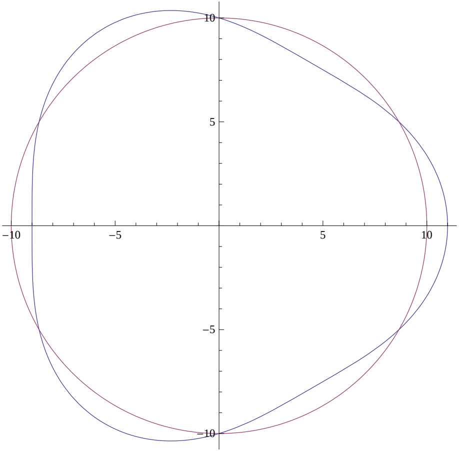
Effect of viscosity



- $\eta = 0$ means $u_1 < u_2 < u_3$ is maintained for a long time
- $\eta \neq 0$ means that $u_1 \simeq u_2 \simeq u_3$ is achieved more quickly
- Shear viscosity smears out flow differences (it's a diffusion)
- Shear Viscosity reduces non-sphericity

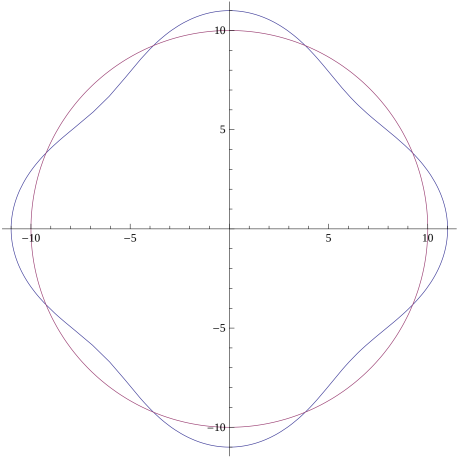


This causes elliptic flow. It is harder to destroy this than

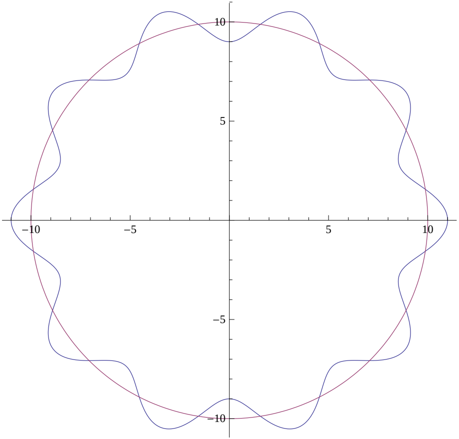


this (v_3) ...

ϵ_n

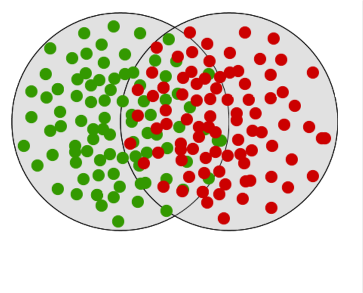


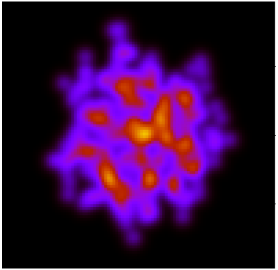
or this (v_4) ...



or this (v_{10}) ...

Initial Conditions

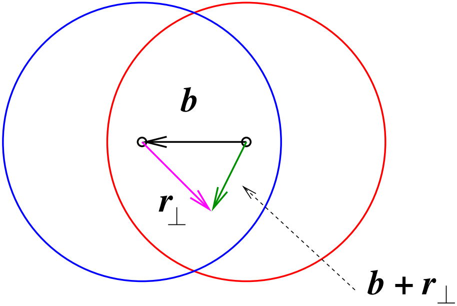




Differences in models:

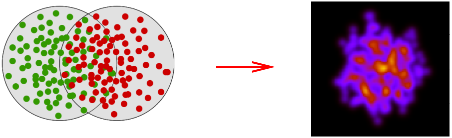
- Position of the energy deposite (collision sites)
- Energy deposit at each collision sites ($xN_{\mathrm{part}} + (1-x)N_{\mathrm{coll}}$)
- Size and spread of the initial lump

Collision Geometry



- **b**: Impact parameter. Vector between two centers in transverse space
- $\mathbf{r}_\perp$: Position vector from the center of the target nucleus
- $\mathbf{b} + \mathbf{r}_\perp$: Position vector from the center of the projectile nucleus

Initial Conditions



MC-Glauber

- Sample Wood-Saxon thickness function

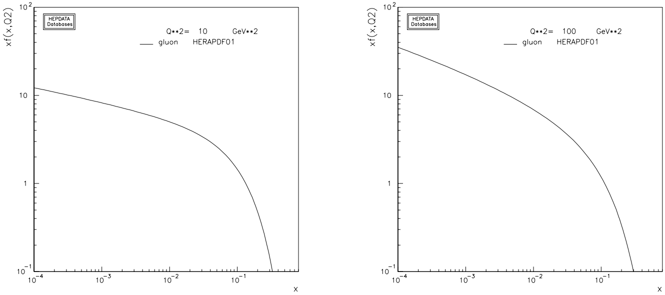
$$
\begin{aligned}
T_A(\mathbf{r}_\perp) &= \int dz\, \frac{\rho_0}{1 + e^{(R-r)/a}} \\
T_A(\mathbf{b} + \mathbf{r}_\perp) &= \int dz'\, \frac{\rho_0}{1 + e^{(R-r')/a}}
\end{aligned}
$$

  for nucleon positions

- *NN* collision occurs if two nucleons are within $D = \sqrt{\sigma_{NN}/\pi}$
- For each wounded nucleon, deposite $\epsilon_0 e^{-(\mathbf{x}-\mathbf{x}_C)^2/2\sigma_0^2}$

Detour: Saturation

[BFKL, JIMWLK, BK]

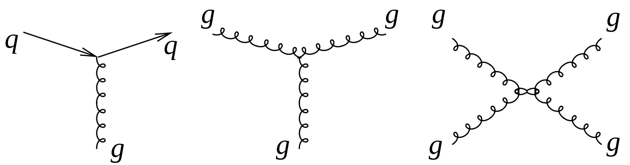


- Gluon distributions for protons for $Q^2 = 10\,\text{GeV}^2$ and $Q^2 = 100\,\text{GeV}^2$.
- Looks like growing indefinitely: Unphysical

Detour: Saturation

[BFKL, JIMWLK, BK]

QCD
– Interaction of quarks and gluons



- Leading order BFKL equation (evolution in *x*) takes into account splitting, but not recombination.
- When density is high, recombination must be taken into account $\Longrightarrow$ JIMWLK & BK
- Density is high: Classical field limit
- Recombination: Non-linear effect

Detour: Saturation

[BFKL, JIMWLK, BK]

- Saturation (or Recombination) scale
  - Transverse gluon density

$$\rho \sim \frac{xg_A(x, Q^2)}{S_\perp} \sim \frac{Axg(x, Q^2)}{A^{2/3}} \sim A^{1/3}xg(x, Q^2)$$

  - Recombination cross-section

$$\frac{\sigma_{gg\to g} \sim \alpha_s^2}{Q^2}$$

  - Saturation when

$$\rho\sigma_{\gg\to g} \sim 1$$

  - Saturation scale

$$Q_s^2 = \alpha_s(Q_s)A^{1/3}xg(x, Q_s^2)$$

Detour: Saturation

- Classical field equation of QCD

$$D_\mu G^{\mu\nu} = J^\nu$$

where

$$D_\mu = \partial_\mu - igA^a_\mu T_a$$

and

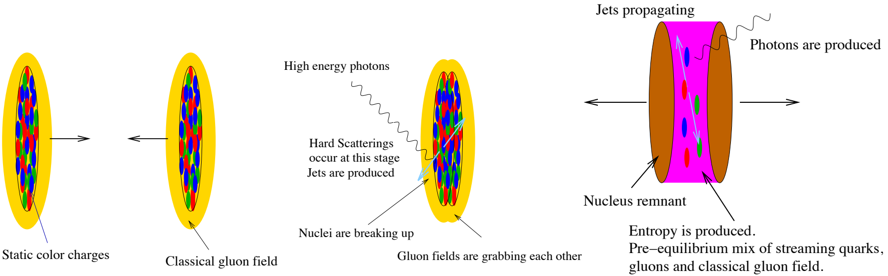
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf_{abc}A^b_\mu A^c_\nu$$

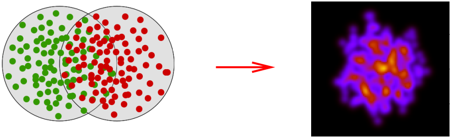
- $J^\mu_a = \rho_A \delta^{\mu+} + \rho_B \delta^{\mu-}$: Color source
- Gluon field

$$A^\mu = A^\mu_A + A^\mu_B + A^\mu_P \theta(\tau)$$

The produced field A_P after the collision is what we are after

Schematically





MC-KLN Drescher and Nara, PRC 75:034905

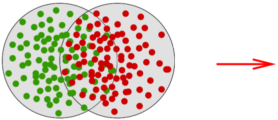
- Sample Wood-Saxon thickness function

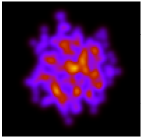
$$T_A(\mathbf{r}_\perp) = \int dz \frac{\rho_0}{1 + e^{(R-r)/a}}$$

$$T_A(\mathbf{b} + \mathbf{r}_\perp) = \int dz' \frac{\rho_0}{1 + e^{(R-r')/a}}$$

for nucleon positions

Initial Conditions





MC-KLN Drescher and Nara, PRC 75:034905

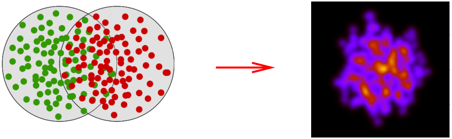
- Calculate thickness function again:

$$t_A(\mathbf{r}_\perp) = \frac{\text{\# of nucleons in the tube at } \mathbf{r}_\perp}{S}$$

$$t_A(\mathbf{b} + \mathbf{r}_\perp) = \frac{\text{\# of nucleons in the tube at } \mathbf{b}{+}\mathbf{r}_\perp}{S}$$

where $S = \sigma_{NN}$ is the cross-section of the tube

Initial Conditions



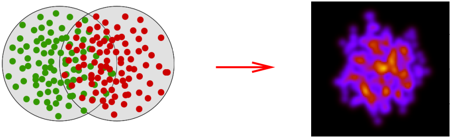
MC-KLN Drescher and Nara, PRC 75:034905

- Calculate the saturation scale

$$Q^2_{s,A}(x, \mathbf{r}_\perp) = 2\,\text{GeV}^2 \left(\frac{t_A(\mathbf{r}_\perp)}{1.53}\right)\left(\frac{0.01}{x}\right)^\lambda$$

- Calculate the unintegrated gluon density function

$$\phi(x, \mathbf{k}^2_\perp; \mathbf{r}_\perp) = \frac{1}{\alpha_s(Q^2_s)} \frac{Q^2_s}{\max(Q^2_s, \mathbf{k}^2_\perp)}$$



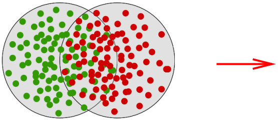
MC-KLN Drescher and Nara, PRC 75:034905

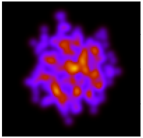
- Deposite energy with an approximate the $g_A g_B \to g_P$ process

$$\frac{dE_g}{d^2\mathbf{r}_\perp dy d^2 p_\perp} = \frac{4N_c}{N_c^2 - 1} \frac{1}{|\mathbf{p}_\perp|} \int d^2k_\perp \, \alpha_s \phi_A((\mathbf{p}_\perp + \mathbf{k}_\perp)^2/4) \phi_A((\mathbf{p}_\perp - \mathbf{k}_\perp)^2/4)$$

Initial Conditions

[Tribedy & Venugopalan, Nucl.Phys.A850 136; Tribedy & Venugopalan, PLB710 125; Schenke, Tribedy & Venugopalan, PRL108 252301; Gale, Jeon, Schenke, Tribedy & Venugopalan, PRL110 012302]





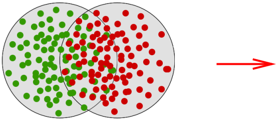
IP-Glasma

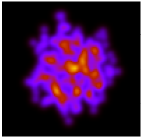
- Sample the position of the nucleons.
- Calcualte the saturation momentum for each nucleon using the IP-Sat model (Kowalski & Teaney, PRD68 114005)
- Calculate the color charge density by summing over all Q_s at the given global position

$$g\mu(x, \mathbf{b}) = c \sum_i Q_s(x, \mathbf{b}_i)$$

Initial Conditions

[Tribedy & Venugopalan, Nucl.Phys.A850 136; Tribedy & Venugopalan, PLB710 125; Schenke, Tribedy & Venugopalan, PRL108 252301; Gale, Jeon, Schenke, Tribedy & Venugopalan, PRL110 012302]





IP-Glasma

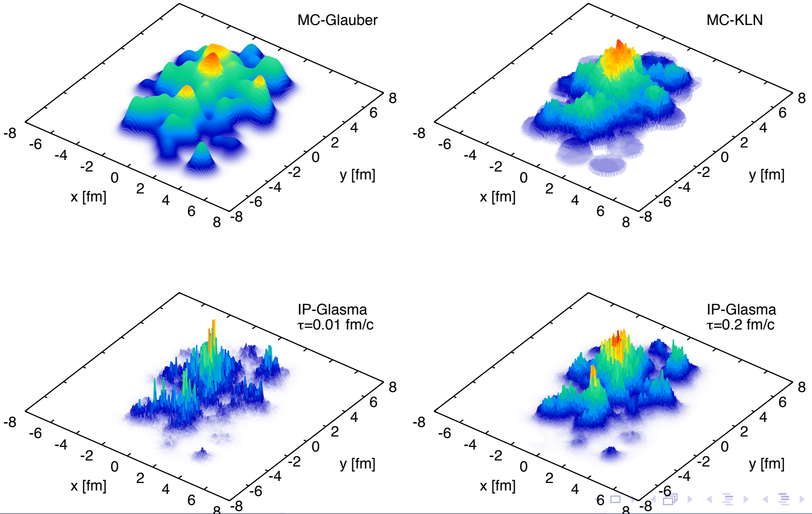
- Sample the color charge distribution of each nucleus using the Gaussian distribution

$$W_A[\rho] = \exp\left(-\rho_a\rho_b/(g^2\mu_A^2)\right)$$

- Solve the Classical Yang-Mills equation
- After evolving for τ_0 calculate $T^{\mu\nu}$
- Connect it to Hydro

Comparison

[Schenke, Tribedy & Venugopalan, PRL108 252301; Schenke, Gale & Jeon, arXiv:1301.5893v1]

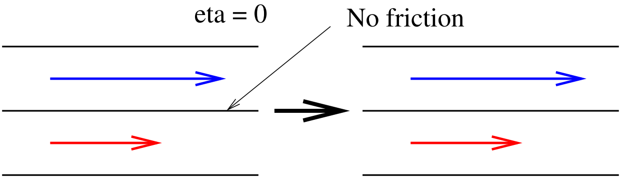


Initial conditions

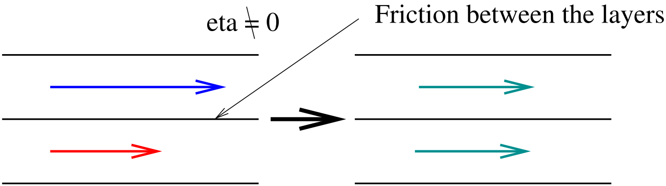
- Different size and distribution of ϵ_n
- Test: Need to get the $v_n(p_T)$ for various centralities
- Test: Need to get the e-by-e distribution of integrated v_n

Physics Issue 2: Viscosity

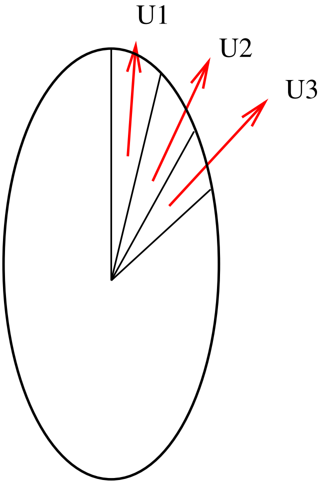
Effect of viscosity



The relative velocity of the two layers does not change.



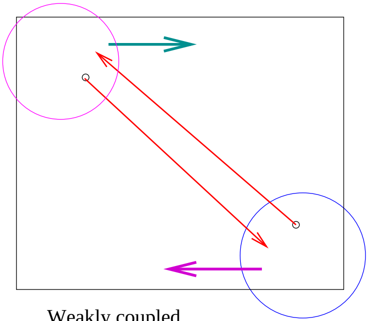
The velocities eventually become the same.



- $\eta = 0$ means $u_1 < u_2 < u_3$ is maintained for a long time
- $\eta \neq 0$ means that $u_1 \simeq u_2 \simeq u_3$ is achieved more quickly
- Shear viscosity smears out flow differences (it's a diffusion)
- Shear Viscosity reduces non-sphericity

Interaction Strength and Viscosity

Weak coupling allows rapid *momentum diffusion*

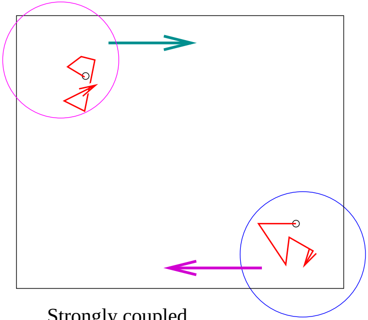


Weakly coupled
Long distance until next collision
Easy mixing

Large η/s: $u_\mu(x)$ changes due to pressure gradient and diffusion

Interaction Strength and Viscosity

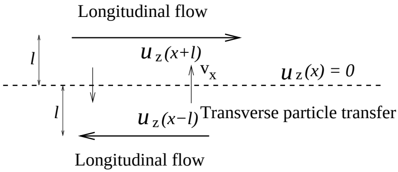
Strong coupling *does not* allow momentum diffusion



Strongly coupled
Very short distance until next collision
Mixing takes very long time

Small η/s: $u_\mu(x)$ changes due to pressure gradient only

Kinetic Theory estimate



u_z: Flow velocity
v_x: Average speed of microscopic particles

- Rough estimate (fluid rest frame, or $u_z(x) = 0$)
  - The momentum density: $T_{0z} = (\epsilon + \mathcal{P})u_0 u_z$ diffuses in the x direction with $v_x = u_x/u_0$. Net change:

$$\begin{aligned} &\langle \epsilon + \mathcal{P} \rangle |v_x| u_0 (u_z(x - l_{\rm mfp}) - u_z(x + l_{\rm mfp})) \\ &\approx -2 \langle \epsilon + \mathcal{P} \rangle |v_x|\, u_0\, l_{\rm mfp} \partial_x u_z(x) \\ &\sim -\eta u_0 \partial_x u_z \end{aligned}$$

  Here $l_{\rm mfp}$: Mean free path
  - Recall thermo. id.: $\langle \epsilon + \mathcal{P} \rangle = sT$

$$\eta \sim \langle \epsilon + \mathcal{P} \rangle\ l_{\rm mfp}\ \langle |v_x| \rangle \sim s\, T\, l_{\rm mfp}\ \langle |v_x| \rangle$$

Perturbative estimate

High Temperature limit: $\langle |v_x| \rangle = O(1)$

- $\eta/s \approx T l_{\rm mfp} \approx \dfrac{T}{n\sigma} \sim \dfrac{1}{T^2\sigma}$
- The only energy scale: T

$$\sigma \sim \frac{(\text{coupling constant})^{\#}}{T^2}$$

Hence

$$\frac{\eta}{s} \sim \frac{1}{(\text{coupling constant})^{\#}}$$

- Perturbative QCD partonic 2-2 cross-section

$$\frac{d\sigma_{\rm el}}{dt} = C\frac{2\pi\alpha_S^2}{t^2}\left(1 + \frac{u^2}{s^2}\right)$$

Perturbative estimate - Cont.

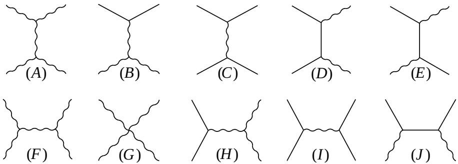
- Naively expect

$$\eta/s \sim \frac{1}{\alpha_s^2}$$

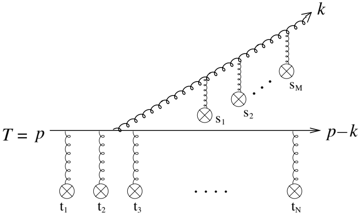
- Coulomb enhancement (cut-off by m_D) leads to

$$\eta/s \sim \frac{1}{\alpha_s^2 \ln(1/\alpha_s)}$$

Relevant processes



($\sim$ 80 %)



($\sim$ 20 %)

Use kinetic theory

$$\frac{df}{dt} = \mathcal{C}_{2\leftrightarrow 2} + \mathcal{C}_{1\leftrightarrow 2}$$

Complication: $1 \leftrightarrow 2$ process needs resummation (LPM effect, AMY)

QCD Estimates of η/s

- Danielewicz and Gyulassy [PRD **31**, 53 (1985)]:
  - η/s bound from the kinetic theory: Recall: $\eta \sim s\, T\, l_{\rm mfp}\, \langle |v_x| \rangle$ Use $l_{\rm mfp}\, \langle |v_x| \rangle \sim \Delta x \Delta p/m$ to get

$$\frac{\eta}{s} \gtrsim \frac{1}{12} \approx 0.08 \approx (1/4\pi)$$

  - QCD estimate in the small α_S limit with $N_f = 2$ and $2 \to 2$ only (min. at $\alpha_S = 0.6$):

$$\eta \approx \frac{T}{\sigma_\eta} \approx \frac{0.57\, T^3}{\alpha_S^2 \ln(1/\alpha_S)} \gtrsim 0.2s \approx (2.5/4\pi)s$$

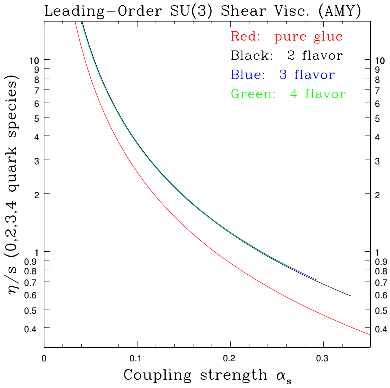
- Baym, Monien, Pethick and Ravenhall [PRL 64, 1867 (1990)]

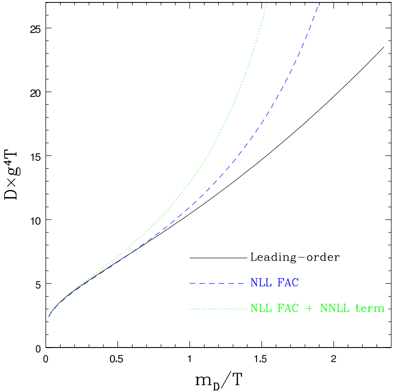
$$\eta \approx \frac{1.16\, T^3}{\alpha_S^2 \ln(1/\alpha_S)} \gtrsim 0.4s \approx (5/4\pi)s$$

- M. Thoma [PLB 269, 144 (1991)]

$$\eta \approx \frac{1.02\, T^3}{\alpha_S^2 \ln(1/\alpha_S)} \gtrsim 0.4s \approx (5/4\pi)s$$

- Arnold-Moore-Yaffe (JHEP **0305**, 051 (2003)) [Plots: Guy]:





Minimum $\eta/s \approx 0.6 \approx 7.5/4\pi$ for $\alpha_S \approx 0.3$

NB: Approximate formula $\eta/s \approx \dfrac{1}{15.4\alpha_S^2 \ln(0.46/\alpha_S)}$

is not good for $\alpha_S > \frac{1}{4\pi(1+N_f/6)}$

Shear viscosity in $\mathcal{N}=4$ SYM

Son, Starinets, Policastro, Kovtun, Buchel, Liu, ...

- Strong coupling limit, 4 ingredients
  - Kubo formula

$$\eta = \lim_{\omega\to 0} \frac{1}{2\omega} \int dt\, d^3x\, e^{i\omega t} \, \langle [T_{xy}(x), T_{xy}(0)] \rangle$$

  - Gauge-Gravity duality

$$\sigma_{\rm abs}(\omega) = \frac{8\pi G}{\omega} \int dt\, d^3x\, e^{i\omega t} \, \langle [T_{xy}(x), T_{xy}(0)] \rangle$$

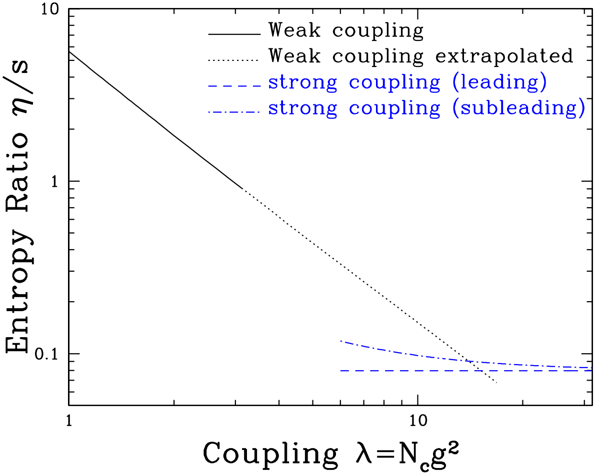
  - $\lim_{\omega\to 0} \sigma_{\rm abs}(\omega) = A_{\rm blackhole}$
  - Entropy of the BH : $s = A_{\rm blackhole}/4G$

Therefore, (including the first order correction)

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 + \frac{7.12}{(g^2 N_c)^{3/2}}\right)$$

Correction is small if $g \gg 1$ (10 % at $g = 2.4$).

$N = 4$ SYM



- Perturbative calculation and the strong coupling calculation behave very differently

S. Carno-Huot, S. Jeon and G. D. Moore, Phys. Rev. Lett. 98, 172303 (2007)

Experimental Evidence for $\eta/s \sim 1/4\pi$

- Theoretical situation:
  - Perturbative calculations: $\eta/s \geq 7.5/(4\pi)$
  - AdS/CFT in the infinite coupling limit: $\eta/s = 1/(4\pi)$
  - Roughly an order of magnitude difference $\Longrightarrow$ Testable!
- A relativistic heavy ion collision produces a complicated system $\Longrightarrow$ Need a hydrodynamics simulation suite
- We use MUSIC (3+1D e-by-e viscous hydrodynamics)
- Viscosity measurement is through the flow coefficients

$$\frac{dN}{dy d^2 p_T} = \frac{dN}{2\pi dy p_T dp_T}\left(1 + 2\sum_{n=1}^{\infty} v_n \cos(n(\phi - \psi_n))\right)$$

- v_n is a translation of the eccentricities ϵ_n via pressure gradient

MUSIC

MUScl for Ion Collisions

MUSCL: Monotone Upstream-centered Schemes for Conservation Laws

Current MUSIC (and MARTINI) Team

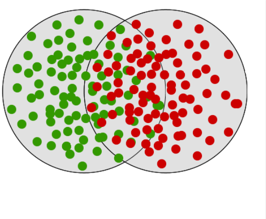
- Charles Gale (McGill)
- Sangyong Jeon (McGill)
- *Björn Schenke* (Formerly McGill, now BNL)
- *Clint Young* (Formerly McGill, now UMN)
- *Gabriel Denicol* (McGill)
- *Matt Luzum* (McGill/LBL)
- Sangwook Ryu (McGill)
- Gojko Vujanovic (McGill)
- Jean-Francois Paquet (McGill)
- Michael Richard (McGill)
- Igor Kozlov (McGill)

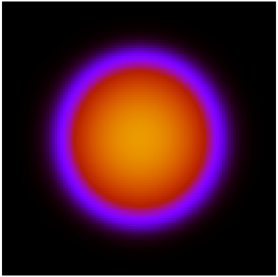
3+1D Event-by-Event Viscous Hydrodynamics

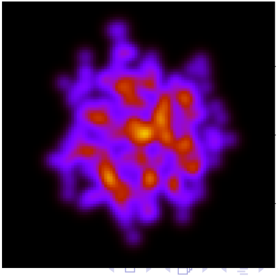
- 3+1D parallel implementation of new *Kurganov-Tadmor Scheme* in (τ, η) with an additional baryon current
  (No need for a Riemann Solver. Semi-discrete method.)
- Ideal *and* Viscous Hydro
- Event-by-Event fluctuating initial condition
- Sophisticated Freeze-out surface construction
- Full resonance decay (3+1D version of Kolb and Heinz)
- Many different equation of states including the newest from Huovinen and Petreczky
- *New Development*: Glasma Initial Conditions & UrQMD after-burner

Fluctuating Initial Condition

Each event is *not* symmetric: Fluctuating initial condition $\Longrightarrow$ All v_n are non-zero.

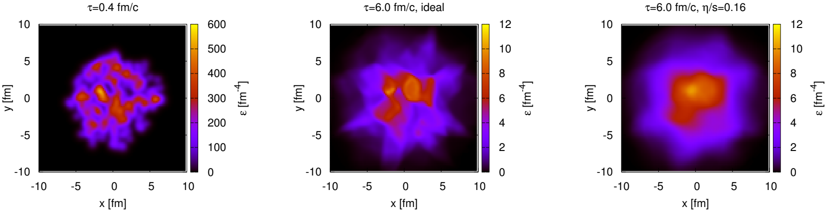






Ideal vs. Viscous

[Movies by B. Schenke]



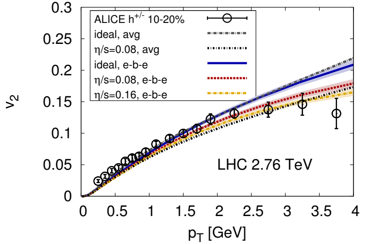
Ideal vs. Viscous

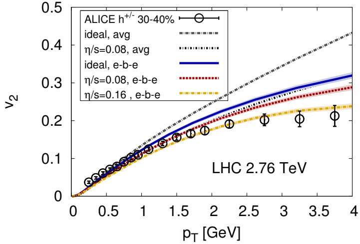
Fluctuations and Viscosity

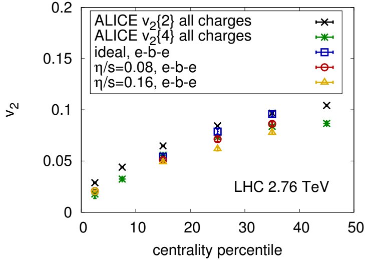
- Magnitude of higher harmonics, $v_3, v_4, \cdots$, (almost) independent of centrality – Local fluctuations dominate
- Higher harmonics are easier to destroy that v_2 which is a *global* distortion – Viscosity effect.
- To get a good handle on flow: Both fluctuations and viscosity are essential

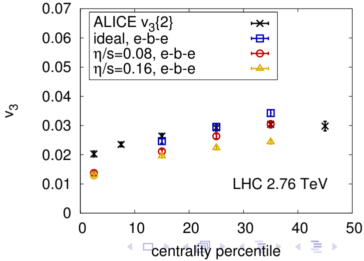
E-by-E MUSIC vs LHC Data

[Schenke, Jeon and Gale, Phys. Rev. C 85, 024901 (2012)]
Best value $\eta/s = 0.16 = 2/(4\pi)$.

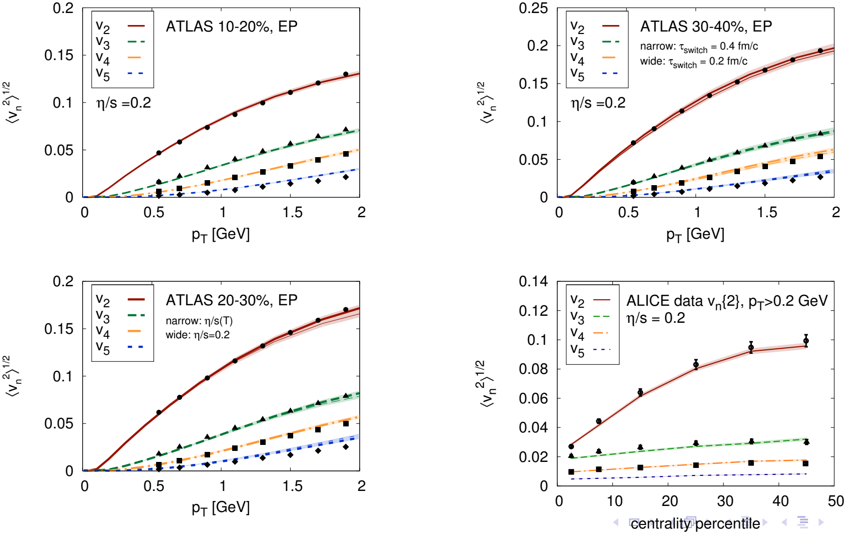




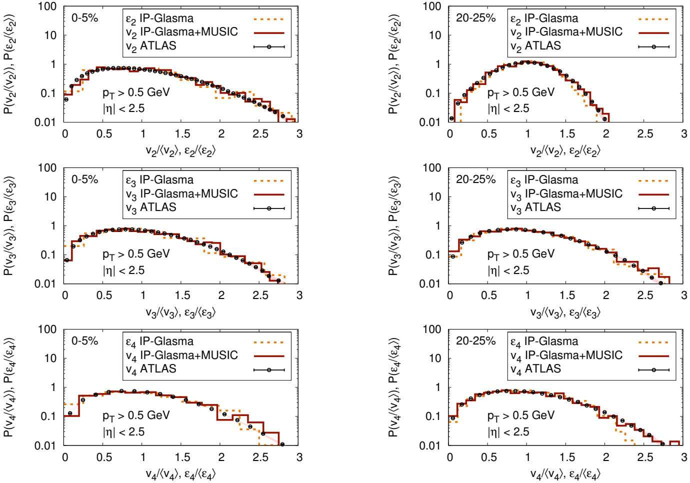




[Gale, Jeon, Schenke, Tribedy and Venugopalan, arXiv:1209.6330]
Best value $\eta/s = 0.2 = 2.5/(4\pi)$.

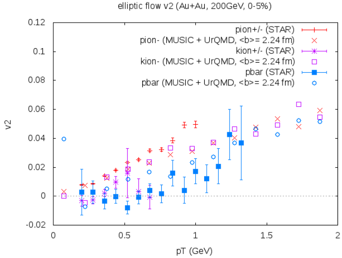


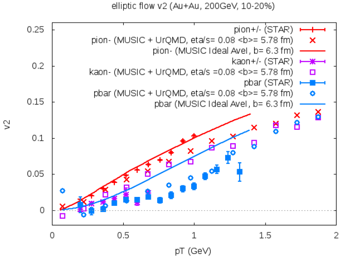
[Gale, Jeon, Schenke, Tribedy and Venugopalan, arXiv:1209.6330]
E-by-E distributions

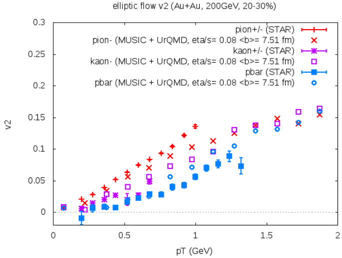


New Development: UrQMD Afterburner

v_2 at RHIC (Midrapidity). In each centrality class: 100 UrQMD times 100 MUSIC events. [Ryu, Jeon, Gale, Schenke and Young, arXiv:1210.4558]



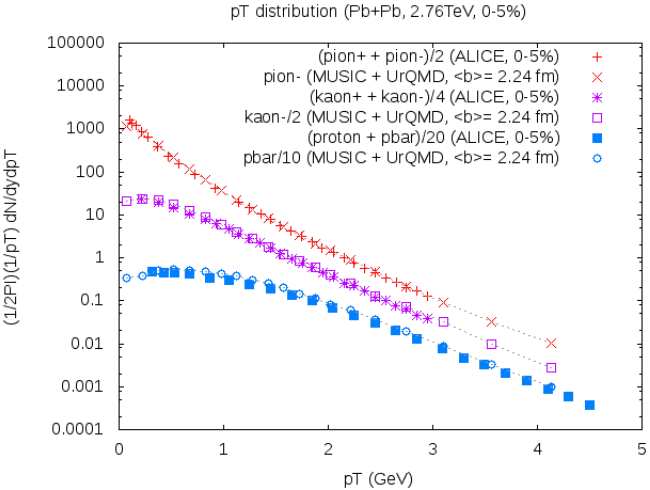




- $\eta/s = 1/4\pi$
- Using previous MUSIC parameters that were tuned to reproduce PHENIX v_n

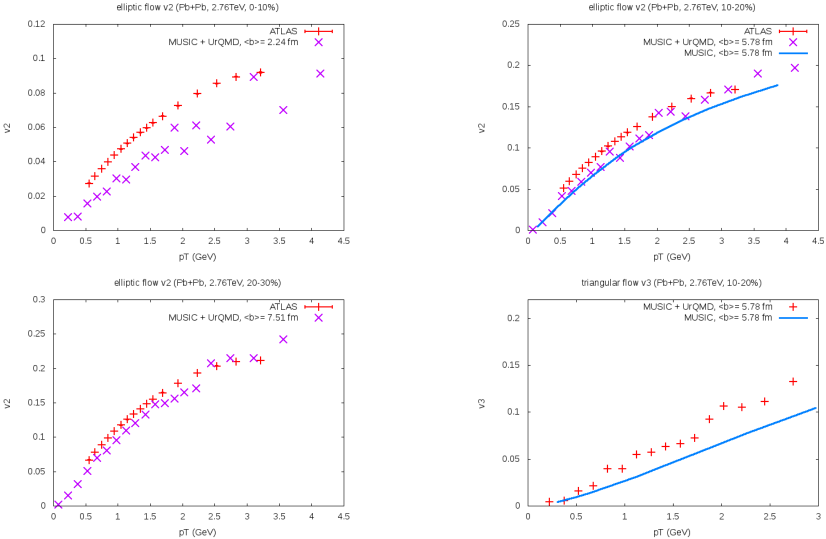
LHC Spectra

In each centrality class: 100 MUSIC times 10 UrQMD events.
$\eta/s = 2/(4\pi)$. ALICE data from QM12.



LHC Flows

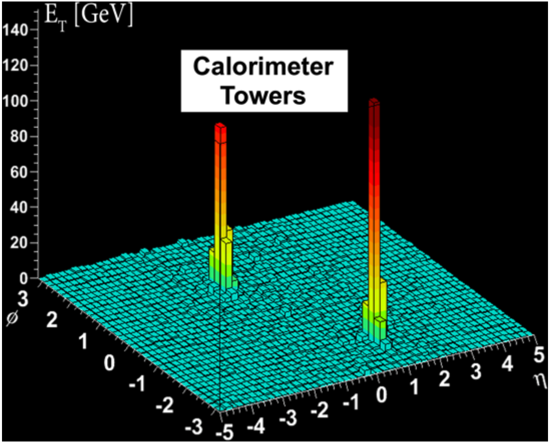
In each centrality class: 100 MUSIC times 10 UrQMD events

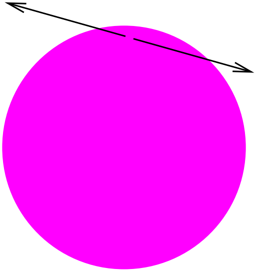


Conclusions and questions for η/s

- Strong flows: Strongest evidence that η/s has to be small
- η/s much larger than 0.2 cannot be accommodated within current understanding of the system.
- Perturbative result of $\eta/s = 0.4 - 0.6$ is out.
- Using the LQCD EoS.
- LQCD estimate $(\eta + 3\zeta/4)/s \approx 0.20 - 0.26$ between $1.58T_c - 2.32T_c$.
  [H. Meyer, Eur.Phys.J.A47:86,2011]
- Does this mean very large coupling?

- Fact: Jets lose energy (ATLAS images).





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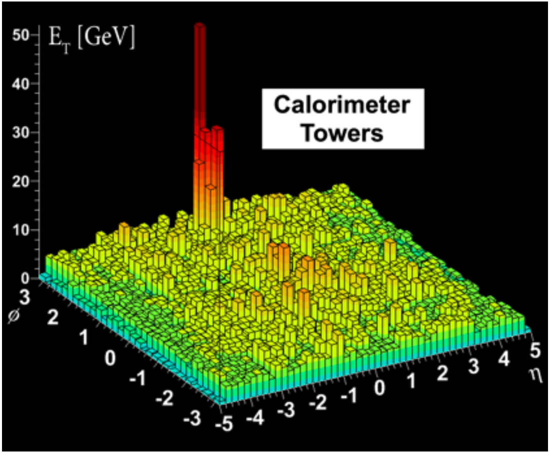
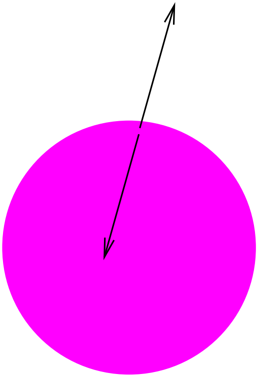




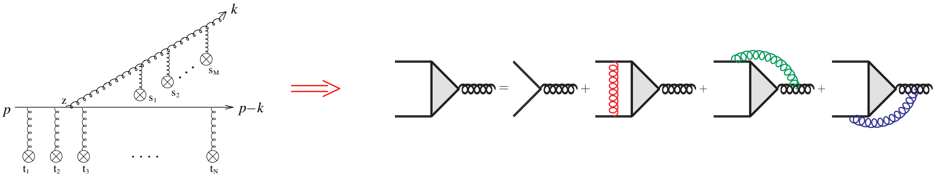
Energy Loss Mechanism

- Collisional energy loss rate [Wicks, Horowitz, Djordjevic and Gyulassy, NPA 784, 426 (2007), Qin, Gale, Moore, Jeon and Ruppert, Eur. Phys. J. C 61, 819 (2009)]:

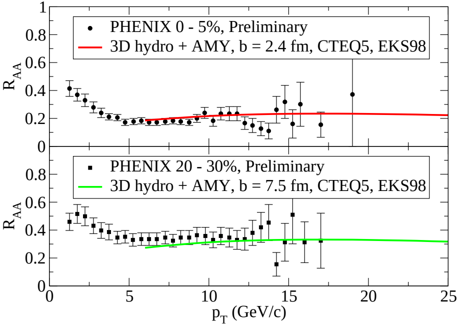
$$\frac{dE}{dx} \approx C_1 \pi \alpha_S^2 T^2 \left[\log\left(\frac{E_p}{\alpha_S T} \right) + C_2 \right]$$

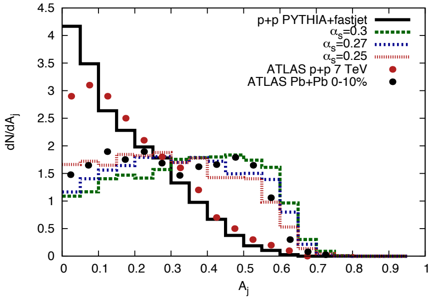
  $C_{1,2}$: Depends on the process. $O(1)$.

- Radiational $\propto \alpha_S^2$ (Arnold, Moore, Yaffe, JHEP 0206, 030 (2002))



- What α_S do we need for these?





$$R_{AA} = \frac{(dN_{AA}/dp_T)}{(N_{\rm coll}dN_{pp}/dp_T)}$$

$$A_j = (E_> - E_<)/(E_> + E_<)$$

Tool

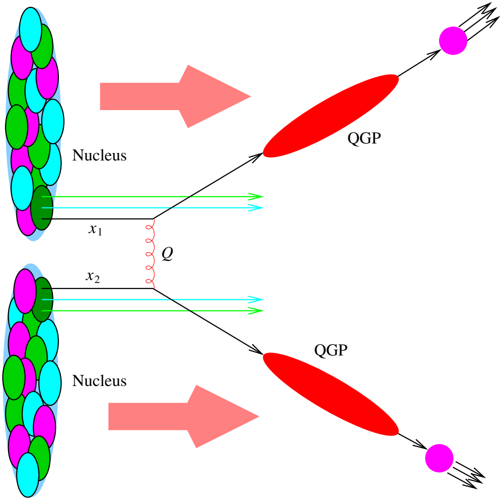
- Event generator
  – Jet propagation through evolving QGP medium.
- Several on the market. We use MARTINI.

MARTINI

MARTINI

- Modular Alogorithm for Relativistic Treatment of Heavy IoN Interactions
- Hybrid approach
  - Calculate Hydrodynamic evolution of the soft mode (MUSIC)
  - Propagate jets in the evolving medium according to McGill-AMY

Heavy Ion Collisions

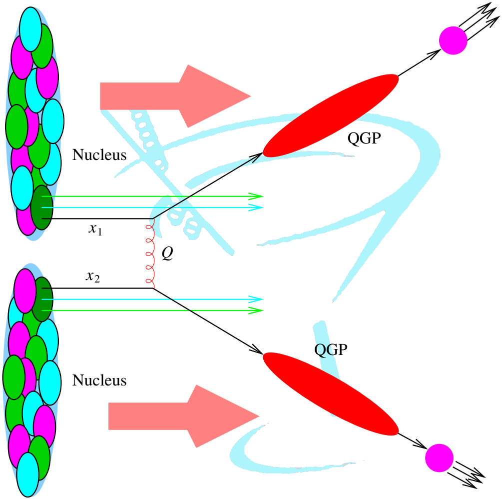


HIC Jet production scheme:

$$\frac{d\sigma_{AB}}{dt} = \int_{\text{geometry}} \int_{abcdc'} \times f_{a/A}(x_a, Q_f) f_{b/B}(x_b, Q_f) \times \frac{d\sigma_{ab\to cd}}{dt} \times \mathcal{P}(x_c \to x'_c | T, u^{\mu}) \times D(z'_c, Q)$$

$\mathcal{P}(x_c \to x'_c | T, u^{\mu})$: Medium modification of high energy parton property

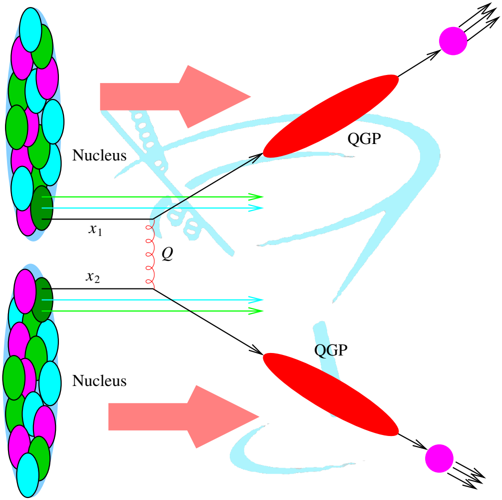
MARTINI - Basic Idea



$$\frac{d\sigma_{AB}}{dt} = \int_{\text{geometry}} \int_{abcdc'}$$
$$\times f_{a/A}(x_a, Q_f) f_{b/B}(x_b, Q_f)$$
$$\times \frac{d\sigma_{ab\to cd}}{dt}$$
$$\times \mathcal{P}(x_c \to x'_c | T, u^\mu)$$
$$\times D(z'_c, Q)$$

- Sample collision geometry using Wood-Saxon

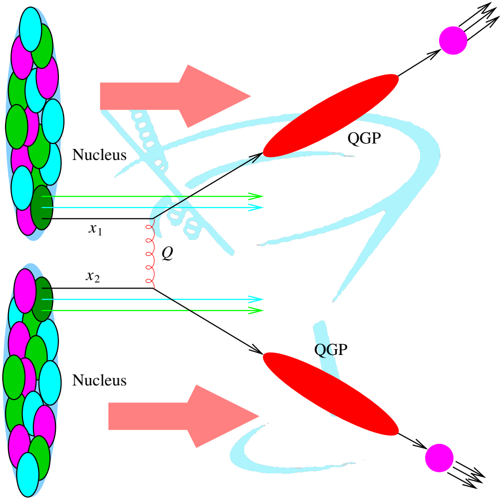
MARTINI - Basic Idea



$$\frac{d\sigma_{AB}}{dt} = \int_{\text{geometry}} \int_{abcdc'}$$
$$\times f_{a/A}(x_a, Q_f) f_{b/B}(x_b, Q_f)$$
$$\times \frac{d\sigma_{ab\to cd}}{dt}$$
$$\times \mathcal{P}(x_c \to x'_c | T, u^\mu)$$
$$\times D(z'_c, Q)$$

- PYTHIA 8.1 generates high p_T partons
- Shadowing included
- Shower (Radiation) stops at $Q = \sqrt{p_T/\tau_0}$

MARTINI - Basic Idea



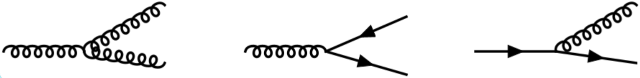
$$\frac{d\sigma_{AB}}{dt} = \int_{\text{geometry}} \int_{abcdc'} \times f_{a/A}(x_a, Q_f) f_{b/B}(x_b, Q_f) \times \frac{d\sigma_{ab\to cd}}{dt} \times \mathcal{P}(x_c \to x'_c | T, u^\mu) \times D(z'_c, Q)$$

- Hydrodynamic phase (MUSIC)
- AMY evolution – MC simulation of the rate equ's.

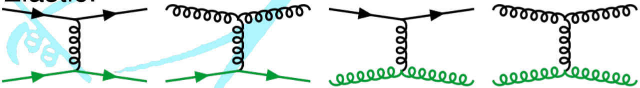
Parton propagation

Process include in MARTINI (all of them can be switched on & off):

- Inelastic:



- Elastic:

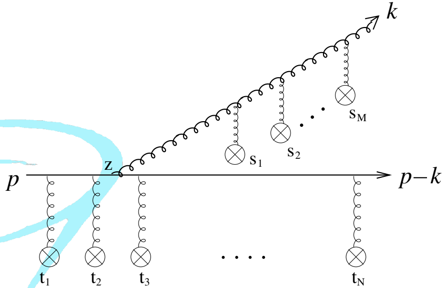


- Conversion:

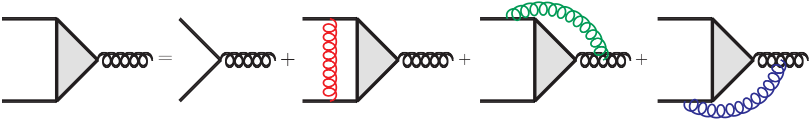
- Photon: emission & conversion

Parton propagation

Resummation for the inelastic processes included:

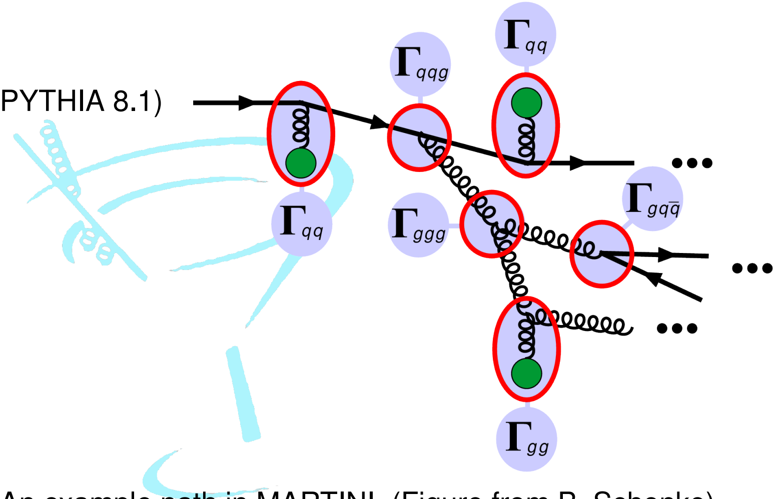


- All such graphs are leading order (BDMPS)
- Full leading order SD-Eq (AMY): (Figure from G. Qin)



Parton propagation

(PYTHIA 8.1)



An example path in MARTINI. (Figure from B. Schenke)

Ideal MUSIC

- While this is happening in the background ...

MARTINI – The Movie

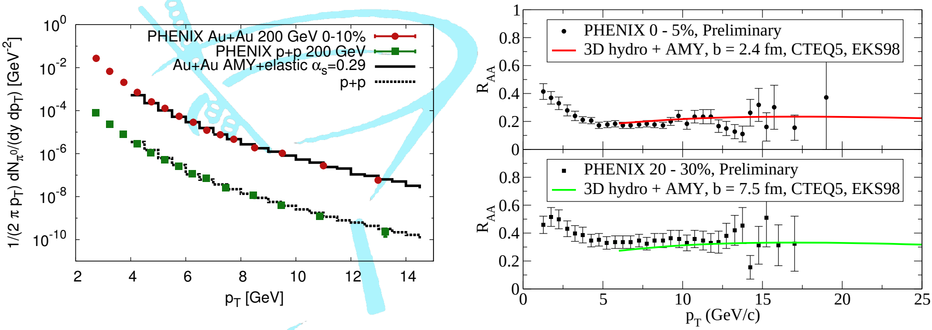
Projection on to the longitudinal plane

MARTINI – The Movie

Projection onto the transverse plane

[Schenke, Jeon and Gale, Phys. Rev. C 80, 054913 (2009)]

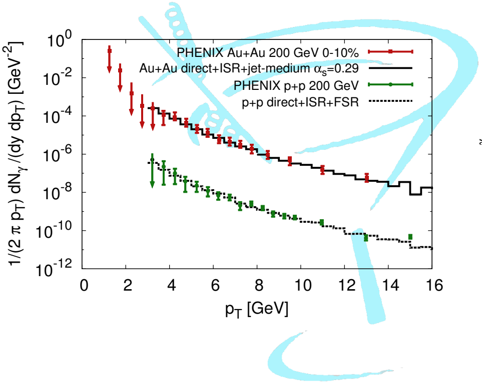
- π^0 spectra and R_{AA}

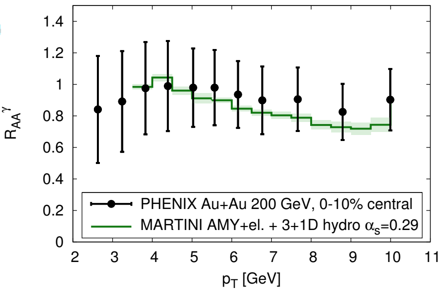


- For RHIC, $\alpha_S = 0.29$

[Schenke, Jeon and Gale, Phys. Rev. C 80, 054913 (2009)]

- Spectra and R_{AA}^{γ}

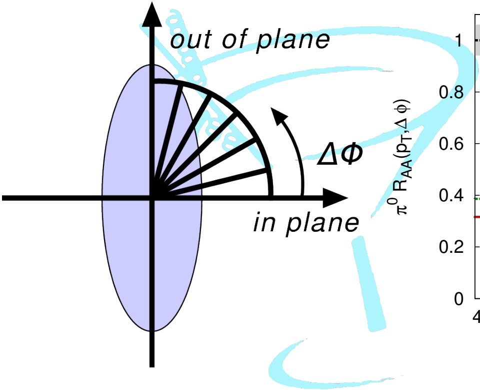


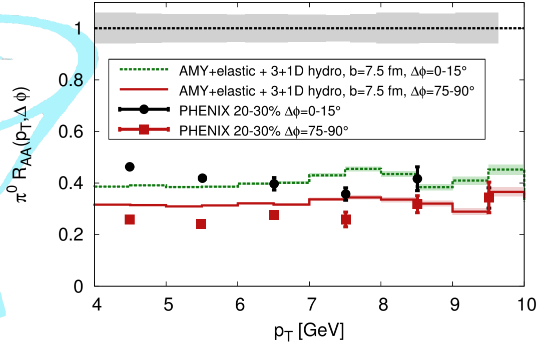


- $\alpha_S = 0.29$

Azimuthal dependence of R_{AA}

- $R_{AA}(p_T, \Delta\phi)$



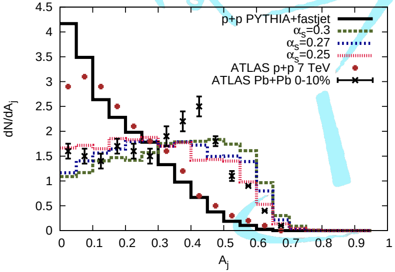


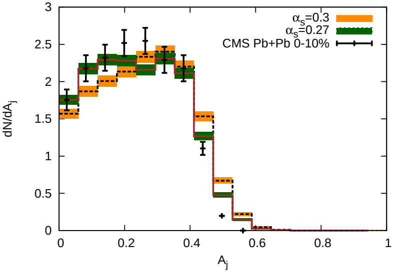
- $\alpha_S = 0.29$

MARTINI – LHC dN/dA

[Young, Schenke, Jeon, Gale, Phys. Rev. C 84, 024907 (2011)].

- $A = (E_t - E_a)/(E_t + E_a)$
- This is with ideal hydro with a smooth initial condition
- Full jet reconstruction with FASTJET
- $\alpha_S = 0.27$ seems to work.





ATLAS, PRL 105 (2010) 252303

CMS, arXiv: 1102.1957 (2011)

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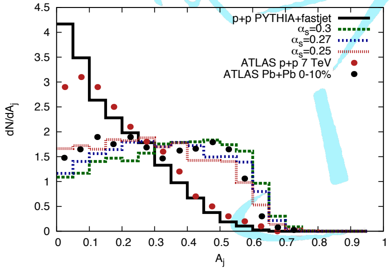
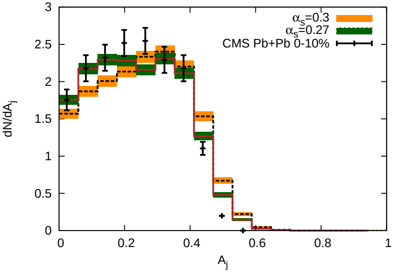


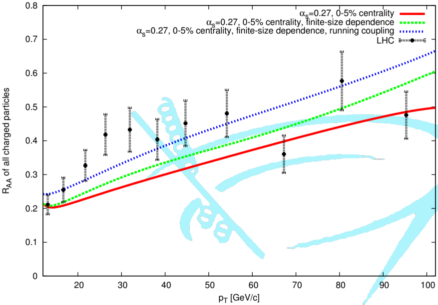


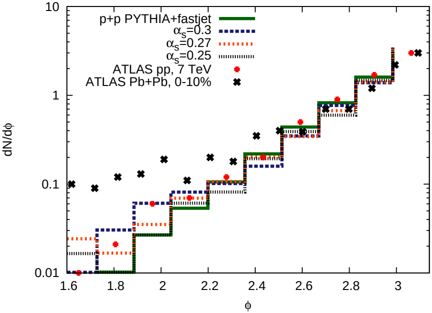
ATLAS, QM 2011

CMS, arXiv: 1102.1957 (2011)

Not the full story

[Clint Young's HP2012 Proceedings]





- R_{AA} – For LHC, constant α_S suppresses jets too much.
- Need to incorporate finite length effect (Caron-Huot-Gale) and running α_s. This is with maximum $\alpha_S = 0.27$.
- Don't quite get azimuthal dependence yet. $\Delta\phi$ broadening may be due to the background fluctuations $\Longrightarrow$ Need to combine UrQMD background?

Few last words

- So many nuclear experiments are being done/planned. – RHIC, LHC, Raon, FRIB, FAIR, JPARC, Dubna, HIRFL-CSR ...
- There never have been a time in history when so much information is so readily available.
- This is a great time to be/become a nuclear physicist.
- Work hard. Think hard. Dream big.

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My email: jeon@physics.mcgill.ca