Independent Particle Model:

1 particle (or hole)

outside closed shell

(very few nuclei)

→ **Multi-particle systems**

Recall mean field approximation

$H = H_{IPM} + H_{Residual}$

↓

Residual Interactions

Effects not included in independent particle model potential

Residual Interactions

Need to consider a more complete Hamiltonian:

$$H = H_0 + H_{residual}$$

$H_{residual}$ reflects interactions not in the single particle potential.

NOT a minor perturbation. In fact, these residual interactions determine almost everything we know about most nuclei.

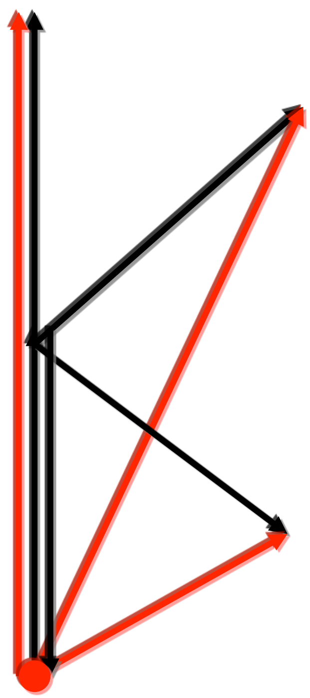
Start with 2- particle system, that is, a nucleus "doubly magic + 2".

$$H_{residual} \quad \text{is} \quad H_{12}(r_{12})$$

Consider two identical valence nucleons with j_1 and j_2.

Two questions: What total angular momenta $j_1 + j_2 = J$ can be formed?
What are the energies of states with these J values?

Coupling of two angular momenta



$j_1 + j_2$ **All values from:** $j_1 - j_2$ to $j_1 + j_2$ $(j_1 \neq j_2)$

Example: $j_1 = 3$, $j_2 = 5$: J = 2, 3, 4, 5, 6, 7, 8

BUT: For $j_1 = j_2$: J = 0, 2, 4, 6, … (2j – 1) (Why these?)

How can we know which total J values are obtained for the coupling of two identical nucleons in the same orbit with total angular momentum j? Several methods: easiest is the "m-scheme".

Table 5.1 *m scheme for the configuration* $|(7/2)^2 J\rangle$*

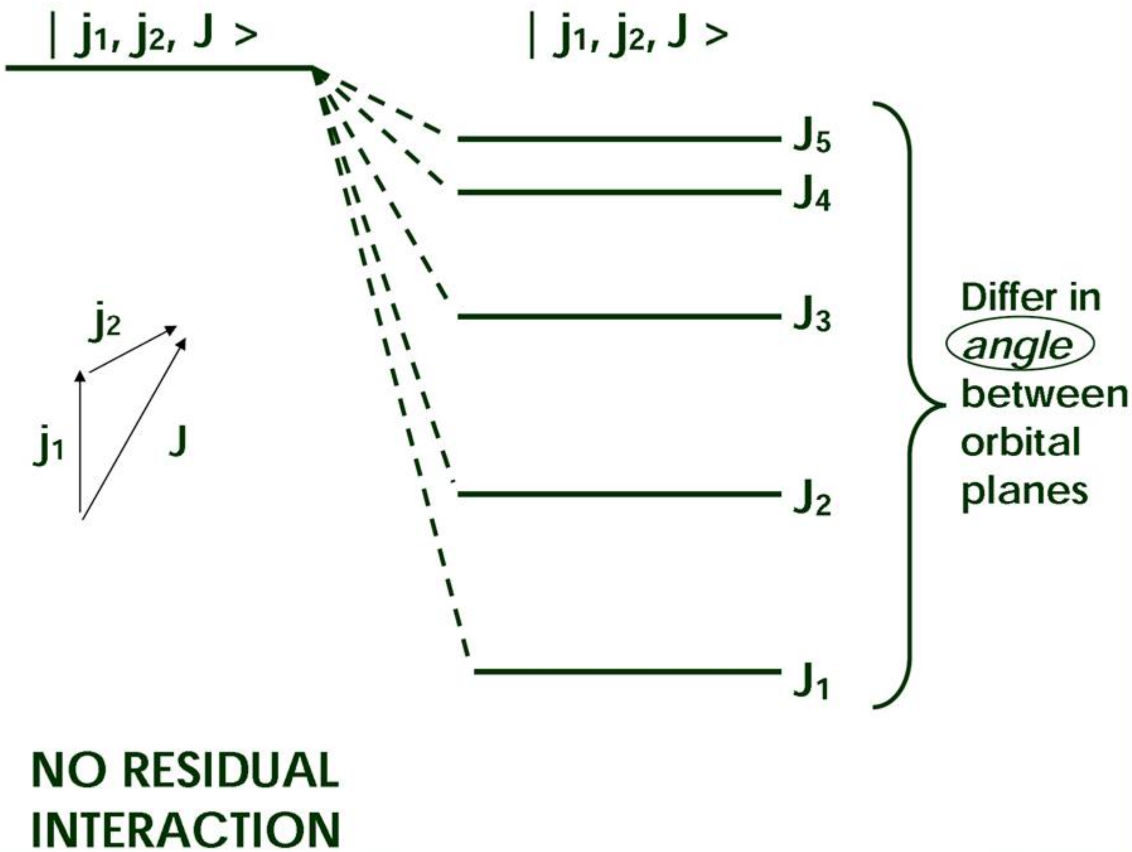
$j_1 = 7/2$ m_1	$j_2 = 7/2$ m_2	M	J
7/2	5/2	6	
7/2	3/2	5	
7/2	1/2	4	
7/2	–1/2	3	6
7/2	–3/2	2	
7/2	–5/2	1	
7/2	–7/2	0	
5/2	3/2	4	
5/2	1/2	3	
5/2	–1/2	2	4
5/2	–3/2	1	
5/2	–5/2	0	
3/2	1/2	2	
3/2	–1/2	1	2
3/2	–3/2	0	
1/2	–1/2	0	0

* Only positive total M values are shown. The table is symmetric for $M < 0$.

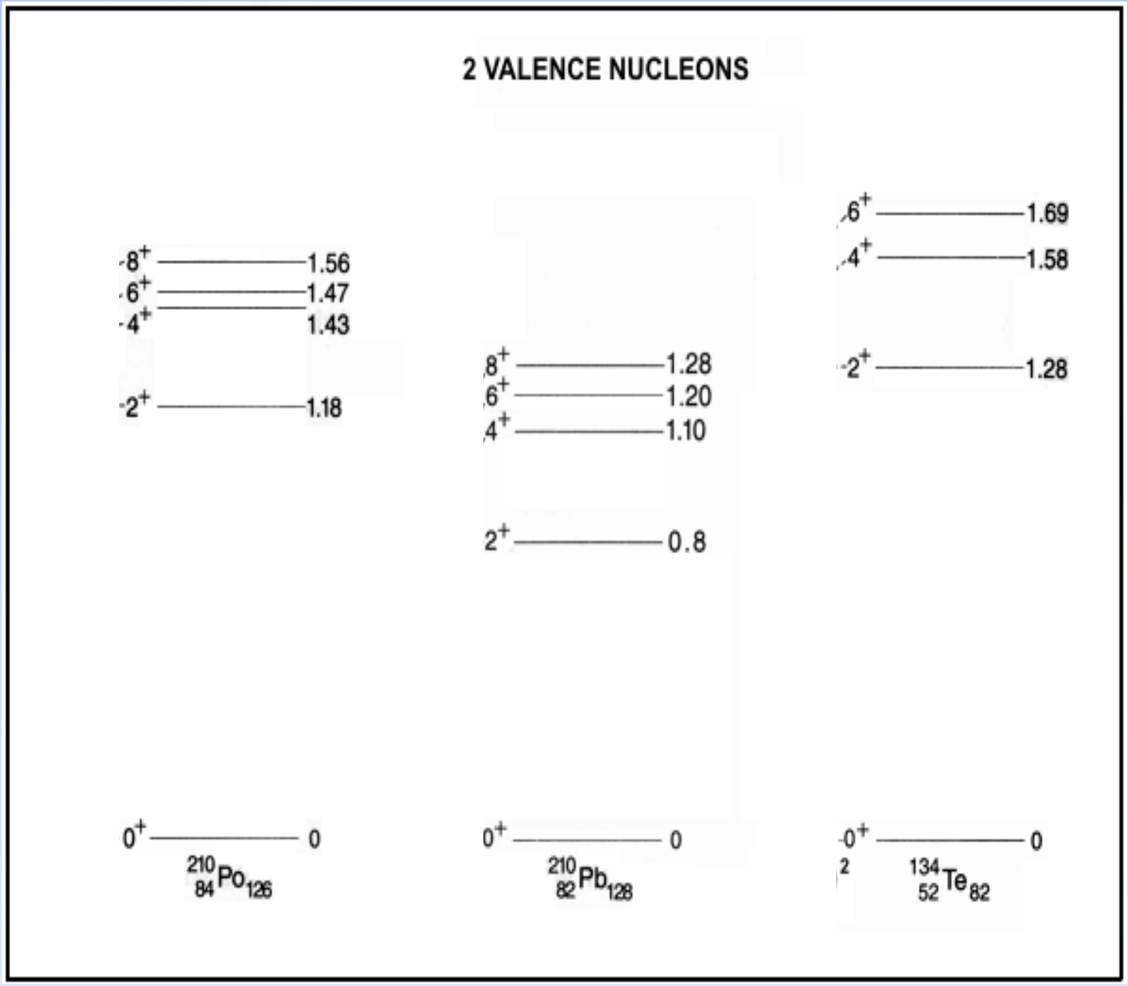
Residual Interactions—Diagonal Effects

Consider 2 particles, in orbits j_1, j_2 coupled to spin J_i, and interacting with a residual interaction, V_{12}.

2 Identical Nucleons



Typical spectra of nuclei with 2 "valence" particles outside doubly magic core. Universal result: Ground state always 0^+



Why? Can we obtain such simple results by considering residual interactions?

What are Energies of 2-particle configurations

$$\Delta E(j_1\ j_2\ J) = \langle j_1\ j_2\ J\ M \,|\, H_{12} \,|\, j_1\ j_2\ J\ M \rangle$$

$$= \frac{1}{\sqrt{2J+1}} \langle j_1\ j_2\ J \,||\, H_{12} \,||\, j_1\ j_2\ J \rangle$$

Separate radial and angular coordinates

$$\Psi = \frac{1}{r} R_{nl}(r)\, Y_{lm}(\theta, \phi)$$

where $$\frac{d^2 R_{nl}}{dr^2} - \frac{l(l+1)}{r^2} R_{nl} + \frac{2m}{\hbar^2}(E_{nl} - V) R_{nl} = 0$$

R_{nl} depends on potential – but generally not very much.

Now, what is H_{resid}?

Many choices possible. Let's start with simplest. Nuclear force is short range and attractive. So, take δ-force

$$V_\delta = \frac{-V_0}{r_1 r_2}\delta(r_1 r_2)\,\delta(\cos\Theta_1, \cos\Theta_2)\,\delta(\Phi_1, \Phi_2)$$

in spherical coordinates

Need to evaluate the matrix element (ME) of the form

$$\left\langle \Psi \middle| ``\delta" \middle| \Psi \right\rangle = \left\langle \frac{1}{r}R_{nl} \middle| V_{\delta_r} \middle| \frac{1}{r}R_{nl} \right\rangle \times \left\langle Y_{lm}(\Phi,\Phi) \middle| V_{\delta_{\Theta,\Phi}} \middle| Y_{lm}(\Phi,\Phi) \right\rangle$$

First factor is just a constant independent of J,

i.e., does not depend on J in $|j_1 j_2 J\rangle$.

<u>So energy shifts for different J's are independent of the form of the radial wave functions and hence of the radial form of the potential</u> ! !

⇒ Great simplification

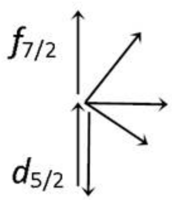
⇒ Typical of many results – radial effects disappear

How can we understand the energy patterns that we have seen for two – particle spectra with residual interactions? Easy – involves a very beautiful application of the Pauli Principle.

Need 2 ideas <u>only</u>

- Nuclear force (including residual interactions) is
  - Short range and attractive
  - Pauli Principle

Physical Interpretation



J depends on angle between the two orbital planes

Interaction strongest when the 2 particles are closest to each other

i.e., when the orbits are co-planar

$\Rightarrow$ strongest interaction either for

J_{min} *or* J_{max}

Which one?

Consider *L*, *S* composition of state *J*

$$\bar{L} = \bar{l}_1 + \bar{l}_2 \qquad S = \bar{\frac{1}{2}} \pm \bar{\frac{1}{2}} = 1 \text{ or } 0$$

Pauli Principle

Fermions:

No two fermions can occupy the same state/place

Wave functions must be totally antisymmetric

$$\Psi(\bar{r}) = -\Psi(-\bar{r}) \qquad \bar{r} = \bar{r}_2 - \bar{r}_1$$

$\therefore$ If particles are at same place ----- $\bar{r}$ = 0-----

then Ψ (0) = -Ψ (0)

$\Rightarrow \Psi$ (0) = 0

so PP is satisfied

We split wave functions into 2 parts - spatial part (L), and spin part (S). PP $\Rightarrow$

$$\Psi_{Tot} = \Psi_{spat} \times \Psi_{spin} = \text{Anti-sym}$$

This is the most important slide: understand this and all the key ideas about residual interactions will be clear !!!!!

PP: **Key Physics Ideas**

$\Psi_{spatial}$	Ψ_{spin}
A	S
S	A

$S = ½ + ½ = 1 = \text{Sym}$

$S = ½ - ½ = 0 = A\text{-Sym}$

$\Psi_{spat}(A) \times \Psi_{spin}(S = 1)$ $\Psi_{spat}(S) \times \Psi_{spin}(S = 0)$

$S = 1$ case $\Psi_{spat} = A$ $\Psi(r_{12}) = -\Psi(-r_{12})$

For δ force, which only acts at $r_{12} = 0$

$\Psi(r_{12} = 0) = 0$! !

So, at the ONLY place where a δ-int acts, the wave fct. vanishes—*i.e.*, No effect of δ fct int on $S = 1$ states ! ! !

$S = 0$ case $\Psi_{spat} = S$

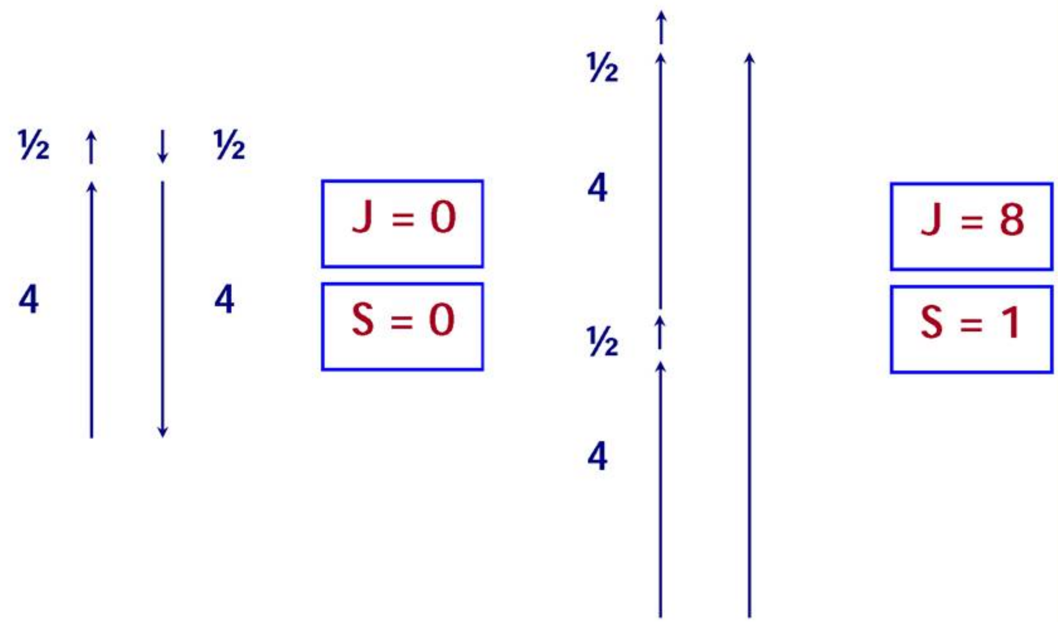
No restriction on $\Psi(r_{12} = 0)$, hence δ-int can have big effect ! ! !

Equivalent Orbits

$j_1 = j_2 \qquad |j^2\, J\rangle$

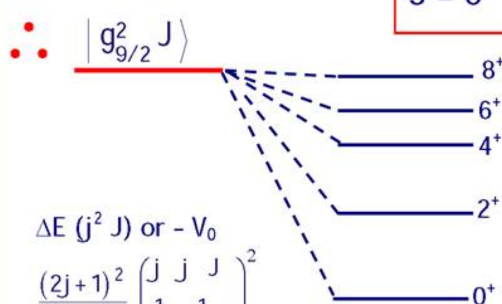
$J = 0,\ 2,\ 4,\ \ldots\ldots\ldots\ldots\ 2j - 1$

e.g. $|g^2_{9/2}\, J\rangle \quad J = 0,\ 2,\ 4,\ 6,\ 8$



S = 0 lowered ⇒ J = 0 lowest

$\therefore$



0^+ lowest
all e - e
nuclei have
0^+ g.s. ***!!!***

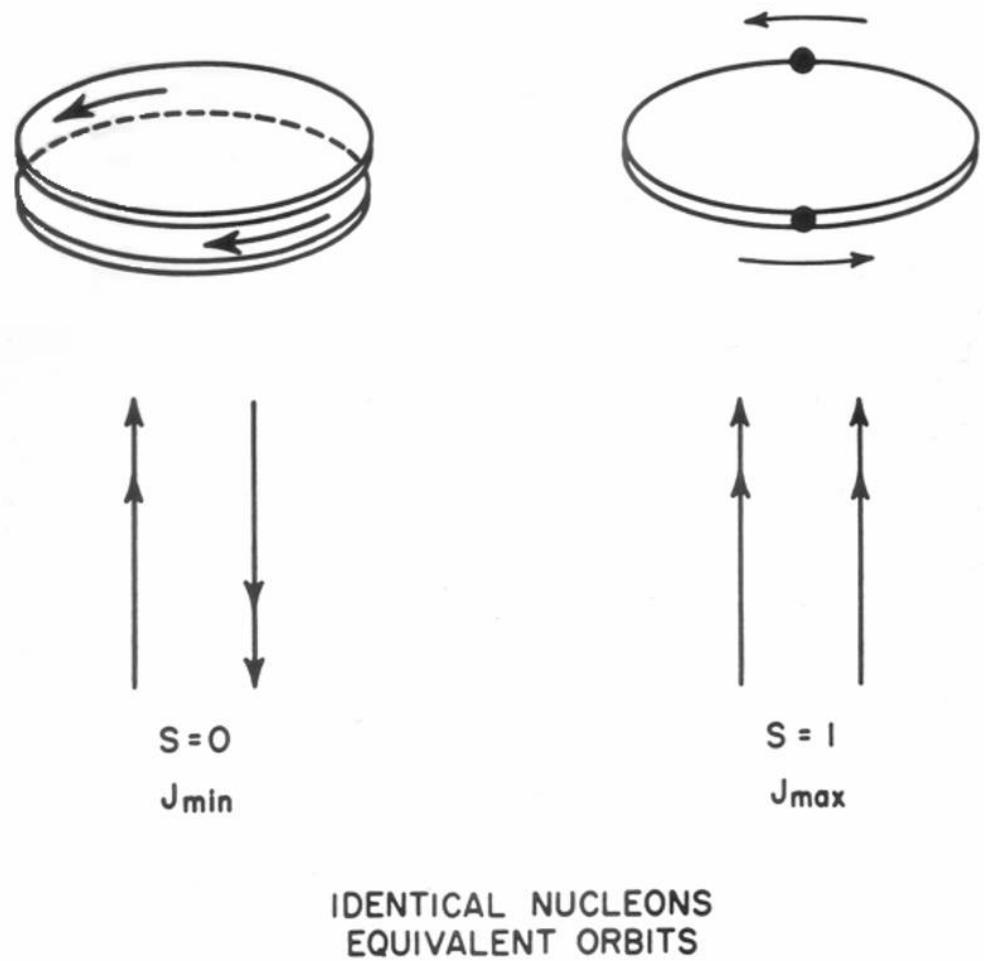
$\Delta E\ (j^2\, J)$ or $-V_0$

$$\frac{(2j+1)^2}{2}\begin{pmatrix} j & j & J \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}^2$$

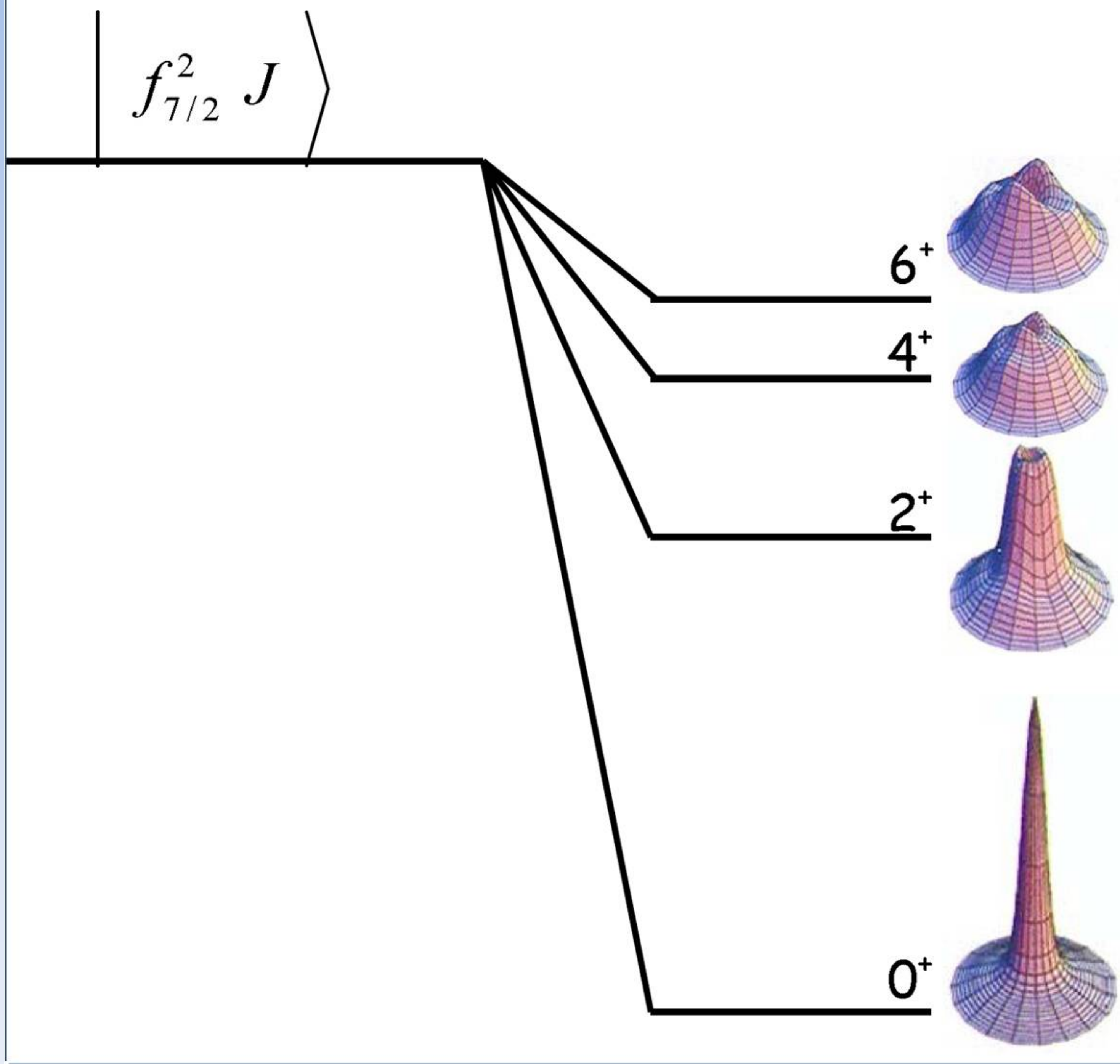
For $J = 0 \quad \Delta E \propto -V_0\, \frac{2j+1}{2}$ $\Rightarrow$ ΔE larger for larger j: Why???

Geometrical Interpretation

for $|j^2 J=0\rangle$ being lowest



Pauli Principle is ~ repulsive interaction !


$\left| f_{7/2}^2 \, J \right\rangle$
6^+
4^+
2^+
0^+

δ Interaction

Analytic formulas

$$V_{12}(\delta) = -V_0\,\delta(r_1 - r_2) = \frac{-V_0}{r_1 - r_2}\delta(r_1 - r_2)\,\delta(\cos\theta_1 - \cos\theta_2)\,\delta(\Phi_1 - \Phi_2)$$

$$\Delta E(j_1 j_2 J) = -V_0 F_R(n_1 l_1 n_2 l_2) A(j_1 j_2 J)$$

where

$$F_R(n_1 l_1 n_2 l_2) = \frac{1}{4\pi}\int \frac{1}{r^2} R^2_{n_1 l_1}(r) R^2_{n_2 l_2}(r)\,dr$$

and

$$A(j_1 j_2 J) = (2j_1+1)(2j_2+1)\begin{pmatrix} j_1 & j_2 & J \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}^2 \quad (\text{if } l_1 + l_2 - J \text{ is even})$$

$$= 0 \quad (\text{if } l_1 + l_2 - J \text{ is odd})$$

(Non-equivalent orbits)

$$\Delta E(j^2 J) = -V_0 F_R(nl) A(j^2 J) \quad (J \text{ even})$$

where

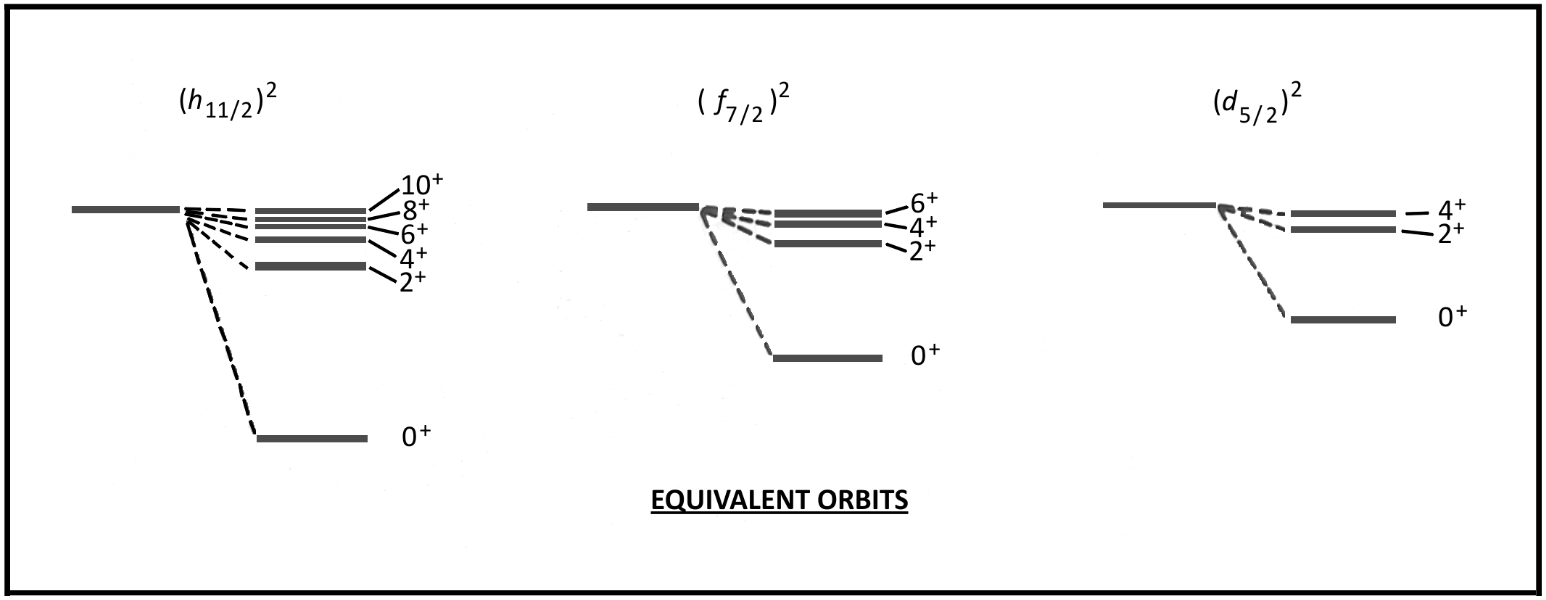
$$F_R(nl) = \frac{1}{4\pi}\int \frac{1}{r^2} R^4_{nl}(r)\,dr$$

and

$$A(j^2 J) = \frac{(2j+1)^2}{2}\begin{pmatrix} j & j & J \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}^2 \quad (J \text{ even})$$

(Equivalent orbits)

MULTIPLET SPLITTINGS; δ INTERACTION (Identical Particles)



NOTE: $R_{4/2} < 2.0$

Simple treatment of residual interactions accounts for universal fact that even-even nuclei have 0^+ ground states.

Note that the 0^+ level is lowered more for higher j orbits

Lowering of $0^{\pm}$ States

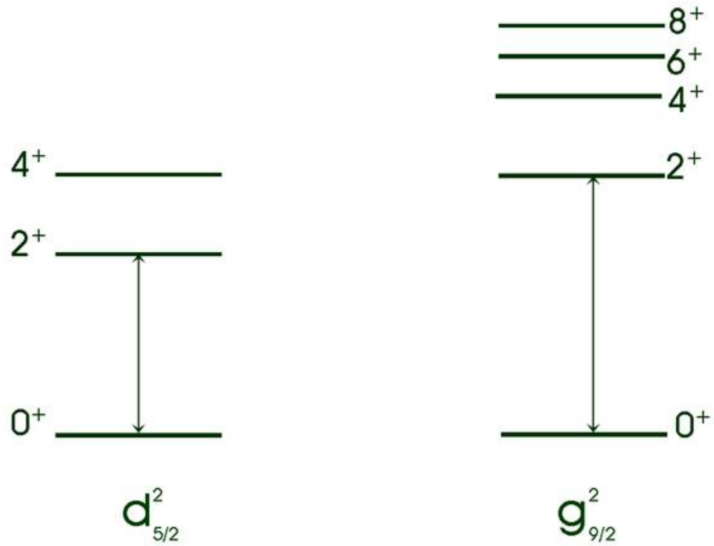
$$\Delta E(j^2 J) \propto -V_0 \frac{(2j+1)^2}{2} \begin{pmatrix} j & j & J \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}$$

For $J = 0$

$$\Delta E(j^2 J = 0) \propto -V_0 \frac{(2j+1)}{2}$$

$\Rightarrow$ $\Delta E \propto 2j + 1$

Energy lowering of 0^+ is larger for larger j



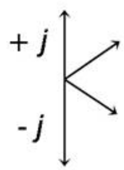
Why ?

Lowering of 0^+ States in $| j^2 J \rangle$

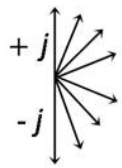
$\Delta E \propto 2j + 1$. Why?

Note: $2j + 1$ = # magnetic substates

low *j*



high *j*



Semi-classical picture-localized

Ψ (*J*, *m*, Θ) is localized to an angular <u>range</u>* centered about normal to ang. mom. vector:

spread of Ψ roughly given by angular "distance" to next substate

*quantum fluctuations

$\therefore$ <u>Larger *j*</u> $\Rightarrow$ more magnetic substates

$\Rightarrow$ greater localization

$\Rightarrow$ greater spatial overlap in $| j, m \rangle$ and $| j, -m \rangle$

$\Rightarrow$ lower energy

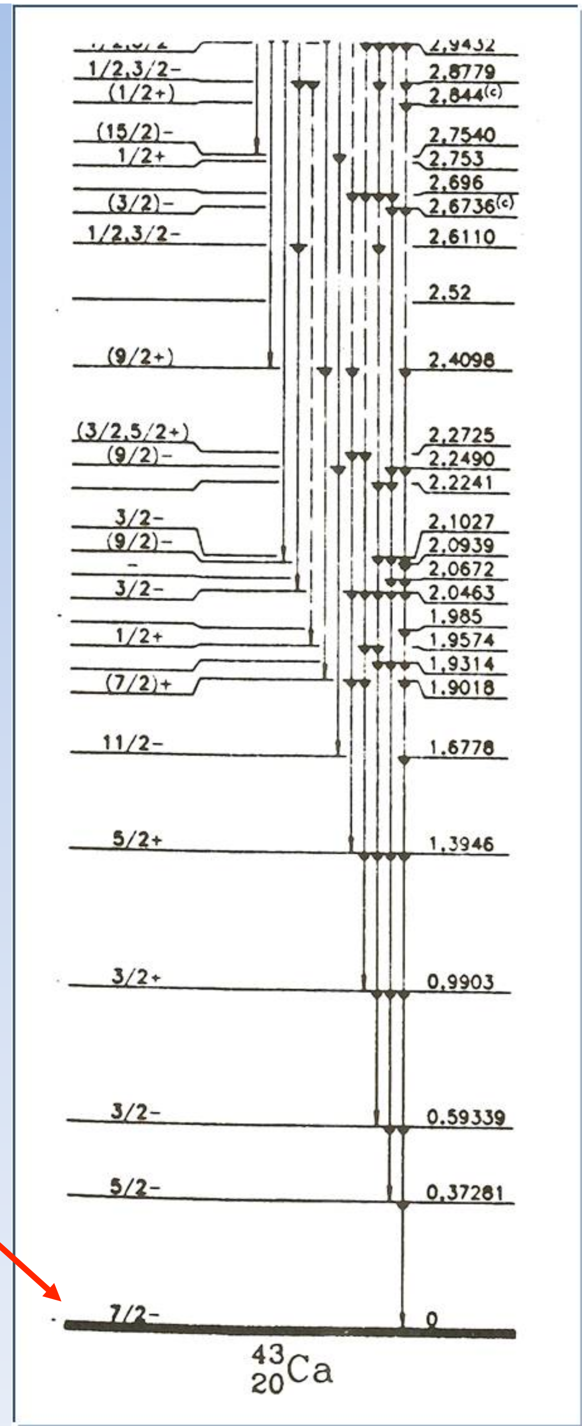
Extending the IPM with residual interactions

- Consider now an extension of, say, the Ca nuclei to ^{43}Ca, with three particles in a j= 7/2 orbit outside a closed shell?
- How do the three particle angular momenta, j, couple to give final total J values?
- If we use the m-scheme for three particles in a 7/2 orbit the allowed J values are 15/2, 11/2, 9/2, 7/2, 5/2, 3/2.
- For the case of J = 7/2, two of the particles must have their angular momenta coupled to J = 0, giving a total J = 7/2 for all three particles.
- For the J = 15/2, 11/2, 9/2, 5/2, and 3/2, there are no pairs of particles coupled to J = 0.
- Since a J = 0 pair is the lowest configuration for two particles in the same orbit, that case, namely total J = 7/2, must lie lowest !!

^{43}Ca

Treat as 20 protons and 20 neutrons forming a doubly magic core with angular momentum J = 0. The lowest energy for the 3-particle configuration is therefore J = 7/2.

Note that the key to this is the results we have discussed for the 2-particle system !!



GROUND STATE J^π VALUES OF SOME ODD MASS NUCLEI

Z = 20	$3/2^+$ ^{37}Ca	$3/2^+$ ^{39}Ca	$7/2^-$ ^{41}Ca	$7/2^-$ ^{43}Ca	$7/2^-$ ^{45}Ca	$7/2^-$ ^{47}Ca	$3/2^-$ ^{49}Ca
Z = 40		$9/2^+$ ^{87}Zr	$9/2^+$ ^{89}Zr	$5/2^+$ ^{91}Zr	$5/2^+$ ^{93}Zr	$5/2^+$ ^{95}Zr	
Z = 39	$1/2^-$ ^{85}Y	$1/2^-$ ^{87}Y	$1/2^-$ ^{89}Y	$1/2^-$ ^{91}Y	$1/2^-$ ^{93}Y	$1/2^-$ ^{95}Y	$1/2^-$ ^{97}Y
Z = 41		$9/2^+$ ^{91}Nb	$9/2^+$ ^{93}Nb	$9/2^+$ ^{95}Nb	$9/2^+$ ^{97}Nb	$9/2^+$ ^{99}Nb	
N = 50	$3/2^-$ $^{85}_{35}Br$	$3/2^-$ $^{87}_{37}Rb$	$1/2^-$ $^{89}_{39}Y$	$9/2^+$ $^{91}_{41}Nb$	$9/2^+$ $^{93}_{43}Tc$	$9/2^+$ $^{95}_{45}Rh$	

But, these were simple cases. As the number of valence nucleons grows, the number of ways of making states of a given J grows hugely.

Those "basis states" will mix. How many states do we need to mix? What are the resulting structures? How difficult a calculation is this? Consider a couple of simple cases and a more typical one.

The Need for Simplification in Multiparticle Spectra

Example: How many 2^+ states?

 # nucl.

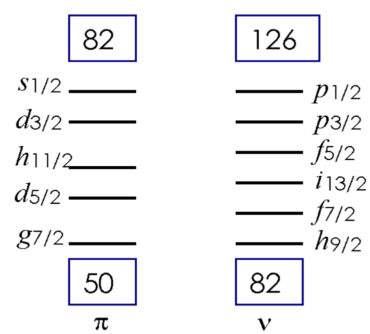
2	$d^2_{5/2}$	1	
4	$d_{5/2}\, g_{7/2}$	≥ 7	$\left\| d^2_{5/2}\, J=2,\, g^2_{7/2}\, J=0 \right\rangle, \left\| d^2_{5/2}\, J=0,\, g^2_{7/2}\, J=2 \right\rangle$
			$\left\| d^2_{5/2}\, J=4,\, g^2_{7/2}\, J=2;\, J=2 \right\rangle,$
			$\left\| d^2_{5/2}\, J=2,\, g^2_{7/2}\, J=4;\, J=2 \right\rangle,$
			$\left\| d^2_{5/2}\, J=4,\, g^2_{7/2}\, J=6;\, J=2 \right\rangle,$
			$\left\| d_{5/2}\, g_{7/2}\, J=1,\, d_{5/2}\, g_{7/2}\, J=1;\, J=2 \right\rangle,$
			$\left\| d^2_{5/2}\, J=4,\, g^2_{7/2}\, J=4;\, J=2 \right\rangle.$

$^{154}_{62}\mathrm{Sm}_{92}$

cl. sh. 50 82

$N_p = 12 \quad N_n = 10$

12 val. π in 50 – 82
10 val. ν in 82 – 126

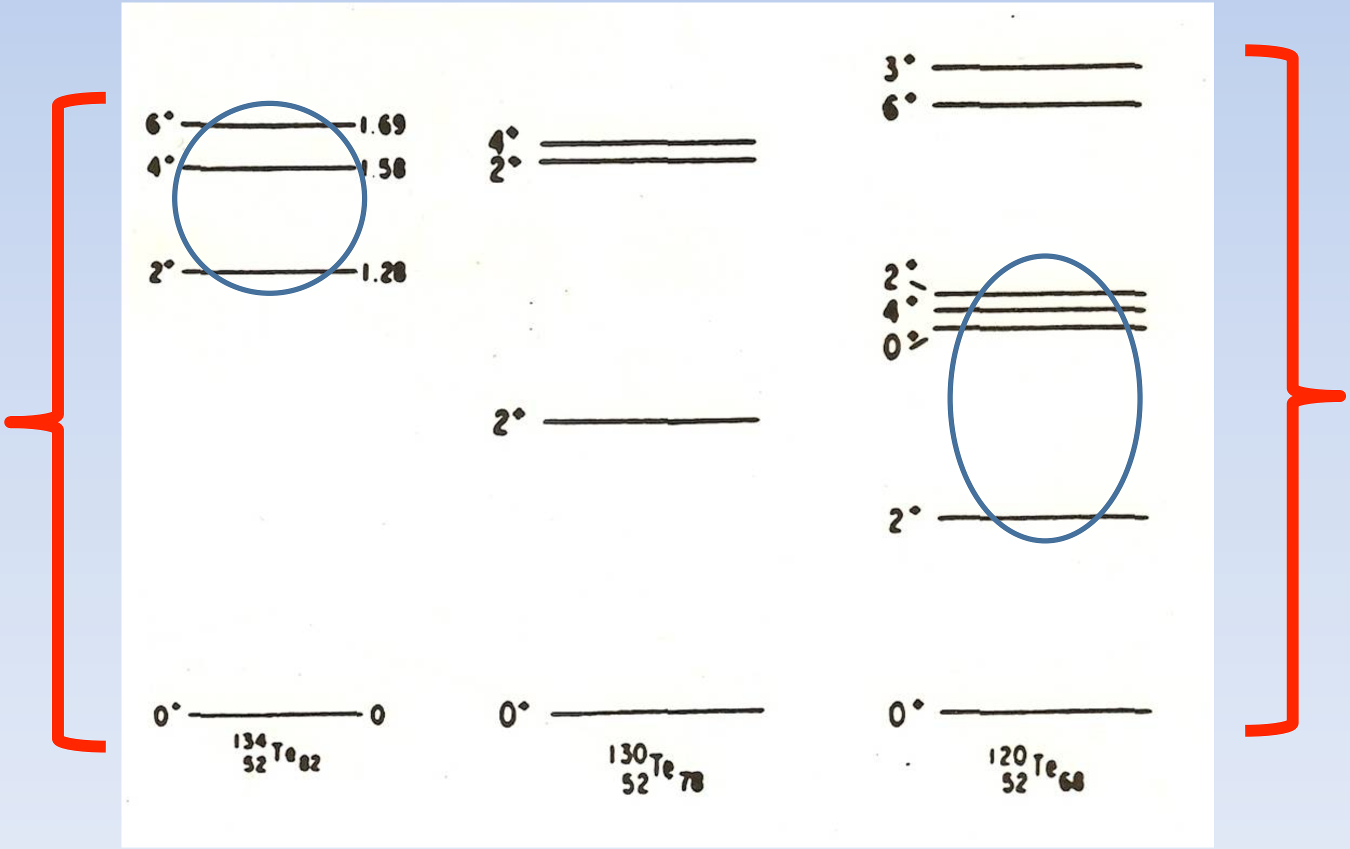


How many 2^+ states subject to Pauli Principle limits?

3×10^{14} !!!

^{154}Sm 2^+ states within the valence shell space

So, with even just a few valence nucleons, such calculations become intractable by simple diagonalization. But yet, nuclei show very simple patterns despite the complexity and chaotic behavior one might expect. Emergence of collective behavior.



How can we understand emergent collectivity?

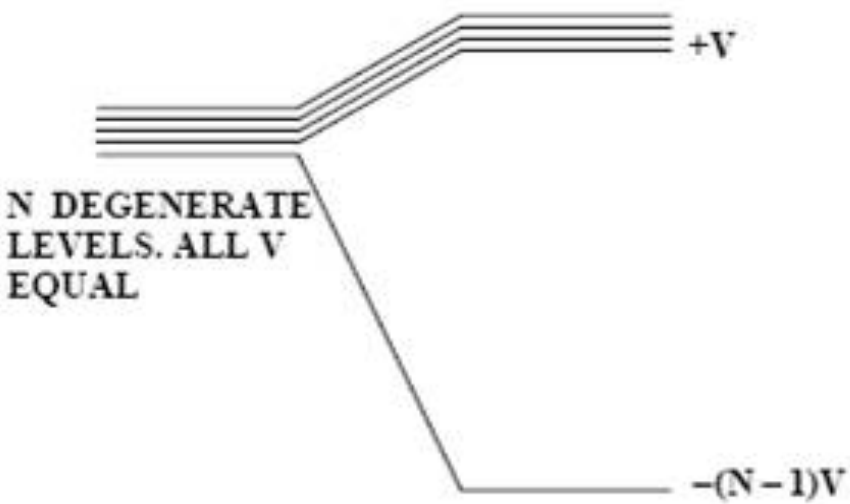
Two approaches

a) **Advanced methods at the level of nucleons and their interactions – See Vary lectures next week**

b) **Collective models that look at the many-body system as a whole, with its shapes, oscillations, quantum numbers, selection rules, etc.**

We will follow this route but then return to ask what the microscopic drivers of structural evolution and emergent collectivity are.

The key concept for Collectivity – Coherent motion of many nucleons. Lowering of collective states



$$\Psi_{LOWEST} = \frac{1}{\sqrt{N}}[\phi_1 + \phi_2 + \ldots + \phi_N]$$

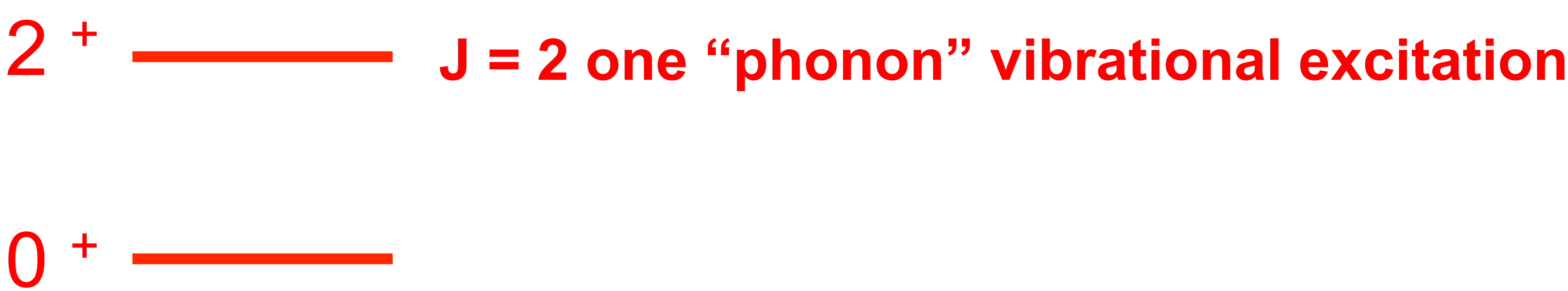
Lowering of one state. Note that the components of its wave function are all equal and in phase

Please think about this carefully – it is one of the most important concepts in all of many-body physics

Consequences of this: Lower energies for collective states, and enhanced transition rates.

First consider nuclei with a moderate number of valence nucleons (~ 6-16).

These nuclei retain the spherical shapes of nuclei near closed shells but are "soft" -- they can take on oscillatory vibrational motion. The lowest lying such excitation is a small amplitude surface quadrupole oscillation with angular momentum 2

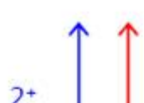


More than one phonon? What angular momenta? M-scheme for phonons

Table 6.1 *m scheme for two-quadrupole phonon states**

$J_1 = 2$ m_1	$J_2 = 2$ m_2	$M = \sum m_i$	J
2	2	4	
2	1	3	
2	0	2	4
2	−1	1	
2	−2	0	
1	1	2	
1	0	1	2
1	−1	0	
0	0	0	0

*Only positive total M values are shown: the table is symmetric for $M < 0$. The full set of allowable m_i values giving $M \geq 0$ is obtained by the conditions $m_1 \geq 0$, $m_2 \leq m_1$.



Types of collective structures

Few valence nucleons of each type:

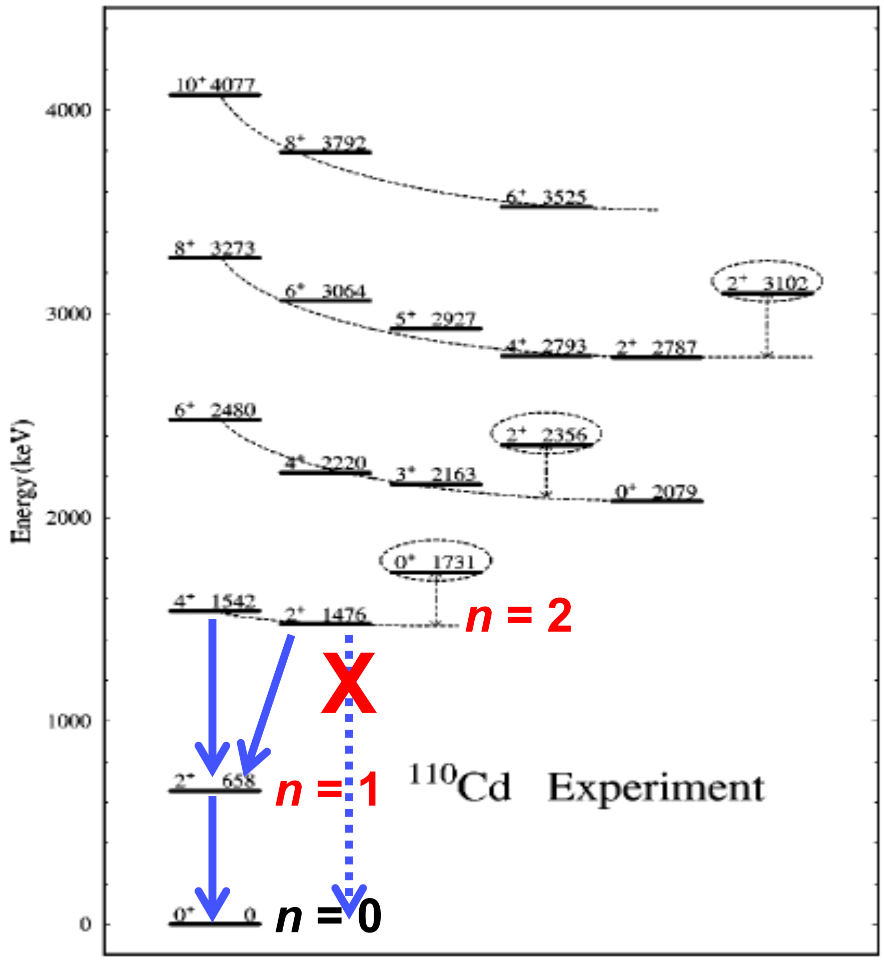
The spherical vibrator

Vibrator (H.O.)

$$E(J) = n\,(\hbar\,\omega_0)$$

$$R_{4/2} = 2.0$$

Gamma-ray transitions:
Selection rule: Can destroy only one phonon



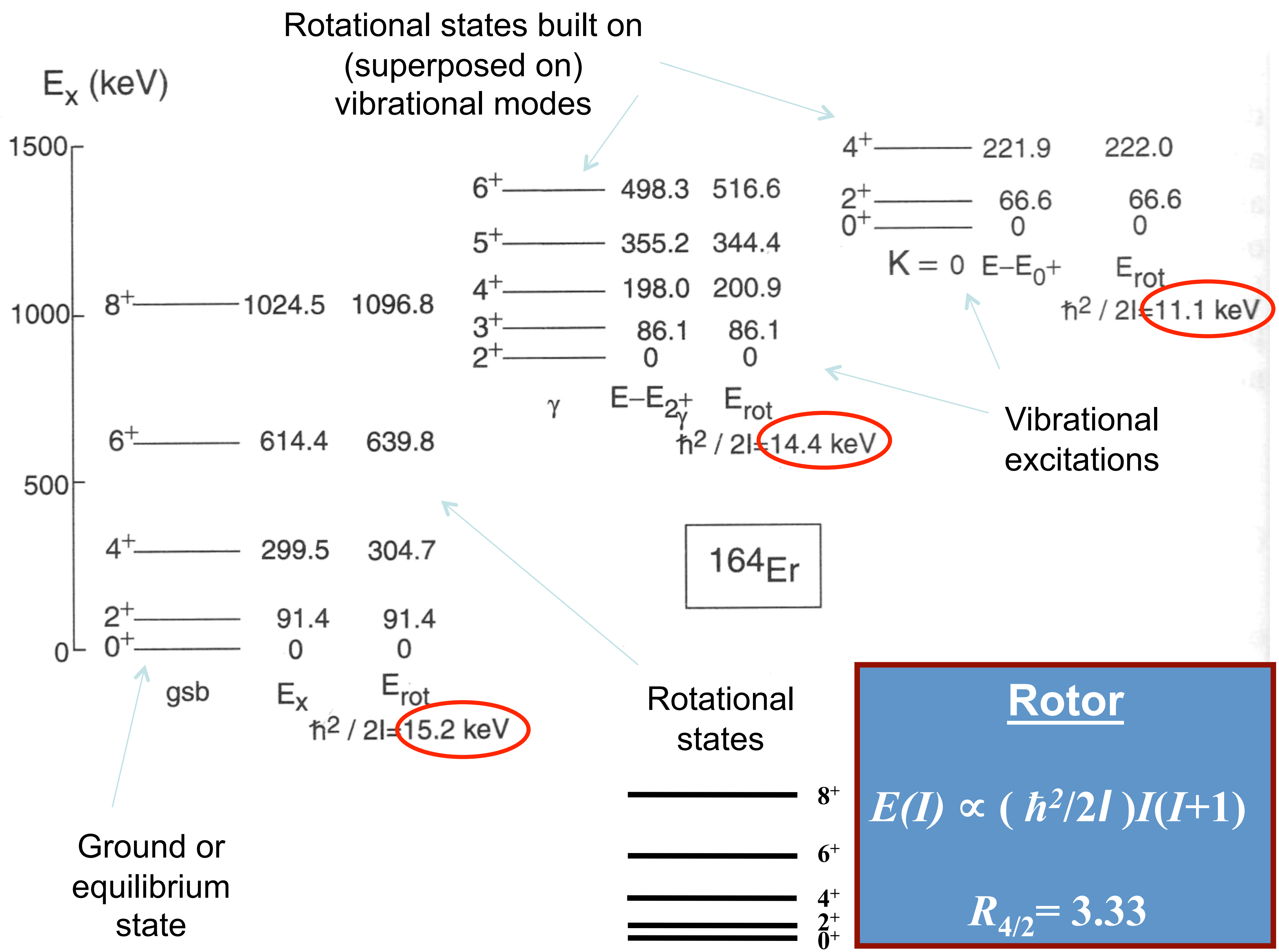
Deformed Nuclei

- What is different about non-spherical nuclei?

- They can ROTATE !!!

- They can also VIBRATE
  - For axially symmetric deformed nuclei there are two low lying vibrational modes called β and γ

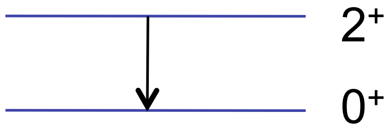
- So, levels of deformed nuclei consist of the ground state, and vibrational states, with rotational sequences of states (rotational bands) built on top of them.

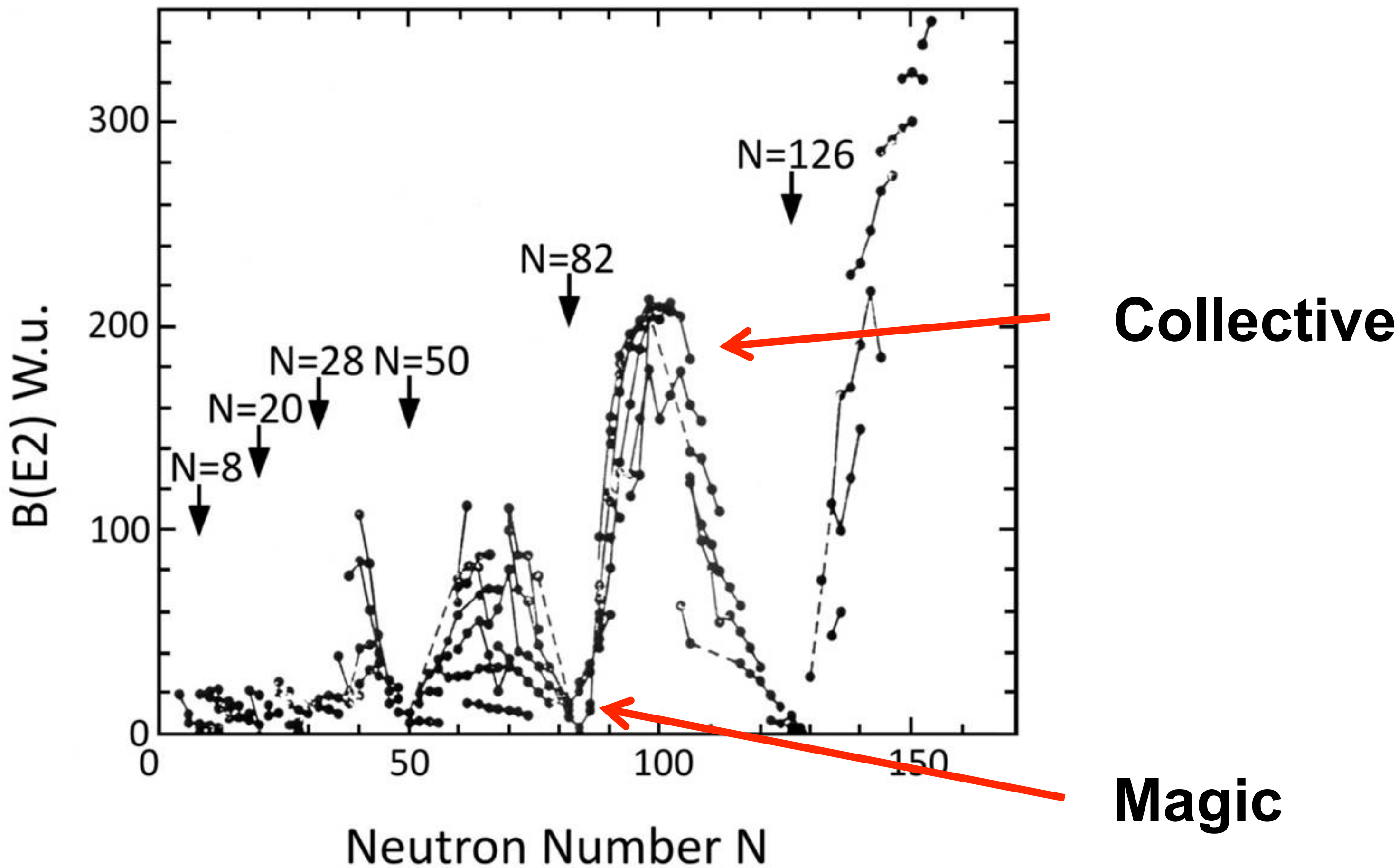


Rotational states built on (superposed on) vibrational modes
Ex (keV)
1500
1000
500
0
8+ 1024.5 1096.8
6+ 614.4 639.8
4+ 299.5 304.7
2+ 91.4 91.4
0+ 0 0
gsb Ex Erot
ħ2 / 2I=15.2 keV
6+ 498.3 516.6
5+ 355.2 344.4
4+ 198.0 200.9
3+ 86.1 86.1
2+ 0 0
γ E–E2γ+ Erot
ħ2 / 2I=14.4 keV
4+ 221.9 222.0
2+ 66.6 66.6
0+ 0 0
K = 0 E–E0+ Erot
ħ2 / 2I=11.1 keV
Vibrational excitations
164Er
Rotational states
8+
6+
4+
2+
0+
Ground or equilibrium state
Rotor
E(I) ∝ (ħ2/2I)I(I+1)
R4/2= 3.33

Transition rates (half lives of excited levels) also tell us a lot about structure

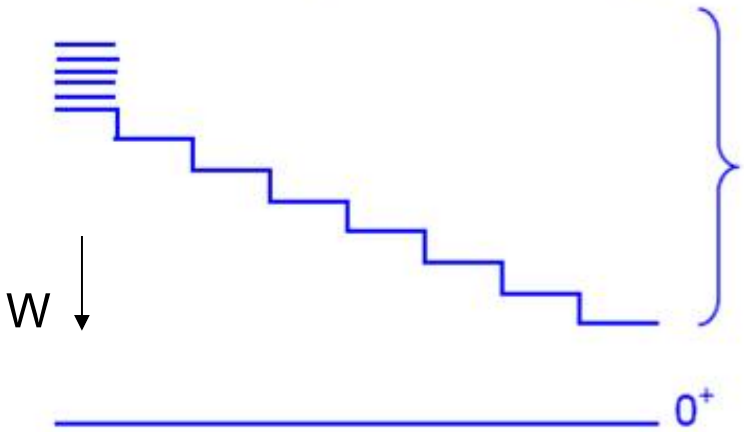
$$B(E2: 0^+_1 \rightarrow 2^+_1) \propto \langle 2^+_1 || E2 || 0^+_1 \rangle^2$$





Coherence and Transition Rates

Consider simple case of N degenerate levels: 2^+



$$\Delta E = (N-1)V$$

$$\Psi = a\varphi_1 + a\varphi_2 + \cdots a\varphi_N$$

where $a = \frac{1}{\sqrt{N}}$

$$\left(\sum_{i=1}^{N} a^2 = \frac{N}{N} = 1 \right)$$

Consider transition rate from $2_1^+ \rightarrow 0_1^+$

$$B(E2;\ 2_1^+ \rightarrow 0_1^+) = \frac{1}{2J_i + 1} \left\langle 0_1^+ \| E2 \| 2_1^+ \right\rangle^2$$

$$\left\langle 0_1^+ \| E2 \| 2_1^+ \right\rangle = \left\langle 0_1^+ \| E2 \| \Psi \right\rangle = a \sum_{i=1}^{N} \left\langle 0_1^+ \| E2 \| \varphi_i \right\rangle$$

Assume all $\left\langle 0_i^+ \| E2 \| \varphi_i \right\rangle$ matrix elements equal.

$$\therefore \left\langle 0_1^+ \| E2 \| 2_1^+ \right\rangle = Na \underbrace{\left\langle 0_1^+ \| E2 \| \varphi_i \right\rangle}_{W} = NaW = \sqrt{N}\,W$$

$$\therefore \boxed{B(E2) \propto NW^2}$$

Transition rate enhanced by factor of N

The more configurations that mix, the stronger the B(E2) value and the lower the energy of the collective state. Fundamental property of collective states.

$\therefore$ Enhanced transition rates are a signature of collectivity, along with low 2_1^+ energies. Lower $E(2_1^+)$, higher $B(E2) \rightarrow$

An algebraic approach

Collective behavior superposed on shell structure

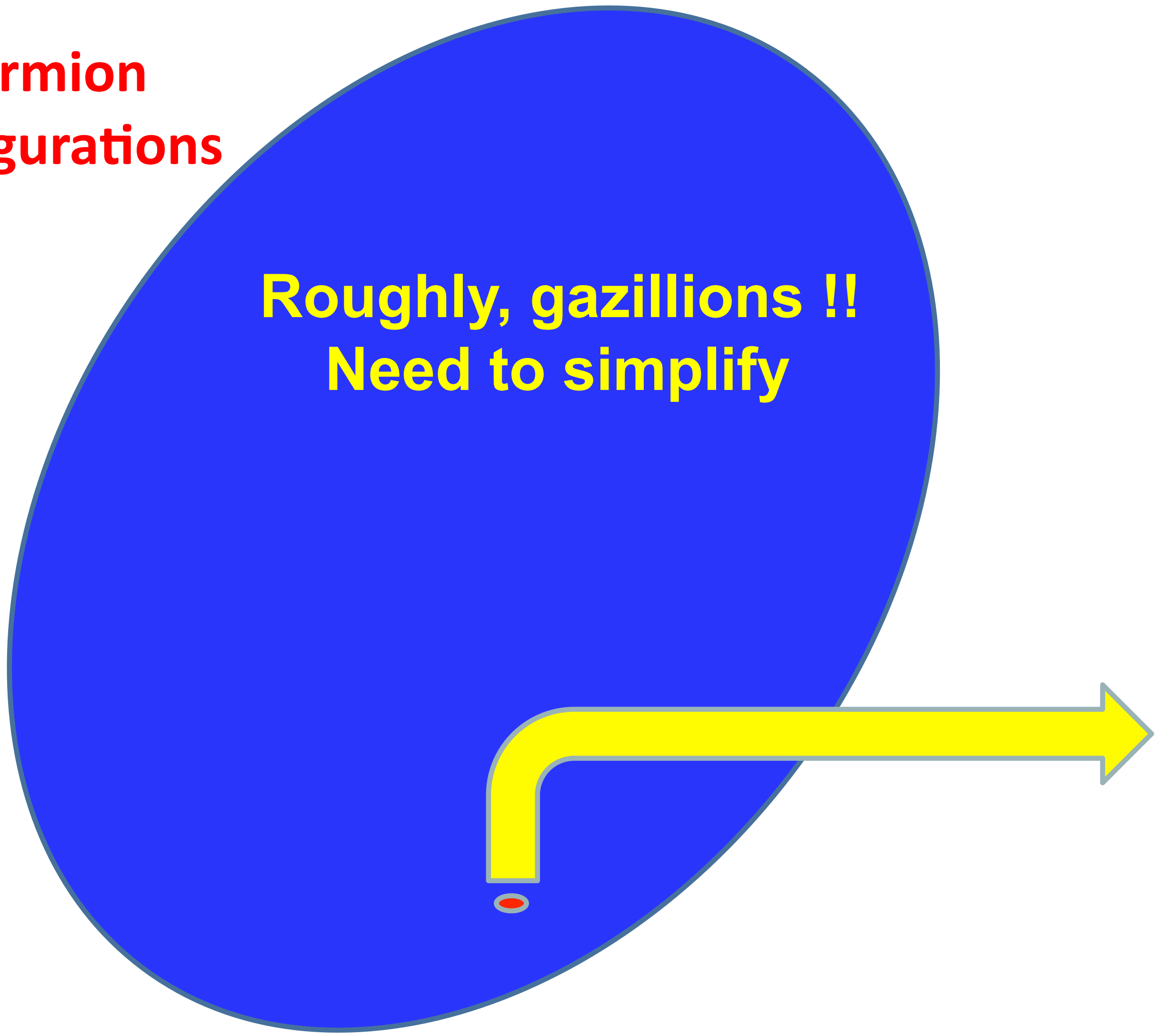
IBA, a symmetry-based model (Iachello and Arima)

Drastic simplification of shell model

- Valence nucleons, in pairs as bosons
- Only certain configurations. Only pairs of nucleons coupled to angular momentum 0(**s**) and 2(**d**). Why?
- Simple Hamiltonian in terms of s an d boson creation, destruction operators – simple interactions
- **Group theoretical** underpinning
- **Why?** Because it works. And extremely parameter-efficient

Shell Model Configurations

Fermion configurations



The IBA

Boson configurations

(by considering only configurations of pairs of fermions with J = 0 or 2.)

Modeling a Nucleus

Why the IBA is the best thing since baseball, a jacket potato, aceto balsamico, Mt. Blanc, raclette, pfannekuchen, baklava, ….

^{154}Sm → Shell model → 3×10^{14} 2^+ states

Need to truncate

IBA assumptions

1. **Only valence nucleons**
2. **Fermions → bosons**

J = 0 (s bosons)

J = 2 (d bosons)

Is it conceivable that these 26 basis states are correctly chosen to account for the properties of the low lying collective states?

IBA: 26 2^+ states

IBA: Truncation of Shell Model with Group Theory structure

IBA has a deep relation to Group theory

That relation is based on the operators that create, destroy *s* and *d* bosons

$s^\dagger, \; s, \quad \underbrace{d^\dagger, \; d}_{\text{Ang. Mom. 2}}$ operators $\qquad N_B = n_s + n_d = s^+s = d^+d$

$d^\dagger_\mu \, , \, d_\mu \quad \mu = 2, 1, 0, -1, -2$

Hamiltonian is written in terms of *s, d* operators

$$H = H_s + H_d + H_{int}(s^\dagger s, \; s^\dagger d, d^\dagger s, d^\dagger d)$$

Since boson number, N_B, is conserved for a given nucleus, *H* can only contain "bilinear" terms: 36 of them.

$s^\dagger s, \; s^\dagger d, d^\dagger s, d^\dagger d$ ⟹ Gr. Theor. classification of Hamiltonian

Group is called **U(6)**

U(6) has three subgroups corresponding to different shapes

Concepts of group theory

First, some fancy words with simple meanings: Generators, Casimirs, Representations, conserved quantum numbers, degeneracy splitting

Generators of a group: Set of operators , O_i that close on commutation.

$[O_i, O_j] = O_i O_j - O_j O_i = O_k$ *i.e.*, their commutator gives back 0 or a member of the set

For IBA, the 36 operators ***$s^\dagger s$, $d^\dagger s$, $s^\dagger d$, $d^\dagger d$*** are generators of the group U(6).

ex:

$$\left[d^\dagger s, s^\dagger s\right]\left|n_d n_s\right\rangle = \left(d^\dagger s s^\dagger s - s^\dagger s d^\dagger s\right)\left|n_d n_s\right\rangle$$

$$= d^\dagger s n_s \left|n_d n_s\right\rangle - s^\dagger s d^\dagger s \left|n_d n_s\right\rangle$$

$$= \left(n_s - s^\dagger s\right) d^\dagger s \left|n_d n_s\right\rangle$$

$$= \left(n_s - s^\dagger s\right)\sqrt{n_d + 1}\sqrt{n_s}\left|n_d + 1, n_s - 1\right\rangle$$

$$= \sqrt{n_d + 1}\sqrt{n_s}\left[n_s - \left(n_s - 1\right)\right]\left|n_d + 1, n_s - 1\right\rangle$$

$$= \sqrt{n_d + 1}\sqrt{n_s}\left|n_d + 1, n_s - 1\right\rangle$$

$$= d^\dagger s \left|n_d n_s\right\rangle$$

or:

$$\left[d^\dagger s, s^\dagger s\right] = d^\dagger s$$

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For IBA, the 36 operators $s^\dagger s, d^\dagger s, s^\dagger d, d^\dagger d$ are generators of the group U(6).

Generators: define and conserve some quantum number.

Ex.: 36 Ops of IBA all conserve total boson number $N = s^\dagger s + d^\dagger \tilde{d} = n_s + n_d$

Casimir: Operator that commutes with all the generators of a group. Therefore, its eigenstates have a specific value of the q.# of that group. The energies are defined **solely** in terms of that q. #. N is Casimir of U(6).

Representations of a group: The set of **degenerate** states with that value of the q. #.

A **Hamiltonian** written solely in terms of Casimirs can be solved analytically

Sub-groups:

Subsets of generators that commute among themselves.

e.g: $d^\dagger d$ 25 generators—span U(5)

They conserve n_d (# *d* bosons)

Set of states with same n_d are the representations of the group [U(5)]

....

Summary to here:

Generators: commute, define a q. #, conserve that q. #

Casimir Ops: commute with a set of generators

∴ Conserve that quantum #

∴ A Hamiltonian that can be written in terms of Casimir Operators is then diagonal for states with that quantum #

Eigenvalues can then be written ANALYTICALLY as a function of that quantum #

Group Structure of the IBA

6-Dim. problem

U(6) →

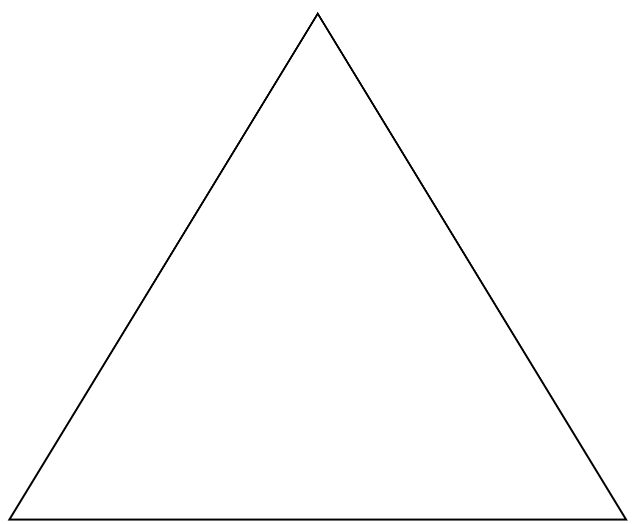
Magical group theory stuff happens here

U(5) **vibrator**

SU(3) **rotor**

O(6) **γ-soft**

Three Dynamic symmetries, nuclear shapes



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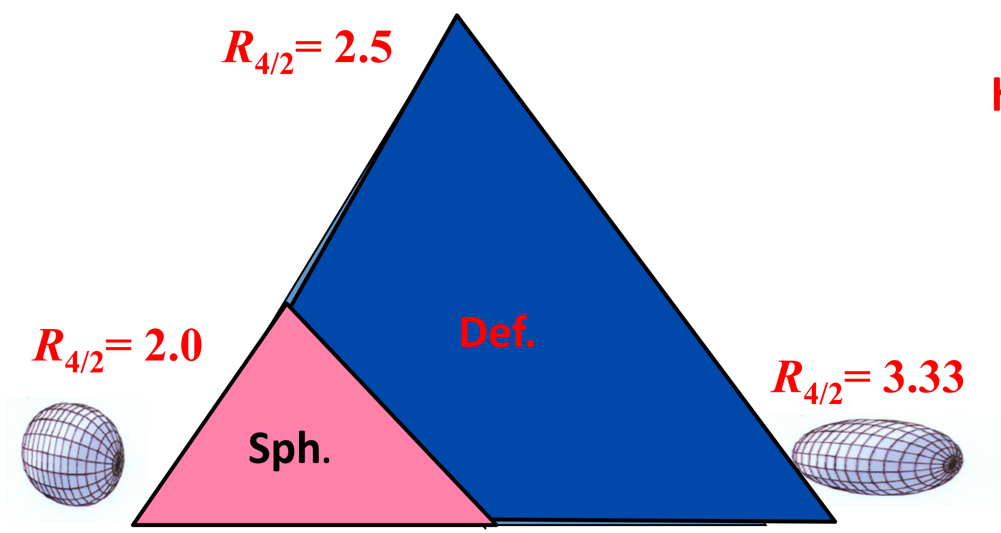


U(5) **vibrator**

SU(3) **rotor**

Symmetry Triangle of the IBA

Let's illustrate group chains and degeneracy-breaking.

Consider a Hamiltonian that is a function ONLY of: $s^\dagger s + d^\dagger d$

Note that $s^\dagger s = n_s$ *and* $d^\dagger d = n_d$ *and that* $n_s + n_d = N = ½$ *val nucleons*

That is: $$H = a(s^\dagger s + d^\dagger d) = a(n_s + n_d) = aN$$

H "counts" the numbers of bosons and multiplies by a boson energy, *a*. The energies depend ONLY on total number of bosons -- the total number of valence nucleons. The states with given N are degenerate and constitute a "representation" of the group U(6) with the quantum number N. U(6) has OTHER representations, corresponding to OTHER values of N, but THOSE states are in DIFFERENT NUCLEI.

Of course, a nucleus with all levels degenerate is not realistic (!!!) and suggests that we should add more terms to the Hamiltonian. I use this example to illustrate the idea of successive steps of degeneracy breaking related to different groups and the quantum numbers they conserve.

$$H' = H + b\, d^{\dagger} d = aN + b\, n_d$$

Now, add a term to this Hamiltonian:

Now the energies depend not only on N but also on n_d

States of a given n_d are now degenerate. They are "representations" of the group U(5). States with different n_d are not degenerate

$$H = aN + b\, d^{\dagger}d = a\,N + b\,n_d$$

$U(6)$ $\quad U(5)$

2a —— N + 2

a —— N + 1

2b —— *2*

b —— *1*

0 —— N

0 —— 0

E

n_d

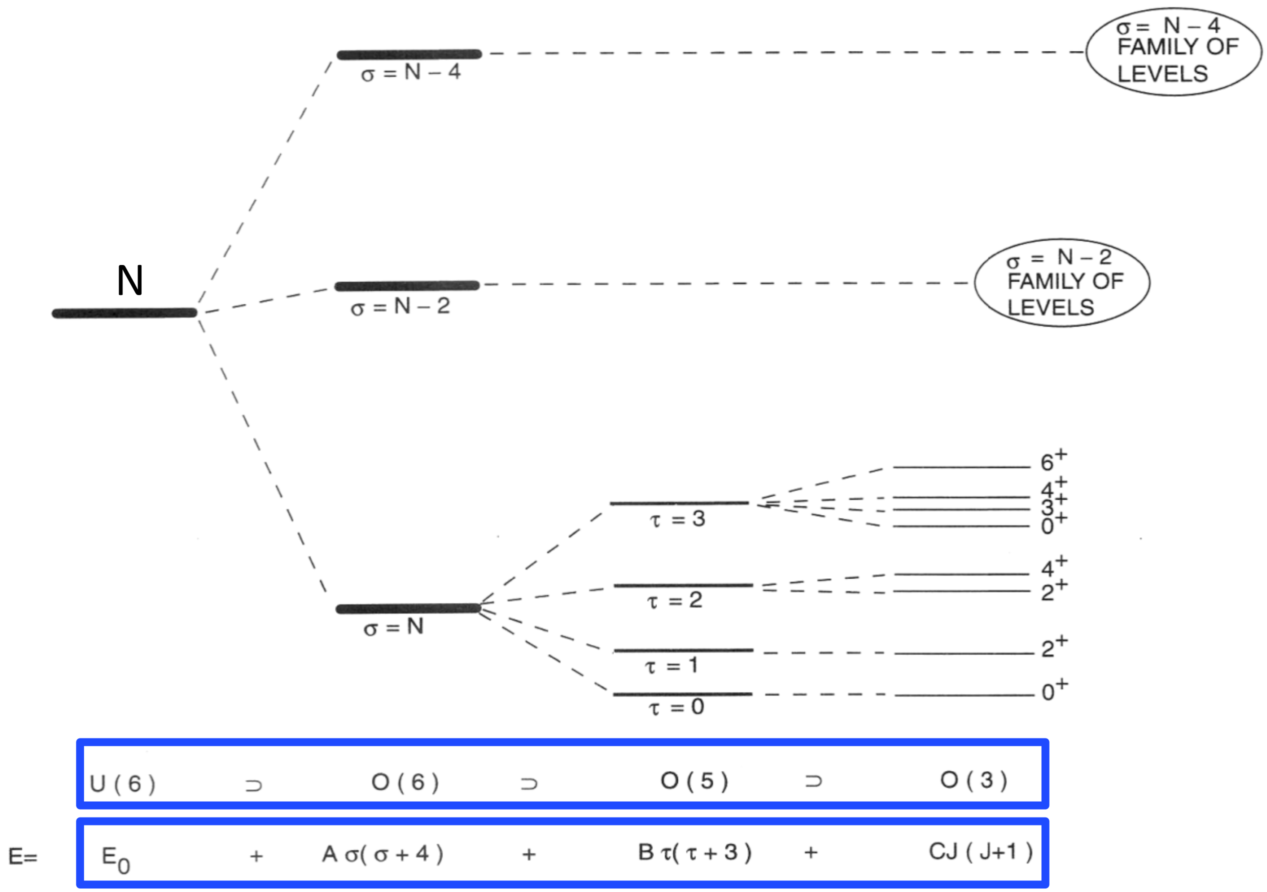
Etc. with further terms

$U(6) \supset U(5)$

$$H = aN + b\, d^{\dagger}d$$

Example of a nuclear dynamical symmetry -- O(6)

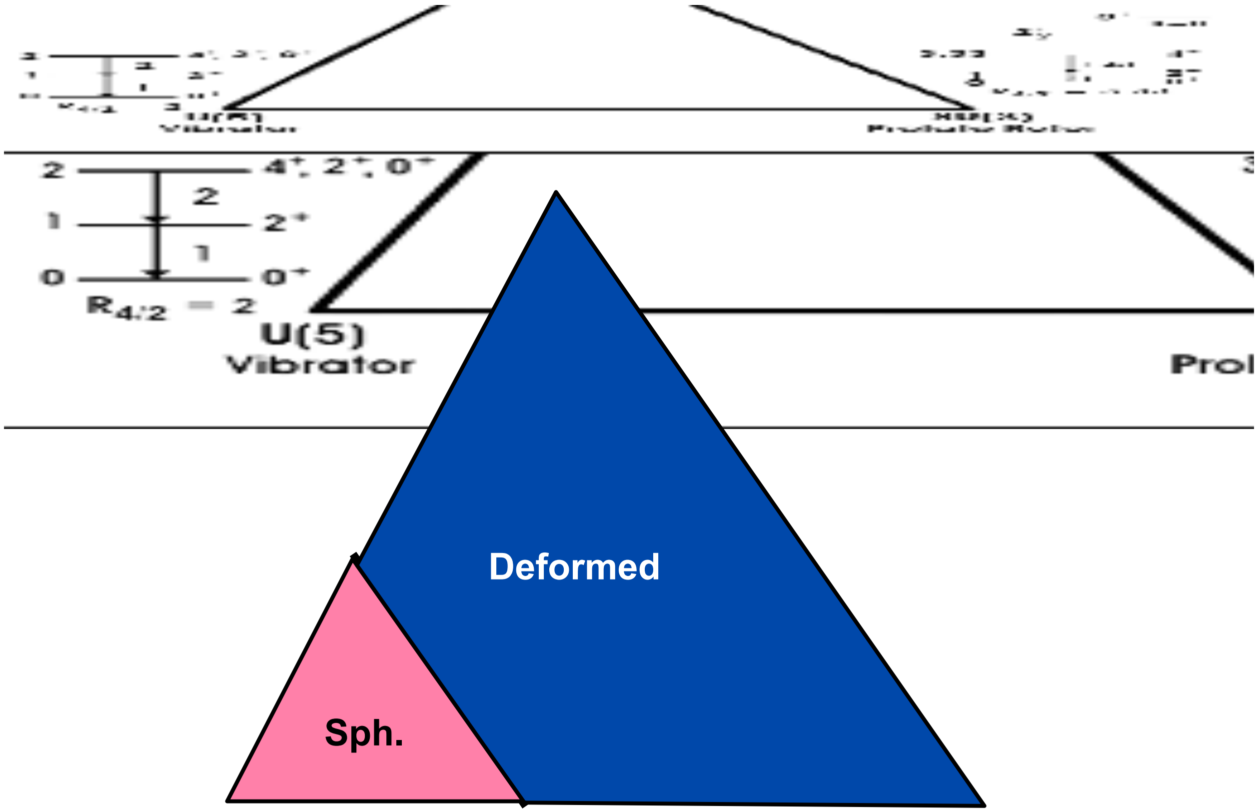
Spectrum generating algebra



Each successive term:

- **Introduces a new sub-group**
- **A new quantum number to label the states described by that group**
- **Adds an eigenvalue term that is a function of the new quantum number, and therefore**
- **Breaks a previous degeneracy**

Classifying Structure -- The Symmetry Triangle

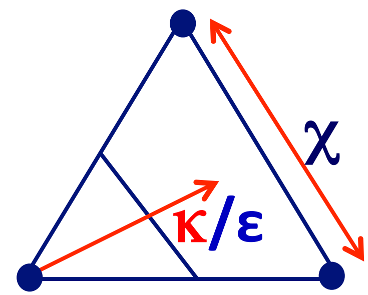


Most nuclei do not exhibit the idealized symmetries but rather lie in transitional regions. Mapping the triangle.

What do you do with all the nuclei that do not manifest a symmetry ? Need a Hamiltonian that breaks the symmetries.

Truncated form of with just two parameters (+ scale):

$$H = \varepsilon n_d - \kappa Q \cdot Q$$



$$Q = e[s^\dagger \tilde{d} + d^\dagger s + \chi (d^\dagger \tilde{d})^{(2)}]$$

Competition:

εn_d	Counts quad bosons: vibrator.
$\kappa Q \cdot Q$	Gives deformed nuclei.
χ	Determines axial asymmetry

Hence structure is given by two parameters, ε / κ and χ

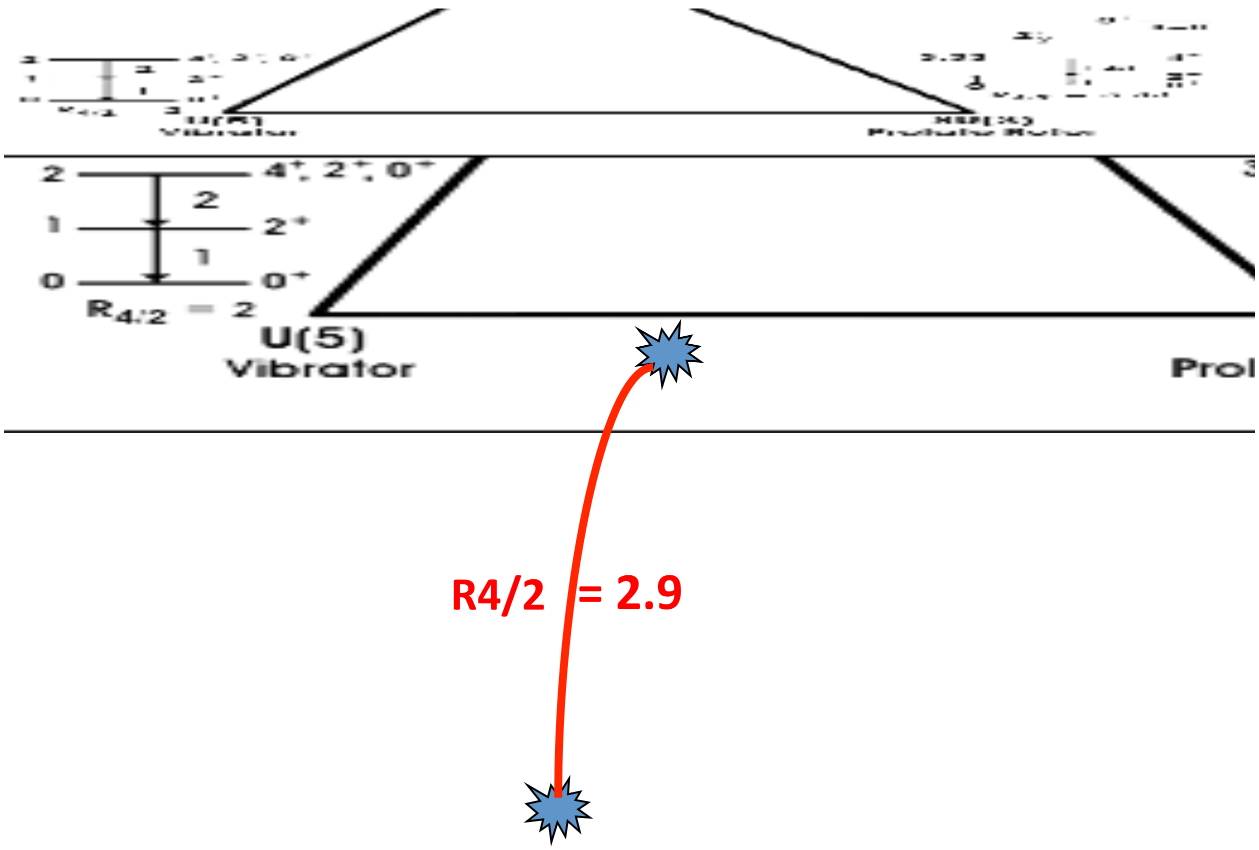
More complicated forms exist but this is the form usually used. It works extremely well in most cases.

2

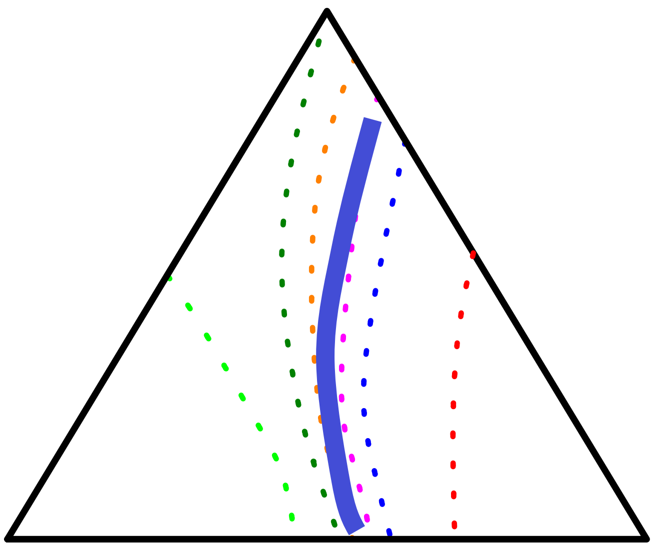
3

5

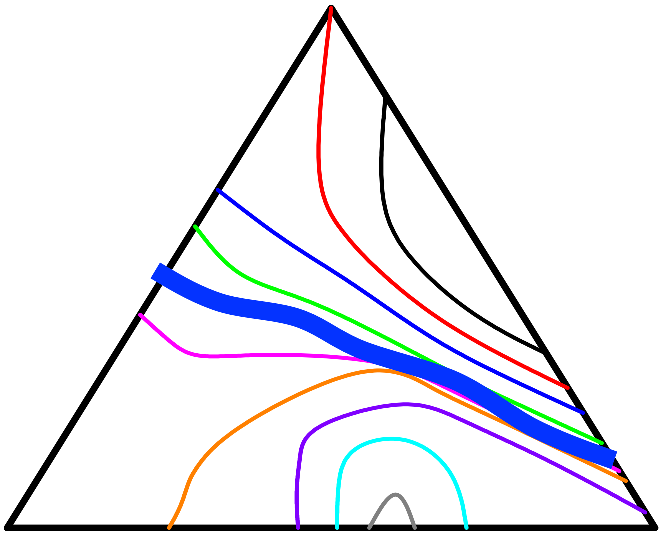
H has two parameters. A given observable can only specify one of them. That is, a given observable has a contour (locus) of constant values within the triangle



Mapping Structure with Simple Observables – Technique of Orthogonal Crossing Contours

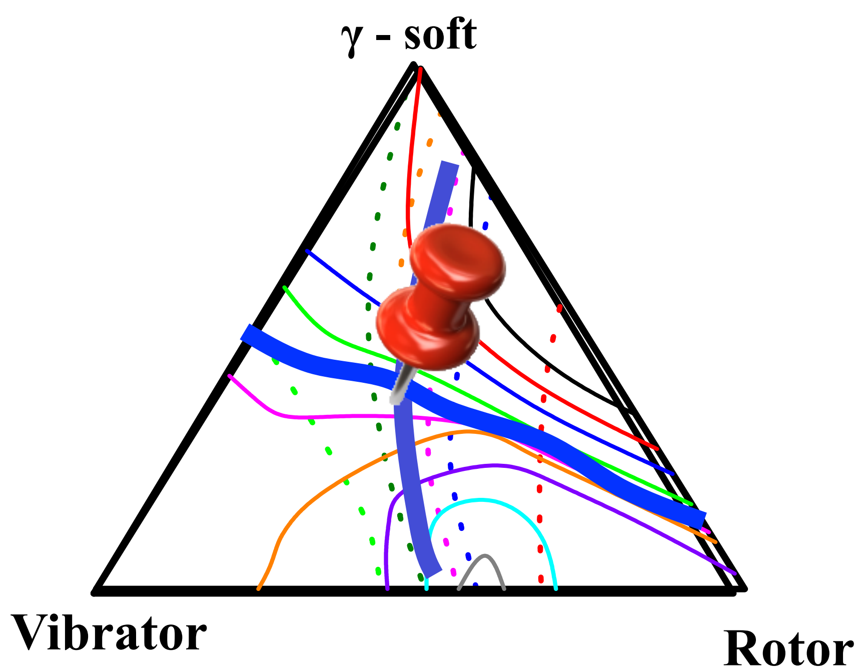


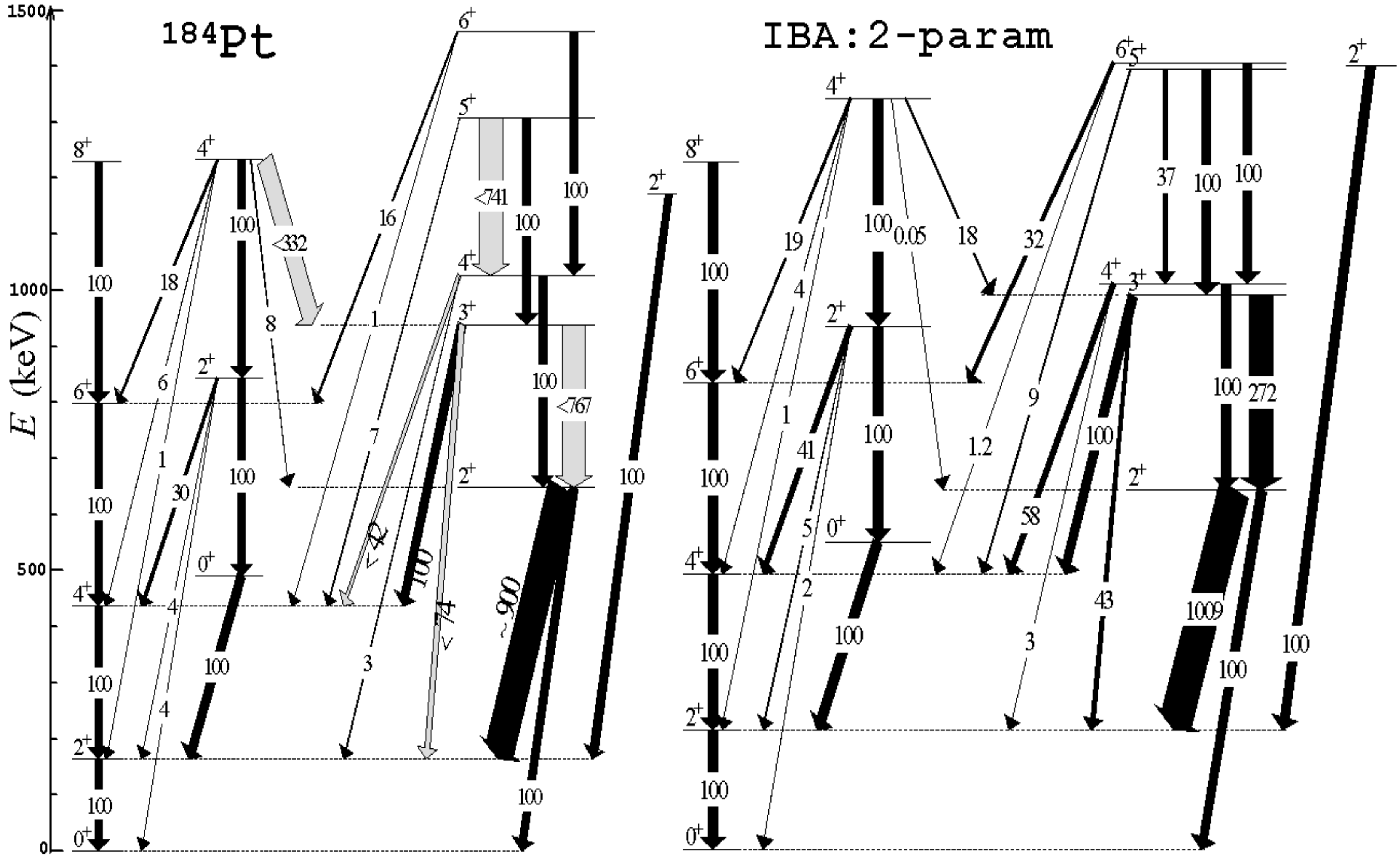
$$\frac{E(4_1^+)}{E(2_1^+)}$$



$$\frac{E(0_2^+) - E(2_2^+)}{E(2_1^+)}$$

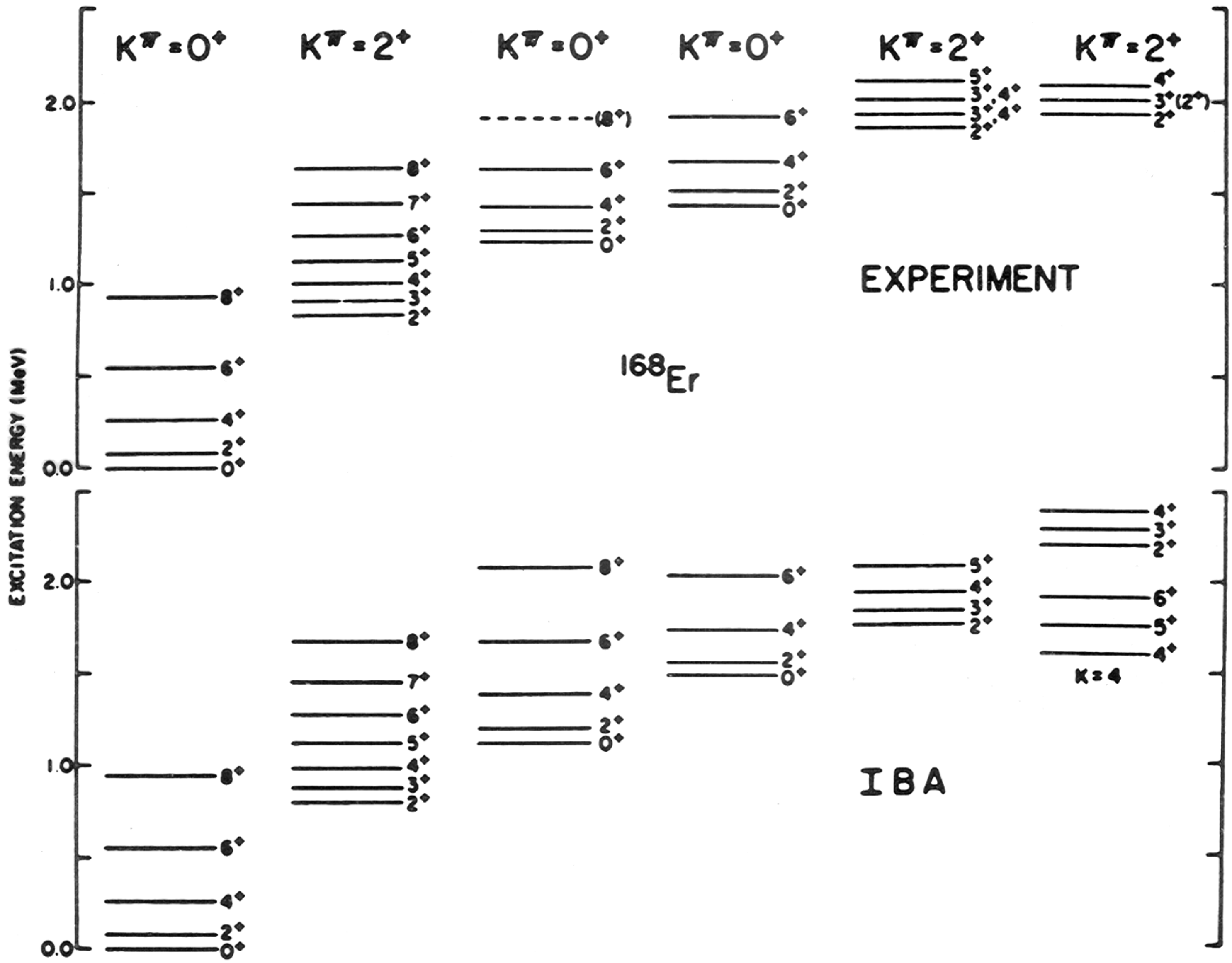
Mapping Structure with Simple Observables – Technique of Orthogonal Crossing Contours





R. Burcu Cakirli *et al.*

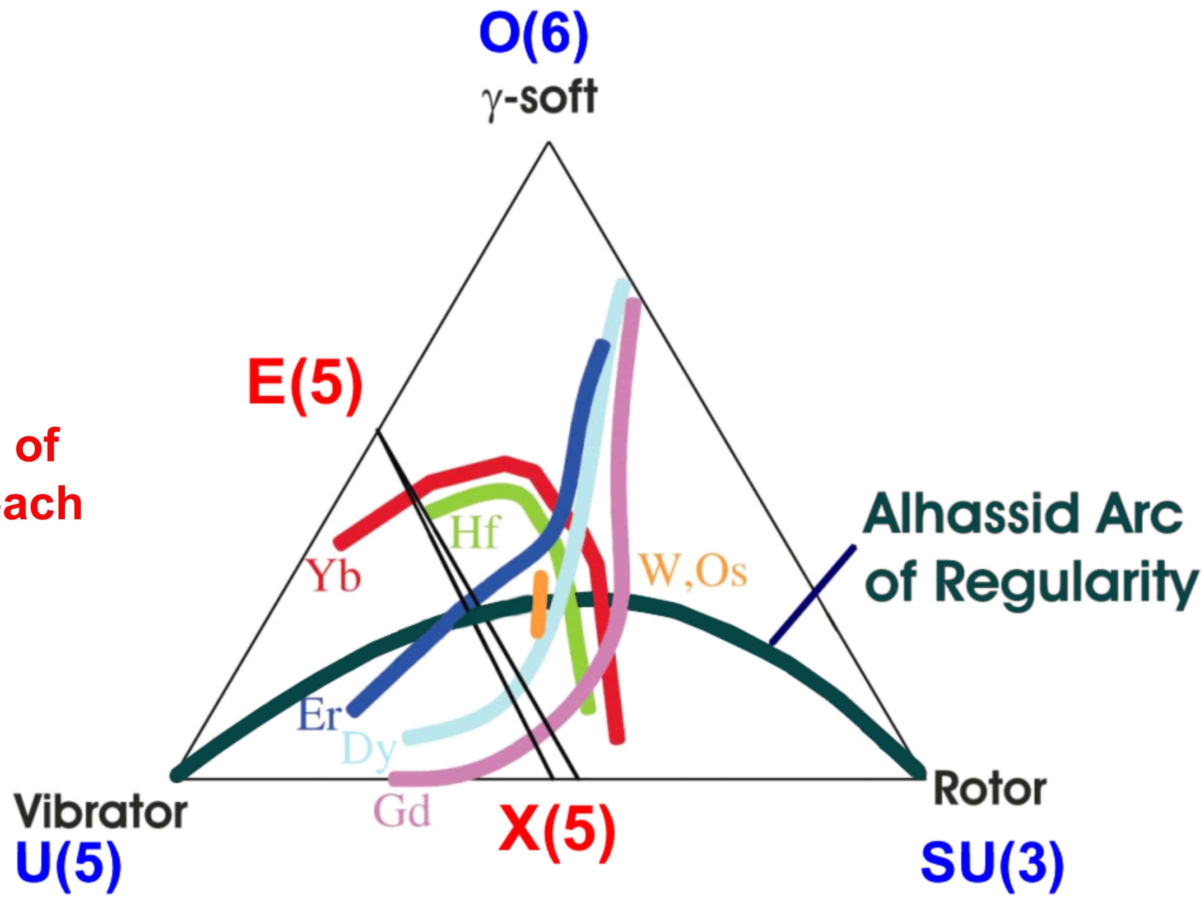
Beta decay exp. + IBA calcs.



Warner, Borner, and Davidson

Evolution of Structure

Trajectories of isotopes of each element



McCutchan, Zamfir

Complementarity of macroscopic and microscopic approaches. Why do certain nuclei exhibit specific symmetries? Why these evolutionary trajectories?

What will happen far from stability in regions of proton-neutron asymmetry and/or weak binding?