LQCD at non-zero temperature : strongly interacting matter at high temperatures and densities

Péter Petreczky

BROOKHAVEN NATIONAL LABORATORY

- QCD and hot and dense matter

- Lattice formulation of QCD

- Deconfinement transition in QCD : EoS, color screening and fluctuations of conserved charges

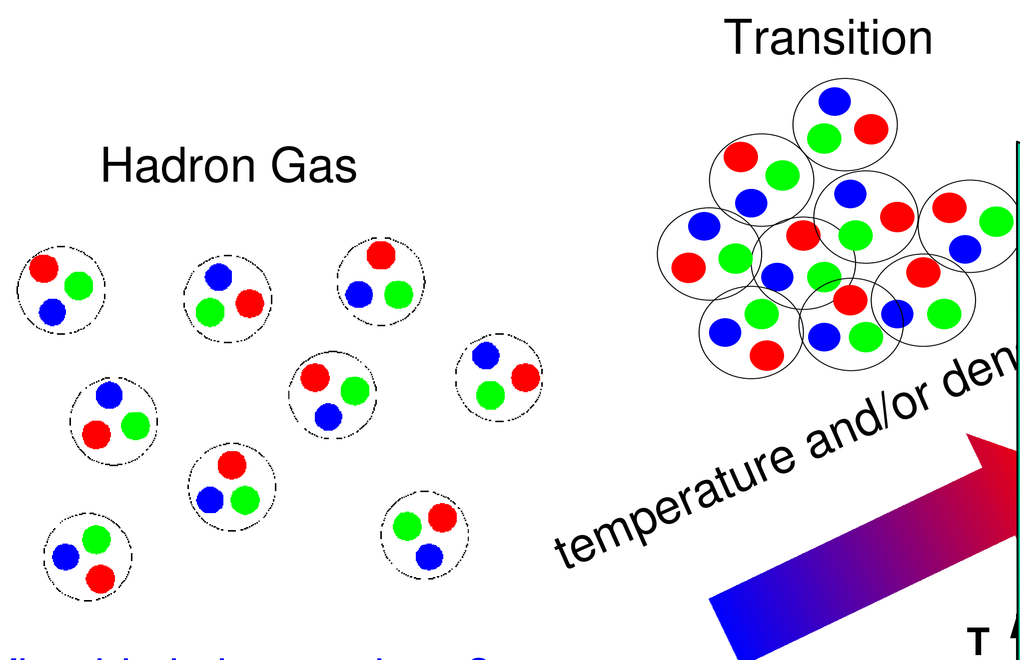
- Chiral transition in QCD and restoration of axial symmetry at high *T*

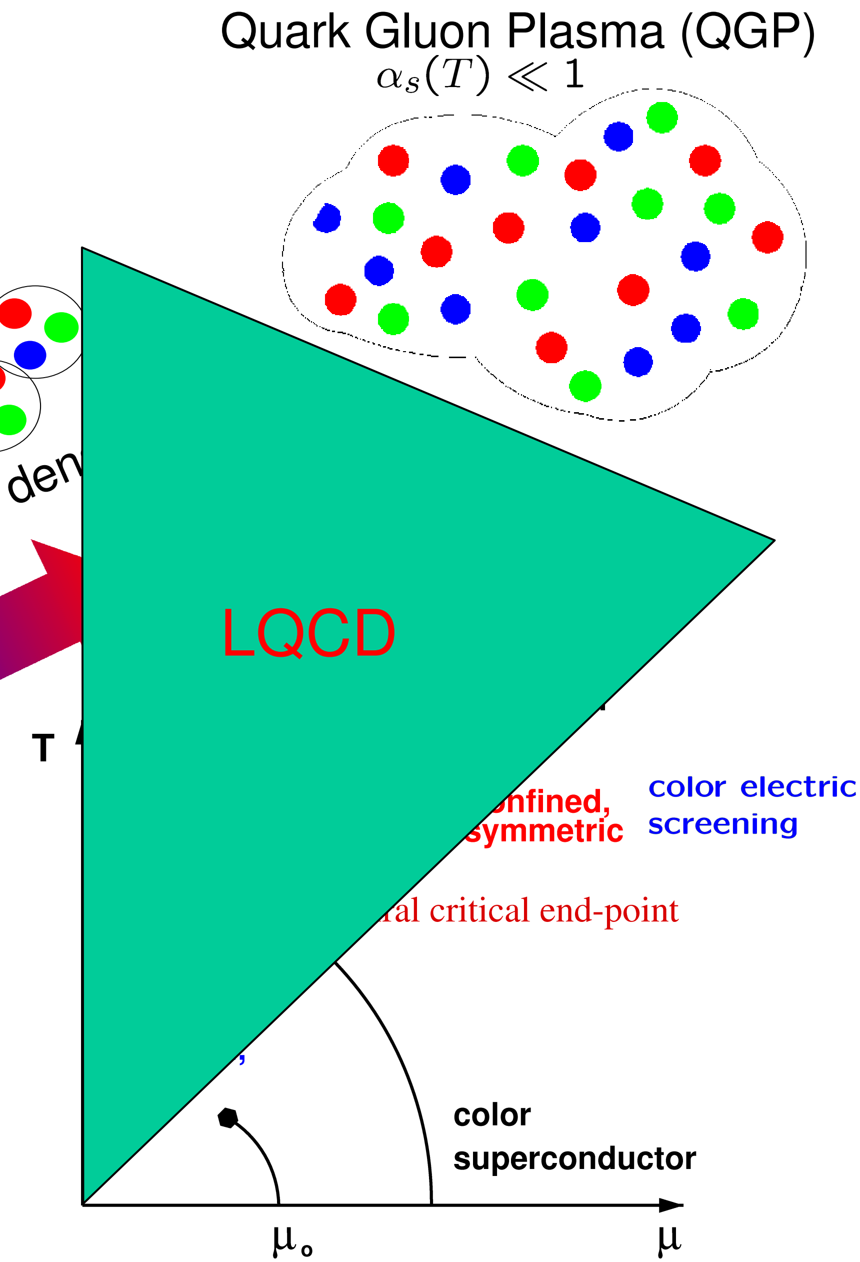
- Meson correlators and spectral function: dilepton rate, electric conductivity

For review see P.P. , arXiv:1203.5320

National Nuclear Physics Summer School 2012, Santa Fe, Friday, July 13, 2012

Deconfinement at high temperature and density





Why this is interesting ? :
basic properties of strong interaction

astrophysical (compact stars): boundaries
of hadronic matter

cosmological consequences
(Early Universe few microseconds after Big Bang)

Chiral symmetry of QCD in the vacuum and for T>0

• Chiral symmetry : For light quarks $m_{u,d} << \Lambda_{QCD}$ QCD Lagrangian has

Nobel Prize 2008

$SU_A(2)$ symmetry $\psi \to e^{i\phi^a T^a \gamma_5}\psi$ $\qquad \psi_{L,R} \to e^{i\phi^a_{L,R}T^a}\psi_{L,R}$

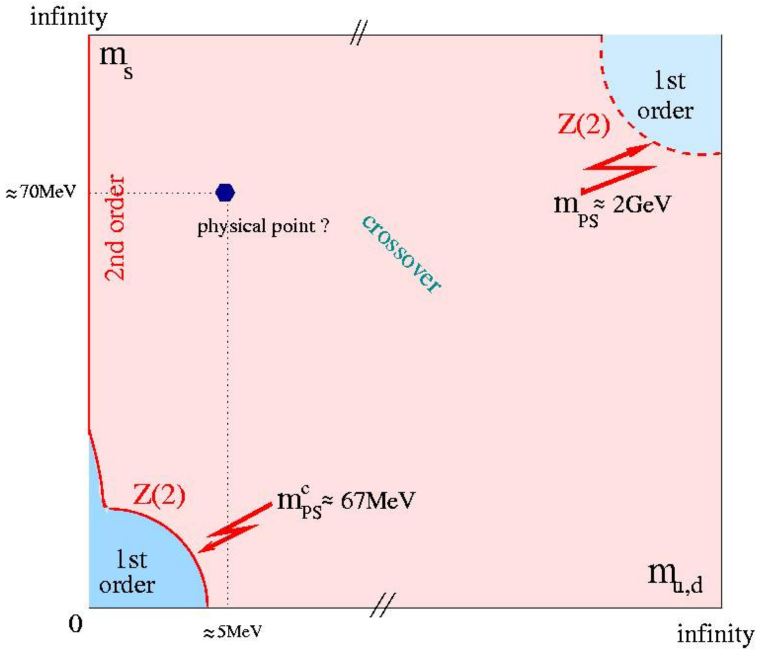
The vacuum breaks the symmetry $\langle\bar{\psi}\psi\rangle = \langle\bar{\psi}_L\psi_R\rangle + \langle\bar{\psi}_R\psi_L\rangle \neq 0$

spontaneous symmetry breaking or Nambu-Goldstone realization of the symmetry

hadrons with opposite parity have very different masses, interactions between hadrons are weak at low E

$U_A(1)$ symmetry $\psi \to e^{i\phi\gamma_5}\psi$ is broken by anomaly (ABJ) : $\langle\partial^\mu j^a_\mu\rangle = -\frac{\alpha_s}{4\pi}\langle\epsilon^{\alpha\beta\gamma\delta}F^a_{\alpha\beta}F^a_{\gamma\delta}\rangle$

$\Rightarrow$ η' meson mass, π-a_0 mass difference



Pisarski, Wilczek, PD29 (1984) 338

$T \gg \Lambda_{QCD}$: $\langle\bar{\psi}\psi\rangle \simeq 0$, $U_A(1)$ symmetry ?

• For vanishing u,d -quark masses the chiral transition is either 1st order or 2nd order phase transition.

• For physical quark masses there could be a 1st order phase transition or crossover

Evidence for 2nd order transition in the chiral limit => universal properties of QCD transition:

$SU_A(2) \sim O(4)$ relation to spin models	transition is a crossover for physical quark masses

Finite Temperature QCD and its Lattice Formulation

$$\langle O \rangle = \mathrm{Tr} O e^{-\beta H - \mu N}$$

$\beta = 1/T$

evolution operator in imaginary time

$$\langle O \rangle = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} O e^{-\int_0^\beta d\tau d^3x \mathcal{L}_{QCD}}$$

$$A_\mu(0,\mathbf{x}) = A_\mu(\beta,\mathbf{x}) \quad \psi(0,\mathbf{x}) = -\psi(\beta,\mathbf{x})$$

Integral over functions → (Lattice) → integral with very large (but finite) dimension (> 1000)

$$\langle O \rangle = \int \prod_x dU_\mu(x) O(det D_q[U,m,\mu]) e^{-\sum_x S_G[U(x)]}, \quad U_\mu(x) = e^{igaA_\mu(x)}$$

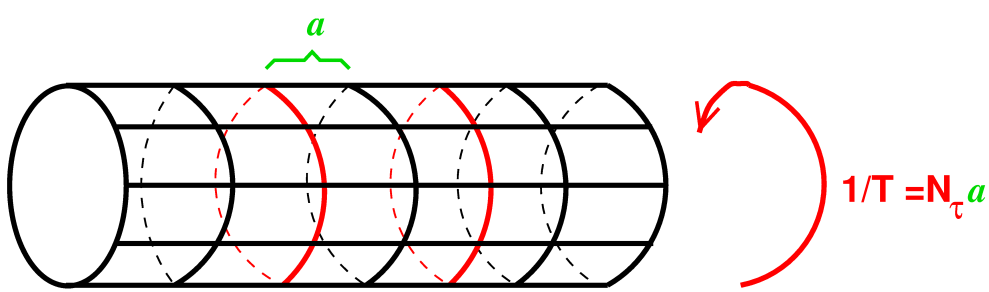
$\mu \neq 0$: $det D_q(U,m,\mu)$ complex → Monte-Carlo Methods / sign-problem

continuum limit

$N_\tau \to \infty$, N_σ / N_τ fixed

Costs :

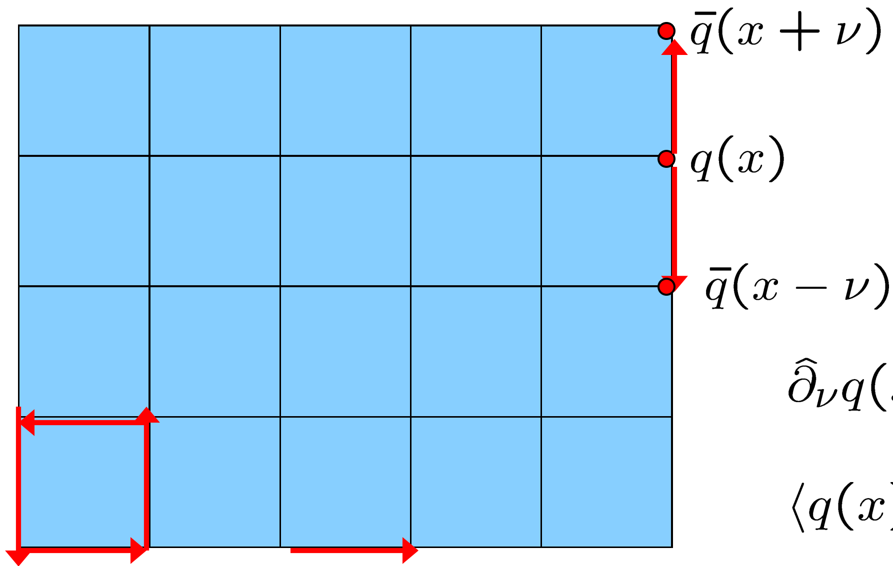
$$\sim a^{-7} \sim N_\tau^7 \; ?$$



Quenched QCD: $det D_q(U,m,\mu) = 1$

improved discretization schemes are needed : p4, asqtad, stout, HISQ

Quarks and gluon fields on a lattice



$\Longrightarrow$ $\bar{q}\;\gamma_\nu D_\nu\;q$

$g = 0$:

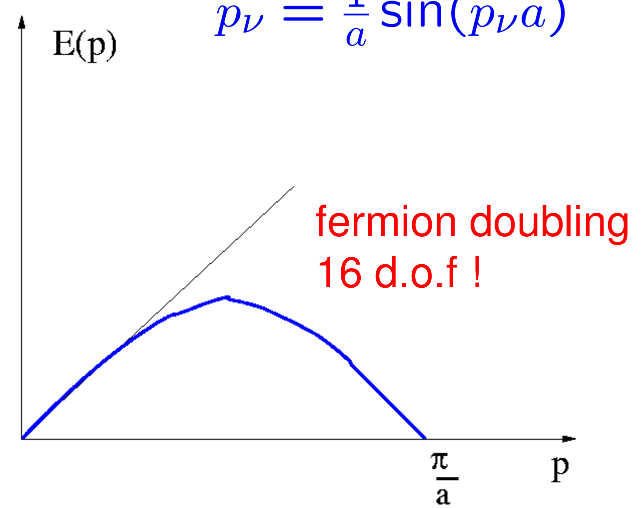
$$\hat{\partial}_\nu q(x) = \frac{1}{2}[q(x+\nu) - q(x-\nu)]$$

$$\langle q(x)\bar{q}(x)\rangle = \int_{-\pi/a}^{\pi/a} \frac{d^4p}{(2\pi)^4} \frac{-i\sum_\nu \gamma_\nu \hat{p}_\nu + m}{\sum_\nu \hat{p}_\nu^2 + m^2}$$

$U_\mu(x) = \exp(igaA_\mu(x))$

$U_\mu(x) \simeq 1 + igaA_\mu(x)$

$$\hat{p}_\nu = \frac{1}{a}\sin(p_\nu a)$$



$$U_P(x) = U_\mu(x)U_\nu(x+\mu)U_\mu^\dagger(x+\nu)U_\nu^\dagger(x)$$

$$S_{Wilson} = \beta\sum_x\left(1 - \frac{1}{3}Re\;trU_P(x)\right),\;\; \beta = \frac{6}{g^2}$$

$$S_{Wilson}|_{a\to 0} = \int d^4x\;trF_{\mu\nu}^2$$

Wilson fermions

$$S_f^W = \int_x [(\bar{q}\ \gamma_\nu D_\nu\ q - a\frac{r}{2}\bar{q}\ \Box\ q]$$

Wilson (1975)

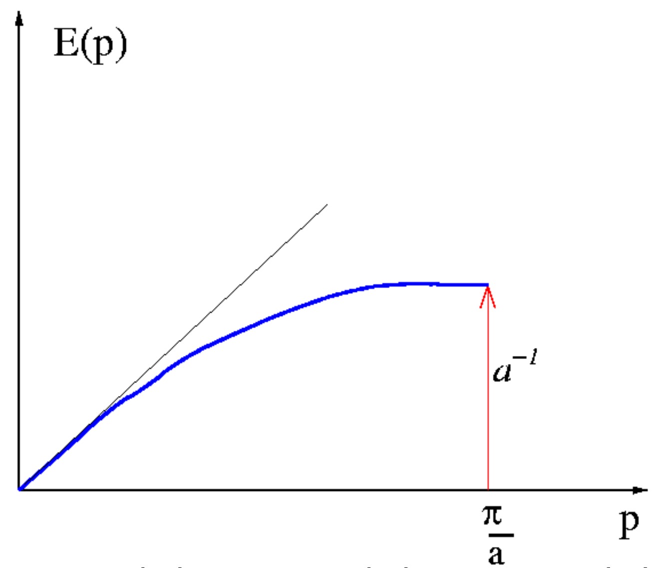
$$S_f^W = \int_x \bar{q}\ D^W\ q, \quad \int_x = \sum_x a^3$$

$$D^W(x,y) = \delta_{x,y}(4+m) + \sum_\mu (r+\gamma_\mu)\delta_{x+\mu,y} + (r-\gamma_\mu)\delta_{x-\mu,y})$$

$U_\mu(x)$ $\qquad U_\mu^\dagger(x)$

$$\langle q(x)\bar{q}(x)\rangle = \int_{-\pi/a}^{\pi/a} \frac{d^4p}{(2\pi)^4} \frac{-i\sum_\nu \gamma_\nu \hat{p}_\nu + m'(p)}{\sum_\nu \hat{p}_\nu^2 + m'^2(p)}$$

$$m'(p) = m + \frac{2r}{a}\sum_\mu \sin^2(\frac{p_\mu a}{2})$$



$$SU(3)_V \times SU(3)_A \to SU(3)_V$$

chiral symmetry is broken even in the massless case !

additive mass renormalization

Wilson Dirac operator is not bounded from below

difficulties in numerical simulations

Hadron properties, spectral functions

Staggered fermions

$$q(x) = T(x)\chi(x), \quad \bar{q}(x) = \bar{\chi}T^\dagger(x)$$ Kogut, Susskid (1975)

$$T(x)\gamma_\mu T^\dagger(x+\mu) = 1 \cdot \eta_\mu(x)$$

$\eta_\mu(x) = (-1)^{x_1+\ldots+x_{\mu-1}}, \eta_1(x) = 1$
$T(x) = \gamma_1^{x_1}\gamma_2^{x_2}\gamma_3^{x_3}\gamma_4^{x_4}$

$$S_f^{stagg} = \sum_x \sum_\alpha [\eta_\mu(x)\bar{\chi}_\alpha(x)\hat{\partial}_\mu \chi_\alpha(x) + m\bar{\chi}_\alpha(x)\chi_\alpha(x)]$$

$\Downarrow$ omit index α and the sum ($16 \to 4$)

$$\sum_{x,y}[\bar{\chi}(x)D_{stagg}(x,y)\chi(y)], \quad D_{stag}(x,y) = \delta_{x,y}m + \sum_\mu \eta_\mu(x)(\delta_{x+\mu,y} - \delta_{x-\mu,y})$$

different flavors, spin components sit in different corners of the Brillouin zone or in 2^4 hypercube

(3+1)-d:

$\sum_{x,y}[\bar{\chi}(x)D(x,y)\chi(y)] \rightarrow$ 4-flavor theory $\quad det D \to (det D)^{1/4}$ rooting trick

$SU(4)_A \to U(1)_A \subset SU_A(4)$ $\quad$ EoS and phase diagram of QCD

(2+1) d: $\rightarrow$ 2-flavor theory

(1+1) d: $\rightarrow$ 1-flavor theory (no doubling)

$||D_{stagg}|| > m$ useful in numerical simulations !

Chiral fermions on the lattice ?

$$S_F = a^4 \sum_{x,y} \bar{\psi}(x) D(x-y) \psi(y)$$

We would like the following properties for the lattice Dirac operator:

1. $D(x)$ should be local, i.e. $||D(x)|| \leq C \exp(-\gamma x)$
2. $D(p) = i \sum_\mu \gamma_\mu p_\mu + O((ap)^2)$ (cubic symmmetry)
3. no doubler exist, i.e. $D(p)$ is invertible for $p \neq 0$
4. $\gamma_5 D + D \gamma_5 = 0$ (chiral symmetry)

$$\gamma_5 D + D \gamma_5 = a D \gamma_5 D$$

Ginsparg, Wilson (1982)

mildest way to break the chiral symmetry on the lattice : physical consequences of the chiral symmetry are maintained
(e.g. soft pion theorem etc.)

Nielsen-Ninomiya no-go theorem :
conditions one 1-4 cannot be satisfied simultaneously

Nielsen, Ninomiya (1981)

Wilson fermion formulation gives up 4)
Staggered fermion formulation gives up 3)

Constructing chiral fermion action

Domain wall fermions : introduce the fictitious 5th dimension of extent N_s :

$$S_{dwf} = -\sum_{x,y,s,s'} \bar{\psi}(D_{x,y}\delta_{s,s'} + D_{s,s'}\delta_{x,y})\psi$$ Shamir (1993)

$$D_{x,y} = \frac{1}{2}\sum_\mu((1+\gamma_\mu)U_{x,\mu}\delta_{x+\mu,y} + (1-\gamma_\mu)U^\dagger_{x,\mu}\delta_{x-\mu,y} + (M-4)\delta_{x,y})$$

$$D_{s,s'} = (P_R\delta_{s+1,s'} + P_L\delta_{s-1,s'} - \delta_{s,s'}) + (P_R\delta_{1,s'} + P_L\delta_{N_s-2,s'}) - (mP_L\delta_{N_s-1,s'} + mP_R\delta_{0,s'} + \delta_{0,s'} + \delta_{N_s-1,s'})$$

$$P_{R,L} = (1 \pm \gamma_5)/2$$

$$q_x = \frac{1+\gamma_5}{2}\psi_{x,0} + \frac{1-\gamma_5}{2}\psi_{x,N_s-1}$$

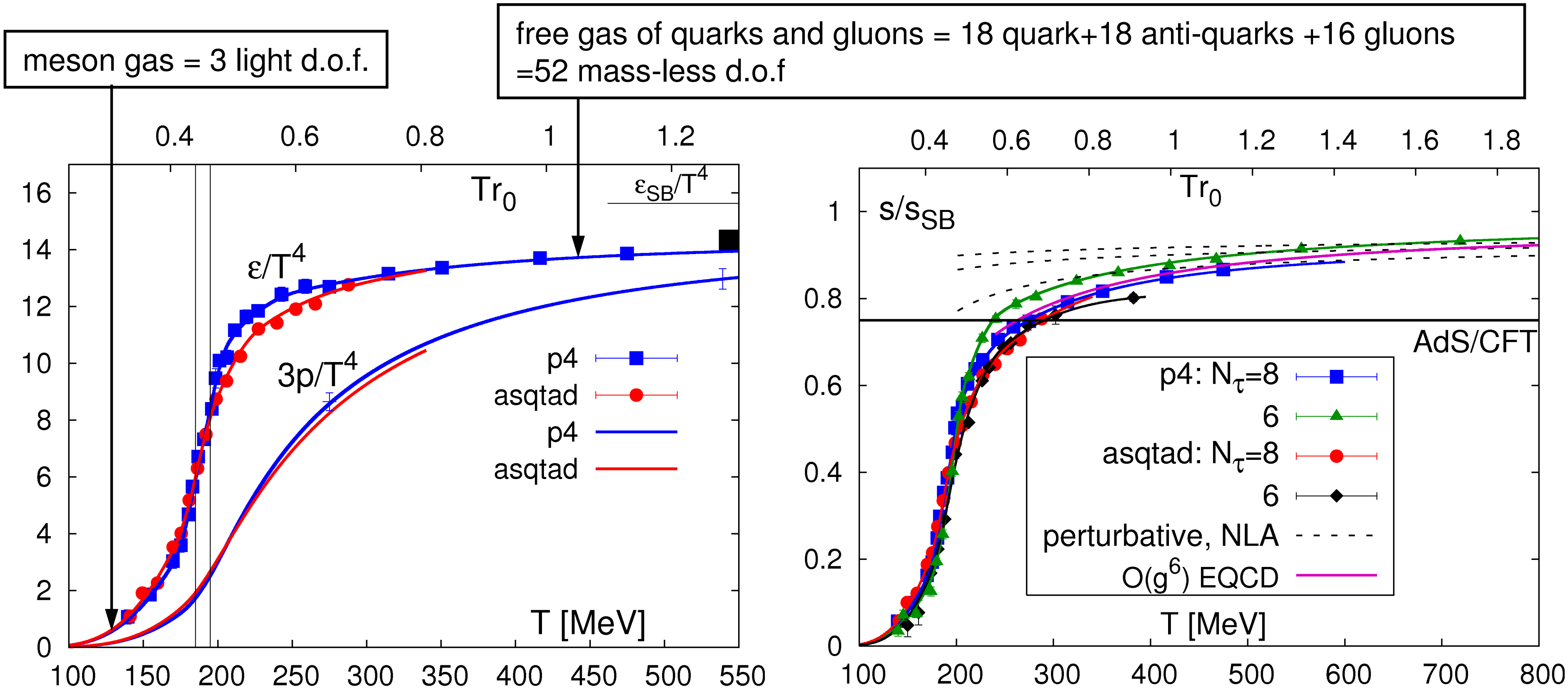
$N_s \to \infty$ two chiral fermions bounded to the 4d walls

$$m_q = mM(M-2)$$

costs $= N_s \times$ costs of Wilson formulations $N_s = 16 - 32$

Extensively used in numerical simulations : (see P. Boyle, 2007 for review)

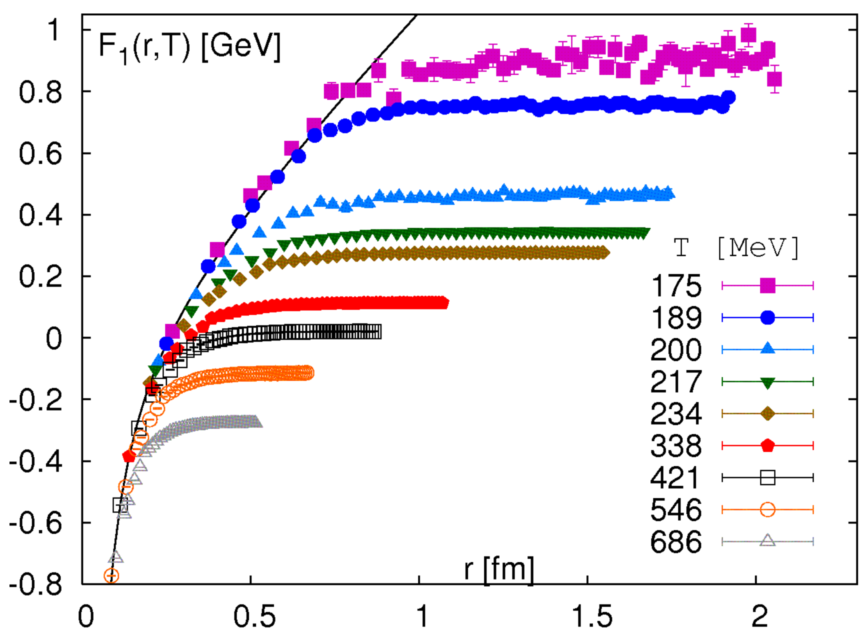
Deconfinement : entropy, pressure and energy density

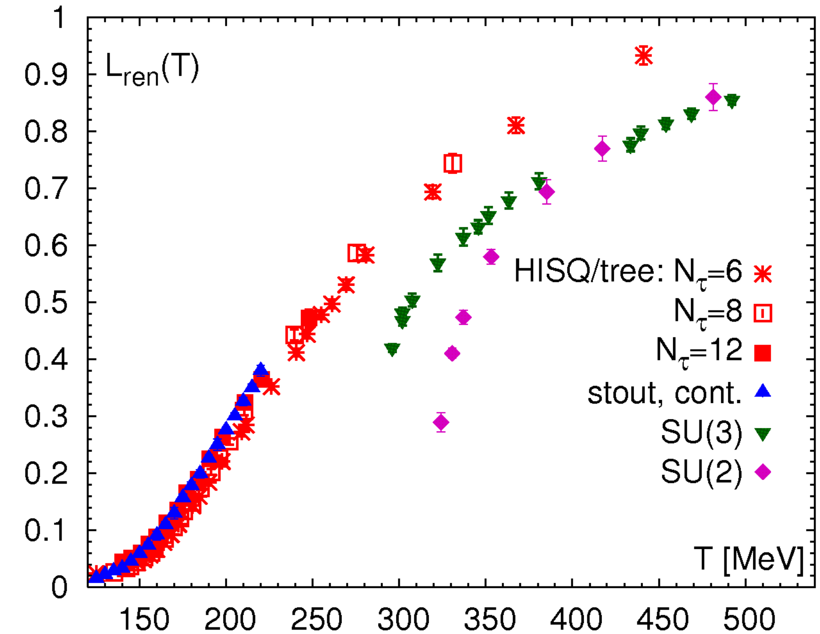


Bazavov et al (HotQCD), PRD 80 (09) 14504 Petreczky, NPA 830 (10) 11c

- rapid change in the number of degrees of freedom at *T=160-200*MeV: deconfinement
- deviation from ideal gas limit is about 10% at high *T* consistent with the perturbative result
- no obviously large discretization errors in the pressure and energy density at high *T*
- no continuum limit ?
- energy density at the chiral transition temperature $\varepsilon(T_c=154MeV)=240$ MeV/fm^3 :

Deconfinement and color screening





free energy of static quark anti-quark pair shows Debye screening at high temperatures

Polyakov loop

$$L = \mathrm{tr}\mathcal{P}e^{ig\int_0^{1/T} d\tau A_0(\tau,\vec{x})} \Rightarrow L_{ren} = \exp(-F_Q(T)/T)$$

$$F_1(r) = -\frac{4}{3}\frac{\alpha_s}{r}\exp(-m_D r) + 2F_Q(T), m_D \sim T$$

free energy of a static quark

$$r_{bound} > 1/m_D$$

melting of bound states of heavy quarks => quarkonium suppression at RHIC

infinite in the pure glue theory or large in the “hadronic” phase ~600MeV

decreases in in the deconfined phase

$$F_Q(T) \simeq \Lambda_{QCD} - C_F \alpha_s m_D$$

QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!l!} \chi_{ijk}^{BQS} \cdot \left(\frac{\mu_B}{T}\right)^i \cdot \left(\frac{\mu_Q}{T}\right)^j \cdot \left(\frac{\mu_Q}{T}\right)^k$$ hadronic

$$\frac{p(T,\mu_u,\mu_d,\mu_s)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{uds} \cdot \left(\frac{\mu_u}{T}\right)^i \cdot \left(\frac{\mu_d}{T}\right)^j \cdot \left(\frac{\mu_s}{T}\right)^k$$ quark

$$\chi_{ijk}^{abc} = T^{i+j+k} \frac{\partial^i}{\partial \mu_a^i} \frac{\partial^j}{\partial \mu_b^j} \frac{\partial^k}{\partial \mu_c^k} \frac{1}{VT^3} \ln Z(T,V,\mu_a,\mu_b,\mu_c)|_{\mu_a=\mu_b=\mu_c=0}$$

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

$$\chi_2^X = \chi_X = \frac{1}{VT^3}(\langle X^2 \rangle - \langle X \rangle^2) \qquad \chi_{11}^{XY} = \frac{1}{VT^3}(\langle XY \rangle - \langle X \rangle \langle Y \rangle)$$

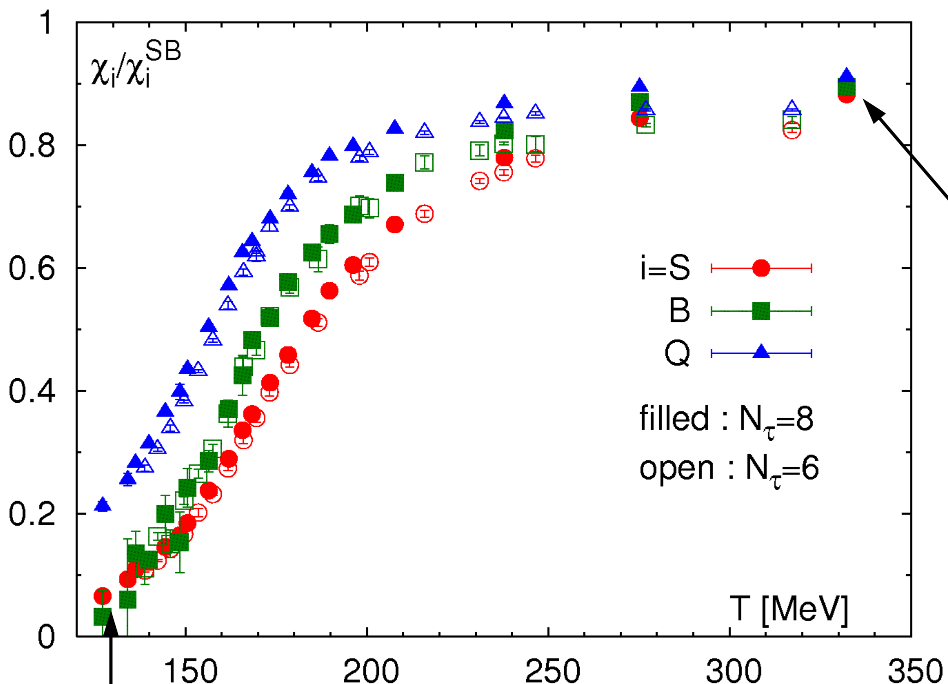
Computation of Taylor expansion coefficients reduces to calculating the product of inverse fermion matrix with different source vectors

Deconfinement : fluctuations of conserved charges

$$\chi_{\mathrm{B}}^{\mathrm{SB}} = \frac{1}{VT^3}(\langle B^2 \rangle - \langle B \rangle^2)$$ baryon number

$$\chi_{\mathrm{Q}}^{\mathrm{SB}} = \frac{1}{VT^3}(\langle Q^2 \rangle - \langle Q \rangle^2)$$ electric charge

$$\chi_{\mathrm{S}}^{\mathrm{SB}} = \frac{1}{VT^3}(\langle S^2 \rangle - \langle S \rangle^2)$$ strange quark number (strangeness)



Ideal gas of massless quarks :

$$\chi_B^{SB} = \frac{1}{3} \qquad \chi_Q^{SB} = \frac{2}{3}$$

$$\chi_B^{SB} = 1$$

conserved charges carried by light quarks

conserved charges are carried by massive hadrons

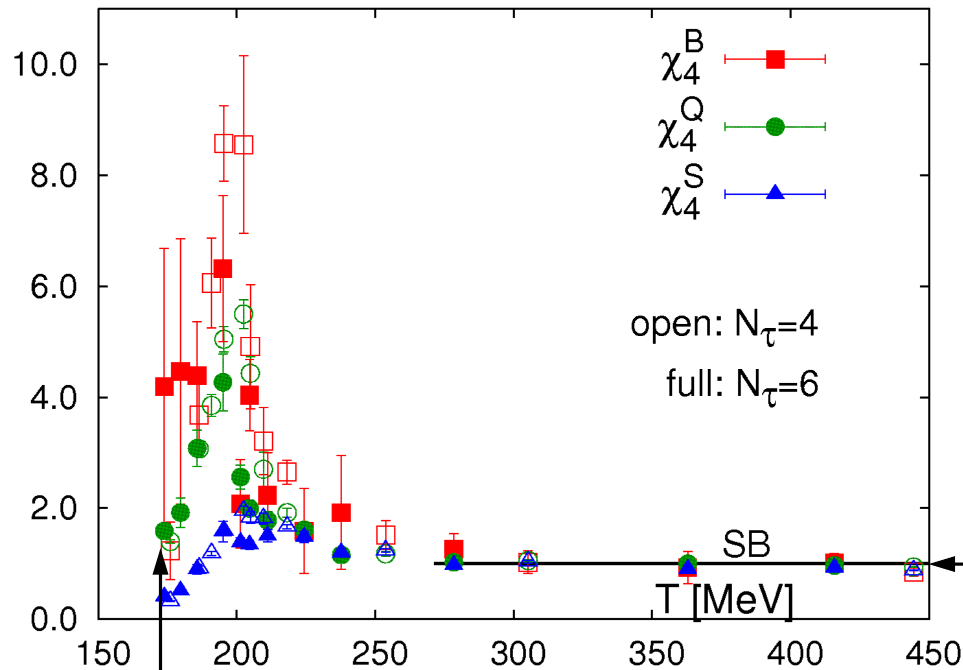
HotQCD: arXiv:1203.0784

Deconfinement : fluctuations of conserved charges

$$\chi_4^B = \frac{1}{VT^3}(\langle B^4\rangle - 2\langle B^2\rangle^2)$$ baryon number

$$\chi_4^Q = \frac{1}{VT^3}(\langle Q^2\rangle - 3\langle Q^2\rangle^2)$$ electric charge

$$\chi_4^S = \frac{1}{VT^3}(\langle S^2\rangle - 3\langle S^2\rangle^2)$$ strange quark number



Ideal gas of massless quarks :

$$\chi_4^B{}_{SB} = \frac{2}{9\pi^2} \qquad \chi_4^Q{}_{SB} = \frac{4}{3\pi^2}$$

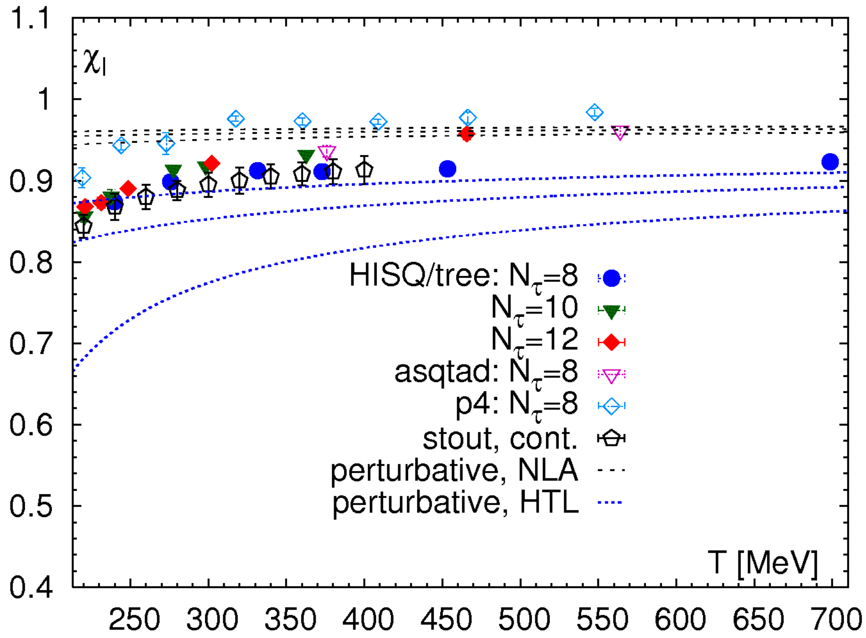
$$\chi_4^S{}_{SB} = \frac{6}{\pi^2}$$

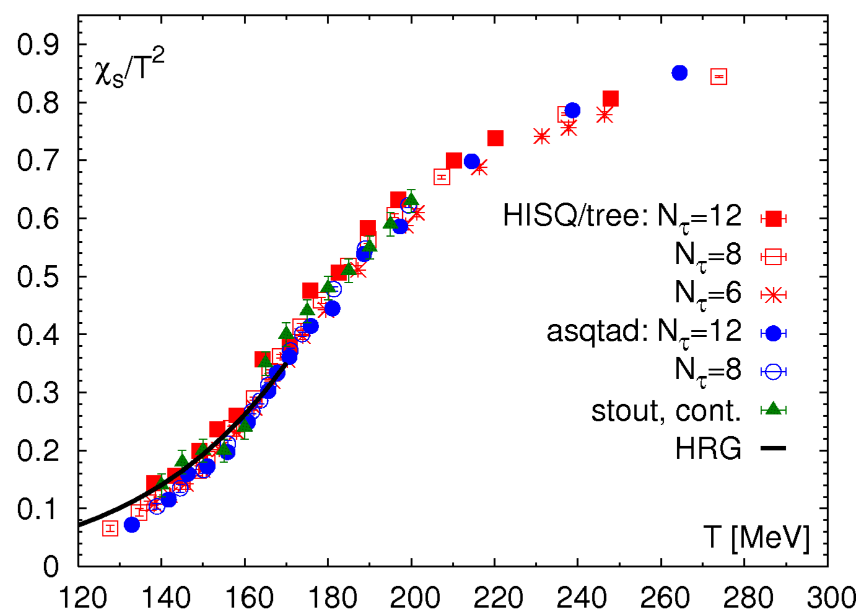
conserved charges carried by light quarks

conserved charges are carried by massive hadrons

Cheng et al, PD79 (09) 074505

Fluctuations at low and high temperatures



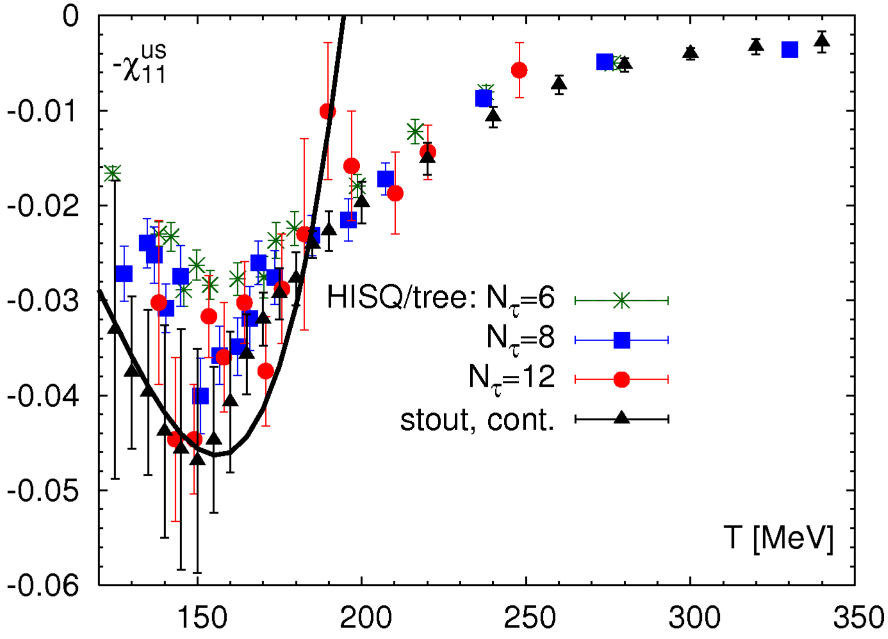


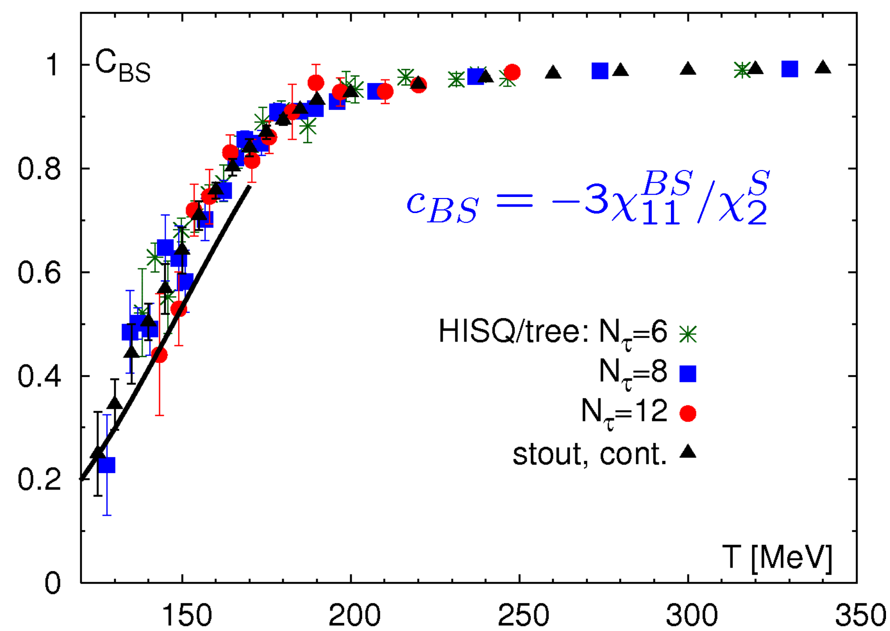
At sufficiently high T fluctuations can be described by perturnation theory because of asymptotic freedom

The quark number susceptibilities for $T>300$MeV agree with resummed petrurbative predictions
A. Rebhan, arXiv:hep-ph/0301130
Blaizot et al, PLB 523 (01) 143

hadrons are the relevant d.o.f. at low T
⇒ hadron gas + interactions
(approximated by s-channel resonances. e.g. $\pi\pi\rightarrow\rho$)
⇒ non-interacting hadron resonance gas (HRG)
Reasonable agreement between lattice results and HRG the remaining discrepancies are due to the lack of continuum extrapolation

Correlations of conserved charges





P.P. arXiv:1203.5320

- Correlations between strange and light quarks at low *T* are due to the fact that strange hadrons contain both strange and light quarks but very small at high *T (>250* MeV*)* => weakly interacting quark gas

- For baryon-strangeness correlations at low temperature agree with HRG result, at *T>250* MeV these correlations are very close to the ideal gas value

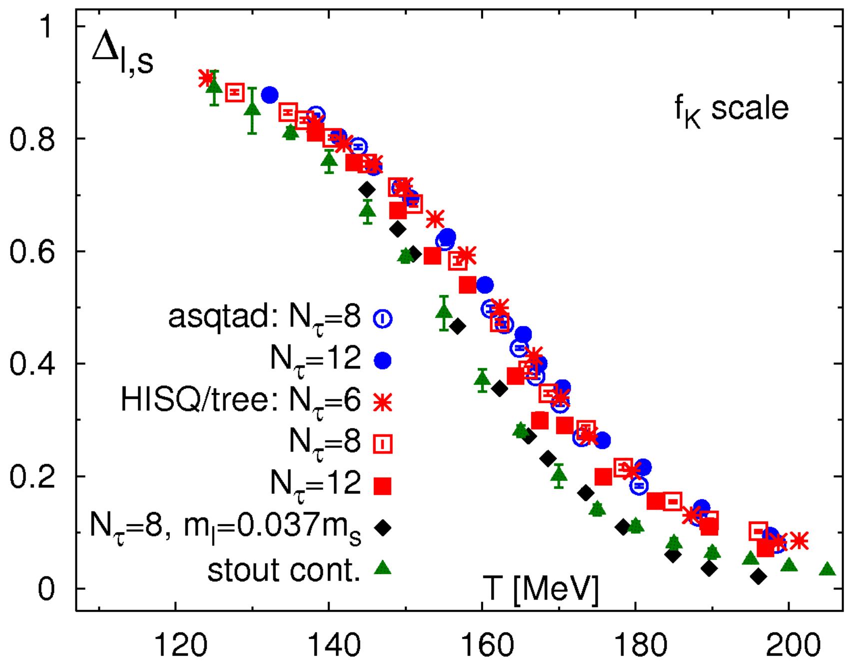
- The transition region where degrees of freedom change from hadronic to quark-like is broad ~ 50 MeV

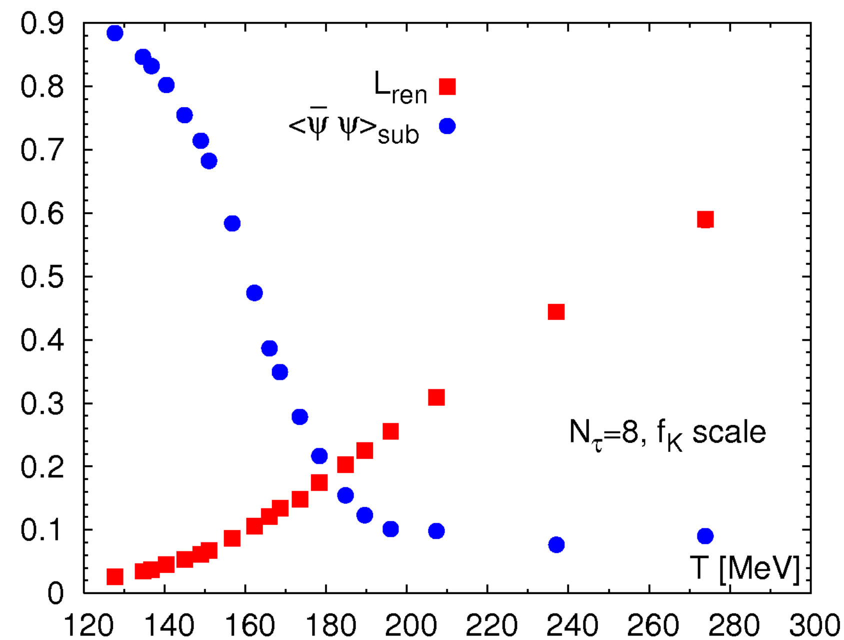
The temperature dependence of chiral condensate

Chiral condensate needs multiplicative and additive renormalization for non-zero quark mass

$$\langle\bar{\psi}\psi\rangle_l \Rightarrow \langle\bar{\psi}\psi\rangle_{sub} = \frac{\langle\bar{\psi}\psi\rangle_{l,T} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,T}}{\langle\bar{\psi}\psi\rangle_{l,T=0} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,T=0}}$$

P.P. arXiv:1203.5320





- Cut-off effects are significantly reduced when f_K is used to set the scale
- After quark mass interpolation based on *O(N)* scaling the HISQ/tree results agree with the stout continuum result !
- The deconfinement in terms of color screening sets in at temperatures higher than the chiral transition temperature

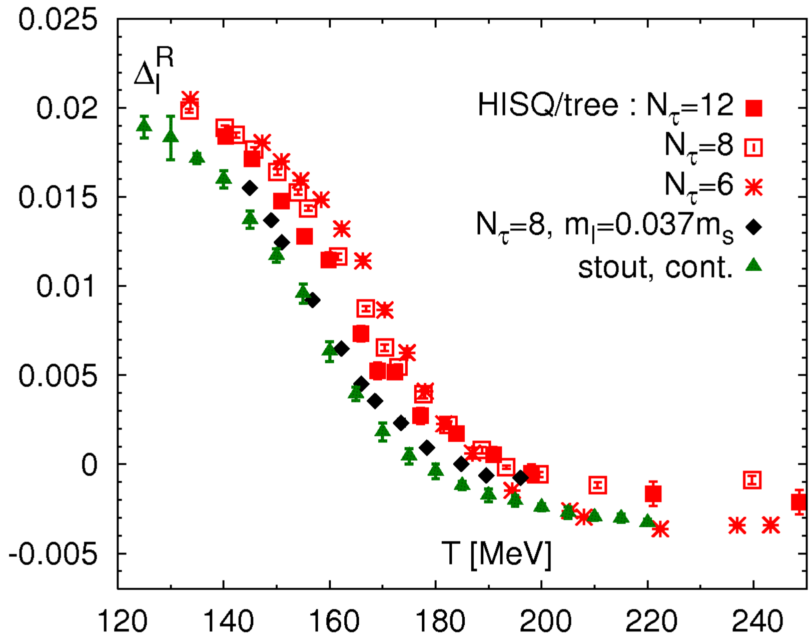
The temperature dependence of chiral condensate

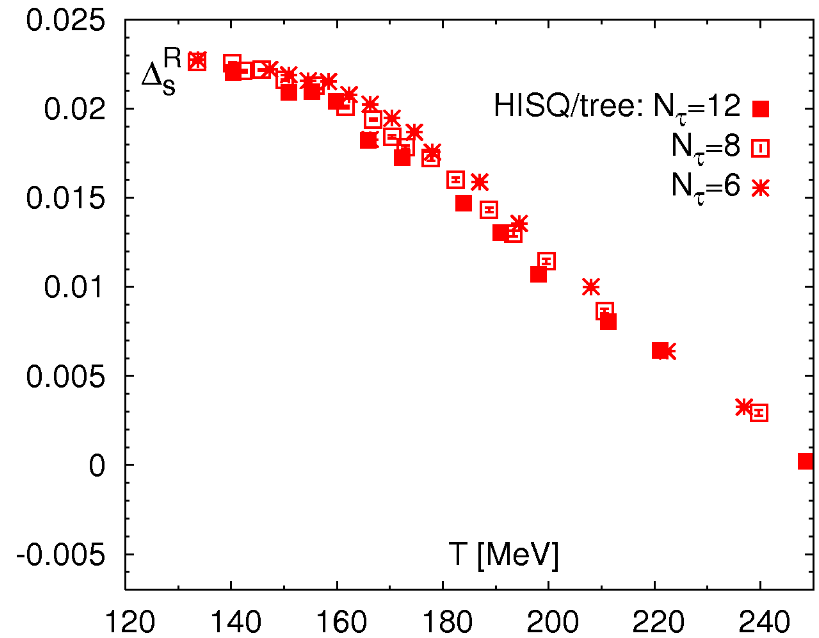
Renormalized chiral condensate introduced by Budapest-Wuppertal collaboration

$$\langle\bar{\psi}\psi\rangle_q \Rightarrow \Delta_q^R(T) = m_s r_1^4 \left(\langle\bar{\psi}\psi\rangle_{q,T} - \langle\bar{\psi}\psi\rangle_{q,T=0}\right) + d, \quad q = l, s$$

with our choice : $d = \langle\bar{\psi}\psi\rangle_{m_q=0}^{T=0}$

HotQCD : Phys. Rev. D85 (2012) 054503





• after extrapolation to the continuum limit and physical quark mass HISQ/tree calculation agree with stout results !

• strange quark condensate does not show a rapid change at the chiral crossover => strange quark do not play a role in the chiral transition

O(N) scaling and the chiral transition temperature

For sufficiently small m_l and in the vicinity of the transition temperature:

$$f(T,m_l) = -\frac{T}{V}\ln Z = f_{reg}(T,m_l)+f_s(t,h),\ t = \frac{1}{t_0}\left(\frac{T-T_c^0}{T_c^0}+\kappa\frac{\mu_q^2}{T^2}\right),\ H = \frac{m_l}{m_s}, h = \frac{H}{h_0}$$

governed by universal *O(4)* scaling $$M = -\frac{\partial f_s(t,h)}{\partial H} = h^{1/\delta} f_G(z),\ \ z = t/h^{1/\beta\delta}$$

T_c^0 is critical temperature in the mass-less limit, h_0 and t_0 are scale parameters

Pseudo-critical temperatures for non-zero quark mass are defined as peaks in the response functions (susceptibilities) :

$$\chi_{m,l} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_l^2} \sim m_l^{1/\delta-1} \qquad \chi_{t,l} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_l \partial t} \sim m_l^{\frac{\beta-1}{\beta\delta}} \qquad \chi_{t,t} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial t^2} \sim |t|^{-\alpha}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$T_{m,l}$ = $T_{t,l}$ = $T_{t,t}$ = $\boldsymbol{T_c^0}$

in the zero quark mass limit

$$\frac{\chi_{l,m}}{T^2} = \frac{T^2}{m_s^2}\left(\frac{1}{h_0}h^{1/\delta-1}f_\chi(z)+reg.\right)$$

universal scaling function has a peak at $z=z_p$

$$T_c(H) = T_{m,l} = T_c^0 + T_c^0\frac{z_p}{z_0}H^{1/(\beta\delta)} + ...$$

Caveat : staggered fermions O(2)

$m_l \to 0,\ a > 0,$

proper limit $a \to 0$, before $m_l \to 0$

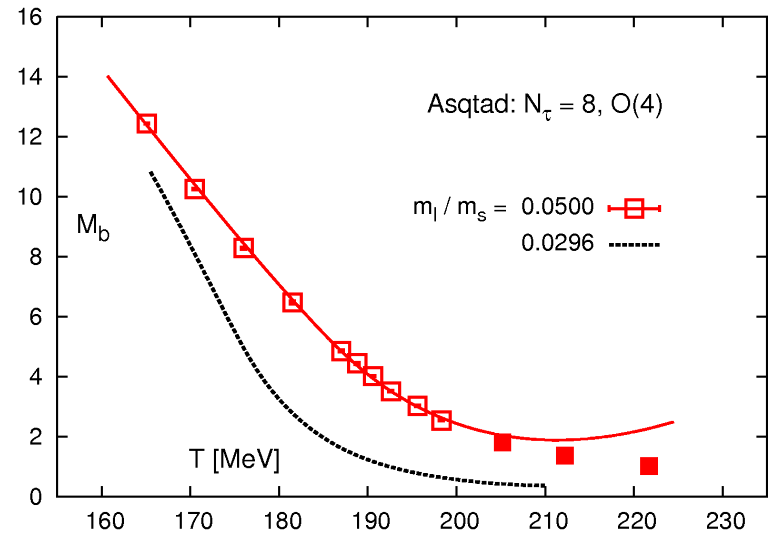
O(N) scaling and the transition temperature

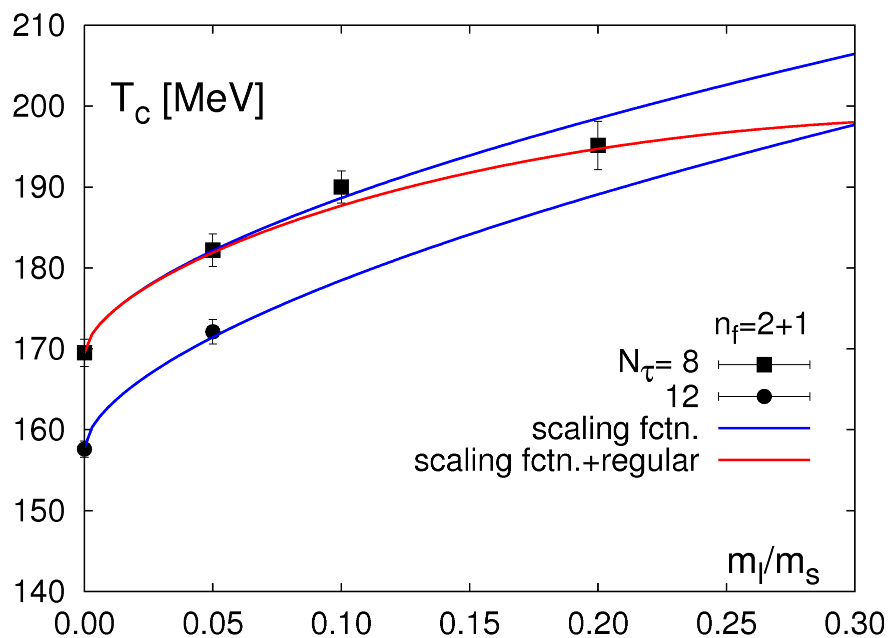
The notion of the transition temperature is only useful if it can be related to the critical temperature in the chiral limit : fit the lattice data on the chiral condensate with scaling form + simple Ansatz for the regular part

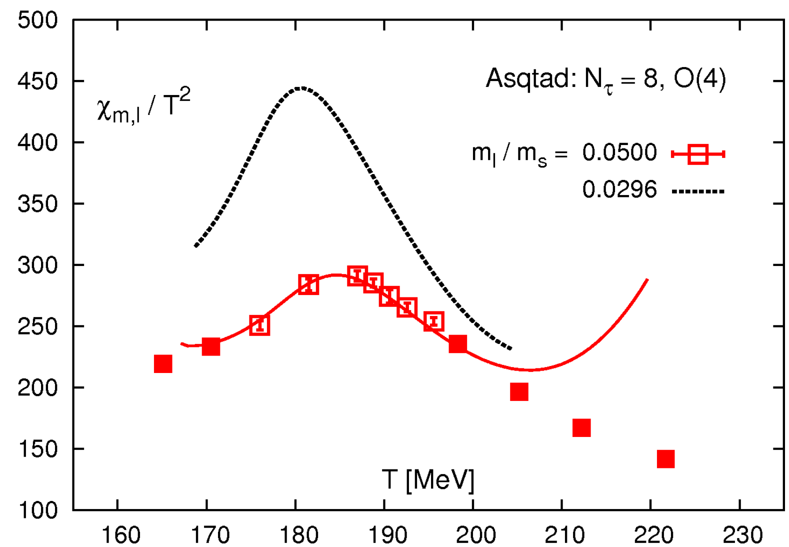
$$M_b = \frac{m_s \langle \bar{\psi}\psi \rangle_l}{T^4} = h^{1/\delta} f_G(t/h^{1/\beta\delta}) + f_{M,reg}(T,H)$$

$$f_{reg}(T,H) = (a_1(T - T_c^0) + a_2(T - T_c^0)^2 + b_1)H$$

6 parameter fit : T_c^0, t_0, h_0, a_1, a_2, b_1

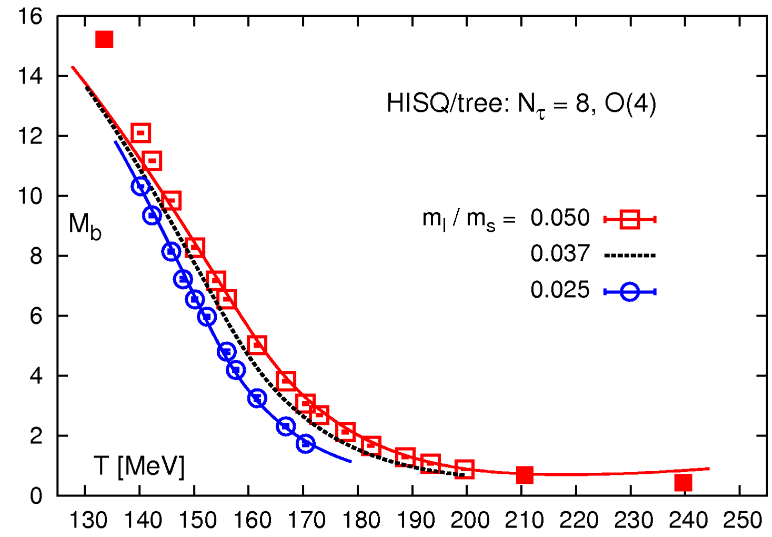






O(N) scaling and the transition temperature

The notion of the transition temperature is only useful if it can be related to the critical temperature in the chiral limit : fit the lattice data on the chiral condensate with scaling form + simple Ansatz for the regular part

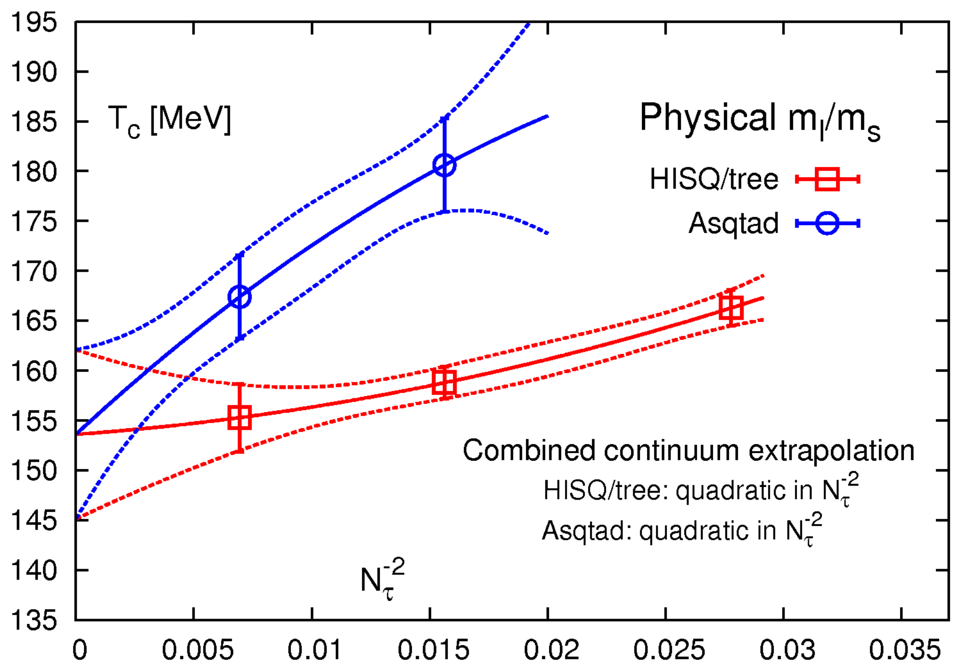


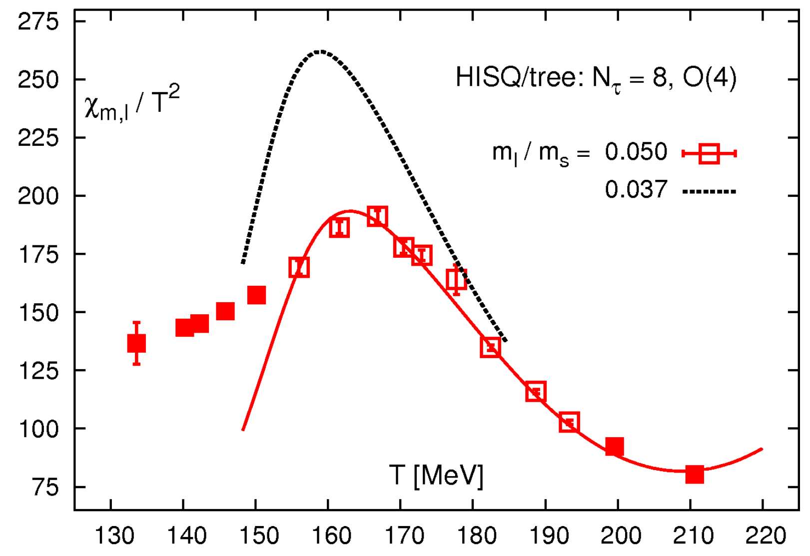
$$M_b = \frac{m_s \langle \bar\psi \psi \rangle_l}{T^4} = h^{1/\delta} f_G(t/h^{1/\beta\delta}) + f_{M,reg}(T,H)$$

$$f_{reg}(T,H) = (a_1(T - T_c^0) + a_2(T - T_c^0)^2 + b_1)H$$

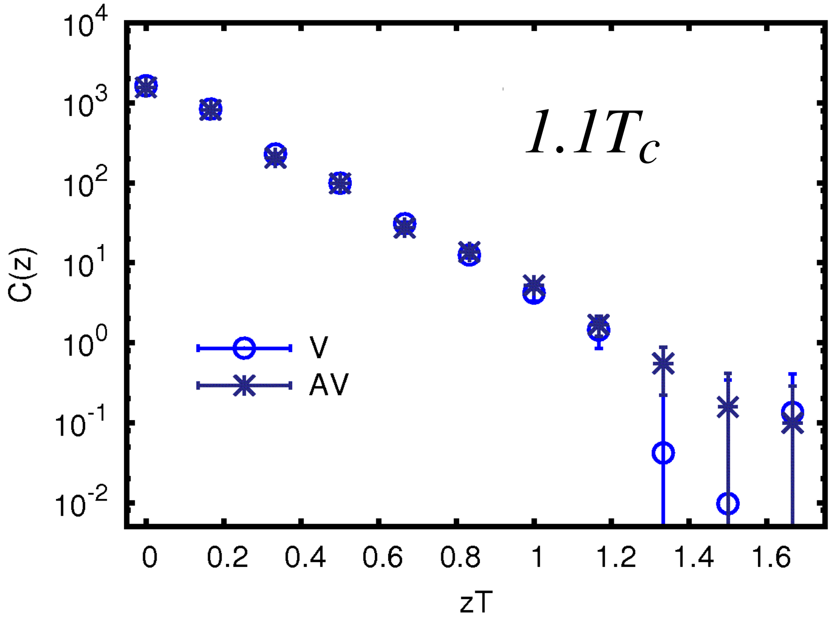
6 parameter fit : $T_c^{\,0}$, t_0, h_0, a_1, a_2, b_1

$$T_c = (154 \pm 8 \pm 1(scale))\text{MeV}$$





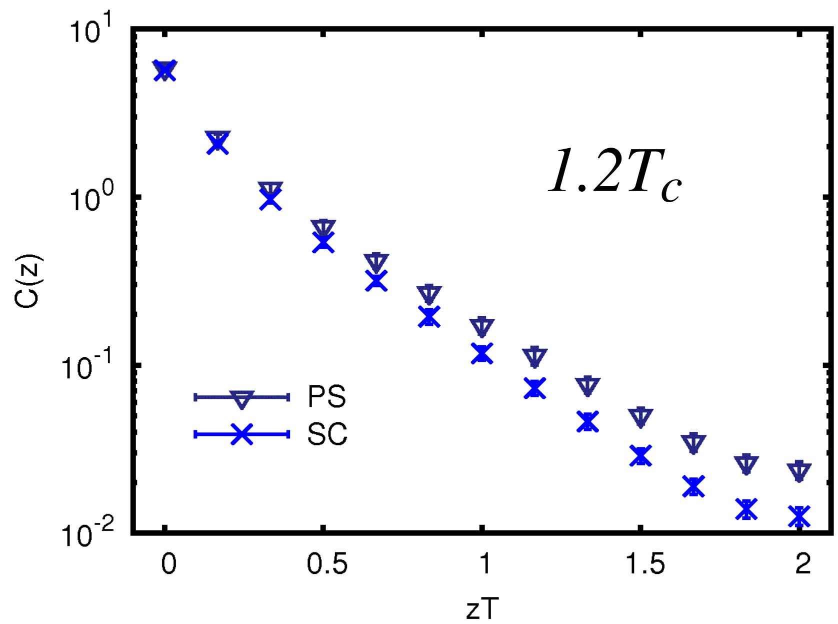
Meson correlators and chiral symmetry

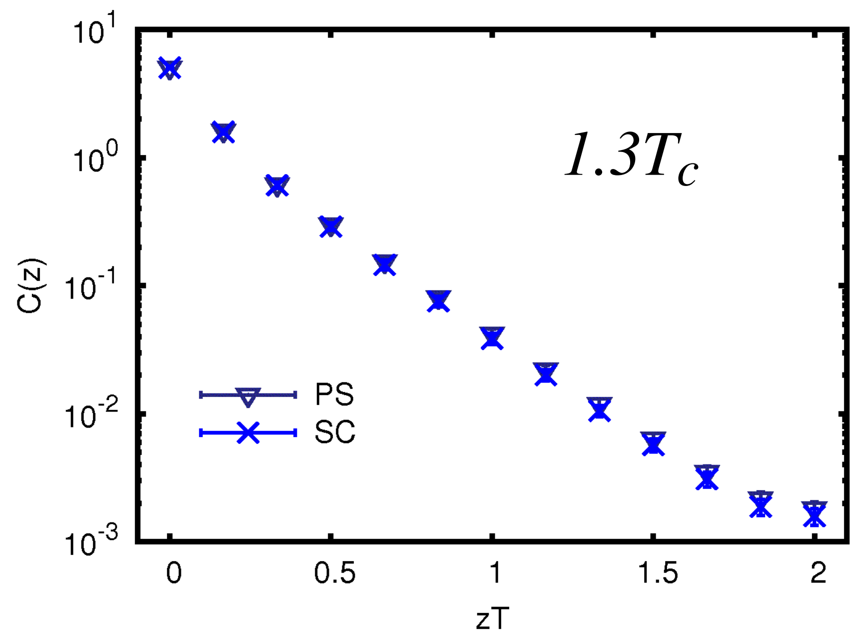


The restoration of the chiral symmetry manifests itself in the degeneracy of vector and axial-vector for $T>T_c$

The flavor non-singlet pseudo-scalar and scalar correlators become degenerate only at $1.3T_c$ => $U_A(1)$ is still broken at $1.2T_c$

Cheng et al, Eur. Phys. J. C71 (2011) 1564



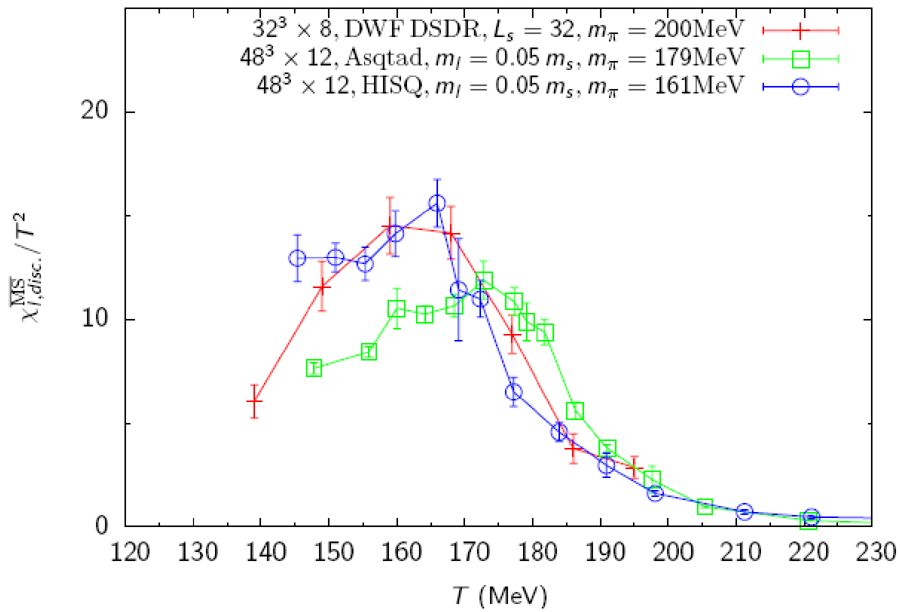


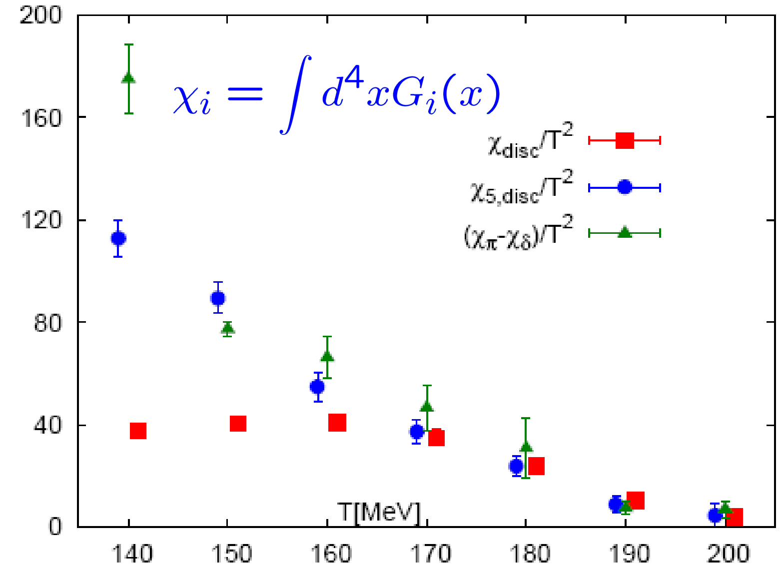
Chiral transition with other fermion formulations

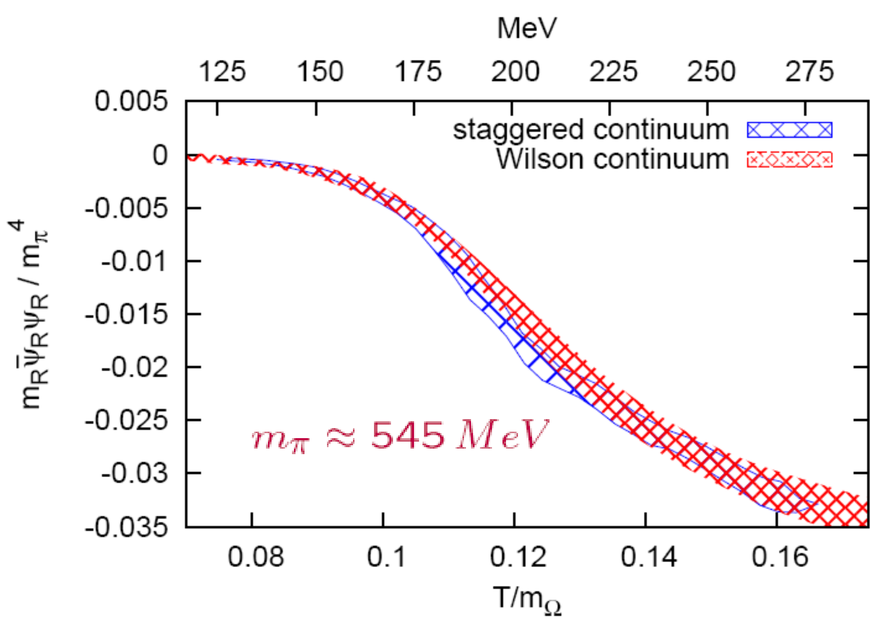
Lin et al (RBC) Lattice 2012

Domain wall fermions

HotQCD 2012







Wilson fermions

Nogradi et al (BW) Lattice 2012

Euclidean correlators and spectral functions

Lattice QCD is formulated in imaginary time

$$G(\tau,\vec{p},T)=\int d^3x\, e^{i\vec{p}\cdot\vec{x}}\left\langle J_H(\tau,\vec{x})\, J_H^{+}(0,0)\right\rangle,$$

$$J_H(\tau,\vec{x})=\overline{\psi}(\tau,\vec{x})\Gamma_H\psi(\tau,\vec{x})$$

$$\Gamma_H=1,\ \gamma_5,\ \gamma_\mu,\ \gamma_5\cdot\gamma_\mu$$

Physical processes take place in real time

$$D^{>}(t,\vec{p},T)=\int d^3x\, e^{i\vec{p}\cdot\vec{x}}\left\langle J_H(t,\vec{x})\, J_H\ (0,0)\right\rangle,$$

$$D^{<}(t,\vec{p},T)=\int d^3x\, e^{i\vec{p}\cdot\vec{x}}\left\langle J_H(0,\vec{0})\, J_H\ (t,\vec{x})\right\rangle$$

$$\frac{D^{>}(\omega)-D^{<}(\omega)}{2\pi}=\frac{1}{\pi}\operatorname{Im}D_R(\omega)=\sigma(\omega)$$

$$R(\omega)=\frac{\sigma_{e^+e^-\to hadrons}}{\sigma_{e^+e^-\to\mu^+\mu^-}}=\frac{\sigma(\omega)}{\omega^2}$$

$$G(\tau,T)=D^{>}(-i\tau)$$

$$G(\tau,T)=\int_0^\infty d\omega\,\sigma(\omega,T)\frac{\cosh(\omega(\tau-1/(2T)))}{\sinh(\omega/(2T))}$$

if $T=0$ and $\sigma(\omega)=\sum_n A_n\delta(\omega-E_n)$ ⇒ $G(\tau)=A_0e^{-E_0\tau}+A_1e^{-E_1\tau}+\ldots$

fit the large distance behavior of the lattice correlation functions

This is not possible for $T>0,\ \tau_{max}=1/T$ ⇒ Maximum Entropy Method (MEM)

Spectral functions at T>0 and physical observables

Heavy meson spectral functions:

$$J_H = \overline{\psi}\,\Gamma_H\,\psi$$

→ quarkonia properties at T>0
heavy quark diffusion in QGP: *D*

Quarkonium suppression (R_{AA})
Open charm/beauty suppression (R_{AA})

thermal dilepton production rate
(# of dileptons/photons per unit 4-volume)

Light vector meson spectral functions:

$$J_\mu = \overline{\psi}\;\gamma_\mu\,\psi$$

→

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha_{em}^2}{27\pi^2}\frac{1}{e^{\omega/T}-1}\frac{\sigma_{\mu\mu}(\omega,p,T)}{\omega^2-p^2}$$

thermal photon production rate :

$$p\frac{dW}{d^3p} = \frac{5\alpha_{em}}{9\pi}\frac{1}{e^{p/T}-1}\sigma_{\mu\mu}(\omega=p,p,T)$$

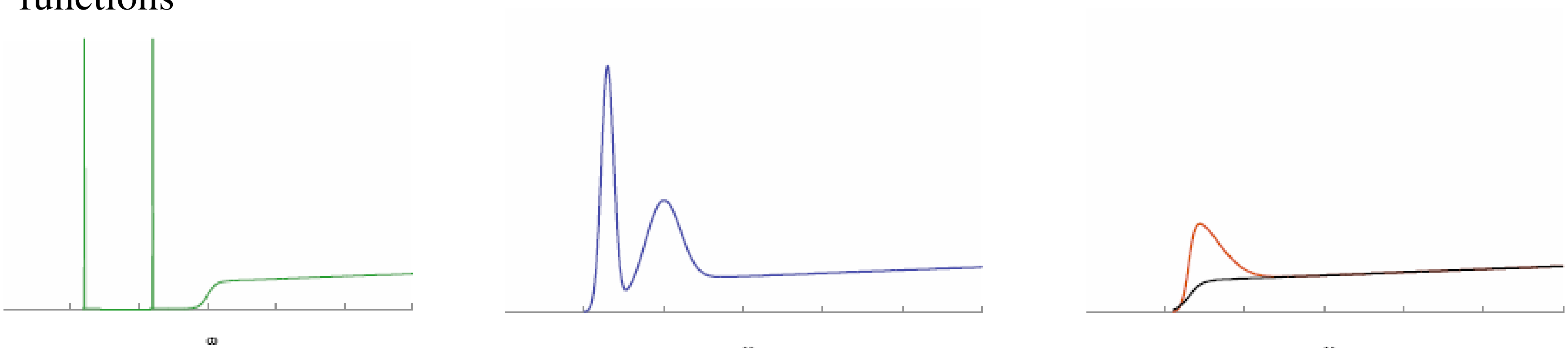
Thermal photons and dileptons provide information about the temperature of the medium produced in heavy ion collisions
Low mass dileptons are sensitive probes of chiral symmetry restoration at $T>0$

2 massless quark (u and d) flavors are assumed; for arbitrary number of flavors $5/9 \rightarrow \Sigma_f Q_f^2$

electric conductivity ζ :

Meson spectral functions and lattice QCD

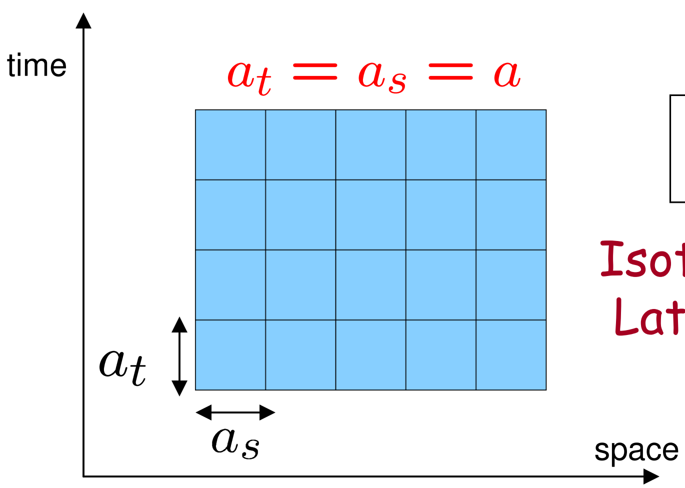
In-medium properties and/or dissolution of quarkonium states are encoded in the spectral functions



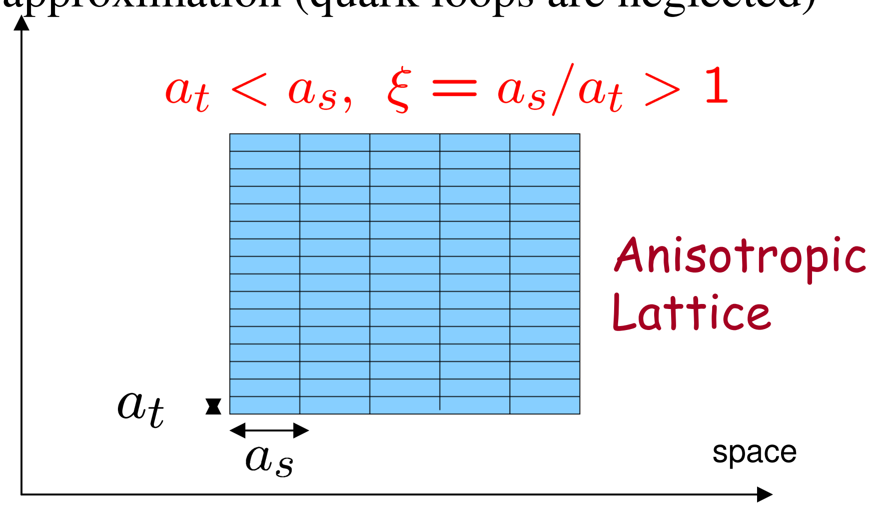
Melting is seen as progressive broadening and disappearance of the bound state peaks

Need to have detailed information on meson correlation functions → large temporal extent N_τ
Good control of discretization effects → small lattice spacing a

Computationally very demanding → use quenched approximation (quark loops are neglected)

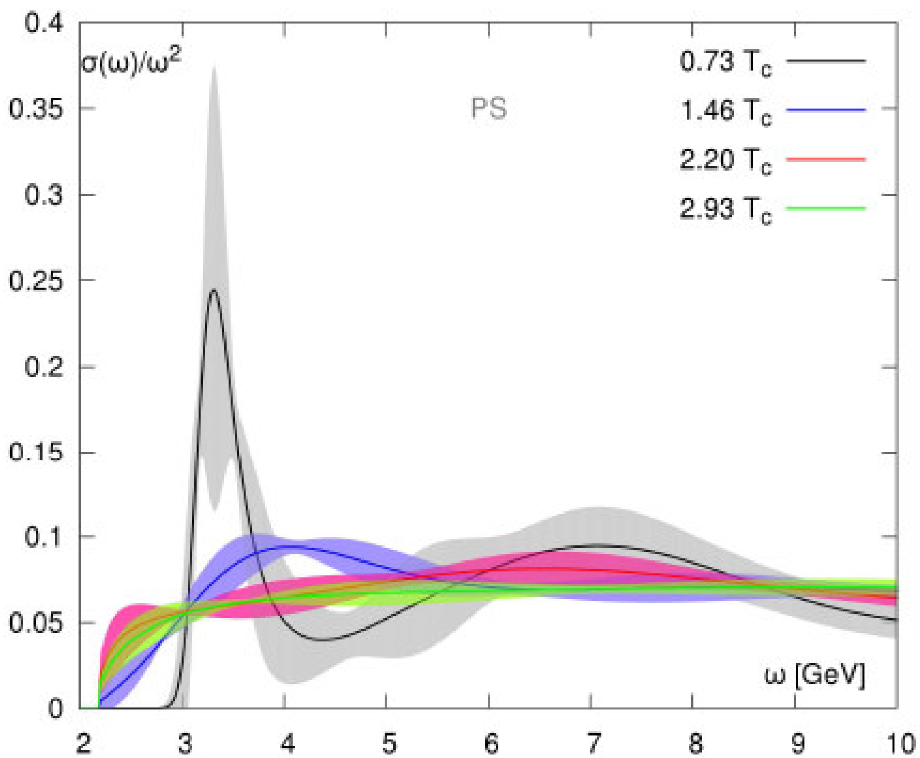


$$T = \frac{1}{N_\tau a_t}$$



Charmonium spectral functions from MEM

Charmonium spectral functions on isotropic lattice in quenched approximation with Wilson quarks:
H.-T. Ding et al, Lattice2010 180
$N_\tau=24\text{-}96,\ a^{-1}=18.97GeV$



No clear evidence for charmonium bound state peaks above T_c
from MEM spectral functions !

Transport contribution to the vector correlators

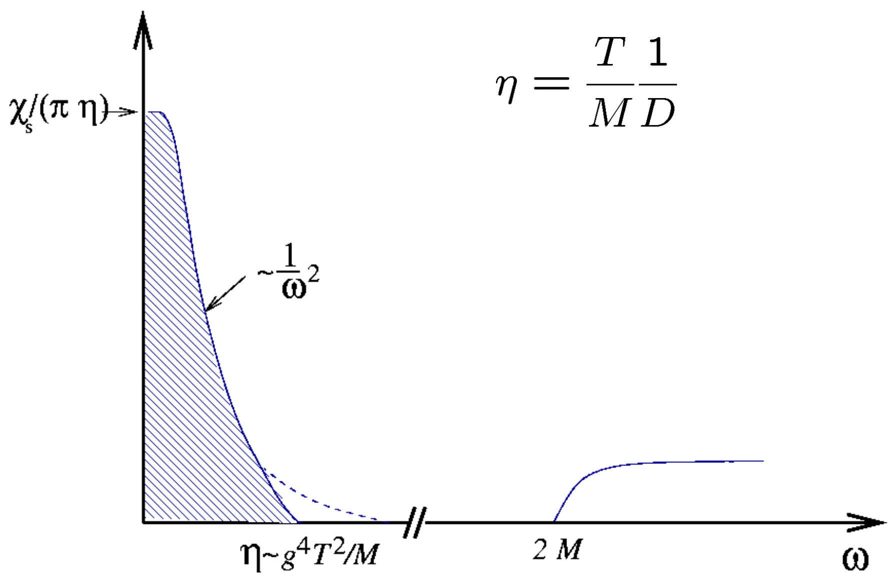
Vector correlator in the free theory for massive quarks :

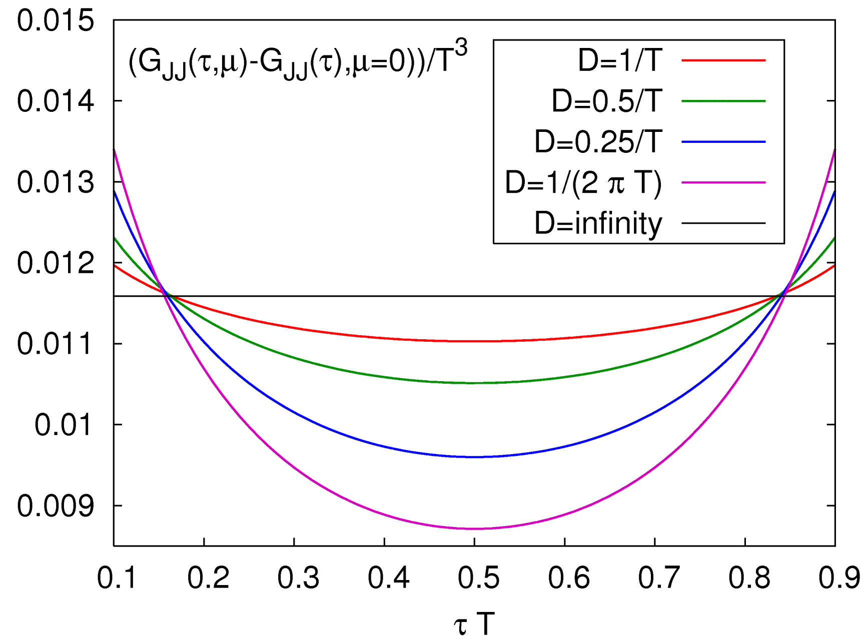
$$\sigma_V^{ii}(\omega) = \frac{1}{4\pi^2}\omega^2\sqrt{1-\frac{4M^2}{\omega^2}}\Theta(\omega-2M) \; + \chi_s(T) v_{therm}^2 \omega\delta(\omega) \qquad v_{therm}^2 \simeq \frac{T}{M}$$

Interactions smear out the $\chi\omega\delta(\omega)$ term $width \sim \eta \sim 1/\tau_{relax}$

$$\delta(\omega) \to \frac{1}{\pi}\frac{T}{M}\frac{\eta}{\omega^2+\eta^2}$$

$$\eta = \frac{T}{M}\frac{1}{D}$$



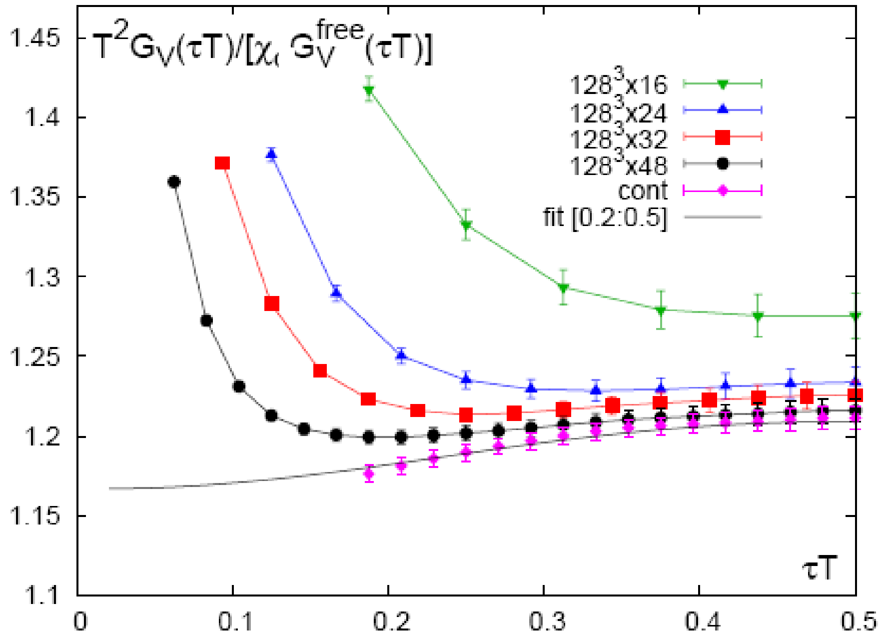


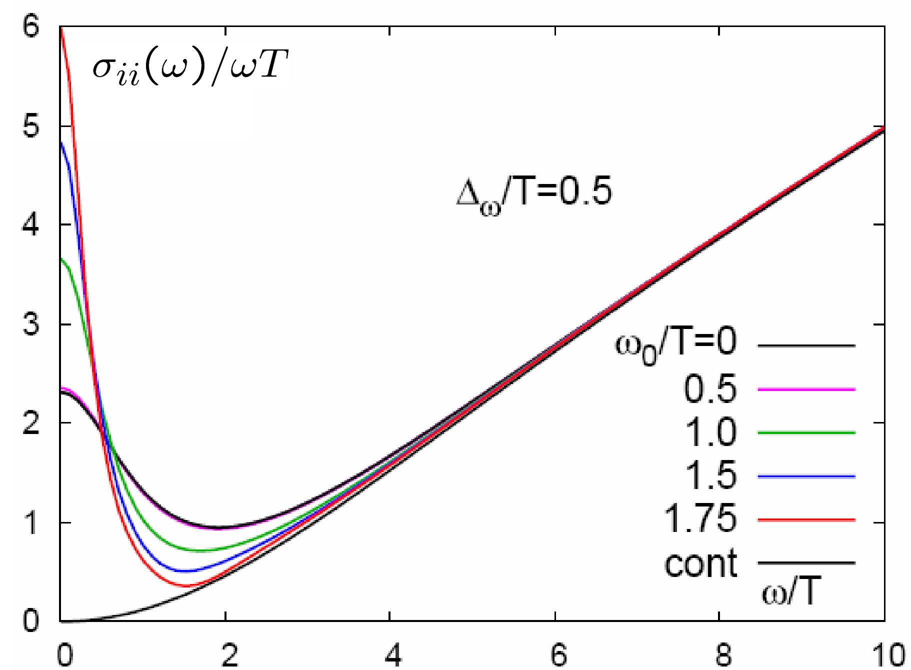
P.P., Teaney, PRD 73 (06) 014508

Lattice calculations of the vector spectral functions

Ding et al, PRD 83 (11) 034504

Isotropic Wilson gauge action, quenched non-perturbatively improved clover fermion action on $128^3 \times N_\tau$ lattices, $T = 1.45T_c$, $m_q^{\overline{MS}}(2\text{GeV}) = 0.1/T$, $N_\tau = 24,\ 32, 48\ (a^{-1} = 9.4 - 18.8\text{GeV})$





$$\sigma_{ii}(\omega) = \chi c_{BW} \frac{1}{\pi} \frac{\omega \Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{4\pi^2}(1+k)\omega^2 \tanh(\omega/4T)\Theta(\omega_0, \Delta_\omega),$$

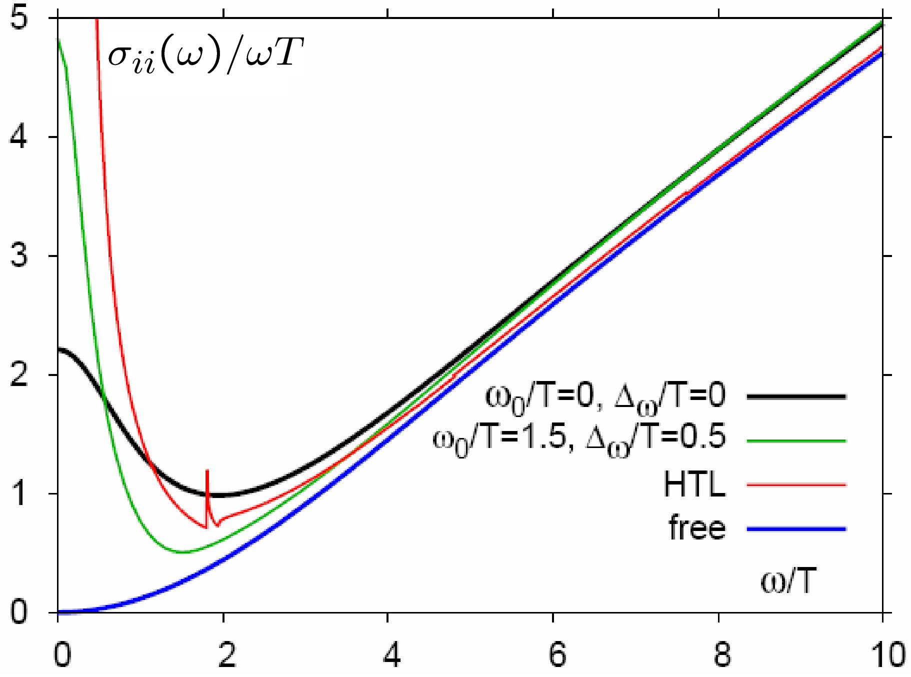
$$\Theta(\omega_0, \Delta_\omega) = (1 + e^{(\omega_0^2 - \omega^2)/\omega\Delta_\omega})^{-1}$$

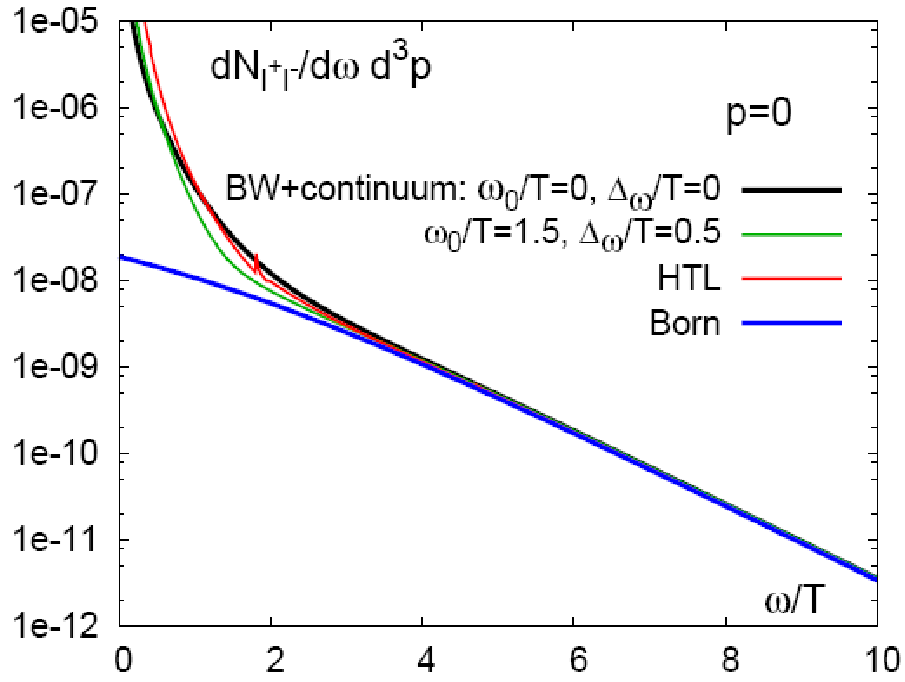
Fit parameters : $c_{BW},\ \Gamma,\ k$ Different choices of : ω_0, Δ_ω

Lattice calculations of the vector spectral functions

Ding et al, PRD 83 (11) 034504

Isotropic Wilson gauge action, quenched non-perturbatively improved clover fermion action on $128^3 \times N_\tau$ lattices, $T = 1.45T_c$, $m_q^{\overline{MS}}(2\text{GeV}) = 0.1/T$,
$N_\tau = 24,\ 32, 48\ (a^{-1} = 9.4 - 18.8\text{GeV})$



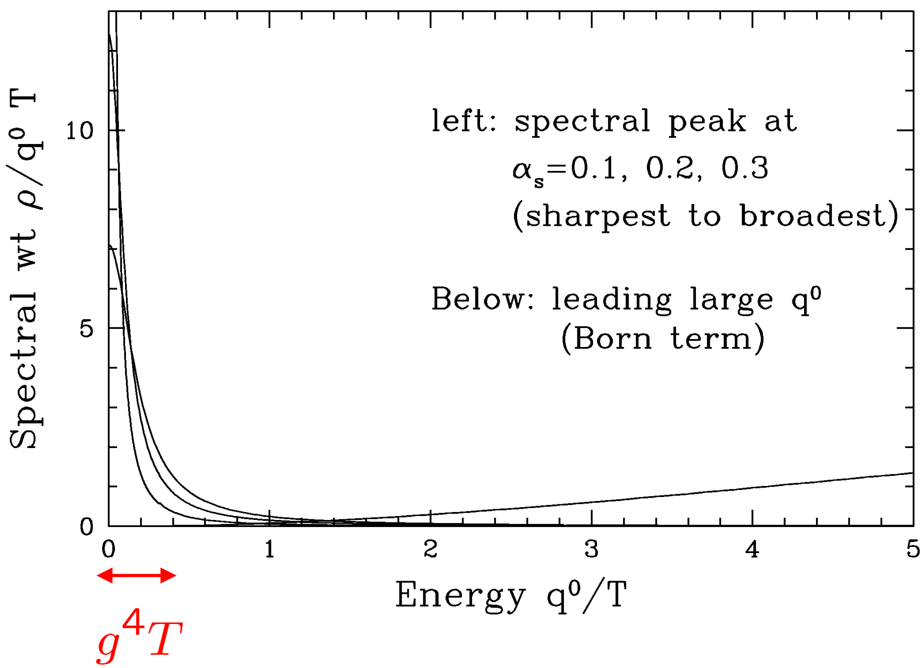


- The HTL resummed perturbative result diverges for $\omega \to 0$ limit
- The lattice results show significant enhancement over the LO (Born) result for small ω
- Electric conductivity: $\zeta = \lim_{\omega\to 0}\sigma_{ii}(\omega)/\omega, \quad 1/3 < \frac{1}{C_{em}}\frac{\zeta}{T} < 1,\ C_{em} = \sum_f Q_f^2$

Strongly coupled or weakly coupled QGP ?

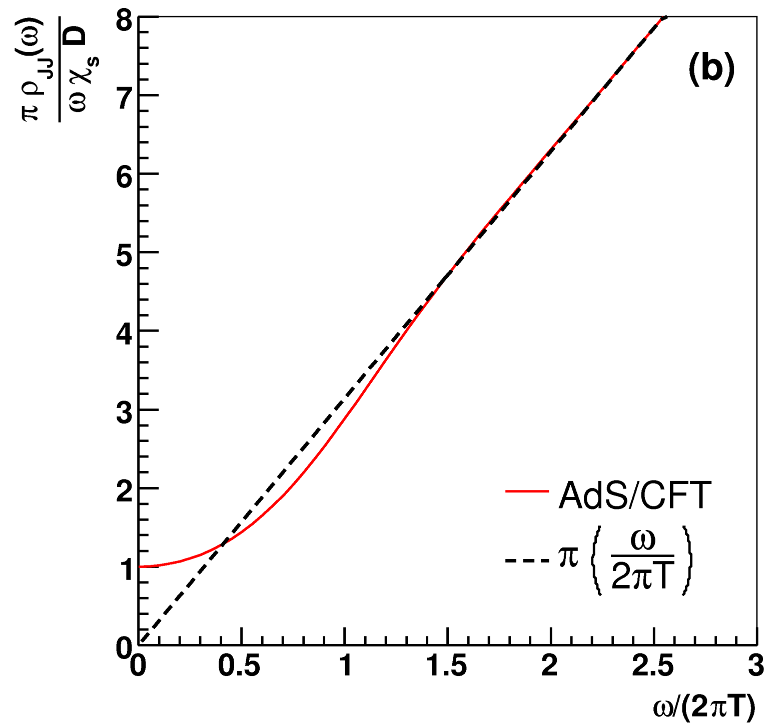
Weak coupling caculation of the vector current spectral function in QCD

Moore, Robert, hep-ph/0607172

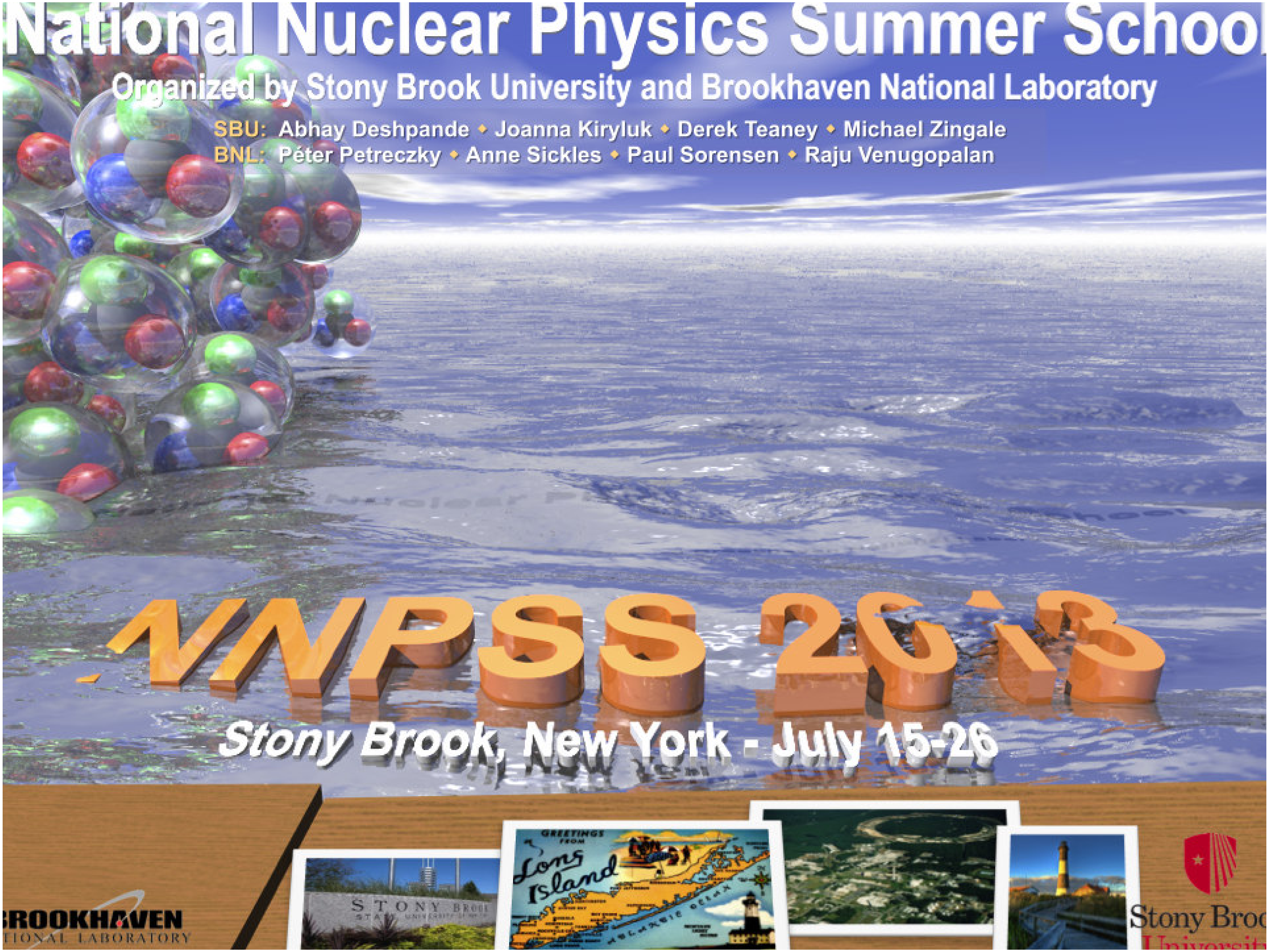


vector current correlator in N=4 SUSY at strong coupling

Teaney, PRD74 (06) 045025



lattice results are closer to the weakly coupled QGP


National Nuclear Physics Summer School
Organized by Stony Brook University and Brookhaven National Laboratory
SBU: Abhay Deshpande • Joanna Kiryluk • Derek Teaney • Michael Zingale
BNL: Péter Petreczky • Anne Sickles • Paul Sorensen • Raju Venugopalan
NNPSS 2013
Stony Brook, New York - July 15-26
GREETINGS FROM
Long Island
STONY
Stony Bro

Summary

• Lattice QCD show that at high temperatures strongly interacting matter undergoes a transition to a new state QGP characterized by deconfinement and chiral symmetry restoration

• We see evidence that provide evidence that the relevant degrees of freedom are quarks and gluons; lattice results agree well with perturbative calculations, while at low *T* thermodynamics can be understood in terms of hadron resonance gas The deconfinement transition can understood as transition from hadron resonance gas to quark gluon gas it is gradual and analogous to ionized gas – plasma transition (implications for sQGP and early thermalization at RHIC ?)

• The chiral aspects of the transition are very similar to the transition in spin system in external magnetic fields: it is governed by universal scaling

• Different calculations with improved staggered actions agree in the continuum limit resulting in a chiral transition temperature (154 ± 9) MeV

Meson spectral functions can also be studied in QCD, calculations of the charmonium spectral functions is consistent with melting of heavy quark bound states, calculations of the light vector current spectral function indicate an existence of a transport peak and provide an estimate of electric conductivity