III: Additional Symmetries

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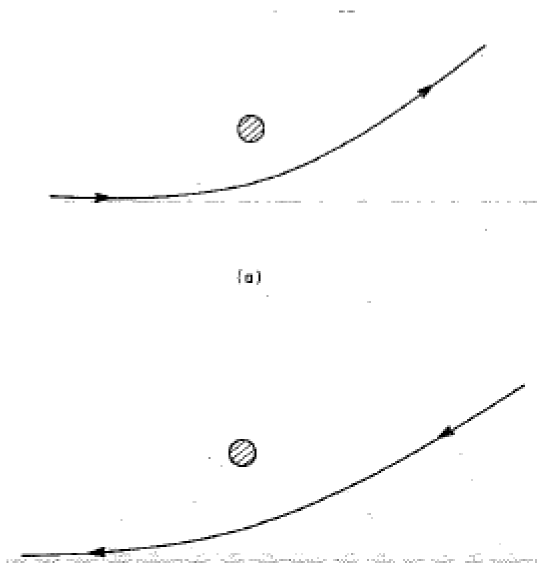
Time Reversal

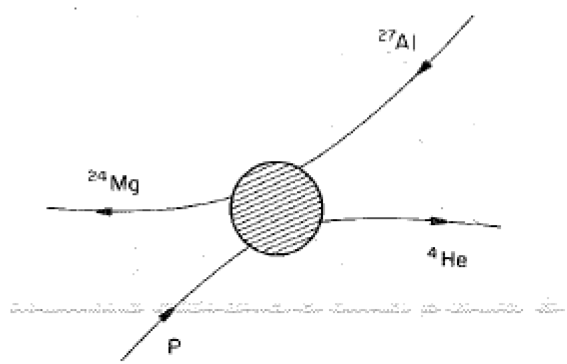
Now look at time reversal. Run movie backward—can you tell?



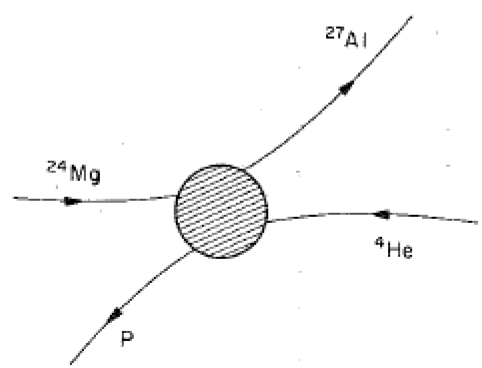
How to test?

i) Detailed Balance: Run reaction backward—

$$p + {}^{27}Al \leftrightarrow \alpha + {}^{24}Mg$$



(a)



(b)

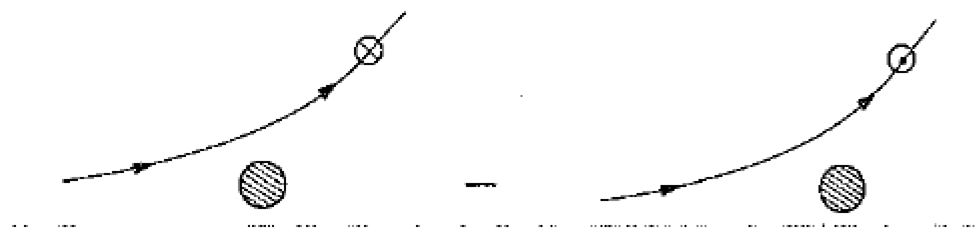
Best such experiments yield

$$\left|\frac{\mathrm{Amp(T-violating)}}{\mathrm{Amp(T-conserving)}}\right| < 5\times 10^{-4}$$

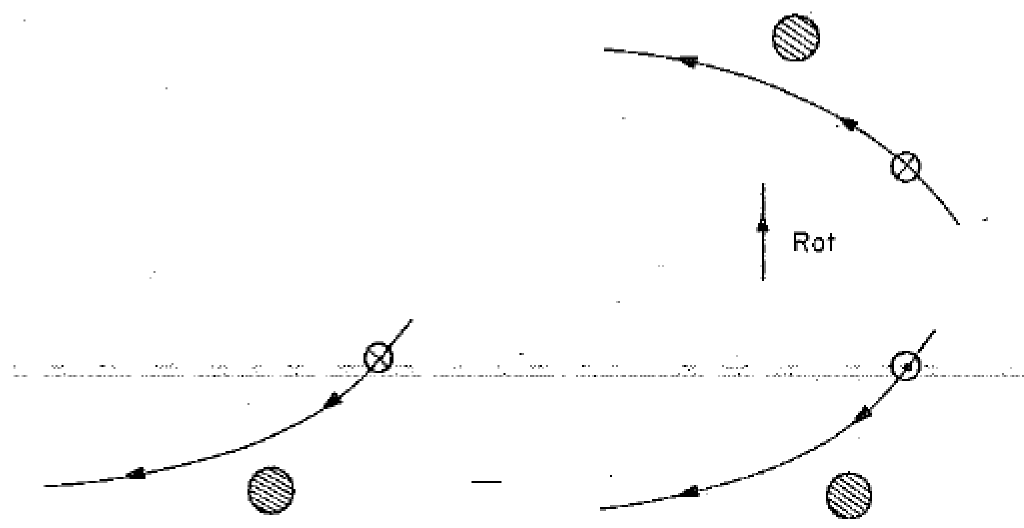
ii) Polarization (P) $=$ Asymmetry (A)

LAMPF np scattering experiment at p_L=800 MeV, $\theta = 133^o$ gave

$$P - A = 0.01 \pm 0.02$$



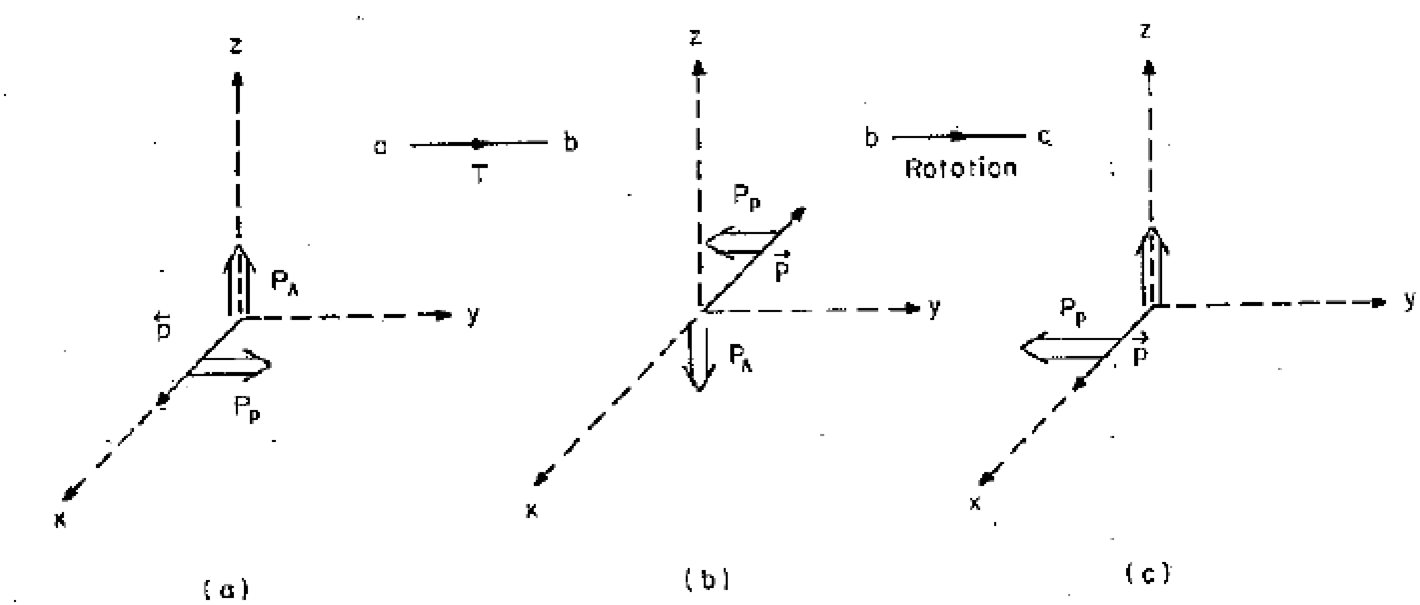
(a)



(b)

iii) Triple Product Correlations: Consider hyperon decay—$\Lambda \to p\pi^-$—and measure K

$$\frac{d\Gamma}{d\Omega_\pi} \sim 1 + K\vec{J}_\Lambda \cdot \vec{J}_p \times \vec{\beta}_\pi + \ldots$$



A subtlety:

$$< p\pi^- |\mathcal{H}_w| \Lambda >= A_s \chi_p^\dagger \chi_\Lambda + A_p \chi^\dagger \vec{\sigma} \chi_\Lambda \cdot \hat{p}_\pi$$

and

$$K = \frac{2\mathrm{Im} A_s * A_p}{|A_s|^2 + |A_p|^2}$$

but Fermi-Watson theorem requires

$$A_s = |A_s| e^{i\delta_s^{p\pi^-}} \quad \text{and} \quad A_p = |A_p| e^{i\delta_p^{p\pi^-}}$$

so $K \neq 0$ even if T is conserved. Must subtract FSI effects.

For this experiment

$$\arg A_s - \arg A_p = 9.0 \pm 5.5^o$$

vs.

$$\delta_s^{p\pi^-} - \delta_p^{p\pi^-} = 6.5 \pm 1.5^o$$

iv) Can also use beta decay via $\vec{J} \cdot \vec{p}_e \times \vec{p}_{\nu_e}$ correlation—

$$\frac{d\Gamma}{d\Omega_\pi} \sim 1 + D\vec{J}_\Lambda \cdot \vec{p}_e \times \hat{p}_{\nu_e}/E_e + \ldots$$

To leading order

$$D = \frac{2\mathrm{Im}a * c}{|a|^2 + |c|^2} + \ldots$$

FSI in this case small from electromagnetic FSI involving c,b.

Results are

$^{19}\mathrm{Ne}: \ D^{exp} = (4 \pm 8) \times 10^{-4} \ D^{FSI} = 2.5 \times 10^{-4}$

$n: \ D^{exp} = (-2.8 \pm 7.1) \times 10^{-4} \ D^{FSI} = 1.1 \times 10^{-5}$

v) Most sensitive test from EDM, *i.e.* $H_E = -d_e \vec{S} \cdot \vec{E}$, analogous to MDM, for which $H_M = -d_M \vec{S} \cdot \vec{B}$. $H_E(H_M)$ is odd(even) under both parity

$$(\vec{S} \to \vec{S}, \vec{E} \to -\vec{E}, \vec{B} \to \vec{B})$$

and time reversal

$$(\vec{S} \to -\vec{S}, \vec{E} \to \vec{E}, \vec{B} \to -\vec{B})$$

Units are ecm. Note

$$d_M \sim \int d^3 r \vec{r} \times \vec{j} \sim e \times 10^{-13}\,\mathrm{cm}$$

In standard model CP violation requires heavy quark and must then be second order, so

$$d_E \sim 10^{-13}\, ecm \times (G m_\pi^2)^2 |V_{td}|^2 \sim 10^{-32}\, ecm$$

However, another standard model source is anomaly

$$\mathcal{L}_{QCD}^{anom} = \frac{\theta}{16\pi^2} F_g^{\mu\nu} \epsilon_{\mu\nu\alpha\beta} F_g^{\alpha\beta}$$

Since

$$F_g^{\mu\nu} \epsilon_{\mu\nu\alpha\beta} F_g^{\alpha\beta} \sim \vec{E}_g \cdot \vec{B}_g$$

is odd under both P and T—can produce EDM via pion loop

$$d_E(n) \sim e \frac{g_{\pi NN} \bar{g}_{\pi NN}}{4\pi^2 M_N} \log \frac{M_N}{m_\pi} \sim 4 \times 10^{-16} \theta$$

Challenge is to get large (and reversible) electric field, since direct field breaks down $\sim 10^4$ V/cm. Solution is to use atomic fields, which can be MUCH larger. Must account for Schiff screening, which completely shields system from external field, except for relativistic and finite size effects. In molecules can use laser excitation to manipulate electric fields and measure electron EDM.

Current limits are

$$d_E(n) < 2 \times 10^{-26} \text{ ecm}$$

$$d_E(e) < 2 \times 10^{-27} \text{ ecm}$$

$$d_E(^{199}Hg) < 2 \times 10^{-29} \text{ ecm}$$

Can get enhancement effects for certain nuclei when near-degenerate opposite parity states involved. Examples:

$$^{161}Dy \sim 10,\ ^{153}Sm \sim 100,\ ^{224}Pa \sim 1000$$

Another Symmetry: Lepton Number— $L_e(e^-, \nu_e) = +1, L_e(e^+, \bar{\nu}_e) = -1.$ Conserved in beta decay since $L_e(e + \bar{\nu}_e) = 0.$ Because of pairing force, energy of even-even nuclei lower than odd-odd. In single beta decay of even-even have $(A, Z) \rightarrow (A, Z + 1)$ so goes to odd-odd. Then odd-odd can decay to even-even. If first decay is kinematically forbidden, then can have double beta decay

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\bar{\nu}_e$$

Nucleus	$T^{2\nu}_{\frac{1}{2}}$(Yr)
^{48}Ca	$(3.9 \pm 0.7 \pm 0.6) \times 10^{19}$
^{76}Ge	$(1.7 \pm 0.2) \times 10^{21}$
^{82}Se	$(9.6 \pm 0.3 \pm 1.0) \times 10^{19}$
^{96}Zr	$(2.0 \pm 0.3 \pm 0.2) \times 10^{19}$
^{100}Mo	$(7.11 \pm 0.02 \pm 0.54) \times 10^{18}$
^{116}Cd	$(2.8 \pm 0.1 \pm 0.3) \times 10^{19}$
^{128}Te	$(2.0 \pm 0.1) \times 10^{24}$
^{130}Te	$(7.6 \pm 1.5 \pm 0.8) \times 10^{20}$
^{150}Nd	$(9.2 \pm 0.25 \pm 0.73) \times 10^{18}$
^{238}U	$(2.0 \pm 0.6) \times 10^{21}$

What do we learn? Rate is

$$\Gamma_{2\nu} = m_e^{11} F_2\left(\frac{Q}{m_e}\right)\left|g_A^2 M_{GT} - g_V^2 M_F\right|^2$$

$$\times\left(\frac{\mathcal{F}(Z)}{E_i - < E_n > -\frac{1}{2}E_0}\right)$$

where

$$F_2(x) = x^7\left(1 + \frac{x}{2} + \frac{x^2}{9} + \frac{x^3}{90} + \frac{x^4}{1980}\right)$$

$$M_F = < f|\frac{1}{2}\sum_{ij}\tau_i^+\tau_j^-|i>$$

$$M_{GT} = < f|\frac{1}{2}\sum_{ij}\tau_i^+\tau_j^-\vec{\sigma}_i\cdot\vec{\sigma}_j|i>$$

More interesting if neutrino is Majorana— $\nu = \bar{\nu}$. Then neutrinoless double beta decay

$$(A, Z) \to (A, Z+2) + 2e^-$$

$$\Gamma_{0\nu} \sim m_e^7 F_) \left(\frac{Q}{m_e}\right) \left| G_A^2 \tilde{M}_{GT} - g_V^2 \tilde{M}_F \right|^2 \frac{< m_\nu >^2}{m_e^2}$$

$$F_0(x) = x\left(1 + 2x + \frac{4x^2}{3} + \frac{x^3}{3} + \frac{x^4}{30}\right)$$

$$\tilde{M}_F =< f|\frac{1}{2}\sum_{ij} \tau_i^+ \tau_j^- \frac{1}{r_{ij}}|i>$$

$$\tilde{M}_{GT} =< f|\frac{1}{2}\sum_{ij} \tau_i^+ \tau_j^- \vec{\sigma}_i \cdot \vec{\sigma}_j \frac{1}{r_{ij}}|i>$$

Nucleus	Q-value (MeV)	$T^{0\nu}_{\frac{1}{2}}$(Yr)	$< m_\nu >$ (eV)
^{48}Ca	4.276	$> 1.14 \times 10^{22}$	< 7.2
^{76}Ge	2.039	$> 1.6 \times 10^{25}$	< 0.33
^{82}Se	2.992	$> 1.9 \times 10^{23}$	< 1.3
^{100}Mo	3.034	$> 5.8 \times 10^{23}$	< 0.8
^{116}Cd	2.804	$> 1.7 \times 10^{23}$	< 1.7
^{128}Te	0.876	$> 7.7 \times 10^{24}$	< 1.1
^{130}Te	2.529	$> 3 \times 10^{23}$	< 0.46
^{136}Xe	2.467	$> 4.4 \times 10^{23}$	< 1.8
^{150}Nd	3.368	$> 1.2 \times 10^{21}$	< 7

$$< m_\nu > = \sum_{i=1}^{3} U_{ie}^2 m_i$$

New Result: EXO experiment in New Mexico

Measured lifetime of ^{136}Xe$=(2.11 \pm 0.04 \pm 0.21) \times 10^{21}$ Yr for two-neutrino beta decay

Measured lower bound for neutrinoless double beta decay—> 1.6×10^{25} Yr—which gives $< m_\nu >< 0.5$ eV

Why are neutrinos so light? Possible answer—seesaw mechanism. No coupling of right-handed neutrinos to gauge fields

$$\mathcal{L}_{\nu_R} = -f_\nu \bar{\ell}_L \Phi_H \nu_R - \frac{m_M}{2} \bar{\nu^c_R} \nu_R + h.c.$$

When Higgs gets vacuum expectation value $v/\sqrt{2}$, define $m_D = f_\nu v/\sqrt{2}$ so

$$\mathcal{L}_{D+M} = -\frac{1}{2} (\bar{\nu}_L \ \bar{\nu^c_R}) \begin{pmatrix} 0 & m_D \\ m_D & m_M \end{pmatrix} \begin{pmatrix} \nu^c_L \\ \nu_R \end{pmatrix}$$

Diagonalize via $\tan 2\theta = 2m_D/m_M$ and

Then

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$$\mathcal{L}_{D+M} = -\frac{m_a}{2}\left[\bar{\nu_a^c}\nu_a + \bar{\nu_a}\nu_a^c\right] - \frac{m_b}{2}\left[\bar{\nu_b^c}\nu_b + \bar{\nu_b}\nu_b^c\right]$$

with

$$m_a = m_M \cos^2\theta + m_D \sin 2\theta$$

$$m_b = m_M \sin^2\theta - m_D \sin 2\theta$$

In limit $m_D << m_M$ have

$$m_a = m_M \qquad m_b = -\frac{m_D^2}{m_M}$$

and both eigenstates are Majorana fields.