Two Lectures on Quark-Gluon Plasma
Lecture 1

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National NP Summer School

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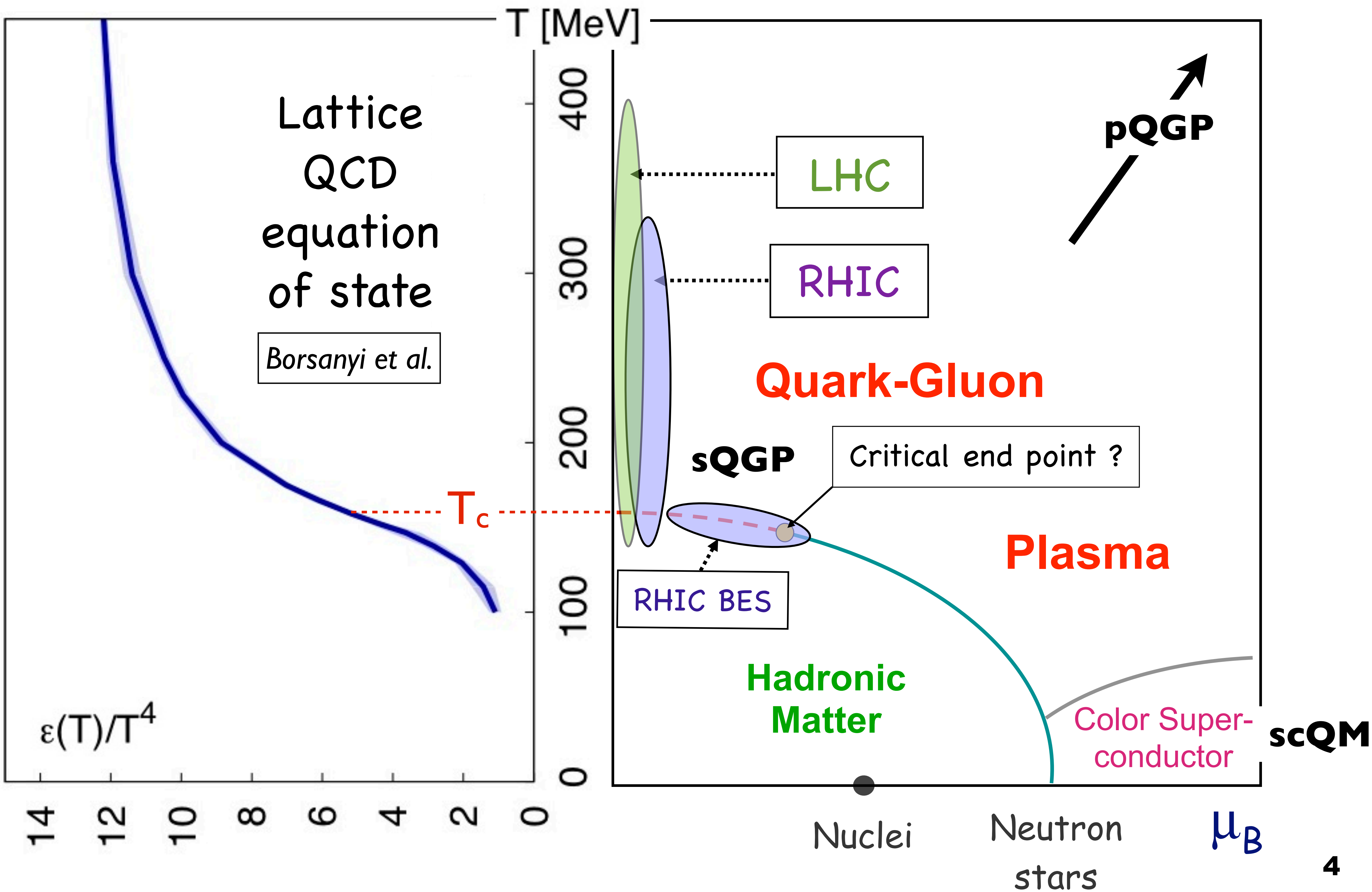
Lecture 1
Quark-Gluon Plasma

What is a QGP?

- Types of strongly interacting matter
- The QCD Vacuum
- The perturbative QGP
- The strongly coupled QGP (sort of)
- The off-equilibrium QGP

QGP Landscape



Part 1
The QCD Vacuum

QCD

Quantum Chromo-Dynamics is the gauge theory of SU(3) color and describes the interactions among particles carrying the color charge.

$$\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\end{pmatrix} \longrightarrow \begin{pmatrix}\psi'_1\\ \psi'_2\\ \psi'_3\end{pmatrix} = U(x)\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\end{pmatrix} \qquad U \in \mathrm{SU}(3)$$

Invariance of derivative ∂ψ/∂x requires introduction of a vector field

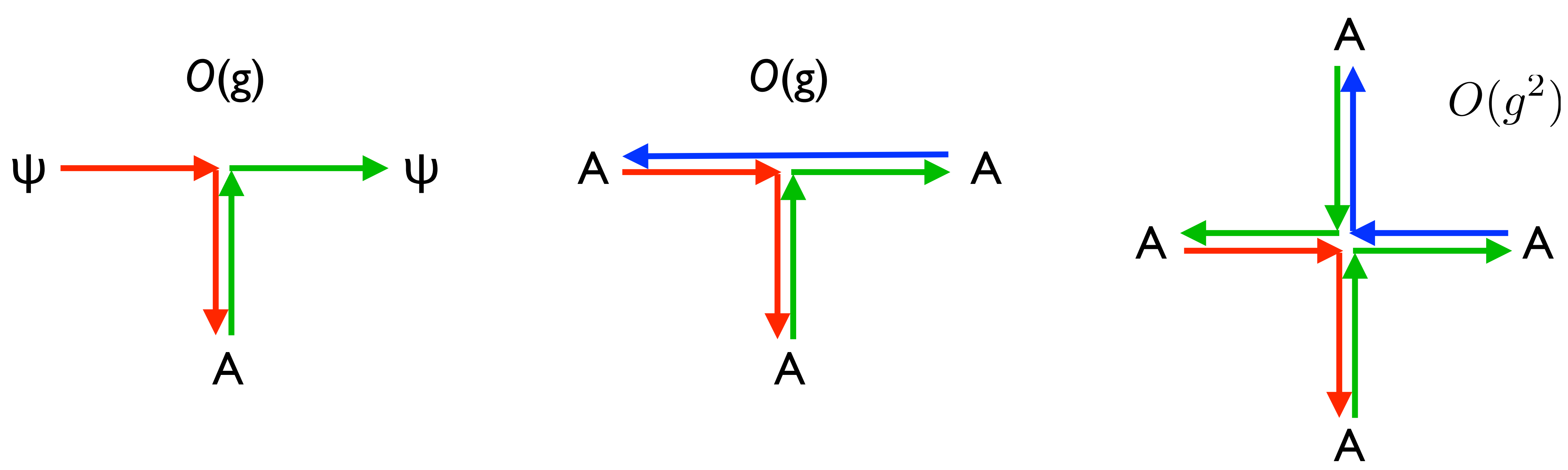
$$\begin{pmatrix}A^\mu_{11} & A^\mu_{12} & A^\mu_{13}\\ A^\mu_{21} & A^\mu_{22} & A^\mu_{23}\\ A^\mu_{31} & A^\mu_{32} & A^\mu_{33}\end{pmatrix} \quad \text{with} \quad A^\mu \longrightarrow A'^\mu = U A^\mu U^{-1} - \frac{i}{g}\frac{\partial U}{\partial x_\mu}U^{-1}$$

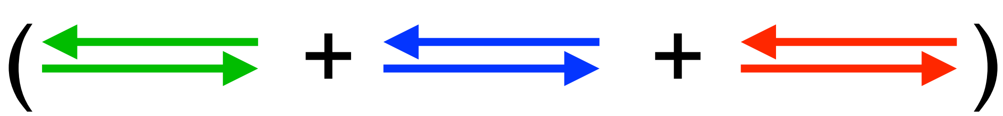
rendering $(i\partial^\mu + gA^\mu)\psi$ invariant under gauge transformations.

QCD in graphs

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{2}\mathrm{Tr_c}\left[F_{\mu\nu}F^{\mu\nu}\right] + \sum_f \overline{\psi}_f\left[\gamma_\mu(i\partial^\mu + gA^\mu) - m_f\right]\psi_f$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig\left[A^\mu, A^\nu\right]$$



$\frac{1}{\sqrt{3}}$  not allowed (color singlet)

The vacuum in QFT

Scalar field: $$\mathcal{S} = \frac{1}{2} \int d^4x \left[(\partial_t \phi(x))^2 - (\nabla \phi(x))^2 - m^2 \phi(x)^2 \right]$$

Quantization condition: $$[\phi(\mathbf{x}, t), \partial_t \phi(\mathbf{x}', t)] = i\hbar\, \delta^3(\mathbf{x} - \mathbf{x}')$$

Mode decomposition: $$\phi(x) = \sum_{\mathbf{k}} \frac{1}{\sqrt{2\omega_k}} \left(a_{\mathbf{k}} e^{-i(\omega t - \mathbf{k}\cdot\mathbf{x})} + a_{\mathbf{k}}^\dagger e^{i(\omega t - \mathbf{k}\cdot\mathbf{x})} \right)$$

with boson operators: $$\left[a_{\mathbf{k}}, a_{\mathbf{k}'}^\dagger \right] = \delta_{\mathbf{k},\mathbf{k}'} \qquad \text{and} \qquad \omega_k = \sqrt{\mathbf{k}^2 + m^2}$$

$$H = \sum_{\mathbf{k}} \omega_k \left(a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + \frac{1}{2} \right) \qquad a_{\mathbf{k}} |0\rangle = 0 \qquad \longrightarrow \qquad H|0\rangle = E_{\text{vac}} |0\rangle$$

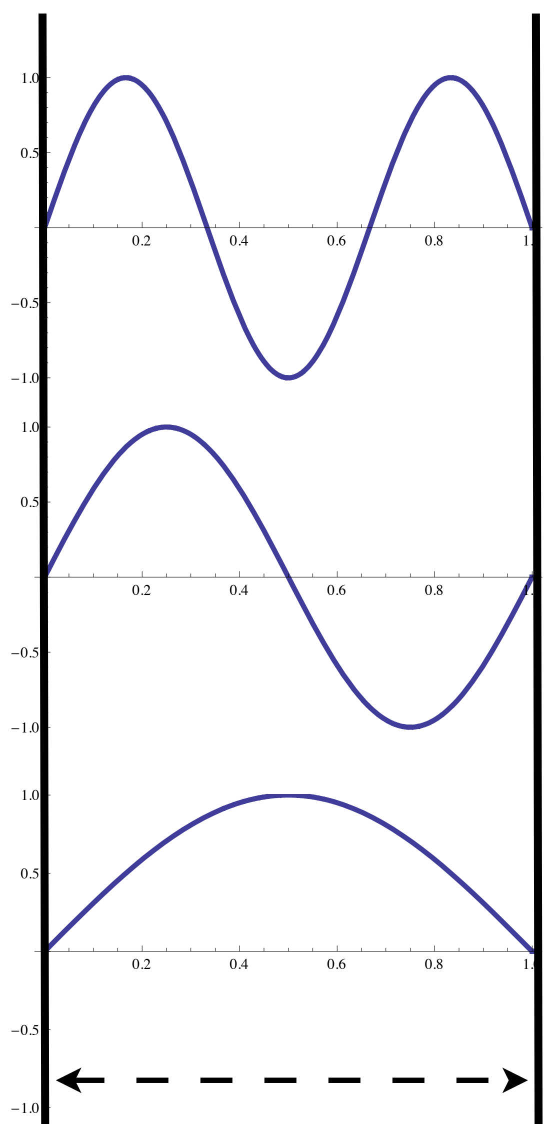
Renormalize: $$H \longrightarrow H_{\text{ren}} = H - E_{\text{vac}} \quad \text{with} \quad E_{\text{vac}} = \frac{1}{2} \sum_{\mathbf{k}} \omega_{\mathbf{k}}$$

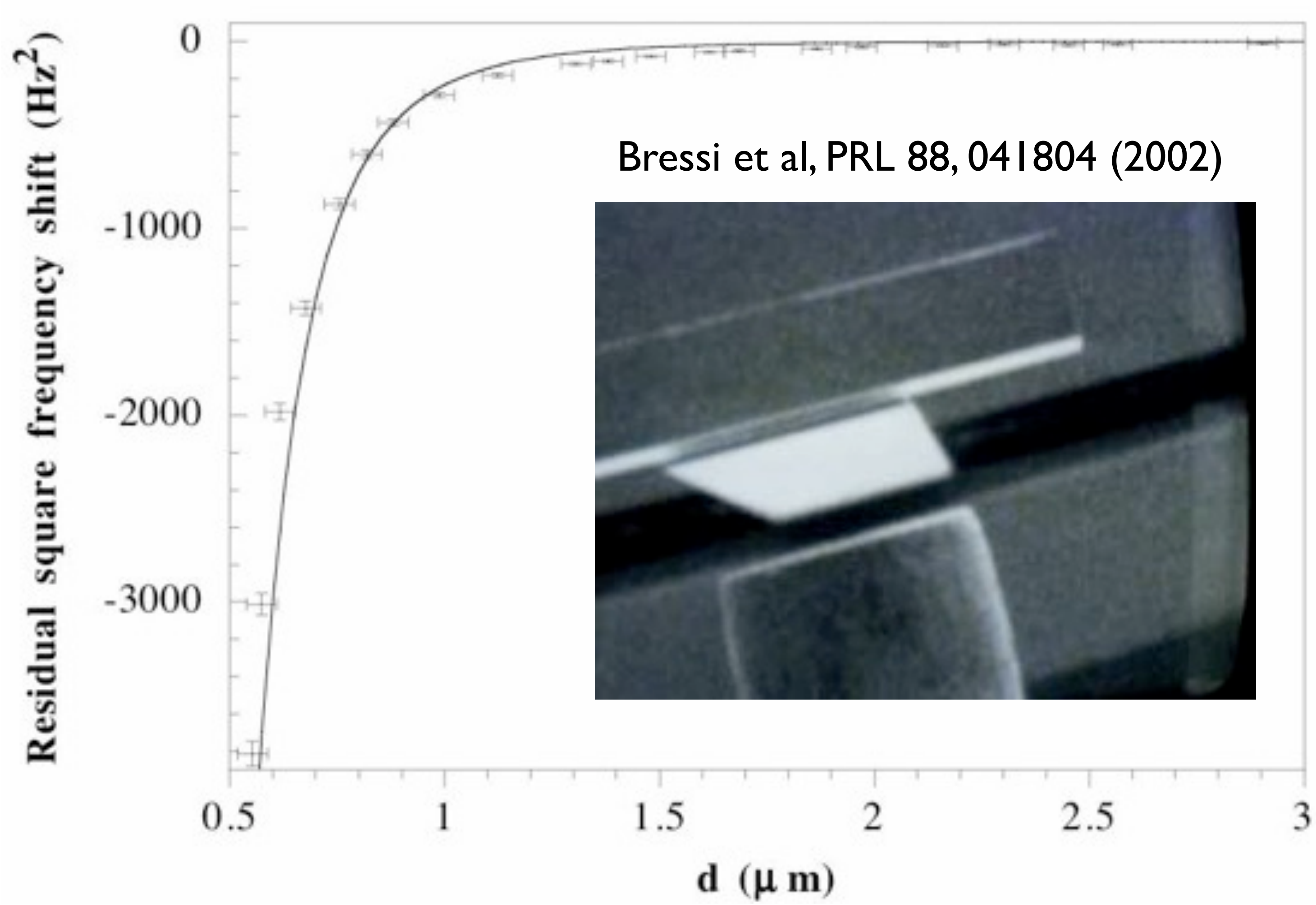
The Casimir effect

Cavity modes

$$\omega_n(\mathbf{k}_\perp) = \sqrt{\left(\frac{n\pi}{d}\right)^2 + \mathbf{k}_\perp^2} \qquad E_{\mathrm{vac}}(d) = \frac{1}{2}\sum_{n,\mathbf{k}_\perp} \omega_n(\mathbf{k}_\perp)$$

$$\varepsilon_{\mathrm{C}}(d) = \varepsilon_{\mathrm{vac}}(d) - \varepsilon_{\mathrm{vac}}(d \to \infty) = -\frac{\pi^2 \hbar c}{360\, d^4}$$

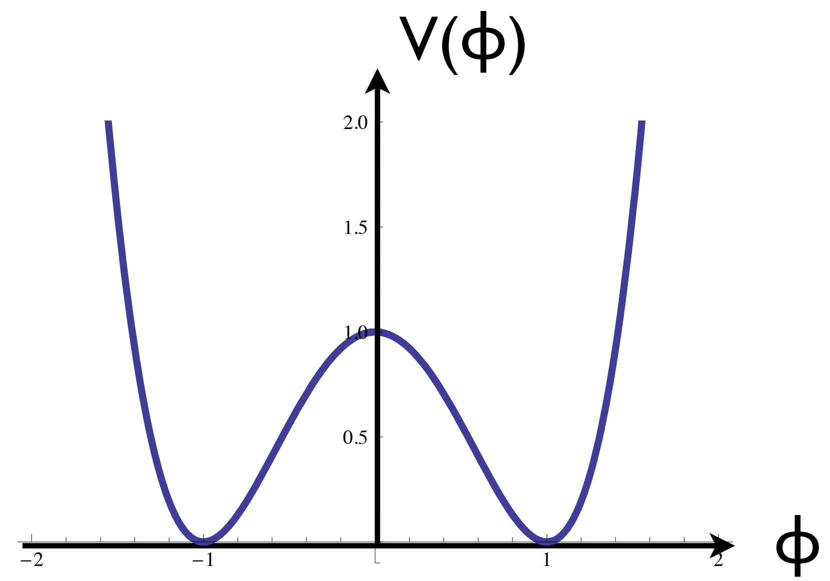


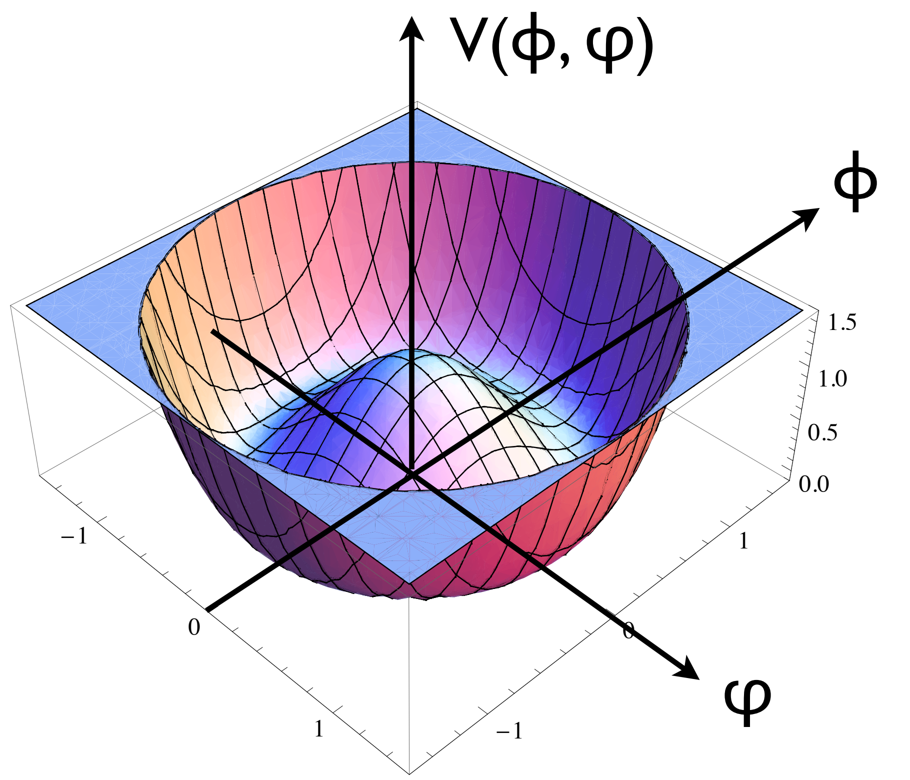


Bressi et al, PRL 88, 041804 (2002)

Symmetry breaking

- Discrete degenerate minima of V(x): Vacuum needs to select one of the minima ➨ symmetry breaking
  - Local transition to other vacuum possible by tunneling
- Continuous symmetry of V(x): Vacuum chooses one point on the minimal surface ("moduli space") ➨ symmetry breaking
  - Local transition to adjacent minimal energy configurations classically allowed: excitations are called *Goldstone bosons*
- In the presence of gauge fields, the Goldstone modes metamorphize into longitudinal modes of the gauge field and generate a mass (so-called *Higgs mechanism*)





Dynamical symmetry breaking

Quantum electrodynamics of a charged scalar field ϕ:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (\partial_\mu + ieA_\mu)\phi^* (\partial^\mu - ieA^\mu)\phi - m_0^2 |\phi|^2$$

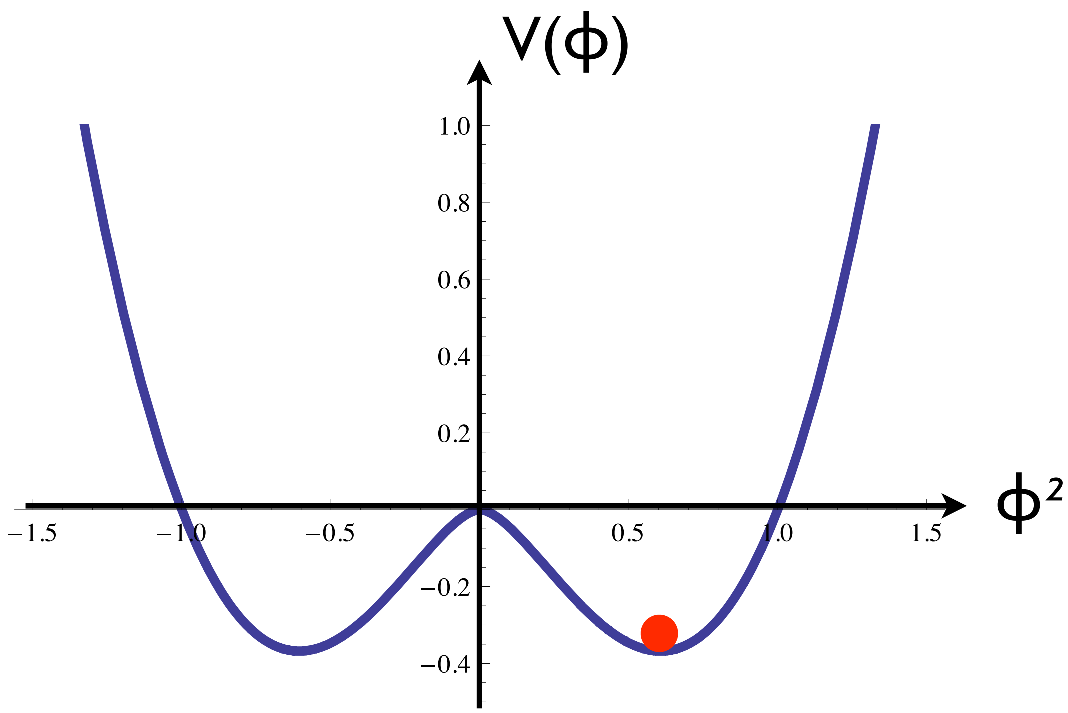
Naively: $V(\phi) = m_0^2 |\phi|^2 + e^2 |\phi|^2 A^2$ implying $\langle |\phi|^2 \rangle = 0$

But vacuum fluctuations modify the effective potential:

$$\delta V(\phi) = \frac{3e^2}{16\pi^2} |\phi|^4 \ln(|\phi|^2/\phi_0^2)$$

implying a shift of the minimum to $\langle |\phi|^2 \rangle \sim \phi_0^2$

In this new minimum the gauge symmetry is spontaneously broken, and both the electromagnetic field acquires a mass $m_A^2 = 2e^2\phi_0^2$.



Monday, July 16, 12

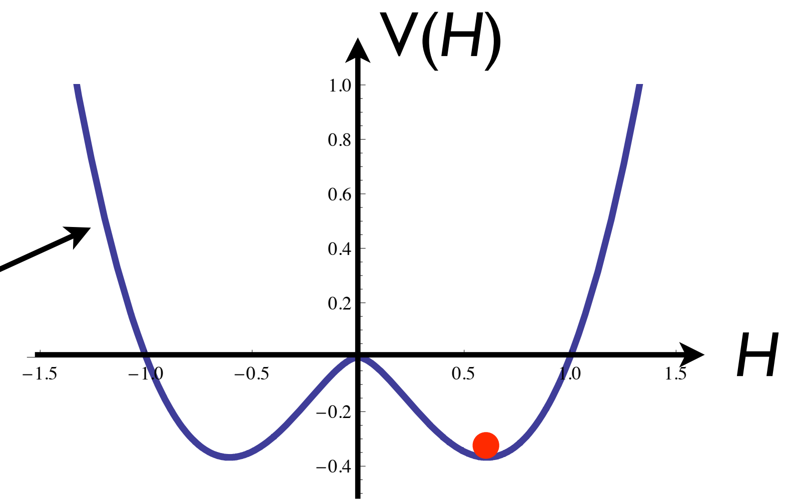
Savvidy's "vacuum"

Constant chromo-magnetic field: $A_{11}^{y} = -A_{22}^{y} = \frac{1}{2}Hx \quad \longrightarrow \quad B_{11}^{z} = -B_{22}^{z} = \frac{1}{2}H$

Modes $A_{12}^{\mu}, A_{21}^{\mu}$ behave like a spin-1 field with charge g in the presence of a homogeneous magnetic field *H*. The Landau levels are:

$$\omega_{n,S_z}(k_z) = \sqrt{k_z^2 + 2\left(n + \frac{1}{2} - S_z\right)gH} \qquad (S_z = \pm 1)$$

$$E_{\rm vac}(H) = \frac{1}{2}\sum_{n,S_z,k_z}^{\rm ren} \omega_{n,S_z}(k_z) = \frac{11g^2}{64\pi^2}H^2 \ln(H^2/H_0^2)$$



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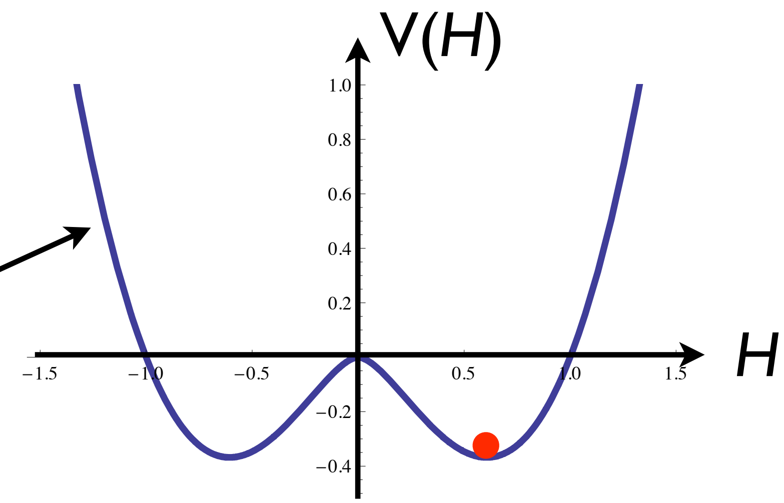
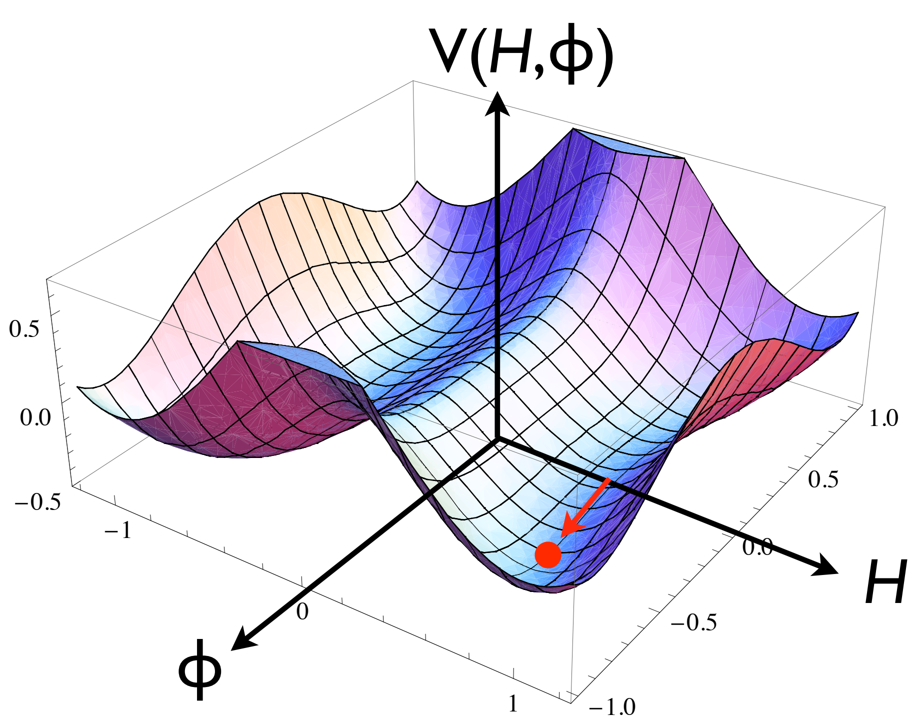


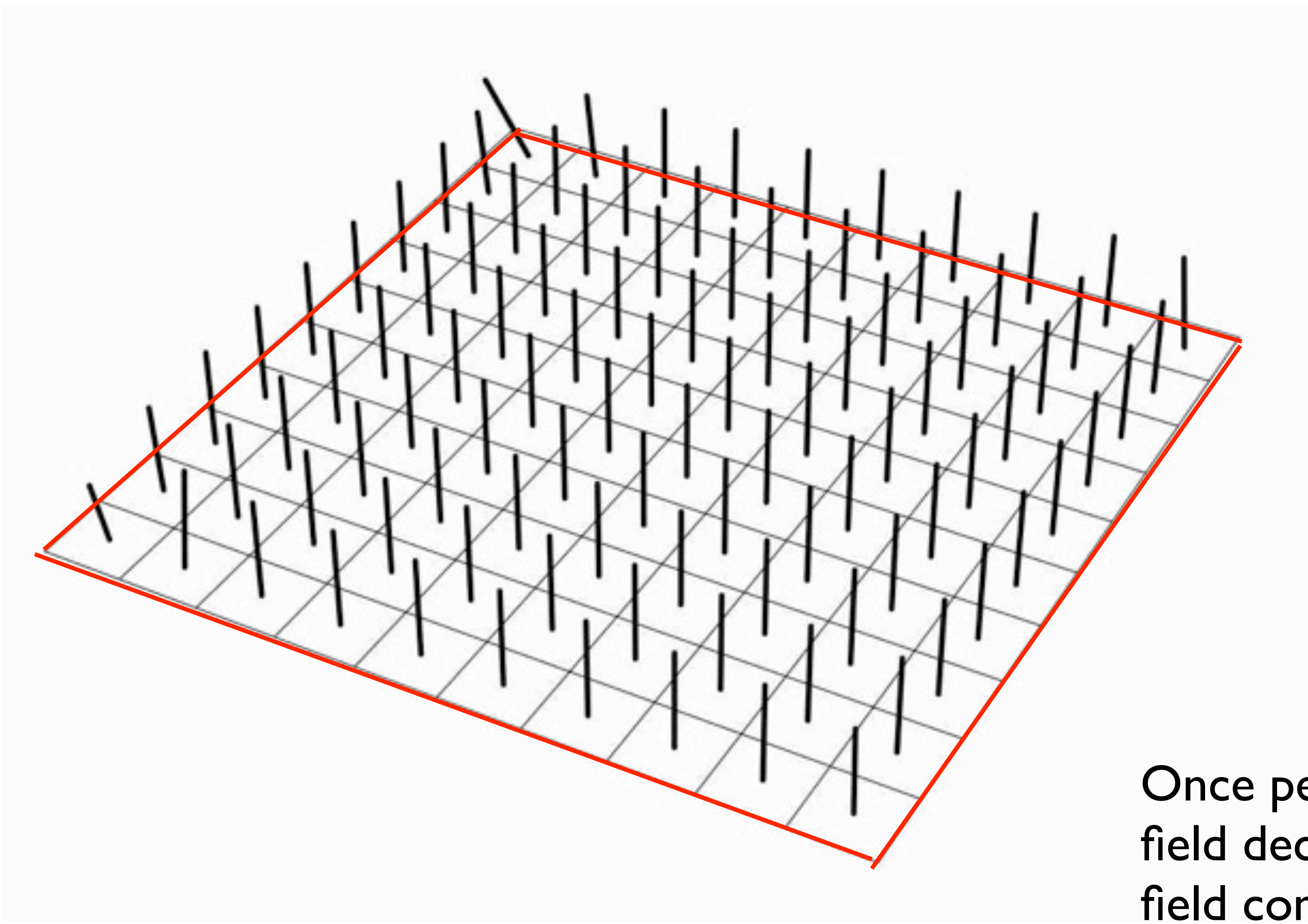


But note:

$$\omega_{n,S_z}(k_z) \text{ for } n = 0, S_z = -1, k_z^2 < gH$$

is imaginary, indicating an instability (Nielsen & Olesen '78). The vacuum energy can be lowered by allowing for a nonzero expectation value of the lowest Landau level ϕ. Unfortunately, this gauge field configuration has another instability, with leads to a lower vacuum energy, which exhibits another instability, and so on, without end....

The NO cascade



A. Trayanov & BM

Once perturbed, the color-magnetic field decays into apparently random field configurations and rapidly reaches equipartition of energy.

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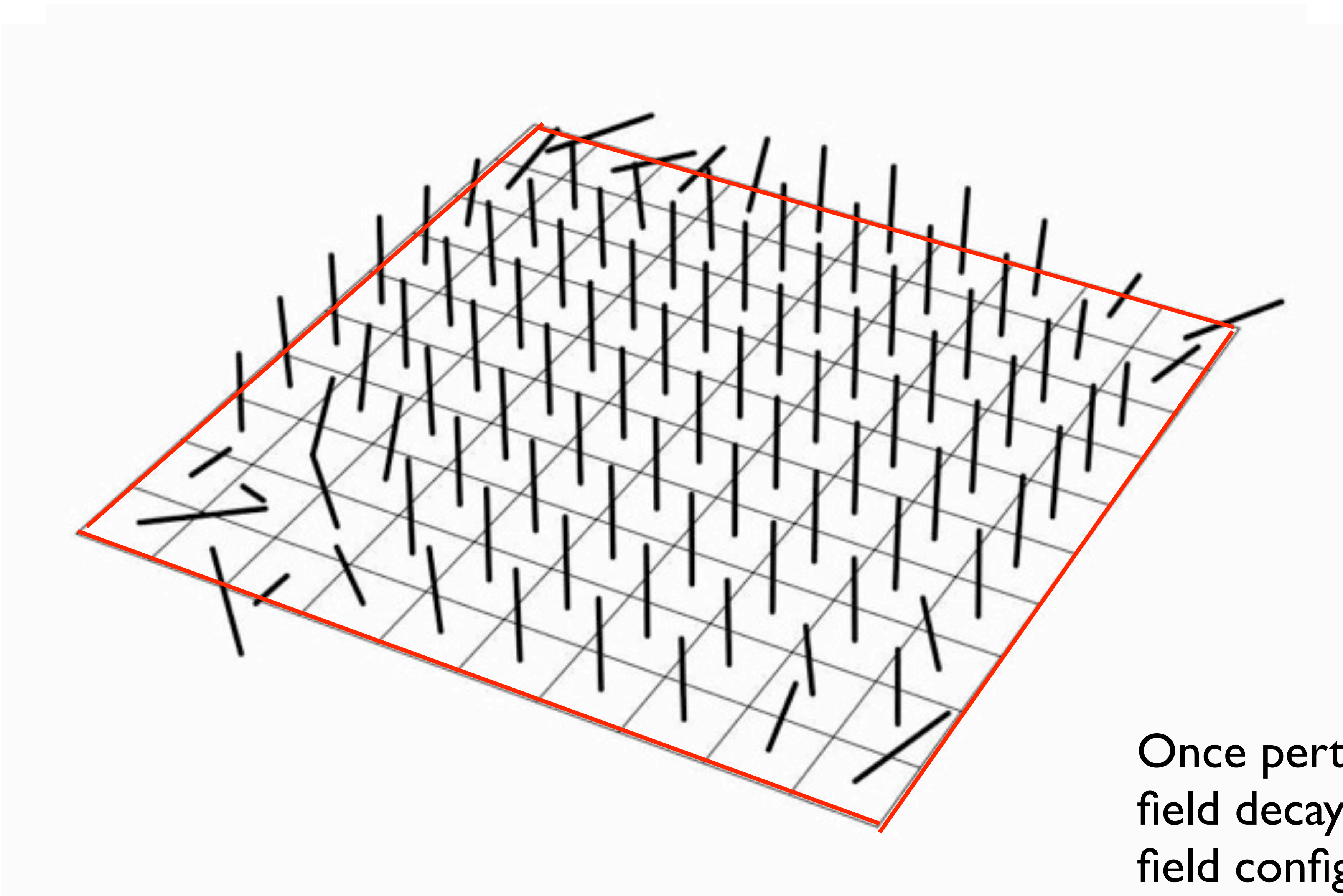

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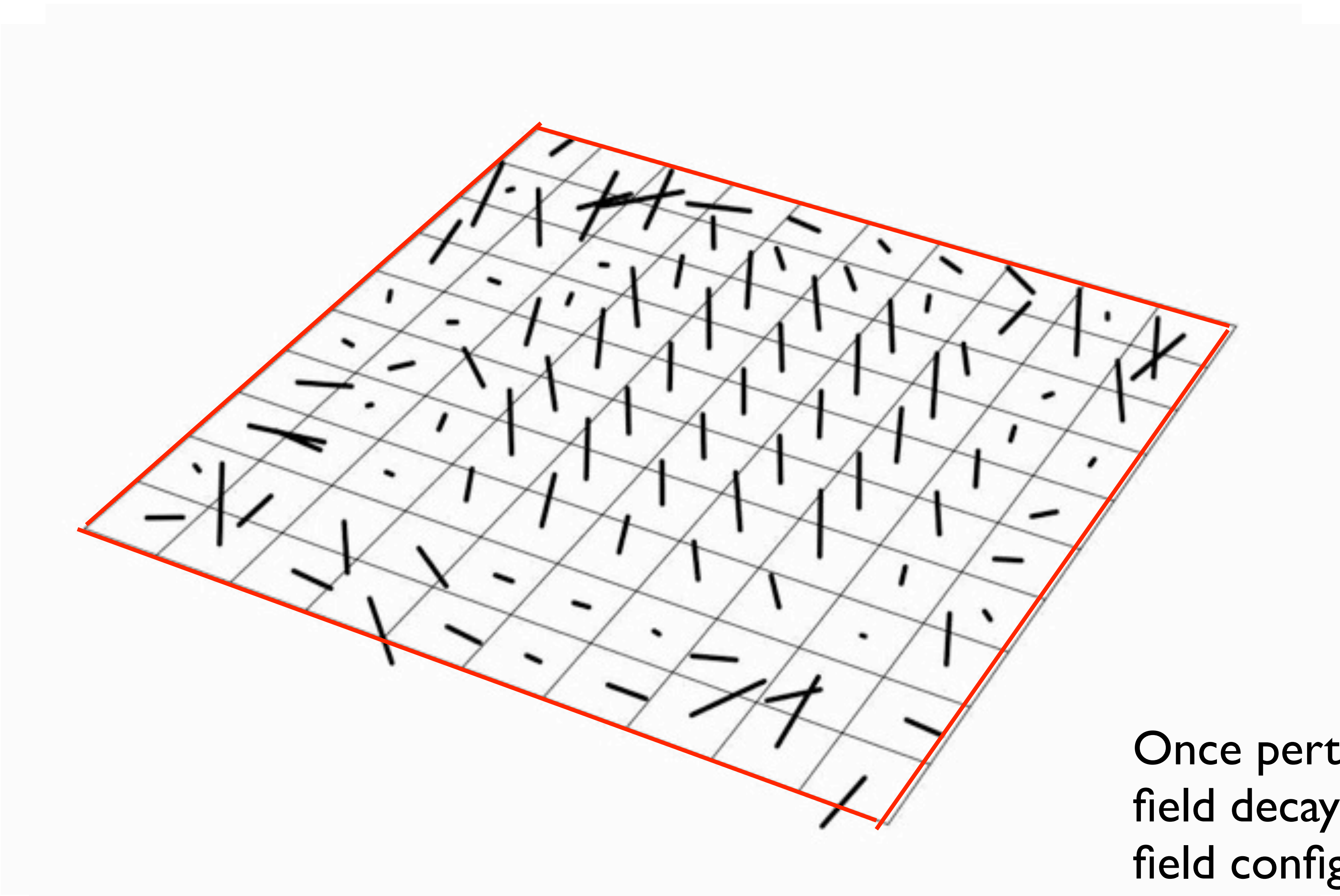

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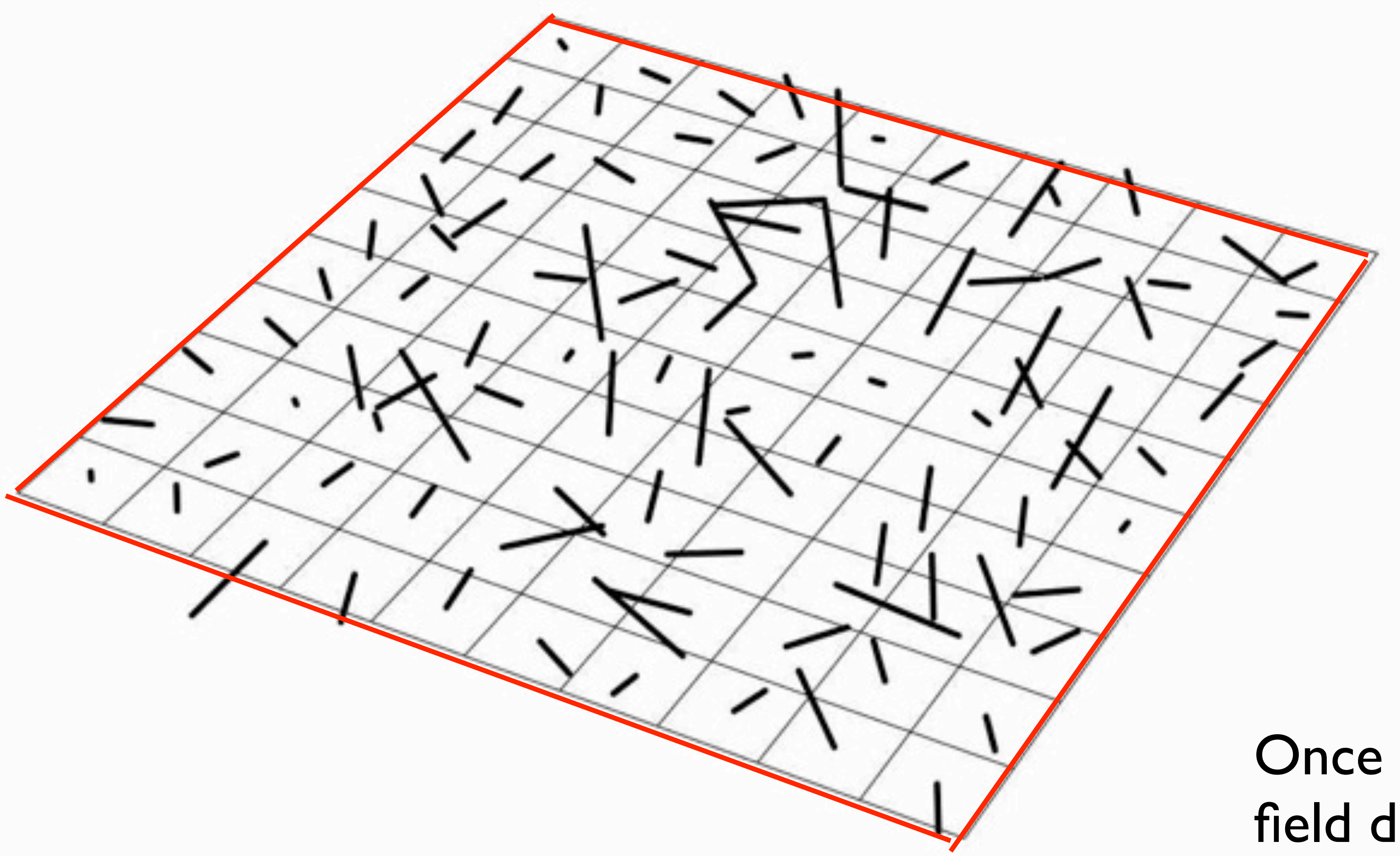

A. Trayanov & BM

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Condensates

S-NO picture suggests that the QCD vacuum is filled with domains of chromo-magnetic field *H*, which have transverse extension $R \sim 1/\sqrt{gH}$ and length $L \sim \pi/\sqrt{gH}$.

Independent of the precise field configuration, the QCD vacuum must have a nonzero expectation value of mean-square chromo-magnetic field $\langle 0 \mid B^2 \mid 0 \rangle \approx (245\ \mathrm{MeV})^4$.

Value from analysis of charmonium spectra (QCD sum rules).

Remarkably, this implies that there is also a nonzero, but negative value for the mean-square chromo-electric field: $\langle E^2 \rangle = \langle -B^2 \rangle$, because the vacuum energy

$$E_{\mathrm{vac}} = \frac{1}{2}\left\langle 0 \middle| E^2 + B^2 \middle| 0 \right\rangle$$

must vanish in the absence of external constraints. This is confirmed by QCD lattice simulations and by phenomenological analyses of heavy quark mesons.

Melting the QCD vacuum

At nonzero temperature, gluonic (and quark) modes get excited:

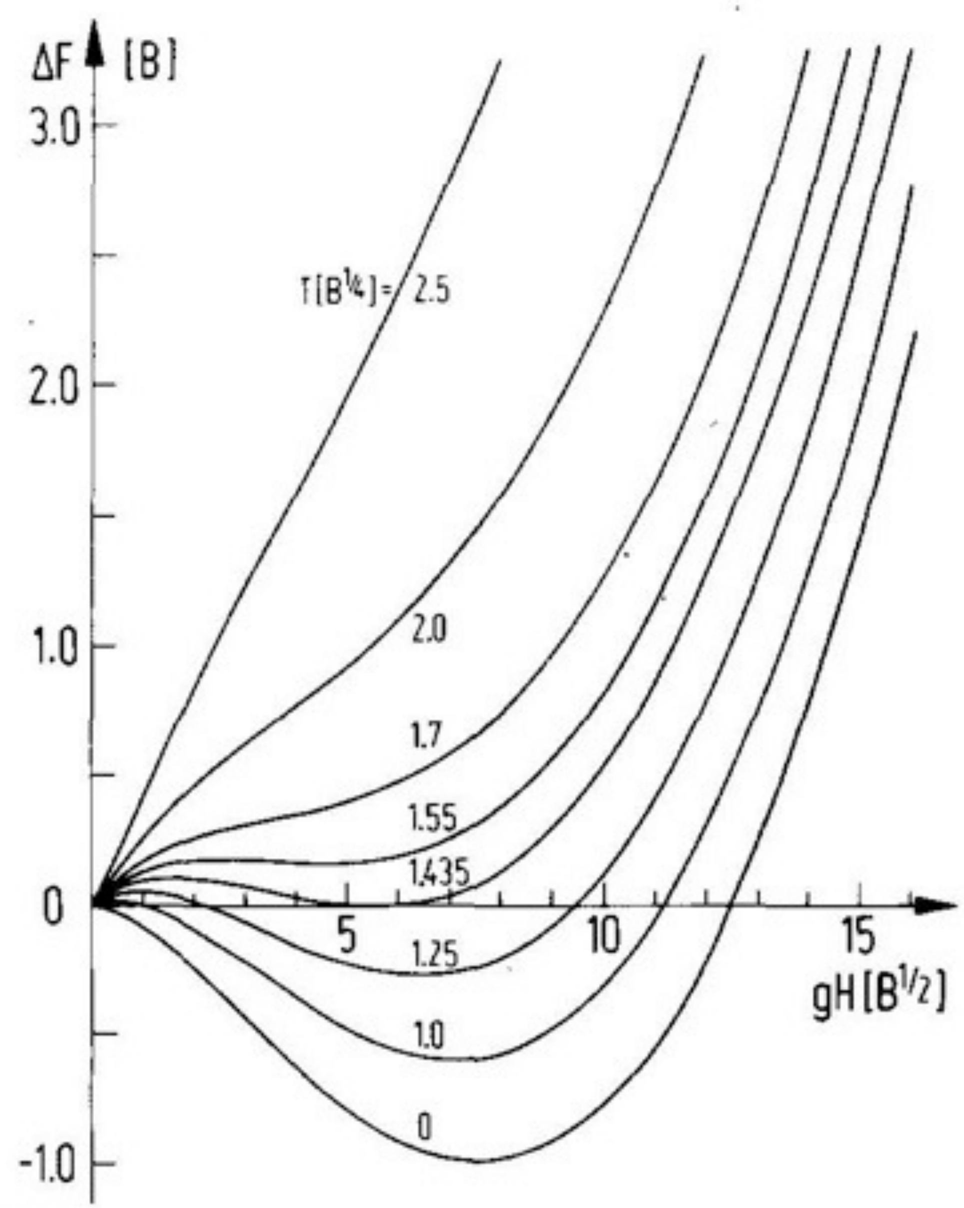
$$-F(H,T) = \frac{1}{2} \sum_{n,k_z,S_z}^{\text{ren}} \omega_{n,S_z}(k_z) \coth \frac{\omega_{n,S_z}(k_z)}{2T}$$

Nontrivial minimum at $H \neq 0$ disappears above a certain critical temperature T (Rafelski & BM, Kapusta '81, ...).

Comparison with empirical value of $\langle B^2 \rangle$ yields:

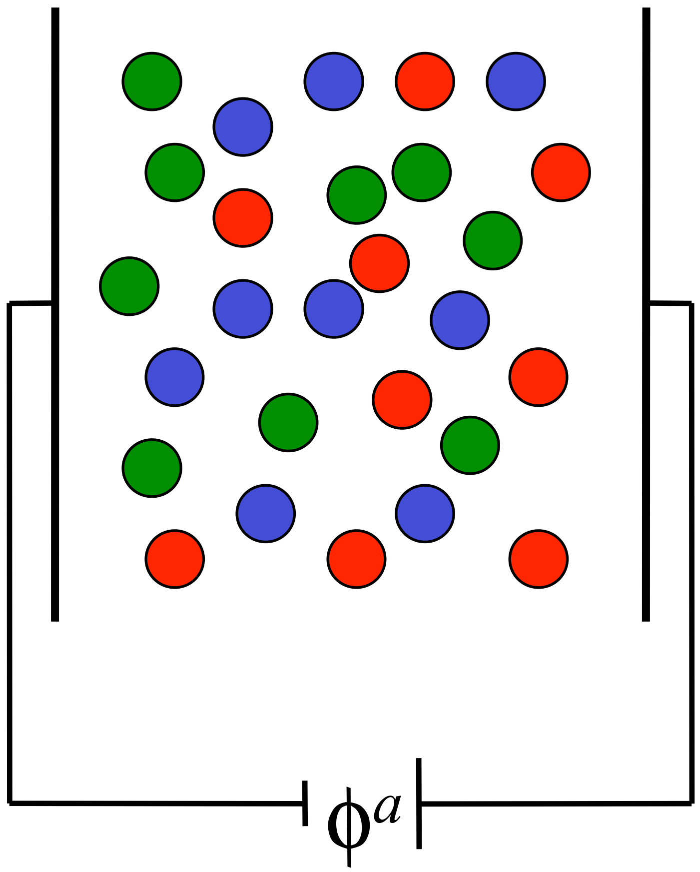
$$\begin{aligned} \varepsilon_{\text{vac}} &\approx (160\ \text{MeV})^4 \\ T_c &= 1.435\, \varepsilon_{\text{vac}}^{-1/4} \approx 230\ \text{MeV} \end{aligned}$$

Critical temperature is lowered when quarks are included → $T_c \sim 155 - 175$ MeV.



Part 2
What is a QGP?

Color screening



Static color charge (heavy quark) generates screened potential

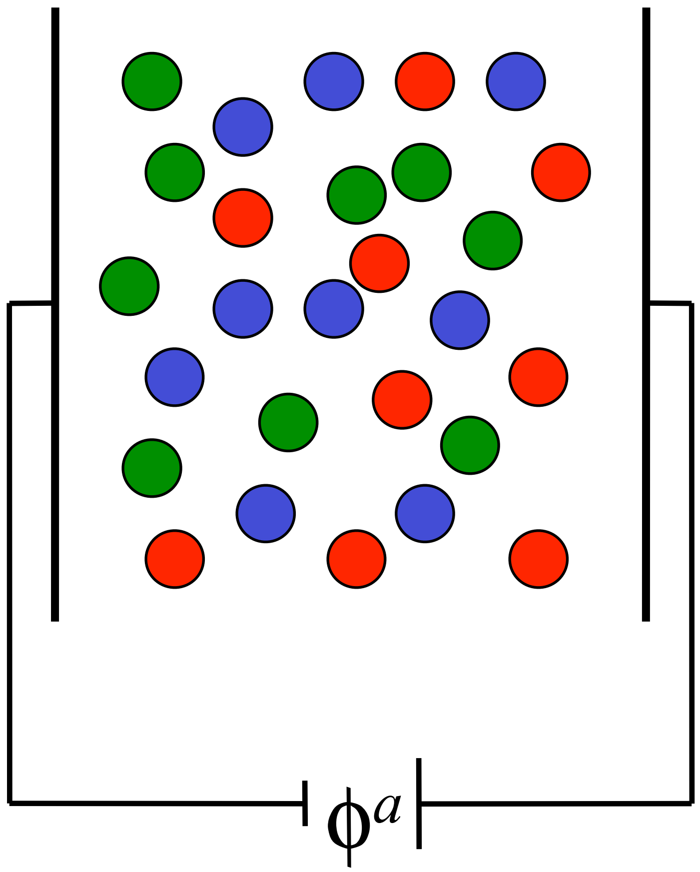
$$\phi^a = t^a \frac{\alpha_s}{r} e^{-\mu r}$$

$$-\nabla^2 \phi^a = g\rho_G^a(\phi^b) + g\rho_Q^a(\phi^b)$$

Induced color density $\quad \rho^a = -\mu^2 \phi^a$

with $\quad \mu_G^2 = (gT)^2, \quad \mu_Q^2 = \frac{N_F}{6}(gT)^2$

Color screening



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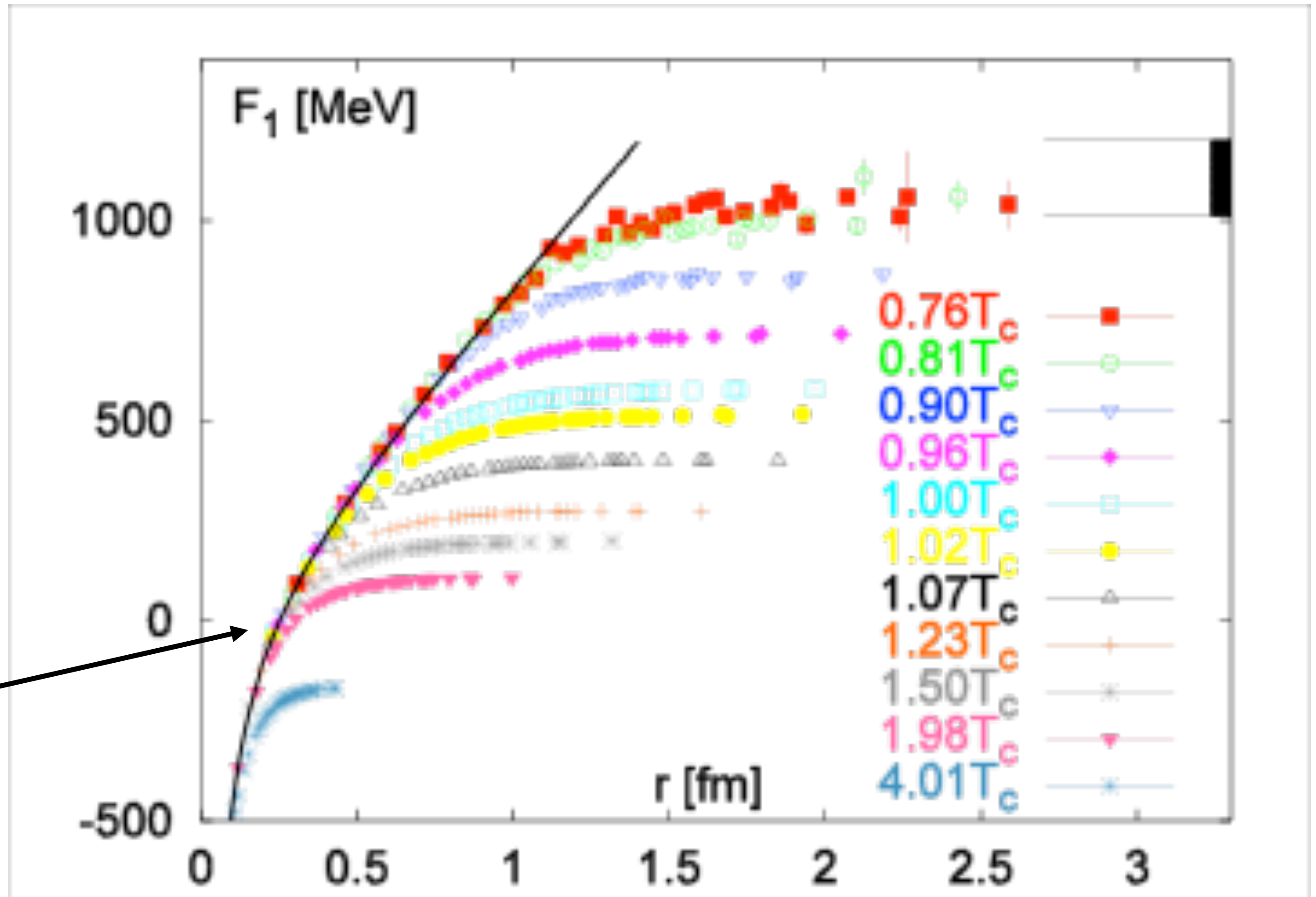

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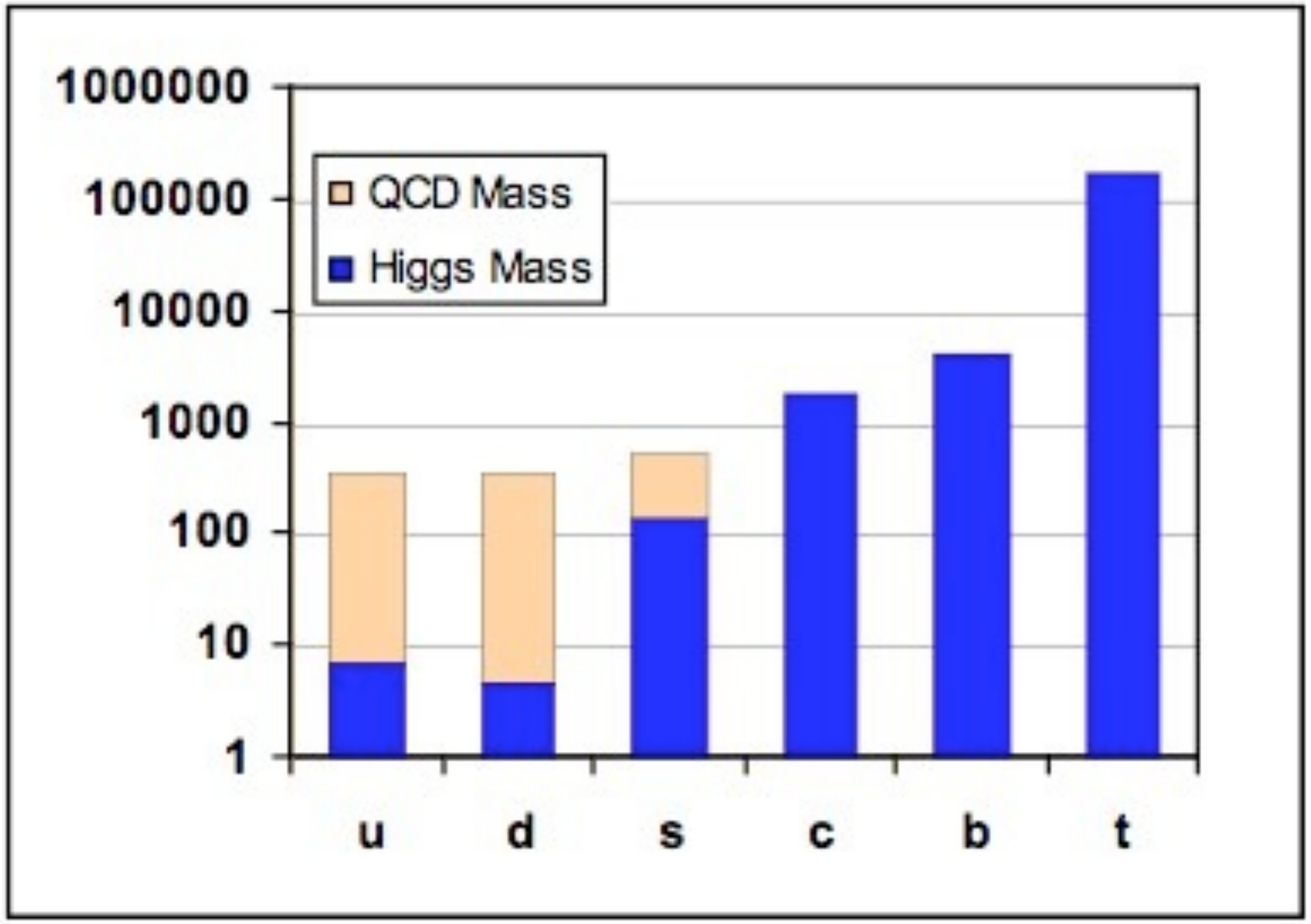
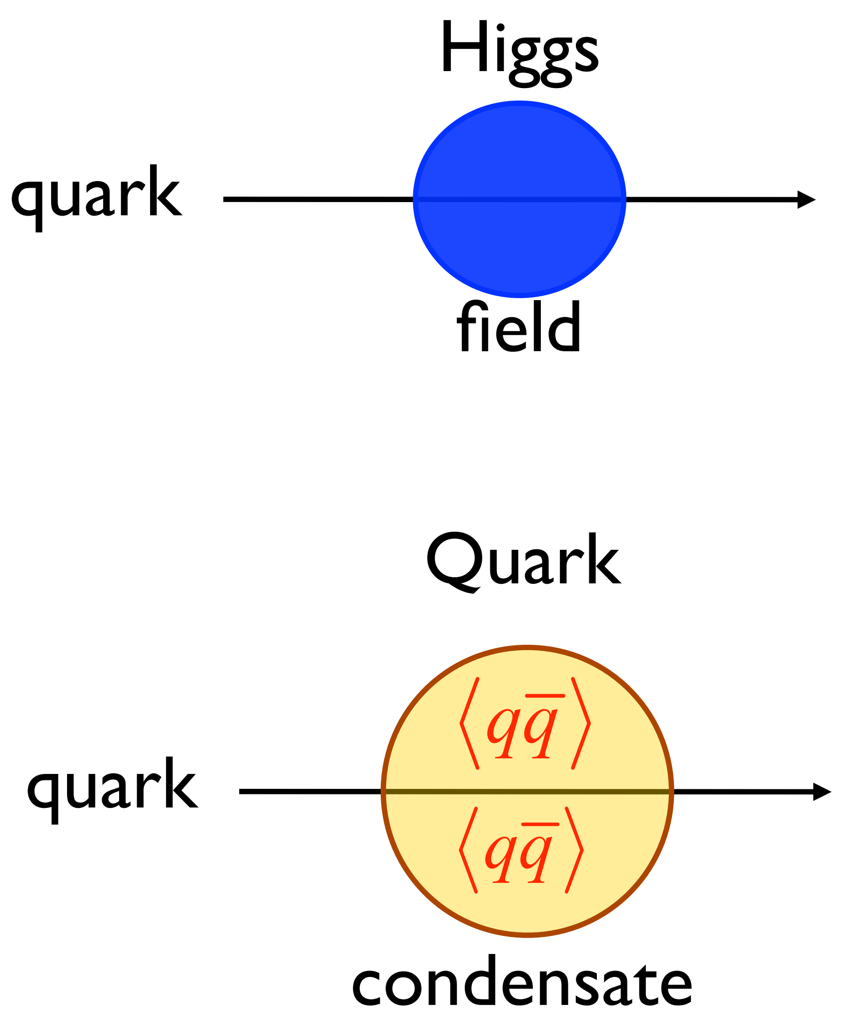
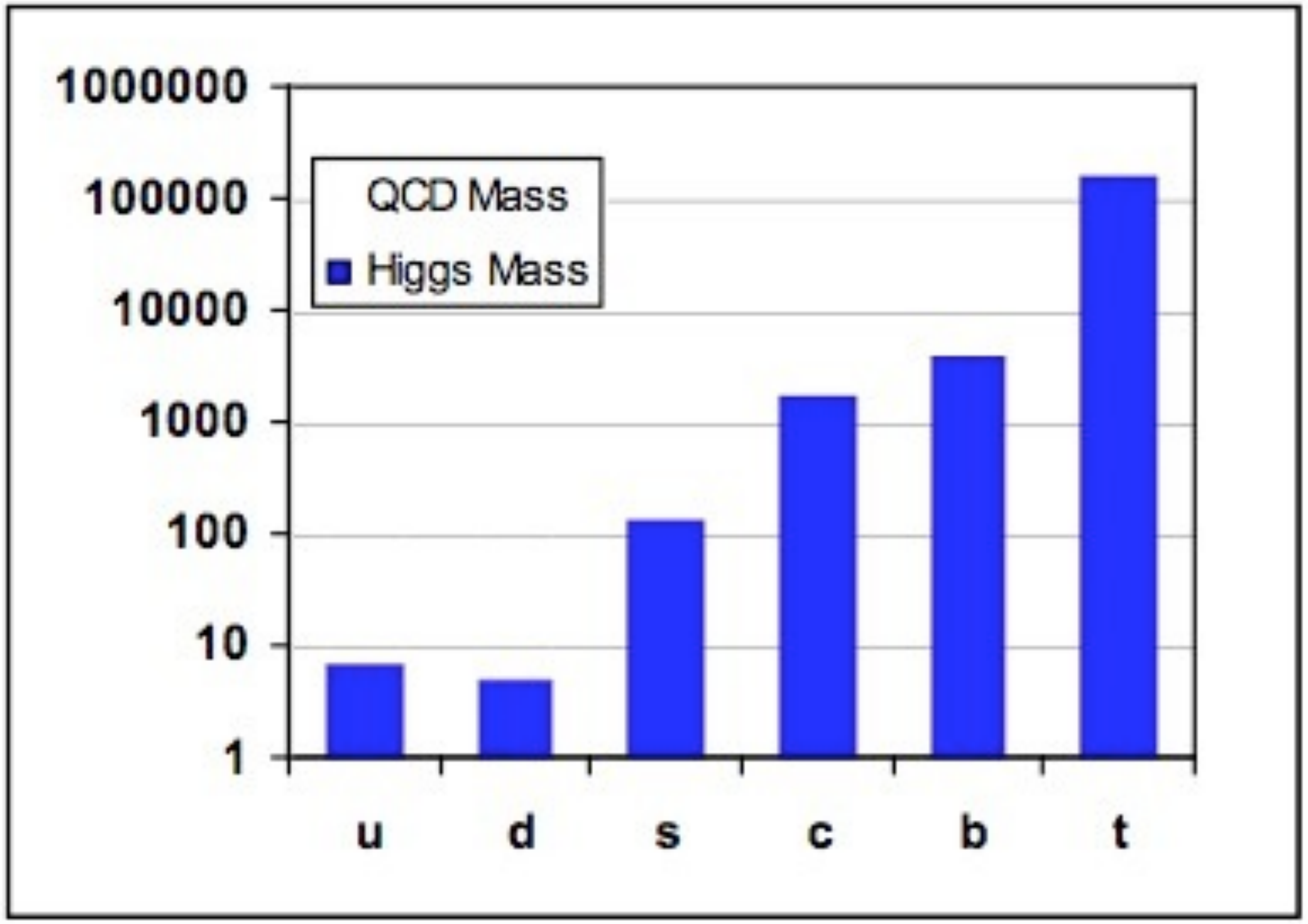
$$\phi^a = t^a \frac{\alpha_s}{r} e^{-\mu r}$$

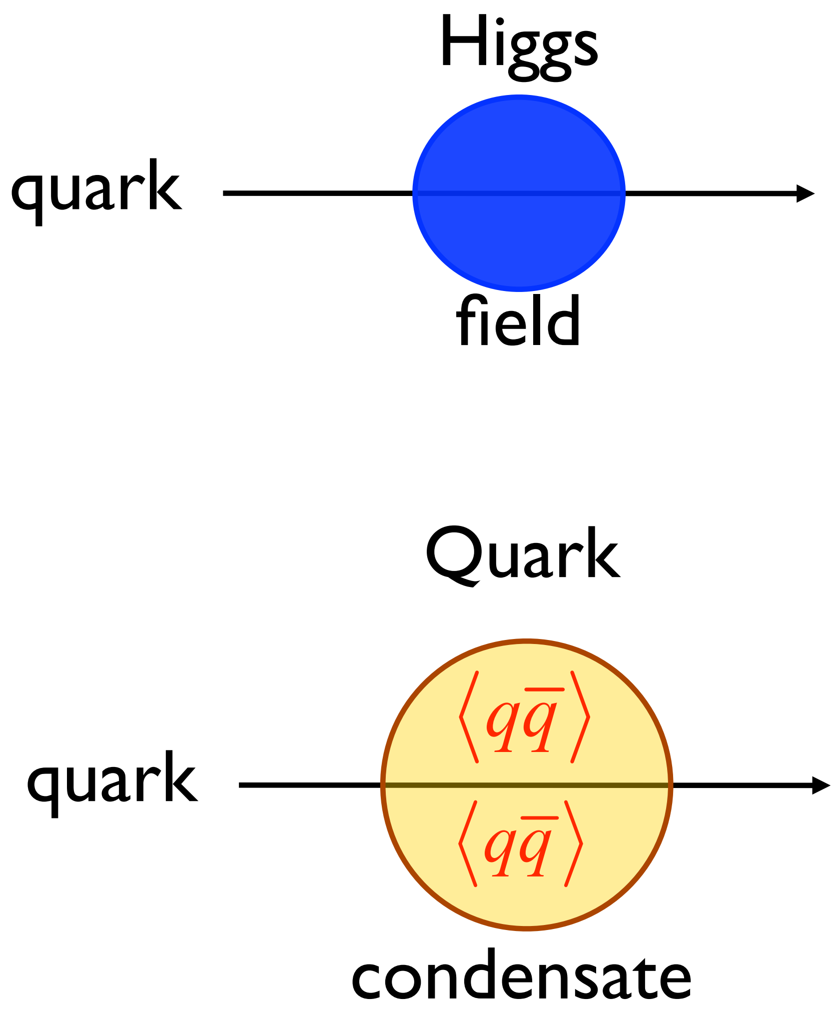
Quark masses





Quark masses

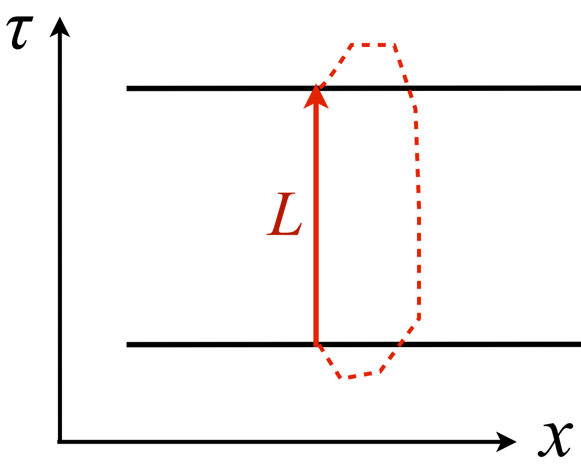




Heat "melts" the quark condensate:
QCD mass disappears above T_c.
(Partial) chiral symmetry restoration

Polyakov Loop

At T > 0 a gauge invariant quantity can be defined, which characterizes the confining properties of the QCD vacuum state: The *Polyakov Loop.*



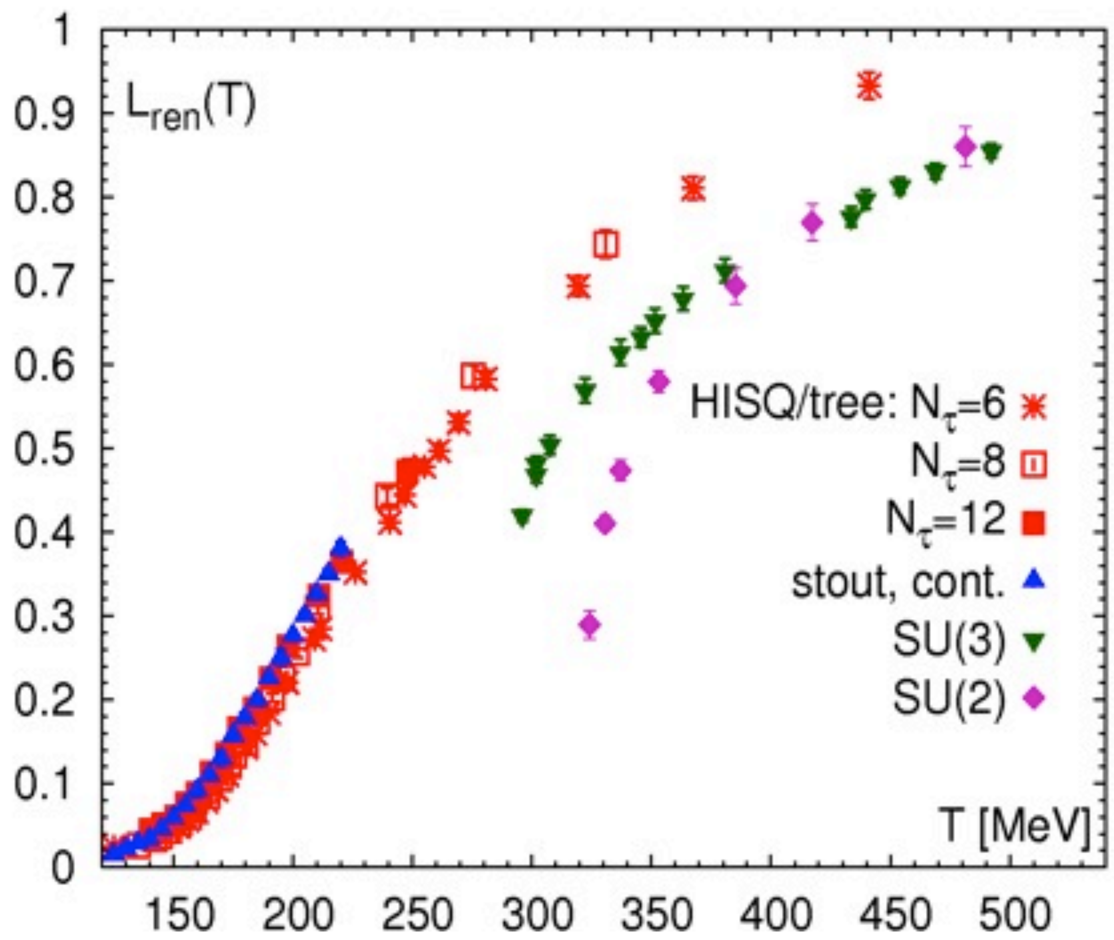
$$L = \frac{1}{N_c} \mathrm{tr}\left[\exp\left(ig \oint d\tau\, A_4^a(x,\tau) \frac{\lambda_a}{2} \right) \right]$$

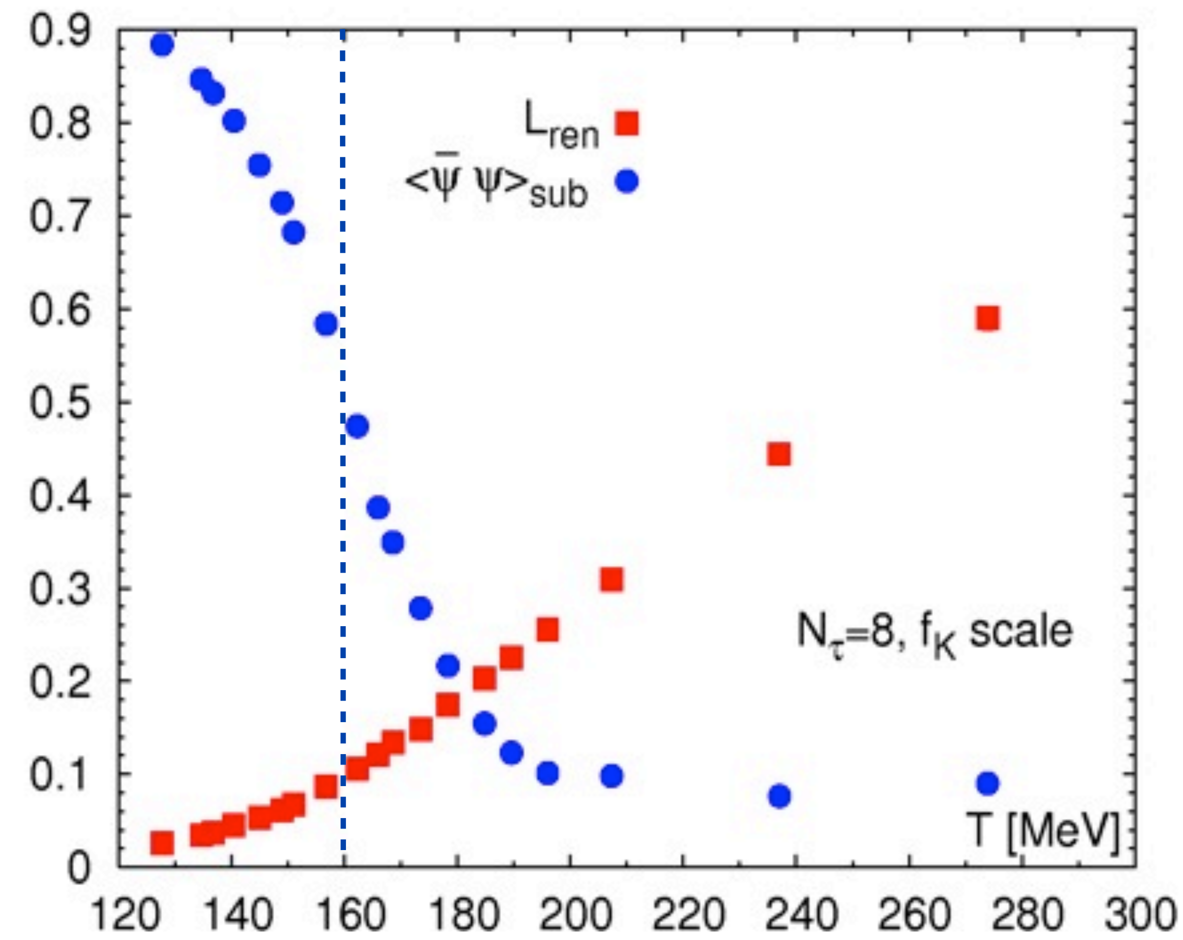
The Polyakov Loop can be interpreted as the fugacity of an isolated static quark due to its interaction with the gauge field:

$$\langle L \rangle = \exp\left(-F_Q / T\right)$$

$\langle L \rangle = 0$ implies absolute quark confinement; $\langle L \rangle < 1$ indicates partial confinement; in a fully developed QGP one expects $\langle L \rangle = 1$.

Lattice QCD





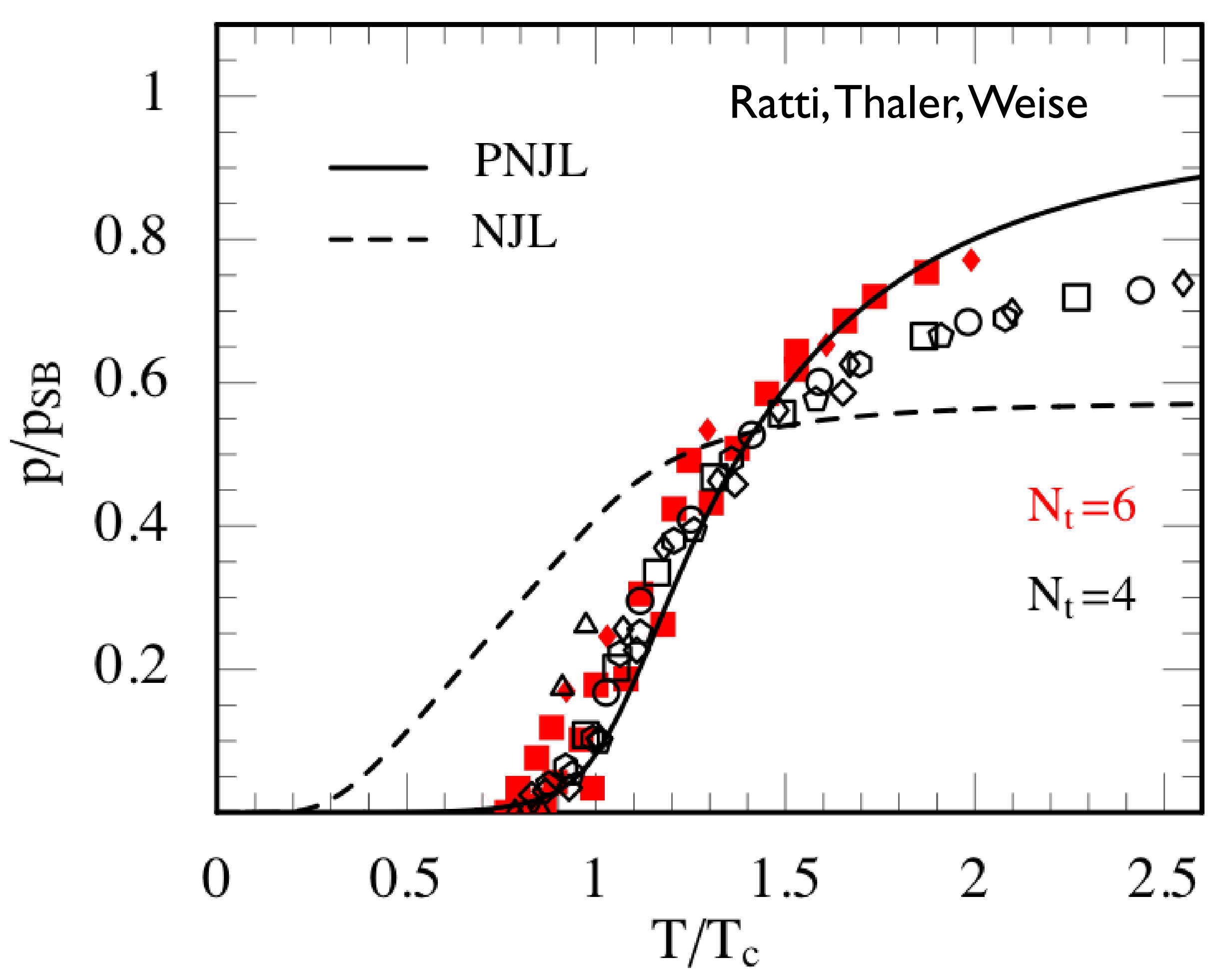
Polyakov loop is true order parameter for pure gauge theory, not for real QCD, because quarks can be produced in pairs, and color can be compensated.

Color is “unthawed” only gradually above T_c; partial color screening. Characteristic for strongly coupled plasmas!

PNJL Model

QCD vacuum transition can be modeled by introducing an effective interaction among quarks which causes a quark condensate to form (Nambu--Jona-Lasinio model) and an effective color phase factor L (Polyakov loop), which enforces quark confinement when $\ell_3 \equiv \langle L \rangle = 0$.

Thermodynamic pressure



$$\mathcal{L} = \bar{\psi}\left(i\gamma\cdot D - \hat{m}\right)\psi - \mathcal{U}(\ell_3, \bar{\ell}_3; T)$$
$$+ \frac{g_S}{2}\sum_{a=0}^{8}\left[\left(\bar{\psi}\lambda^a\psi\right)^2 + \left(\bar{\psi}i\gamma_5\lambda^a\psi\right)^2\right]$$
$$+ g_D\left[\det\bar{\psi}\left(1-\gamma_5\right)\psi + h.c.\right]$$

$$D_\mu = \partial_\mu - g\delta_{\mu 4}A_4$$

current quark mass matrix

$$\hat{m} = \mathrm{diag}\left(m_u, m_d, m_s\right)$$

Part 3
The perturbative QGP

The perturbative QGP

Perturbative expansion of the equation of state:

$$\varepsilon = \left(1 - \frac{15\alpha_s}{4\pi}\right)\frac{16\pi^2}{30}T^4 + \left(1 - \frac{50\alpha_s}{21\pi}\right)\frac{21\pi^2}{30}N_F T^4 + \sum_q \left(1 - \frac{2\alpha_s}{\pi}\right)\frac{3}{\pi^2}\mu_q^2(\pi^2 T^2 + \tfrac{1}{2}\mu_q^2) + \cdots$$

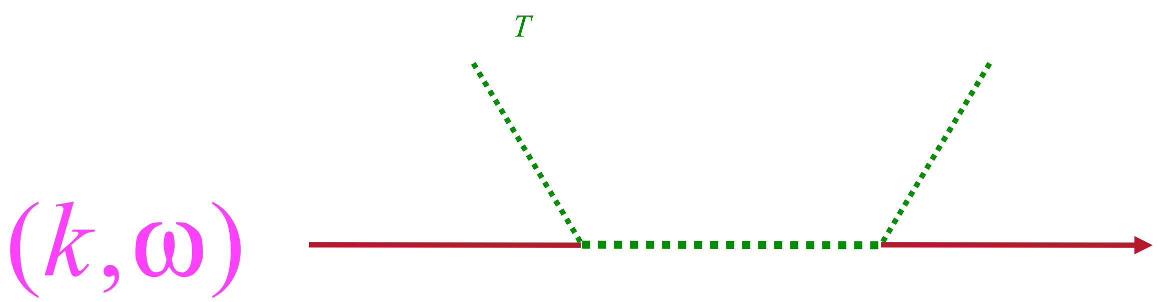
has been calculated up to order $\alpha_s^3 \ln(\alpha_s)$ [Kajantie et al., hep-ph/0211321].

Expansion in α_s does *not* converge, one must include interactions in the particle modes ("quasiparticles") as basis for the expansion:

Hard-thermal loop perturbation theory [Braaten & Pisarski 1990].

Quasiparticles in the QGP

Physical excitation modes at high T are not elementary quarks and gluons, but "dressed" quarks and gluons:



Screening of longitudinally polarized gluons (Debye mass): $m_D \xrightarrow[k \to 0]{\omega = 0} gT$

Propagator of transversely polarized gluons $$D(k,\omega)^{-1} = \omega^2 - k^2 - \frac{1}{2}(gT)^2 \left[1 - \frac{1}{2}\left(\frac{\omega}{k} - \frac{k}{\omega} \right) \ln \frac{\omega + k}{\omega - k} \right]$$

→ Effective gluon mass:

$$m_{\text{Plasmon}} \xrightarrow[k \to 0]{} \frac{1}{\sqrt{3}} gT$$

$$m_{Gluon} \xrightarrow[k \to \infty]{} \frac{1}{\sqrt{2}} gT$$

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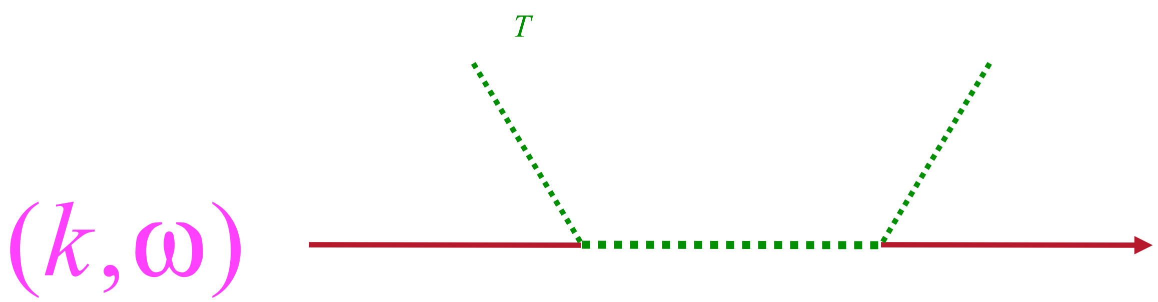

Compton scattering on a thermal gluon!

Screening of longitudinally polarized gluons (Debye mass): $m_D \xrightarrow[k\to 0]{\omega=0} gT$

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Screened perturbation theory

Andersen, Leganger, Strickland & Su [hep-ph/1103.2528]:

Add a gauge invariant mass term, that vanishes for δ→1

$$\mathcal{L} = \left(\mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm HTL}\right)\Big|_{g\to\sqrt{\delta}g}$$

with

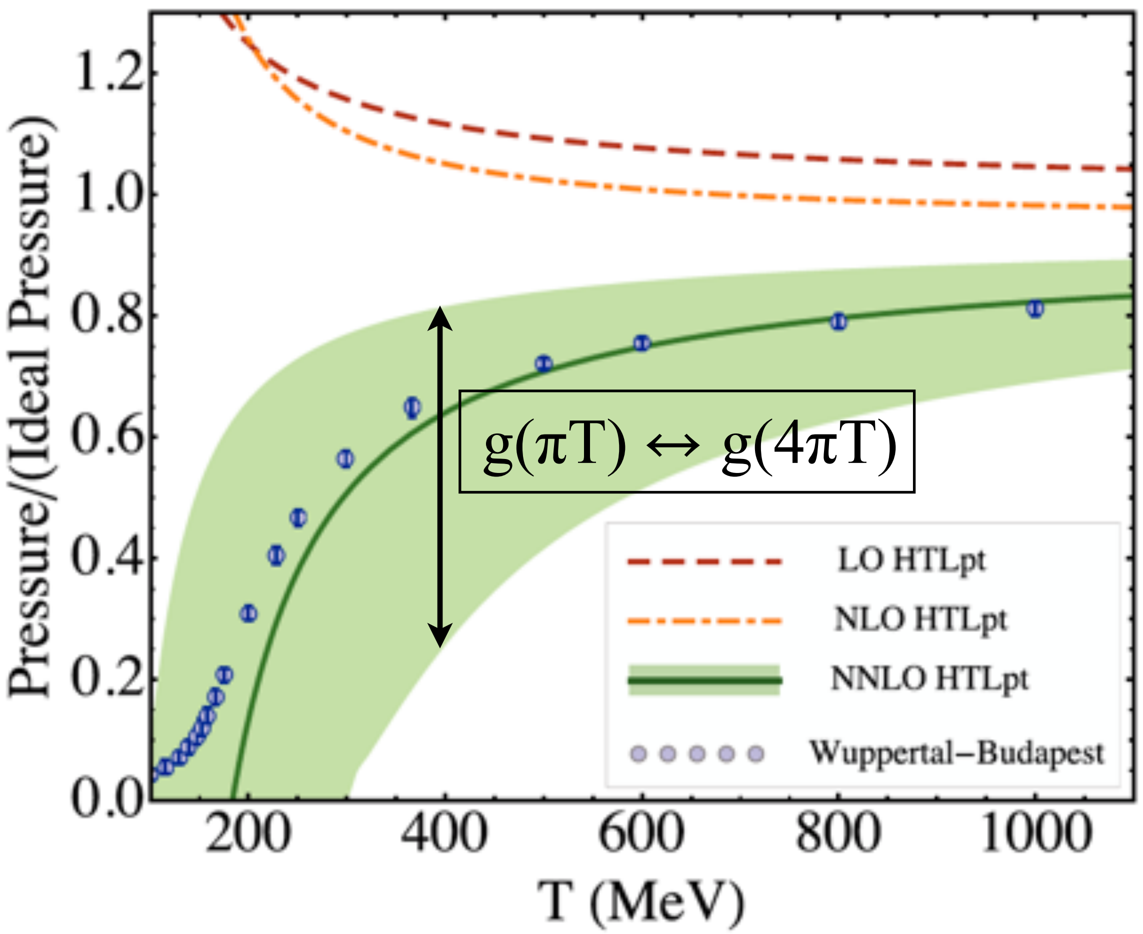
$$\mathcal{L}_{\rm QCD} = -\frac{1}{2}{\rm Tr}\left[G_{\mu\nu}G^{\mu\nu}\right] + i\bar\psi\gamma^\mu D_\mu\psi + \mathcal{L}_{\rm gf} + \mathcal{L}_{\rm gh}$$

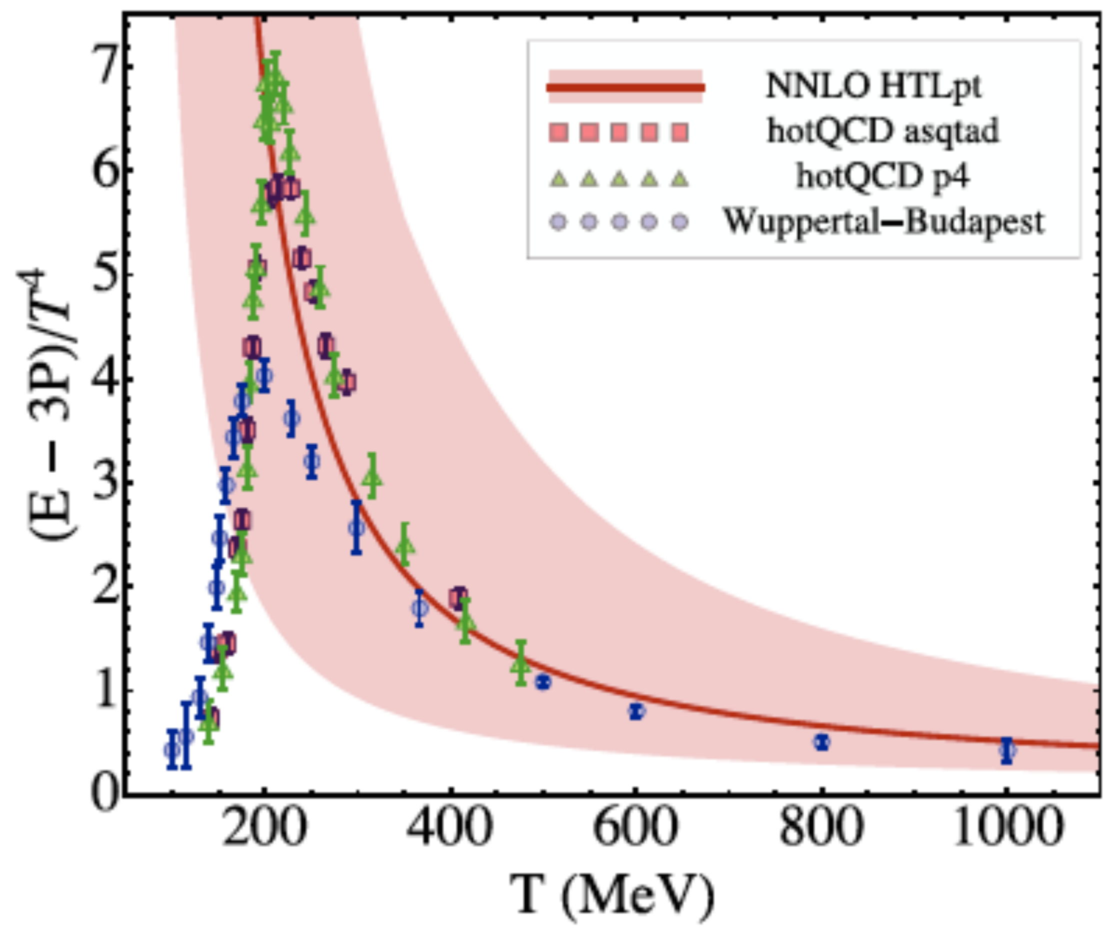
$$\mathcal{L}_{\rm HTL} = -\frac{1}{2}(1-\delta)m_D^2{\rm Tr}\left(G_{\mu\alpha}\left\langle\frac{y^\alpha y^\beta}{(y\cdot D)^2}\right\rangle_y G^\mu{}_\beta\right) + (1-\delta)\,im_q^2\bar\psi\gamma^\mu\left\langle\frac{y_\mu}{y\cdot D}\right\rangle_y\psi$$

ALSS obtained an analytic result for the equation of state at NNLO order.

m_D and m_q are fixed by minimizing the thermodynamic potential

HTL PT results





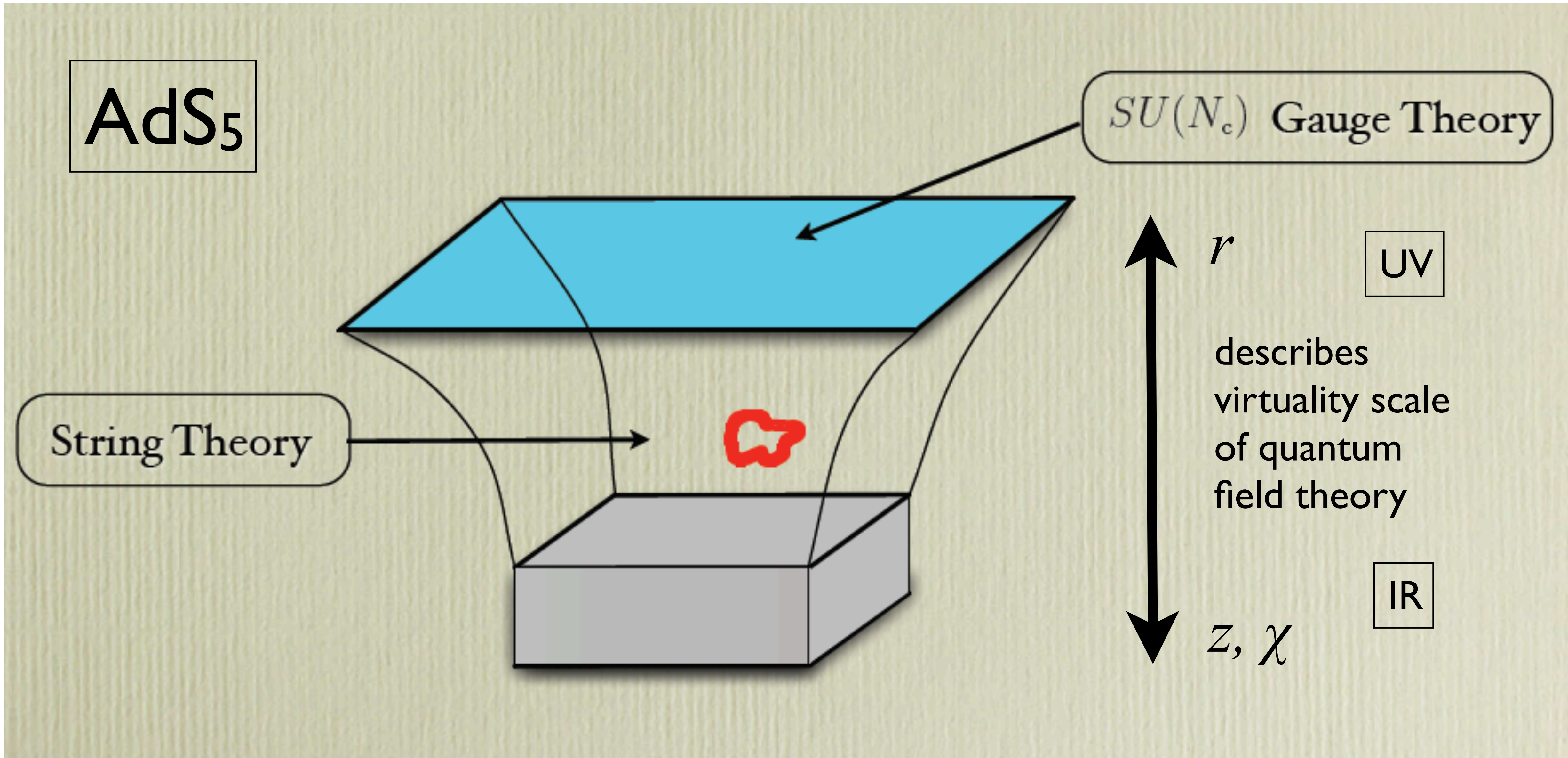
The HTL perturbation theory at NNLO seems to work well above T ≈ 300 MeV, but with very large systematic uncertainties from setting the coupling scale. The QGP in the RHIC/LHC domain is not weakly coupled, and even the most sophisticated perturbative approach available is unreliable!

Part 4

The strongly coupled QGP

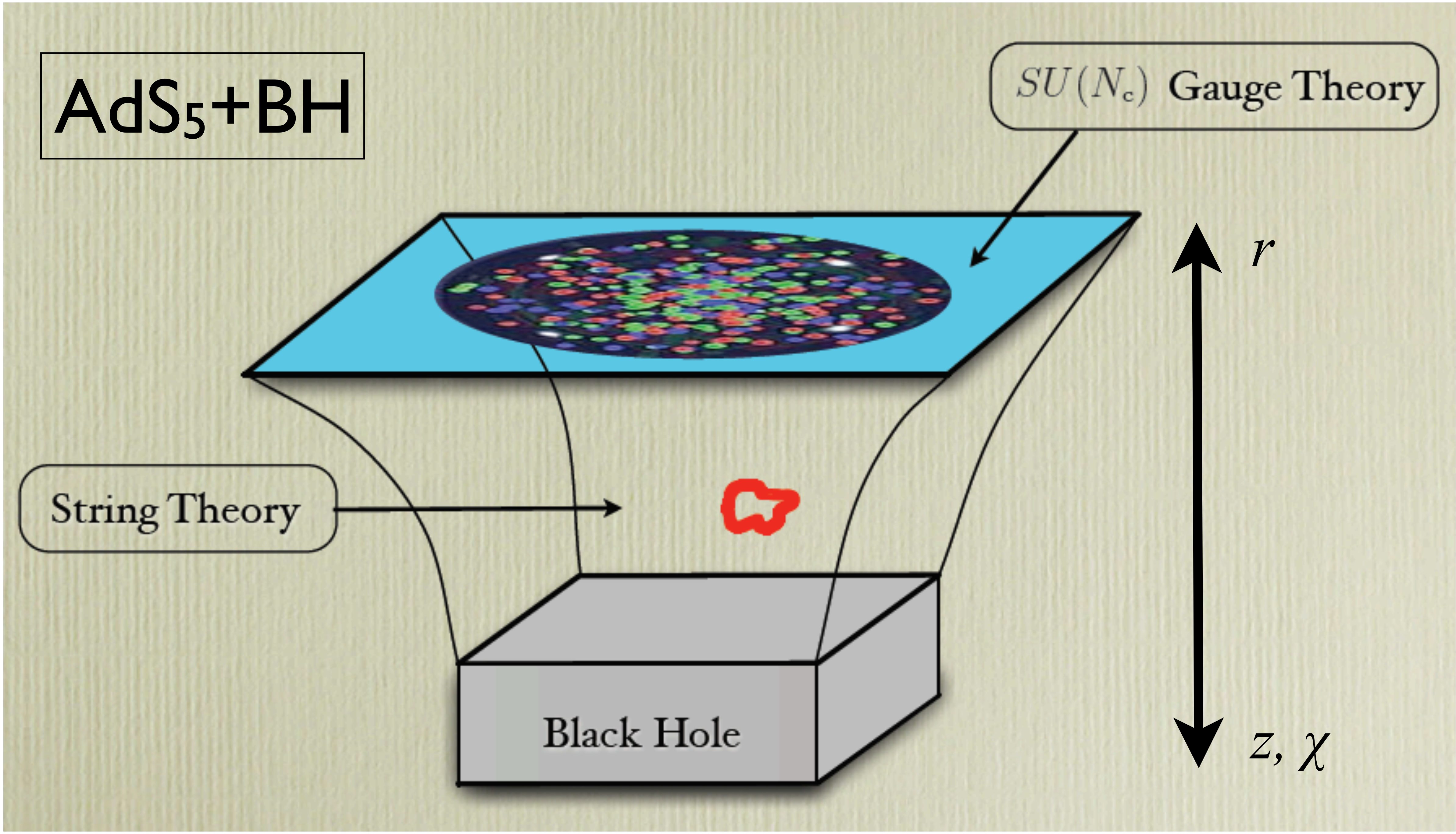
(sQGP)

Gauge-string duality

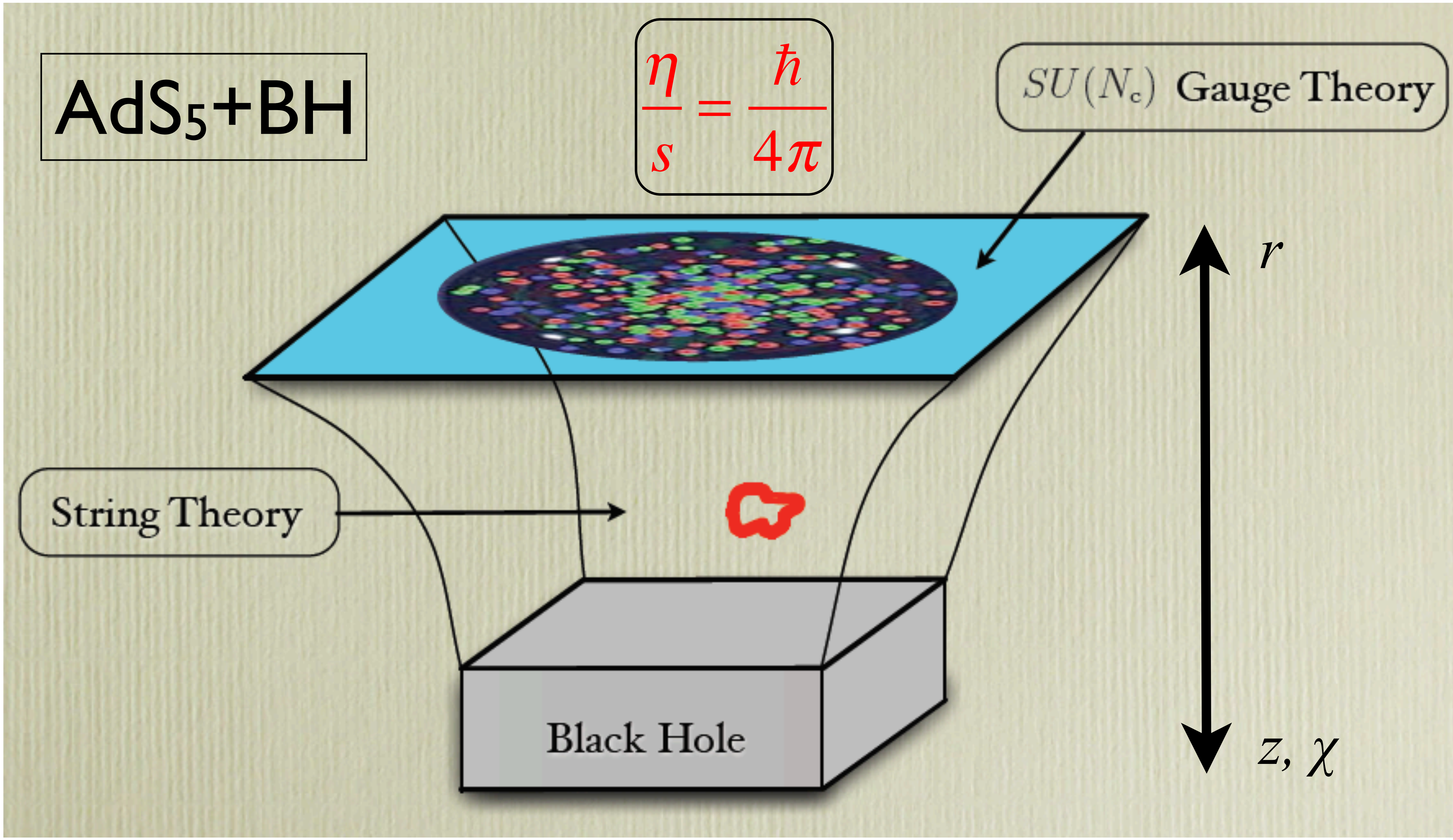


- Exact equivalence at *all* energies, N_c and $\lambda = g^2_{\rm YM} N_c$!

Thermal holography



Thermal holography

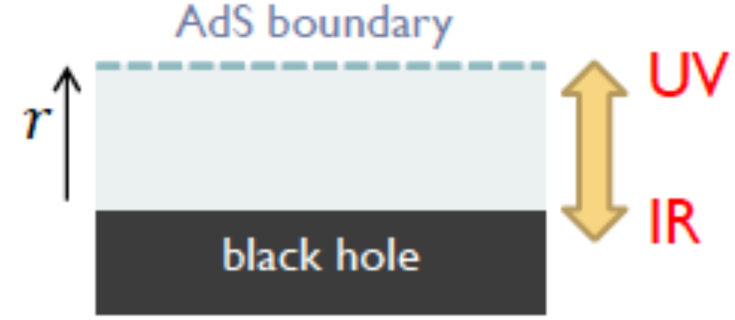


AdS/CFT dictionary

- Want to study strongly coupled phenomena in QCD
- Toy model: $\mathcal{N} = 4\ SU(N_c)$ SYM

$\mathcal{N} = 4$ SYM	holo. dual ⟷ AdS/CFT	String theory in AdS$_5$
Strong coupling	⟷	Weak coupling
Vacuum	⟷	Empty AdS$_5$
Thermal state (equilibrated plasma)	⟷	AdS BH
Thermalization	⟷	BH formation
HI collision	⟷	Energy injection

$$\left(\frac{R_{AdS}}{l_s}\right)^4 = g_{YM}^2 N_c \equiv \lambda$$



Boundary-bulk correspondence

- Local and nonlocal operators on the boundary
  - Equation of state

$$\langle T_{\mu\nu} \rangle \ \text{etc.}$$

  - Correlators & Green functions:

$$\langle O(x) O(x') \rangle \ \text{etc.}$$

- What are the spectral functions of QFT modes?
  - Find Green functions from bulk and analyze

Spectral functions

Start from retarded Green function

$$iD_{\mathrm{R}}(\mathbf{x},t) = \theta(t)\mathrm{Tr}\,\langle\hat{\rho}\,[\mathcal{O}(\mathbf{x},t),\mathcal{O}(0,0)]\rangle$$

Fourier transform and identify the imaginary part:

$$\rho(\mathbf{k},\omega) = -2\,\mathrm{Im}\,D_{\mathrm{R}}(\mathbf{k},\omega)$$

For free massive particle:

$$\rho(k,\omega) = 2\pi\,\mathrm{sgn}(\omega)\delta(\omega^2 - k^2 - m^2)$$

For an unstable particle:

$$\rho(k,\omega) = \frac{2\pi\Gamma/\omega}{\left(\omega^2 - k^2 - m^2\right)^2 + \frac{1}{4}\Gamma^2}$$

Spectral functions in AdS/CFT

Analytically known in $d = 1+1$ dimensions for massless field:

$$G_R(\omega, k) \sim \frac{\omega^2 - k^2}{(2\pi)^2}\left[\psi\left(1 - i\frac{\omega - k}{4\pi T}\right) + \psi\left(1 - i\frac{\omega + k}{4\pi T}\right)\right]$$

Spectral density is completely different - no peak, just a step:

$$\rho(k,\omega) = 2\pi\,\mathrm{sgn}(\omega)\left(\omega^2 - k^2\right)^{\nu}\theta\left(\omega^2 - k^2\right)$$

For $d = 3+1$ dimensions for massless field no complete analytical solution, except for spectral function of fluctuations of $T_{\mu\nu}$ [Kovtun & Starinets]:

$$\rho_4(\omega, k) \sim \frac{(\omega^2 - \vec{k}^2)^2}{(4\pi)^2}\,\pi\theta(\omega^2 - k^2)\mathrm{sign}(\omega)$$

Conclusion: No quasi-particle like modes at all - the strongly coupled gauge plasma behaves like “mush” or more correctly, like a “perfect” liquid.

Sound

Conclusion: No quasi-particle like modes at all - the strongly coupled gauge plasma behaves like “mush” or more correctly, like a “perfect” liquid.

But there exists one nicely propagating mode: Sound!

Green’s function for (longitudinal) fluctuations of the energy density / pressure:

$$g_L(k,\omega) \sim \frac{1}{\omega^2 - c_s^2 k^2 + i\Gamma_s \omega k^2}$$

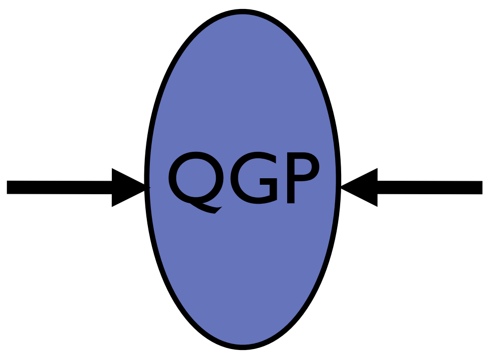
$\Gamma_s = \dfrac{\frac{4}{3}\eta + \varsigma}{s_0 T}$ is called the *sound attenuation length*; describes damping of high frequency - short wavelength sound.

But note that sound is not a “quasi-particle” in the usual sense, because the dispersion relation $\omega = c_s k < k$ is space-like!

Part 5
The unstable QGP

Expansion → Anisotropy

X-space



P-space

Perturbed equilibrium distribution:

$$f(p) = f_0(p)\left[1 + f_1(p)\left(1 \pm f_0(p)\right)\right]$$

$$f_0(p) = \exp[-u_\mu p^\mu / T]$$

For shear flow of ultrarelat. fluid:

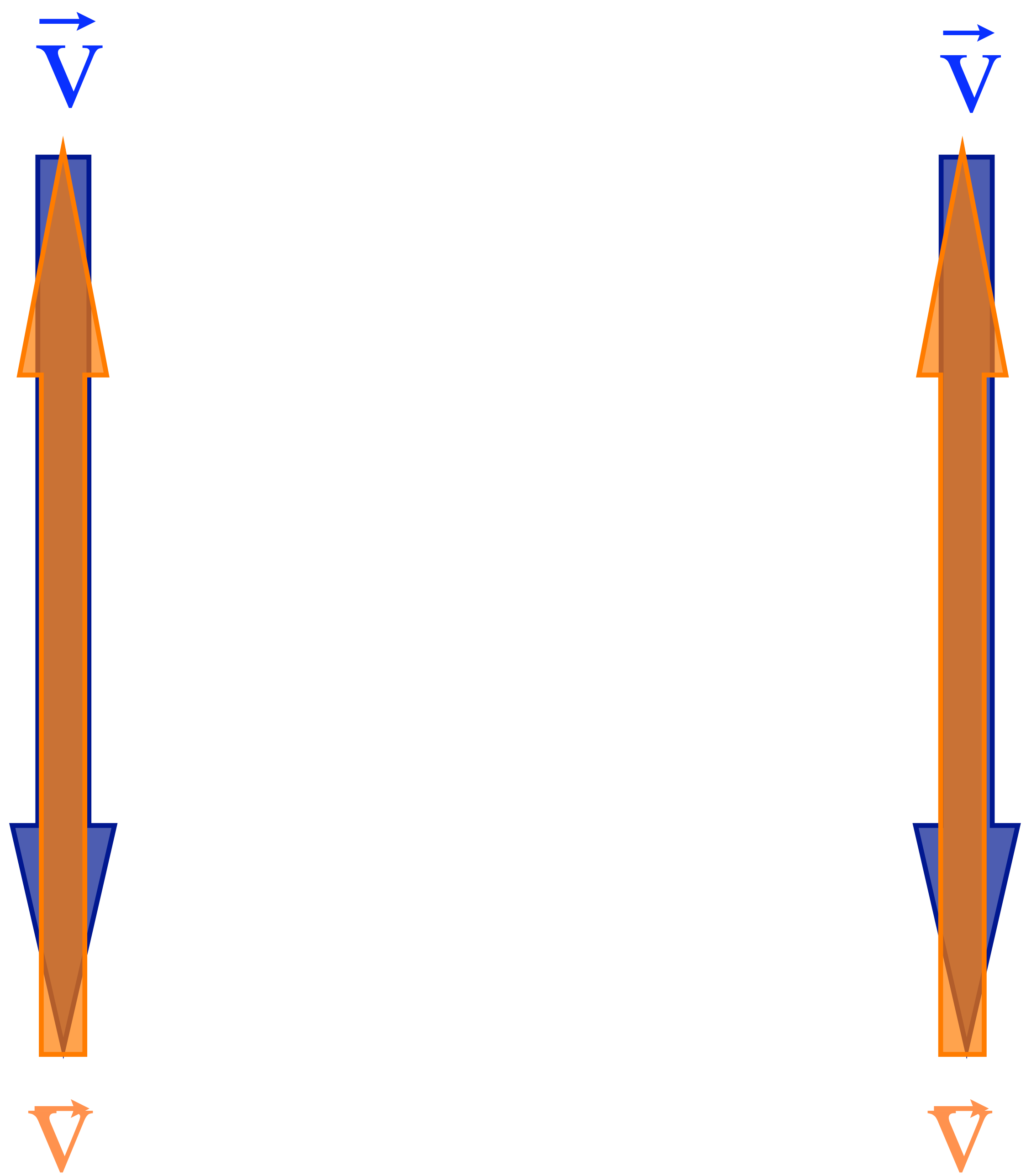
$$f_1(p) = -\frac{5\eta / s}{2ET^2}\left(p^i p^j - \tfrac{1}{3}\delta_{ij}\right)\Delta_{ij}(u)$$

$$\Delta_{ij}(u) = \nabla_i u_j + \nabla_j u_i - \tfrac{2}{3}\delta_{ij}\nabla \cdot u$$

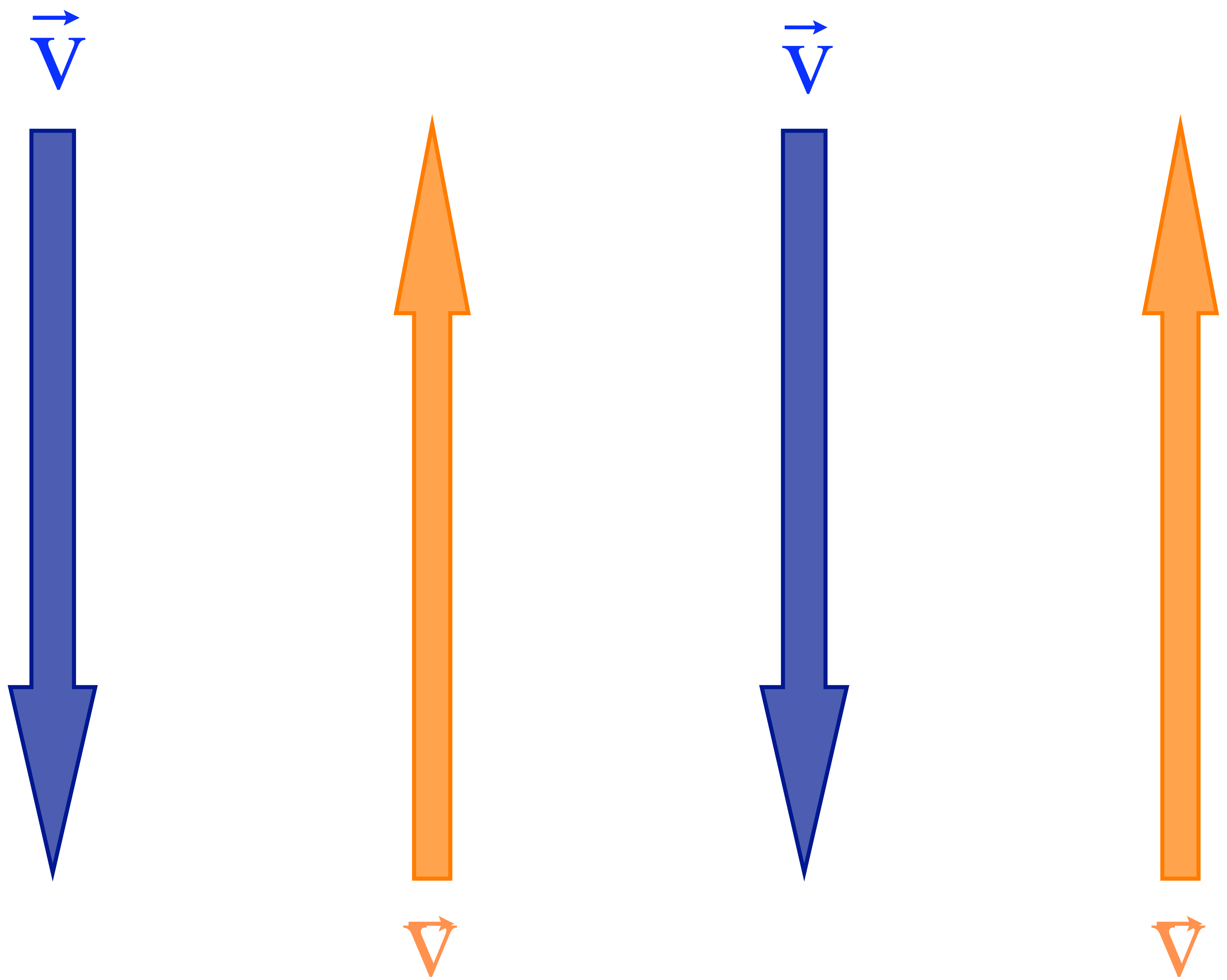
Momentum space anisotropy of expanding fluid is a measure of the ratio: shear viscosity / entropy density.

Small η/s implies nearly isotropic local momentum distribution; large η/s implies strong local momentum anisotropy.

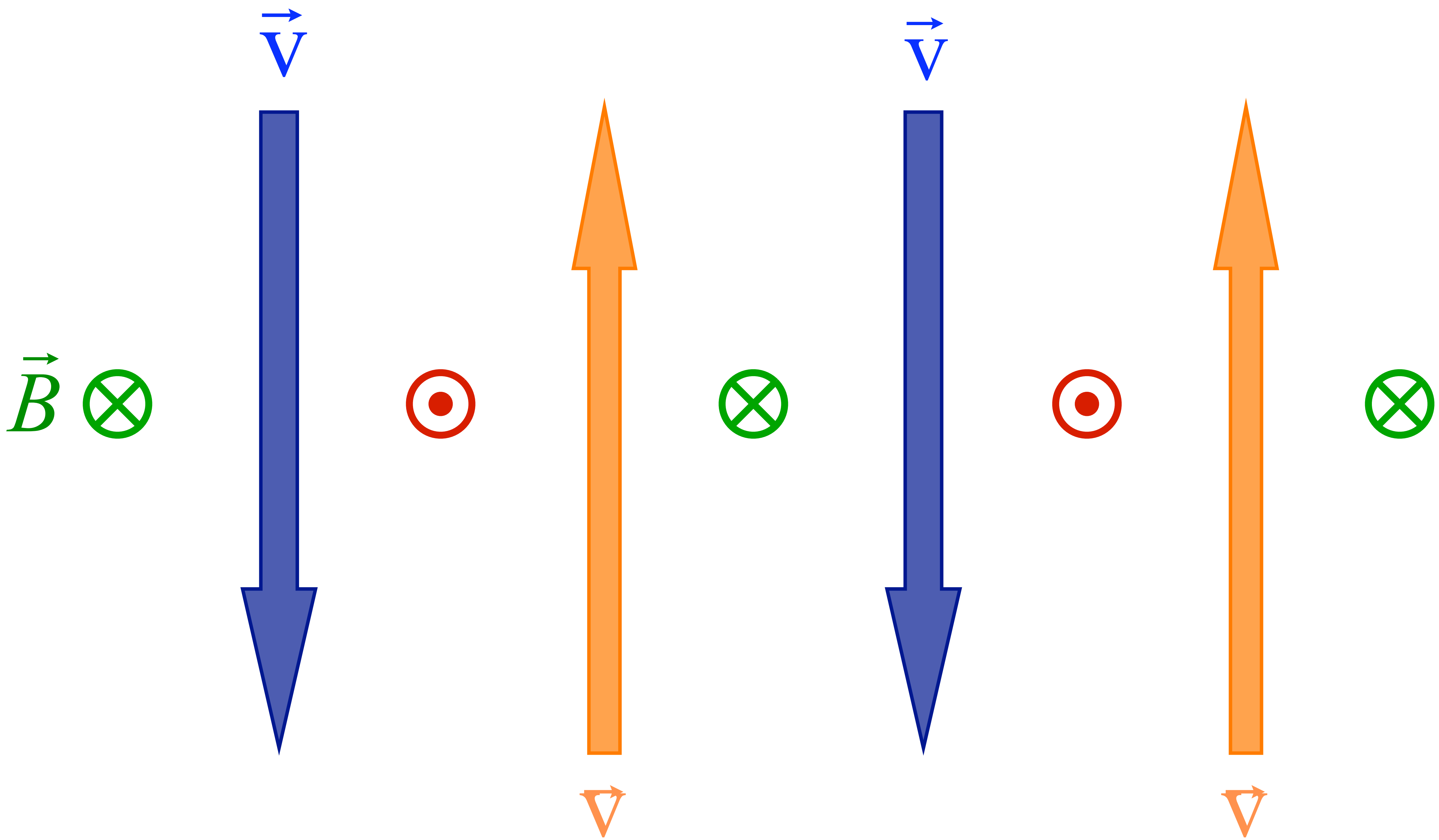
Plasma two-stream instability



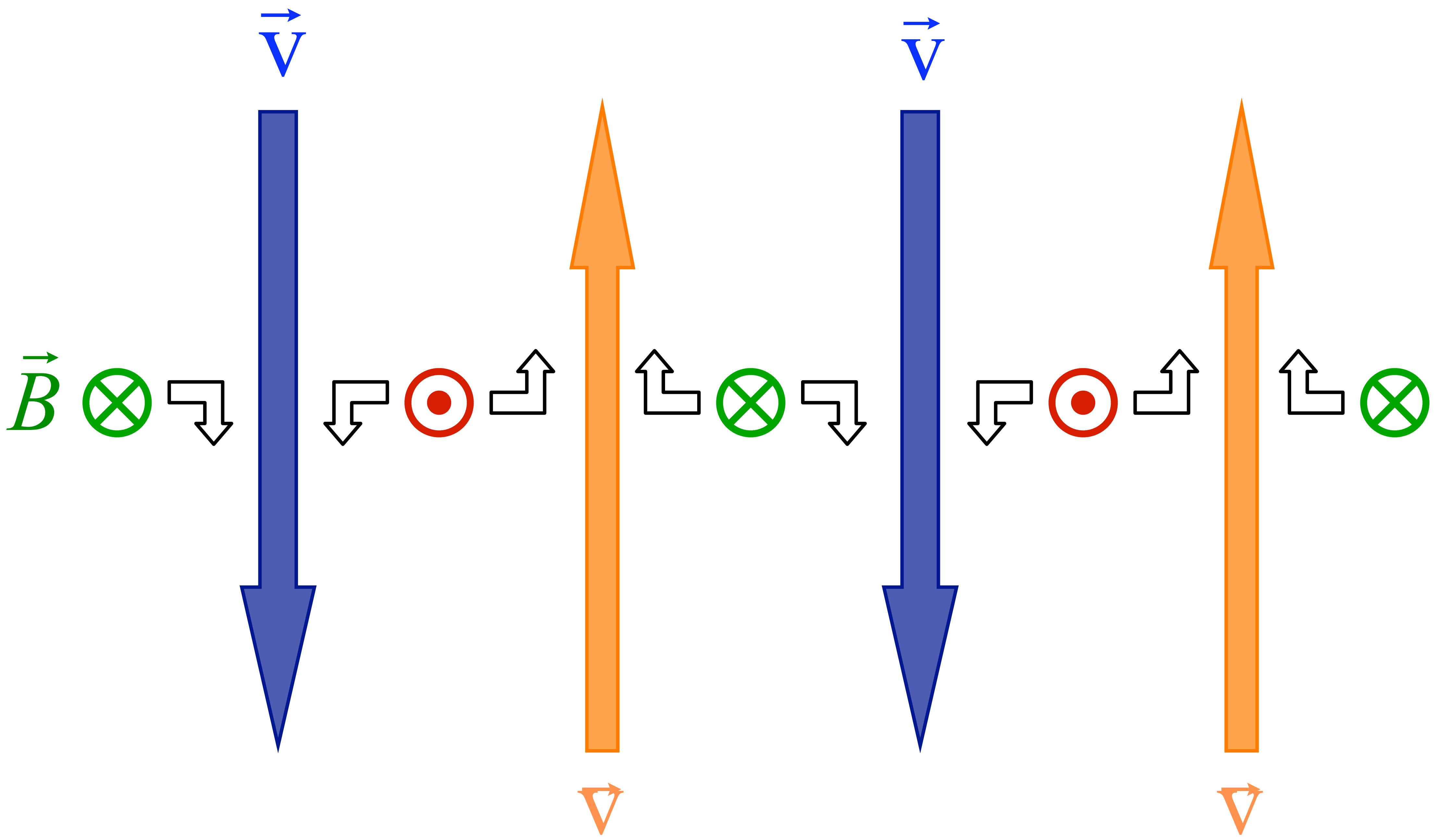
Plasma two-stream instability



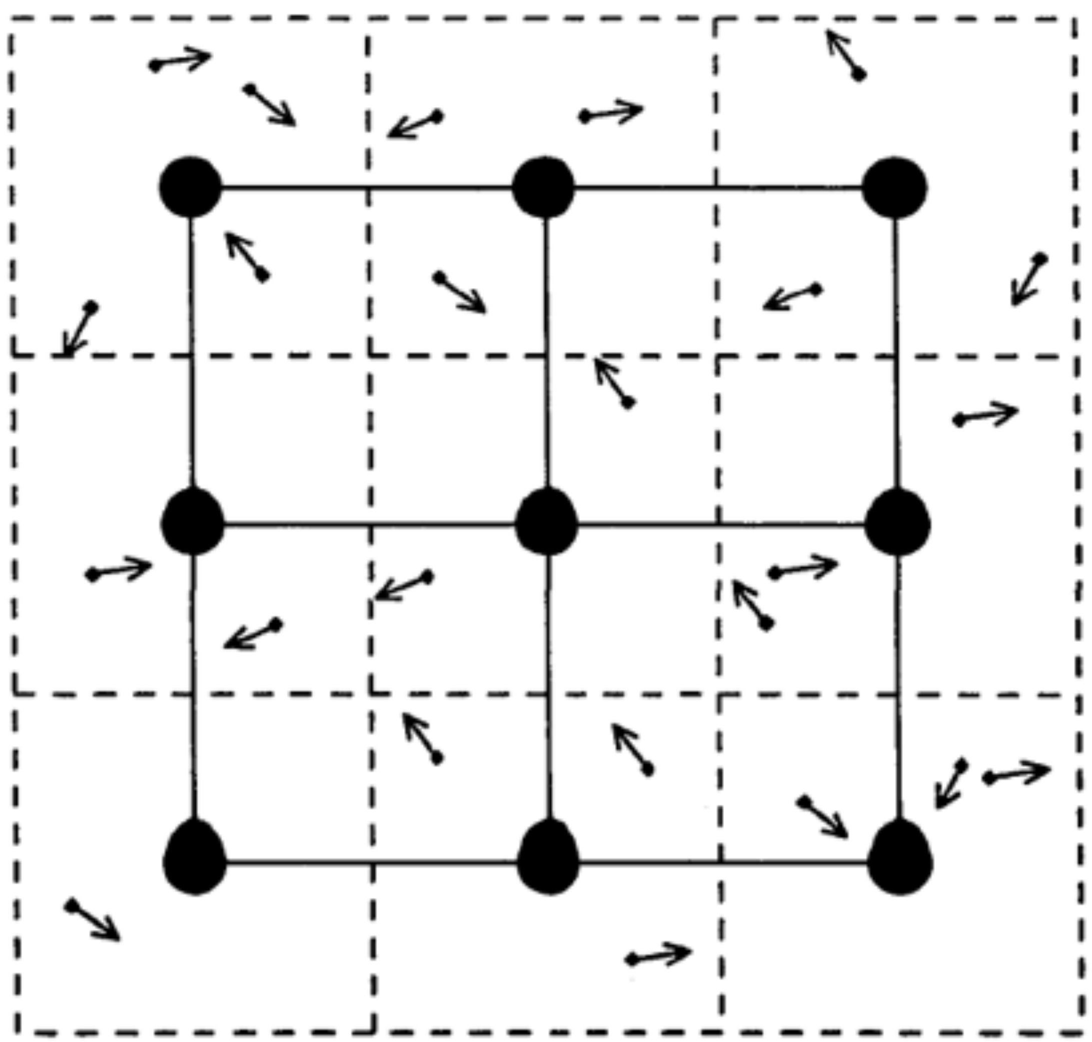
Plasma two-stream instability



Plasma two-stream instability



CPIC method



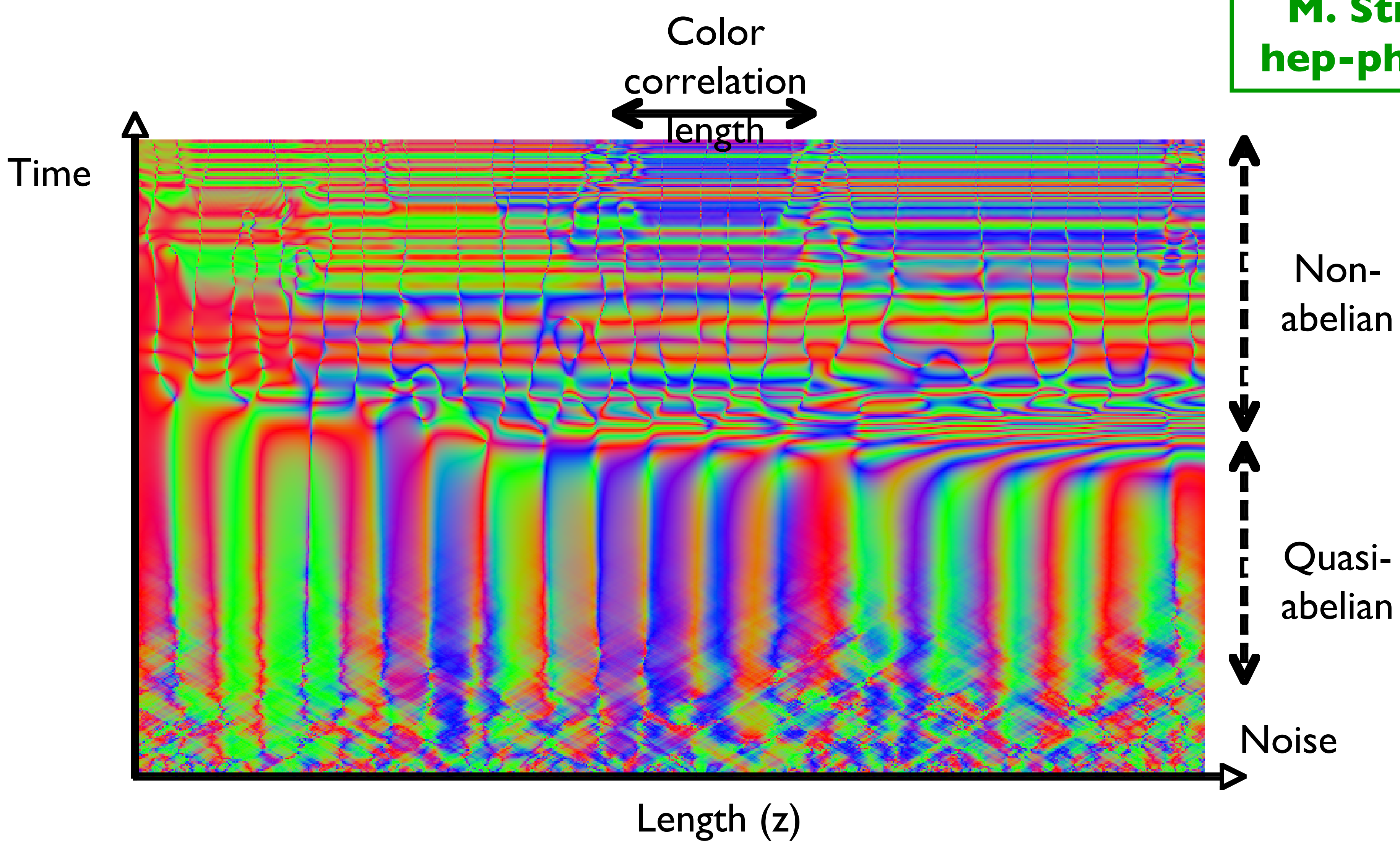
Simulate "soft" collectice modes of gauge field using a spatial lattice, the "hard" thermal modes using classical colored particles.

Hu & BM hep-ph/9611292

Schenke et al. hep-ph/0603029

Turbulent color fields

M. Strickland, hep-ph/0511212

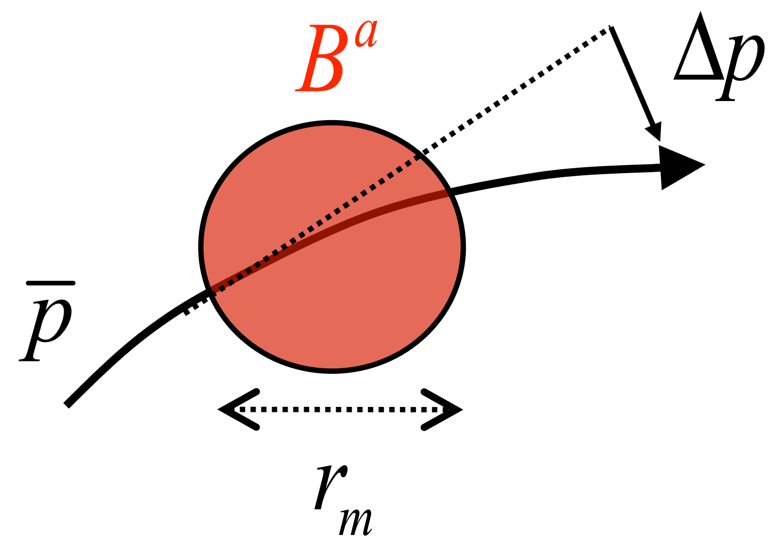


Extended domains of coherent color field can create "anomalous" contributions to transport coefficients and accelerate equilibration (as in EM plasmas).

QGP viscosity – anomalous

Classical expression for shear viscosity:

$$\eta \approx \tfrac{1}{3} n \bar{p} \lambda_f$$



Momentum change in one coherent domain:

$$\Delta p \approx g Q^a B^a r_m$$

Anomalous (collisionless) mean free path:

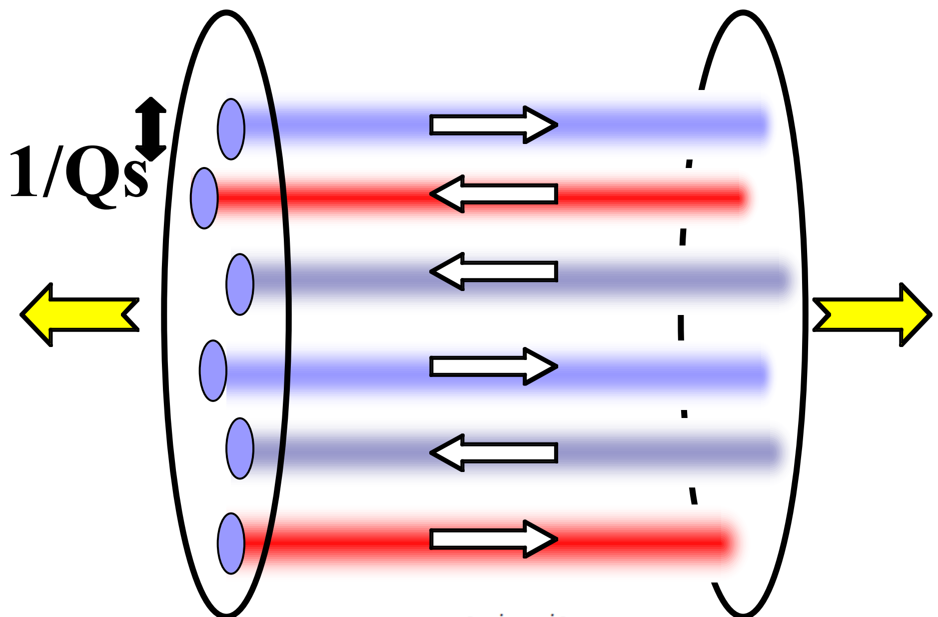
$$\lambda_f^{(A)} \approx r_m \left\langle \frac{\bar{p}^2}{(\Delta p)^2} \right\rangle \approx \frac{\bar{p}^2}{g^2 Q^2 \langle B^2 \rangle r_m}$$

Anomalous viscosity due to random color fields:

$$\eta_A \approx \frac{n \bar{p}^3}{3 g^2 Q^2 \langle B^2 \rangle r_m} \approx \frac{\frac{9}{4} s T^3}{g^2 Q^2 \langle B^2 \rangle r_m}$$

Asakawa, Bass & BM,
PRL 96: 252301 (2006)
PTP. 116: 725 (2006)

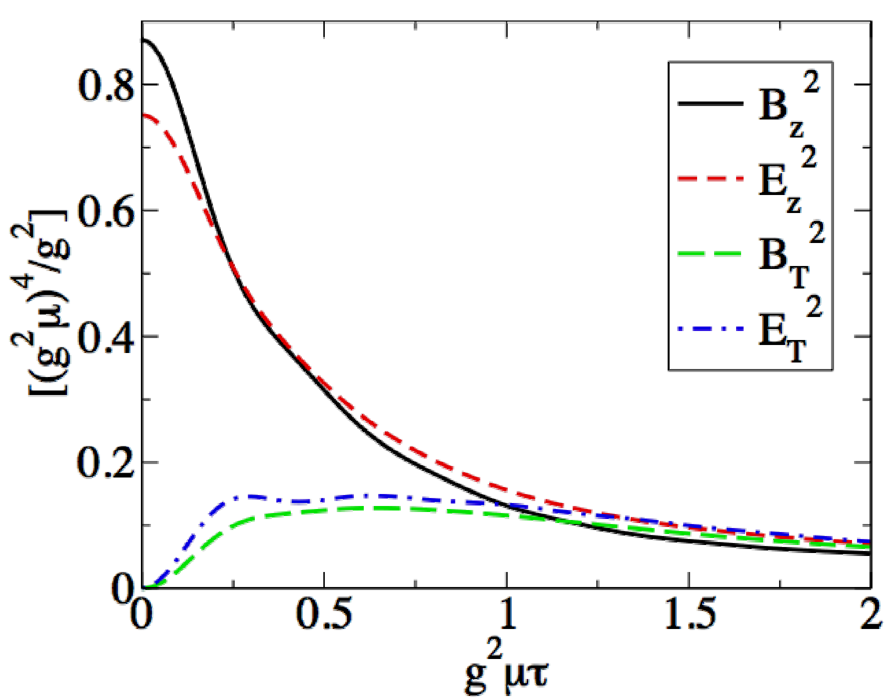
Glasma instabilities

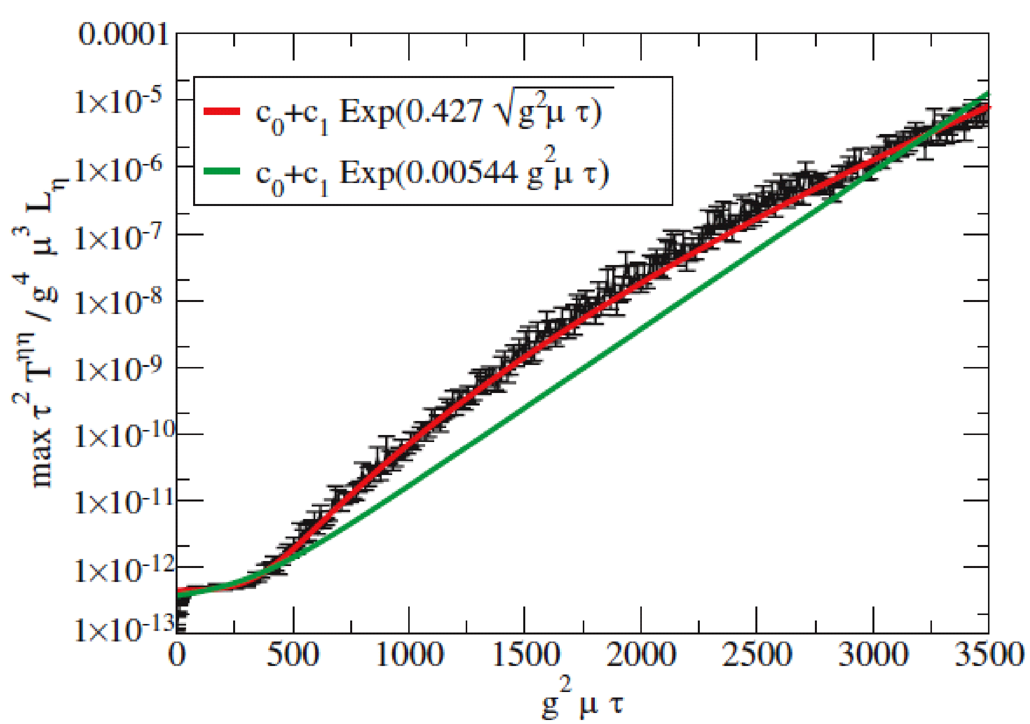


$E^z = ig[\alpha_1^i, \alpha_2^i]$
$B^z = ig\epsilon^{ij}[\alpha_1^i, \alpha_2^j].$

Nielsen-Olesen instability of longitudinal color-magnetic field (Itakura & Fujii, Iwazaki)

$$\frac{\partial^2 \phi}{\partial \tau^2} + \frac{1}{\tau}\frac{\partial \phi}{\partial \tau} + \left(\frac{(k_z - gA_\eta)^2}{\tau^2} - gB_z \right)\phi = 0$$





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