Lattice QCD for Nuclear Physics

Silas Beane

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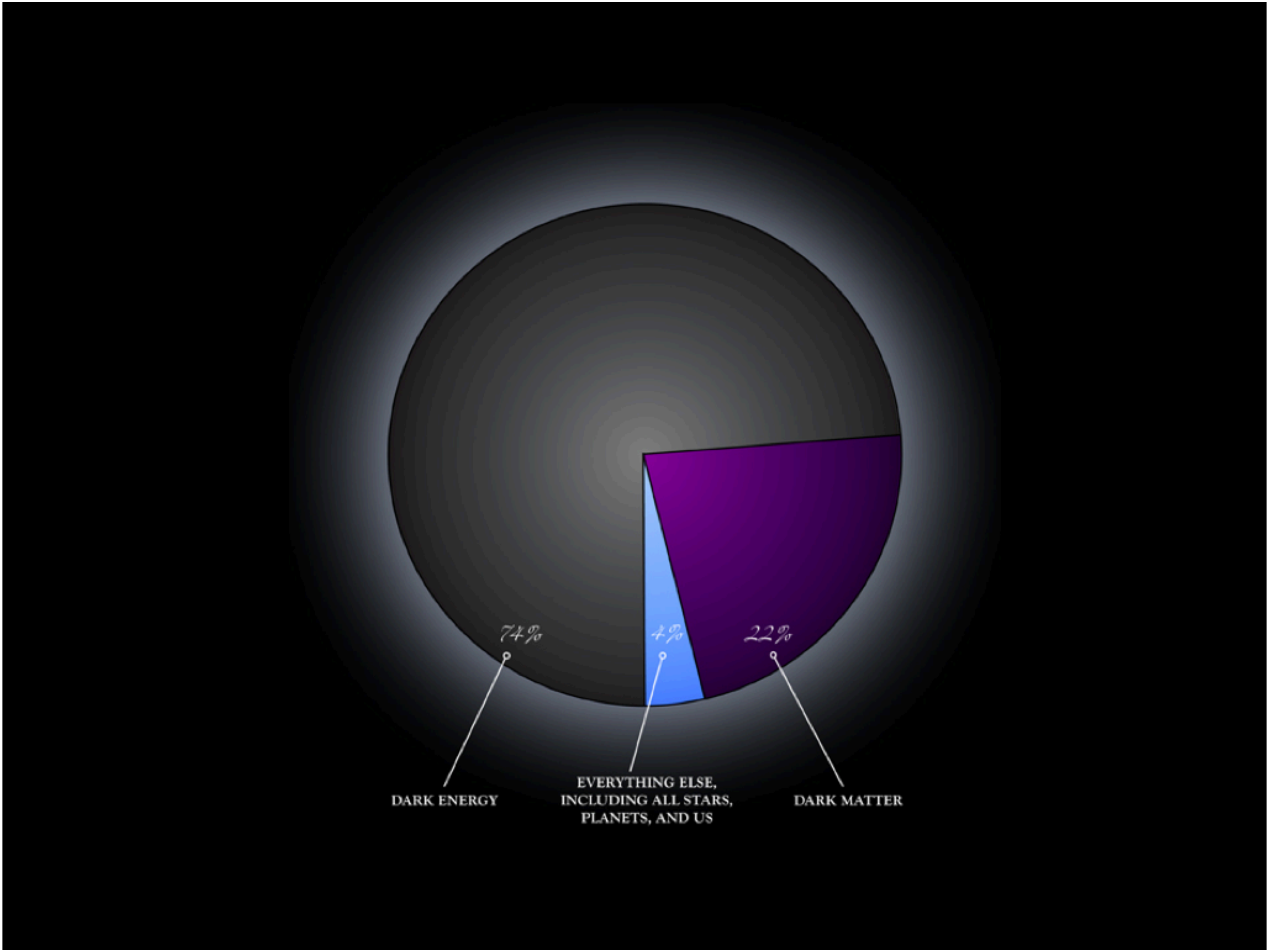
Outline

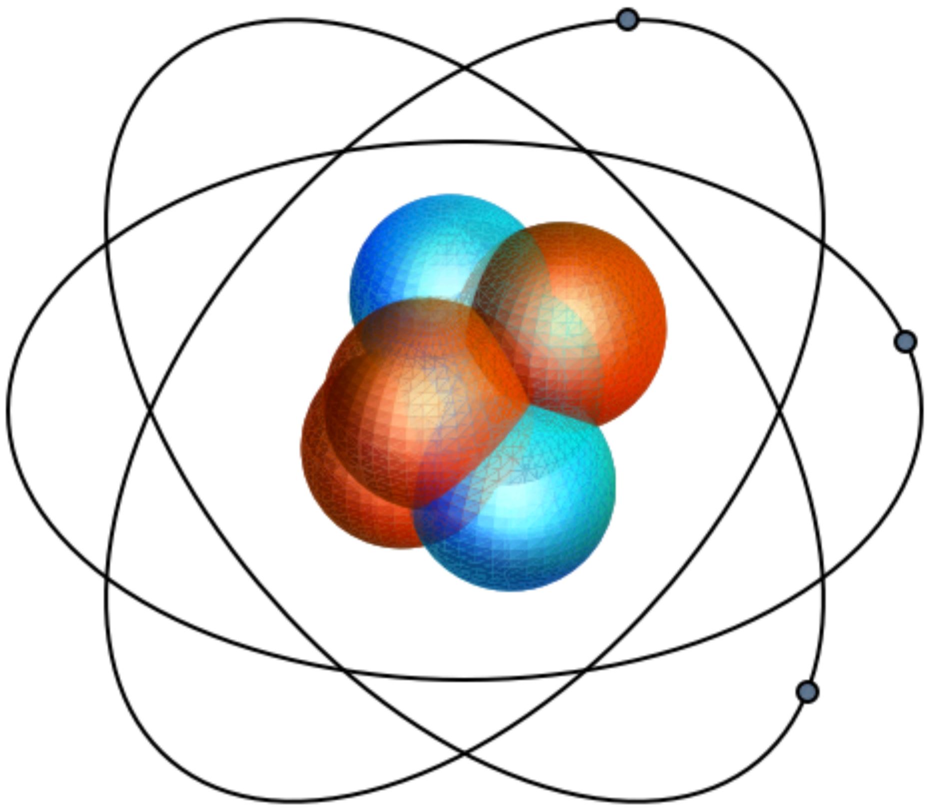
- QCD and Nuclear physics
- Why it's hard (i.e. expensive)
- Road-map to the exa-scale
- Scattering in a finite volume: $\pi\pi$
- The deuteron and fine tuning
- Hypernuclear physics
- Conclusion

Review Articles

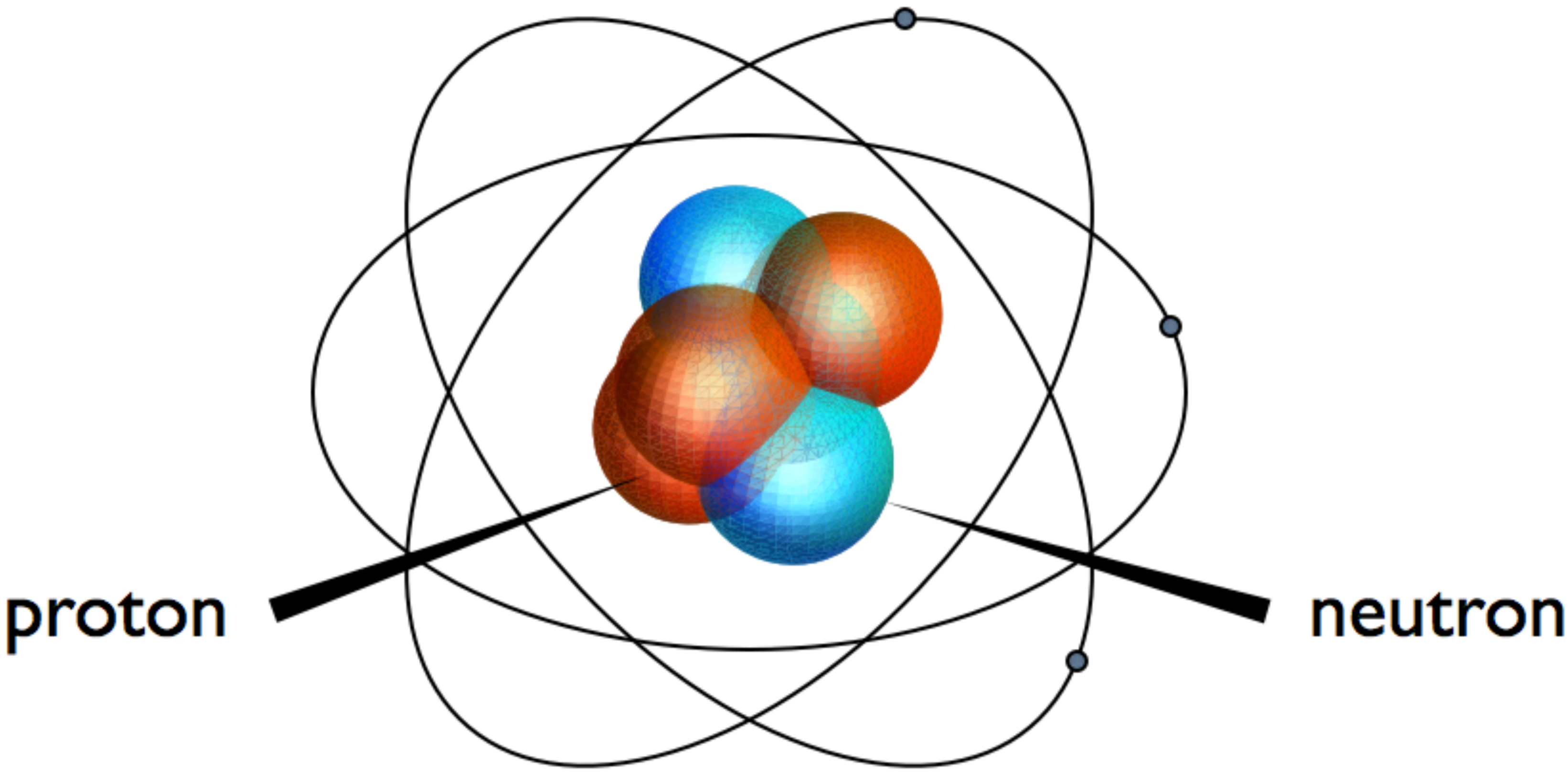
- NPLQCD, arXiv:1004.2935
- NPLQCD, arXiv:0805.4629
- HALQCD, arXiv:1206.5088


74%
4%
22%
DARK ENERGY
EVERYTHING ELSE, INCLUDING ALL STARS, PLANETS, AND US
DARK MATTER

4% is ATOMS!



4% is ATOMS!



Lithium-6

The Nature of the Visible Matter in the Universe

Dictated by the Standard Model:

gravity

electromagnetism:QED

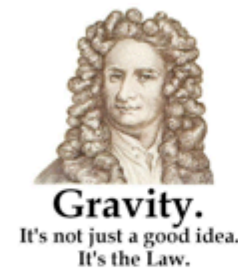
weak interaction

strong interaction: QCD

The Nature of the Visible Matter in the Universe

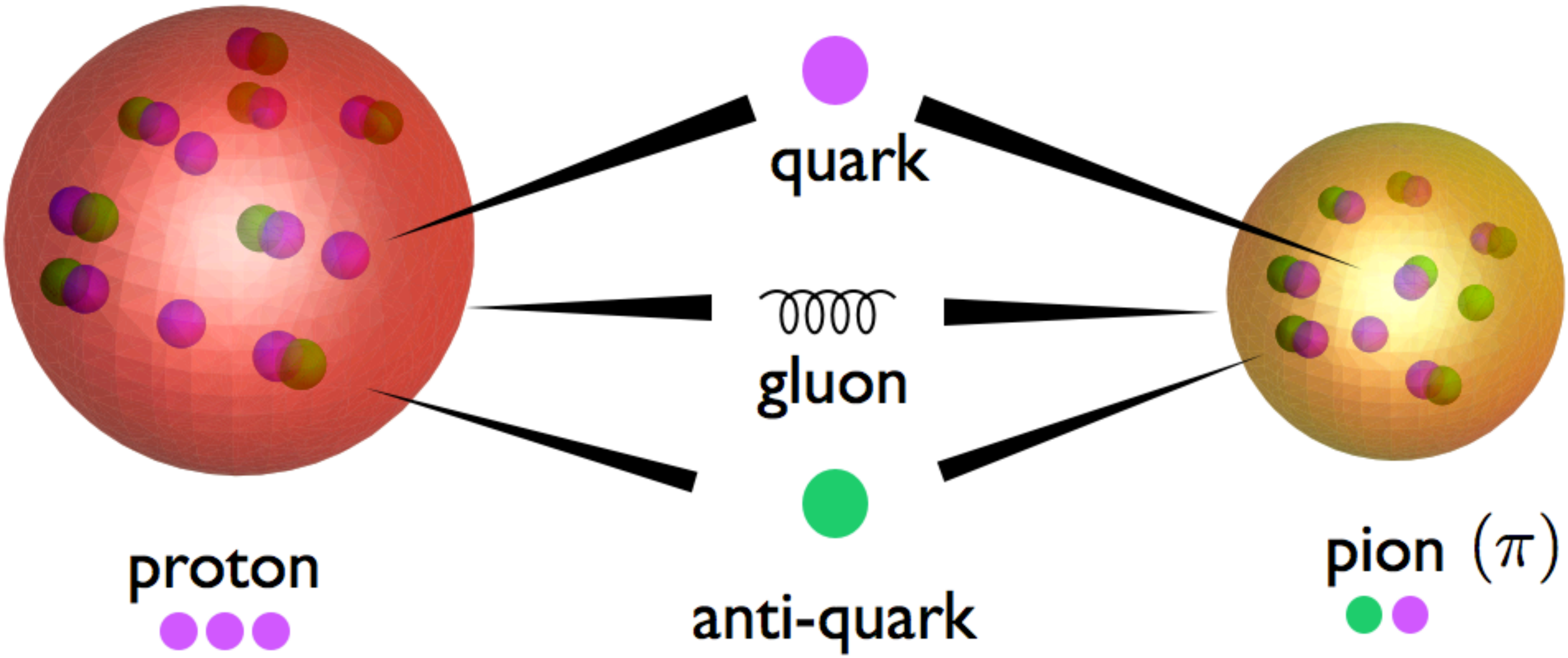
Dictated by the Standard Model:

easy!
- gravity
- electromagnetism:QED
- weak interaction



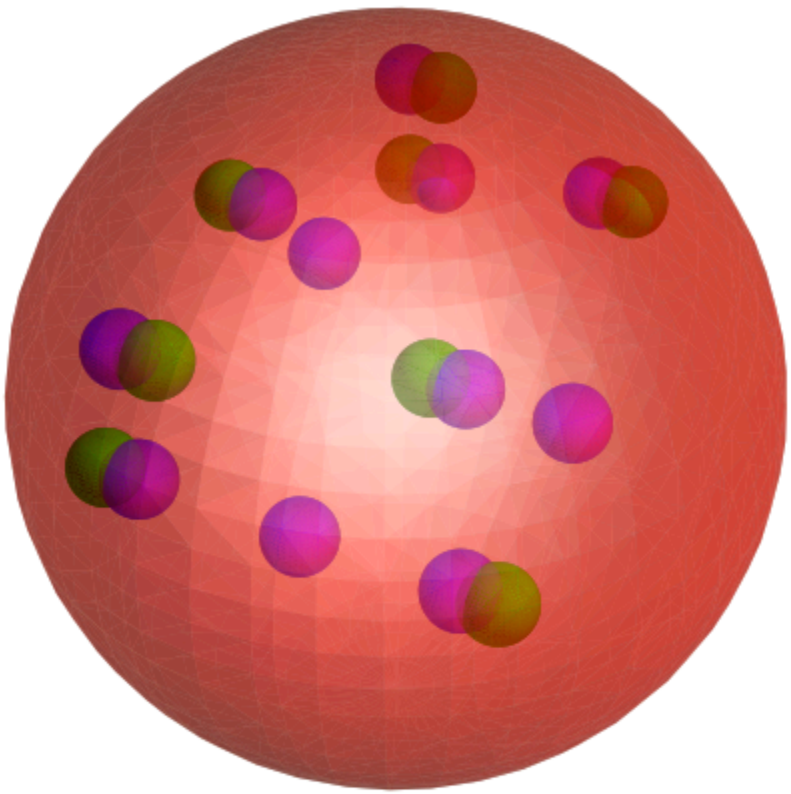
hard!
- strong interaction: QCD

Quantum Chromodynamics



QCD = rules for making hadrons from quarks and gluons

Complicated strong coupling problem!



proton

Mass Budget

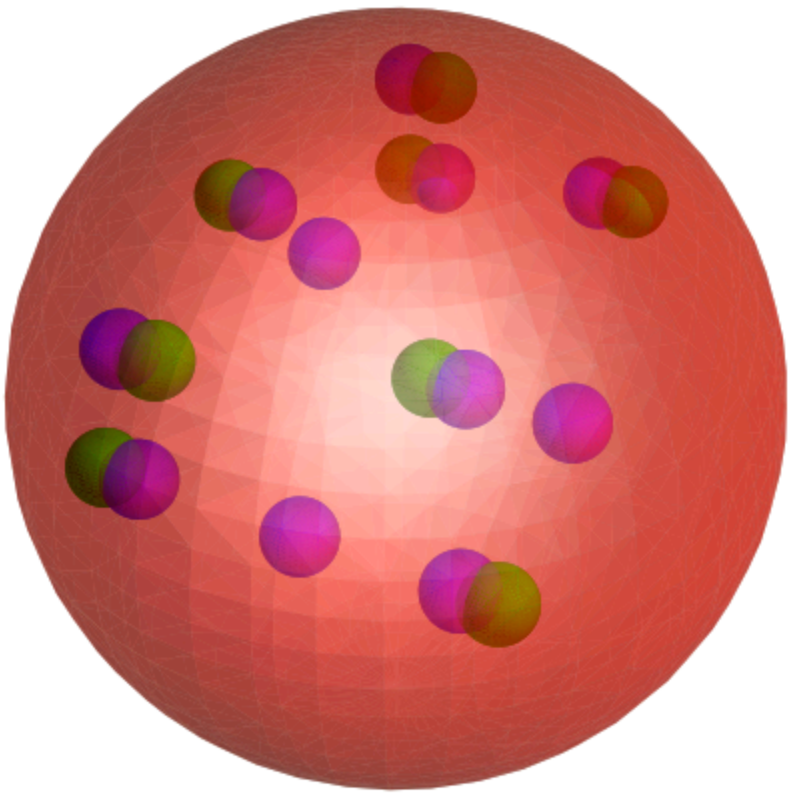
gluon mass $= 0$

up quark mass ~ 3 MeV

down quark mass ~ 5 MeV

proton mass ~ 940 MeV

Complicated strong coupling problem!



proton

Mass Budget

gluon mass $= 0$

up quark mass ~ 3 MeV

down quark mass ~ 5 MeV

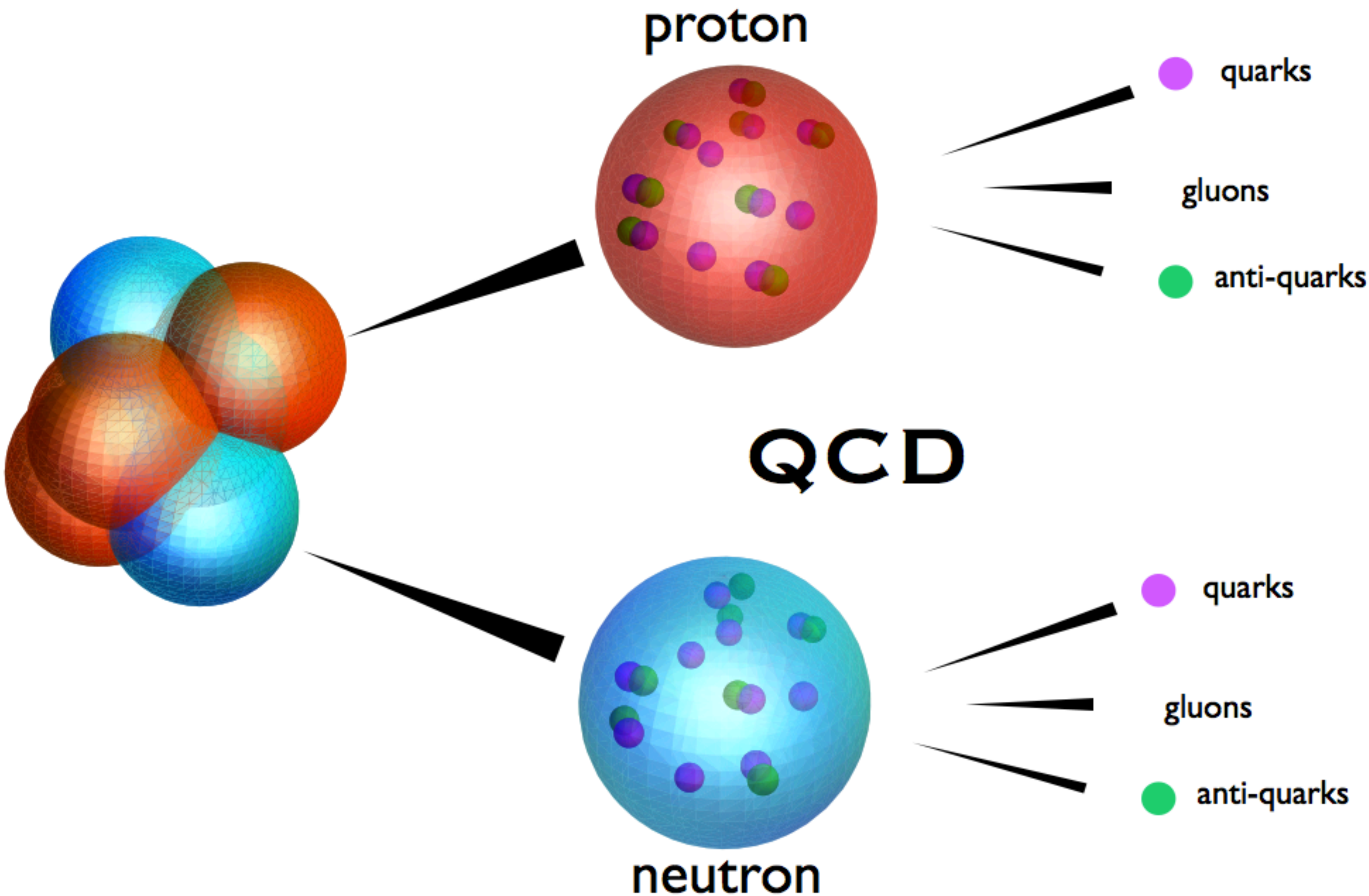
proton mass ~ 940 MeV

~ 4% is QCD interactions!

Is Higgs the origin of mass??

Is Higgs the origin of mass??

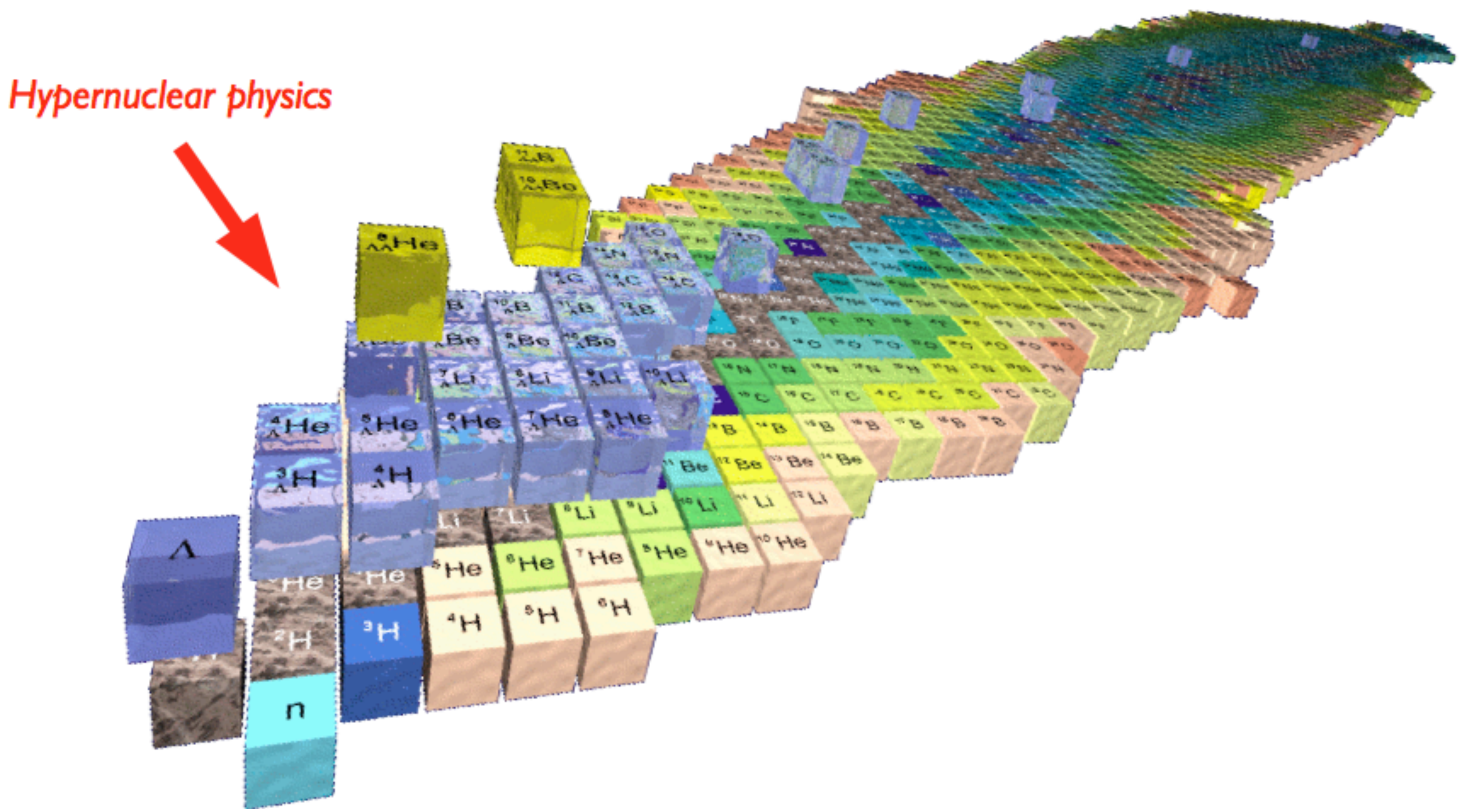
Nuclear Physics: two layers of complexity!!



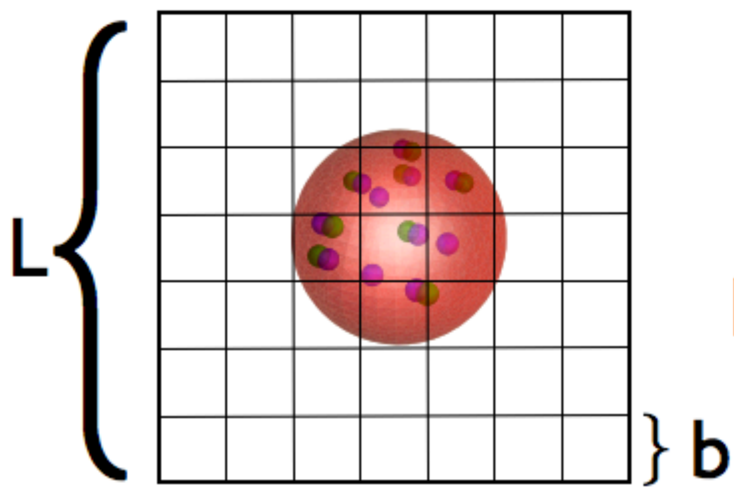
Why do it?

- ★ Nuclear physics with quantifiable uncertainties!
- ★ Dependence on fundamental parameters of nature:

$$\alpha_s \qquad \alpha_e \qquad m_u \qquad m_d \qquad m_s$$



Lattice QCD = QCD on a grid or lattice



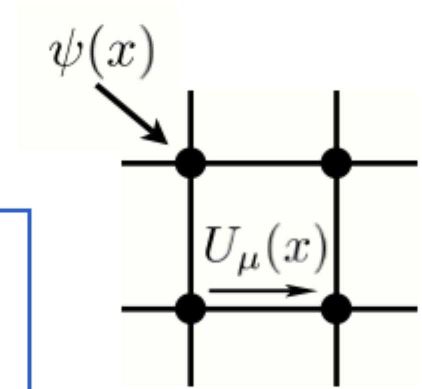
volume: $M_\pi L \gg 1$

lattice spacing: $b \ll M_N^{-1}$

Can use Effective Field Theory to extrapolate in L and b!

(systematic uncertainties fully controlled)

QCD path integral with Montecarlo



$$\langle \mathcal{O} \rangle \sim \int dU_\mu \, d\bar{\psi} \, d\psi \; \mathcal{O}\left(U, \psi, \bar{\psi}\right) \; e^{-S_g(U) - \bar{\psi} D(U) \psi}$$

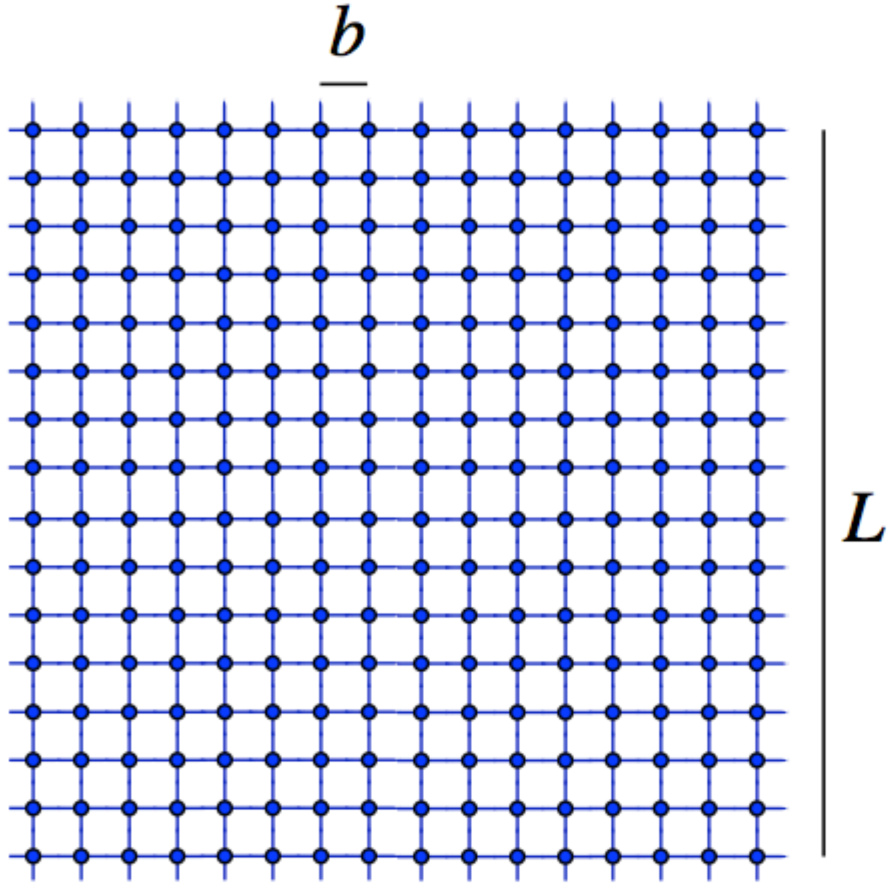
propagators

N gauge configurations

$$\langle \mathcal{O} \rangle \sim \int dU_\mu \, \mathcal{O}(D(U)^{-1}) \; \det\left(f(U)\right) \; e^{-S_g(U)} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(D(U_i)^{-1})$$

Estimate of $\mathcal{O}$ with $\sigma_{\mathcal{O}} \sim 1/\sqrt{N}$

Lattice QCD Economics



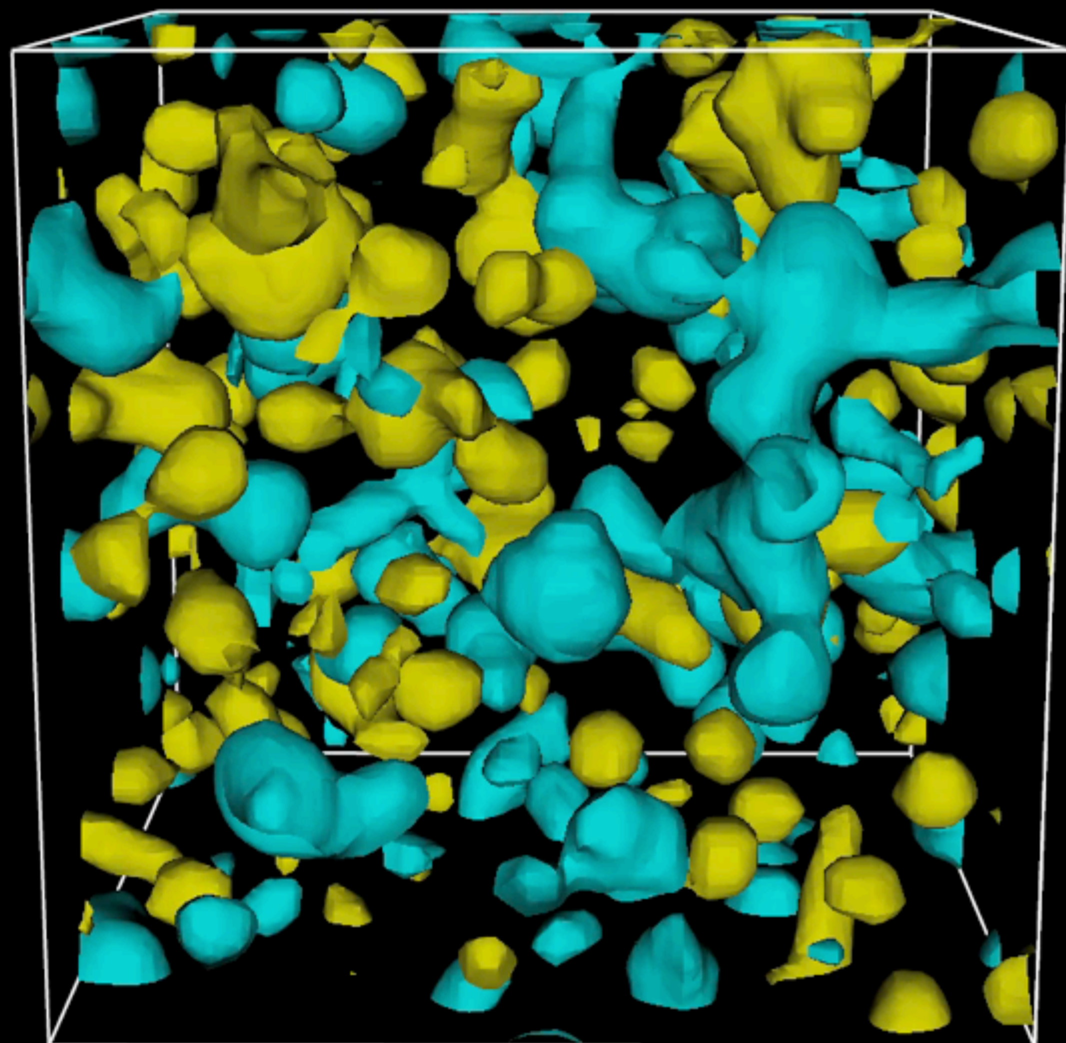
$$\text{COST} \sim (L)^4\ (b)^{-6.5}\ (M_q)^{-2.5}$$

The QCD vacuum

(Massimo DiPierro)

$\mathcal{O} \sim F\tilde{F}(0)F\tilde{F}(x)$

$L \sim 4$ fm



"pixelation" $\sim (0.12 \times 10^{-15}\ \mathrm{m})^3$

$\Delta t \sim 6 \times 10^{-24}$ s

Why is it hard?

Why is it hard?

- Signal/noise and statistics
- Number of contractions

Correlators in Euclidean Space

$$\pi^+(\mathbf{x},t) = \overline{u}(\mathbf{x},t)\gamma_5 d(\mathbf{x},t)$$

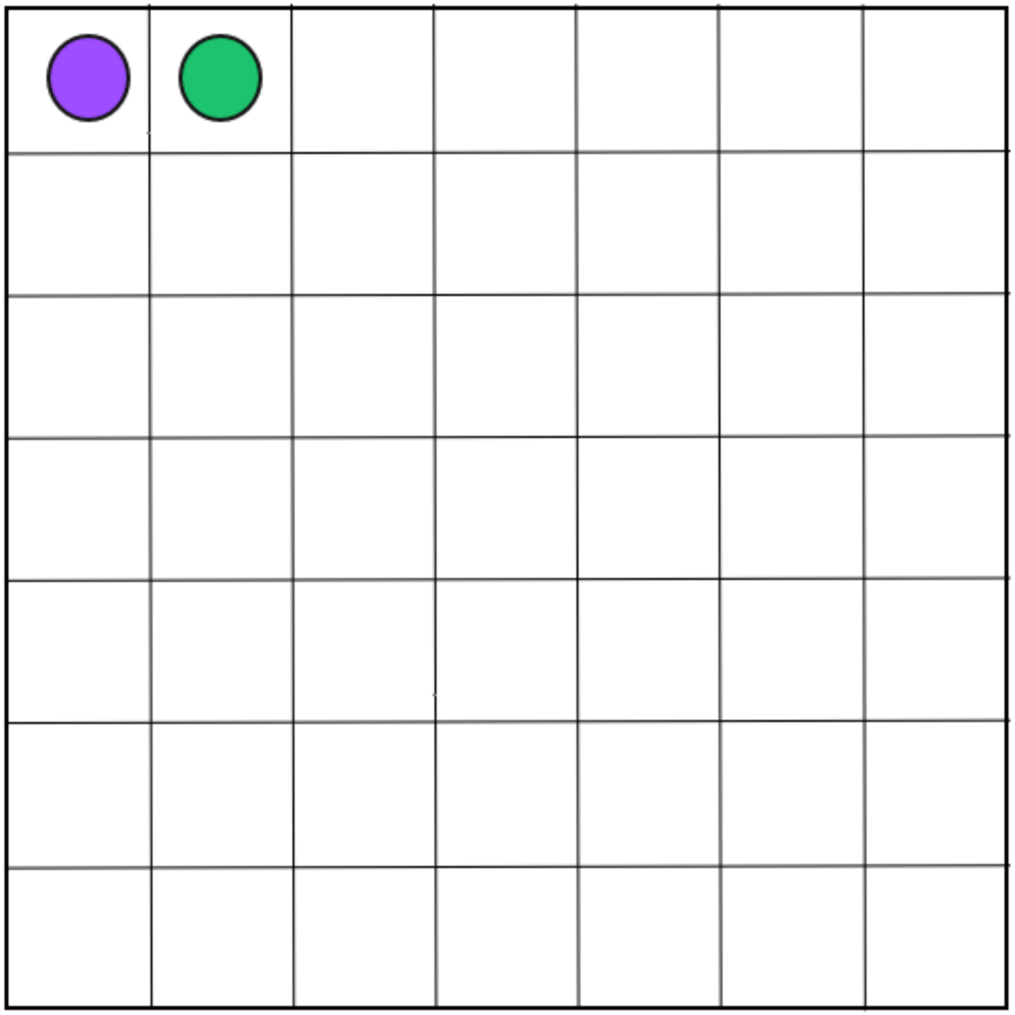
$$C_{\pi^+}(t) \;=\; \sum_{\mathbf{x}} \langle 0|\; \pi^-(\mathbf{x},t)\; \pi^+(\mathbf{0},0)\; |0\rangle$$

$$\pi^+(\mathbf{x},t) = e^{\hat{H}t}\pi^+(\mathbf{x},0)e^{-\hat{H}t}$$

$$C_{\pi^+}(t) \;=\; \sum_n \frac{e^{-E_n t}}{2E_n} \sum_{\mathbf{x}} \langle 0|\; \pi^-(\mathbf{x},0)|n\rangle\langle n|\pi^+(\mathbf{0},0)|0\rangle \;\to\; A_0\,\frac{e^{-m_\pi t}}{2m_\pi}$$

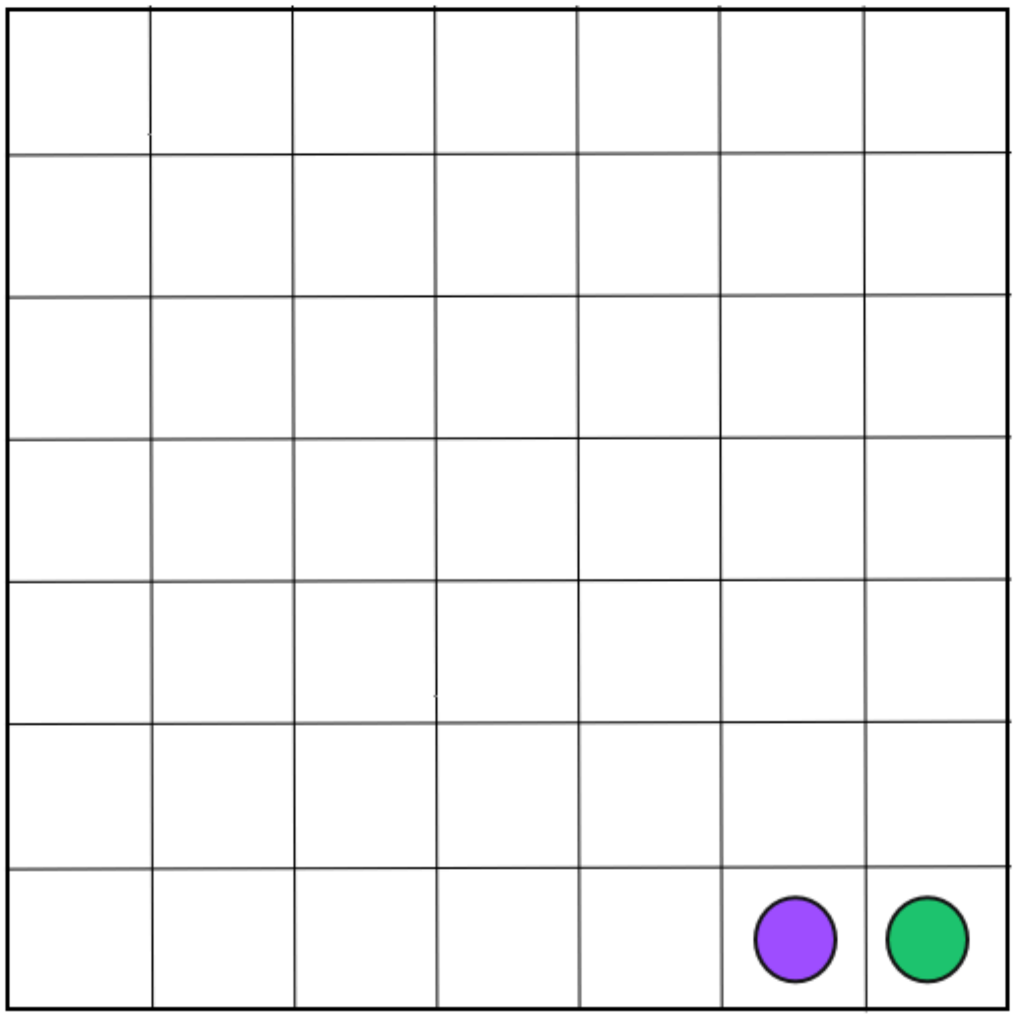
Infinite sum of exponentials: $E_0 = m_\pi = \frac{1}{t_J}\log\left(\frac{C_{\pi^+}(t)}{C_{\pi^+}(t+t_J)}\right)$

SIGNAL $C(t) = \langle \theta(t) \rangle$ $\sum_{\mathbf{x}} \langle 0| \; \pi^-(\mathbf{x}, t) \; \pi^+(\mathbf{0}, 0) \; |0\rangle$



SIGNAL

$$C(t) = \langle \theta(t) \rangle \qquad \sum_{\mathbf{x}} \langle 0|\ \pi^-(\mathbf{x}, t)\ \pi^+(\mathbf{0}, 0)\ |0\rangle$$

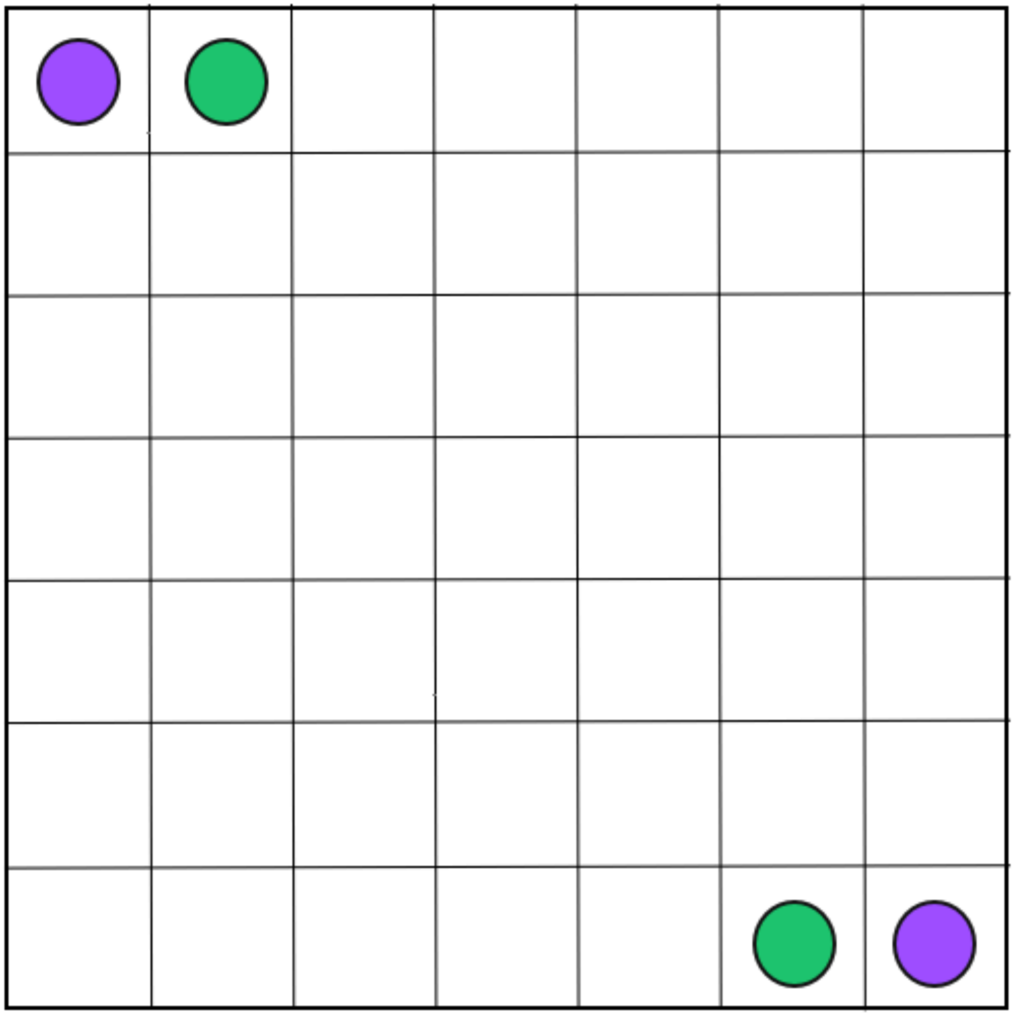


$$\sim e^{-M_\pi t}$$

NOISE

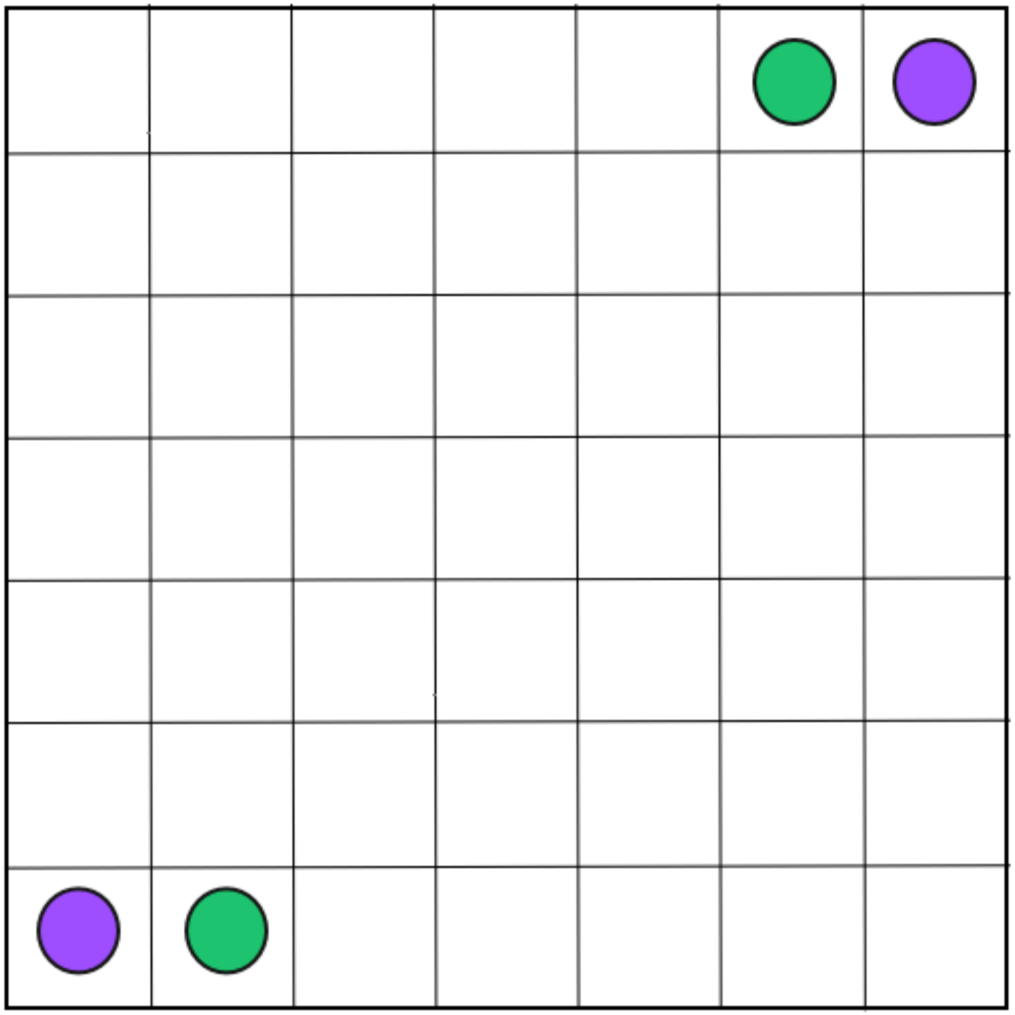
$$\langle \theta(t)\theta^\dagger(t)\rangle - \langle \theta(t)\rangle^2$$

$$\langle \sum_{\mathbf{x}} \sum_{\mathbf{y}} \pi^-(\mathbf{x},t)\pi^+(\mathbf{y},t)\pi^-(\mathbf{0},0)\pi^+(\mathbf{0},0)\rangle$$



NOISE $\langle\theta(t)\theta^\dagger(t)\rangle - \langle\theta(t)\rangle^2$

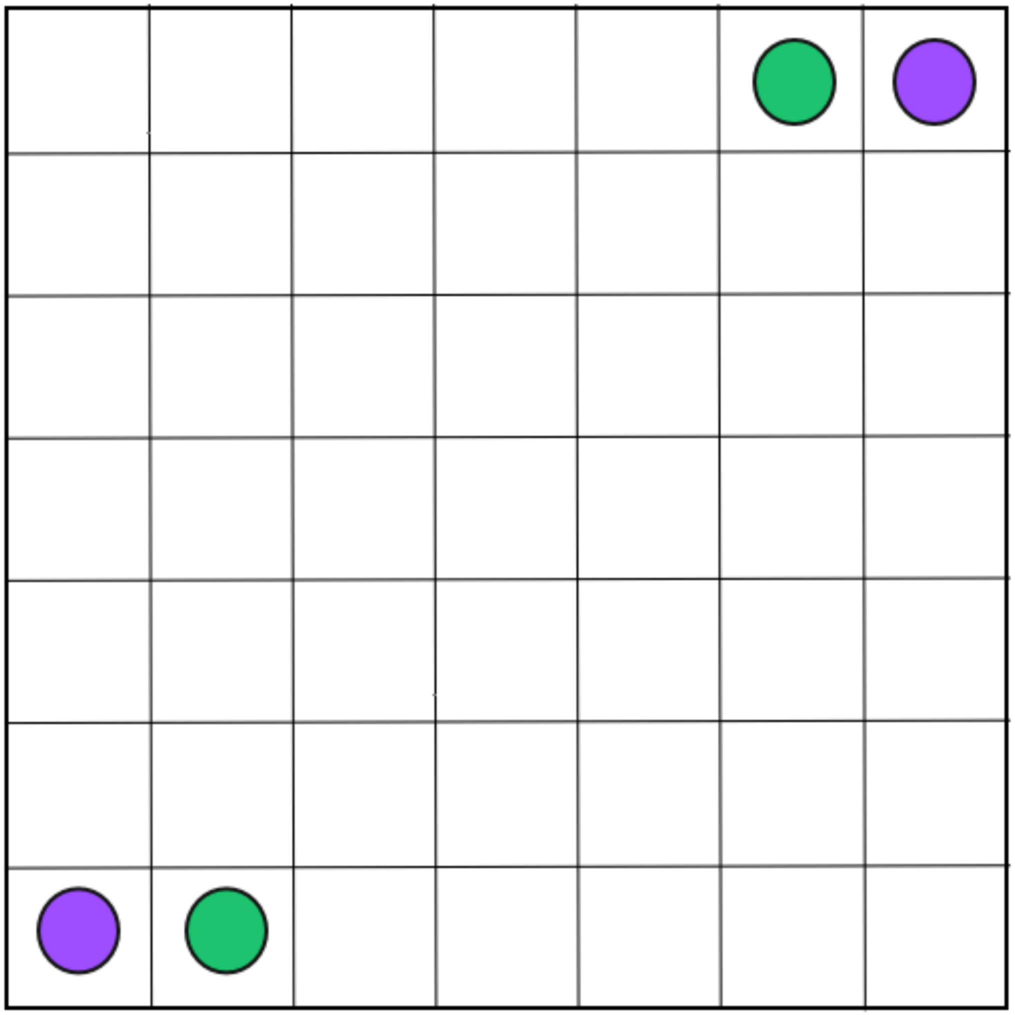
$$\langle\sum_{\mathbf{x}}\sum_{\mathbf{y}}\pi^-(\mathbf{x},t)\pi^+(\mathbf{y},t)\pi^-(\mathbf{0},0)\pi^+(\mathbf{0},0)\rangle$$



$$\sim e^{-2M_\pi t}$$

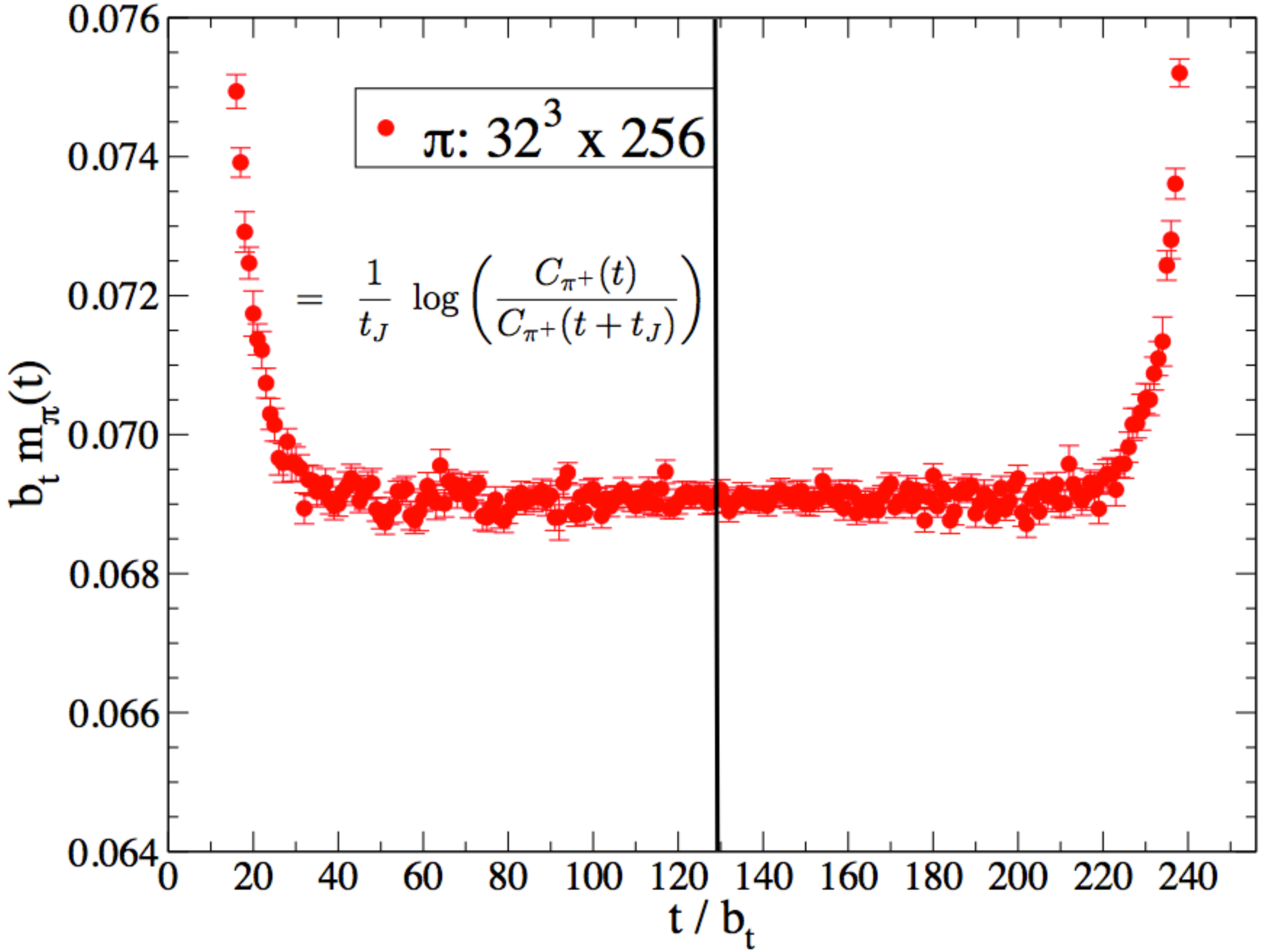
NOISE $\langle \theta(t)\theta^\dagger(t)\rangle - \langle \theta(t)\rangle^2$

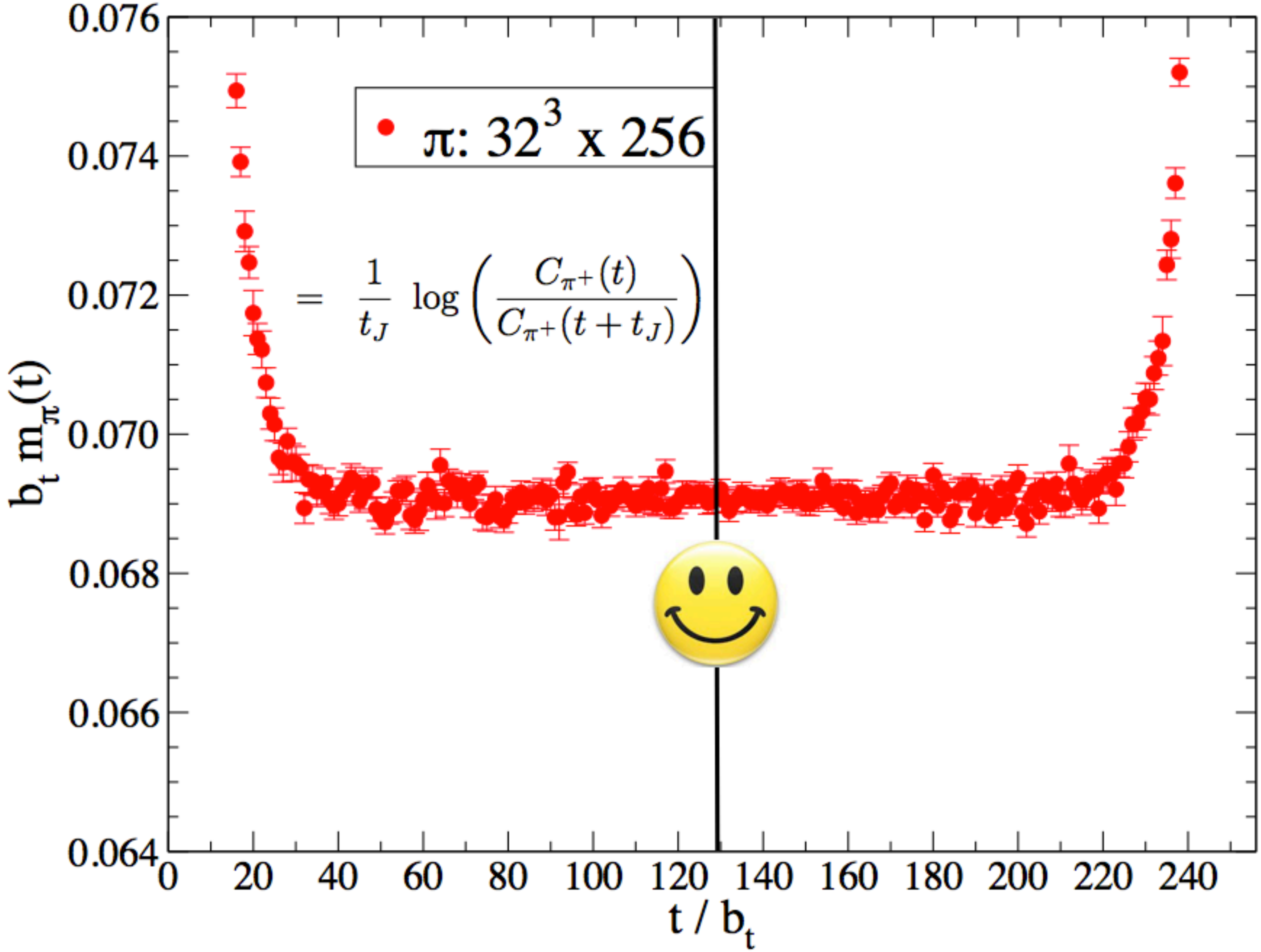
$$\langle \sum_{\mathbf{x}} \sum_{\mathbf{y}} \pi^-(\mathbf{x},t)\pi^+(\mathbf{y},t)\pi^-(\mathbf{0},0)\pi^+(\mathbf{0},0)\rangle$$



$$\sim e^{-2M_\pi t}$$

$$\text{signal/noise} \ \sim \ 1$$

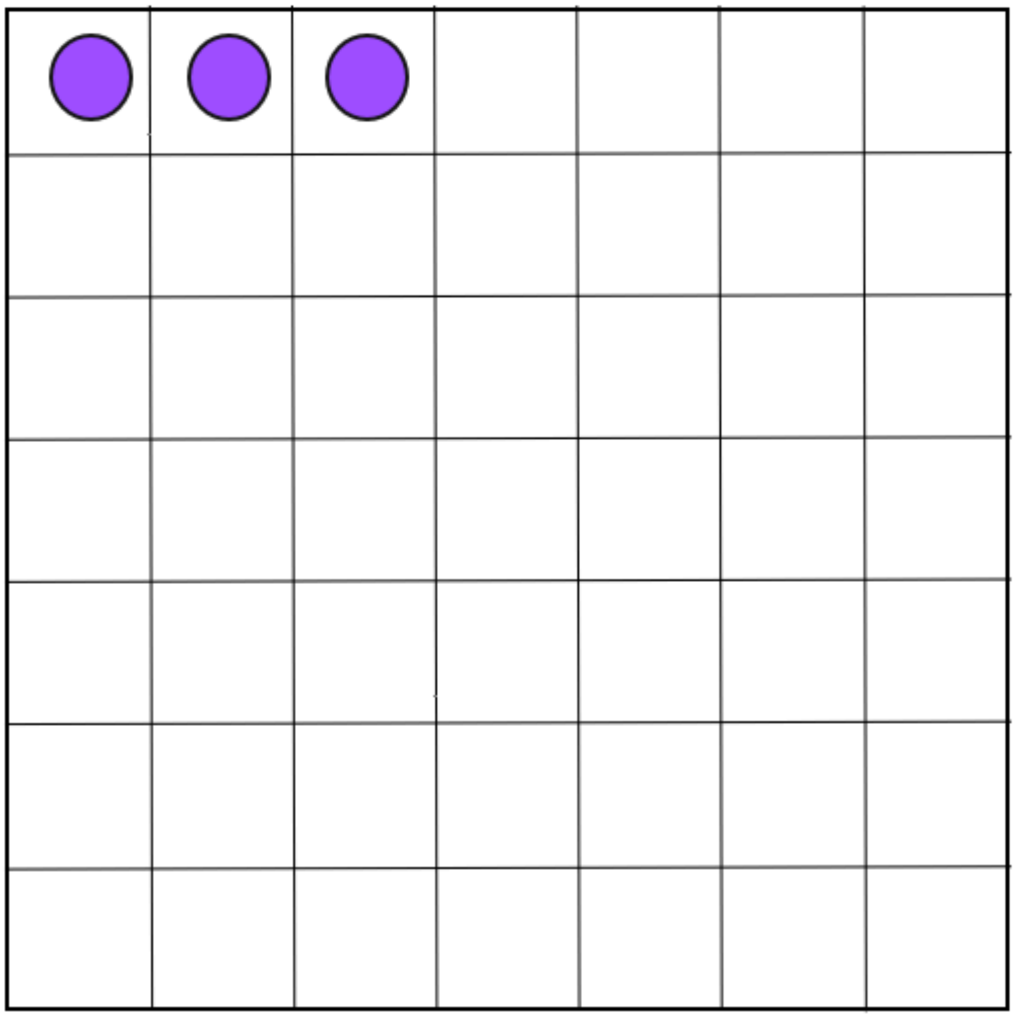

π: 32³ x 256
$= \frac{1}{t_J} \log\left(\frac{C_{\pi^+}(t)}{C_{\pi^+}(t+t_J)}\right)$
$b_t\, m_\pi(t)$
$t\, /\, b_t$
0.076
0.074
0.072
0.070
0.068
0.066
0.064
0
20
40
60
80
100
120
140
160
180
200
220
240



π: 32^3 x 256
= 1/t_J log(C_{π+}(t) / C_{π+}(t + t_J))
b_t m_π(t)
t / b_t
0.064
0.066
0.068
0.070
0.072
0.074
0.076
0
20
40
60
80
100
120
140
160
180
200
220
240

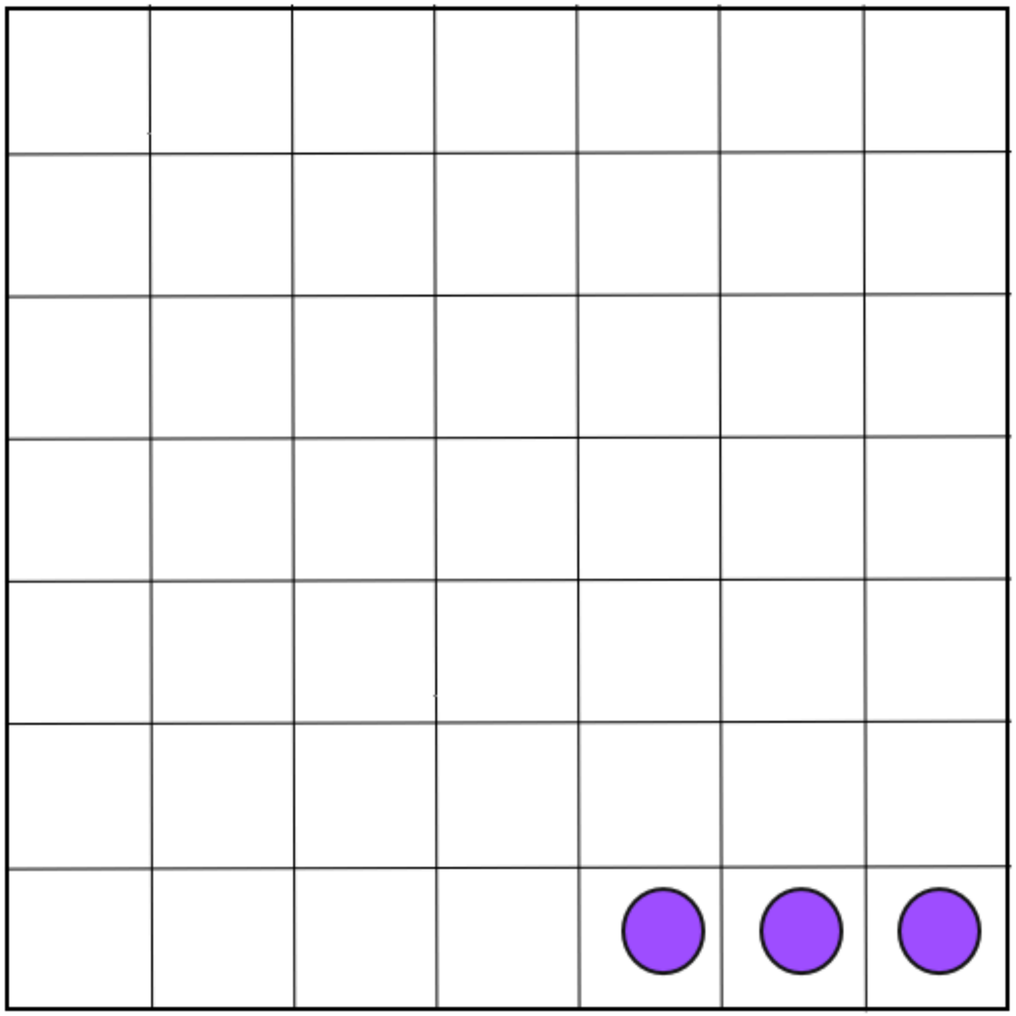
SIGNAL

$$\sum_{\mathbf{x}} \Gamma_{+}^{\beta\alpha} \langle 0|\, N^{\alpha}(\mathbf{x},t)\overline{N}^{\beta}(\mathbf{0},0)\, |0\rangle$$



SIGNAL

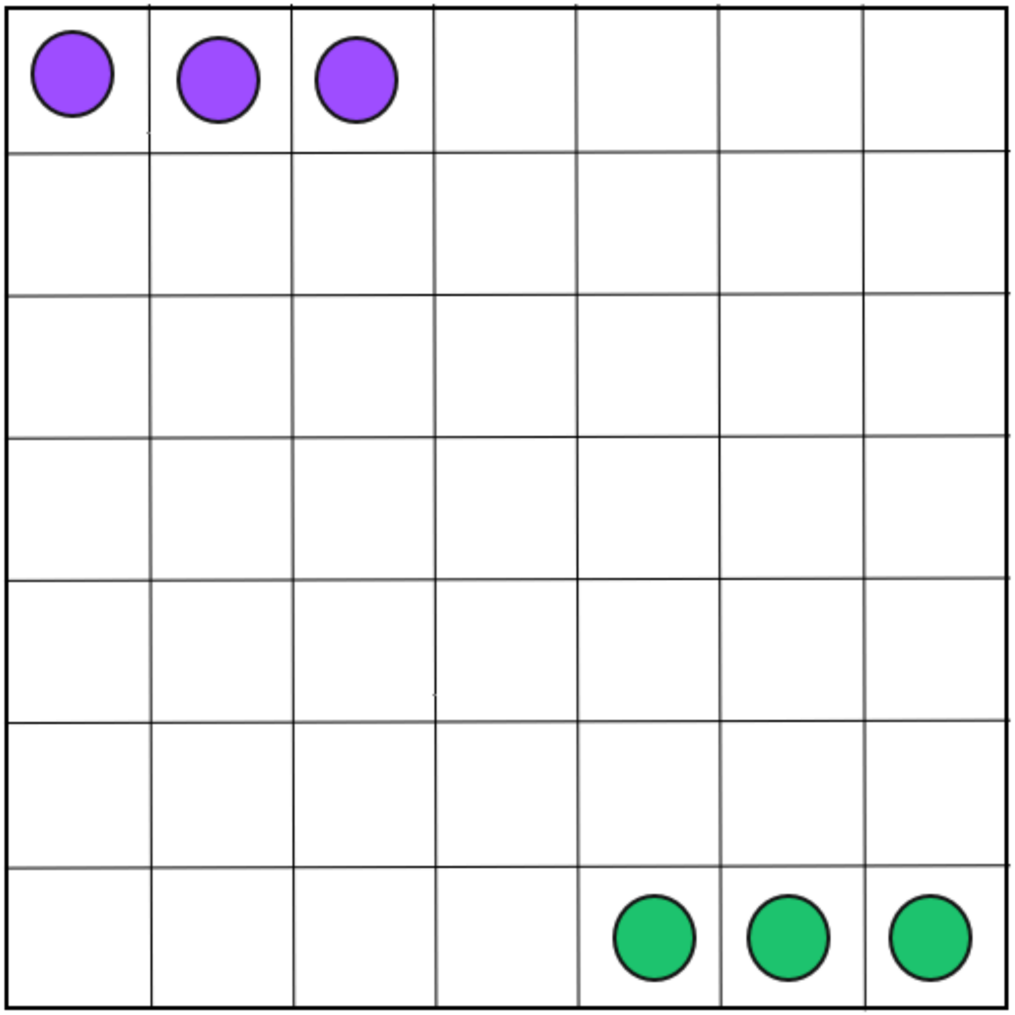
$$\sum_{\mathbf{x}} \Gamma_{+}^{\beta\alpha} \langle 0|\ N^{\alpha}(\mathbf{x},t)\overline{N}^{\beta}(\mathbf{0},0)\ |0\rangle$$



$$\sim e^{-M_N t}$$

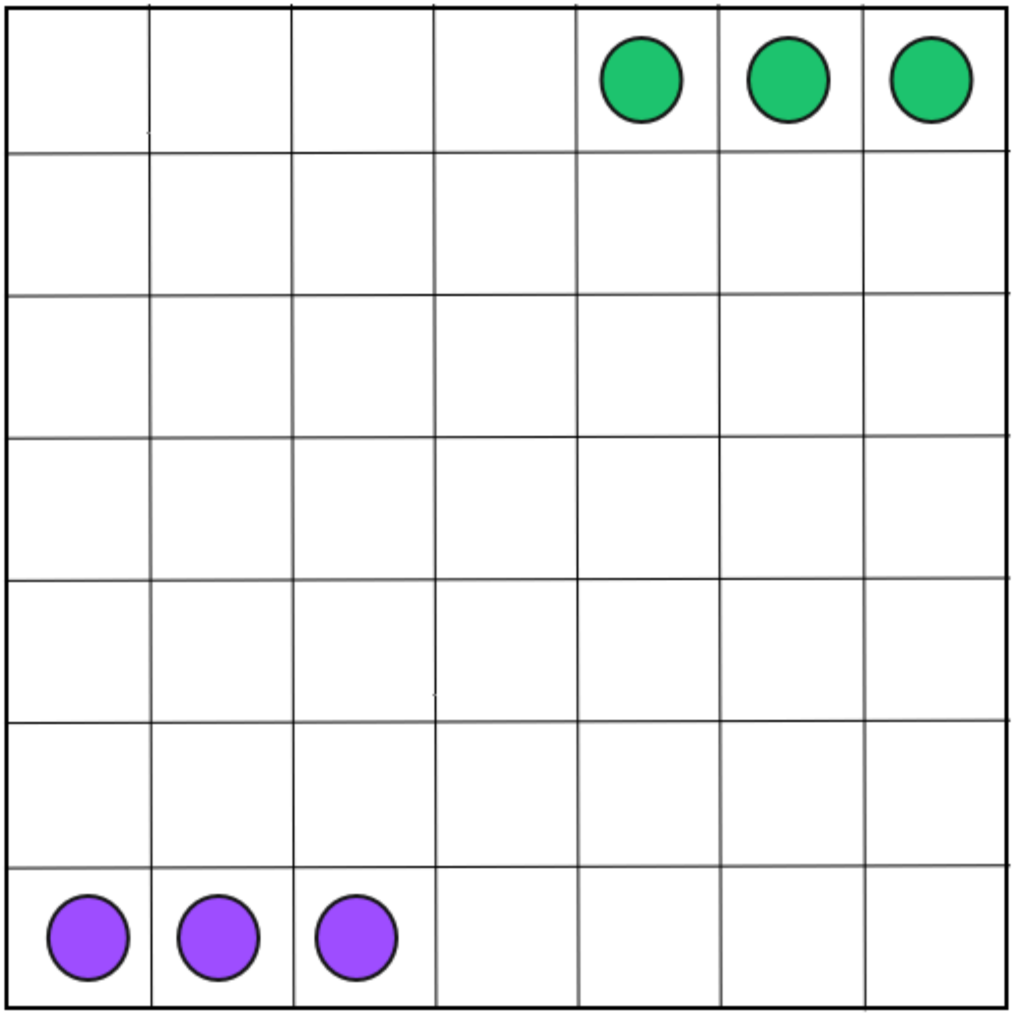
NOISE

$$\sum_{\mathbf{x},\mathbf{y}} \Gamma_+^{\delta\alpha}\Gamma_+^{\gamma\beta\dagger} \langle 0|\; N^\alpha(\mathbf{x},t)\overline{N}^\beta(\mathbf{y},t)N^\gamma(\mathbf{0},0)\overline{N}^\delta(\mathbf{0},0)\; |0\rangle$$



NOISE

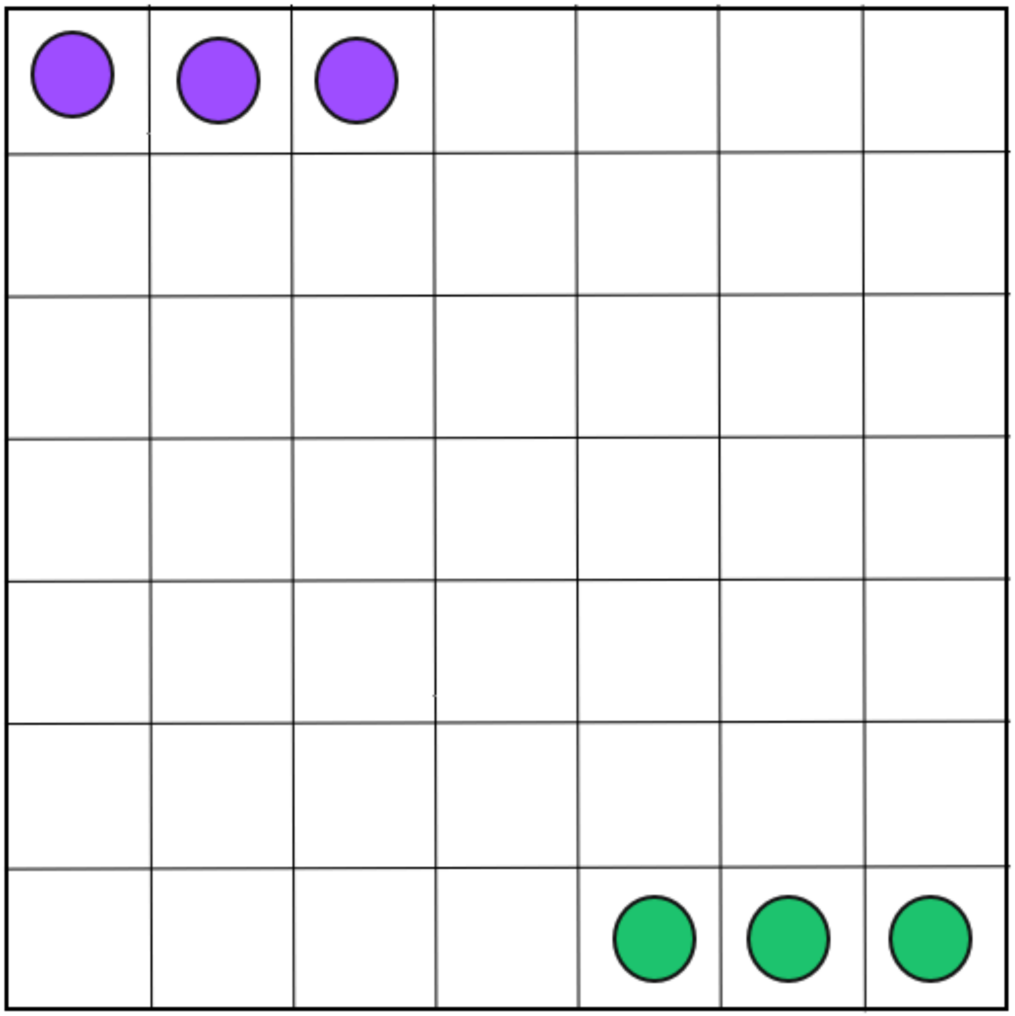
$$\sum_{\mathbf{x},\mathbf{y}} \Gamma_{+}^{\delta\alpha}\Gamma_{+}^{\gamma\beta\dagger} \langle 0|\ N^{\alpha}(\mathbf{x},t)\overline{N}^{\beta}(\mathbf{y},t)N^{\gamma}(\mathbf{0},0)\overline{N}^{\delta}(\mathbf{0},0)\ |0\rangle$$



$$\sim e^{-2M_N t}$$

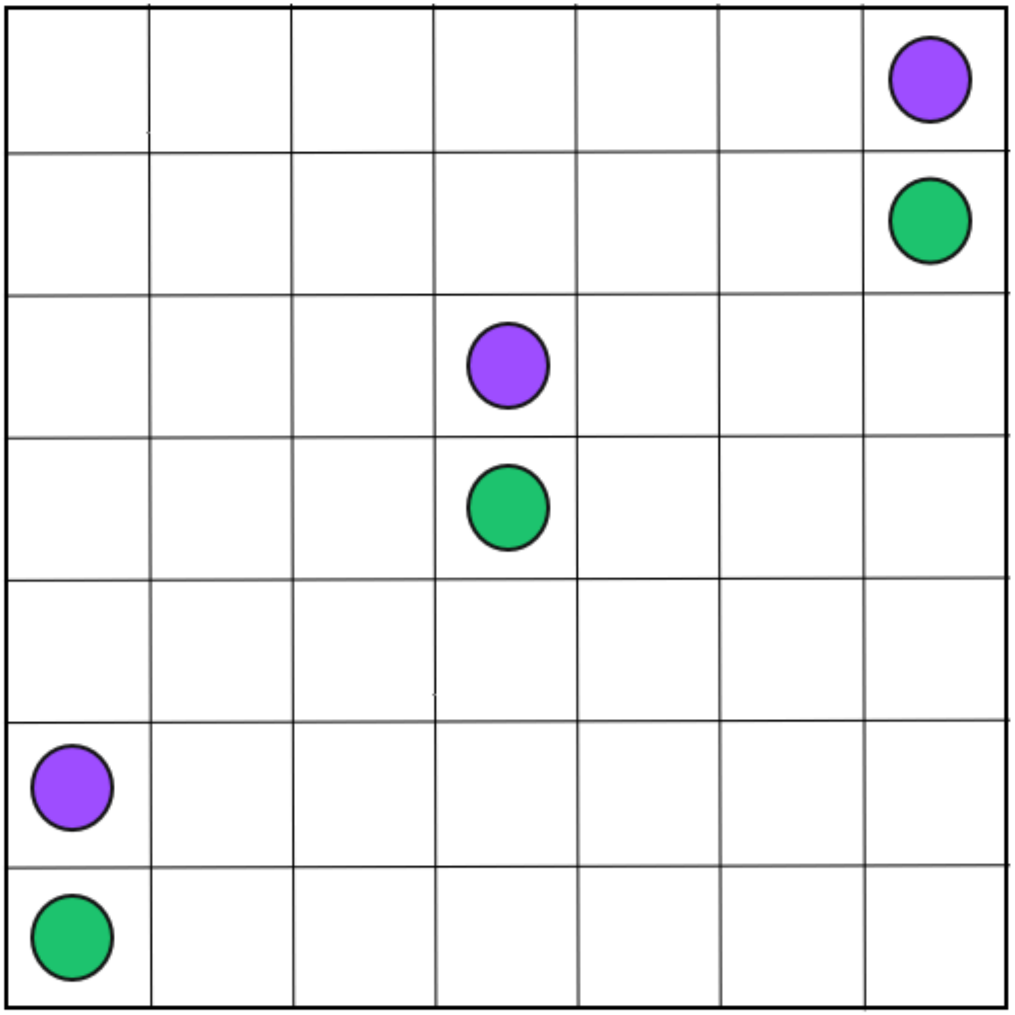
NOISE

$$\sum_{\mathbf{x},\mathbf{y}} \Gamma_+^{\delta\alpha}\Gamma_+^{\gamma\beta\dagger} \langle 0|\; N^\alpha(\mathbf{x},t)\overline{N}^\beta(\mathbf{y},t)N^\gamma(\mathbf{0},0)\overline{N}^\delta(\mathbf{0},0)\;|0\rangle$$



NOISE

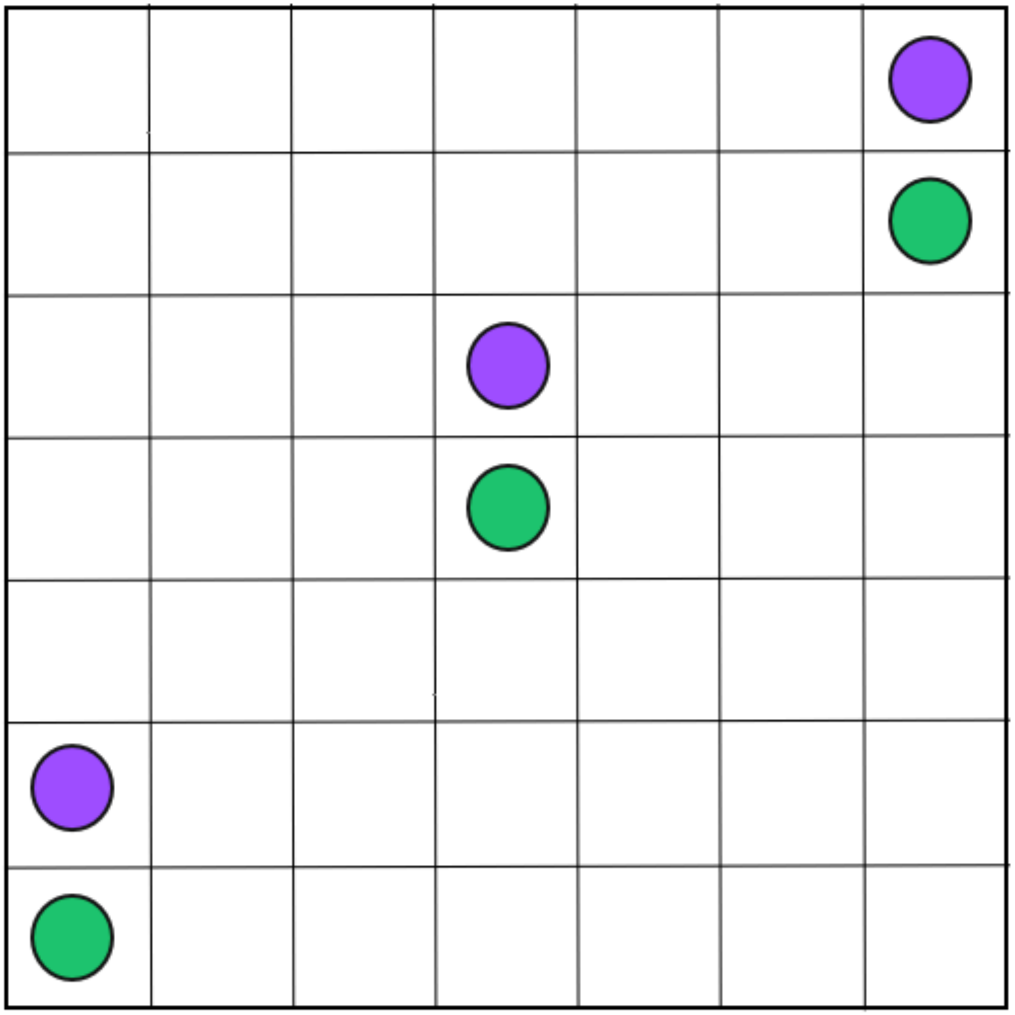
$$\sum_{\mathbf{x},\mathbf{y}} \Gamma_{+}^{\delta\alpha}\Gamma_{+}^{\gamma\beta\dagger} \langle 0|\ N^{\alpha}(\mathbf{x},t)\overline{N}^{\beta}(\mathbf{y},t)N^{\gamma}(\mathbf{0},0)\overline{N}^{\delta}(\mathbf{0},0)\ |0\rangle$$



$$\sim\ e^{-3M_{\pi}t}$$

NOISE

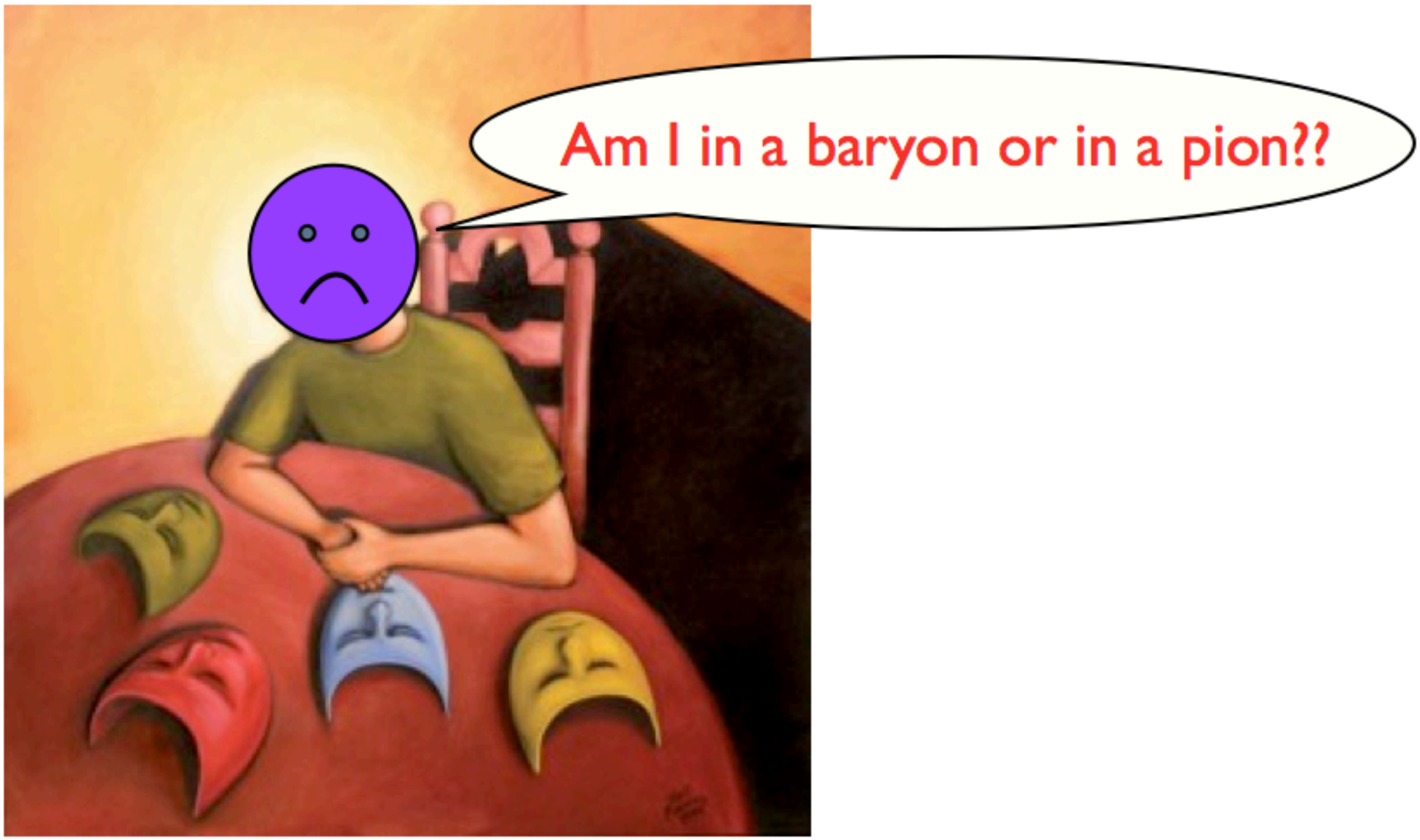
$$\sum_{\mathbf{x},\mathbf{y}} \Gamma_+^{\delta\alpha}\Gamma_+^{\gamma\beta\dagger} \langle 0|\ N^\alpha(\mathbf{x},t)\overline{N}^\beta(\mathbf{y},t)N^\gamma(\mathbf{0},0)\overline{N}^\delta(\mathbf{0},0)\ |0\rangle$$

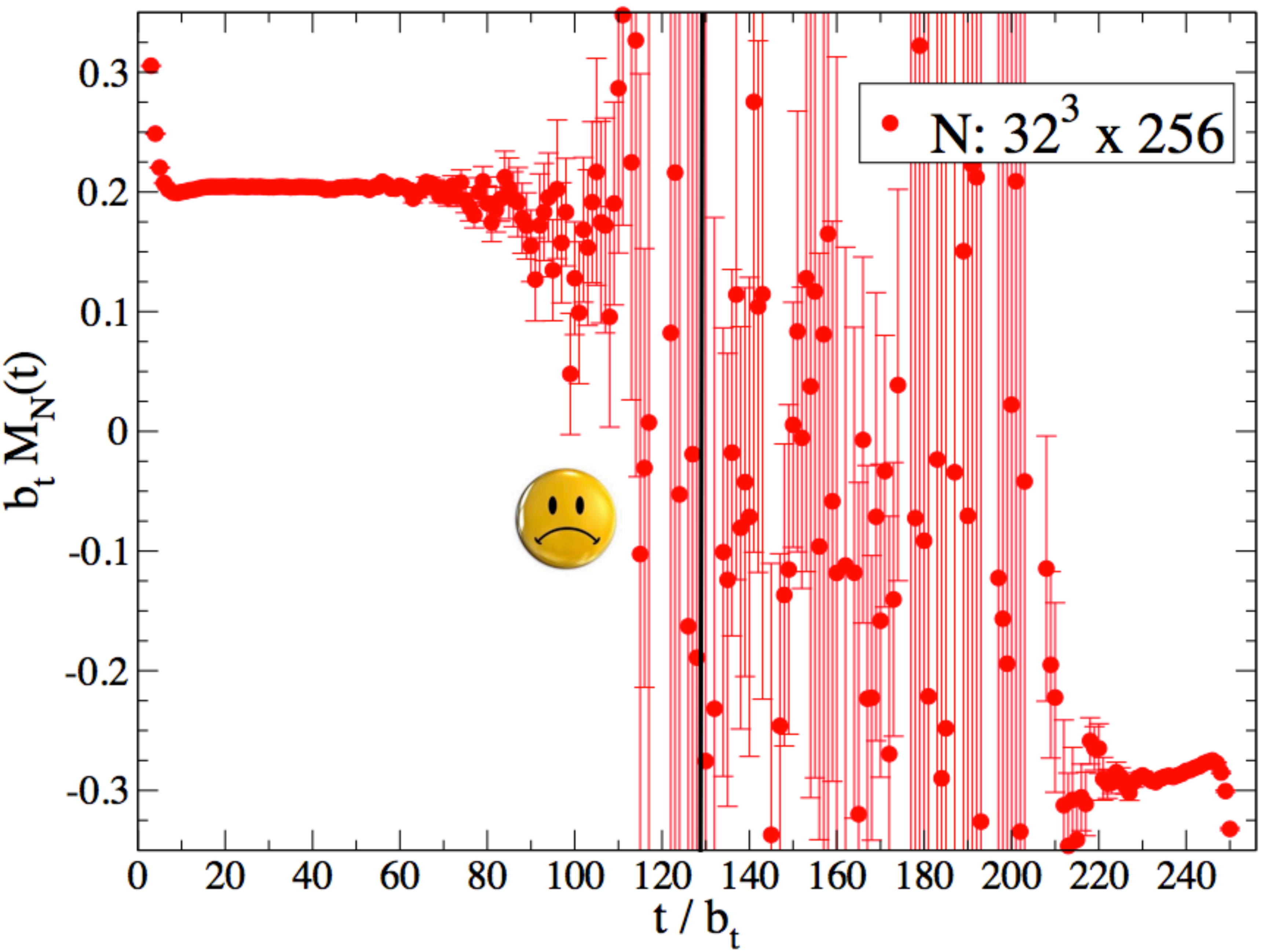


$$\sim\ e^{-3M_\pi t}$$

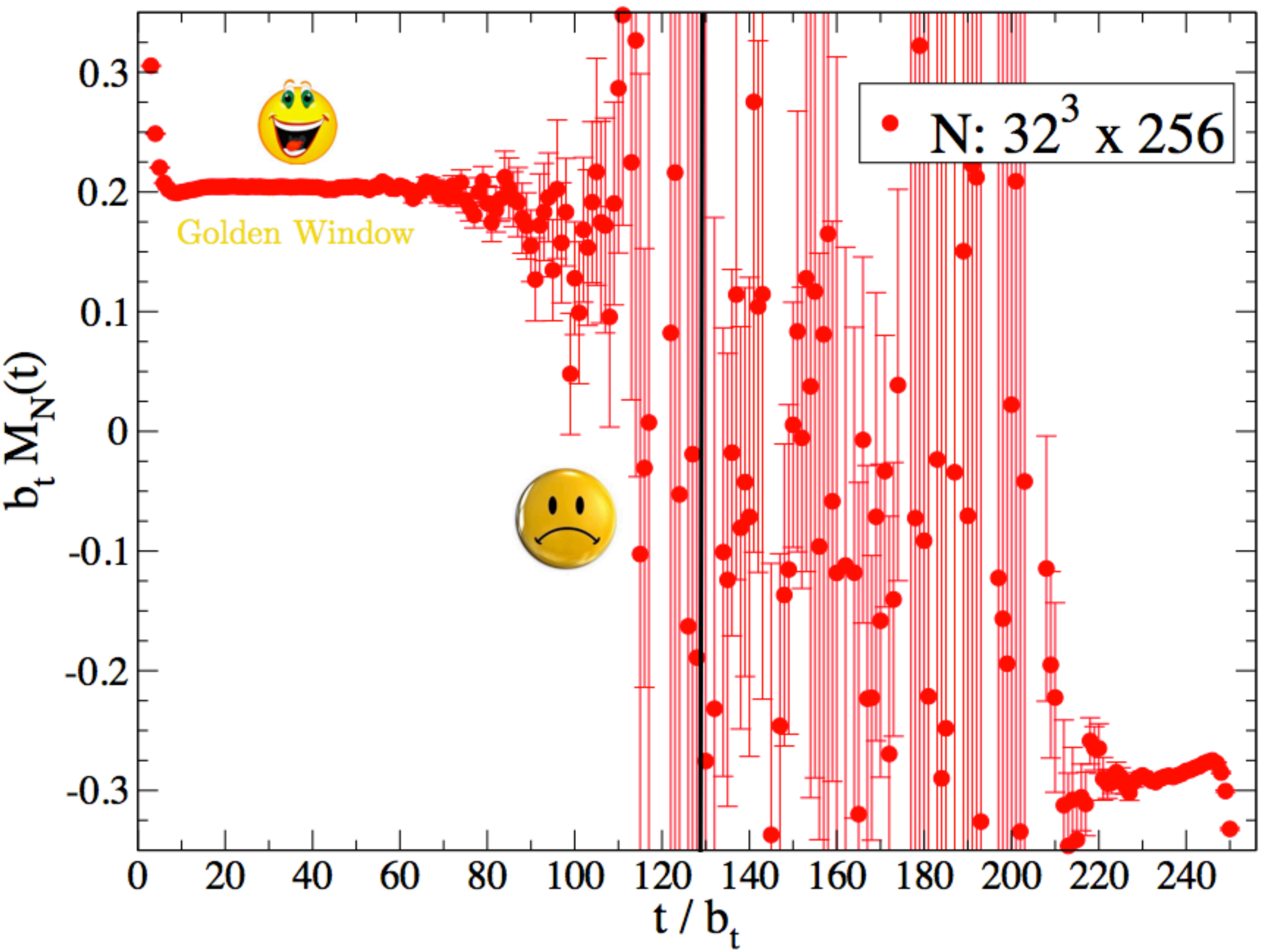
$$\text{signal/noise}\ \sim\ e^{-(M_N-\frac{3}{2}M_\pi)t}$$

signal/noise problem = quark identity crisis!



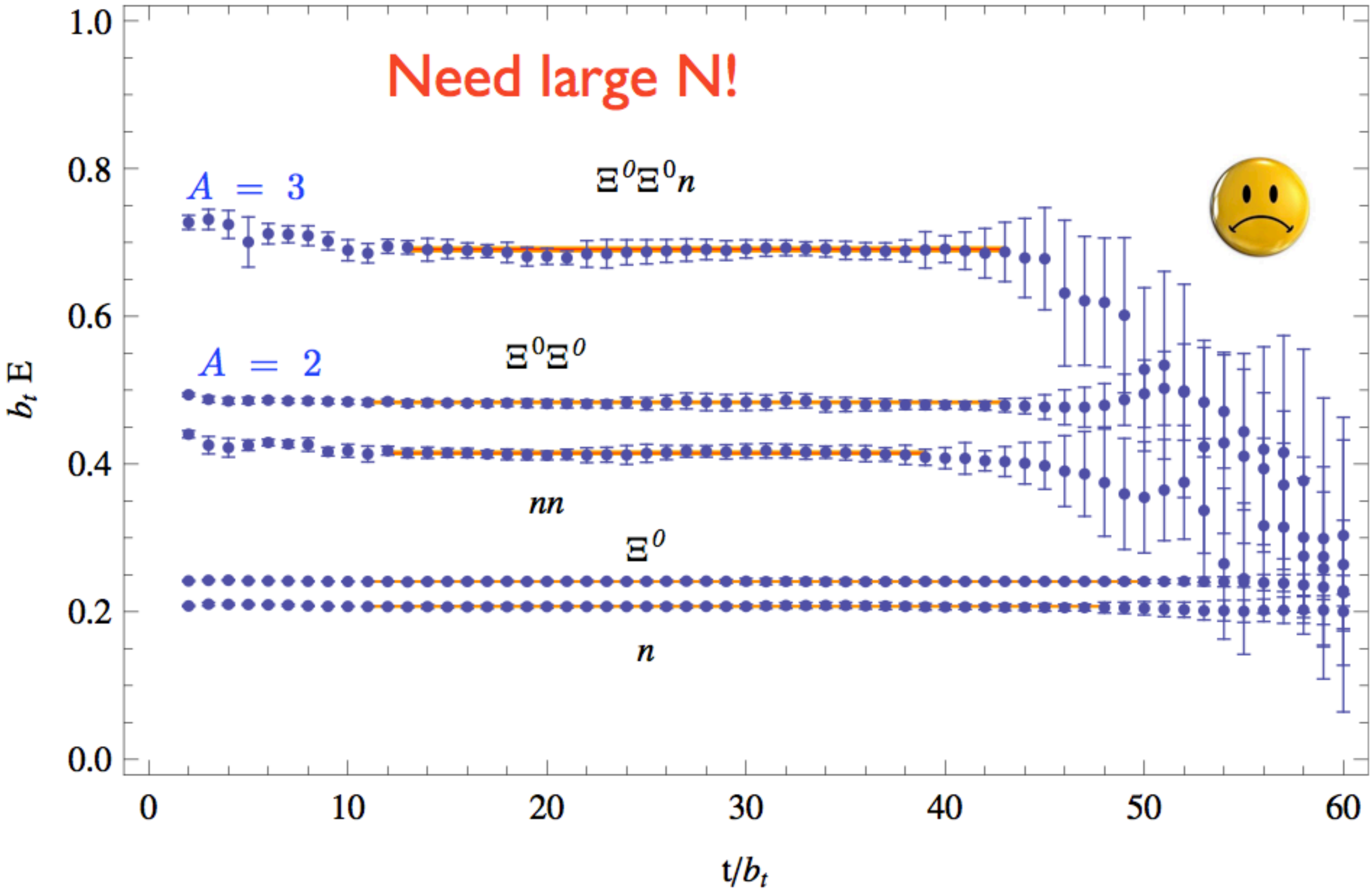


N: 32^3 x 256
$b_t M_N(t)$
t / b_t
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
20
40
60
80
100
120
140
160
180
200
220
240



N: 32^3 x 256
Golden Window
b_t M_N(t)
t / b_t
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
20
40
60
80
100
120
140
160
180
200
220
240

"Golden window" persists to $A > 1$



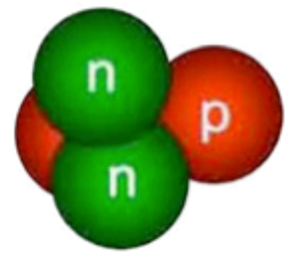
Contraction bottleneck for $A \gg 2$?

Naive factorial growth!

np: 36

nnp: 2880

npnp: 518400

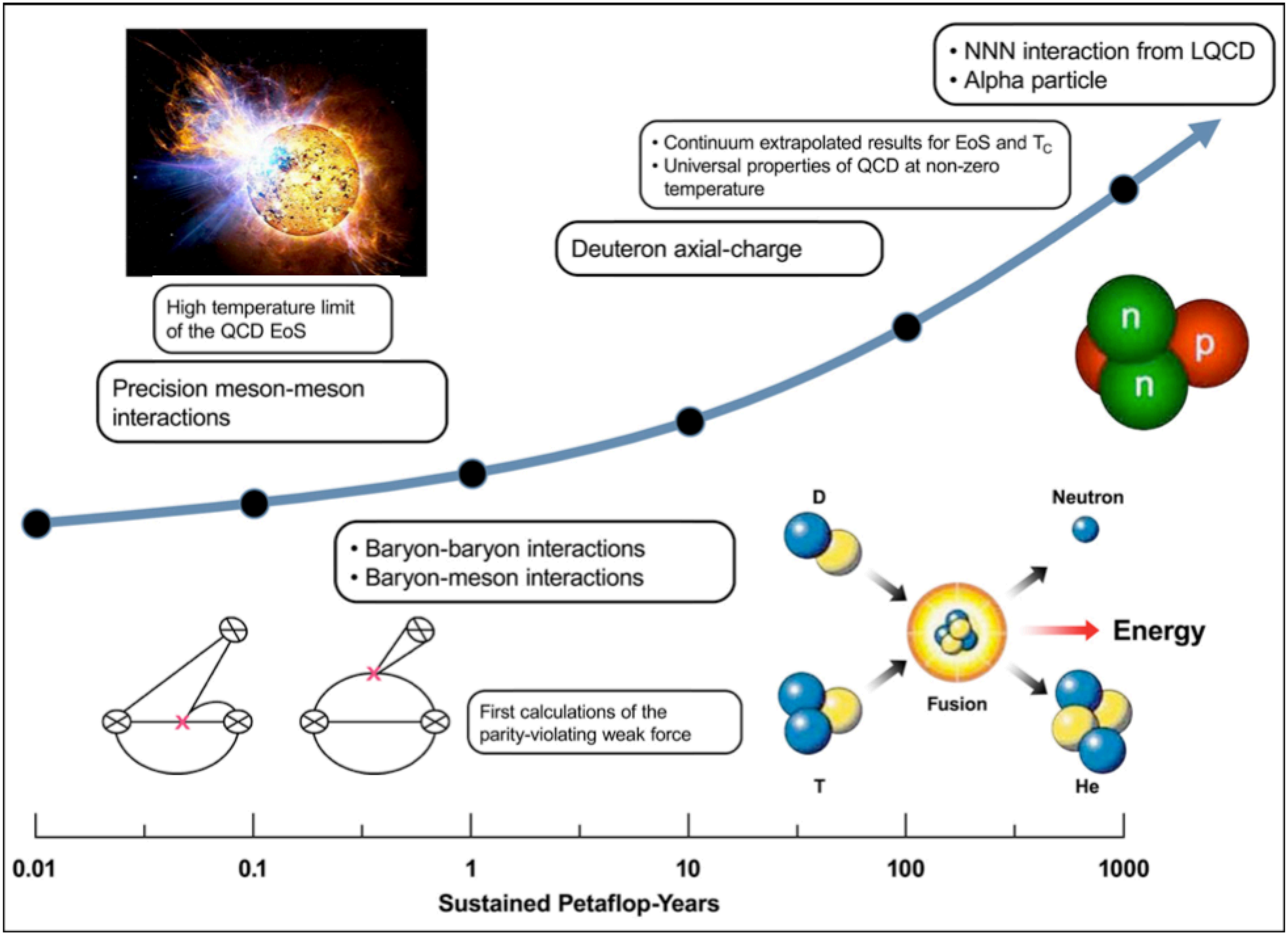


$\vdots$

(A,Z): (A+Z)! (2A-Z)!

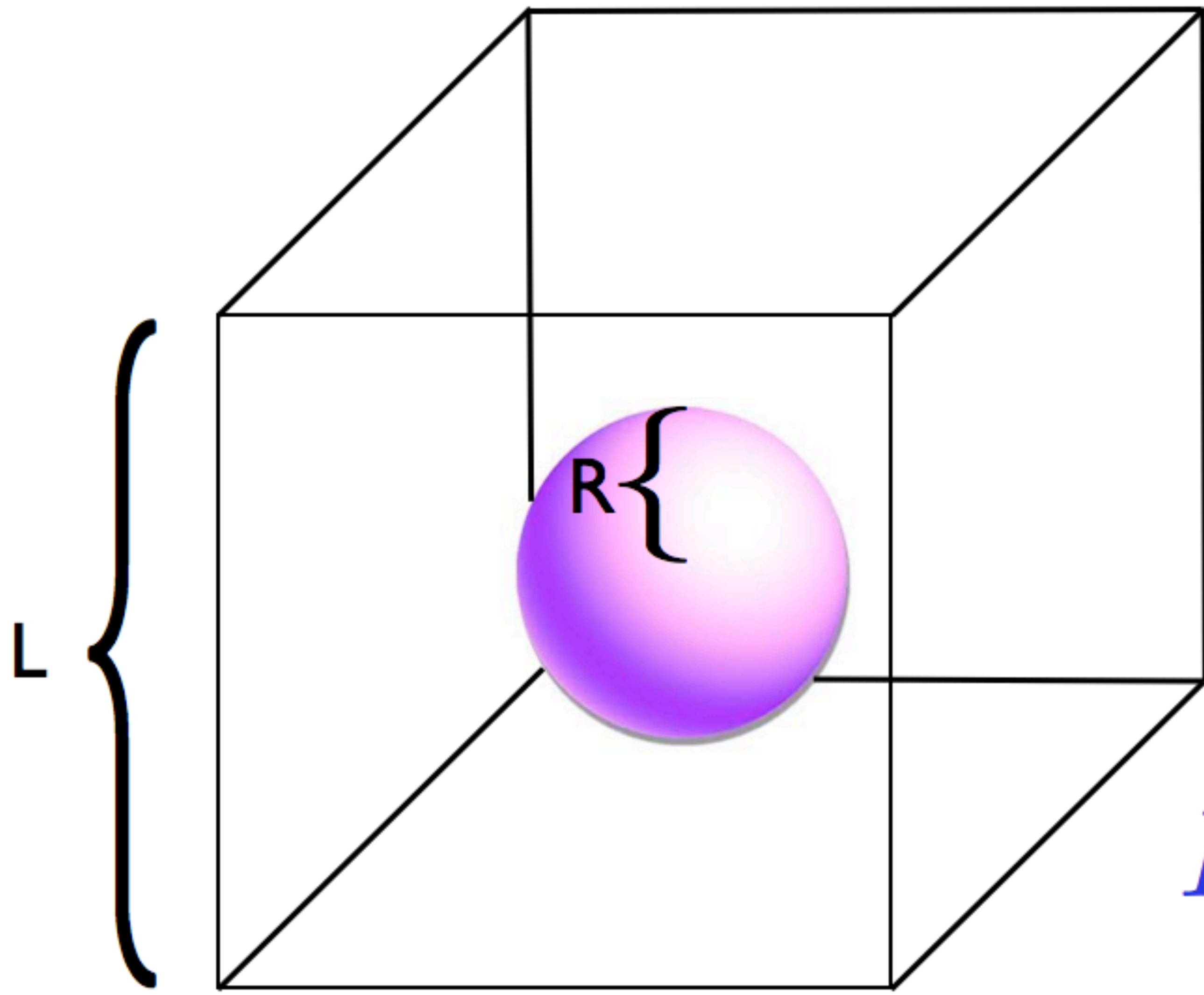
Recursion relations for mesons $\rightarrow A$ growth!

Very recent progress for Baryons!
(Detmold,Orginos)



NNN interaction from LQCD
Alpha particle
Continuum extrapolated results for EoS and T_C
Universal properties of QCD at non-zero temperature
Deuteron axial-charge
High temperature limit of the QCD EoS
Precision meson-meson interactions
n
p
n
Baryon-baryon interactions
Baryon-meson interactions
D
Neutron
Energy
Fusion
T
He
First calculations of the parity-violating weak force
0.01
0.1
1
10
100
1000
Sustained Petaflop-Years

Scattering in a finite volume



$$R < L/2$$

Recall NR scattering

$$\mathcal{A}_2(p) = \frac{4\pi}{M}\frac{1}{p\cot\delta(p) - ip}$$

=

sum of poles in a Finite Volume!

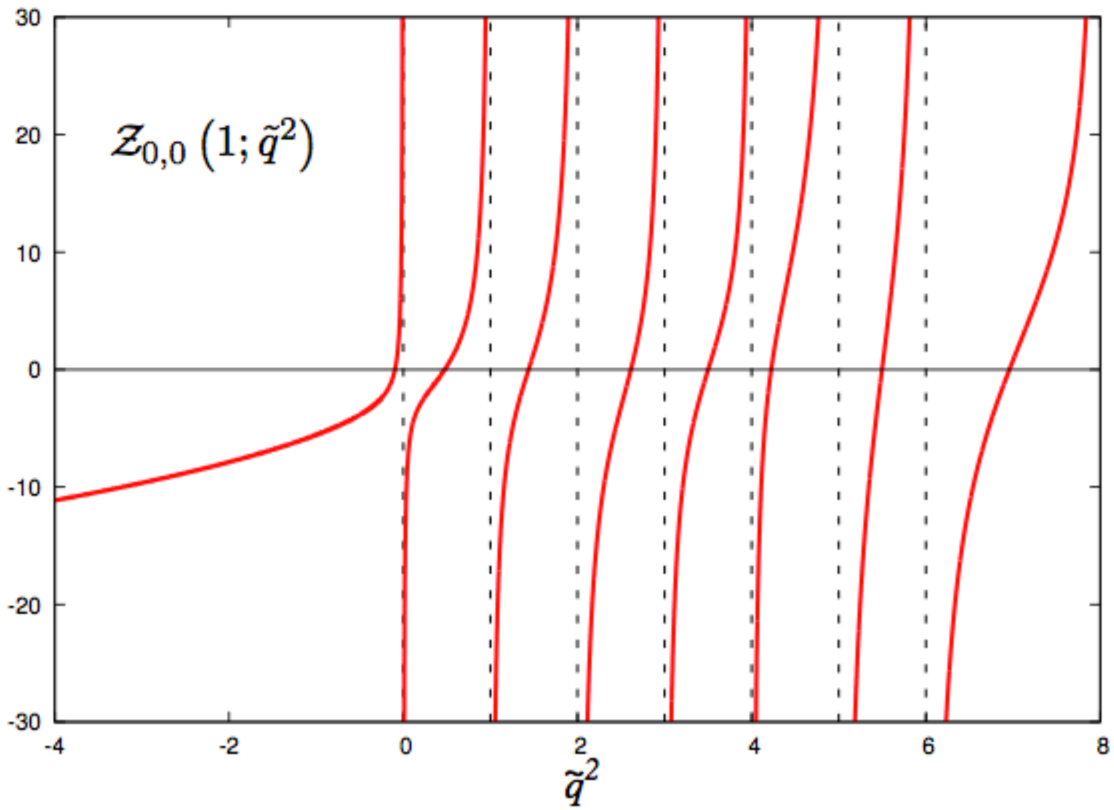
$$\boxed{\mathcal{A}_2^{-1}(p) = 0}$$

eigenvalue equation

S-wave at Finite Volume

$$q\cot\delta_0 = \frac{2}{\sqrt{\pi}L} \mathcal{Z}_{0,0}\left(1;\tilde{q}^2\right) \qquad \mathcal{Z}_{0,0}(1;\tilde{q}^2) = \frac{1}{\sqrt{4\pi}} \lim_{\Lambda_{\mathbf{n}}\to\infty} \left[\sum_{\mathbf{n}}^{\Lambda_{\mathbf{n}}} \frac{1}{|\mathbf{n}|^2 - \tilde{q}^2} - 4\pi\Lambda_{\mathbf{n}} \right]$$

$$E_2^{(AB)} = \sqrt{q^2 + m_A^2} + \sqrt{q^2 + m_B^2} + \mathcal{O}\left(e^{-M_\pi L}\right)$$



(Luscher, 1990)

Weak coupling expansion:

$$\Delta E_0(2,L) = \frac{4\pi\, a_{\pi\pi}}{m_\pi L^3}\left\{1 - \left(\frac{a_{\pi\pi}}{\pi L}\right)\mathcal{I} + \left(\frac{a_{\pi\pi}}{\pi L}\right)^2\left[\mathcal{I}^2 - \mathcal{J}\right] + \left(\frac{a_{\pi\pi}}{\pi L}\right)^3\left[-\mathcal{I}^3 + 3\mathcal{I}\mathcal{J} - \mathcal{K}\right]\right\} + \frac{8\pi^2 {a_{\pi\pi}}^3}{m_\pi L^6}\, r_{\pi\pi} + \mathcal{O}\left(L^{-7}\right)$$

Calculated on the lattice!

phase shift

$$\mathcal{I} = \lim_{\Lambda_j \to \infty} \sum_{\mathbf{i} \neq \mathbf{0}}^{|\mathbf{i}| \leq \Lambda_j} \frac{1}{|\mathbf{i}|^2} - 4\pi\Lambda_j = -8.91363291781$$

$$\mathcal{J} = \sum_{\mathbf{i} \neq \mathbf{0}} \frac{1}{|\mathbf{i}|^4} = 16.532315959$$

$$\mathcal{K} = \sum_{\mathbf{i} \neq \mathbf{0}} \frac{1}{|\mathbf{i}|^6} = 8.401923974433$$

$$\Delta E = 2\sqrt{m_\pi^2 + q^2} - 2m_\pi$$

What about bound states?

$$\mathcal{A}_2(p) = \frac{8\pi}{M}\frac{1}{p\cot\delta(p) - ip} \longrightarrow \cot\delta(i\gamma) = i$$

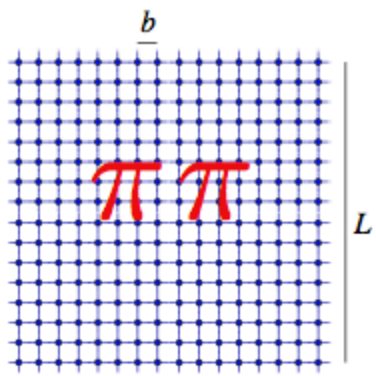
Finite-V:

$$\cot\delta(i\kappa) = i - i\sum_{\mathbf{m}\neq 0}\frac{e^{-|\mathbf{m}|\kappa L}}{|\mathbf{m}|\kappa L}$$

$$\kappa = \gamma + \frac{6}{L}\frac{e^{-\gamma L}}{1 - \gamma r_3} + \mathcal{O}(e^{-\sqrt{2}\gamma L})$$

Need several volumes!

$\pi\pi$ scattering in lattice QCD



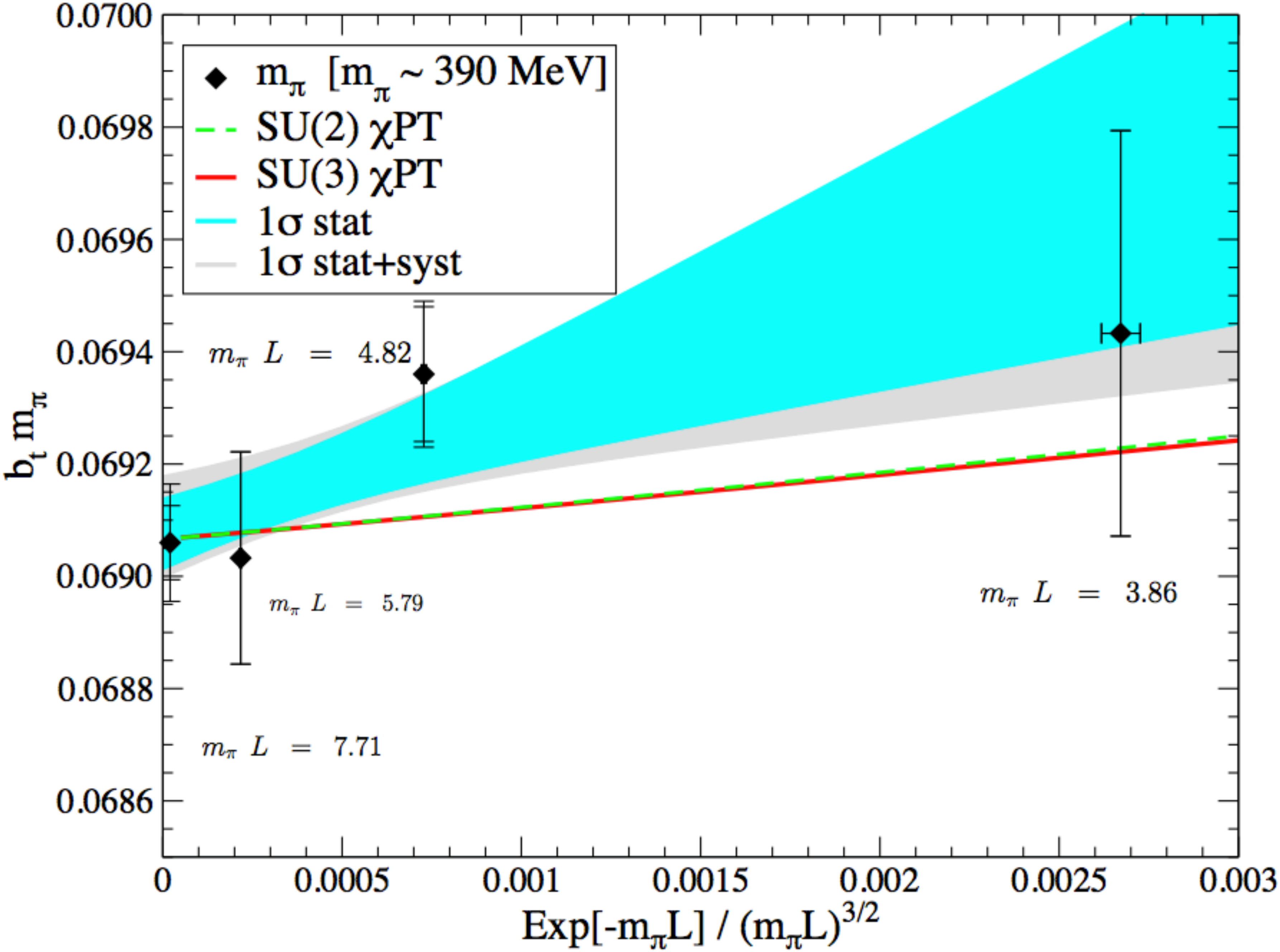
$$\mathcal{O}_{\pi^+}(t,\vec{x}) = \overline{u}(t,\vec{x})\gamma_5 d(t,\vec{x})$$

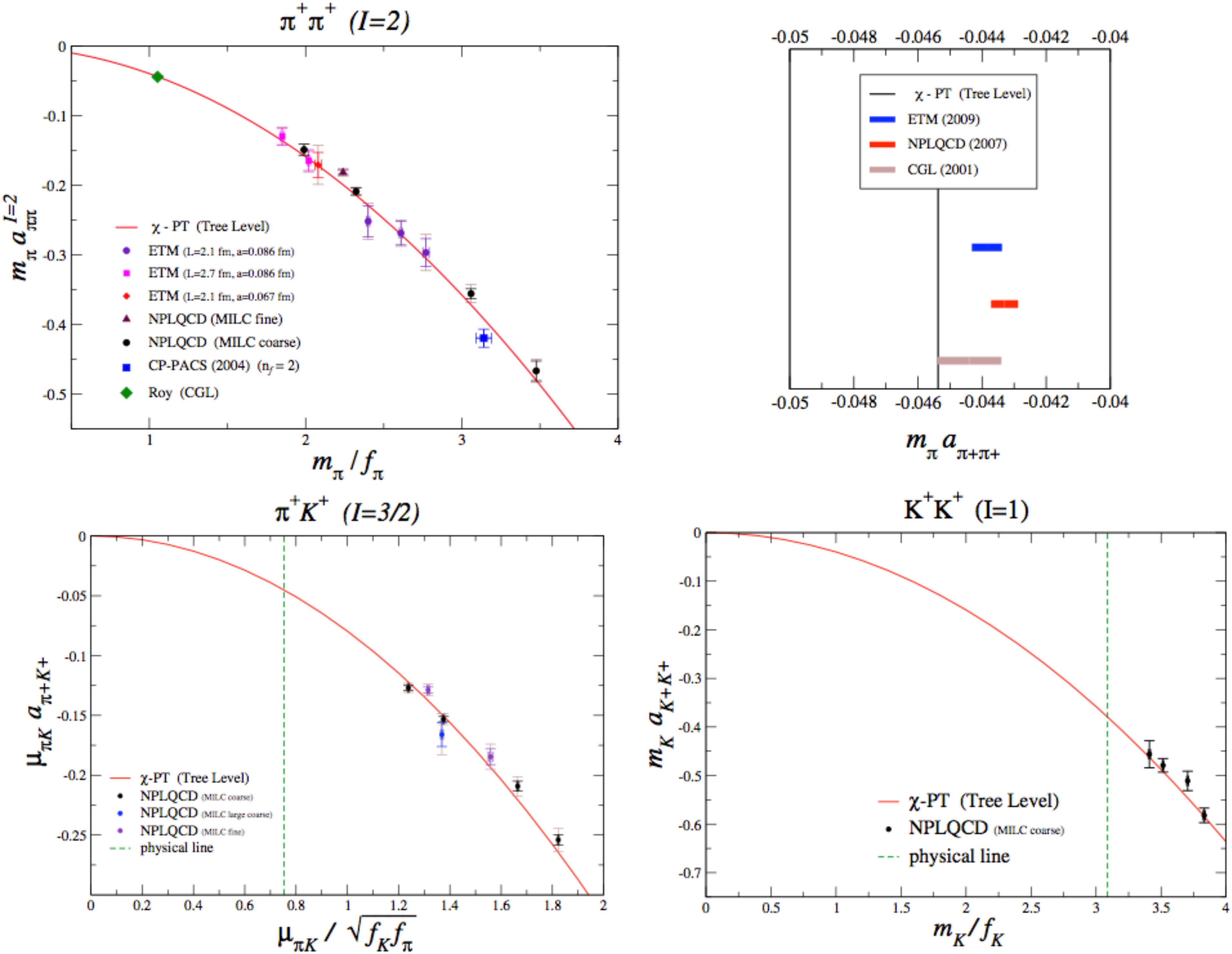
$$C(p,t) = \langle 0| \sum_{|\mathbf{p}|=p} \sum_{\mathbf{x},\mathbf{y}} e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \mathcal{O}_{\pi^-}(t,\mathbf{x})\, \mathcal{O}_{\pi^-}(t,\mathbf{y})\, \mathcal{O}_{\pi^+}(0,\mathbf{0})\, \mathcal{O}_{\pi^+}(0,\mathbf{0})|0\rangle$$

$$C(p,t) \longrightarrow \sum_{n=0}^{\infty} A_n\, e^{-E_n t}$$

Need several levels to get phase shift!

FV Corrections?

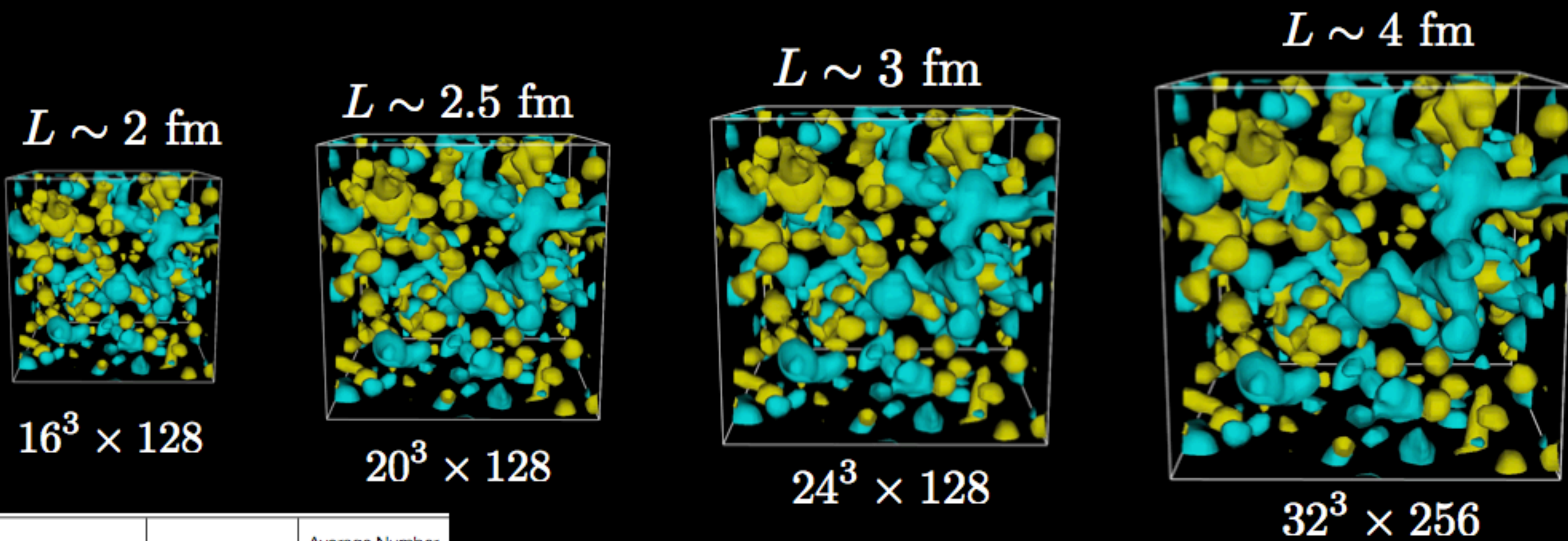




π+π+ (I=2)
0
-0.1
-0.2
-0.3
-0.4
-0.5
1
2
3
4
m_π a_ππ^(I=2)
m_π / f_π
χ - PT (Tree Level)
ETM (L=2.1 fm, a=0.086 fm)
ETM (L=2.7 fm, a=0.086 fm)
ETM (L=2.1 fm, a=0.067 fm)
NPLQCD (MILC fine)
NPLQCD (MILC coarse)
CP-PACS (2004) (n_f = 2)
Roy (CGL)
-0.05
-0.048
-0.046
-0.044
-0.042
-0.04
χ - PT (Tree Level)
ETM (2009)
NPLQCD (2007)
CGL (2001)
m_π a_π+π+
π+K+ (I=3/2)
0
-0.05
-0.1
-0.15
-0.2
-0.25
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
μ_πK a_π+K+
μ_πK / √(f_K f_π)
χ-PT (Tree Level)
NPLQCD (MILC coarse)
NPLQCD (MILC large coarse)
NPLQCD (MILC fine)
physical line
K+K+ (I=1)
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
0
0.5
1
1.5
2
2.5
3
3.5
4
m_K a_K+K+
m_K / f_K
χ-PT (Tree Level)
NPLQCD (MILC coarse)
physical line

$N_f = 2 + 1$ *Anisotropic Clover*

$m_\pi \sim 389$ MeV $\qquad b_s \sim 0.1227(8)$ fm $\qquad b_s/b_t = 3.500(32)$



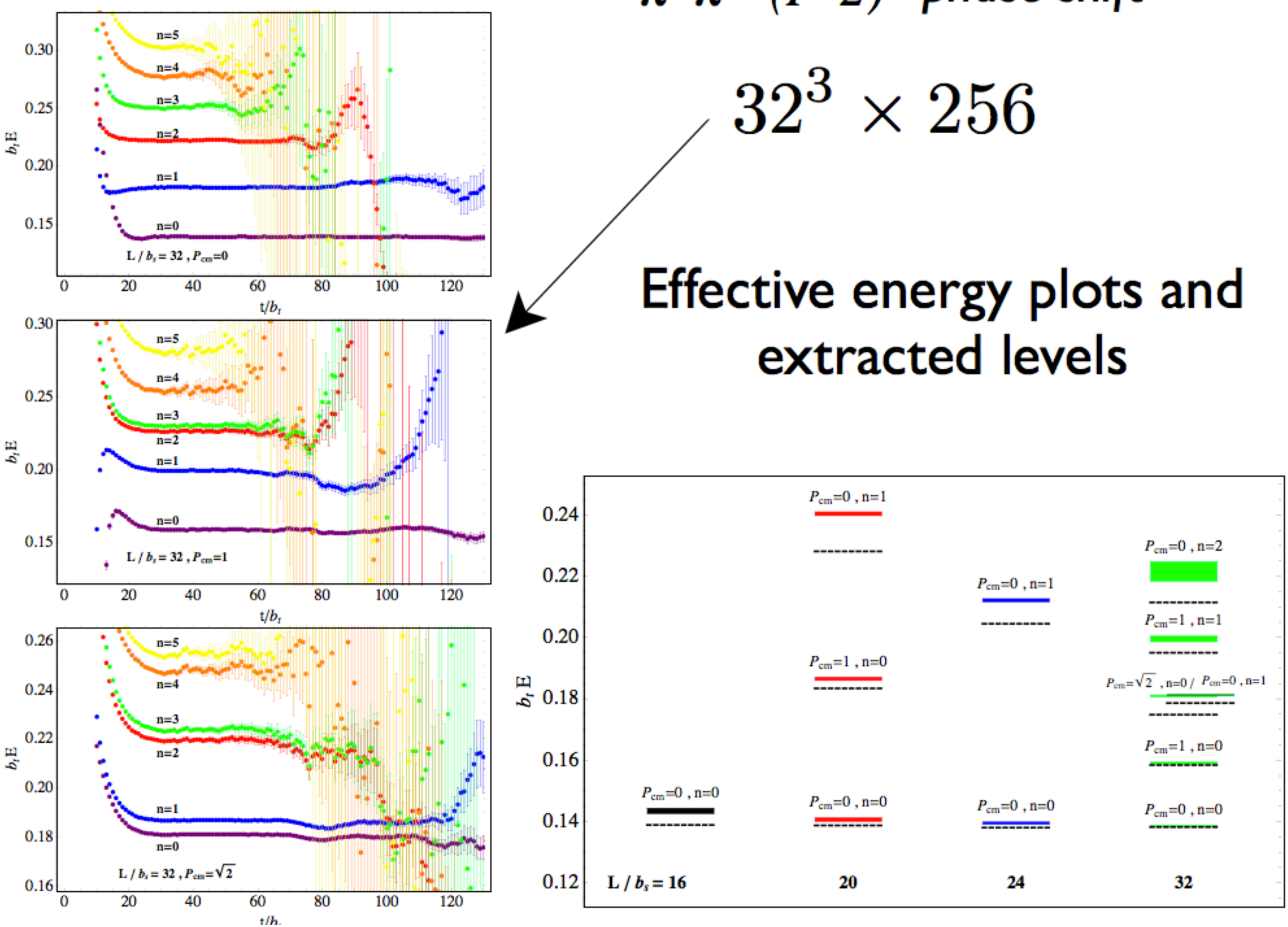
Ensemble	Number of cfgs	Average Number of Sources per cfg
16^3 X 128	2001	224
20^3 X 128	1195	364
24^3 X 128	2215	180
32^3 X 256	774	174

High statistics!

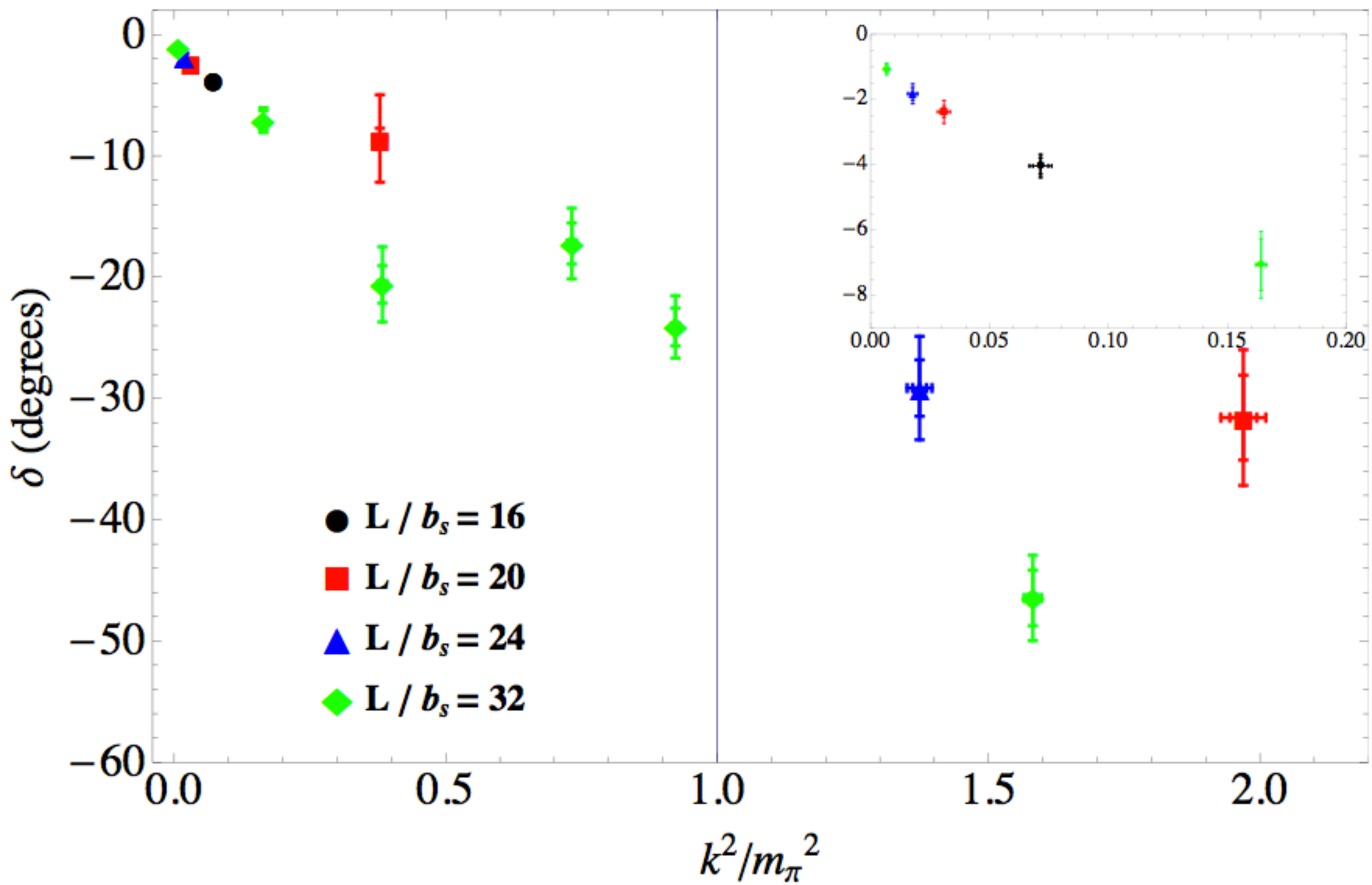
$\pi^+\pi^+$ *(I=2)* *phase shift*

$32^3 \times 256$

Effective energy plots and extracted levels



$m_\pi \sim 389$ MeV **PHASE SHIFT DATA**



$$t(s) = \left(\frac{s}{s-4}\right)^{1/2} \frac{1}{2i}\{e^{2i\delta(s)} - 1\}$$

$$\frac{k\,\cot\delta}{m_\pi} = -\frac{1}{m_\pi a} + \frac{1}{2}\, m_\pi r \left(\frac{k^2}{m_\pi^2}\right) + P\,(m_\pi r)^3 \left(\frac{k^2}{m_\pi^2}\right)^2 + \ldots$$

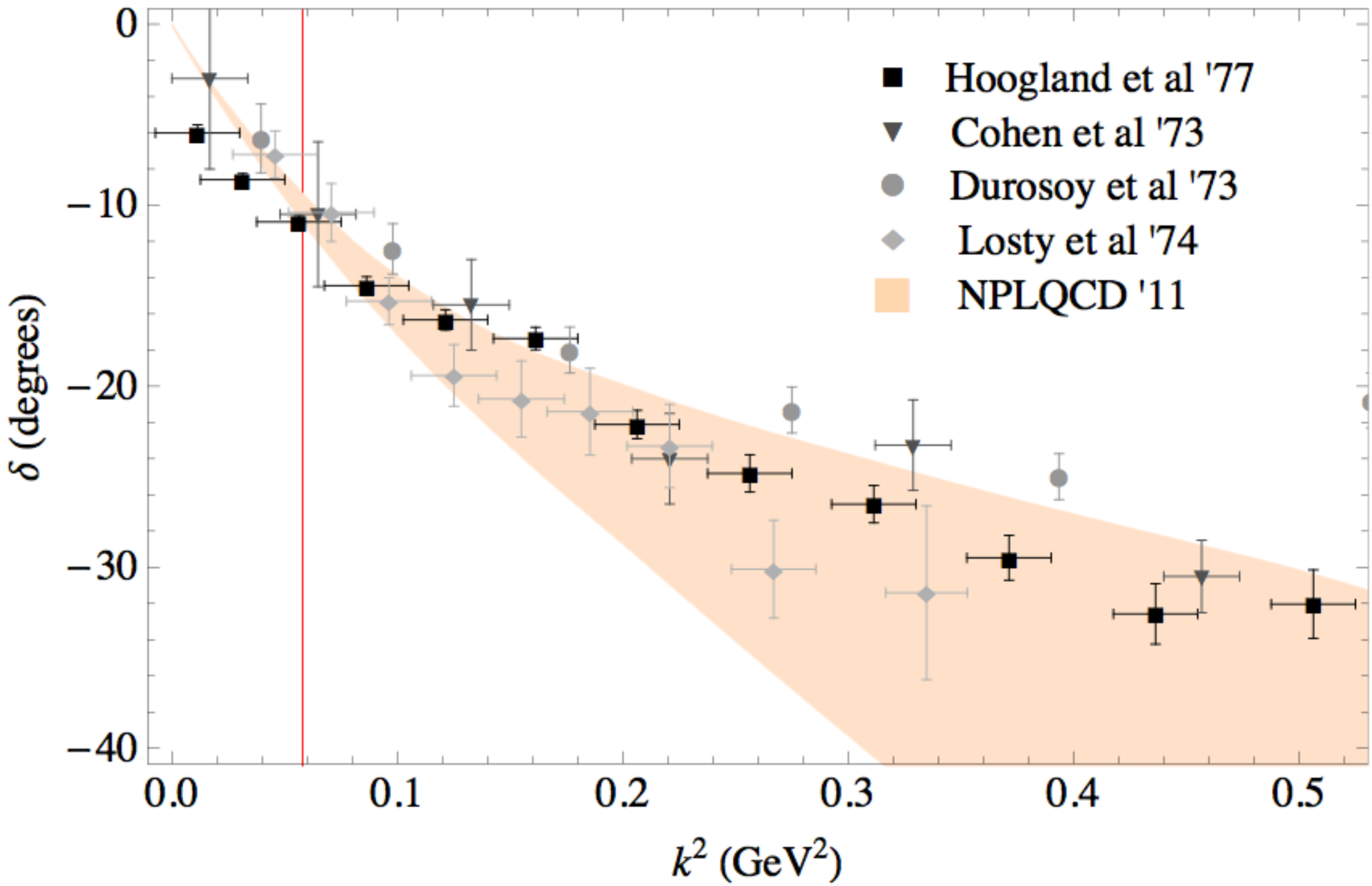
E.R.T.

NLO:

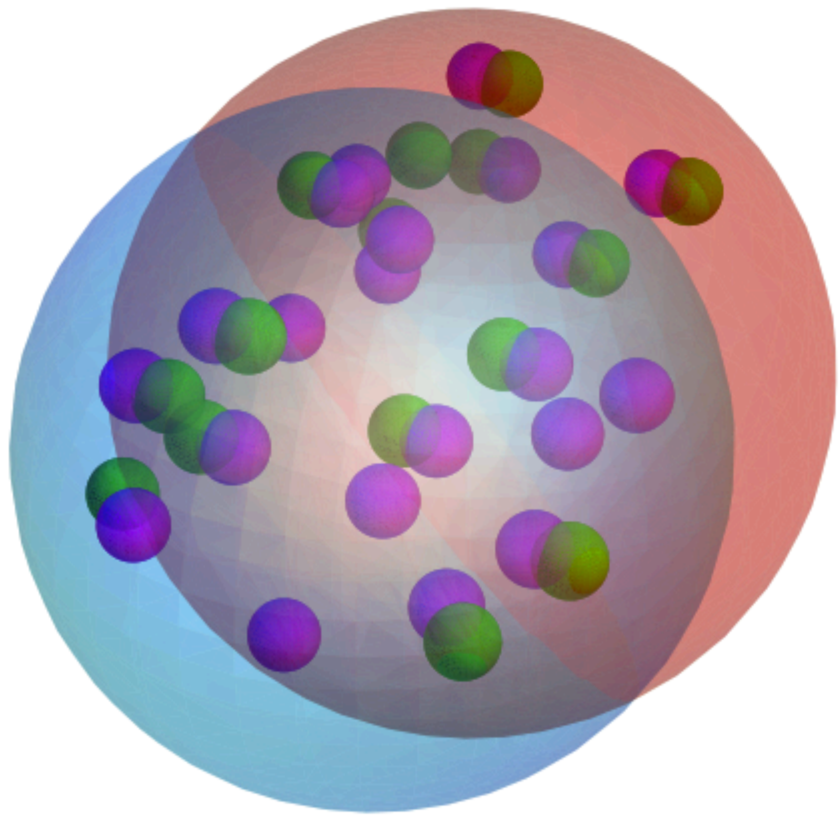
$\chi - PT$

$$\begin{aligned}
t(k) =& -\frac{m_\pi^2}{8\pi f_\pi^2} - \frac{m_\pi^4}{f_\pi^4}\left(C_1 - \frac{31}{384\pi^3}\right) \\
&+ \frac{k^2}{f_\pi^2}\left[-\frac{1}{4\pi} + \frac{m_\pi^2}{f_\pi^2}\left(\frac{301}{1152\pi^3} - \frac{1}{128\pi^2}C_2 - \frac{7}{2}C_1\right)\right] \\
&+ \frac{k^4}{f_\pi^4}\left[\frac{14}{45\pi^3} - \left(\frac{19}{8}C_1 - \frac{9}{512\pi^2}C_2 + 216\pi C_4\right)\right] \\
&- \frac{1}{4\pi^3 f_\pi^4}\left(\frac{3}{32}m_\pi^4 + \frac{5}{12}m_\pi^2 k^2 + \frac{5}{9}k^4\right)\log\left(\frac{m_\pi^2}{f_\pi^2}\right) \\
&+ \frac{1}{16\pi^3 f_\pi^4}\left(\frac{1}{4}m_\pi^4 + m_\pi^2 k^2 + k^4\right)\sqrt{\frac{k^2}{k^2+m_\pi^2}}\log\left(\frac{\sqrt{\frac{k^2}{k^2+m_\pi^2}} - 1}{\sqrt{\frac{k^2}{k^2+m_\pi^2}} + 1}\right) \\
&+ \frac{1}{8\pi^3 f_\pi^4}\left(\frac{3}{16}m_\pi^4 + \frac{7}{9}m_\pi^2 k^2 + \frac{11}{18}k^4\right)\sqrt{\frac{k^2+m_\pi^2}{k^2}}\log\left(\frac{\sqrt{\frac{k^2+m_\pi^2}{k^2}} - 1}{\sqrt{\frac{k^2+m_\pi^2}{k^2}} + 1}\right) \\
&- \frac{m_\pi^4}{128\pi^3 f_\pi^4}\left(1 + \frac{13}{12}\frac{m_\pi^2}{k^2}\right)\log^2\left(\frac{\sqrt{\frac{k^2+m_\pi^2}{k^2}} - 1}{\sqrt{\frac{k^2+m_\pi^2}{k^2}} + 1}\right) .
\end{aligned}$$

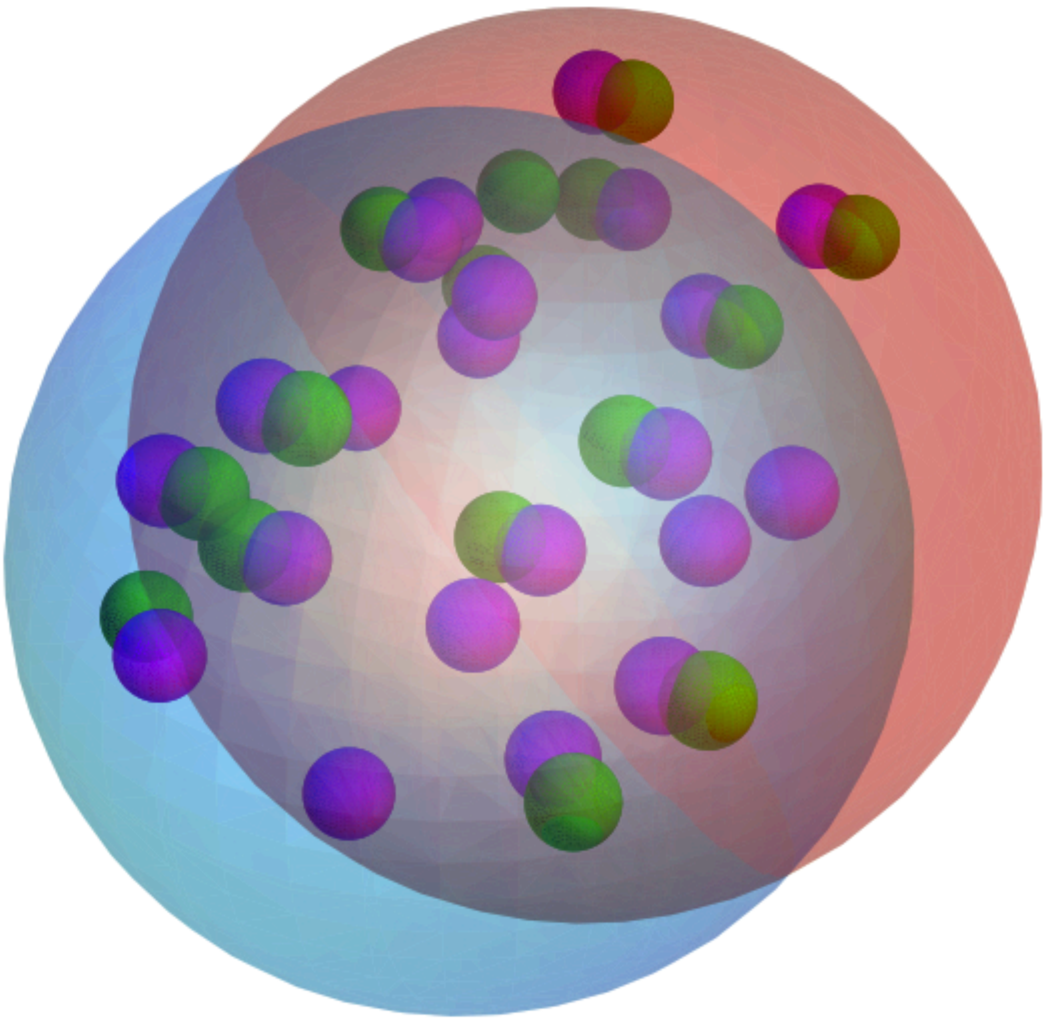
CHIPT EXTRAPOLATION



Baryon-baryon interactions



The simplest nucleus:

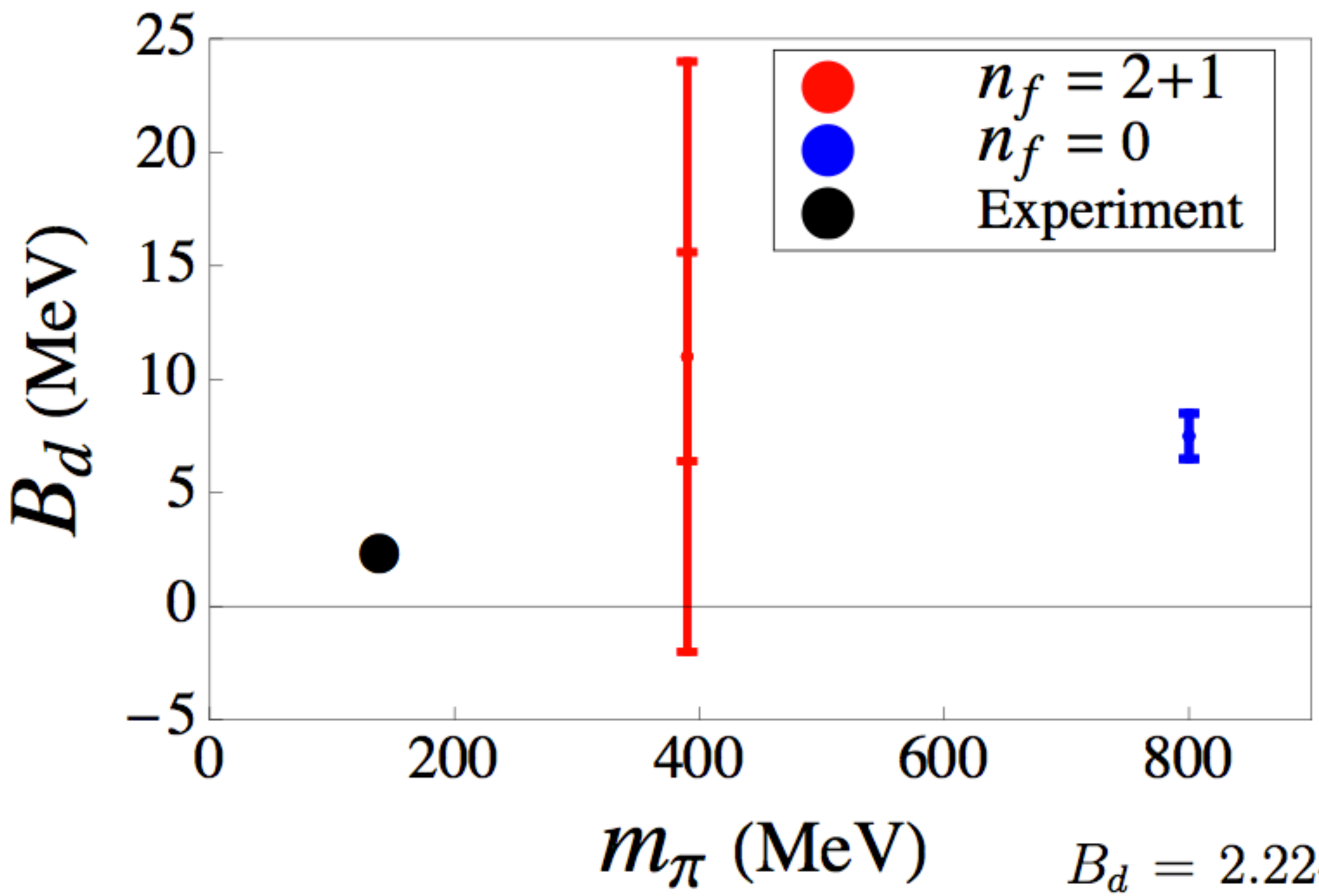


Fundamental benchmark for Lattice QCD!

$$B_d = 2.224644(34)\ \text{MeV}$$

deuteron

Deuteron binding still ambiguous..



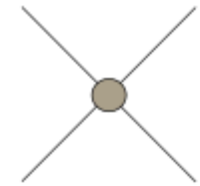
$$B_d = 2.224644(34)\ \text{MeV}$$

Fundamental benchmark for Lattice QCD!

Nuclear physics

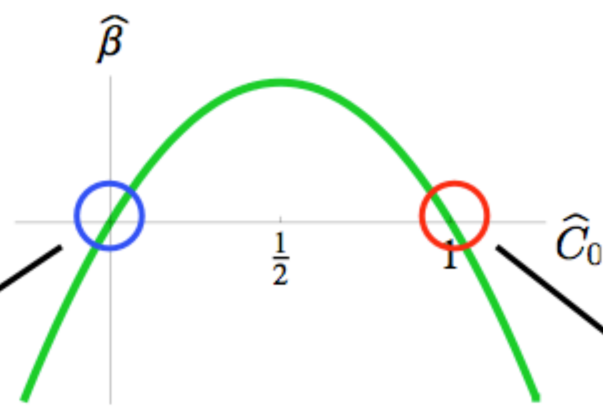
Experiment:

$$a_s^{^1S_0} = -23.714 \text{ fm} \qquad r_s^{^1S_0} = 2.73 \text{ fm}$$
$$a_s^{^3S_1} = 5.425 \text{ fm} \qquad r_s^{^3S_1} = 1.749 \text{ fm}$$



$$a_s \gg \Lambda_{QCD}^{-1}$$

$$\hat{\beta}_0 = \mu \frac{d}{d\mu} \hat{C}_0 = -\hat{C}_0(\hat{C}_0 - 1)$$

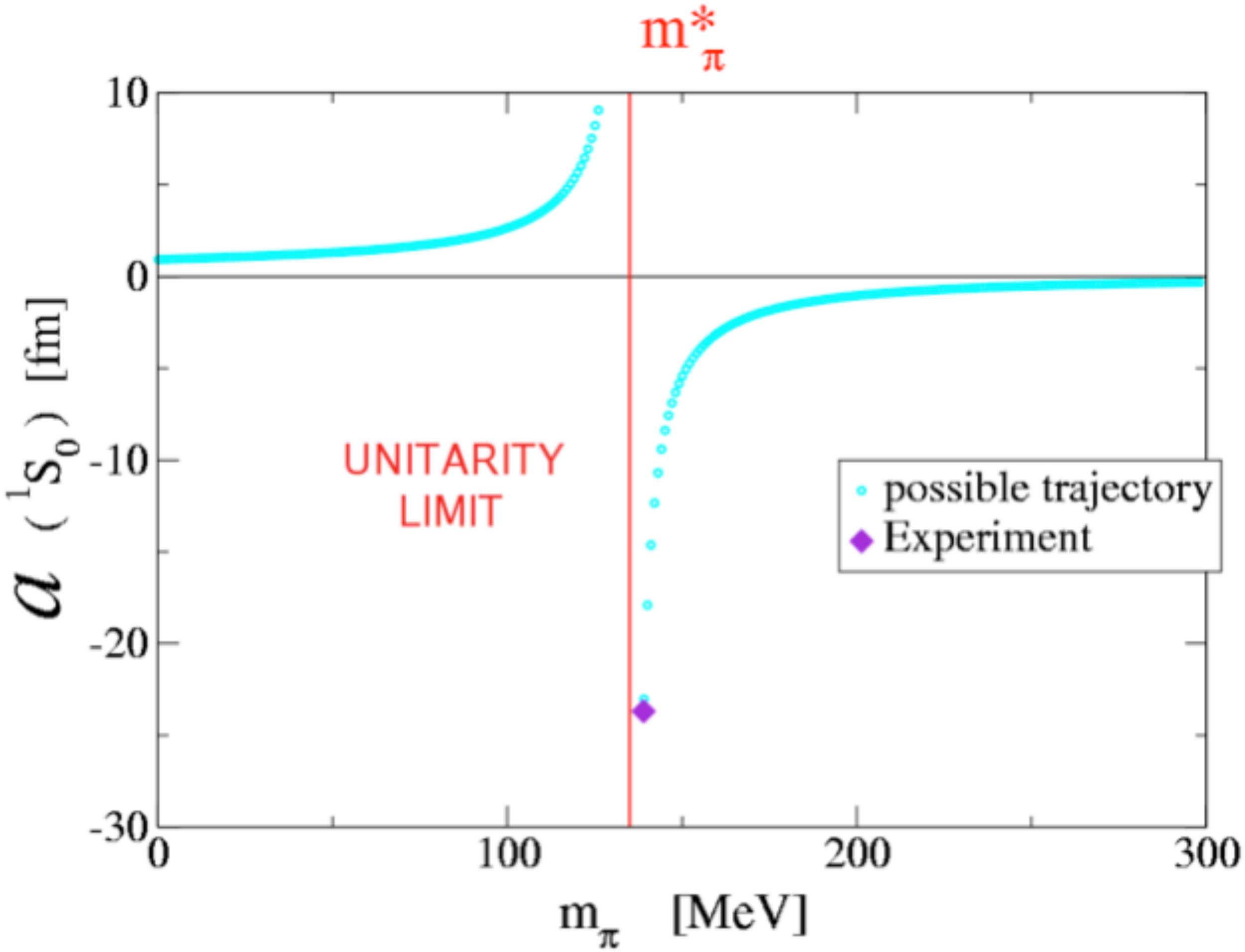


Trivial IR fixed point:
"natural case"

Nontrivial UV fixed point: "unnatural case"

"Unitarity"

Why is nuclear physics near this UV fixed point??

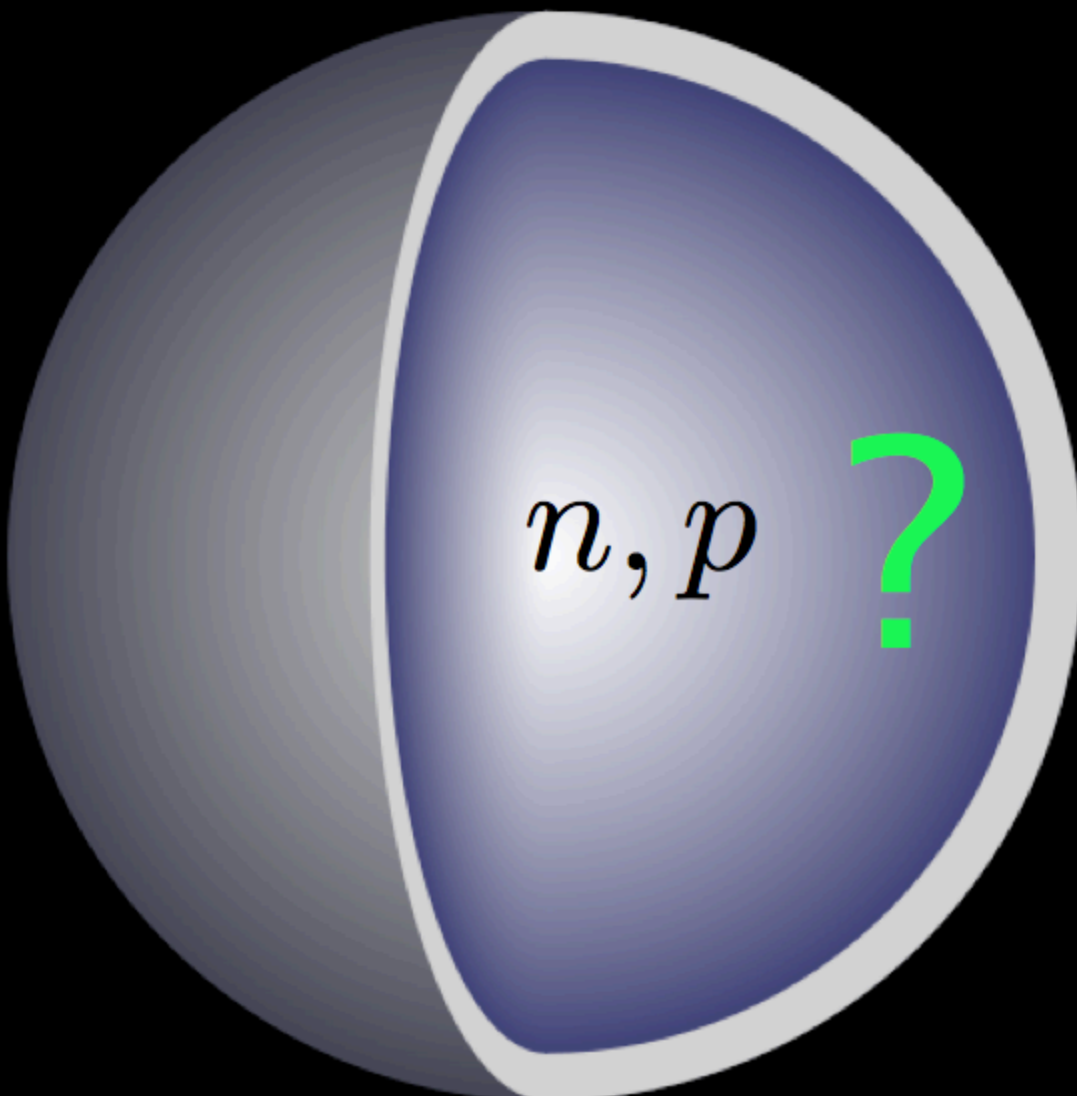


$$a_s^{-1} \sim \frac{m_\pi - m_\pi^*}{m_\pi} \Lambda_{QCD}$$

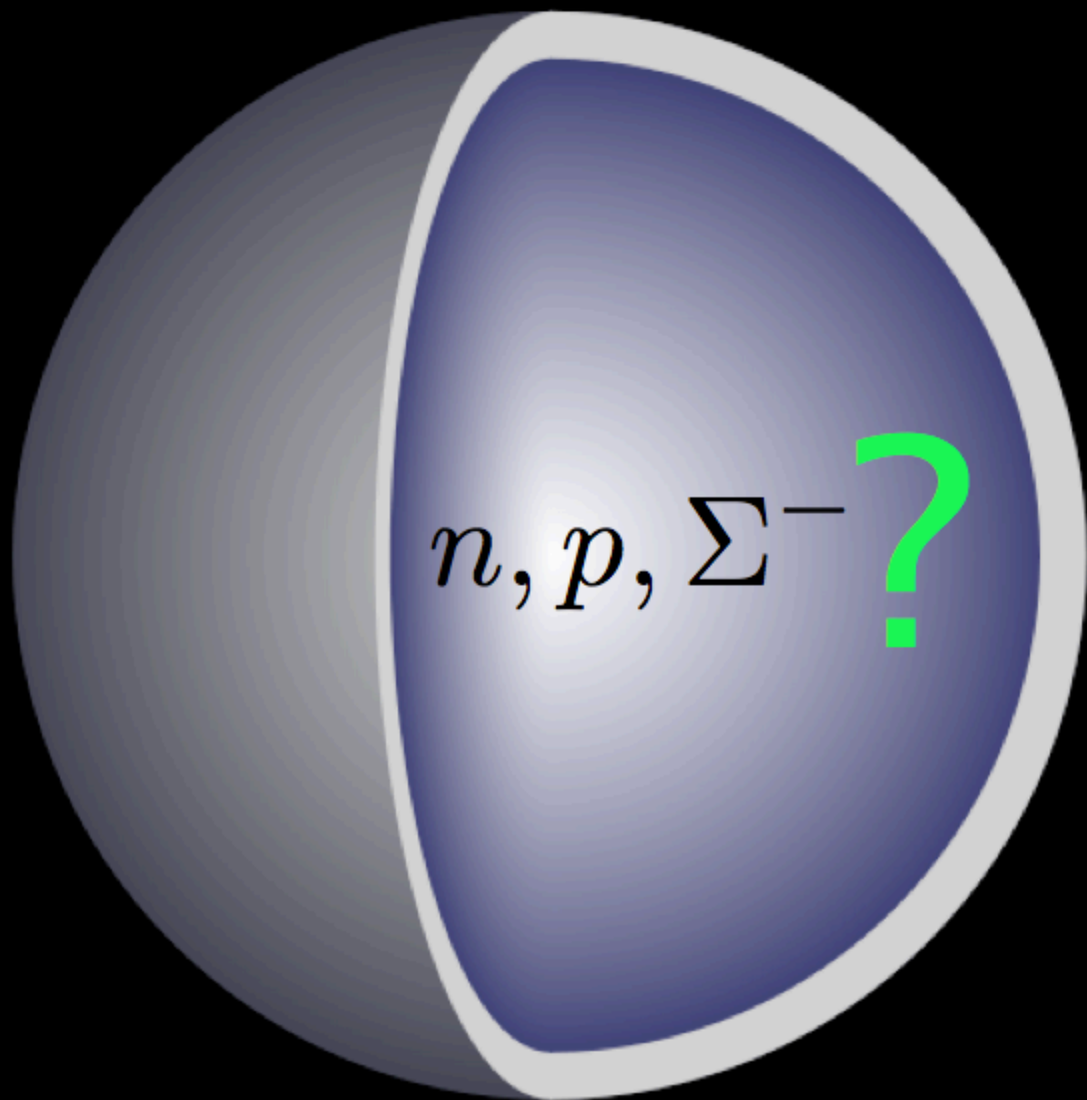
Lattice QCD will answer this question!

What can we measure that we do not know?

Neutron Star Core



Neutron Star Core



BB $SU(3)$ flavor decomposition

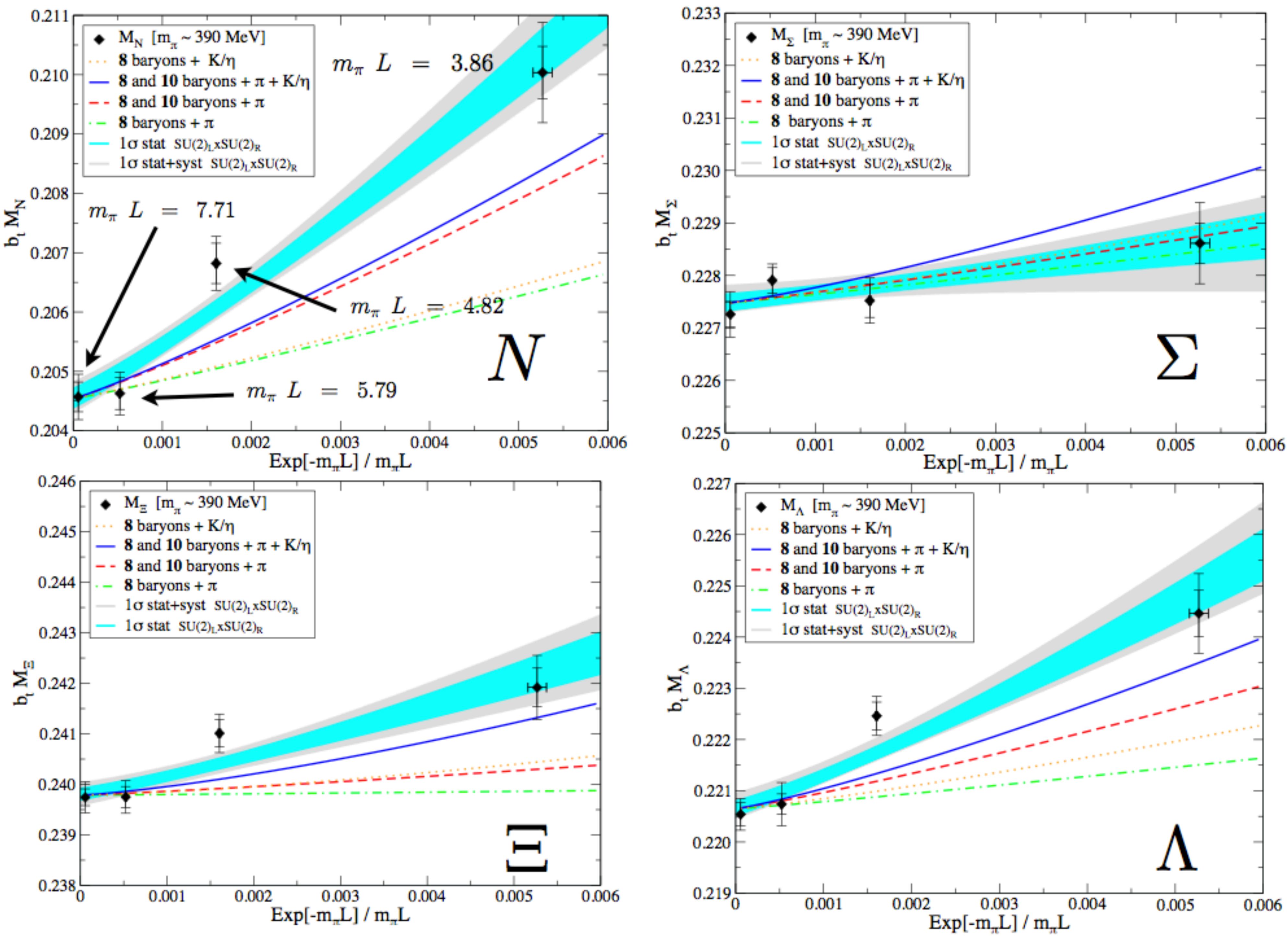
$$\mathbf{8} \otimes \mathbf{8} = \mathbf{27} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1}$$

$\overline{\mathbf{10}}$: ${}^3S_1 \;:\; NN,\ \ldots$

$\mathbf{10}$: ${}^3S_1 \;:\; \Sigma N,\ \ldots$

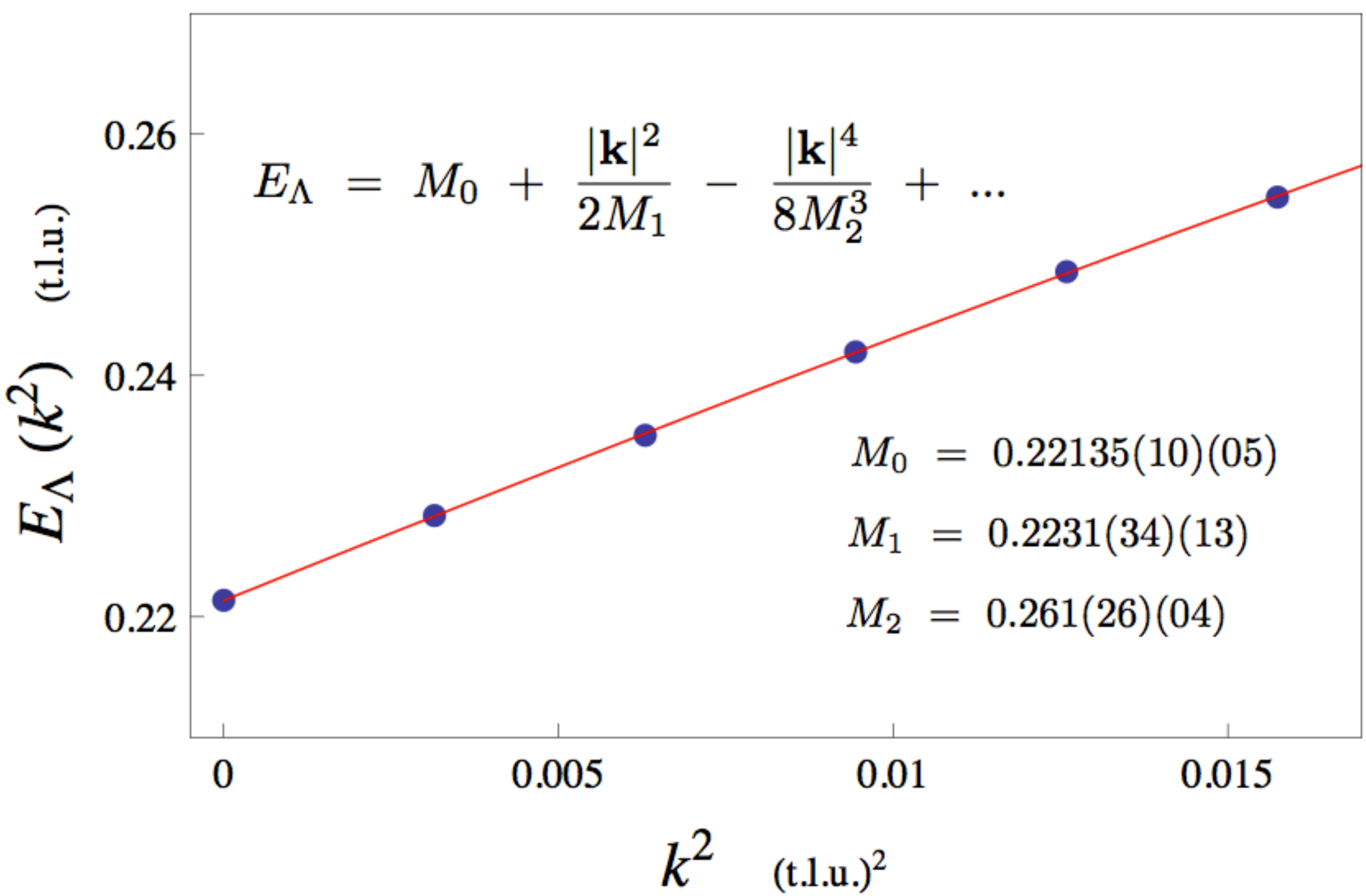
$\mathbf{27}$: ${}^1S_0 \;:\; NN,\ \Sigma\Sigma,\ \Xi\Xi,\ \Sigma N,\ \ldots$

$\mathbf{1}$: ${}^1S_0 \;:\; -\frac{1}{\sqrt{3}}\Lambda\Lambda + \Sigma\Sigma + \frac{2}{\sqrt{3}}\Xi N$



M_N [m_π ~ 390 MeV]
8 baryons + K/η
8 and 10 baryons + π + K/η
8 and 10 baryons + π
8 baryons + π
1σ stat SU(2)_L xSU(2)_R
1σ stat+syst SU(2)_L xSU(2)_R
m_π L = 3.86
m_π L = 7.71
m_π L = 4.82
m_π L = 5.79
N
b_t M_N
Exp[-m_πL] / m_πL
M_Σ [m_π ~ 390 MeV]
Σ
b_t M_Σ
M_Ξ [m_π ~ 390 MeV]
Ξ
b_t M_Ξ
M_Λ [m_π ~ 390 MeV]
Λ
b_t M_Λ

Energy-Momentum Relation



Special relativity satisfied!

Is there an H-dibaryon?

In SU(3) limit:

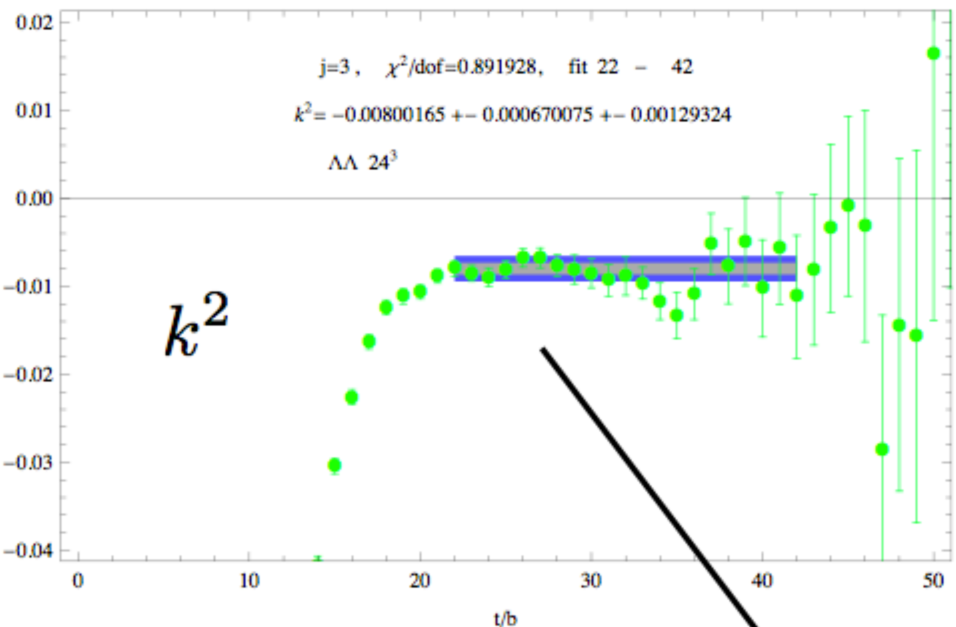
$$(\mathbf{8} \otimes \mathbf{8})_S = -\frac{1}{\sqrt{3}}\Lambda\Lambda + \Sigma\Sigma + \frac{2}{\sqrt{3}}\Xi N$$

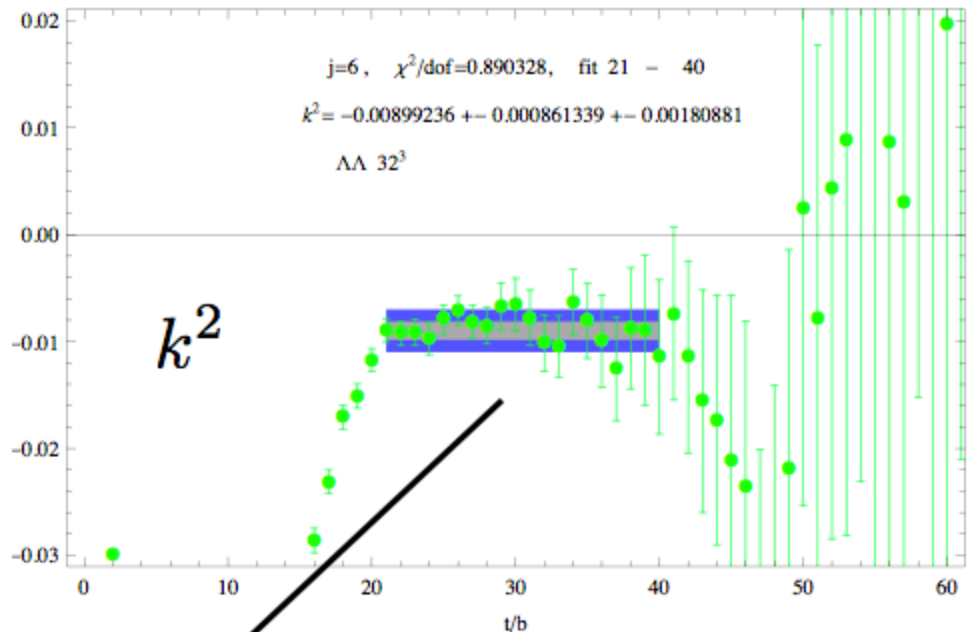
With SU(3) breaking:

$\Lambda\Lambda - \Sigma\Sigma - N\Xi$ coupled channels!

Need isolated G.S.! ✓

$24^3 \times 128$ $\Lambda\Lambda$ $32^3 \times 256$





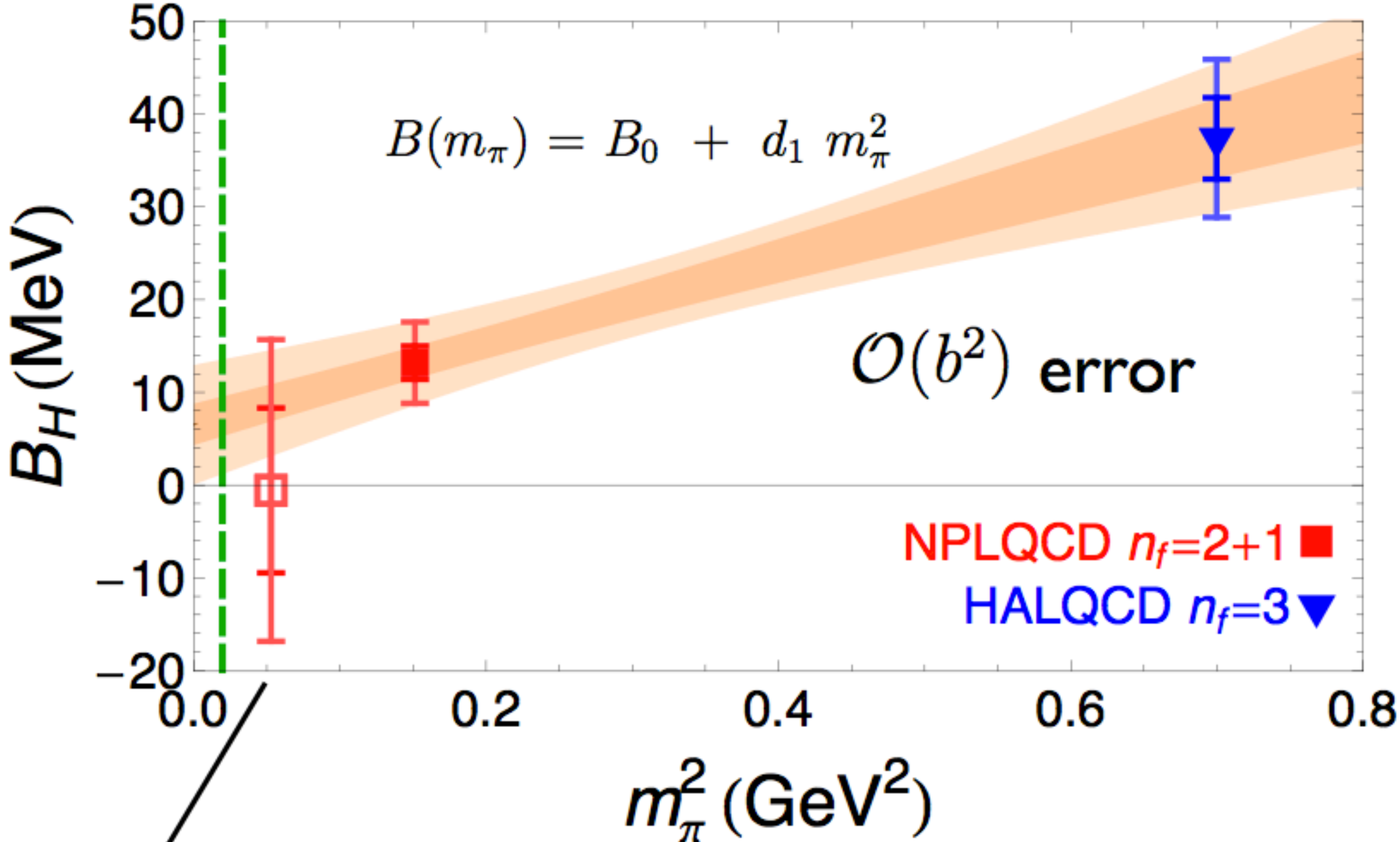
$$k = i\kappa$$

$$\kappa = \gamma + \frac{6}{L}\frac{e^{-\gamma L}}{1 - \gamma r_3} + \mathcal{O}(e^{-\sqrt{2}\gamma L})$$

$m_\pi \sim 389$ MeV

$$B_H = \frac{\gamma^2}{M_\Lambda} = 16.6 \pm 2.1 \pm 4.5 \pm 1.0 \pm 0.6 \text{ MeV}$$

The Bottom Line



preliminary!

$$B_H^{\text{quadratic}} = 7.4 \pm 2.1 \pm 5.8 \text{ MeV}$$

What about the continuum limit?

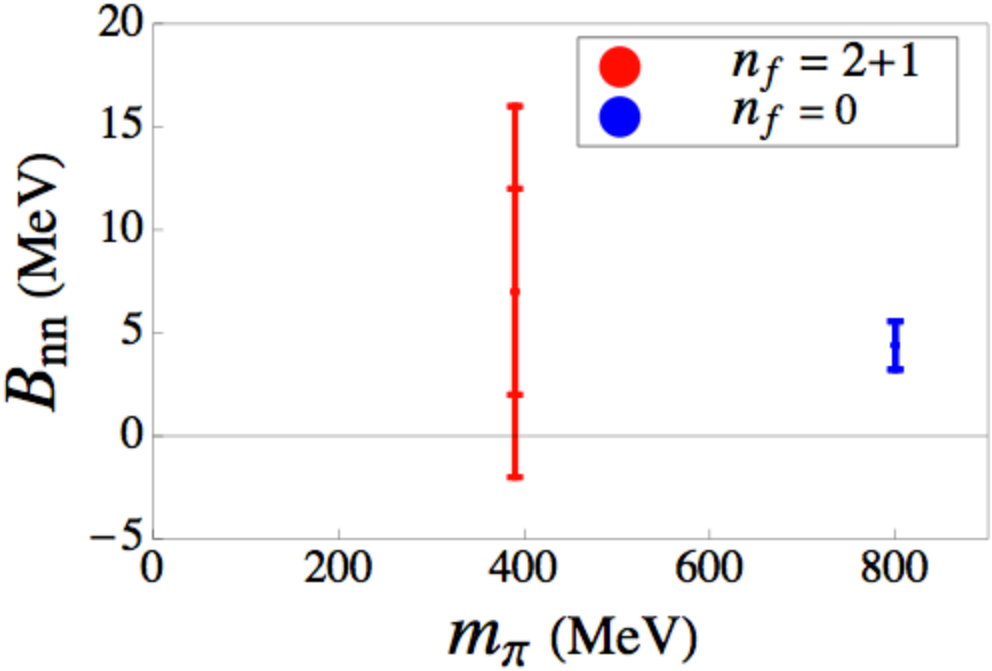
- errors scale as $\mathcal{O}(b^2)$
- coefficients of Symanzik action are different!
- Lorentz breaking operators are small!

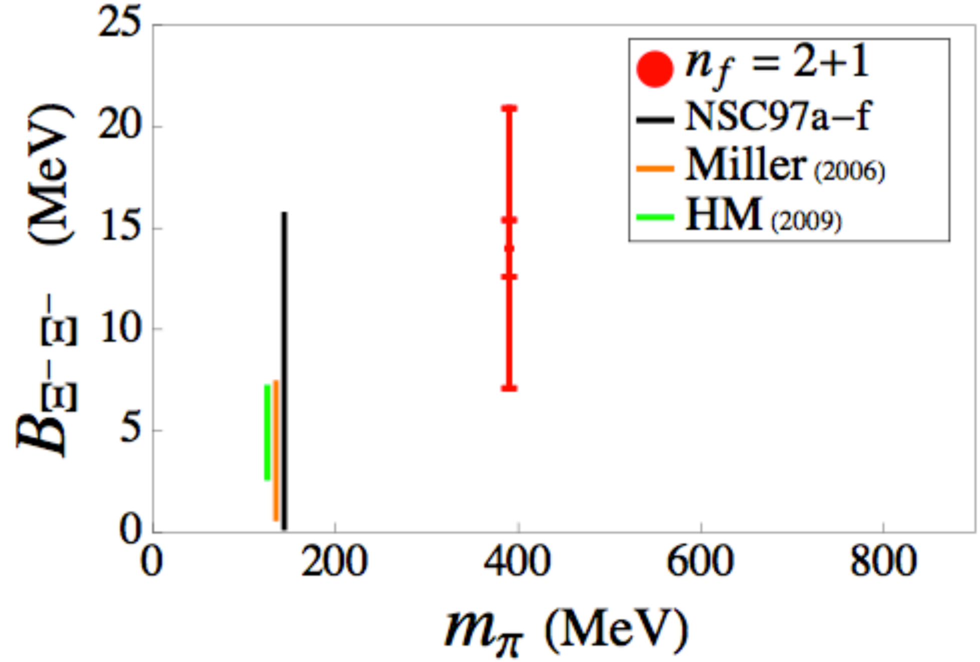
Need smaller lattice spacing!

27 of $SU(3)$

$^1S_0$: $NN,\ \Sigma\Sigma,\ \Xi\Xi,\ \Sigma N,\ \ldots$

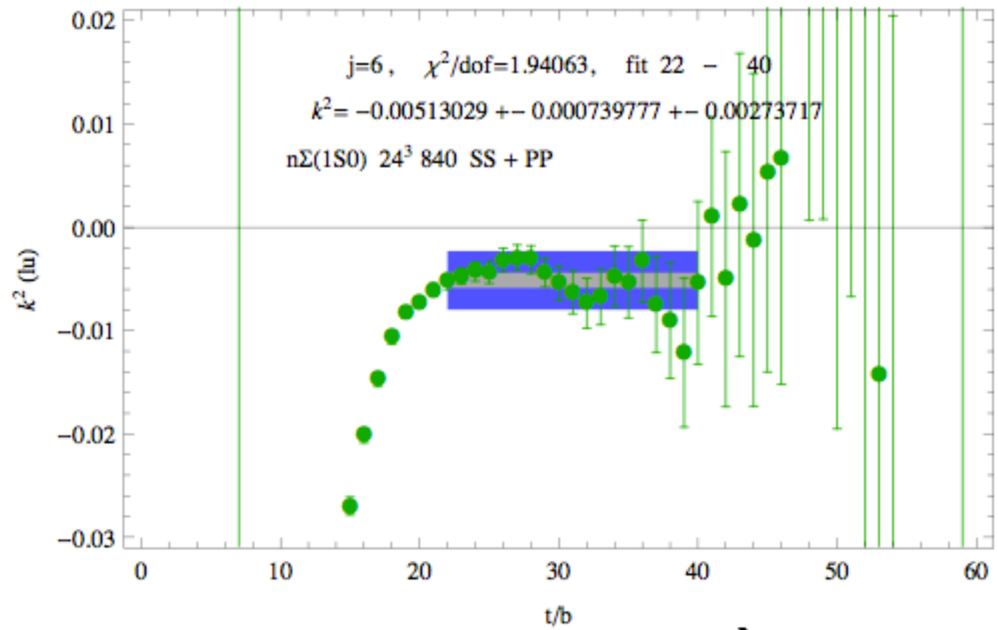
inconclusive



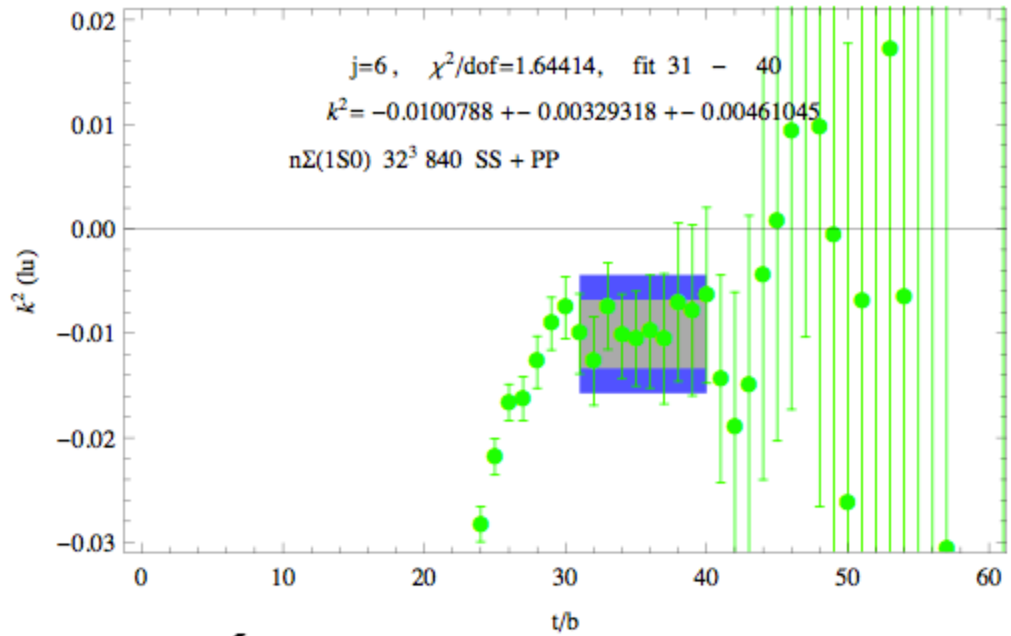


$^1S_0 \; n\Sigma^-$

$24^3 \times 128$



$32^3 \times 256$



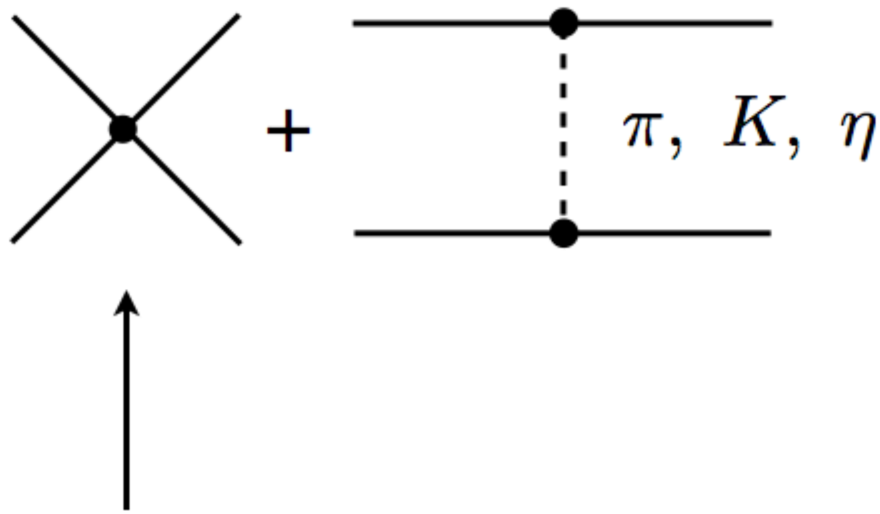
Lüscher's relation

↓ $m_\pi \sim 389$ MeV

$$B_{n\Sigma} = \frac{\gamma^2}{2\mu_{n\Sigma}} = 25 \pm 9.3 \pm 11 \text{ MeV}$$

Match to Effective Field Theory! (Polinder,Haidenbauer,Meissner)

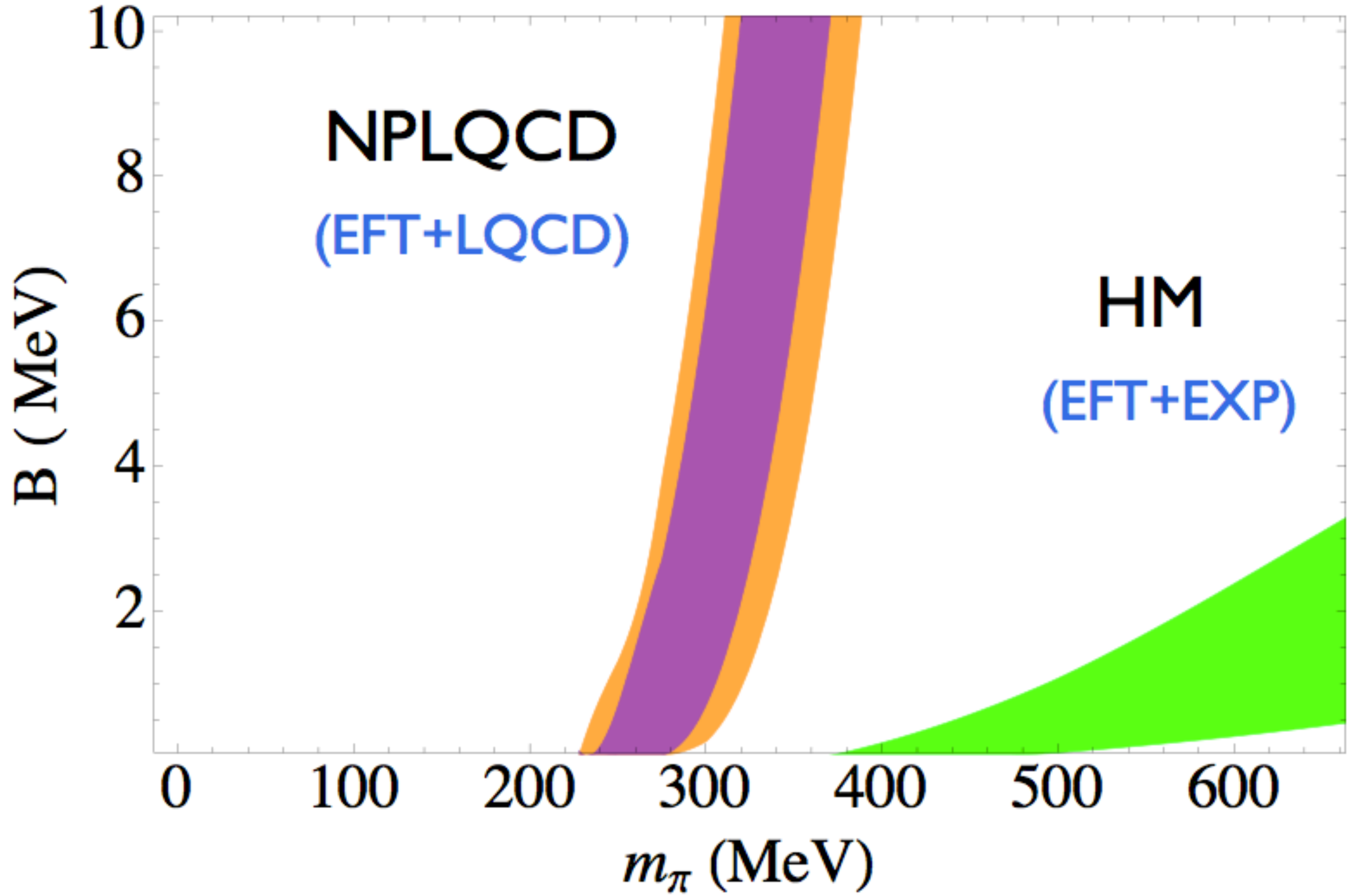
LO potential:



Fit LEC to binding energy $B_{n\Sigma^-}$

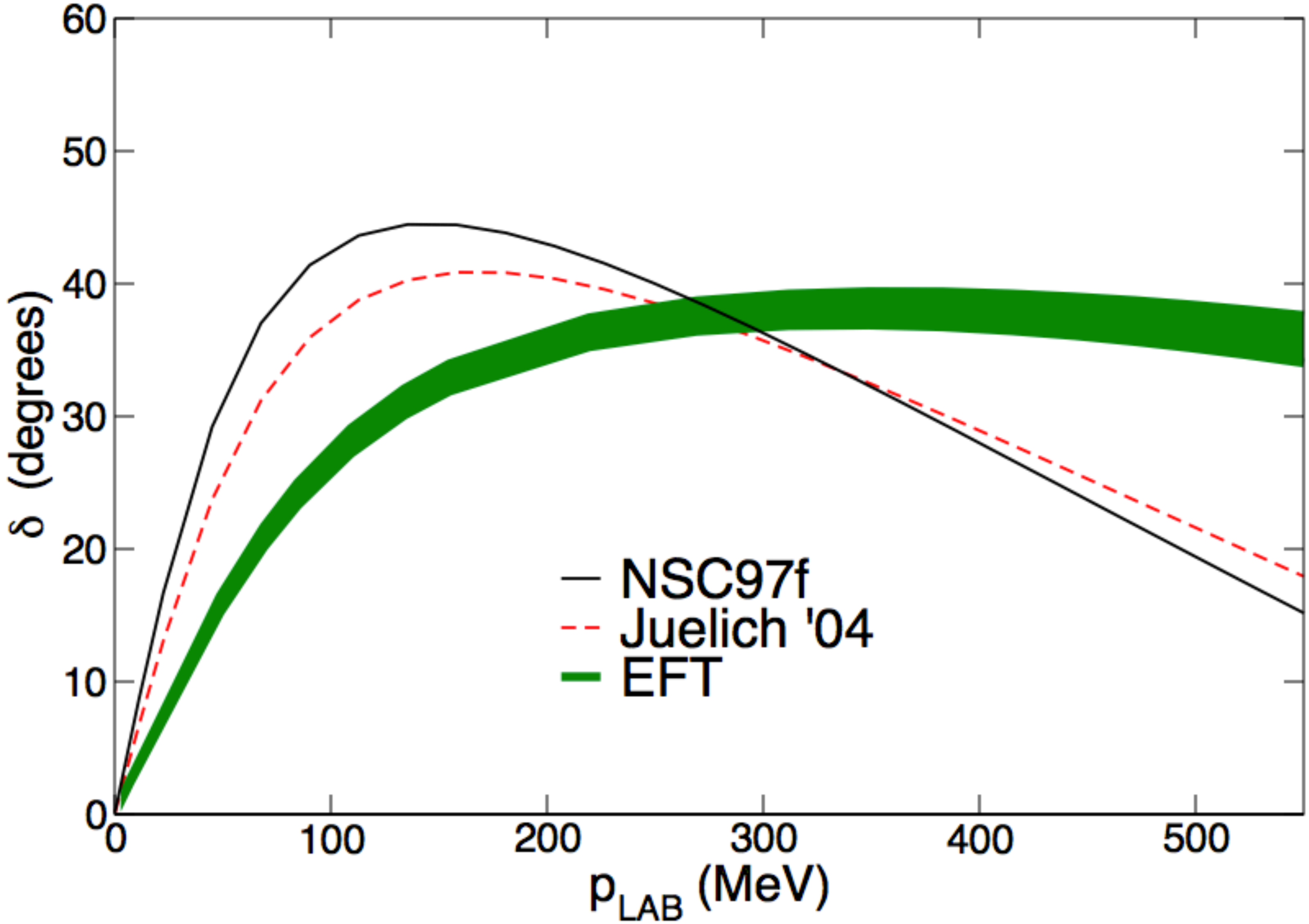
Now we have LO EFT potential at ALL pion masses!

$^1S_0 \; n\Sigma^-$

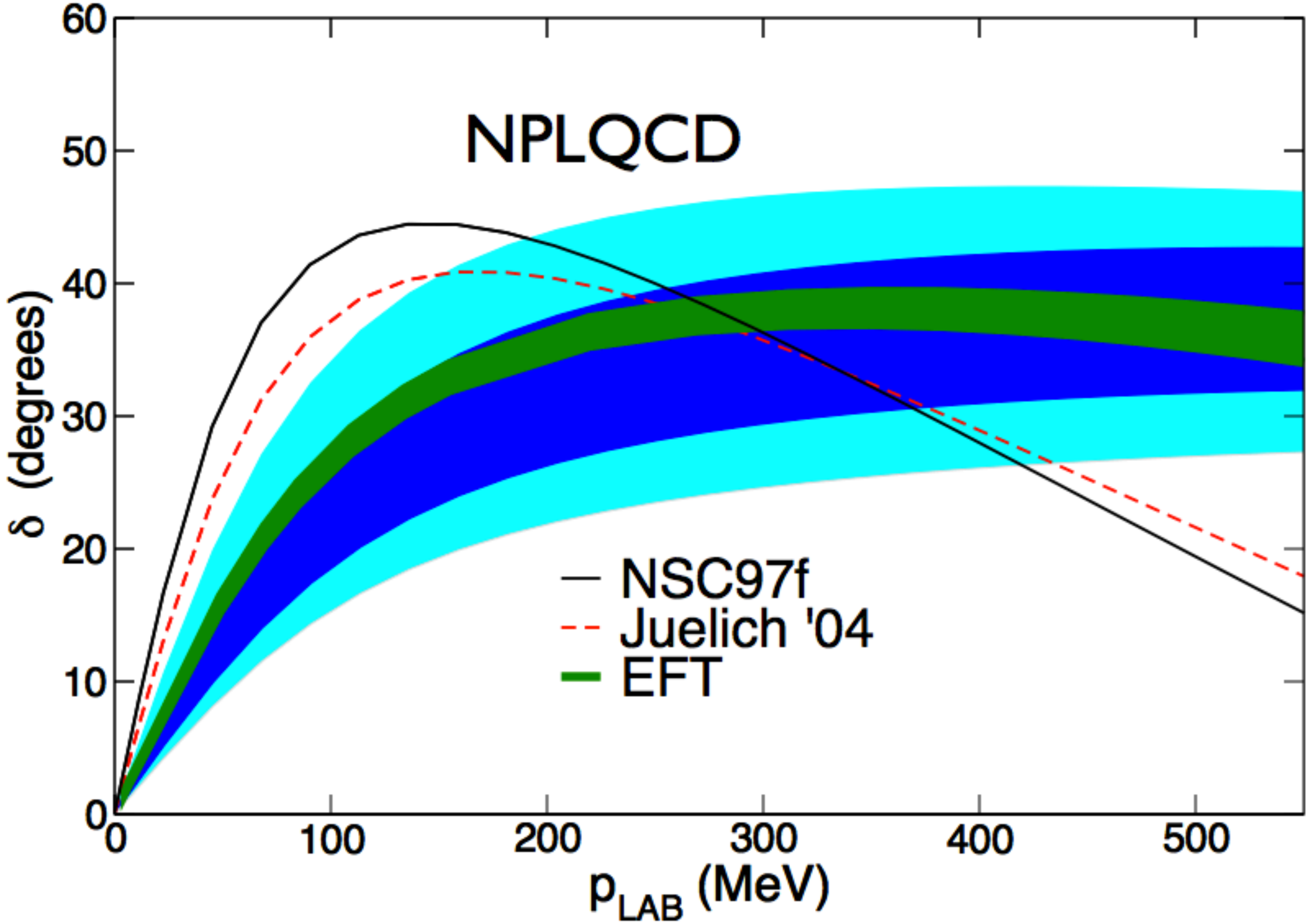


(NOTE: HM run only pion mass)

$^1S_0 \;\; n\Sigma^-$


60
50
40
30
20
10
0
0
100
200
300
400
500
δ (degrees)
p_LAB (MeV)
NSC97f
Juelich '04
EFT

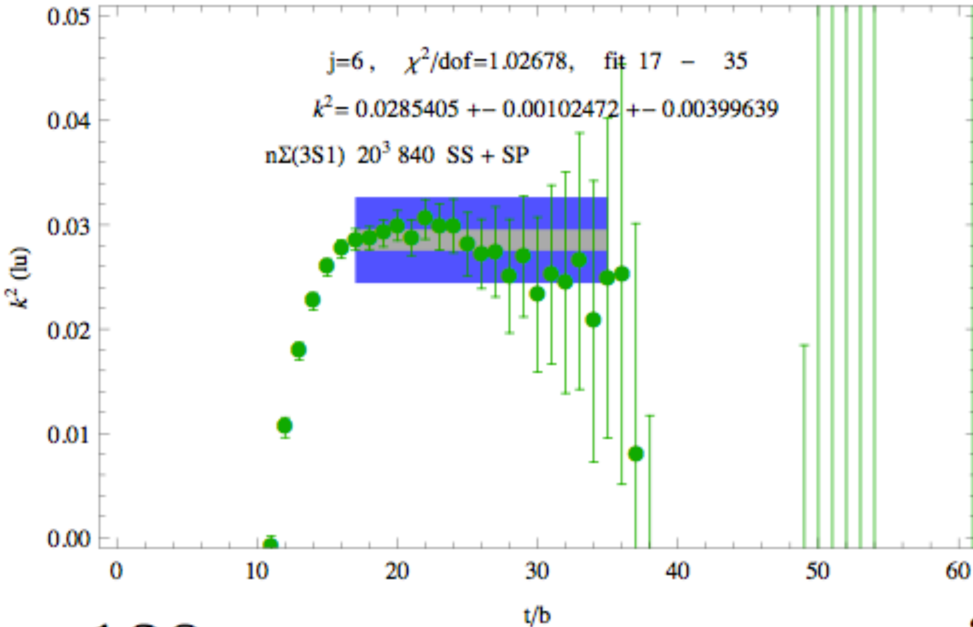
$^1S_0\ n\Sigma^-$


NPLQCD
NSC97f
Juelich '04
EFT
δ (degrees)
p_LAB (MeV)
0
10
20
30
40
50
60
0
100
200
300
400
500

$^3S_1\ n\Sigma^-$

10 of $SU(3)$

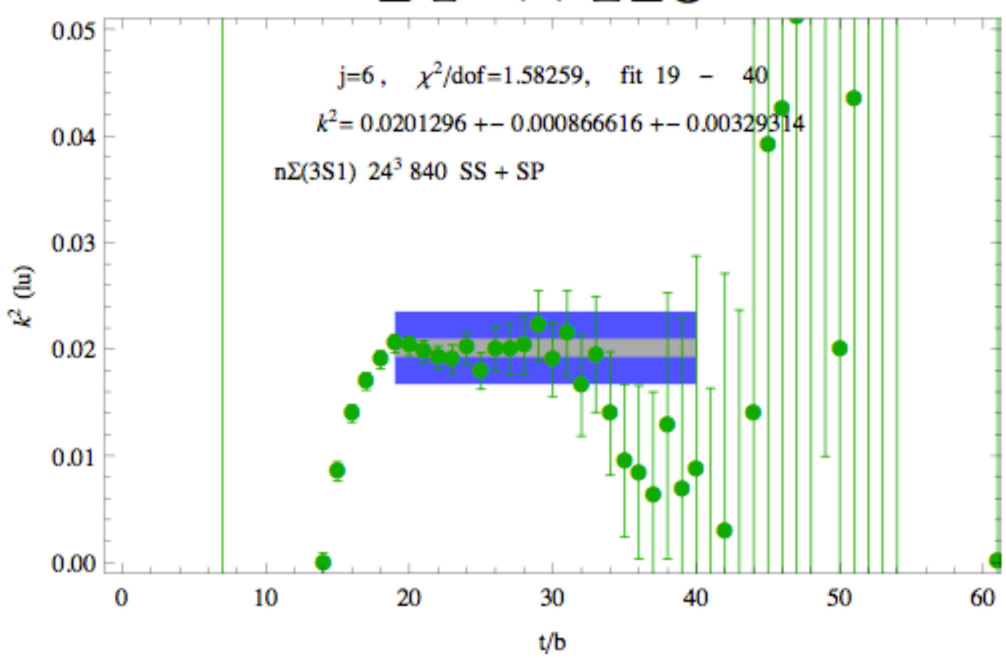
$20^3 \times 128$



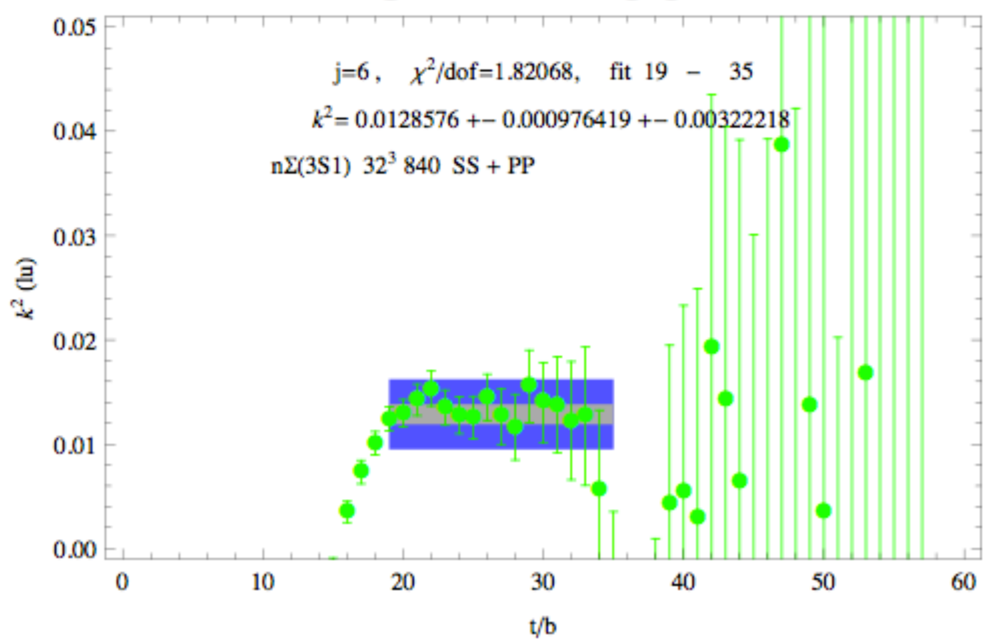
Extreme Repulsion!!

Scaling is violated!

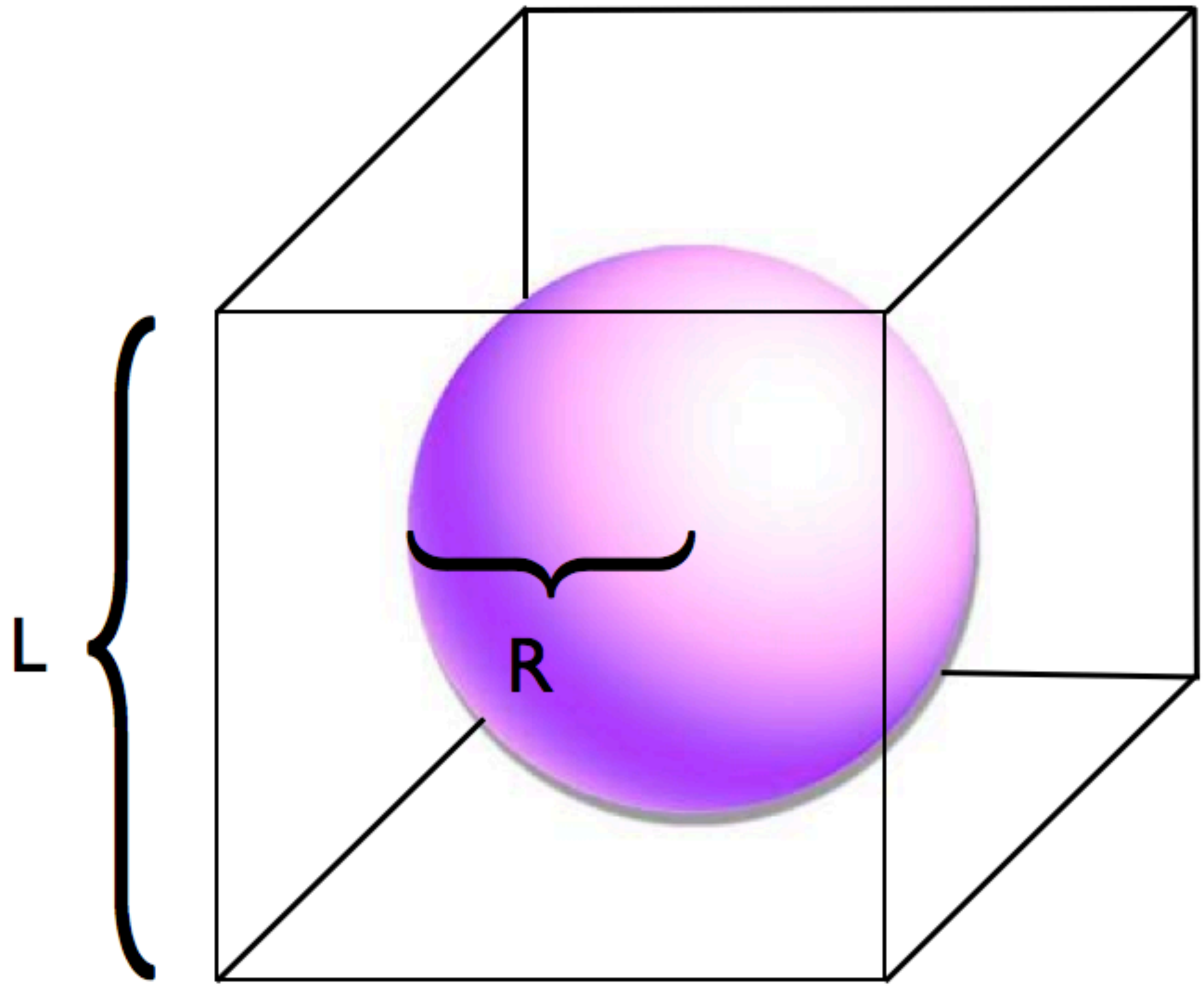
$24^3 \times 128$



$32^3 \times 256$

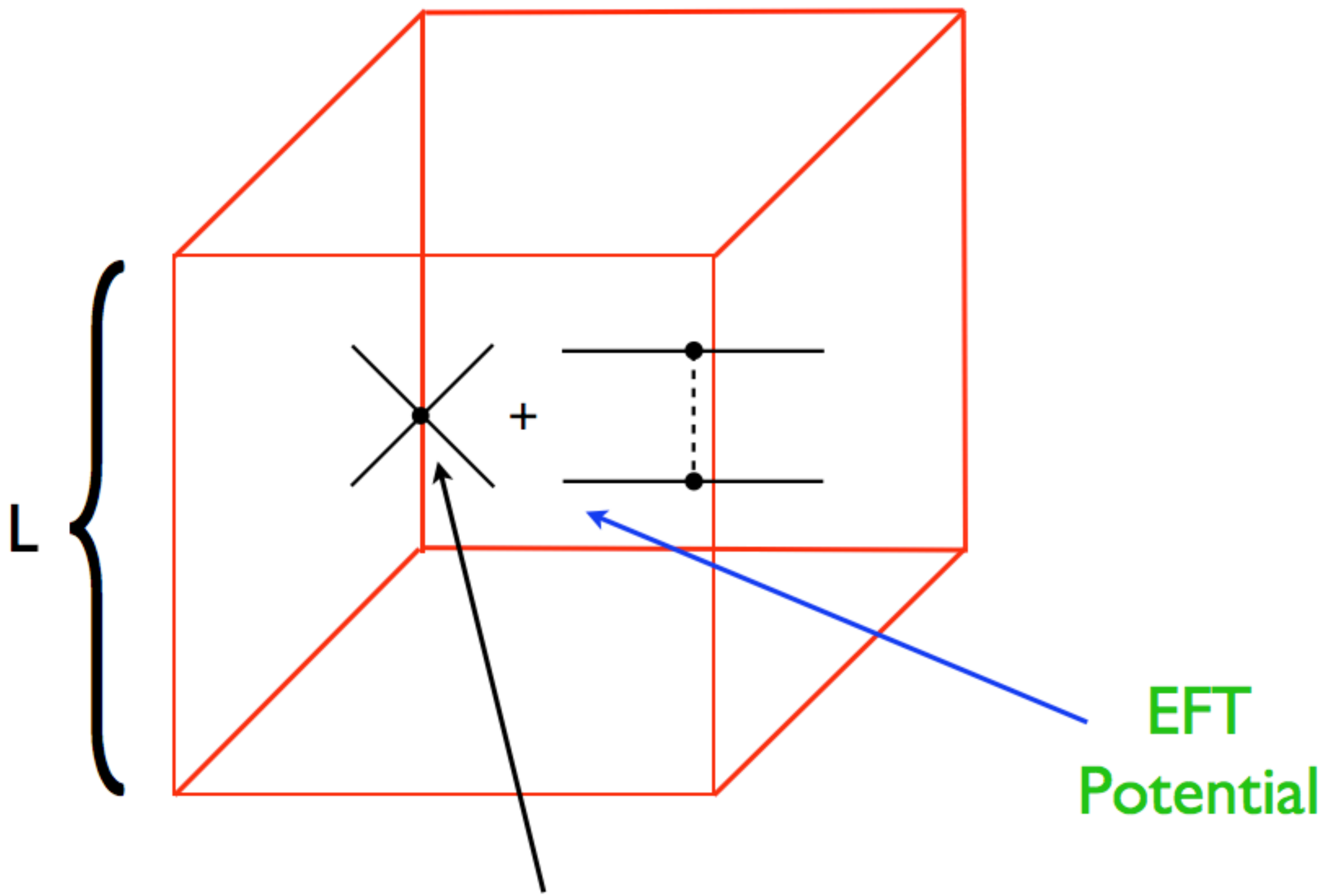


Lüscher's method valid ??



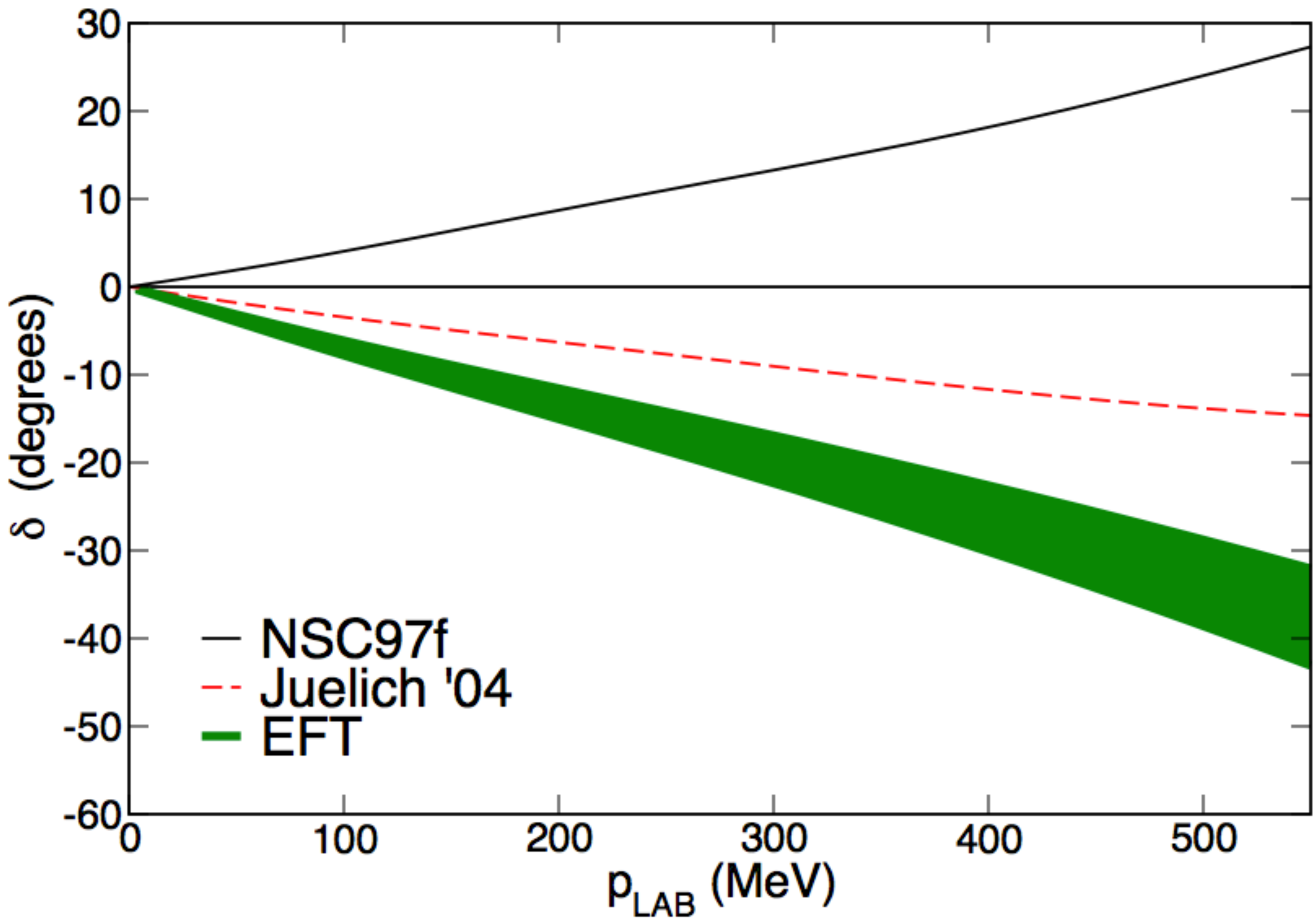
$R < L/2$??

3-dimensional Schrödinger equation in finite volume



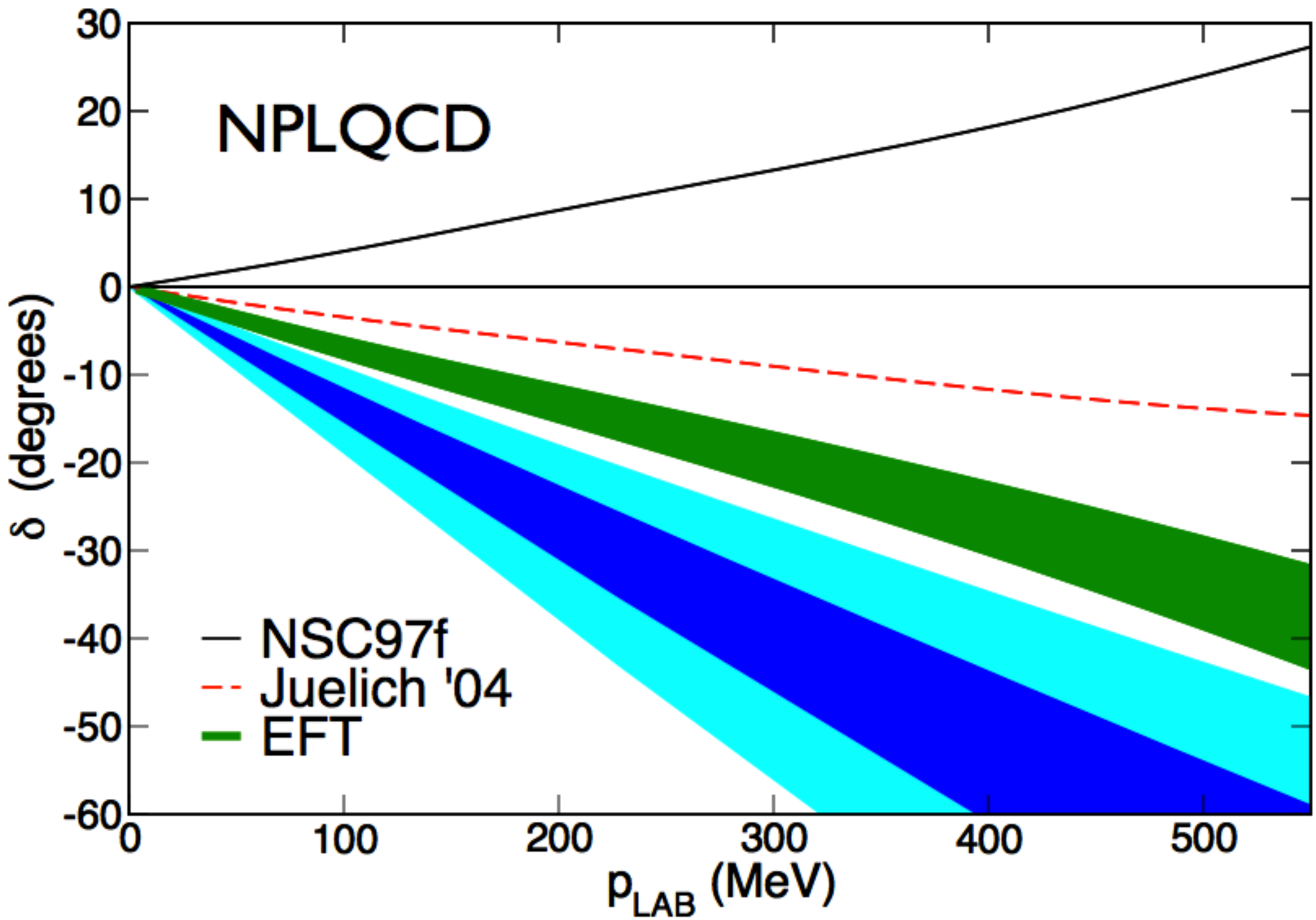
Fit to Lattice QCD energy levels directly!!

$^3S_1\ n\Sigma^-$



30
20
10
0
-10
-20
-30
-40
-50
-60
0
100
200
300
400
500
δ (degrees)
p_LAB (MeV)
NSC97f
Juelich '04
EFT

$^3S_1\ n\Sigma^-$


NPLQCD
NSC97f
Juelich '04
EFT
δ (degrees)
p_LAB (MeV)
30
20
10
0
-10
-20
-30
-40
-50
-60
0
100
200
300
400
500

NPLQCD

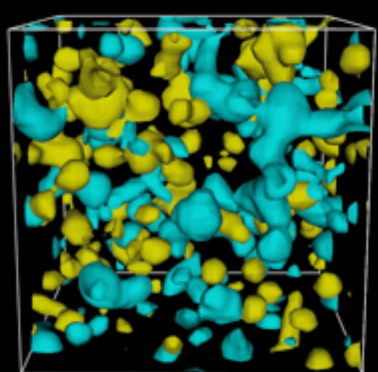
$SU(3)$ Isotropic Clover

Jefferson Lab

$N_f = 3$

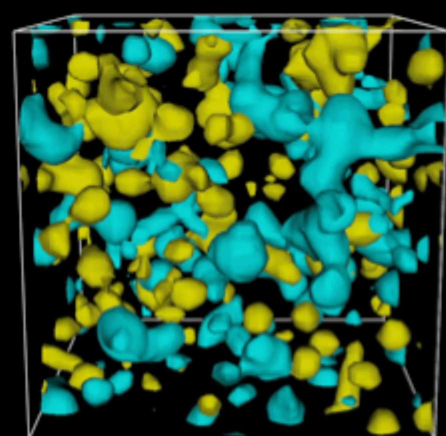
$m_\pi \sim 800$ MeV $\qquad b \sim 0.145$ fm

$L \sim 3.4$ fm



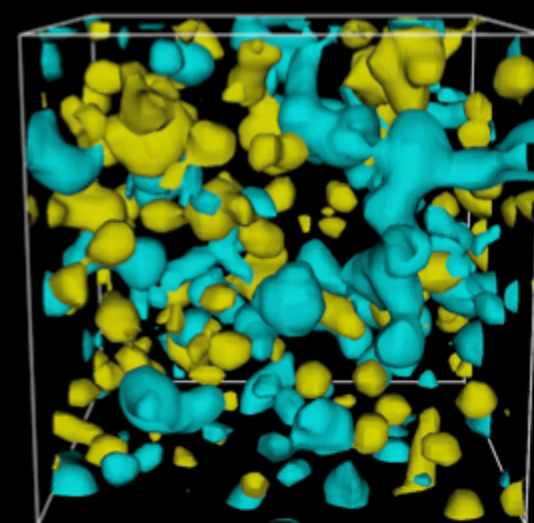
$24^3 \times 48$

$L \sim 4.5$ fm



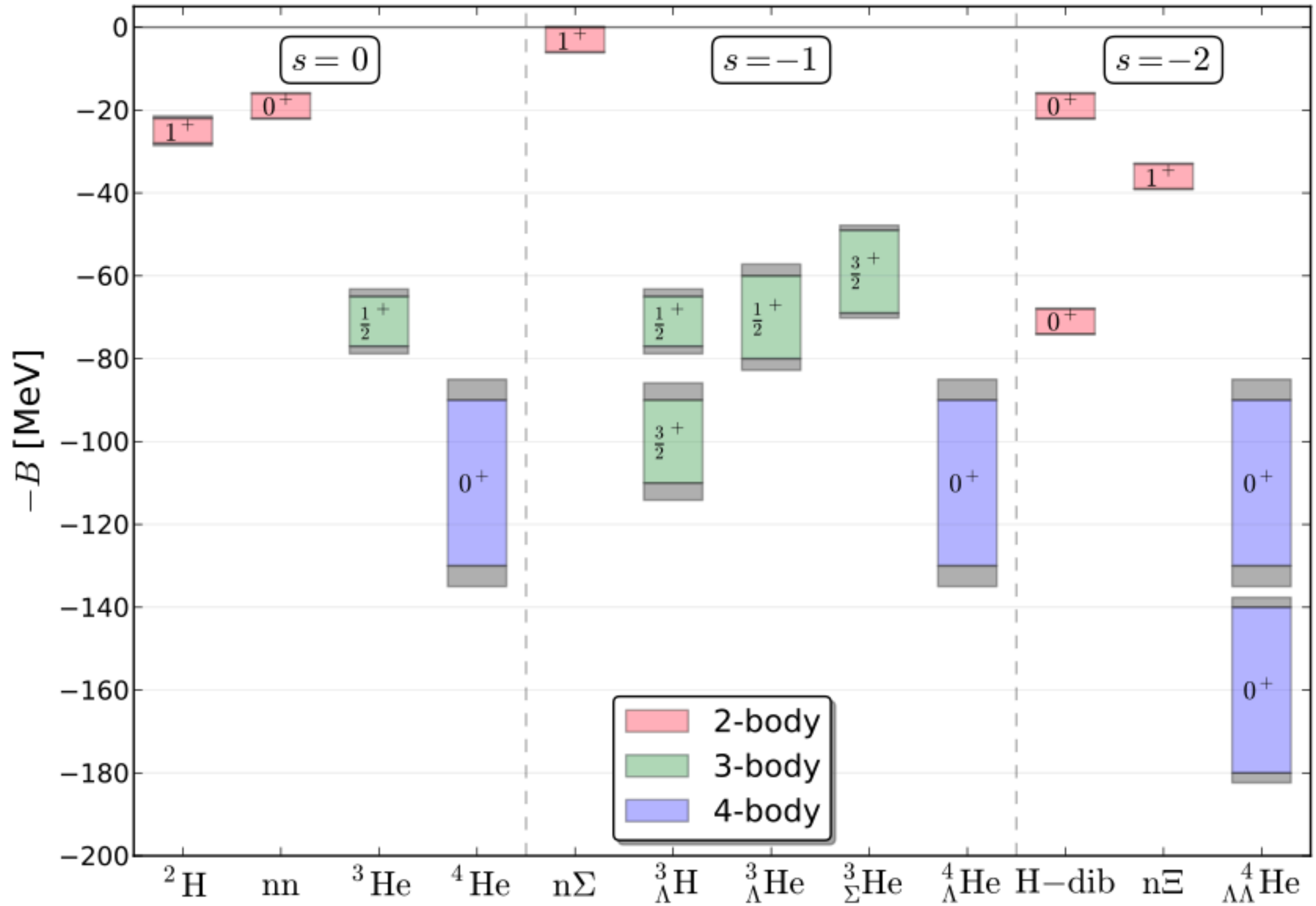
$32^3 \times 48$

$L \sim 6.7$ fm



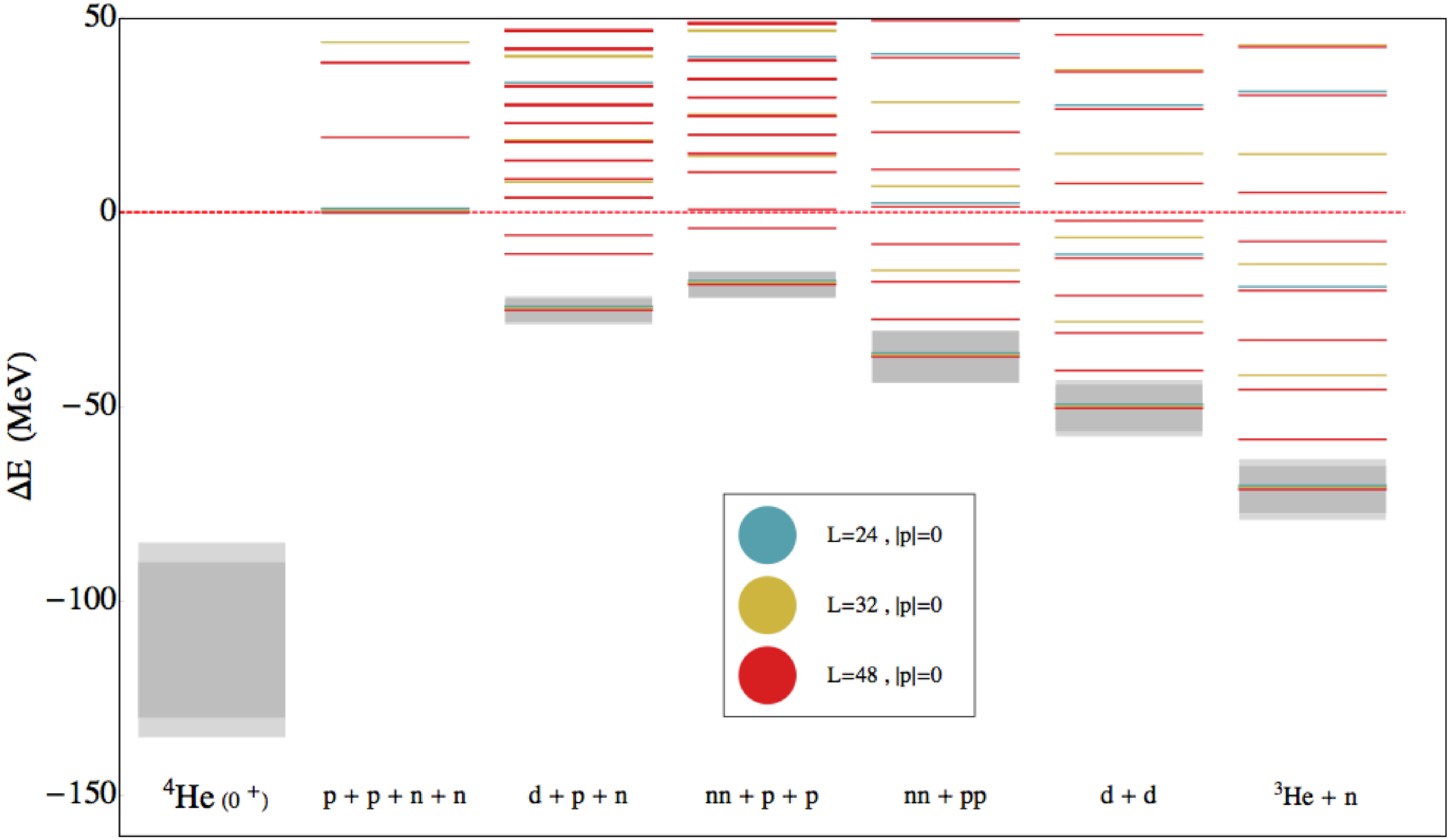
$48^3 \times 64$

(Hyper)Nuclei at the SU(3) Limit



Everything is bound!

Expected continuum levels:



High density of levels at the symmetry point!

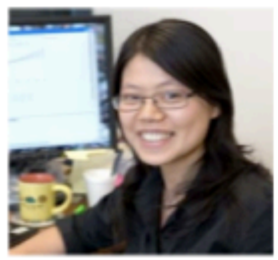
Conclusion

- The H-dibaryon is bound at unphysical quark masses. Naive chiral extrapolation of the existing lattice data indicate that at 2-sigma level H can be unbound or independent of the quark masses. Need smaller pion masses and smaller lattice spacing!
- The deuteron is unbound at one sigma. Basic expectations of flavor SU(3) are realized in the BB system.
- Physical predictions of the I=3/2 $n\Sigma^-$ phase shifts are in agreement with EFT results, and can be used to study the relevance of hyperons in dense nuclear matter.
- Spectrum of light (Hyper)Nuclei at the SU(3) point are now being mapped out.
- Lattice QCD is making predictions for nuclear physics!!

Silas Beane
New Hampshire

Emmanuel Chang
Barcelona

William Detmold
Willam+Mary



Huey-Wen Lin
U. of Washington

Tom Luu
LLNL

Kostas Orginos
William+Mary

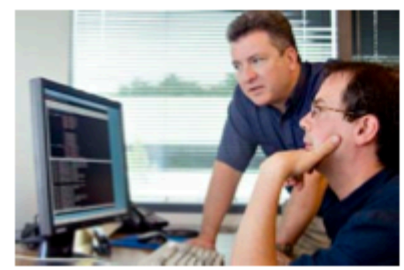
Assumpta Parreno
Barcelona

Marton Savage
U. of Washington

Aaron Torok
Indiana

Andre Walker-Loud
LBNL

+



Parikshit Junnarkar
New Hampshire