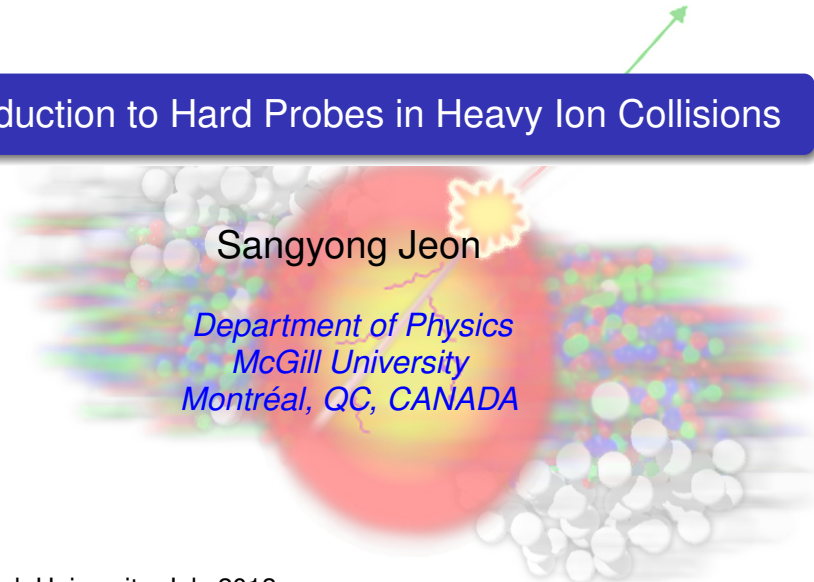


# Introduction to Hard Probes in Heavy Ion Collisions



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NNSPP

Stonybrook University, July 2013

# McGill is in Montréal, Québec, Canada



Montreal  
QC  
Canada

# McGill is in Montréal, Québec, Canada



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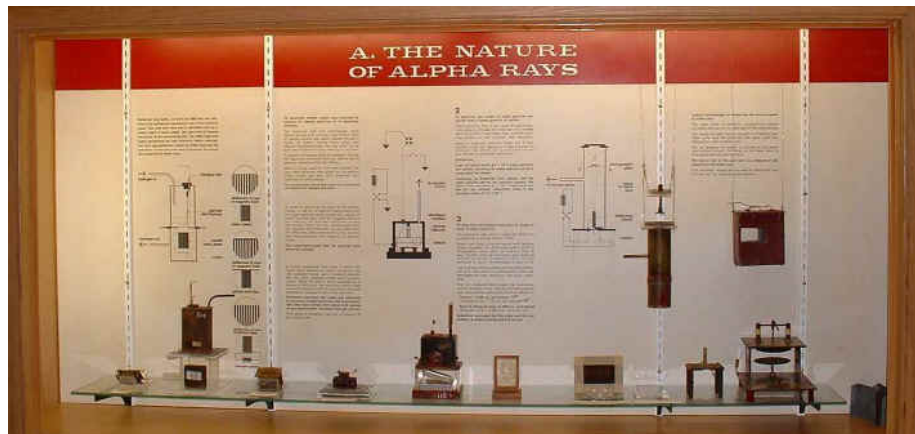


# McGill is in Montréal, Québec, Canada



Mr. McGill going home after a hard day's work.

# McGill is in Montréal, Québec, Canada



Rutherford carried out his Nobel (1908) winning work at McGill (1898-1907).

His *original* equipments on display

- Charles Gale
- Sangyong Jeon
- *Björn Schenke*  
(Formerly McGill, now BNL)
- *Clint Young*  
(Formerly McGill, Now UMinn)
- *Gabriel Denicol*
- *Matt Luzum*
- Sangwook Ryu
- Gojko Vujanovic
- Jean-Francois Paquet
- Michael Richard
- Igor Kozlov
- Khadija El Berhoumi
- Jean-Bernard Rose

# Relativistic Heavy Ion Collisions

- Why do it?
  - To study QGP
  - Most extreme environment ever created:  $T \sim 1 \text{ GeV}$ .  
This existed only at around 1 microsecond after the Big Bang
- How do we understand it?
  - Theory: Many-body QCD
  - Experimental probes:
    - Soft
    - Hard

# Hard Probes are useful

- Hard Probes  $\sim$  Large momentum/energy phenomena
- pQCD applies – We know how to do this
- Produced *before* QGP is formed in the same way as in hadron-hadron collisions
- Difference between  $pp$ ,  $pA$  and  $AA$  tells us about the medium.
- Caveat: How well do we know the *nuclear initial state*?



# What do we want to learn?

- Medium properties
  - What is it made of? Quarks? Gluons? Hadrons?
  - Thermodynamic properties – Temperature, Equation of state, etc.
  - Transport properties – Mean-free-path, transport coefficients, etc.
- Tools
  - Jets
  - Hard Photons

- 1 pQCD
  - 2 Jet Quenching
  - 3 Hard Photons
- My goal for these lectures: Qualitative understanding

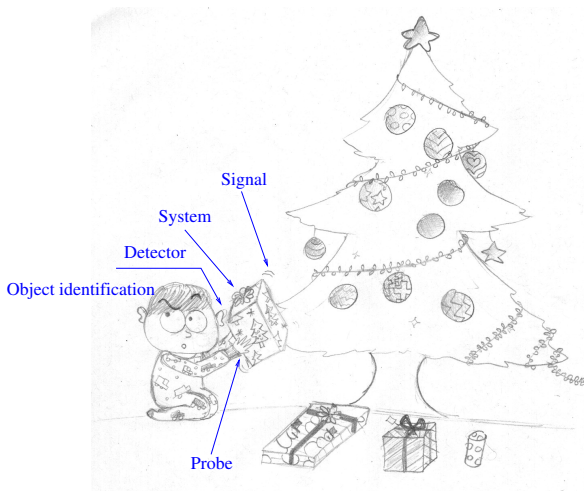
# What is a hard probe?

- Early hard probe experiments



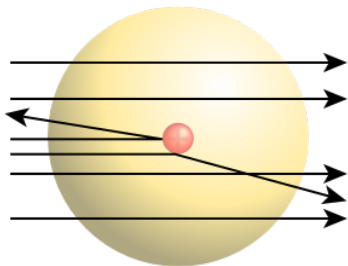
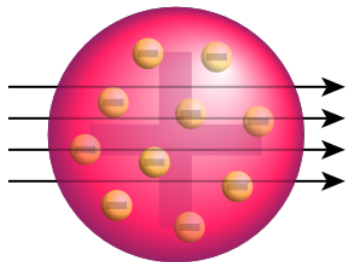
# What is a hard probe?

- Early hard probe experiments



# What is a hard probe?

- Early hard probe experiments



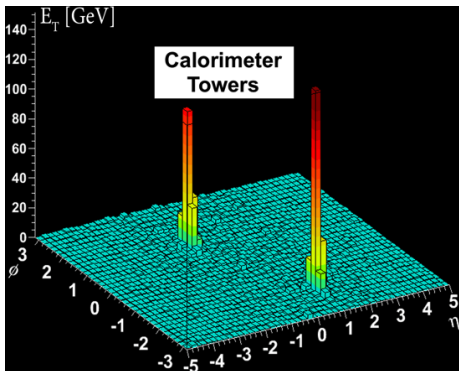
- Rutherford's  $\alpha$  scattering experiment (1911)

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi}{2} Z^2 \alpha_{\text{EM}}^2 \left( \frac{\hbar c}{E_{\text{kin}}} \right)^2 \times \frac{1}{(1 - \cos\theta)^2}$$

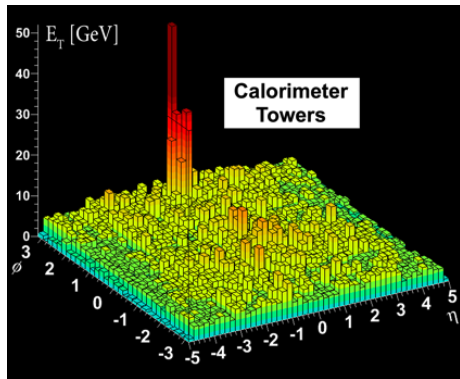
- Small angle scattering dominates  $d\sigma/d\cos\theta \propto 1/\theta^4$
- But backscattering prob. is finite, favoring Rutherford's model over Thompson's (which causes no backscattering)



# Fast-forward to the present



ATLAS: Intact dijets in Pb+Pb



ATLAS: One jet is fully quenched in Pb+Pb

- Simplest conclusion to draw: The medium is *opaque*.
- We want to know much more than that!

# Hard Probe Requirements

- Must be known & calculable using pQCD.
- Must be created *before* QGP forms
- Both requirements satisfied if the energy scale is much large compared to  $\Lambda_{\text{QCD}} \approx 200 \text{ MeV}$  and the length (time) scale is much shorter than  $\sim 1 \text{ fm}$ .
- Example: Jets (high energy partons) with  $E \gg 1 \text{ GeV}$  and Heavy quarks ( $c, b$ ) with  $M \gg 1 \text{ GeV}$

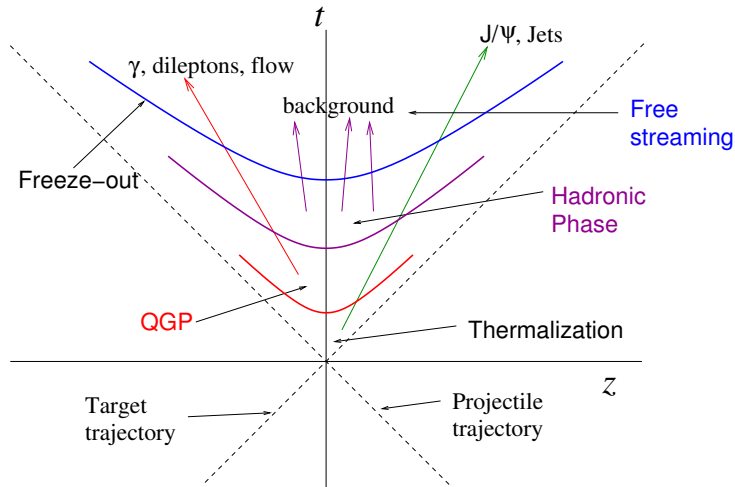
## Probes

- Propagation of hard partons or “Jets”
- Quarkonium suppression
- High  $p_T$  electromagnetic probes (real and virtual photons)

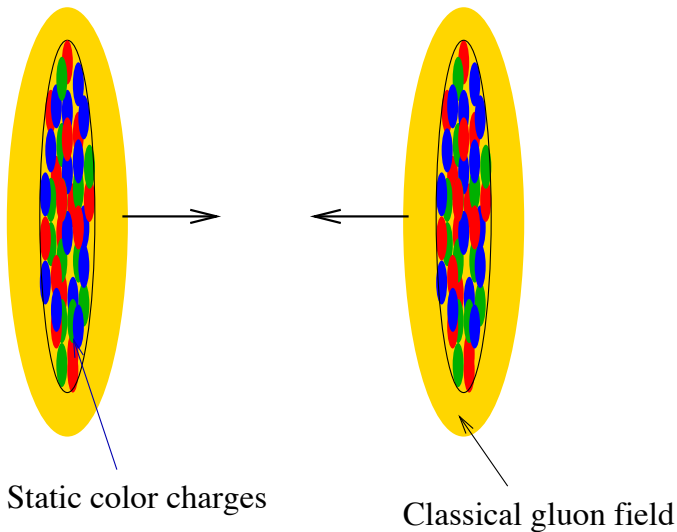
## Goal

- To characterize *QGP*
- To characterize initial state (nPDF, CGC?)

# (Very) Schematic view of heavy ion collisions



# (Very) Schematic view of heavy ion collisions





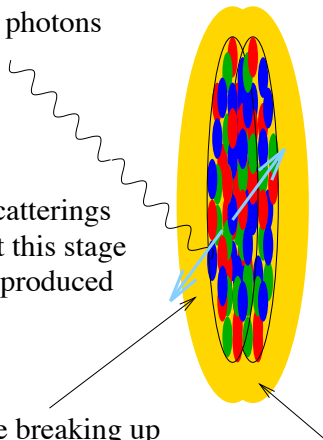
# (Very) Schematic view of heavy ion collisions

High energy photons

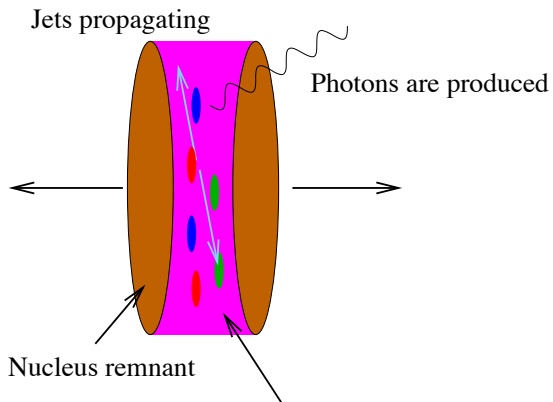
Hard Scatterings  
occur at this stage  
Jets are produced

Nuclei are breaking up

Gluon fields are grabbing each other

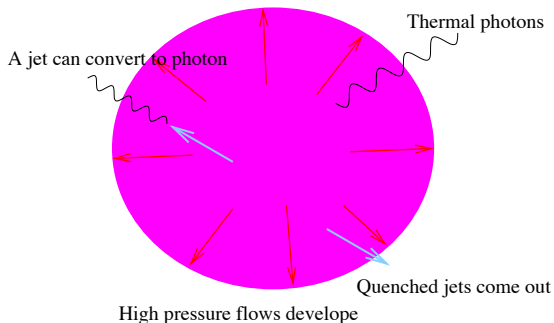


# (Very) Schematic view of heavy ion collisions



Entropy is produced.  
Pre-equilibrium mix of streaming quarks,  
gluons and classical gluon field.

# (Very) Schematic view of heavy ion collisions



# Review of some basic concepts - Feynman Rules

(a) Propagators: Gluon, quark, and ghost lines of momentum  $k$

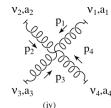
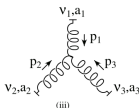
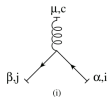
$$v, a \text{ --- } \text{Gluon} \text{ --- } \mu, b \quad i \frac{\delta_{ab}}{k^2+i\epsilon} [-g^{\mu\nu} + (1 - \lambda) \frac{k^\mu k^\nu}{k^2}] \quad \text{covariant gauge}$$

$$i \frac{\delta_{ab}}{k^2+i\epsilon} [-g^{\mu\nu} + \frac{k^\mu k^\nu}{\omega^2} - n^2 \frac{k^\mu k^\nu}{(n \cdot k)^2}] \quad \text{physical gauge}$$

$$\alpha, i \xrightarrow{k} \beta, j \quad i \frac{\delta_{ij}}{k \not{+} m + i\epsilon} [k + m]_{\beta\alpha}$$

$$a \text{ --- } b \quad i \frac{\delta_{ab}}{k^2+i\epsilon}$$

(b) Vertices (all momenta defined to flow in)



(i)  $-ig[T_c^F]_{\beta\alpha} \gamma_\mu$

(ii)  $gC_{abc}k'_\mu$

(iii)  $-gC_{\alpha_1\alpha_2\alpha_3} [g^{\alpha_1\alpha_2}(p_1 - p_2)^{\alpha_3} + g^{\alpha_2\alpha_3}(p_2 - p_3)^{\alpha_1} + g^{\alpha_3\alpha_1}(p_3 - p_1)^{\alpha_2}]$

(iv)  $-ig^2 [ C_{\alpha_1\alpha_2\alpha_3} C_{\alpha_3\alpha_4\alpha_1} (g^{\alpha_1\alpha_2} g^{\alpha_3\alpha_4} - g^{\alpha_1\alpha_4} g^{\alpha_2\alpha_3}) + C_{\alpha_1\alpha_3\alpha_2} C_{\alpha_2\alpha_4\alpha_1} (g^{\alpha_1\alpha_2} g^{\alpha_3\alpha_4} - g^{\alpha_1\alpha_4} g^{\alpha_2\alpha_3}) + C_{\alpha_1\alpha_4\alpha_2} C_{\alpha_2\alpha_3\alpha_1} (g^{\alpha_1\alpha_2} g^{\alpha_3\alpha_4} - g^{\alpha_1\alpha_4} g^{\alpha_2\alpha_3}) ]$

Figure 1: Perturbation theory rules for QCD.

$$\bullet G_{ba}^{\mu\nu} = \frac{i\delta_{ba}}{k^2+i\epsilon} \left( -g^{\mu\nu} + (1 - 1/\lambda) \frac{k^\mu k^\nu}{k^2+i\epsilon} \right)$$

$$\bullet S_{ij}^{\beta\alpha} = i \frac{\delta_{ij}}{k^2 - m^2 + i\epsilon} (k_\mu \gamma^\mu + m)_{\beta\alpha}$$

$$\bullet -ig[T_c^F]_{\beta\alpha} [\gamma_\mu]_{\beta\alpha}$$

$$\bullet -gf_{a_1 a_2 a_3} (g^{\nu_1 \nu_2} (p_1 - p_2)^{\nu_3} + \text{perm.})$$

$$\bullet -ig^2 (f_{ea_1 a_2} f_{ea_3 a_4} (g^{\nu_1 \nu_3} g^{\nu_2 \nu_4} - g^{\nu_1 \nu_4} g^{\nu_2 \nu_3}) + \text{perm. on } 234)$$

# Review of some basic concepts

- Basic unit:

$$\hbar c = 197.3 \text{ MeV} \cdot \text{fm} \approx 0.2 \text{ GeV} \cdot \text{fm}$$

- With  $\hbar = c = 1$

- Units

Mass:  $\text{GeV}/c^2$

Momentum:  $\text{GeV}/c$

Energy:  $\text{GeV}$

Length:  $\hbar c/\text{GeV}$

- $200 \text{ MeV} \leftrightarrow 1/\text{fm}$

- $1 \text{ fm} \leftrightarrow 1/(200 \text{ MeV})$

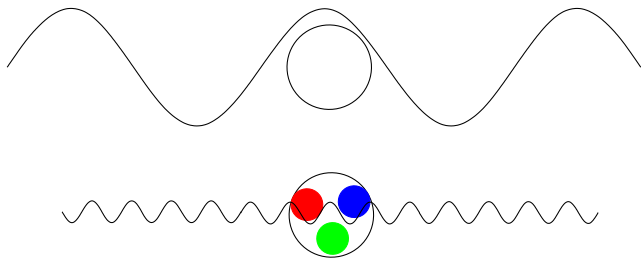
- Thermal energy  $k_B = 8.617 \times 10^{-5} \text{ eV K}^{-1}$

With  $k_B = 1$ ,

$1 \text{ eV} = 11,605 \text{ K}$  or  $290 \text{ K} \approx \frac{1}{40} \text{ eV}$

# Review of some basic concepts

- Spatial resolution:  $\Delta x \Delta p \geq 1/2$



- Shorter the wavelength (larger the momentum) sees spatial details up to  $\Delta x \approx \lambda$ .

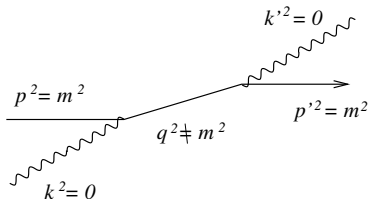
# Review of some basic concepts

Energy-Time uncertainty:  $|\Delta E|\Delta t \geq 1/2$

- $\Delta E = p^0 - \sqrt{\mathbf{p}^2 + m^2}$ .
- If  $\Delta E = 0$ , then  $p^\mu p_\mu = m^2$ : On-shell
- If  $\Delta E \neq 0$ , the  $p^\mu p_\mu \neq m^2$ : Off-shell

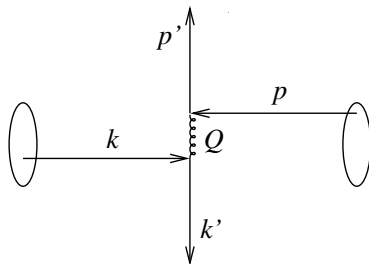
Interpretation

- An off-shell state can exist only for  $\Delta t \sim 1/|\Delta E|$ .



This interaction lasts  $\Delta t \sim 1/(|\mathbf{p}| + |\mathbf{k}| - \sqrt{(\mathbf{p} + \mathbf{k})^2})$

# Hard Probe time scale



Off-shell scale with  $k' = k + Q$

$$Q^2 = (k - k')_\mu (k - k')^\mu = (|\mathbf{k}| - |\mathbf{k}|)^2 - (|\mathbf{k}| - 0)^2 - (0 - |\mathbf{k}|)^2 \propto \sqrt{s}$$

Time scale:

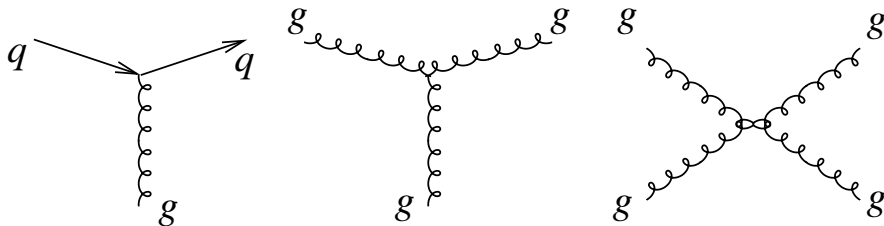
$$\Delta\tau \sim 1/\sqrt{|(k - k')^2|} \sim 1/\sqrt{s}$$



# Perturbative QCD

## QCD

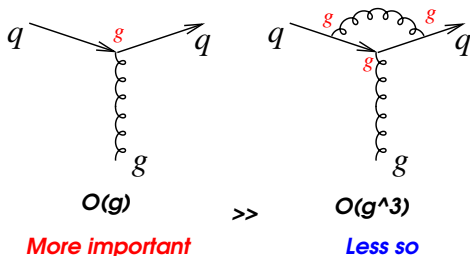
### – Interaction of quarks and gluons



- $N_f$  flavors of quarks
- $N_c^2 - 1$  gluons

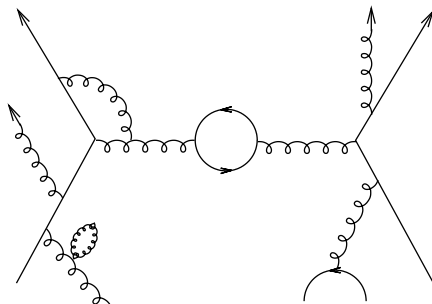
# Perturbative QCD (pQCD)

*Perturbation Theory when  $g \ll 1$*



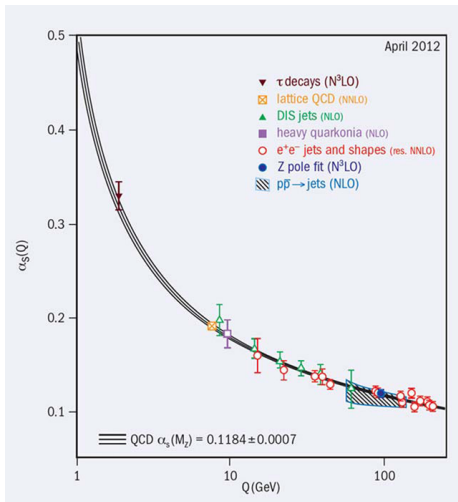
- Calculate physical quantities as an expansion in the small coupling constant  $g$
- Corrections to vertices
- Corrections to propagators

# Perturbative QCD (pQCD)



- Calculate physical quantities as an expansion in the small coupling constant  $g$
- Corrections to vertices
- Corrections to propagators

# Perturbative QCD (pQCD)



- Perturbative expansion possible because of asymptotic freedom

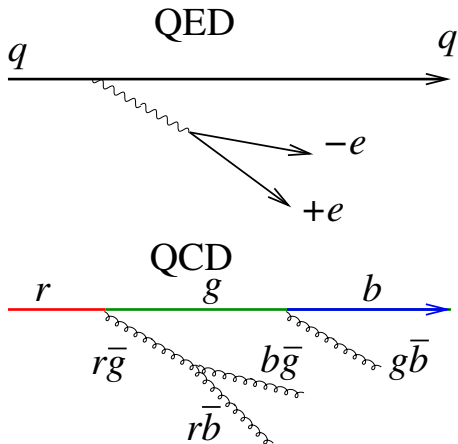
- $$Q^2 \frac{\partial \alpha_S}{\partial Q^2} = -\beta_0 \alpha_S^2 - \beta_1 \alpha_S^3 + \dots$$

- $$\alpha_S(Q^2) \approx \frac{1}{((33 - 2n_f)/12\pi) \ln(Q^2/\Lambda_{\text{QCD}}^2)}$$

- pQCD reliable for  $Q \gtrsim 1 \text{ GeV}$

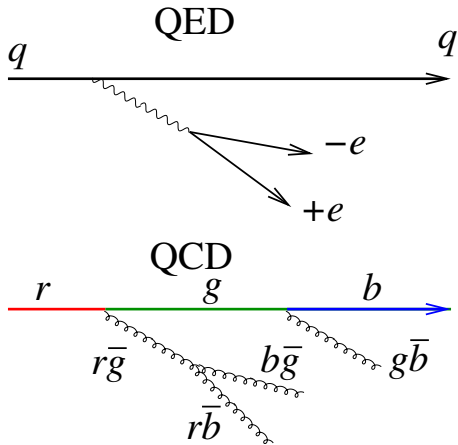
S. Bethke, arXiv:1210.0325.

# Intuitive understanding of asymptotic freedom



- QED: Surrounded by virtual  $e\bar{e}$  cloud
- Virtual  $-e$  cloud drawn closer to  $q > 0 \implies$  Screening
- Larger  $Q \implies$  smaller distance  $\implies$  Sees less of the cloud  $\implies$  Closer to bare charge
- Possible because the original  $q$  never changes and photons do not carry charges

# Intuitive understanding of asymptotic freedom



- QCD: Can resolve more soft virtual gluons at larger  $Q$
- The color of the real particle can change whenever a gluon is emitted.
- Larger  $Q \implies$  More frequent changes  $\implies$  Less average color charge  $\implies$  Asymptotic freedom

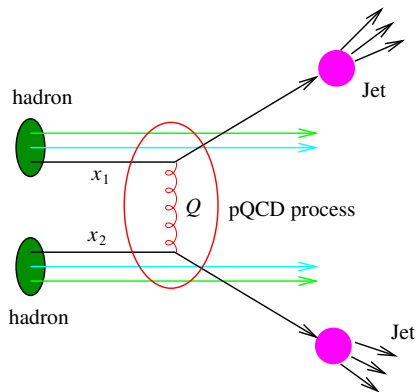
- As  $Q \rightarrow \Lambda_{\text{QCD}}$ ,

$$\alpha_S(Q^2) \approx \frac{1}{((33 - 2n_f)/12\pi) \ln(Q^2/\Lambda_{\text{QCD}}^2)} \rightarrow \infty$$

- Hadrons are  $O(\Lambda_{\text{QCD}})$  objects.
- Anything that has to do with hadron properties such as color confinement and hadronization is *non-perturbative*.
- In the IR limit, perturbation theory does not work  $\implies$  Factorize what can be calculated with pQCD (UV) and what cannot be calculated (IR)



# Factorization Theorem



Hadron-Hadron Jet production scheme:

$$\sigma = \int_{abcd} f_{a/A}(x_a, Q_f) f_{b/B}(x_b, Q_f) \times \sigma_{ab \rightarrow cd} D_{C/c}(z_C, Q)$$

# Factorization Theorem

- How realistic pQCD calculations are done

$$\sigma_{hh' \rightarrow C+X} = \int_{abcd} dx_1 dx_2 f_{a/h}(x_1, Q_f) f_{b/h'}(x_2, Q_f) \sigma_{ab \rightarrow cd}(Q_R) D_{C/c}(z_C, Q_f')$$

- $f_{a/h}(x_1, Q_f)$ : Parton distribution function. Probability to have a parton type  $a$  with the momentum fraction  $x_1$  in a hadron  $h$ . Depends on the factorization scale  $Q_f$ .
- $D_{C/c}(z_C, Q_f')$ : Fragmentation function. Probability to create a hadron type  $C$  out of parton type  $c$  carrying the momentum fraction  $z_C$ .
- $\sigma_{ab \rightarrow cd}(Q_R)$ : Parton-parton scattering cross-section.

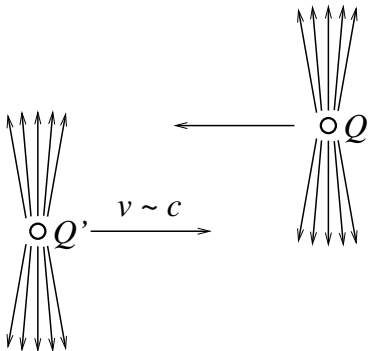
# Factorization Theorem

- How realistic pQCD calculations are done

$$\sigma_{hh' \rightarrow C+X} = \int_{abcd} dx_1 dx_2 f_{a/h}(x_1, Q_f) f_{b/h'}(x_2, Q_f) \sigma_{ab \rightarrow cd}(Q_R) D_{C/c}(z_C, Q'_f)$$

- pQCD controls the *evolutions* of  $f_{a/h}(x_1, Q_f)$  and  $D_{C/c}(z_C, Q'_f)$ . But pQCD cannot determine the initial data because this is dominated by IR processes.
- pQCD *can* calculate  $\sigma_{ab \rightarrow cd}(Q_R)$  when the renormalization scale  $Q_R$  can be set high (that is, when  $\sqrt{s}$  is large)

# How to think about the initial state factorization – QED analogy



- Weizsäcker-Williams field – Highly contracted in the  $z$  direction
- Coulomb potential in the rest frame of the charge

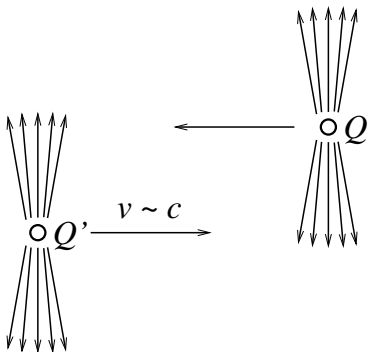
$$\varphi = Q/|\mathbf{r}|$$

- In the moving frame

$$A^\mu(x') = \Lambda_\nu^\mu A^\nu(x(x'))$$

- The coordinate in the moving frame  $x' = (t, x, y, z)$ . This corresponds to the rest frame position  $x = (t\gamma - z\gamma v, x, y, z\gamma - t\gamma v)$ .

# How to think about the initial state factorization – QED analogy



- Weizsäcker-Williams field – Highly contracted in the  $z$  direction
- Coulomb potential in the rest frame of the charge

$$\varphi = Q/|\mathbf{r}|$$

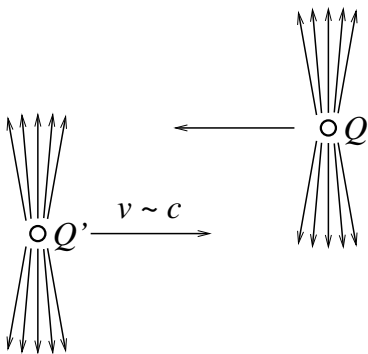
- In the moving frame

$$A^\mu = \frac{Q(\gamma, 0, 0, \gamma v)}{\sqrt{(z - vt)^2 \gamma^2 + \Delta \mathbf{x}_\perp^2}}$$

- Pure gauge in the  $v \rightarrow 1$  limit

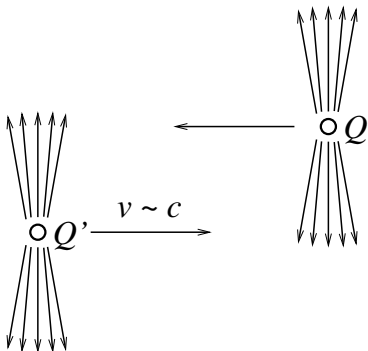
$$A^\mu \approx \frac{Q(1, 0, 0, 1)}{|z - vt|} = Q \partial_\mu \ln |z - vt|$$

# How to think about the initial state factorization – QED analogy



- Weizsäcker-Williams field – Highly contracted in the  $z$  direction  
 $F^{\mu\nu} \approx 0$  unless  $z \approx vt$
- In the rest frame: Coulomb field is made up of space-like virtual photons  
 $q^\mu q_\mu = -\mathbf{q}^2$  with  $q_0 = 0$ .
- In the Lab frame:  
 $q'^\mu = (q^z \sinh \eta, \mathbf{q}_\perp, q^z \cosh \eta)$
- For large  $\eta$ ,  
 $|\Delta E| = |q^- - |\mathbf{q}|| \sim e^{-\eta} \mathbf{q}^2 / q_z$   
 $\implies \Delta t \sim 1/|\Delta E| \sim e^\eta q_z / \mathbf{q}^2 \implies$  virtual photons look almost like real photons.

# How to think about the initial state factorization – QED analogy



- Weizsäcker-Williams field – Highly contracted in the  $z$  direction  
 $F^{\mu\nu} \approx 0$  unless  $z \approx vt$
- To a first approximation, the approaching particles *do not* know about each other until they are on top of each other.
- Initial photon momentum distribution *factorizes*:  $F(x_1, x_2) = f(x_1)f(x_2)$  but this is not exact.
- In QCD, color neutrality of hadrons help.

# DGLAP Equation

- $f(x, Q_f)$ : Probability density of partons with the virtuality *less than*  $Q_f$ .

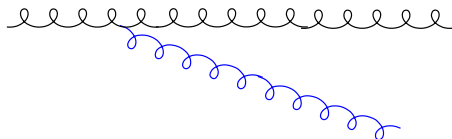


$Q_0$ : Coarse grained. You see one almost on-shell parton.



# DGLAP Equation

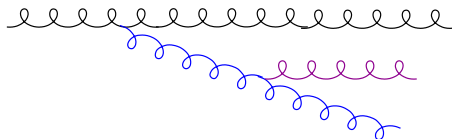
- $f(x, Q_f)$ : Probability density of partons with the virtuality *less than*  $Q_f$ .



$Q_0 < Q_1$ : Start to resolve another parton

# DGLAP Equation

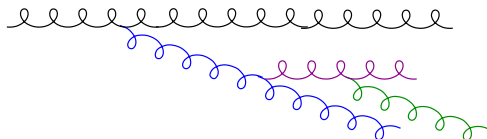
- $f(x, Q_f)$ : Probability density of partons with the virtuality *less than*  $Q_f$ .



$Q_0 < Q_1 < Q_2$ : And another

# DGLAP Equation

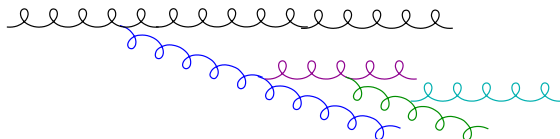
- $f(x, Q_f)$ : Probability density of partons with the virtuality *less than*  $Q_f$ .



$Q_0 < Q_1 < Q_2 < Q_3$ : And another

# DGLAP Equation

- $f(x, Q_f)$ : Probability density of partons with the virtuality *less than*  $Q_f$ .



You get the idea

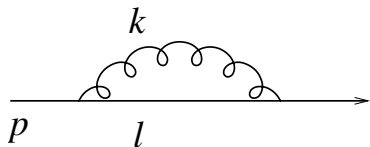
# DGLAP Equation

- $f(x, Q_f)$ : Probability density of partons with the virtuality *less than*  $Q_f$ .

$$Q^2 \frac{\partial}{\partial Q^2} \begin{pmatrix} q^S \\ g \end{pmatrix} = \frac{\alpha_S(Q^2)}{2\pi} \begin{pmatrix} P_{qq} & 2n_f P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q^S \\ g \end{pmatrix}$$

where  $P_{ij}$ : Splitting function  $\sim$  Probability to end up with  $ij$  in the final state.

# Splitting can cause IR divergence



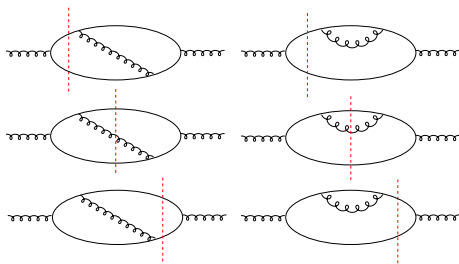
- $p$  is on-shell:  $p^2 = 0$
- Diverges when either  $k$  or  $l$  is on-shell
- This happens either  $k$  is very soft so that

$$l^2 = (p - k)^2 \approx p^2$$

- or  $p$  and  $k$  are almost collinear

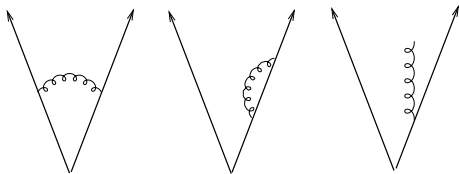
$$\begin{aligned} l^2 &= (p - k)^2 = p^2 + k^2 - 2pk \\ &\approx 0 \end{aligned}$$

# Splitting can cause IR divergence



- $g \rightarrow q\bar{q}$  and  $g \rightarrow q\bar{q}g$
- Only the *sum* is IR finite because soft and collinear divergences
- Splitting functions know about this

# Splitting can cause IR divergence



- Observables must be IR safe.
- 3rd diagram must be treated as 2-jet when the radiation is soft or collinear  $\Rightarrow$  IR-safe Jet definitions



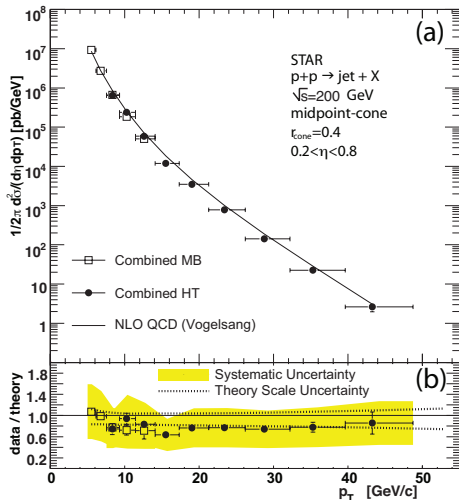
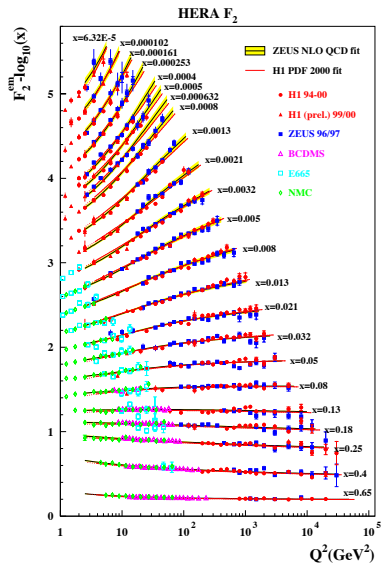
# Factorization Theorem

- Fragmentation function similarly runs
- 3 different scales:  $Q_f$  for the pdf,  $Q_R$  for  $\sigma(Q_R)$  and  $Q'_f$  for the fragmentation function
- In principle, physical observables should not depend on these scales. However, factorization theorem is only *approximate*.
- Lots of freedom to choose the scales. Usually something like

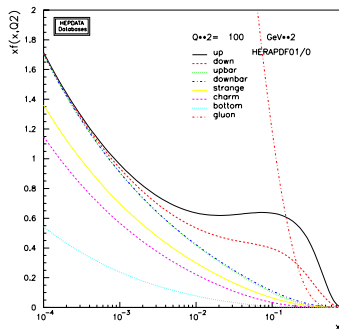
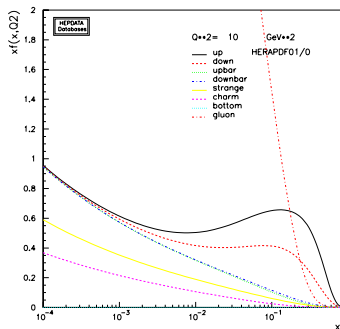
$$Q_f = Q_R = Q'_f = \#p_T$$

works OK where  $p_T$  is the momentum of the *final* state particle.

# pQCD & Factorization at work

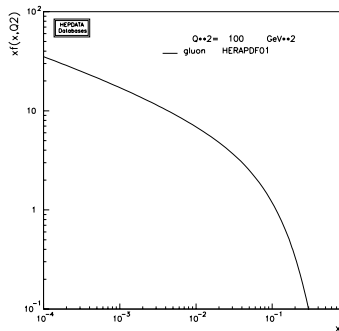
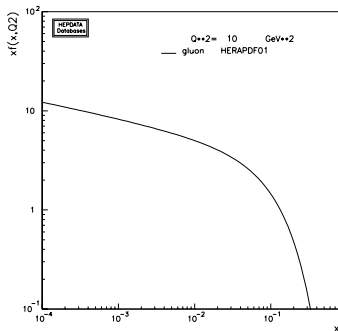


# pQCD & Factorization at work



- CTEQ 06 Proton PDF's
- Larger  $Q \implies$  More soft partons

# pQCD & Factorization at work



- Gluon distributions for  $Q^2 = 10 \text{ GeV}^2$  and  $Q^2 = 100 \text{ GeV}^2$ .