

Experimental results on nucleon structure

Lecture II

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Outline

- 1 Course literature
- 2 Introduction
- 3 Nucleon elastic form factors
 - Basic formulae
 - Form factor measurements
 - Radiative corrections
- 4 Parton structure of the nucleon
 - Feynman parton model
 - Partons vs quarks
 - Introducing gluons
 - Parton distribution functions
 - EMC effect
 - Fragmentation functions
 - Sum rules
- 5 “Forward” physics
 - Phenomena at low x
 - Diffraction

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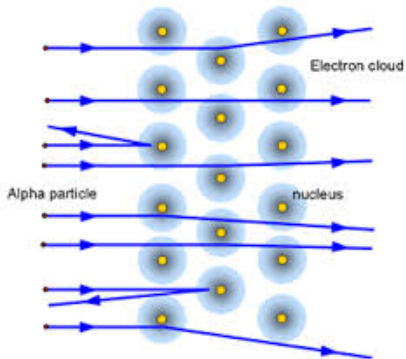
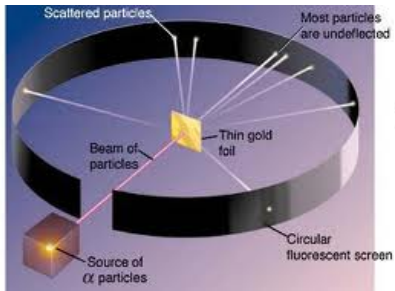
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Rutherford scattering

In 1910 – 1911, Ernest Rutherford + students: H. Geiger and E. Marsden

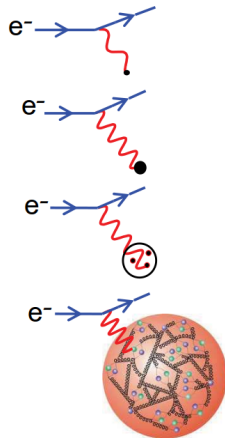


First exploration of atomic structure:

small, massive, positive nucleus and negative charge around it

Probing the structure of the proton

- At **very low** electron energies $\lambda \gg r_p$:
the scattering is equivalent to that from a
"point-like" spin-less object
- At **low** electron energies $\lambda \sim r_p$:
the scattering is equivalent to that from a
extended charged object
- At **high** electron energies $\lambda < r_p$:
the wavelength is sufficiently short to
resolve sub-structure. Scattering from
constituent quarks
- At **very high** electron energies $\lambda \ll r_p$:
the proton appears to be a sea of
quarks and gluons.



From: M.A. Thomson, Michaelmas Term 2011

Elastic electron – nucleon scattering

- **Rutherford scattering:** a particle ze of E scatters off Ze at rest and changes its momentum vector by ϑ :

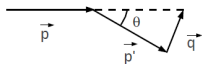
$$\left(\frac{d\sigma}{d\Omega}\right)_R = \frac{(zeZe)^2}{(4\pi\epsilon_0)^2(4E)^2 \sin^4 \frac{\vartheta}{2}}$$

This formula is **nonrelativistic**; target recoil is neglected (= target is very heavy), i.e.

$$E = E', \quad |\vec{p}| = |\vec{p}'|, \quad |\vec{q}| = |\vec{k}| = 2|\vec{p}| \sin \vartheta/2$$

A **relativistic formula** ($E \approx |\vec{p}|c$) and $z = 1$:

$$\left(\frac{d\sigma}{d\Omega}\right)_R = \frac{Z^2\alpha^2(\hbar c)^2}{4E^2 \sin^4 \frac{\vartheta}{2}} \quad (4)$$



At relativistic energies, Rutherford formula is modified by spin effects.

- If electron **relativistic and its spin** included \implies Mott cross-section (still no recoil):

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott}^* = \left(\frac{d\sigma}{d\Omega}\right)_R \cdot \left(1 - \beta^2 \sin^2 \frac{\vartheta}{2}\right) = \frac{4Z^2\alpha^2(\hbar c)^2 E'^2}{|\vec{q}c|^4} \cos^2 \frac{\vartheta}{2} \quad (5)$$

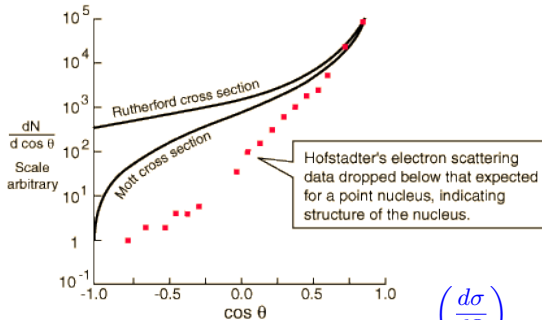
(an asterisk means the recoil of the nucleus is neglected)

For $\beta \rightarrow 1$, $\vartheta = \pi$ suppressed on spinless target

(a consequence of a helicity, h , conservation, $h = \vec{s}\vec{p}/|\vec{s}||\vec{p}|$).

Elastic electron – nucleon scattering...cont'd

- Mott expression agrees with the data for $|\vec{q}| \rightarrow 0 (\vartheta \rightarrow 0)$ but at higher $|\vec{q}|$ experimental cross-sections are smaller: **a form factor!**



$$\left(\frac{d\sigma}{d\Omega}\right)_{exp} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}^* \cdot |F(\vec{q}^2)|^2 \quad (6)$$

(for spherically symmetric systems, the form factor depends on \vec{q} only!

- Determination of a form factor: measure of $d\sigma/d\Omega$ at fixed E and different ϑ (= various $|\vec{q}|$) and divide by the Mott cross-section.

Elastic electron – nucleon scattering...cont'd

- First measurements at SLAC in early 50-ties, $E_e = 0.5$ GeV.

- **Define** a charge distribution function f by $\rho(\vec{r}) = Ze f(\vec{r})$ so that $\int f(\vec{r}) d^3r = 1$. Then the **form factor**:

$$F(\vec{q}^2) = \int e^{i\vec{q}\vec{r}/\hbar} f(\vec{r}) d^3r \quad (7)$$

but only under conditions: no recoil, Ze small (or $Z\alpha \ll 1$)!

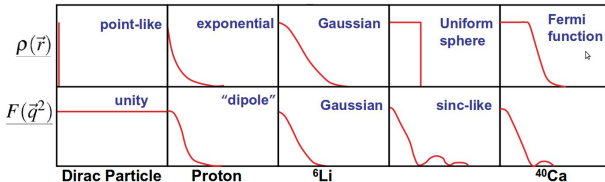
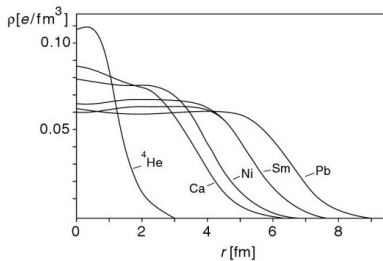
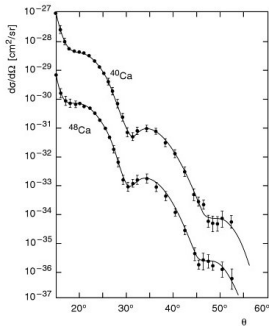
- For spherically symmetric cases f depends only on $r = |\vec{r}|$. Then

$$F(\vec{q}^2) = 4\pi \int f(r) \frac{\sin(|\vec{q}|r/\hbar)}{|\vec{q}|r/\hbar} r^2 dr \quad 1 = 4\pi \int_0^\infty f(r) r^2 dr$$

- The radial charge distribution, $f(r)$ cannot be determined from the inverse Fourier transform of $F(\vec{q}^2)$ due to limited interval of measured values of $|\vec{q}|$.
- Thus a **procedure of finding $F(\vec{q}^2)$** : choose parameterisation of $f(r)$, calculate $F(\vec{q}^2)$ and vary its parameters to get best fit to data.
- Observe:

$$\langle r^2 \rangle = 4\pi \int_0^\infty r^2 f(r) dr = -6\hbar^2 \left. \frac{dF(\vec{q}^2)}{d\vec{q}^2} \right|_{\vec{q}^2=0} \quad (8)$$

Examples of form factors and charge densities



From: M.A. Thomson, Michaelmas Term 2011

Form factors of the nucleons

- Studies of nucleon structure ($r \sim 0.8$ fm) demand $E \gtrsim 1$ GeV; thus nucleon recoil can no longer be neglected.

- With recoil:

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}^* \cdot \frac{E'}{E} \quad (9)$$

Also: a $Q^2 = -q^2 = 4EE' \sin^2 \vartheta/2$ needed instead of \vec{q}^2 in the Mott cross-section

- Apart of e-N Coulomb interaction, now also electron current - nucleon's magnetic moment.

Reminder: a pointlike, charged particle of spin 1/2 has a magnetic moment: $\mu = g \frac{e \hbar}{2M} \frac{1}{2}$ ($g=2$ from relativistic Q.M.). [Magnetic interaction associated with a flip of the nucleon spin.](#)

- Scattering at $\vartheta = 0$ not consistent with helicity (and angular momentum) conservation; scattering at $\vartheta = \pi$ is preferred. Thus:

$$\left(\frac{d\sigma}{d\Omega}\right)_{point;spin1/2} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left[1 + 2\tau \tan^2 \frac{\vartheta}{2} \right], \quad \tau = \frac{Q^2}{4M^2} \quad (10)$$

New, magnetic term (above) is large at large Q^2 and ϑ .

Form factors of the nucleons...cont'd

- Anomalous magnetic moments for nucleons:

$$\mu_p = +2.79... \cdot \frac{e\hbar}{2M} = +2.79... \cdot \mu_N \quad \mu_n = -1.91... \cdot \frac{e\hbar}{2M} = -1.91... \cdot \mu_N \quad (11)$$

where $\mu_N = 3.1525 \cdot 10^{-14}$ MeV/T = nuclear magneton.

- Charge and current distributions described by **two** (Sachs) form factors (**Rosenbluth**):

$$\left(\frac{d\sigma}{d\Omega} \right) = \left(\frac{d\sigma}{d\Omega} \right)_{Mott} \cdot \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1 + \tau} + 2\tau G_M^2(Q^2) \tan^2 \frac{\vartheta}{2} \right] \quad (12)$$

Here: $G_E(Q^2)$ and $G_M(Q^2)$ are the **electric and magnetic form factors**.

At very low Q^2 , $G_E(Q^2)$ and $G_M(Q^2)$ are Fourier transforms of the charge and magnetization current densities inside the nucleon.

- At $Q^2 \rightarrow 0$:

$$\begin{aligned} G_E^p &= 1, & G_E^n &= 0 \\ G_M^p &= 2.79, & G_M^n &= -1.91 \end{aligned} \quad (13)$$

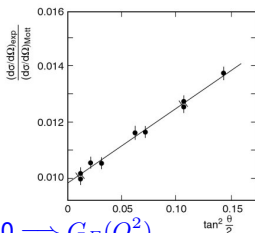
- How do we measure (separate) $G_E(Q^2)$ and $G_M(Q^2)$?

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Measurement of form factors – Rosenbluth method

- Independent measurement of $G_E(Q^2)$, $G_M(Q^2)$ needed. This is done at fixed Q^2 and different ϑ (or energies E). The measured cross-section is then divided by σ_{Mott} . Example of results:



Slope of the line yields $G_M(Q^2)$, intercept at $\vartheta = 0 \Rightarrow G_E(Q^2)$

- For a long time it seemed that:

$$G_E^p(Q^2) = \frac{G_M^p(Q^2)}{2.79} = \frac{G_M^n(Q^2)}{-1.91} = G_D(Q^2) = \left(1 + \frac{Q^2}{0.71(\text{GeV}/c)^2}\right)^{-2}$$

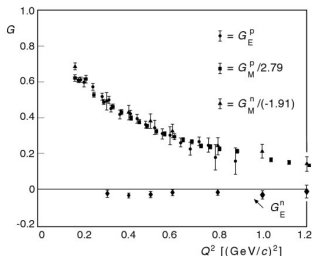
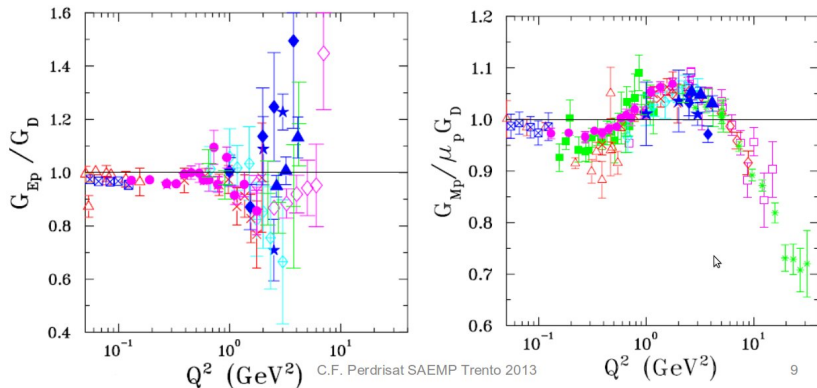


Fig. from the book of B.Povh et al.

Measurement of form factors – Rosenbluth method...cont'd

All published Rosenbluth separation data for the proton:

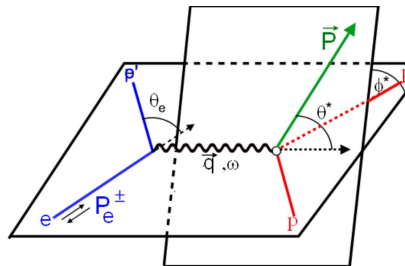
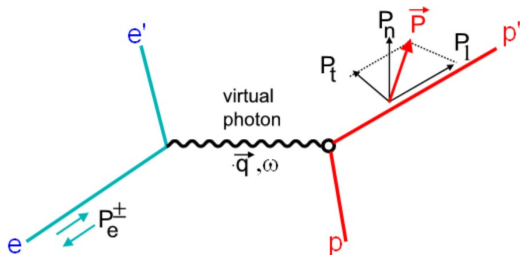


- At $Q^2 \gtrsim 1$ GeV², G_E^p inaccurately determined...
- ...but G_M^p errors small up to $Q^2 \sim 30$ GeV².
- Neutron data (from elastic and break-up *ed* scattering) of poorer quality.
- Old paradigm (until 1998 → JLAB): proton from factors similar, and close to G_D .

Measurement of form factors – polarisation transfer method

1998: a new method of form factors determination (coincided with opening of the JLAB): polarisation observables instead of cross-sections.

$$1) \vec{e}p \rightarrow e\vec{p} \quad \text{or} \quad 2) \vec{e}\vec{p} \rightarrow ep$$



www.scholarpedia.org/article/Nuclear_Form_factors

Measurement of form factors – polarisation transfer...cont'd

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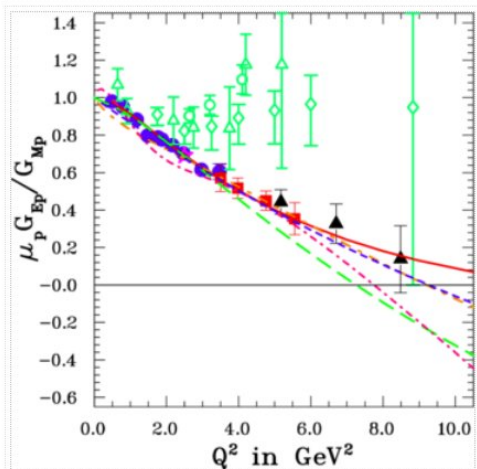
- Polarisation of the recoil proton contains terms proportional to $G_E^p G_M^p$ so that G_E^p may be determined even when it is small. Also radiative corrections minimised (polarisation observables are ratios of cross sections).
- In reaction (1): measurement of 2 components of the proton polarisation, e.g. longitudinal (P_l) and transverse (P_t) to the proton momentum in the scattering plane.
- If only polarisations measured in reaction (1) then only

$$\frac{G_E^p}{G_M^p} = -\frac{P_t}{P_l} \frac{(E + E')}{2M} \tan \frac{\vartheta}{2}$$

determined; separation of form factors need cross-section measurements.

- Independently of beam polarisation, a small normal component, P_n , is introduced by a double-photon exchange.

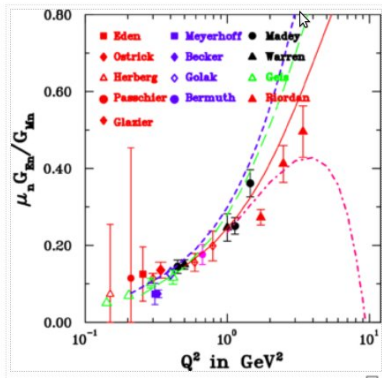
Measurement of form factors – polarisation method results



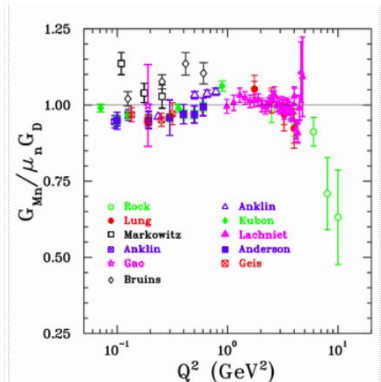
Green points - Rosenbluth method; other colours - recoil polarisation results.

www.scholarpedia.org/article/Nuclear_Form_factors

Measurement of form factors – results for neutron



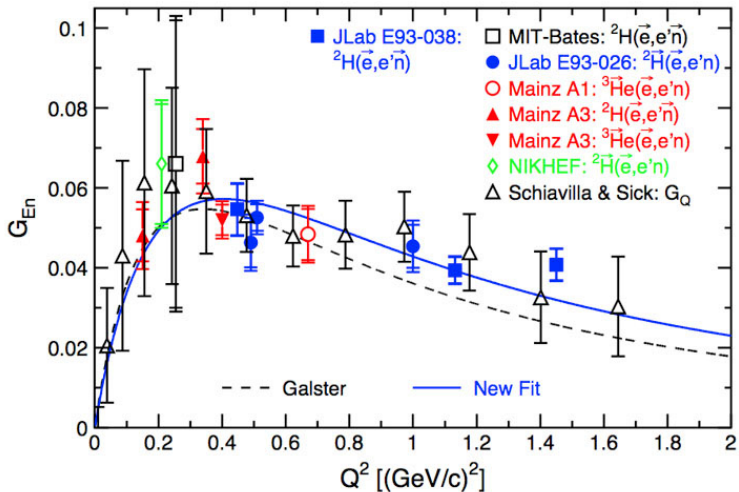
Double polarisation measurements



Cross-section measurements

www.scholarpedia.org/article/Nuclear_Form_factors

Measurement of form factors – results for neutron,...cont'd

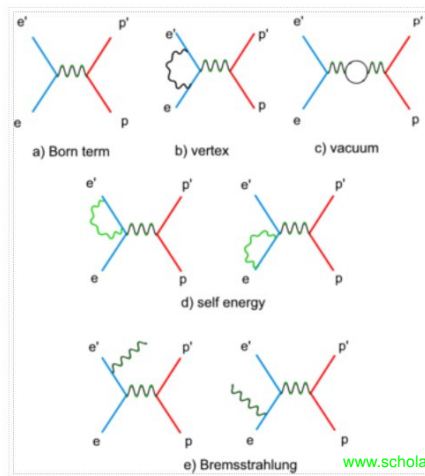


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Corrections to data

Lowest order QED corrections to data (“radiative corrections”):

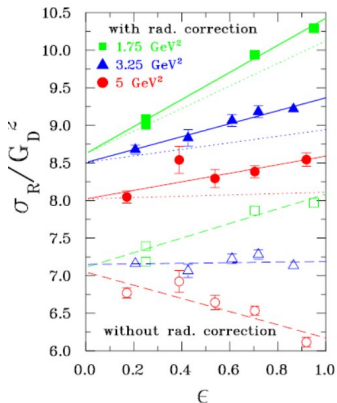


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Change of paradigm ???

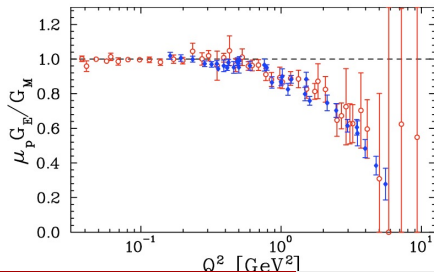
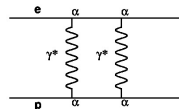
- Which approach is correct?
- And what is a reason of discrepancy?

$$R = \mu \frac{G_E}{G_M} \quad \text{vs} \quad \epsilon = \left[1 + 2(1 + \tau) \tan^2 \frac{\vartheta}{2} \right]^{-1}$$



Change of paradigm ???...cont'd

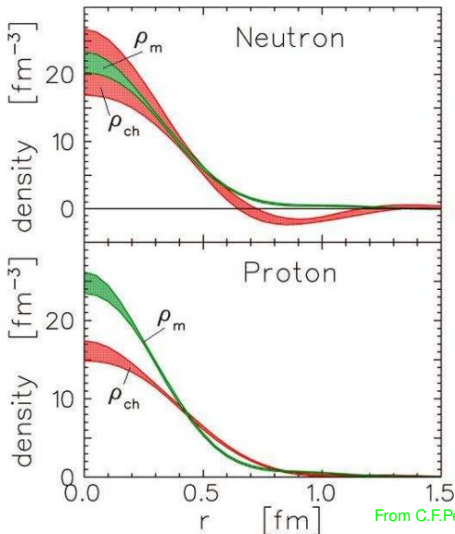
- Which approach is correct?
- And what is a reason of discrepancy? **Missing physics?**
- **Most probable culprit: neglecting 2γ exchange in radiative corrections!**
- Rosenbluth method: σ very sensitive to ϑ dependence \implies dramatic effect; polarisation (ratio) method: few percent effect.
- Results from both methods agree if 2γ exchange contribution accounted for:



red – Rosenbluth method
blue – polarisation transfer method

J. Arrington et al., Phys.Rev. C76 (2007) 035205

From form factors to charge/magnetisation densities

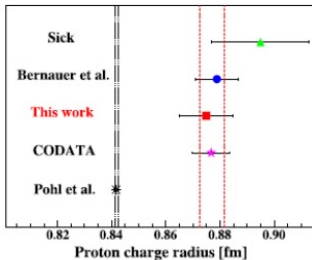


neutron charge distr.
multiplied by a factor of 6 !

From C.F.Perdrisat *et al.*, Prog.Part.Nucl.Phys. 59 (2007) 694

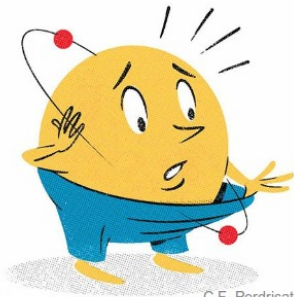
Proton Charge Radius Puzzle

Mainz
 JLab
 Mostly Hydrogen
 Lamb shift
 Muonic Hydrogen
 Lamb shift



The figure is from X. Zhan et al.,
 PLB 705, 59 (2011)

Dotted red lines combined CODATA,
 Bernauer (Mainz).



From the New York Times, July
 13, 2010.

"For a Proton, a Little Off the Top (or
 Side) Could Be Big Trouble"

It went from 0.8768 ± 0.0069 fm
 to 0.8418 ± 0.0007 fm.

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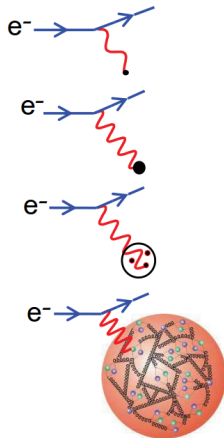
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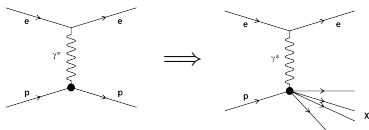
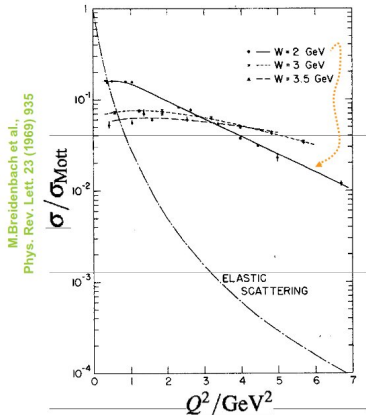
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Towards inelastic electron – nucleon scattering

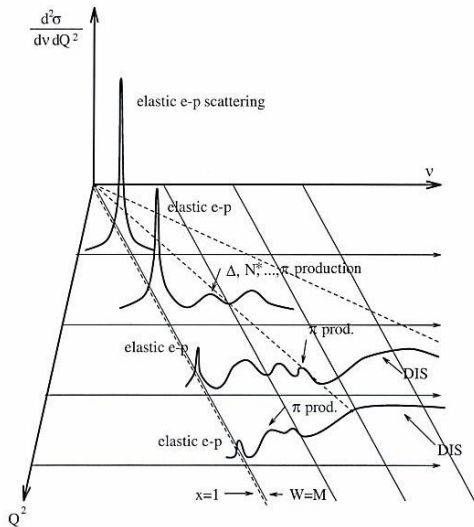
- At large scattering angles ϑ (i.e. large Q^2 or large ν): $F(Q^2) \rightarrow 0$ and **inelastic** scattering becomes more probable than the elastic.
- Now $Q^2 \neq 2M\nu$ (or $x \neq 1$) or: $Q^2 = M^2 + 2M\nu - W^2$ and a second variable, apart of Q^2 is needed, e.g. ν or x .



Scattering from point-like components
in the proton!

From: M.A. Thomson, Michaelmas Term 2011

Towards inelastic electron – nucleon scattering, ...cont'd



Radial, broken lines: $x = \text{const.}$
 Parallel, continuous lines: $W = \text{const.}$

Low x – large parton (gluon) densities.
 Low Q^2 – nonperturbative effects.

DIS = Deep Inelastic Scattering
 (large Q^2, ν)

Inelastic electron – nucleon scattering

From Eq. (12):

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1 + \tau} + 2\tau G_M^2(Q^2) \tan^2 \frac{\vartheta}{2} \right] \quad (14)$$

But

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{dQ^2} \cdot \frac{EE'}{\pi}$$

Then:

$$\left(\frac{d\sigma}{dQ^2}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \frac{\pi}{EE'} \cdot \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1 + \tau} + 2\tau G_M^2(Q^2) \tan^2 \frac{\vartheta}{2} \right] \quad (15)$$

Therefore the result in Eq. (12) may be written as:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[\left(1 - y - \frac{M^2 y^2}{Q^2}\right) f_2(Q^2) + \frac{1}{2} y^2 f_1(Q^2) \right] \quad (16)$$

This may be compared with an inelastic cross-section:

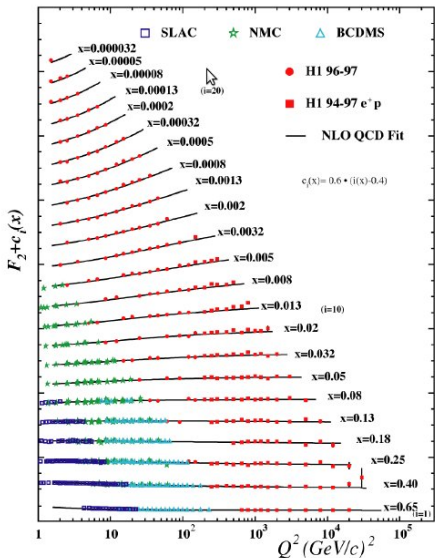
$$\frac{d^2\sigma}{dQ^2 dx} = \frac{4\pi\alpha^2}{Q^4} \left[\left(1 - y - \frac{M^2 y^2}{Q^2}\right) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2) \right] \quad (17)$$

where $y = \nu/E$

Structure functions $F_1(x, Q^2)$ and $F_2(x, Q^2)$

- Instead of two elastic form factors, $G_E(Q^2), G_M(Q^2)$ we have two structure functions $F_1(x, Q^2), F_2(x, Q^2)$.
- As the form factors, the structure functions cannot be obtained from theory; must be **measured**.
- Experimentally: both F_1 and F_2 are only weakly dependent on Q^2 .
- To determine F_1 and F_2 and for a given x and Q^2 need measurements of the differential cross section at several different scattering angles and incoming electron beam energies.

Structure functions,... cont'd



Bjorken scaling hypothesis

- If leptons scatter from point-like components then structure functions cannot depend on any dimensioned variable, e.g. Q^2 or ν .
- Bjorken: if for $Q^2 \rightarrow \infty$ and $\nu \rightarrow \infty$, $F_2(Q^2, \nu)$ is finite then it may depend only on dimensionless, finite ratio of these variables, i.e. on $x = \frac{Q^2}{2M\nu}$.
This is called **scale invariance** or “**scaling**”.
- Scaling holds already for $Q^2 \approx \text{few } M^2$ or $\sim 1 \text{ GeV}^2 \dots$
- ...but it is slightly ($\sim 10\%$) violated, especially at low x .
- SLAC experiments 1967 – 1973; luckily run at $x \sim 0.2$.
- Physics interpretation of scaling
 \implies **Feynman's parton model (1969)**.

Feynman parton model

- Assume a coordinate system where a proton target has ∞ momentum. There: $M \sim 0$ and all 4-momenta: $P = (p, 0, 0, p)$.
- Every parton in a proton has 4-momentum uP where $0 < u < 1$.
- At large P , masses (m_p) and \perp momenta of partons are ≈ 0 .
- Thus a proton \equiv a parallel stream of partons, each with 4-momentum uP .
- A scattered parton absorbs Q^2 ; thus:

$$(uP + Q)^2 = -m_p^2 \approx 0$$

- If $u^2 P^2 = u^2 M^2 \ll Q^2$ then we get: $u^2 P^2 + 2uPQ + Q^2 \approx 0$

$$2uPQ + Q^2 = 0 \implies u = \frac{-Q^2}{2PQ}$$

- In the lab. system: $P = (0, 0, 0, M)$ and $Q = (\bar{q}, \nu)$ and

$$u = \frac{-Q^2}{-2M\nu} = \frac{Q^2}{2M\nu} \equiv x$$

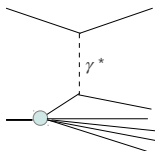
- Thus a meaning of x : a fraction of proton (three-)momentum carried by a struck quark (in an infinite proton momentum frame).

Feynman parton model,... cont'd

- However, free partons are not observed in nature; therefore it is assumed that a scattering is a two-step process

- 1 a photon-parton collision which occurs in $t_1 \sim \hbar/\nu$
- 2 a final state parton recombination into a hadron of mass W which occurs in

$$t_2 \sim \frac{\hbar}{W} \implies \gamma_L \frac{\hbar}{W} = \frac{\nu}{W} \frac{\hbar}{W} = \frac{\nu \hbar}{W^2} = (W^2 = M^2 + 2M\nu - Q^2 \approx 2M\nu) = \frac{\nu \hbar}{2M\nu} = \frac{\hbar}{2M} \gg \frac{\hbar}{\nu} = t_1$$



- We then assume that the cross-section will depend on the initial state dynamics and will be almost independent of the final state interaction (a good approximation except when $\nu \sim M$).
- To summarise: **in the Feynman model, the $ep \rightarrow eX$ interaction is an incoherent sum of electron-parton interactions.** Scaling is a direct consequence of that (elastic scattering is described by one variable only).
- **Important:** during the photon-parton interaction, remaining (spectator) partons do not interact with each other!

How do we measure structure functions $F_{1,2}(x, Q^2)$

- Observables: E, E', ϑ measured in detectors.
- From the above observables we reconstruct kinematic variables, e.g.:

$$Q^2 \approx 4EE' \sin^2 \vartheta / 2 \qquad x = \frac{Q^2}{2M\nu} = \frac{4EE' \sin^2 \frac{\vartheta}{2}}{2M(E - E')}$$

These formulae valid for a fixed-target experiment; in a collider, definition of ν is different.

- $F_{1,2}(x, Q^2)$ is determined for fixed (x, Q^2) values by a method similar to the Rosenbluth method of separating two form factors: **measurements must be done at different values of energy, E .**
- Traditionally, instead of F_1 one rather measures a function $R(x, Q^2)$ defined as:

$$R(x, Q^2) = \frac{F_2(x, Q^2)}{2xF_1(x, Q^2)} \left(1 + \frac{4M^2x^2}{Q^2} \right) - 1 = \frac{\sigma_L}{\sigma_T} \quad (18)$$

where $\sigma_{L,T}$ are cross sections for γ^* -parton interactions for longitudinal/transverse polarised virtual photons. Observe that $\lim_{Q^2 \rightarrow 0} \sigma_L = 0$ and $\lim_{Q^2 \rightarrow 0} \sigma_T = \sigma^{\text{tot}}(\gamma p)$ and that 50% of transverse photons is left- and 50% right polarised (electromagnetic interactions do not tell between left and right (they conserve parity)).

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Partons – what are they? a) partons' spin

- Compare two equations: for scattering of electron on a point-like target of spin 1/2, charge ze and mass m , Eq. (10):

$$\left(\frac{d\sigma}{dQ^2}\right) = \frac{4\pi^2 z^2}{q^4} \left(\frac{E'}{E}\right)^2 \left(\cos^2 \frac{\vartheta}{2} + \frac{Q^2}{2m^2} \sin^2 \frac{\vartheta}{2}\right)$$

and for the inelastic electron scattering on a target of mass M , Eq. (17):

$$\frac{d^2\sigma}{dQ^2 dx} = \frac{4\pi\alpha^2}{Q^4} \frac{E'}{Ex} \left[F_2 \cos^2 \frac{\vartheta}{2} + \frac{Q^2}{2M^2 x^2} 2xF_1 \sin^2 \frac{\vartheta}{2} \right]$$

- Coefficients in front of $\cos^2 \frac{\vartheta}{2}$ and $\sin^2 \frac{\vartheta}{2}$ should be the same, thus:

$$z^2 \frac{E'}{E} = \frac{1}{x} F_2, \quad z^2 \frac{E'}{E} \frac{Q^2}{2m^2} = \frac{1}{x} \frac{Q^2}{2M^2 x^2} 2xF_1$$

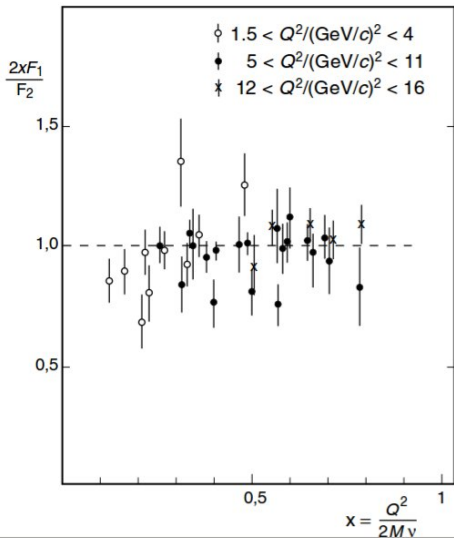
dividing the two equations by each other:

$$\frac{Q^2}{2m^2} = \frac{Q^2}{2M^2 x^2} \frac{2xF_1}{F_2} \quad \Rightarrow \quad \frac{2xF_1}{F_2} = 1 \quad (\text{if } m = Mx) \quad (19)$$

This is the Callan–Gross relation, valid if scattering occurs on a point-like nucleon components, of spin 1/2 and “normal” magnetic moments: $\mu = (ze\hbar)/(2mc)$.

- Zero spin partons would have: $(2xF_1)/(F_2) = 0$
($F_1 = 0$ and it corresponds to a magnetic interaction).

Partons – what are they? a) partons' spin,...cont'd



Evidence for partons' spin $1/2 \hbar$

Figure from the book by B. Povh et al.

Partons – what are they? b) partons' charge

Let us now use the formula for an inelastic cross-section, Eq. (17):

$$\frac{d^2\sigma}{dQ^2 dx} = \frac{4\pi\alpha^2}{Q^4} \left[\left(1 - y - \frac{M^2 y^2}{Q^2}\right) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2) \right]$$

noticing that $\frac{M^2 y^2}{Q^2} \approx 0$:

$$\frac{d^2\sigma}{dQ^2 dx} = \frac{4\pi\alpha^2}{Q^4} \left[(1 - y) \frac{F_2(x, Q^2)}{x} + \frac{y^2}{2} \frac{2xF_1(x, Q^2)}{x} \right]$$

Now we take the $y \rightarrow 0$ limit of it:

$$\frac{d^2\sigma}{dQ^2 dx} \rightarrow \frac{4\pi\alpha^2}{Q^4} \frac{F_2}{x} \quad \Longrightarrow \quad \frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \int \frac{F_2}{x} dx$$

But in the Rutherford scattering:

$$\left(\frac{d\sigma}{dQ^2} \right)_{\text{Ruth}} \sim \frac{(Ze \cdot e)^2}{Q^4}$$

which means that $\int \frac{F_2}{x} dx$ must have a meaning of a sum of squares of parton charges. Thus

$F_2^{\text{ep}}(x)/x$ is expressed through quark densities in the proton, weighted by squares of charges.

Therefore:

$$F_2(x) = x \sum_{i=1}^6 e_i^2 q_i(x) \quad (20)$$

Partons – what are they? b) partons' charge,...cont'd

Why only quarks and not gluons? And WHICH quarks (remember quark ($q\bar{q}$) pair sea)?

E.g. proton: $p \equiv (u, d, u\bar{u}, d\bar{d}, s\bar{s}, \dots)$

$$F_2^{\text{ep}}(x) = x \left\{ \frac{4}{9} [u^P(x) + \bar{u}^P(x)] + \frac{1}{9} [d^P(x) + \bar{d}^P(x)] + \frac{1}{9} [s^P(x) + \bar{s}^P(x)] + \dots \right\}$$

Strong interactions do not see electric charges, i.e. for them **a proton \equiv a neutron** or a “u” quark \equiv a “d” quark; thus:

$$u^P \equiv d^N = u$$

$$d^P \equiv u^N = d$$

$$s^P \equiv s^N = s$$

(same for antiquarks).

Thus we get:

$$\frac{1}{x} F_2^{\text{ep}} = \frac{4}{9}(u + \bar{u}) + \frac{1}{9}(d + \bar{d} + s + \bar{s})\dots \quad (21)$$

$$\frac{1}{x} F_2^{\text{en}} = \frac{4}{9}(d + \bar{d}) + \frac{1}{9}(u + \bar{u} + s + \bar{s})\dots$$

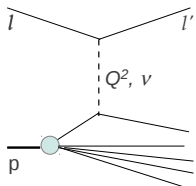
For clarity we neglect a contribution from $s\bar{s}$ (at $x \sim 0.03$ it is about 6% error):

$$\frac{1}{x} F_2^{\text{eN}} = \frac{1}{x} \frac{(F_2^{\text{ep}} + F_2^{\text{en}})}{2} = \frac{5}{18}(u + \bar{u} + d + \bar{d})$$

Partons – what are they? b) partons' charge,...cont'd

To determine separately a charge of a “u” and “d” quark, from the F_2 measurements we need another piece of information \implies **neutrino scattering**.

Summarising lepton–nucleon scattering:



| l | l' | exchanged boson | interaction | example |
|------------------------------|------------------------------|----------------------|-----------------------------|--|
| e^\pm μ^\pm | e^\pm μ^\pm | γ γ | electromagnetic | $e^-p \rightarrow e^-X$ $\mu^+p \rightarrow \mu^+X$ |
| ν_μ $\bar{\nu}_\mu$ | μ^- μ^+ | W^\pm W^\pm | weak, charge currents (CC) | $\nu_\mu d \rightarrow \mu^- u$ $\bar{\nu}_\mu u \rightarrow \mu^+ d$ |
| ν_μ $\bar{\nu}_\mu$ | ν_μ $\bar{\nu}_\mu$ | Z^0 Z^0 | weak, neutral currents (NC) | $\nu_\mu d \rightarrow \nu_\mu d$ $\bar{\nu}_\mu u \rightarrow \bar{\nu}_\mu u$ |

In weak interactions, W^\pm, Z^0 do not couple to electric charges.

This means that:

$$F_2^{\nu P} = 2x(d + \bar{u}) \quad F_2^{\nu n} = 2x(u + \bar{d})$$

or

$$F_2^{\nu N} = x [u + \bar{u} + d + \bar{d}]$$

(22)

Partons – what are they? b) partons' charge,...cont'd

Finally we get for the nucleon:

$$\frac{F_2^{eN}}{F_2^{\nu N}} = \frac{1}{2} (e_u^2 + e_d^2) = \frac{5}{18} \approx 0.28 \quad \text{or more accurately: } F_2^{eN} \geq \frac{5}{18} F_2^{\nu N}$$

EMC measurements @ CERN gave:

$$\frac{F_2^{eN}}{F_2^{\nu N}} = 0.29 \pm 0.02$$

We need to separately determine e_u and e_d . We have to use neutron (or deuteron) and assume that sea distributions are the same for proton and neutron. Then any difference will result from valence partons.

$$\frac{F_2^P - F_2^n}{x} = e_u^2 u_v^P + e_d^2 d_v^P - e_u^2 u_v^n - e_d^2 d_v^n = (e_u^2 - e_d^2) (u_v - d_v)$$

$$\int \frac{F_2^P - F_2^n}{x} dx = (e_u^2 - e_d^2) \left[\int u_v dx - \int d_v dx \right]$$

EMC gave: $(e_u^2 - e_d^2) = 0.24 \pm 0.11$; but we also have: $\int u_v dx = 2$ and $\int d_v dx = 1$ and thus:

$$e_u = 0.64 \pm 0.05$$

$$e_d = 0.41 \pm 0.09$$

Partons – what are they?

If we identify partons with quarks, then the following integral:

$$\frac{18}{5} \int F_2^{\text{eN}}(x) dx = \int F_2^{\nu\text{N}}(x) dx = \int [u(x) + \bar{u}(x) + d(x) + \bar{d}(x)] x dx \quad (23)$$

should be ≈ 1 . **Yet measurements give ≈ 0.5 !!! \implies gluons (a POSTULATE!)**

Summary of basic quark properties

- 1 Approximate scale invariance, $F_2(x, Q^2) \approx F_2(x) \implies$ the nucleon has point-like components
- 2 $2xF_1 \approx F_2 \implies$ these components have spin $(1/2)\hbar$
- 3 Electromagnetic and weak interaction cross-sections point towards identifying active partons with quarks of fractional charges
- 4 $\frac{18}{5} \int F_2^{\text{eN}}(x) dx = \int F_2^{\nu\text{N}}(x) dx \approx 0.5 \implies$ quarks carry about 50% of nucleon momentum; the rest is attributed to gluons.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{\nu^2 + Q^2}} \approx \frac{h}{\nu} = \frac{h2Mx}{Q^2} \approx 10^{-3} \text{ fm}$$

$$(\text{for } x = 0.2, Q^2 = 100 \text{ GeV}^2)$$

Quark model of hadrons

- All hadron properties should be reproducible from quark properties
- Charges - OK, np.: proton \equiv (uud), $\frac{2}{3}e + \frac{2}{3}e - \frac{1}{3}e = +1e$
- Magnetic moments - OK:

TABLE 6.5 A comparison of the observed magnetic moments of the $\frac{1}{2}^+$ baryon octet, and the predictions of the simple quark model, Eqs. (6.25a) and (6.26), for $m_u = m_d = 336 \text{ MeV}/c^2$ and $m_s = 510 \text{ MeV}/c^2$

| Particle | Prediction (μ_N) | Experiment (μ_N) |
|-------------------|------------------------|------------------------|
| p(938) | 2.79 | 2.793 ^a |
| n(940) | -1.86 | -1.913 ^a |
| Λ (1116) | -0.61 | -0.613 ± 0.004 |
| Σ^+ (1189) | 2.69 | 2.458 ± 0.010 |
| Σ^- (1197) | -1.04 | -1.160 ± 0.025 |
| Ξ^0 (1315) | -1.44 | -1.250 ± 0.014 |
| Ξ^- (1321) | -0.51 | -0.651 ± 0.003 |

^aThe errors on the proton and neutron magnetic moments are of the order 6×10^{-8} and 5×10^{-7} respectively.

- What about SPINS ?

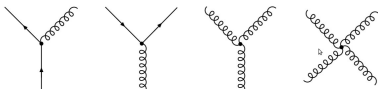
Table from the book of Martin and Shaw

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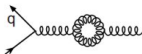
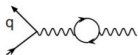
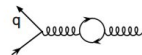
Strong vs electromagnetic interactions

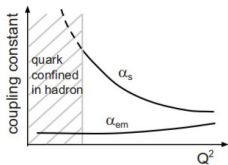
Basic strong interactions diagrams:



QCD

QED





First order approximation:

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - n_f) \cdot \ln \frac{Q^2}{\Lambda^2}}$$

n_f = number of quark types;

Λ = (the only) free parameter of QCD,

$\Lambda \approx 250 \text{ MeV}/c$.

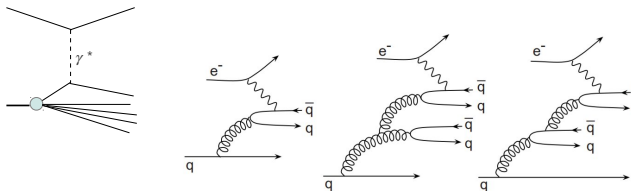
Perturbative approach to QCD valid only if:

$\alpha_s \ll 1$ i.e. $Q^2 \gg \Lambda^2 \approx 0.06 \text{ (GeV}/c)^2$.

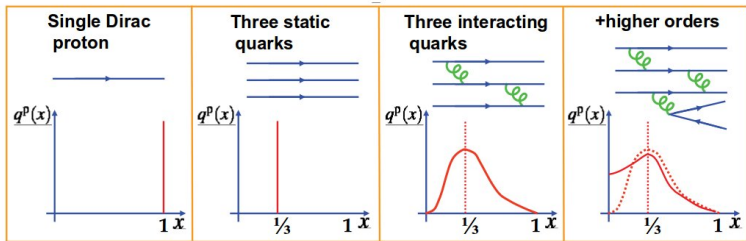
Figures from the book of B.Povh et al.

Strong vs electromagnetic interactions in DIS

Quark-Parton Model (QPM) becomes complicated...

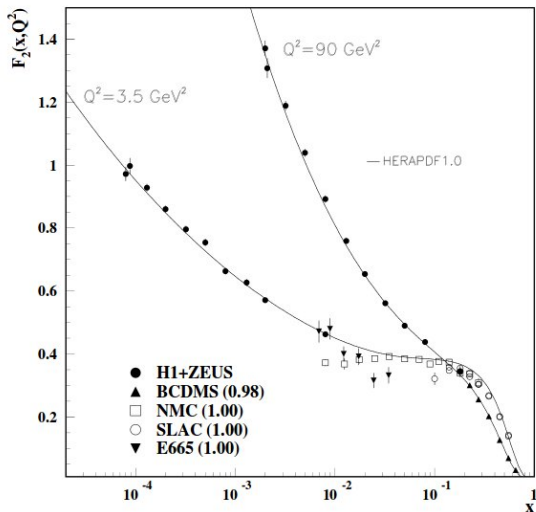


From the book of Povh et al.



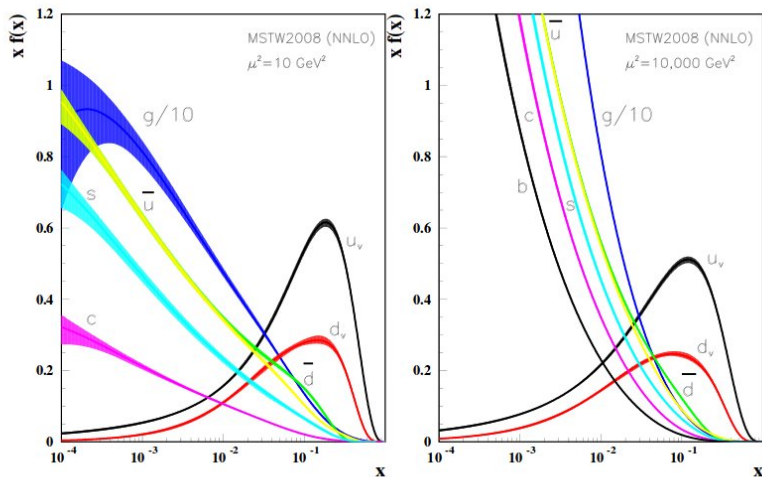
From M.A. Thomson, Michaelmas Term 2011

Scaling violation



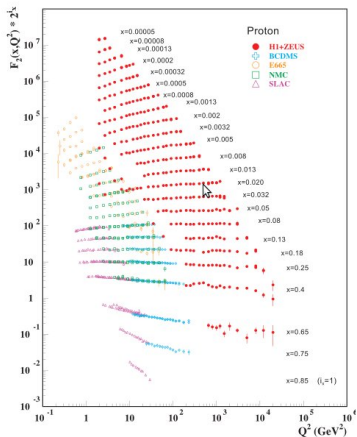
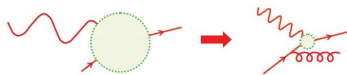
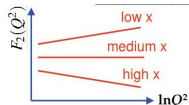
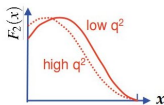
From Particle Data Tables, 2012

Scaling violation, ...cont'd



From Particle Data Tables, 2012

Scaling violation, ...cont'd

low Q^2 high Q^2 

QCD evolution equation (at lowest order):

$$\frac{dq(x, t)}{dt} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 dy \frac{q(y, t)}{y} \cdot P\left(\frac{x}{y}\right)$$

$$t = \ln \frac{Q^2}{\Lambda^2} \quad (24)$$

QCD can predict the Q^2 dependence of $F_2(x, Q^2)$

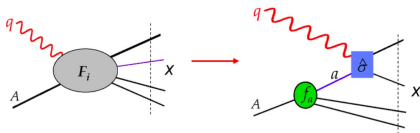
From Particle Data Tables, 2012 and from M.A. Thomson, Michaelmas Term 2011

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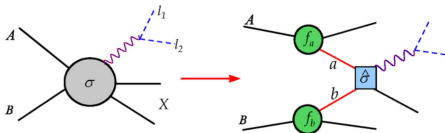
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Factorization theorem

$$\lim_{Q^2 \rightarrow \infty, x = \text{finite}} F_i(x, Q^2) = f_a \otimes \hat{\sigma}_i^a$$



$$\sigma \sim f_a \otimes \hat{\sigma}^{ab} \otimes f_b$$

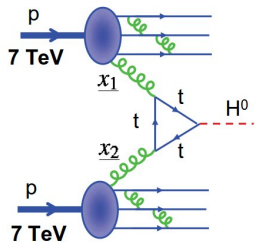


Figures from Scholarpedia

Universality of parton distributions

PDFs are universal!

Example of the LHC Higgs particle production in a “gluon–gluon fusion”:



$$\sigma(pp \rightarrow HX) \sim \int_0^1 \int_0^1 g(x_1)g(x_2)\sigma(gg \rightarrow H)dx_1dx_2$$

Observe: uncertainty in $g(x)$ leads to 5% uncertainty in the cross section!

How do we get PDFs? Measure $F_2(x, Q_0^2)$ for “all” values of x and assume a functional x dependence. Fit its coefficients at any Q^2 from QCD predictions of the Q^2 dependence of F_2 (“QCD evolution”).

From M.A. Thomson, Michaelmas Term 2011

pdf determination “industry”

Current status on PDFs

| | MSTW08 | CTEQ6.6/CT10 | NNPDF2.1/2.3 | HERAPDF1.0/1.5 | ABKM09/ABM11 | GJR08/JR09 |
|------------------|---------------|---------------|---------------|----------------|--------------|------------|
| PDF order | LO, NLO, NNLO | LO, NLO, NNLO | LO, NLO, NNLO | NLO, NNLO | NLO, NNLO | NLO, NNLO |
| HERA DIS | ✓ (old) | ✓ (old/new) | ✓ (new) | ✓ (new/newest) | ✓ (new) | ✓ (new) |
| Fixed target DIS | ✓ | ✓ | ✓ | - | ✓ | ✓ |
| Fixed target DY | ✓ | ✓ | ✓ | - | ✓ | ✓ |
| Tevatron W, Z | ✓ | ✓ | some | - | some | some |
| Tevatron jets | ✓ | ✓ | ✓ | - | ✓ | ✓ |
| LHC | - | - | -/W,Z+jets | - | - | - |
| HF Scheme | RTGMVF | SACOT GMVFN | FONLL GMVFN | RT GMVFN | BMSN FFNS | FFNS |
| Alphas (NLO) | 0.120 | 0.118(f) | 0.119 | 0.1176(f) | 0.1179 | 0.1145 |
| Alphas (NNLO) | 0.1171 | 0.118(f) | 0.1174 | 0.1176(f) | 0.1135 | 0.1124 |

The analyses differ in many areas:

- different treatment of heavy quarks
- inclusion of various data sets and account for possible tensions
- different alphas assumption

HERAFitter is an Open Source QCD Platform which can be used for benchmarking and understanding such differences

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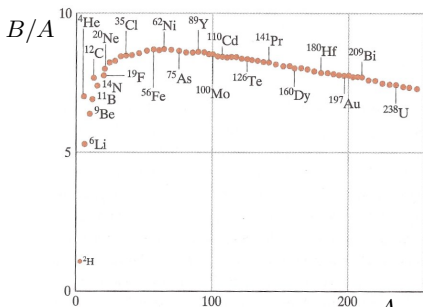
Quarks in a nucleon (nucleus)

- Elastic ep maxima in Fig. page 32 smeared around $x = Q^2/2M\nu = 1$ since nucleons are confined in a nucleus of radius $R_A \sim 1$ fm. Thus a Fermi momentum:

$$p_F \sim \frac{\hbar}{R_A} \approx 0.2 \text{ GeV}$$

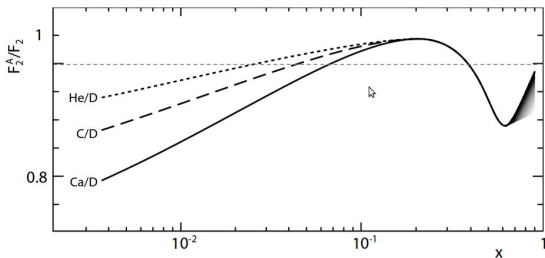
which is a few % of a typical ν .

- Remember also the nuclear binding energy, $B \sim 8$ MeV/nucleon (can be neglected as compared with ν).



$$B = \sum_{i=1}^A M_i c^2 - M_A c^2$$

Nuclear effects in parton distributions



- Here: $R \equiv \frac{F_2^A}{F_2^d} \equiv \frac{(F_2^A)/A}{(F_2^d)/2}$, i.e. nuclear structure functions “per nucleon”.
- For $x \lesssim 0.8$, “the EMC effect” (a shift in the quark momentum distributions towards lower x when nuclens are bound).
Observe a nuclear “shadowing” for $R < 1$, at lowest x
- At largest $x \implies$ scattering on a nucleon cluster?

EMC effect in the CERN Courier, May 2013

CERN Courier May 2013

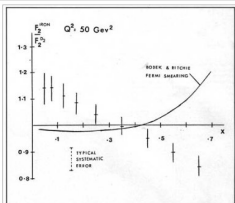
EMC effect

The EMC effect still puzzles after 30 years

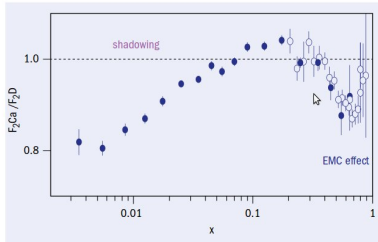
Thirty years ago, high-energy muons at CERN revealed the first hints of an effect that puzzles experimentalists and theorists alike to this day.

Contrary to the stereotype, advances in science are not typically about shouting "Eureka!". Instead, they are about results that make a researcher say, "That's strange". This is what happened 30 years ago when the European Muon collaboration (EMC) at CERN looked at the ratio of their data on per-nucleon deep-inelastic muon scattering off iron and compared it with that of the much smaller nucleus of deuterium.

The data were plotted as a function of Bjorken- x , which in deep-inelastic scattering is interpreted as the fraction of the nucleon's



original EMC plot for $F_2^{\text{Fe}} / F_2^{\text{D}}$



NMC (filled symbols) and SLAC data for $F_2^{\text{Ca}} / F_2^{\text{D}}$

Outline

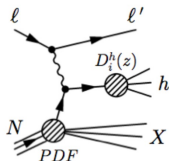
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 - Fragmentation functions**
 - Sum rules
- 5 “Forward” physics
 - Phenomena at low x
 - Diffraction

Other universal functions: fragmentation functions, $D_q^h(z, Q^2)$

- Studied through measurements of charged (single-) hadron multiplicities.
At LO:

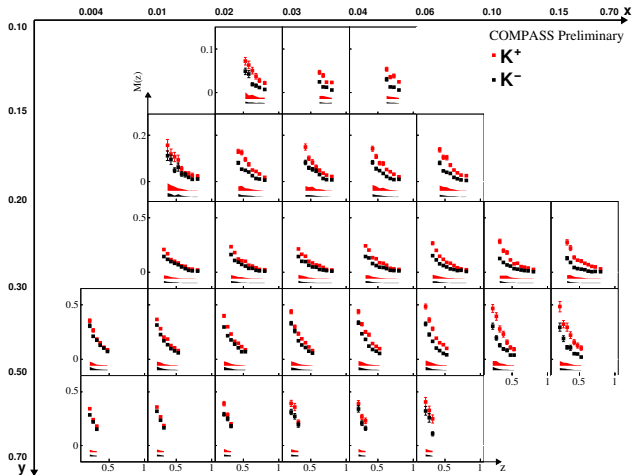
$$M^h(x, z) = \frac{\frac{d\sigma_{\text{SIDIS}}}{dx dz}}{\frac{d\sigma_{\text{DIS}}}{dx dz}} = \frac{\sum_q e_q^2 [q(x) D_q^h(z) + \bar{q}(x) D_{\bar{q}}^h(z)]}{\sum_q e_q^2 [q(x) + \bar{q}(x)]}$$

$$z = \frac{E_h}{\nu}, \quad \text{SIDIS} = \text{semi-inclusive DIS}$$



- High precision Single Inclusive e^+e^- Annihilation data do not separate q and \bar{q} and only access charge sum of FF for a hadron h .
- Measurements at a fixed, large ($\sim M_Z$), scale, except BELLE ($Q^2 \sim 10 \text{ GeV}^2$).
- Inclusive single hadron production by RHIC \implies improve constraints on gluon FF.
- Lepton-nucleon DIS: lower values and wide range of scales, sensitivity to parton flavour and hadron charge (\implies new data of HERMES, COMPASS).
- Global NLO analyses, e.g.: [DSS, Phys. Rev. D 75 \(2007\) 114010](#).

Charged (single-) hadron multiplicities; identified kaons



From N. Makke (COMPASS), DISXXI, 2013

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Sum rules (examples)

- Hide a very important physics! Recall that for parton distributions (3 valence quarks):

$$\int_0^1 dx u_v(x, Q^2) \equiv \int_0^1 dx [u(x, Q^2) - \bar{u}(x, Q^2)] = 2$$

$$\int_0^1 dx d_v(x, Q^2) \equiv \int_0^1 dx [d(x, Q^2) - \bar{d}(x, Q^2)] = 1$$

$$\int_0^1 dx [s(x, Q^2) - \bar{s}(x, Q^2)] = 0$$

In this form they are subject to QCD corrections involving powers of $\alpha_s(Q^2)$!

- Recall the **quark momentum sum rule**, Eq. (23), (**gluon existence**):

$$\frac{18}{5} \int F_2^{\text{eN}}(x) dx = \int F_2^{\nu\text{N}}(x) dx = \int [u(x) + \bar{u}(x) + d(x) + \bar{d}(x)] x dx \approx 0.5$$

- **Gottfried sum rule** (first checked by the NMC). From Eq. (21) we get:

$$\int_0^1 [F_2^{\text{ep}}(x) - F_2^{\text{en}}(x)] \frac{dx}{x} = \frac{1}{3} + \frac{2}{3} \int_0^1 [\bar{u}(x) - \bar{d}(x)] dx < \frac{1}{3} \quad (25)$$

which means:

- $q\bar{q}$ sea is not flavour symmetric
- more \bar{d} than \bar{u} in the proton: $\int_0^1 [\bar{u}(x) - \bar{d}(x)] dx = -0.118 \pm 0.012$

Outline

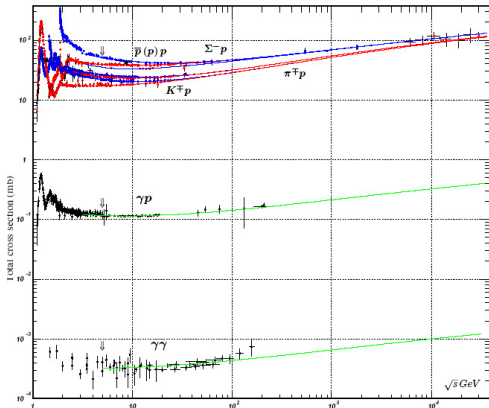
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Nucleon structure at low values of x ; γ^* behaviour

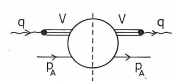
Experimental fact: photon interactions are often similar to those of a hadron



Contributions to the self-energy of a physical photon:

$$\begin{aligned}
 x \text{---} \text{[hadron loop]} \text{---} x &= x \text{---} \text{[vacuum loop]} \text{---} x + \alpha \left(x \text{---} \text{[hadron loop]} \text{---} x \right) \\
 &+ \alpha \left(x \text{---} \text{[electron loop]} \text{---} x \right) + O(\alpha^2)
 \end{aligned}$$

The hadron-type interaction:



V (\equiv a Vector Meson) has quantum numbers of the photon!

From Particle Data Tables, 2012

Nucleon structure at low values of x , ...cont'd

Hadrons in the γ fluctuation: either a pair of $q\bar{q}$ or a hadron of $J^P = 1^-$ (i.e. $\rho, \omega, \Phi, J/\Psi, \dots$). Observe that if E_γ is much larger than mass of the fluctuation, m , then the hadronic fluctuation traverses

$$d(E_\gamma, Q^2) \sim \frac{2E_\gamma}{Q^2 + m^2} \approx 80 \text{ fm!!!} \quad (\text{for } Q^2 = 0, E_\gamma = 100 \text{ GeV}, m^2 = 0.5 \text{ GeV}^2). \quad (26)$$

But a highly virtual γ^* , $Q^2 \rightarrow \infty$, may have no time to develop a structure before the interaction:

$$d(E_\gamma, Q^2) \sim \frac{2E_\gamma}{Q^2 + m^2} \rightarrow \frac{2E_\gamma}{Q^2} \rightarrow 0 \quad (27)$$

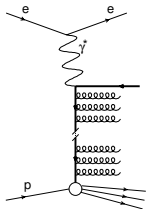
However the γ^* structure is visible! Observe that

$$\frac{2E_\gamma}{Q^2} = \frac{1}{Mx} \quad (28)$$

and if $x \ll 1$ then $d(E_\gamma, Q^2)$ may be very high independently of Q^2
(e.g. @ $x=0.001$, $d \sim 200 \text{ fm!}$ **proton sea quarks outside proton ???**)

Nucleon structure at low values of x , ...cont'd

Low $x \equiv$ large parton densities, due to QCD processes, e.g.:

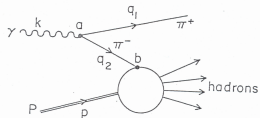


Who is probing whom?? (A. Levy)

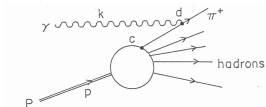
Solution: cross section is Lorentz invariant
but time development is not. (L. Frankfurt)

So γ^* and proton are probing each other and we are measuring the interaction as a whole. A consequence: @ low x , F_2^P and F_2^γ are related!

Two ways of γ interactions (observe time ordering!)

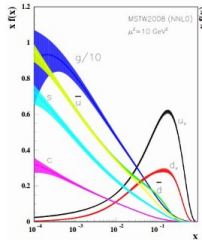
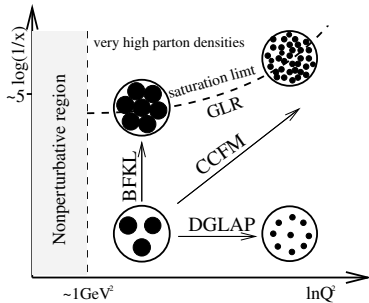


dominant if $\nu \rightarrow \infty$ and target at rest
photon structure



dominant in the ∞ target momentum system and finite ν
proton structure (DIS)

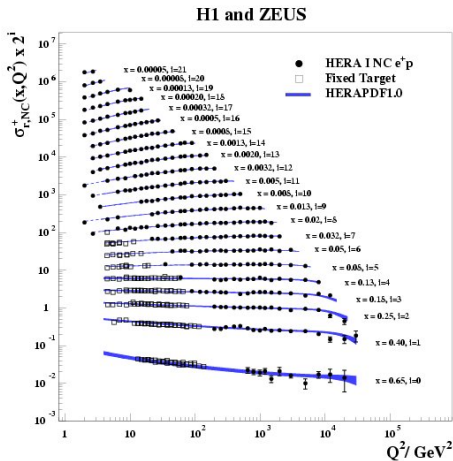
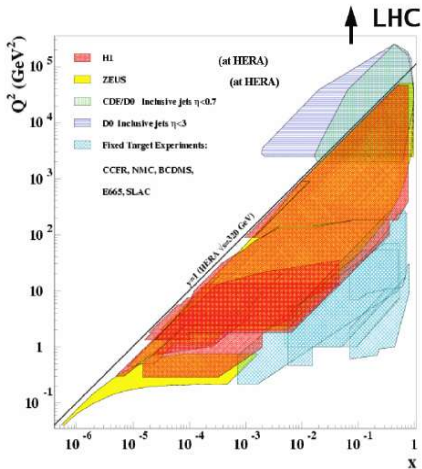
Physics domains: "Kwieciński plot"



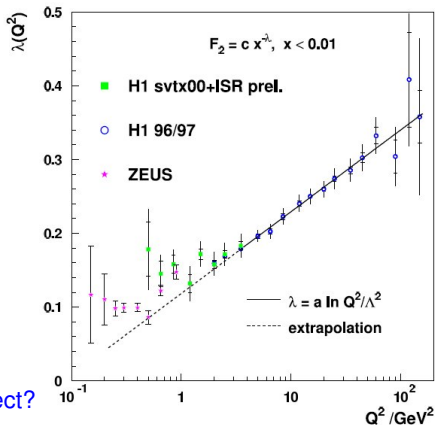
BFKL: $[\alpha_s \ln(1/x)]^n$
 DGLAP: $[\alpha_s \ln(Q^2)]^n$

- At low x , energy in the γ^*p cms is large (large gluon cascades): $W_{\gamma^*p}^2 = Q^2(1-x)/x$.
- Contributions from large $\alpha_s \ln \frac{1}{x}$ terms \Rightarrow new evolution equations: BFKL, CCFM.
- At low x : strong increase of gluon density with decreasing x (cf. HERA data) \Rightarrow gluon recombination (saturation).
- At $Q^2 \ll Q_{sat}^2$ nonlinear effects of parton saturation must be considered.

HERA data at low x . Inclusive measurements



From arXiv: 0911.0884

HERA data at low x . Inclusive measurements...cont'd

What should we expect?

- From DGLAP (DLA approximation): $xg(x)$ grows faster than any power of $\ln \frac{1}{x}$; partons do not necessarily overlap.
- From BFKL: $xg(x) \sim x^{-\lambda} \Rightarrow F_2 \sim x^{-\lambda}, \lambda \approx 0.5$

Fig. from hep-ex/0211051

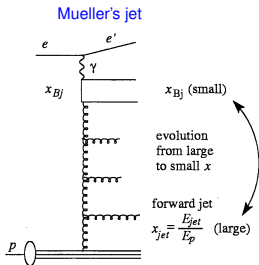
HERA data at low x . Hadron final states

- At low x parton probed by γ^* comes from a cascade initiated by a parton of a large longitudinal momentum.
- No k_t ordering of this cascade in BFKL
 \implies more hard gluons (\rightarrow hadrons) in the forward and central region.
- Measured: transverse energy flow, p_T hadrons, forward hadrons and jets, multijets, azimuthal correlations between energetic jets,...

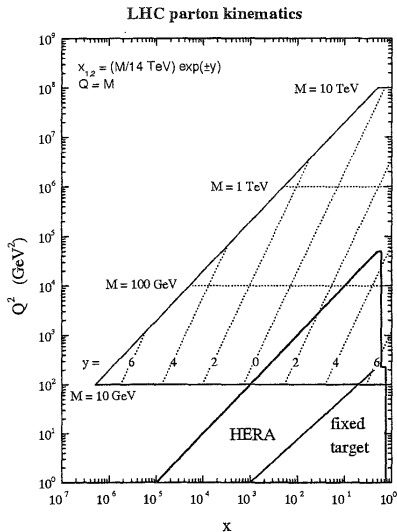
- **Conclusions:**

NLO DGLAP + resolved photon describe data fairly well.

BFKL effects not conclusive (too short cascade?)



Low x @ LHC



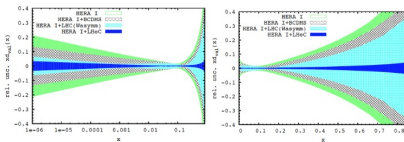
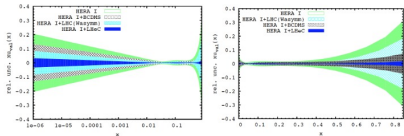
Expectations from the LHC

- Signatures for the BFKL evolution
- Parton saturation taming (especially on nuclei)
- Colour Glass Condensates ?
- Meaning of geometric scaling...

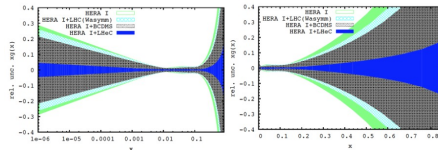
via observation of
jets, dijets, (semi-)inclusive reactions,...

Low x @ future e-p colliders (LHeC, EIC)

Example: $xq(x)$ at $Q^2 = 1.9 \text{ GeV}^2$ at LHeC



valence quarks (u and d)



gluons

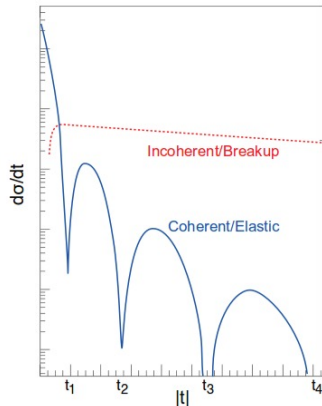
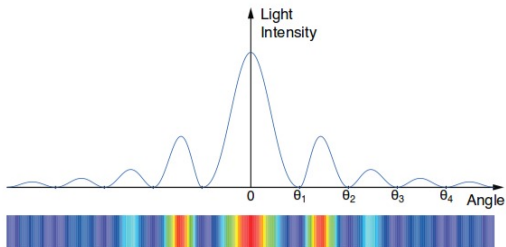
From LHeC CDR arXiv:1206.2913

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Diffraction in optics

Diffractive pattern of light on a circular disk and diffractive cross-section in HEP;
 $\vartheta_i \sim 1/(kR)$, $|t| \approx k^2 \vartheta^2$ (k - wave number, R - radius)



From EIC White Paper [arXiv:1212.1701](https://arxiv.org/abs/1212.1701)



Digression: rapidity, y

- Definition of **rapidity, y** :

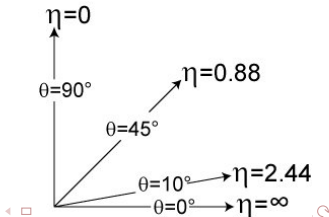
$$y = \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right)$$

(particle production is constant as a function of y).

- Under a boost in z to a frame with velocity β , $y \rightarrow y - \tanh^{-1} \beta \dots$
- ...hence shape of rapidity, dN/dy is invariant as are differences in rapidity $\implies y$ is preferred over polar angle θ in hadron collider physics.
- "Forward" in a hadron-hadron collider experiment means close to the beam axis, i.e. **high pseudorapidity, $|\eta|$** , where

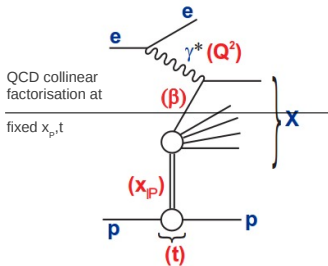
$$\eta = \frac{1}{2} \ln \left(\frac{|\vec{p}| + p_z}{|\vec{p}| - p_z} \right) = - \ln \left[\tan \left(\frac{\theta}{2} \right) \right]$$

- $\eta \Rightarrow y$ for $v \approx c$ or $m \approx 0$,
 η can be measured even if m and p unknown!



Definition of diffraction in high energy physics

- No quantum number exchange in a process.
Target (or both hadrons) emerges intact. "Pomeron, \mathbb{P} , exchange".
- Cross section not decreasing with energy.
- Secondary features: small t and large Δy (forward !) in final state hadrons.



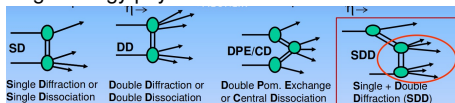
reaction described by 4 variables:

$$Q^2, x, \beta = x/x_{\mathbb{P}}, t$$

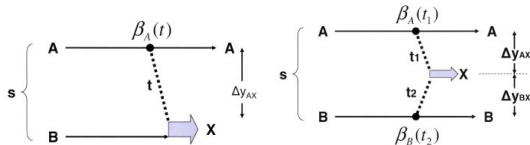
- Soft/hard diffraction \implies diffractive parton distributions! Universality?
Rapidity gap survival probability for hadron-hadron.
- Diffractive PDF, $f_i^D = f_i^D(x, Q^2, x_{\mathbb{P}}, t)$.
Within "vertex factorisation", $f_i^D(x, Q^2, x_{\mathbb{P}}, t) = f_{\mathbb{P}/p}(x_{\mathbb{P}}, t) \cdot f_i(\beta = x/x_{\mathbb{P}}, Q^2)$

Diffraction: brief experimental status

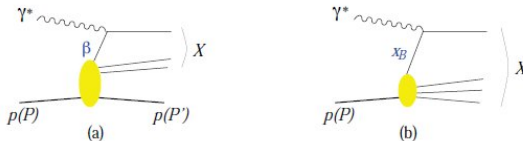
- Types of diffraction in high-energy physics:



- ISR @ CERN** (pp , $\sqrt{s} = 23 - 63$ GeV): σ_{e1} had exponential slope and diffractive minimum; shrinkage; pomeron/double pomeron exchange.

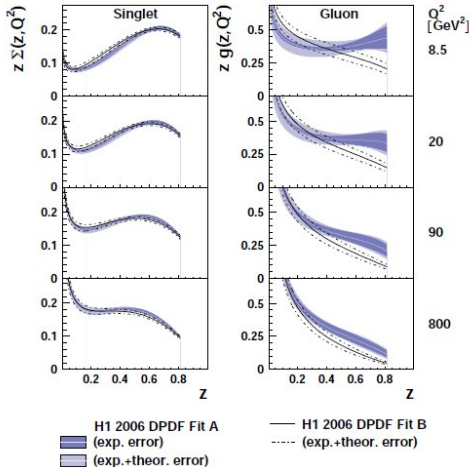
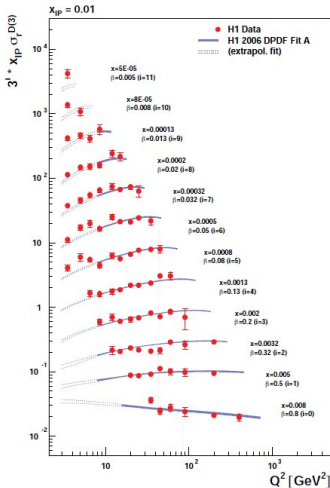


- $Spp\bar{p}S$ @ CERN** ($\sqrt{s} = 630 - 900$ GeV): first observation of hard diffraction.
- HERA @ DESY** (ep , $\sqrt{s} = 320$ GeV): factorisation(s) holds; DPDFs.



From K. Goulianos, [Low x, 2013 and arXiv:1606.1289](https://arxiv.org/abs/1606.1289)

Diffraction: brief experimental status, ...cont'd



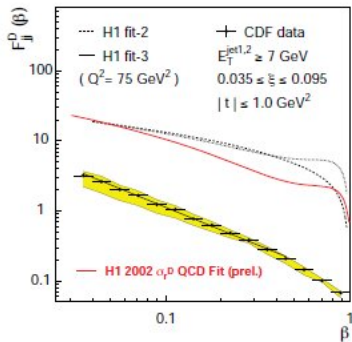
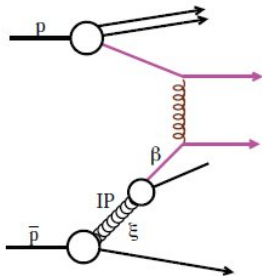
(here $z \equiv \beta$ at lowest order)

From hep-ex/0606004



Diffraction: brief experimental status,...cont'd

- **Tevatron @ FNAL ($p\bar{p}$, $\sqrt{s} = 550 - 1960$ GeV):** diffraction in hadron-hadron scatt. is more complicated; hard diffractive factorisation broken by multiple interactions.

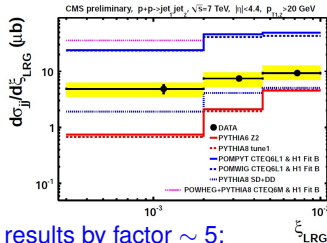
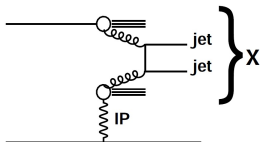
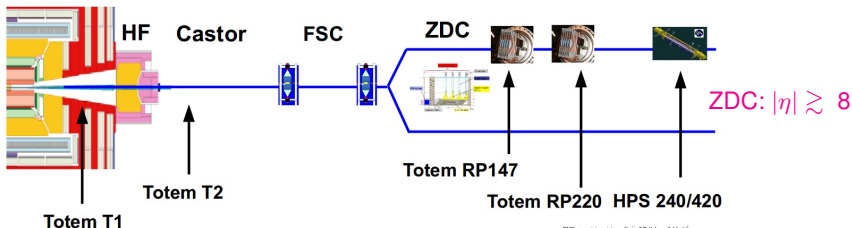


Gap survival problem??

- **LHC predictions (pp , $\sqrt{s} = 14\,000$ GeV):** inclusive single diffraction and double pomeron exchange also with dijets, vector bosons, heavy quarks.

Forward physics at LHC

- LHC "forward" arrangements (not to scale):



- Diffractive MC models overestimate the results by factor ~ 5 ;
gap survival probability ?

After G. Brona (2012) and CMS PAS FWD-10-004