Selected Topics in Nuclear Astrophysics

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Overview

I. A brief primer on stellar physics

II. Neutron stars and nuclear physics

III. Observing neutron stars in the wild

Basics of stellar physics

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Basics of stellar physics

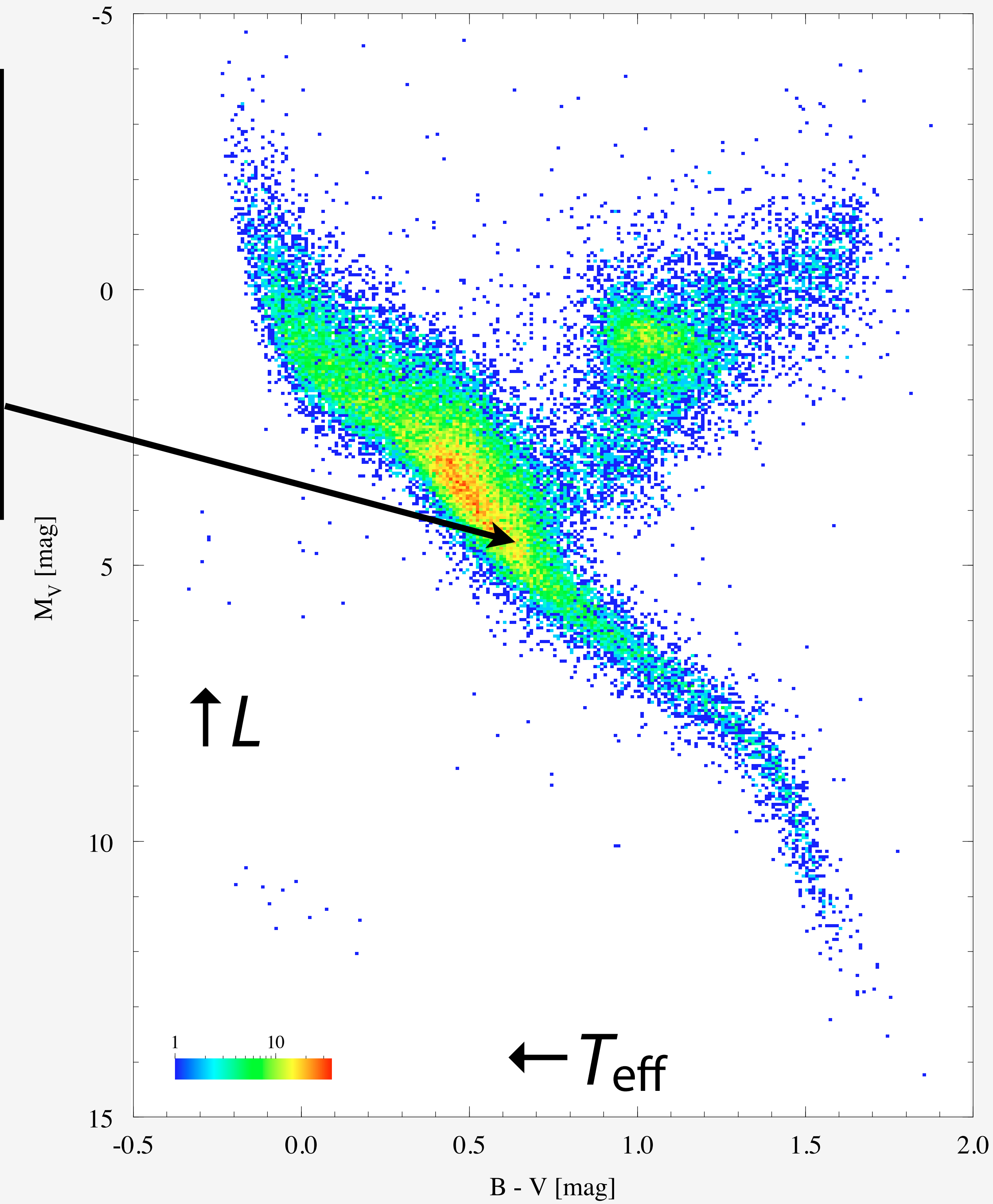
Deduce basic properties from mechanics

Role of heat transport

Evolution of star: pressure v. gravity

The local population as observed by *Hipparcos*

$$
\begin{aligned}
M_\odot &= 1.99 \times 10^{33}\ \mathrm{g} \\
R_\odot &= 6.96 \times 10^{10}\ \mathrm{cm} \\
L_\odot &= 3.86 \times 10^{33}\ \mathrm{erg\ s^{-1}} \\
T_{\mathrm{eff}} &= 5780\ \mathrm{K} \\
\tau_\odot &= 4.6\ \mathrm{Gyr.}
\end{aligned}
$$



NB. Magnitudes are a logarithmic flux (integrated over some wavelength range) scale:

$$m_2 - m_1 = -\frac{5}{2} \log \left(\frac{F_2}{F_1} \right) .$$

Backwards, and 5 magnitudes is a factor of 100 in flux

Hydrostatics: $F = ma$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{\mathrm{d}\Phi}{\mathrm{d}r} - \frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r}$$

$$\frac{U}{R/U} \sim \frac{U^2}{R} \qquad \sim \frac{GM}{R^2}$$

Dimensional analysis: terms on r.h.s. scale as U^2/R.
What are their relevant velocity scales for each term?

Hydrostatics: $F = ma$

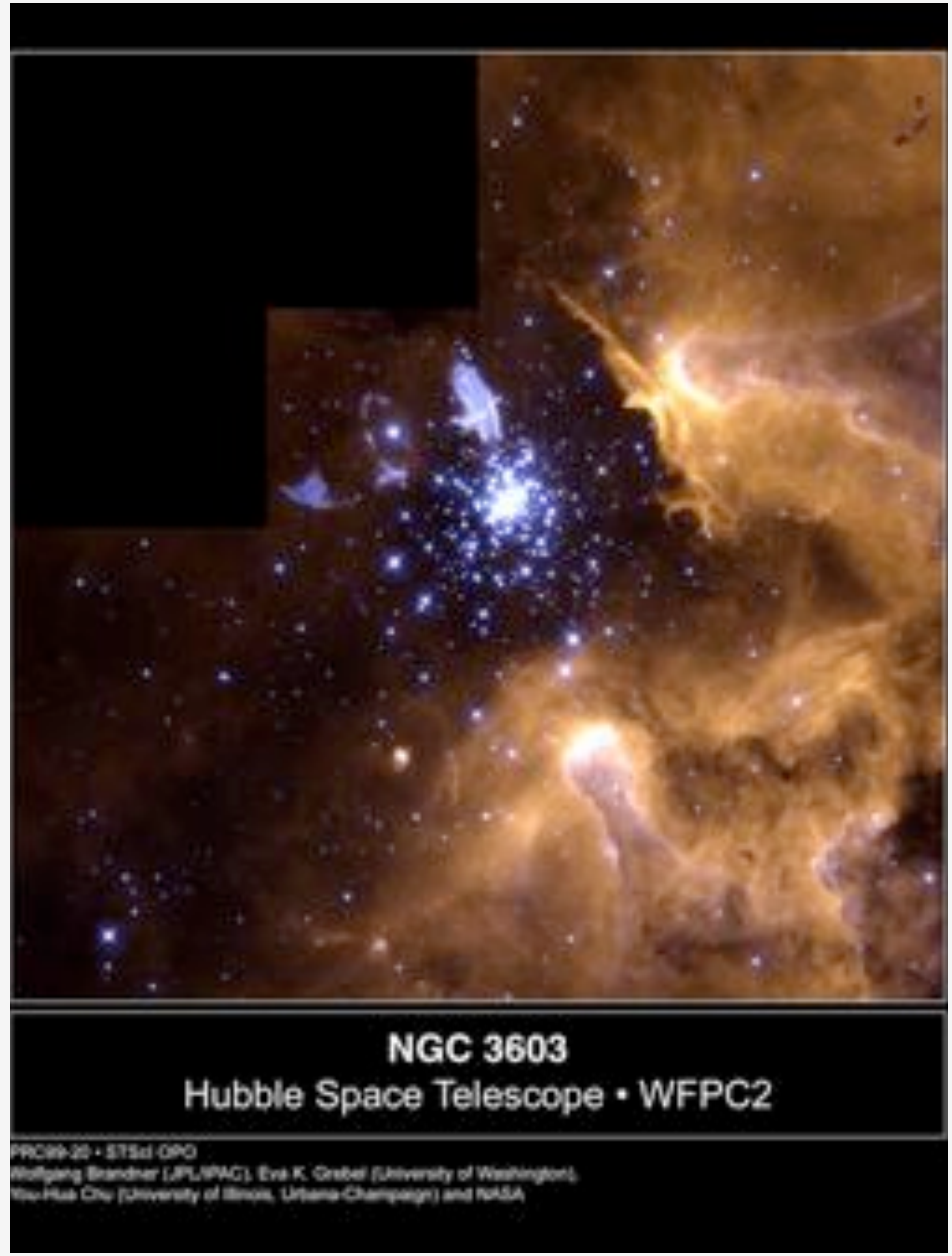
$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{\mathrm{d}\Phi}{\mathrm{d}r} - \frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r}$$

$$\frac{U}{R/U} \sim \frac{U^2}{R} \qquad \frac{GM}{R^2} \sim \frac{v_{\mathrm{esc}}^2}{R} \qquad \frac{c_s^2}{R} \sim \frac{kT}{m_{\mathrm{H}}}\frac{1}{R}$$

- L.H.S. typically very small; $c_s \sim v_{esc}$
- dynamical timescale $\tau_{\mathrm{dyn}} = (G\bar{\rho})^{-1/2}$ *this is ~ 1 hr for sun*
- "virial" scaling: $P \sim GM^2/R^4$, $\rho \sim M/R^3$
- Temperature at sun's center ~ GMm/R ~ keV ~ 10^7 K (assuming ideal gas!)

Star formation

Occurs when a portion of a dense cloud becomes unstable to collapse.



Contraction time of sun

When the star first begins to contract, it is too cold for nuclear reactions to occur. The surface is warm and radiates, so to conserve energy, the star must contract. How long would this take for a star like our sun?

Answer: time ~ total energy/luminosity,

$$\frac{GM^2}{RL_\odot} \approx 30 \text{ Myr}.$$

Question: What happens to T_{center} during this contraction? What happens to ρ_{center}?

Heat transport: thermal diffusion

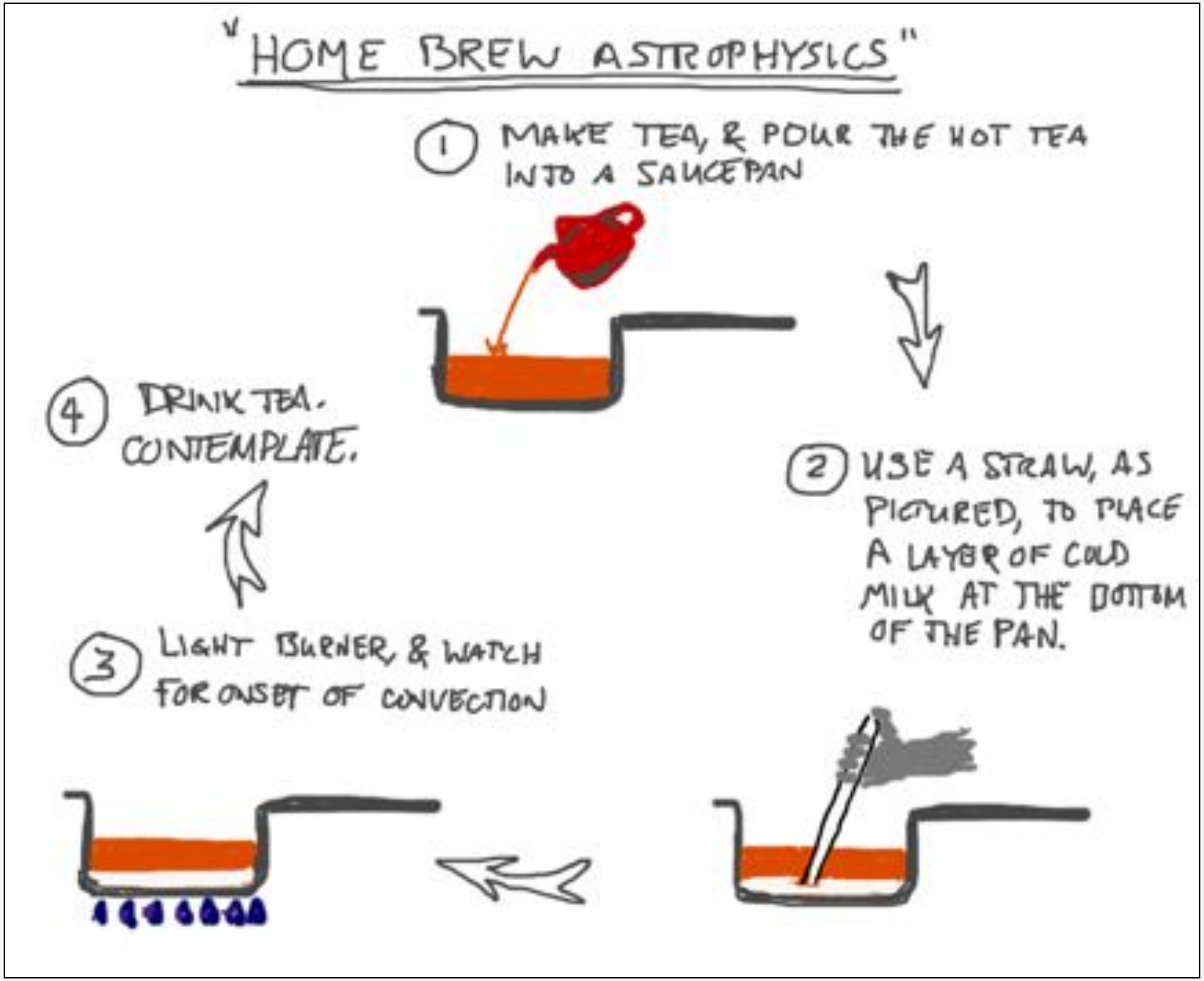
The mean-free path of photons much less than the stellar radius. As a result, photons must random-walk to the surface: heat transport is a diffusive process, with

$$F = -\frac{1}{3} v \ell \frac{\mathrm{d}U}{\mathrm{d}r}$$

For photons,

$$F = -\frac{1}{3} \frac{c}{\rho\kappa} \frac{\mathrm{d}aT^4}{\mathrm{d}r}$$

with $\frac{1}{\rho\kappa} = \frac{1}{n\sigma} = \ell$.



Heat transport: convection—onset when $dS/dr < 0$

What sets the radius?

The entropy per particle of an ideal gas is

$$s = k\left\{\frac{3}{2}\ln\left(\frac{P}{\rho^{5/3}}\right) + \text{const.}\right\}.$$

Suppose we have a star the interior of which lies along an adiabat. What happens to the radius as heat is added? What happens to the central temperature as heat is added?

$$s = \frac{3k}{2} \ln\left(\frac{P}{\rho^{5/3}}\right) + s_0$$

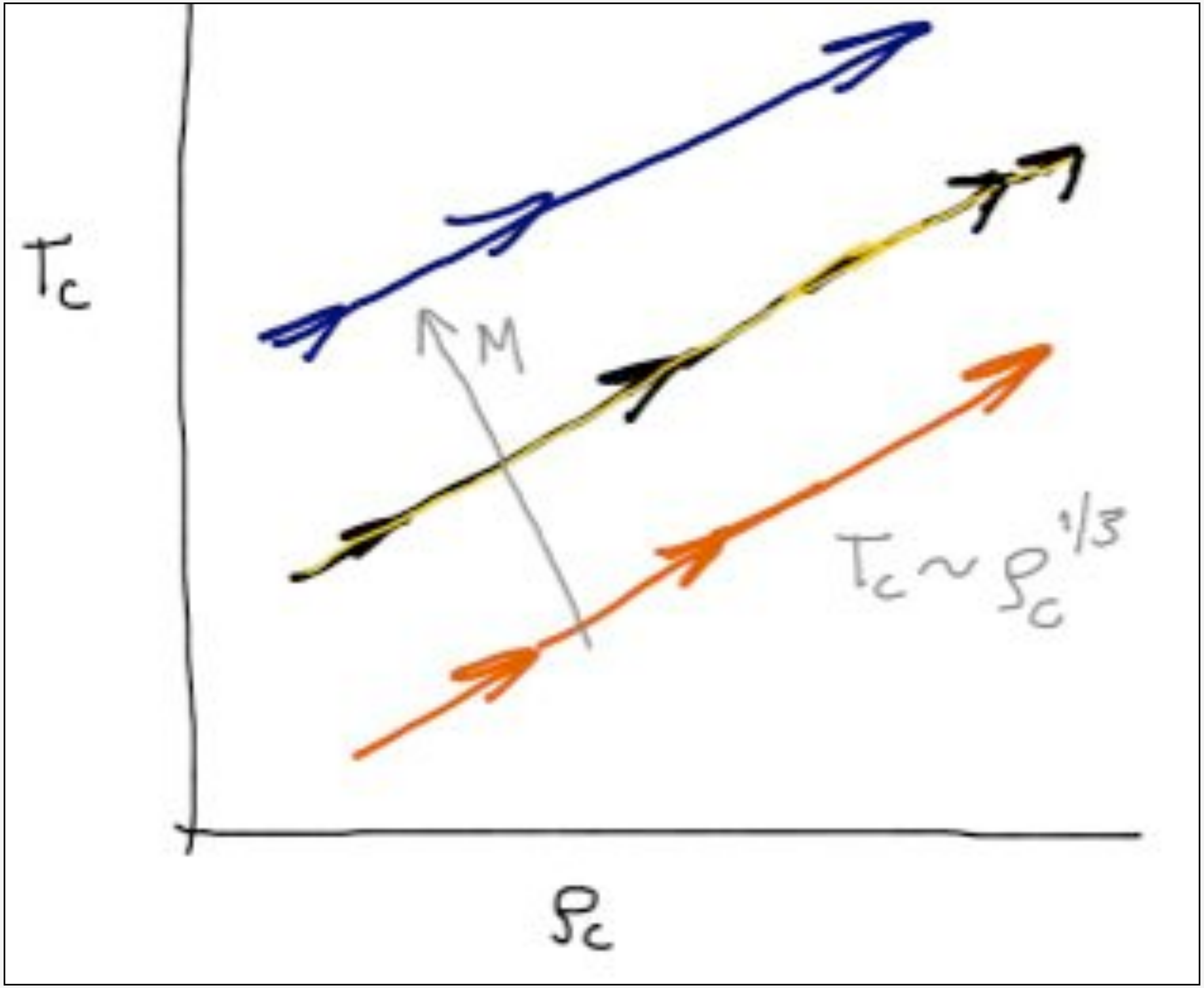
$$P \sim \frac{M^2}{R^4} \qquad \rho \sim \frac{M}{R^3}$$

$$\frac{P}{\rho^{5/3}} \sim M^{2-5/3} \; R^{-4+5}$$

$$\sim M^{1/3} R$$

$$R \sim \exp(\cdots[S - S_0])$$

$$kT \sim \frac{GMm_p}{R}, \quad R\uparrow, \; T_c \downarrow$$



Central temperature, density during contraction

Onset of degeneracy

The gas becomes degenerate when the electrons are about one wavelength apart,

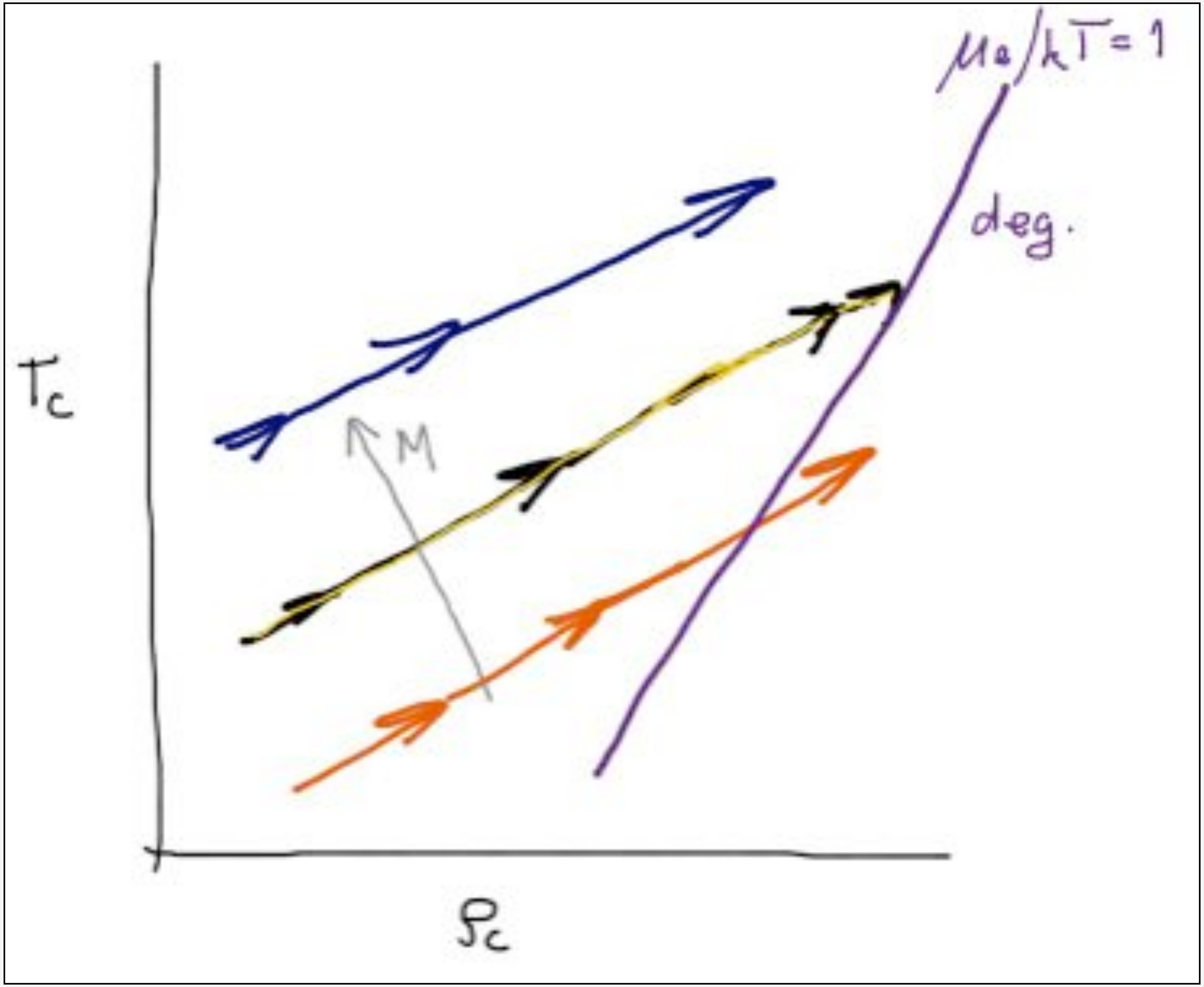
$$\Delta x \sim \frac{\hbar}{\Delta p} \sim \frac{\hbar}{\sqrt{mkT}},$$

$$n_e \approx \left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2}.$$

Cold degenerate objects have a definite mass-radius relation for a given composition. The pressure is

$$P = \frac{2}{5} n_e \mu_e = \frac{2}{5}\frac{(3\pi^2)^{2/3}\hbar^2}{2m_e}\left(\frac{Y_e\rho}{m_u}\right)^{5/3},$$

so $R \sim M^{-1/3}$ from hydrostatic balance and continuity.



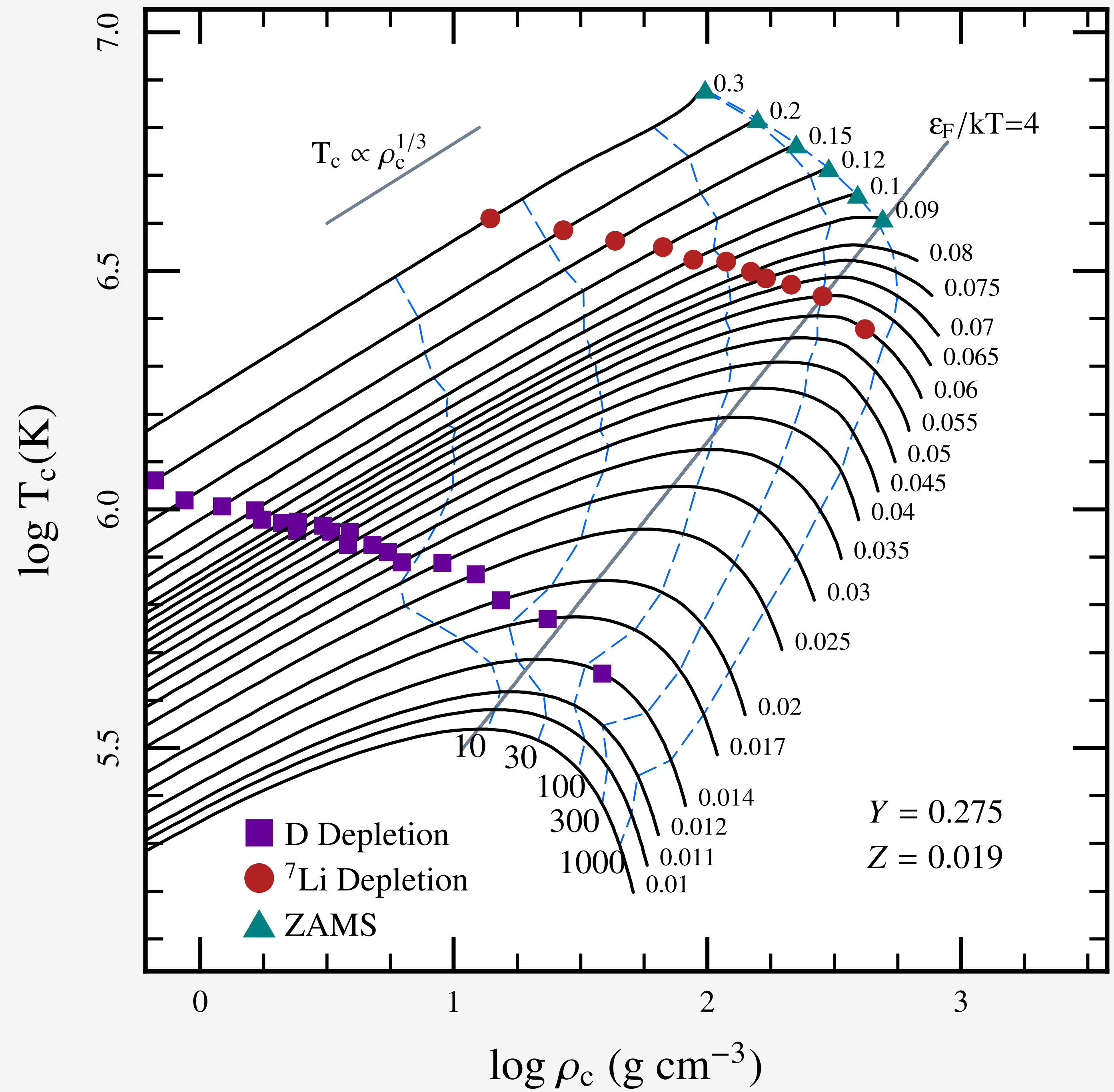
What happens when EOS becomes degenerate?

Low-mass stellar tracks with the open-source *MESA* stellar evolution code (Paxton et al. 2010)

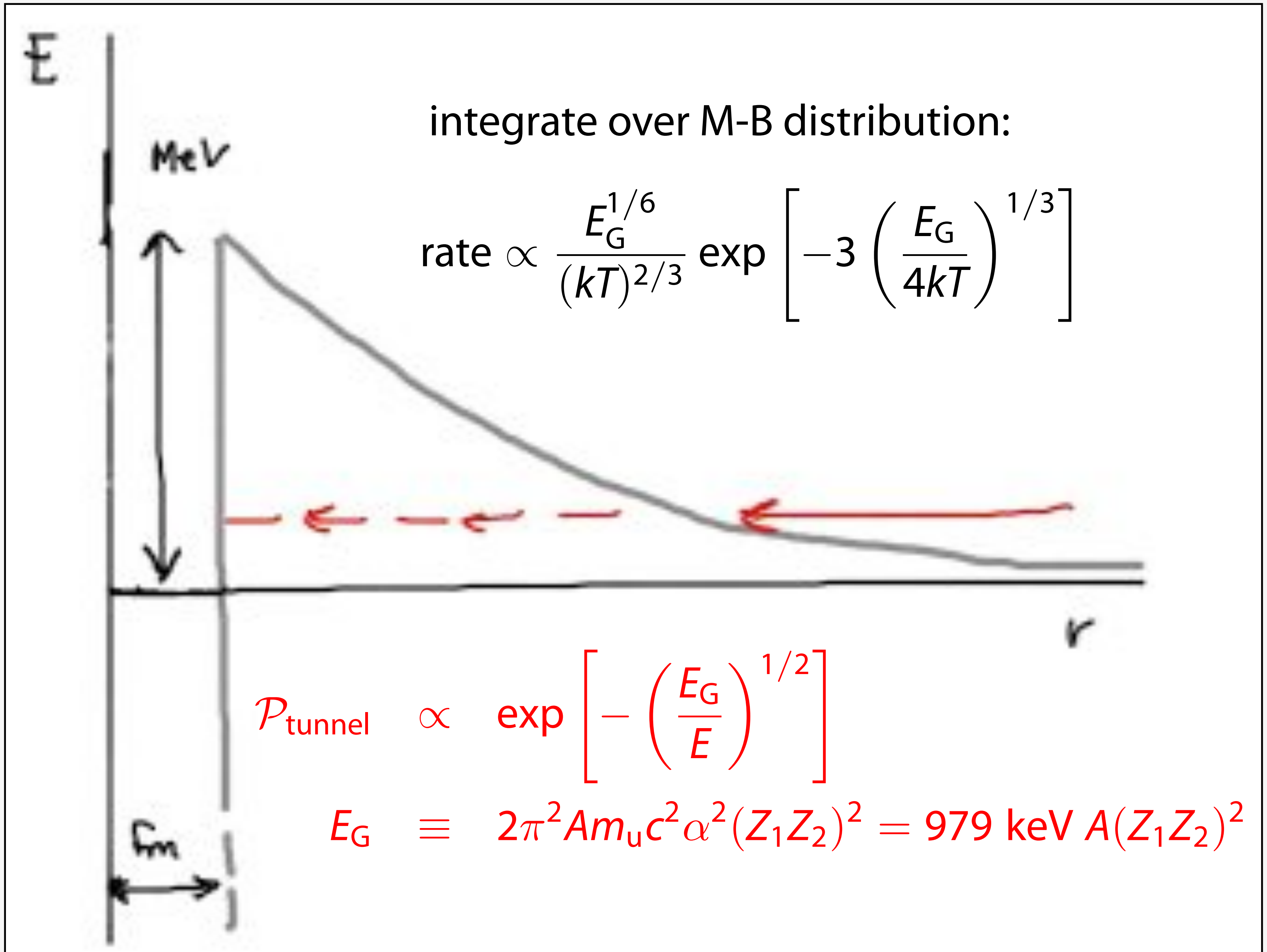
Notice temperature, density scalings

"ZAMS"—Zero-Age Main-Sequence—indicates ignition of p+p→d leading to synthesis of ^{4}He.

Why are d, ^{7}Li depleted so early?



Nuclear reactions are strongly temperature-sensitive at stellar energies



integrate over M-B distribution:

$$\text{rate} \propto \frac{E_G^{1/6}}{(kT)^{2/3}} \exp\left[-3\left(\frac{E_G}{4kT}\right)^{1/3}\right]$$

$$\mathcal{P}_{\text{tunnel}} \propto \exp\left[-\left(\frac{E_G}{E}\right)^{1/2}\right]$$

$$E_G \equiv 2\pi^2 A m_u c^2 \alpha^2 (Z_1 Z_2)^2 = 979\ \text{keV}\ A(Z_1 Z_2)^2$$

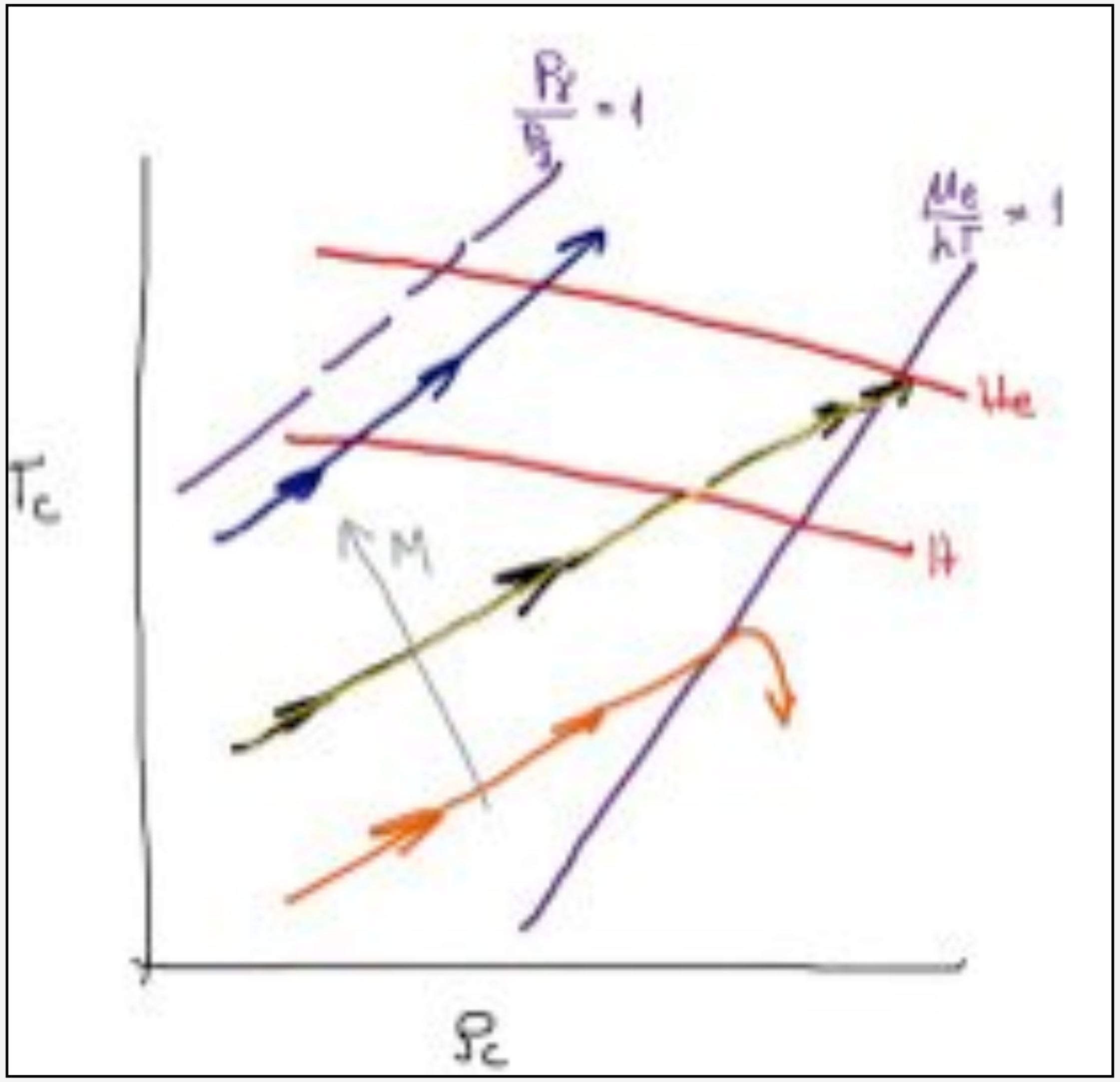
strong T-sensitivity:

$$\left.\frac{\partial \ln r(p + {}^{12}\text{C})}{\partial \ln T}\right|_{T=10^7\ \text{K}} \approx 21$$

rates "turn on" at certain temperature

Nucleosynthesis set by central temperature reached before degeneracy.

- Larger *Z* requires higher temperature to overcome Coulomb barrier
- E.g., solar-like stars can ignite $3\,{}^{4}He \rightarrow {}^{12}C$, but not ${}^{12}C \rightarrow {}^{12}C$.



Exercise

Stars more massive than the sun consume H via the CNO cycle. The rate-limiting step is p + ^{14}N, which is very temperature sensitive. Use this fact and the scalings from hydrostatic balance to determine how the central pressure changes as the mass increases.

Why are stars stable (when burning hydrogen)?

Nuclear reactions are very temperature-sensitive. What prevents thermonuclear runaway? Let's look at heat balance. Normally we would write something like

$$T\frac{\partial S}{\partial T}\bigg|_{?} = \varepsilon_{\mathrm{nuc}} - \frac{1}{\rho}\nabla \cdot F,$$

but what is held constant?

"Gravithermal" specific heat

If we perturb the star, the mass is fixed, so the heat term becomes

$$T\frac{\partial S}{\partial T}\bigg|_M = C_P\left[1 - \left(\frac{\partial T}{\partial P}\right)_S \left(\frac{\partial P}{\partial T}\right)_M\right]$$

Structure of star: $P(M, R)$, $\rho(M, R)$. EOS: $P(\rho,T)$

“Gravithermal” specific heat-2

Equation of state: $\ln P = \chi_\rho \ln \rho + \chi_T \ln T$

$$\left(\frac{\partial \ln P}{\partial \ln R}\right)_M = -4 \qquad \left(\frac{\partial \ln \rho}{\partial \ln R}\right)_M = -3$$

hydrostatic balance

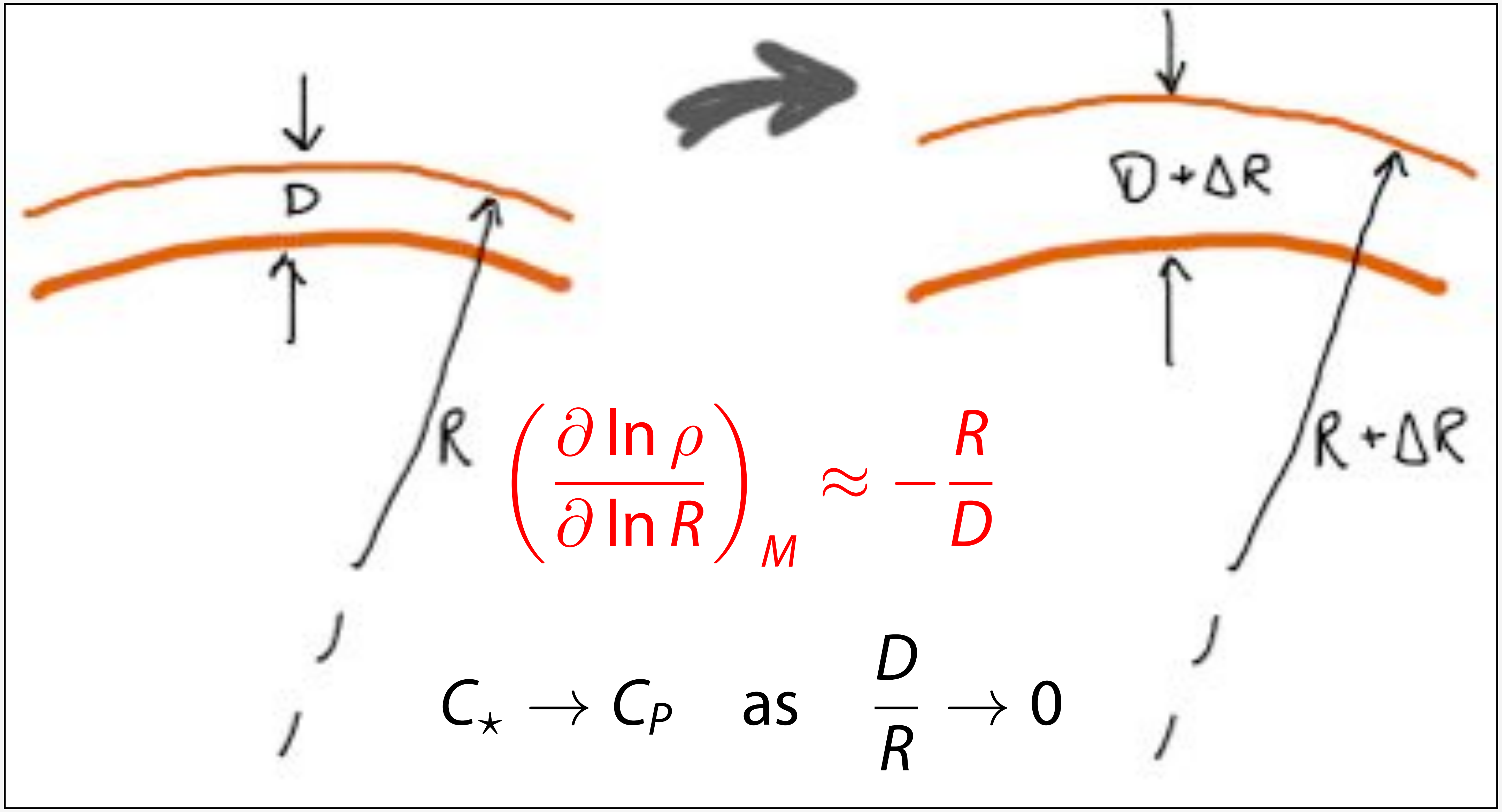
continuity

$$C_\star = T\frac{\partial S}{\partial T}\bigg|_M = C_P\left[1 - \left(\frac{\partial \ln T}{\partial \ln P}\right)_S \frac{4\chi_T}{4 - 3\chi_\rho}\right]$$

ideal gas: $\chi_T = \chi_\rho = 1$ and $C_\star < 0$

What happens if gas is degenerate?

What happens if burning is in a thin layer?



$$\left(\frac{\partial \ln \rho}{\partial \ln R}\right)_M \approx -\frac{R}{D}$$

$$C_\star \to C_P \quad \text{as} \quad \frac{D}{R} \to 0$$

So if heating beats cooling, the layer is thermally unstable.

Exercise

What is the thermal stability of the following scenarios?

1. $3\,^4\text{He} \rightarrow {}^{12}\text{C}$ in a degenerate stellar core
2. $p + {}^{12}\text{C}$ in a thin shell surrounding stellar core
3. ${}^{12}\text{C} + {}^{12}\text{C}$ in a thin layer on a neutron star
4. Neutrino cooling ($\ell \gg R$) in a thin layer on a neutron star

Terminus, $M < 8\text{–}10\ M_{sun}$

The cores of low-mass stars become degenerate and composed of C, O (Ne, Mg). Strong luminosity of H, He burning shells add fresh C to the core and expel outer envelope.

NGC 2440; note hot core visible at center