Lecture 2:
Elliptic, higher-order flow

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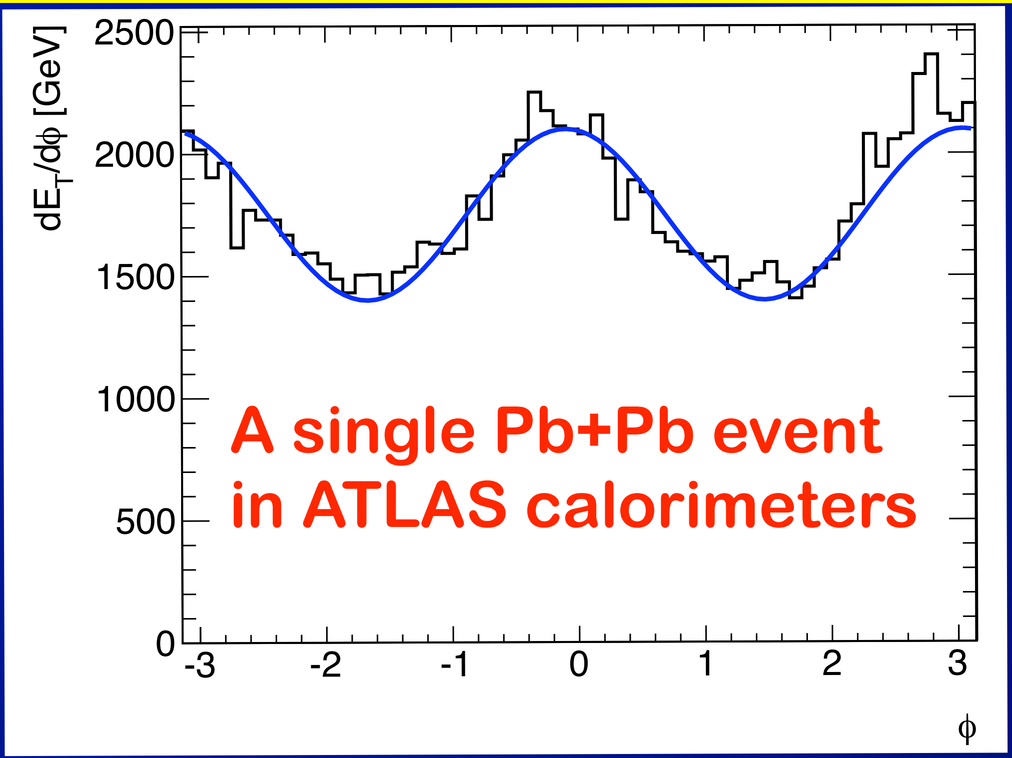
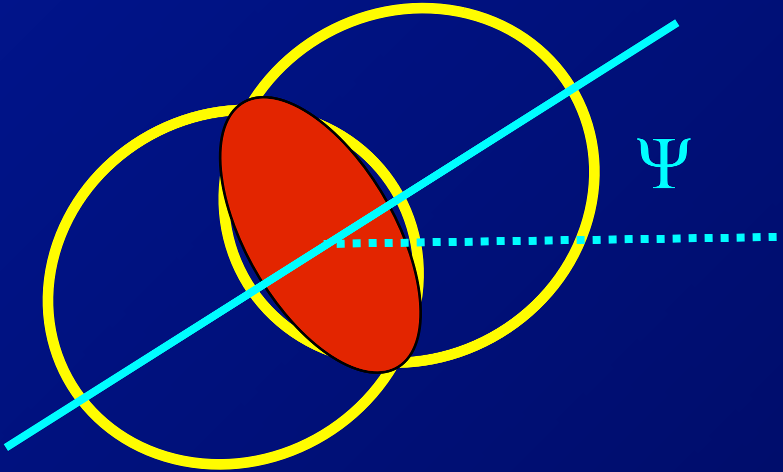
Elliptic flow

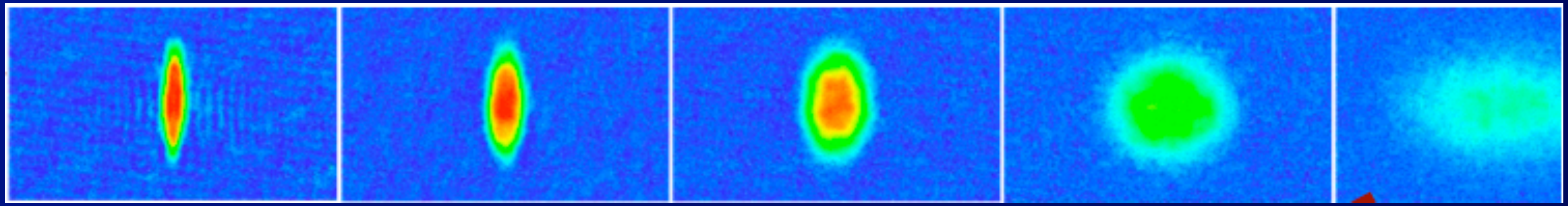
Collective Motion: Elliptic Flow



A single Pb+Pb event in ATLAS calorimeters

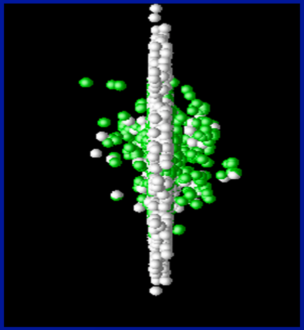


- Pressure-driven expansion converts spatial anisotropy to momentum anisotropy.



Hydrodynamics (ideal)

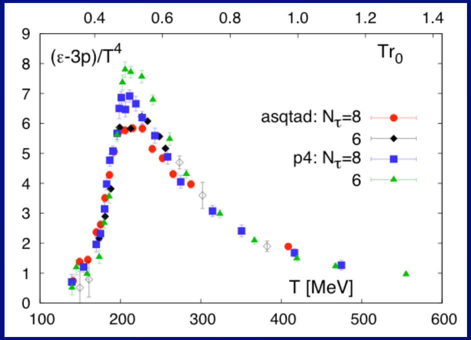
Initial conditions



E, p conservation

$$+ \quad \partial_\mu T^{\mu\nu} = 0 \quad +$$

equation of state

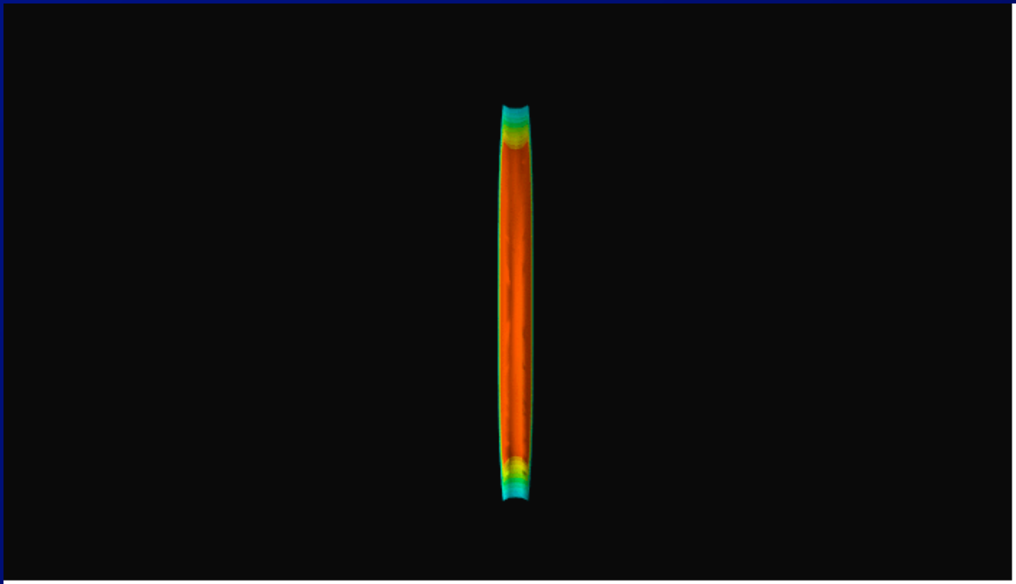


Ideal $T^{\mu\nu} \equiv (\varepsilon + p)\, u^\mu u^\nu - g^{\mu\nu} p$

energy density

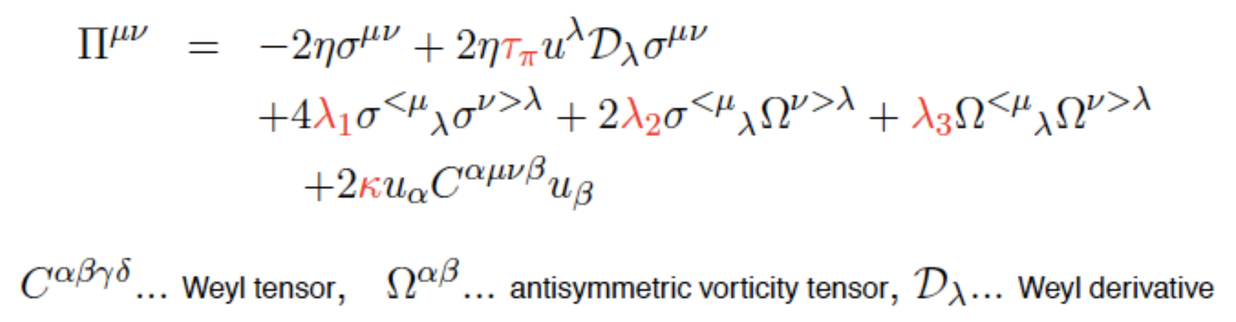
pressure

- Hydrodynamics can be viewed as long wave-length effective theory applicable when $\lambda_{MFP} \ll L$



Relativistic viscous hydrodynamics

- **Major theoretical breakthrough in last 5 years**
  - Partially driven by AdS/CFT
- **Viscous hydrodynamics can be viewed as an expansion in velocity gradients**
  - $T^{\mu\nu} \equiv (\varepsilon + p)\, u^\mu u^\nu + g^{\mu\nu} p + \Pi^{\mu\nu}$
  - $\Pi^{\mu\nu} = \eta\sigma^{\mu\nu} + 2^{\text{nd}}\ \text{order terms}$ with $\sigma^{\mu\nu} = \langle \nabla^\mu u^\nu \rangle$
- **First order expansion violates causality**
  - Need to go to second order
    - ⇒But, many possible terms at second order
- **Using AdS/CFT to constrain possibilities:**

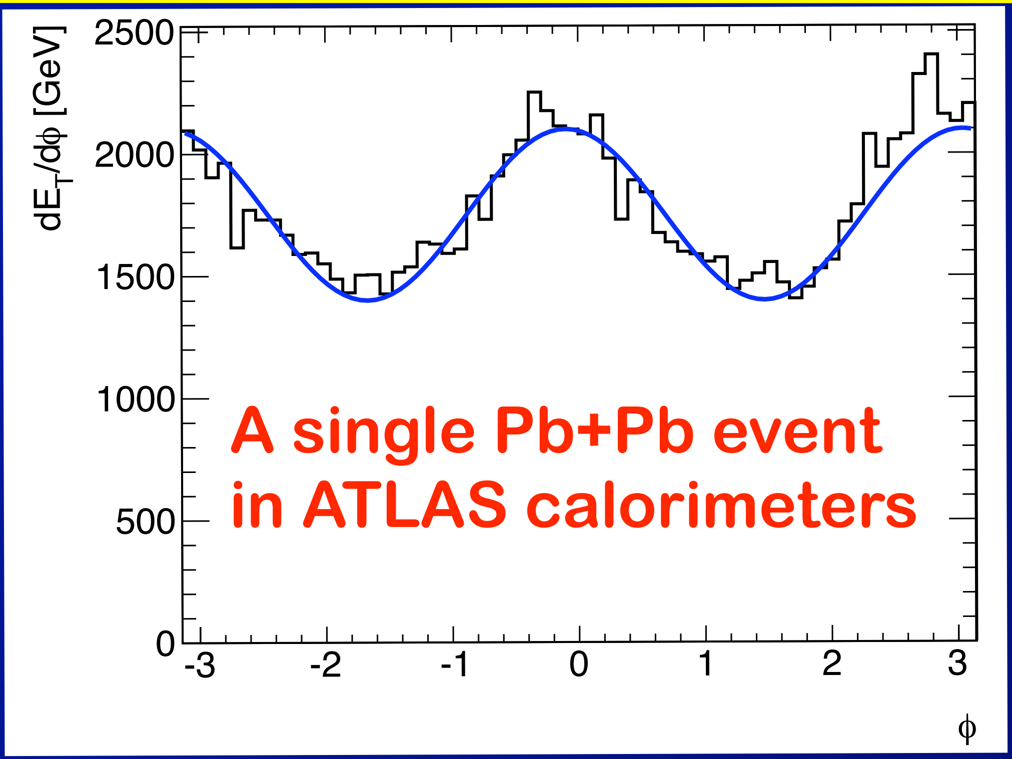


$$\begin{aligned}\Pi^{\mu\nu} \;=\;& -2\eta\sigma^{\mu\nu} + 2\eta\tau_\pi u^\lambda \mathcal{D}_\lambda \sigma^{\mu\nu} \\ & +4\lambda_1 \sigma^{<\mu}{}_\lambda \sigma^{\nu>\lambda} + 2\lambda_2 \sigma^{<\mu}{}_\lambda \Omega^{\nu>\lambda} + \lambda_3 \Omega^{<\mu}{}_\lambda \Omega^{\nu>\lambda} \\ & +2\kappa u_\alpha C^{\alpha\mu\nu\beta} u_\beta\end{aligned}$$

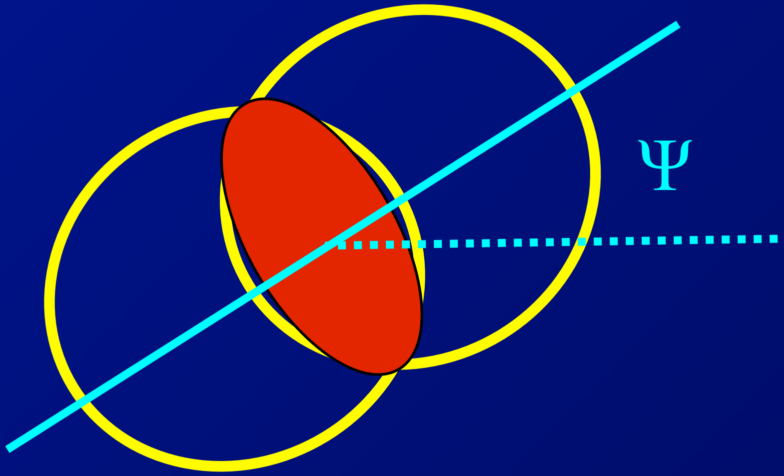
$C^{\alpha\beta\gamma\delta}$… Weyl tensor, $\Omega^{\alpha\beta}$… antisymmetric vorticity tensor, $\mathcal{D}_\lambda$… Weyl derivative

From an excellent colloquium at the Univ of Frankfurt by R. Baier

Collective Motion: Elliptic Flow



A single Pb+Pb event in ATLAS calorimeters

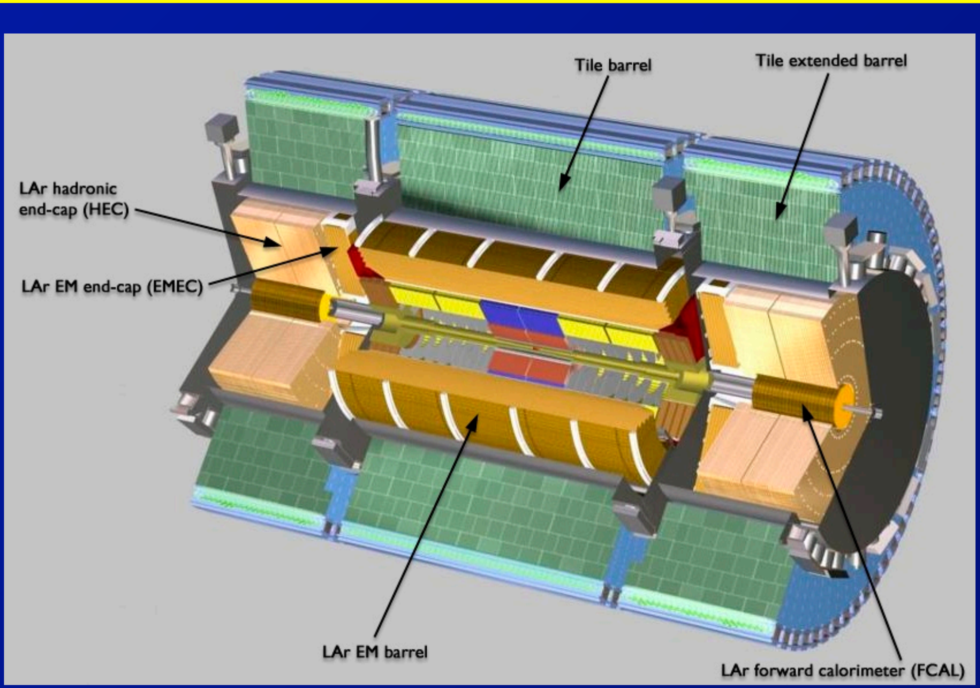


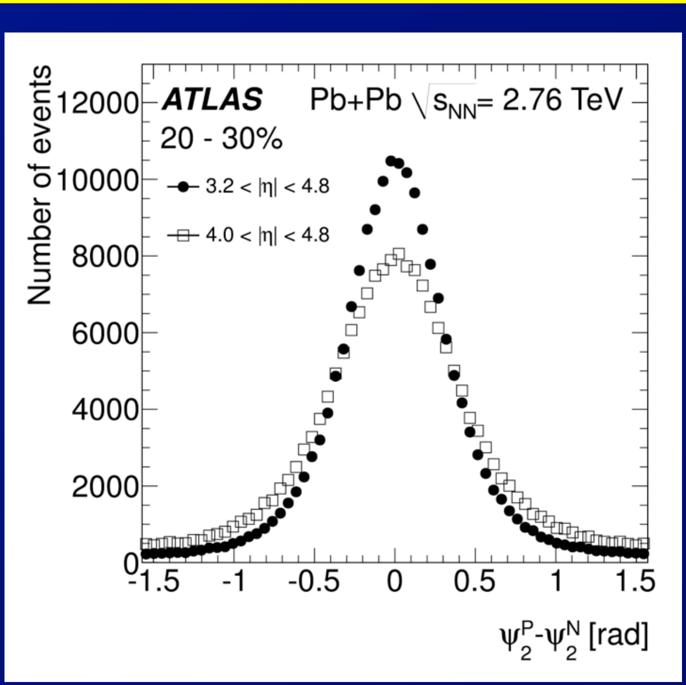
- Characterize the modulation in terms of 2nd Fourier coefficient

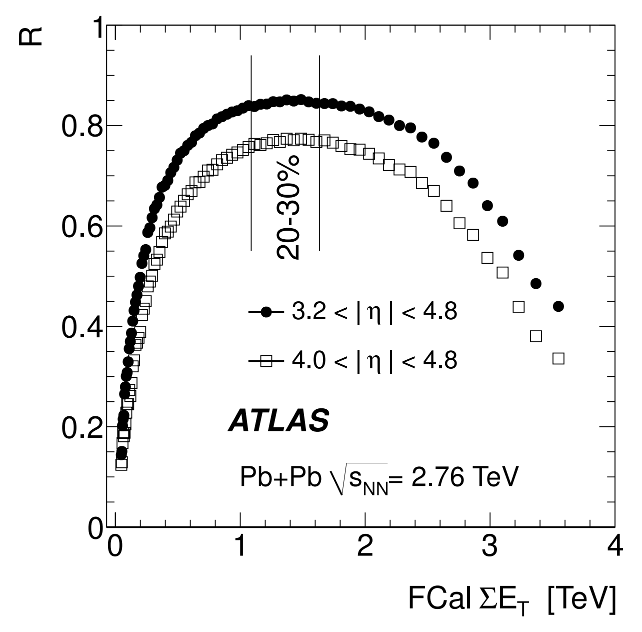
$$\frac{dN}{d\phi} = A\left(1 + 2v_2 \cos\left[2(\phi - \psi_{\mathrm{evt}})\right]\right)$$

  - v_2 measures relative modulation of particle yield or transverse energy as a function of ϕ

Event plane technique (e.g. ATLAS)







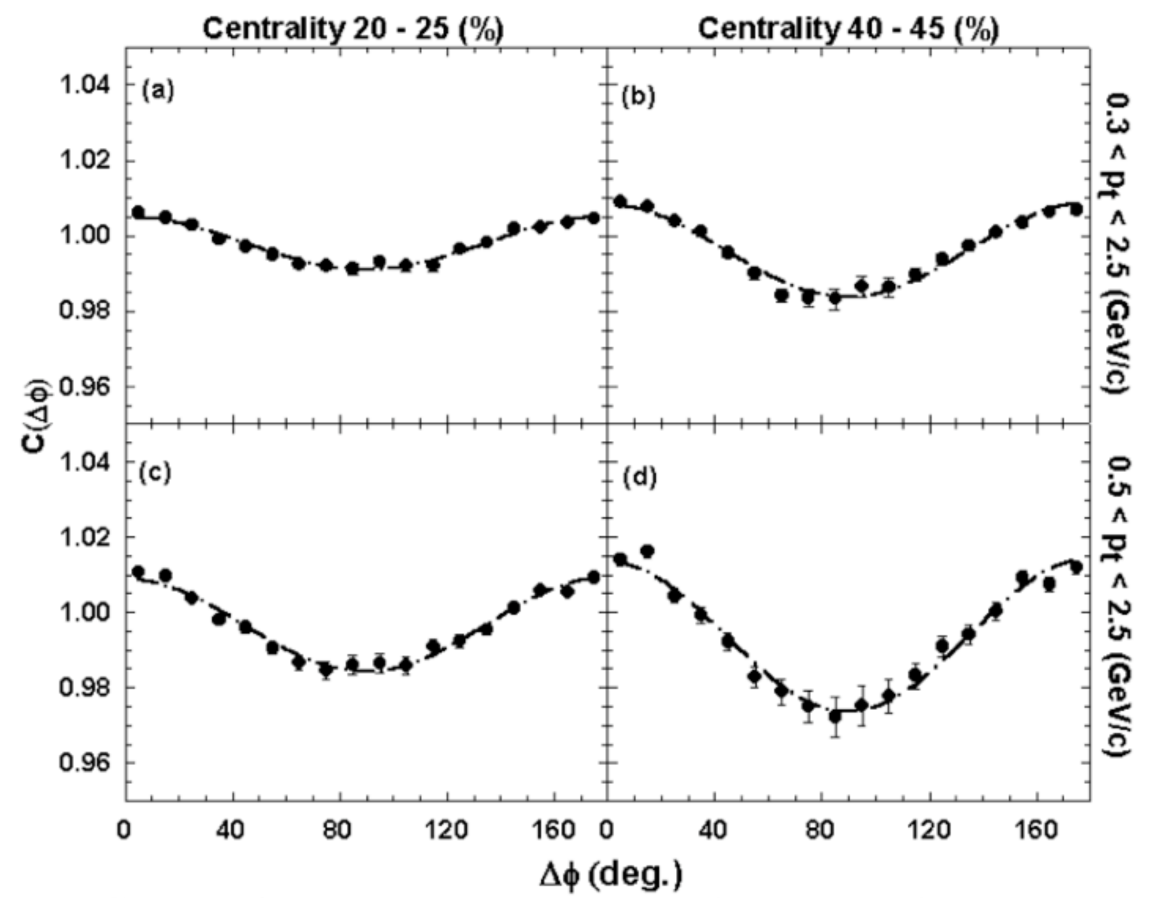
- Measure event plane in 2 or more "sub-events"
  - ATLAS: two sides of FCal
- Evaluate resolution using distribution of $\Delta\psi = \psi_1 - \psi_2$
- Calculate correction for v_2 measured using combined ψ

2-particle correlation method (PHENIX)

PHENIX
Phys. Rev. Lett. 89, 212301 (2002)

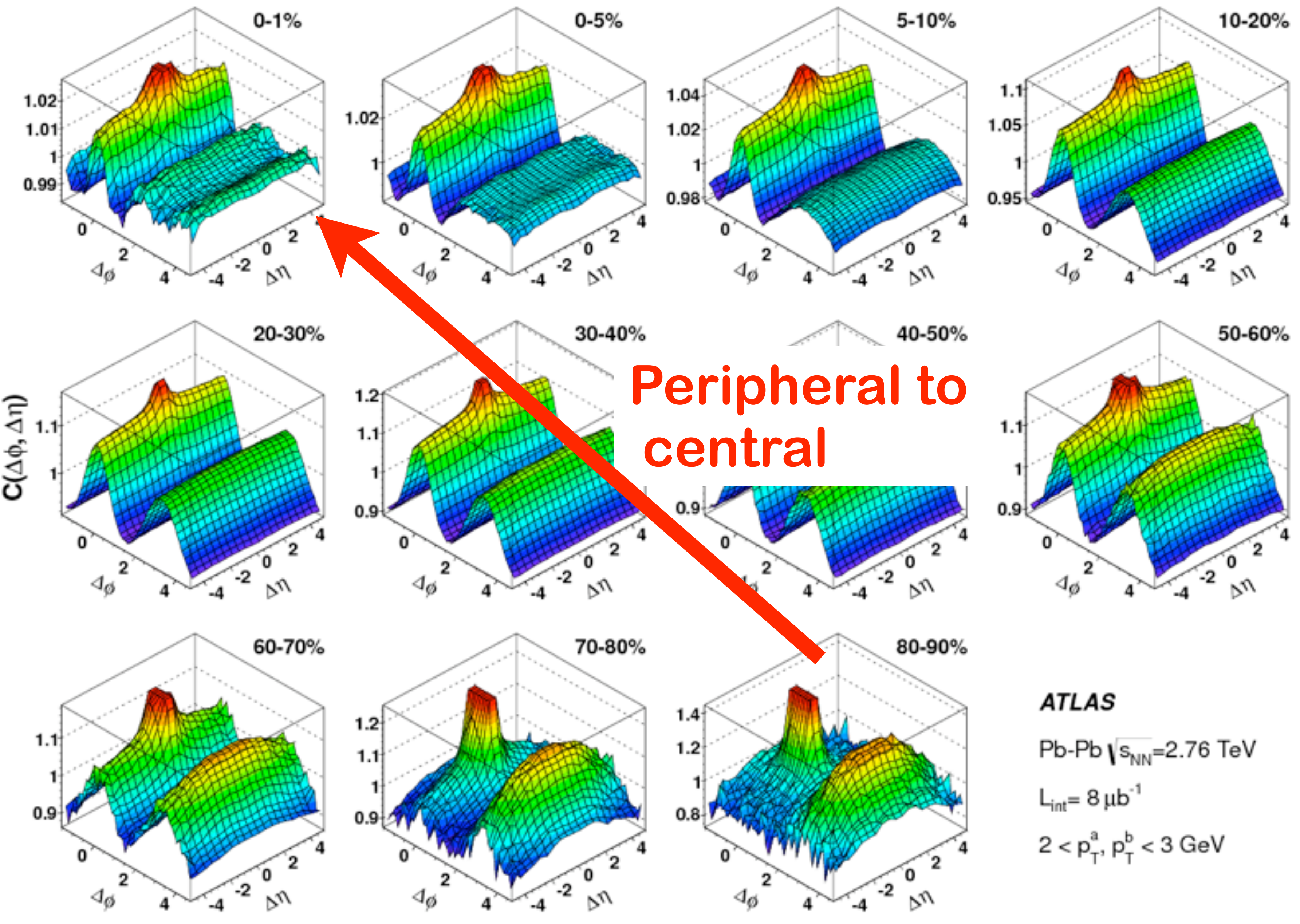
$$C(\Delta\phi) = \frac{N_{cor}(\Delta\phi)}{N_{uncor}(\Delta\phi)}$$

$$\frac{dN}{d\Delta\phi} \propto \left(1 + \sum_{n=1}^{\infty} 2v_n^2 \cos(n\Delta\phi)\right)$$



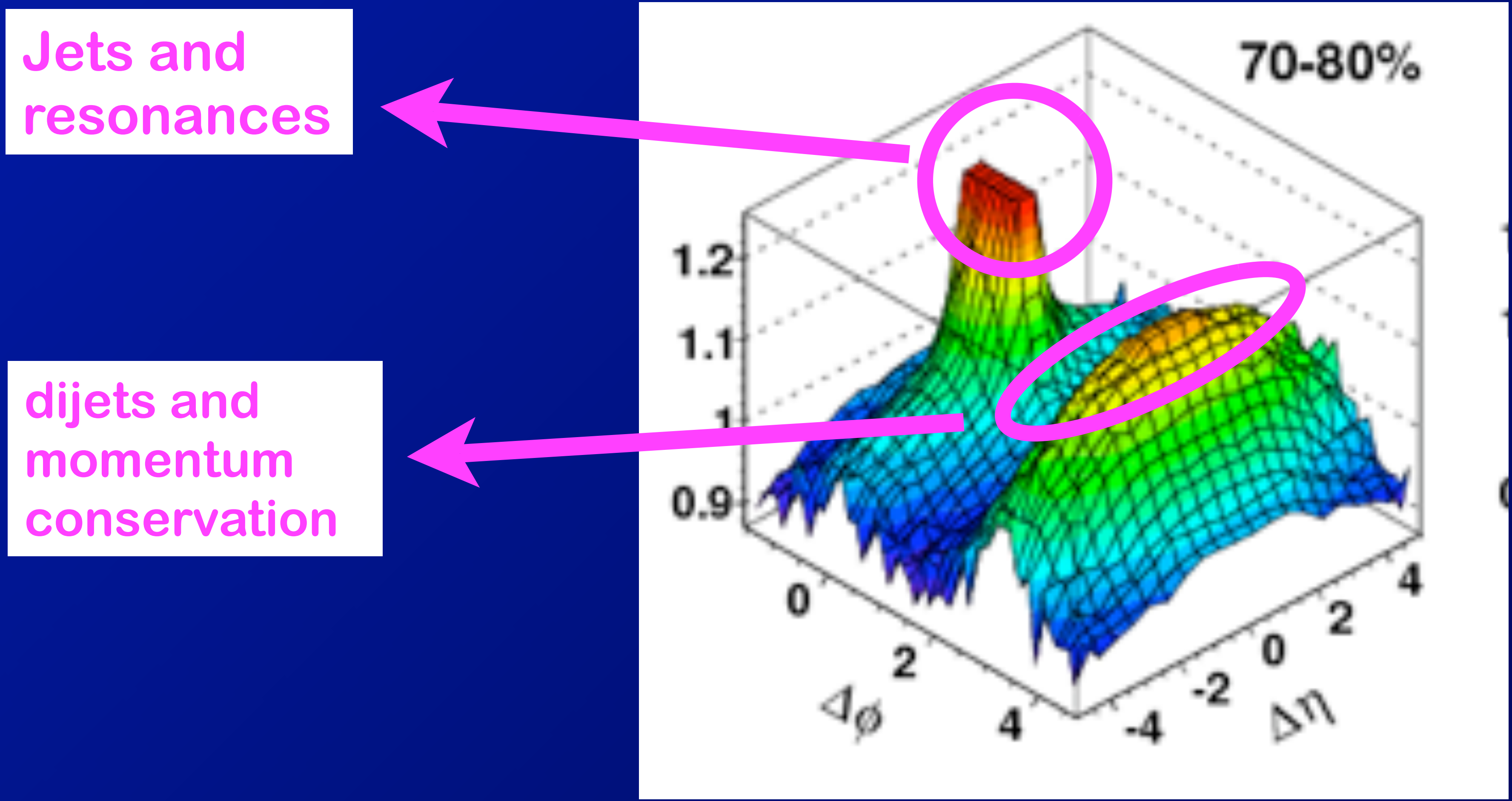
- 2-particle correlation function, C(Δφ), has Fourier coefficient $v_{21} \times v_{22}$
  - Doesn't need event plane, resolution correction
  - May be more sensitive to non-flow correlations

ATLAS: 2-particle correlations



Peripheral to central

ATLAS: 2-particle correlations



Jets and resonances

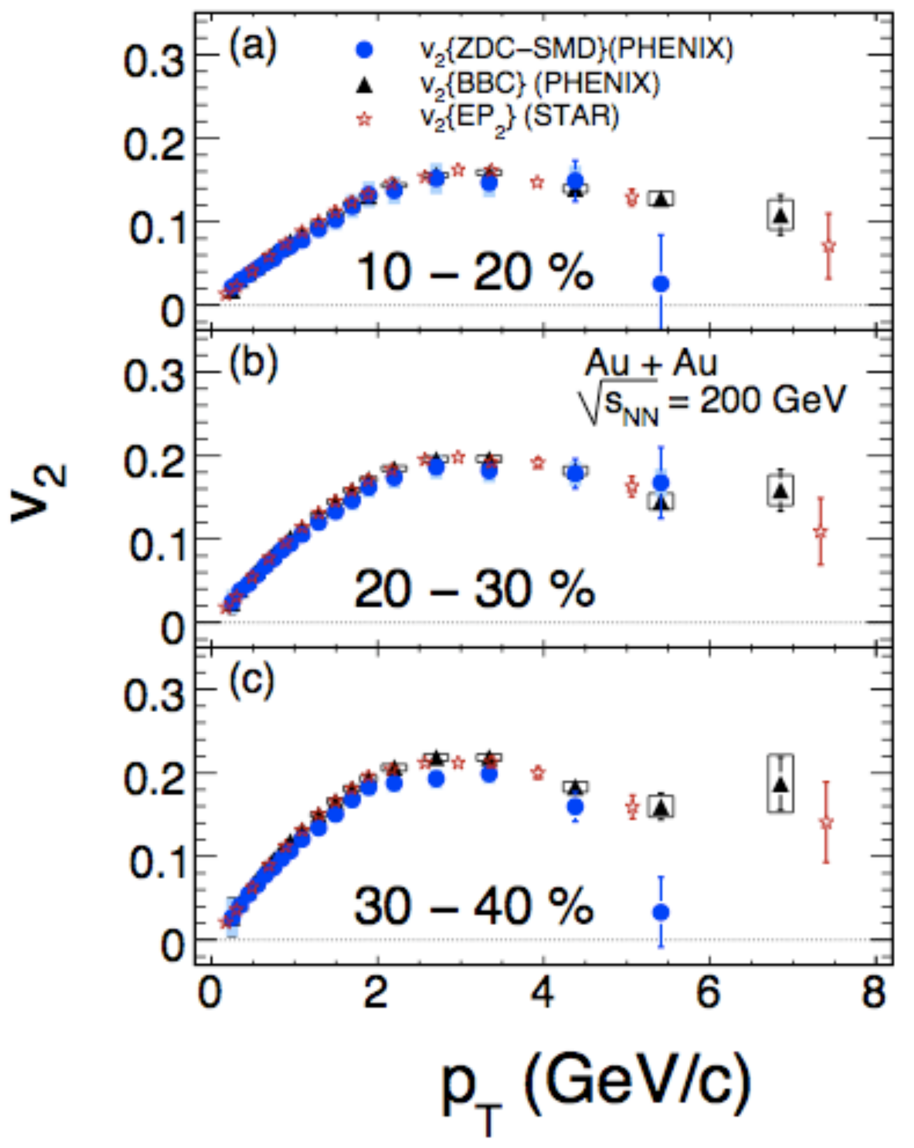
dijets and momentum conservation

- Jet contributions to 2-particle correlations can be suppressed with Δη cut
  - But dijets and momentum conservation can produce long-range non-flow effects

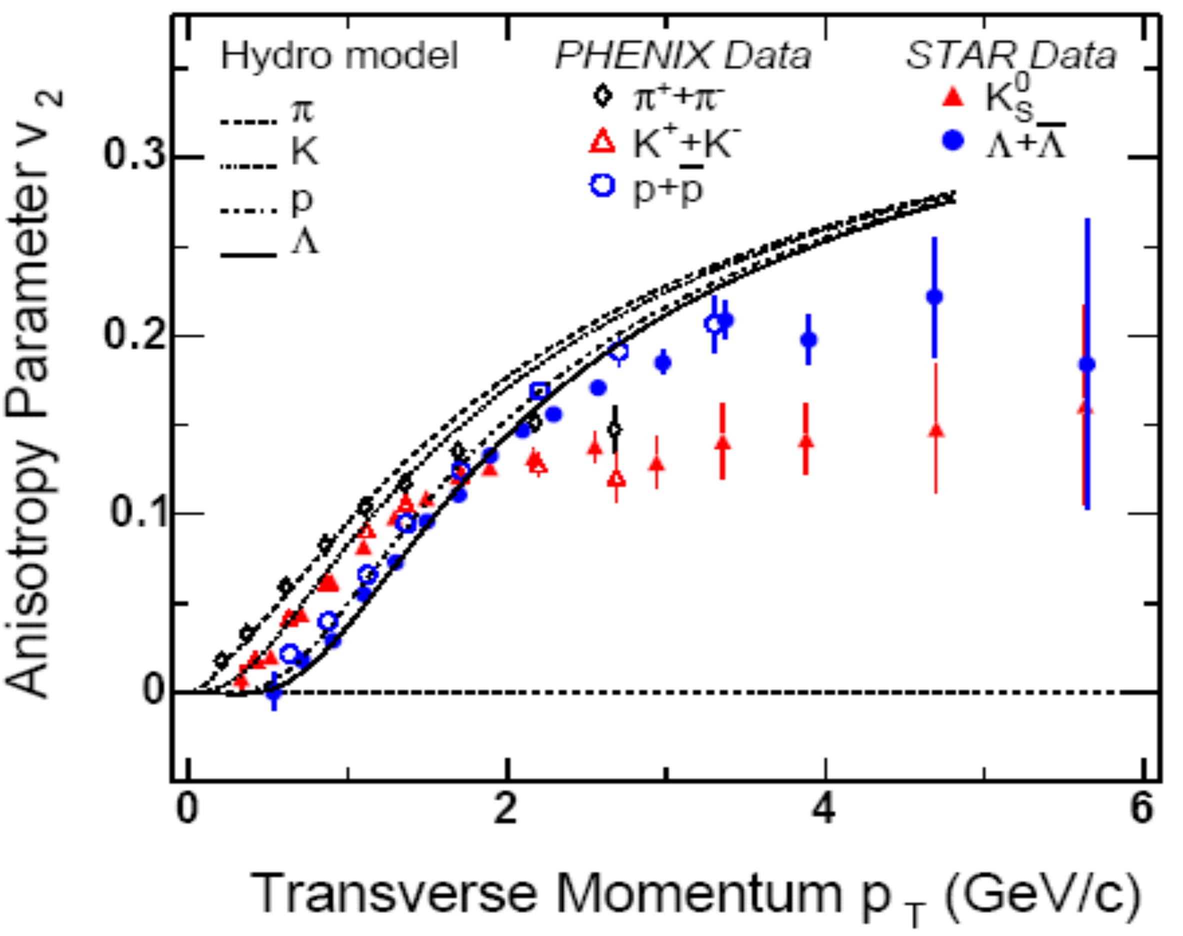
$v_2(p_T)$ at RHIC

- charged particle v_2 as a function of p_T
  - Characteristic, ~ linear dependence at low p_T

    ⇒kinematic effect, flow arises from velocity boost to particles
- Good agreement between PHENIX, STAR using event plane method
  - Measure ψ_2 event-by-event, determine v_2 with respect to that direction



Particle identified v_2 at RHIC (2003)

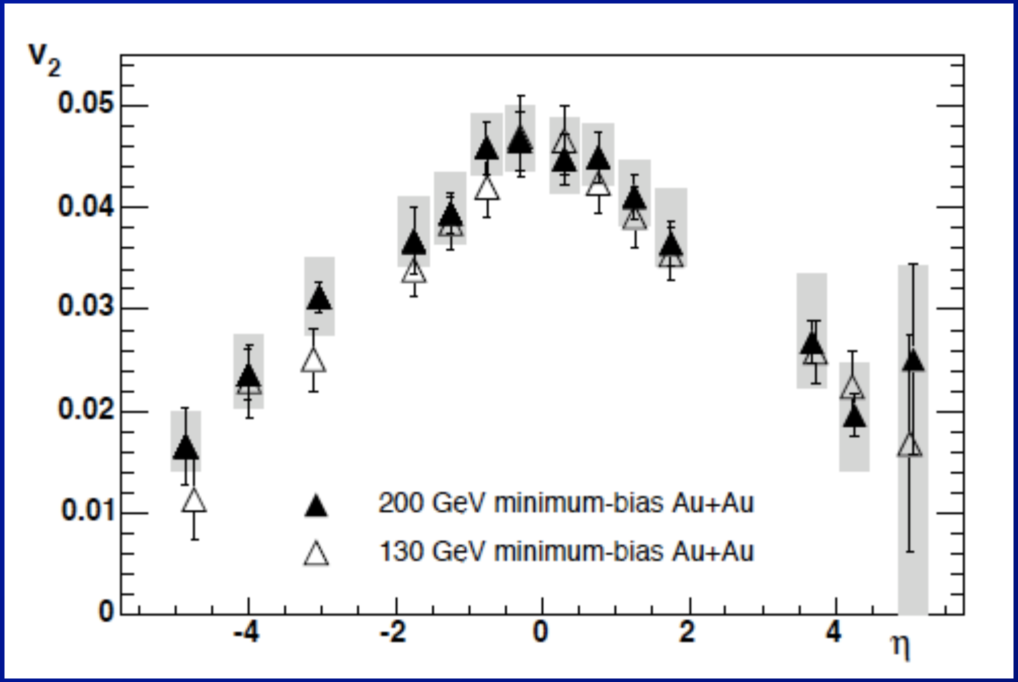


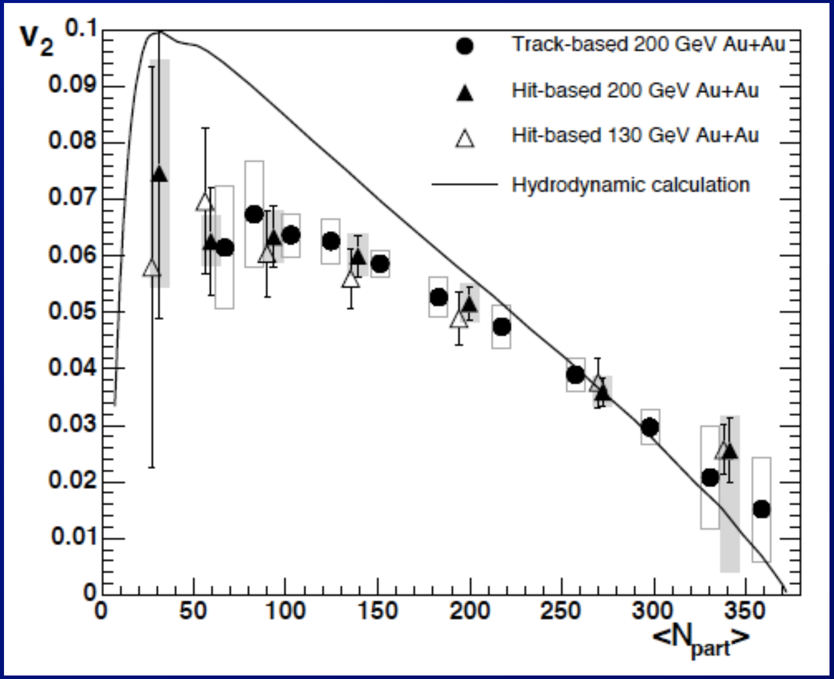
- **Characteristic variation with particle mass**
  - Successfully described by hydrodynamics for not too high p_T

⇒Crucially, reproduced by hydrodynamics

Elliptic flow at RHIC (2005)

PHOBOS, Phys.Rev. C72 (2005) 051901





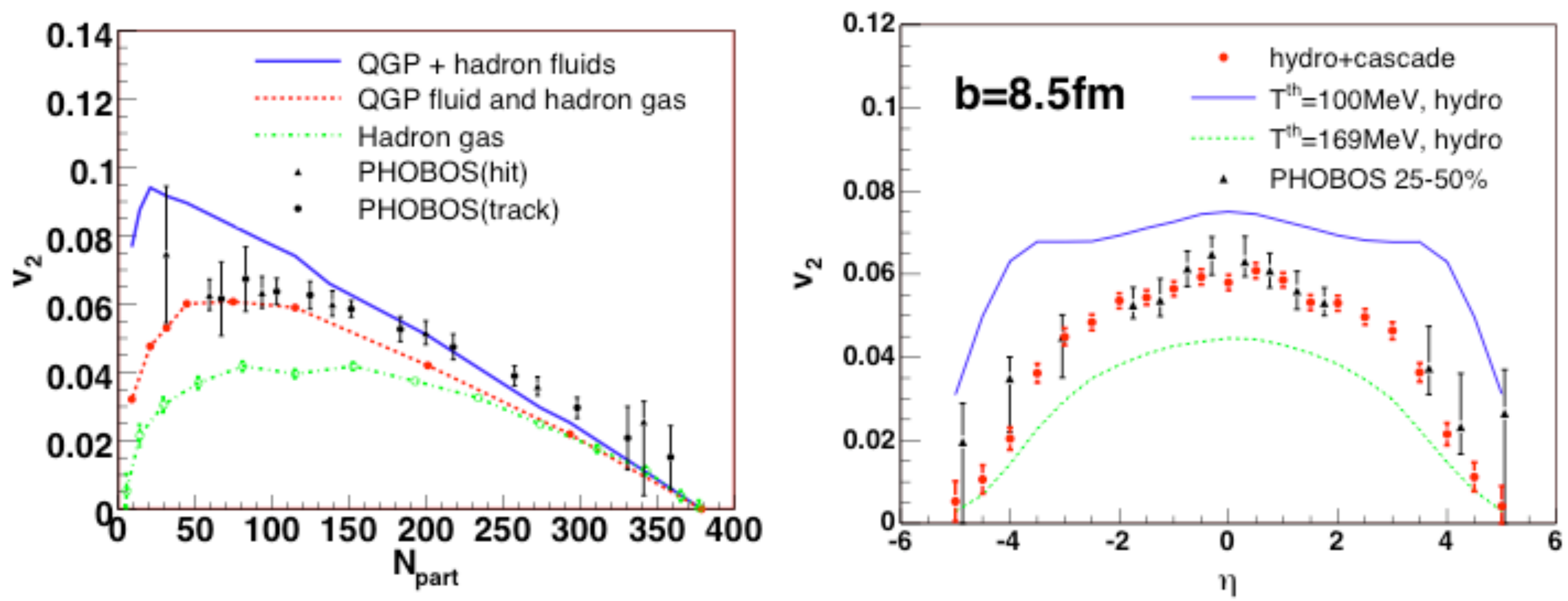
- Rapid variation of (p_T-averaged) v_2 as a function of η

  ⇒Challenge to theoretical calculations?

- Centrality dependence not completely consistent with theoretical calculation

  ⇒hydrodynamics

Role of hadronic viscosity

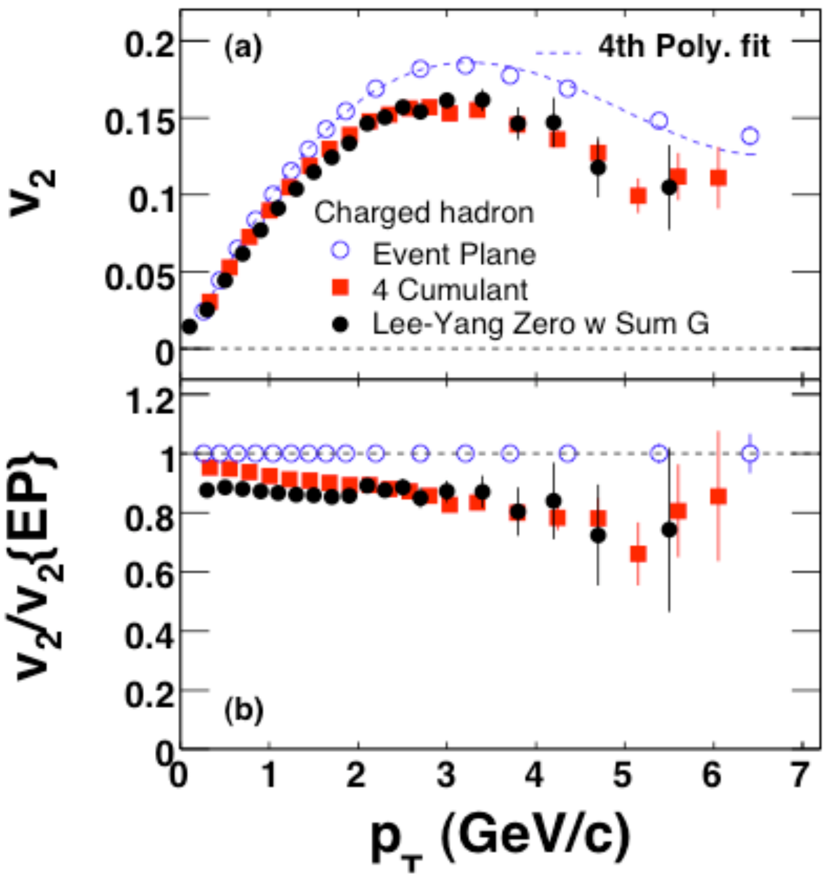


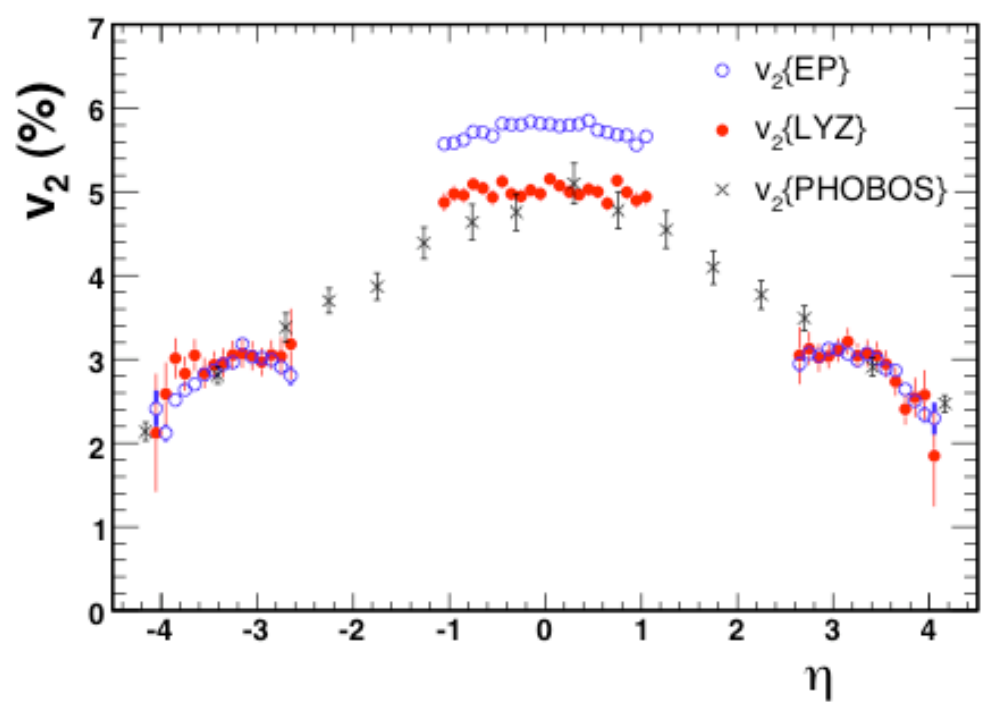
- **Hirano *et al*:**
  - Hadronic dissipative effects important @ RHIC for non-central collisions and for η ≠ 0.

    ⇒Explains pseudorapidity and centrality (?) dependence measured by PHOBOS

    ⇒ But, beware, many details.

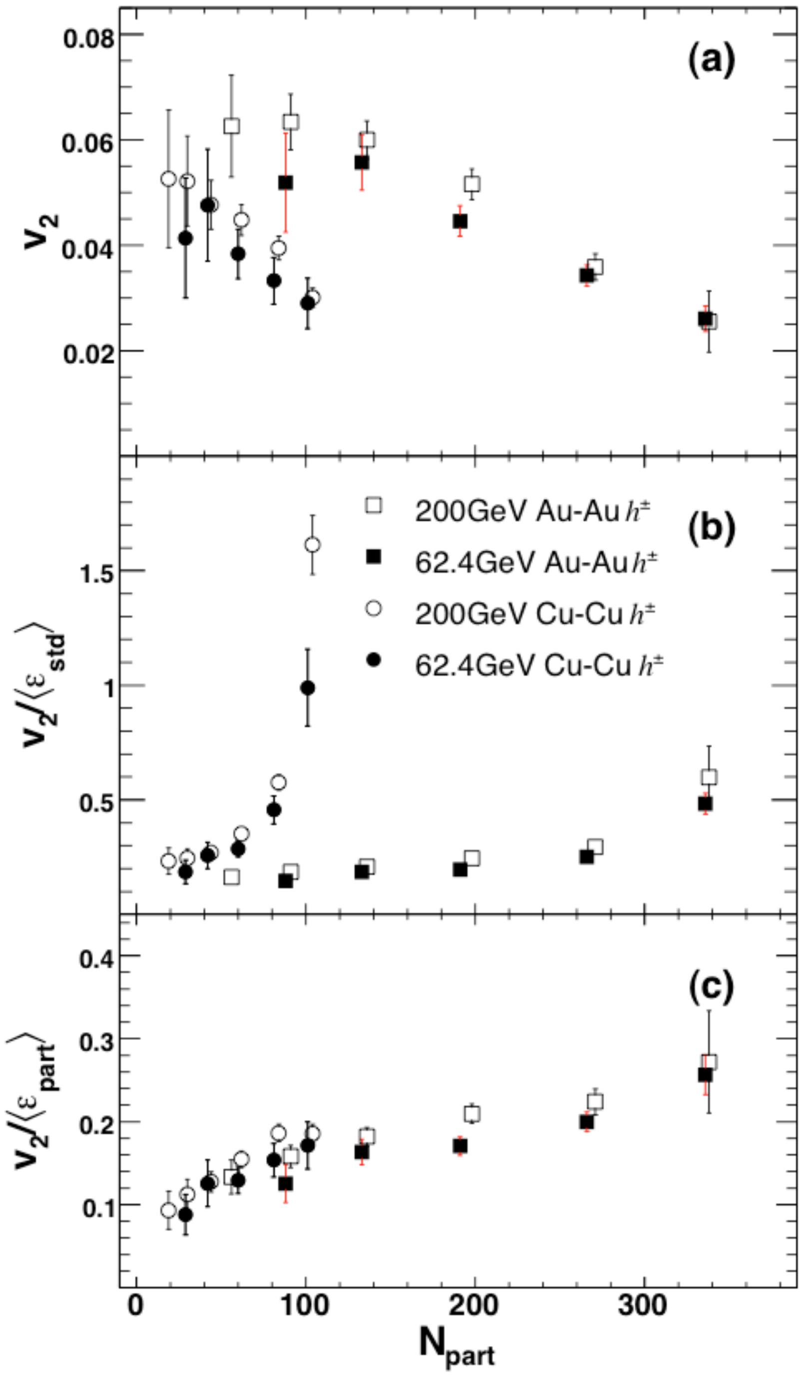
Non-flow effects (STAR)

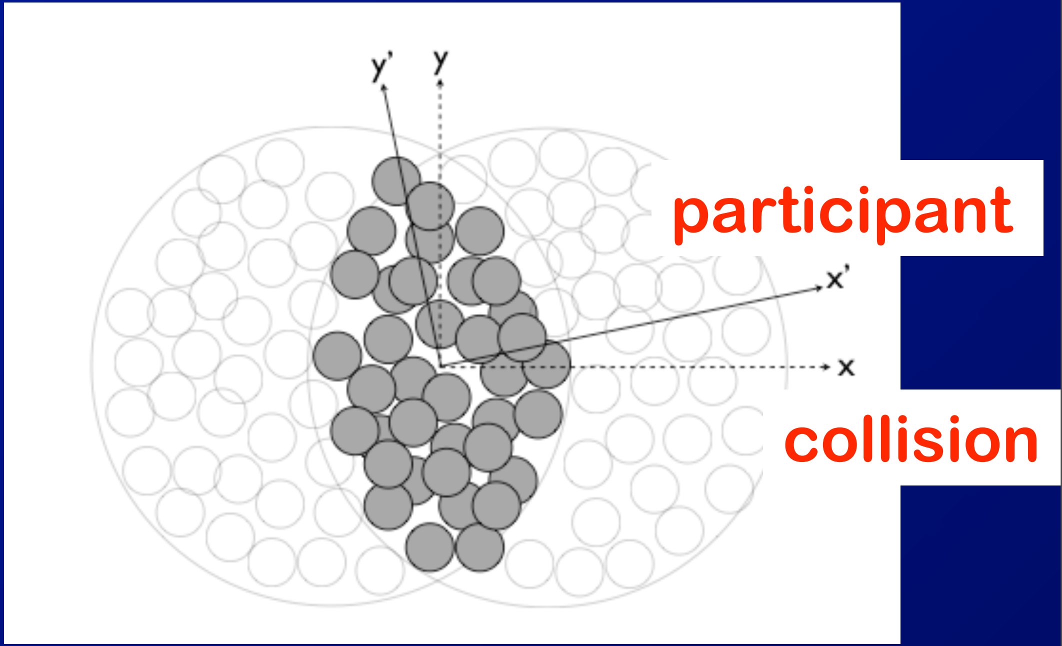




- Event plane determination potentially sensitive to non-flow azimuthal correlations.
  - Particularly when EP measured near mid-rapidity
  - 4 or multi-particle (LYZ) correlations less sensitive
    - ⇒Clearly seen in STAR comparison to event-plane v_2

Eccentricity Fluctuations





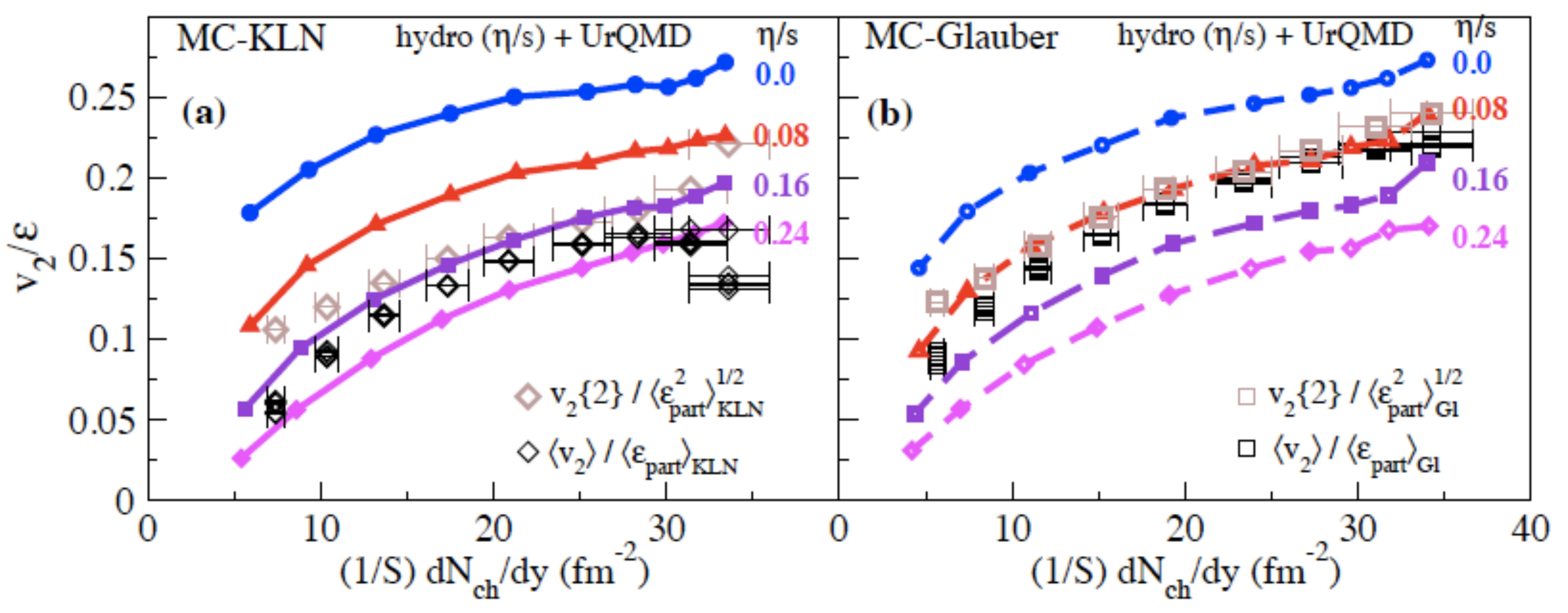
- 1st results from PHOBOS on Cu+Cu v_2 yielded v_2/ε values > v_2/ε in Au+Au
  - But, use of eccentricity calculated using Glauber eccentricity more sensible

    ⇒v_2 inpretation must account for fluctuations

Hydro comparisons to RHIC data

Saturation initial conditions

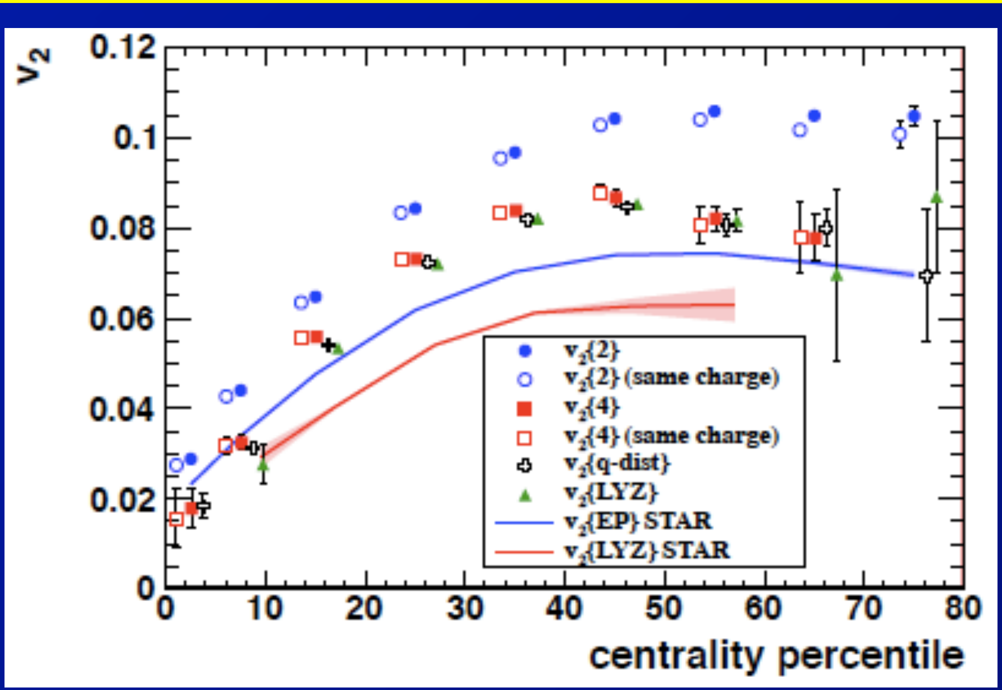
Non-saturation initial conditions

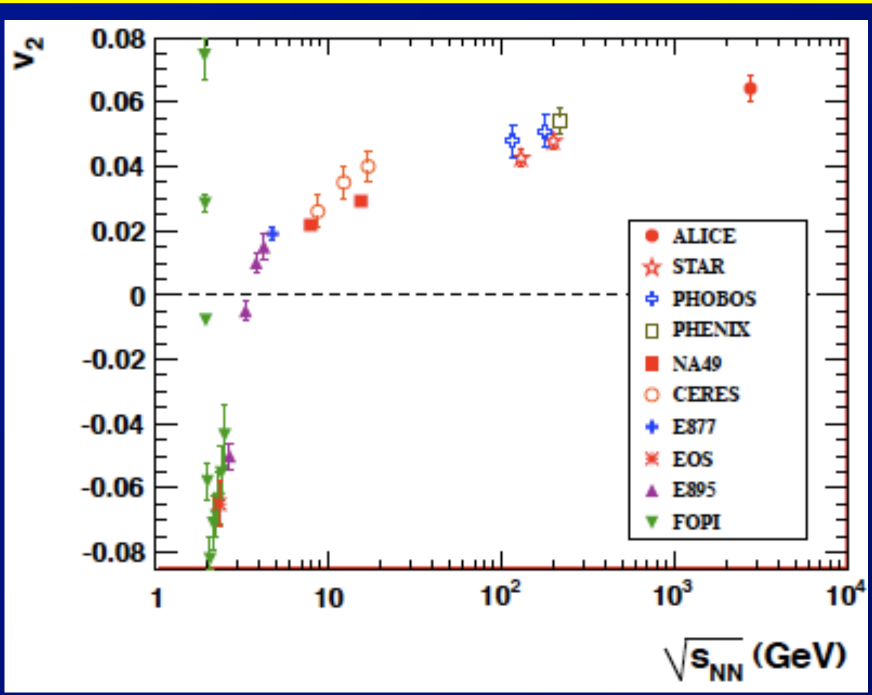


- Unfortunately, extraction of η/s from data very sensitive to initial conditions (ε)

  ⇒Factor of 2.5 difference between saturation (η/s = 0.2) and non-saturation (η/s = 0.08)

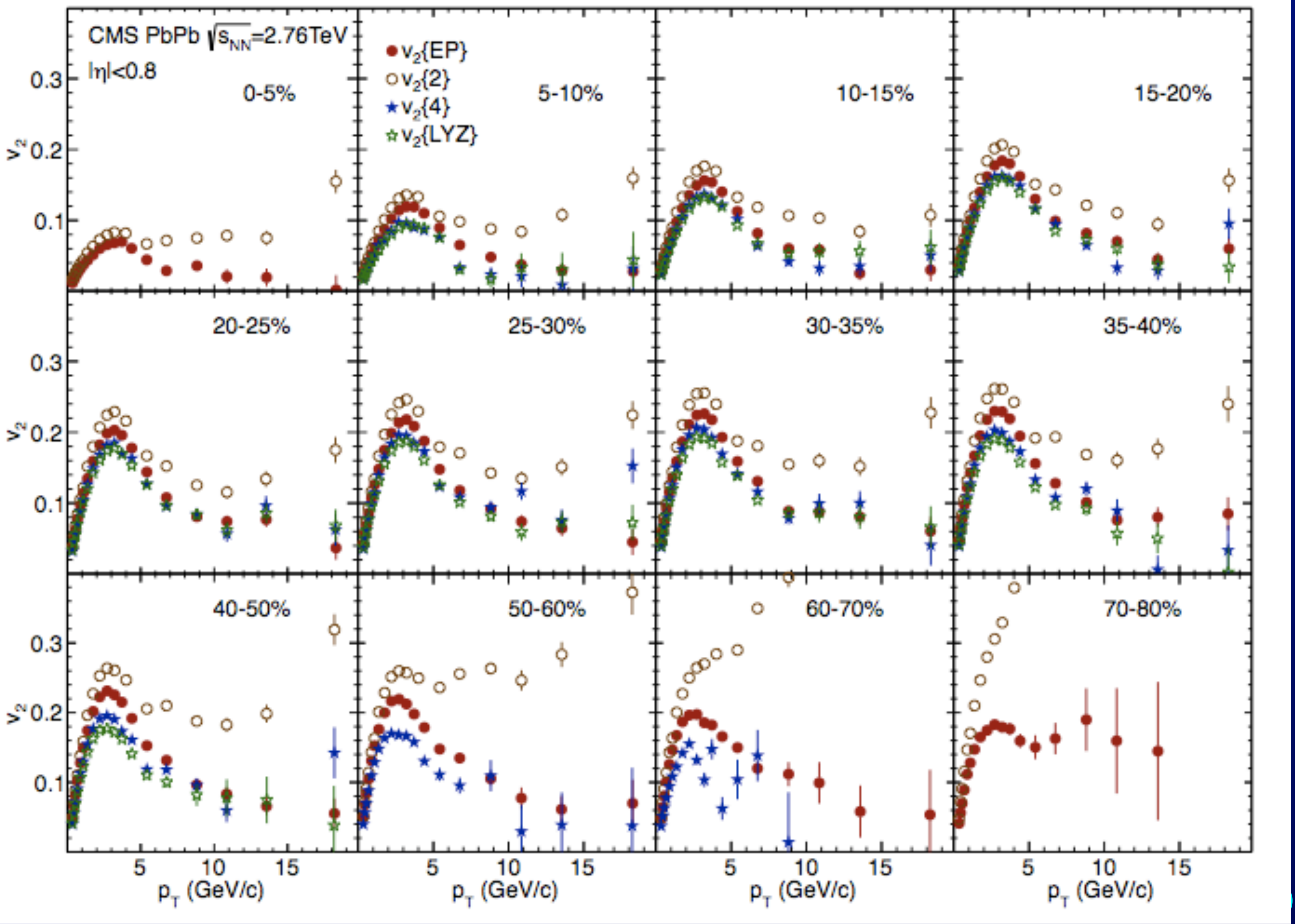
LHC v_2 ALICE



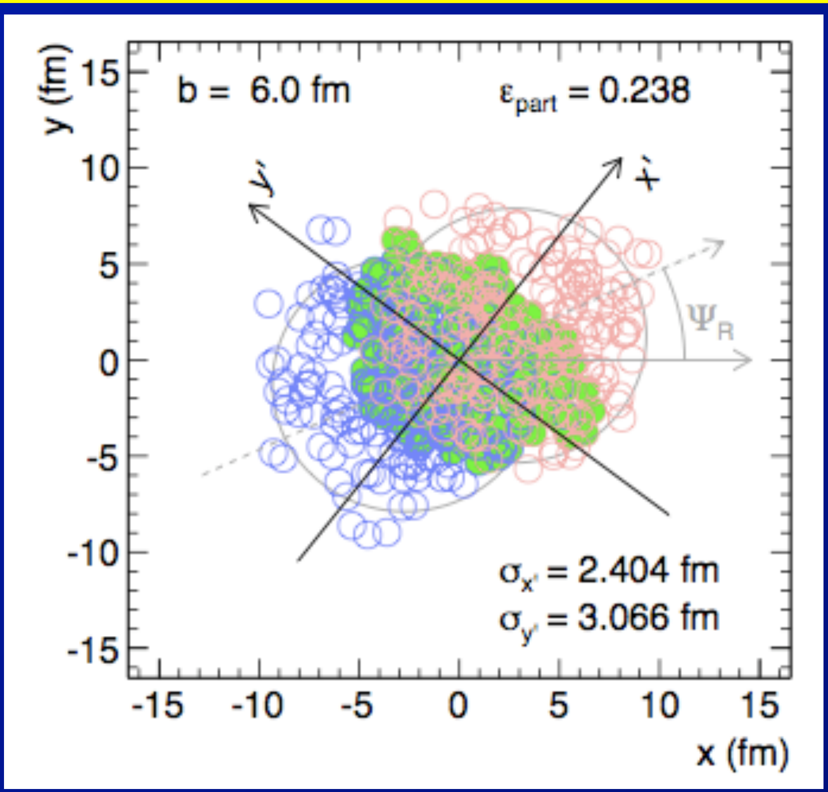


- ALICE 2, 4, many (LYZ) particle correlations
  - ⇒Good agreement between 4 and many particle methods
- Center of mass energy dependence shows little growth in integrated v_2
  - ⇒But many details …

CMS v2, method comparison



CMS ε calculation



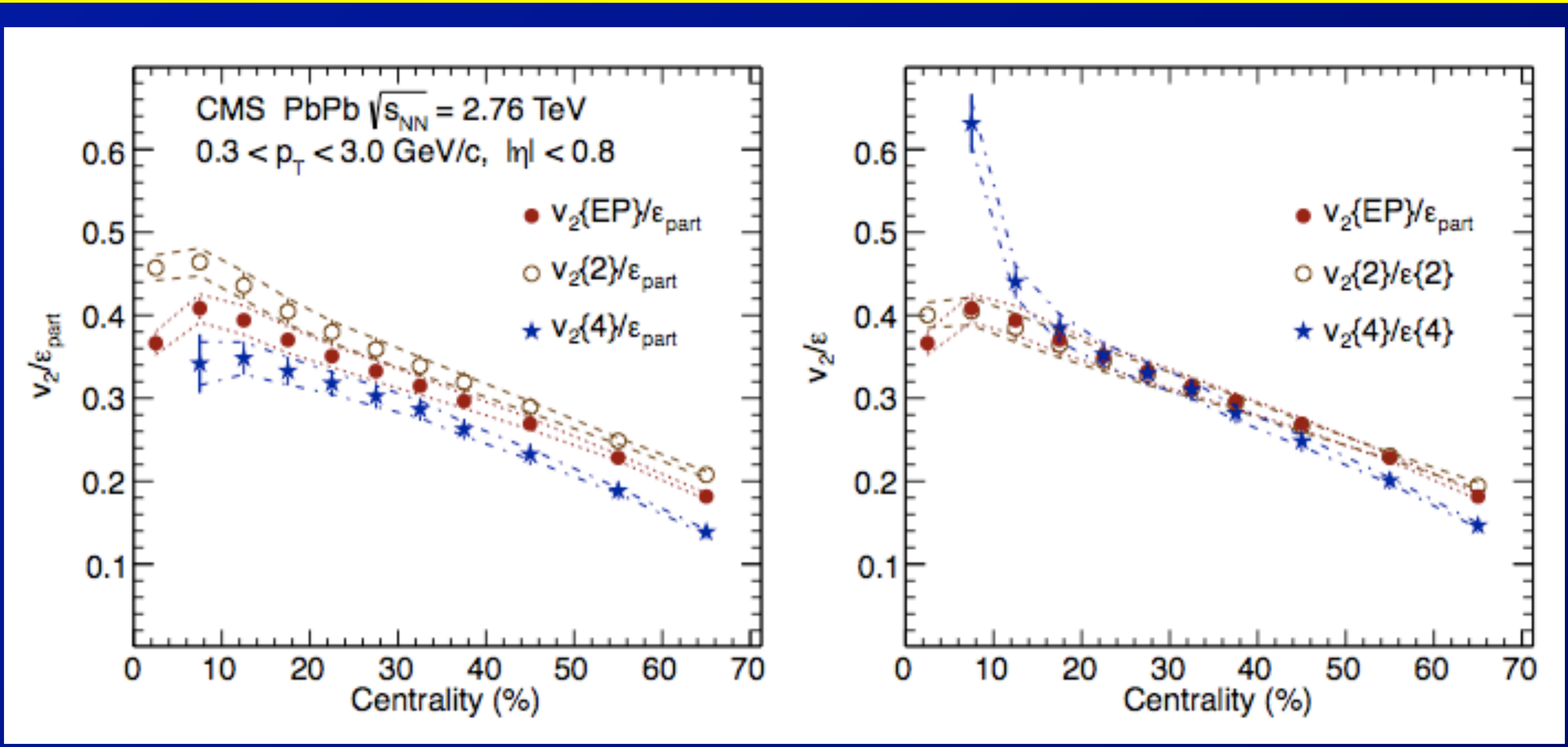
$$\epsilon_{\rm part} \equiv \frac{\sigma_{y'}^2-\sigma_{x'}^2}{\sigma_{y'}^2+\sigma_{x'}^2} = \frac{\sqrt{\left(\sigma_y^2-\sigma_x^2\right)^2+4\sigma_{xy}^2}}{\sigma_y^2+\sigma_x^2},$$

$$\epsilon\{2\}^2 \equiv \langle \epsilon_{\rm part}^2 \rangle,$$

$$\epsilon\{4\}^4 \equiv 2\langle \epsilon_{\rm part}^2 \rangle^2 - \langle \epsilon_{\rm part}^4 \rangle, \text{ and}$$

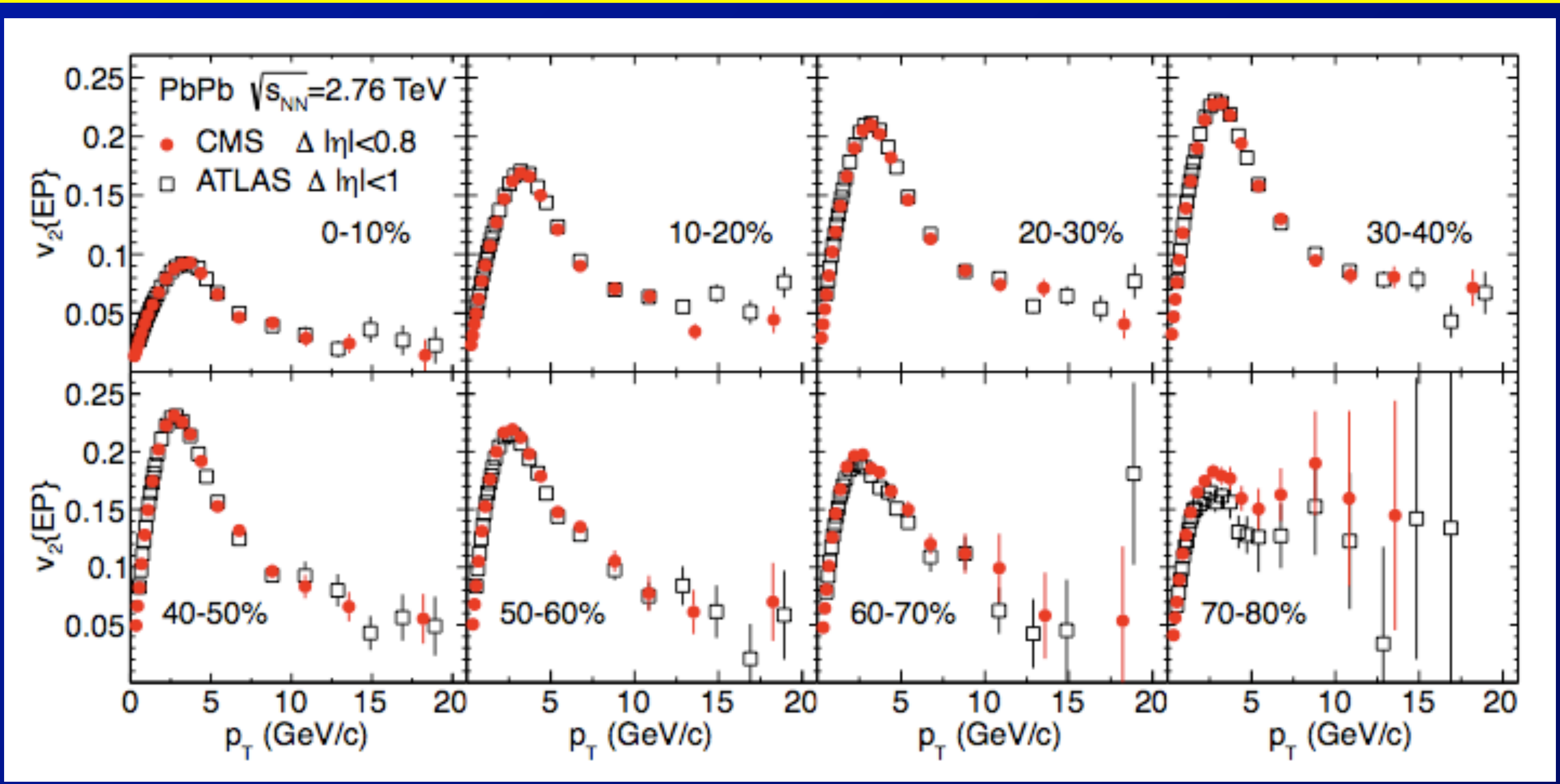
- Different v_2 measurement techniques have different sensitivity to the event-to-event fluctuations
  - ⇒ Use corresponding statistic when calculating the eccentricity.
  - ⇒ Here, based on Glauber model

CMS v2/ε, role of fluctuations



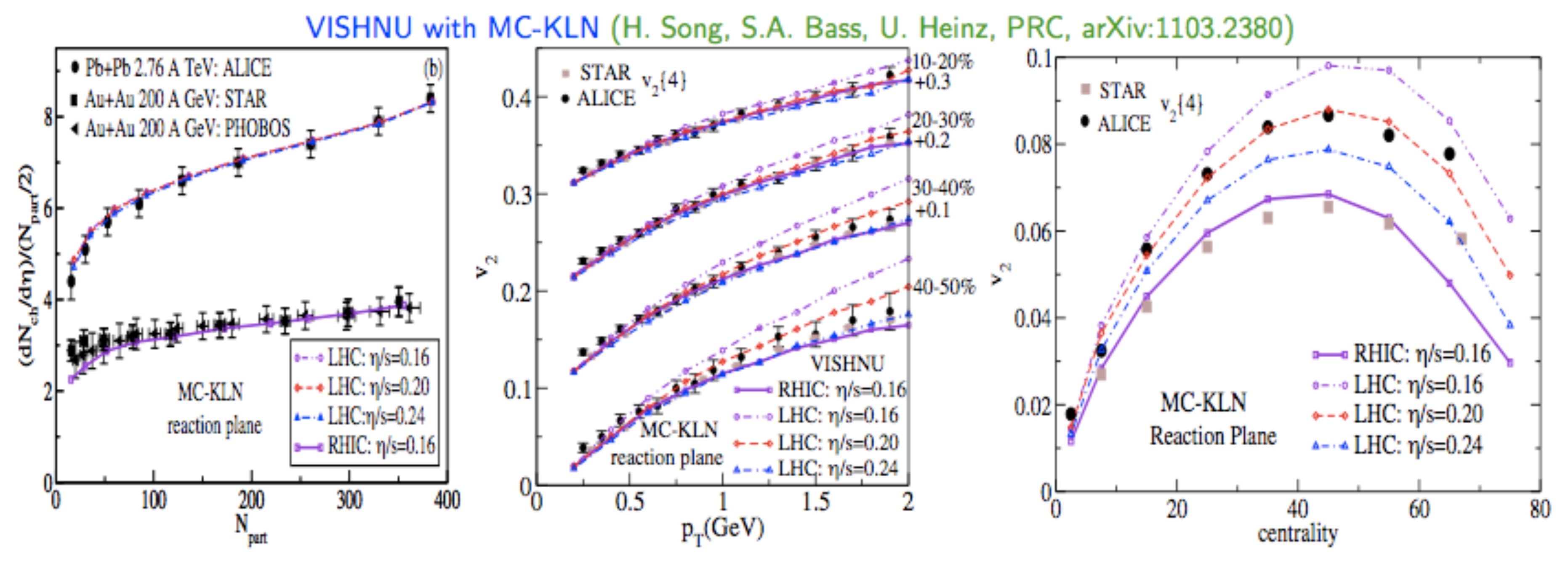
- **Better agreement between v2/ε values for different v2 methods when using corresponding statistic on ε**
  - But, in central collisions $\varepsilon\{4\}$ is small
  - Non-flow for v2{2}, v2{EP} in peripheral ?

LHC CMS, ATLAS $v_2(p_T)$



- See non-zero v_2 over a wide range of p_T
  - But at high p_T (how high?), not due to collective expansion of quark gluon plasma
    - ⇒Due to jet quenching (tomorrow)
    - ⇒$5 < p_T < 10$ GeV range complicated

(Viscous) hydrodynamics applied

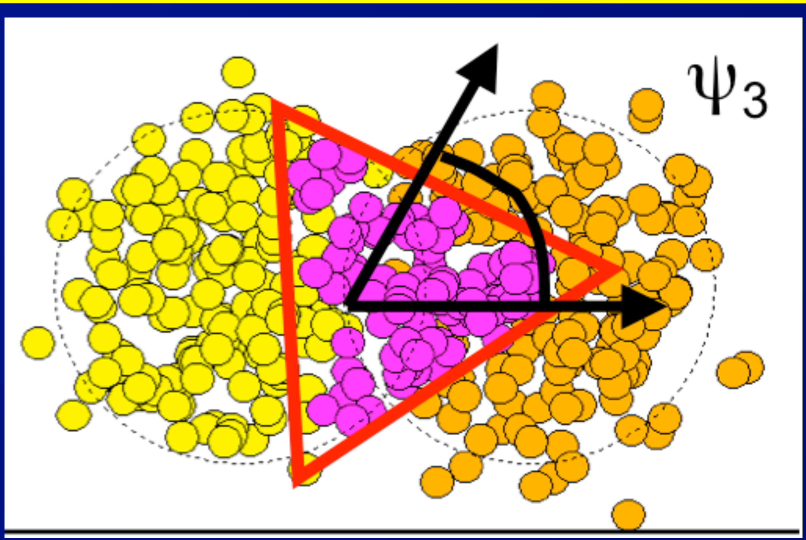

VISHNU with MC-KLN (H. Song, S.A. Bass, U. Heinz, PRC, arXiv:1103.2380)

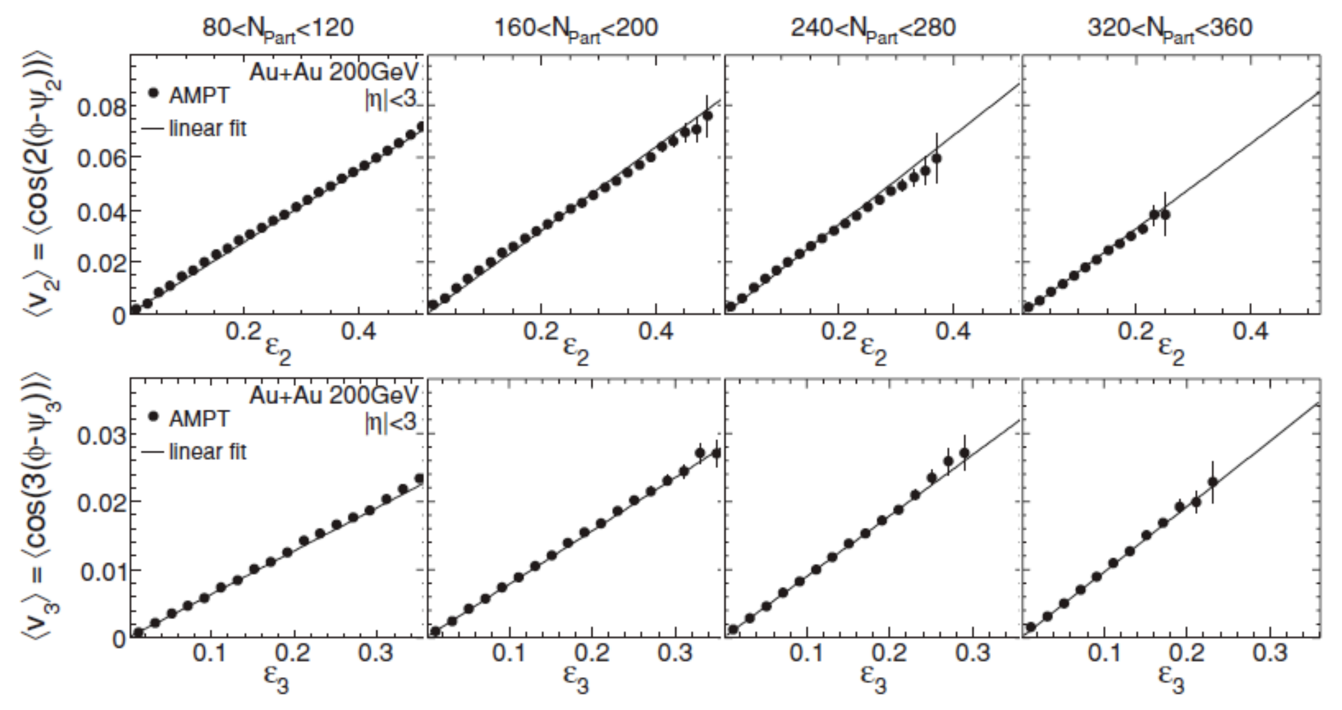
- Viscous hydro + hadronic cascade (VISHNU)
  - Compare to RHIC and LHC $dN_{chg}/d\eta$, $v_2(p_T)$, v_2(cent)
  - Specific choice of initial conditions (CGC)
    - ⇒η/s ~ 1-3 x 1/4π
    - ⇒possibly larger at LHC than RHIC
      - » But beware use of constant η/s

Higher-order flow

Triangularity

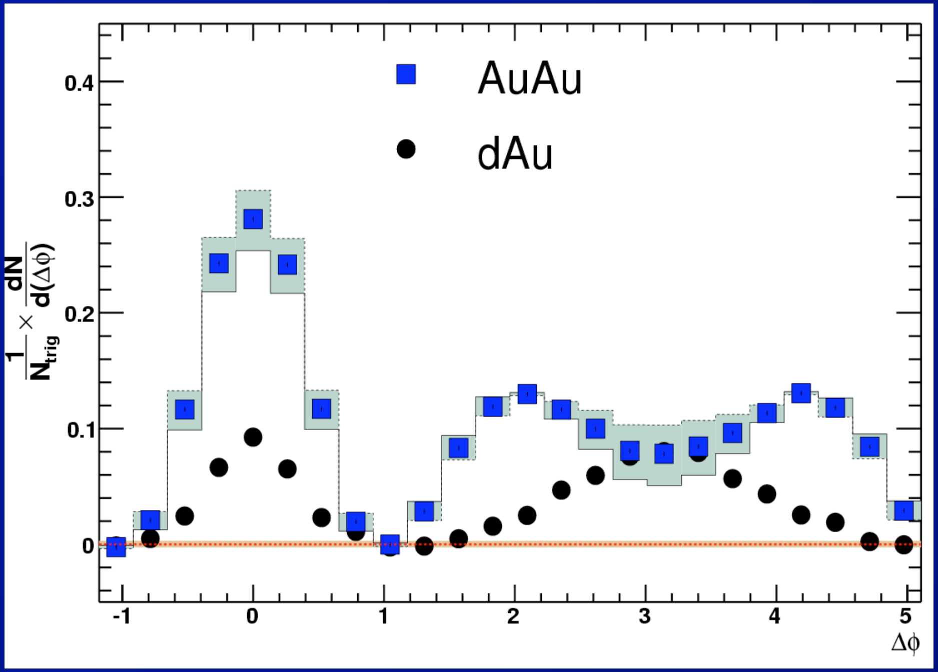
- Big surprise:
  - Initial-state fluctuations can generate odd and higher-order harmonics
    - ⇒e.g. ε_3
- v_3 demonstrated in a Monte Carlo cascade model (AMPT)



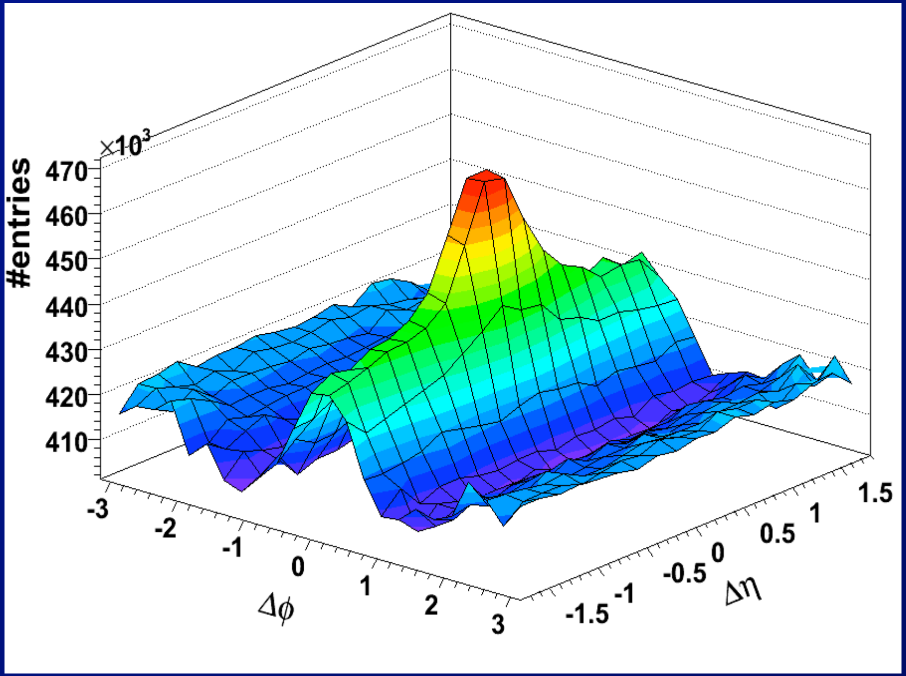


Experimental archaeology

“Mach cone”



“Ridge”

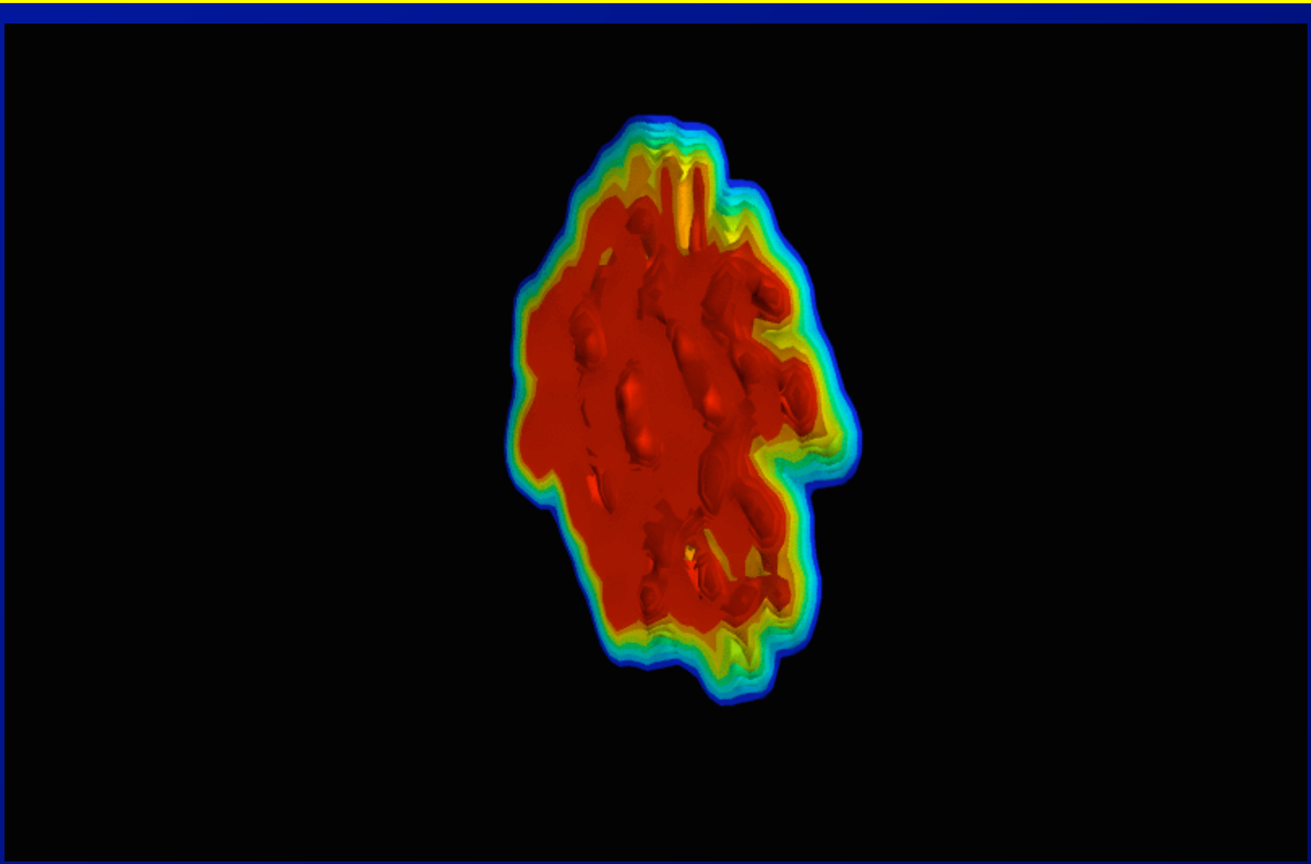


- The understanding that initial-state fluctuations can generate triangular and higher-order flow solved two long-standing (2005-2010) experimental puzzles in two-particle correlations

  ⇒ hypothesized mach shock from jets

  ⇒long-range (in η) near-side correlation

Fluctuations, higher-order flow



- Initial state of Pb+Pb (or Au+Au) collision is not necessarily smooth
  - Fluctuations in transverse plane (hot spots) will generate higher frequency (in φ) flow components

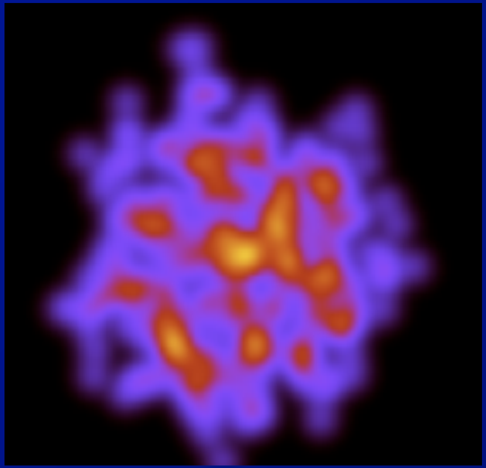
    ⇒Higher frequencies more sensitive to η/s

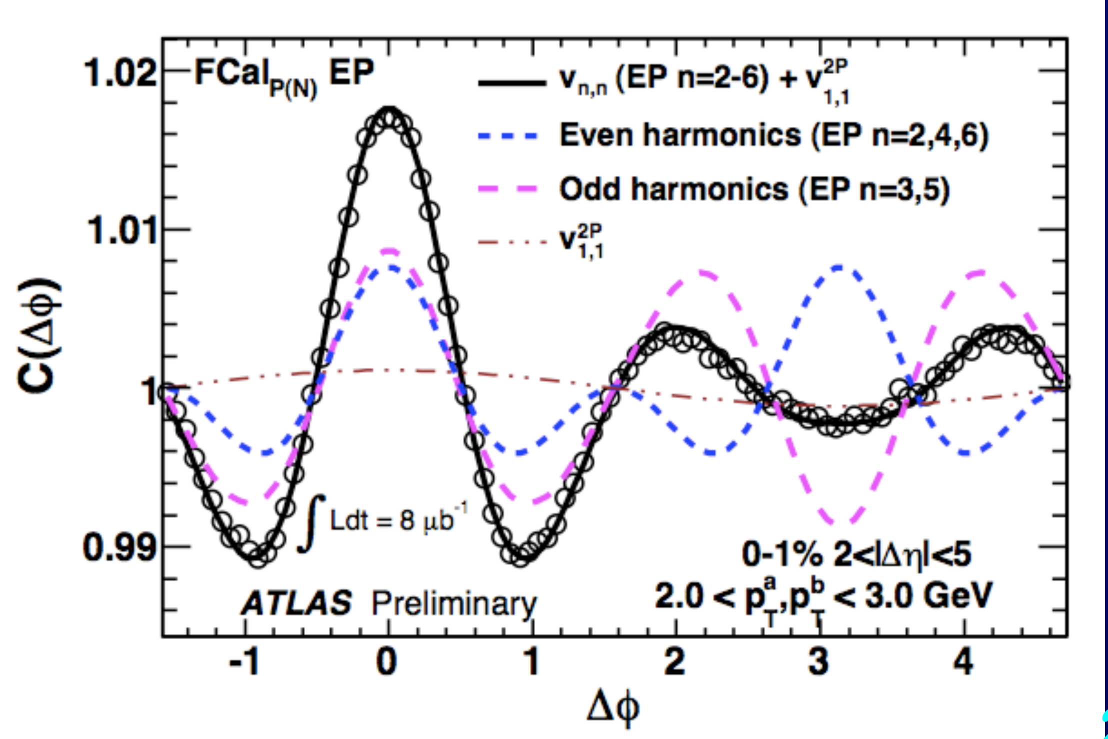
Higher Flow Harmonics

- Major paradigm shift in the field in last 3 years
  - Higher flow harmonics arising from initial-state fluctuations in transverse positions of participants

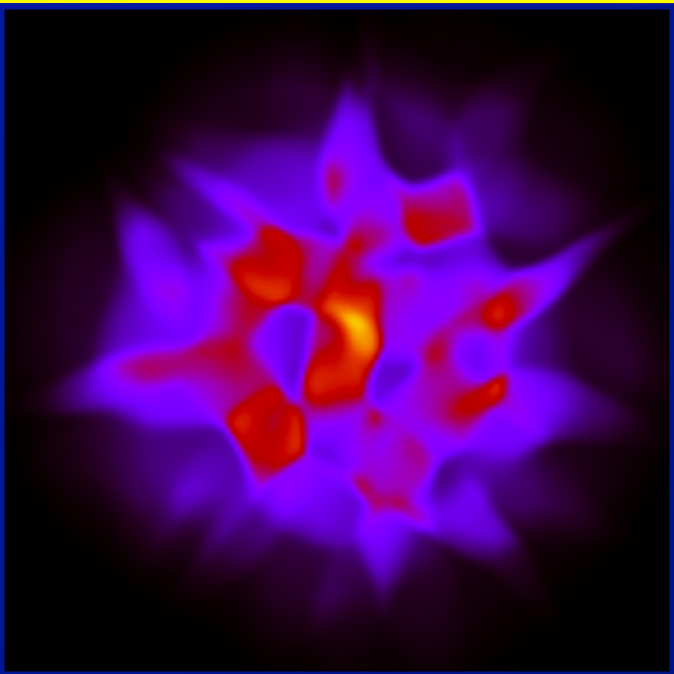
$$\frac{dN}{d\phi dp_T d\eta} = \frac{dN}{2\pi dp_T d\eta}\left(1 + \sum_n 2v_n \cos\left[n(\phi - \psi_n)\right]\right)$$

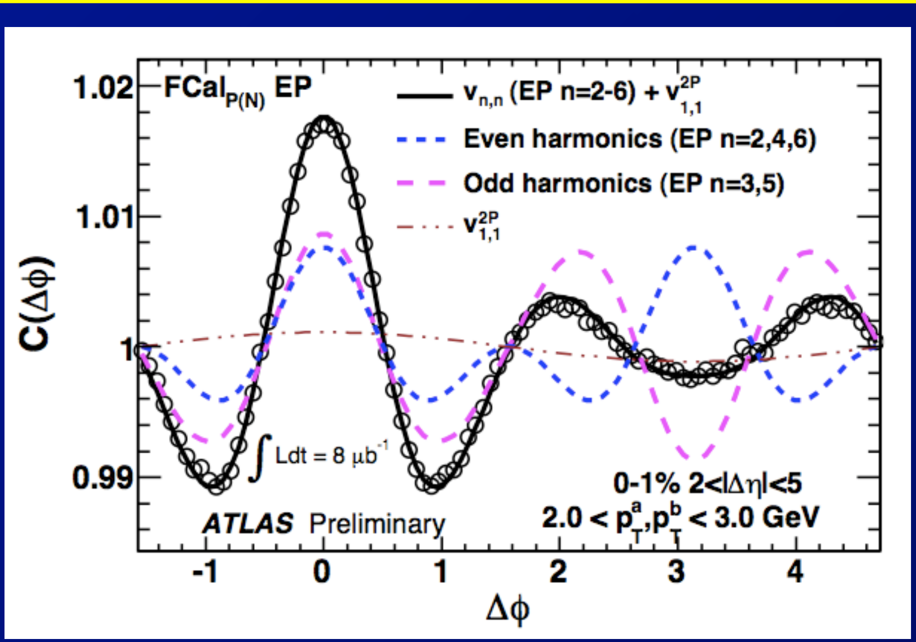
- Frequently measured using pairs of particles



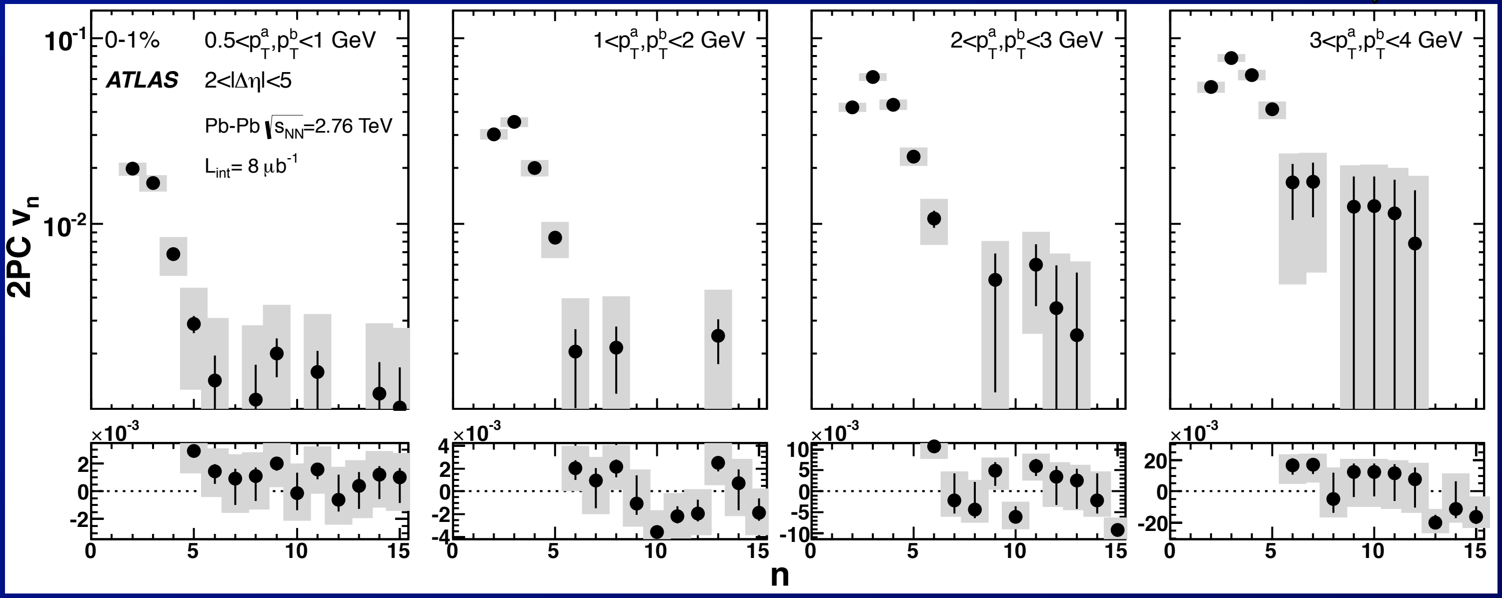


Fluctuations, Fourier amplitudes



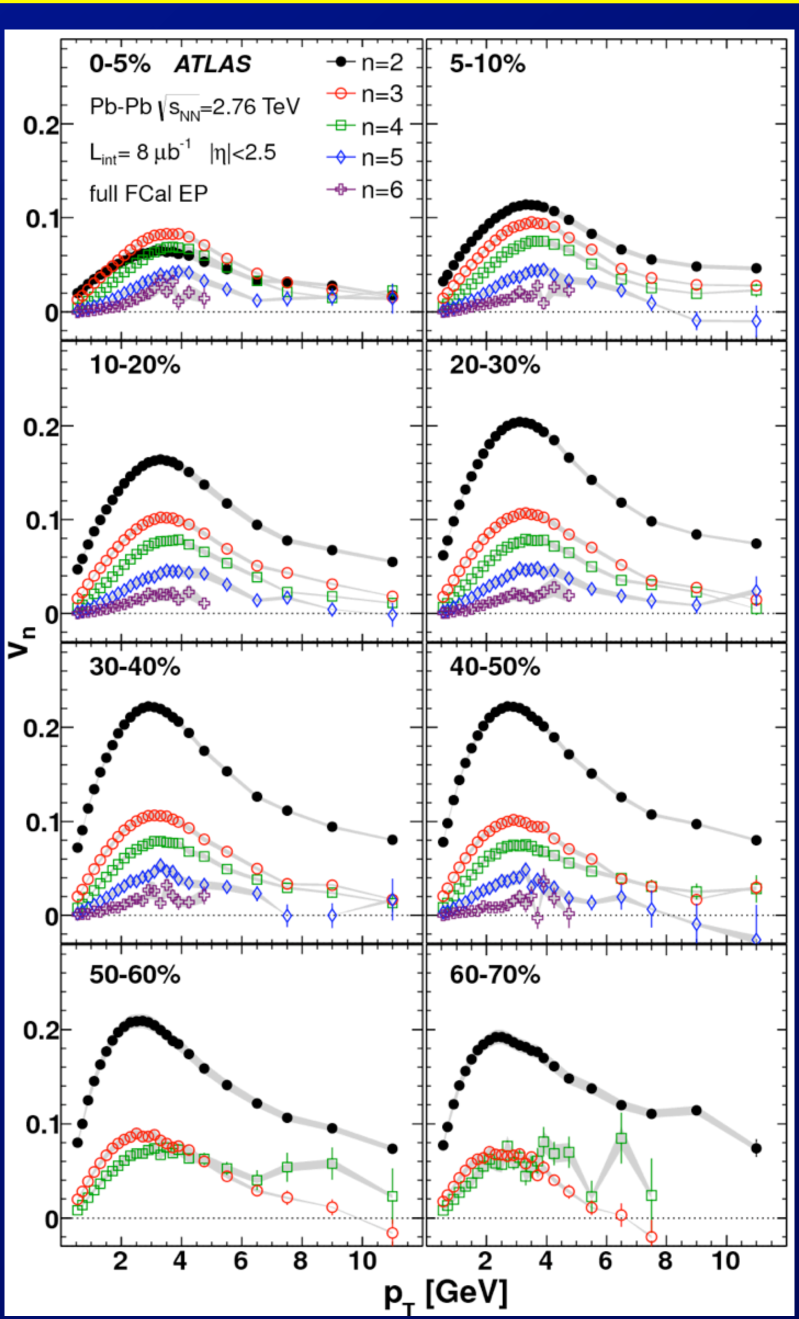


Increasing momenta



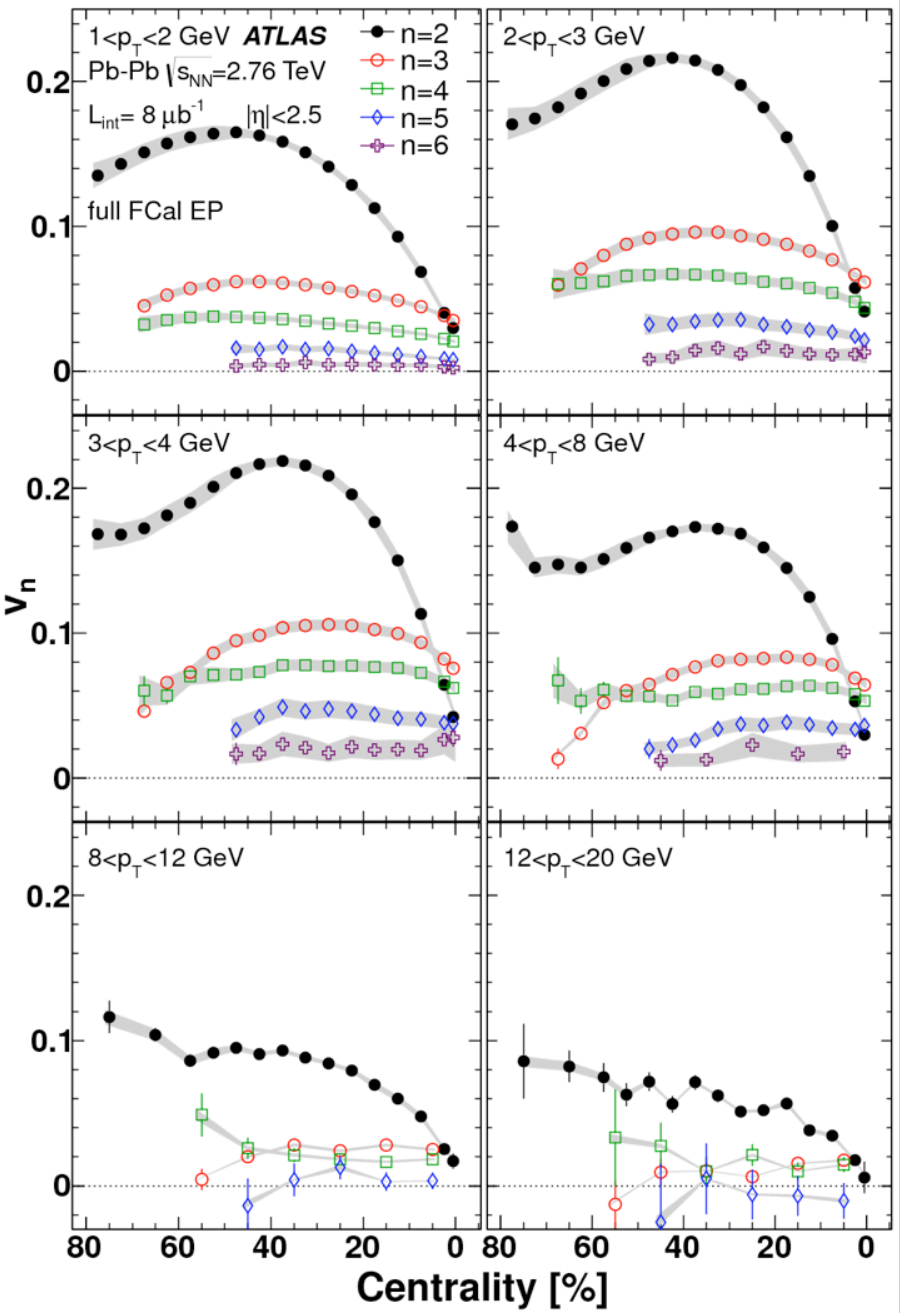
Event plane v_n measurements

- **Different v_n's have similar p_T distribution**
  - understood to result from interplay of soft and hard contributions



Event plane v_n measurements

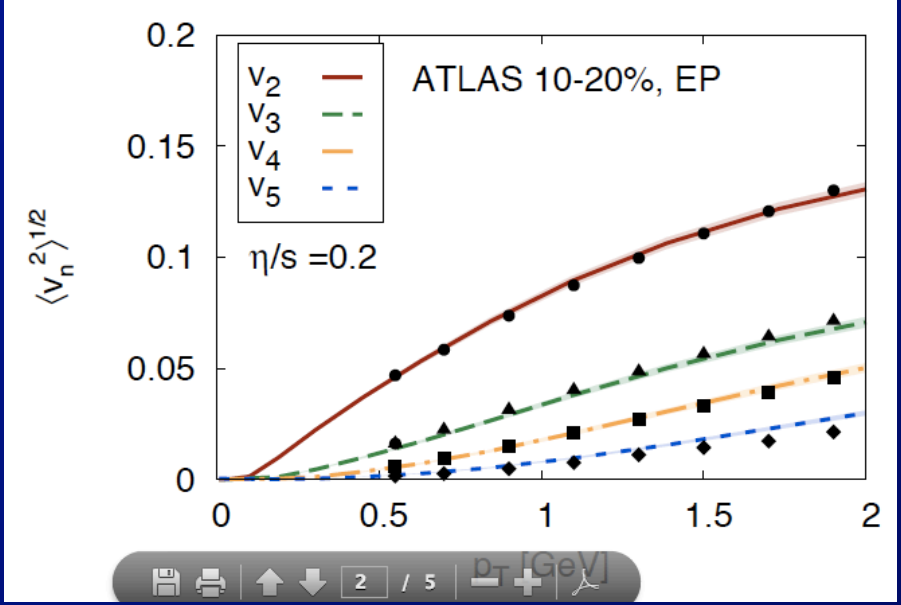
- n = 2 has a natural geometric variation with centrality
- for n > 2, a weak centrality dependence due to the dominance of fluctuations
- in very central collisions, n = 3, 4 larger than n = 2.
  - ⇒partly because fluctuations tend not to respect symmetry
  - ⇒requires minimal dispersion during system evolution

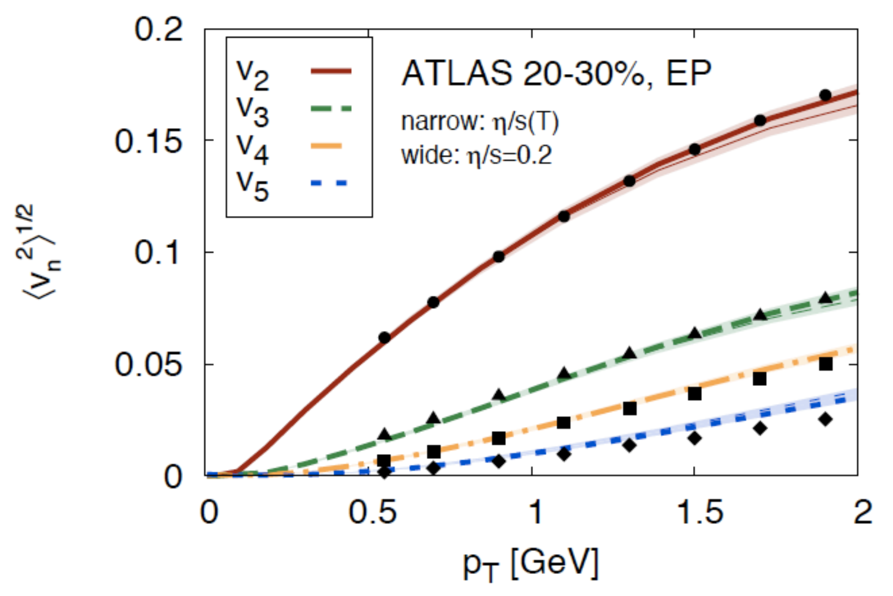


Hydrodynamic model comparisons

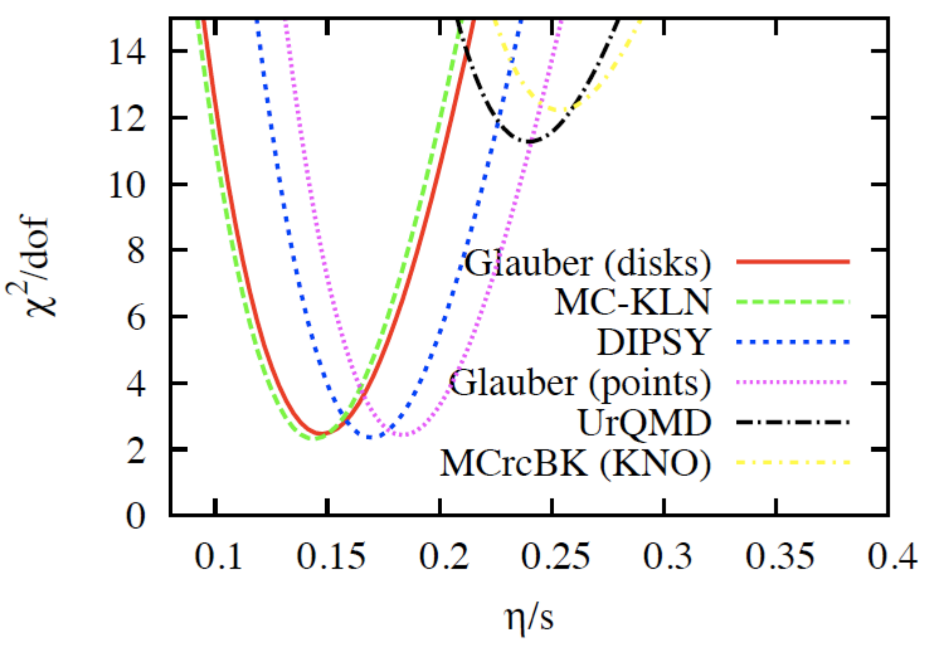
- Viscous hydrodynamic calculation with IP-sat saturation in initial state, with GQP and hadron gas EOS
  - Bottom: constant (0.2) and temperature dependent η/s
    - ⇒ Not yet able to test T dependence in LHC data alone
- But, good description of v_n's
  - detailed evaluation of η/s not attempted here

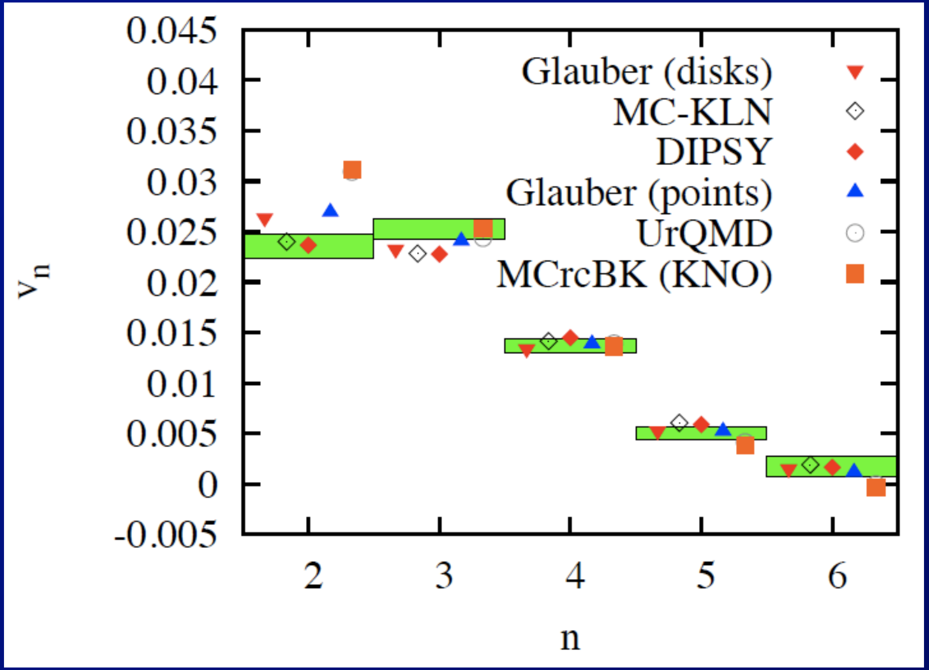
MUSIC: Gale et al, arXiv:1209.6330





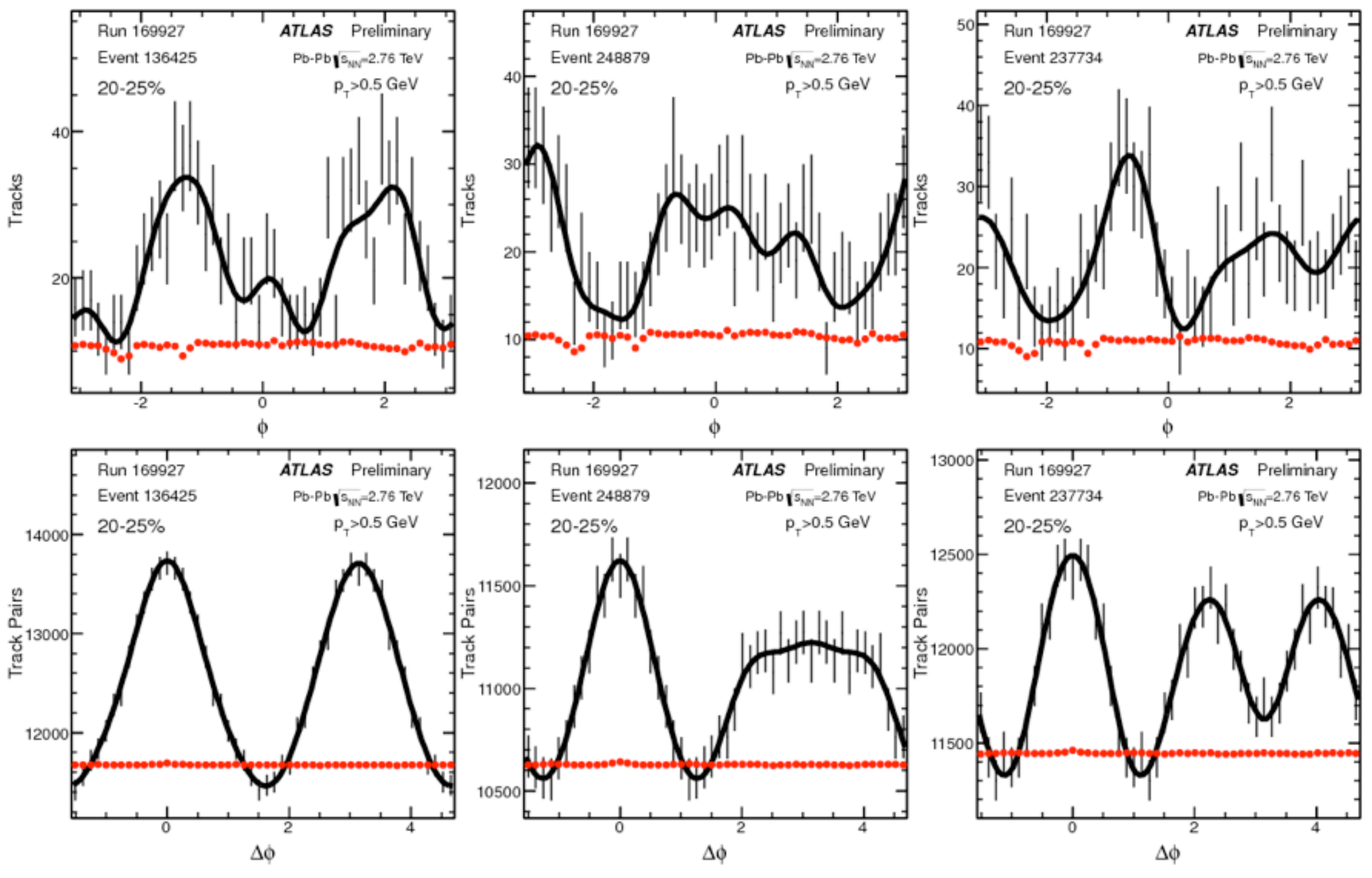
Fits to (ATLAS) data





- Most rigorous attempt so far to extract η/s from LHC (ATLAS) v_n data
  - Saturation vs non-saturation differences substantially reduced by including higher harmonics.

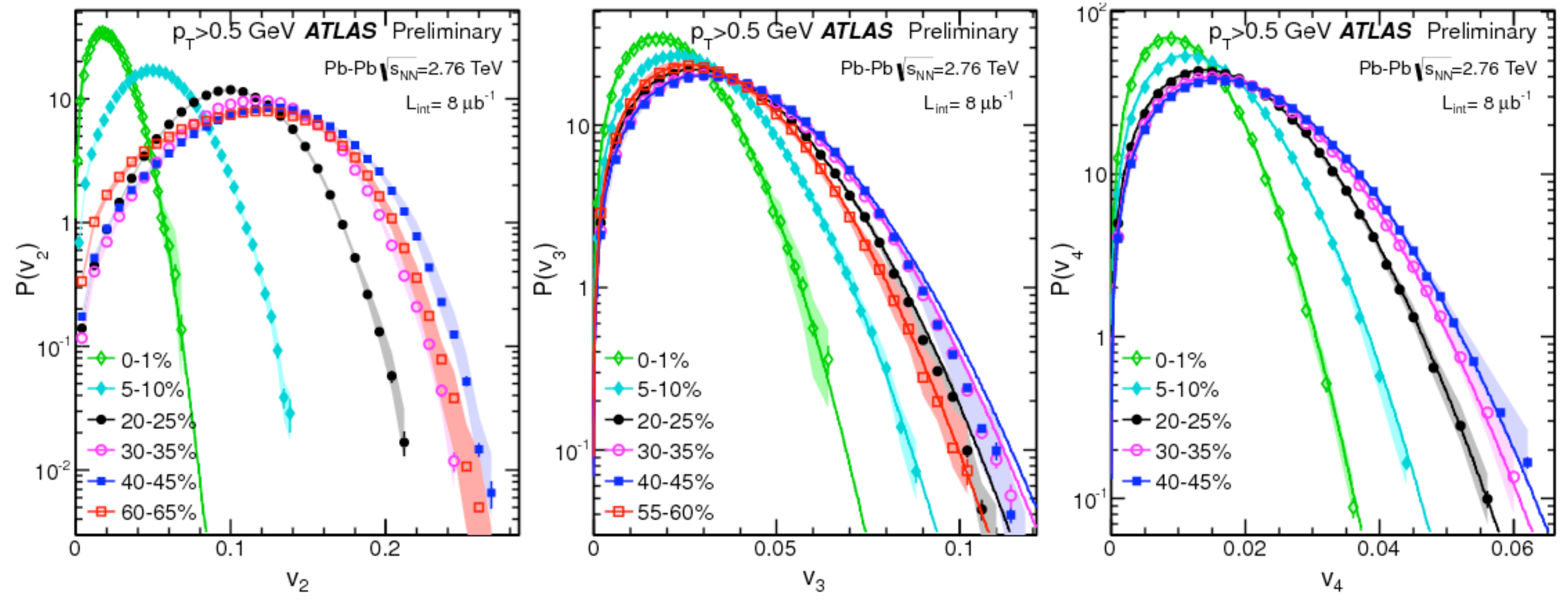
Event-by-event v_n



- **Experimental breakthrough by ATLAS**

  ⇒event-by-event v_n measurement

Event-by-event v_n



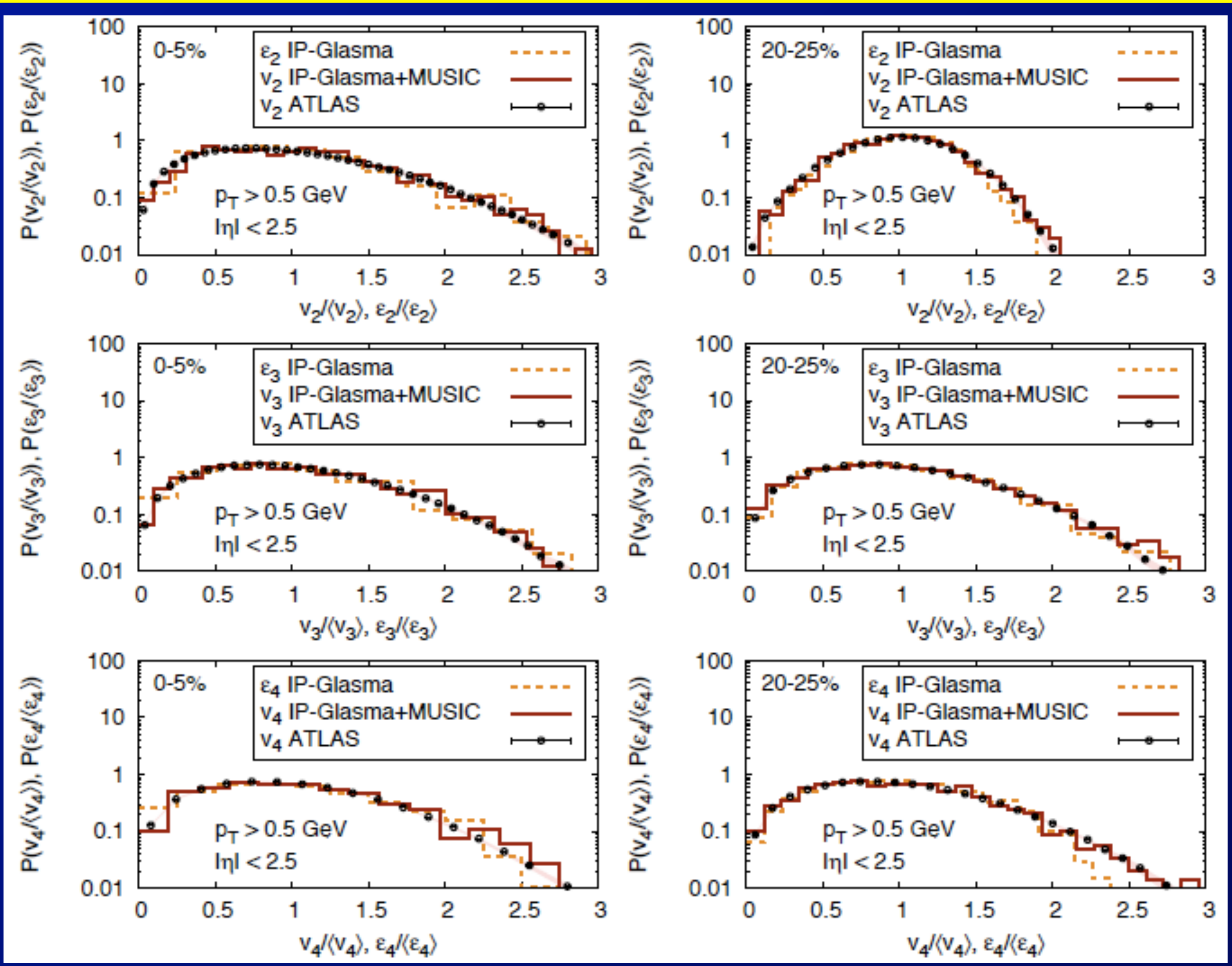
- Probability density distributions for obtaining v_2, v_3, v_4 in a given event
  - ⇒Distributions sensitive to both initial-state fluctuations and hydrodynamic evolution
  - ⇒Allow detailed tests of theoretical calculations

Event-by-event: hydro comparisons

MUSIC: Gale et al, arXiv: 1210.5144

Saturated initial conditions + viscous hydrodynamics

lattice + hadron gas equation of state



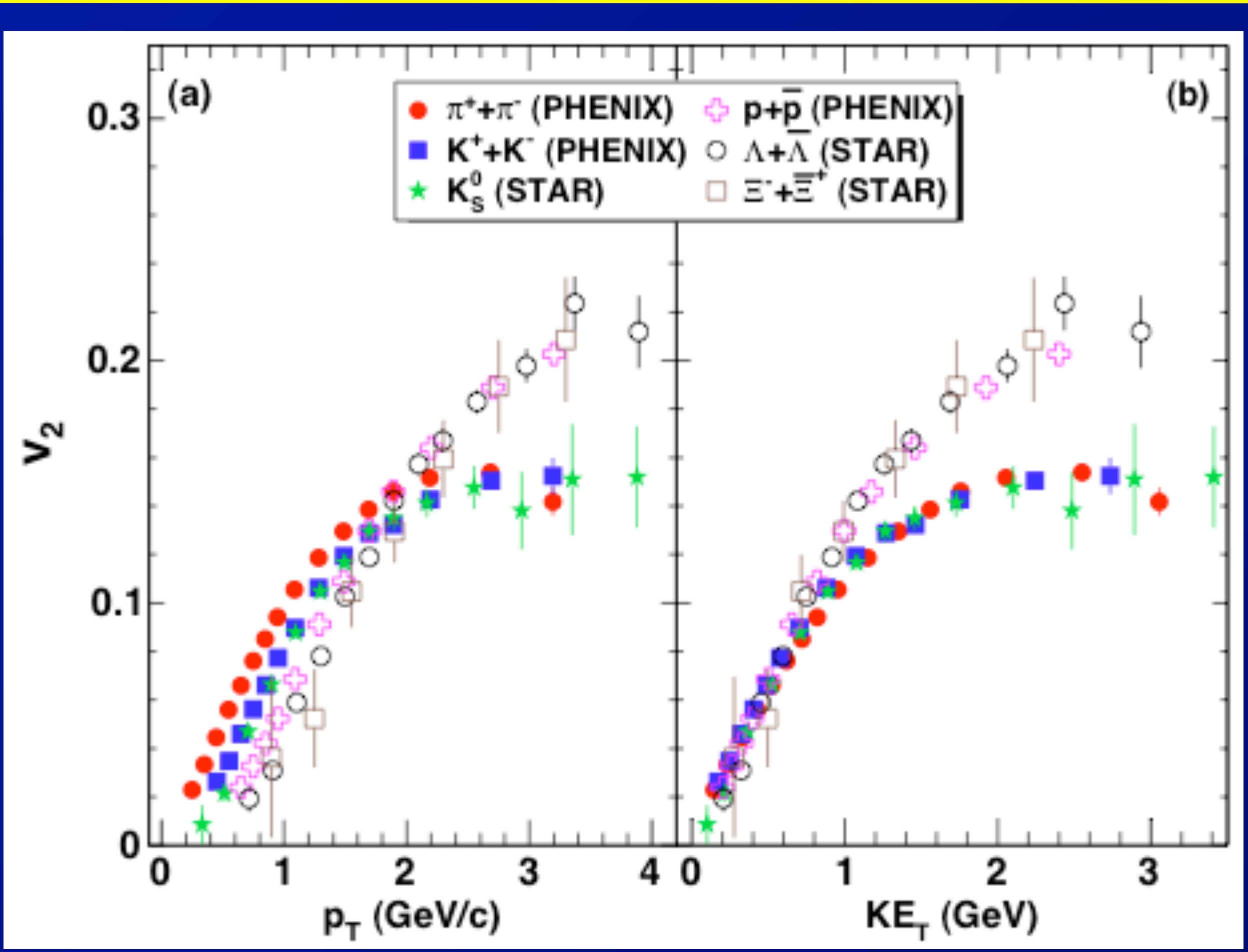
- **(Implausibly?) good agreement with data**

  ⇒Event-by-event v_n probing both initial state and hydrodynamic evolution (here $\eta/s = 0.2 \approx 2.5/4\pi$)

Pb+Pb Flow: summary

- Collective expansion of the quark gluon plasma provides experimentally accessible signatures
  - ⇒Probe transport properties of the plasma
  - ⇒e.g. η/s
- Measurements at RHIC and the LHC together with viscous hydrodynamics calculations yield $\eta/s <\sim 2.5 \times 1/4\pi$
  - ⇒Very close to conjectured lower bound
- Dominant systematic uncertainty is due to uncertainty over initial-state eccentricities
  - But, higher order flow results, including event-by-event measurements provide constraints
    - ⇒And provide better sensitivity to η/s
- We may soon be able to start testing models of temperature dependence of η/s.

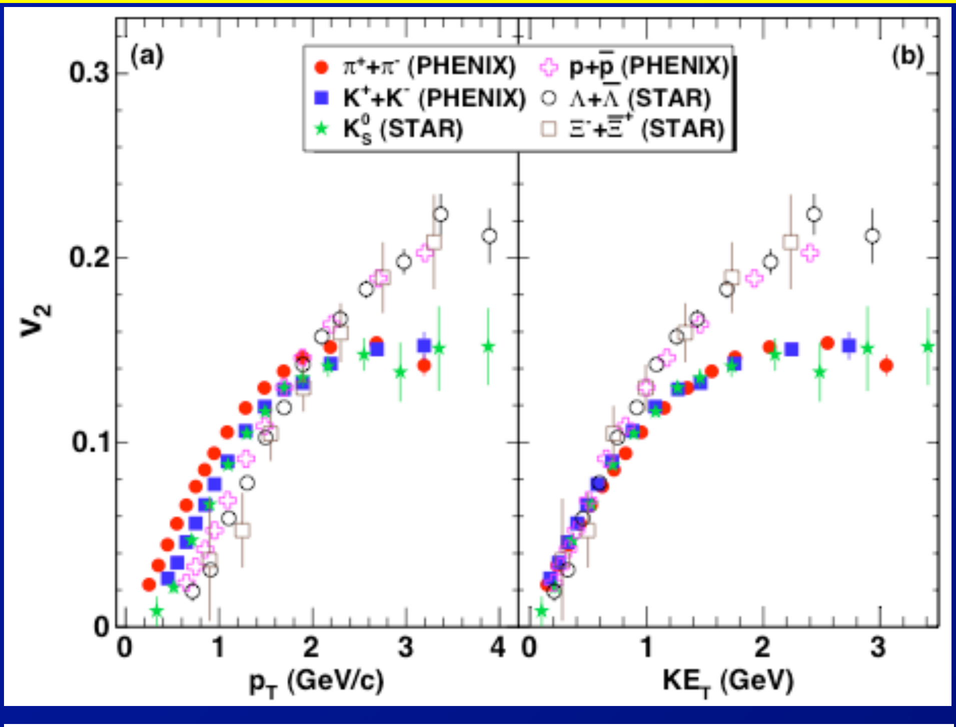
particle identified v_2

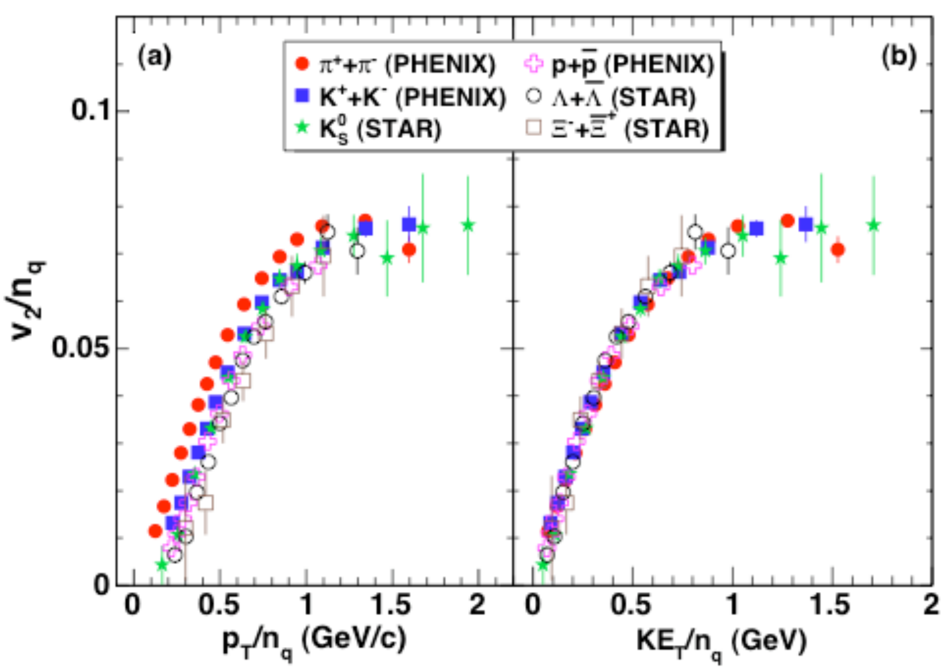


Au+Au minimum-bias @ η=0 (important)

- Mass splitting at low p_T due to “wrong” choice of kinematic variable
  - plotting vs $KE_T \equiv m_T - m$ removes mass dependence

v_2, n quark scaling @ RHIC ?





- Departure from mass independent $v_2(KE_T)$ from recombination?
  - Hadrons formed at hadronization by combining n quarks from QGP

$$\Rightarrow v_2 \propto n_q$$

$$\Rightarrow KE_T \propto n_q$$

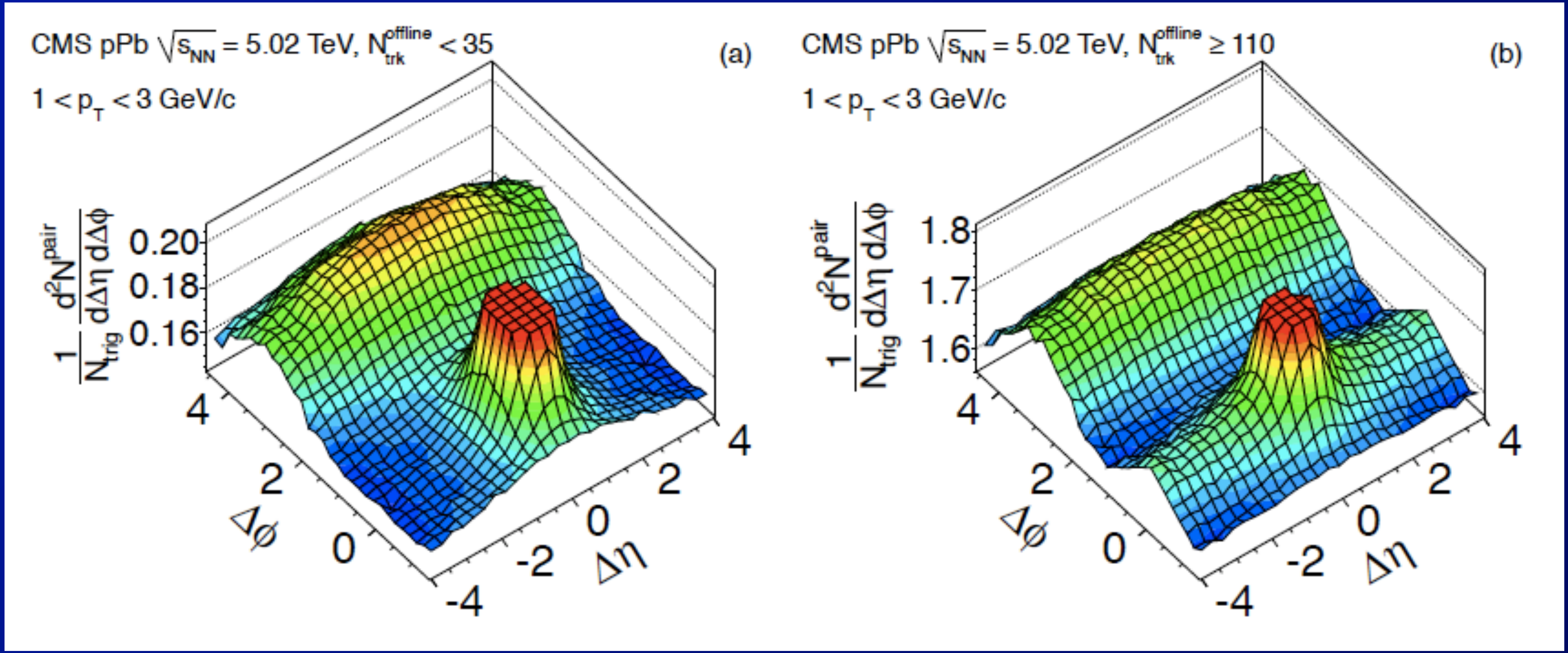
- Plot: $\frac{v_2}{n_q}$ vs $\frac{KE_T}{n_q}$

$\Rightarrow$Observe universal curve!

p/d-A collisions

CMS: ridge in p+Pb collisions

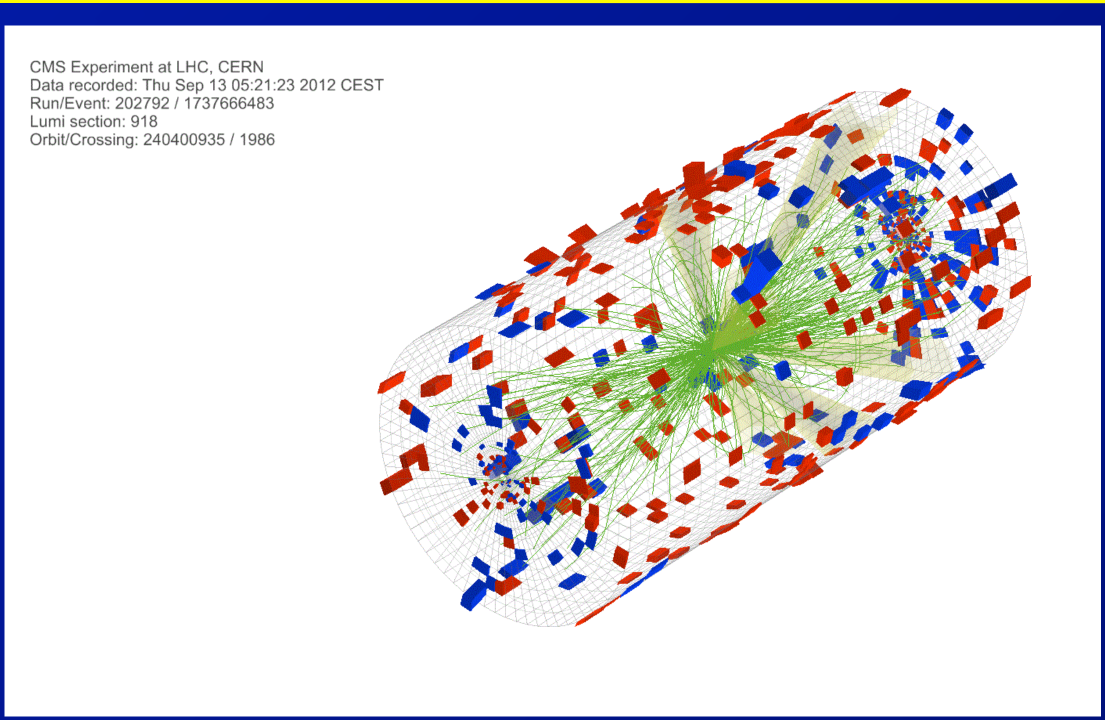
Low multiplicity **High multiplicity**

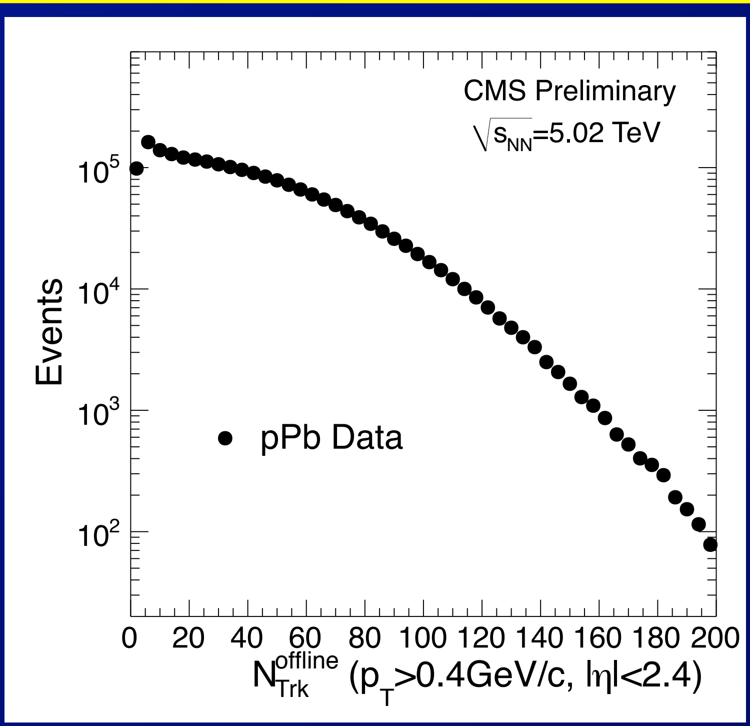


- Observe long-range near-side correlation in high-multiplicity p+Pb collisions (ridge)

  ⇒Also seen by CMS in high mult. p-p collisions

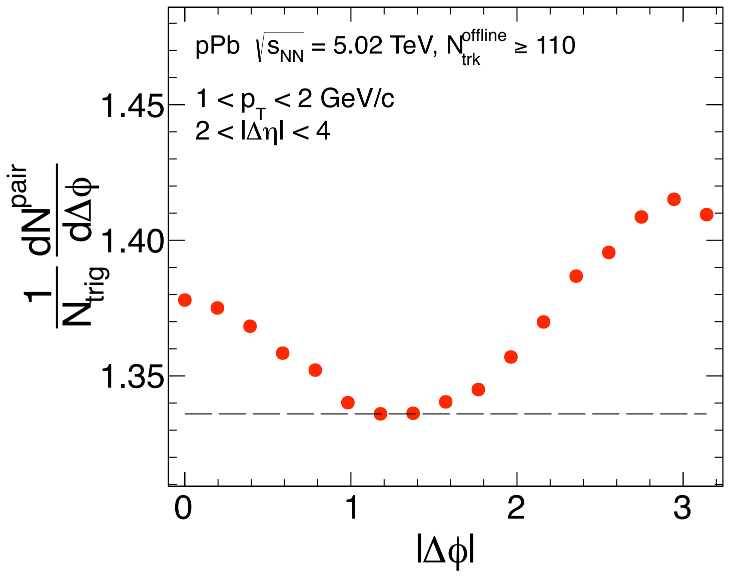
CMS: background on p+Pb collisions

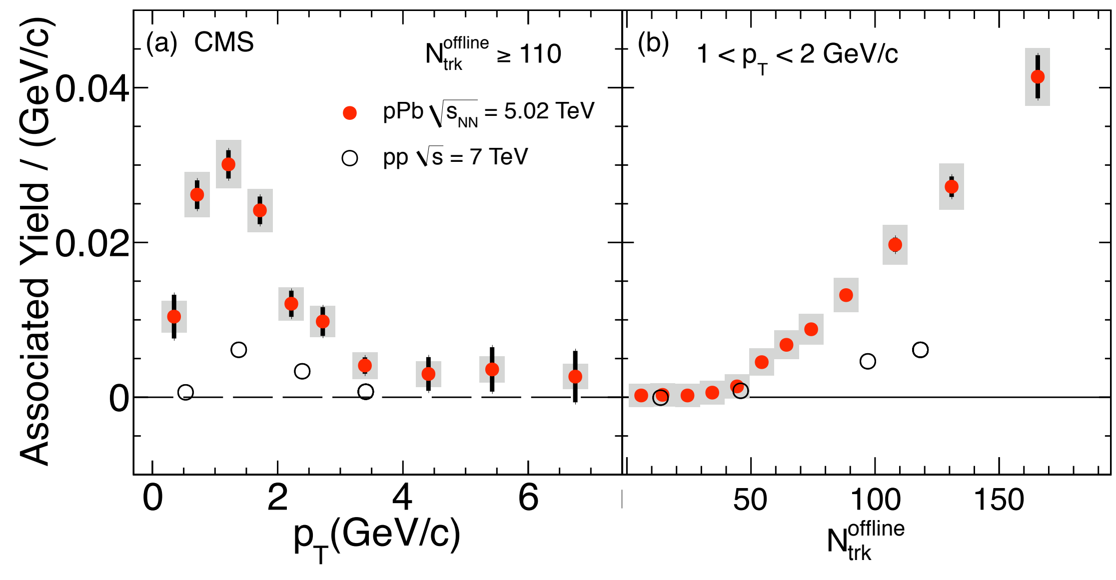




- **p+Pb collisions @ 5.02 TeV**
  - LHC design: beams must have same q/p
    - ⇒ 4 TeV p on 1.58 TeV/nucleon Pb
    - ⇒Center of mass has rapidity 0.47 *wrt* lab
  - Multiplicity distribution of reconstructed charge particles in $|\eta| < 2.5$ extends to > 200

CMS: ridge in p+Pb collisions

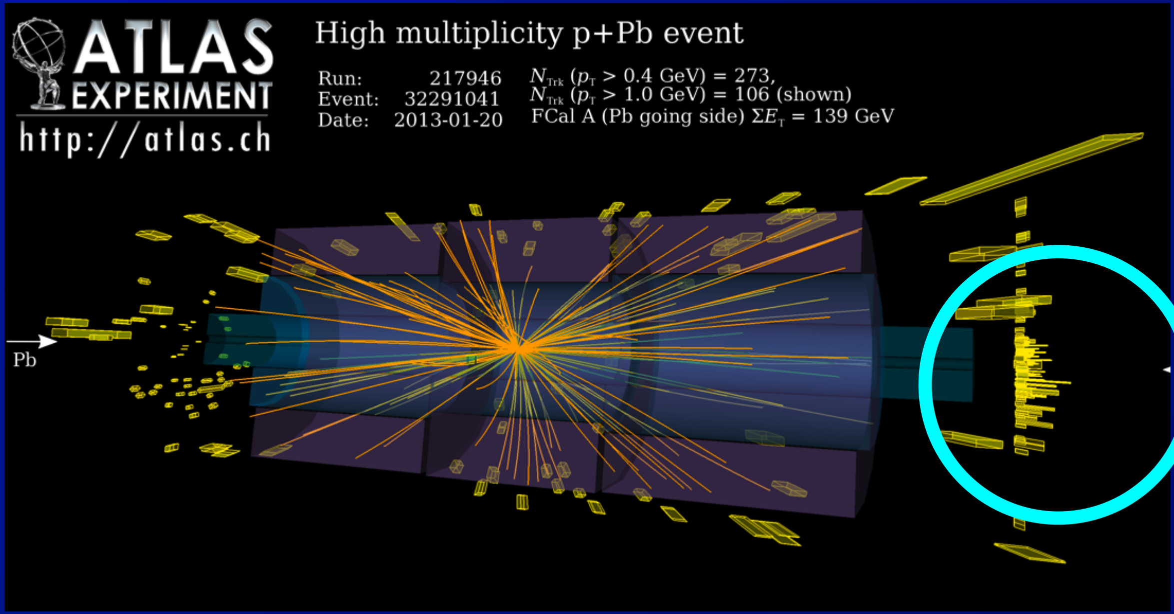


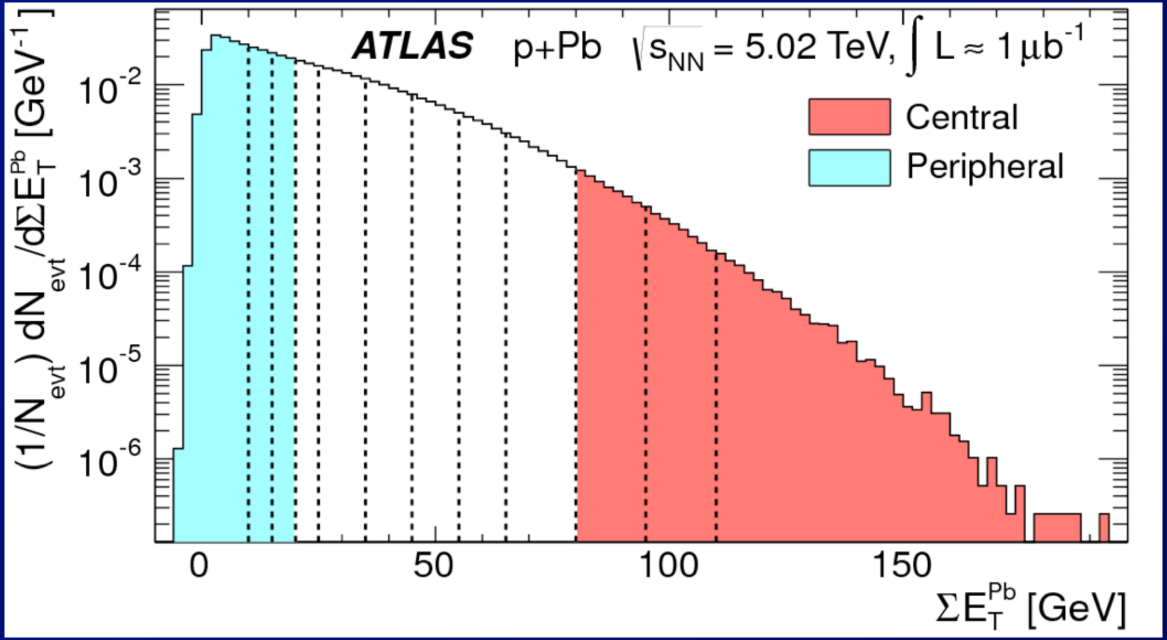


Dirty little secret:

- **Two-particle correlation contains 'signal' on top of a pedestal of uncorrelated pairs.**
  - – No way to, *a priori*, determine how much pedestal
- **Prescription used by all experiments**
  - – "Zero Yield (in correlation) At the Minimum"
    - ⇒Subtract constant to make it so
  - – (e.g.) calculate conditional yield over $|\Delta\phi| < 1.2$

ATLAS p+Pb collisions

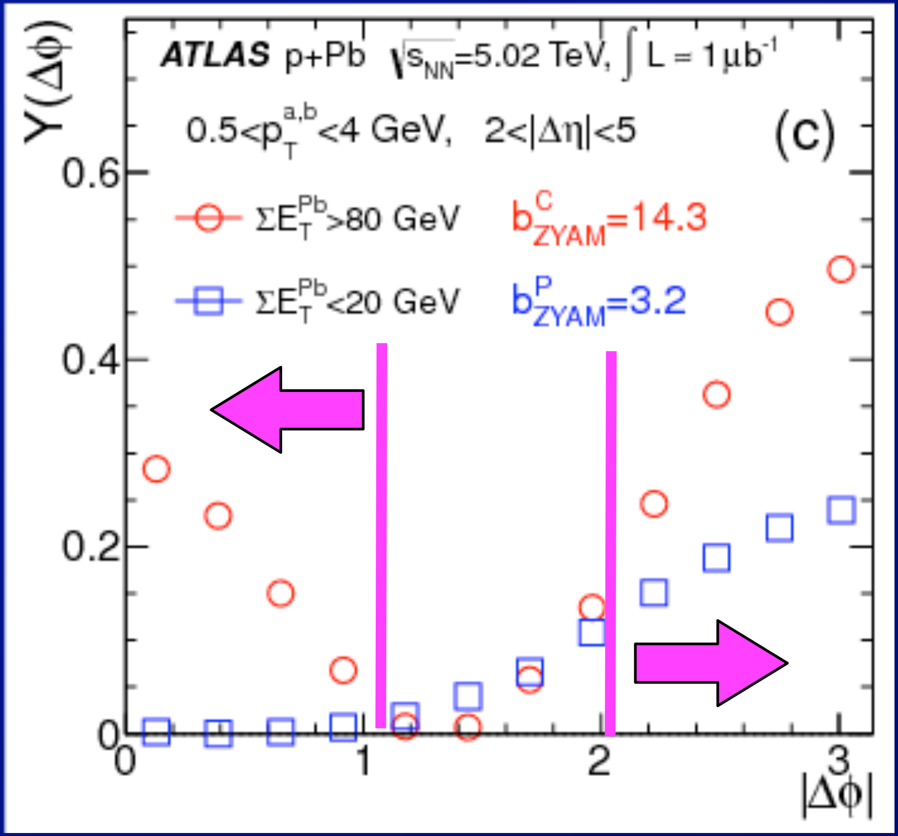


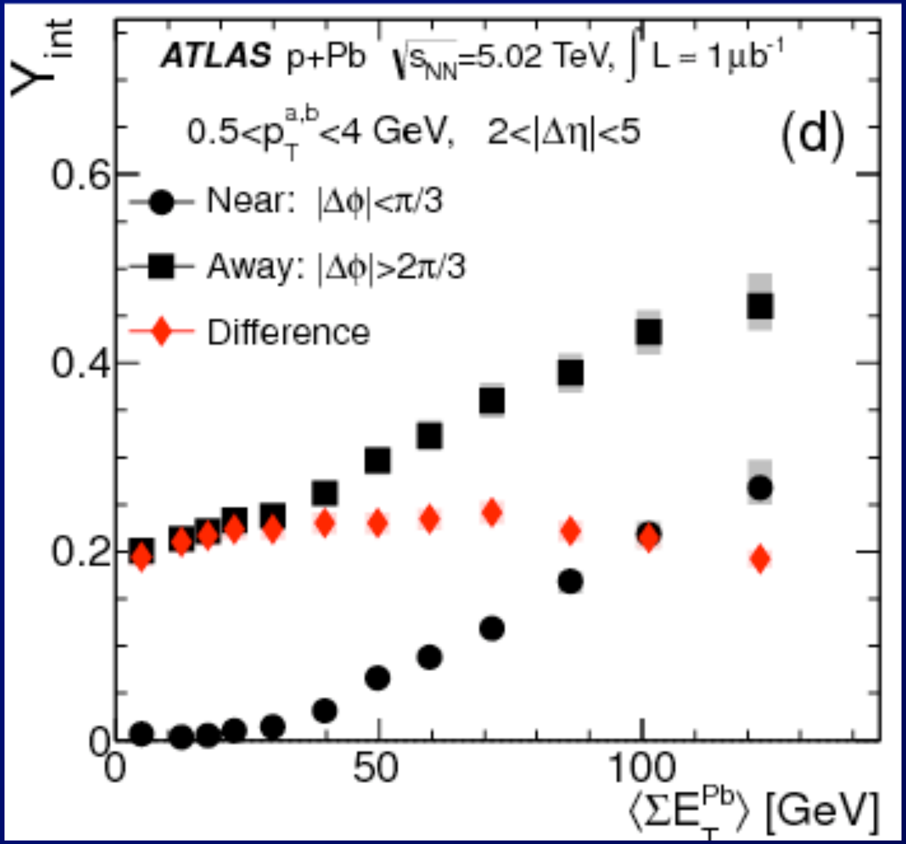


Characterize "multiplicity" or event activity using forward calorimeter on Pb-going side

ATLAS 2-particle correlations (3)

- Associated yields, Y(Δφ), integrated over η
  - peripheral and central
    - ⇒“Ridge” clearly present in central
    - ⇒Similar increase in the away side yield between peripheral, and central collisions

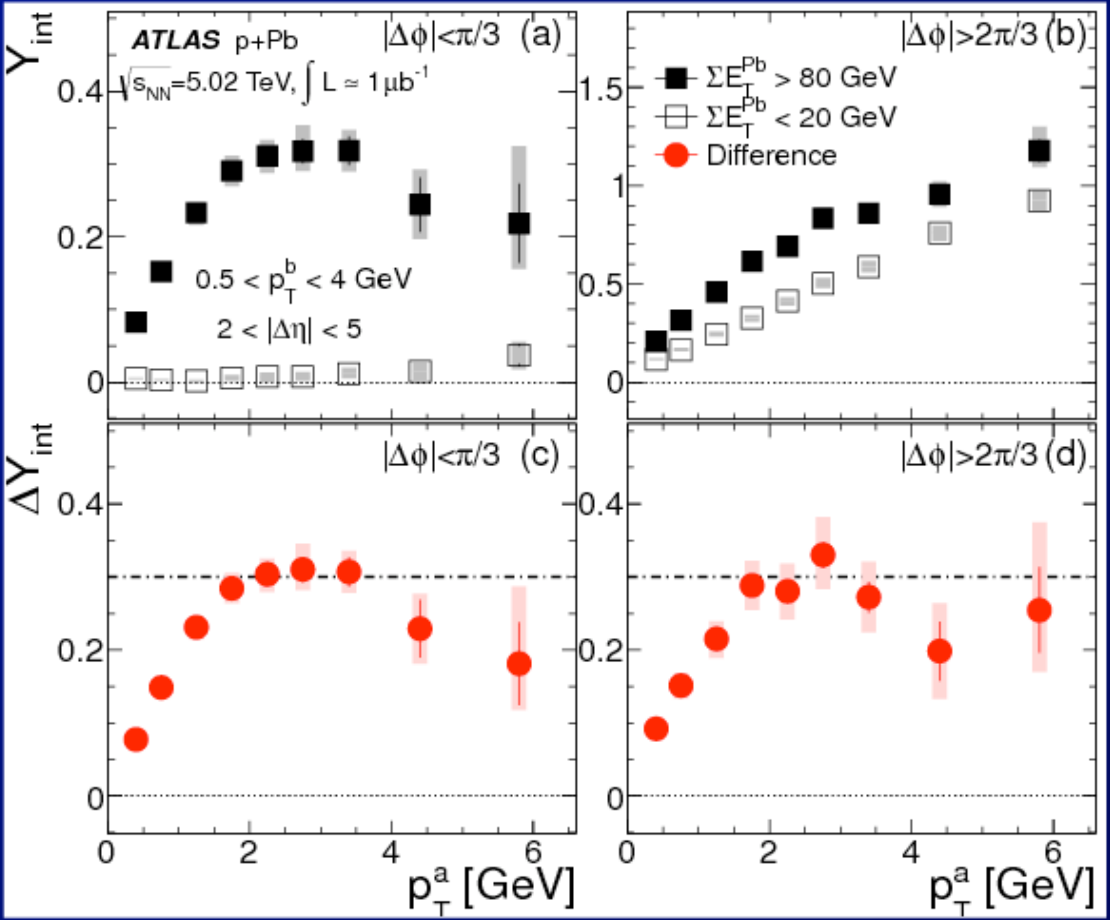




ATLAS 2-particle correlations (5)

- Study variation of integrated per-trigger yields with trigger p_T
  - For associated $0.5 < p_T < 4$ GeV
- Evaluate difference between peripheral and central
  - difference ≈ same on near and away sides, and similar p_T dependence

near away

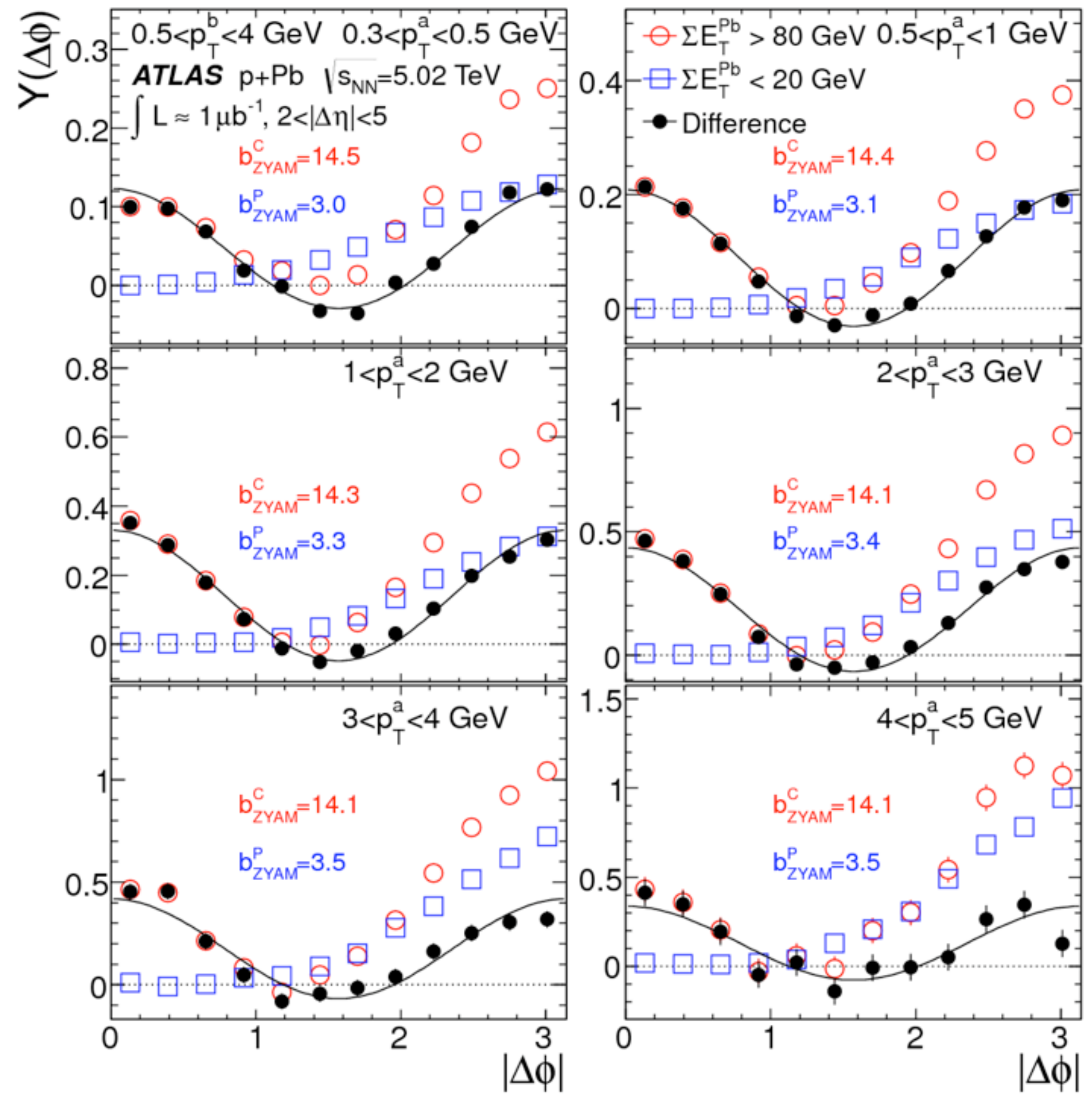


Beware different vertical scales on top panels

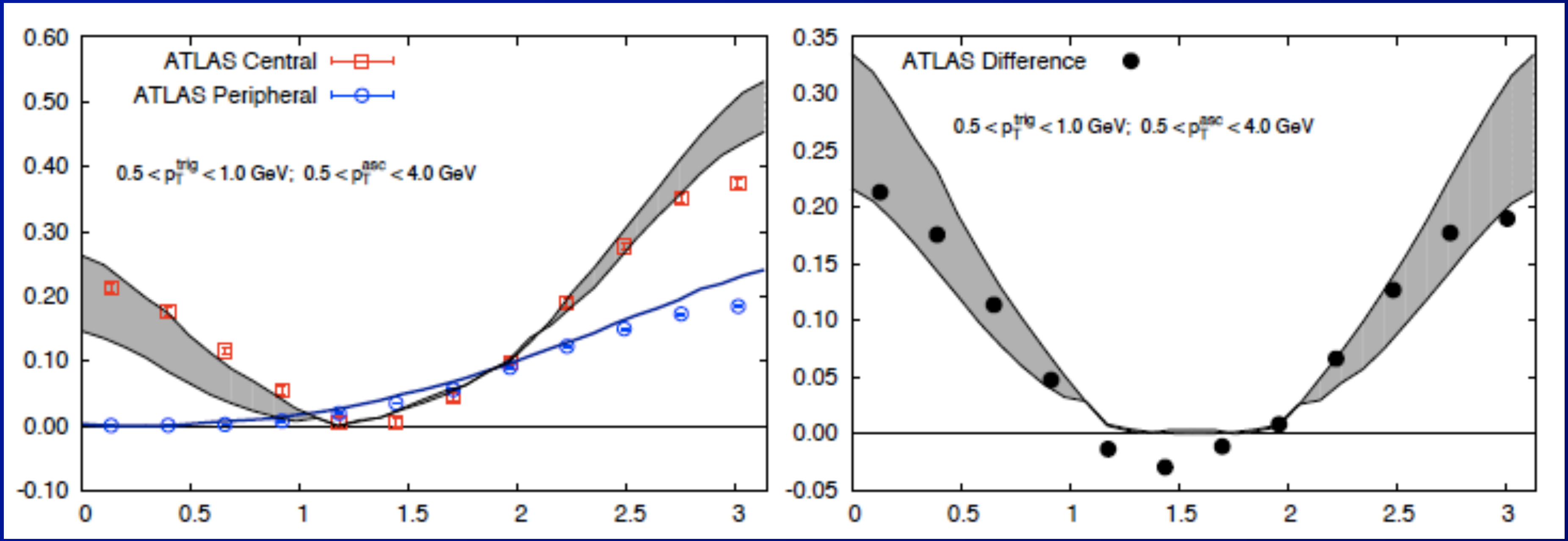
ATLAS 2-particle correlations (6)

- Motivated by above observations subtract peripheral Y(Δφ) from central Y(Δφ)
  - With associated $0.5 < p_T < 4$ GeV
  - In different trigger p_T bins

    ⇒Observe an approximately symmetric modulation in all bins

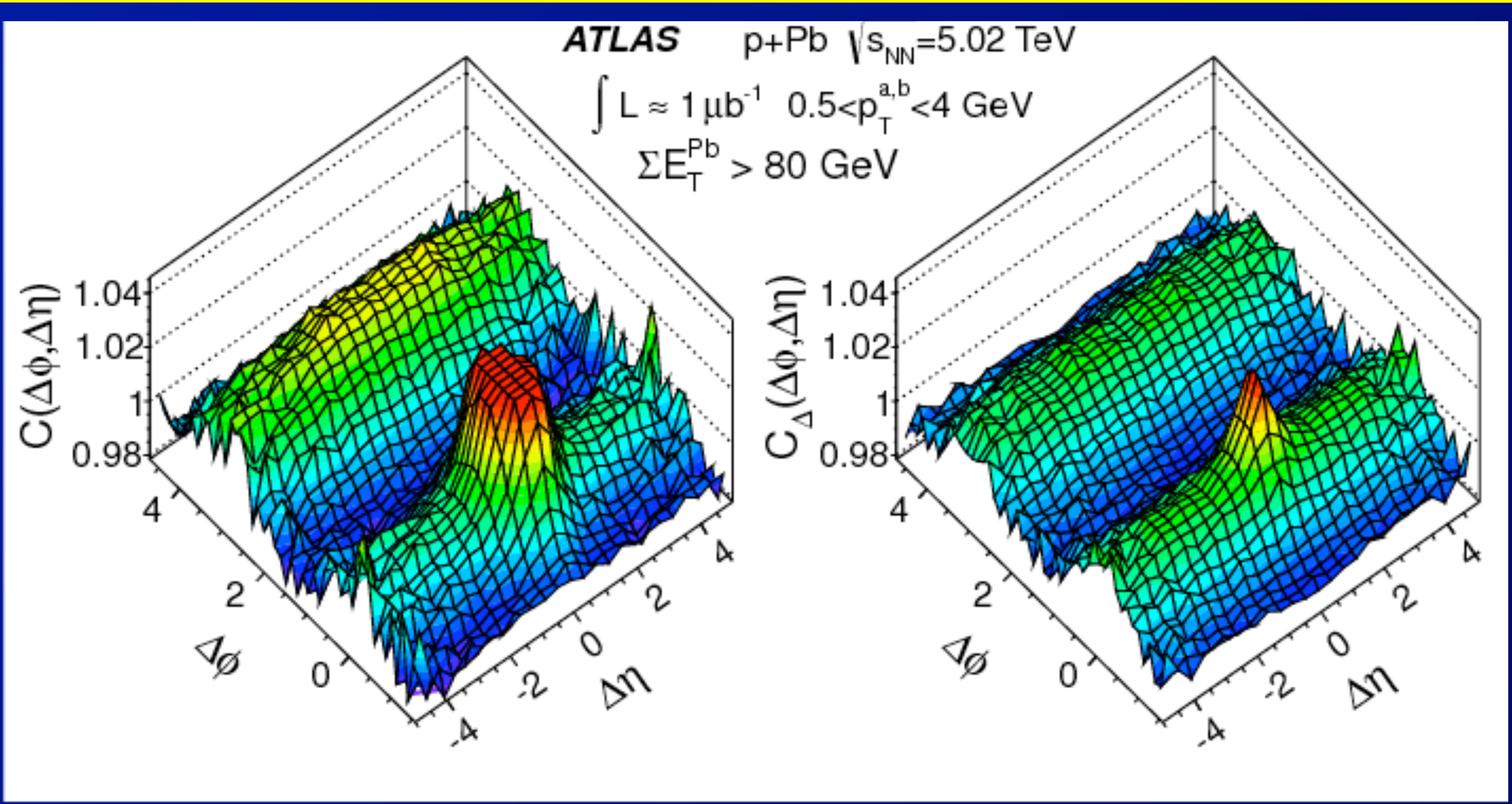


Explained by saturation?



- Theoretical calculations of the effects of saturation can reproduce the ATLAS (e.g.) data.

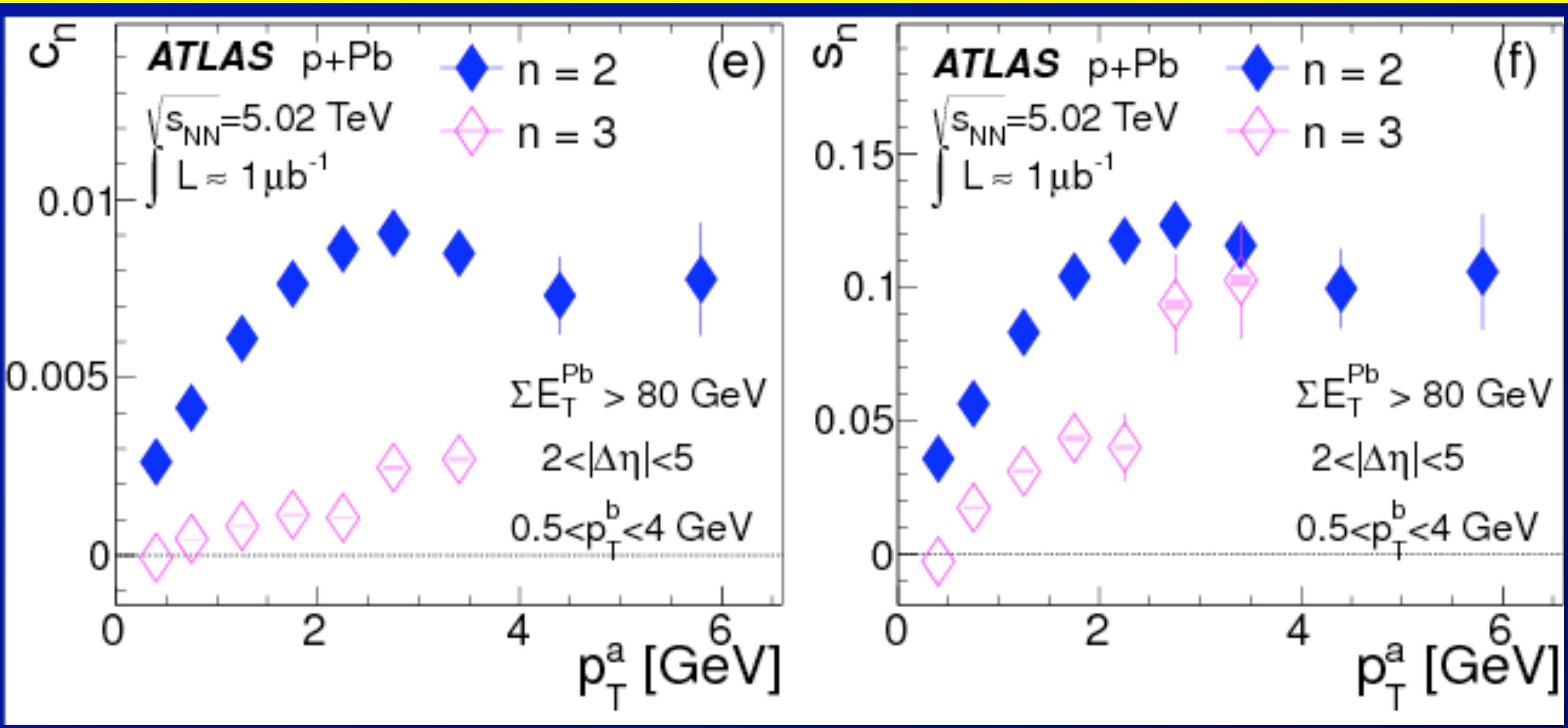
ATLAS 2-particle correlations (7)



- Central correlation function before and after subtraction of peripheral per-trigger yields, and converting back to C(Δφ,Δη)

  ⇒Long-range modulation

Fourier decomposition



- Extract Fourier coefficients for the pair distributions (c_2, c_3)
  - analog of 2-particle $v_{2,2}$, $v_{3,3}$
- Assume factorization $c_2(p_T^a, p_T^b) = s_2(p_T^a)\, s_2(p_T^b)$
  - checked

$\Rightarrow$To obtain s_2, s_3 → if flow, v_2, v_3