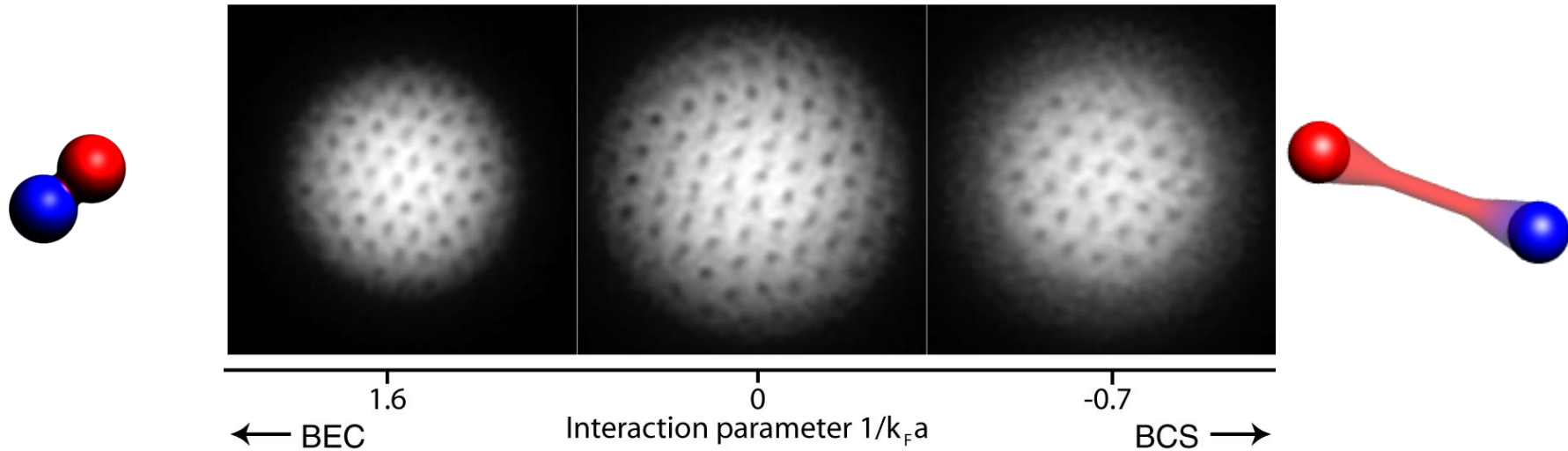


Strongly Interacting Fermi Gases

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Massachusetts Institute of Technology
Center for Ultracold Atoms at MIT and Harvard



Biblio- (or Webbo-)graphy

Varena Notes:

on Fermi Gases:

Ketterle, Zwierlein, *Making, Probing and Understanding Ultracold Fermi Gases*

<http://arxiv.org/abs/0801.2500>

on BEC:

Ketterle, Durfee, Stamper-Kurn, *Making, Probing and Understanding BEC*

http://cua.mit.edu/ketterle_group/Projects_1999/Pubs_99/kett99varena.pdf

Stefano Giorgini, Lev P. Pitaevskii, Sandro Stringari

The theory of Fermi gases

<http://arxiv.org/abs/0706.3360>

Immanuel Bloch, Jean Dalibard, Wilhelm Zwerger

Many-Body Physics with Ultracold Gases:

<http://arxiv.org/abs/0704.3011>

Lecture Notes “The BEC-BCS crossover and the Unitary Fermi Gas”

Edited by W. Zwerger, Springer, 2012

Indistinguishable Particles

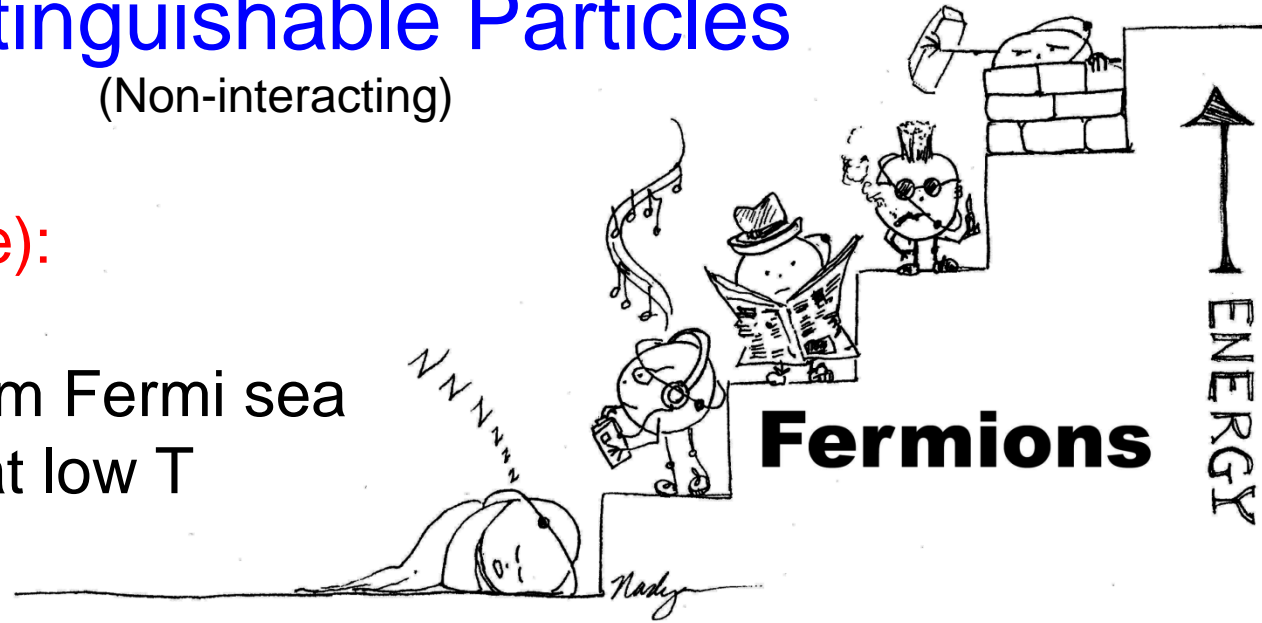
(Non-interacting)

Fermions (unsociable):

Half-Integer Spin

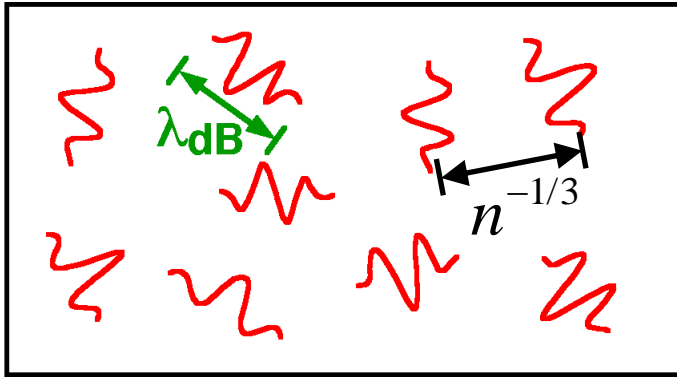
Pauli blocking \rightarrow Form Fermi sea

No phase transition at low T



Degenerate gases

de Broglie wavelength ~ Interparticle spacing



$$\lambda_{dB} \approx n^{-1/3}$$

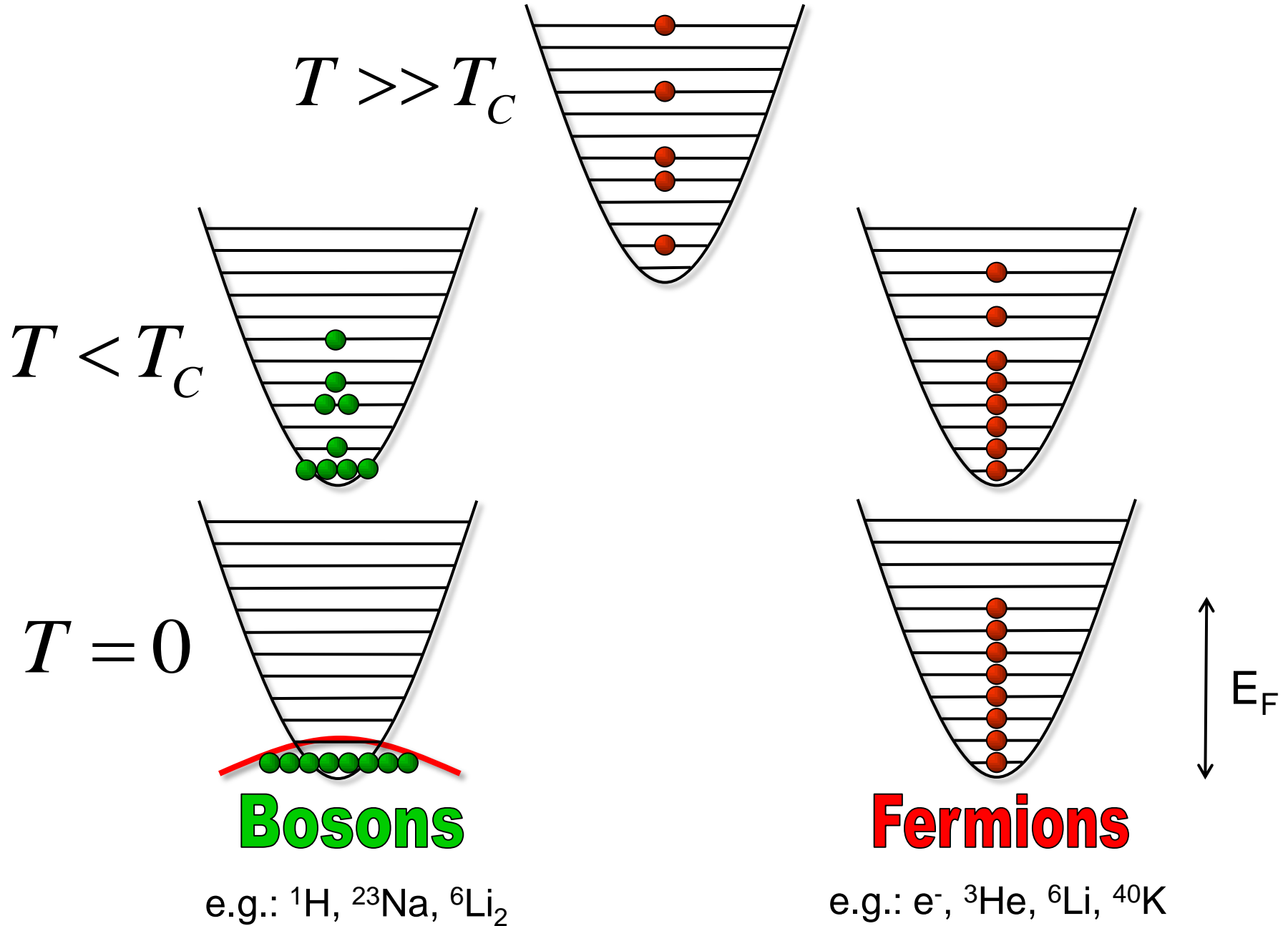
Want lifetime $> 1\text{s}$ $\Rightarrow n < 10^{15} \text{ cm}^{-3}$ *Ultradilute*

$$T_F \approx \frac{\hbar^2}{k_B m} n^{2/3} \approx 1 \mu\text{K} \quad \textit{Ultracold}$$

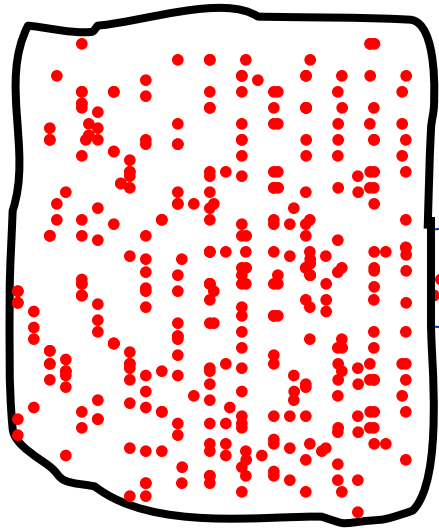
Good news: **Bosons** condense at

$$T_C \approx T_F$$

Bosons vs Fermions

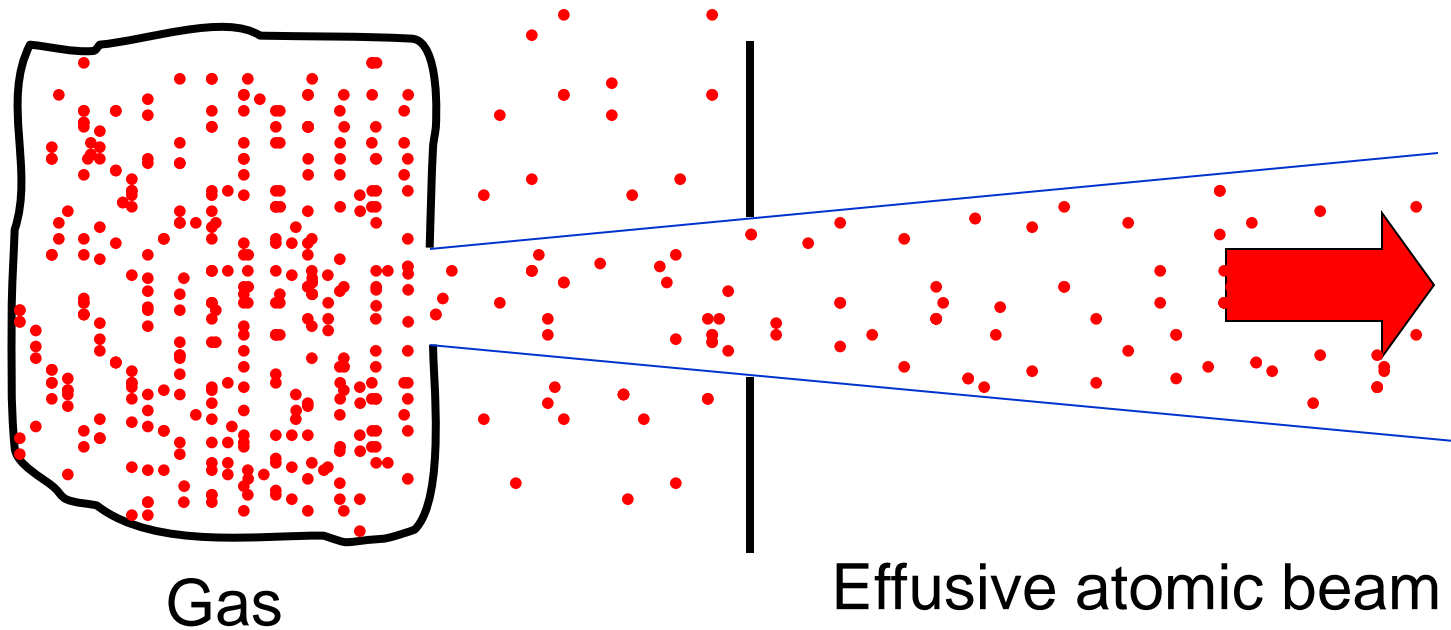


How to measure temperature?

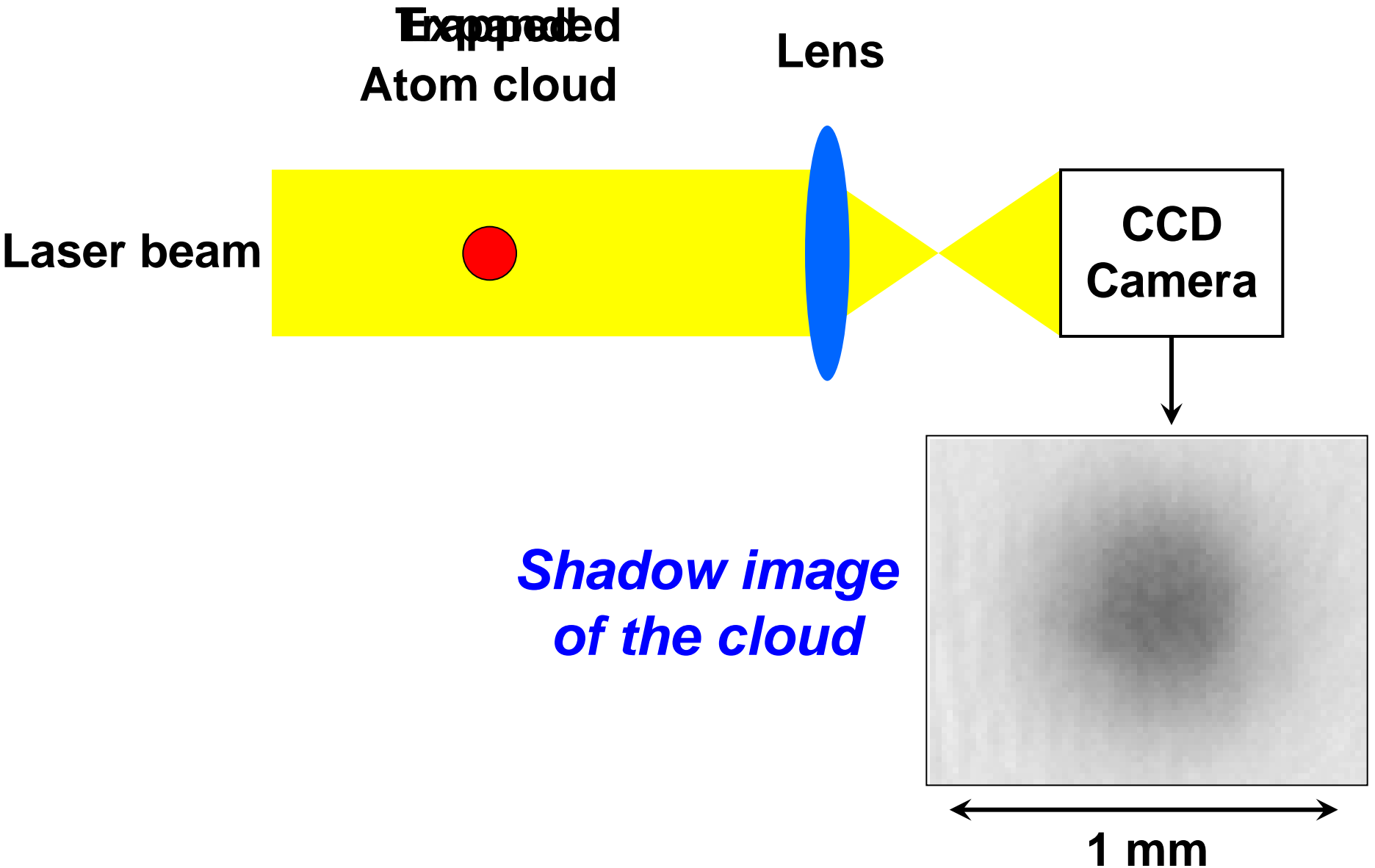


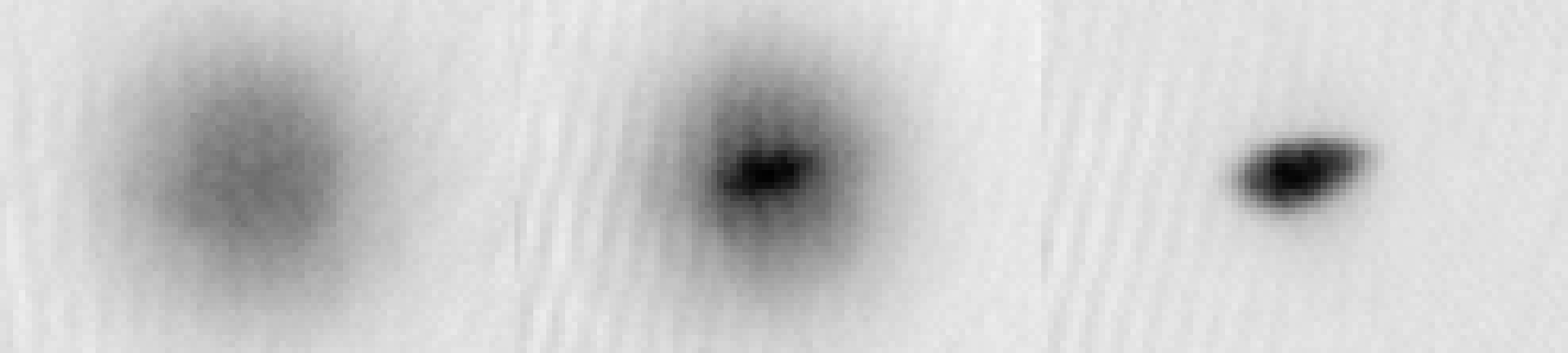
Gas

How to measure temperature?

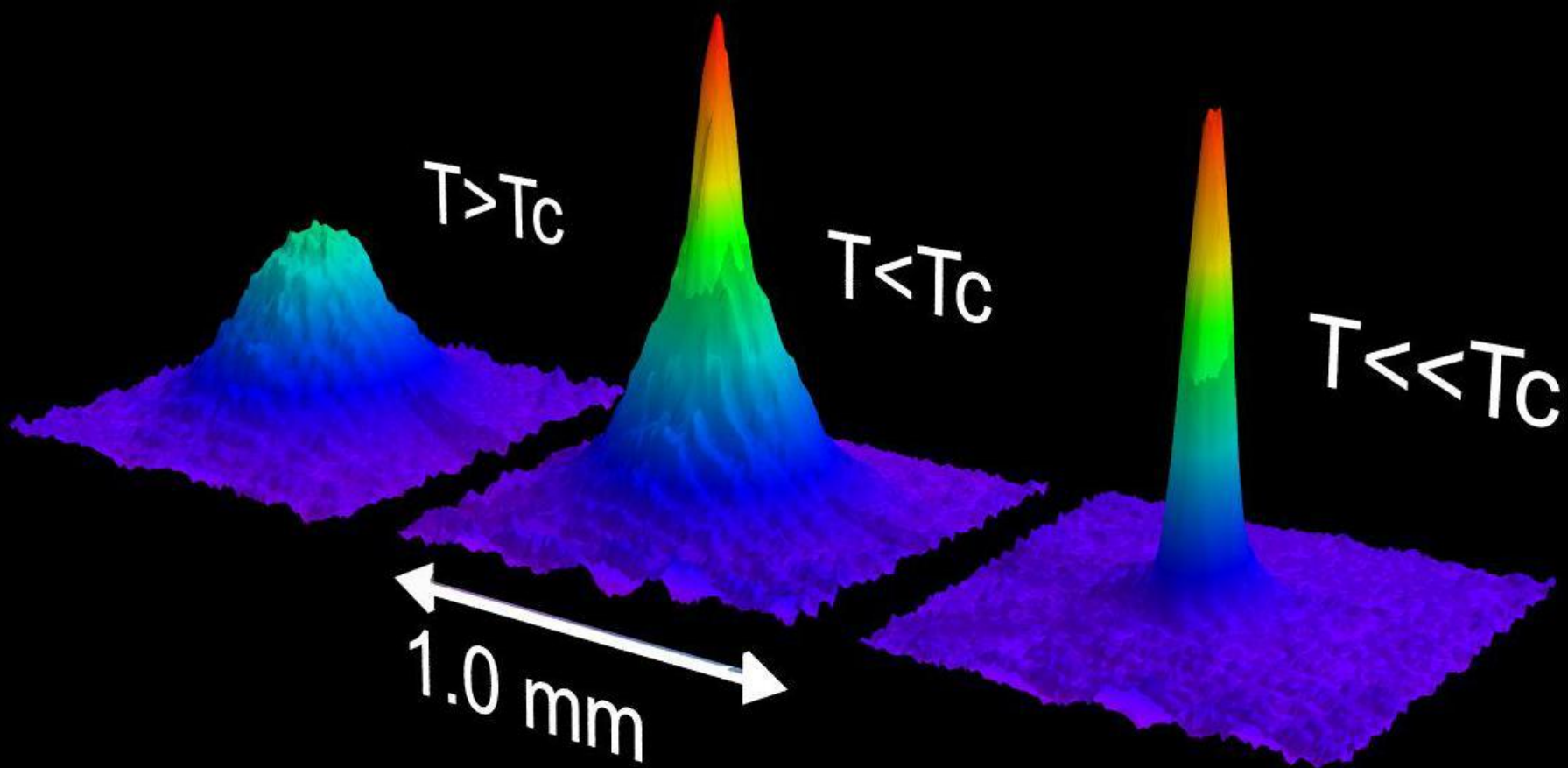


Observation of the atom cloud





BEC @ MIT, 1995 (Sodium)

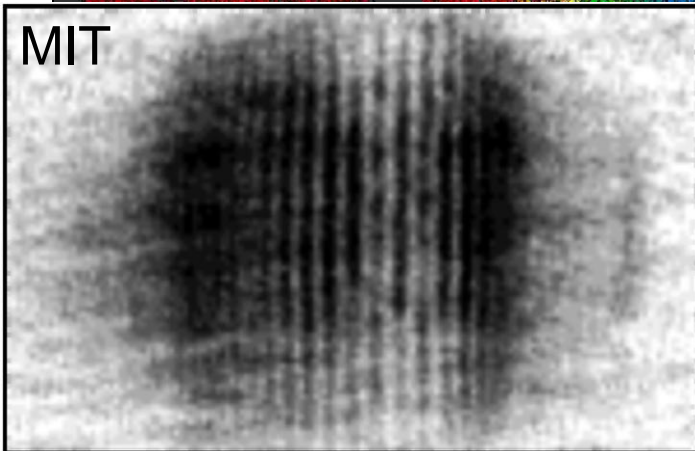
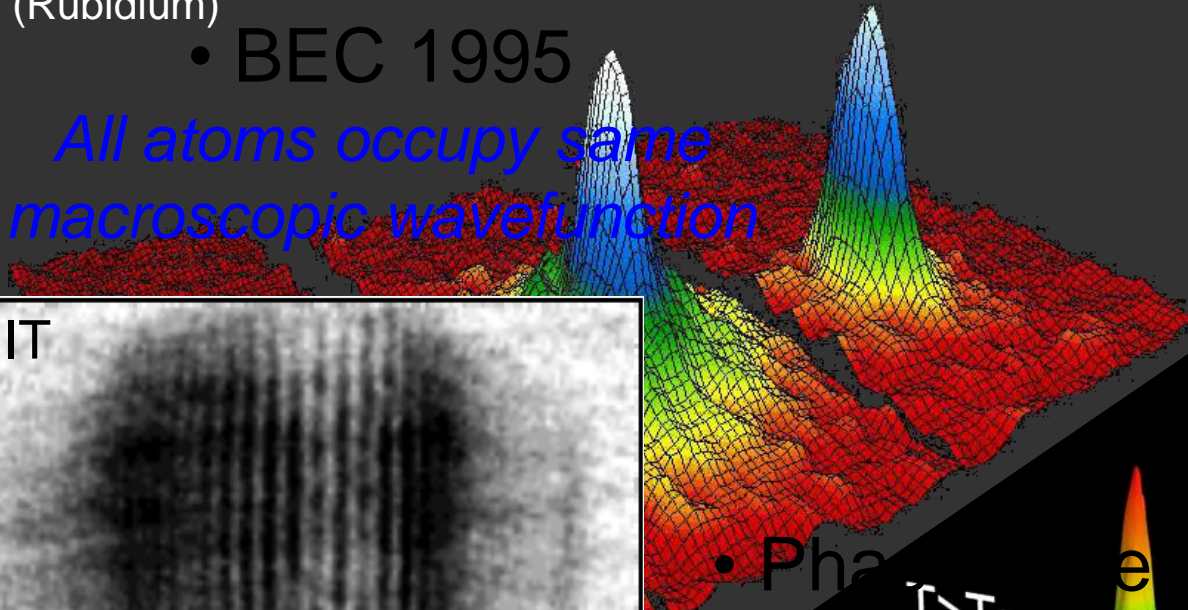


Superfluidity in Bosonic Gases

BEC @ JILA, Juni '95
(Rubidium)

- BEC 1995

*All atoms occupy same
macroscopic wavefunction*



• Phase

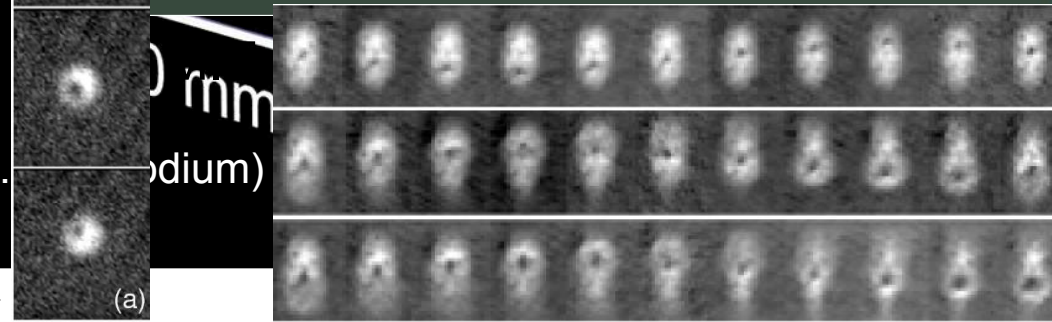


- Superfluid

*Frictionless flow,
quantized vorticity*

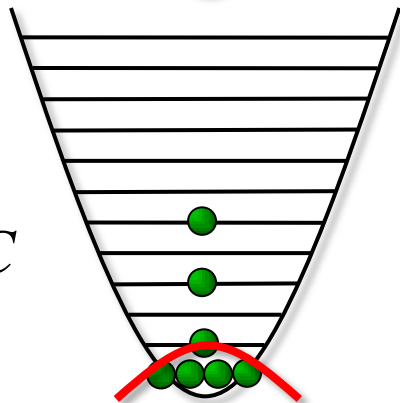
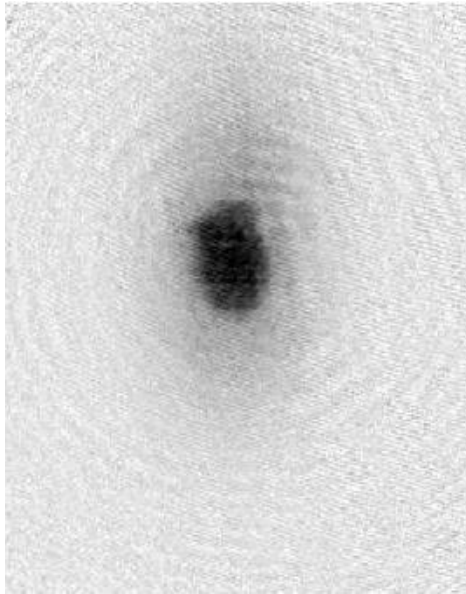
BEC @ MIT, Sept.

JILA



Bosons vs Fermions

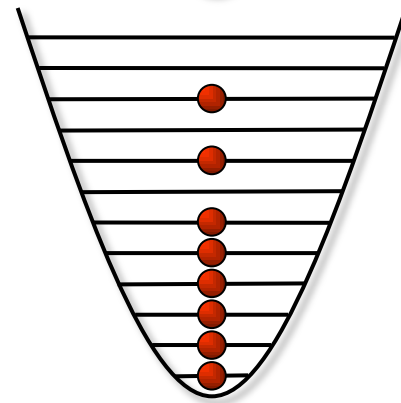
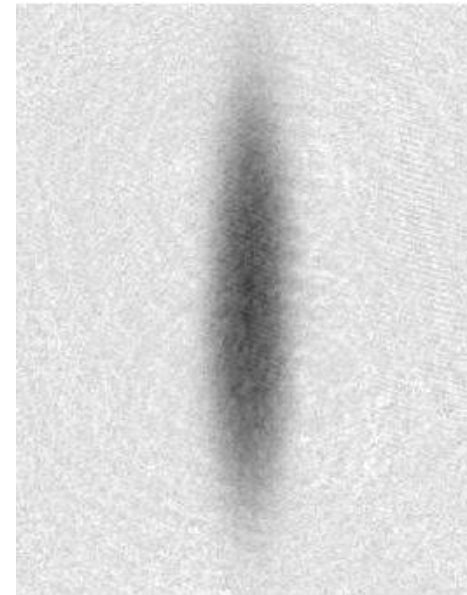
^{23}Na



Bosons

e.g.: ^1H , ^{23}Na , $^6\text{Li}_2$

^6Li



Fermions

e.g.: e^- , ^3He , ^6Li , ^{40}K

$T < T_c$

E_F

Non-interacting Fermi gas

- Fermi-Dirac Statistics:

$$f(\epsilon) = \frac{1}{e^{\beta(\epsilon - \mu)} + 1}$$

At $T = 0$:

$$f(\epsilon) \rightarrow \theta(E_F - \epsilon)$$

- Fermi gas in a box:

$$\epsilon = \frac{\hbar^2 k^2}{2m}$$

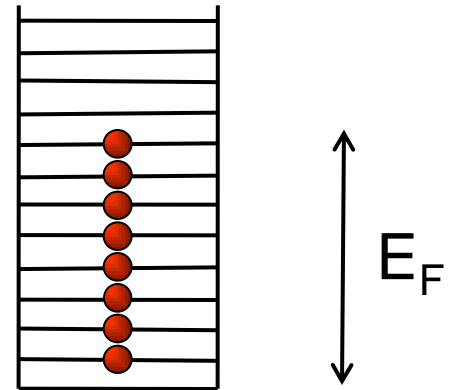
$$N = \int d^3x \int \frac{d^3k}{(2\pi)^3} \theta\left(E_F - \frac{\hbar^2 k^2}{2m}\right)$$

$$n = \frac{1}{(2\pi)^3} \text{Volume of sphere in } k\text{-space with radius } k_F$$

$$= \frac{1}{6\pi^2} k_F^3$$

$$E_F = \frac{\hbar^2 k_F^2}{2m}$$

$$k_F = (6\pi^2 n)^{-1/3} \sim 1/\text{Interparticle spacing}$$



Non-interacting Fermi gas

- Fermi-Dirac Statistics:

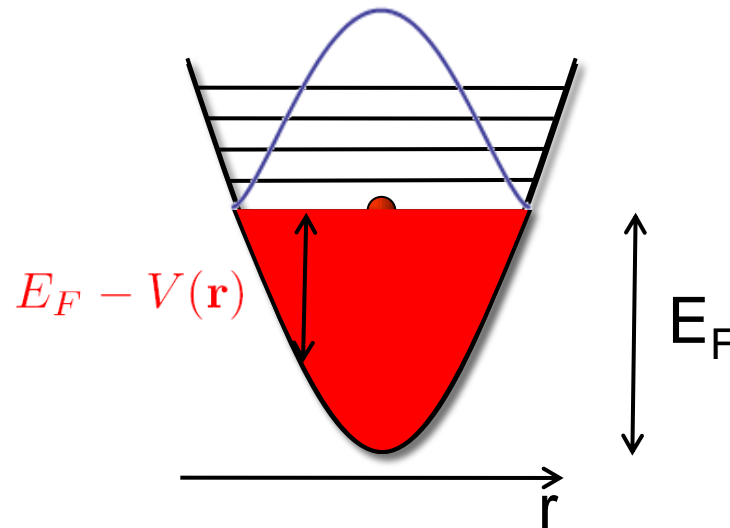
$$f(\epsilon) = \frac{1}{e^{\beta(\epsilon - \mu)} + 1}$$

- Fermi gas in a trap:

$$\epsilon_F(\mathbf{r}) = E_F - V(\mathbf{r})$$

At $T = 0$:

$$f(\epsilon) \rightarrow \theta(E_F - \epsilon)$$

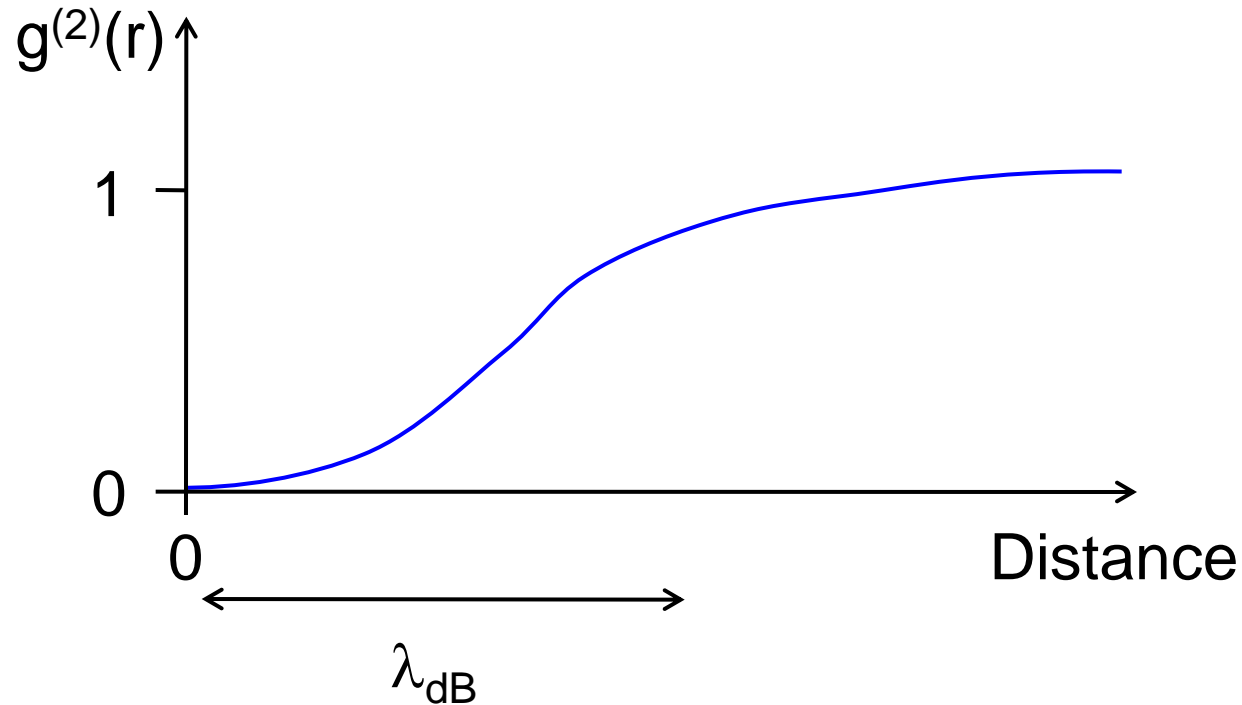


$$n(\mathbf{r}) = \frac{1}{6\pi^2} k_F(\mathbf{r})^3 = \frac{1}{6\pi^2 \hbar^3} (2m(E_F - V(\mathbf{r})))^{3/2}$$

(Local density approximation)

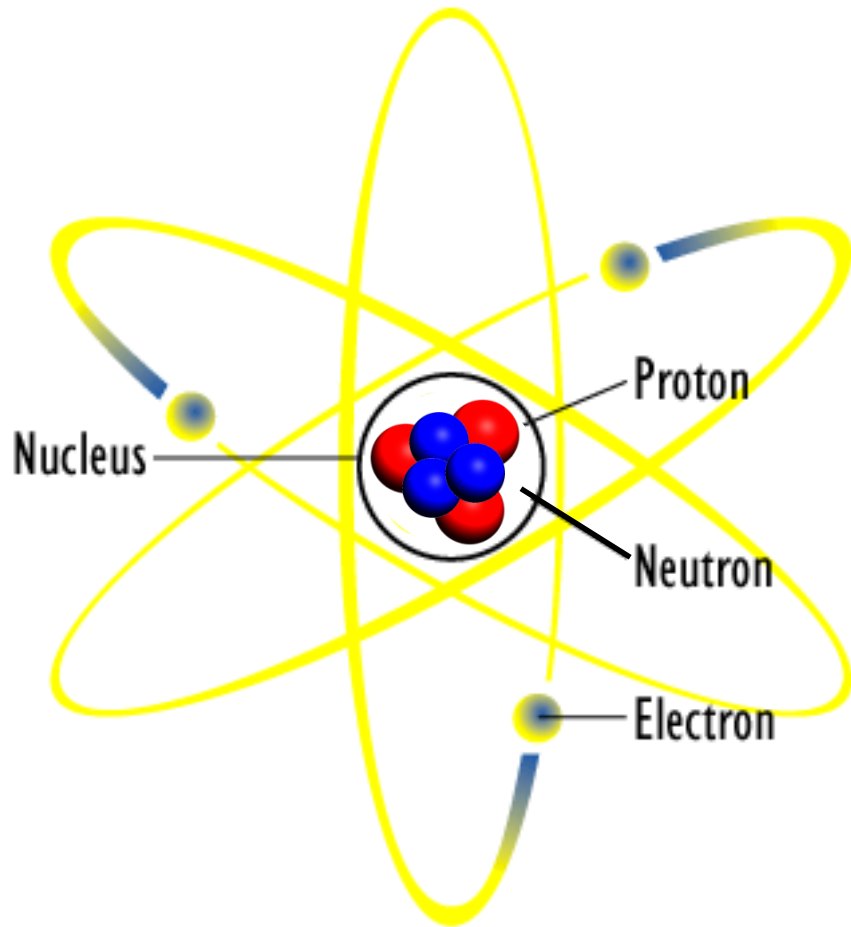
Freezing out of collisions

Pair correlations in a Fermi gas:

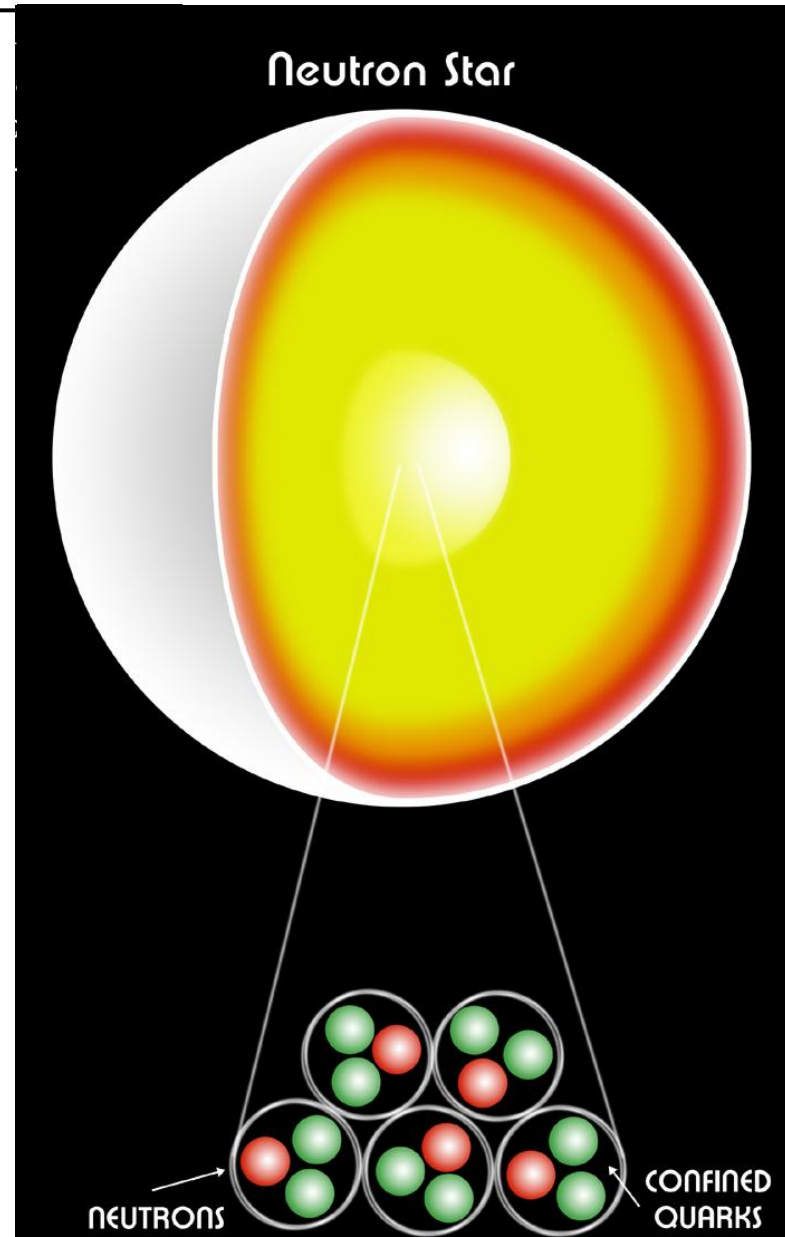


No interactions if range of potential is $< \lambda_{dB}$

Fermions – The Building Blocks of Matter



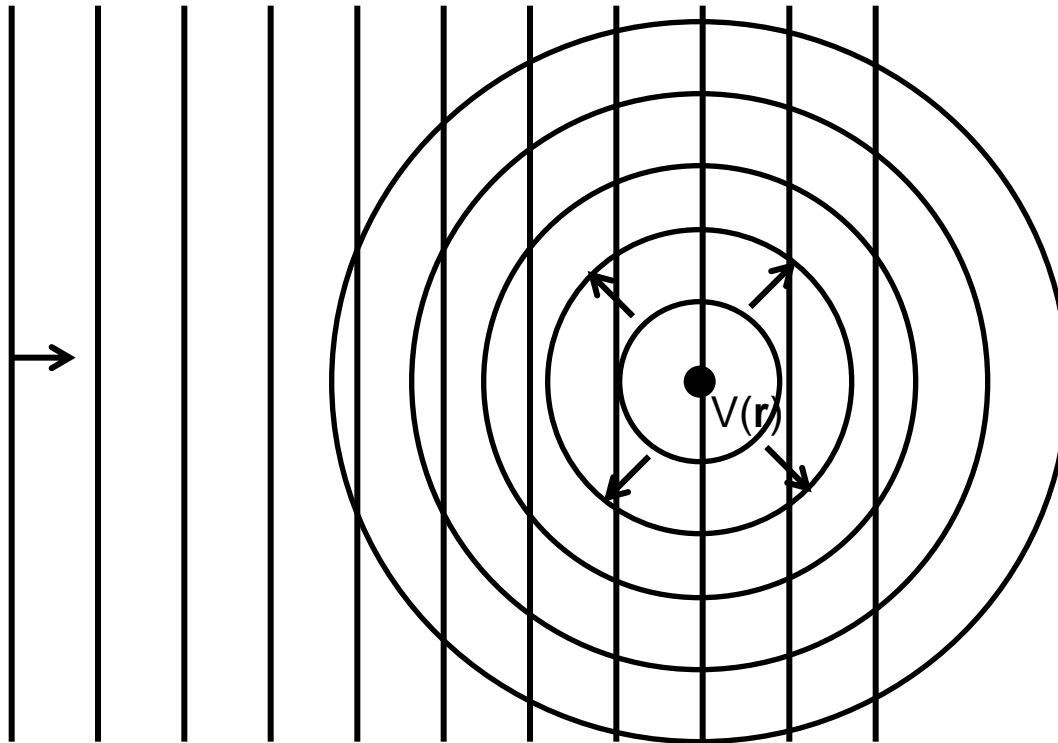
Lithium-6



Scattering Theory

Incoming plane wave

$$e^{ikz - i\omega t}$$



Radially outgoing wave

$$f_k(\vartheta, \varphi) \frac{1}{r} e^{ikr - i\omega t}$$

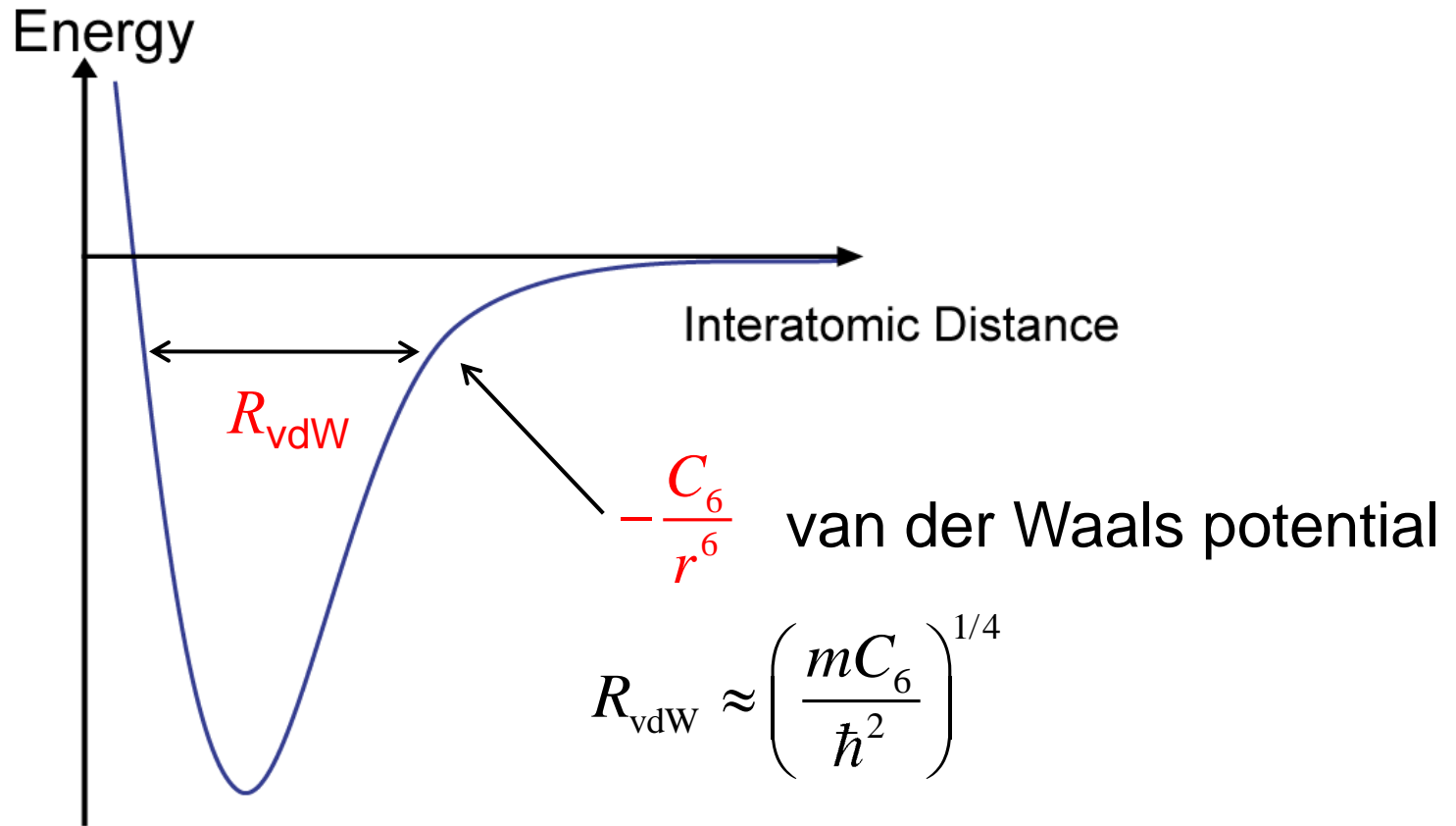
For s-wave:

$$f_k \frac{1}{r} e^{ikr - i\omega t}$$

$$e^{ikz} + f_k \frac{1}{r} e^{ikr} \approx \frac{1}{kr} e^{i\delta(k)} \sin(kr + \delta(k))$$

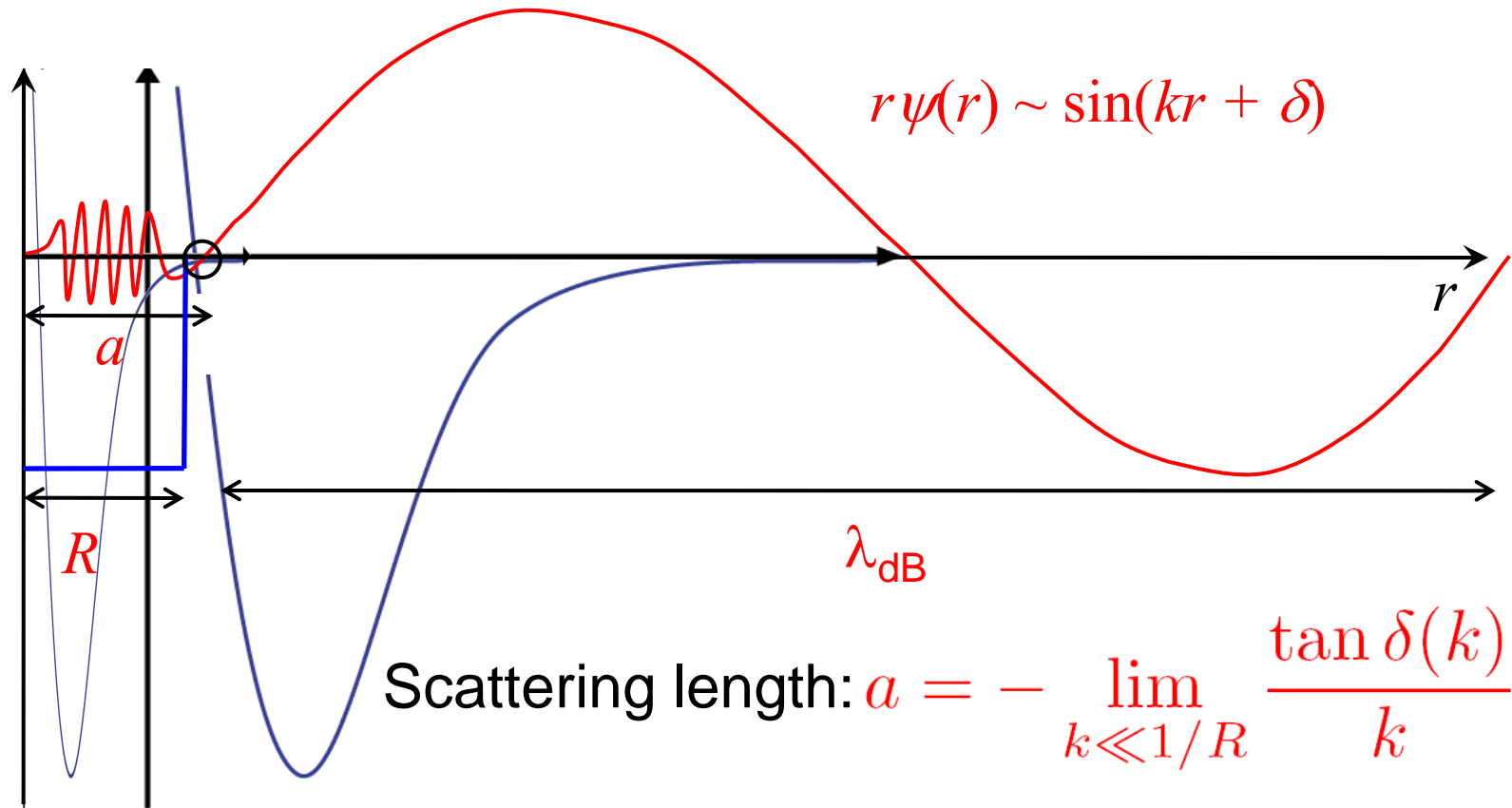
$$f_k = \frac{1}{k} e^{i\delta(k)} \sin(\delta(k))$$

Interatomic interactions



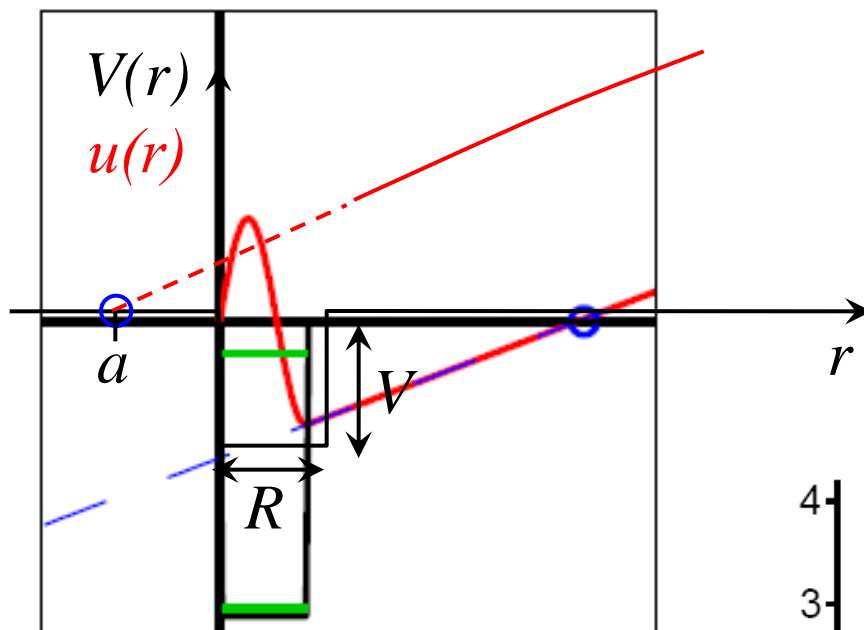
- For Alkali atoms: $R_{\text{vdW}} \sim 50\text{-}200 a_0$
 - Ultracold collisions: $\lambda_{dB} \approx 1 \mu\text{m} \gg R$
- atoms do not probe the details of the potential

Interatomic interactions



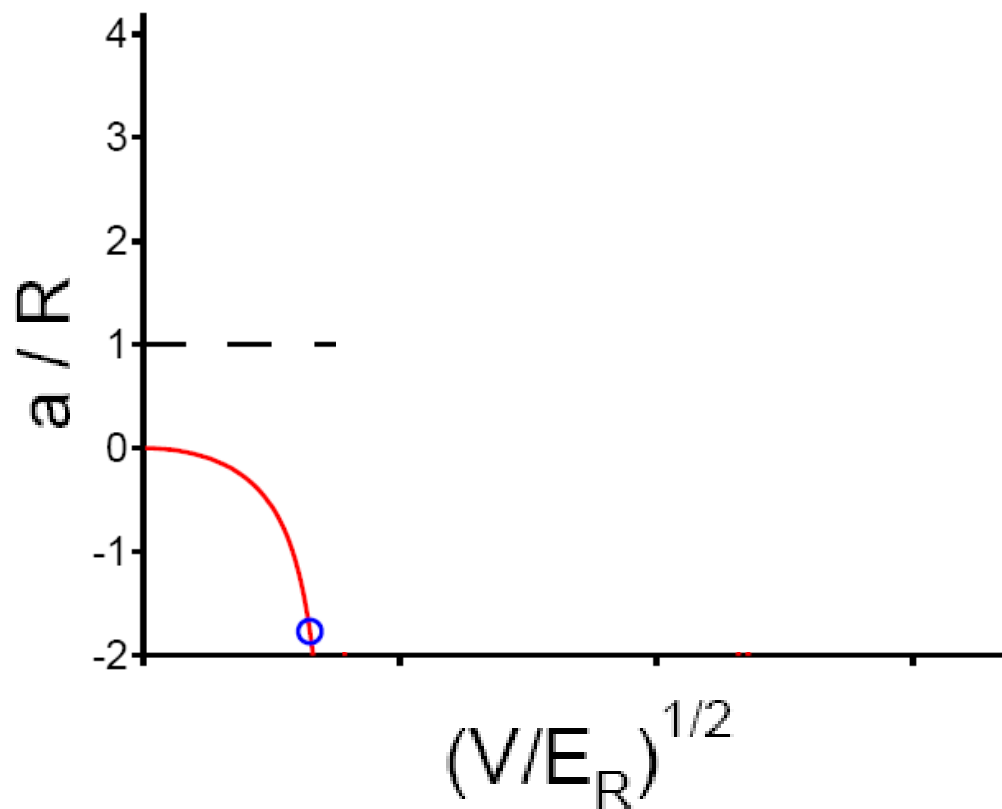
- For Alkali atoms: $R_{\text{vdW}} \sim 50\text{-}200 a_0$
 - Ultracold collisions: $\lambda_{dB} \approx 1 \mu\text{m} \gg R$
- atoms do not probe the details of the potential

Scattering Resonances



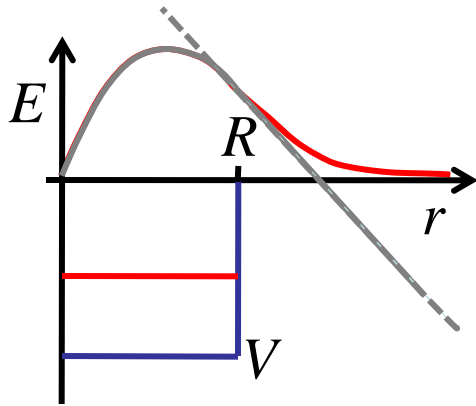
$$u(r) = \sin(k(r - a)) \quad r > R$$

(s-wave) scattering length



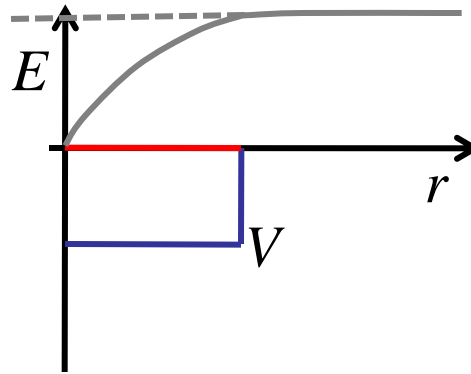
Tunable Interactions

Vary interaction strength between spin up and spin down
Example: tunable square well (with $k_F R \ll 1$):



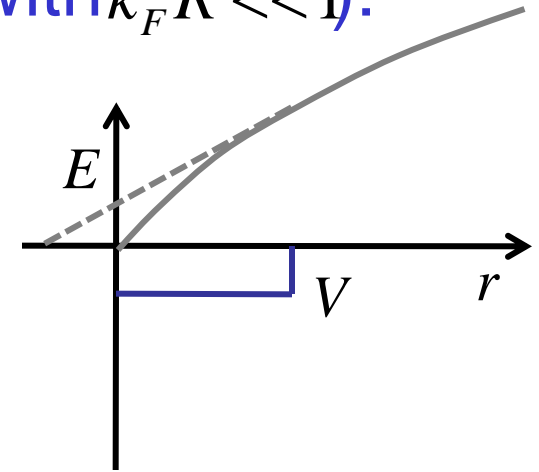
strong attraction
deep bound state

$$a > 0$$



Resonance
bound state appears

$$a \rightarrow \pm\infty$$



weak attraction
no bound state

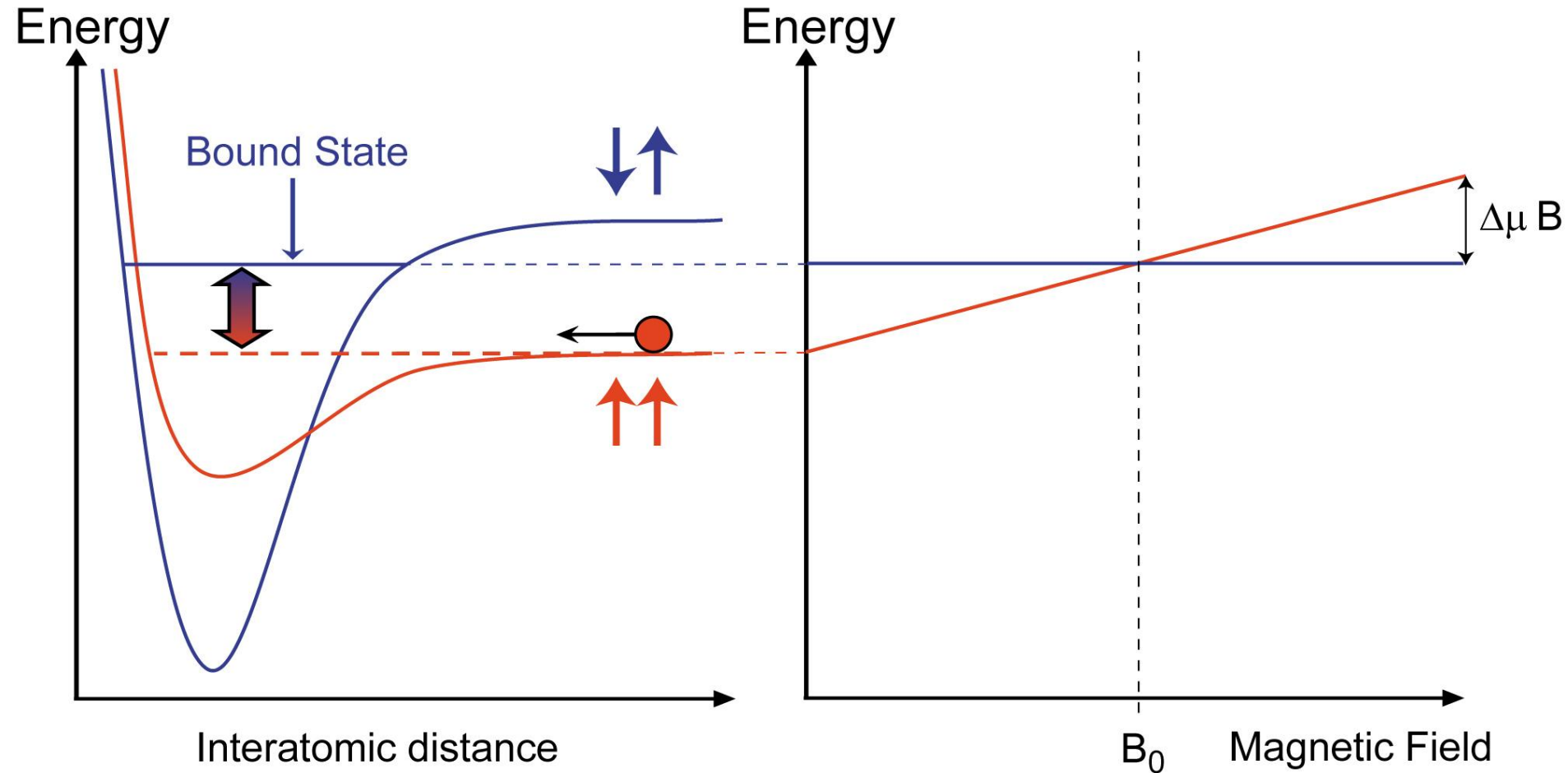
$$a < 0$$

scattering length

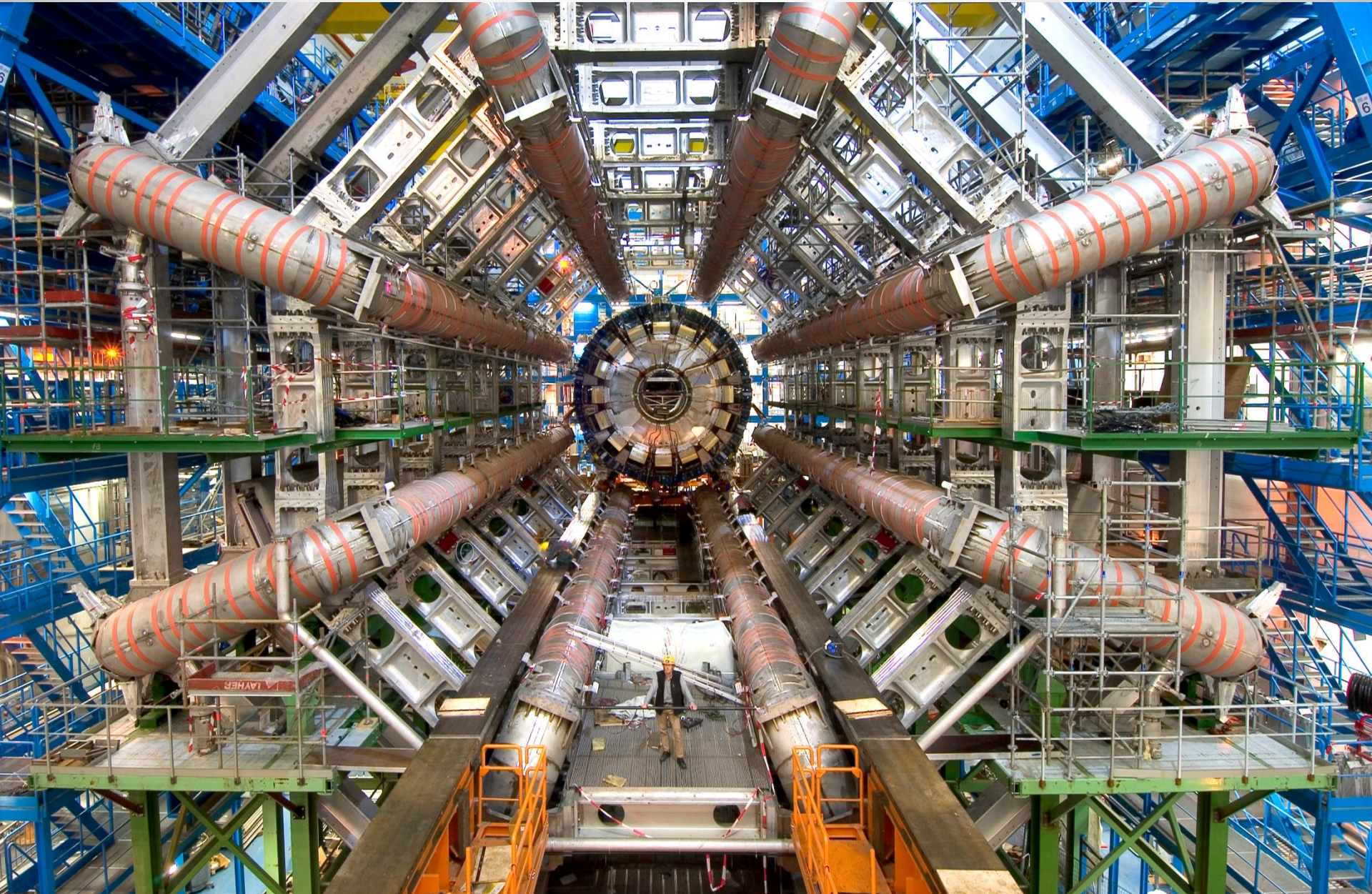
Important for Many-Body
Physics:

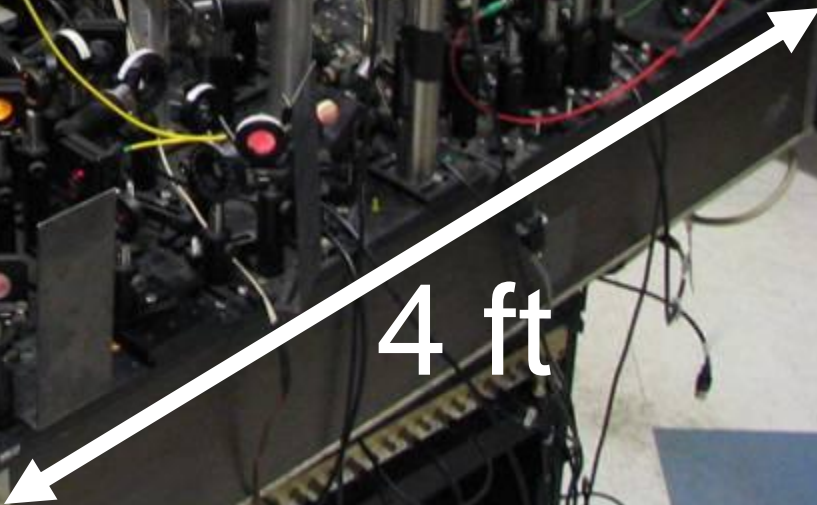
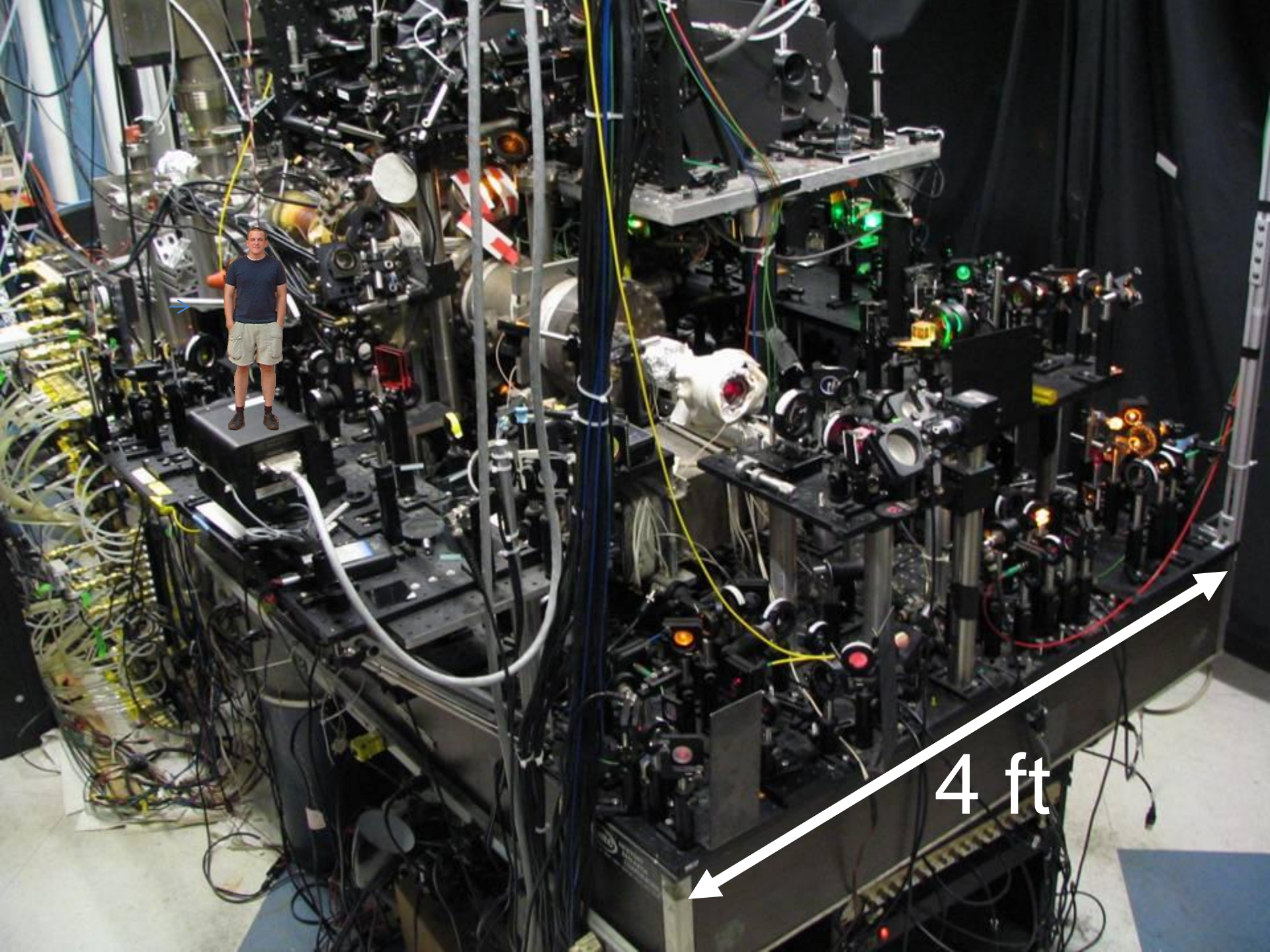
$$\frac{\text{Interparticle distance}}{\text{Scattering length}} = \frac{1}{k_F a}$$

Feshbach resonances: Tuning the interactions



Large Hadron Collider (LHC)





4 ft

Little Fermi Collider (LFC)

A \downarrow Fermi gas collides with a \uparrow cloud
with resonant interactions



Collision at 100 peV

23 orders of magnitude smaller than LHC

Harmonic
Trap

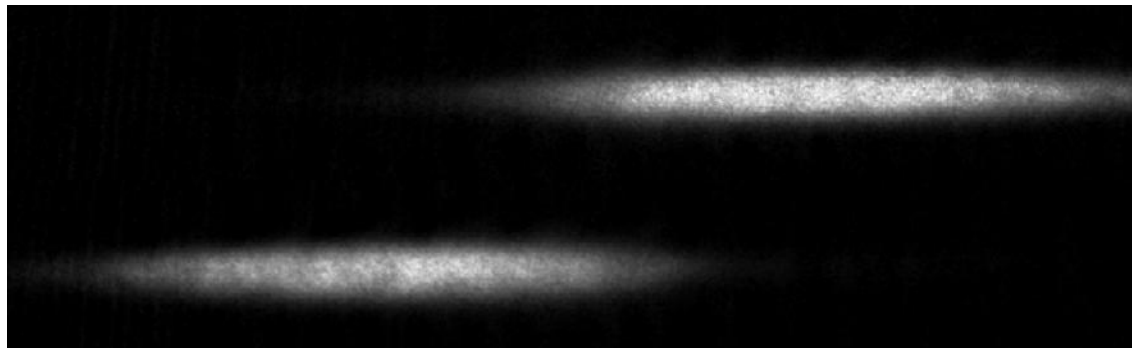
A.T. Sommer, M.J.H. Ku, G. Roati, M.W. Zwierlein, Nature 472, 201 (2011)

First study of spin currents in degenerate Fermi gases: Jin, DeMarco, 2001

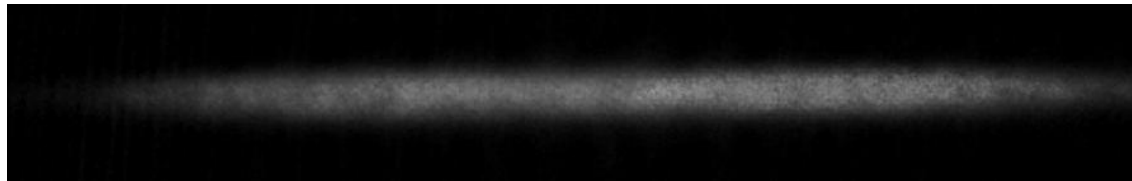
Little Fermi Collider

Preparation: Mix, cool, kick, and rush to resonance

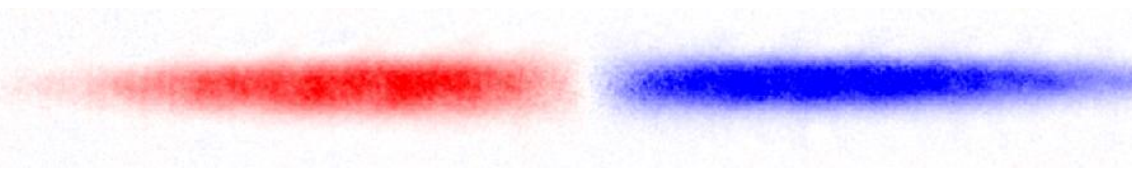
Rapid (10 us) probing of spin up and down



Total OD



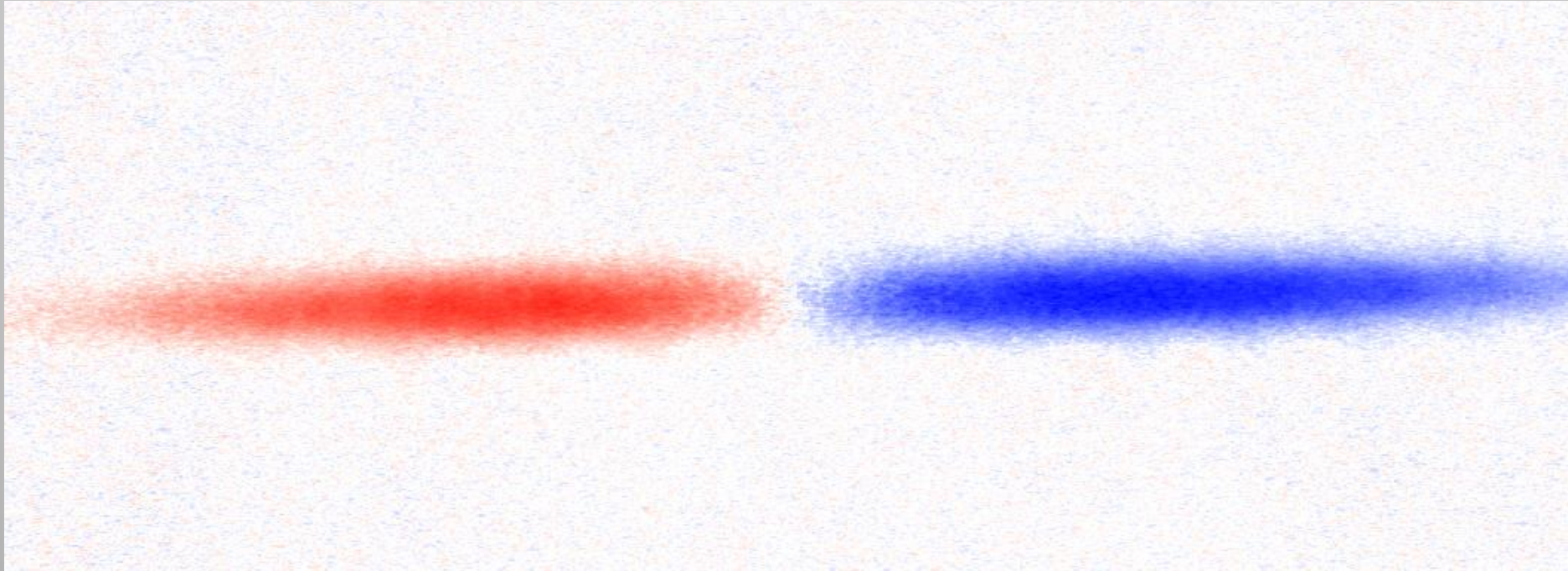
Difference



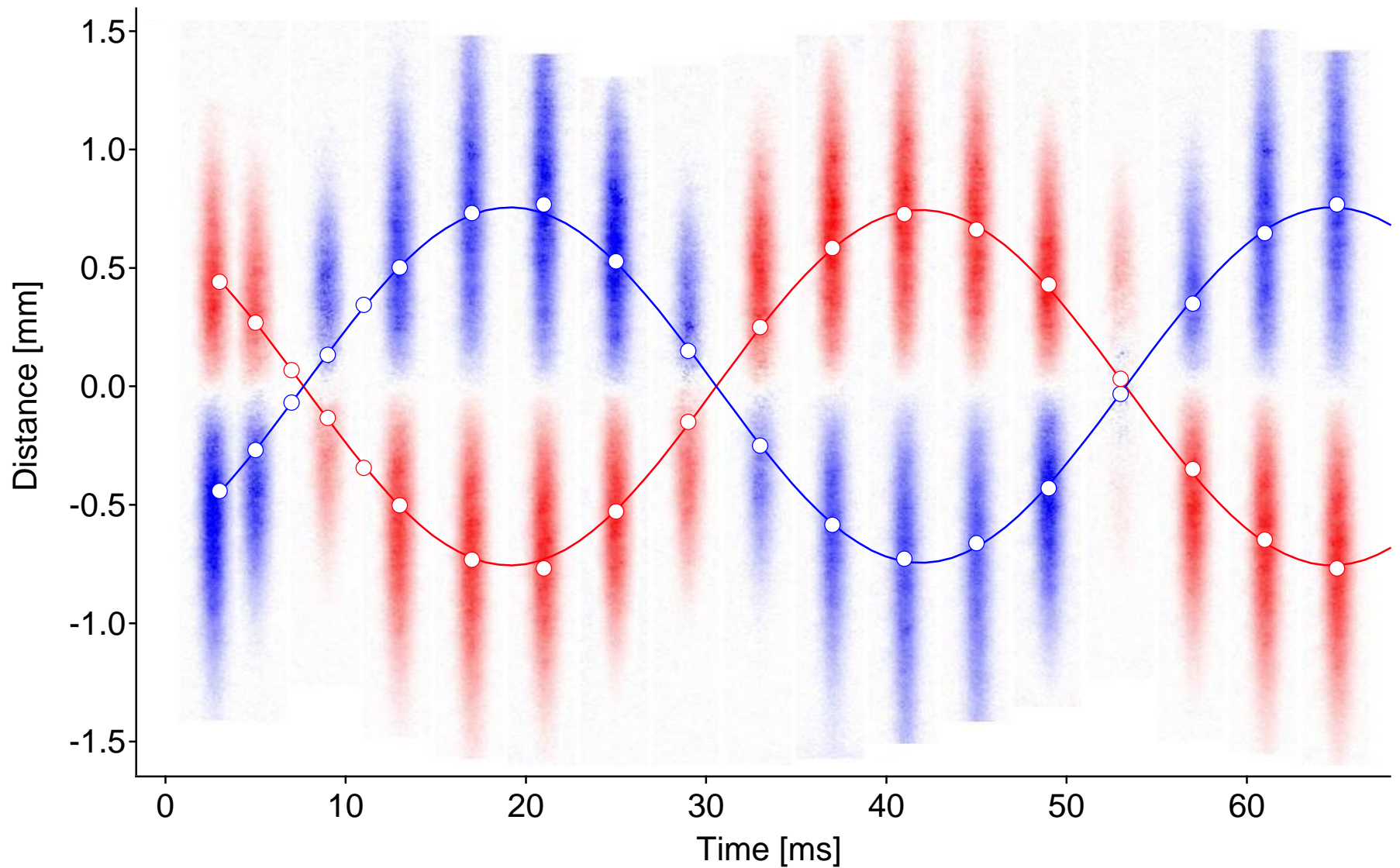
Earlier spin excitation experiments: DeMarco and Jin, PRL 88, 040405 (2002)

Little Fermi Collider (LFC)

Without Interactions

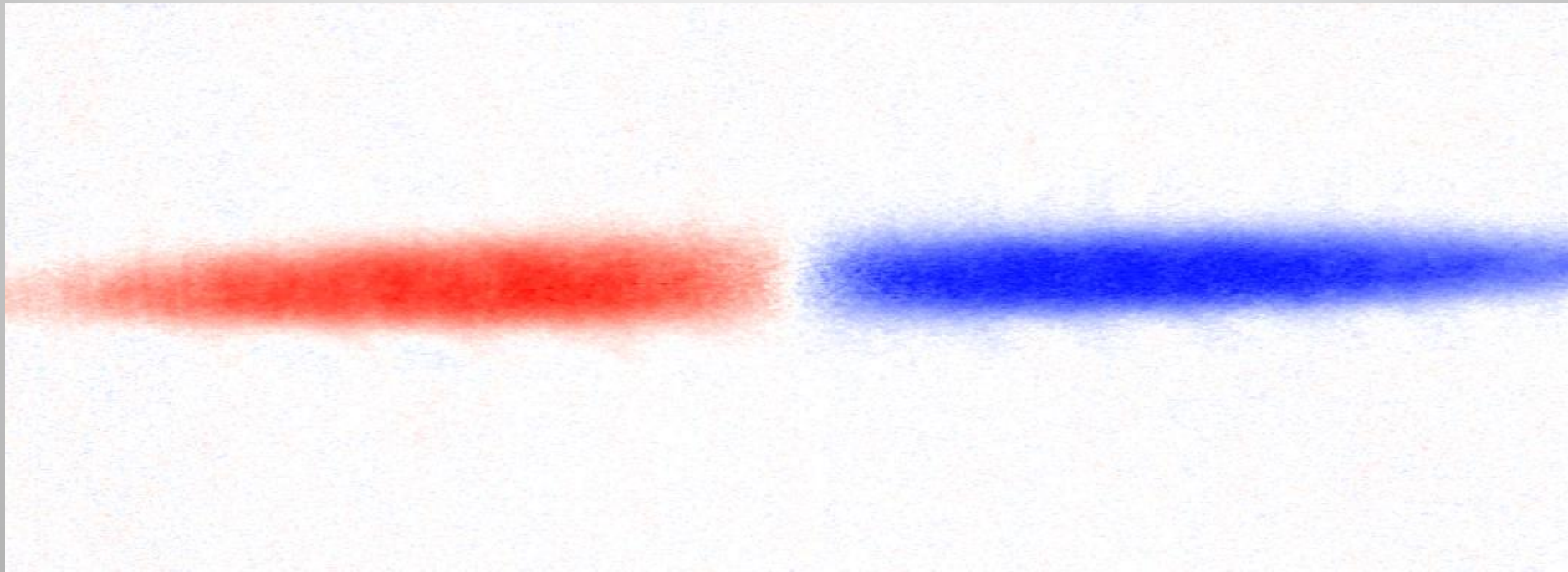


Evolution without interactions



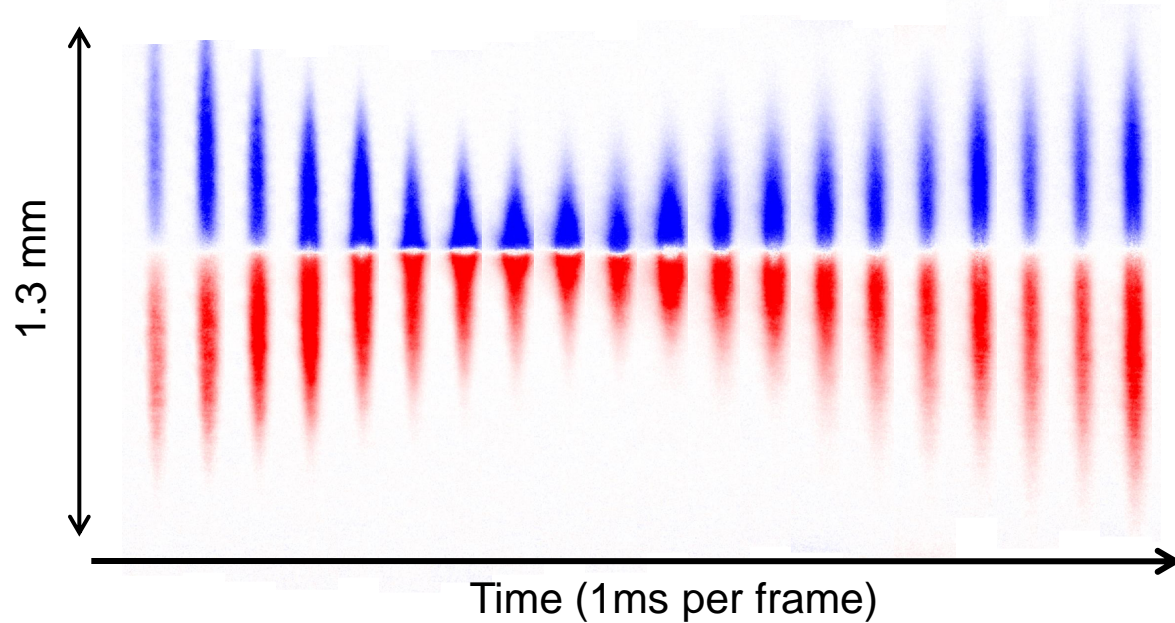
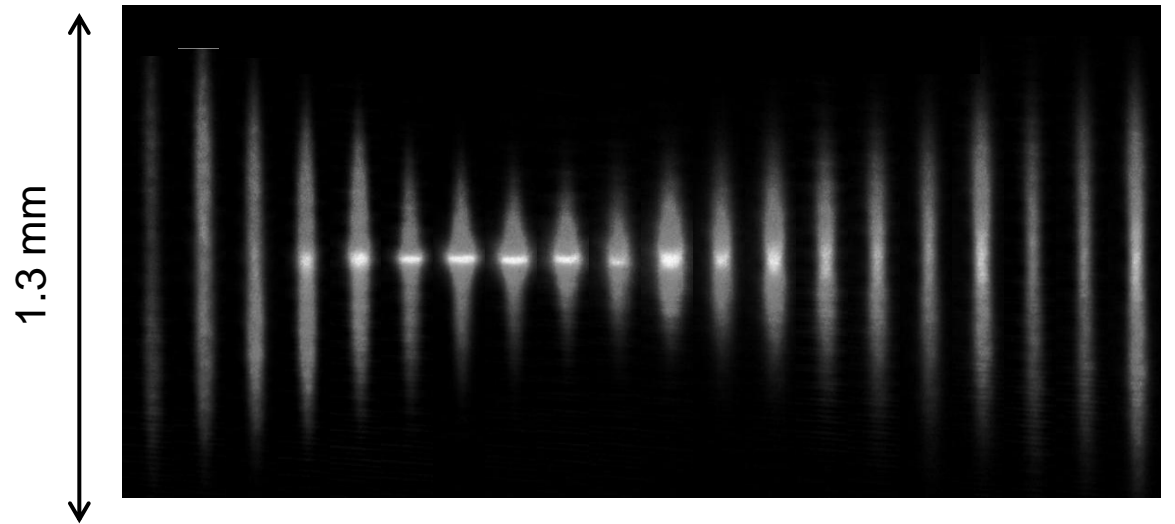
Little Fermi Collider (LFC)

With resonant interactions

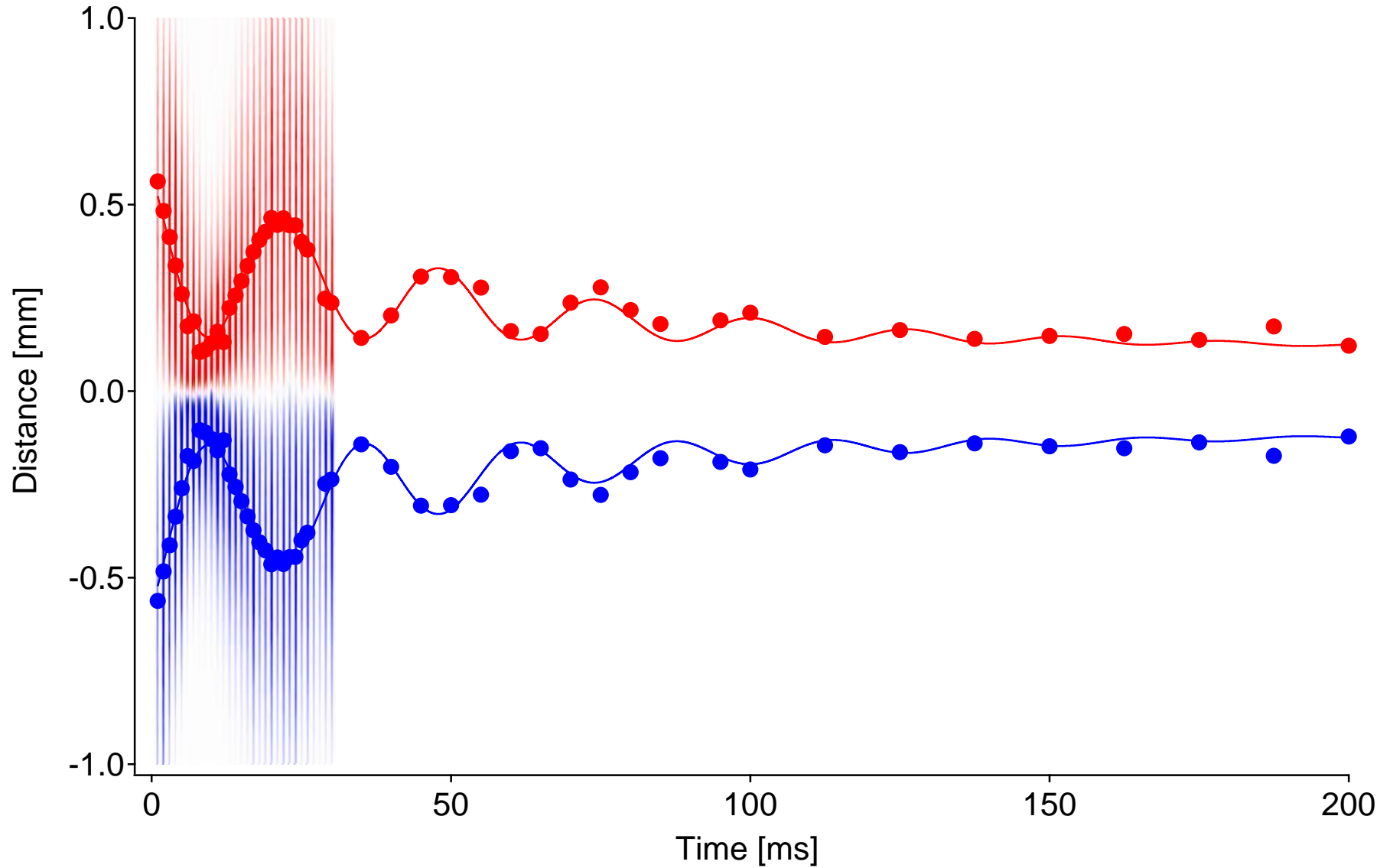


First collision

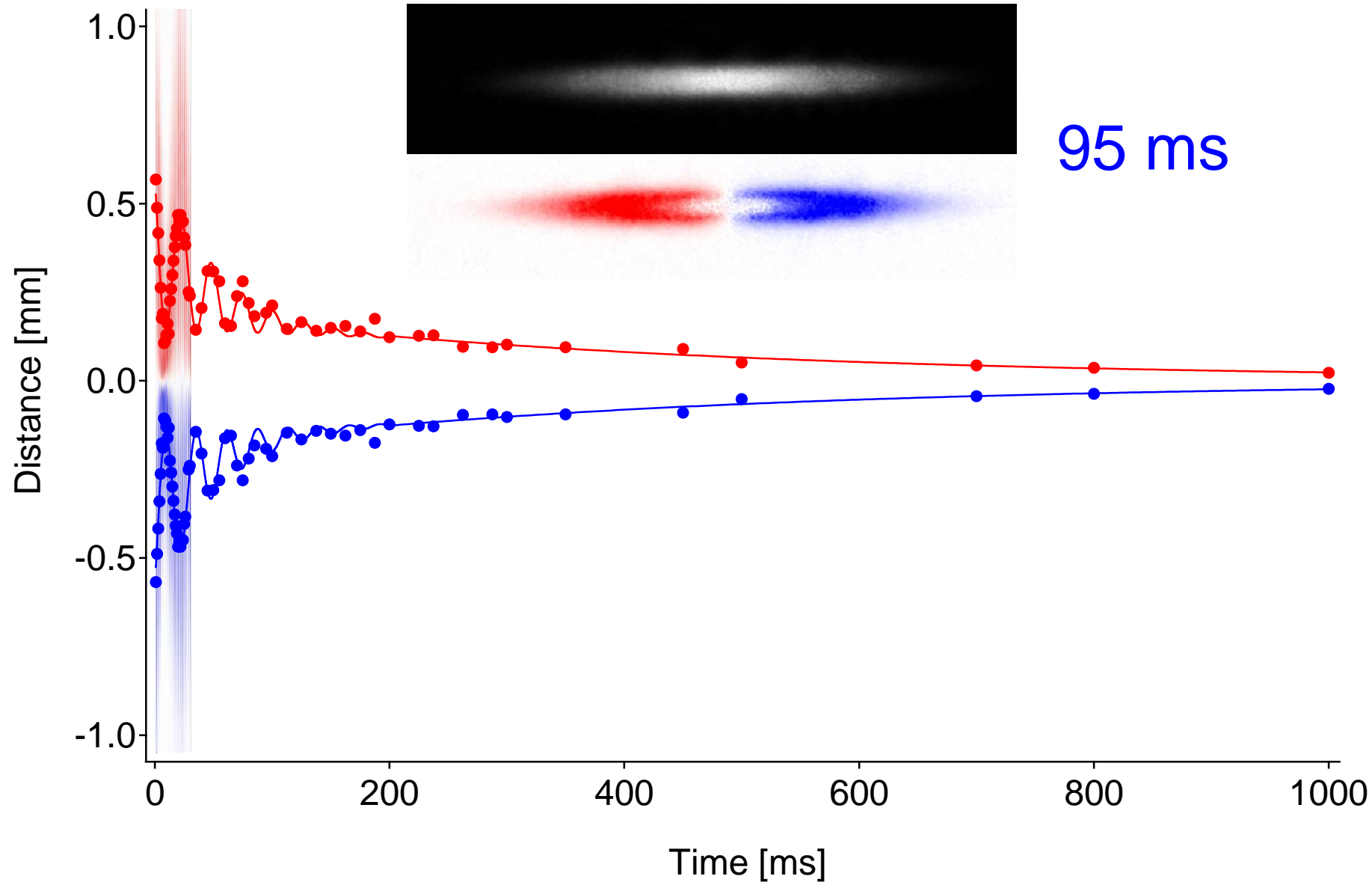
initial 20 ms



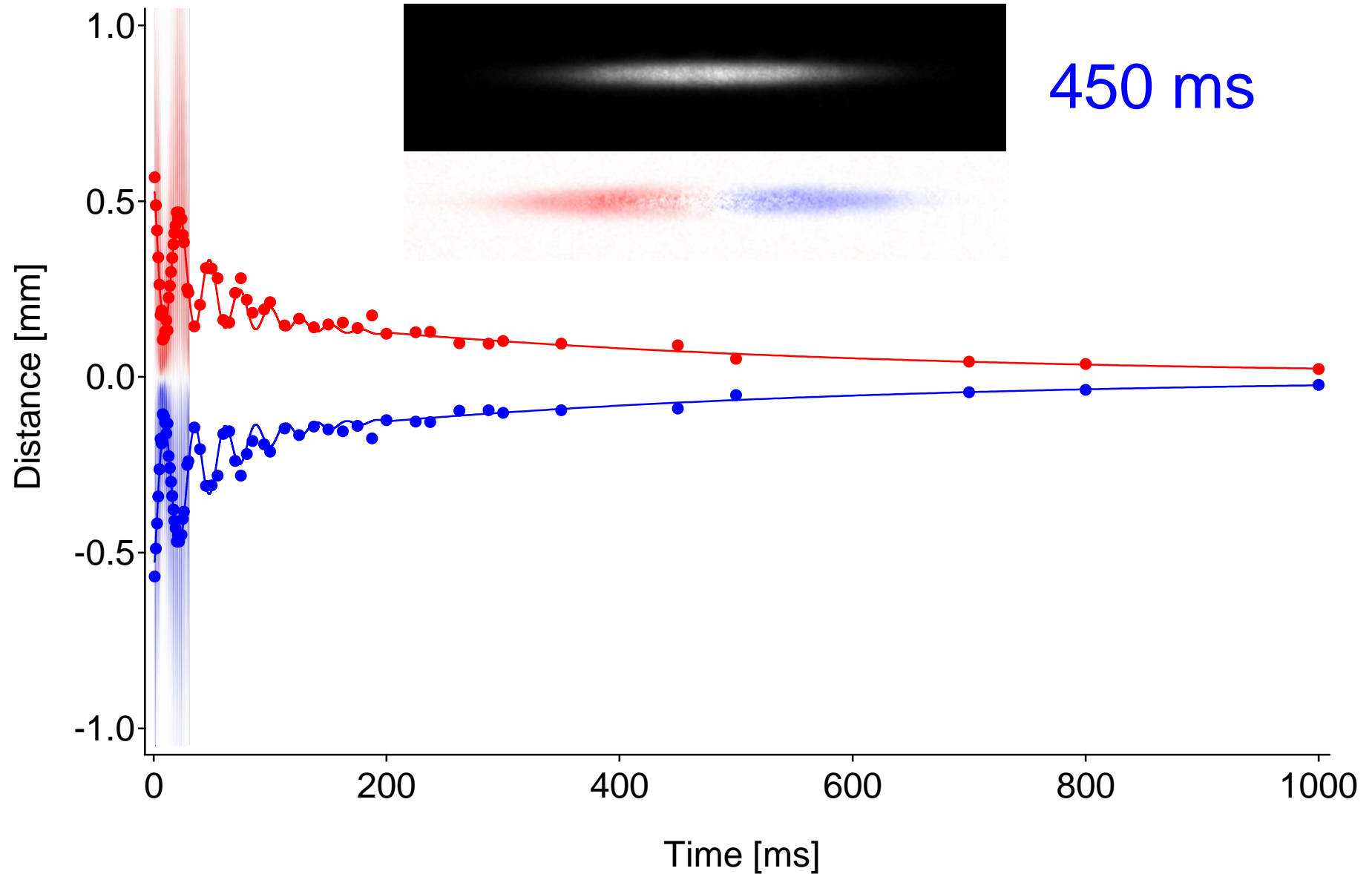
Later times



Much later times



Much later times



Universal Spin Transport

Relaxation of spin current only due to $\downarrow \uparrow$ collisions

Resonant scattering cross section: $\sigma \sim \frac{1}{k_F^2}$

Mean free path: $l = \frac{1}{n\sigma} \sim \frac{1}{k_F} = \text{interparticle spacing}$

“A perfect liquid”

Spin drag coefficient
(\propto Collision rate) $\Gamma_{SD} \sim n\sigma v \sim \frac{\hbar}{m} k_F^2 \sim E_F / \hbar$

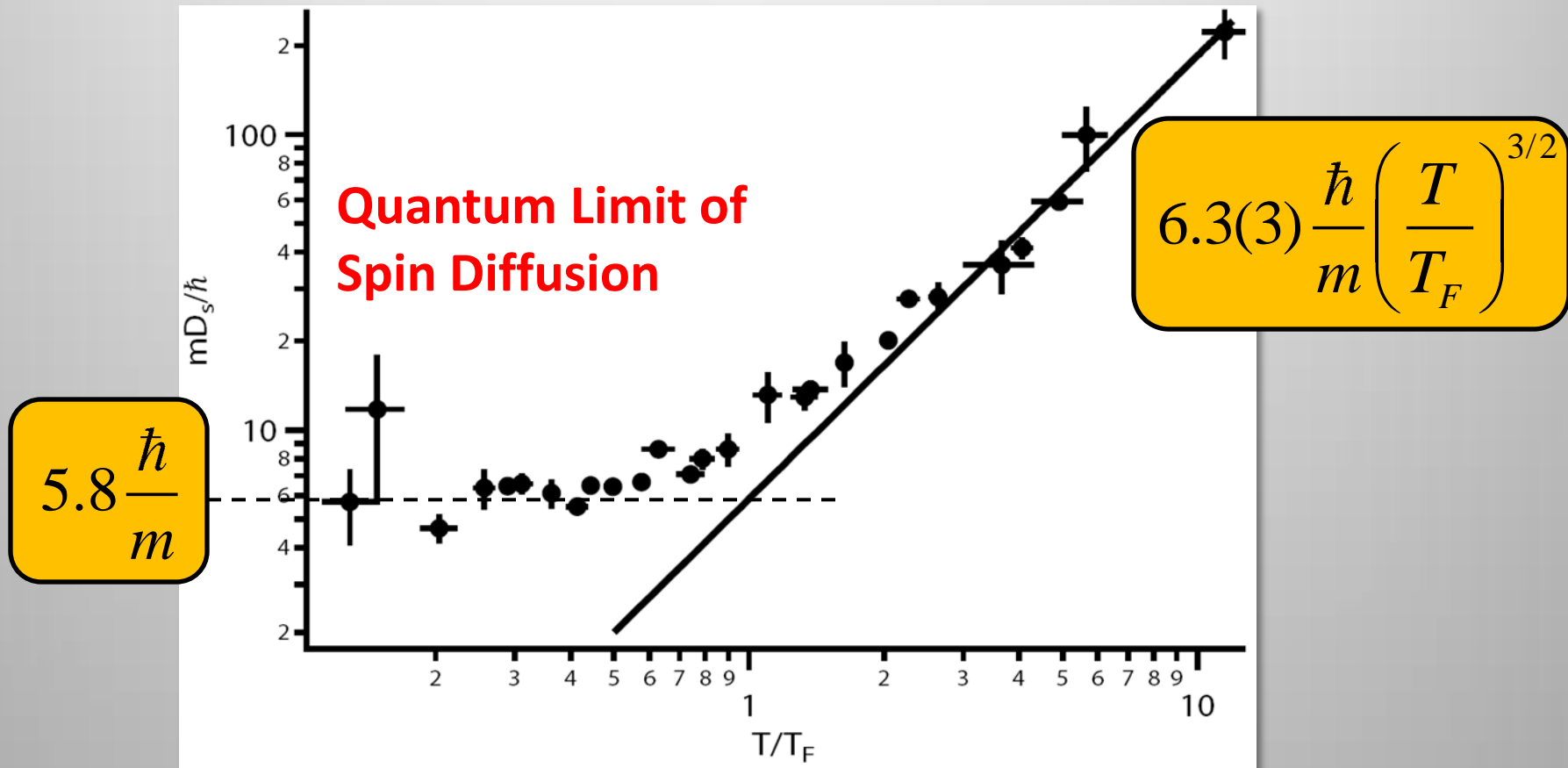
Diffusion constant: $D \sim \frac{(\text{mean free path})^2}{\text{collision time}} \sim \frac{\hbar}{m}$

$$\frac{\hbar}{m} = \frac{(100 \mu\text{m})^2}{1 \text{ s}}$$

Spin Diffusion vs Temperature

Spin current = $-D \cdot$ Spin density gradient

Universal high-T behavior:



**Can Fermi Gases become
superfluid?**

Superconductivity

Electrons are Fermions

Discovery of superconductivity 1911



Heike
Kamerlingh-Onnes
1911

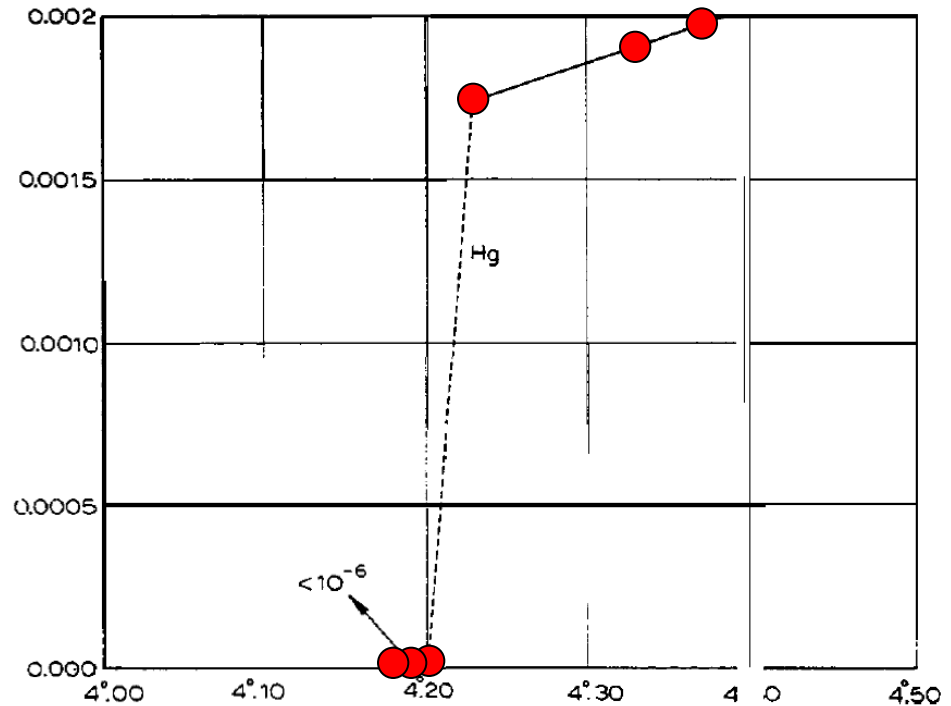


Fig. 17.

Fermionic Superfluidity

Condensation of Fermion Pairs

- **Helium-3** (Lee, Osheroff, Richardson 1971)
- **Superconductors: *Charged* superfluids of electron pairs**
Frictionless flow \Leftrightarrow Resistance-less current



John Bardeen



Leon N. Cooper



John R. Schrieffer

- **Neutron stars**
In the core: **Quark superfluid**

BCS Wavefunction

- Many-body wavefunction for a condensate of Fermion Pairs:

$$\Psi(\mathbf{r}_1, \dots, \mathbf{r}_N) = \varphi(|\mathbf{r}_1 - \mathbf{r}_2|) \chi_{12} \dots \varphi(|\mathbf{r}_{N-1} - \mathbf{r}_N|) \chi_{N-1,N}$$

↑
Spatial pair wavefunction

↑
Spin wavefunction

$$\chi_{ij} = \frac{1}{\sqrt{2}} (|\uparrow\rangle_i |\downarrow\rangle_j - |\downarrow\rangle_i |\uparrow\rangle_j)$$

- Second quantization:

$$|\Psi\rangle_N = \int \prod_i d^3 r_i \varphi(\mathbf{r}_1 - \mathbf{r}_2) \Psi_{\uparrow}^{\dagger}(\mathbf{r}_1) \Psi_{\downarrow}^{\dagger}(\mathbf{r}_2) \dots \varphi(\mathbf{r}_{N-1} - \mathbf{r}_N) \Psi_{\uparrow}^{\dagger}(\mathbf{r}_{N-1}) \Psi_{\downarrow}^{\dagger}(\mathbf{r}_N) |0\rangle$$

- Fourier transform: Pair wavefunction: $\varphi(\mathbf{r}) = \sum_k \varphi_k \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{\Omega}}$
- Operators: $\Psi_{\sigma}^{\dagger}(\mathbf{r}) = \sum_k c_{k\sigma}^{\dagger} \frac{e^{-i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{\Omega}}$

- Pair creation operator: $b^{\dagger} = \sum_k \varphi_k c_{k\uparrow}^{\dagger} c_{-k\downarrow}^{\dagger}$

- Many-body wavefunction: $|\Psi\rangle_N = b^{\dagger N/2} |0\rangle$
a fermion pair condensate

$|\Psi\rangle_N$ is not a Bose condensate

$$|\Psi\rangle_N = b^\dagger{}^{N/2} |0\rangle$$

- Commutation relations for pair creation/annihilation operators

$$[b^\dagger, b^\dagger]_- = \sum_{kk'} \varphi_k \varphi_{k'} [c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger, c_{k'\uparrow}^\dagger c_{-k'\downarrow}^\dagger]_- = 0 \quad \checkmark$$

$$[b, b]_- = \dots = 0 \quad \checkmark$$

$$[b, b^\dagger]_- = \dots = \sum_k |\varphi_k|^2 (1 - n_{k\uparrow} - n_{k\downarrow}) \neq 1 \quad \times$$

Occupation of momentum k

- pairs do not obey Bose commutation relations, *unless* $n_k \ll 1$

$$[b, b^\dagger]_- \approx \sum_k |\varphi_k|^2 = 1$$

**BEC limit of
tightly bound molecules**

BCS Wavefunction

- Introduce coherent state / switch to grand-canonical description:

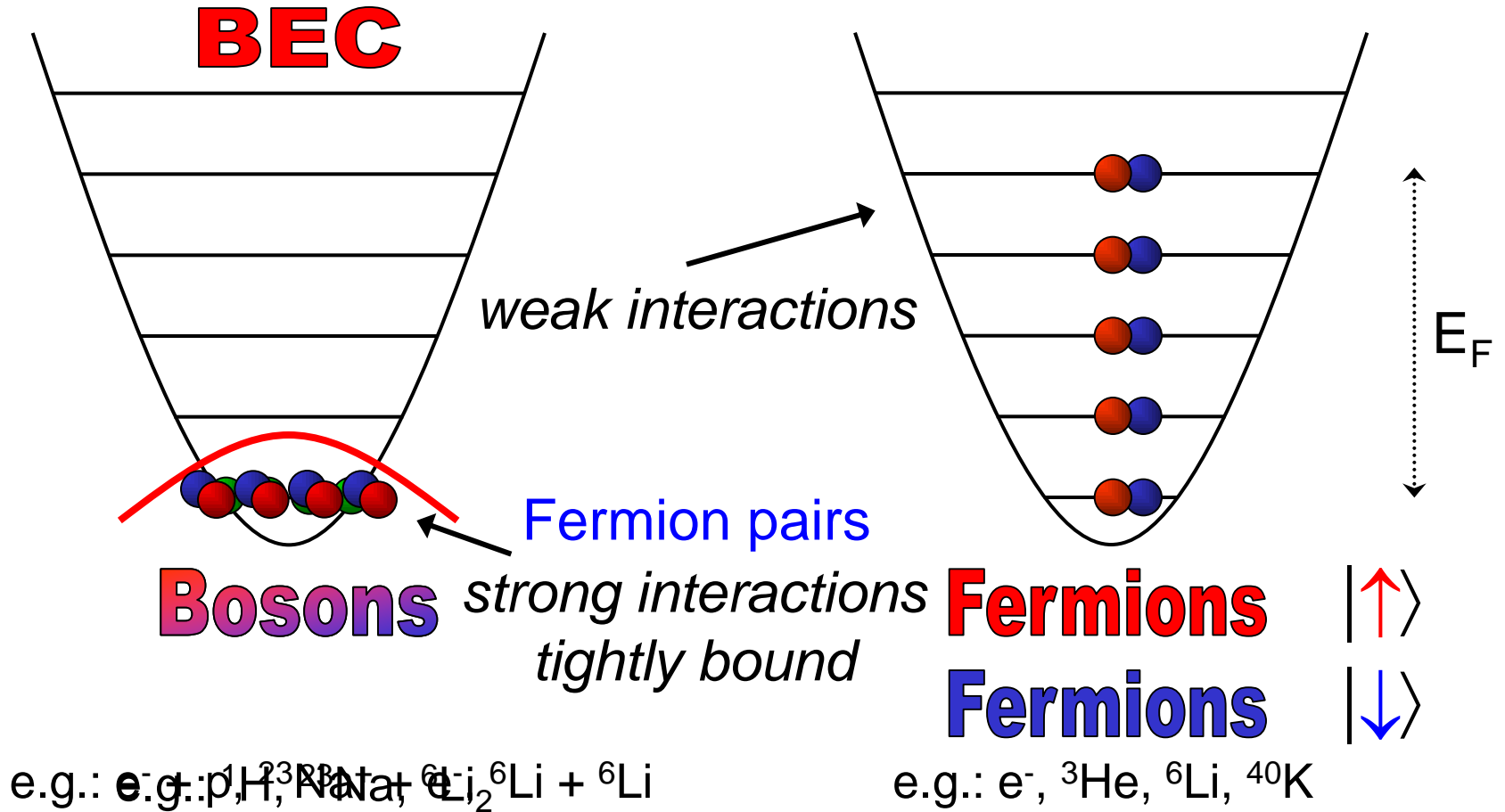
$$\begin{aligned}\mathcal{N} |\Psi\rangle &= \sum_{J_{\text{even}}} \frac{N_p^{J/4}}{(J/2)!} |\Psi\rangle_J = \sum_M \frac{1}{M!} N_p^{M/2} b^{\dagger M} |0\rangle \\ &= e^{\sqrt{N_p} b^{\dagger}} |0\rangle \\ &= \prod_k e^{\sqrt{N_p} \varphi_k c_{k\uparrow}^{\dagger} c_{-k\downarrow}^{\dagger}} |0\rangle \quad c_k^{\dagger} \text{ and } c_k^{\dagger} \text{ commute} \\ &= \prod_k (1 + \sqrt{N_p} \varphi_k c_{k\uparrow}^{\dagger} c_{-k\downarrow}^{\dagger}) |0\rangle \quad \text{because } c_k^{\dagger 2} = 0\end{aligned}$$

- Normalization: $\mathcal{N} = \prod_k \frac{1}{u_k} = \prod_k \sqrt{1 + N_p |\varphi_k|^2}$

- BCS wavefunction: $|\Psi_{\text{BCS}}\rangle = \prod_k (u_k + v_k c_{k\uparrow}^{\dagger} c_{-k\downarrow}^{\dagger}) |0\rangle$

with $v_k = \sqrt{N_p} \varphi_k u_k$ and $|u_k|^2 + |v_k|^2 = 1$

Bosons vs Fermions



$$T_C \approx T_F$$

Interatomic interactions

- Mean-field interaction energy: $E_{\text{MF}} = \frac{4\pi\hbar^2 n a}{m}$

- Weak or strong interaction? $\frac{E_{\text{MF}}}{E_F} \simeq k_F a$

$$k_F = (6\pi^2 n)^{1/3} \sim 1/2000 a_0$$

$$a \sim 50 - 100 a_0$$

$$k_F a \lesssim 5\%$$

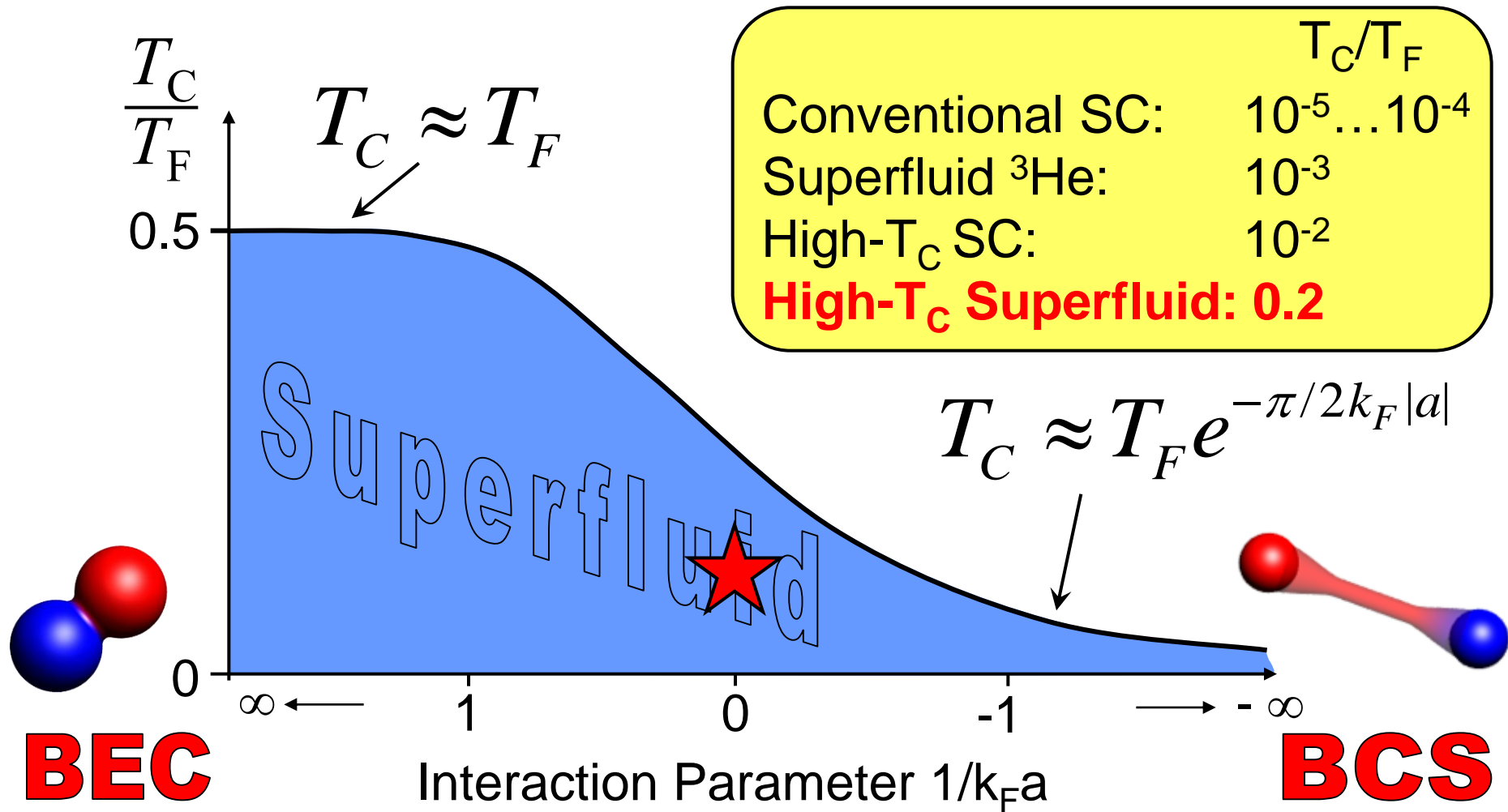
- Typically weak interaction

- Superconductors: Electron-Phonon interaction

also weak: $\frac{\hbar\omega_D}{E_F} = \frac{100 \text{ K}}{10\,000 \text{ K}} \sim 1\%$

How can we have atom pairs with arbitrarily weak interaction?

Critical Temperature for Fermionic Superfluidity



Scaled to the density of electrons in solids:

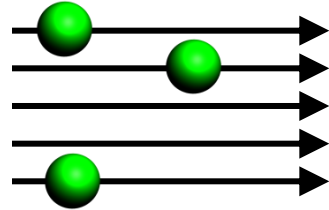
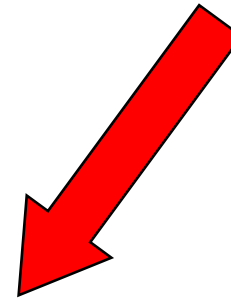
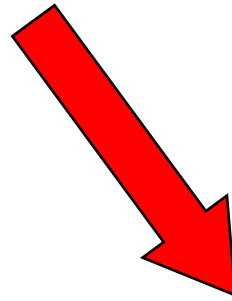
Superconductivity far above room temperature!

Experimental realization
of the
BEC-BCS Crossover

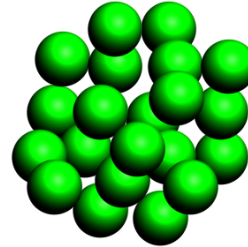
The cooling methods

- **Laser cooling**
- **Evaporative cooling**

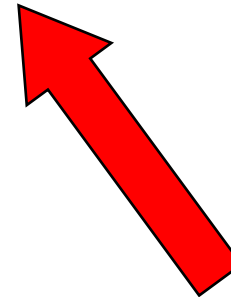
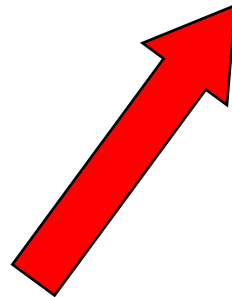
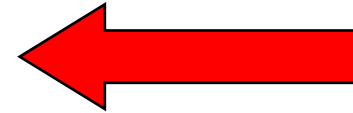
Techniques: Laser cooling



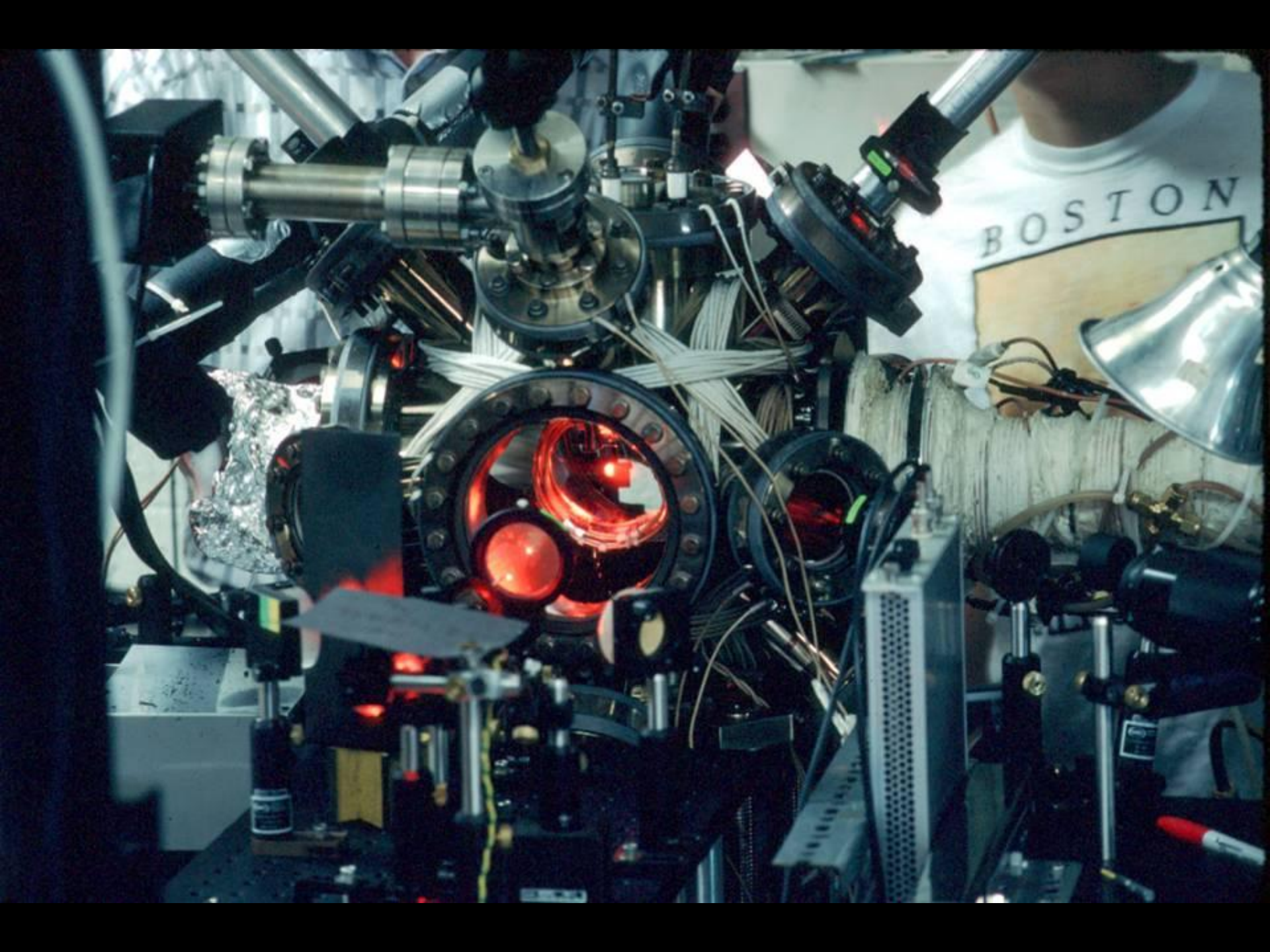
Hot atomic beam
700 K



Laser cooled cloud
 $100 \mu\text{K} - 1 \text{ mK}$

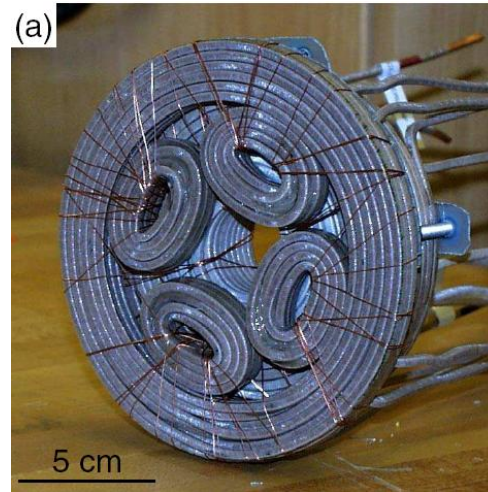
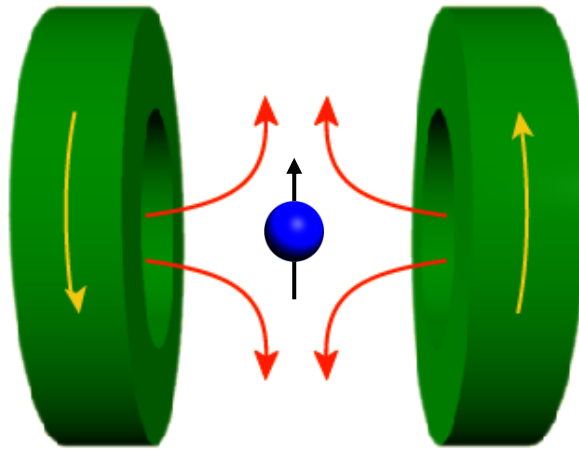


Chu, Cohen-Tannoudji, Phillips, Pritchard, Ashkin, Lethokov, Hänsch,
Schawlow, Wineland ...

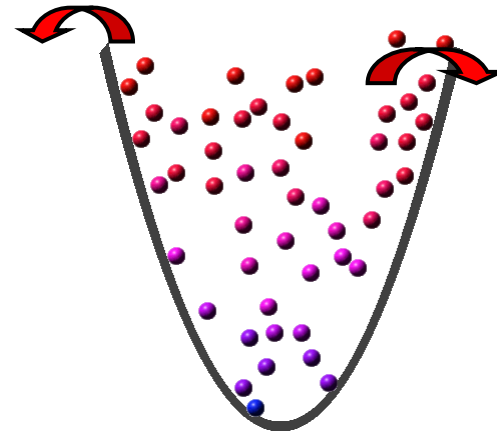


Techniques

Magnetic Trapping



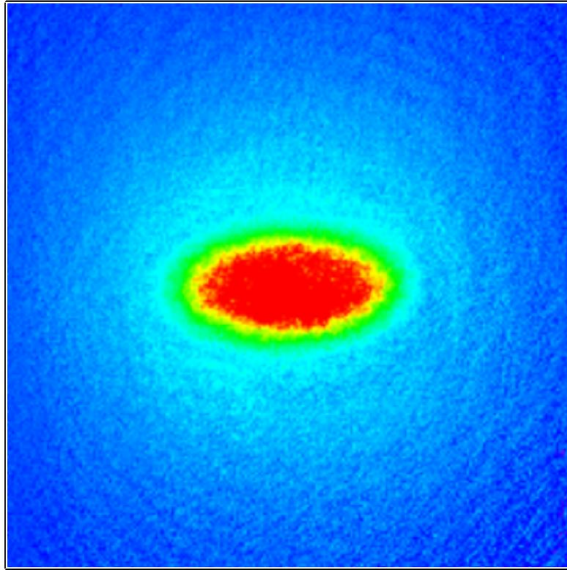
Evaporative Cooling



Source of ultracold fermions

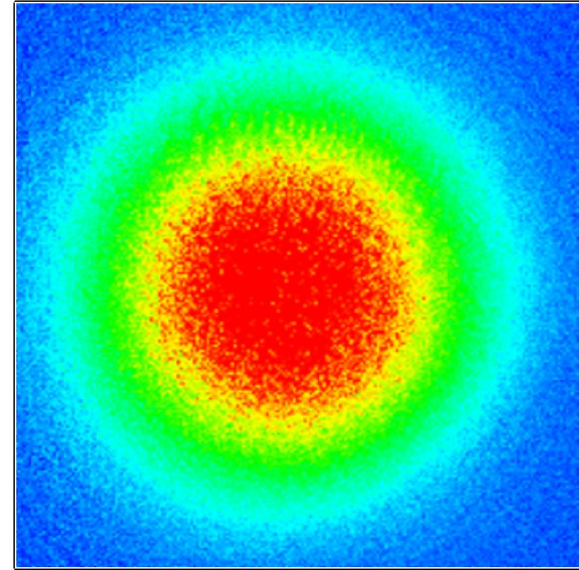
Cool fermionic lithium-6
using sodium as a refrigerator

10^7 atoms in BEC (w/o Li)



Bosons

5×10^7 Li atoms at $\frac{T}{T_F} < 0.3$

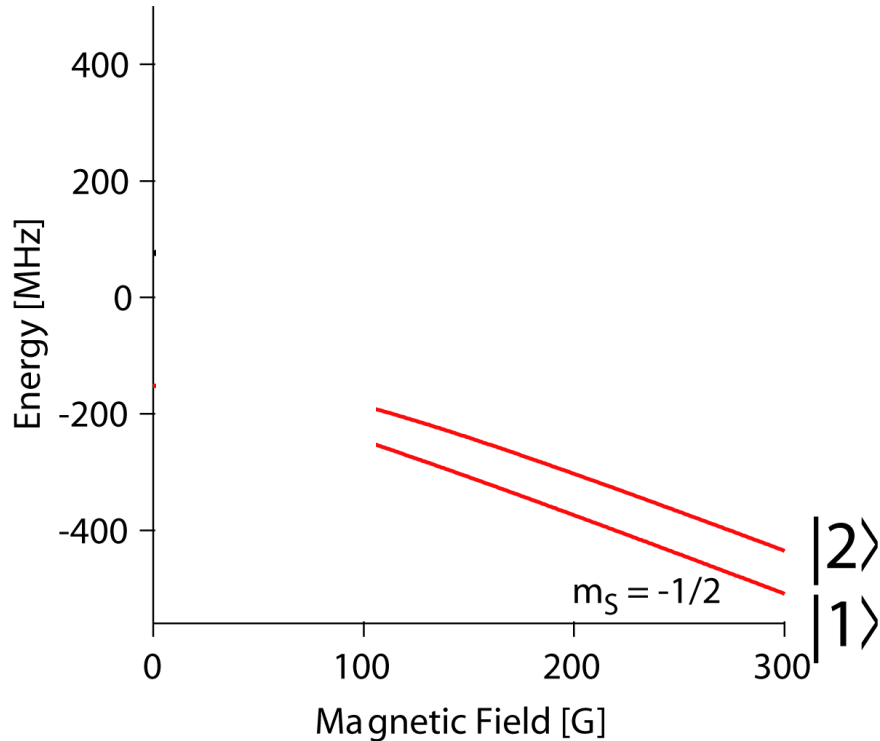


Fermions

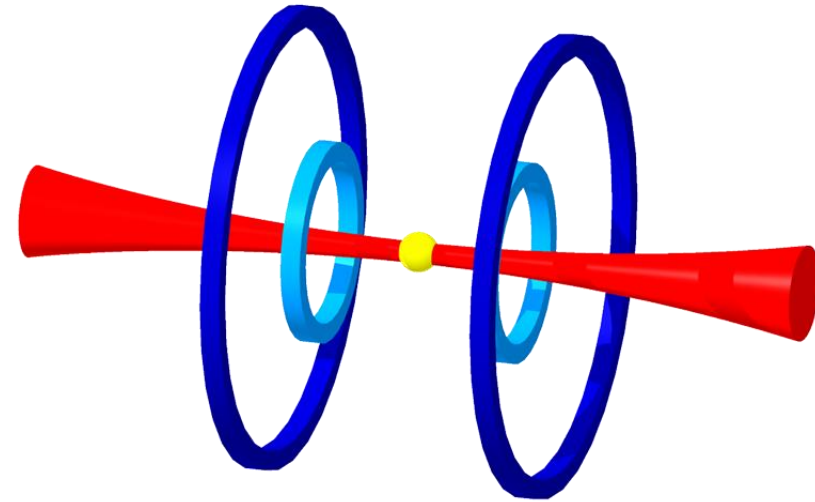
Preparation of an interacting Fermi system in Lithium-6

Electronic spin: $S = \frac{1}{2}$, Nuclear Spin: $I = 1$

→ $(2I+1)(2S+1) = 6$ hyperfine states



Optical trapping @ 1064 nm



$$v_{\text{axial}} = 10\text{-}20 \text{ Hz}$$

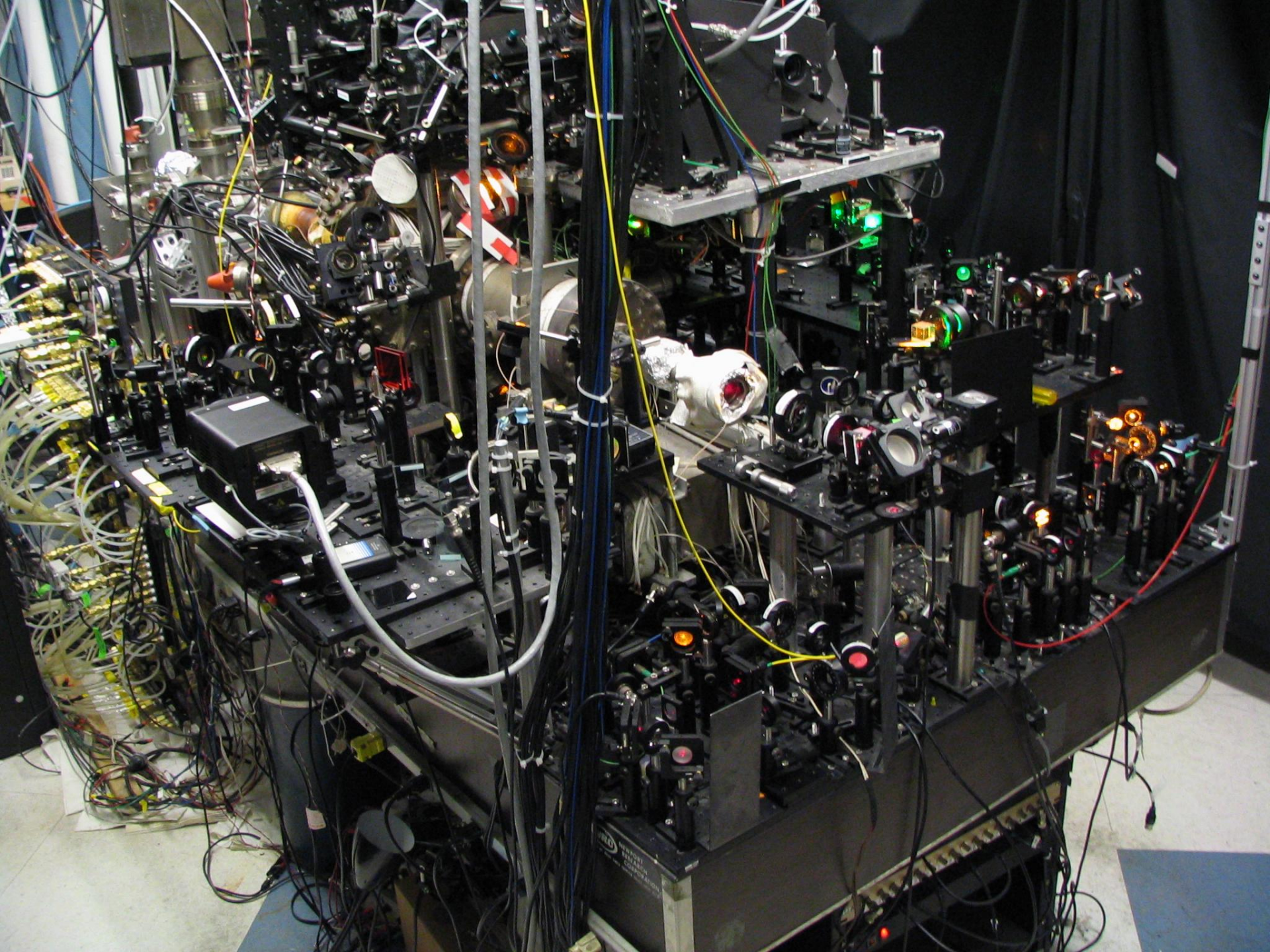
$$v_{\text{radial}} = 50\text{-}200 \text{ Hz}$$

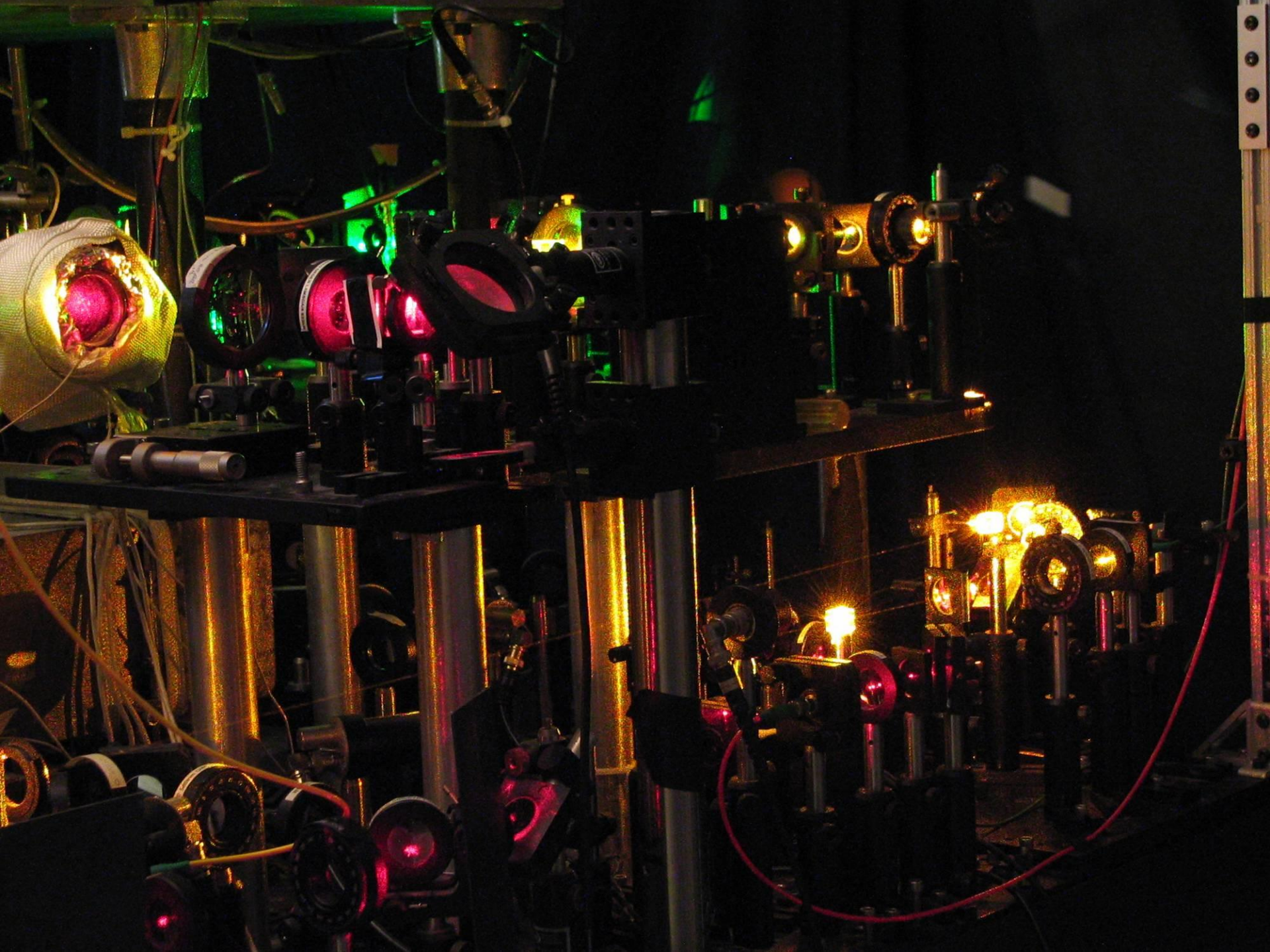
$$E_{\text{trap}} = 0.5 - 5 \mu\text{K}$$

High T_C !

At high fields, states $|1\rangle$ and $|2\rangle$ have large and negative scattering length

$$a_{12} = -2100 a_0$$

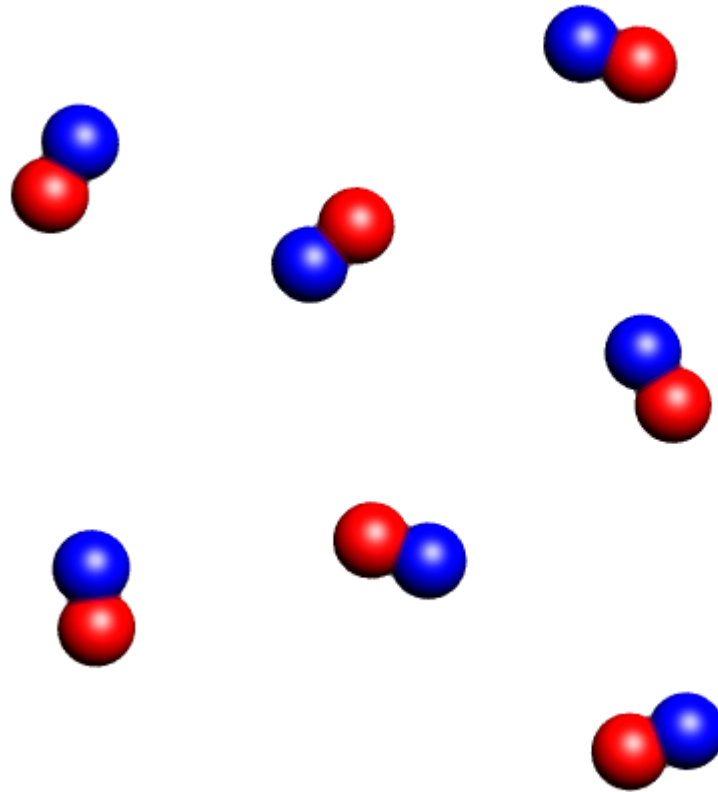




Feshbach resonances: Tuning the interactions

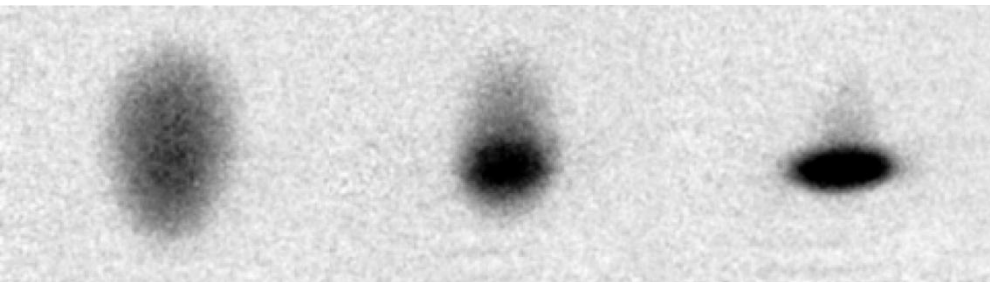
$$T_C \approx T_F$$

Tune!



Disclaimer: That's a cartoon picture

BEC of Fermion Pairs (Molecules)



$T > T_C$

$T < T_C$

$T \ll T_C$

These days: Up to 10 million
condensed molecules

Boulder

Nov '03

Innsbruck

Nov '03, Jan '04

MIT

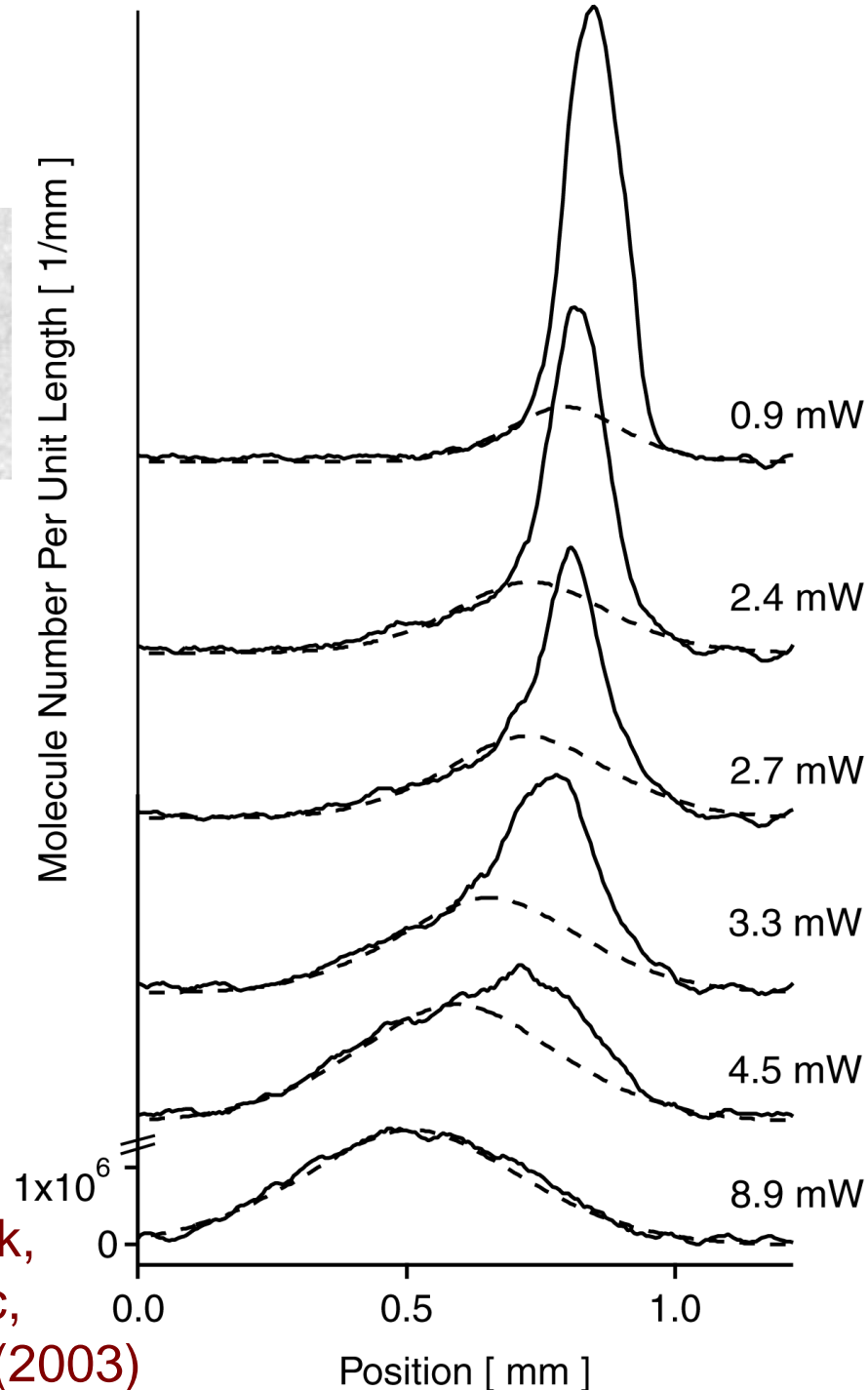
Nov '03

Paris

March '04

Rice, Duke

M.W. Zwierlein, C. A. Stan, C. H. Schunck,
S.M.F. Raupach, S. Gupta, Z. Hadzibabic,
W. Ketterle, Phys. Rev. Lett. 91, 250401 (2003)



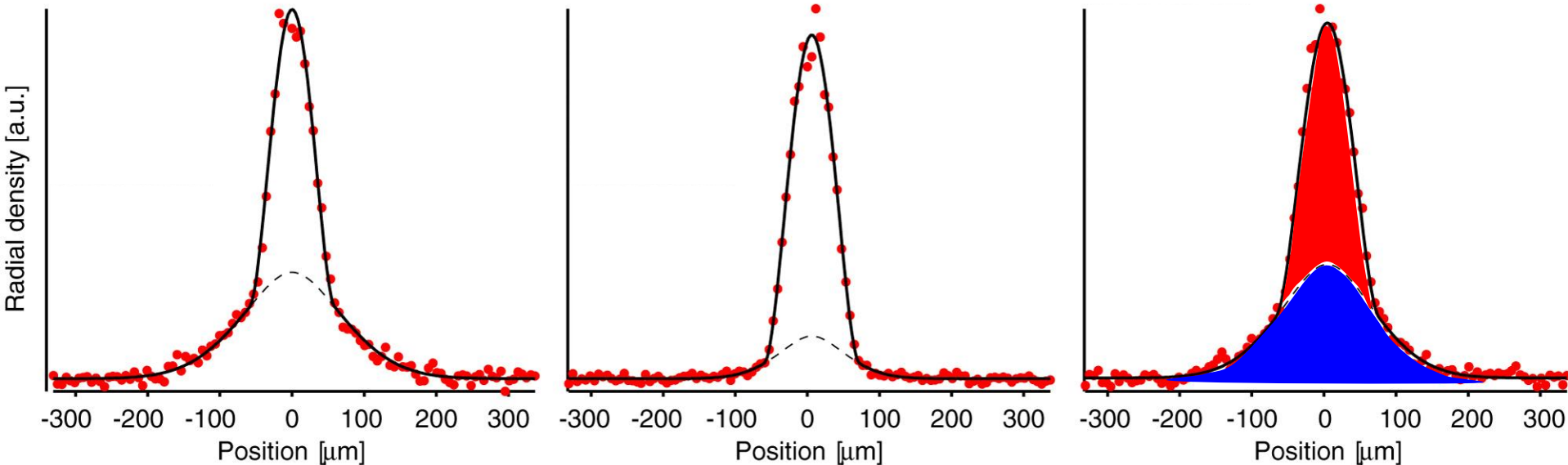
Observation of Pair Condensates

BEC-Side

Resonance

BCS-Side

(above dissociation
limit for molecules)

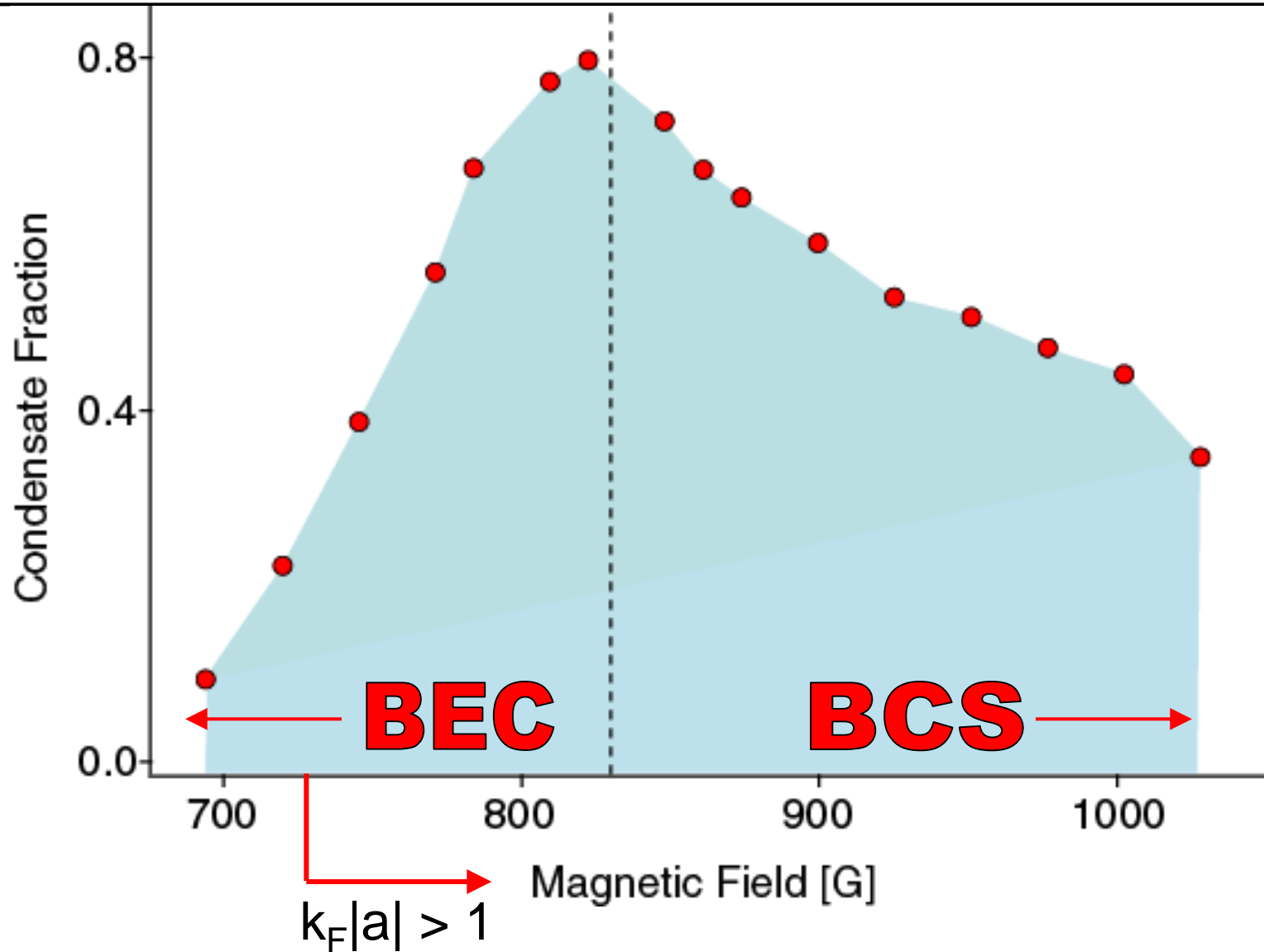


Thermal + condensed pairs

First observation: C.A. Regal et al., Phys. Rev. Lett. **92**, 040403 (2004)

M.W. Zwierlein, C.A. Stan, C.H. Schunck, S.M.F. Raupach, A.J. Kerman,
W. Ketterle, Phys. Rev. Lett. **92**, 120403 (2004).

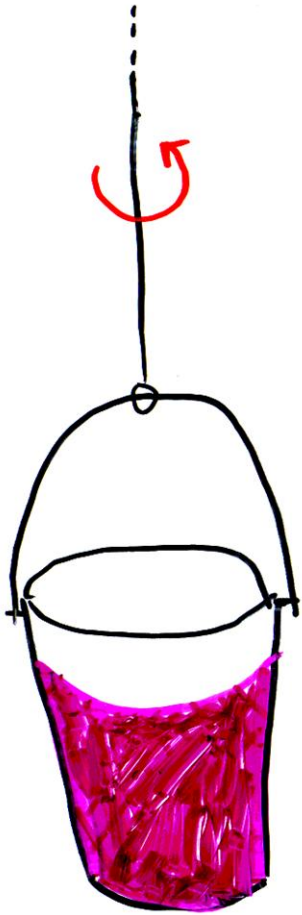
Condensate Fraction vs Magnetic Field



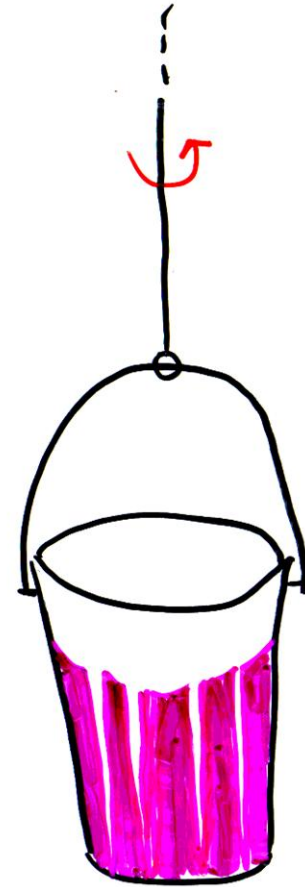
M.W. Zwierlein, C.A. Stan, C.H. Schunck, S.M.F. Raupach, A.J. Kerman, W. Ketterle, Phys. Rev. Lett. **92**, 120403 (2004).

**How can we show that these
gases are superfluid?**

Rotating Fluids



Normal

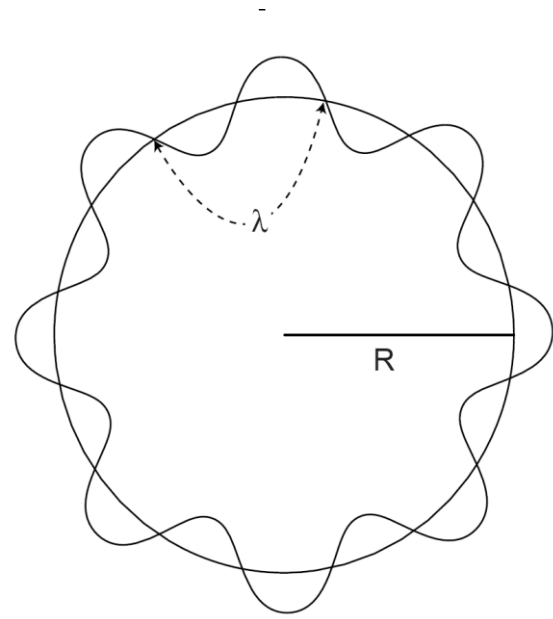
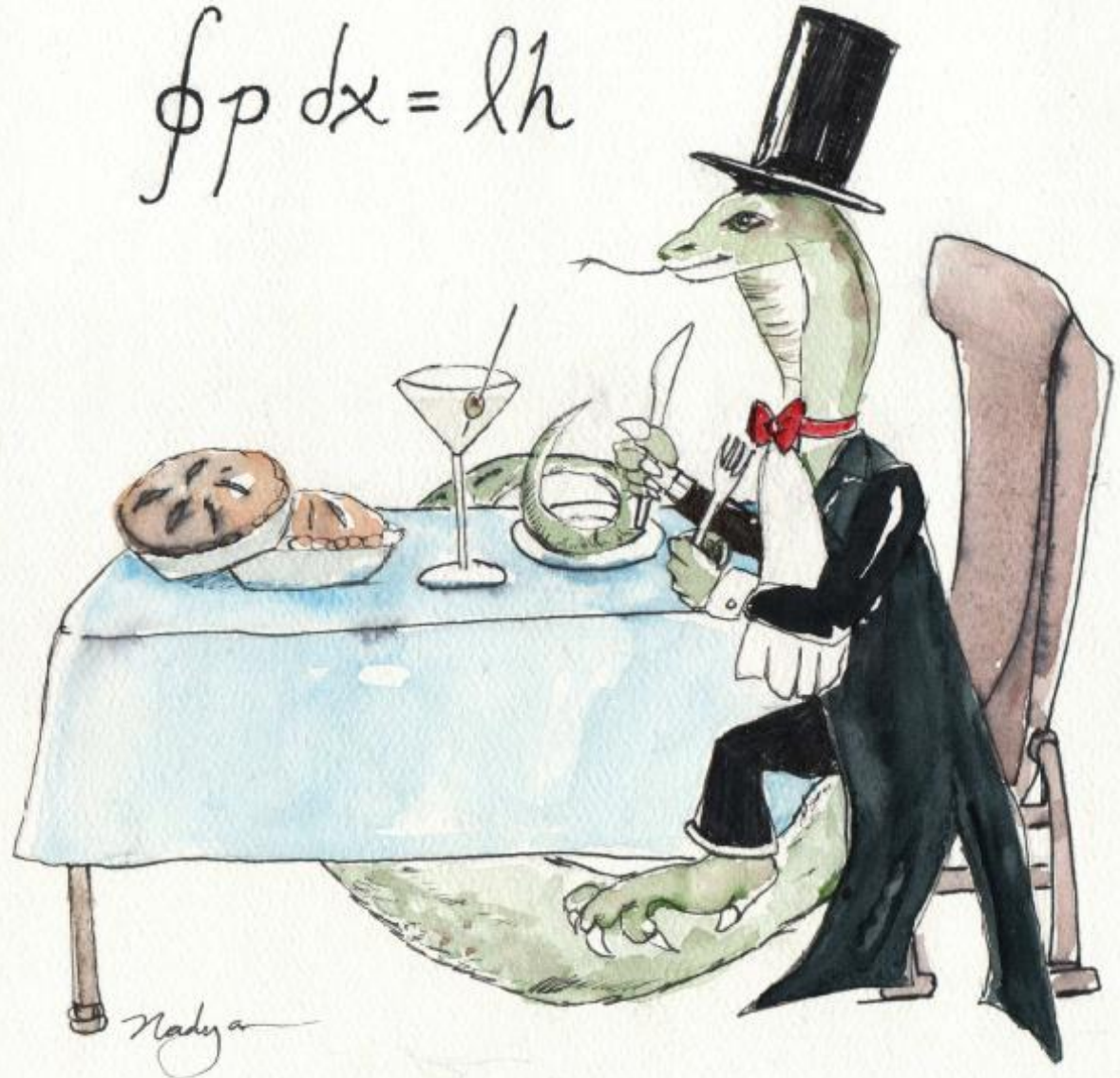


Quantum

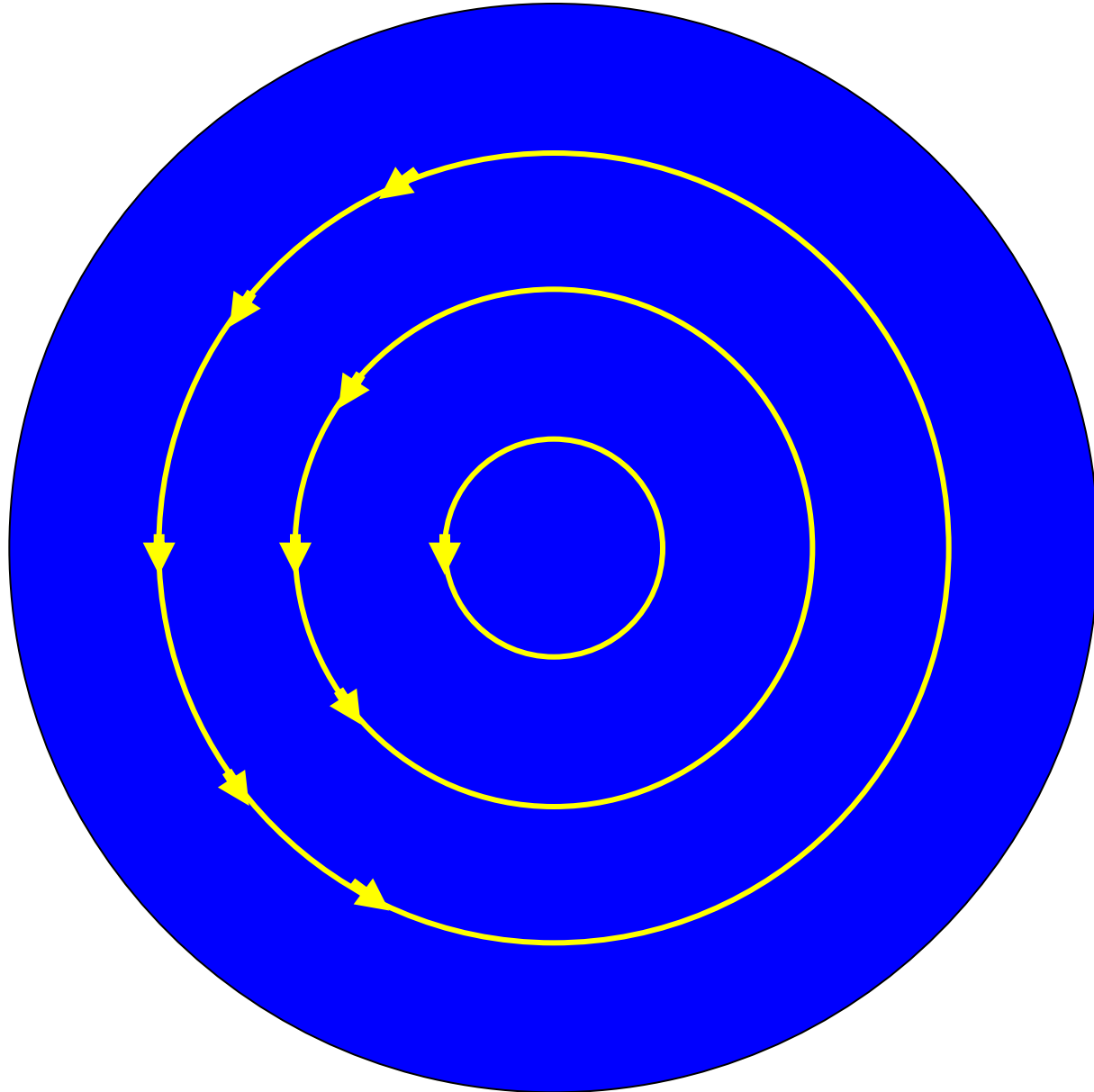
Fluid

Rotating superfluid

$$\oint p dx = \hbar h$$



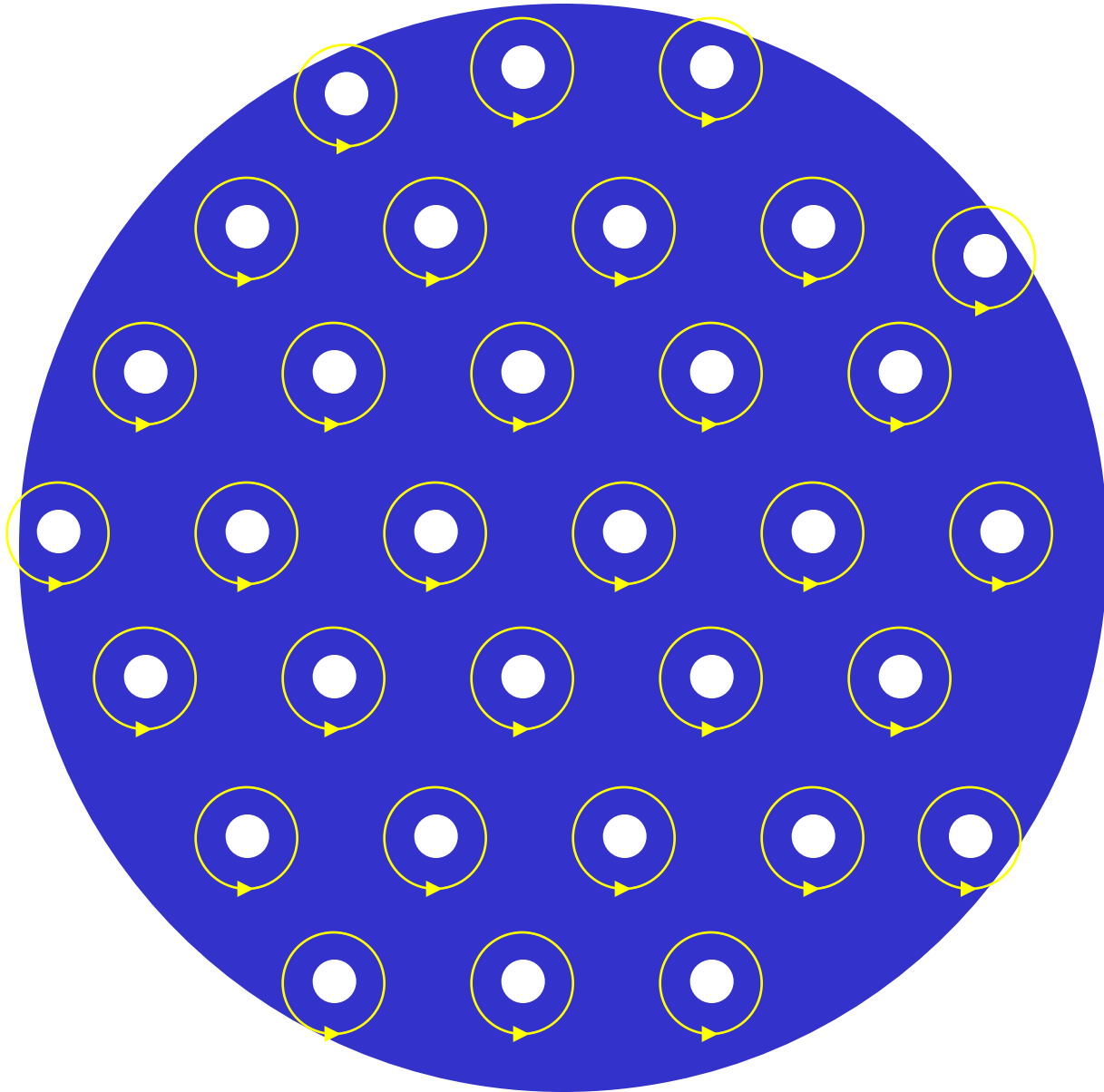
Look from top into the bucket



Look from top into the bucket



Aleksei A.
Abrikosov



Abrikosov lattice (honeycomb lattice)

Vortex Arrays *in Bosonic Gases / Fluids*

Berkeley

(R.E. Packard, 1979)

Helium-4

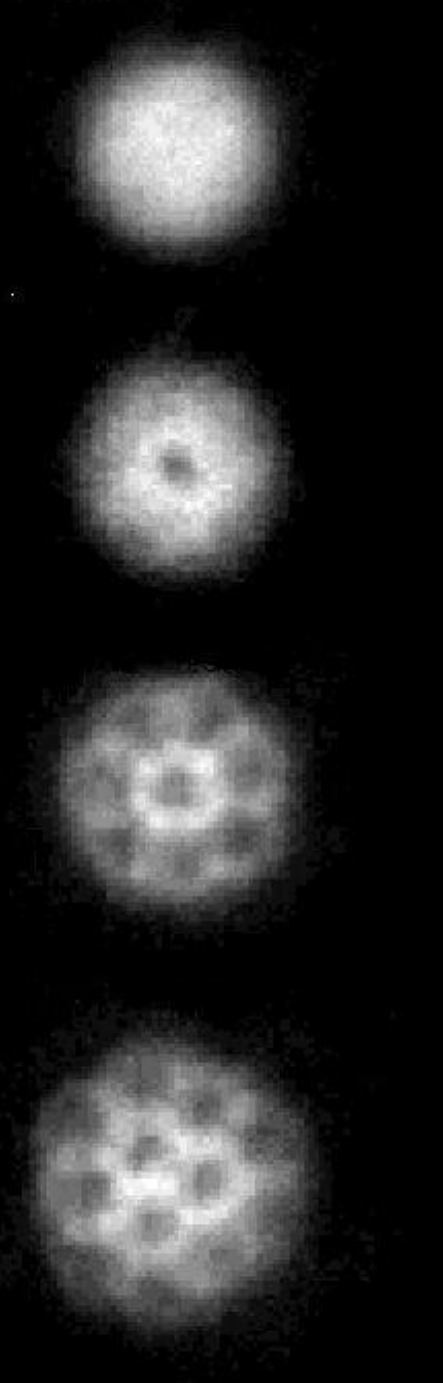
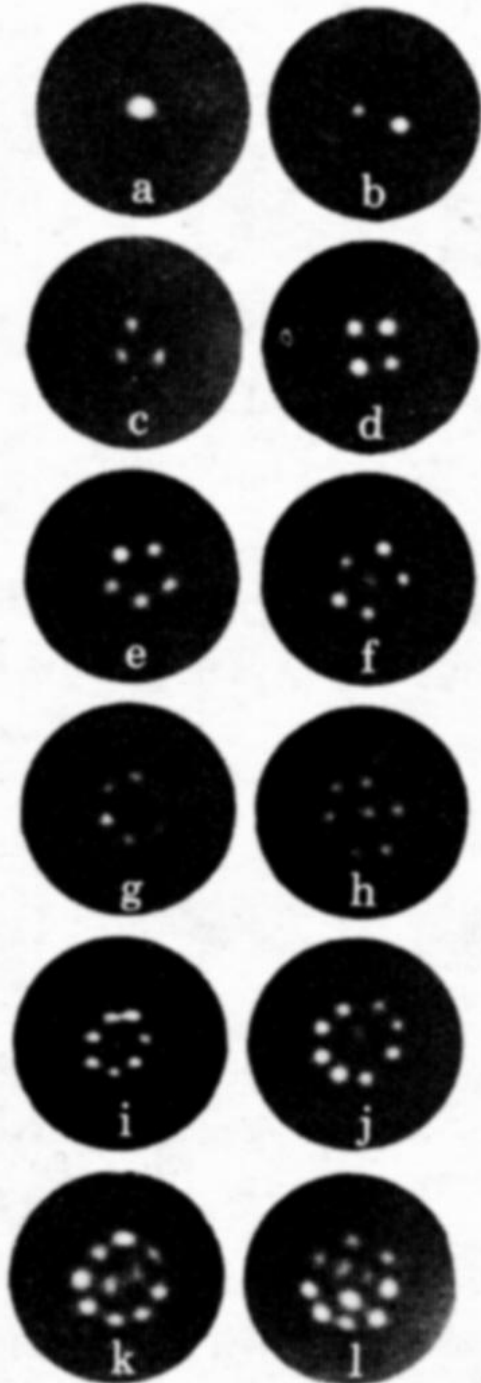
ENS

(J. Dalibard, 2000)

Rubidium BEC

*Also: Phase engineering
of single vortices in BEC:*

JILA (1999)



THE DIRECT OBSERVATION OF INDIVIDUAL FLUX LINES IN TYPE II SUPERCONDUCTORS

U. ESSMANN and H. TRÄUBLE

*Institut für Physik am Max-Planck-Institut für Metallforschung, Stuttgart and
Institut für theoretische und angewandte Physik der Technischen Hochschule Stuttgart*

Received 4 April 1967

Neutral superfluids under rotation

$$\vec{F} = 2m\vec{v} \times \vec{\omega}$$

Coriolis force in rotating frame



Superconductors in magnetic field

$$\vec{F} = q\vec{v} \times \vec{B}$$

Lorentz Force

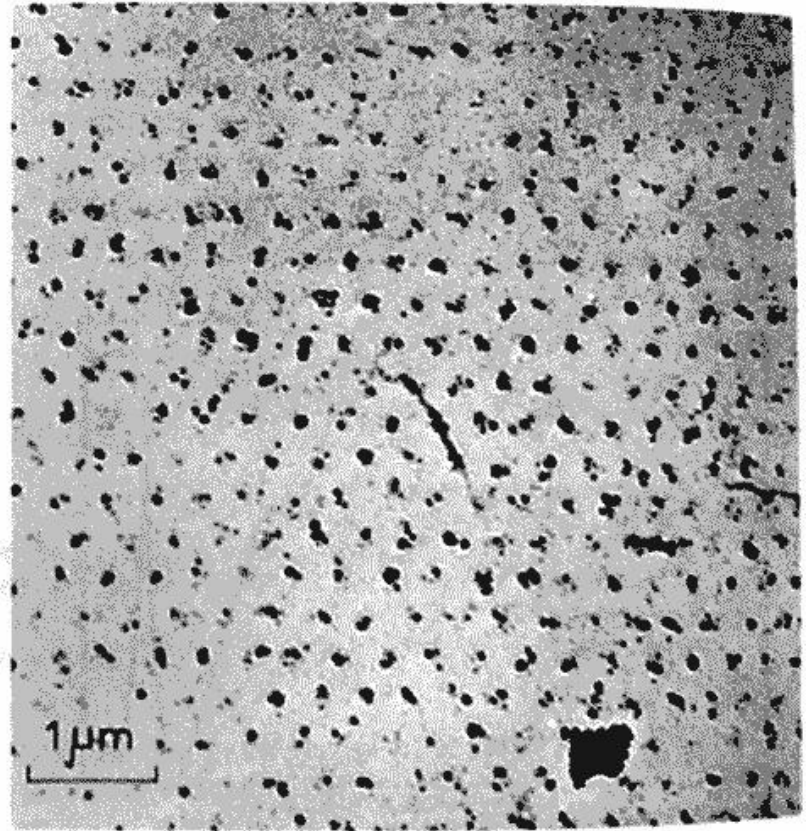
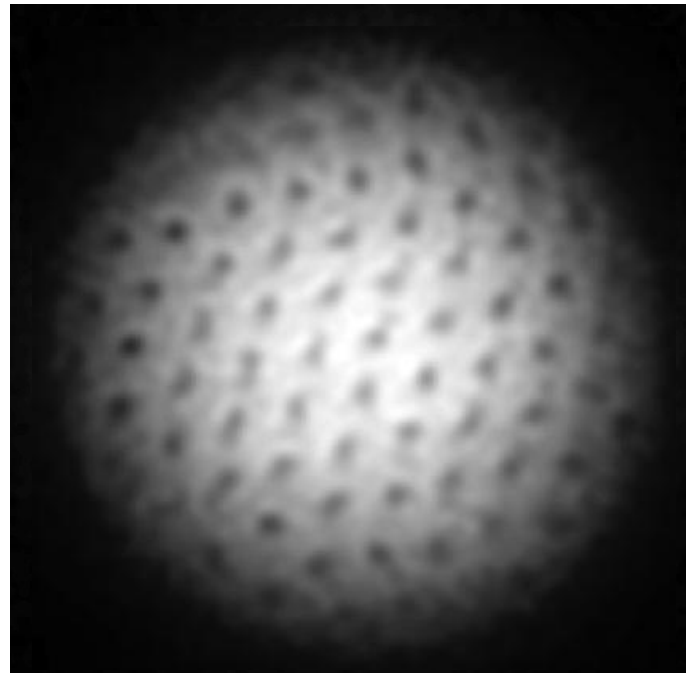
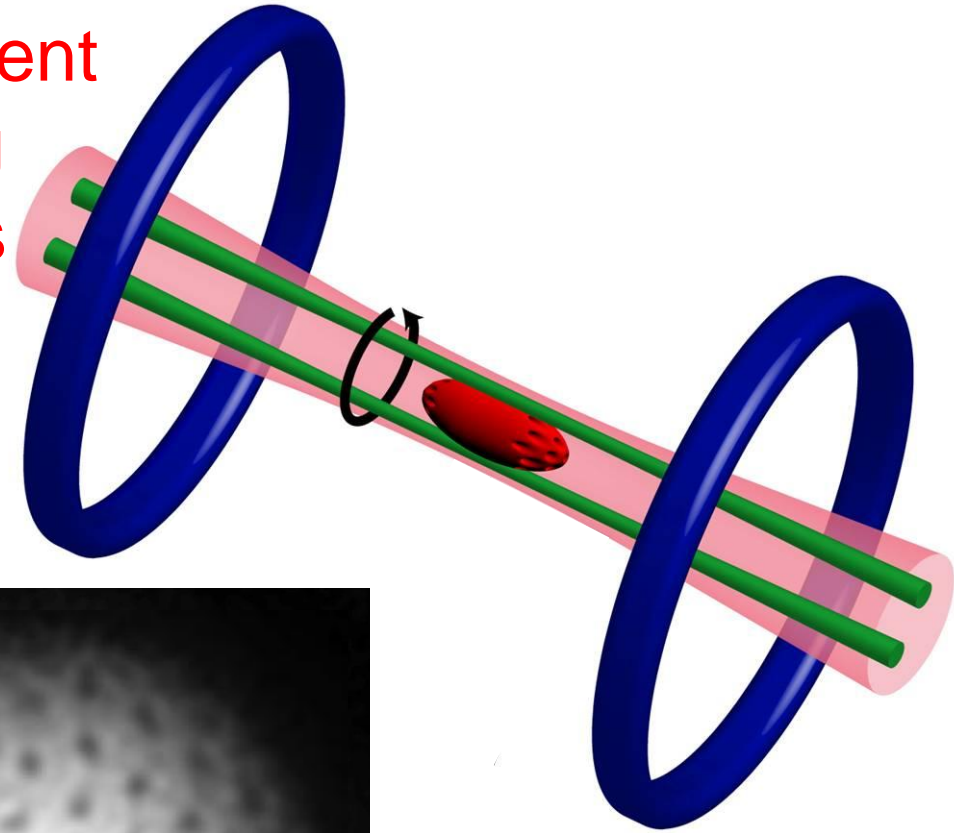


Fig. 1. "Perfect" triangular lattice of flux lines on the surface of a lead-4at% indium rod at 1.1°K. The black dots consist of small cobalt particles which have been stripped from the surface with a carbon replica.

U. Essmann and H. Träuble,
Physics Letters A, **24**, 526 (1967)

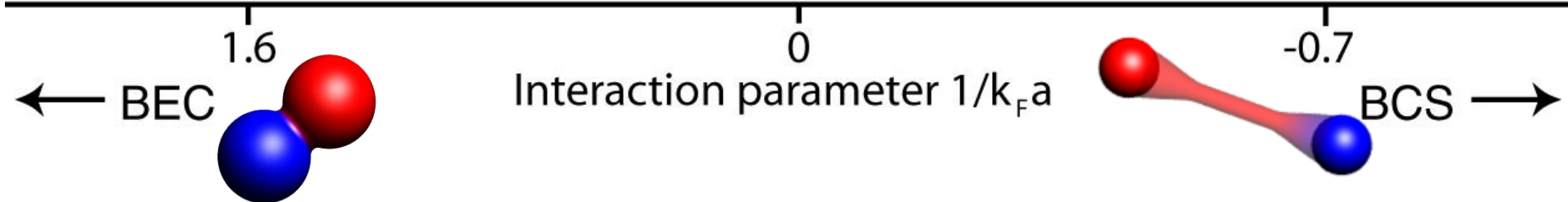
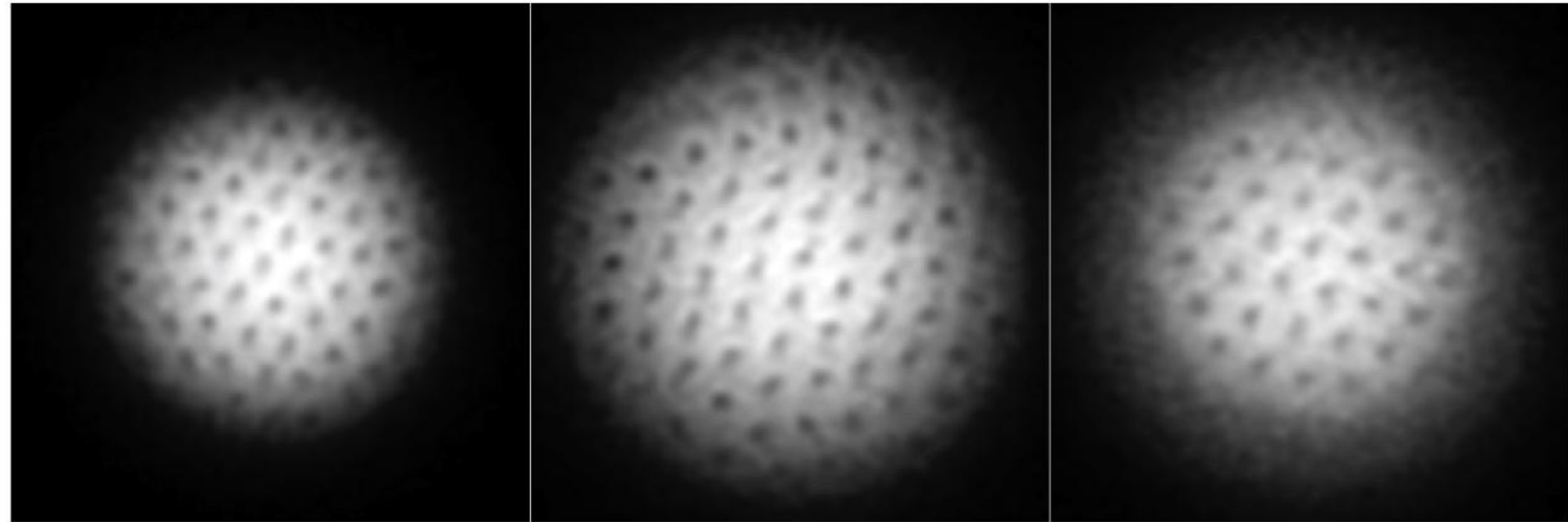
Spinning a strongly interacting Fermi gas

The rotating bucket experiment
with a strongly interacting
Fermi gas, a million times
thinner than air



Vortex lattices in the BEC-BCS crossover

Establishes *superfluidity* and *phase coherence*
in gases of **fermionic atom pairs**



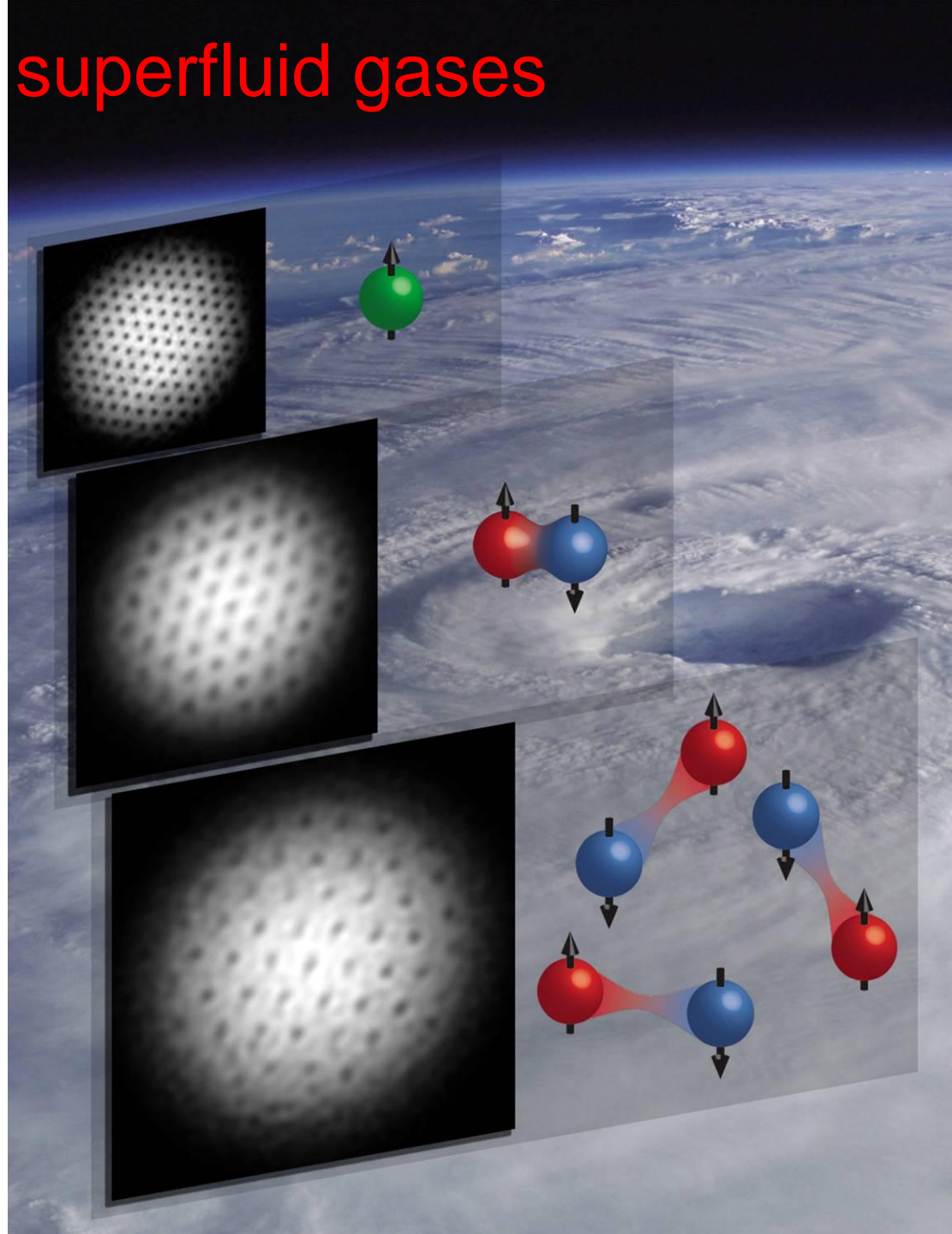
M.W. Zwierlein, J.R. Abo-Shaeer, A. Schirotzek, C.H. Schunck, W. Ketterle,
Nature 435, 1047-1051 (2005)

Gallery of superfluid gases

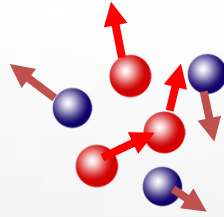
Atomic Bose-Einstein condensate (sodium)

Molecular Bose-Einstein condensate (lithium ${}^6\text{Li}_2$)

Pairs of fermionic atoms (lithium-6)

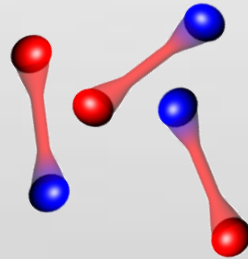


Thermodynamics



Classical gas Equation of State (EoS):

$$P = nk_B T$$



Jason asks:

What is the EoS of a strongly interacting Fermi gas?

$$P(n, T)$$

Lots of expts.: Thomas, Jin, Salomon, Grimm, Ketterle, Hulet, Mukaiyama, Vale, ...

Influential proposal: Tin-Lun Ho, Qi Zhou, Nature Physics 6, 131 (2010)

Thermodynamics

Equilibrium Thermodynamics
as baseline for non-equilibrium studies

→ Establish Equation of State

Classical gas

$$P = nk_B T$$

Generally need

$$P(n, T)$$

Or fixing chemical potential

$$P(\mu, T)$$

Or replacing

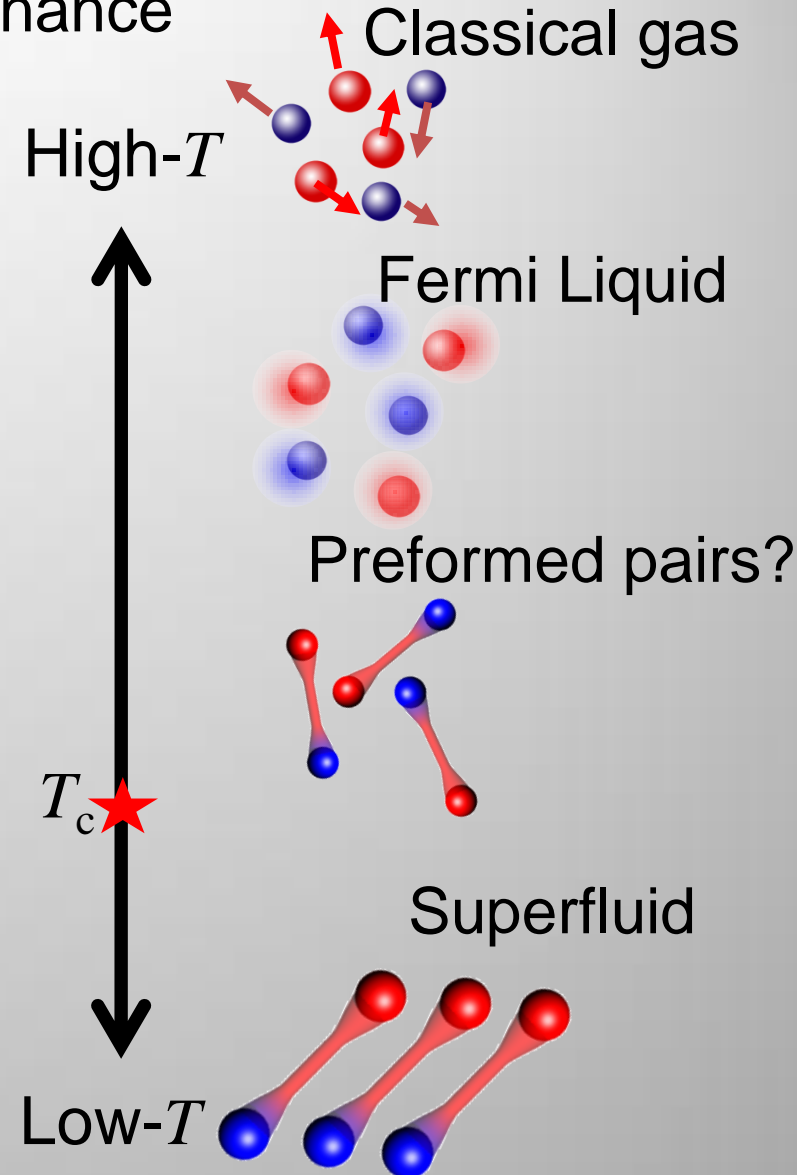
$$n = \left. \frac{\partial P}{\partial \mu} \right|_{T, a, \dots}$$

$$n(\mu, T)$$

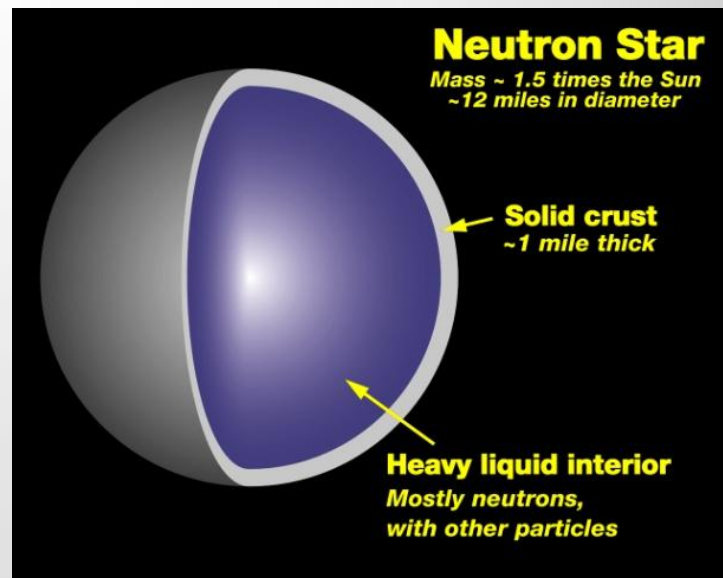
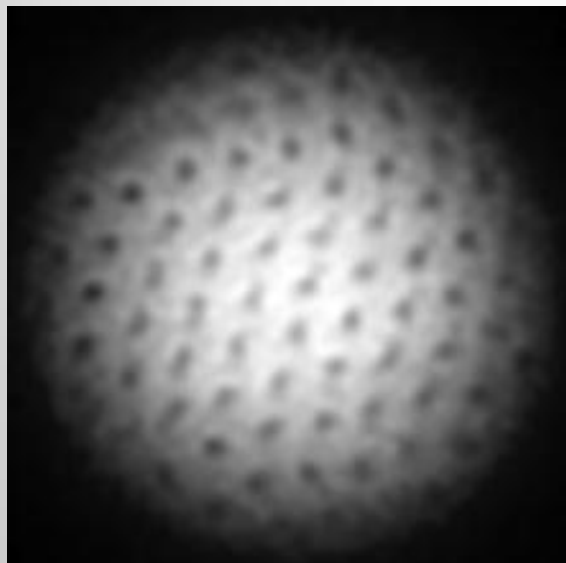
Thermodynamics of the Unitary Fermi Gas

Spin $\frac{1}{2}$ - Fermi gas at a Feshbach resonance

- *Normal state:*
 - Is it a Fermi liquid?
 - Are there preformed pairs (pseudogap regime)?
- *Superfluid properties:*
 - Transition temperature
 - Critical Entropy
 - Energy of the superfluid
 - ...



Relation to equation of state of a neutron star



Property	Atoms	Neutrons
Spin	Pseudospin $\frac{1}{2}$	Spin $\frac{1}{2}$
Interparticle distance $n^{-1/3}$	1 μm	1 fm
Density	10^{13} cm^{-3}	10^{38} cm^{-3}
Fermi Energy	1 $\mu\text{K} = 10^{-10} \text{ eV}$	$10^{12} \text{ K} = 150 \text{ MeV}$
Scattering length a	freely tunable	-19 fm

Both systems lie in universal regime: $a \gg n^{-1/3}$

Measuring the Equation of State

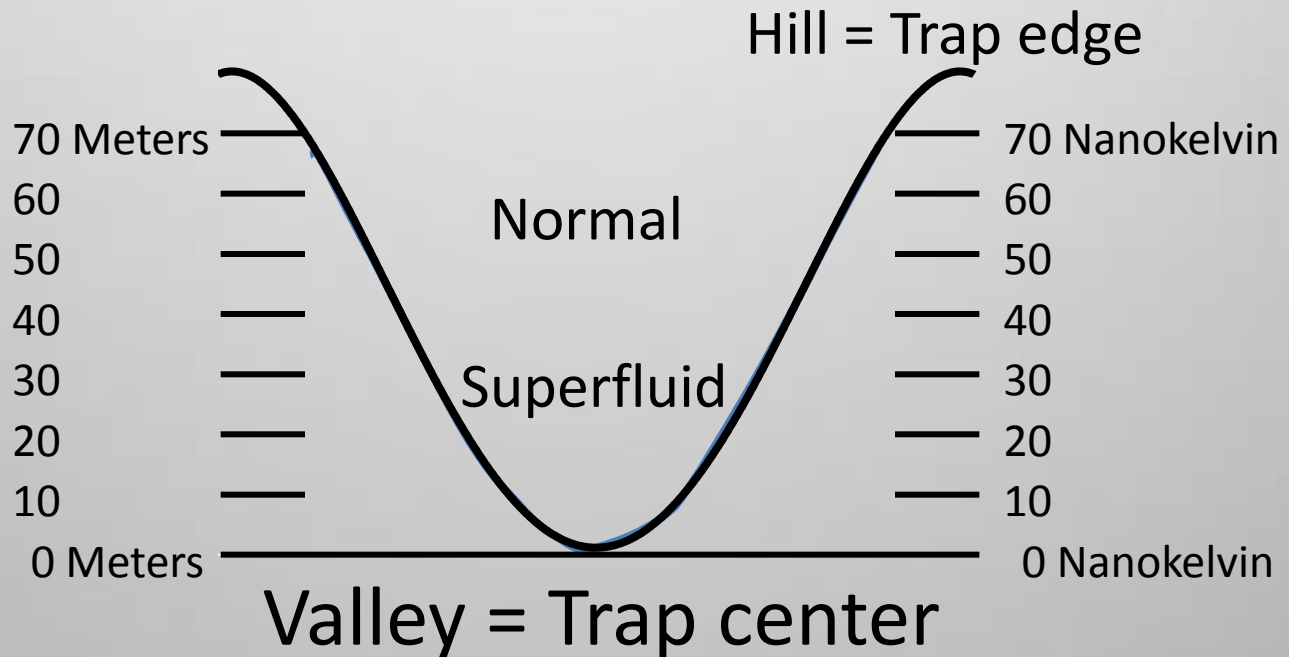
When climbing a mountain, the air gets thinner...

Equation of state \rightarrow density as a function of height

The inverse works as well!

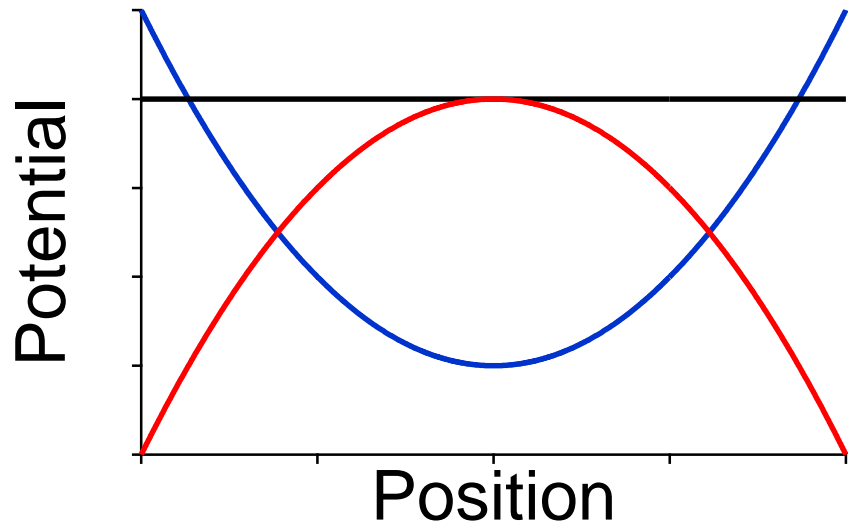
Density as a function of height \rightarrow equation of state

Atoms in our trapping potential $\hat{=}$ air particles in gravitational potential



Equation of State

Equation of state from density distribution in a trap

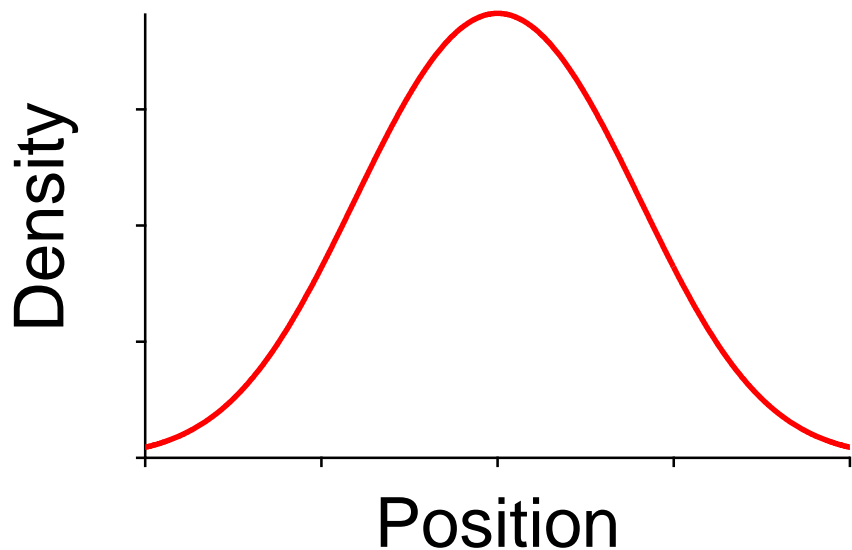


$V(\mathbf{r})$

μ_0

Local chemical potential

$$\mu(\vec{r}) = \mu_0 - V(\vec{r})$$

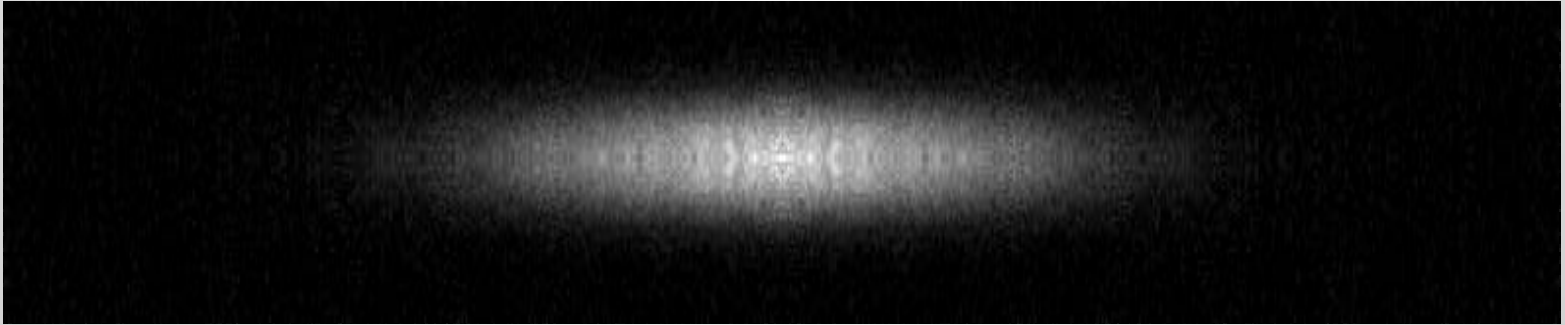


Local density

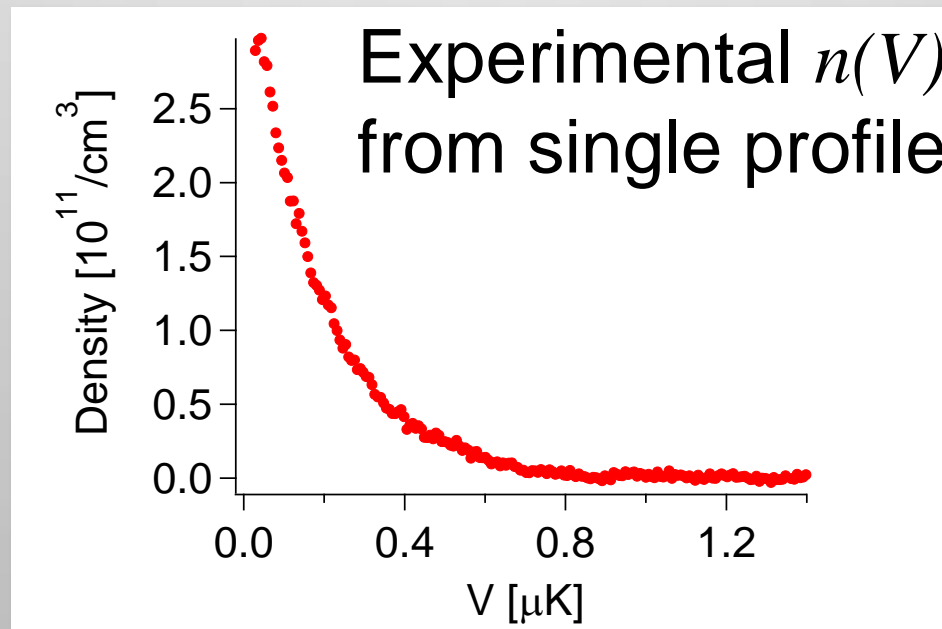
$$n(V) = n(\mu_0 - V, T)$$

Density profile provides a scan through the equation of state

Equation of State: Measuring density



Exploiting cylindrical symmetry and careful characterization of trapping potential:



Equation of State: Measuring pressure

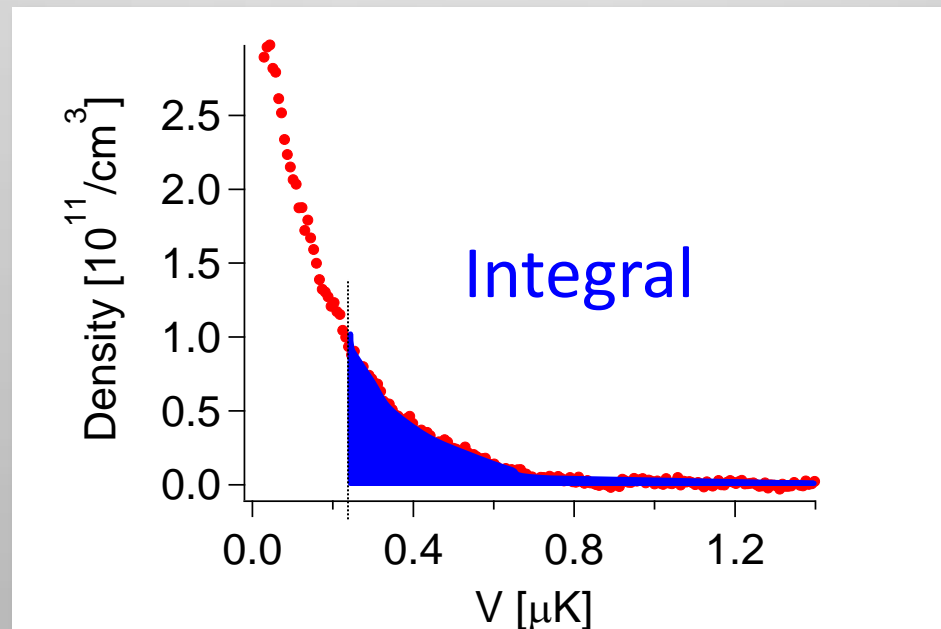
Pressure = weight / unit area of air above you

For atom trappers: replace $m g h \rightarrow V$

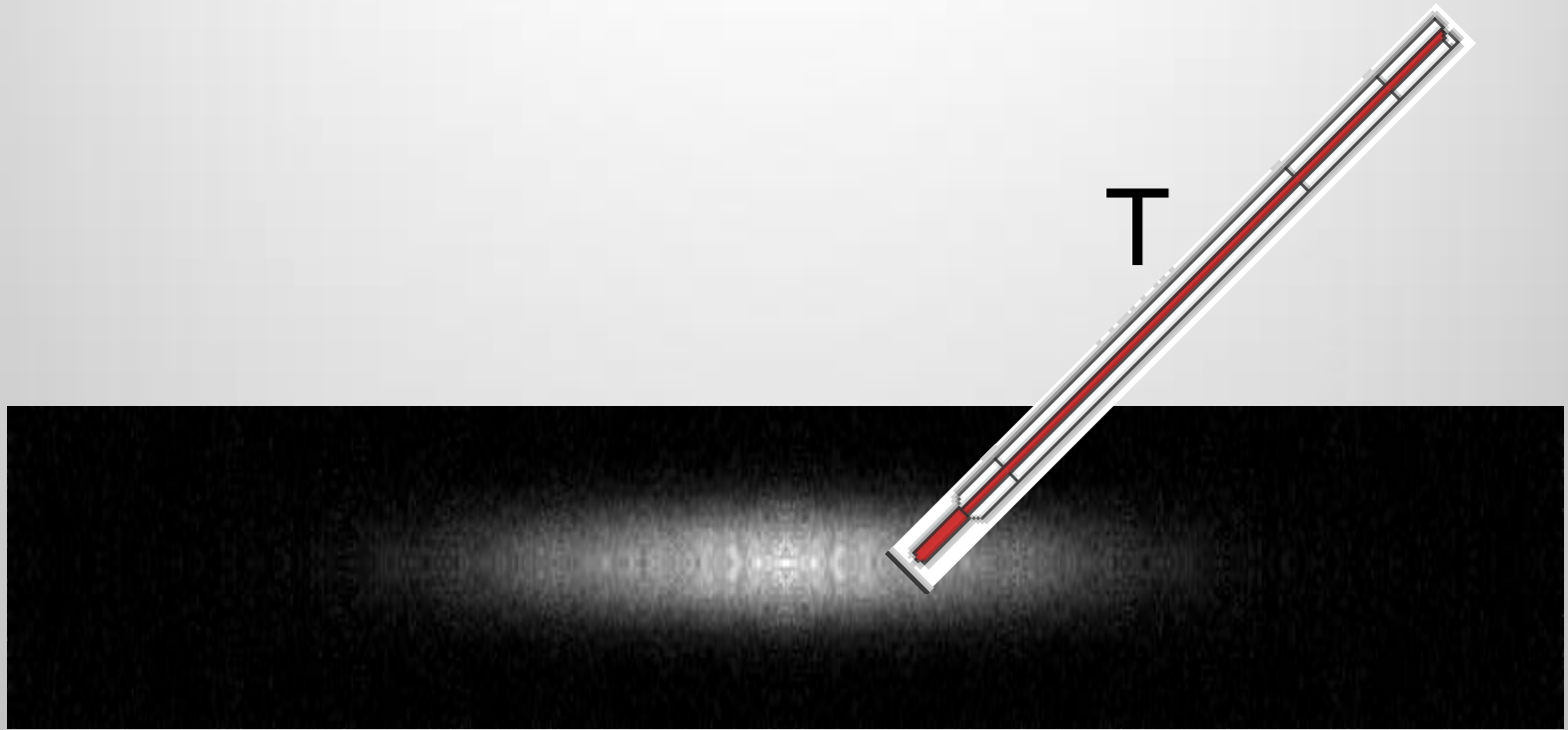
Pressure = integrated density over potential

Local pressure

$$P = \int_V^{\infty} dV' n(V')$$



How to get T?

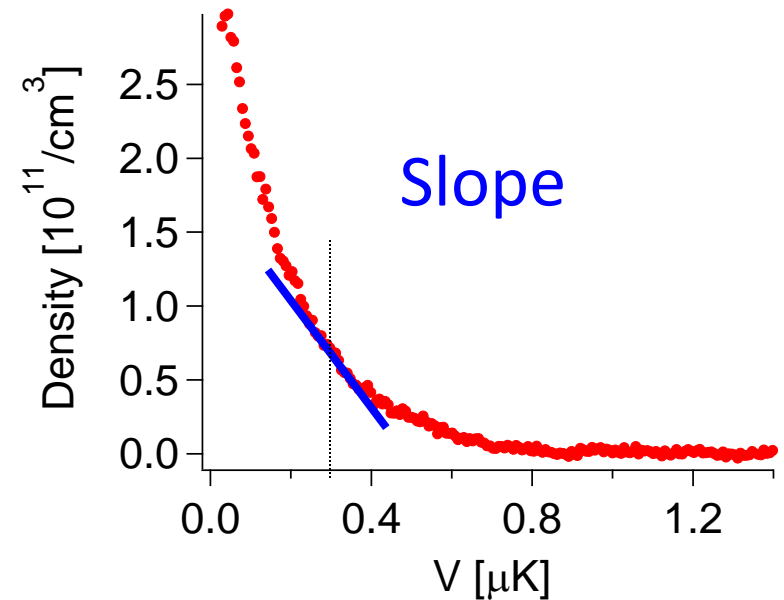


...Not impossible, but it's very difficult, so...

Don't! Instead:

Local compressibility

$$\kappa = -\frac{1}{n^2} \frac{dn}{dV}$$



Compressibility Equation of State

$$\kappa(n, P)$$

All other thermodynamic quantities follow!

No-fit Equation of State

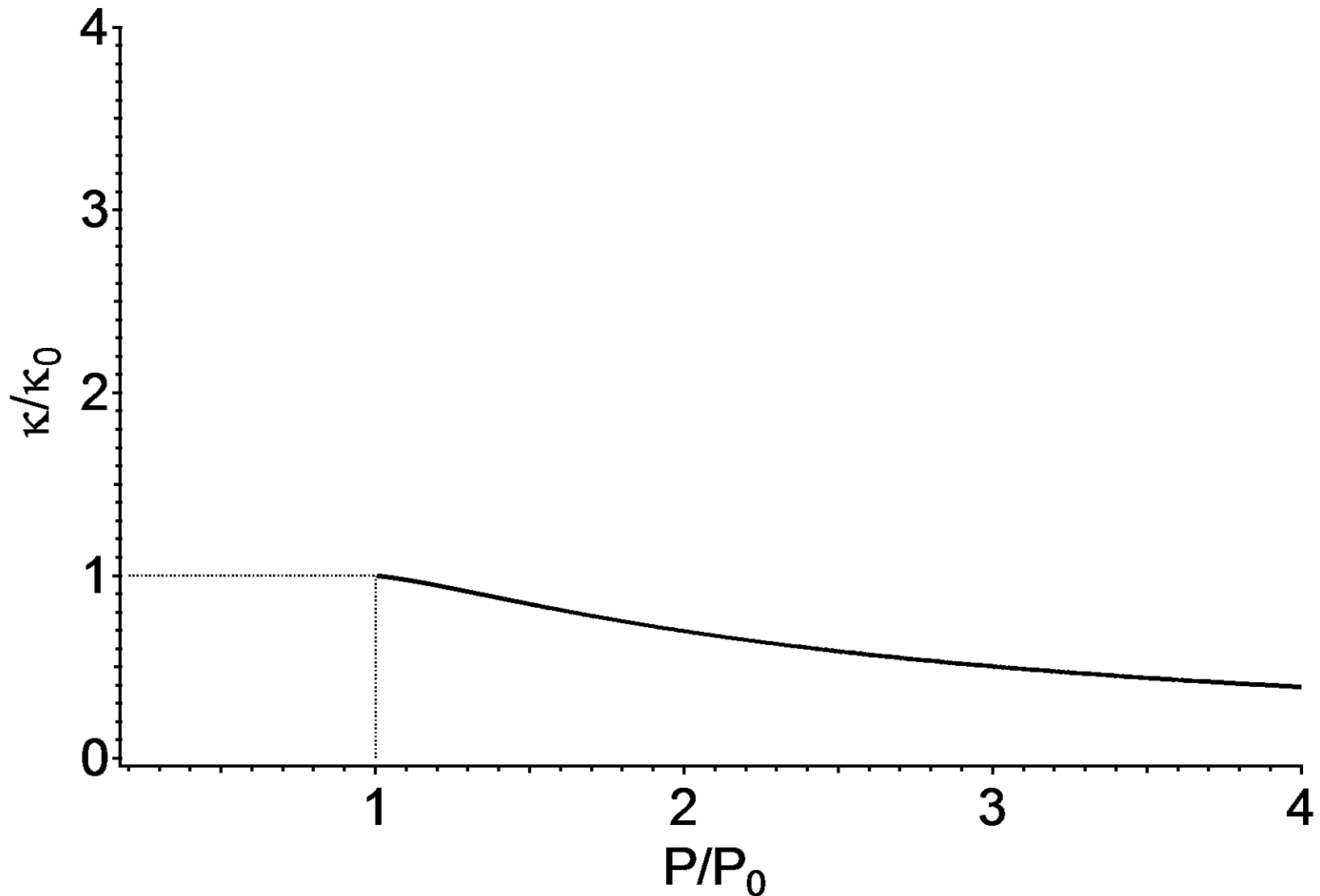
Normalize compressibility & pressure via known density:

$$\tilde{\kappa} = \frac{\kappa}{\kappa_0} = \frac{d\varepsilon_F}{d\mu} = -\frac{d\varepsilon_F}{dV} \qquad \tilde{p} = \frac{P}{P_0}$$
$$\kappa_0 = \frac{3}{2} \frac{1}{n\varepsilon_F} \qquad P_0 = \frac{2}{5} n\varepsilon_F$$

For a scale-invariant system

$$\tilde{\kappa} = \tilde{\kappa}(\tilde{p})$$

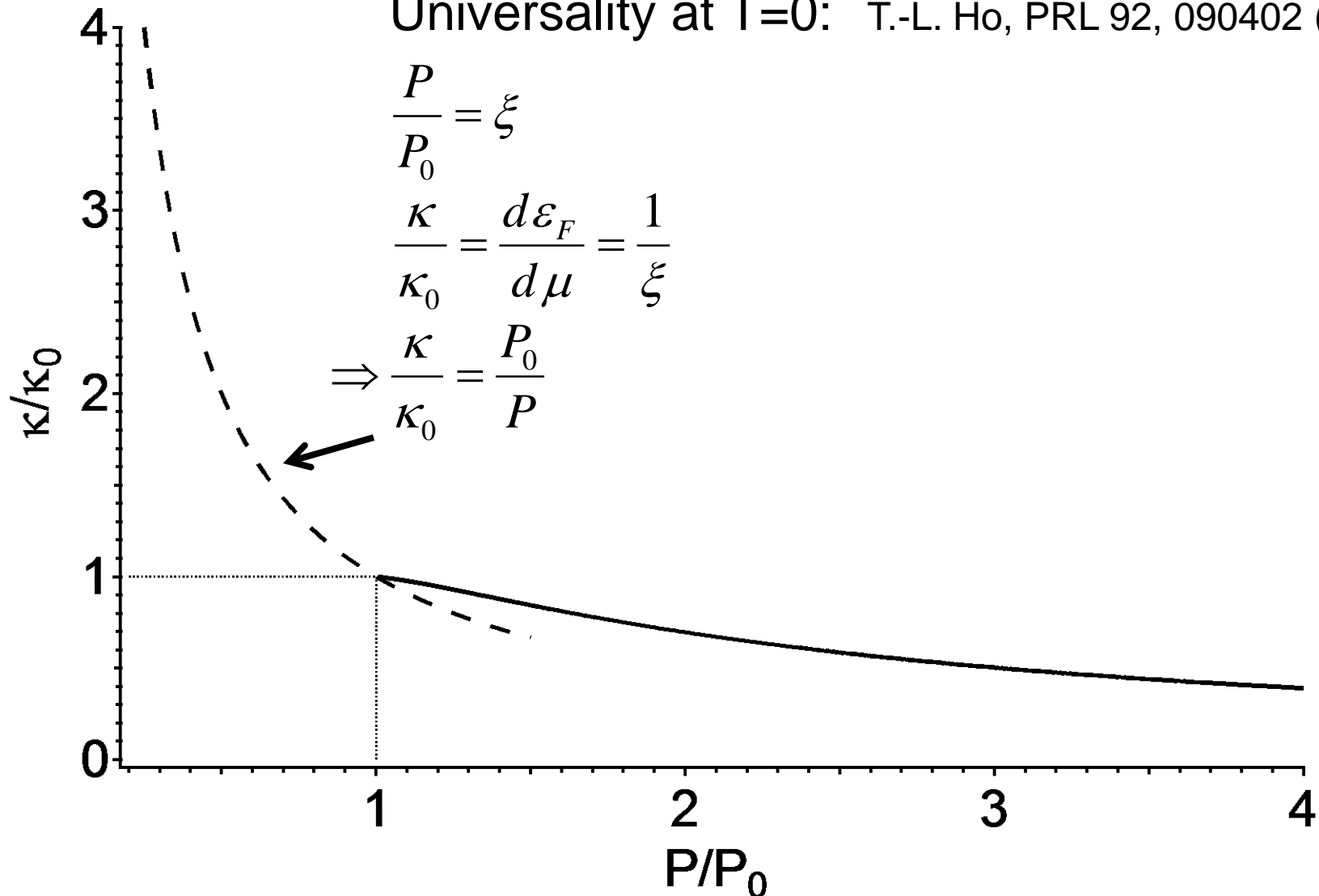
Compressibility Equation of State



Mark J. H. Ku, Ariel T. Sommer, Lawrence W. Cheuk, Martin W. Zwierlein
Science **335**, 563-567 (2012)

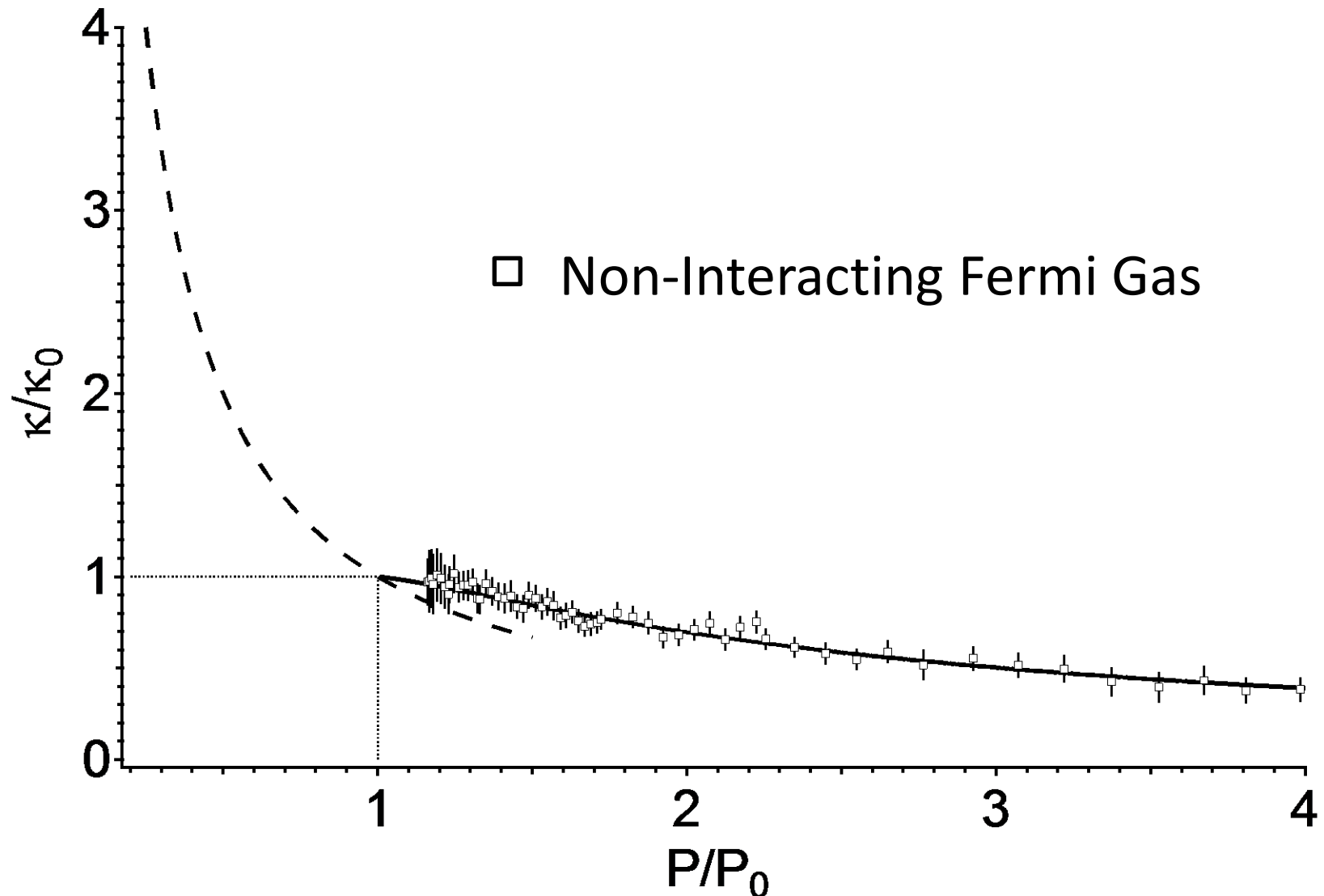
Compressibility Equation of State

Universality at $T=0$: T.-L. Ho, PRL 92, 090402 (2004)



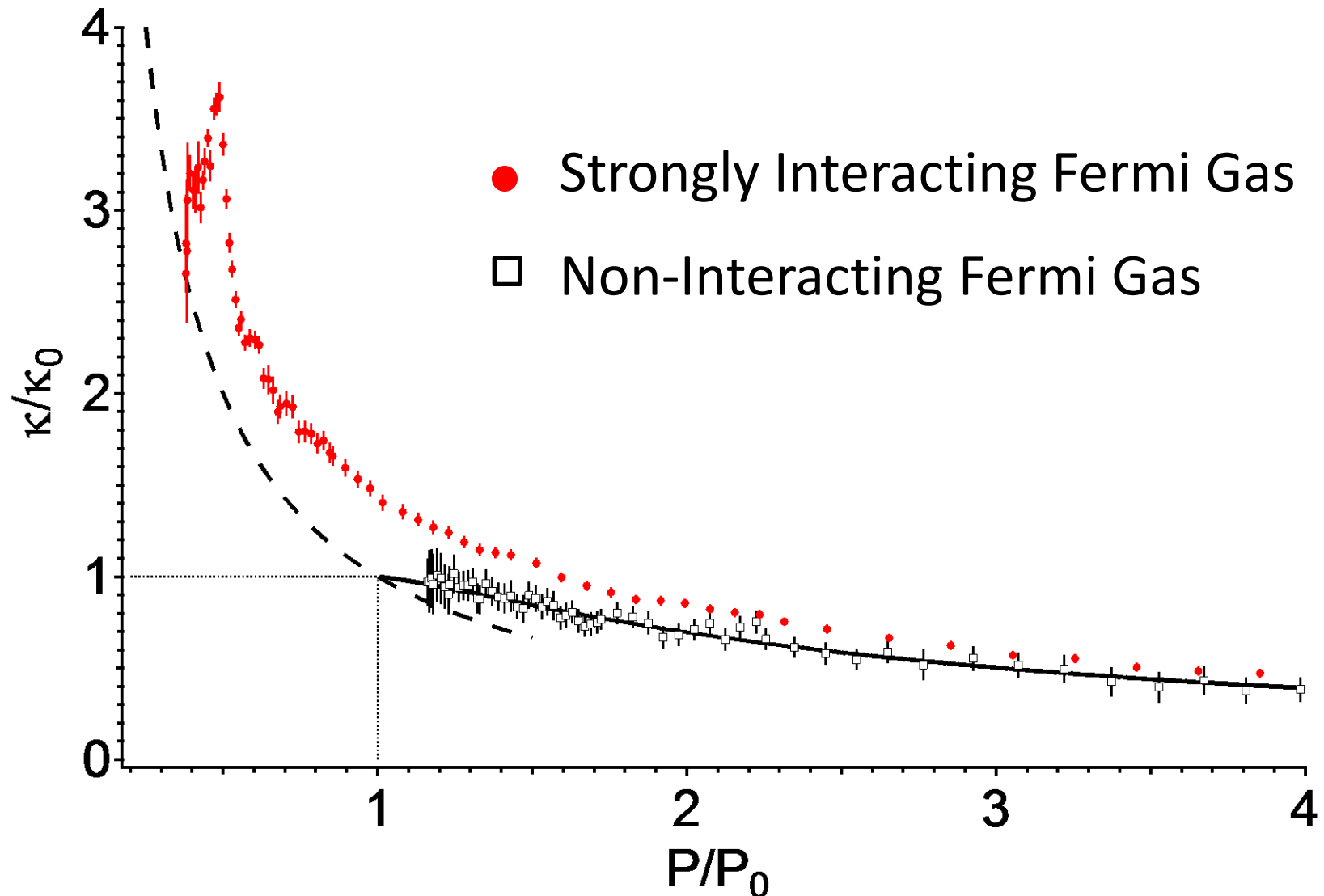
Mark J. H. Ku, Ariel T. Sommer, Lawrence W. Cheuk, Martin W. Zwierlein
Science **335**, 563-567 (2012)

Compressibility Equation of State

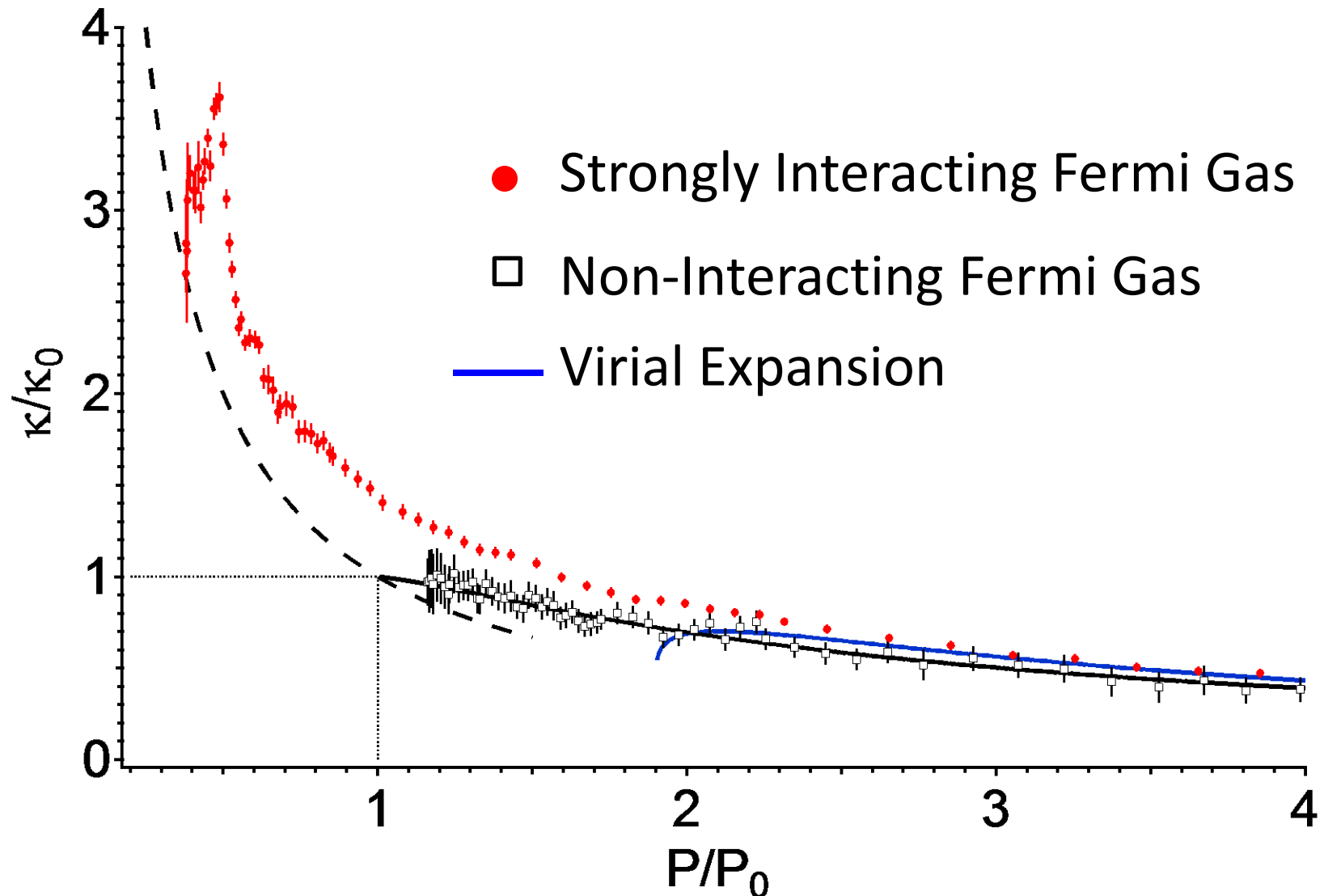


Mark J. H. Ku, Ariel T. Sommer, Lawrence W. Cheuk, Martin W. Zwierlein
Science **335**, 563-567 (2012)

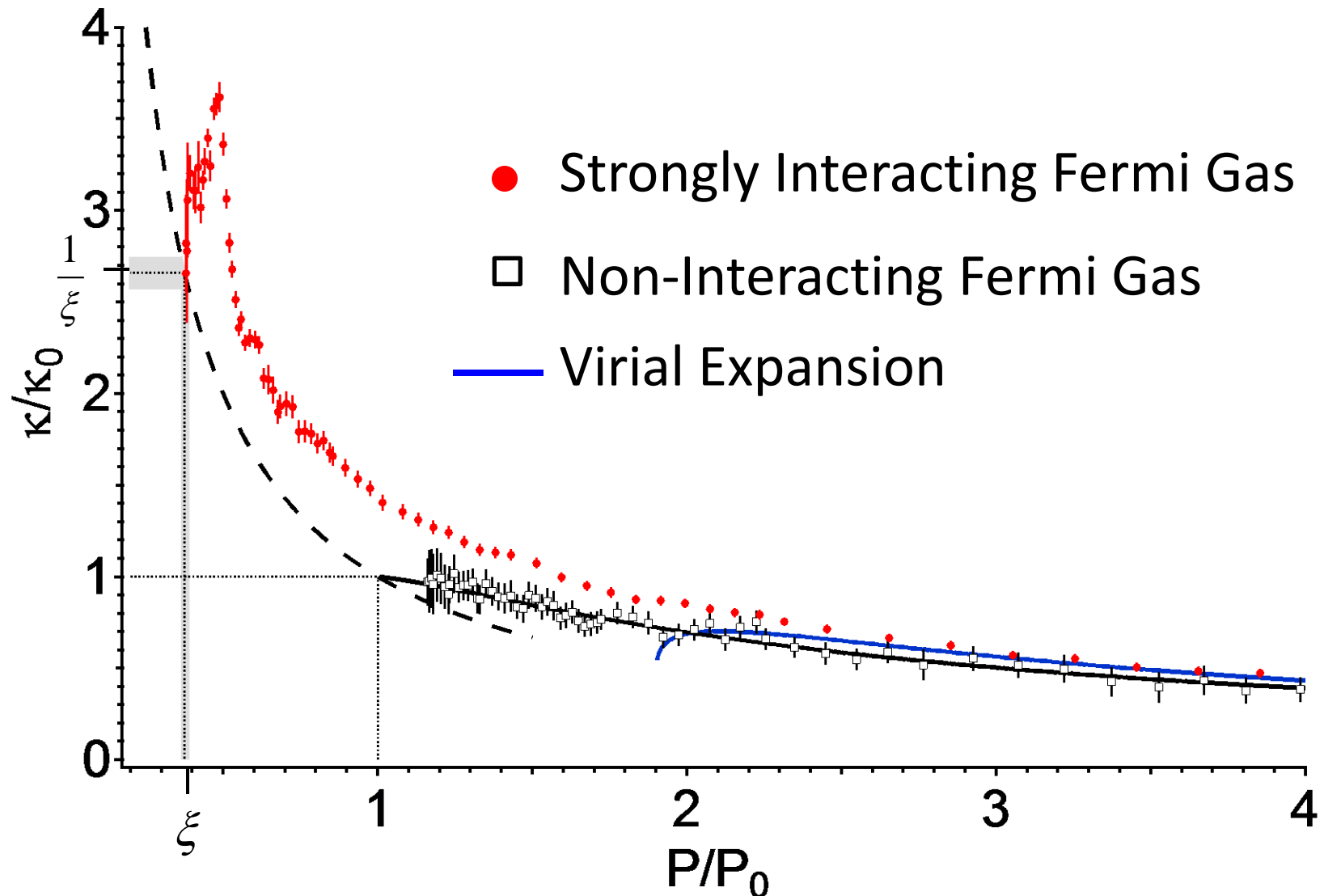
Compressibility Equation of State



Compressibility Equation of State



Compressibility Equation of State



Compressibility Equation of State

$$\kappa(n, P) \longrightarrow \kappa(n, T)$$

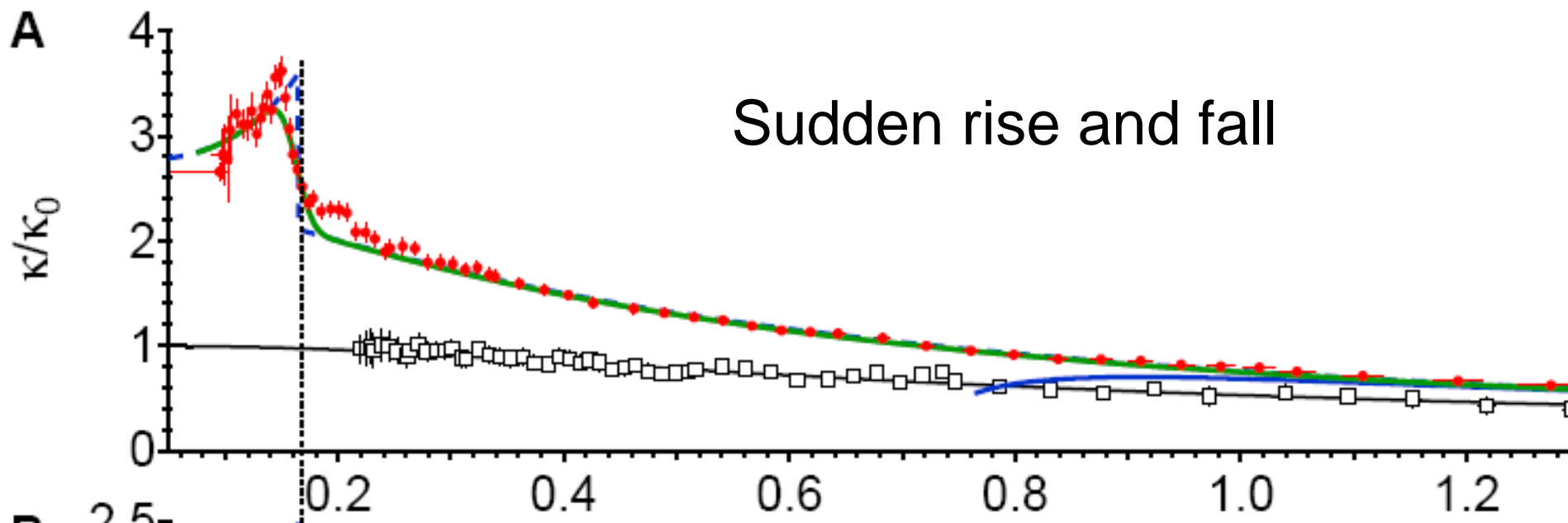
Getting the temperature scale:

$$\left. \frac{d(P/P_0)}{d(T/T_F)} \right|_T = \frac{5 T_F}{2 T} \left(\frac{P}{P_0} - \frac{\kappa_0}{\kappa} \right)$$

$$\frac{T}{T_F} = \frac{T_i}{T_F} \exp \left\{ \frac{2}{5} \int_{\tilde{p}_i}^{\tilde{p}} d\tilde{p} \frac{1}{\tilde{p} - \frac{1}{\tilde{\kappa}}} \right\}$$

Compressibility Equation of State

Compressibility

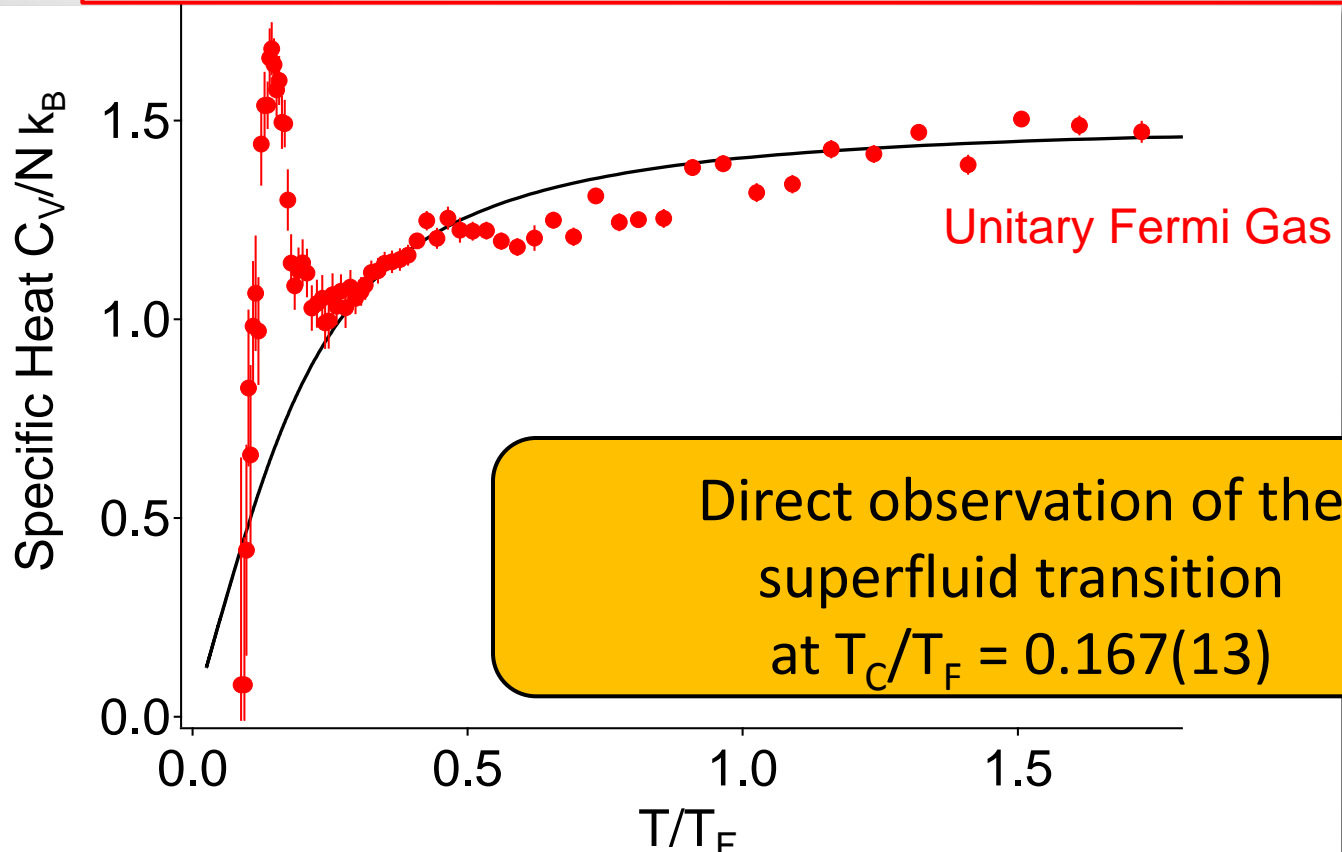


Heat capacity

For a resonant gas:

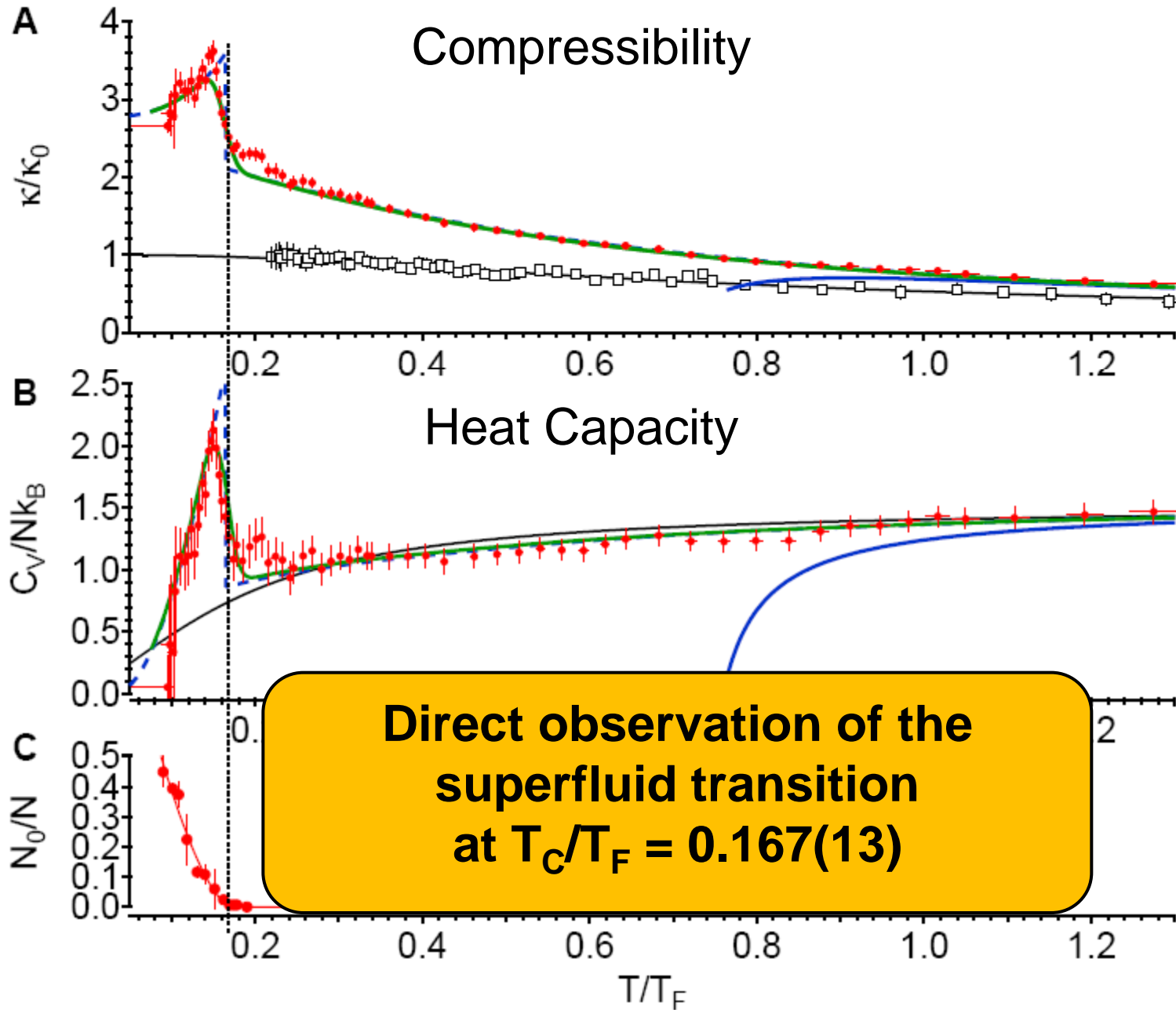
$$P = \frac{2}{3} \frac{E}{V}$$

$$\frac{C_V}{Nk_B} = \frac{d(E/Nk_B)}{dT} \Big|_{N,V} = \frac{d(P/nE_F)}{d(T/T_F)} = \frac{3}{2} \frac{T_F}{T} \left(\frac{P}{P_0} - \frac{\kappa_0}{\kappa} \right)$$

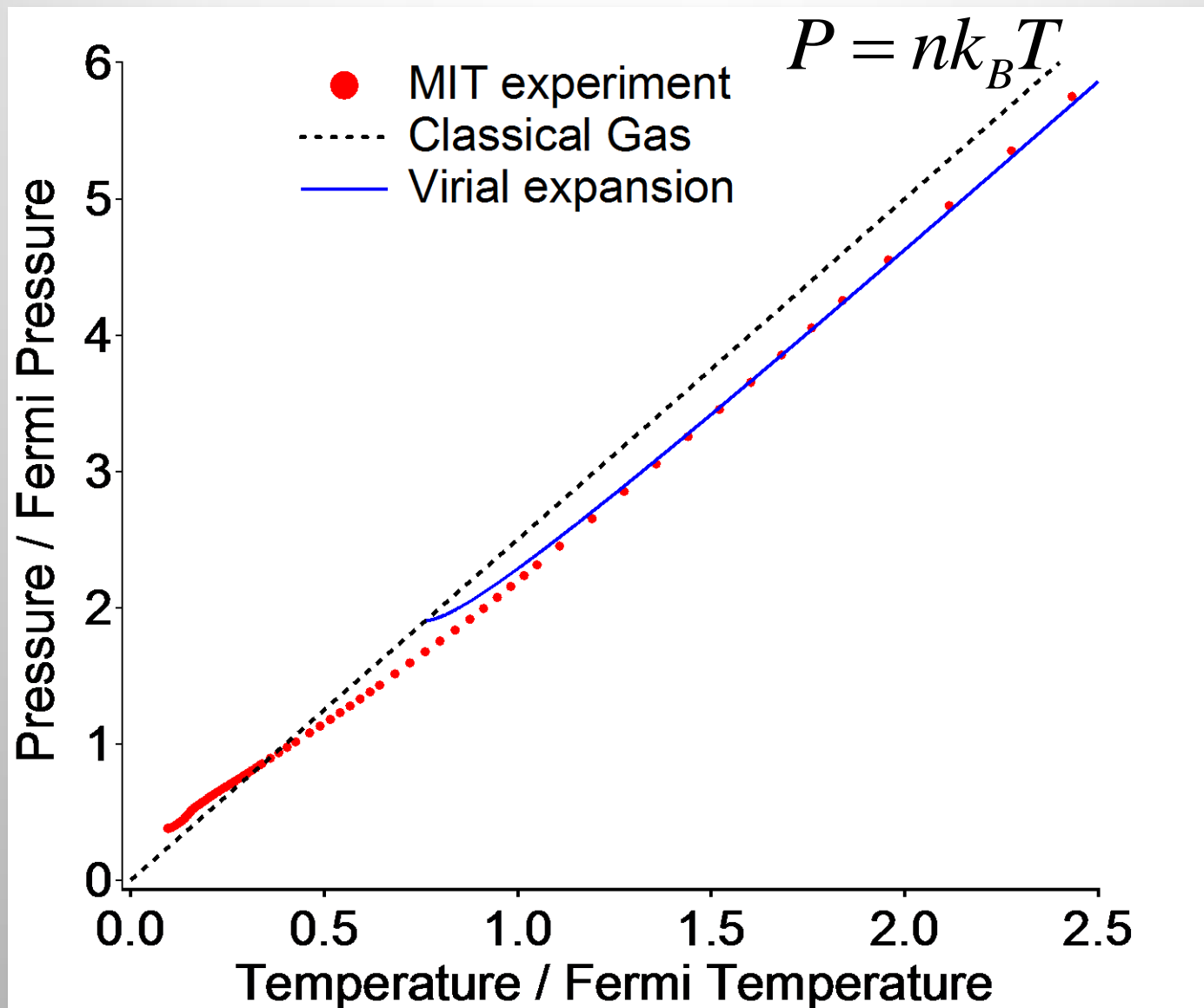


Scaled to the density of electrons in a solid, superfluidity would occur far above room temperature

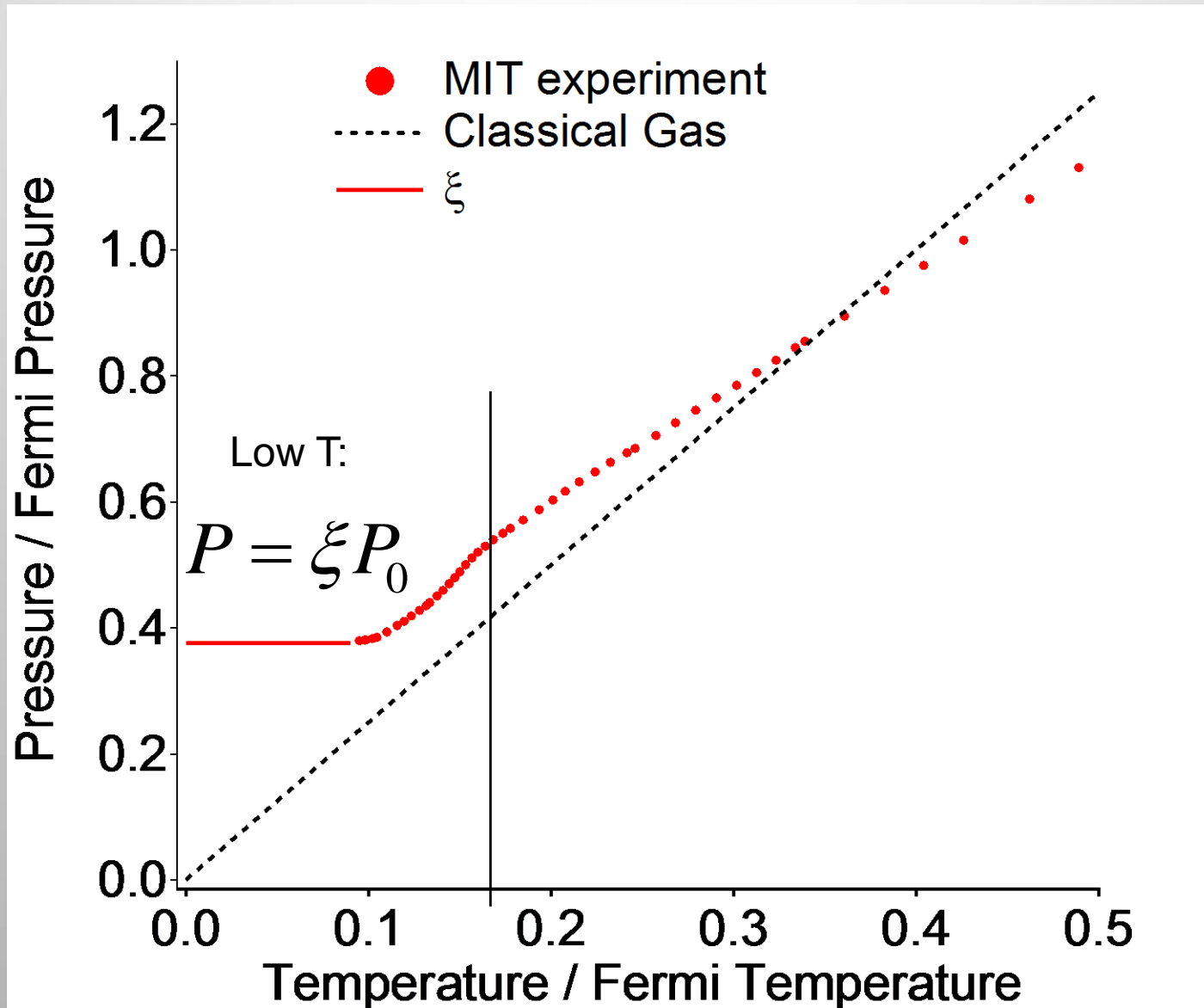
Compressibility, Heat Capacity, Condensates



Equation of state of a resonant Fermi gas



Equation of state of a resonant Fermi gas



New value for ξ : $\xi = 0.376(5)[8]$

Compressibility Equation of State

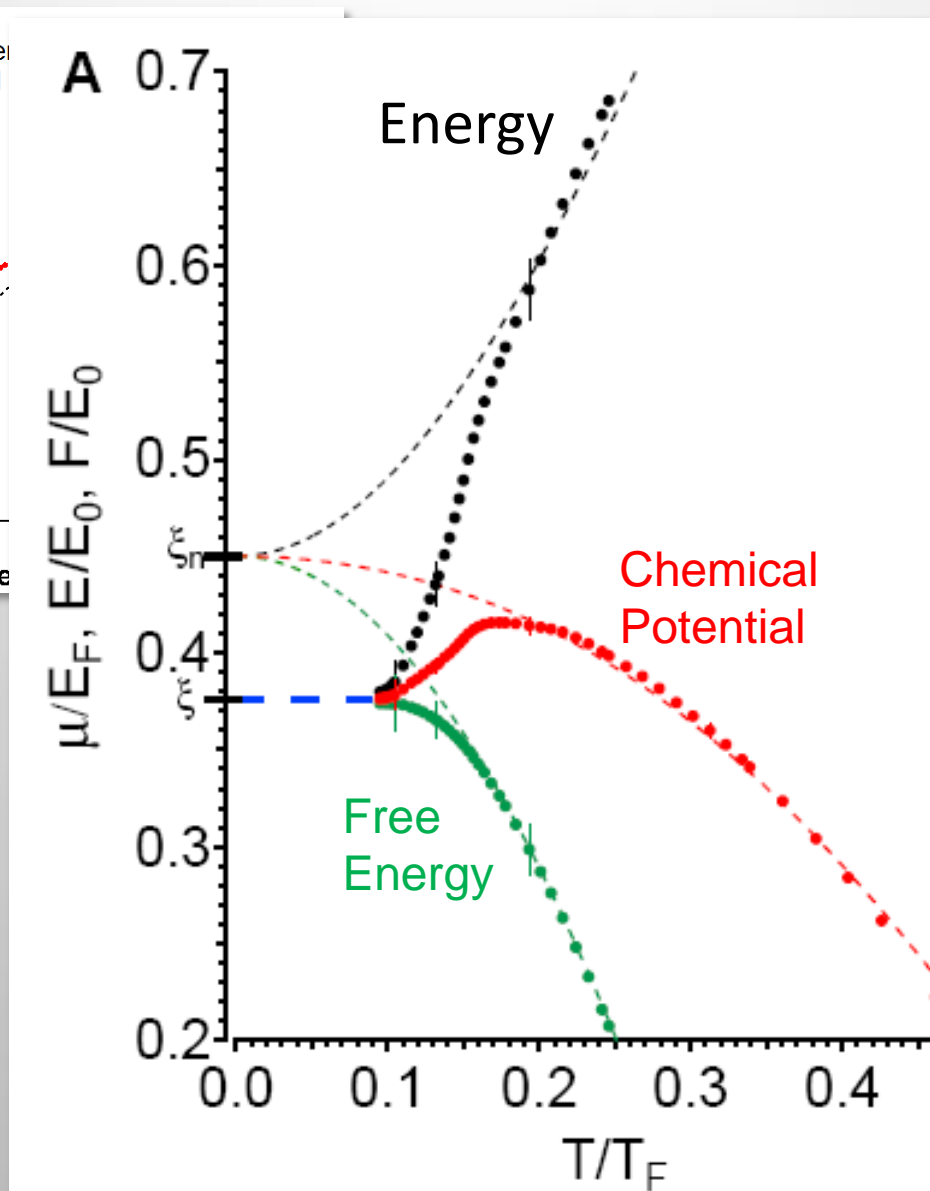
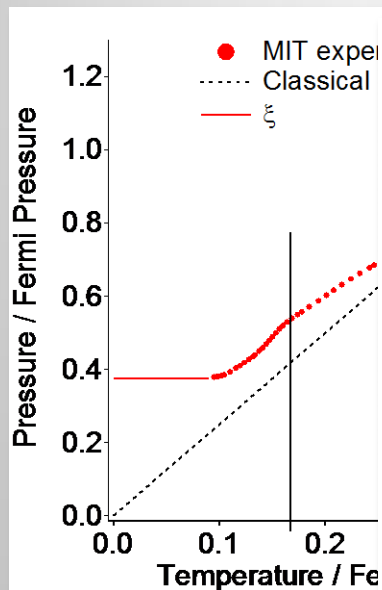
Going back to Density Equation of State

$$\tilde{\kappa} = \frac{d\varepsilon_F}{d\mu} = -\frac{T_F^2}{T^2} \frac{d(T/T_F)}{d(\beta\mu)}$$

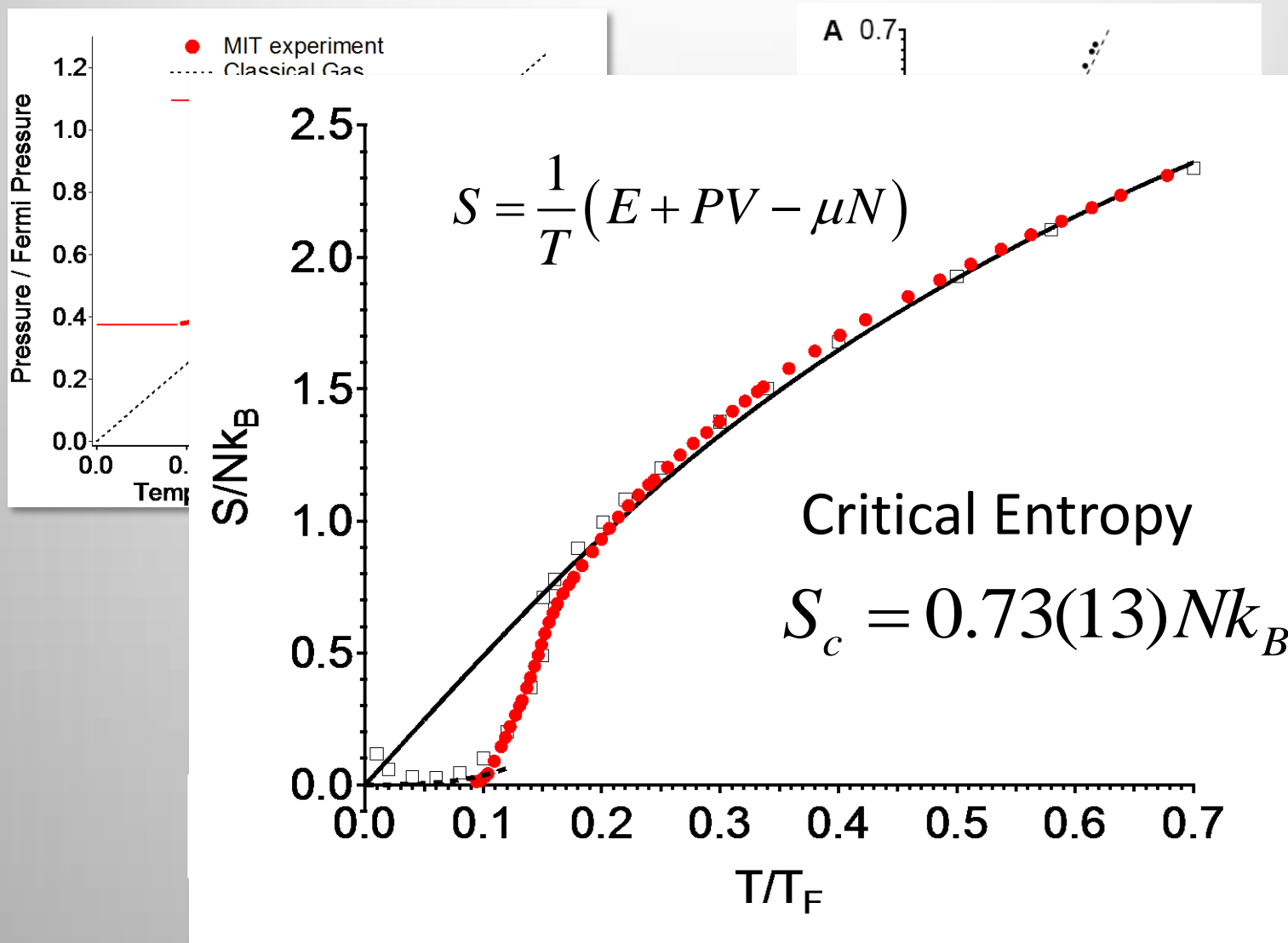
$$\beta\mu = \beta\mu_i - \int_{T_i/T_F}^{T/T_F} d(T/T_F) \frac{T_F^2}{T^2} \frac{1}{\tilde{\kappa}}$$

with $\beta\mu_i$ and T_i known at high temperatures

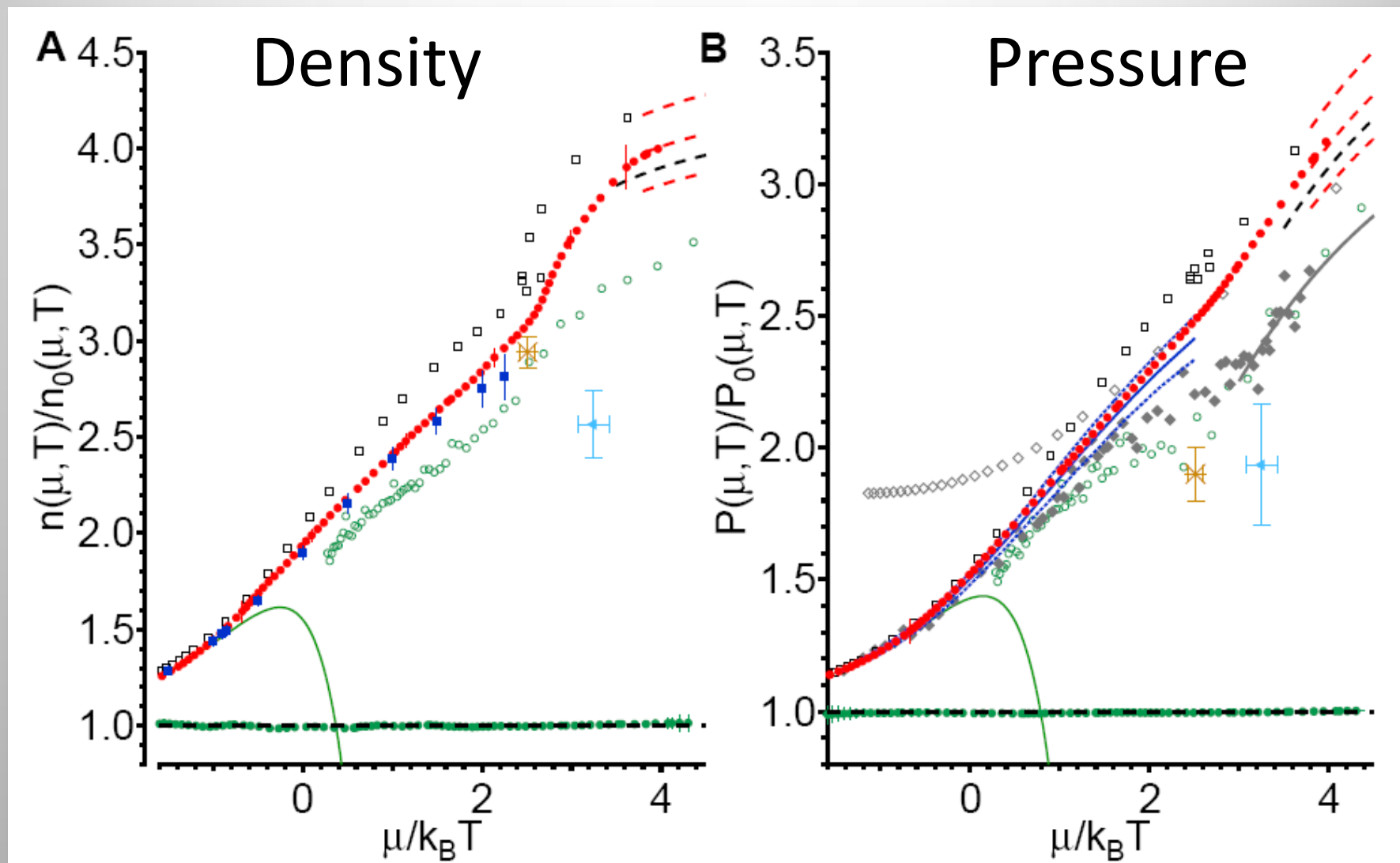
Equation of state of a resonant Fermi gas



Equation of state of a resonant Fermi gas



Equation of state of a resonant Fermi gas



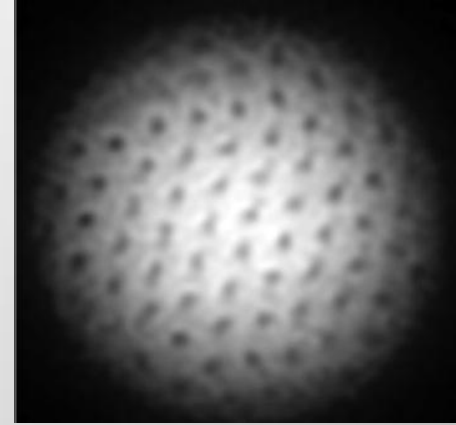
Mark Ku, Ariel Sommer, Lawrence Cheuk, MWZ, Science **335**, 563-567 (2012)

K. Van Houcke, F. Werner, E. Kozik, N. Prokofev, B. Svistunov,

M. Ku, A. Sommer, L. Cheuk, A. Schirotzek, MWZ, Nature Physics **8**, 366 (2012)

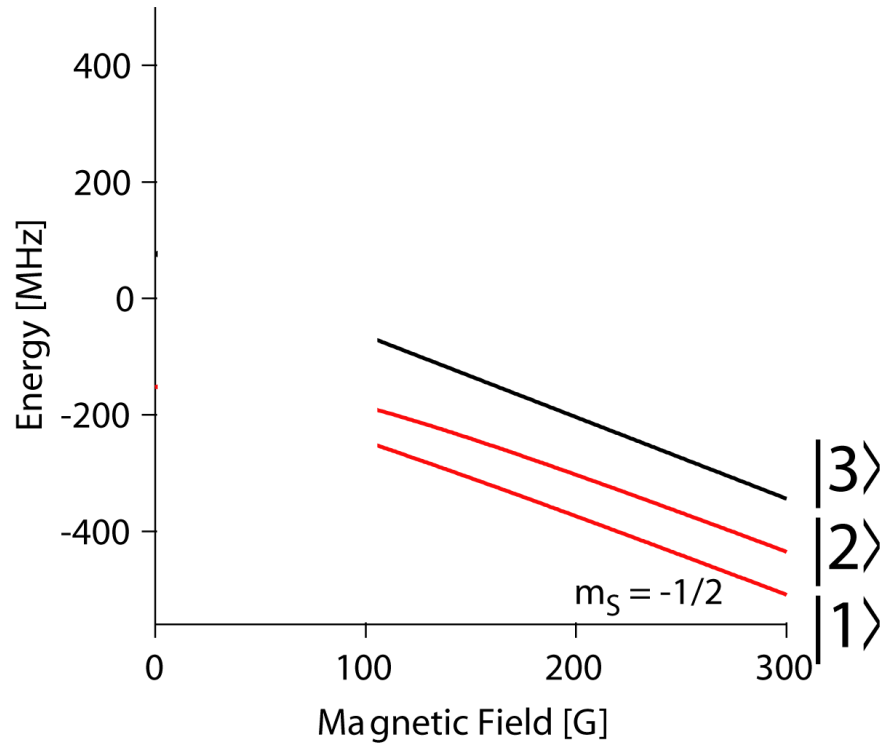
Binding Energy of Pairs

A Gedanken experiment



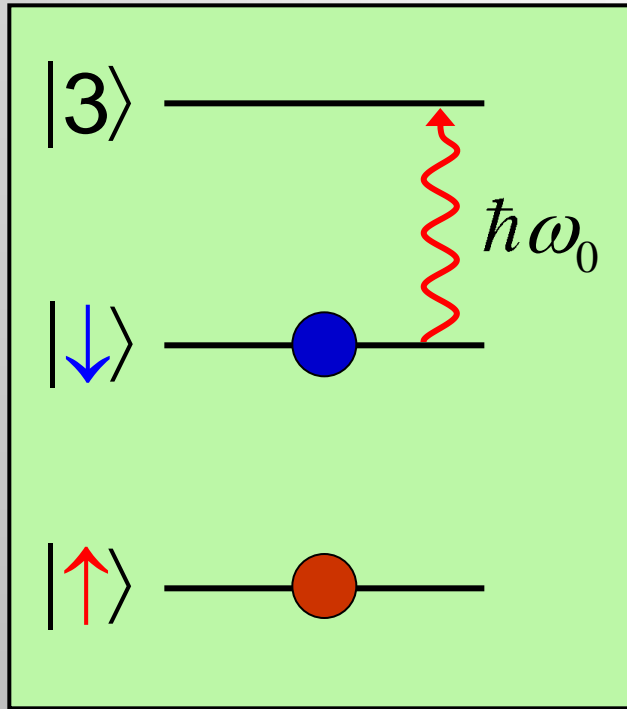
What is the energy cost of removing one fermion from the superfluid?

Radiofrequency spectroscopy

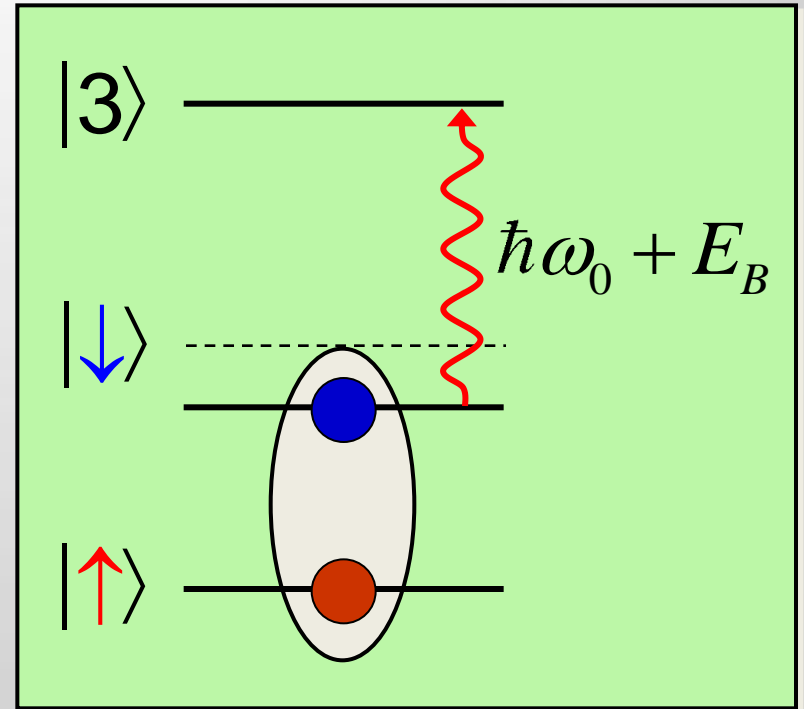


Radiofrequency spectroscopy

No interactions



Molecular Pairing

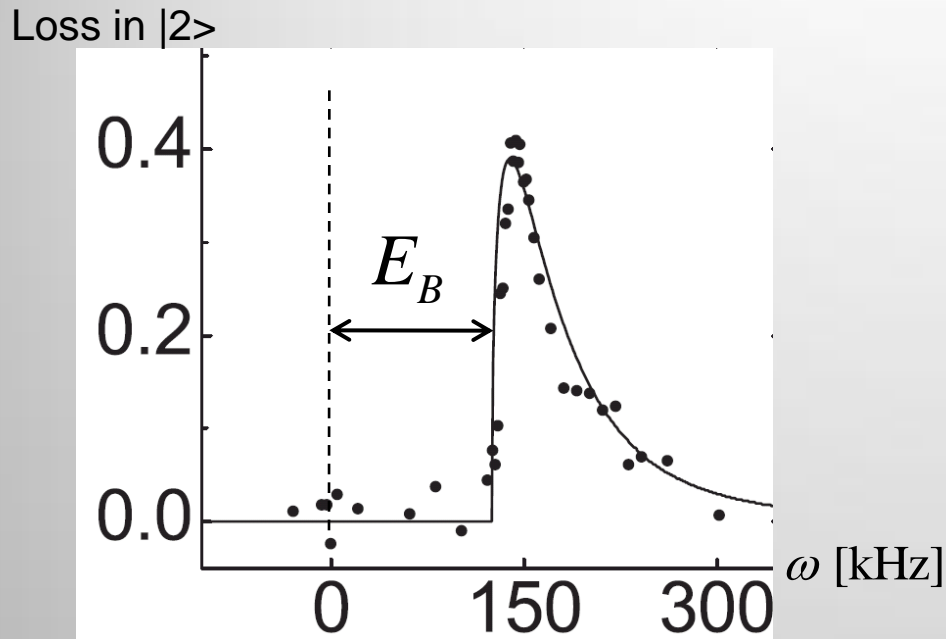


Photon energy = Zeeman + Binding + Kinetic energy

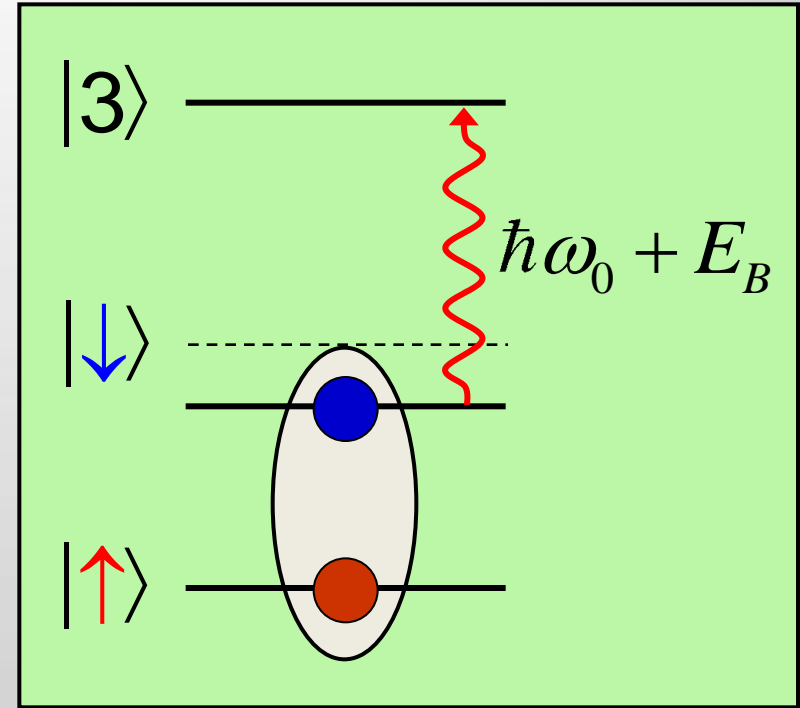
$$\hbar\omega = \hbar\omega_0 + E_B + 2\varepsilon_k$$

Radiofrequency spectroscopy

Molecular Pairing



C.Chin et al. Science, **305**,
1128 (2004)

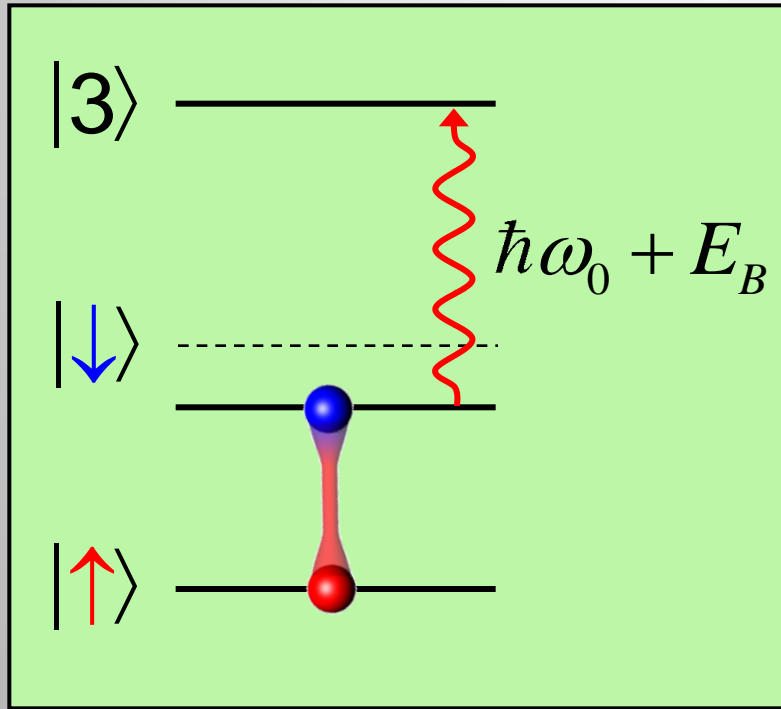


Photon energy = Zeeman + Binding + Kinetic energy

$$\hbar\omega = \hbar\omega_0 + E_B + 2\varepsilon_k$$

Radiofrequency spectroscopy

BCS pairing is a many-body affair. Does the picture still hold?



Energy gain due to pairing (BCS):

$$\delta E(N) = E_{SF}(N) - E_{Normal}(N) = -\frac{3}{8} N \frac{\Delta^2}{E_F}$$

Binding energy per particle:

$$E_B = \frac{\delta E(N+2) - \delta E(N)}{2} = -\frac{\Delta^2}{2E_F}$$

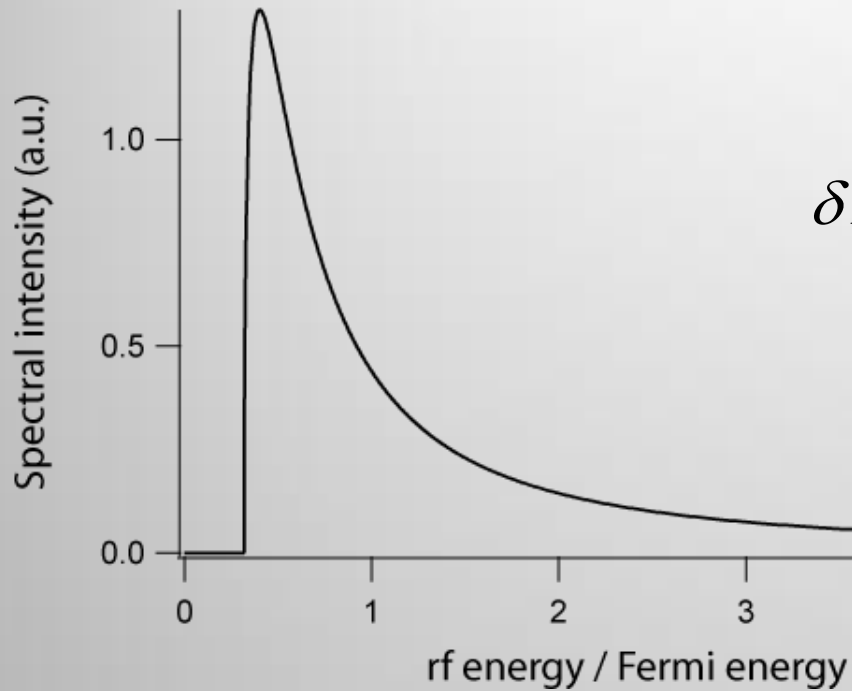
Photon energy = Zeeman + Quasiparticle + Kinetic energy

$$\hbar\omega = \hbar\omega_0 + E_k - \mu + \varepsilon_k$$

Onset at
$$\hbar\omega = \hbar\omega_0 + \frac{\Delta^2}{2E_F}$$

Radiofrequency spectroscopy

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$$\hbar\omega = \hbar\omega_0 + \frac{\Delta^2}{2E_F}$$

Radiofrequency spectroscopy

Fermi's Golden Rule:

$$\Gamma(\omega) \equiv \frac{2\pi}{\hbar} \sum_f \left| \langle f | \hat{V} | \Psi_{\text{BCS}} \rangle \right|^2 \delta(\hbar\omega - E_f)$$

Interaction:

$$\hat{V} = V_0 \sum_k c_{k3}^\dagger c_{k\uparrow} + c_{k\uparrow}^\dagger c_{k3}$$

Insert quasiparticle operators:

$$c_{k\uparrow} = u_k \gamma_{k\uparrow} + v_k \gamma_{-k\downarrow}^\dagger$$

Act on BCS-state:

$$c_{k3}^\dagger c_{k\uparrow} | \Psi_{\text{BCS}} \rangle = v_k c_{k3}^\dagger \gamma_{-k\downarrow}^\dagger | \Psi_{\text{BCS}} \rangle$$

And thus:

$$\hat{V} | \Psi_{\text{BCS}} \rangle = V_0 \sum_k v_k c_{k3}^\dagger \gamma_{-k\downarrow}^\dagger | \Psi_{\text{BCS}} \rangle$$

Radiofrequency spectroscopy

Fermi's Golden Rule:

$$\Gamma(\omega) \equiv \frac{2\pi}{\hbar} \sum_f \left| \langle f | \hat{V} | \Psi_{\text{BCS}} \rangle \right|^2 \delta(\hbar\omega - E_f)$$

Possible final states: $|k\rangle \equiv c_{k3}^\dagger \gamma_{-k\uparrow} |\Psi_{\text{BCS}}\rangle$

RF Photon provides: $\hbar\Omega(k) = \hbar\omega_{\uparrow 3} + E_k + \epsilon_k - \mu$

Invert: $\hbar\Omega(k) = \hbar\omega_{\uparrow 3} + E_k + \epsilon_k - \mu$ to get ϵ_k in terms of ω

Then: $\delta(\hbar\omega - \hbar\Omega(k)) = \frac{1}{\hbar} \frac{d\epsilon_k}{d\Omega} \delta(\epsilon_k - \epsilon(\omega))$

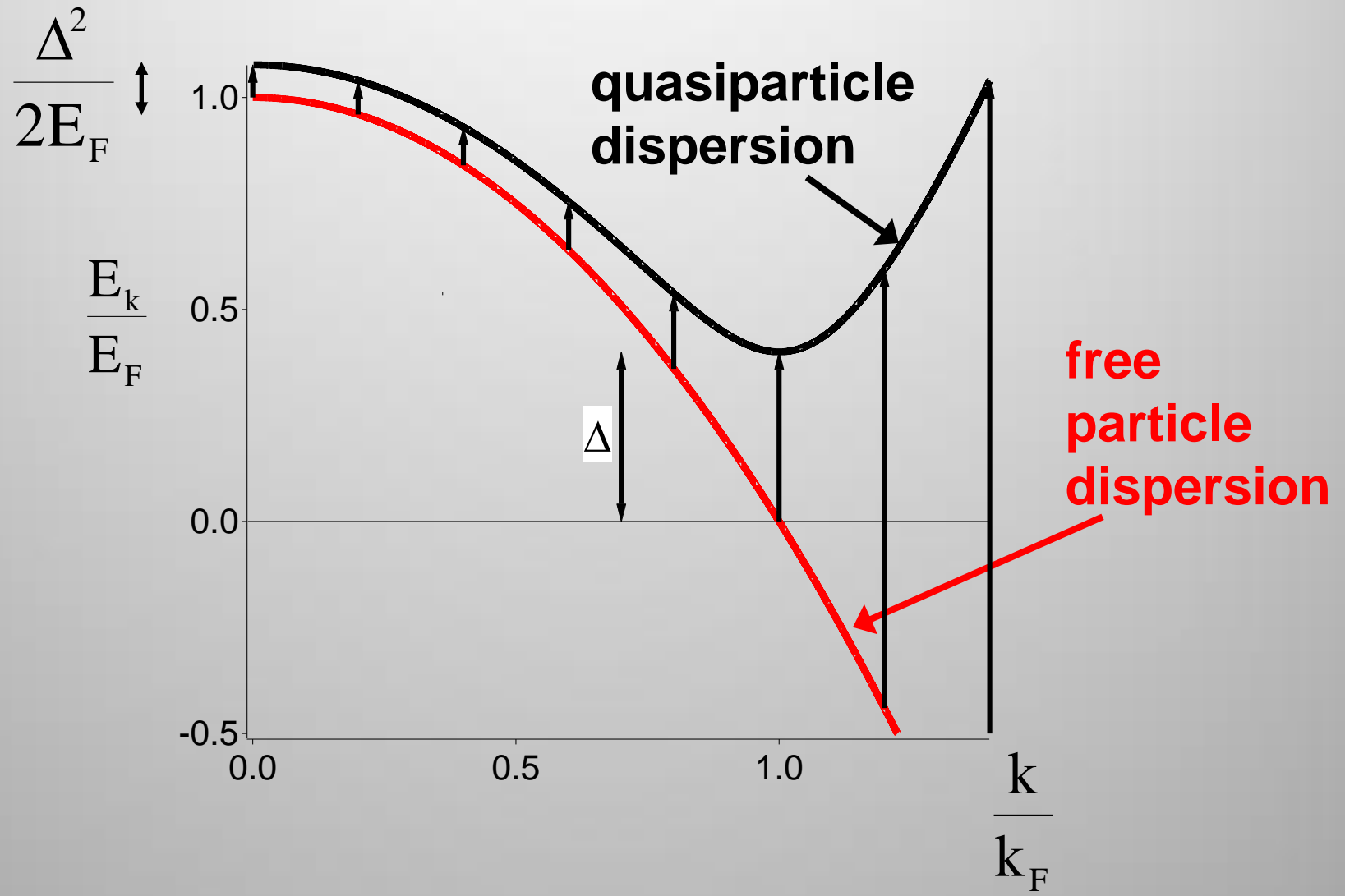
But: $\frac{d\Omega}{d\epsilon_k} = \frac{\xi_k}{E_k} + 1 = 2u_k^2$

And the final RF spectrum becomes:

$$\Gamma(\omega) = \frac{\pi}{\hbar} V_0^2 \rho(\epsilon_k) \left. \frac{v_k^2}{u_k^2} \right|_{\epsilon_k = \epsilon(\omega)} = \pi N_p V_0^2 \rho(\epsilon_k) |\varphi_k|^2 \Big|_{\epsilon_k = \epsilon(\omega)}$$

Radiofrequency spectroscopy

$$\hbar\Omega = \hbar\omega_{\uparrow 3} + E_k - (\mu - \varepsilon_k)$$



Radiofrequency spectroscopy

$$\Gamma(\omega) = \pi N_p V_0^2 \rho(\epsilon_k) |\varphi_k|^2 \Big|_{\epsilon_k = \epsilon(\omega)}$$

$$\hbar\omega_{\text{th}} = \sqrt{\mu^2 + \Delta^2} - \mu \rightarrow \begin{cases} \frac{\Delta^2}{2E_F} & \text{in the BCS-limit} \\ 0.31E_F & \text{on resonance} \\ |E_B| = \frac{\hbar^2}{ma^2} & \text{in the BEC-limit} \end{cases}$$

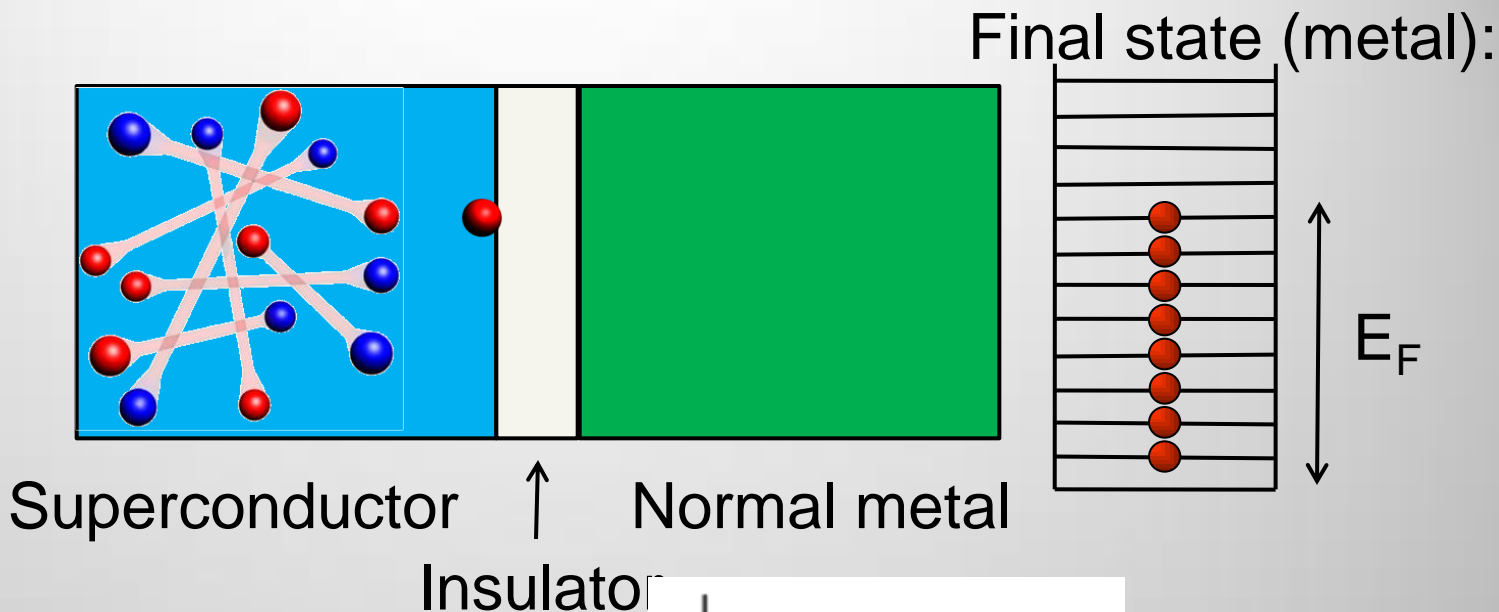
Explicitly:

$$\Gamma(\omega) = \frac{3\pi}{4\sqrt{2}\hbar} \frac{N V_0^2 \Delta^2}{E_F^{3/2}} \frac{\sqrt{\hbar\omega - \hbar\omega_{\text{th}}}}{\hbar^2\omega^2} \sqrt{1 + \frac{\omega_{\text{th}}}{\omega} + \frac{2\mu}{\hbar\omega}}$$

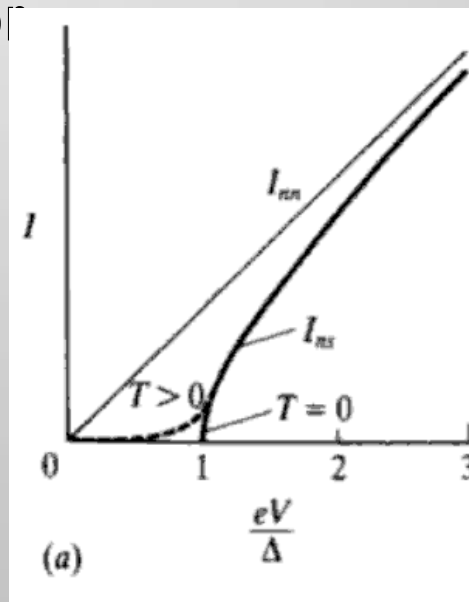
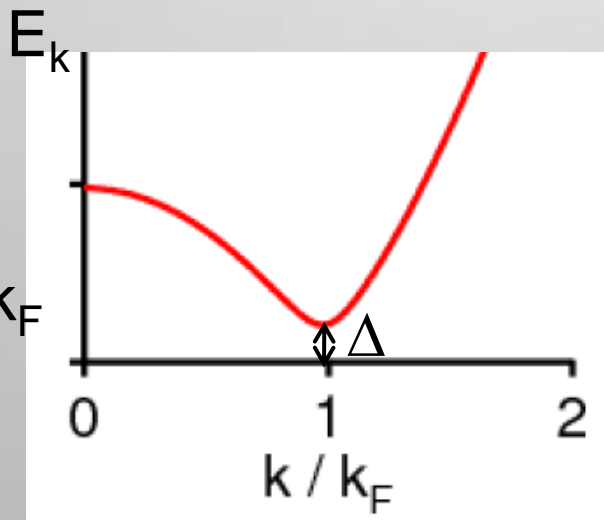
BEC-limit:

$$\Gamma_{\text{BEC}}(\omega) = \frac{4}{\hbar} N_M V_0^2 \sqrt{|E_B|} \frac{\sqrt{\hbar\omega - |E_B|}}{\hbar^2\omega^2}$$

Connection to tunneling experiments



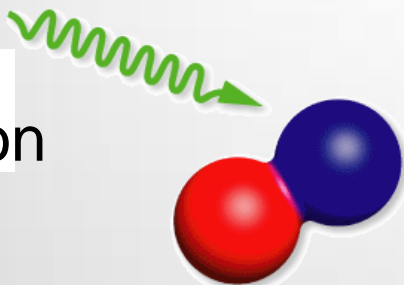
Transfer
at Fermi
surface $k=k_F$



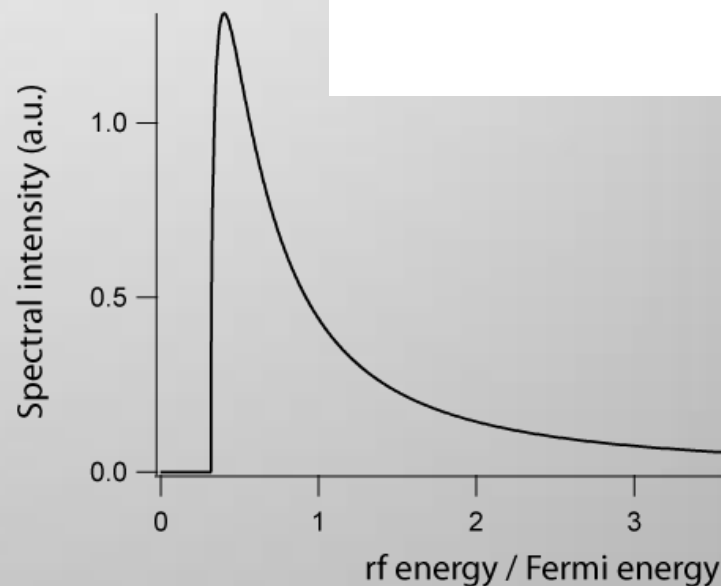
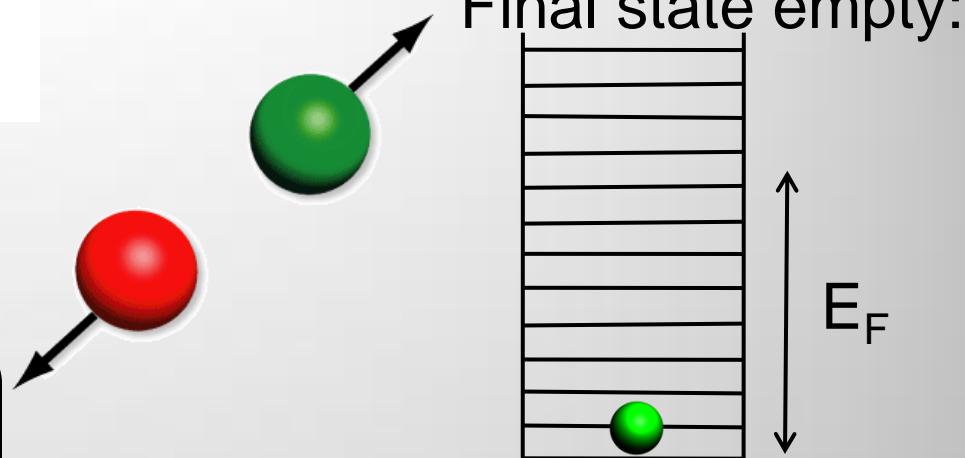
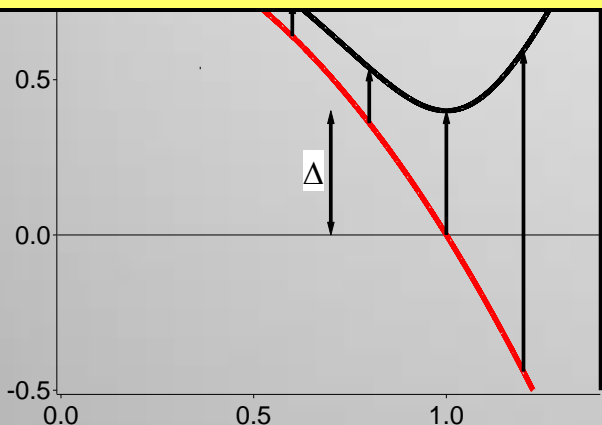
Onset at: $eV = \Delta$

Connection to tunneling experiments

RF Photon



RF spectroscopy directly measures the binding energy of fermion pairs (not the gap)

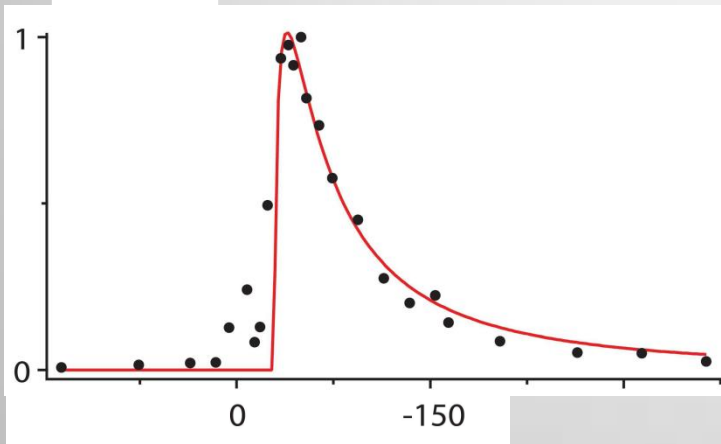


BCS limit, onset at:

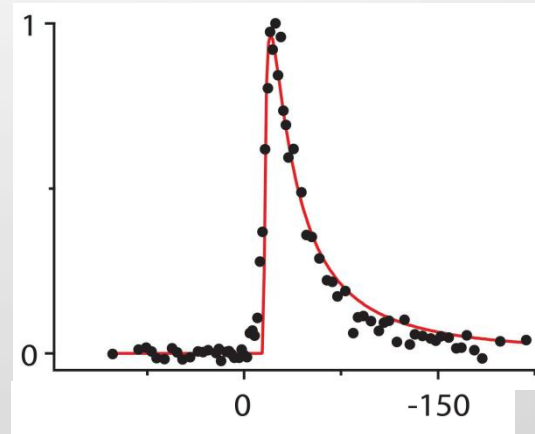
$$\hbar\omega - \hbar\omega_0 = \frac{\Delta^2}{2E_F}$$

Rf Spectra in the BEC-BCS Crossover

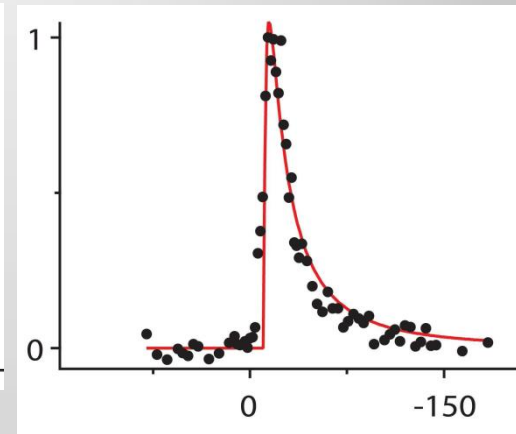
BEC



Unitarity



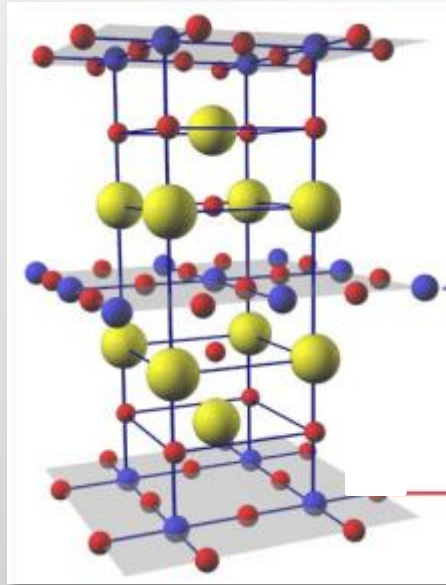
BCS



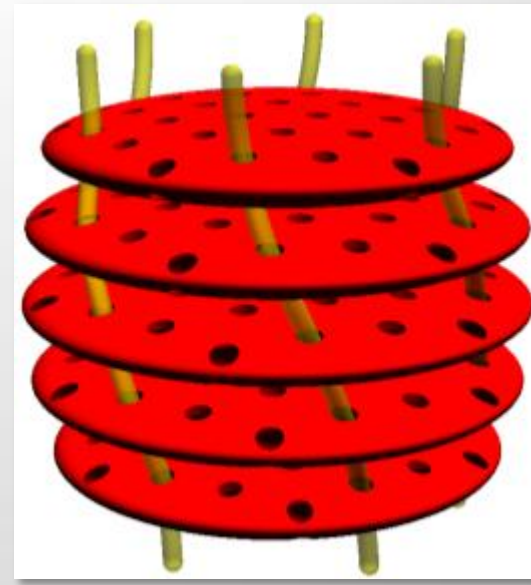
- Determine binding energy spectroscopically
- Infer size of the pairs at unitarity:
about half the interparticle spacing

Christian H. Schunck, Yong-il Shin, Andre Schirotzek,
Wolfgang Ketterle, *Nature* **454**, 739 (2008)

Fermions entering Flatland



**High- T_c Superconductor
with stacks of CuO planes**



**Stacks of 2D coupled
fermionic superfluids**

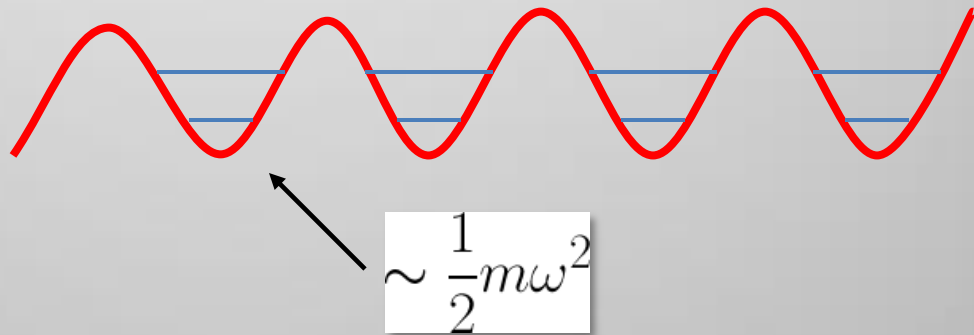
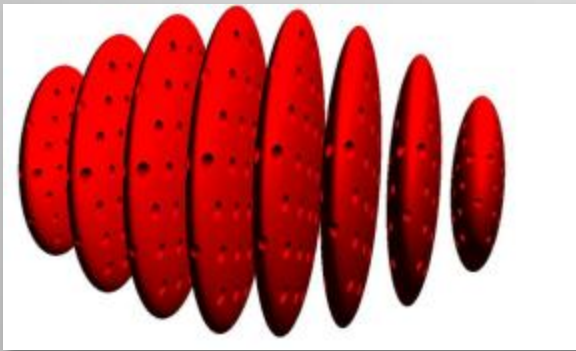
- 2D Fermi Gases: A paradigm of condensed matter physics
- Access physics of layered superconductors
- Evolution of Fermion Pairing from 3D to 2D
- Study superfluidity in lower dimensions

Expts on 2D Bose gases: Ketterle, Dalibard, Cornell, Phillipps, Chin

Expts on 2D Fermi gases: Turlapov, Koehl, Thomas, Vale

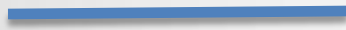
Making quasi-2D Fermi gases

- Confine atoms tightly in one direction until only the ground state is occupied
- Our setup: 1D lattice (retro-reflected laser beam)
- 2D-ness tuned by lattice depth
- Deep lattice: $\frac{\varepsilon_F}{\hbar\omega} \sim 0.1$, aspect ratio of $\sim 1:1000$



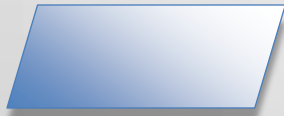
Pairing in 1D, 2D, 3D

- In 1D



Two particles bind for the slightest attraction

- In 2D



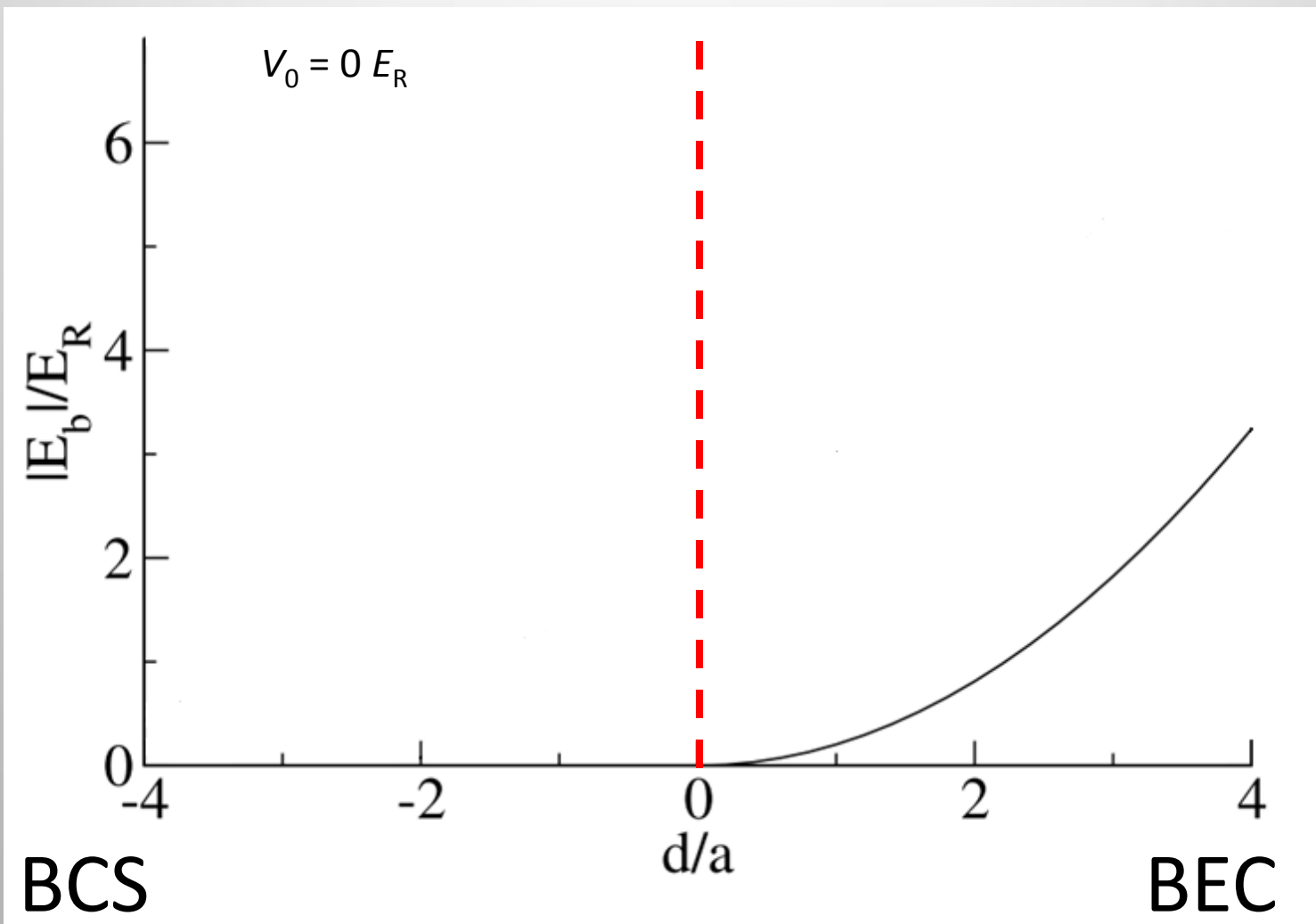
Two particles bind for the slightest attraction
... but binding is exponentially weak

- In 3D

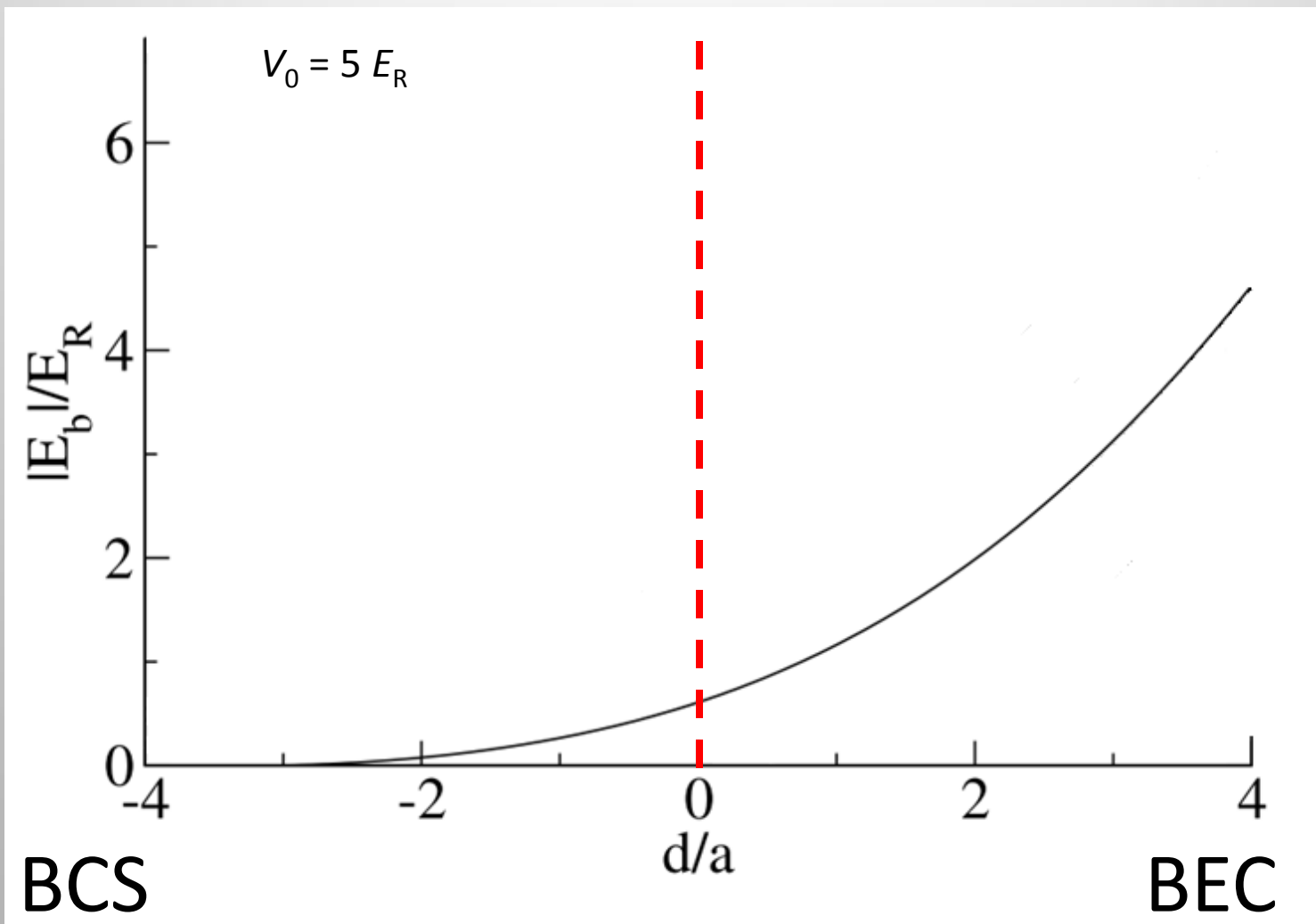


Pairing requires strong attraction, or many-body physics:
The presence of a Fermi sea (Cooper pairing)

Binding Energy from 3D to 2D (1D lattice)

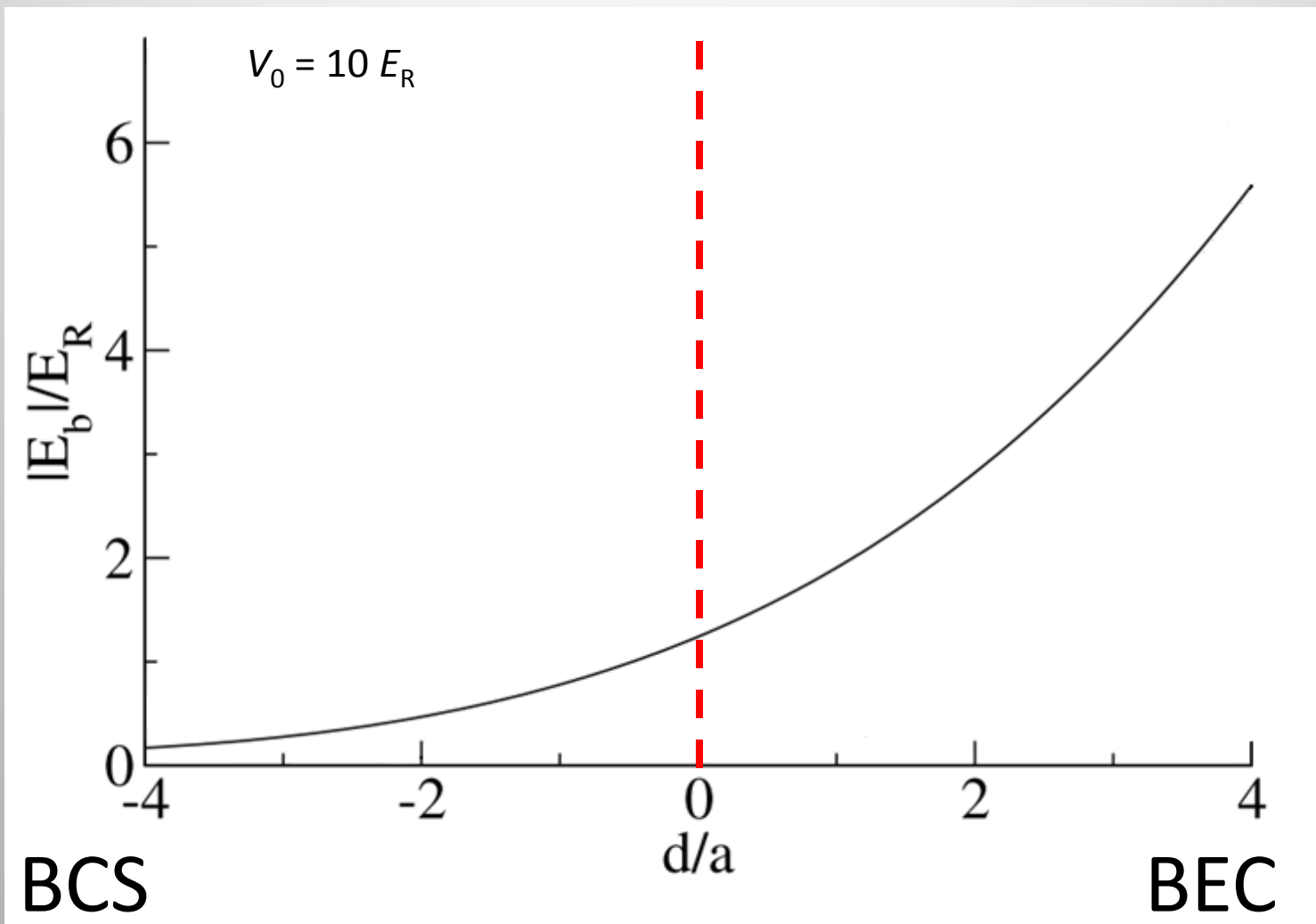


Binding Energy from 3D to 2D (1D lattice)



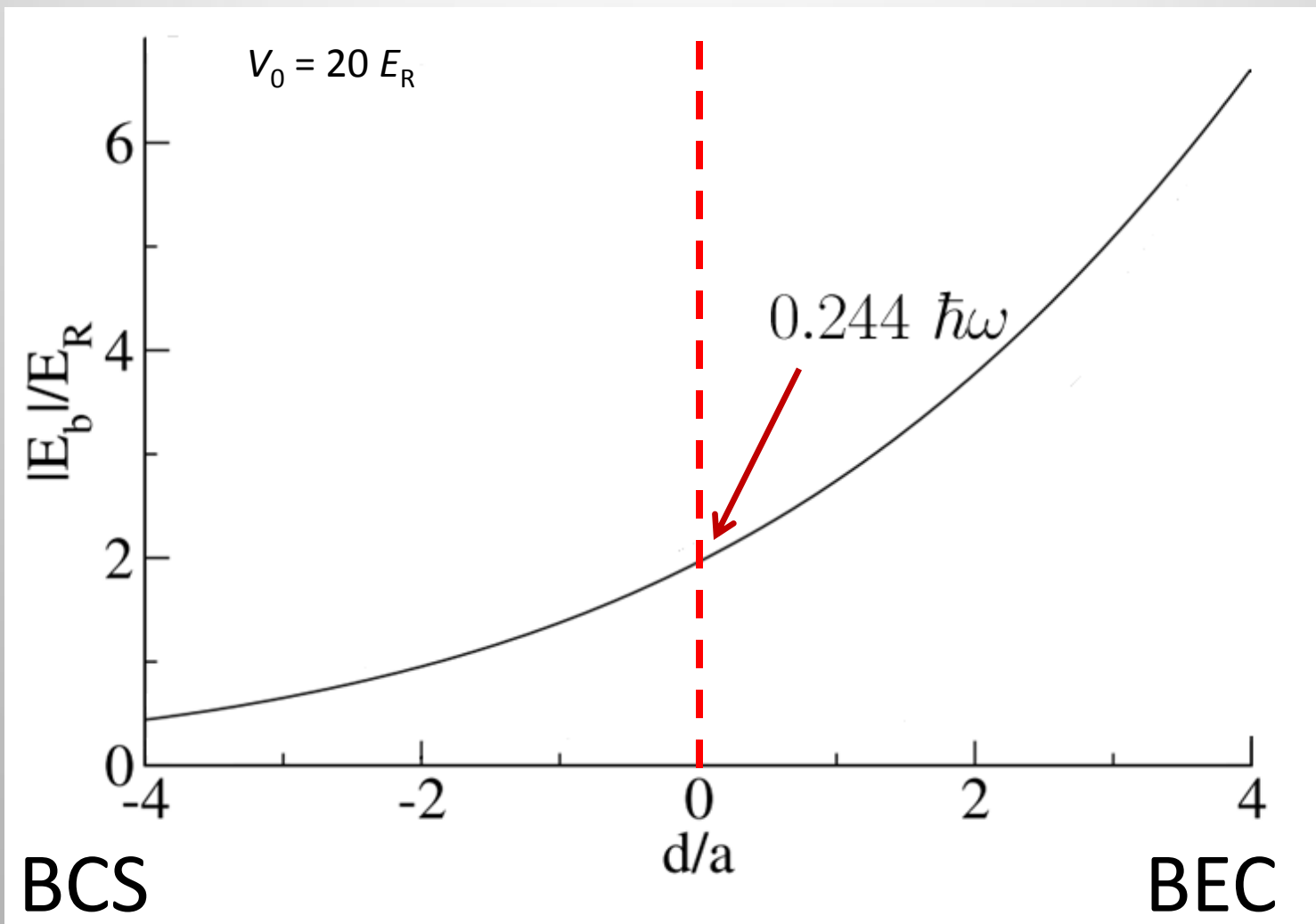
Orso & Wouters. PRL **95** 060402 (2005)

Binding Energy from 3D to 2D (1D lattice)



Orso & Wouters. PRL **95** 060402 (2005)

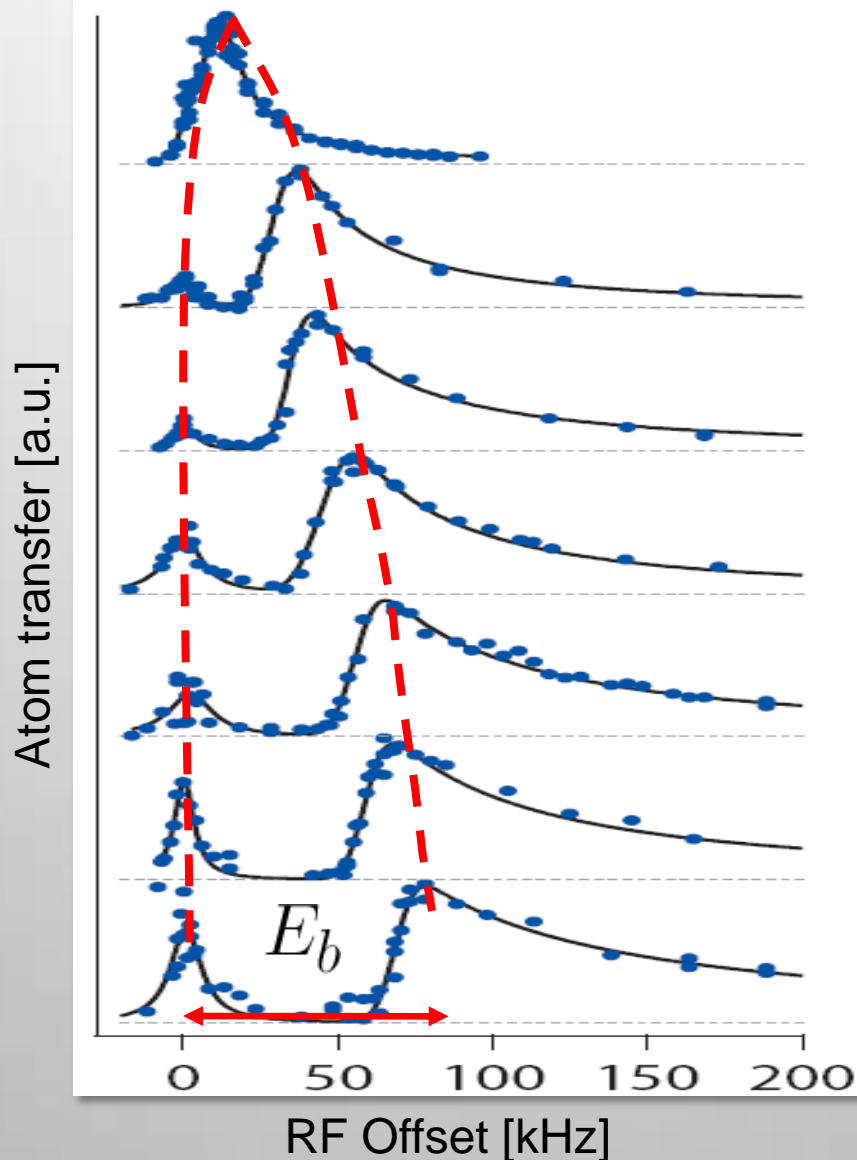
Binding Energy from 3D to 2D (1D lattice)



Orso & Wouters. PRL **95** 060402 (2005)

Evolution of Fermion Pairing from 3D to 2D

3D



Lattice Depth

$$V_0 = 2 E_R$$

$$V_0 = 5 E_R$$

$$V_0 = 6 E_R$$

$$V_0 = 10 E_R$$

$$V_0 = 12 E_R$$

$$V_0 = 19 E_R$$

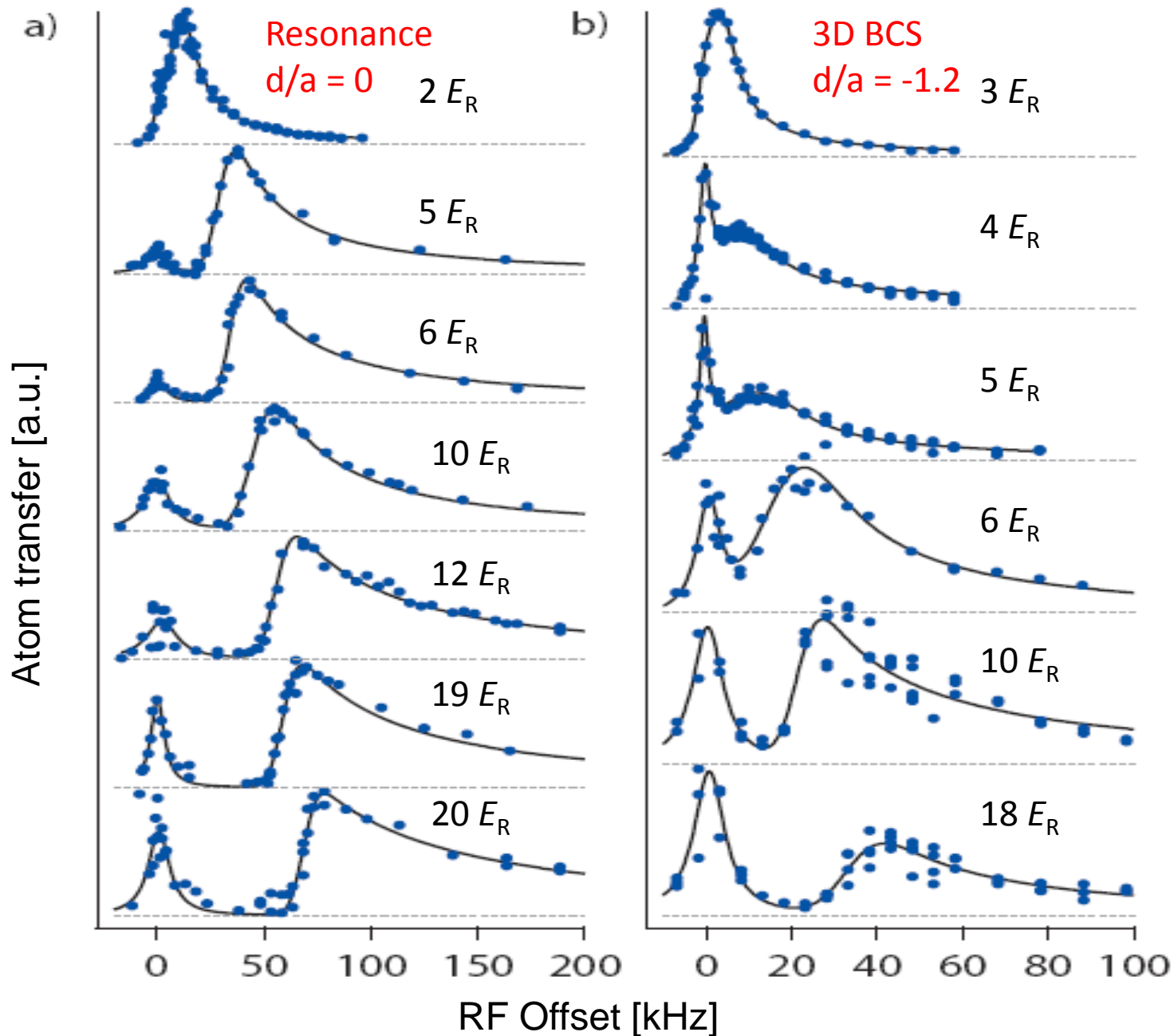
$$V_0 = 20 E_R$$

2D



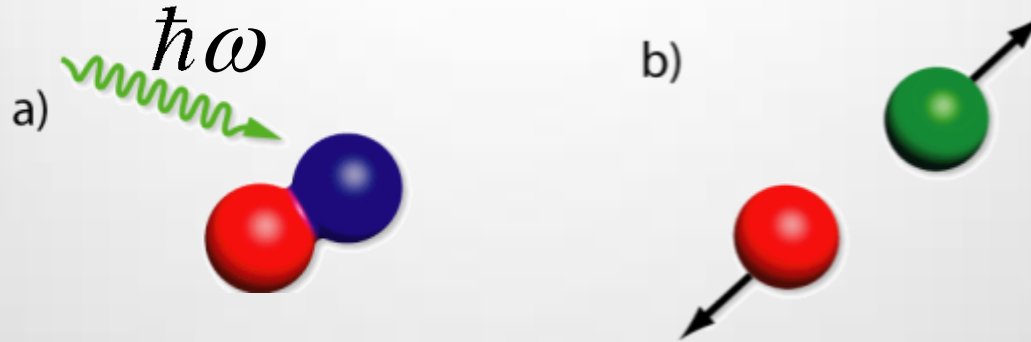
Evolution of Fermion Pairing from 3D to 2D

3D



2D

RF spectrum of fermion pairs



$$I(\omega) \sim \rho(\varepsilon_k) |\psi(k)|^2_{|\varepsilon_k = \varepsilon(\omega)}$$

Density of States: 3D $\sim \sqrt{\varepsilon}$
 2D $const.$

$$3D: I(\omega) \sim \frac{\sqrt{\omega - \omega_{th}}}{\omega^2}$$

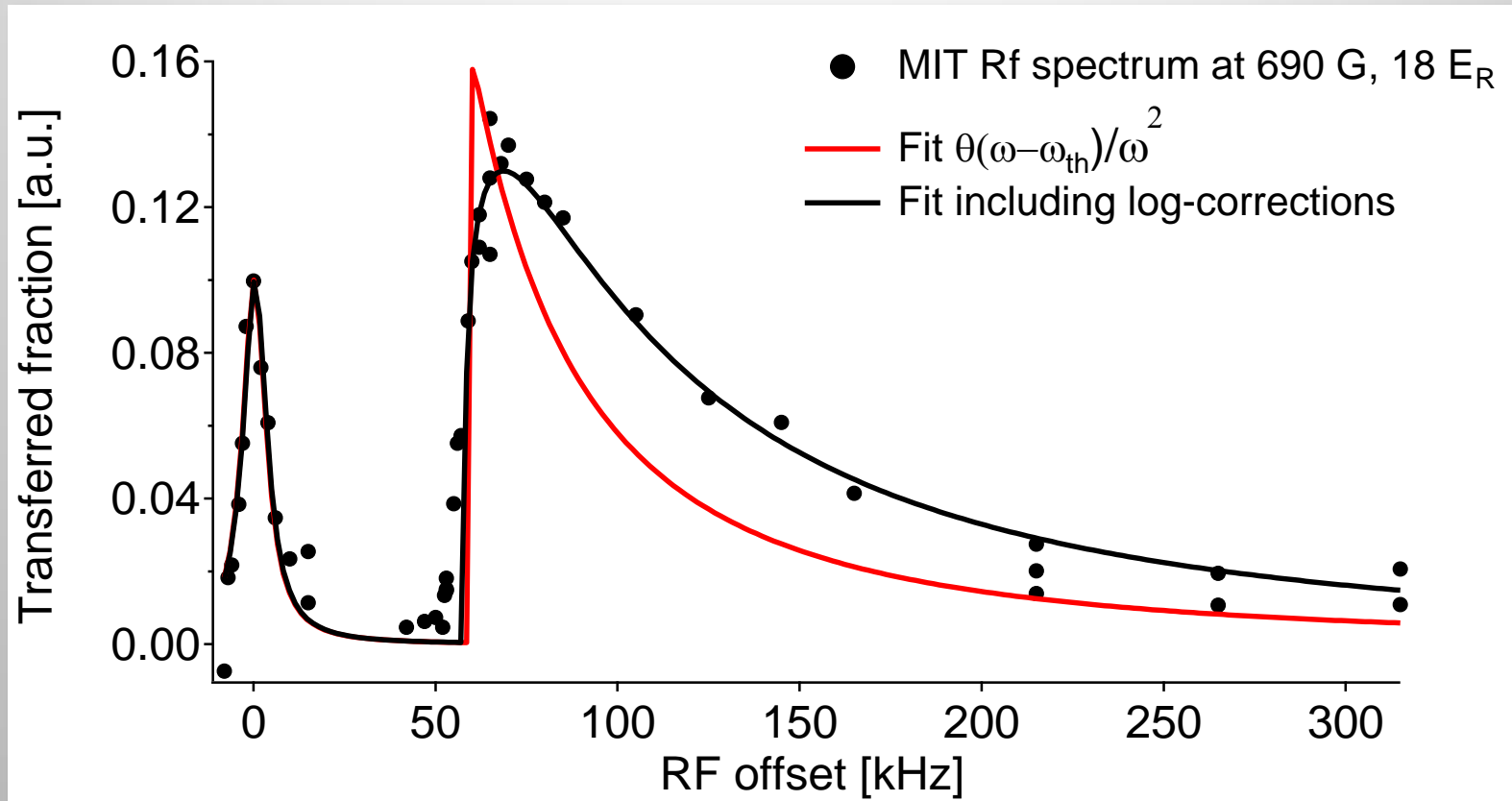
$$2D: I(\omega) \sim \frac{\theta(\omega - \omega_{th})}{\omega^2}$$

Interactions in final state:
 Langmack, Barth, Zwerger, Braaten
 Phys. Rev. Lett. **108**, 060402 (2012)

$$\frac{\ln^2(E_b^f / E_b)}{\ln^2((\omega - E_b) / E_b^f) + \pi^2}$$

Logarithmic Corrections in 2D Spectra

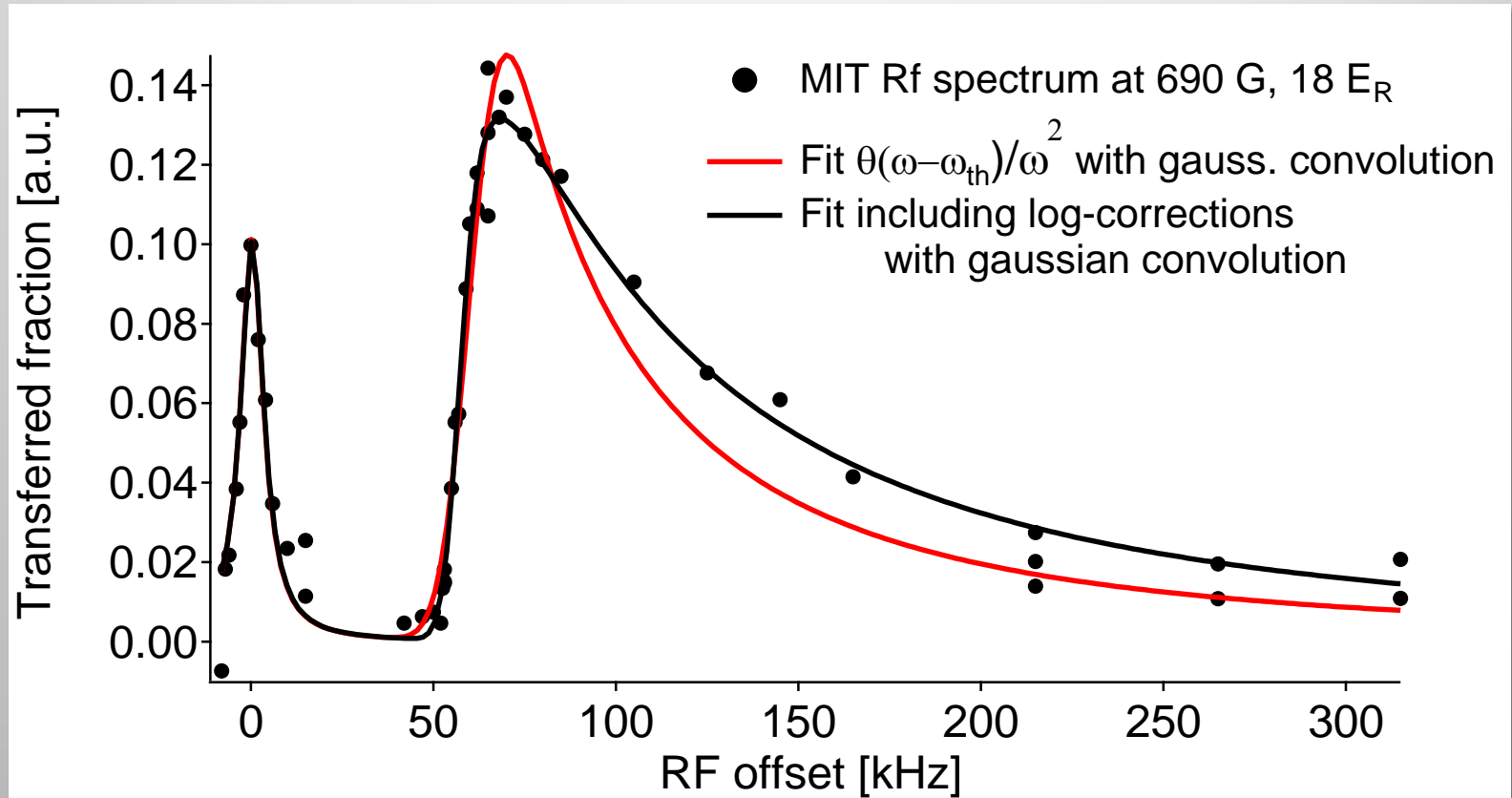
2D nature of interactions strongly influence “naïve” spectra



$$I_{Pair}(\omega) \sim \frac{\theta(\omega - \omega_{th})}{\omega^2} \frac{\ln^2(E_b^f / E_b)}{\ln^2((\omega - E_b) / E_b^f) + \pi^2}$$

Logarithmic Corrections in 2D Spectra

2D nature of interactions strongly influence “naïve” spectra

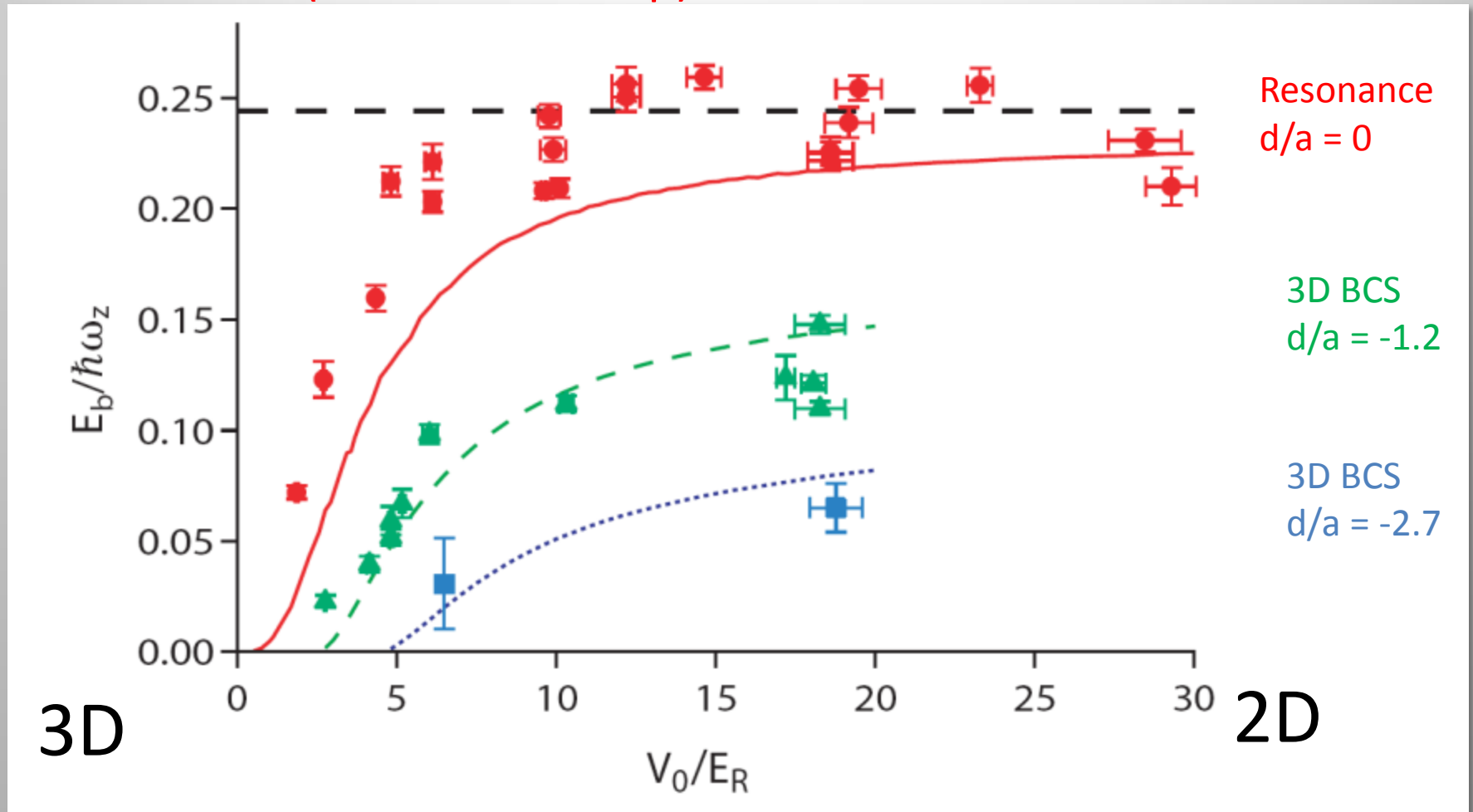


$$I_{Pair}(\omega) \sim \frac{\theta(\omega - \omega_{th})}{\omega^2} \frac{\ln^2(E_b^f / E_b)}{\ln^2((\omega - E_b) / E_b^f) + \pi^2}$$

Evolution of Fermion Pairing from 3D to 2D

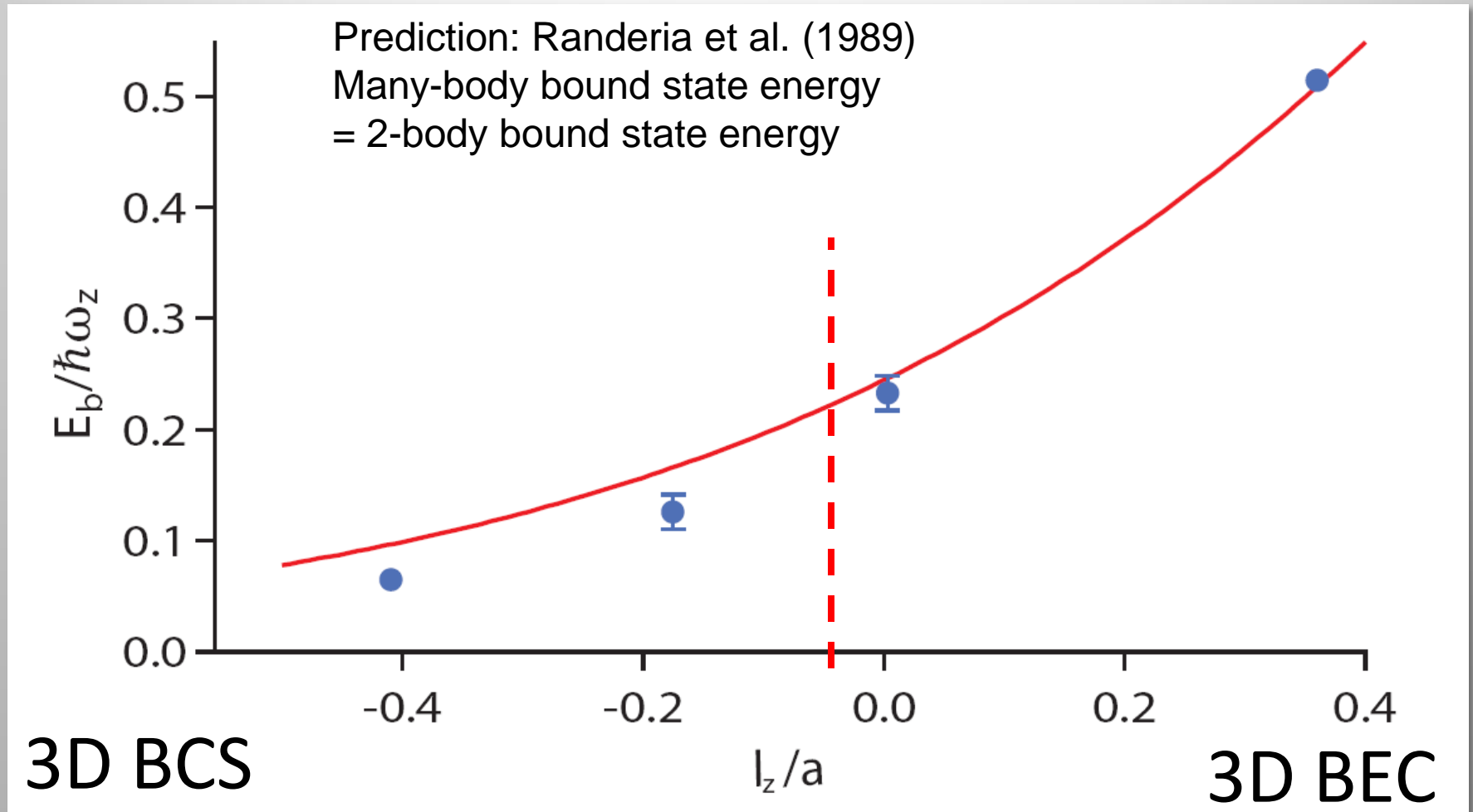
On Resonance

(in harmonic trap): $E_{B,th} = 0.244\hbar\omega_z$



Deep 2D regime

Comparison with mean-field BEC-BCS in 2D



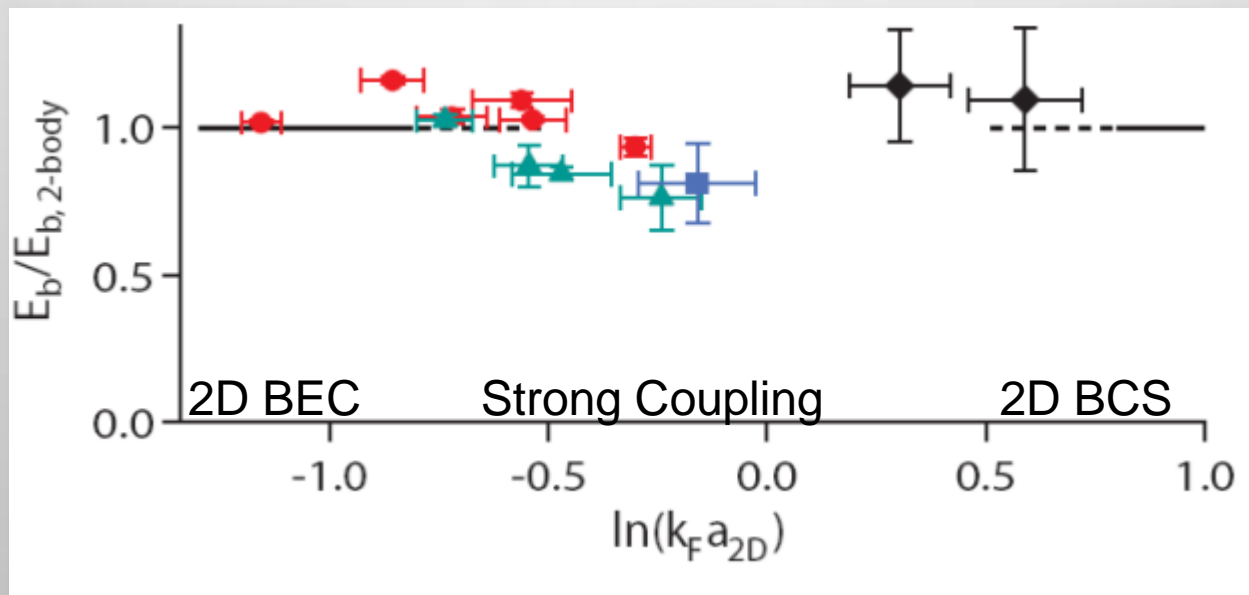
Deep 2D regime

Comparison with mean-field BEC-BCS in 2D

Prediction: Randeria et al. (1989)

Many-body bound state energy

= 2-body bound state energy



Only small deviation observed between many-body and 2-body bound-state energy

Same conclusion in M. Koehl group: Feld et al., Nature 480, 75 (2011)

A. T. Sommer, L. W. Cheuk, M. J.-H. Ku, W. S. Bakr, M. W. Zwierlein
PRL **108**, 045302 (2012)



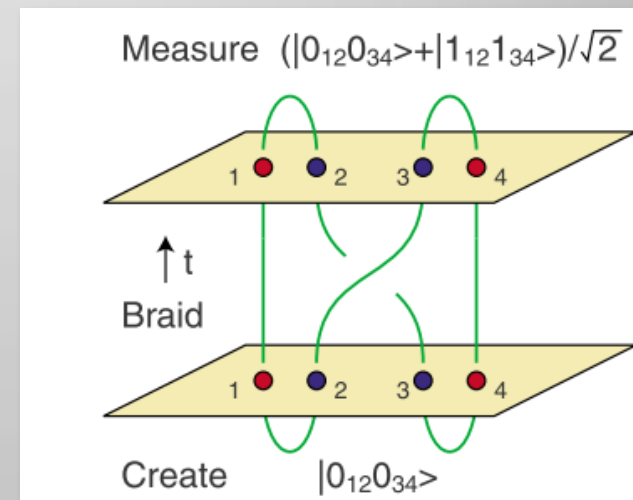
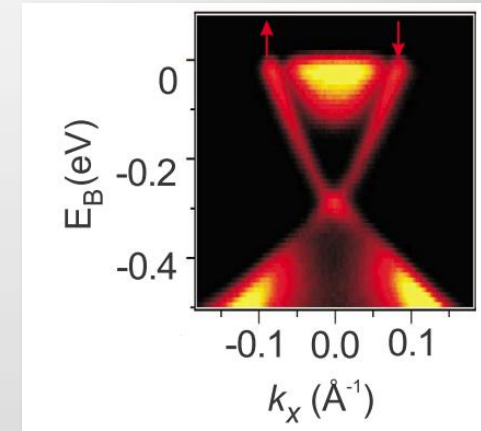
Selected for Viewpoint in Physics, Jan '12

Spin-Orbit Coupled Fermi Gases

Spin-Orbit Coupling

Motivation:

- Possible Ingredient for Topological Phases of Matter
- Induces p- (and higher-order) partial wave interactions + p-wave pairing
- Topological Superconductors:
 - Majorana Edge States
 - Topological protection
- Quantum computation with Majorana fermions (Kitaev)



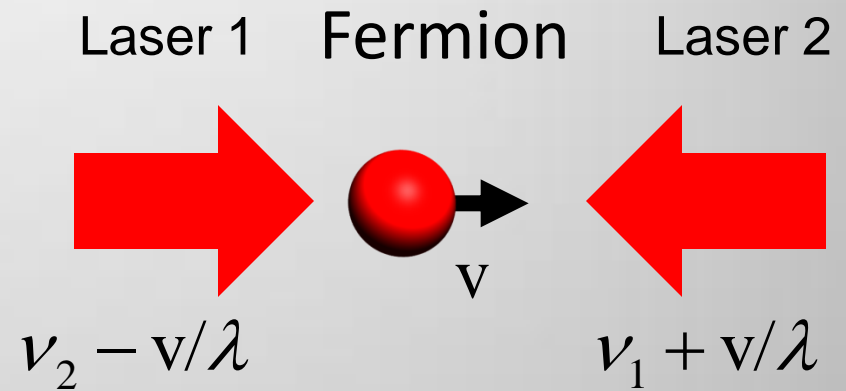
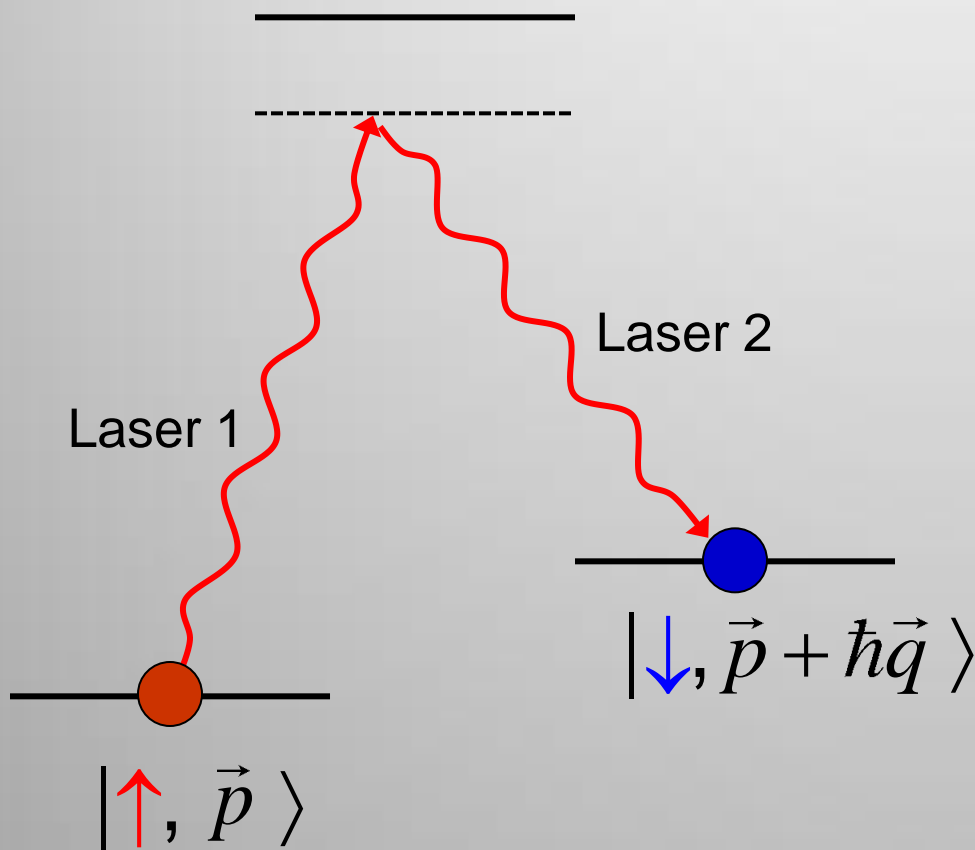
M.Z. Hasan and C.L. Kane
RMP **82**, 3045 (2010)

Spin-Orbit Coupling

Raman transition:

Couple different spin (hyperfine) states

Doppler effect causes momentum-dependent coupling



Pioneering work with bosons:
Ian Spielman, NIST

Recent work with fermions:
Zhang et al., Shanxi

The spin-orbit Hamiltonian

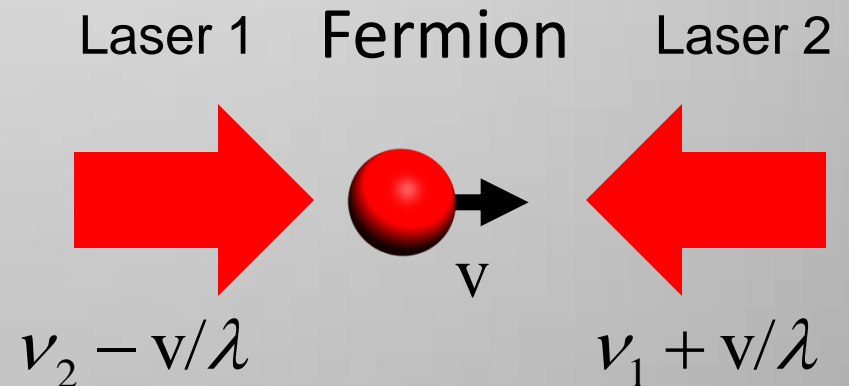
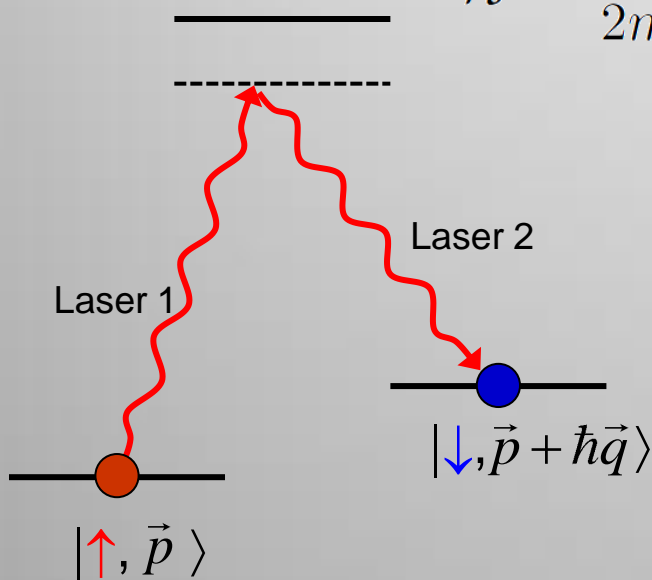
- The SO Hamiltonian

$$\mathcal{H} = \frac{\hbar^2 k^2}{2m} - \frac{g\mu_B}{\hbar} \mathbf{S} \cdot (\mathbf{B}^{(D)} + \mathbf{B}^{(R)} + \mathbf{B}^{(Z)})$$

$$\mathbf{B}^{(R)} = \alpha(k_y, -k_x, 0) \quad \mathbf{B}^{(D)} = \beta(k_y, k_x, 0)$$

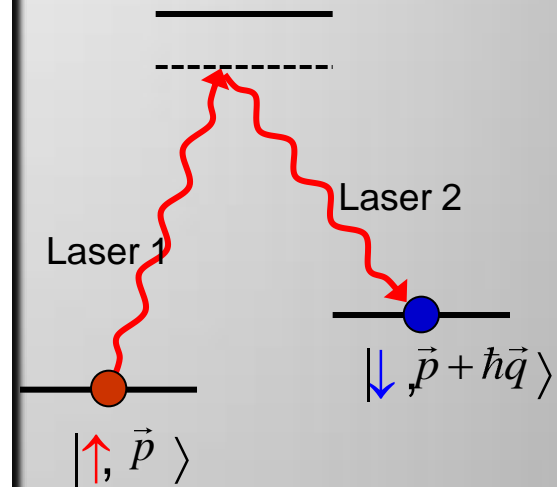
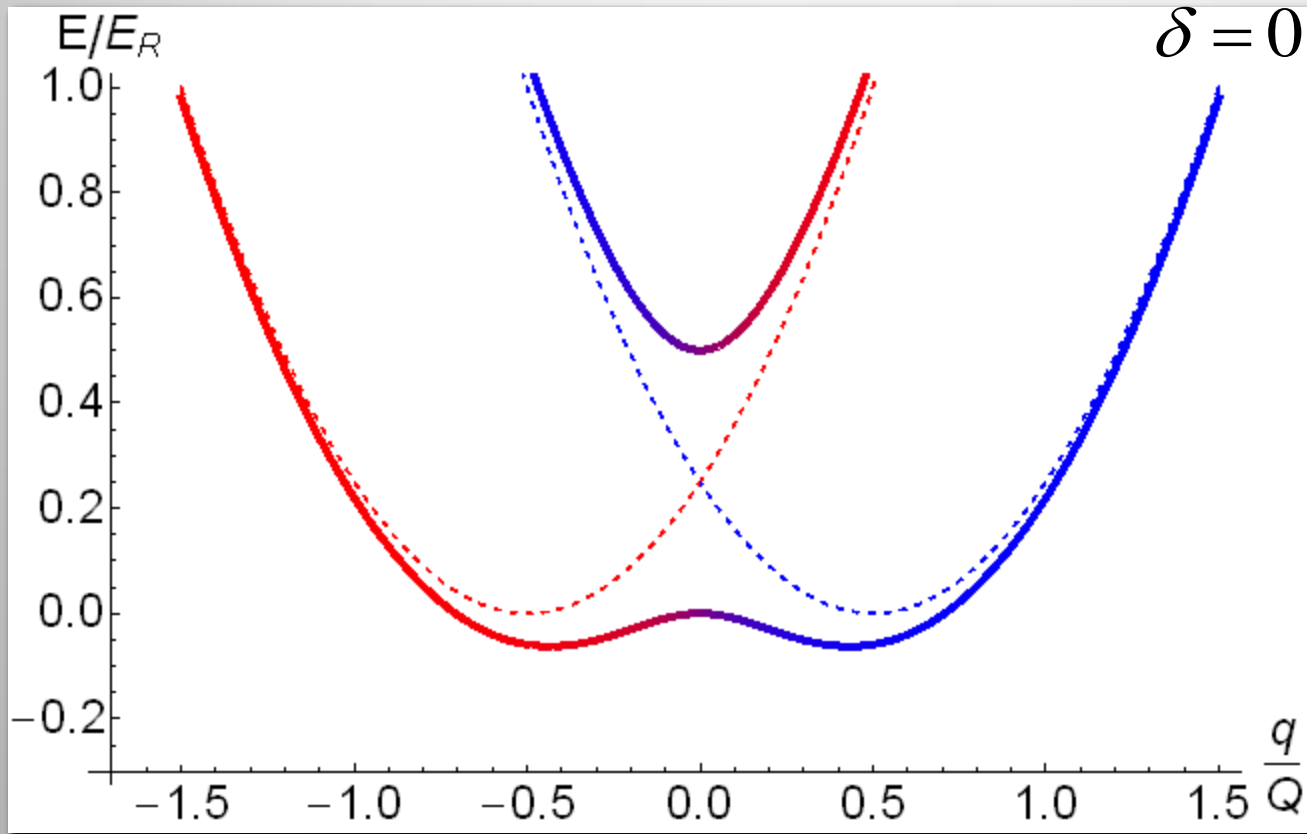
- 1D SO Hamiltonian

$$\mathcal{H} = \frac{\hbar^2 k^2}{2m} + 2\alpha k \sigma_z + \frac{\delta}{2} \sigma_z + \frac{\hbar \Omega_R}{2} \sigma_x$$

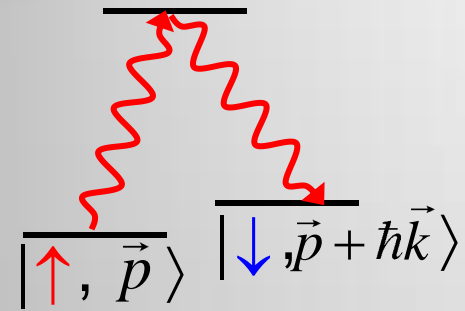


The spin-orbit gap

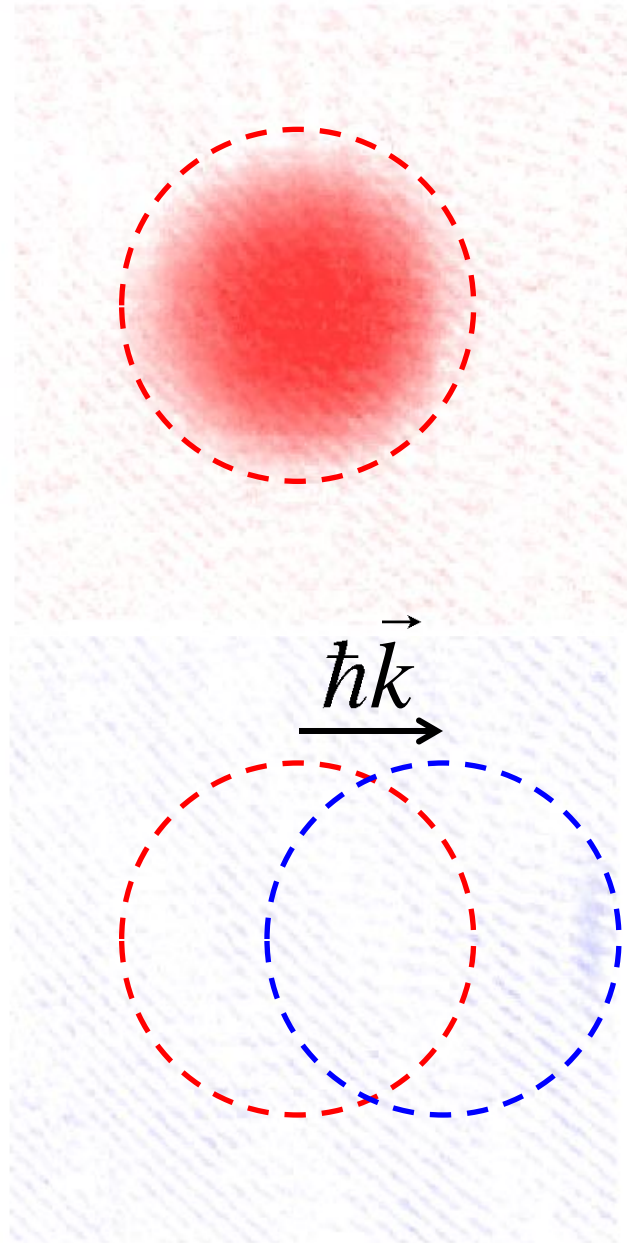
$$\mathcal{H} = \frac{\hbar^2 k^2}{2m} + 2\alpha k \sigma_z + \frac{\delta}{2} \sigma_z + \frac{\hbar \Omega_R}{2} \sigma_x$$



Coupling Spin and Momentum via Raman

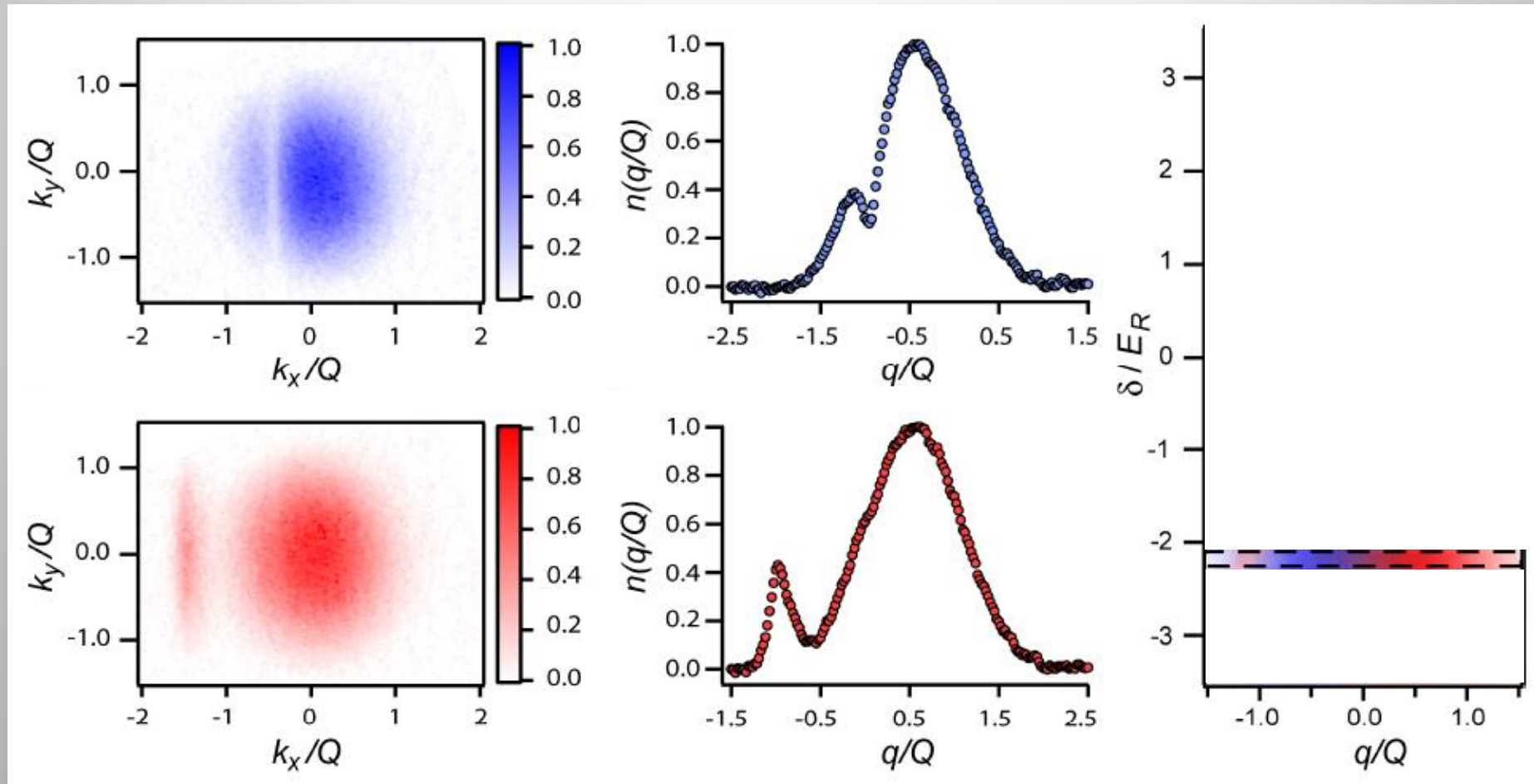


Vary detuning
Short pulse



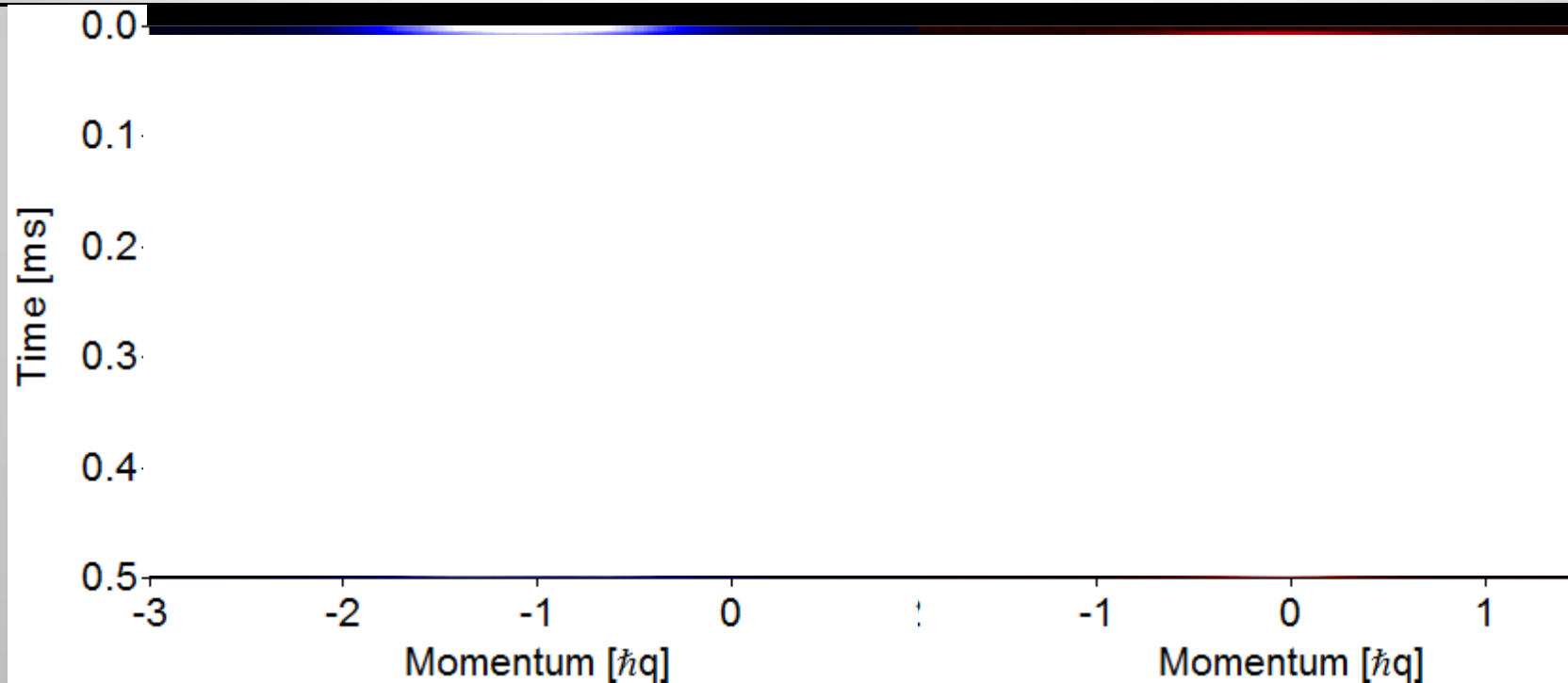
See also: Zhang et al.
arXiv:1204.1887

Coupling Spin and Momentum via Raman

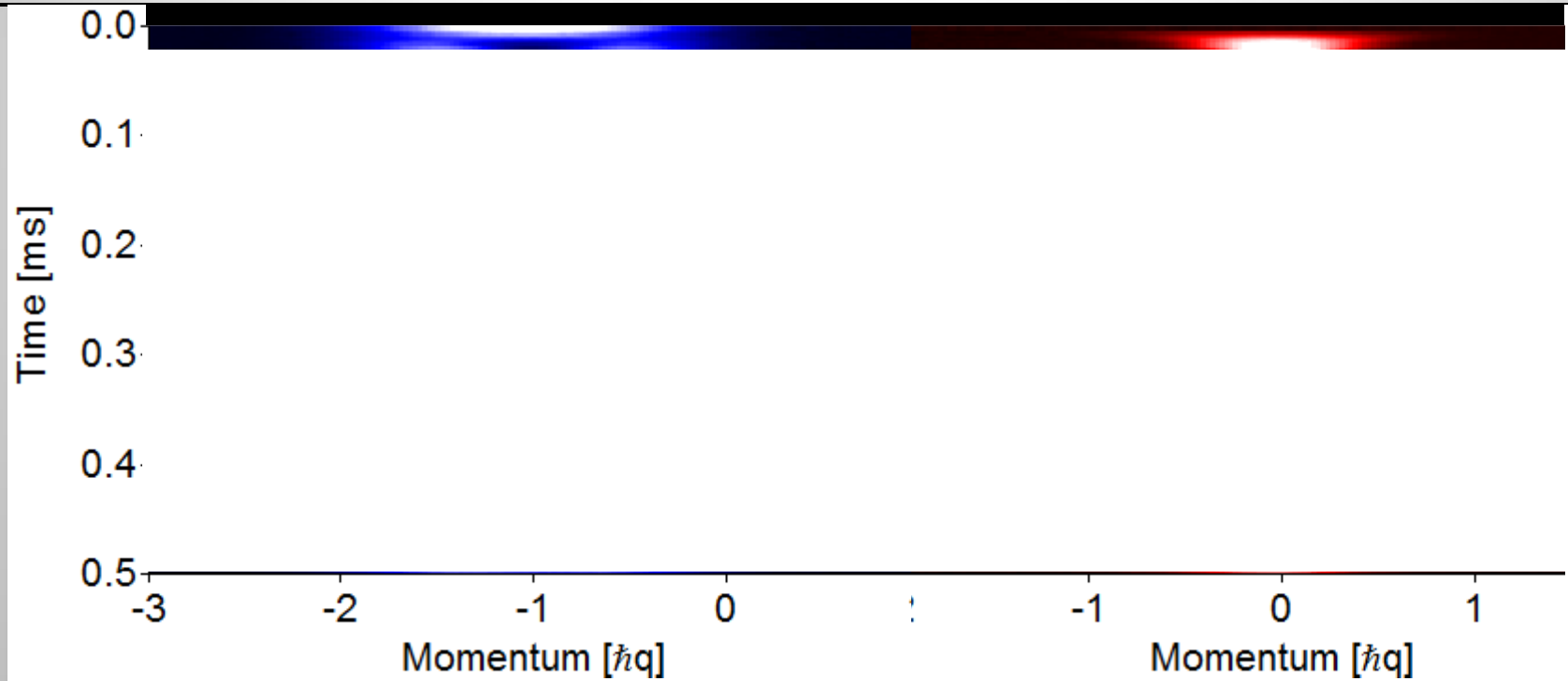


$$\begin{aligned}
 & |\tilde{k}\rangle \\
 &= \alpha_k |k, \uparrow\rangle + \beta_k |k+q, \downarrow\rangle
 \end{aligned}$$

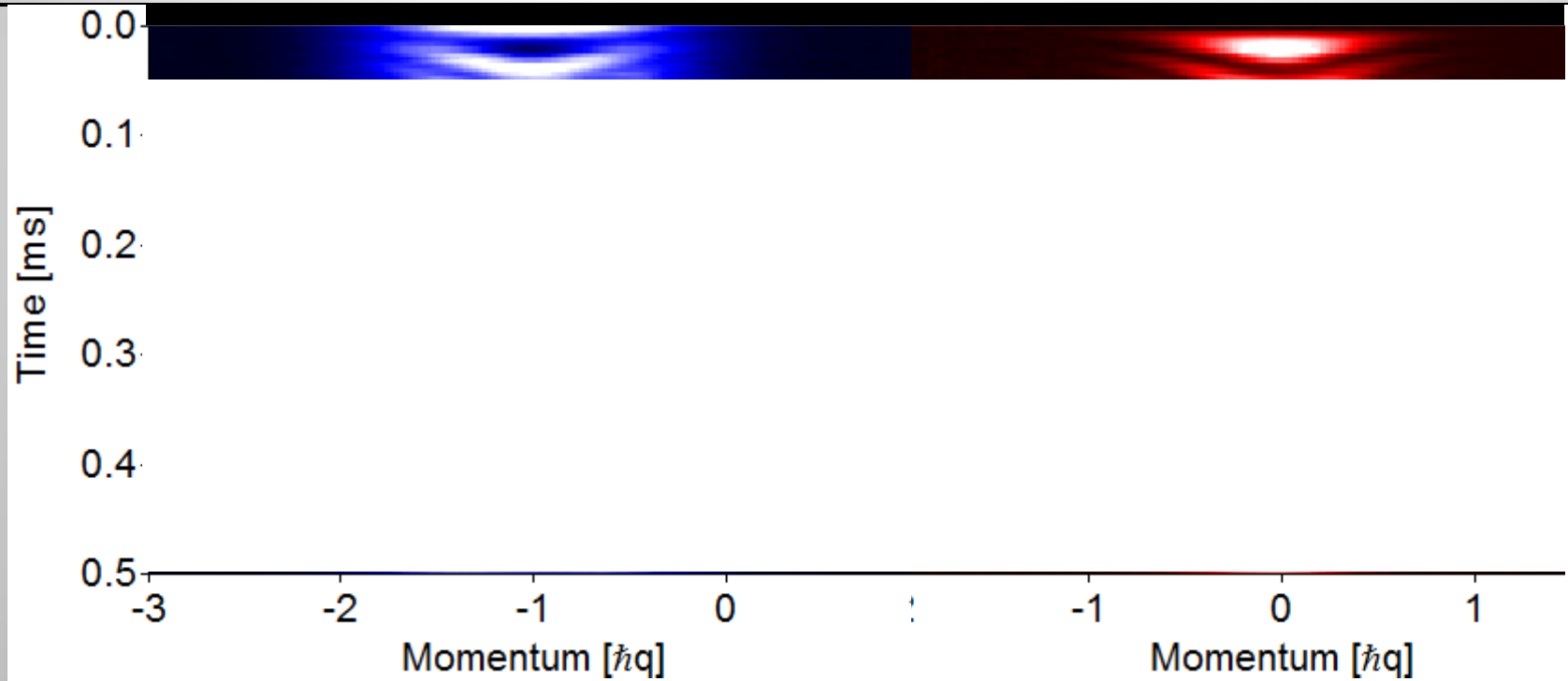
Rabi Oscillations of Spin-Momentum Coupled States



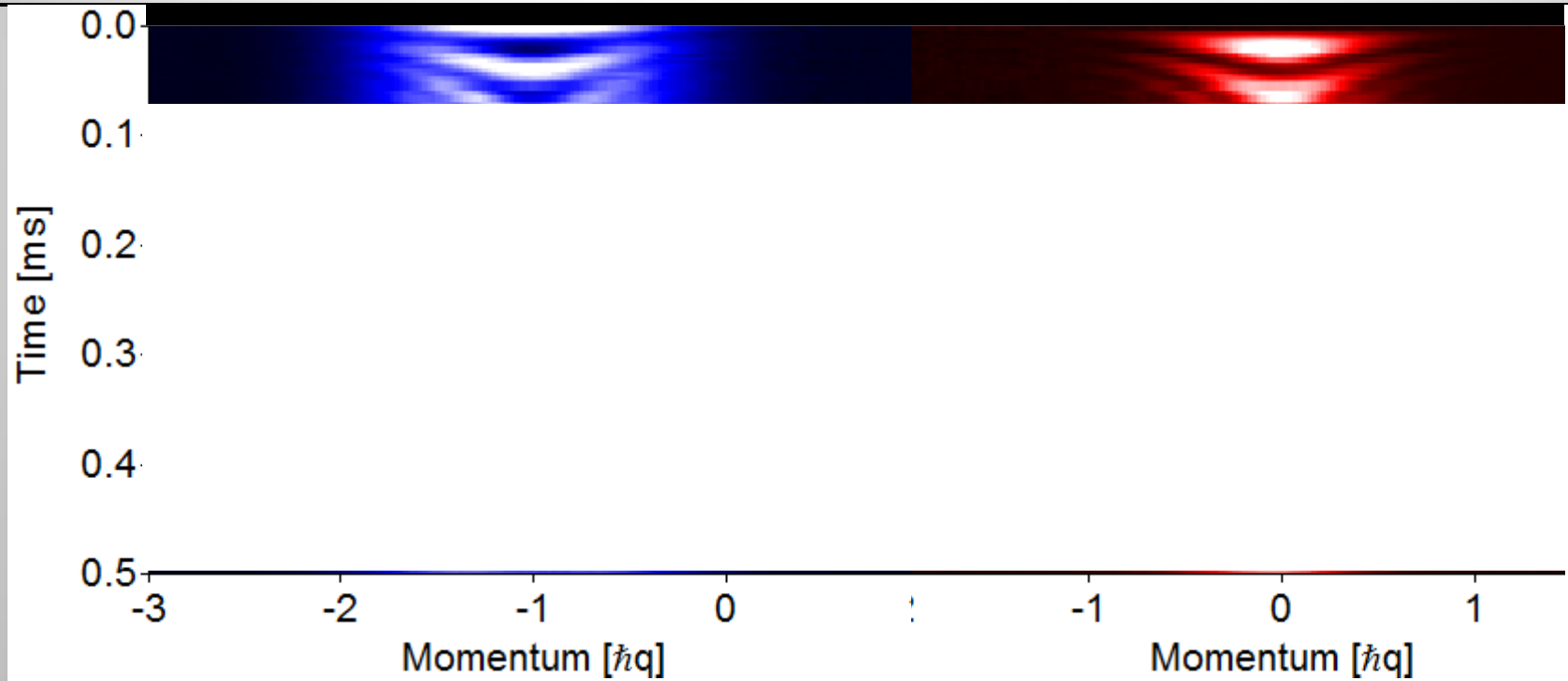
Rabi Oscillations of Spin-Momentum Coupled States



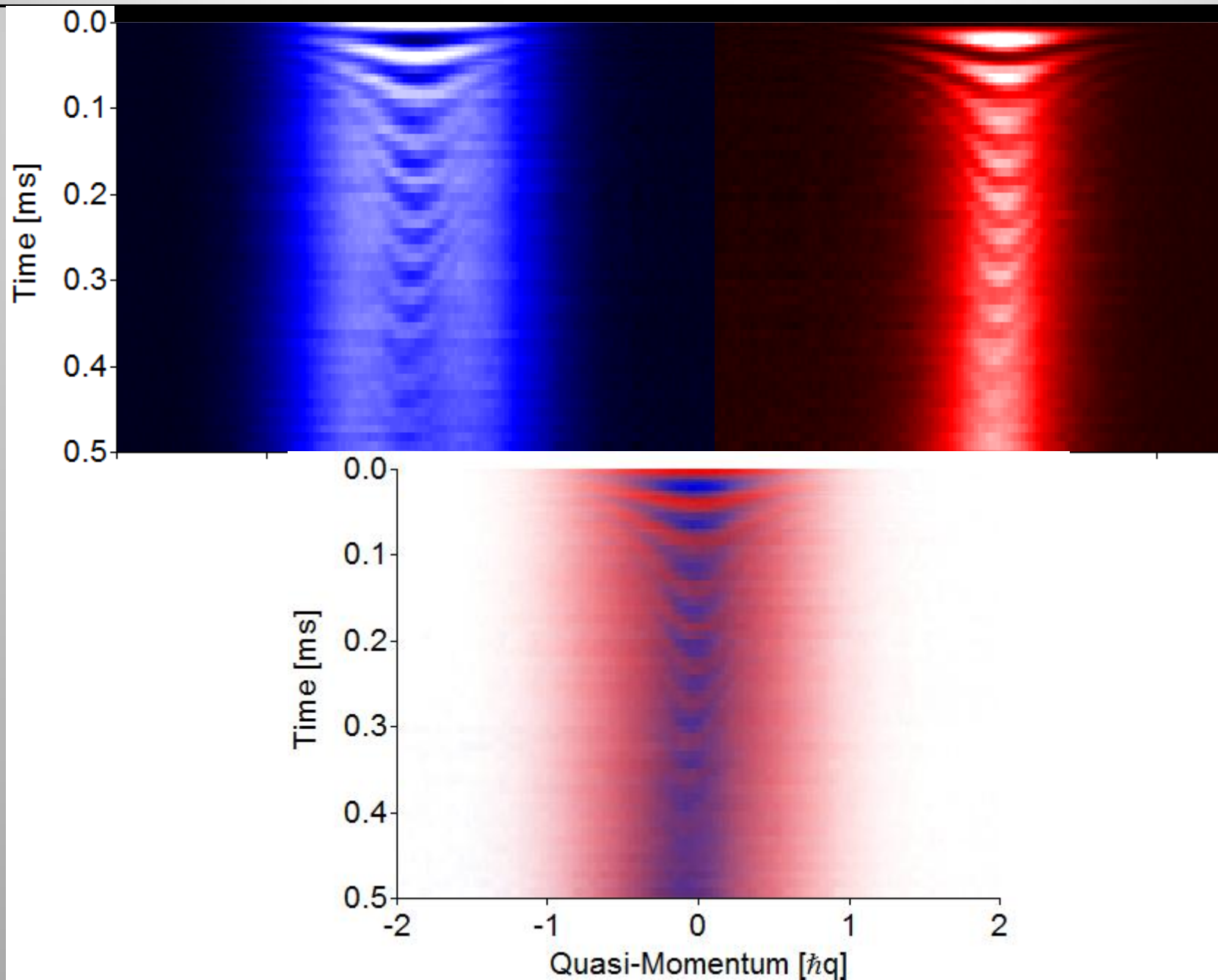
Rabi Oscillations of Spin-Momentum Coupled States



Rabi Oscillations of Spin-Momentum Coupled States

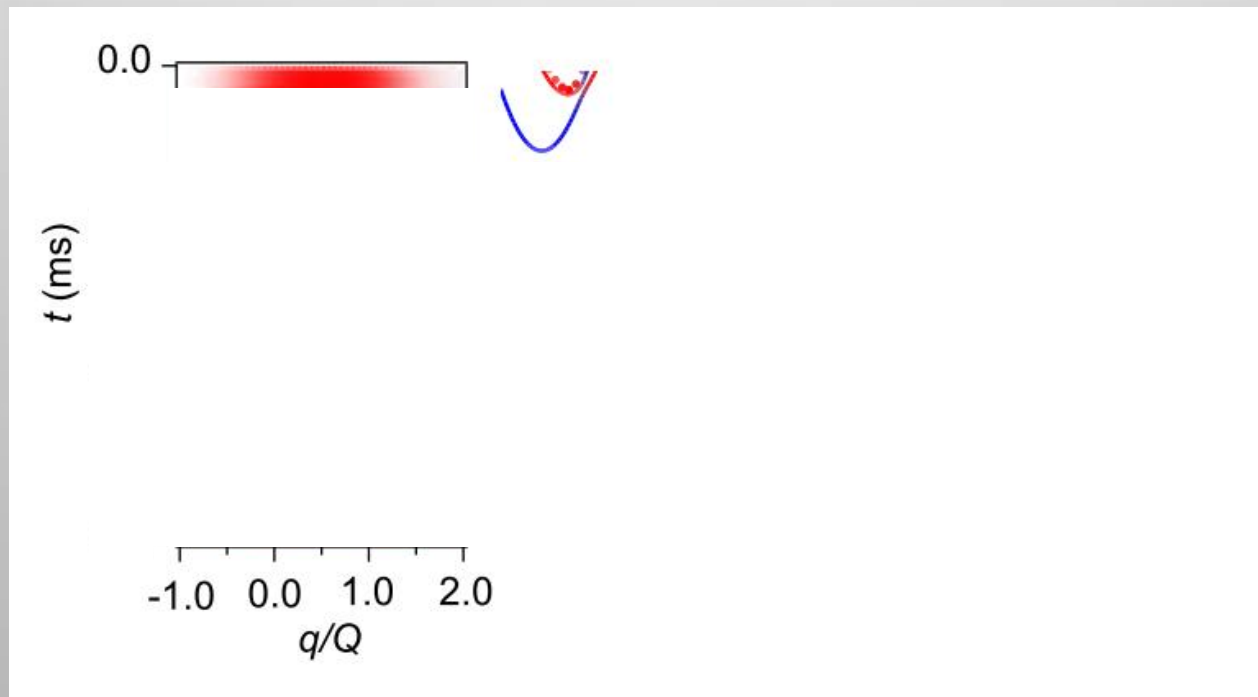


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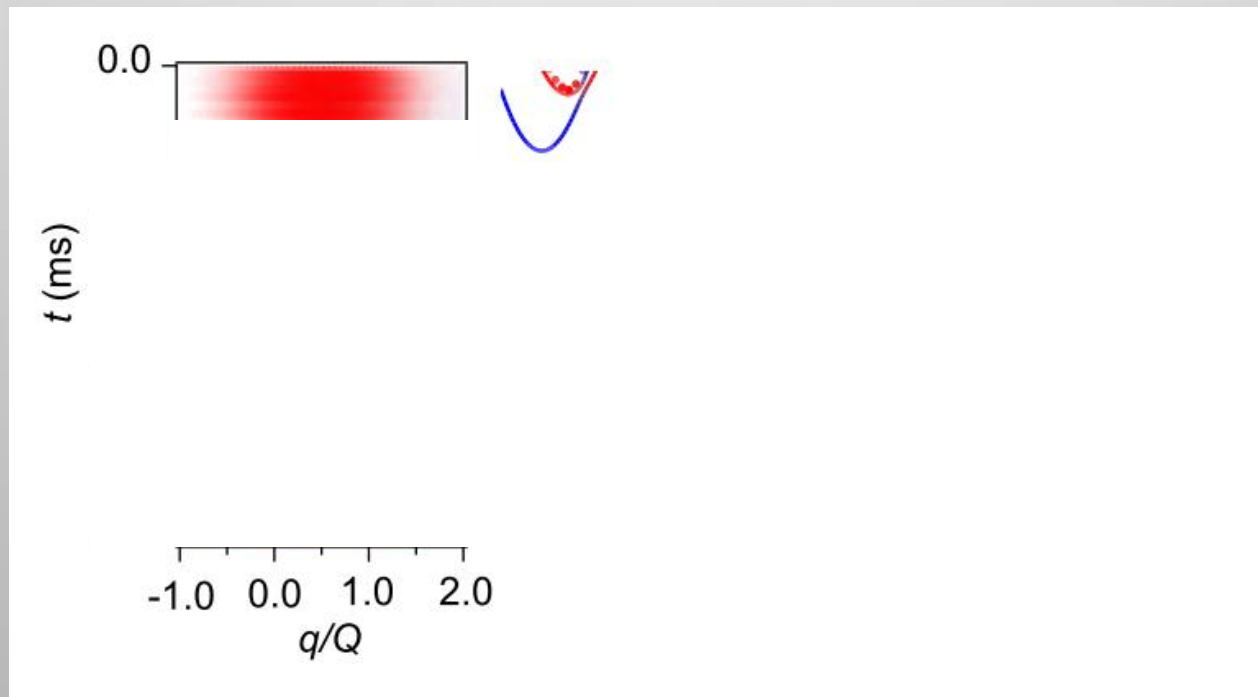
Raman Dressing

- When SO coupling is ramped slowly:
 - Spin composition follows effective magnetic field
 - Process is reversible
- By changing detuning, dress either into upper band or lower band



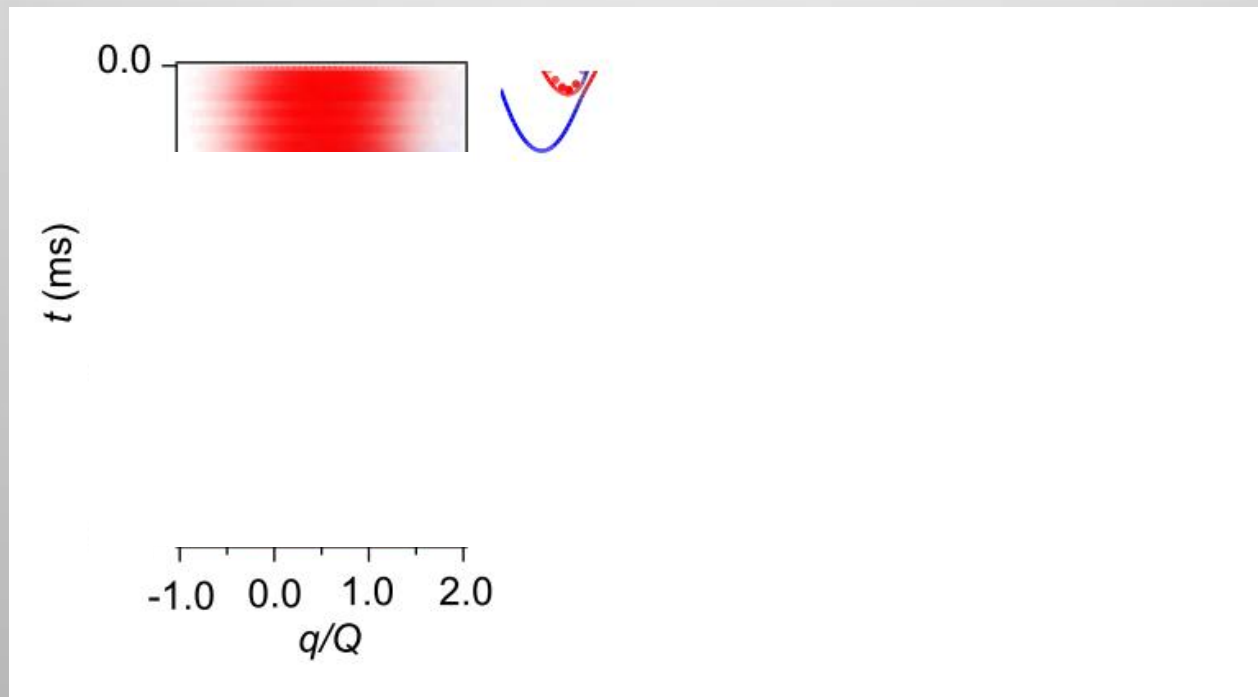
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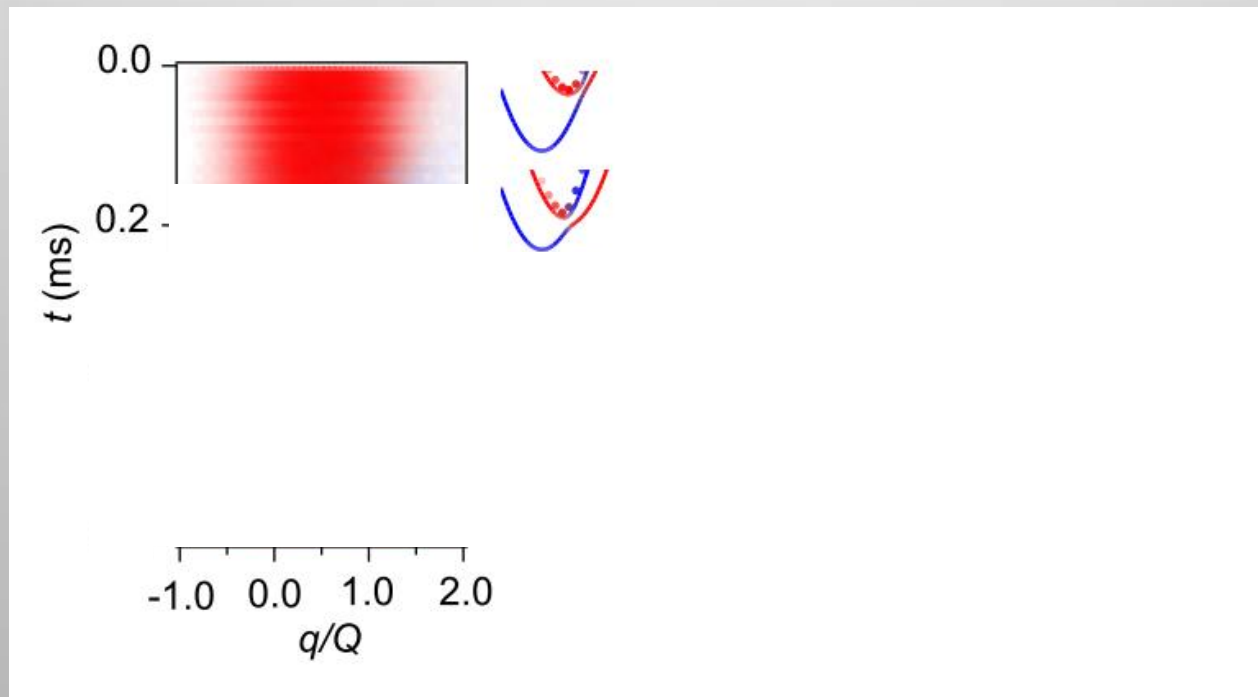
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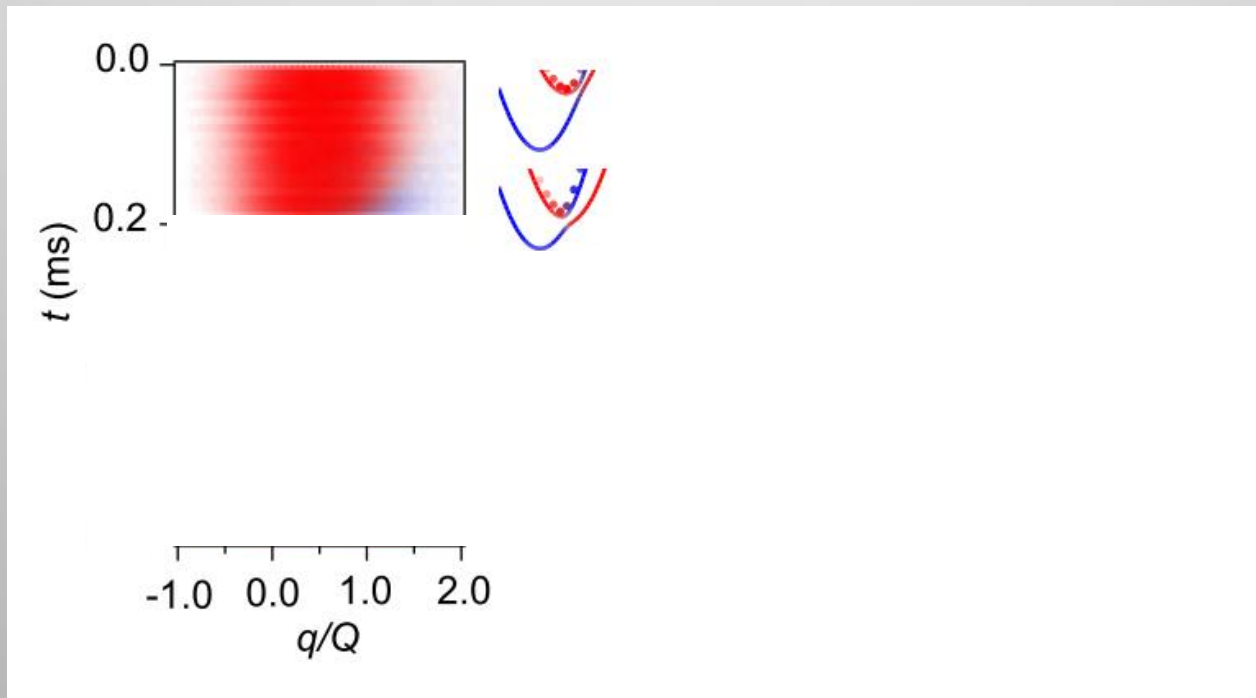
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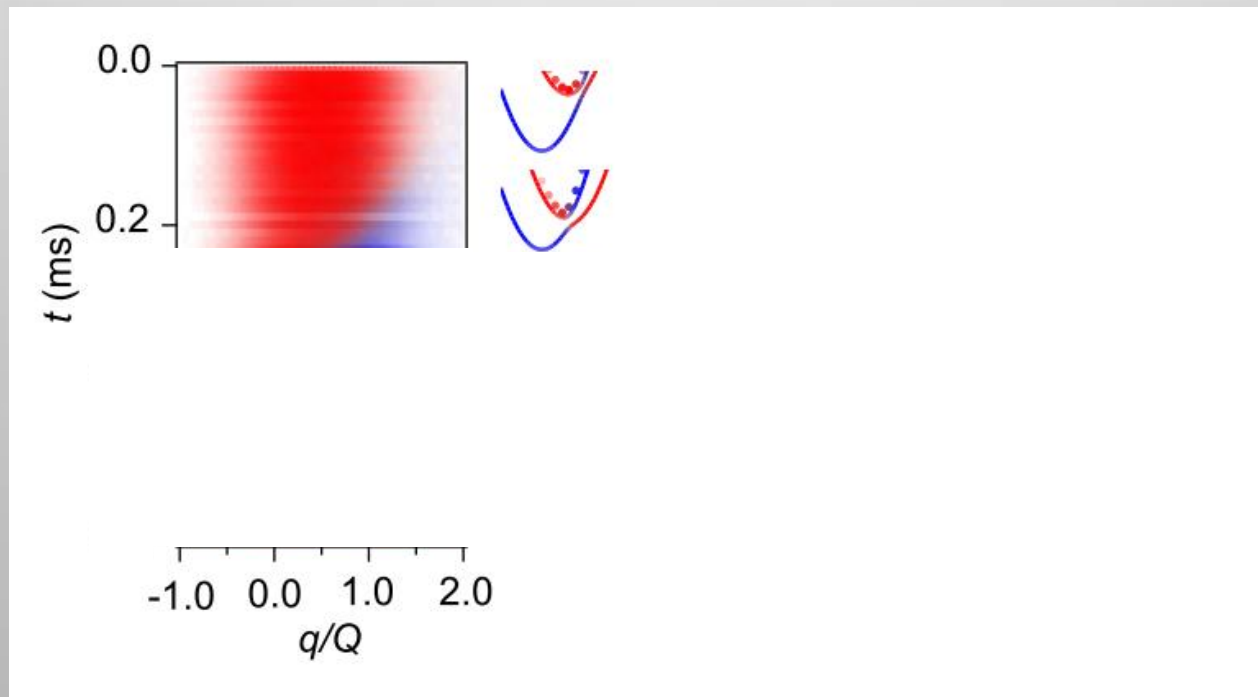
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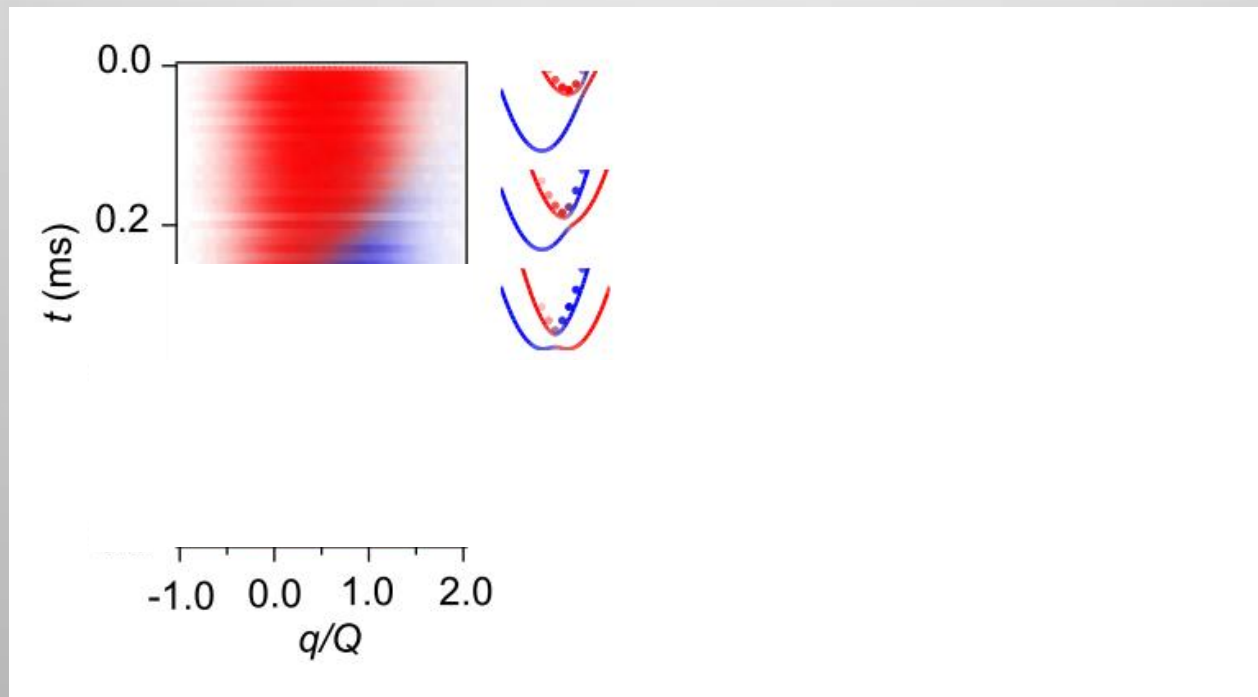
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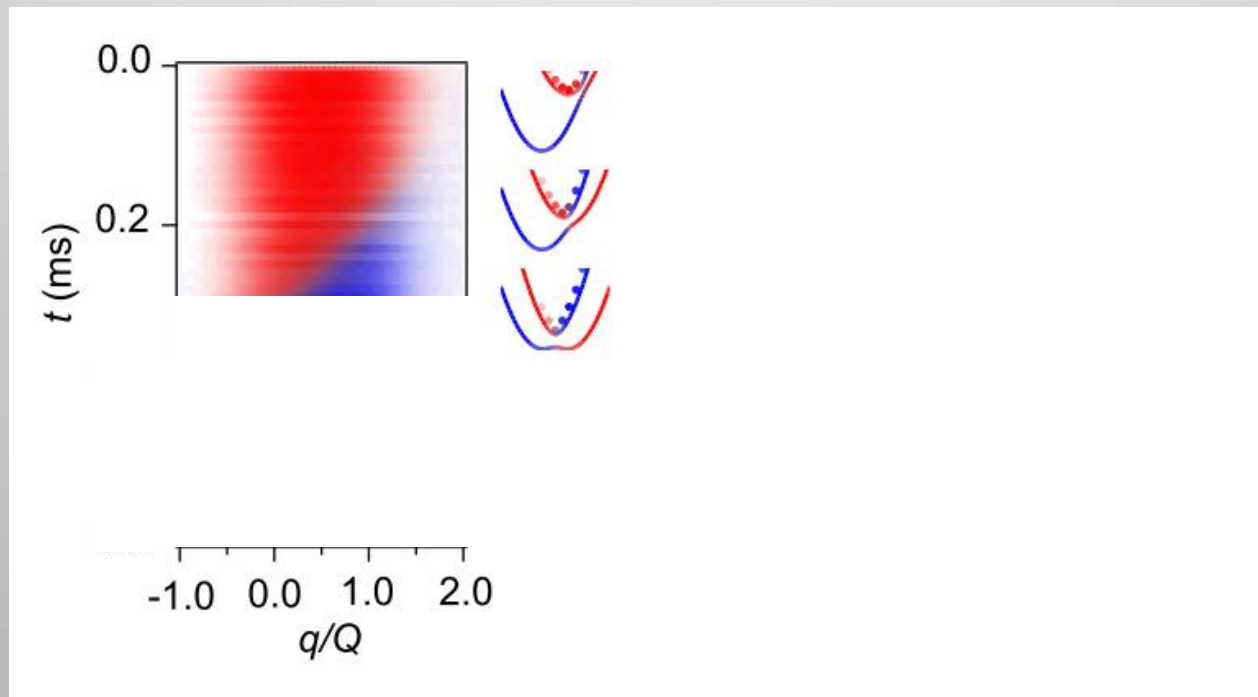
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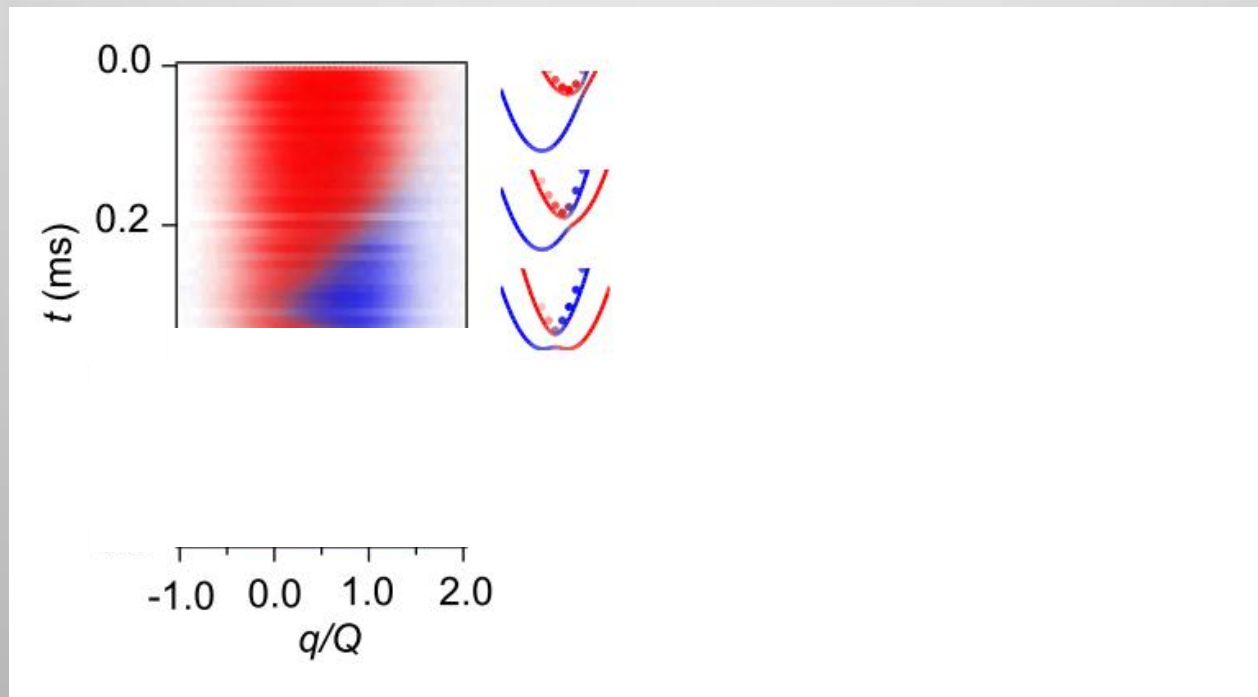
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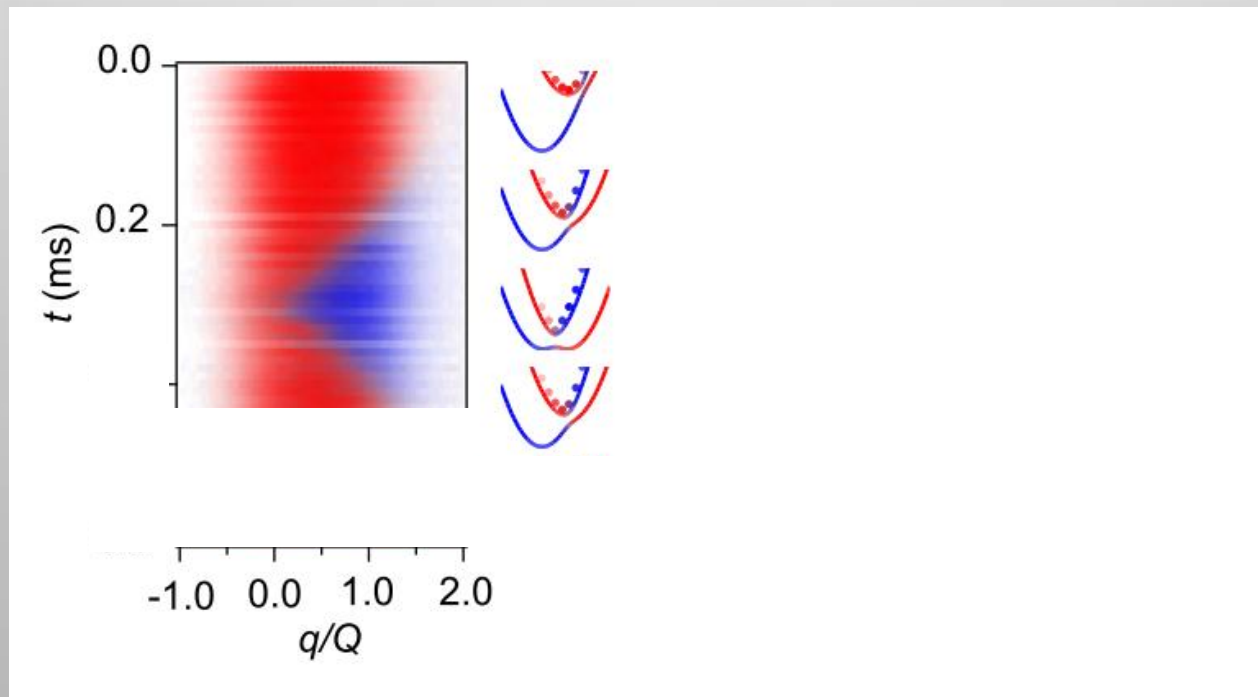
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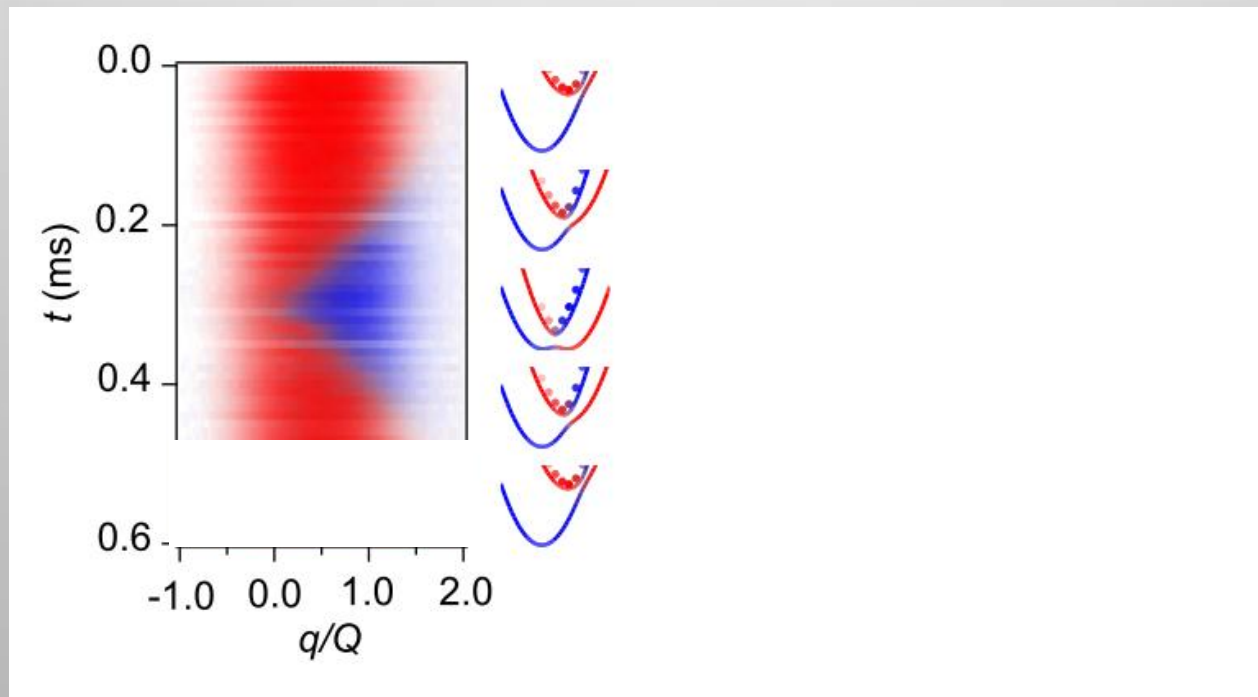
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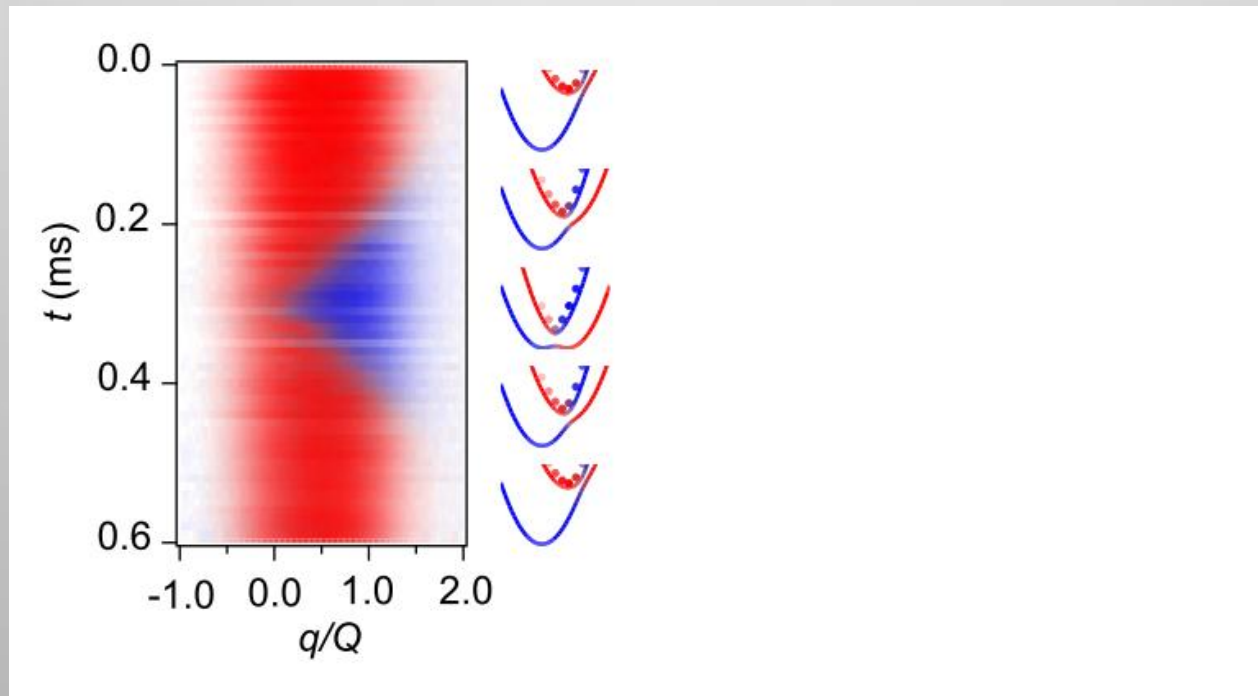
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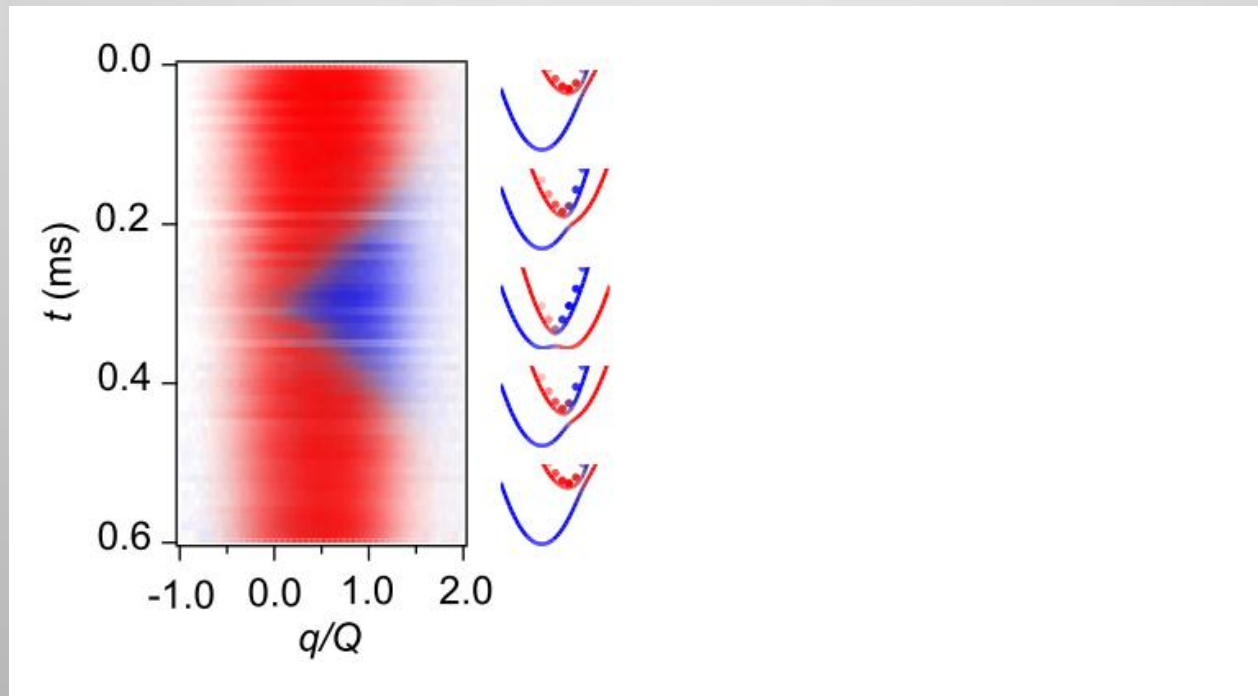
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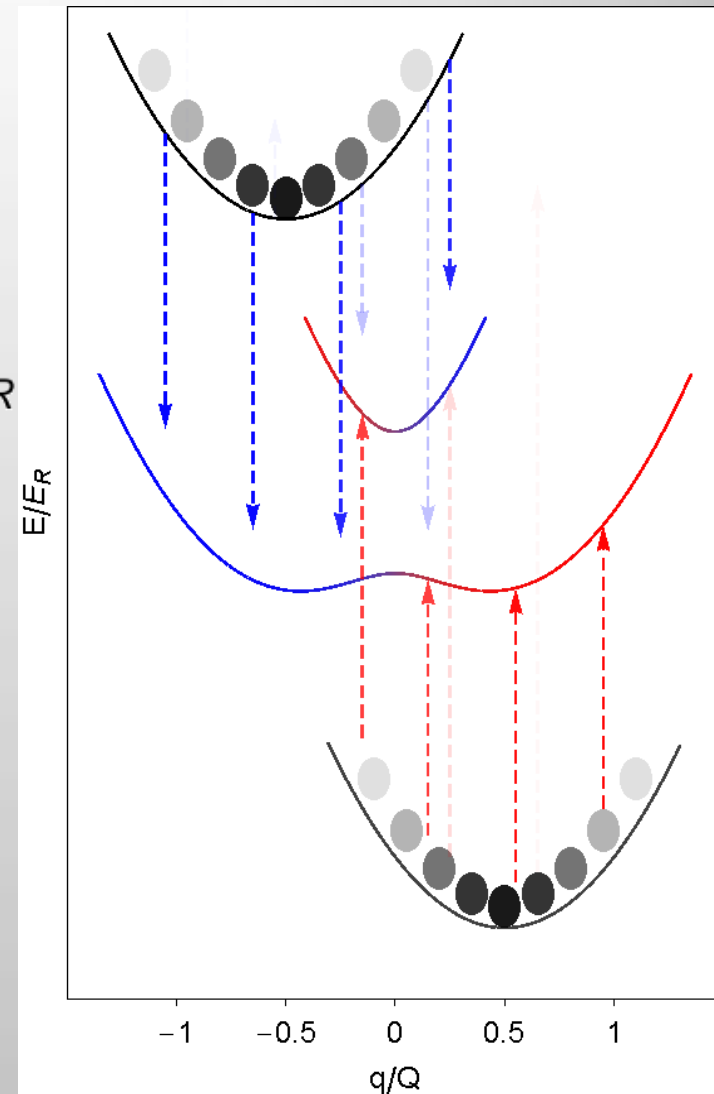


Spin-injection spectroscopy

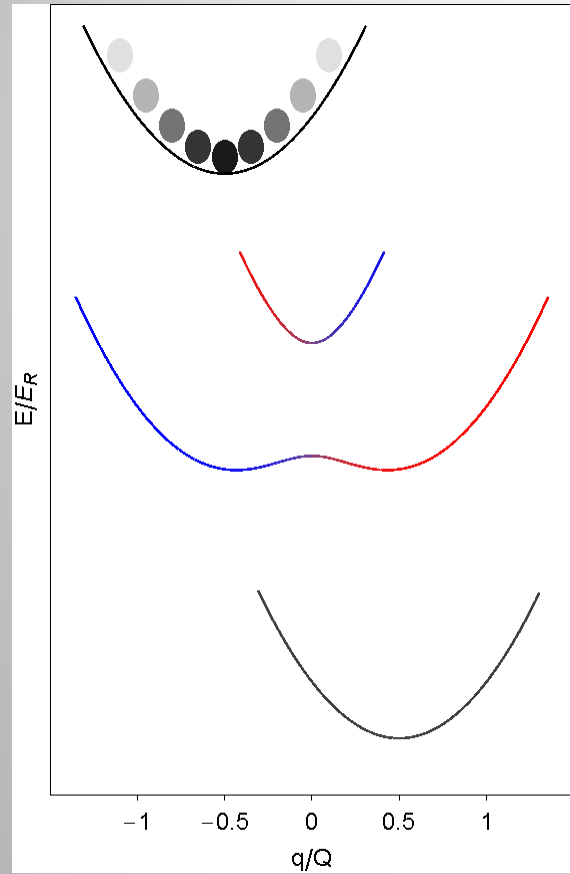
- How to characterize Hamiltonian?
 - Can topology be measured?
- Condensed matter: transport, (spin-)ARPES, STM ...
- Cold atom analog: momentum resolved RF (Jin, Koehl)
(=photoemission spectroscopy)
- Photoemission Spectroscopy probes dispersion $E(k)$

Spin-injection spectroscopy

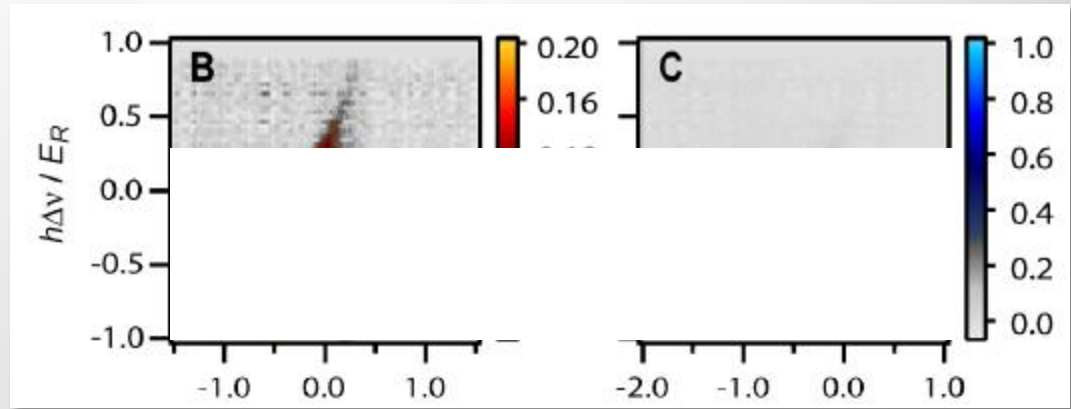
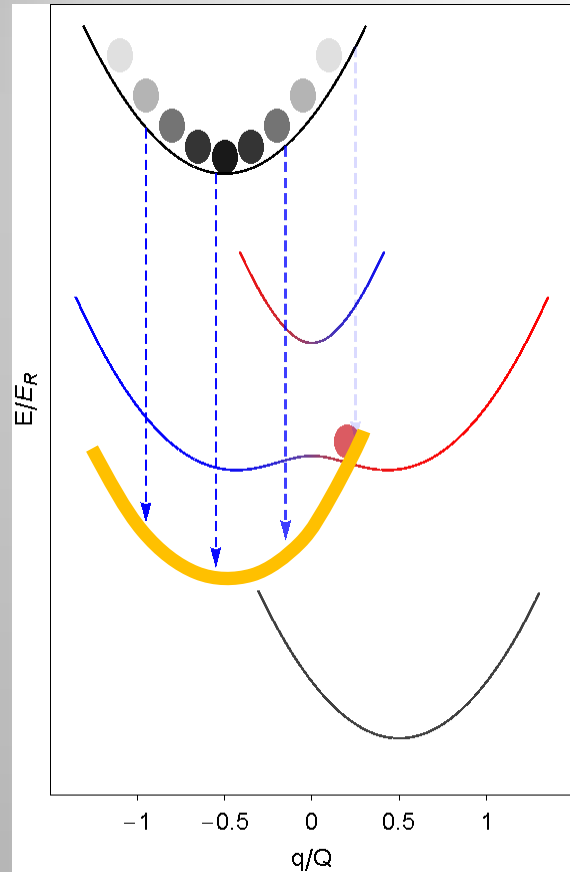
- Spin-injection spectroscopy
 - Measures $E(k)$ and spin texture.
 - Start from reservoir states: $|\uparrow\rangle_R, |\downarrow\rangle_R$
 - Transfer into SO coupled system with RF pulse
 - Project into free space, give TOF
 - Spin-selective imaging gives spin/momentum
 - Reconstruct $E(k)$ along with “color” of band



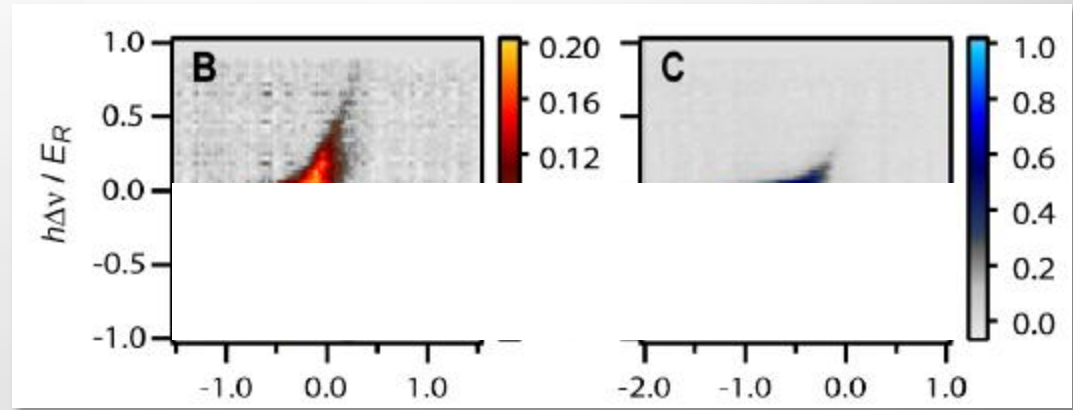
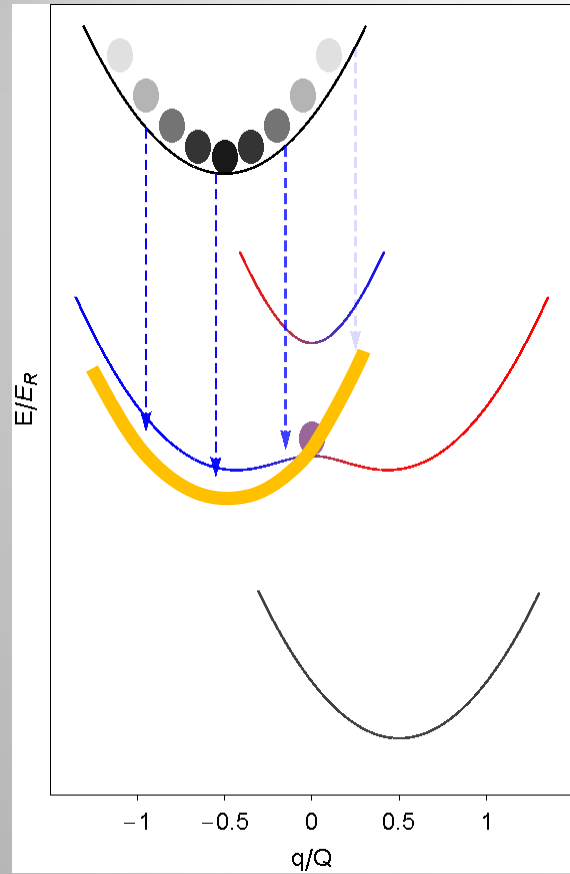
Spin-injection spectroscopy



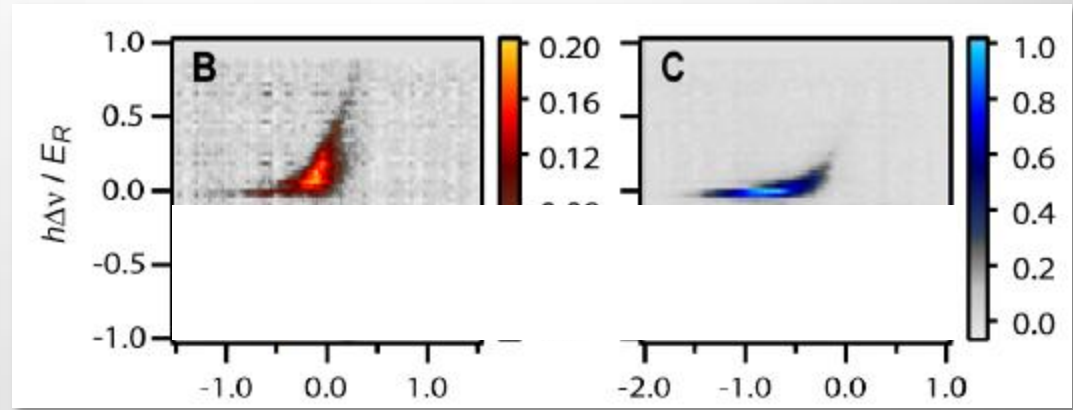
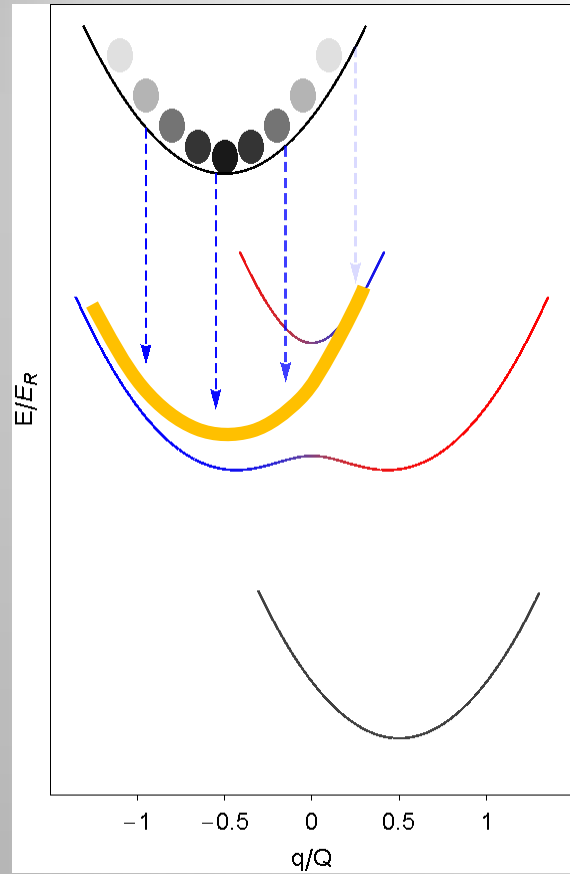
Spin-injection spectroscopy



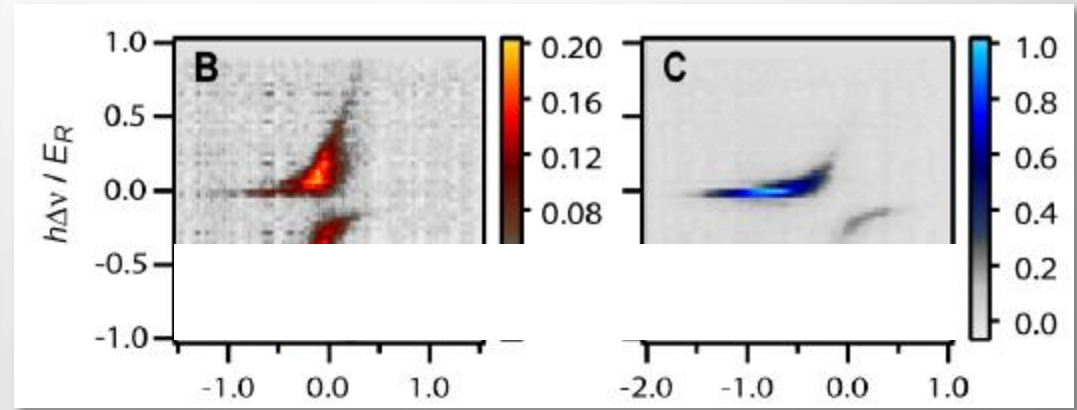
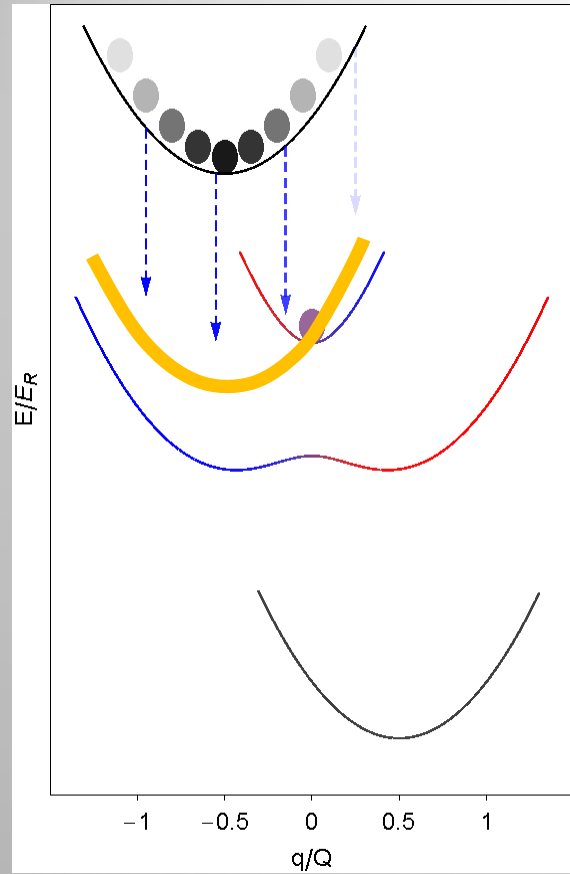
Spin-injection spectroscopy



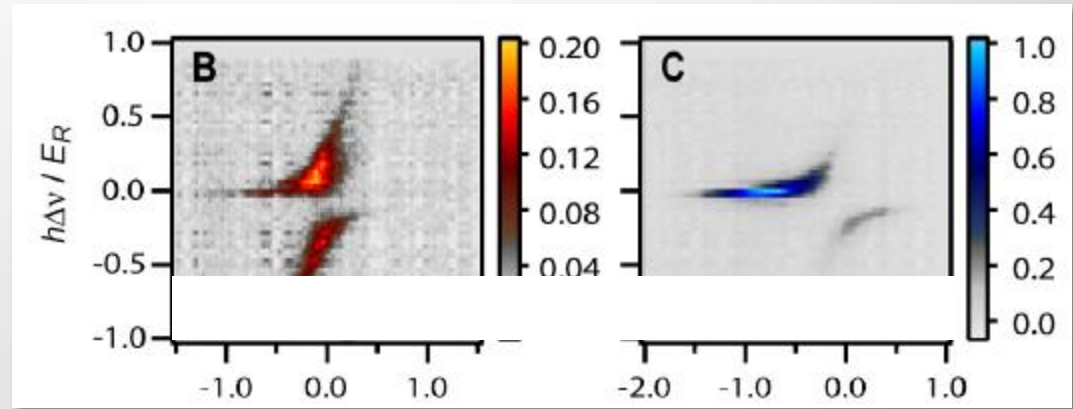
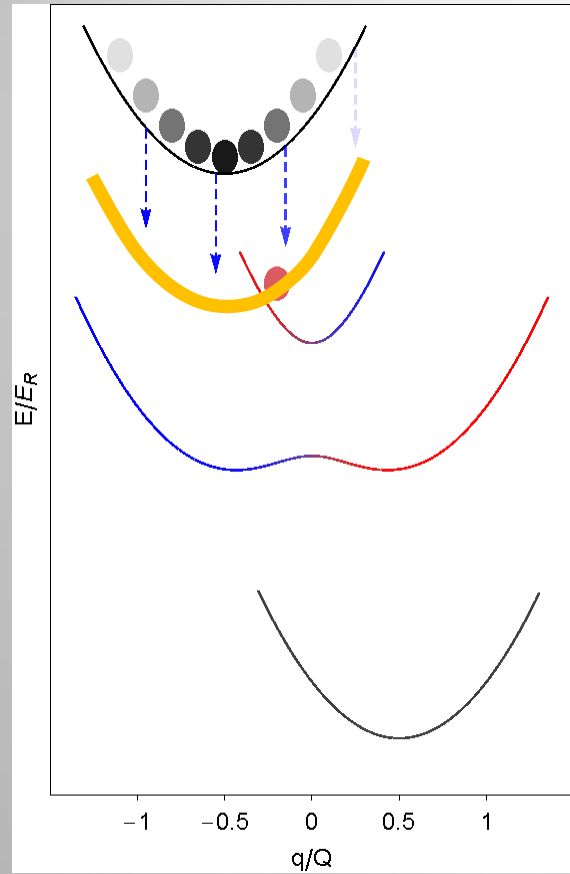
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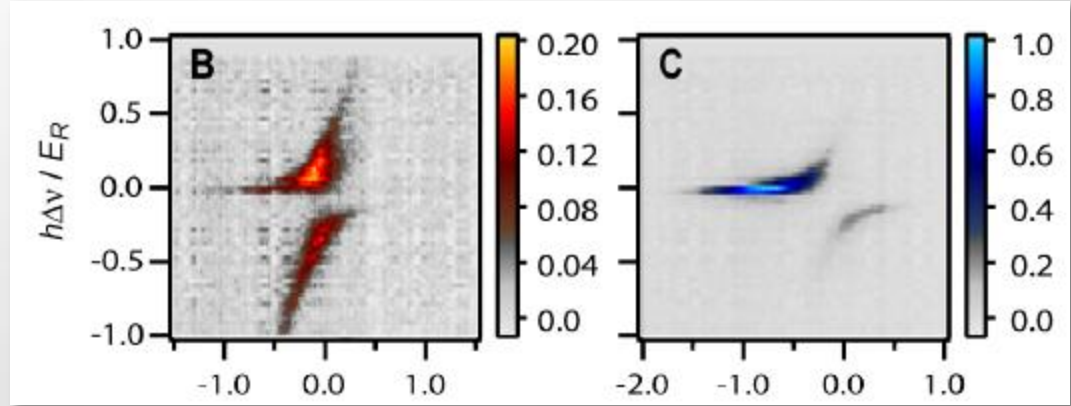
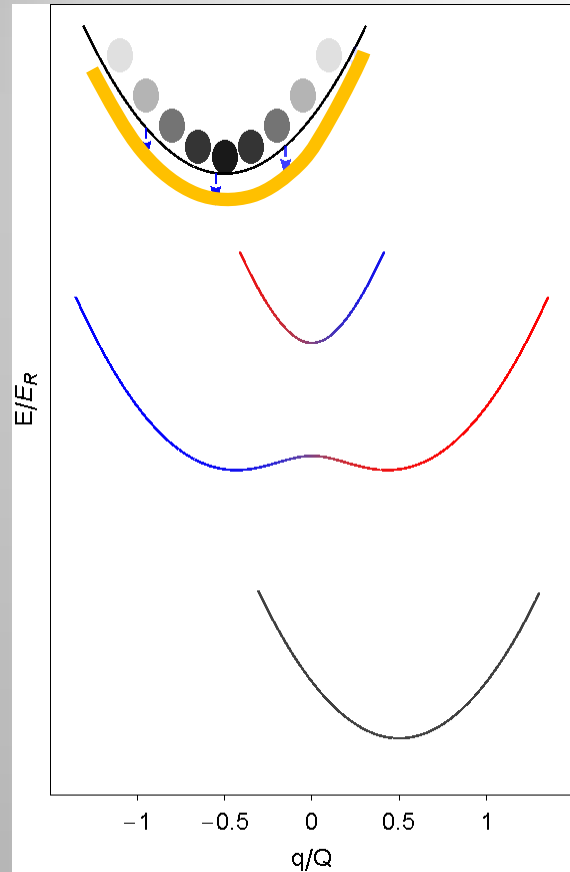
Spin-injection spectroscopy



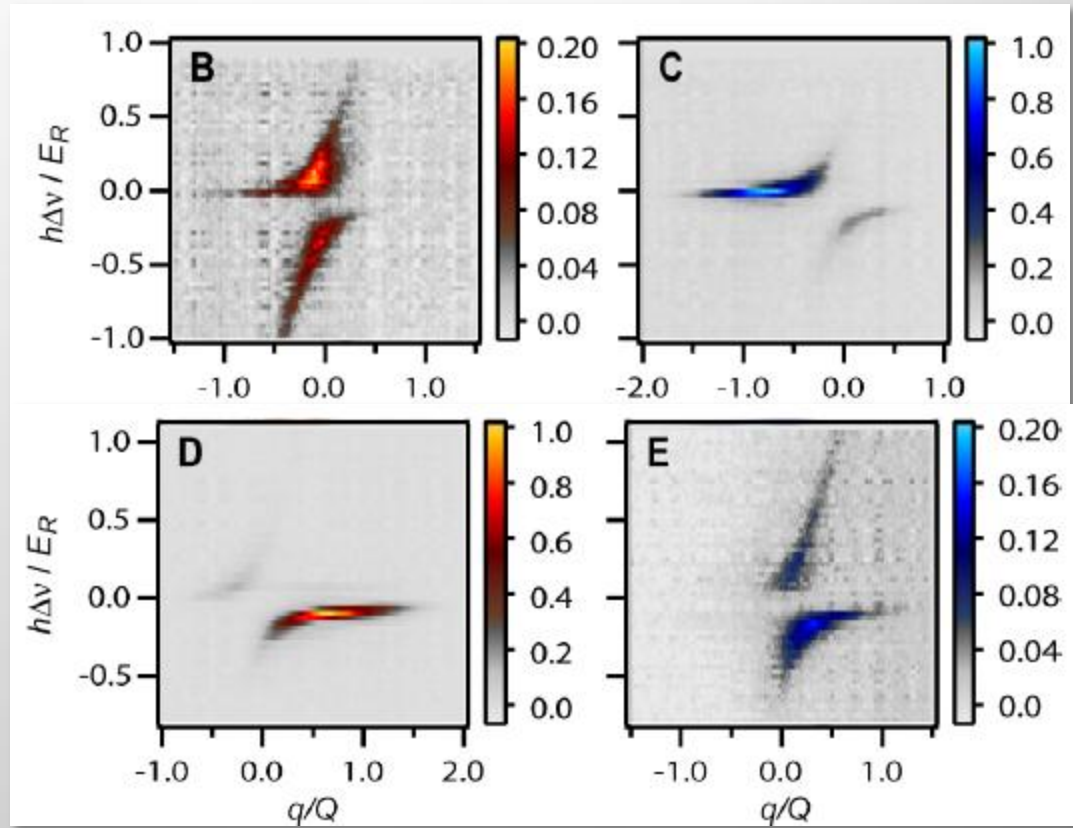
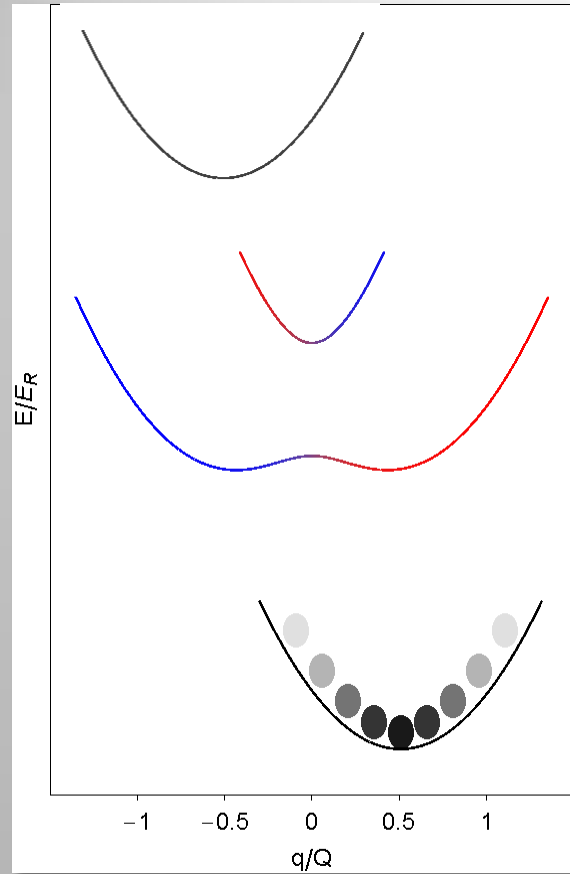
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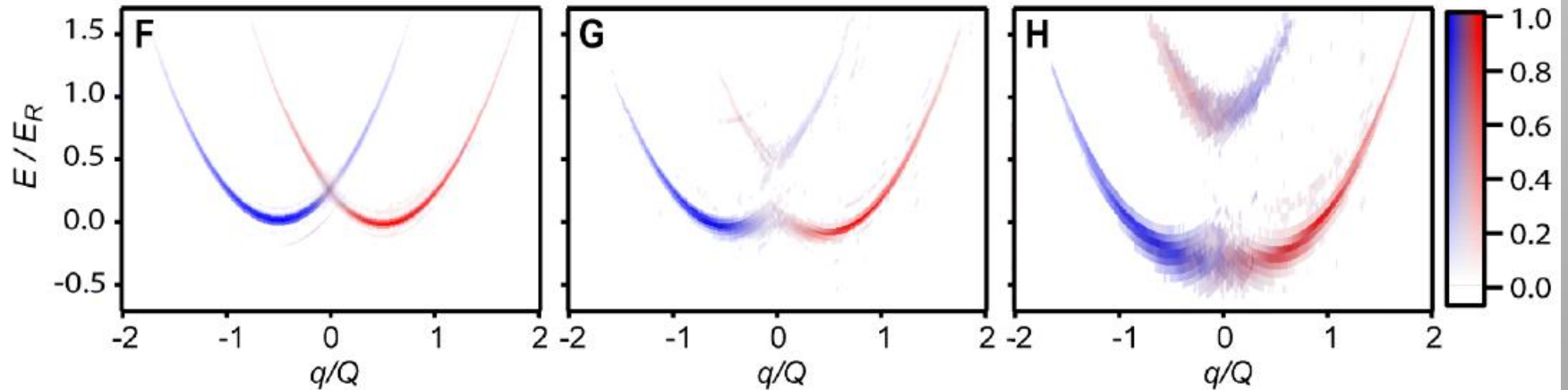
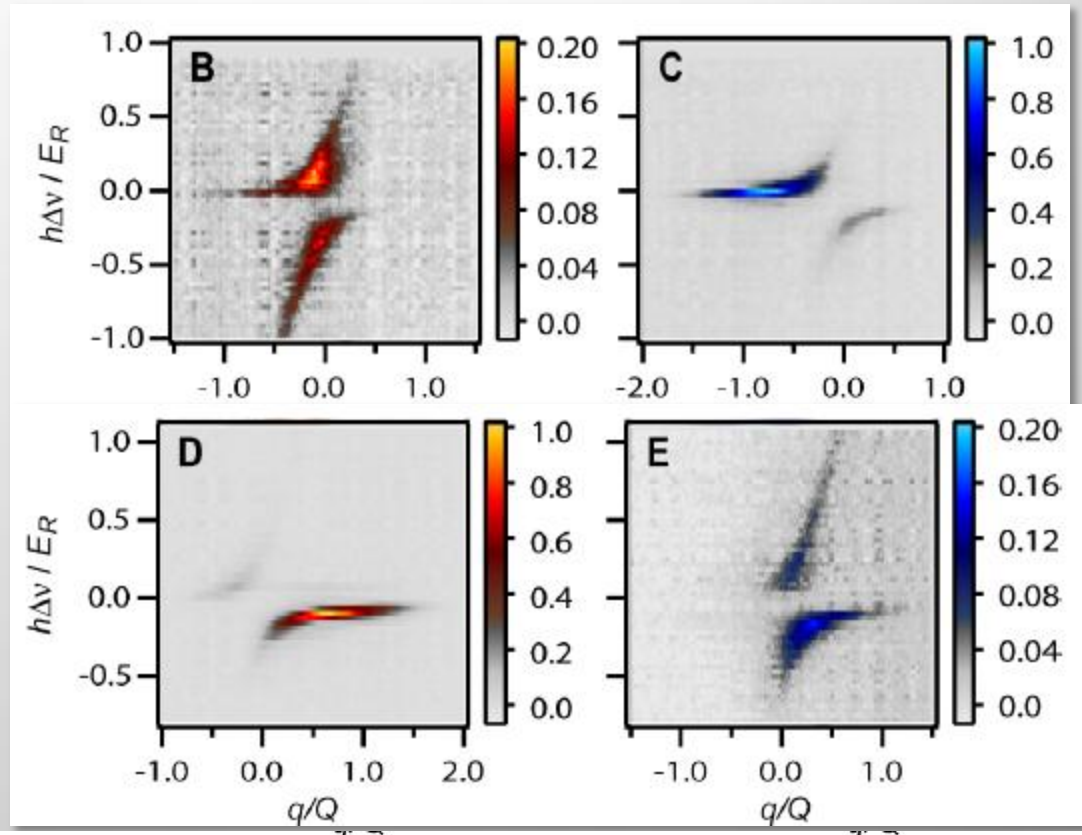
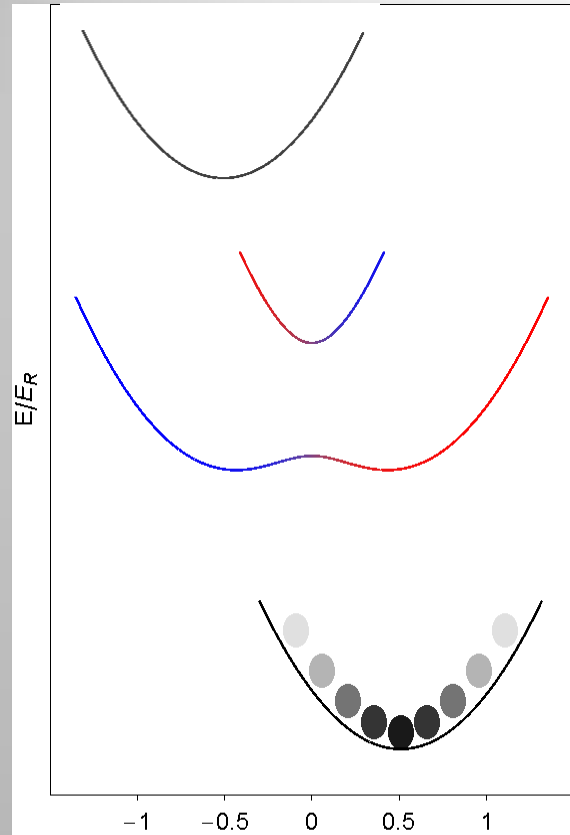
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Spin-injection spectroscopy

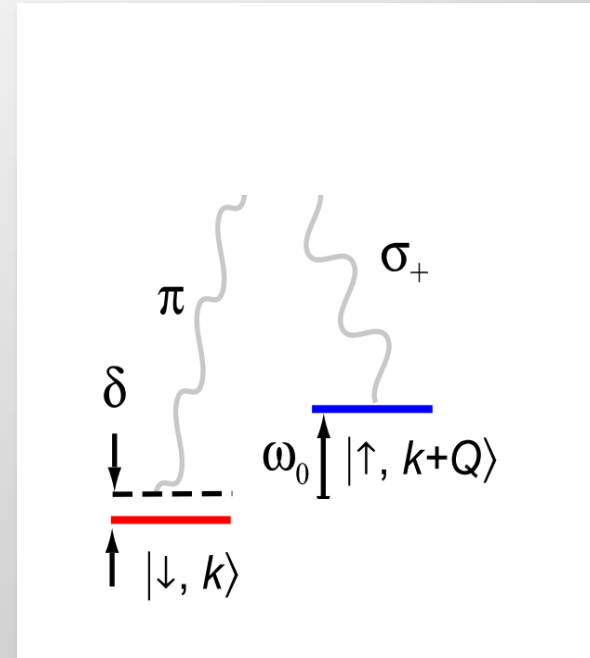
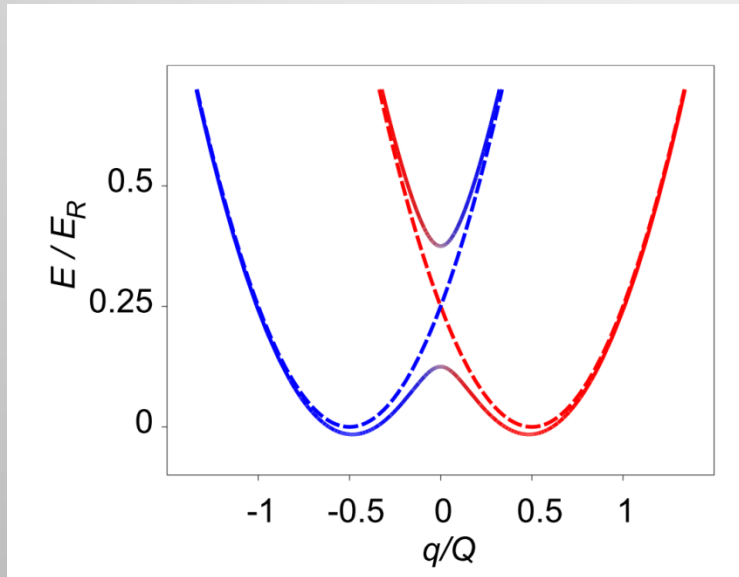


Direct Observation of the Spin-Orbit Gap



Creating a spinful lattice

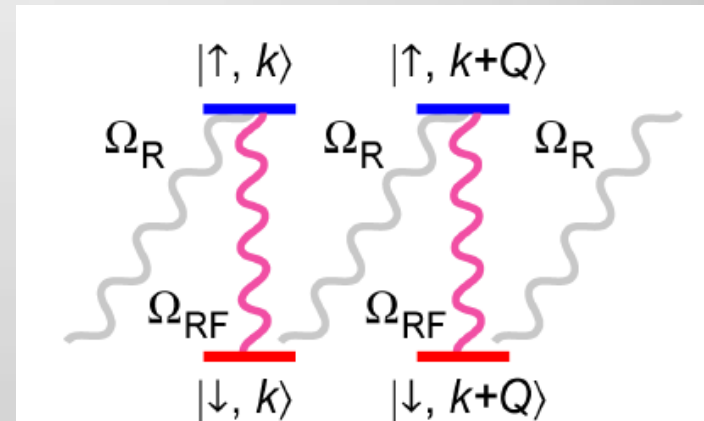
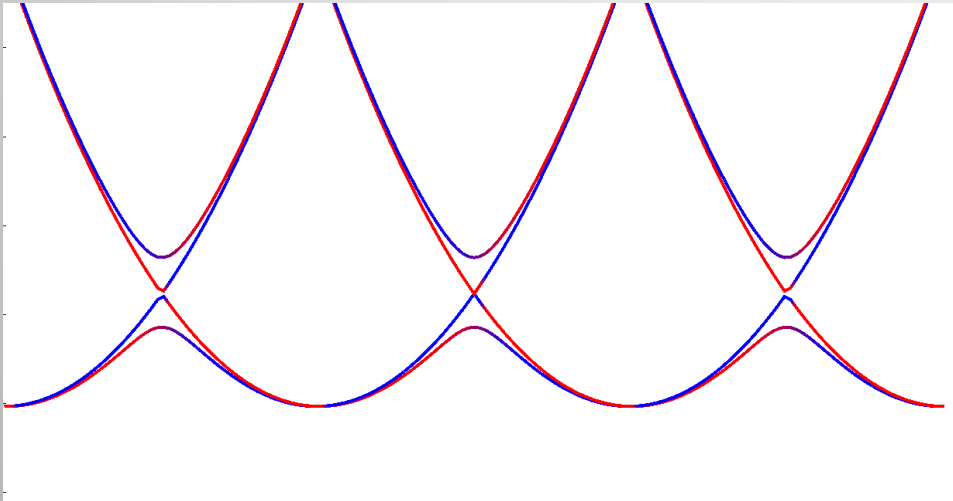
- With both Raman and RF, we obtain a spin-orbit coupled lattice (see Jiménez-García et al., arXiv:1201.6630 (2012))



Creating a spinful lattice

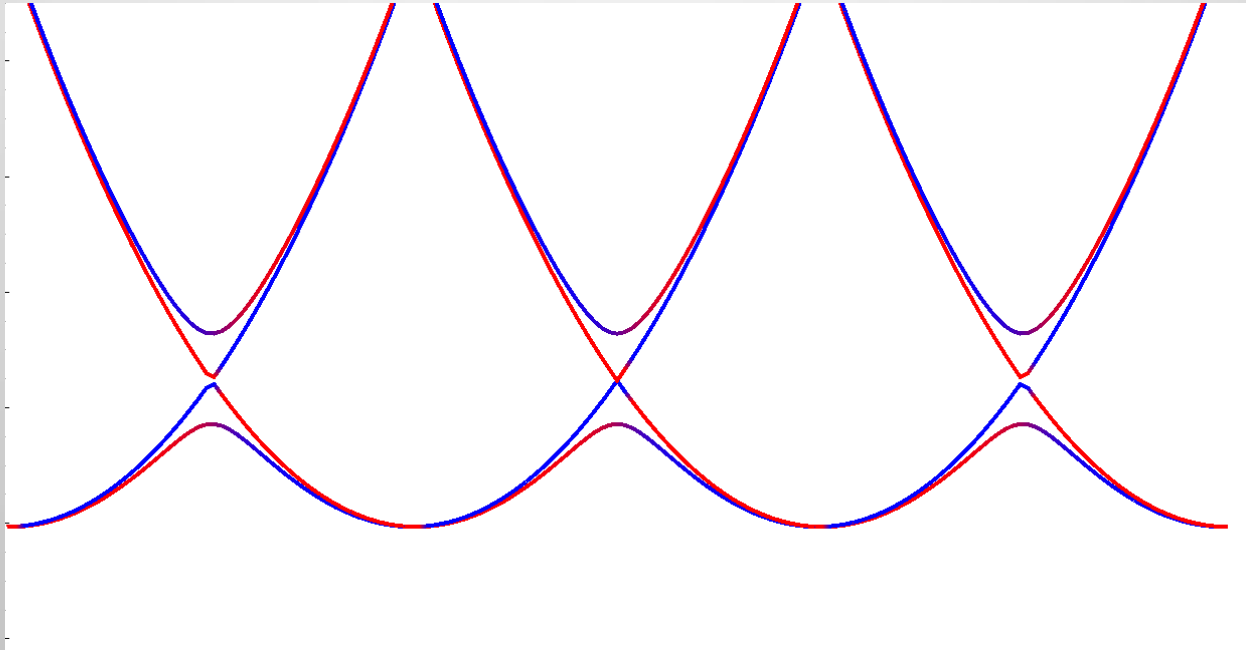
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Repeated scheme:



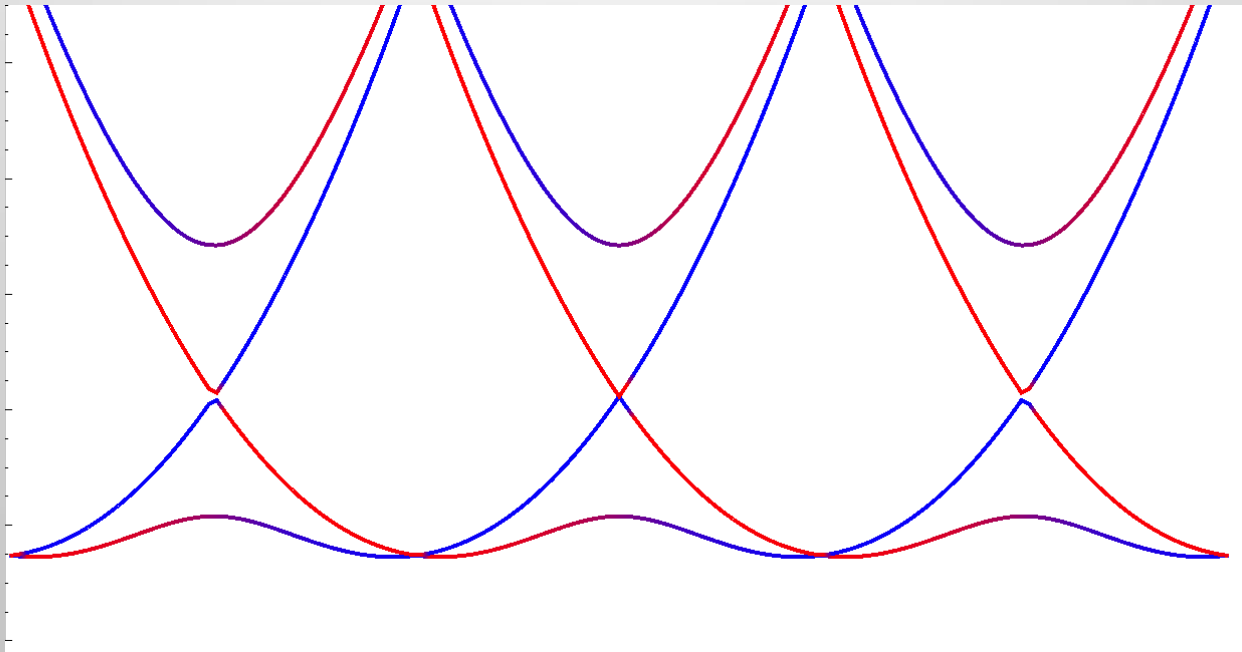
Creating a spinful lattice

- In repeated scheme



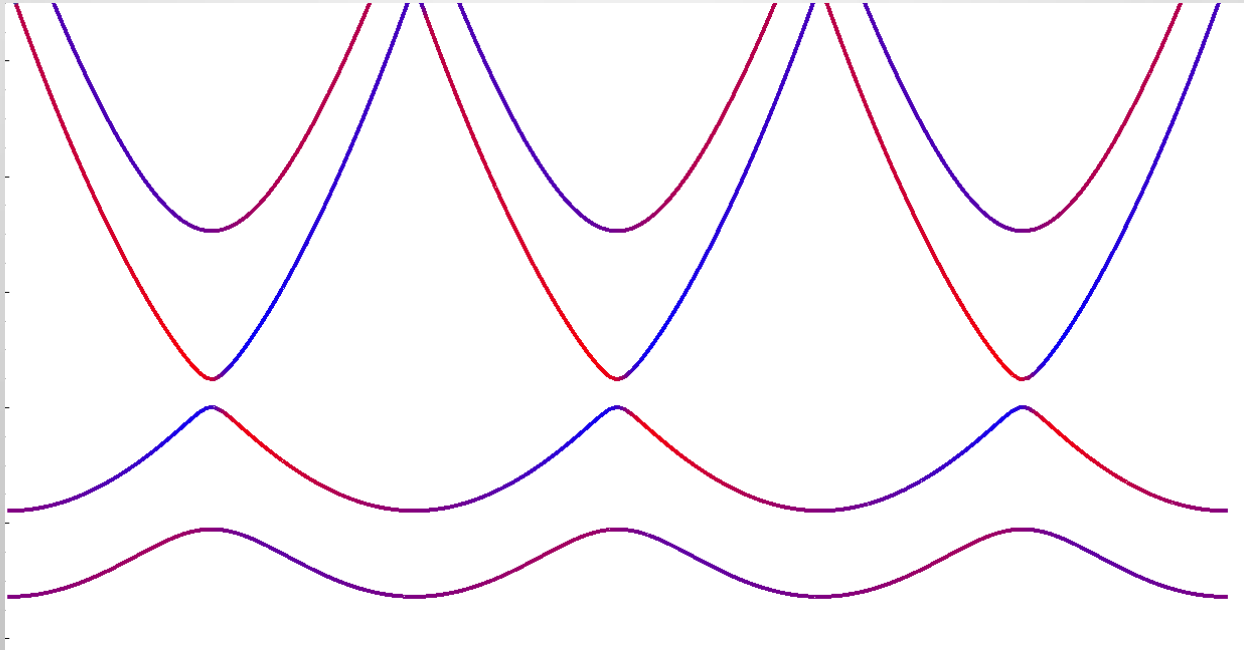
Creating a spinful lattice

- Degenerate point inside spin orbit gap



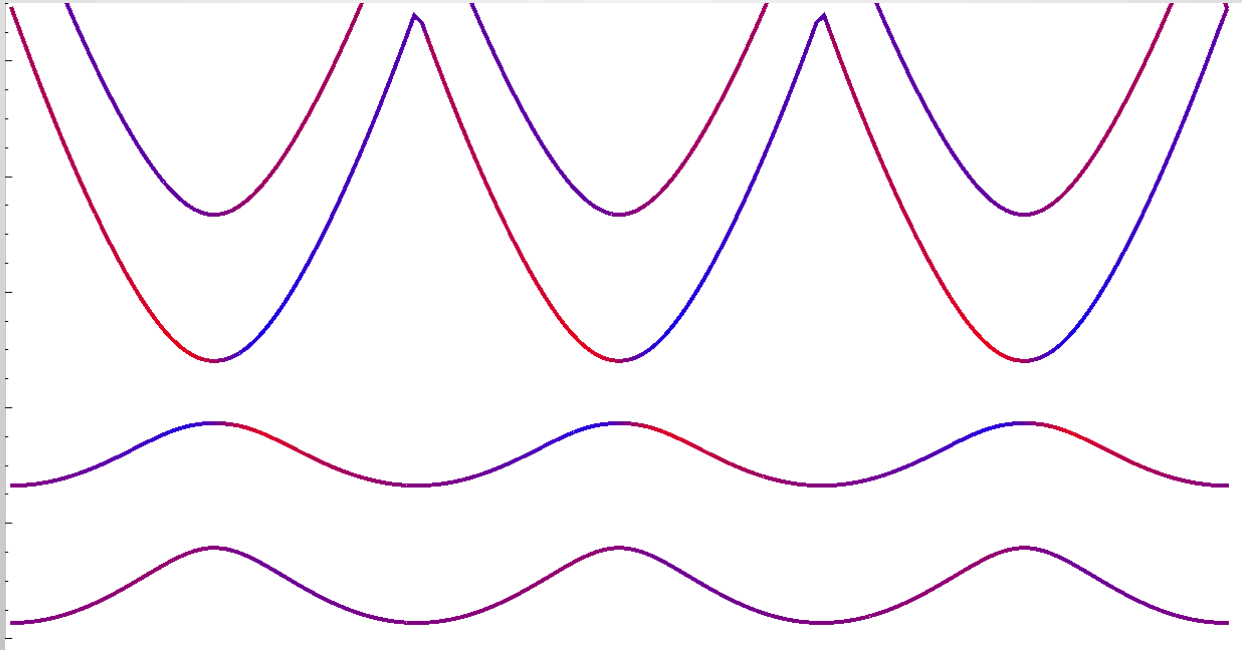
Creating a spinful lattice

- Bandgap opens between 2nd and 3rd band

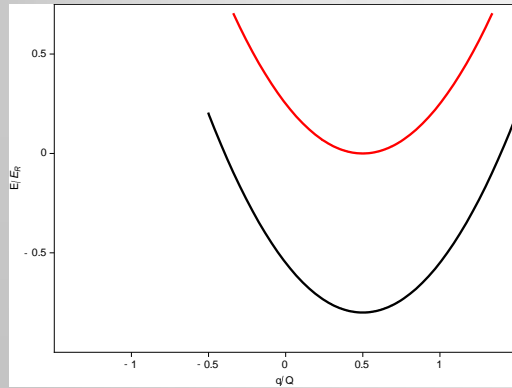


Creating a spinful lattice

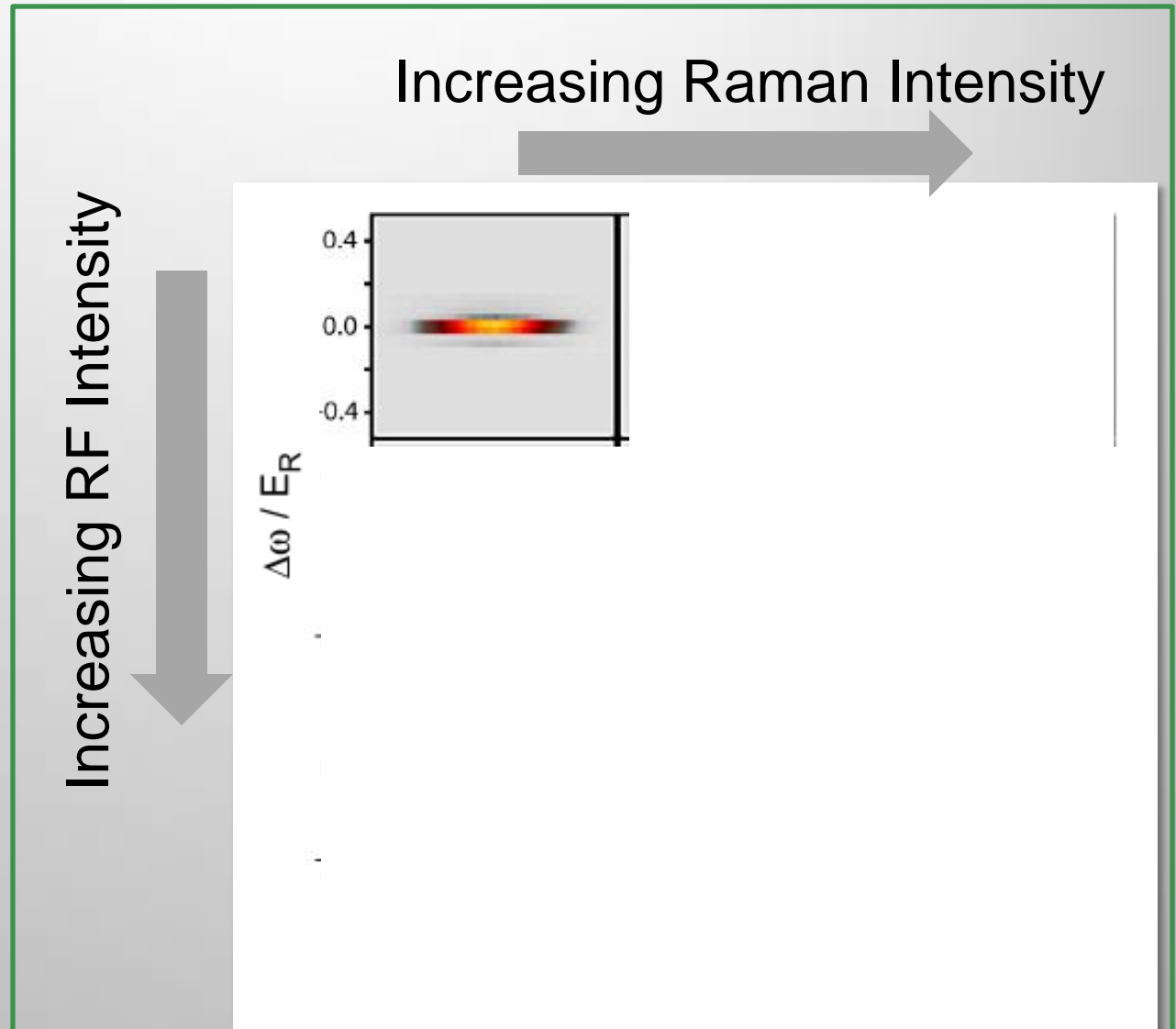
- Larger RF, gap between lowest bands



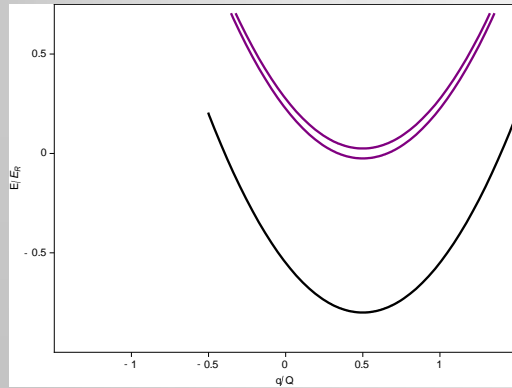
Spin-injection spectroscopy



- Transition at hyperfine splitting



Spin-injection spectroscopy

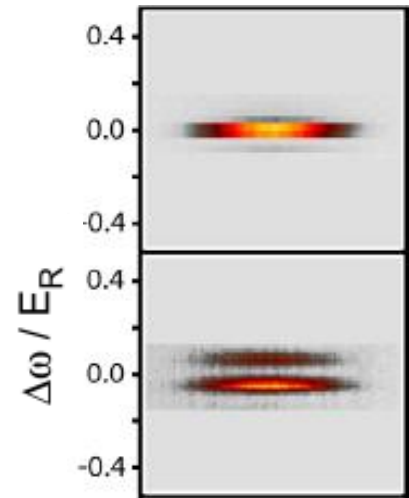
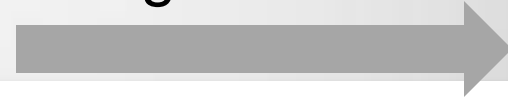


- Autler-Townes doublet

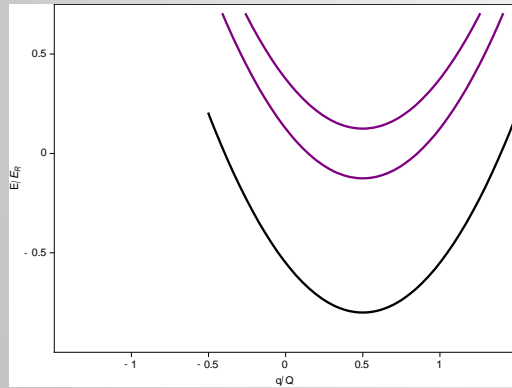
Increasing RF Intensity



Increasing Raman Intensity



Spin-injection spectroscopy

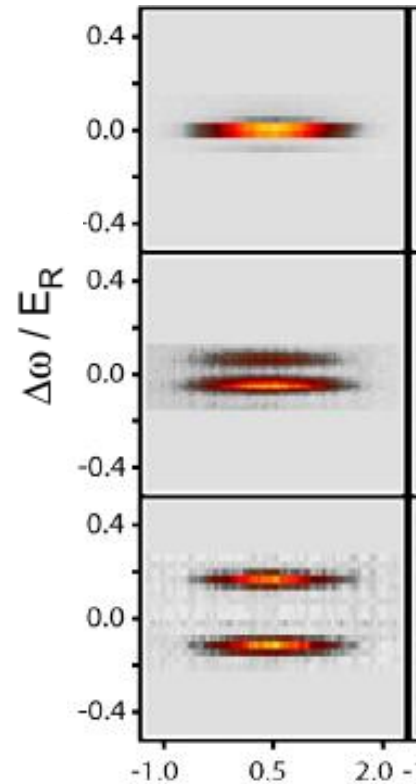
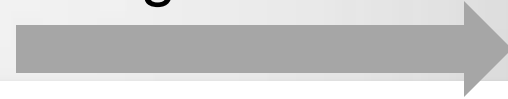


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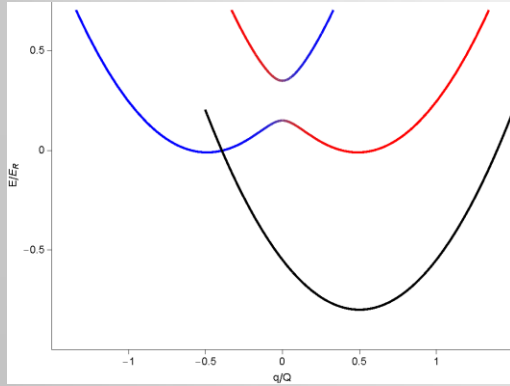
Increasing RF Intensity



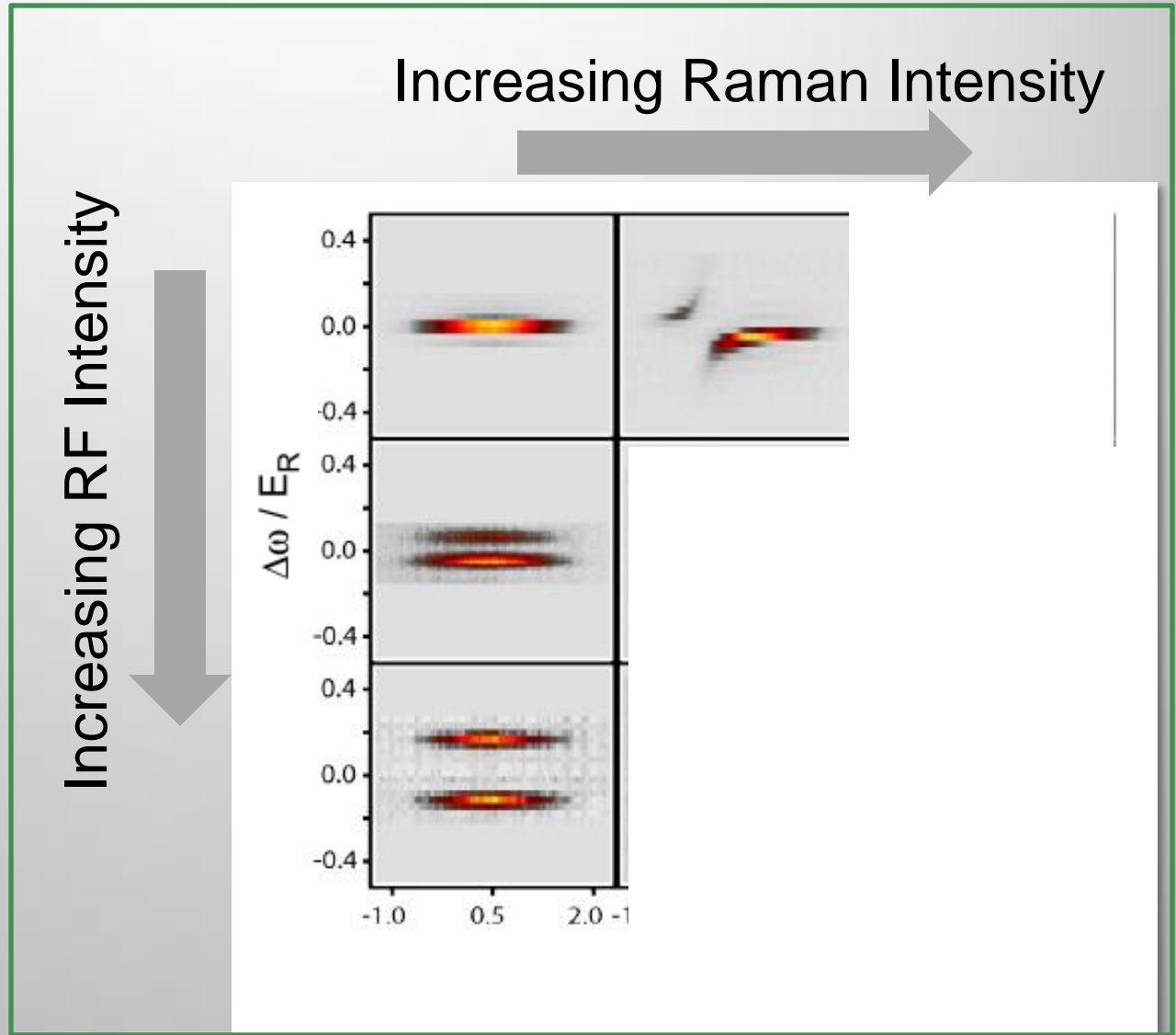
Increasing Raman Intensity



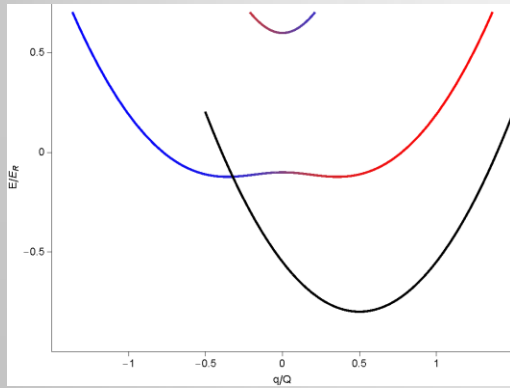
Spin-injection spectroscopy



- Momentum-dependent Autler-Townes doublet

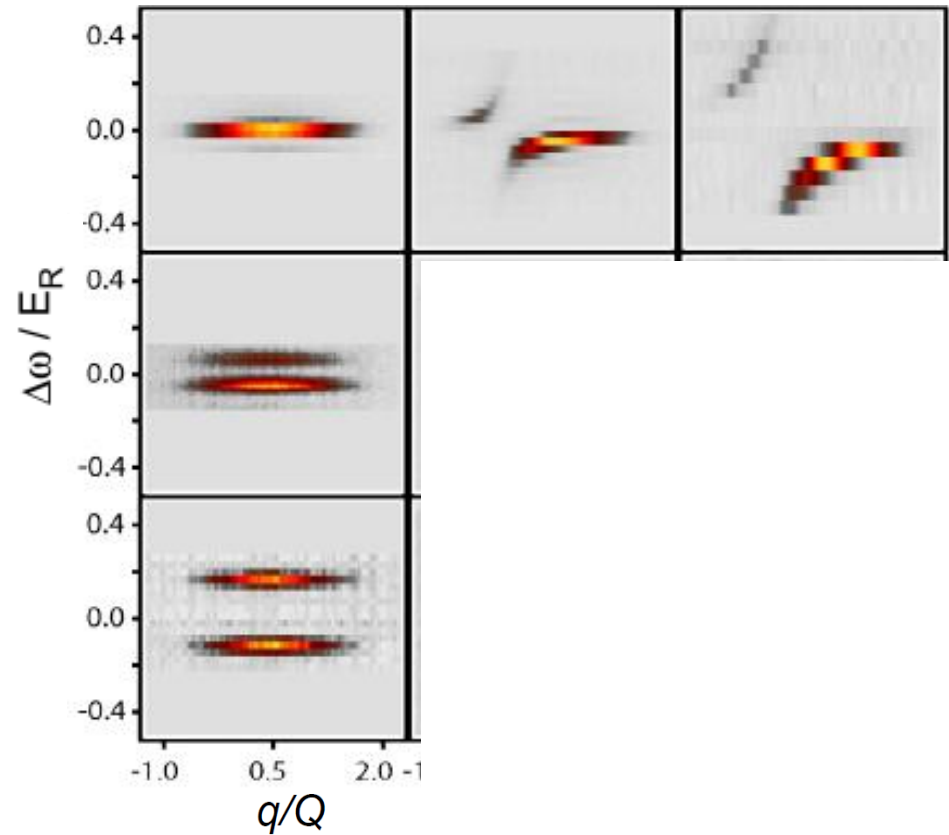


Spin-injection spectroscopy of spinful lattice

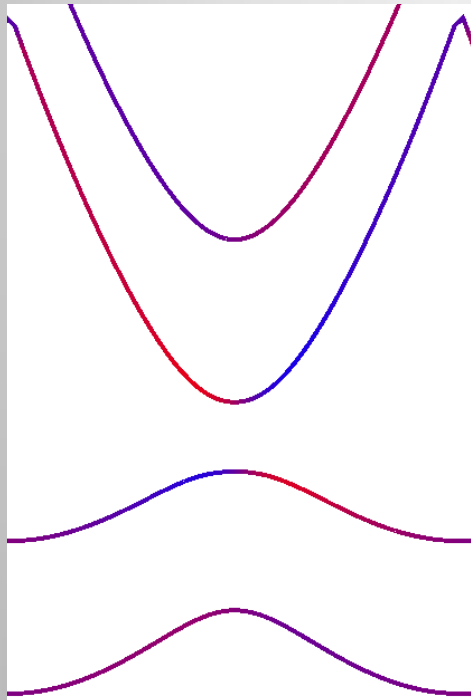


Increasing RF Intensity

Increasing Raman Intensity

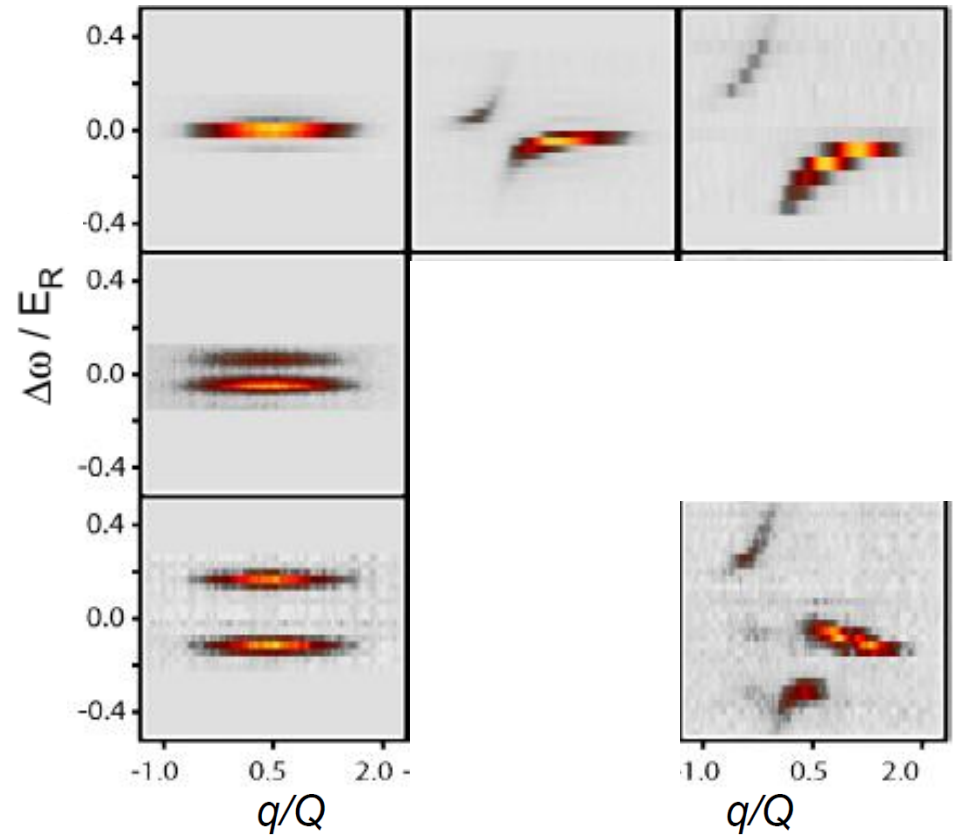


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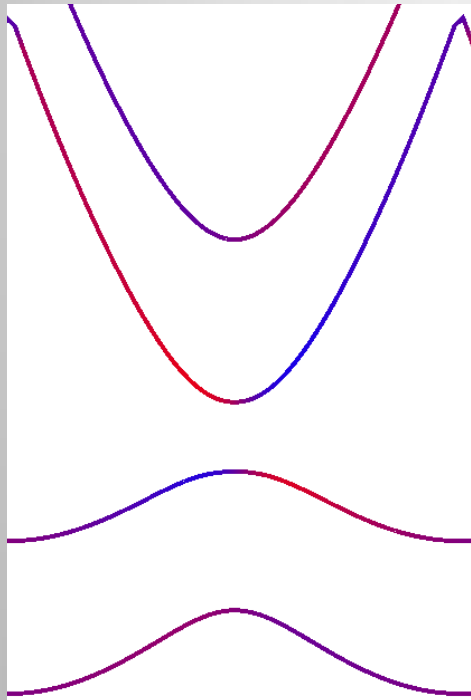


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Increasing Raman Intensity

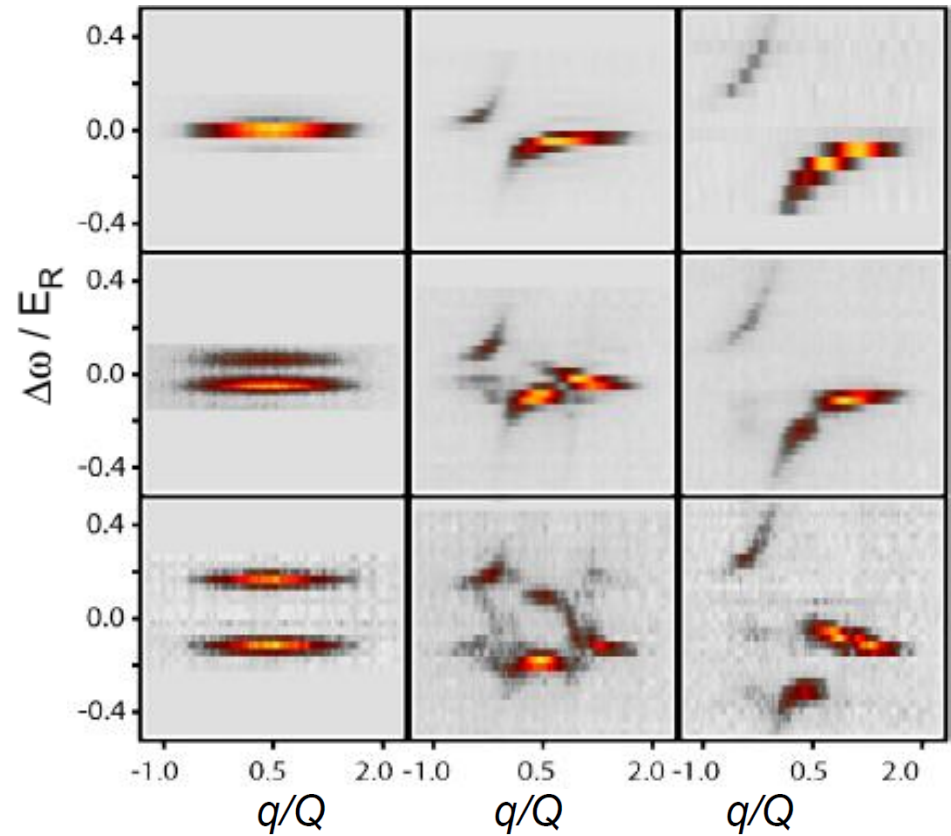


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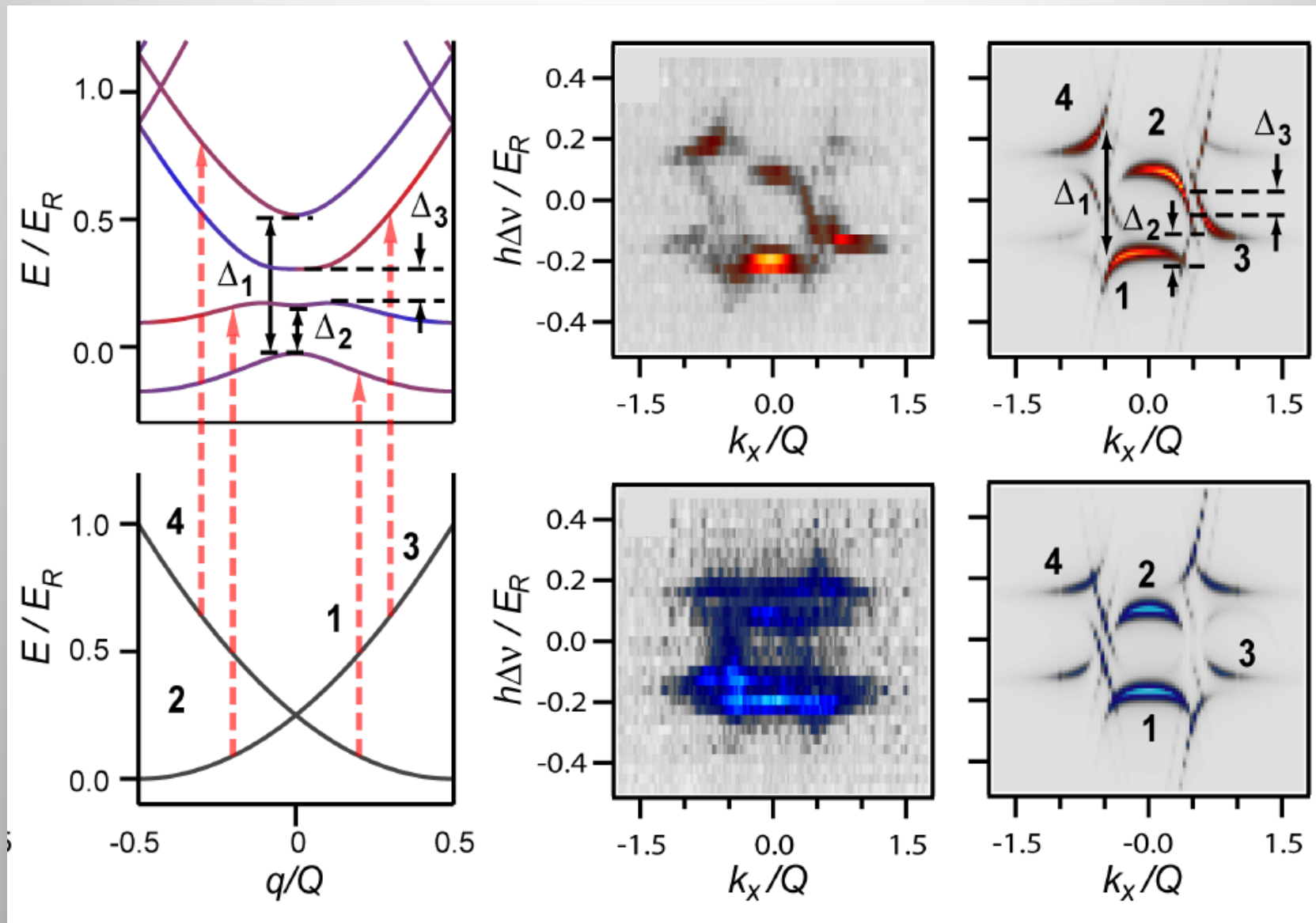


Increasing RF Intensity

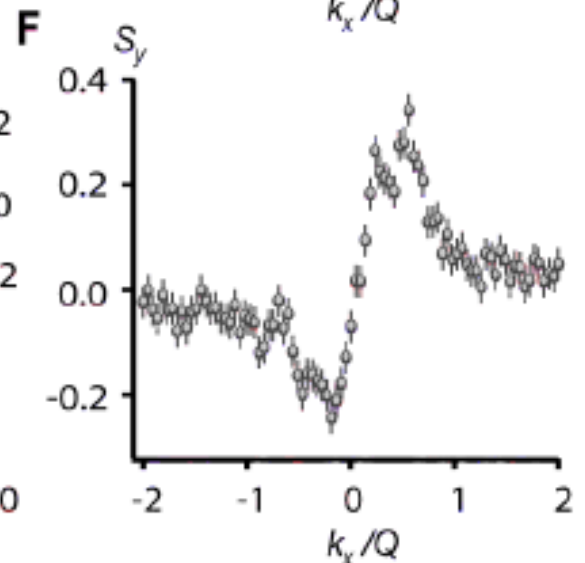
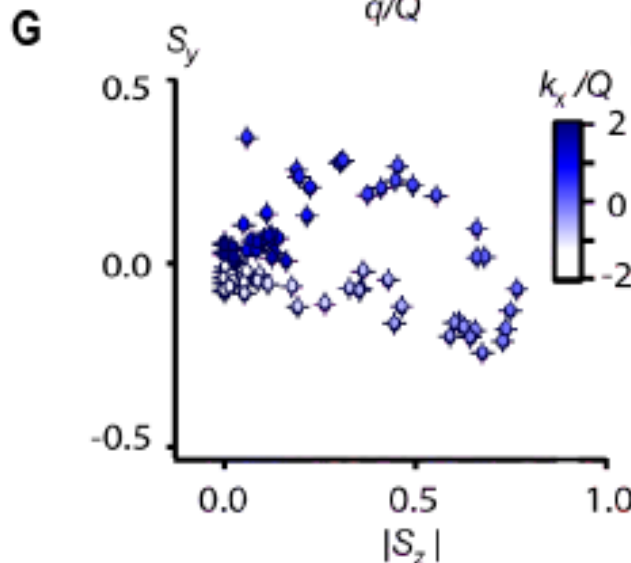
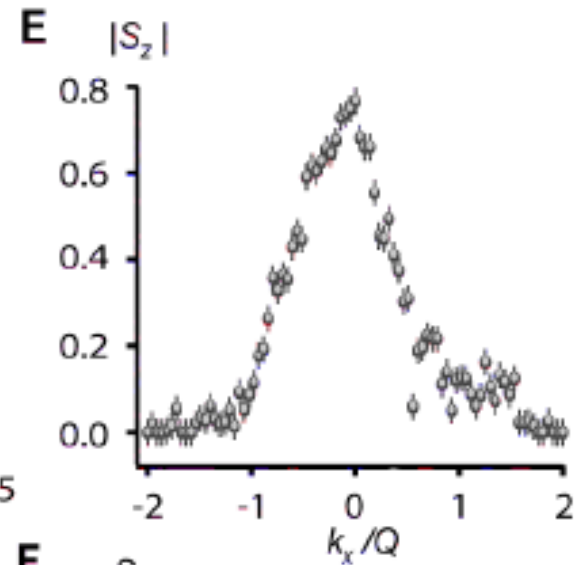
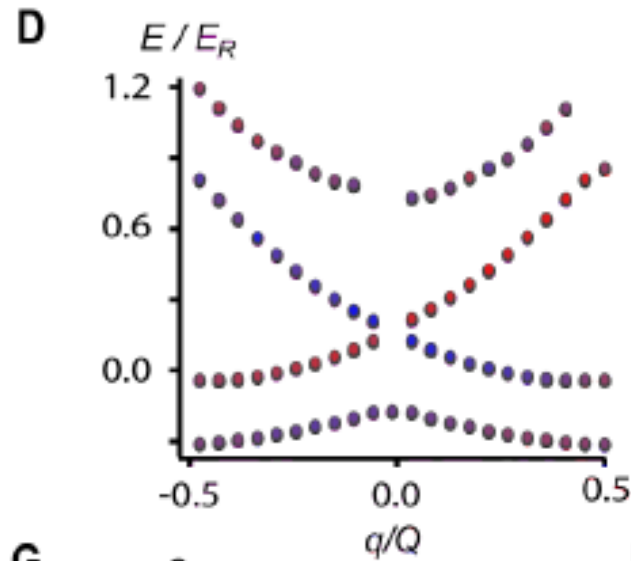
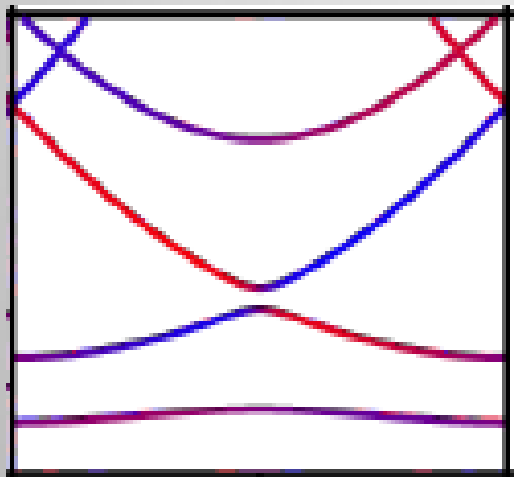
Increasing Raman Intensity



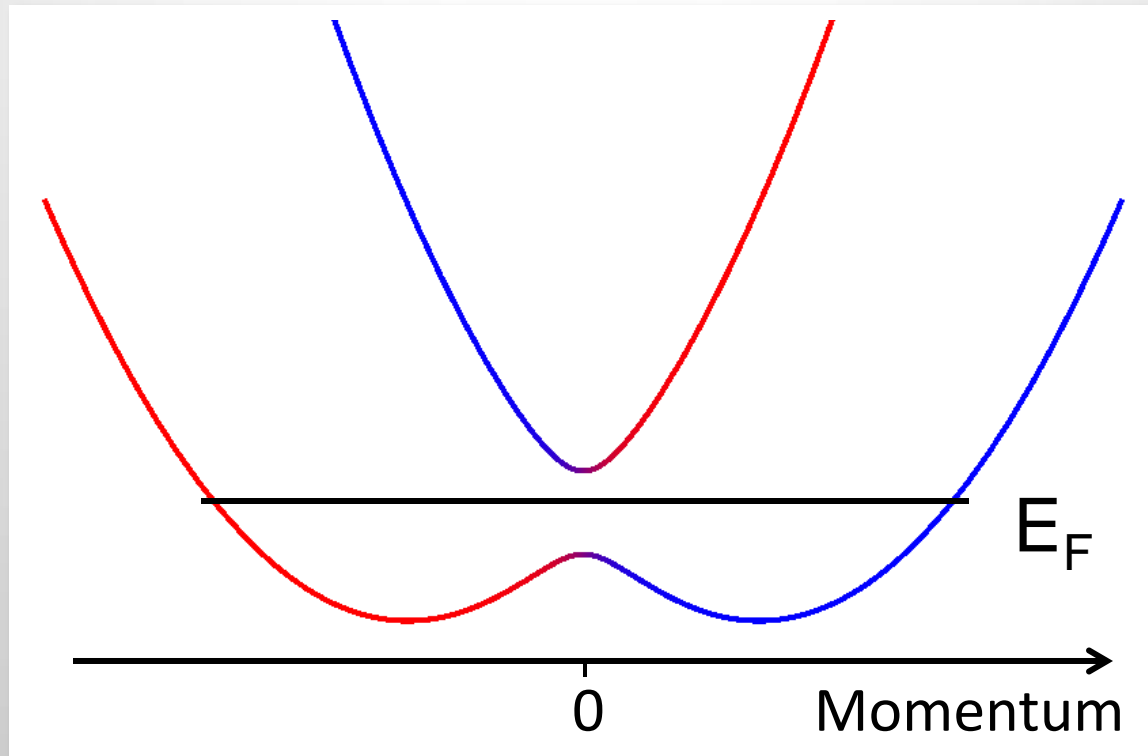
Spin-injection spectroscopy of spinful lattice



Determination of Spinful Band Structure



Spin-orbit Coupling of a Fermi sea



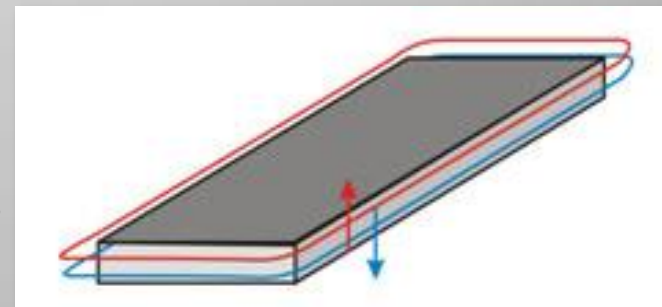
Spin-Orbit coupled Fermi sea \uparrow moves to the left, and \downarrow to the right

With extensions of these techniques:

Topological insulators, edge states

With interactions:

Topological superfluids, majorana fermions?



Fermions and Bosons

BEC I

Fermi Gases in 3D and 2D

Synthetic Gauge Fields

A.T. Sommer

M.J.H. Ku

L.W. Cheuk

Dr. T. Yefsah

Dr. W. Bakr

visitor spring 2012: Zoran Hadzibabic

Fermi I

LiNaK

Fermi-Fermi mixtures

C.-H. Wu

J.W. Park

J. Santiago

S. Will

Fermi II

Fermi Gas

Microscope

T. Gersdorf

D. Reens

M. Okan

V. Ramasesh

Dr. W. Bakr



ction: Felix Werner, Kris van Houcke, Evgeny Kozik,
Nikolay Prokof'ev, Boris Svistunov

