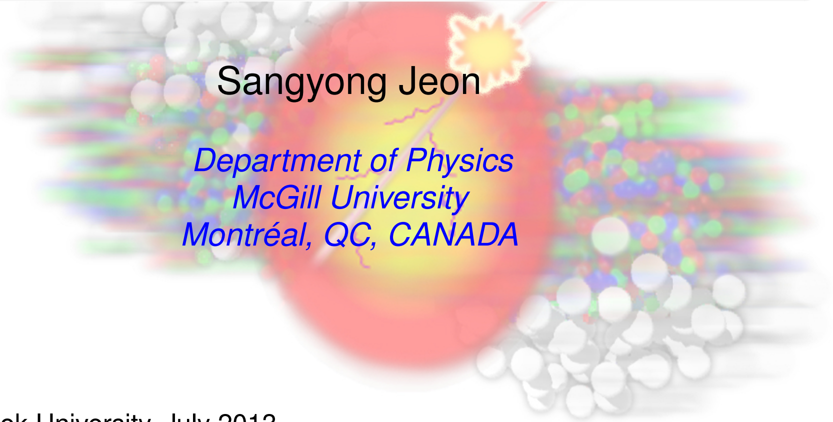
Introduction to Hard Probes in Heavy Ion Collisions



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NNSPP
Stonybrook University, July 2013

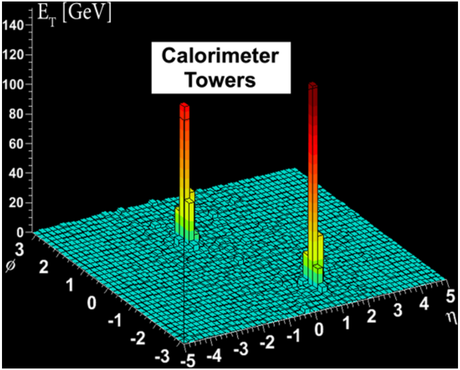
Day 2

Jet Quenching

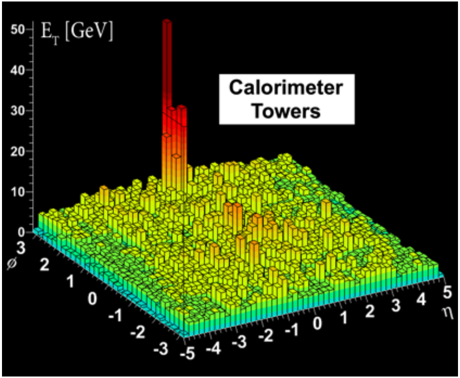
What do we want to learn?

- Medium properties
  - What is it made of? QGP or HG?
  - Thermodynamic properties – Temperature, Equation of state, etc.
  - Transport properties – Mean-free-path, transport coefficients, etc.
- Tools – Change in jet properties
  - Jet Quenching
  - Jet Broadening

Away side jet disappears! – Proof of principle

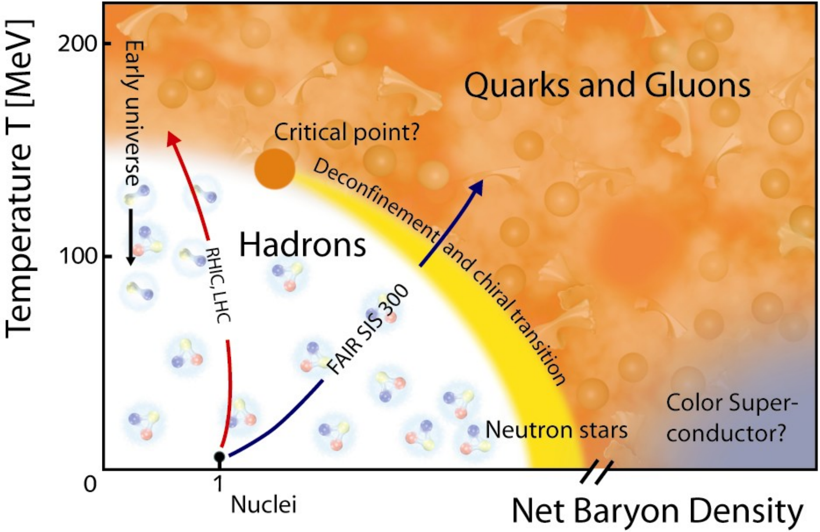


ATLAS: Intact dijets in Pb+Pb



ATLAS: One jet is fully quenched in Pb+Pb

QCD Phase Diagram



Picture credit: GSI (www.gsi.de)

Nobelprize.org

The Nobel Prize in Physics 2004

"for the discovery of asymptotic freedom in the theory of the strong interaction"

David J. Gross

1/3 of the prize

USA

University of California, Kavli Institute for Theoretical Physics
Santa Barbara, CA, USA

b. 1941

H. David Politzer

1/3 of the prize

USA

California Institute of Technology (Caltech)
Pasadena, CA, USA

b. 1949

Frank Wilczek

1/3 of the prize

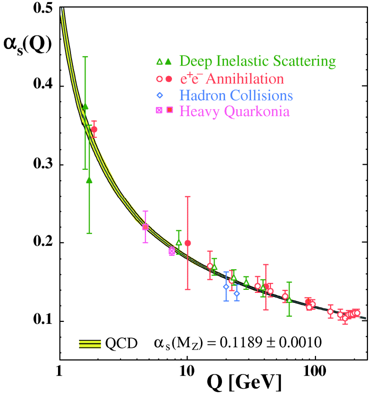
USA

Massachusetts Institute of Technology (MIT)
Cambridge, MA, USA

b. 1951

Titles, data and places given above refer to the time of the award.

QCD is asymptotically free.



Bethke, hep-ex/0606035

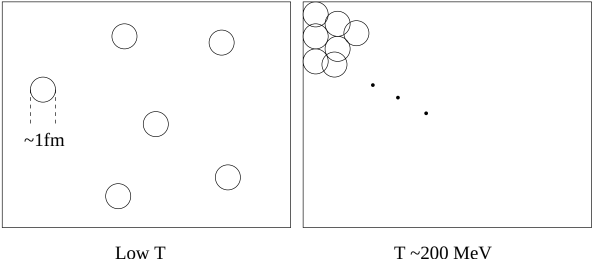
At high T

- Running coupling

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2N_f)\ln(Q^2/\Lambda^2_{\rm QCD})}$$

  - When $Q \sim \Lambda_{\rm QCD} \sim 200\,\text{MeV}$, the above expression blows up: Not physical. Indicates breakdown of perturbation theory. Hadrons.
  - Perturbative QCD is a theory of quarks and gluons *not* hadrons.
  - At high T, $Q \sim T$.
  - Possible phase transition around $T \sim \Lambda_{\rm QCD}$?
  - If $Q \sim T \to \infty$, $\alpha_s \to 0$: Weakly coupled
  - At $Q \sim$ few GeV, $\alpha_s \sim 0.2 - 0.4$

Another estimate of $T_{\text{transition}}$



- Density: Consider a pion gas.

$$n = 3 \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{E_p/T} - 1} = 0.37\, T^3$$

As T becomes larger, more and more pair creation results.

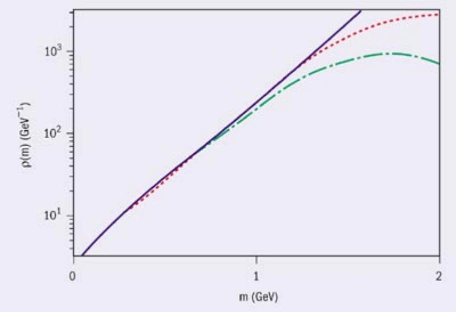
- Inter particle distance:

$$l_{\text{inter}} = n^{1/3} = 1.4/T$$

At $T = 200\,\text{MeV}$, $l_{\text{inter}} \approx 1.4\,\text{fm} \approx d_\pi$

Hagedorn Temperature

Hadronic density of states $\rho(m) \sim e^{m/T_H}$:



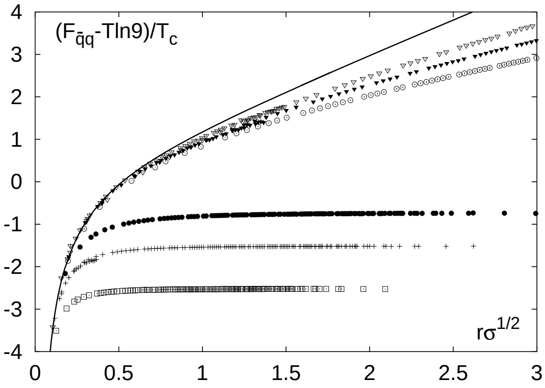
The smoothed mass spectrum of hadronic states as a function of mass. Experimental data: long-dashed green line with the 1411 states known in 1967; short-dashed red line with the 4627 states of 1996. The solid blue line represents the exponential fit yielding T_H=158 MeV. *CERN Courier, Sept, 2003*

- $\sum_m \int_p \rho(m) e^{-E_p/T}$: Not well defined when $T > T_H$ for hadronic matter.
- Phase transition around T_H: Hagedorn temperature $\approx 160\,\mathrm{MeV}$

Story so far

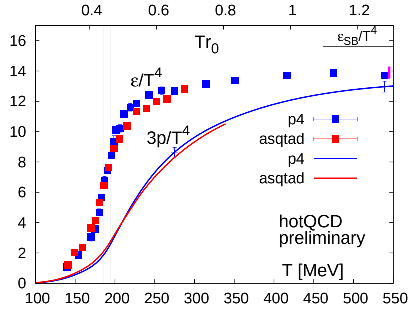
- Perturbative calculation possible much above $Q = \Lambda_{\rm QCD}$
- $Q \sim T$ at high T
- If T is much above the binding energy of hadrons
  $\Longrightarrow$ Deconfinement
- At high enough T, the system is a plasma of weakly interacting quarks and gluons
- All the above arguments are plausible but not a proof

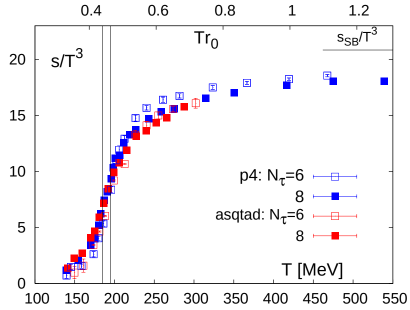
Lattice QCD Evidence



- F. Karsch, hep-lat/0403016. The color averaged heavy quark free energy at temperatures $T/T_c = 0.9,\ 0.94,\ 0.98,\ 1.05,\ 1.2,\ 1.5$ (from top to bottom) obtained in quenched QCD.

Lattice QCD Evidence of QGP

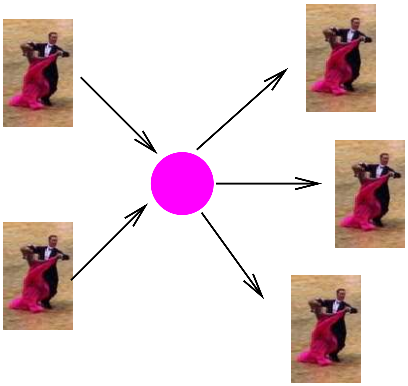




- From HotQCD Collaboration (C. DeTar, arXiv:0811.2429)
- "Cross-over" between 185 - 195 MeV

Ordinary low T matter has paired up quarks

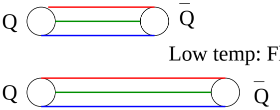
Collision can create more pairs



When $T \gtrsim Mc^2$ (Recall: $E = Mc^2$)
Relativistic equilibrium: Both
$n = \int \frac{d^3p}{(2\pi)^3}\, e^{-E_p/T}$ and $\varepsilon = \int \frac{d^3p}{(2\pi)^3}\, e^{-E_p/T}\, E_p$ are fixed by T.

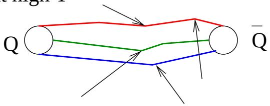
Pump up the volume (I mean, energy)!

What it means

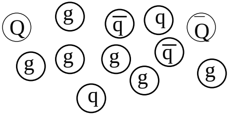


Low temp: Flux tube extends: F = const

At high T



Kicks from thermal particles



Can't maintain the flux tube

http://www.physics.adelaide.edu.au/

∼dleinweb/VisualQCD/Nobel/

Expected properties

- High number density

$$n \approx (24+16)\int \frac{d^3p}{(2\pi)^3}\, e^{-p/T} \approx 4\,T^3$$
$$= 4\left(\frac{T}{200\,\mathrm{MeV}}\right)^3 \mathrm{fm}^{-3}$$

- High energy density

$$\varepsilon \approx (24+16)\int \frac{d^3p}{(2\pi)^3}\, p\, e^{-p/T} \approx 12\,T^4$$
$$= 2.4\left(\frac{T}{200\,\mathrm{MeV}}\right)^4 \mathrm{GeV/fm}^3$$

Simple Estimates

With $\hbar = c = 1$

- 1 mole of hydrogen atom: 6.02×10^{23} atoms $= 1$ g (Avogadro's number)
- 1 hydrogen atom $m_p \approx (1/6) \times 10^{-23}\,\text{g} = (1/6) \times 10^{-26}\,\text{kg}$
- $m_p = 940\,\text{MeV} \approx 1\,\text{GeV}$
- $E = mc^2$: $1\,\text{GeV} \approx (1/6) \times 10^{-26}\,\text{kg}$

$$\begin{aligned} 2.4\,\text{GeV}/\text{fm}^3 &= 0.4 \times 10^{-26}\,\text{kg}/(10^{-13}\,\text{cm})^3 \\ &= 0.4 \times 10^{-26+39}\,\text{kg/cm}^3 \\ &= 4 \times 10^{12}\,\text{kg/cm}^3 \end{aligned}$$

- Typical human: $\sim 100\,\text{kg}$

$$2.4\,\text{GeV}/\text{fm}^3 \sim 4 \times 10^{10}\,\text{human/cm}^3$$

Simple Estimates

With $\hbar = c = 1$

- Another way of looking at the energy density

$$2.4\,\mathrm{GeV}/\mathrm{fm}^3 = 4 \times 10^{12}\,\mathrm{kg}/\mathrm{cm}^3$$

- Restoring $c = 3 \times 10^8$ m/s,

$$2.4\,\mathrm{GeV}/\mathrm{fm}^3 = 4 \times 10^{12} \times (9 \times 10^{16})\,\mathrm{J/cm}^3 = 3.6 \times 10^{29}\,\mathrm{J/cm}^3$$

- World energy consumption (2008):

$$144\,\mathrm{pWh} = 144 \times 10^{15} \times 3.6 \times 10^3\,\mathrm{J} = 5.2 \times 10^{20}\,\mathrm{J}$$

- A cubic centimeter of QGP can power the world for about 70 million years.

Simple Estimates

With $\hbar = c = 1$

- Pressure $P \approx \epsilon/3$

$$P = 0.8\,\text{GeV}/\text{fm}^3 \approx 1.3 \times 10^{12}\,\text{kg}/\text{cm}^3 = 1.3 \times 10^{18}\,\text{kg}/\text{m}^3$$

- SI Unit for pressure: $\text{Pa} = \text{N/m}^2 = \text{kg/m/s}^2$
- Restoring $c = 3 \times 10^8\,\text{m/s}$,

$$P \approx 1.3 \times 10^{18} \times (9 \times 10^{16})\,\text{kg}/\text{m/s}^2 \approx 10^{35}\,\text{Pa} \approx 10^{30}\,\text{atm}$$

How do you achieve high temperature?

- Temperature = energy (1 eV $\approx 12,000$K)
- More usefully, the energy density:

$$\varepsilon = g \int \frac{d^3p}{(2\pi)^3} E_p \, e^{-E_p/T} \approx \frac{3g}{\pi^2} T^4$$

- To get high temperature: Get high energy density $\Longrightarrow$ Cram maximum possible energy into the smallest possible volume while randomizing the momenta $\Longrightarrow$ Relativistic heavy ion collisions.
- What to expect: $dN/d\eta$ and $dE/d\eta$ grow something like $(\ln s)^n$ with $n \sim 1$ $\Longrightarrow$ T should behave something like $(\ln s)^n$ with $n \sim 1$

Observable Consequence

- High temperature $\Longrightarrow$ Thermal photons
- High density $\Longrightarrow$ *Jet quenching*
- High pressure $\Longrightarrow$ Hydrodynamic flow
  - The size of the eliptic flow depends on the shear viscosity η.
  - If weakly coupled, $\eta/s \gg 1$: $\approx$ Ideal gas
  - If stronlgy coupled, $\eta/s \ll 1$: $\approx$ Perfect (Ideal) fluid.
- Neutrality $\Longrightarrow$ Tight unlike-sign correlation
- Critical point $\Longrightarrow$ Large momentum fluctuations

Back in 1990...

In fig. 4 EMC- and SLAC-data on the ratio of integrated particle yields

$$\overline{\mathscr{R}}_A \equiv \int_{x_{\min}}^{1} dx \frac{1}{\sigma_{eA}} \frac{d\sigma_{eA}}{dx} \Big/ \int_{x_{\min}}^{1} dx \frac{1}{\sigma_{eN}} \frac{d\sigma_{eN}}{dx} \quad (7)$$

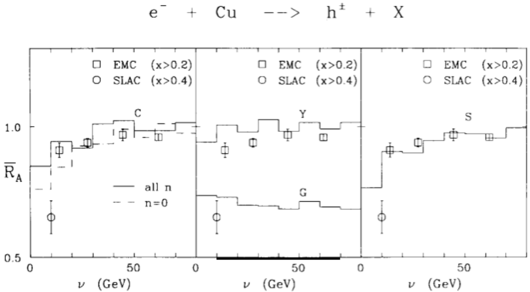


Fig. 4. The ν-dependence of the ratio $\mathscr{R}_A$ of hadrons produced in the forward region. The histograms labeled Y, C, G and S correspond to the yoyo formation model, the constituent formation model, the Glauber limit ($l \equiv 0$) and to the string-flip model, respectively. For the constituent formation model, the zero scattering component has been included (dashed histogram). The data are from refs. [1, 2].

Miklos Gyulassy and Michael Plümer
Jet quenching in lepton nucleus scattering
in Nuclear Physics B Volume 346, 1 (1990).

Key Idea: Compare high p_T spectrum in sth-N and sth-A by plotting the ratio.

How jets are disappearing in hot/dense medium can tell us about the medium

Back in 1990...

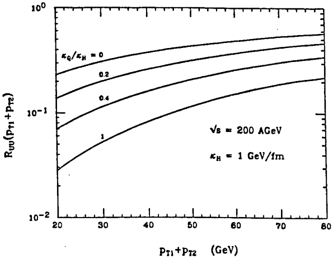


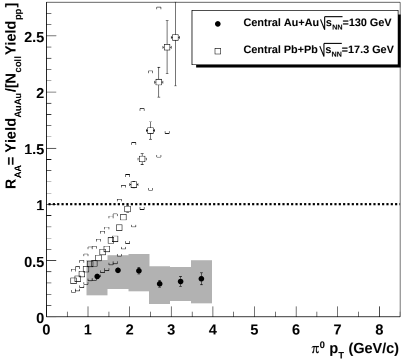
Fig. 7 Dijet reduction factor for central $U+U$ collisions at $\sqrt{s}$ =200 GeV/n as a function of the dijet energy $E = P_{T1} + P_{T2}$, for different values of κ_Q/κ_H assuming κ_H =1 GeV/fm.

transverse coordinate, ϕ the azimuthal angle of the jet and $\tau_f(r,\phi)$ the escape time. Assuming only Bjorken[31] scaling longitudinal expansion and a Bag model equation of state[31], one can find the time dependence of $dE(\tau)/dx$ and get the reduction rate of jet production at fixed P_T by averaging over the initial coordinates (r,ϕ)[22],

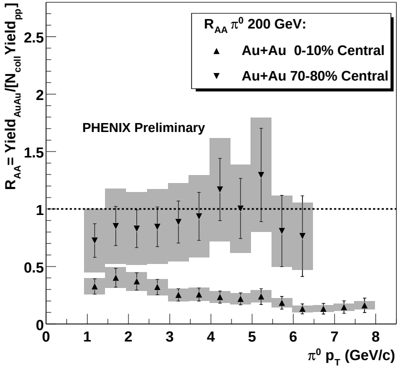
$$R_{AA}(E) = \frac{\sigma^{jet}(E)_{quenching}}{\sigma^{jet}(E)_{no-quenching}}. \qquad (11)$$

In the plasma phase, the temperature decreases as $T(\tau)/T_c = (\tau_Q/\tau)^{1/3}$. According to Eq. 9, $dE/dx \approx \kappa_Q(\tau_Q/\tau)^{2/3}$, denoting the energy loss in the plasma phase by

Xin-Nian Wang and Miklos Gyulassy,
Jets in relativistic heavy ion collisions
in BNL RHIC Workshop 1990:0079-102 (QCD199:R2:1990)

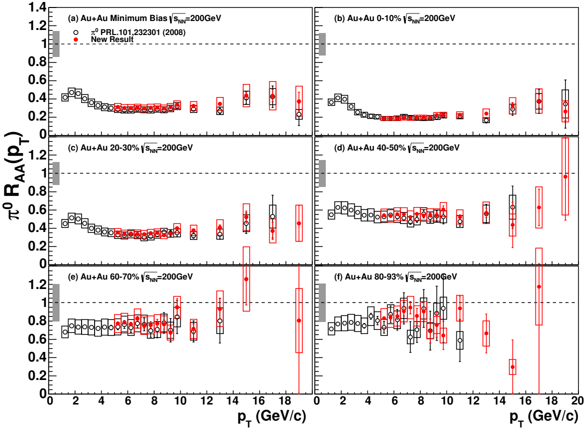


R_{AA} (π^0) for central Pb+Pb collisions at $\sqrt{s_{NN}} = 17$ GeV and central Au+Au collisions at $\sqrt{s_{NN}} = 130$ GeV.



R_{AA} (π^0) for central and peripheral Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

Presented by S. Mioduszewski at QM 2002

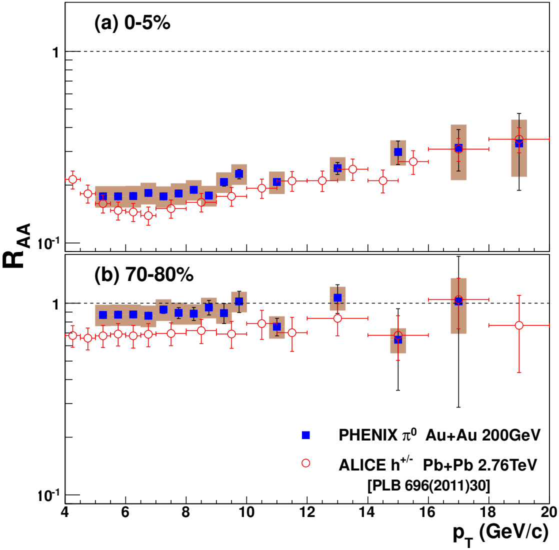


PHENIX, arXiv:1208.2254

$$\frac{dN_{AA}/dp_T}{N_{\rm coll}\,dN_{pp}/dp_T} \approx \text{Const.}$$

Slight rising is becoming evident at high p_T.

In 2012

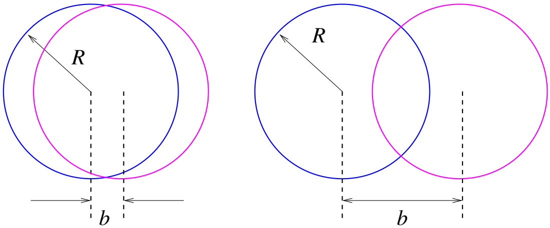


PHENIX, arXiv:1208.2254

$$\frac{dN_{AA}/dp_T}{N_{\rm coll}\, dN_{pp}/dp_T} \approx \text{Const.}$$

Slight rising is becoming evident at high p_T.

Centrality



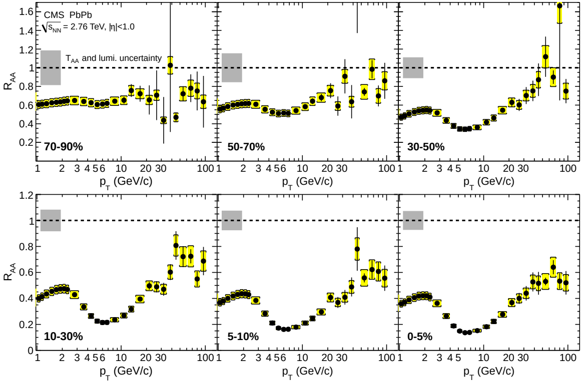
Central collisions
0 % means b = 0

Peripheral collisions
100 % means b = 2R
That is, they missed.

For instance:

- 0 – 5 % means top 5 % of all collisions in terms of the number of particles produced (multiplicity).
- 70 – 80 % means the collection of events whose multiplicity ranks between bottom 30 % and bottom 20 %.
- Centrality and impact parameter *b* not strictly 1 to 1, but very close.

In 2012



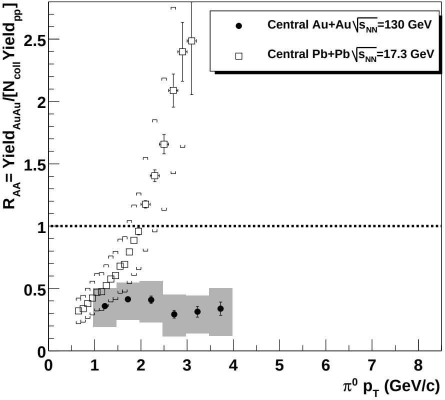
$$R_{AA} = \frac{dN_{AA}/dp_T}{N_{\rm coll}\, dN_{pp}/dp_T}$$

No longer flat. Slow rise for $p_T \gtrsim 10\,\text{GeV}$.

CMS, 1208.6218v1

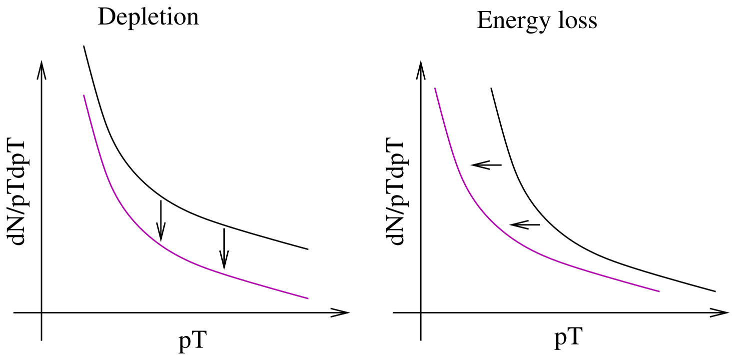
Can we understand the features?

R_{AA}



- $R_{AA} < 1$: Energy loss
- $R_{AA} > 1$: Energy gain

Two ways to understand $R_{AA} < 1$



- The spectrum can shift down when particles actually disappear (depletion)
- The spectrum can shift to the left by energy loss – *This is the more realistic scenario.*

Very Rough Understanding

- For high p_T, $dN_{\rm pp}/dp_T \approx 1/p_T^n$.
- Suppose, on average, a particle with p_T loses Δp_T while traversing QGP.
- Then the number of particles with p_T in AA is the same as the number of particles with $p_T + \Delta p_T$ in pp.

$$R_{AA} = \frac{dN_{AA}/dp_T}{N_{\rm col} dN_{pp}/dp_T} \approx \frac{dN_{pp}/dp_T|_{p_T+\Delta p_T}}{dN_{pp}/dp_T|_{p_T}}$$

- What we want to learn: Behavior of Δp_T in the medium
- Shape of R_{AA} depends very much on the shape of dN_{pp}/dp_T

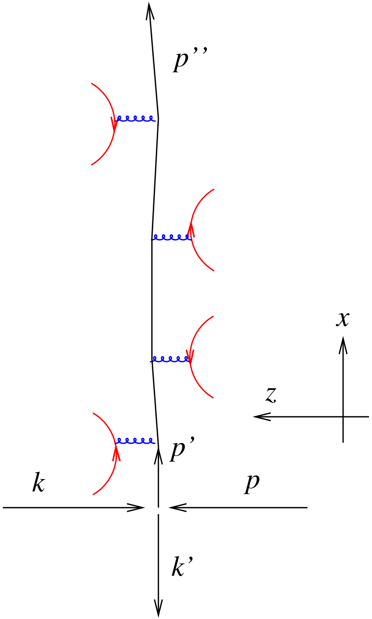
Very Rough Understanding

- Suppose $dN_{pp}/dp_T = 1/p_T^n$ (realistic for high p_T)

$$R_{AA} \approx \left(\frac{p_T}{p_T + \Delta p_T}\right)^n = \left(\frac{1}{1 + \Delta p_T/p_T}\right)^n$$

- Suppose $\Delta p_T/p_T = 0.2$: $R_{AA} = 0.2$ for $n = 8$, $R_{AA} = 0.5$ for $n = 4$.
- Let $\Delta p_T \propto p_T^s$.
- R_{AA} constant if $s = 1$
- R_{AA} approaches 1 as $p_T \to \infty$ if $s < 1$.
- R_{AA} approaches 0 as $p_T \to \infty$ if $s > 1$.

$R_{AA} < 1$ – Final state energy loss

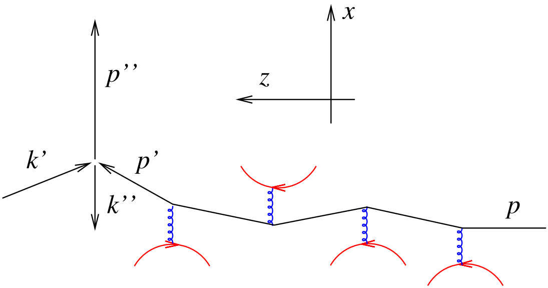


- High energy particle
- Initial energy $E_p = p_z$
- Just after collision: $p'_x = p_z$
- *Final state interactions* with the QGP medium add little bits to p'_z but *subtract little bits* from p'_x.
- Resulting in:

  $E_{\rm jet} = \sqrt{p''^2_x + p''^2_z} \approx p''_x < E_p$

  $\Longrightarrow$ Energy loss

$R_{AA} > 1$ – Initial state energy gain

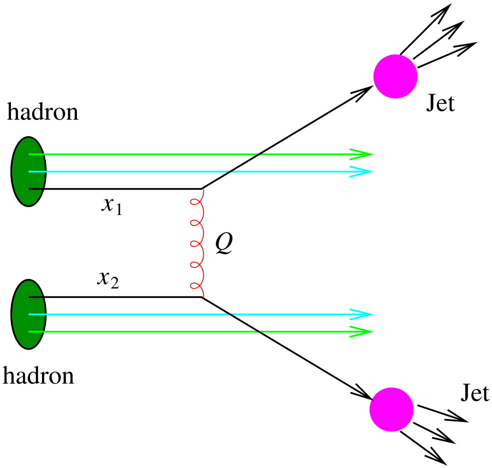


- Low energy particle
- *Initial state interactions* with other nucleons add not-so-small momentum (compared to the original energy) in both directions.
- $|p'| > |p|$
- After the hard collision:
  $p''_x \approx |p'| > p_z \Longrightarrow$ Energy gain

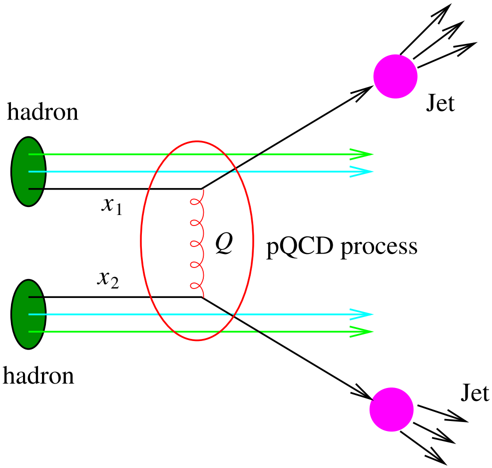
Jet Quenching

– Schematic Ideas

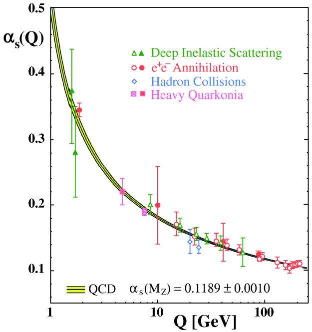
Hadronic Jet production



Hadronic Jet production

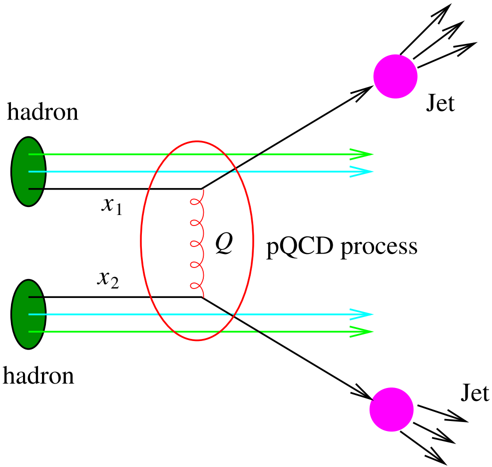


If $Q \gg \Lambda_{\mathrm{QCD}}$, $\alpha_s(Q) \ll 1$:
Jet production is perturbative.



Bethke, hep-ex/0606035

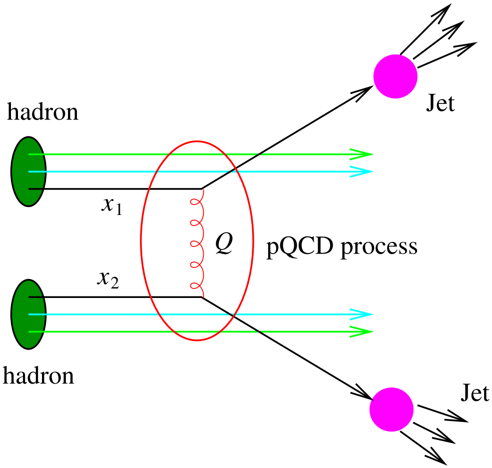
Hadronic Jet production



If $Q \gg \Lambda_{\mathrm{QCD}}$, $\alpha_s(Q) \ll 1$:
Jet production is perturbative.

➥ Calculation is possible.

Hadronic Jet production

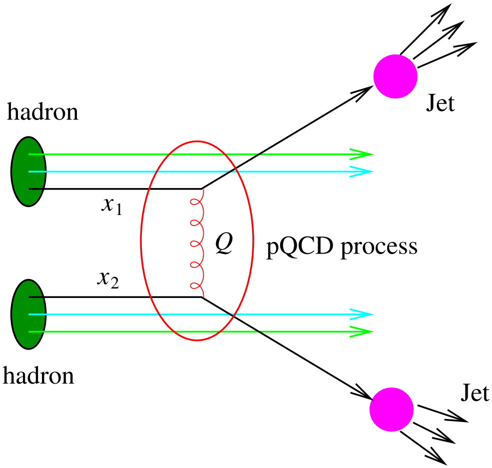


If $Q \gg \Lambda_{\rm QCD}$, $\alpha_s(Q) \ll 1$:
Jet production is perturbative.

➥ Calculation is possible.

➥ We understand this process in hadron-hadron collisions.

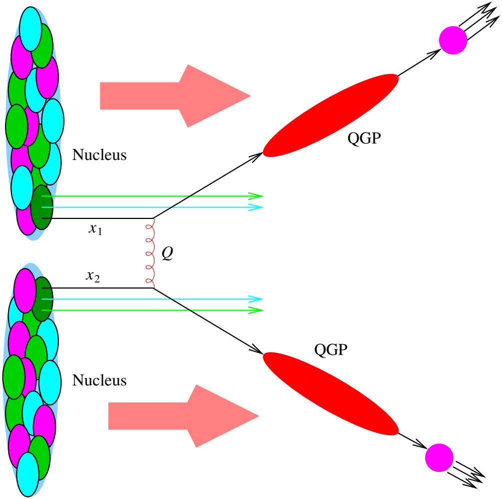
Hadronic Jet production



Hadron-Hadron Jet production scheme:

$$\frac{d\sigma}{dt} = \int_{abcd} f_{a/A}(x_a, Q_f) f_{b/B}(x_b, Q_f) \times \frac{d\sigma_{ab\to cd}}{dt} D(z_c, Q)$$

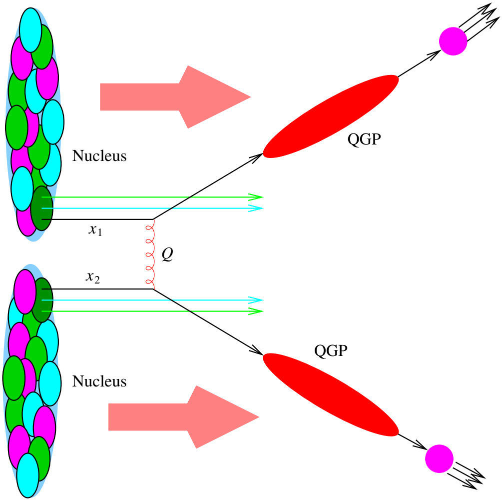
Heavy Ion Collisions



What we want to study:

- How does QGP modify jet property?

Heavy Ion Collisions

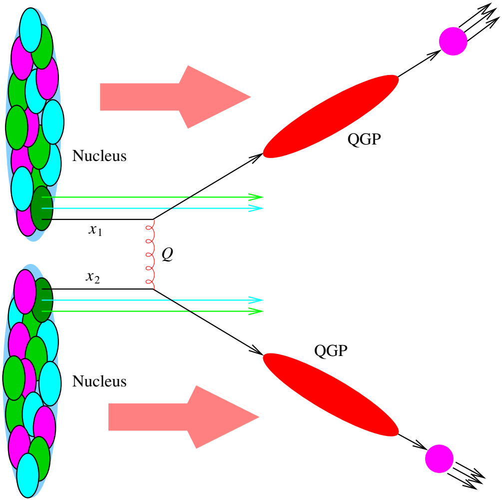


What we want to study:

- How does QGP modify jet property?

Complications:
How well do we know the *initial condition?*

- Nuclear initial condition?
- What happens to a jet between the production and the formation of (hydrodynamic) QGP?

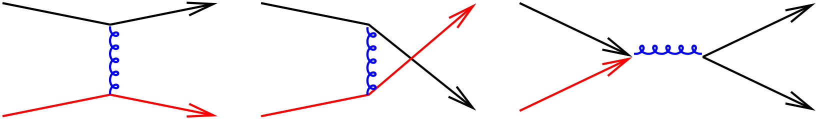


Schematically,

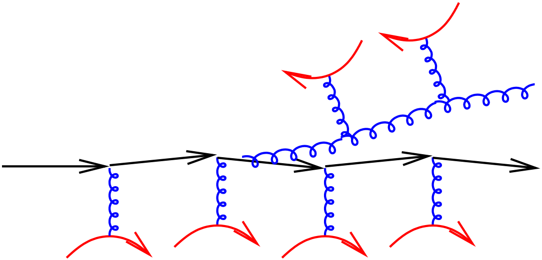
$$\frac{d\sigma_{AB}}{dt} = \int_{\text{geometry}} \int_{abcd} \times f_{a/A}(x_a, Q_f) f_{b/B}(x_b, Q_f) \times \frac{d\sigma_{ab \to cd}}{dt} \times \mathcal{P}(x_c \to x'_c | T, u^\mu) \times D(z'_c, Q)$$

$\mathcal{P}(x_c \to x'_c | T, u^\mu)$: Medium modification of high energy parton property $\Longrightarrow$ Jet quenching

Relevant processes for E-loss



Elastic scatterings with thermal particles



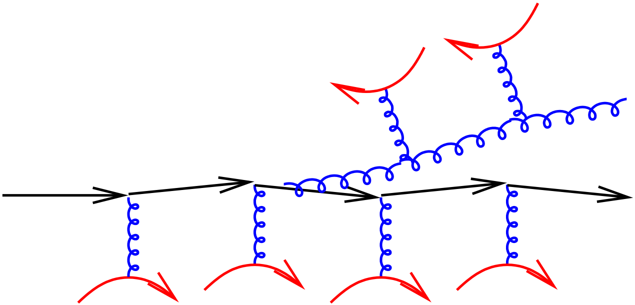
Collinear radiation

Why it is not-trivial

- Hot and dense system – Requires resummation: HTL & LPM
- Finite size system
- System is evolving

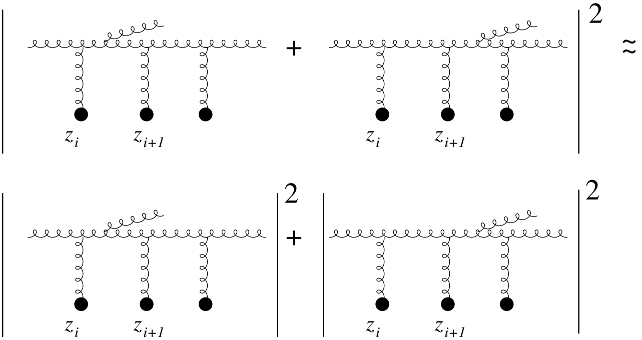
Radiational Energy Loss – Why coherence matters

Process to study



- Radiative (Inelastic) energy loss via collinear gluon emission

Incoherent emission

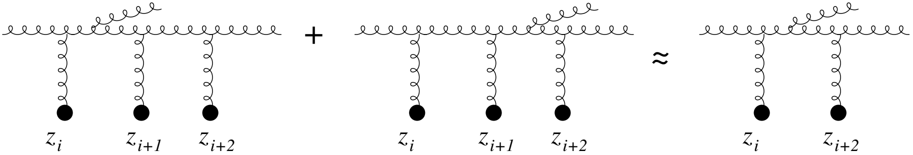


- $|\sum_n T_n|^2 \approx \sum |T_n|^2$
- Interference terms $T_n^* T_m$ with $n \neq m$ negligible.
- Single emission probabilist scales like the number of scatterers:

$$\mathcal{P}_{\mathrm{N_{sc}}} \approx N_{\mathrm{sc}} \mathcal{P}_1$$

Coherent emission

- If there is a destructive interference,



- Single emission probability scales like

$$\mathcal{P}_{N_{\rm sc}} \approx \frac{N_{\rm sc}}{N_{\rm coh}} \mathcal{P}_1$$

where $N_{\rm coh}$ is the number of scattering centers that destructively interfere.

- The medium's power to induce radiation is *reduced*.
- In the unit length, there are effectively,

$$N_{\rm eff.\ sc} = \frac{1}{l_{\rm coh}} = \frac{1}{l_{\rm mfp}} \frac{1}{N_{\rm coh}} = \frac{1}{l_{\rm coh}}$$

Effective Emission rate

- Coherent Emission rate:

$$\frac{d\mathcal{P}}{dt} \approx \frac{c}{l_{\rm coh}} \mathcal{P}_1$$

- Incoherent Emission rate:

$$\frac{d\mathcal{P}}{dt} \approx \frac{c}{l_{\rm mfp}} \mathcal{P}_1$$

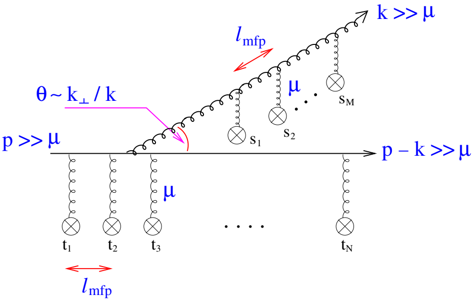
- Here, $\mathcal{P}_1$: Bethe-Heitler

$$\mathcal{P}_1 \approx \frac{\alpha_S N_c}{\pi\omega}$$

  for small ω

Coherent scattering can be important

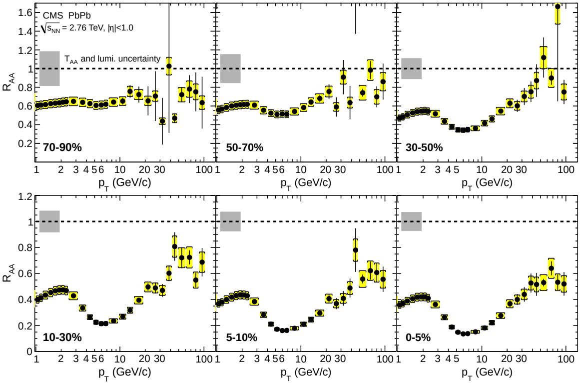
Following BDMPS



- What we need to calculate R_{AA}: Differential gluon radiation rate $\omega \dfrac{dN_g}{d\omega dz}$
  Medium dependence comes through a scattering length scale $l \approx t$

$$\omega \frac{dN_g}{d\omega dz} \approx \frac{1}{l} \omega \left. \frac{dN_g}{d\omega} \right|_{\rm BH}$$

Goal for today



CMS, 1208.6218v1

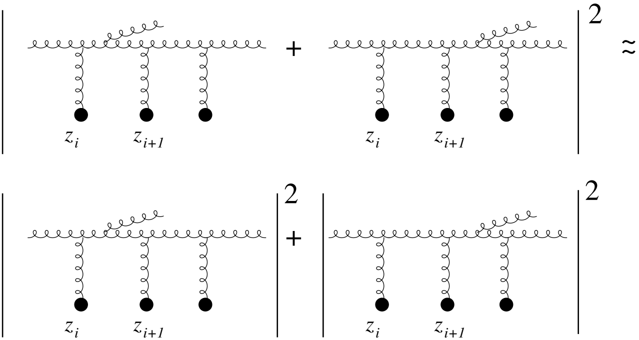
$$R_{AA} = \frac{dN_{AA}/dp_T}{N_{\mathrm{coll}}\, dN_{pp}/dp_T}$$

Goal for the day: Roughly Understand these features from the behavior of the unit scattering length l in

$$\omega \frac{dN_g}{d\omega dz} \approx \frac{1}{l} \omega \left. \frac{dN_g}{d\omega} \right|_{\mathrm{BH}}$$

Length Scales

Following BDMPS

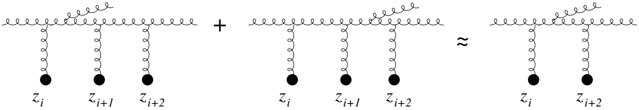


- If all scatterings are incoherent ($l_{\text{mfp}} > l_{\text{coh}}$),

$$l = l_{\text{mfp}} = 1/\rho\sigma$$

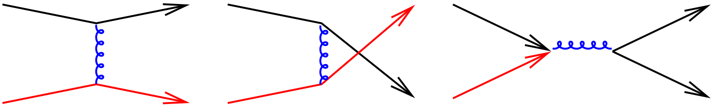
Length Scales

Following BDMPS



- If $l_{\rm coh} \geq l_{\rm mfp} \implies$ LPM effect:
  All scatterings within $l_{\rm coh}$ effectively count as a single scattering.
- $l = l_{\rm coh}$

Estimation of $l_{\rm mfp}$



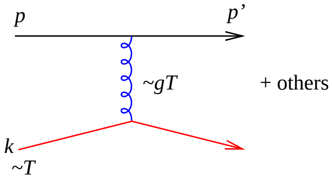
- Mean free path (textbook definition)

$$\frac{1}{l_{\rm mfp}} \equiv \int d^3k\, \rho(k) \int dq^2\, (1 - \cos\theta_{pk}) \frac{d\sigma^{\rm el}}{dq^2}$$

where

- $\rho(k)$: density, $(1 - \cos\theta_{pk})$: flux factor
- Elastic cross-section (Coulombic) $\frac{d\sigma}{dq^2} \approx C_R \frac{2\pi\alpha_s^2}{(q^2)^2}$

Estimation of $l_{\rm mfp}$



- Mean free path (textbook definition)

$$\frac{1}{l_{\rm mfp}} \equiv \int d^3k\,\rho(k) \int dq^2\,(1 - \cos\theta_{pk})\frac{d\sigma^{\rm el}}{dq^2}$$

where

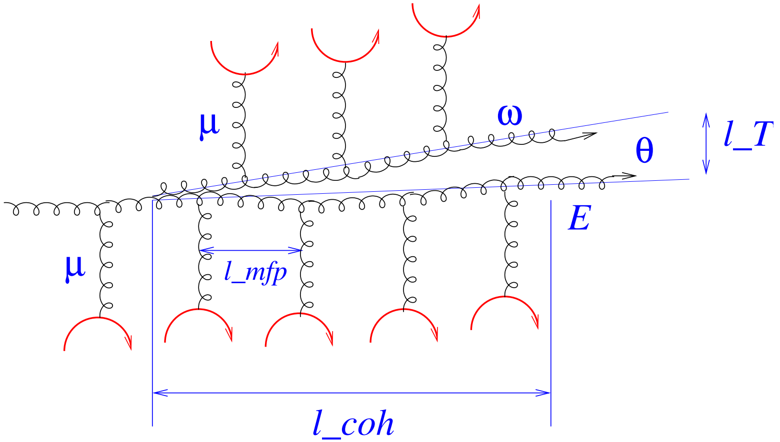
- $\rho(k)$: density, $(1 - \cos\theta_{pk})$: flux factor
- Elastic cross-section (Coulombic) $\dfrac{d\sigma}{dq^2} \approx C_R \dfrac{2\pi\alpha_s^2}{(q^2)^2}$

Estimation of $l_{\rm mfp}$

- With thermal $\rho(k)$, this yields

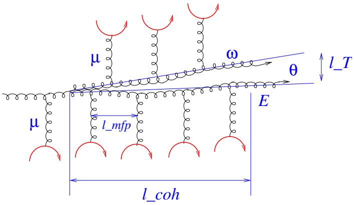
$$\frac{1}{l_{\rm mfp}} \sim \int d^3k\rho(k)\int_{m_D^2}^{\infty} dq^2\frac{\alpha_S^2}{q^4} \sim T^3\alpha_S^2/m_D^2 \sim \alpha_S T$$

Estimation of $l_{\rm coh}$



- $E \gg \omega_g \gg \mu$

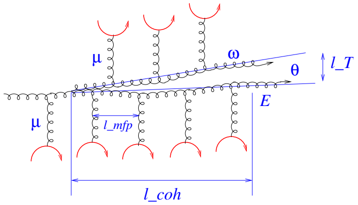
Estimation of $l_{\rm coh}$



- $\omega \ll E \implies$ The radiated gluon random walks away from the original parton. Original parton's trajectory is less affected.
- From the geometry $\frac{\omega_g}{k_T^g} \approx \frac{l_{\rm coh}}{l_T}$
- Separation condition: l_T is longer than the transverse size of the radiated gluon. $l_T \approx 1/k_T^g$
- Putting together,

$$l_{\rm coh} \approx \frac{\omega_g}{(k_T^g)^2}$$

Estimation of $l_{\rm coh}$



- We have: $l_{\rm coh} \approx \dfrac{\omega_g}{(k_T^g)^2}$
- After suffering $N_{\rm coh}$ collisions (random walk),

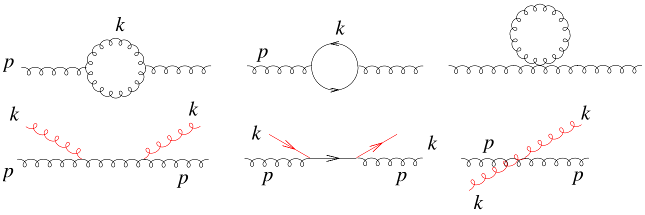
$$\left\langle (k_T^g)^2 \right\rangle = N_{\rm coh}\mu^2 = \frac{l_{\rm coh}}{l_{\rm mfp}}\mu^2$$

- Becomes, with $\hat{q} = \mu^2/l_{\rm mfp}$ and $E_{\rm LPM} = \mu^2 l_{\rm mfp}$,

$$l_{\rm coh} \approx l_{\rm mfp}\sqrt{\frac{\omega_g}{E_{\rm LPM}}} = \sqrt{\frac{\omega_g}{\hat{q}}}$$

Estimation of μ^2

- Debye mass

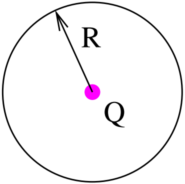


- Second row: Physical forward scattering with particles in the medium
- The last term is easiest to calculate:

$$m_D^2 \propto g^2 \int \frac{d^3k}{E_k} f(k) \propto g^2 T^2$$

- Effectively, this adds $m_D^2 A_0^2$ to the Lagrangian $\Longrightarrow$ NOT gauge invariant $\Longrightarrow$ Gauge invariant formulation: Hard Thermal Loops

Physical origin of Debye mass



- E & M
- Let $Q > 0$. Within the range R
  - Positive charges are pushed away: $Q_+ = Q_0 - \delta Q$
  - Negative charges are pulled in: $Q_- = Q_0 + \delta Q$
- At position R, apparent net charge is *reduced*

$$Q_{\rm net} = Q + (Q_0 - \delta Q) - (Q_0 + \delta Q) = Q - 2\delta Q$$

  This is screening.
- When it's moving, there is a net potential energy associated with Q even in charge neutral medium $\Longrightarrow$ Acts like a "mass"

Physical origin of Debye mass

- E & M
- Potential in a thermal system

$$\nabla^2\Phi(\mathbf{r}) = -\rho(\mathbf{r})$$

- Medium composed of many charged particles

$$\rho(\mathbf{r}) = qn_+(\mathbf{r}) - qn_-(\mathbf{r})$$

- Boltzmann Density:

$$\begin{aligned} n_\pm(\mathbf{r}) &= \int \frac{d^3k}{(2\pi)^3}\, e^{-E/T} \\ &= \int \frac{d^3k}{(2\pi)^3}\, e^{-\sqrt{k^2+m^2}} e^{\mp q\Phi(\mathbf{r})/T} \\ &= n_0(T) e^{\mp q\Phi(\mathbf{r})/T} \\ &\approx n_0(T)(1 \mp q\Phi(\mathbf{r})/T) \end{aligned}$$

Physical origin of Debye mass

- E & M
- Boltzmann Density:

$$n_{\pm}(\mathbf{r}) \approx n_0(T)(1 \mp q\Phi(\mathbf{r})/T)$$

- Linearized equation for the potential:

$$\nabla^2\Phi - m_D^2\Phi \approx 0$$

where

$$m_D^2 = 2q^2(n_0(T)/T)$$

What we learned so far

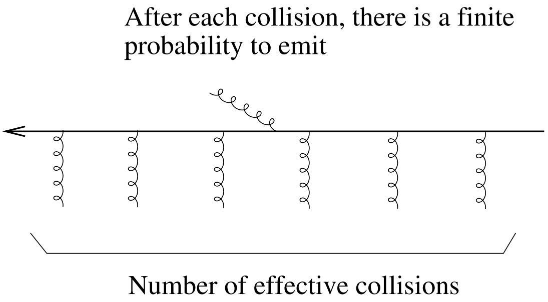
Coherence length

$$l_{\rm coh} \approx l_{\rm mfp}\sqrt{\frac{\omega_g}{E_{\rm LPM}}} = \sqrt{\frac{\omega_g}{\hat{q}}}$$

where $\hat{q} = \mu^2/l_{\rm mfp}$ (average momentum transfer squared per collision)
If your chosen process is

- Soft gluon emission, $\omega_g < \mu^2 l_{\rm mfp}$,
  $\Longrightarrow$ Coherence matters not. BH should suffice. No need to resum.
- Hard gluon emission, $E \gg \omega_g > \mu^2 l_{\rm mfp}$,
  $\Longrightarrow$ Coherence matters. Resummation needed.
- Both
  $\Longrightarrow$ Need the cross-section that is correct in both limits.
- Key quantity: $E_{\rm LPM} = \mu^2 l_{\rm mfp} \sim T$ in pert. thermal QCD
- Key quantity: $\hat{q} \sim \alpha_S^2 T^3$ in pert. thermal QCD

Rough Idea – Multiple Emission (Poisson ansatz)



- Let the emission probability be p
- Total number of *effective* collisions $N_{\rm trial}$ taking into account of $l_{\rm mfp}$ and $l_{\rm coh}$.
- Average number of emissions $\langle n \rangle = N_{\rm trial} p$
- Probability to emit n gluons

$$P(n) = \frac{N_{\rm trial}!}{n!(N_{\rm trial} - n)!} p^n (1-p)^{N_{\rm trial} - n}$$

Rough Idea – Multiple Emission (Poisson ansatz)

- Poisson probability: Limit of binary process as $\lim_{N_{\rm trial}\to\infty} N_{\rm trial} p \to \langle n \rangle$

$$P(n) = e^{-\langle n \rangle} \frac{\langle n \rangle^n}{n!}$$

- Average number of gluons emitted up to $t_i < t$

$$\langle n \rangle = \int_{-\infty}^{E} d\omega \int_{t_i}^{t} dz \frac{dN}{dz d\omega} = \int_{-\infty}^{E} d\omega \frac{dN}{d\omega}(t)$$

- Probability to lose ϵ amount of energy by emitting n gluons:

$$\begin{aligned} \langle n \rangle^n & \to D(\epsilon, t) \\ & = \int_{-\infty}^{E} d\omega_1 \frac{dN}{d\omega_1} \int_{-\infty}^{E} d\omega_2 \frac{dN}{d\omega_2} \cdots \int_{-\infty}^{E} d\omega_n \frac{dN}{d\omega_n} \delta(\epsilon - \sum_{k=1}^{n} \omega_k) \end{aligned}$$

Rough Idea – Multiple Emission (Poisson ansatz)

Parton spectrum at t

$$P(p,t) = \int d\epsilon\, D(\epsilon,t)\, P_0(p+\epsilon)$$

where

$$D(\epsilon,t) = e^{-\int d\omega \frac{dN}{d\omega}(\omega,t)} \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^{n} \int d\omega_i\, \frac{dN}{d\omega_i}(\omega_i,t) \right] \delta\left(\epsilon - \sum_{i=1}^{n} \omega_i \right)$$

Can easily show that this Poisson ansatz solves:

$$\frac{dP(p,t)}{dt} = \int d\omega\, \frac{dN_{\text{Poiss.}}}{d\omega dt}(\omega) P(p+\omega,t) - P(p,t) \int d\omega\, \frac{dN_{\text{Poiss.}}}{d\omega dt}(\omega)$$

with the p (jet energy) independent rate

$$\frac{dN}{d\omega}(\omega,t) = \int_{t_0}^{t} dt'\, \frac{dN_{\text{Poiss.}}}{d\omega dt}(\omega,t')$$

Rough Idea - The behavior of R_{AA}

Use $R_{AA} \approx 1/(1+\epsilon/p)^n \approx e^{-n\epsilon/p}$ when $n \gg 1$. Include gain by absoprtion or $\omega < 0$:

$$R_{AA}(p) = \frac{P(p)}{P_0(p)} \approx \exp\left(-\int_{-\infty}^{\infty} d\omega \int_0^t dt' \, (dN_{\rm inel+el}/d\omega dt)(1 - e^{-\omega n/p})\right)$$

For the radiation rate, use simple estimates

$$\frac{dN}{d\omega dt} \approx \frac{\alpha}{\pi\omega}\frac{N_c}{l_{\rm mfp}} \quad \text{for} \quad 0 < \omega < l_{\rm mfp}\mu^2$$

$$\frac{dN}{d\omega dt} \approx \frac{\alpha}{\pi\omega} N_c \sqrt{\frac{\mu^2}{l_{\rm mfp}\omega}} \quad \text{for} \quad l_{\rm mfp}\mu^2 < \omega < l_{\rm mfp}\mu^2 (L/l_{\rm mfp})^2$$

$$\frac{dN}{d\omega dt} \approx \frac{\alpha}{\pi\omega}\frac{N_c}{L} \quad \text{for} \quad l_{\rm mfp}\mu^2 (L/l_{\rm mfp})^2 < \omega < E$$

$$\frac{dN}{d\omega dt} \approx \frac{\alpha}{\pi|\omega|}\frac{N_c}{l_{\rm mfp}} e^{-|\omega|/T} \quad \text{for} \quad \omega < 0$$

Rough Idea - The behavior of R_{AA}

For elastic energy loss,

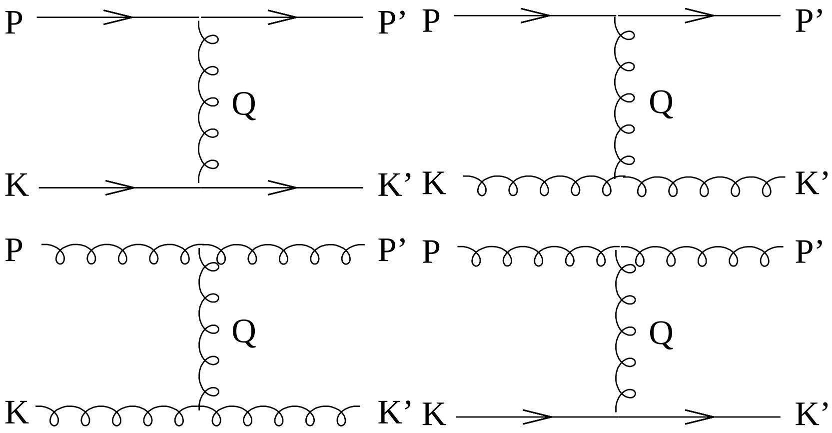
$$
\begin{aligned}
R_{AA}^{\rm el} &\approx \exp\left(-\int_{-\infty}^{\infty} d\omega \int_0^t dt'\,(d\Gamma_{\rm el}/d\omega dt)(1-e^{-\omega n/p})\right) \\
&\approx \exp\left(-t\left(\frac{dE}{dt}\frac{K(\omega_0)}{|\omega_0|}\right)\right) \\
&\approx \exp\left(-t\left(\frac{dE}{dt}\right)\left(\frac{n}{p}\right)\left(1-\frac{nT}{p}\right)\right)
\end{aligned}
$$

valid for $p > nT$ and we used

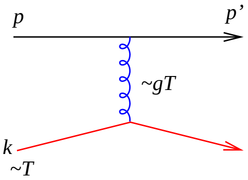
$$
\begin{aligned}
K(\omega_0) &= (1+n_B(|\omega_0|))(1-e^{-|\omega_0|n/p}) + n_B(|\omega_0|)(1-e^{|\omega_0|n/p}) \\
&\approx |\omega_0|\left(\frac{n}{p}\right)\left(1-\frac{nT}{p}\right) \quad \text{for small } \omega_0
\end{aligned}
$$

where ω_0 is the typical gluon energy

Coulombic t-channel dominates



Rough Idea - Elastic energy loss(Following Bjorken)



+ others

- Mean free path (textbook definition)

$$\frac{1}{l_{\rm mfp}} \equiv \int d^3k\, \rho(k) \int dq^2\, (1 - \cos\theta_{pk}) \frac{d\sigma^{\rm el}}{dq^2}$$

- Energy loss per unit length

$$\frac{dE}{dz} = \int d^3k\, \rho(k) \int dq^2\, (1 - \cos\theta_{pk}) \Delta E \frac{d\sigma^{\rm el}}{dq^2}$$

where

  - $\rho(k)$: density, $(1 - \cos\theta_{pk})\Delta E \approx q^2/2k$: flux factor
  - Elastic cross-section (Coulombic) $\frac{d\sigma}{dq^2} \approx C_R \frac{2\pi\alpha_s^2}{(q^2)^2}$

Rough Idea - Elastic energy loss(Following Bjorken)

- With thermal ρ, this yields

$$\left(\frac{dE}{dz}\right)_{\rm coll} \sim \int d^3k\rho(k)/k \int dq^2\alpha_S^2/q^2 \sim \alpha_S^2 T^2 \ln(ET/m_D^2)$$

Upper limit determined by

$$q^2 = (p-k)^2 = p^2 + k^2 - 2pk \approx -2pk \sim ET$$

when $|\mathbf{p}| = E$ (emitter) and $|\mathbf{k}| = O(T)$ (thermal scatterer)
Lower limit determined by the Debye mass $m_D = O(gT)$.

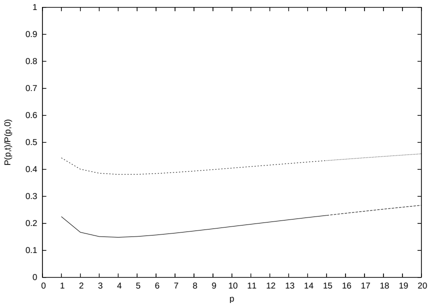
Elastic scattering rate

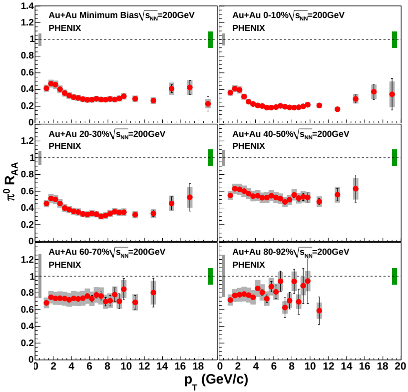
More precisely,

$$\begin{aligned}\frac{dE}{dt} &= \frac{1}{2E}\int_{k,k',p'} \delta^4(p+k-p'-k')\,(E-E')|M|^2\, f(E_k)[1 \pm f(E'_k)] \\ &= C_r \pi\, \alpha_s^2 T^2 \left[\ln(ET/m_g^2) + D_r\right]\end{aligned}$$

where C_r and D_r are channel dependent $O(1)$ constants.

Rough Idea - The behavior of R_{AA}

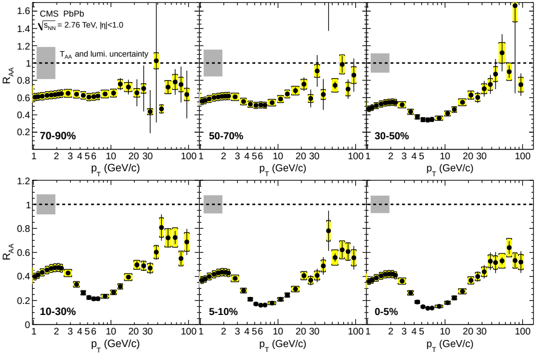




- Upper line: Without elastic
- Lower line: With elastic
- Flat R is produced in both cases up to $O(10\,T)$.
- R just not that sensitive to p in the RHIC-relevant range.

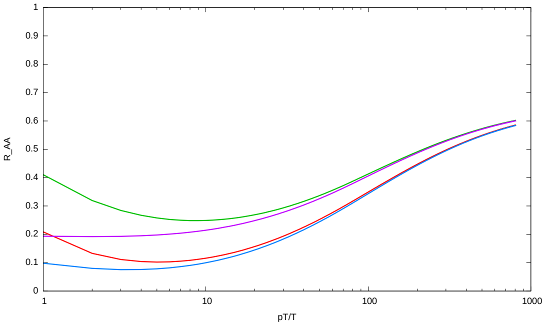
Rough Idea - The behavior of R_{AA}

CMS: Up to $p_T = 100\,\text{GeV}$



No longer flat. Slow rise for $p_T \gtrsim 10\,\text{GeV}$.
Can we understand these features?

Rough Idea - The behavior of R_{AA}



- Red: Elastic on, thermal absorption on
- Blue: Elastic on, thermal absorption off
- Green: Elastic off, thermal absorption on
- Magenta: Elastic off, thermal absorption off
- Dip, rise, leveling-off roughly reproduced
- *No dip if thermal absorption is turned off*

For other features, first recall

Use $R_{AA} \approx 1/(1+\epsilon/p)^n \approx e^{-n\epsilon/p}$ when $n \gg 1$. Include gain by absoprtion or $\omega < 0$:

$$R_{AA}(p) = \frac{P(p)}{P_0(p)} \approx \exp\left(-\int_{-\infty}^{\infty} d\omega \int_0^t dt' \, (dN_{\rm inel+el}/d\omega dt)(1 - e^{-\omega n/p})\right)$$

For the radiation rate, use simple estimates

$$\frac{dN}{d\omega dt} \approx \frac{\alpha}{\pi\omega}\frac{N_c}{l_{\rm mfp}} \quad \text{for} \quad 0 < \omega < l_{\rm mfp}\mu^2$$

$$\frac{dN}{d\omega dt} \approx \frac{\alpha}{\pi\omega} N_c \sqrt{\frac{\mu^2}{l_{\rm mfp}\omega}} \quad \text{for} \quad l_{\rm mfp}\mu^2 < \omega < l_{\rm mfp}\mu^2 (L/l_{\rm mfp})^2$$

$$\frac{dN}{d\omega dt} \approx \frac{\alpha}{\pi\omega}\frac{N_c}{L} \quad \text{for} \quad l_{\rm mfp}\mu^2 (L/l_{\rm mfp})^2 < \omega < E$$

$$\frac{dN}{d\omega dt} \approx \frac{\alpha}{\pi|\omega|}\frac{N_c}{l_{\rm mfp}} e^{-|\omega|/T} \quad \text{for} \quad \omega < 0$$

Flat then slow rise

With $E = p$ (original parton energy) and the system size L and $(1 - e^{-n\omega/E}) \approx n\omega/E$:

- If $E < E_{\rm LPM} = \mu^2 l_{\rm mfp}$,

$$\ln R_{AA} \approx -L\int_0^E d\omega\, \frac{dN}{d\omega dt}\left(\frac{n\omega}{E}\right) \approx \frac{nL}{E}\int_0^E d\omega\omega\, \left(\frac{\alpha_S}{\pi\omega}\frac{N_c}{l_{\rm mfp}}\right) \sim \text{Const.}$$

  Flat R_{AA}

- If $E_{\rm LPM} < E < E_L = L^2\mu^2/l_{\rm mfp}$,

$$\begin{aligned}
\ln R_{AA} &\approx -\frac{nL}{E}\int_0^{E_{\rm LPM}} d\omega\omega\, \left(\frac{\alpha_S}{\pi\omega}\frac{N_c}{l_{\rm mfp}}\right) \\
&\quad -\frac{nL}{E}\int_{E_{\rm LPM}}^{E} d\omega\omega\, \left(\frac{\alpha_S}{\pi\omega}N_c\sqrt{\frac{\mu^2}{l_{\rm mfp}\omega}}\right) \\
&\approx -\frac{nL\alpha_S N_c}{\pi l_{\rm mfp}}\left(2\sqrt{\frac{E_{\rm LPM}}{E}}\right)
\end{aligned}$$

  Slowly rising R_{AA}

Plateau at high p_T

- If $l_{\rm coh} > L$, effectively only a single scattering happens. $\Longrightarrow$ Goes back to BH

If $E > E_L = L^2\mu^2/l_{\rm mfp}$,

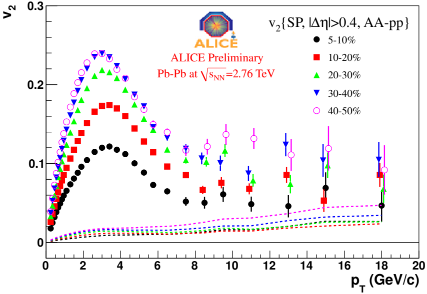
$$
\begin{aligned}
\ln R_{AA} &\approx -\frac{nL}{E}\int_0^{E_{\rm LPM}} d\omega\omega \left(\frac{\alpha_S}{\pi\omega}\frac{N_c}{l_{\rm mfp}}\right) \\
&\quad - \frac{nL}{E}\int_{E_{\rm LPM}}^{E_L} d\omega\omega \left(\frac{\alpha_S}{\pi\omega}N_c\sqrt{\frac{\mu^2}{l_{\rm mfp}\omega}}\right) \\
&\quad - \frac{nL}{E}\int_{E_L}^{E} d\omega\omega \left(\frac{\alpha_S}{\pi\omega}\frac{N_c}{L}\right) \\
&\approx -n\frac{\alpha_S N_c}{\pi}\left(1 + \frac{E_L}{E}(1 - l_{\rm mfp}/L)\right)
\end{aligned}
$$

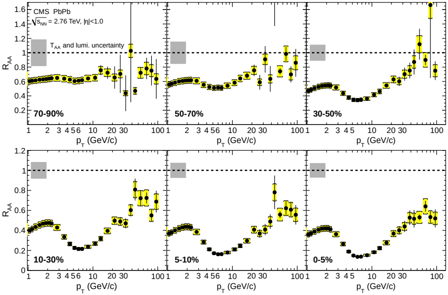
This is approximately constant for large E.

What is R_{AA} telling us?

- Dip-rise-flat feature qualitatively understandable
- Opaque medium
- Density of the medium
- Dip in R_{AA}: Could be an indirect indication of the initial temperature.
- Plateau at high p_T: Could be an indication that $l_{\rm coh} > L$ is reached.
  $\Longrightarrow$ Extract $\hat{q}$ from $l_{\rm coh} \approx \sqrt{\omega/\hat{q}}$?

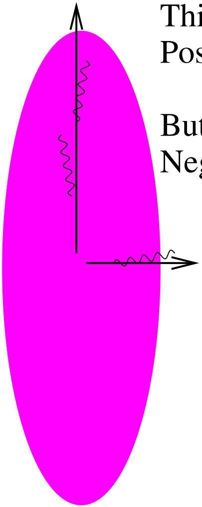
Understanding high p_T part of v_2





- v_2 and R_{AA}: Is there a relationship?

Understanding high p_T part of v_2



This jet loses more energy:
Positive v2

But it radiates more photons:
Negative photon v2

Understanding high p_T part of v_2

- Start with an isotropic distribution of high energy particles
- After going through the almond:

$$p_x = E - \Delta E_x$$
$$p_y = E - \Delta E_y$$

That is,

$$p_x^2 \approx E^2 - 2\Delta E_x E$$

- Elliptic flow definition:

$$v_2 = \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} \sim \frac{2\Delta E_y E - 2\Delta E_x E}{2E^2} = \left(\frac{\Delta E_y - \Delta E_x}{E}\right)$$

Approx. relationship between R_{AA} and v_2 at high p_T

- BDMPS: If $dN/p_T dp_T \approx 1/p_T^n$, $\ln R_{AA} \approx -n\frac{\Delta E}{E}$
- If $E < E_{\rm LPM} = \mu^2 l_{\rm mfp}$, $\ln R_{AA} \approx -\frac{nL}{E}\frac{\alpha_S N_c}{\pi_{\rm mfp}}$

$$v_2 \sim \left(\frac{\Delta E_y - \Delta E_x}{E}\right) \propto (L_y - L_x)$$

Flat v_2

- If $E_{\rm LPM} < E < E_L = L^2\mu^2/l_{\rm mfp}$,

$$\ln R_{AA} \sim -\frac{nL\alpha_S N_c}{\pi l_{\rm mfp}}\left(2\sqrt{\frac{E_{\rm LPM}}{E}} - \frac{E_{\rm LPM}}{E}\right)$$

$$v_2 \sim \left(\frac{\Delta E_y - \Delta E_x}{E}\right) \propto (L_y - L_x)\sqrt{\frac{\hat{q}}{E}}$$

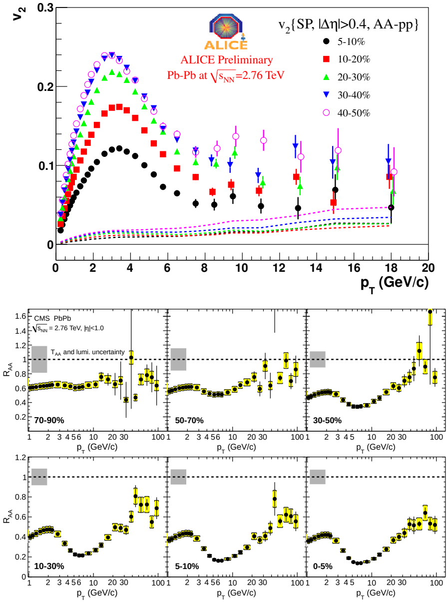
Slowly falling v_2

Very high p_T

- If $E > E_L = L^2\mu^2/l_{\rm mfp}$, $\ln R_{AA} \approx -n\frac{\alpha_S N_c}{\pi}\left(1 + \frac{E_L}{E}(1 - l_{\rm mfp}/L)\right)$

$$v_2 \sim \left(\frac{\Delta E_y - \Delta E_x}{E}\right) \propto (L_y^2 - L_x^2)\frac{\hat{q}}{E}$$

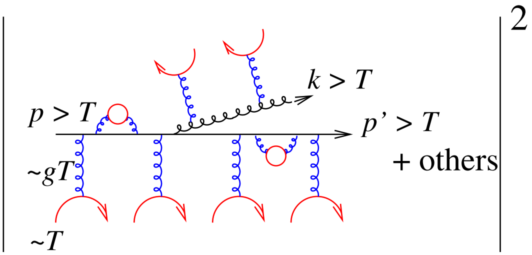
Faster falling v_2



- Data: ALICE and CMS
- High p_T v_2: Flat, then falls like $1/\sqrt{p_T}$ and then $1/p_T$.
- Can understand high p_T data qualitatively although $1/p_T$ behavior may not be visible since this is for $E > E_L$.
- The slope $dv_2/dp_T \propto -\sqrt{\hat{q}}$
- Of course, this is very rough: Viscosity also curves it down and $p_T \gtrsim 3\,\text{GeV}$ may not be high enough.

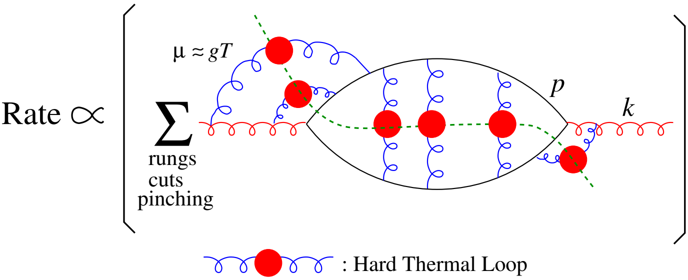
Thermal QCD calculation of the radiation rate

McGill-AMY



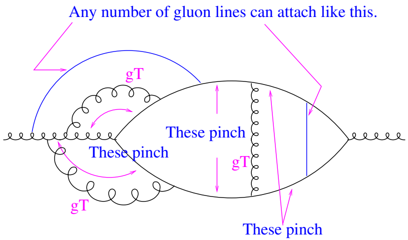
- Medium is weakly coupled QGP with thermal quarks and gluons
- Requires $g \ll 1$, $p > T$, $k > T$

McGill-AMY



- Medium is weakly coupled QGP with thermal quarks and gluons
- Requires $g \ll 1$, $p > T$, $k > T$
- Sum all interactions with the medium including the self-energy

McGill-AMY



Adding one more rung = O(1).
Need to resum.

- Medium is weakly coupled QGP with thermal quarks and gluons
- Requires $g \ll 1$, $p > T$, $k > T$
- Sum all interactions with the medium including the self-energy
- Leading order: 3 different kinds of collinear pinching poles

McGill-AMY

- What pinching does: Let

$$P = \left(\frac{i}{p_1^2 + m_2^2 + 2iE_1\Gamma_1} \right)^* \frac{i}{p_2^2 + m_1^2 + 2iE_2\Gamma_2}$$

- Poles for positive energies at $p_1^0 = E_1 - i\Gamma_1$ and $p_2^0 = E_2 + i\Gamma_2$
- If $p_1^0 = E_1 - i\Gamma_1$ puts p_2 also almost on-shell,

$$P \propto \frac{1}{E_1 E_2} \delta(p_1^0 - E_1) \frac{1}{\delta E + i\Gamma_2 + i\Gamma_1}$$

  where δE: difference in the real part of the energy
- Physically, this means that an almost on-shell particle lives a long time $\Delta t \sim 1/\delta E \sim 1/\Gamma$ $\Longrightarrow$ Introduces a secular divergence

McGill-AMY

- Pinching poles occur when
  - $p_1 \approx p_2$: Soft momentum exchange or radiation.
    If $p_1^2 + m^2 = O(g^2T^2)$, so is $p_2^2 + m^2 = O(g^2T^2)$.
  - $p_2 = xp_1$: Collinear radiation.
    When $p_1^2 + m^2 = O(g^2T^2)$,

$$p_2^2 + m^2 = x^2p_1^2 + m^2 + O(g^2T^2) = (1 - x^2)m^2 + O(g^2T^2)$$

    When $m \approx gT$, the whole expression is $O(g^2T^2)$.

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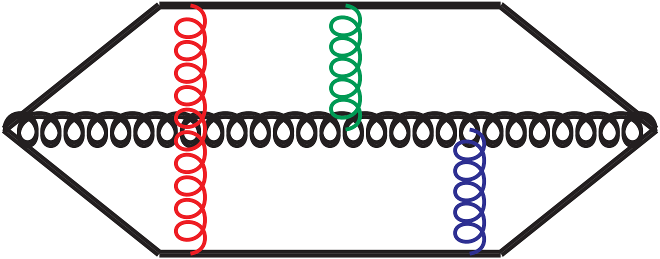
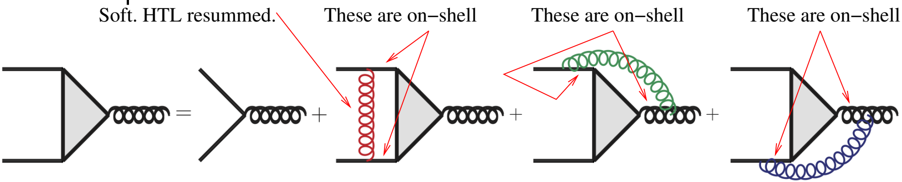


- SD-Eq:



Figures from G. Qin

- SD Equation for the vertex **F**

$$
\begin{aligned}
2\mathbf{h} &= i\delta E(\mathbf{h},p,k)\mathbf{F}_s(\mathbf{h}) + g^2 \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} C(\mathbf{q}_\perp) \times \\
&\quad \times \Big\{ (C_s - C_A/2)[\mathbf{F}_s(\mathbf{h}) - \mathbf{F}_s(\mathbf{h}{-}k\,\mathbf{q}_\perp)] \\
&\qquad + (C_A/2)[\mathbf{F}_s(\mathbf{h}) - \mathbf{F}_s(\mathbf{h}{+}p\,\mathbf{q}_\perp)] \\
&\qquad + (C_A/2)[\mathbf{F}_s(\mathbf{h}) - \mathbf{F}_s(\mathbf{h}{-}(p{-}k)\,\mathbf{q}_\perp)] \Big\}, \\
\delta E(\mathbf{h},p,k) &= \frac{\mathbf{h}^2}{2pk(p{-}k)} + \frac{m_k^{g2}}{2k} + \frac{m_{p-k}^{s2}}{2(p{-}k)} - \frac{m_p^{s2}}{2p}\,.
\end{aligned}
$$

- $\mathbf{h} = (\mathbf{p}\times\mathbf{k})\times\mathbf{e}_{||}$ — Must keep track of both $\mathbf{p}_\perp$ and $\mathbf{k}_\perp$ now. For photons, we could just set $\mathbf{k}_\perp = 0$.

McGill-AMY

- SD Equation for the vertex **F**

$$
\begin{aligned}
2\mathbf{h} &= i\delta E(\mathbf{h},p,k)\mathbf{F}_s(\mathbf{h}) + g^2 \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} C(\mathbf{q}_\perp) \times \\
&\times \Big\{ (C_s - C_{\rm A}/2)[\mathbf{F}_s(\mathbf{h}) - \mathbf{F}_s(\mathbf{h}{-}k\,\mathbf{q}_\perp)] \\
&\quad +(C_{\rm A}/2)[\mathbf{F}_s(\mathbf{h}) - \mathbf{F}_s(\mathbf{h}{+}p\,\mathbf{q}_\perp)] \\
&\quad +(C_{\rm A}/2)[\mathbf{F}_s(\mathbf{h}) - \mathbf{F}_s(\mathbf{h}{-}(p{-}k)\,\mathbf{q}_\perp)] \Big\}, \\
\delta E(\mathbf{h},p,k) &= \frac{\mathbf{h}^2}{2pk(p{-}k)} + \frac{m_k^{g2}}{2k} + \frac{m_{p-k}^{s2}}{2(p{-}k)} - \frac{m_p^{s2}}{2p} .
\end{aligned}
$$

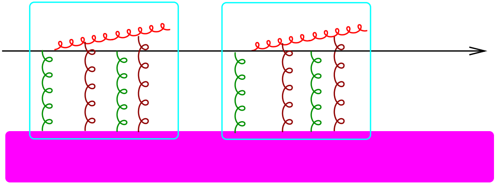
- s: Process dependence. $q \to qg$, $g \to gg$, $g \to q\bar{q}$.
- $g \to q\bar{q}$: Exchange coeff. of the first and second line
- m_s^2: Medium induced thermal masses of the emitter.

McGill-AMY

- Rate for $p > T, k > T$ (valid for $p \gg T$ and $k \gg T$ as well)

$$
\begin{aligned}
\frac{dN_g(p,k)}{dkdt} &= \frac{C_s g_s^2}{16\pi p^7} \frac{1}{1 \pm e^{-k/T}} \frac{1}{1 \pm e^{-(p-k)/T}} \times \\
&\times \left\{ \begin{array}{cc} \frac{1+(1-x)^2}{x^3(1-x)^2} & q \to qg \\ N_f \frac{x^2+(1-x)^2}{x^2(1-x)^2} & g \to q\bar{q} \\ \frac{1+x^4+(1-x)^4}{x^3(1-x)^3} & g \to gg \end{array} \right\} \\
&\times \int \frac{d^2\mathbf{h}}{(2\pi)^2} 2\mathbf{h} \cdot \mathrm{Re}\, \mathbf{F}_s(\mathbf{h}, p, k) \,,
\end{aligned}
$$

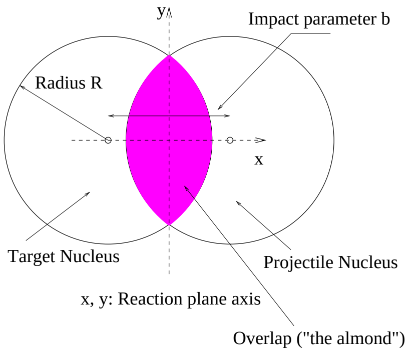
- s: Process dependence.



- Evolution - Medium enters through $T(t, \mathbf{x})$ and $u^\mu(t, \mathbf{x})$

$$
\begin{aligned}
\frac{d\mathcal{P}_q(p)}{dt} &= \int_k \mathcal{P}_q(p+k) \frac{dN^q_{qg}(p+k,k)}{dkdt} - \mathcal{P}_q(p) \int_k \frac{dN^q_{qg}(p,k)}{dkdt} \\
&\quad + \int_k 2\mathcal{P}_g(p+k) \frac{dN^g_{q\bar{q}}(p+k,k)}{dkdt}\,, \\
\frac{d\mathcal{P}_g(p)}{dt} &= \int_k \mathcal{P}_q(p+k) \frac{dN^q_{qg}(p+k,p)}{dkdt} + \int_k \mathcal{P}_g(p+k) \frac{dN^g_{gg}(p+k,k)}{dkdt} \\
&\quad - \mathcal{P}_g(p) \int_k \left(\frac{dN^g_{q\bar{q}}(p,k)}{dkdt} + \frac{dN^g_{gg}(p,k)}{dkdt} \Theta(k-p/2) \right)
\end{aligned}
$$

McGill-AMY

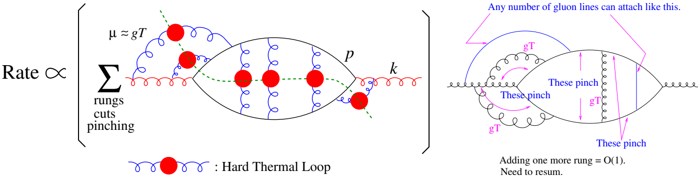


- Modified fragmentation function with jet initial condition $\mathbf{s}$, $\mathbf{n}$, p_i

$$\bar{D}_{\pi^0,c}(z,Q;\mathbf{s},\mathbf{n}) = \int dp_f \frac{z'}{z}\left(\mathcal{P}_{qq/c}(p_f;p_i)D_{\pi^0/q}(z',Q) + \mathcal{P}_{g/c}(p_f;p_i)D_{\pi^0/g}(z',Q)\right),$$

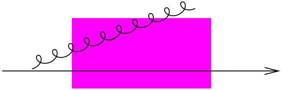
$$\tilde{D}(z,Q) = \int d^2s\, \frac{T_A(\mathbf{s})T_B(\mathbf{s}+\mathbf{b})}{T_{AB}(\mathbf{b})}\, \bar{D}_{\pi^0,c}(z,Q;\mathbf{s},\mathbf{n})$$

McGill-AMY

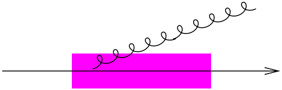


- Collision geometry including path length fluctuations are all included.
- Both BH and LPM limits included
- Includes all leading order splittings
- Includes thermal absorption
- All produced quarks and gluons fragment
- Medium evolution ($T(t, \mathbf{x})$, $u_\mu(t, \mathbf{x})$) fully taken into account including the effect of flow vector
- Easy to add other process such as elastic coll. γ production within leading order QCD/QED.

What is not included yet (vacuum-medium interference)



Included in the PDF scale dependence



Correctly dealt with in the AMY−McGill approach

Part of this in the fragmentation function

These two can interfere.

What is not included yet (vacuum-medium interference)

- The L^2 dependence in the heuristic BDMPS expression we got before

$$\ln R_{AA} \approx -n\frac{\alpha_S N_c}{\pi}\left(1 - \frac{L\mu^2}{E} + \frac{E_L}{E}\right)$$

  cannot be reproduced since original AMY always assumes $L > l_{\rm coh}$.
- Finite size effect is being worked on (Caron-Huot and Gale).