Lectures on Nuclear Structure – An empirical overview from a simple perspective

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Outline

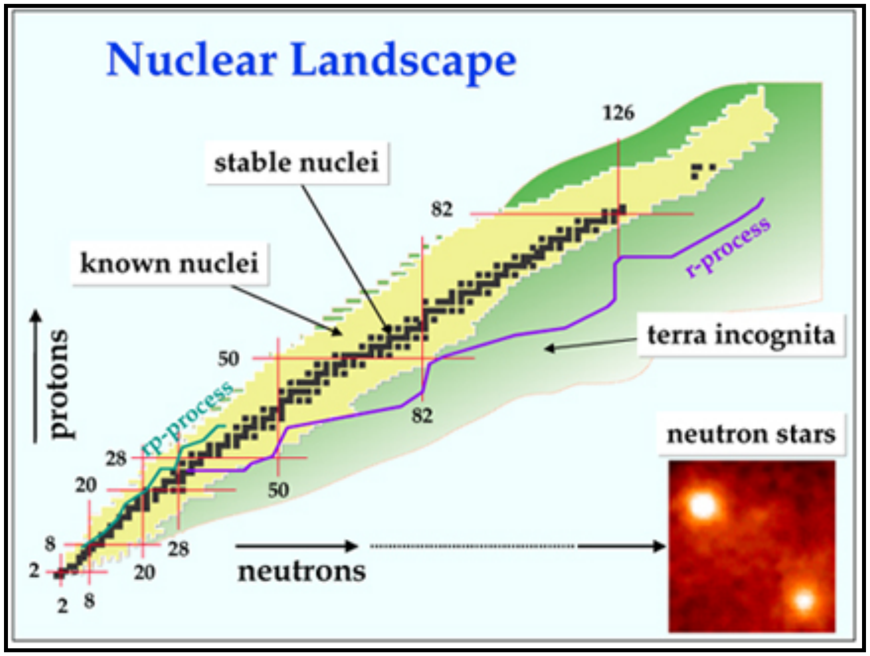
- **Introduction, empirical survey – what nuclei do**
- **Independent particle model and Residual interactions**
  - **Particles in orbits in the nucleus**
  - **Residual interactions: results and simple physical interpretation**
- **Collective models -- Geometrical, algebraic (The IBA)**
- **Linking microscopic and macroscopic – measuring the p-n interaction. Competition with pairing.**
- **Exotic Nuclei – FRIB et al.**

Not so much experimental techniques as a perspective on the data on nuclei and a simple physical picture of what is going on.

The scope of Nuclear Structure Physics

The Four Frontiers

1. Proton Rich Nuclei
2. Neutron Rich Nuclei
3. Heaviest Nuclei
4. Evolution of structure within these boundaries



Terra incognita — huge gene pool of new nuclei

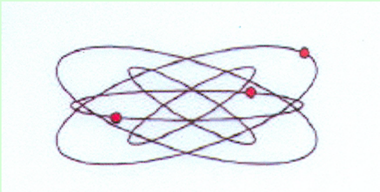
We can customize our system – fabricate "designer" nuclei to *isolate and amplify* specific physics or interactions

Themes and challenges of Modern Science

•Complexity out of simplicity -- Microscopic

How the world, with all its apparent complexity and diversity can be constructed out of a few elementary building blocks and their interactions

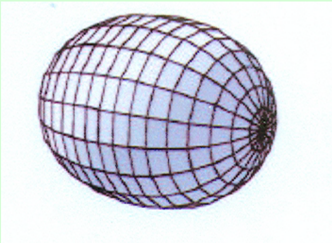
What is the force that binds nuclei?
Why do nuclei do what they do?



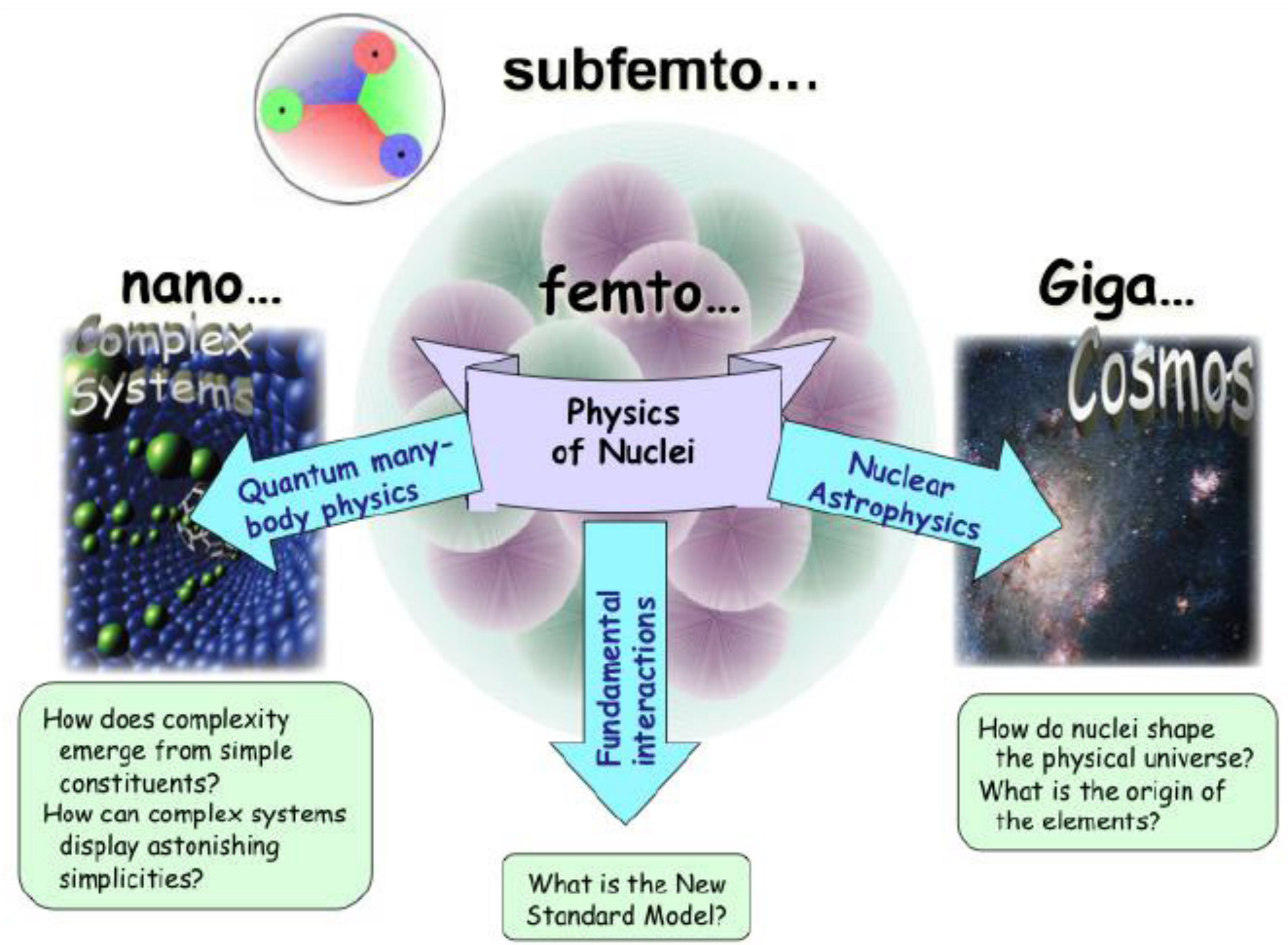
•Simplicity out of complexity – Macroscopic

How the world of complex systems can display such remarkable regularity and simplicity

What are the simple patterns that nuclei display and what is their origin ?



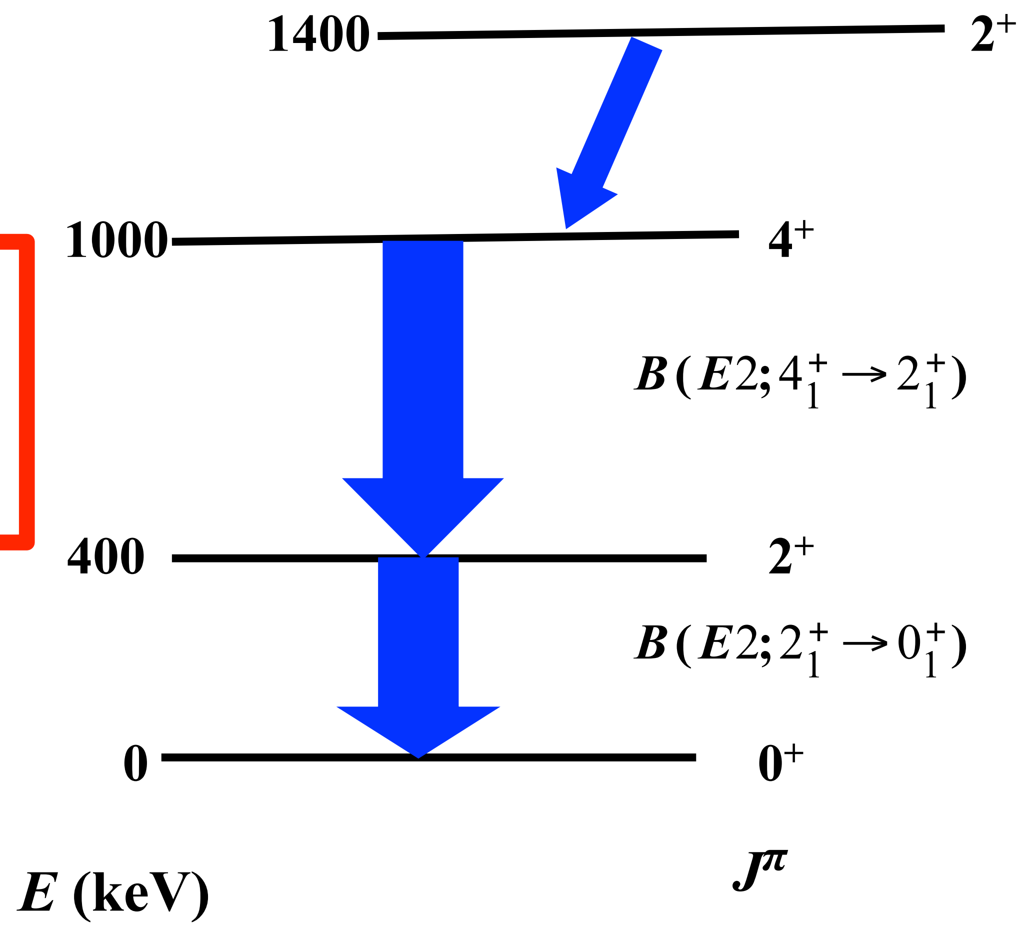
Where do nuclei fit into the overall picture?



Simple Observables - Even-Even Nuclei

$$R_{4/2} = \frac{E(4_1^+)}{E(2_1^+)}$$

Masses

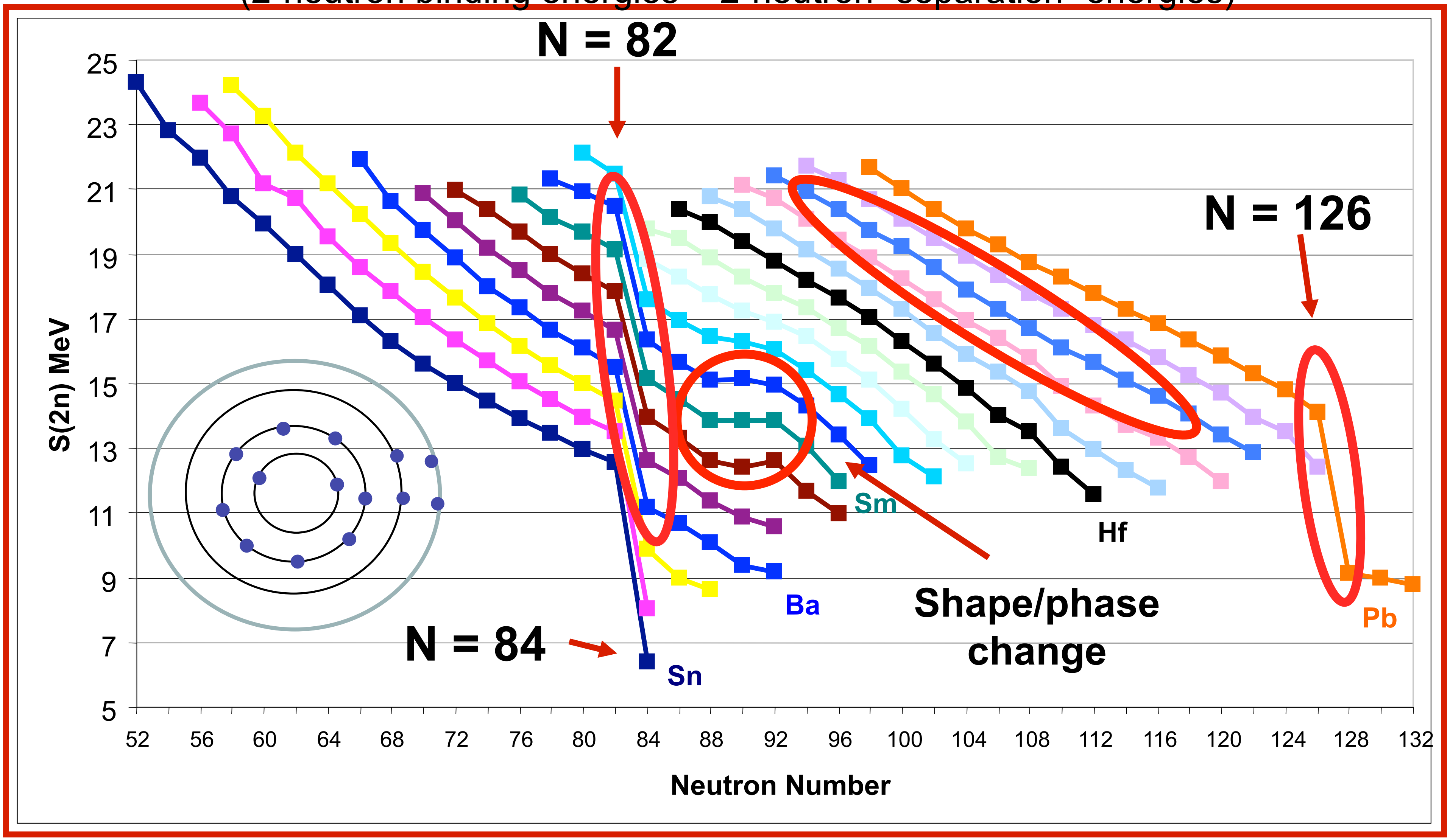


$$B(E2; J_i \rightarrow J_f) \equiv \frac{1}{2J_i + 1} \left\langle \Psi_i \| E2 \| \Psi_f \right\rangle^2$$

Masses: reflect all interactions. ~ 100GeV.

Separation energies to remove two neutrons ~ 16MeV

(2-neutron binding energies = 2-neutron “separation” energies)



$$S_{2n} = A + BN + S_{2n}\text{ (Coll.)}$$

Spectroscopic observables

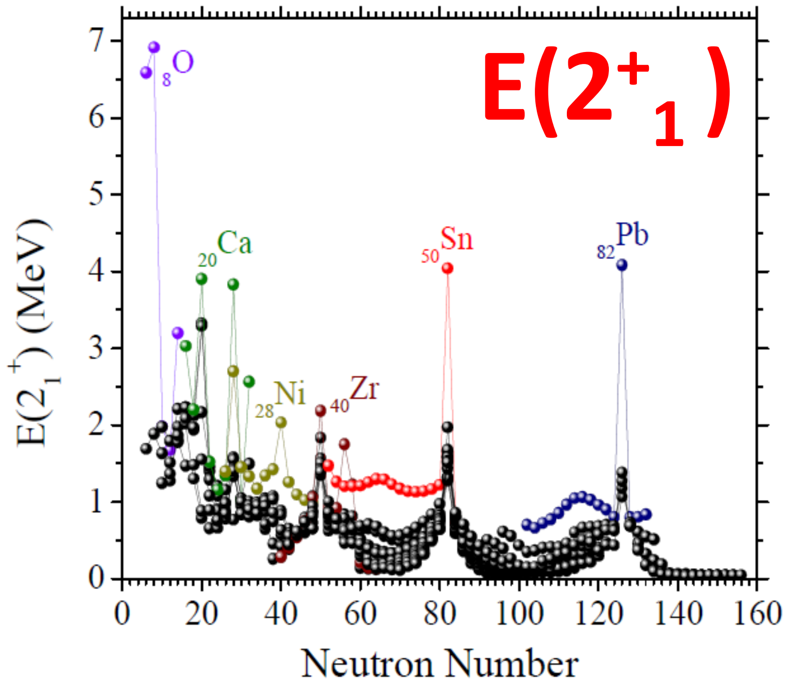
Two obvious features which capture much of the physics:

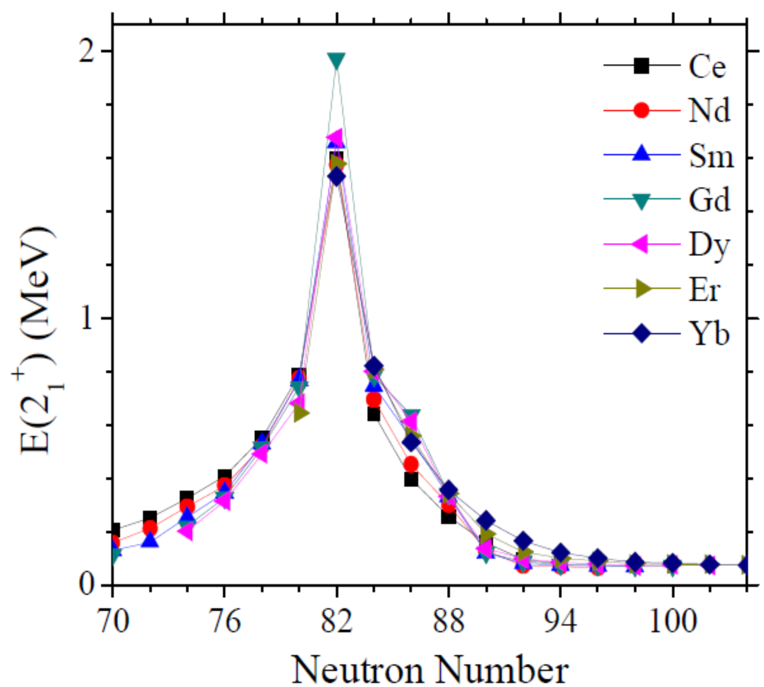
- **High values** at certain (magic) numbers, 2, 8, 20, 50, 82, 126…

These show the rigidity to excitation of nuclei with these special numbers of nucleons

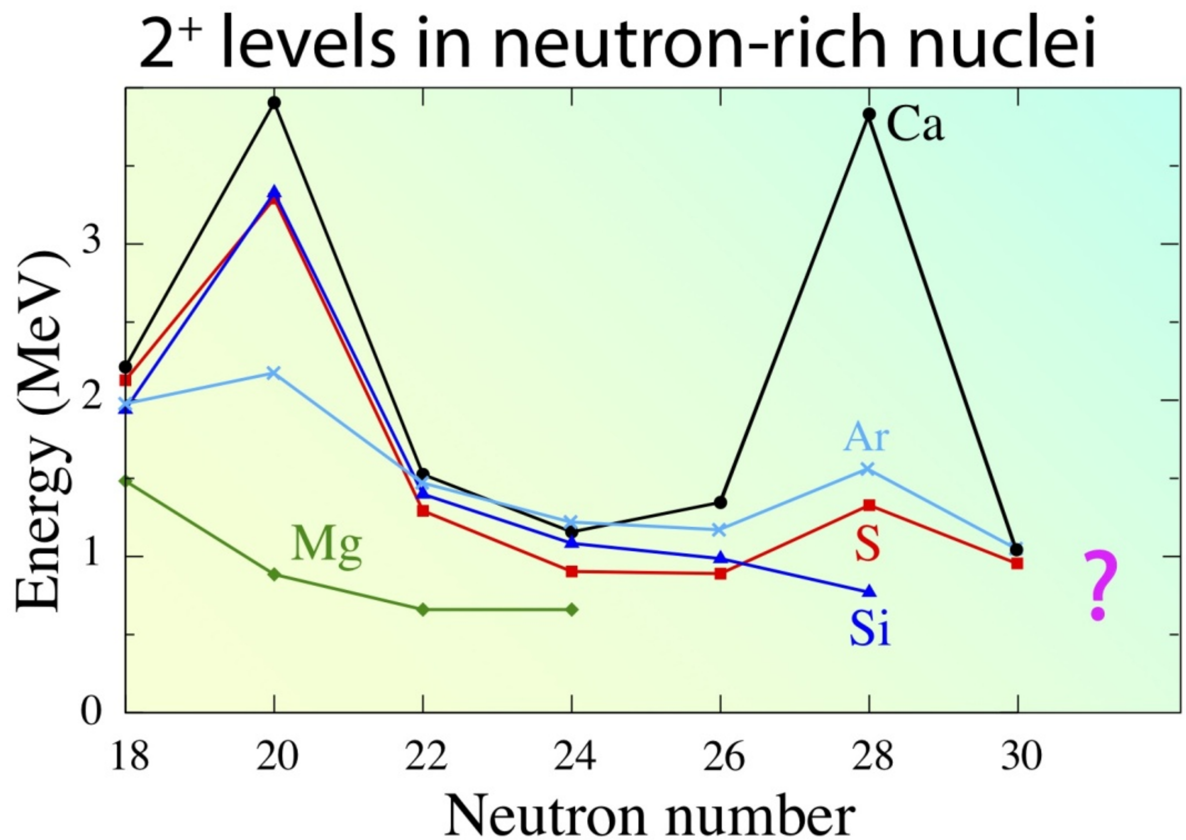
- **Sharp drops** thereafter.

This shows the emergence of collectivity as a general feature of non-magic nuclei





BUT, trouble looming:



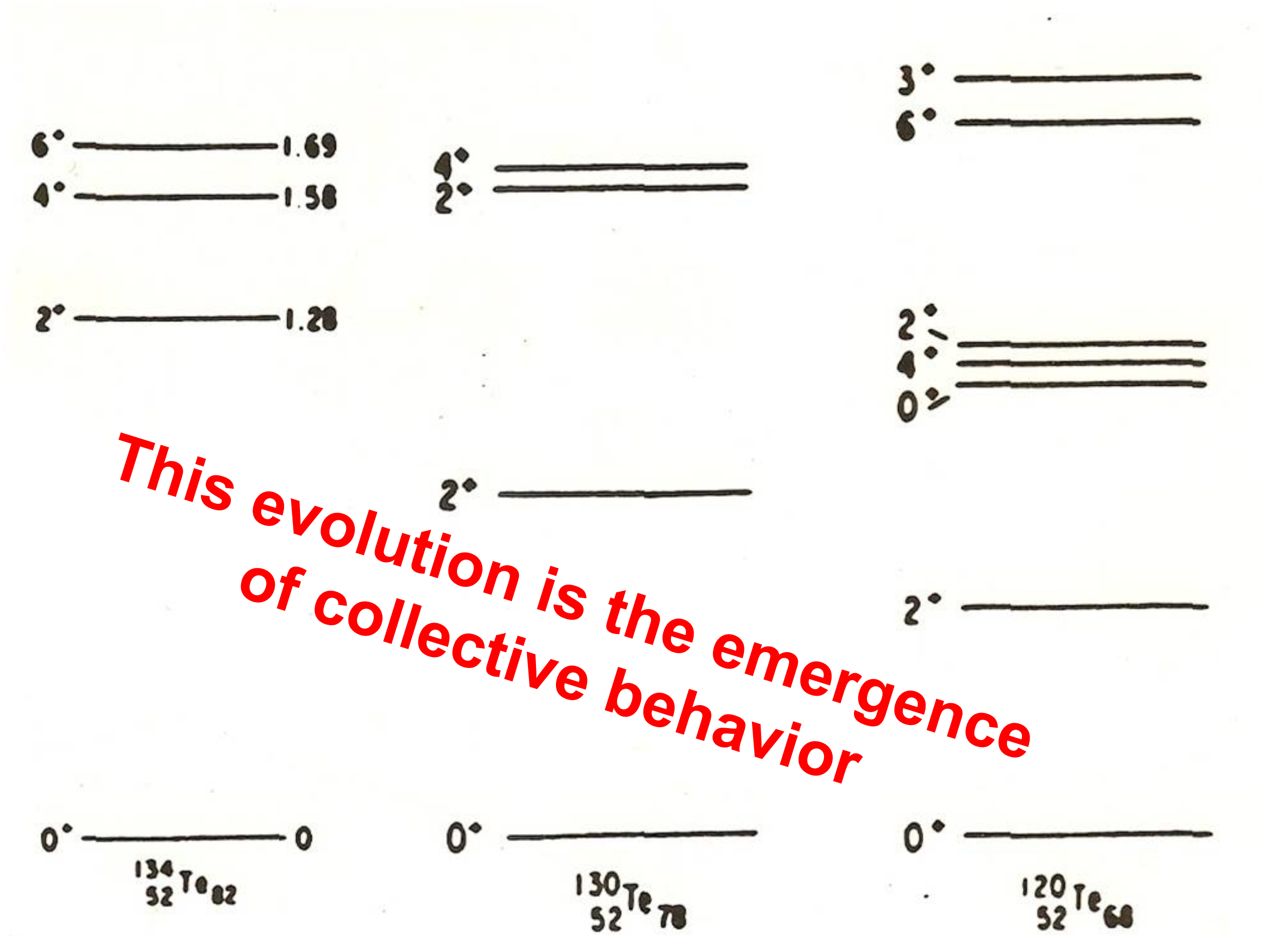
Migration of magicity: N = 20 is NOT magic for Mg and N = 28 is NOT magic for Si and S !!!! Evolution of shell structure -- one of the most active, important areas of nuclear structure research today.

Starting from a doubly magic nucleus, what happens as the numbers of valence neutrons and protons **increase**?

Case of **few** valence nucleons:

Lowering of energies, development of multiplets.

Two nucleons of one type



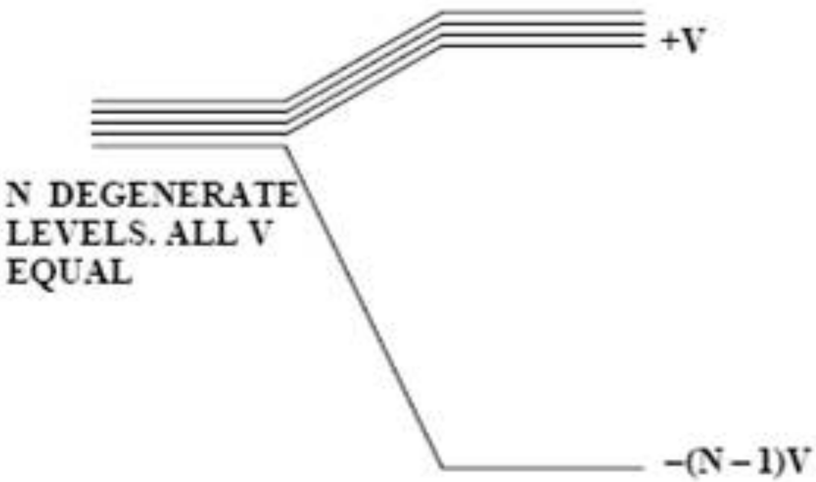
This evolution is the emergence of collective behavior

Few nucleons of both types

Development of collective behavior in nuclei

- Results primarily from correlations among valence nucleons.
- Instead of pure "Independent Particle model" configurations (see later discussion), the wave functions are mixed – linear combinations of many components.
- Leads to a lowering of the collective states and to enhanced transition rates as characteristic signatures.
- How does this happen? Consider mixing of states.

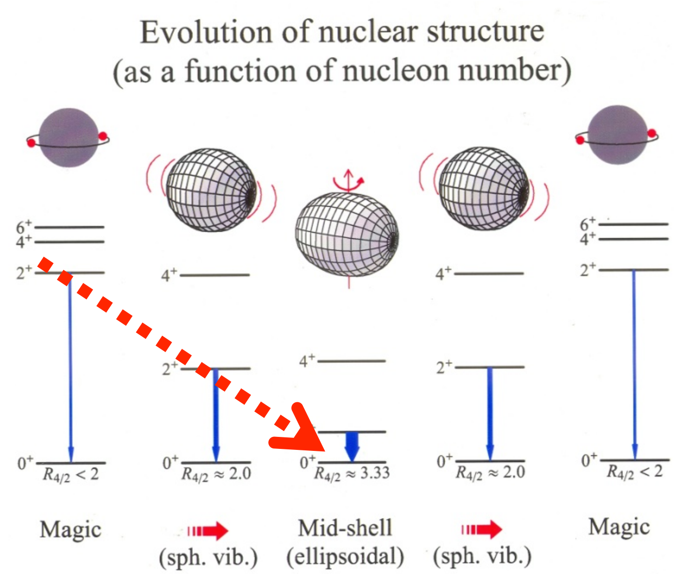
Recall: Microscopic origins of collectivity correlations, configuration mixing and deformation: Residual interactions



$$\Psi_{LOWEST} = \frac{1}{\sqrt{N}}[\phi_1 + \phi_2 + \ldots + \phi_N]$$

We will come back to this idea many times.

Crucial for structure



$R_{4/2}$

How does it vary, and why, and why do we care

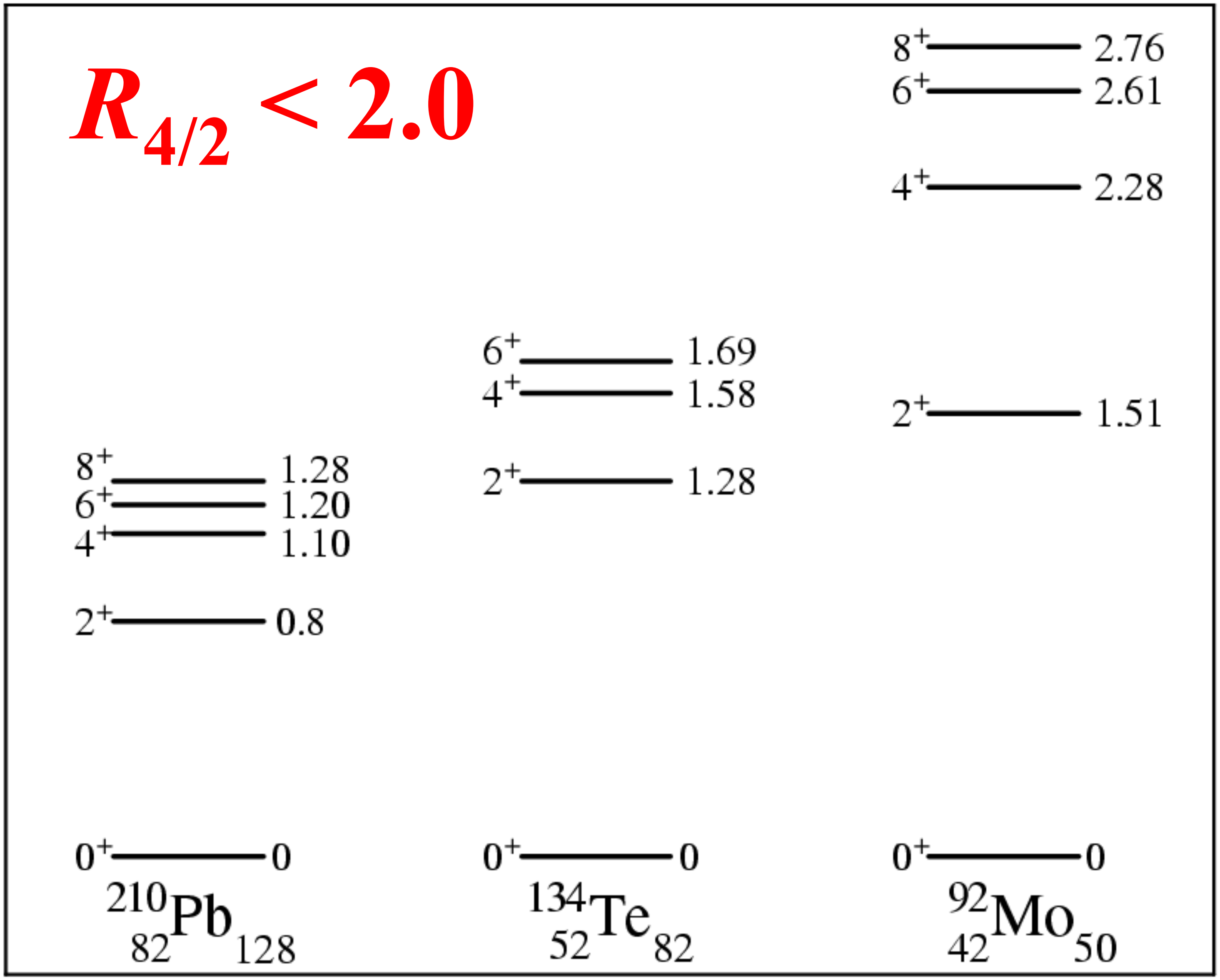
- We care because it is the almost the only observable whose value immediately tells us something about structure.
- We care because it is easy to measure.
- Why: It reflects the emergence of nuclear collectivity: 4 cases

- **Nuclei w/ two nucleons outside doubly magic: <2**

Three "collective" structures

- **Spherical vibrational nuclei: ~2**
- **Axial symmetric ellipsoidal deformed nuclei: ~3.33**
- **(Non-Axial ellipsoidal deformed nuclei: ~2.5)**

Spectra of “2 valence nucleon” nuclei



Types of collective structures

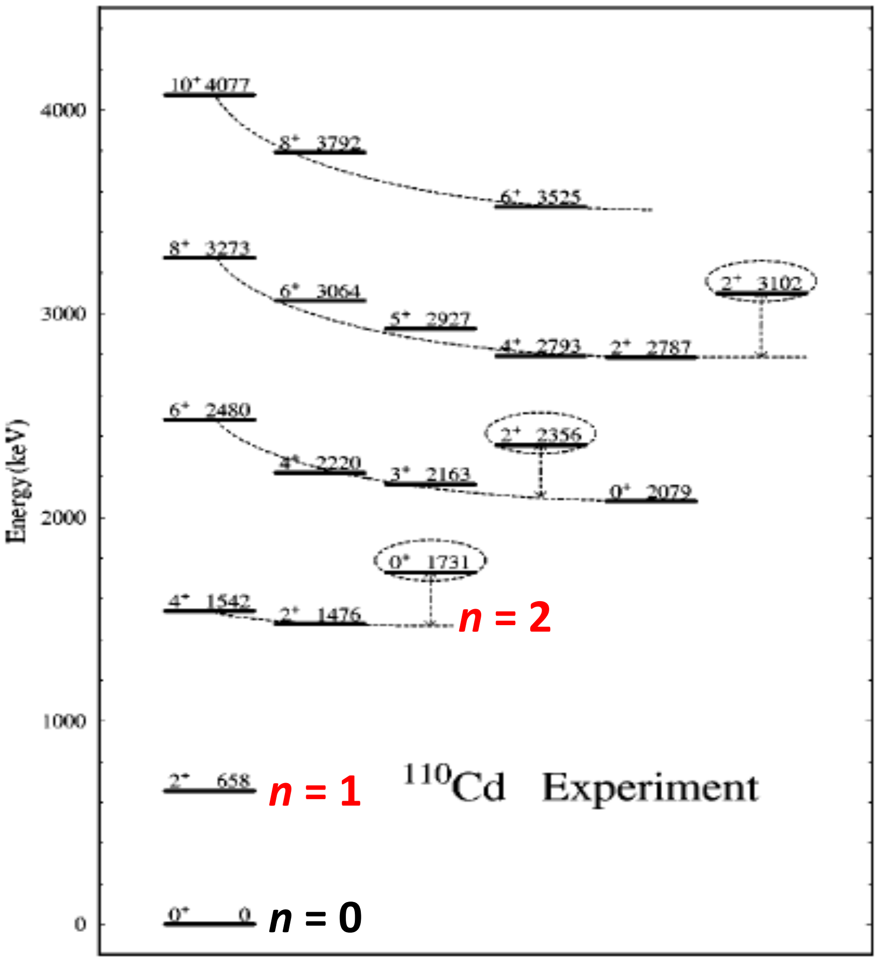
Few valence nucleons of each type:

The spherical vibrator

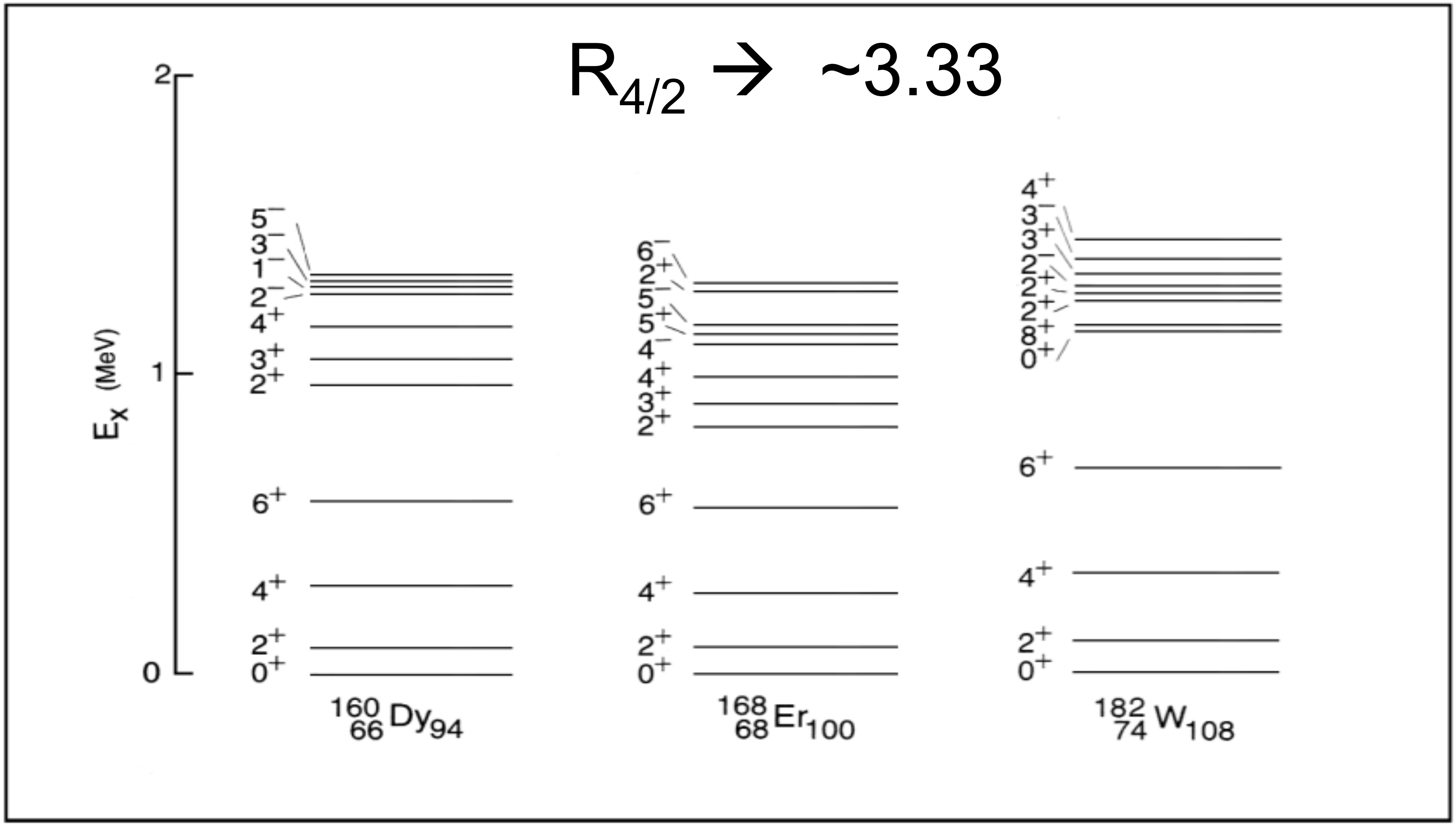
Vibrator (H.O.)

$$E(J) = n\,(\hbar\,\omega_0)$$

$$R_{4/2} = 2.0$$



Lots of valence nucleons of **both** types: emergence of deformation and therefore rotation (nuclei live in the world, not in their own solipsistic enclaves)



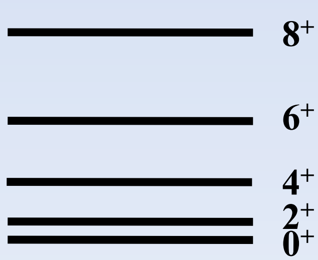
Deformed nuclei – rotational spectra

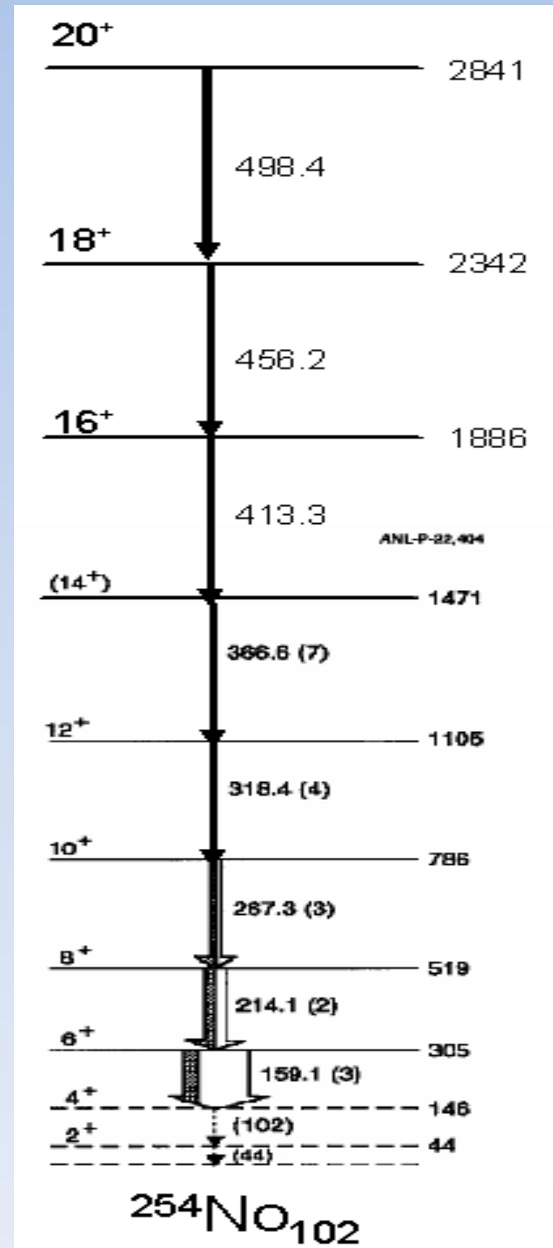
Rotor

$$E(I) \propto (\hbar^2/2I)\,I(I+1)$$

$$R_{4/2} = 3.33$$

BTW, note value of paradigm in spotting physics (otherwise invisible) from deviations





Value of paradigms

J	E (keV)
6^+	690
4^+	330
2^+	100
0^+	0

?

Without rotor paradigm

Paradigm Benchmark

700
333
100
0

Rotor $J(J + 1)$

Amplifies structural differences

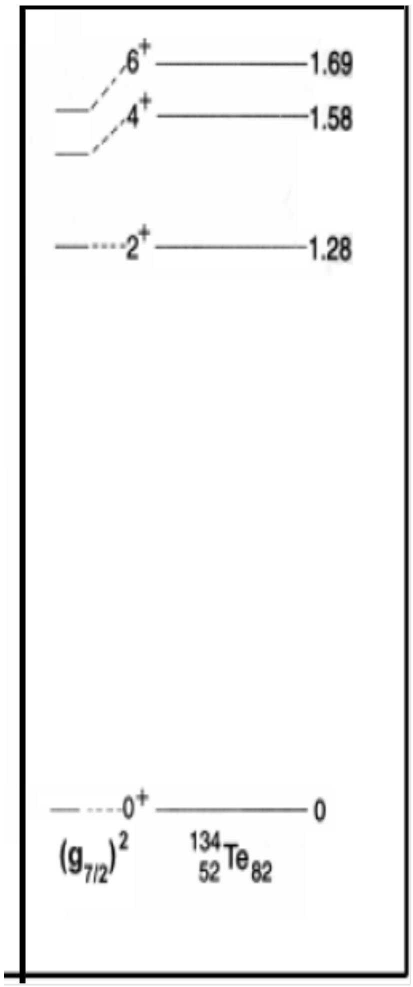
Centrifugal stretching

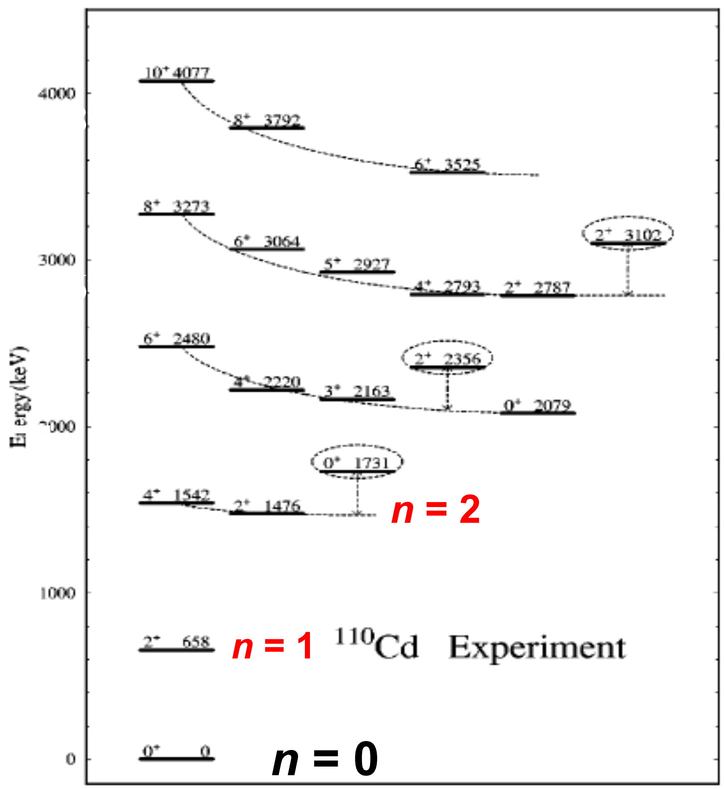
Deviations

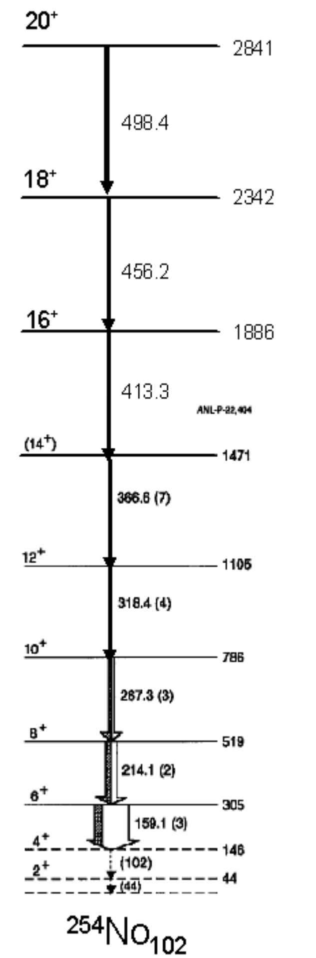
↓

Identify additional degrees of freedom

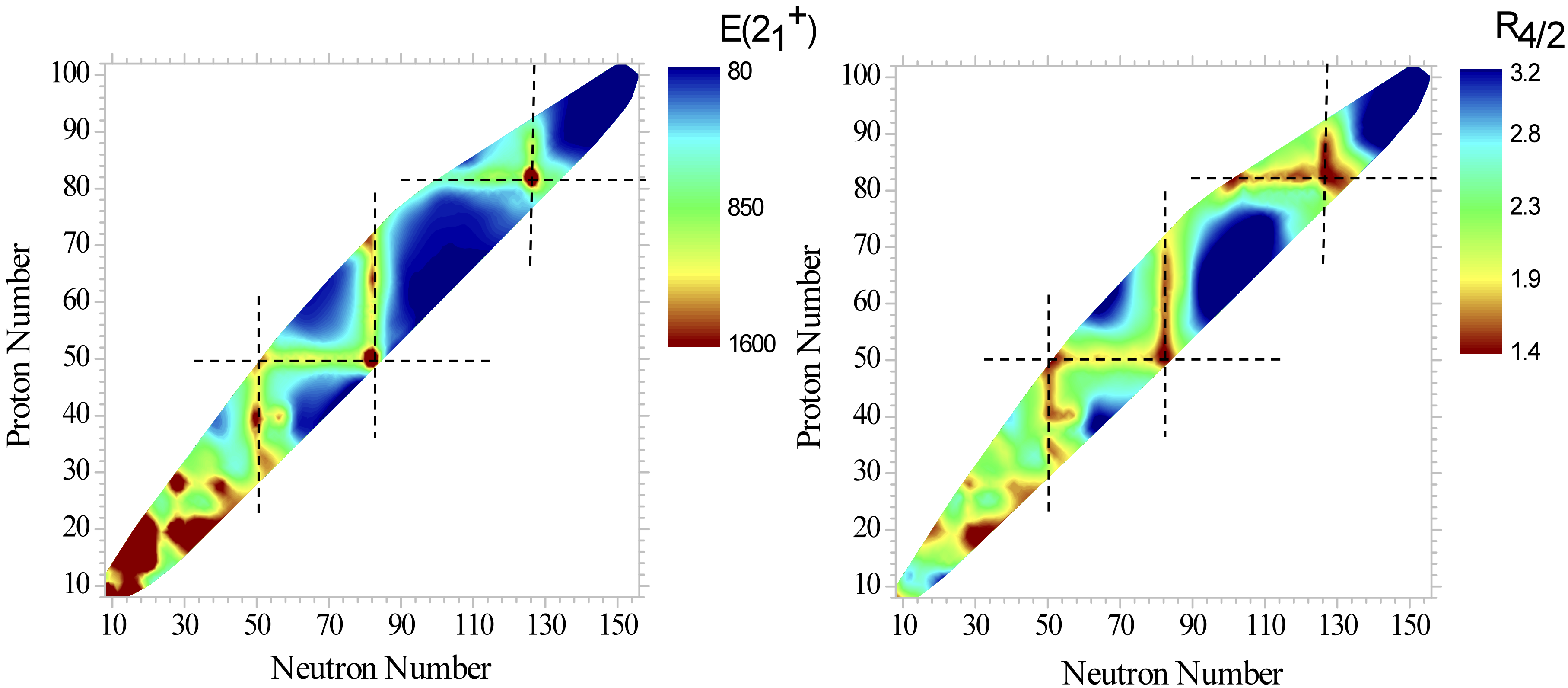
Doubly magic plus 2 nucleons	Vibrator (H.O.)	Rotor
	$E(J) = n\ (\hbar\ \omega_0\ )$	$E(J) \propto (\ \hbar^2/2I\ )J(J+1)$
$R_{4/2} < 2.0$	$R_{4/2} = 2.0$	$R_{4/2} = 3.33$







Broad perspective on structural evolution

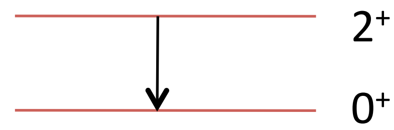


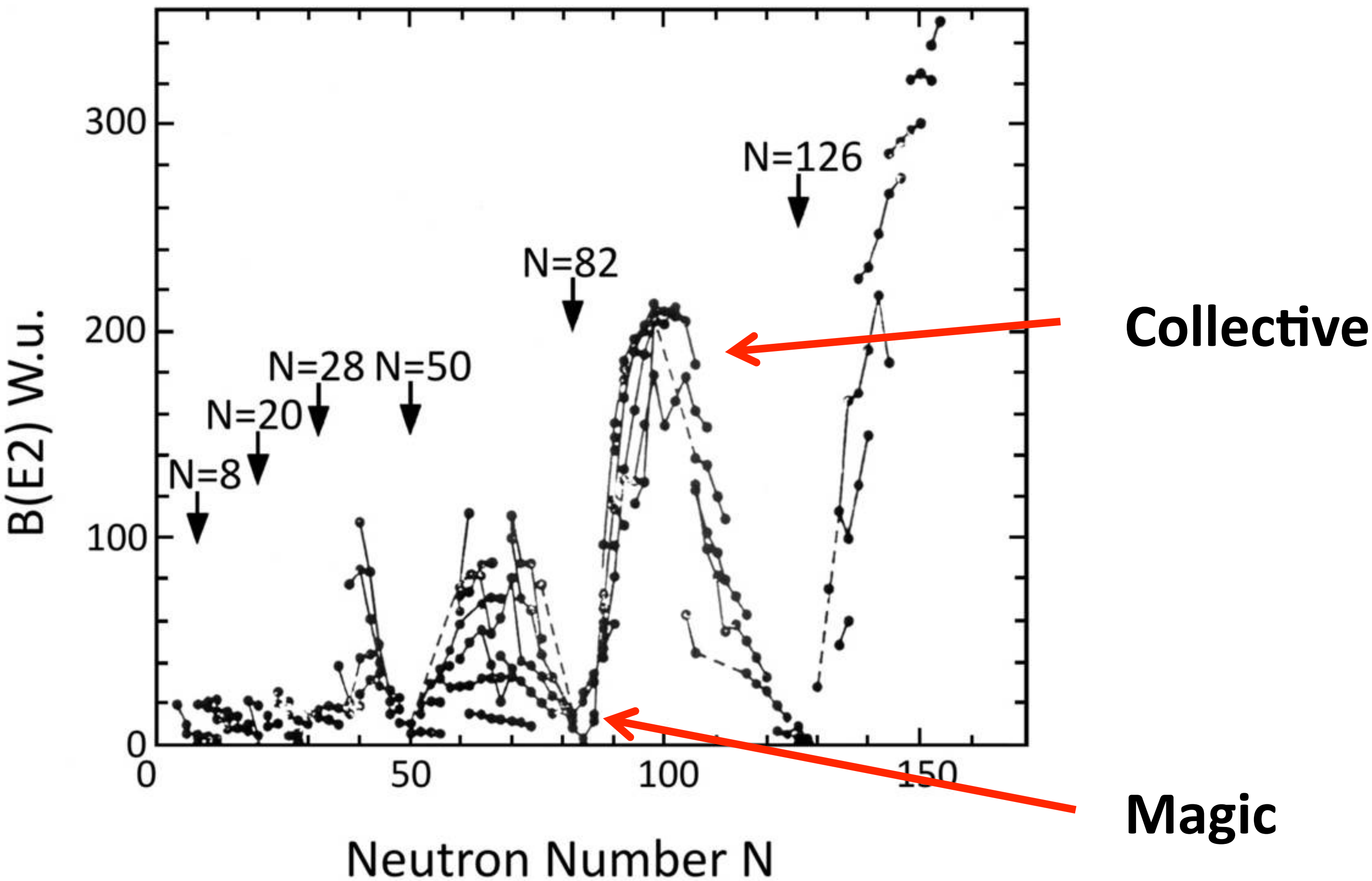
The remarkable regularity of these patterns is one of the beauties of nuclear systematics and one of the challenges to nuclear theory.

Whether they persist far off stability is one of the fascinating questions for the future

Transition rates (half lives of excited levels) also tell us a lot about structure

$$B(E2: 0^+_1 \rightarrow 2^+_1) \propto \langle 2^+_1 || E2 || 0^+_1 \rangle^2$$

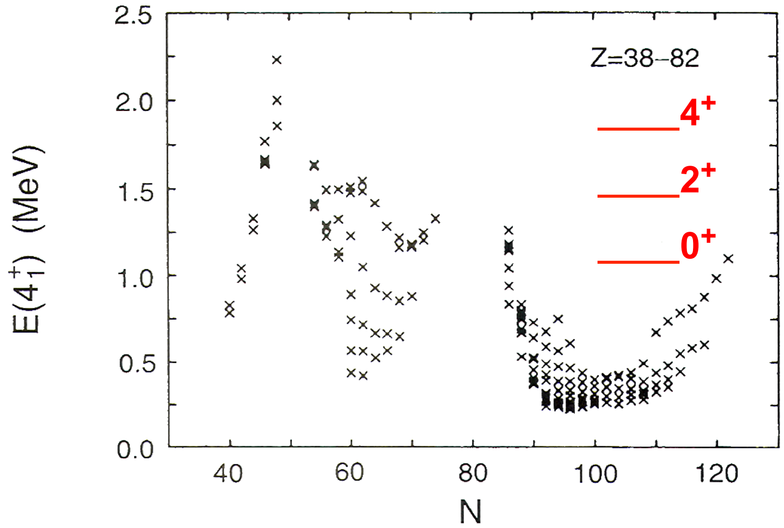




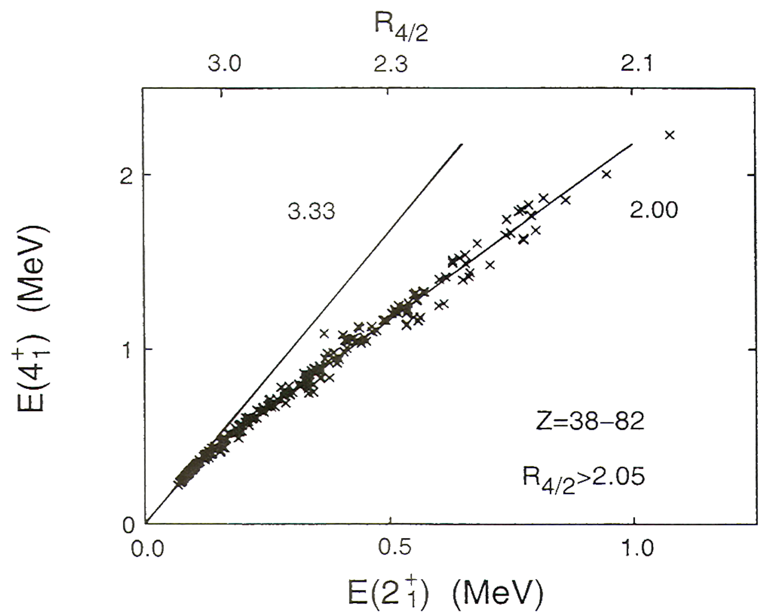
So far, everything we have plotted has been an individual observable against N or Z (or A)

Now we introduce the idea of correlations of different observables with each other.

Correlations of Collective Observables



There is only one appropriate reaction to this result ….



Wow !!!! !!!!

There is only one worry, however …. accidental or false correlations. Beware of lobsters !!!

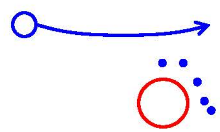
BEWARE OF FALSE CORRELATIONS!

How do we study nuclei experimentally?

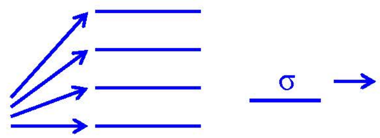
2 general classes of approaches:

- bottom up
- top down

- **Bottom up:** excite nucleus from g.s. with some projectile



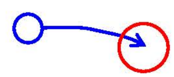
Can be scattering or transfer of nucleons. Cross section directly dependent on level structure.



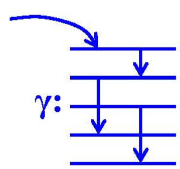
selective ⟶ σ gives structure info

- **Top down:** form nucleus at high E_{ex}

  nucleus decays to lower states



(capture)



or radioactive decay (α, β)

Population of levels is ~ statistical and J-dependent, but <u>not</u> structure-dependent. Get structure from <u>γ's</u>

A couple of semi-randomly chosen experimental techniques

- Absolute transition matrix elements: Coulomb excitation and Doppler techniques

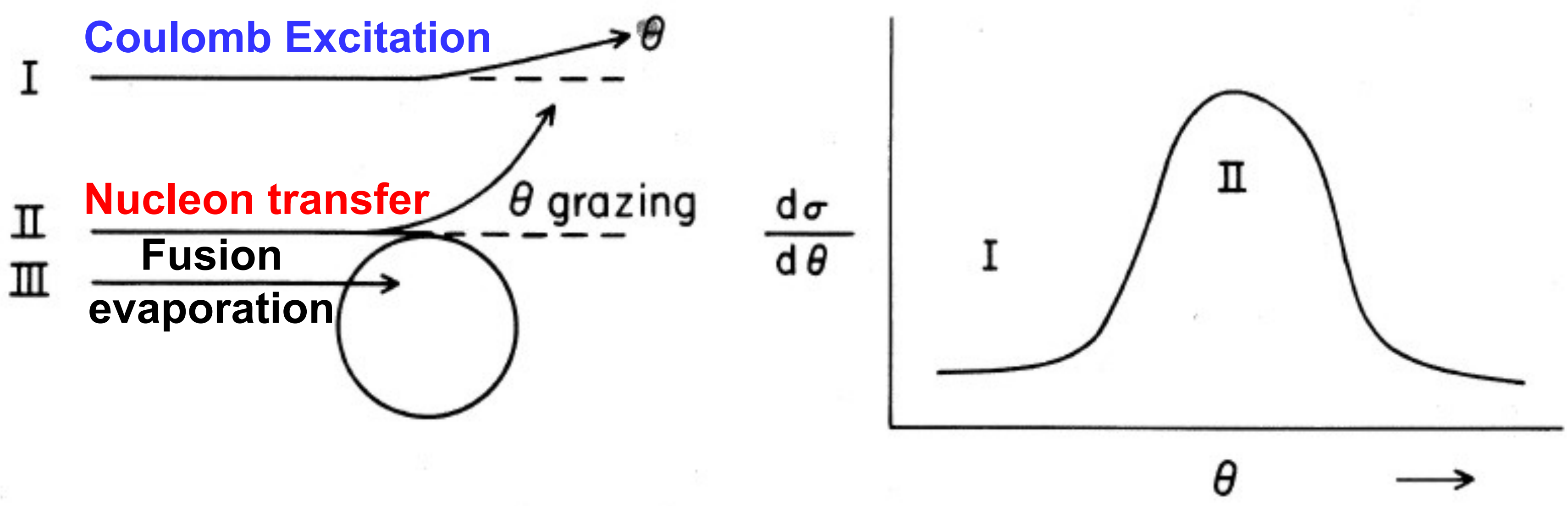
- Transfer reactions

- Fusion evaporation reactions and coincidence gating with large arrays

- Later – experiments with exotic nuclei

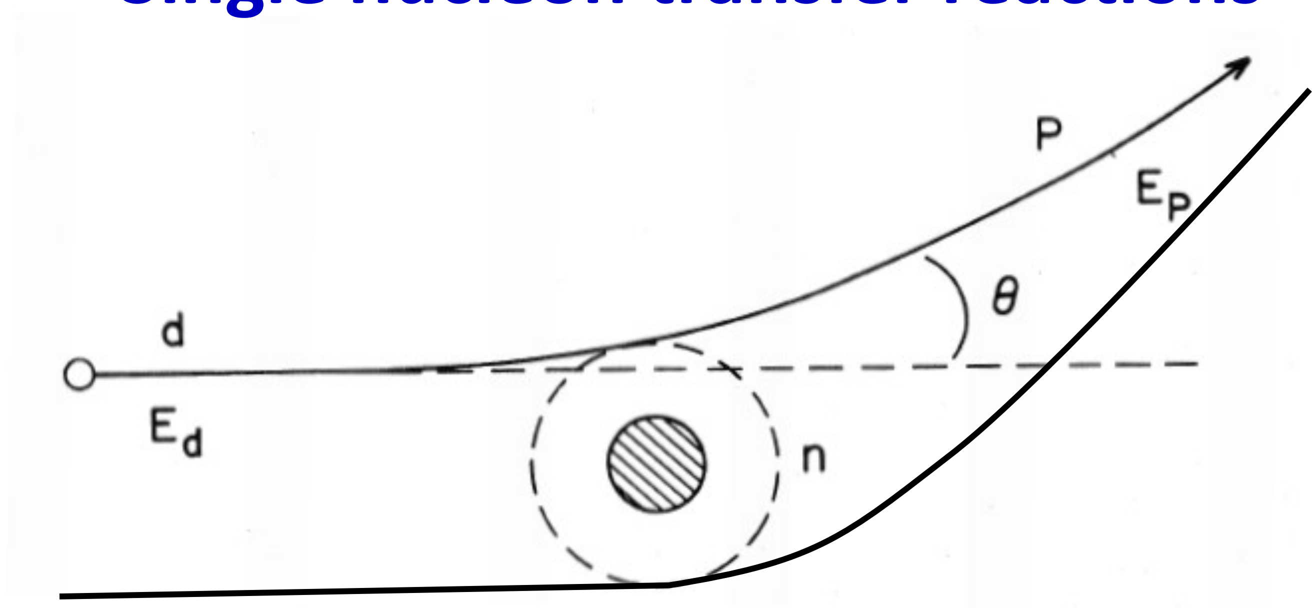
Reactions

Role of impact parameter and relation to scattering angle

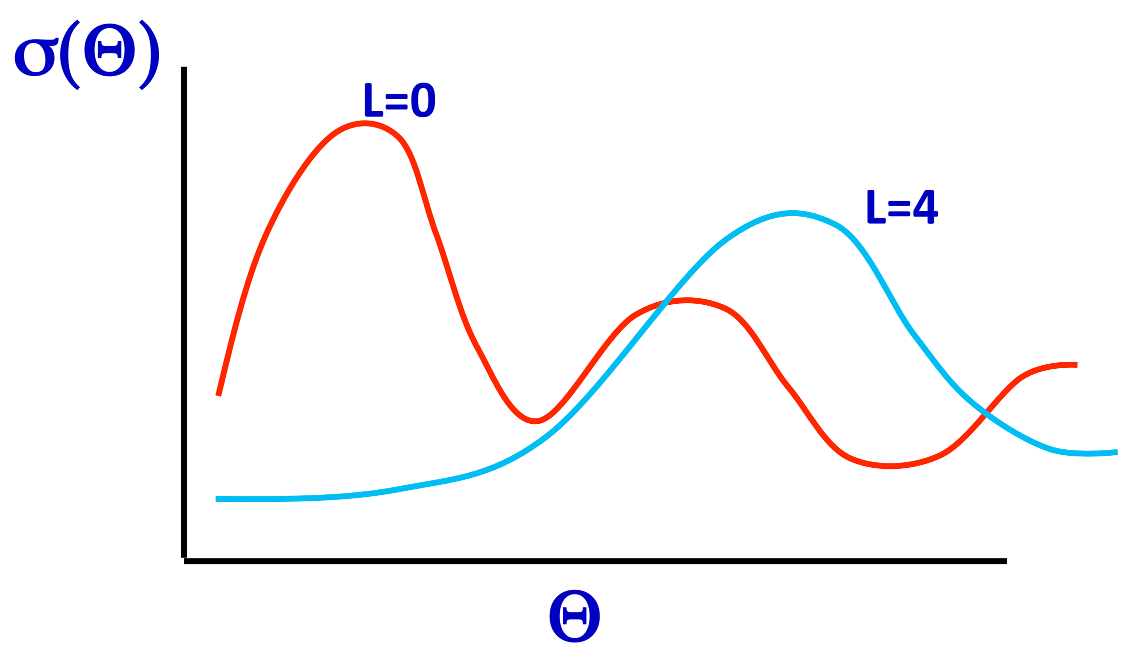


II

Single nucleon transfer reactions



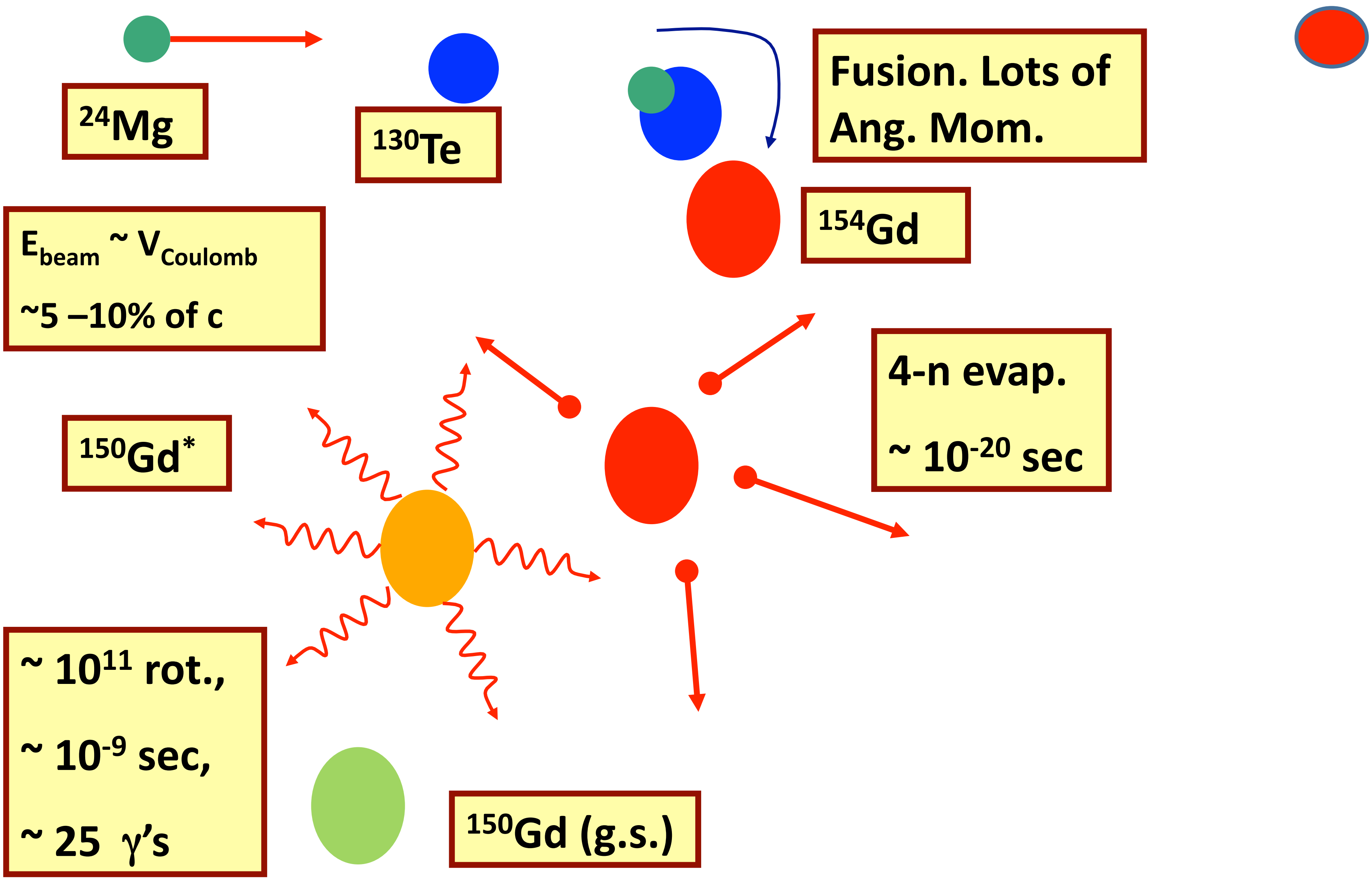
These two paths are indistinguishable and interfere either constructively or destructively depending on the difference in path length, which is a function of angle



III

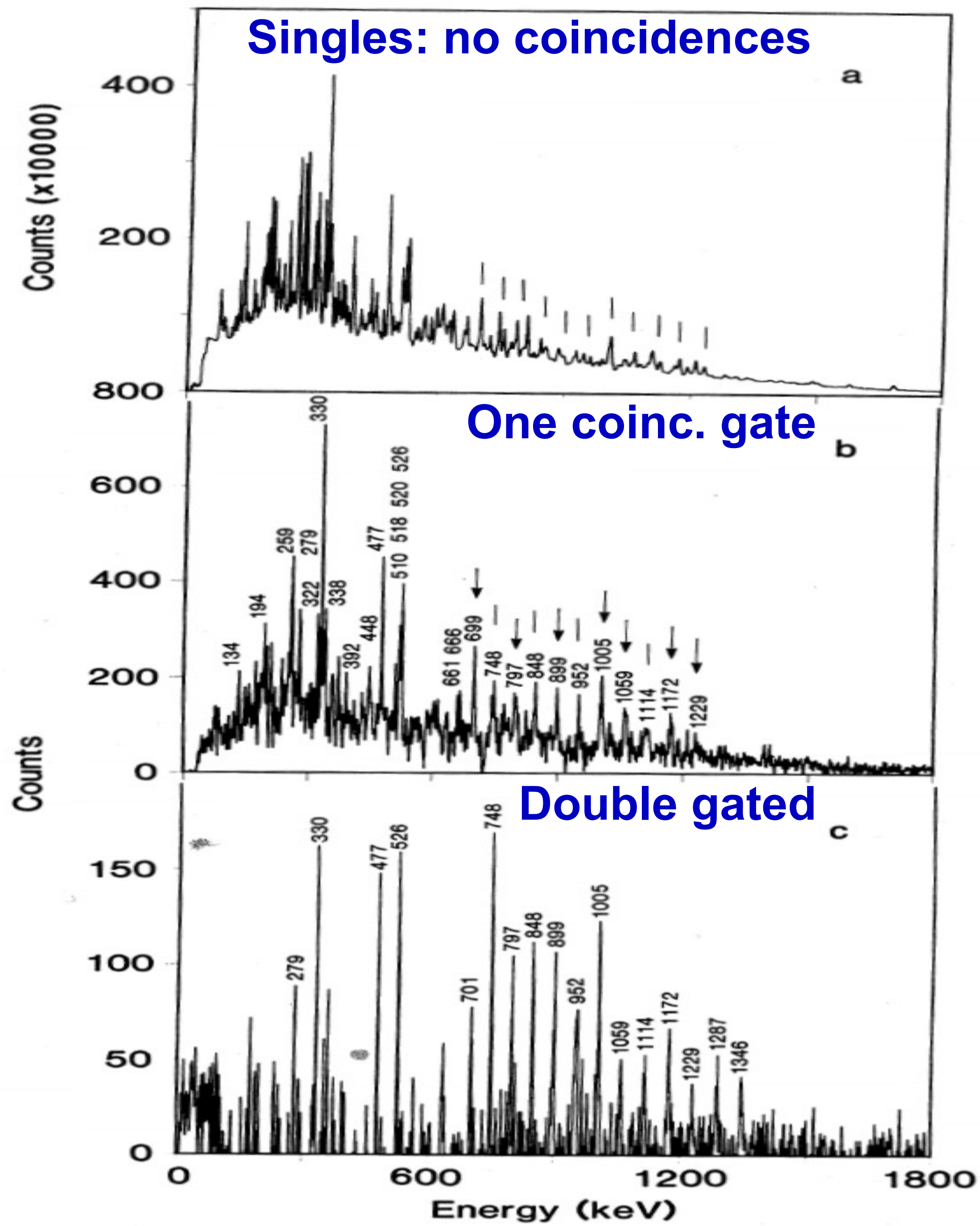
Heavy-ion Fusion Evaporation Reaction

Production Mechanism



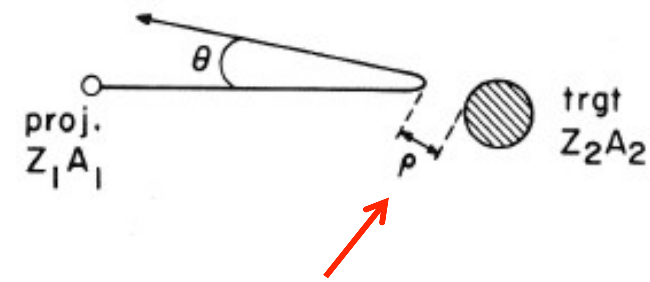
Advantages of mutli-fold coincidence gating in fusion evaporation reactions: high spin states in neutron deficient nuclei

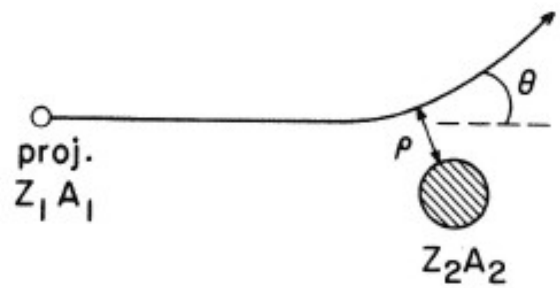
$^{124}Sn(^{29}Si, 5n\gamma)^{148}Gd$



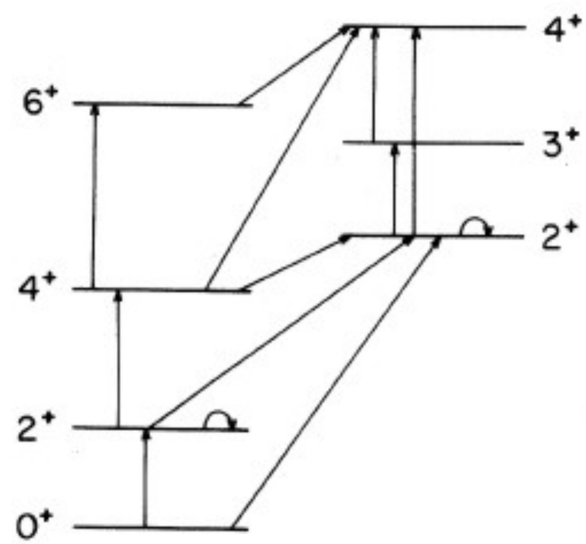
Coulomb excitation

$$V_{\text{Coul}}(r_{12}) = \frac{Z_1 Z_2 e^2}{r_{12}}$$

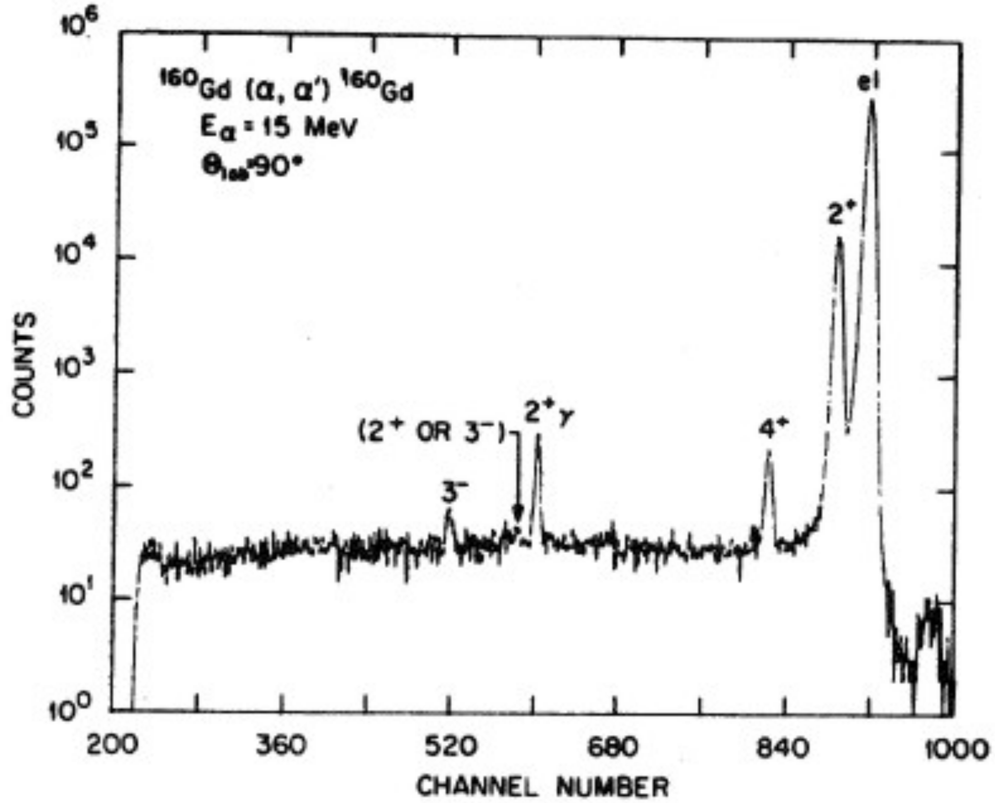




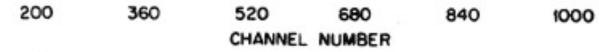
$$\rho_{\min} = \frac{2 Z_1 Z_2 e^2}{m_0 v^2}$$



Light projectiles: one- or two-step excitation.
Step by step up the level scheme



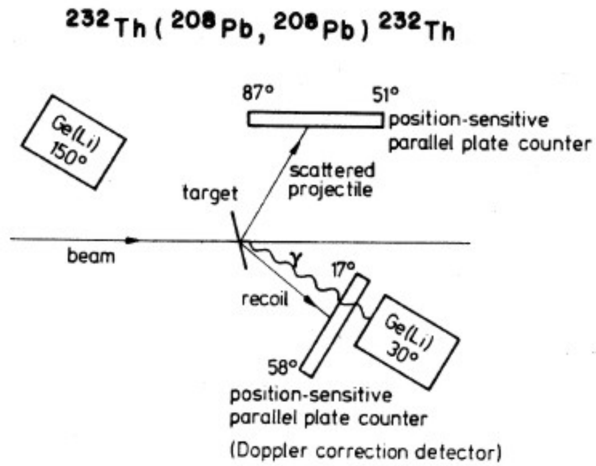
Interfering routes: Need to be very careful. Matrix element signs

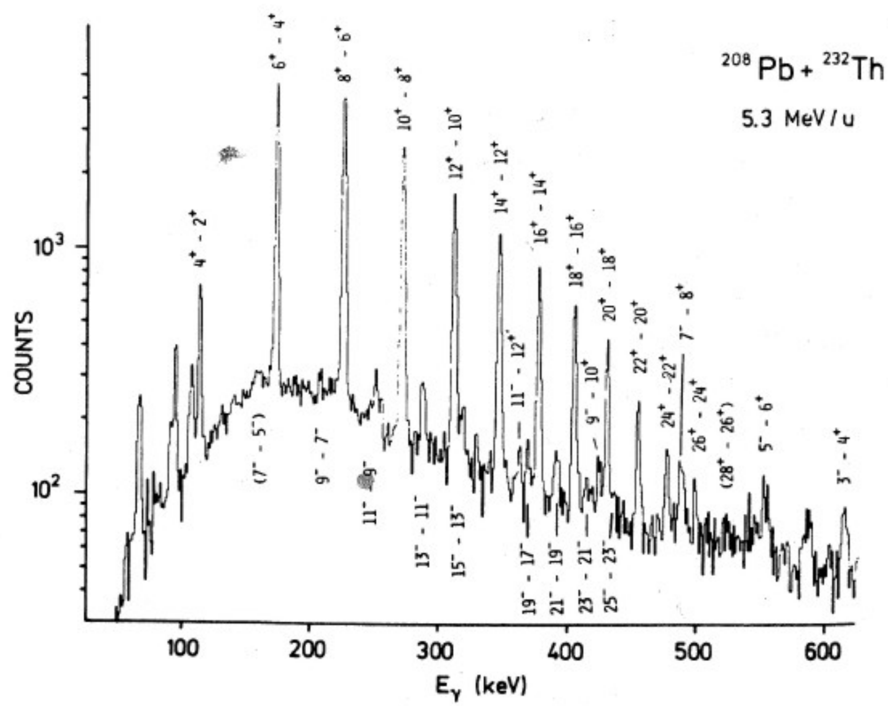


Heavy projectiles: multi-step excitation. Vary beam (Z) or beam energy or detected angle.

Many (really many !!!!) matrix elements to extract. Opportunity but need lots of data, not just one spectrum.

Signs of matrix elements !!!





Intermediate energy CE

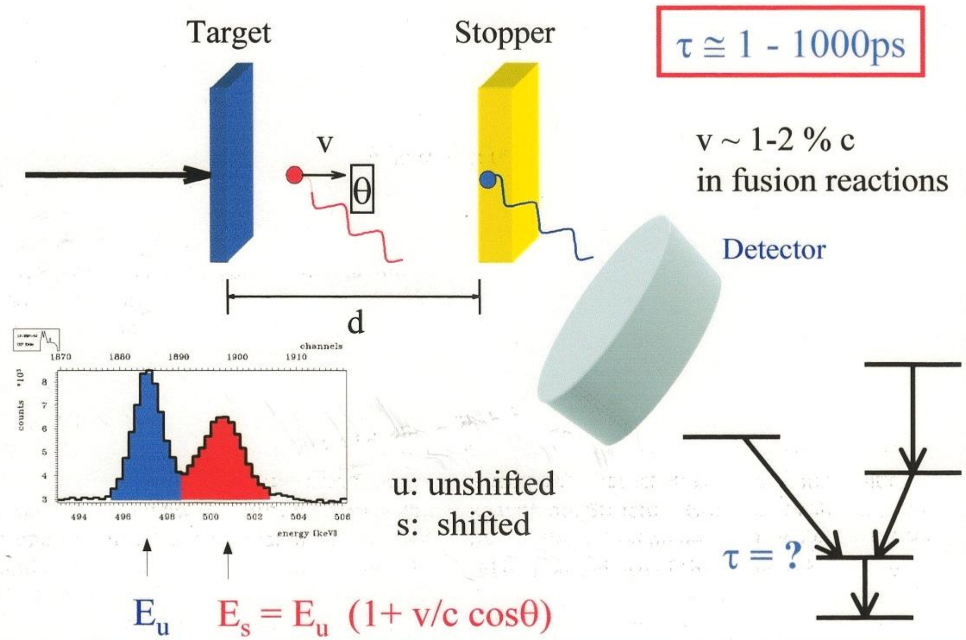
What happens? Projectile passes by the nucleus so quickly that there is time for only one excitation. NO multiple excitation.

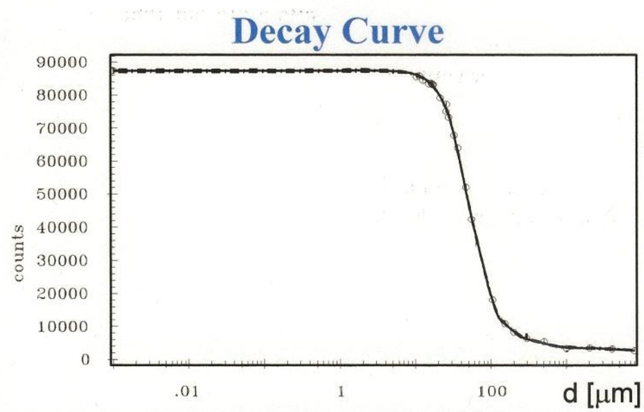
Projectile goes forward. Nuclear reactions involve closer contact, more interaction, bigger change in angle, and more backward-going projectile (if projectile comes out at all).

Even even nuclei, E2 excitation: Single step excitation makes 2+ states -- ONLY !!!!

So Int. En. CE is a "meter" for 2 + states and their B(E2) values

The Recoil Distance Doppler-Shift Method





Standard Analysis:

χ^2-fit with set of exponential functions.

Feeding behavior as input of fit.

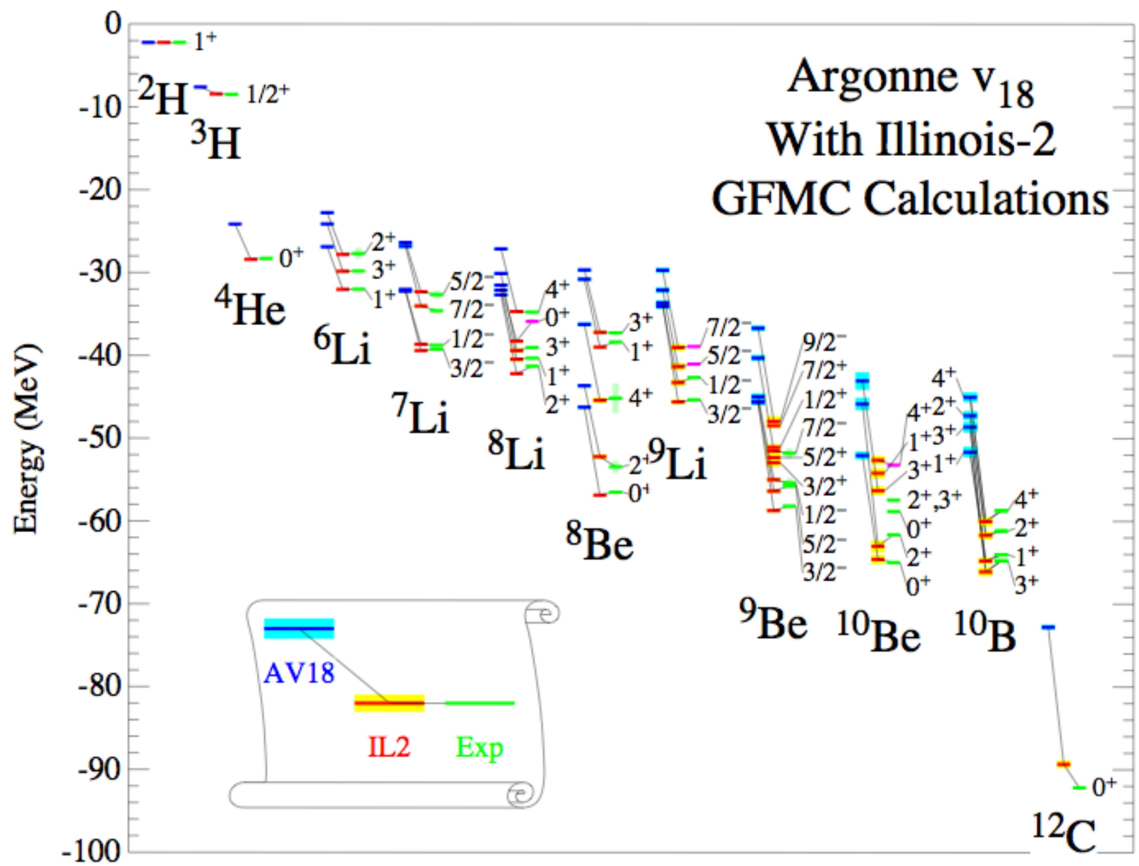
No feedback of fit results.

How can we understand nuclear behavior

- Do microscopic calculations, in the Shell Model or its modern versions, such as with density functional theory, ab initio, or Monte Carlo methods. These approaches are making amazing progress in the last few years with advances in computing power.

Ab initio calculations: An on-going success story

James Vary Lectures next week



How can we understand nuclear behavior

- Do microscopic calculations, in the Shell Model or its modern versions, such as with density functional theory, ab initio, or Monte Carlo methods. These approaches are making amazing progress in the last few years with advances in computing power.

Nevertheless, such approaches often do not give an intuitive feeling for the structure involved.

- Collective models, which focus on the structure and symmetries of the many-body, macroscopic system itself. Two classes: Geometric and Algebraic

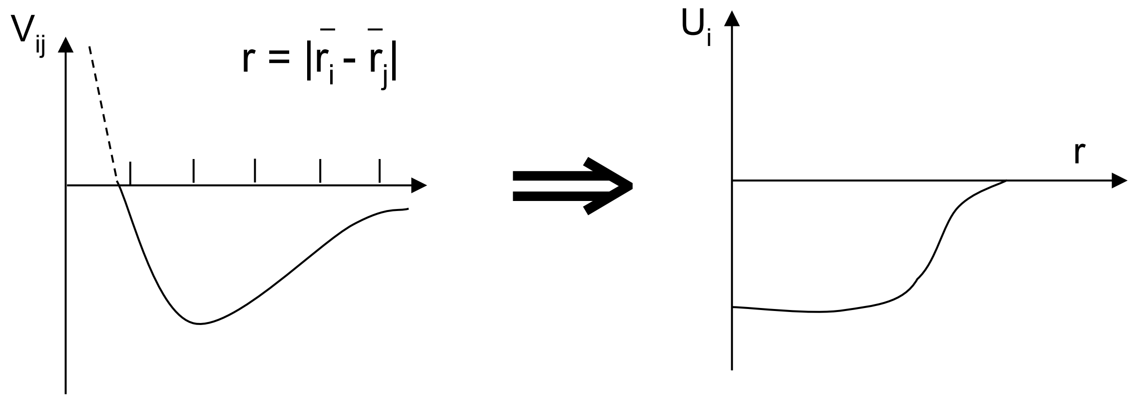
Geometrical models introduce a potential which depends on the shape of the nucleus. One can then have rotations and vibrations of that shape.

Algebraic models invoke symmetries of the nucleus and use group theoretical approaches to solve as much as possible analytically.

We will discuss both approaches from a simple perspective.

Start with Independent particle model:
magic numbers, shell gaps, valence nucleons.
Three key ingredients

First:

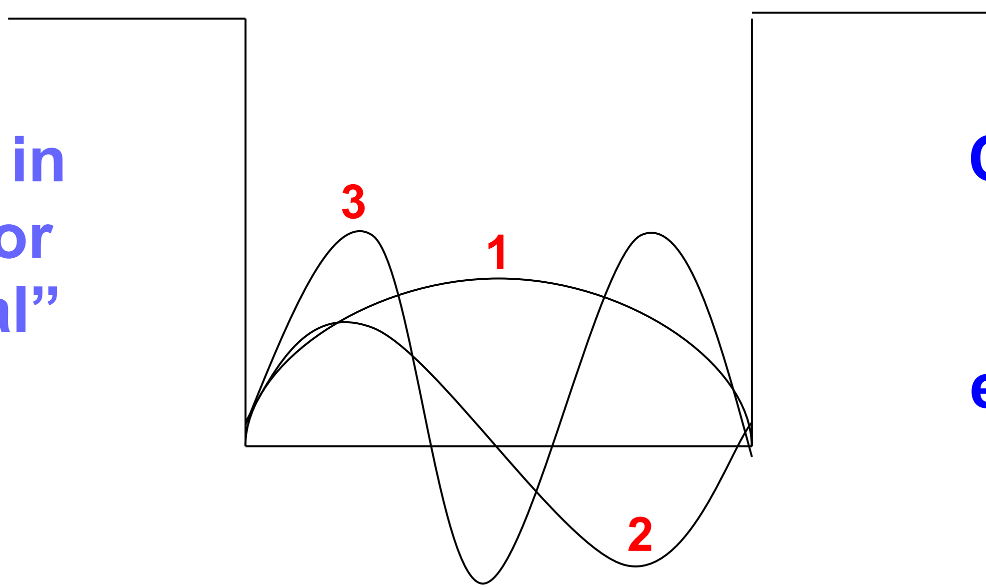


Nucleon-nucleon force – very complex ~ **One-body potential – very simple: Particle in a box**

This extreme approximation cannot be the full story. Will need "residual" interactions. But it works surprisingly well in special cases.

Second key ingredient: Quantum mechanics

Particles in a “box” or “potential” well



Confinement is origin of quantized energies levels

Energy ~ 1 / wave length

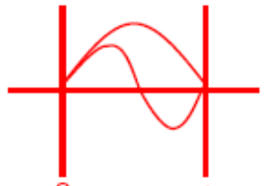
n = 1,2,3 is principal quantum number

E ↑ up with *n* because wave length is shorter

Energies in an Infinite Square Well

(box)

Simple Derivation



$\Psi(0) = \Psi(d) = 0$

for containment

$\therefore \ \dfrac{n\lambda}{2} = d \qquad n = 1, 2, \cdots$

Now, use de Broglie relation

$P = \dfrac{h}{\lambda}$ and $E = \frac{1}{2} m v^2 = p^2/2m$

or $p = \sqrt{2mE}$

$\therefore \ \dfrac{nh}{2p} = \dfrac{nh}{2\sqrt{2mE}} = d$

$\therefore \ \dfrac{n^2 h^2}{8mE} = d^2$

or $\boxed{E = \dfrac{n^2 h^2}{8 m d^2}} \qquad n = 1, 2, \cdots$

Zero point motion

a) confinement

b) wave/particle relation

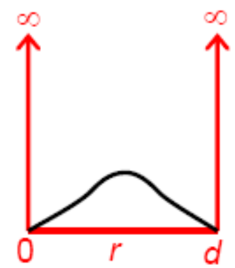
$\Big\} \rightarrow$ quantization

Potential Wells (I – d)

a) Infinite

$V = 0 \qquad 0 < r < d$

$V = \infty \qquad r \le 0,\ r \ge d$



Schrödinger Eq.

$$\frac{-\hbar^2}{2m}\frac{d^2\Psi}{dr^2} + V(=0) = E\,\Psi$$

OR $$\frac{d^2\Psi}{dr^2} = -\frac{1}{\hbar^2}\,2mE\ \ \Psi \equiv -k^2\,\Psi \qquad k = \frac{1}{\hbar}\sqrt{2mE} = \frac{2\pi}{\lambda}$$

Double difference $\Psi \rightarrow \Psi \Rightarrow \Psi$ is "exponential"

Double difference $\Psi \rightarrow$ neg # $\Rightarrow$ exponent contains i

$\Psi = A \sin kr + B \cos kr = A \sin kr \qquad \Psi(0) = 0$

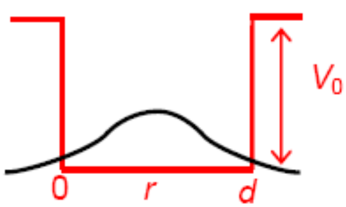
$\Psi(d) = 0 \ \Rightarrow\ k\,d = n\,\pi$ or $k = n\,\pi/d$

$\Psi = A \sin \frac{n\pi r}{d} \qquad \boxed{E} = \frac{k^2\hbar^2}{2m} = \frac{n^2\pi^2 h^2}{2md^2(2\pi)^2} = \boxed{\frac{n^2 h^2}{8md^2}}$

---

b) Finite

$V = V_0 \qquad r \le 0\ \ r \ge d$



Inside: as above

Outside: $$\frac{d^2\Psi}{dr^2} = -\frac{2m}{\hbar^2}\left(E - V\right)\Psi = k^2\,\Psi \qquad k = \frac{1}{\hbar}\sqrt{2m\left(V_0 - E\right)}$$

Double difference $\Psi \rightarrow +\Psi \Rightarrow$ exponent without i

$\Psi = Ae^{-kr}$ (+ $k\,r$ grows without bound) $\qquad \boxed{E \text{ lower}}$

Uncertainty Principle and Zero Point Motion

Uncertainty Principle ⇒ particle cannot fall to bottom of well

If $\Delta p = 0$ (*i.e.*, stopped in bottom of pot)
then $\Delta x \to \infty$ (particle is <u>un</u>bound)

OR:

If $\Delta x = 0$ (particle exactly localized)
then $\Delta p \to \infty$ (particle jumps out of potential)

Example: Particle in well (3 - d)

$$\Delta p_x = \frac{\hbar}{2\,\Delta x} = \frac{\hbar}{2r} \qquad r = \text{radius of well}$$

$$\therefore \quad \boxed{\Delta E_x} = \frac{\Delta P_2^2}{2m} = \boxed{\frac{\hbar^2}{8mr^2}}$$

Compare e^- in atom, p in nucleus. Take $m_e = 1$, $m_p = 10^3\, m_e$

<u>Atom</u> $\quad \Delta E_{min} \sim \dfrac{1}{\left(10^{-8}\right)^2} = \dfrac{1}{10^{-16}\,\text{cm}^2}$

<u>Nucleus</u> $\quad \Delta E_{min} \sim \dfrac{1}{10^3\left(10^{-12}\right)^2} = \dfrac{1}{10^{-21}\,\text{cm}^2}$

Nuclear scale is ~ 10^5 times larger

(10 eV) $_{\text{atom}}$ → (1 MeV) $_{\text{nucleus}}$

Nuclei are 3-dimensional

- What is new in 3 dimensions?
  - Angular momentum
  - Centrifugal effects

Central Potential U(r) U(r) · 0 at r = 0

Schrödinger eq.:

$$H\,\Psi = \left(\frac{P^2}{2M} + U(r) \right) \Psi_{nlm}(r) = E_{nlm}\,\Psi_{nlm}(r)$$

Separable into radial and angular parts:

$$\Psi_{nlm}(r) = \Psi_{nlm}(r\,\theta\,\Phi) = \frac{1}{r}\,R_{nl}(r)\,\Psi_{nl}(\theta\,\Phi)$$

n = radial q. # = # nodes
l = orbital angular momentum
m = substate (direction in space)

Nomenclature:

l = 0, 1, 2, 3, 4, 5… s, p, d, f, g, h…

$m = l, l-1, \ldots\ldots\ldots\ldots\ldots\ldots(-l)$. $2l + 1$ m substates

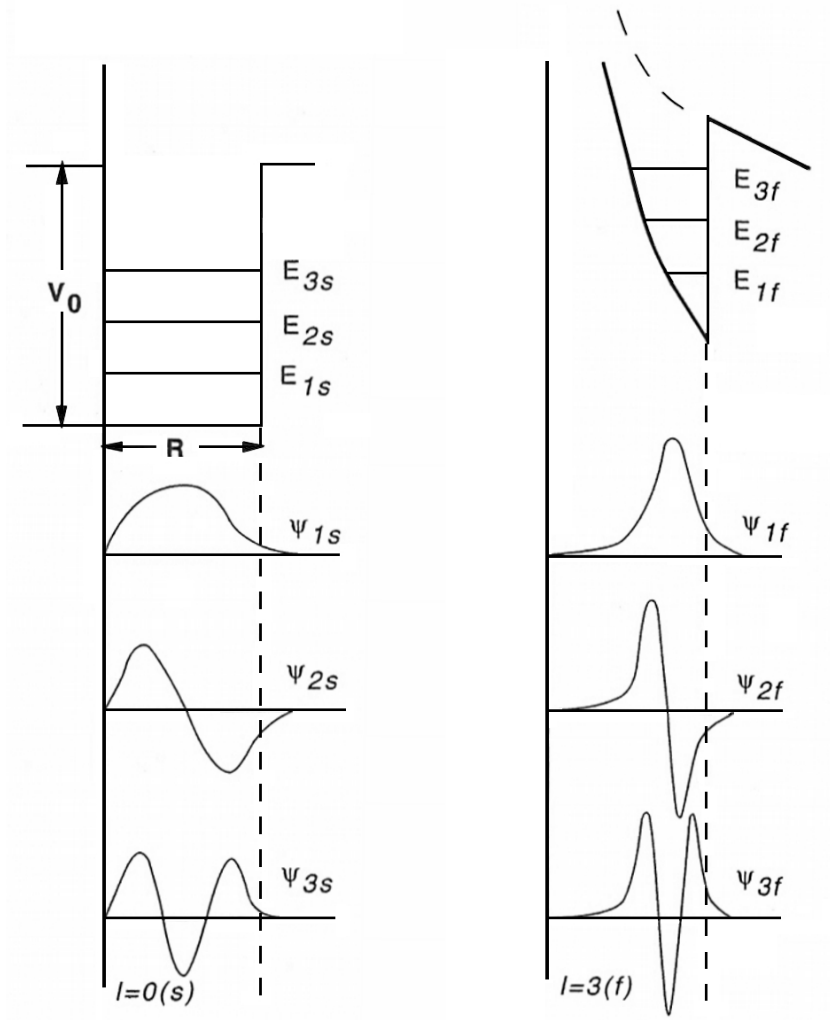
E($l\ m_i$) = E($l\ m_j$)

Radial Schrödinger eq.:

$$\frac{\hbar^2}{2M}\frac{d^2 R_{nl}(r)}{d\,r^2} + \left[E_{nl} - U(r) - \frac{\hbar^2}{2M}\frac{l\,(l+1)}{r^2} \right] R_{nl}(r) = 0$$

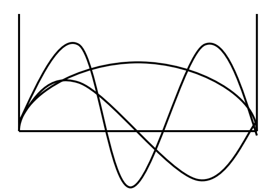
Radial Schroedinger wave function

$$\frac{h^2}{2m}\frac{d^2R_{nl}(r)}{dr^2} + \left[E_{nl} - U(r) - \frac{h^2}{2m}\frac{l(l+1)}{r^2} \right] R_{nl}(r) = 0$$



Higher Ang Mom: potential well is raised and squeezed. Wave functions have smaller wave lengths. Energies rise

Energies also rise with principal quantum number, *n*.



Raising one, lowering the other can give similar energies – "level clustering":

H.O: E = ħω (2n+l)

E (n,l) = E (n-1, l+2)

e.g., E (2s) = E (1d)

Properties of the Solutions

1) Higher n ⟹ higher E (more KE)

2) Higher l ⟹ higher E (larger radius, less bound)

⟹ lowest state is $1s_{1/2}$ $n = 1, l = 0$

(Explains gr. st. deuteron: $L = l_1 + l_2 = 0$)

1) and 2) ⟹

3) Can have similar energies for 2 orbits if one has larger n and smaller l (or vice versa)

Third key ingredient

Pauli Principle

- **Two fermions, like protons or neutrons, can NOT be in the same place at the same time: can NOT occupy the same orbit.**
- **Orbit with total Ang Mom, j, has 2j + 1 substates, hence can only contain 2j + 1 neutrons or protons.**

This, plus the clustering of levels in simple potentials, gives nuclear SHELL STRUCTURE

Consider SHO Levels

nlj: Pauli Prin. 2j + 1 nucleons

Levels	# nucleons (either *p* or *n*)	Total up to *E*
$3s_{1/2}$, $2d_{3/2}$, $2d_{5/2}$, $1g_{7/2}$, $1g_{9/2}$	2 + 4 + 6 + 8 + 10 = 30	70
$2p_{1/2}$, $2p_{3/2}$, $1f_{5/2}$, $1f_{7/2}$	2 + 4 + 6 + 8 = 20	40
$2s_{1/2}$, $1d_{3/2}$, $1d_{5/2}$	2 + 4 + 6 = 12	20
$1p_{1/2}$, $1p_{3/2}$	2 + 4 = 6	8
$1s_{1/2}$	2	2

(Next higher is 112)

First few are known magic numbers

Higher ones are not. And known magic numbers

50, 82, 126 do not appear

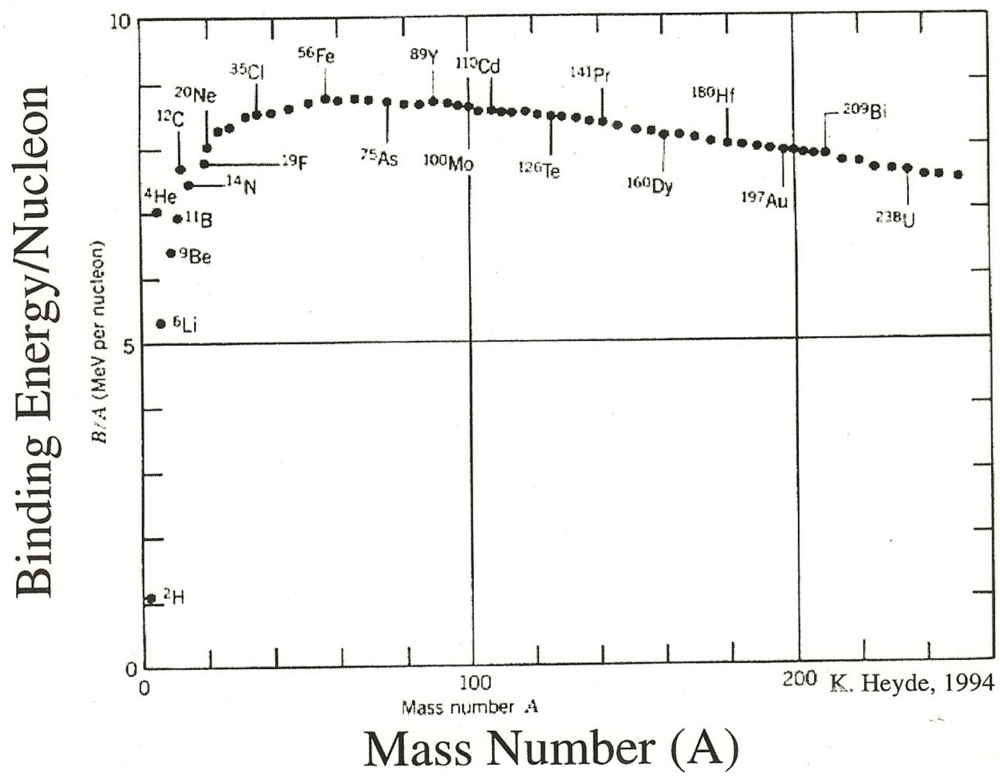
We can see how to improve the potential by looking at nuclear Binding Energies.

The plot gives B.E.s **PER nucleon**.

Note that they saturate. What does this tell us?

Nuclear Force Saturates

Binding Energies



Constant Nuclear Density

Nuclear Radius: $R = R_o A^{1/3}$ $R_o \sim 1.2$ fm

C.J. Barton - Clark University

Consider the simplest possible model of nuclear binding.

Assume that each nucleon interacts with *n* others. Assume all such interactions are equal.

Look at the resulting binding as a function of *n* and A. Compare this with the B.E./A plot.

Each nucleon interacts with 10 or so others. Nuclear force is short range – shorter range than the size of heavy nuclei !!!

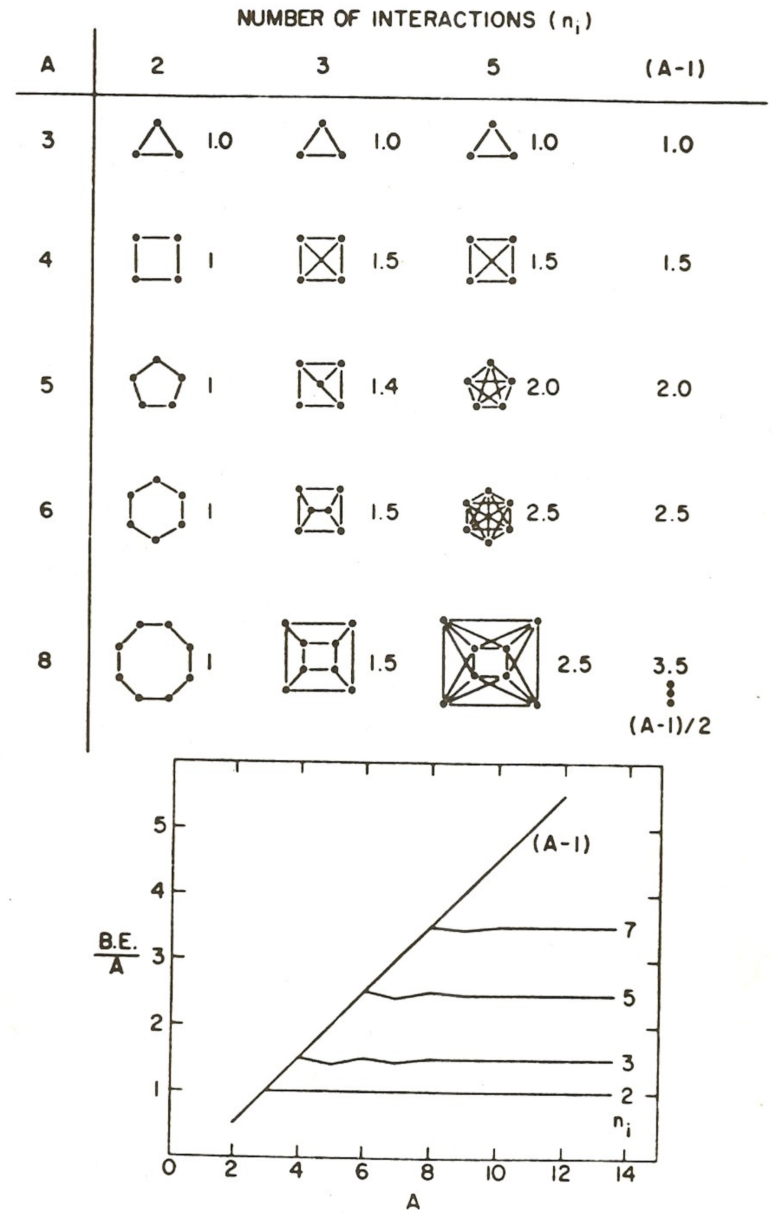
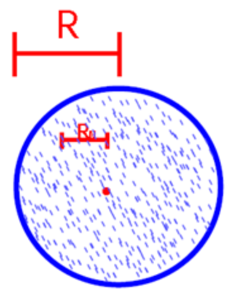


Fig.1.4. Highly schematic calculation of the binding energy per nucleon under different "saturation" assumptions concerning nuclear forces. The number of connections indicated, n_i, is the number of nucleons with which each other nucleon is assumed to interact. All such interactions are considered to be of equal strength. The lower part shows a plot of the resulting binding energies per nucleon.

Form of Central Potential

Consider nucleus of radius $R \gg R_{nucl.\ force}$
and nucleon in "interior"



Surrounded more or less uniformly by nucleons on all sides within range of force

$\therefore$ **No net force,** $V_{int} \sim$ **const**

Perhaps square well might be better approximation

Compared to SHO, will mostly affect orbits at large radii – higher angular momentum states

Comparison of energy levels in H.O. and square well

Sq. well:

Potential deeper (more attractive) at large radii.

Lowers E of higher l states whose probability density is, on average, at larger radius.

H.O. Sq. well

So, square off the potential, add in a spin-orbit force that lowers states with angular momentum

$J = l + ½$

compared to those with

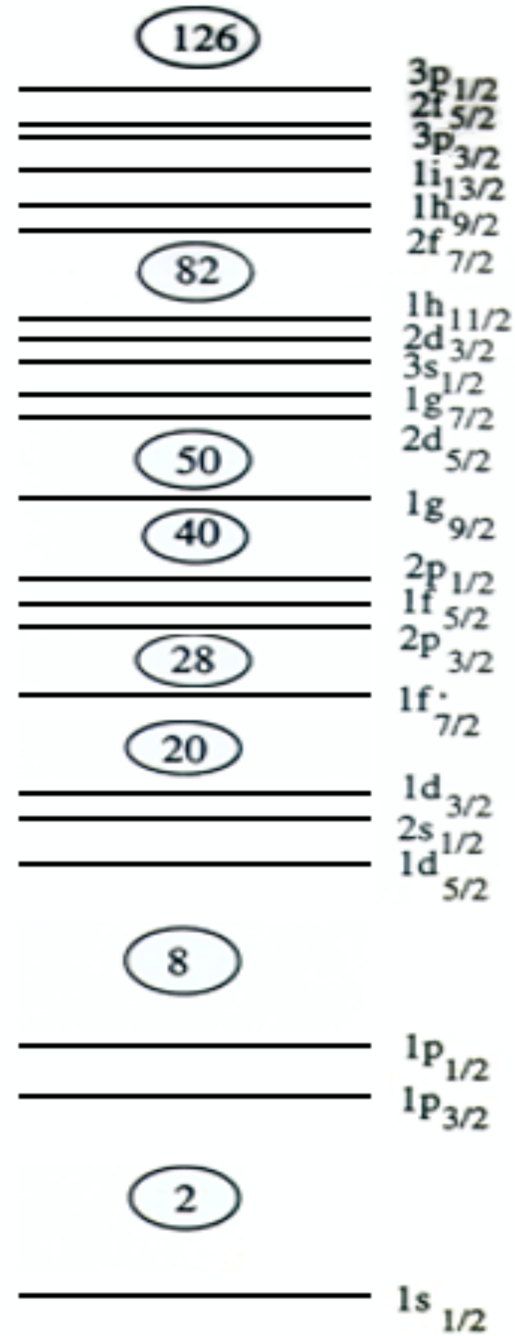
$J = l – ½$

→ Clustering of levels.

Add in Pauli Principle → magic numbers, inert cores

Concept of valence nucleons – key to structure. Many-body → few-body: each body counts.

Addition of 2 neutrons in a nucleus with 150 can drastically alter structure



Independent Particle Model

- Put nucleons (protons and neutrons separately) into orbits.
- Key question – how do we figure out the total angular momentum of a nucleus with more than one particle? Need to do vector combinations of angular momenta subject to the Pauli Principal. More on that later. However, there is one trivial yet critical case.
- Put **2j + 1** identical nucleons (fermions) in an orbit with angular momentum **j**. Each one MUST go into a different magnetic substate. Remember, angular momenta add vectorially but projections (m values) add algebraically.
- So, total M is sum of m's

$$M = j + (j - 1) + (j - 2) + \ldots + 1/2 + (-1/2) + \ldots + [-(j - 2)] + [-(j - 1)] + (-j) = 0$$

M = 0. **So, if the only possible M is 0, then J= 0**

Thus, a full shell of nucleons always has total angular momentum 0. This simplifies things enormously !!!

INDEPENDENT PARTICLE MODEL

Consider $^{41}_{20}Ca_{21}$

^{40}Ca is doubly magic

Proton, neutron shells filled: All magnetic substates filled, hence $\sum_i m_i = M = 0$ **Hence J = 0**

Ground state of ^{41}Ca determined by last odd neutron in

$1f_{7/2}$ orbit

$\therefore$ g.s. ^{41}Ca is $\frac{7}{2}^-$ **Agrees with experiment**

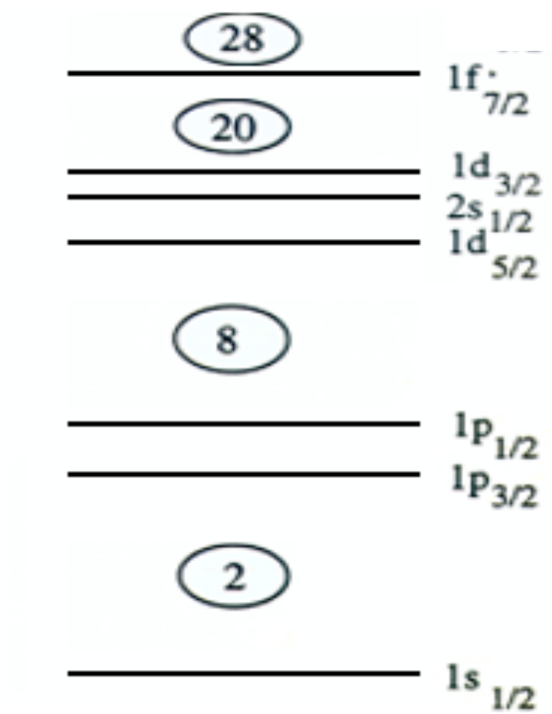
Spectroscopic factors in simple nuclei

Cross section ~ (matrix element)^2 specifying how much the initial and final systems "look" like each other and the ease of putting a particle into an orbit. Consider the reaction: ^{40}Ca (d,p) ^{41}Ca to the ground state 7/2$^-$ state of ^{41}Ca. Cross section goes as:

$S_{7/2} \sim < {}^{41}Ca | \; n \; x \; {}^{40}Ca >^2 \sim 8$

because there are 8 empty places to put a single $f_{7/2}$ particle.

S is spectroscopic factor



Spectroscopic factors in simple nuclei

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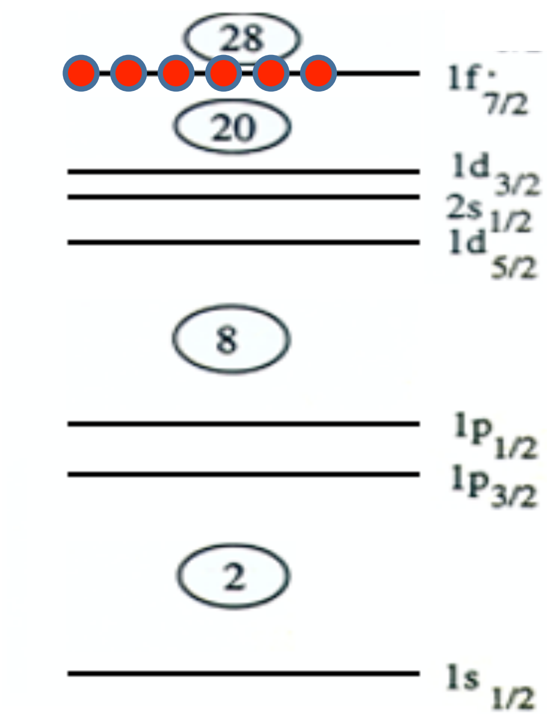
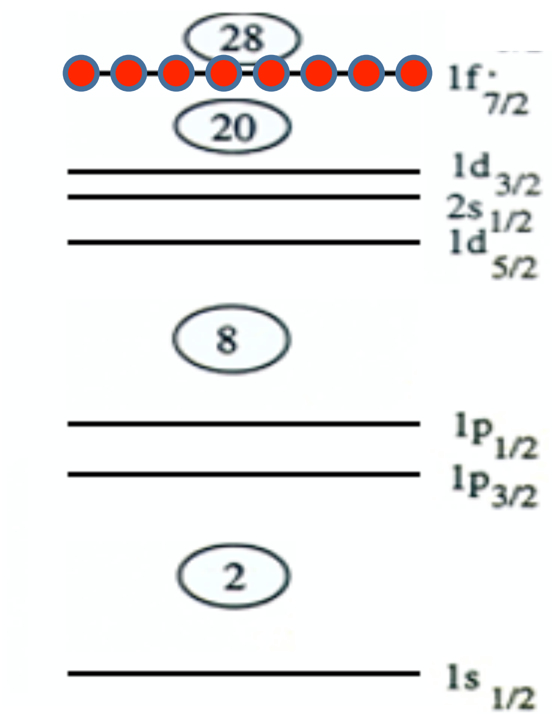


Now consider the reactions:

^{46}Ca (d,p) ^{47}Ca $S_{7/2} \sim 2$;

Spectroscopic factors in simple nuclei

Cross section ~ (matrix element)^2 specifying how much the initial and final systems "look" like each other and the ease of putting a particle into an orbit. Consider the reaction: ^{40}Ca (d,p) ^{41}Ca to the ground state $7/2^-$ state of ^{41}Ca. Cross section goes as:



$S_{7/2} \sim < {}^{41}\text{Ca} |\ n \times {}^{40}\text{Ca}>^2 \sim 8$

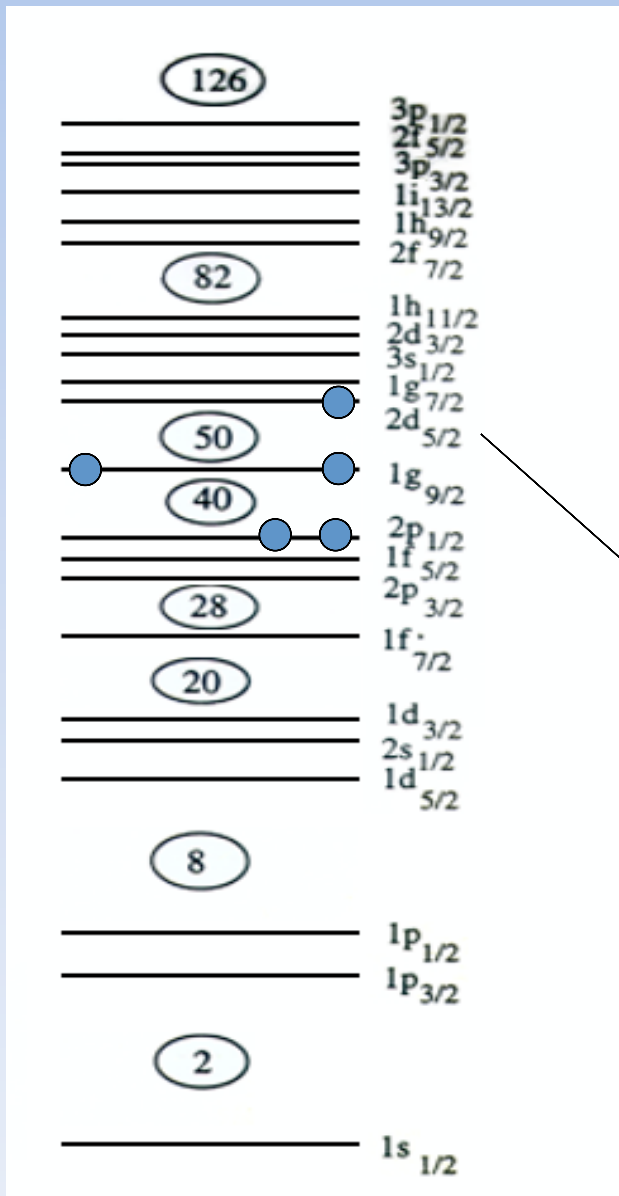
because there are 8 empty places to put a single $f_{7/2}$ particle.

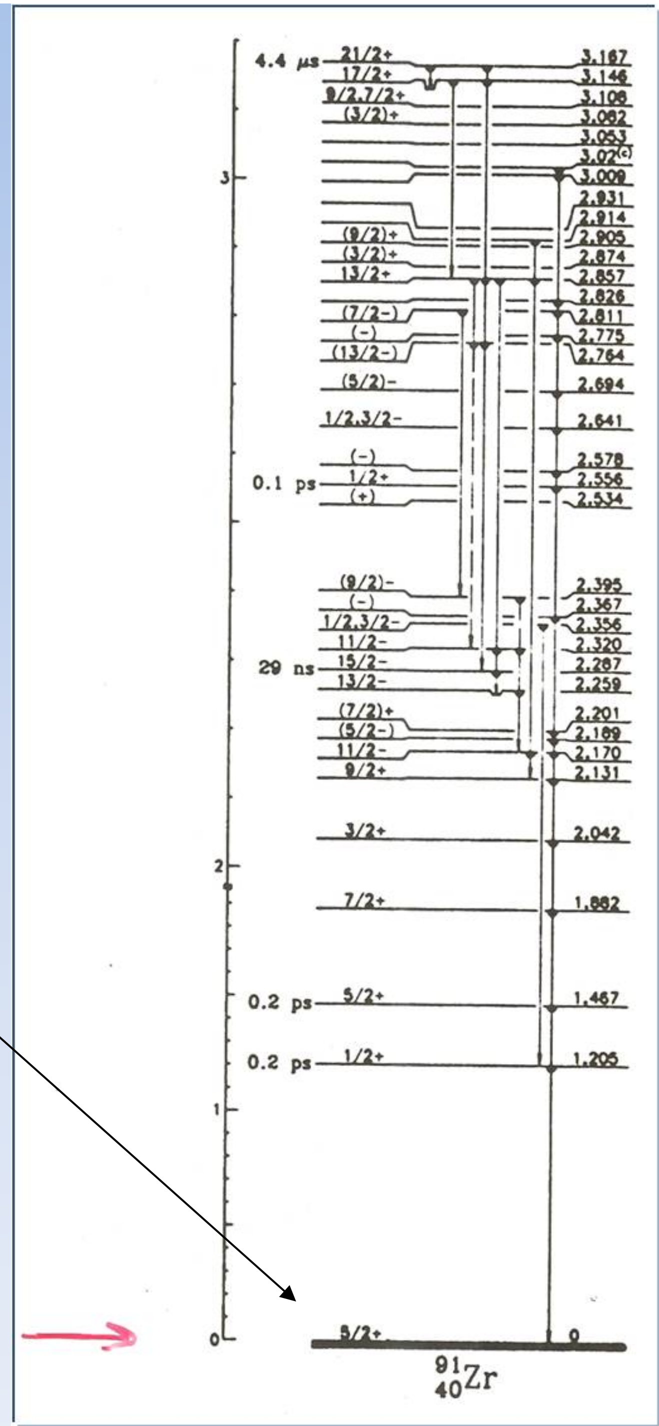
S is spectroscopic factor

Now consider the reactions:

^{46}Ca (d,p) ^{47}Ca $S_{7/2} \sim 2$; ^{48}Ca (d,p) ^{49}Ca $S_{7/2} \sim 0$

Let's do $^{91}{}_{40}Zr_{51}$





Paradigm Shifts in Physics/ Nuclear Physics

("Impossible" $\xrightarrow{PS}$ "Possible")

Changes of co-ordinate systems

Lab to center-of-mass

Masses to separation energies

Masses to excitation energies

Complex systems to simpler approximations

Most nuclear models represent some sort of paradigm shift (up to 50 orders simpler!)

Most simplifications of complex systematics of nuclear properties involve paradigm shifts.

Look for, exploit, paradigm shifts

Shell Model:	P.S. 1	2-body → 1-body Pot.
	P.S. 2	All nucleons → valence

^{91}Zr:

From incredibly complex situation of 91 particles interacting with strong and Coulomb forces → 90 + 1 particles and then → 1 !!!

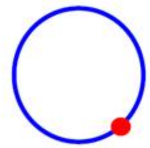
GROUND STATE J^{π} VALUES OF SOME ODD MASS NUCLEI

Z = 20	$3/2^+$ ^{37}Ca	$3/2^+$ ^{39}Ca	$7/2^-$ ^{41}Ca	$7/2^-$ ^{43}Ca	$7/2^-$ ^{45}Ca	$7/2^-$ ^{47}Ca	$3/2^-$ ^{49}Ca
Z = 40		$9/2^+$ ^{87}Zr	$9/2^+$ ^{89}Zr	$5/2^+$ ^{91}Zr	$5/2^+$ ^{93}Zr	$5/2^+$ ^{95}Zr	
Z = 39	$1/2^-$ ^{85}Y	$1/2^-$ ^{87}Y	$1/2^-$ ^{89}Y	$1/2^-$ ^{91}Y	$1/2^-$ ^{93}Y	$1/2^-$ ^{95}Y	$1/2^-$ ^{97}Y
Z = 41		$9/2^+$ ^{91}Nb	$9/2^+$ ^{93}Nb	$9/2^+$ ^{95}Nb	$9/2^+$ ^{97}Nb	$9/2^+$ ^{99}Nb	
N = 50	$3/2^-$ $^{85}_{35}Br$	$3/2^-$ $^{87}_{37}Rb$	$1/2^-$ $^{89}_{39}Y$	$9/2^+$ $^{91}_{41}Nb$	$9/2^+$ $^{93}_{43}Tc$	$9/2^+$ $^{95}_{45}Rh$	

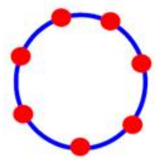
Particle-hole sym try
(basic ideas—it ca.. get more subtle)

Consider 1 particle in $f_{7/2}^{-}$ orbit

Then $J^{\pi} = j^{\pi} = 7/2^{-}$



Consider 7 particles in $f_{7/2}$ orbit



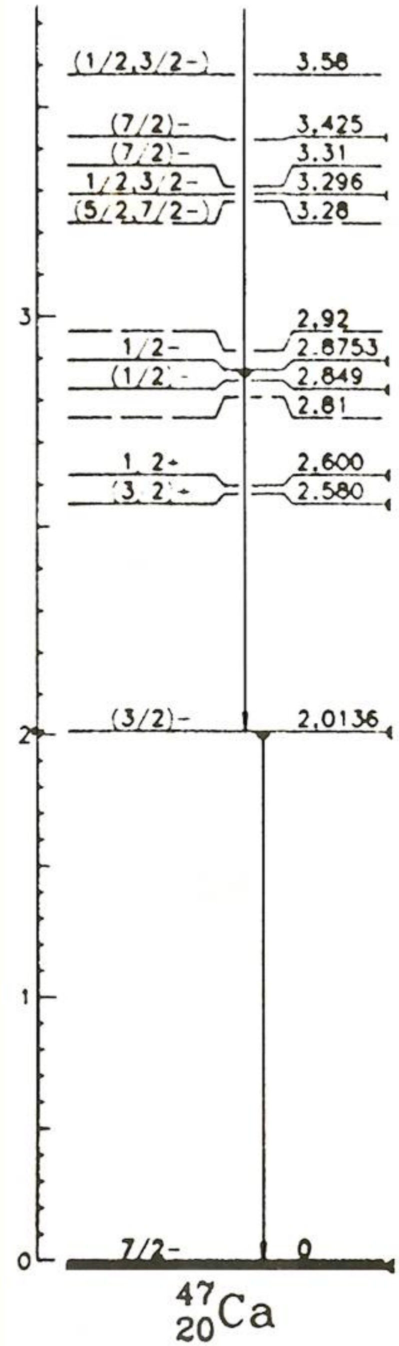
When shell is full (8 particles) $J = 0$

$$\therefore \quad \bar{J}_{\text{full shell}} = \bar{J}_{\text{full shell -1}} + \bar{7/2} = 0$$

$$\Rightarrow \quad J_{\text{full shell -1}} = 7/2$$

e.g.: $^{47}_{20}\text{Ca}_{27}$

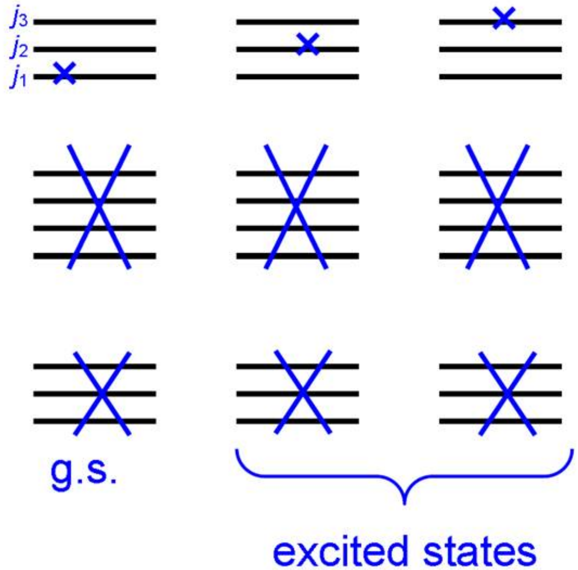
- 1 hole has same J as one particle



GROUND STATE J^{π} VALUES OF SOME ODD MASS NUCLEI

Z = 20	$3/2^+$ ^{37}Ca	$3/2^+$ ^{39}Ca	$7/2^-$ ^{41}Ca	$7/2^-$ ^{43}Ca	$7/2^-$ ^{45}Ca	$7/2^-$ ^{47}Ca	$3/2^-$ ^{49}Ca
Z = 40		$9/2^+$ ^{87}Zr	$9/2^+$ ^{89}Zr	$5/2^+$ ^{91}Zr	$5/2^+$ ^{93}Zr	$5/2^+$ ^{95}Zr	
Z = 39	$1/2^-$ ^{85}Y	$1/2^-$ ^{87}Y	$1/2^-$ ^{89}Y	$1/2^-$ ^{91}Y	$1/2^-$ ^{93}Y	$1/2^-$ ^{95}Y	$1/2^-$ ^{97}Y
Z = 41		$9/2^+$ ^{91}Nb	$9/2^+$ ^{93}Nb	$9/2^+$ ^{95}Nb	$9/2^+$ ^{97}Nb	$9/2^+$ ^{99}Nb	
N = 50	$3/2^-$ $^{85}_{35}Br$	$3/2^-$ $^{87}_{37}Rb$	$1/2^-$ $^{89}_{39}Y$	$9/2^+$ $^{91}_{41}Nb$	$9/2^+$ $^{93}_{43}Tc$	$9/2^+$ $^{95}_{45}Rh$	

nucleon



$E_{ex} = E_{sm}$ (all filled orbits)

$- E_{sm}$ (g.s.)

$$E_{ex} = E_{j_i} - E_{j_0}$$