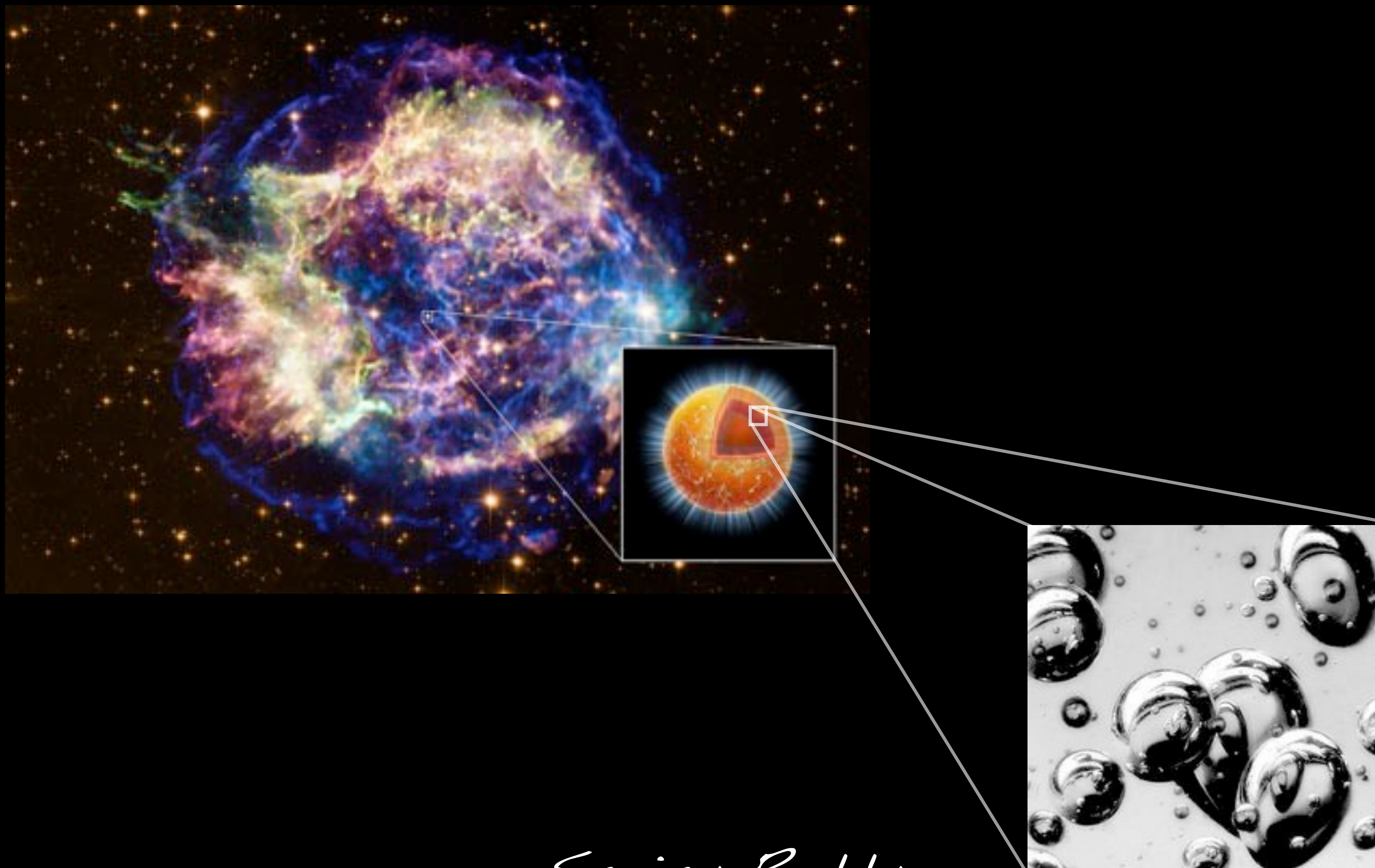
Unravelling the neutron star interior: Prospects and challenges



Sanjay Reddy
INT, Univ. of Washington

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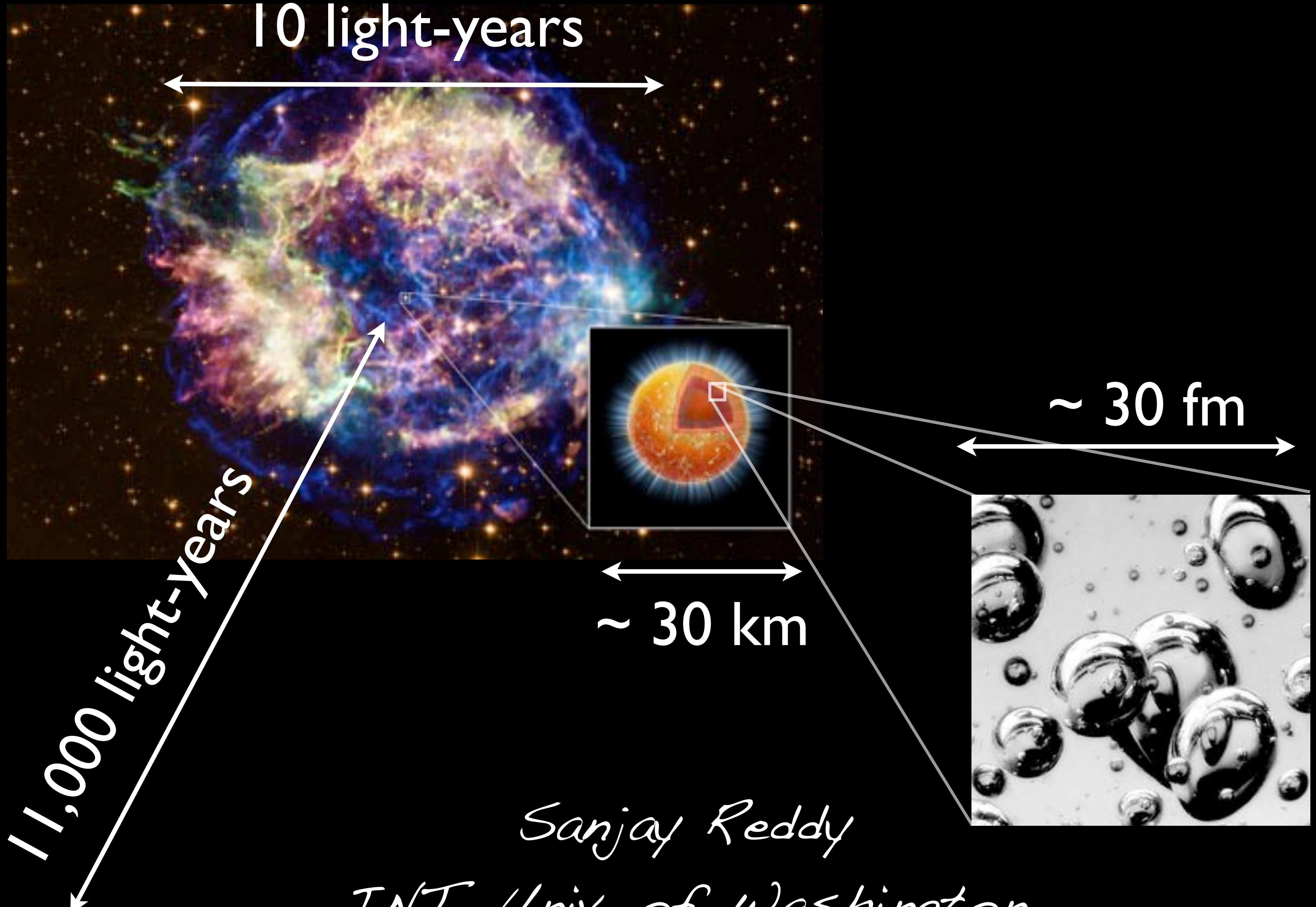

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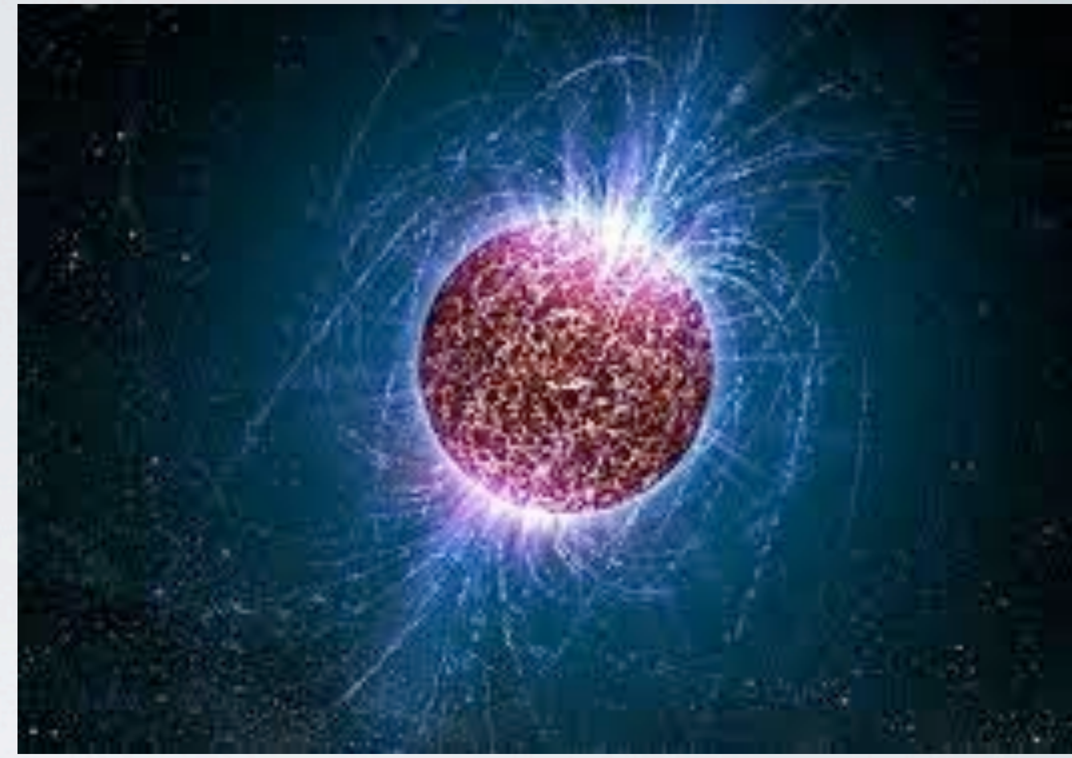
Cold Compression

Cold Compression

$$x \Rightarrow 10^{-4}\ x$$



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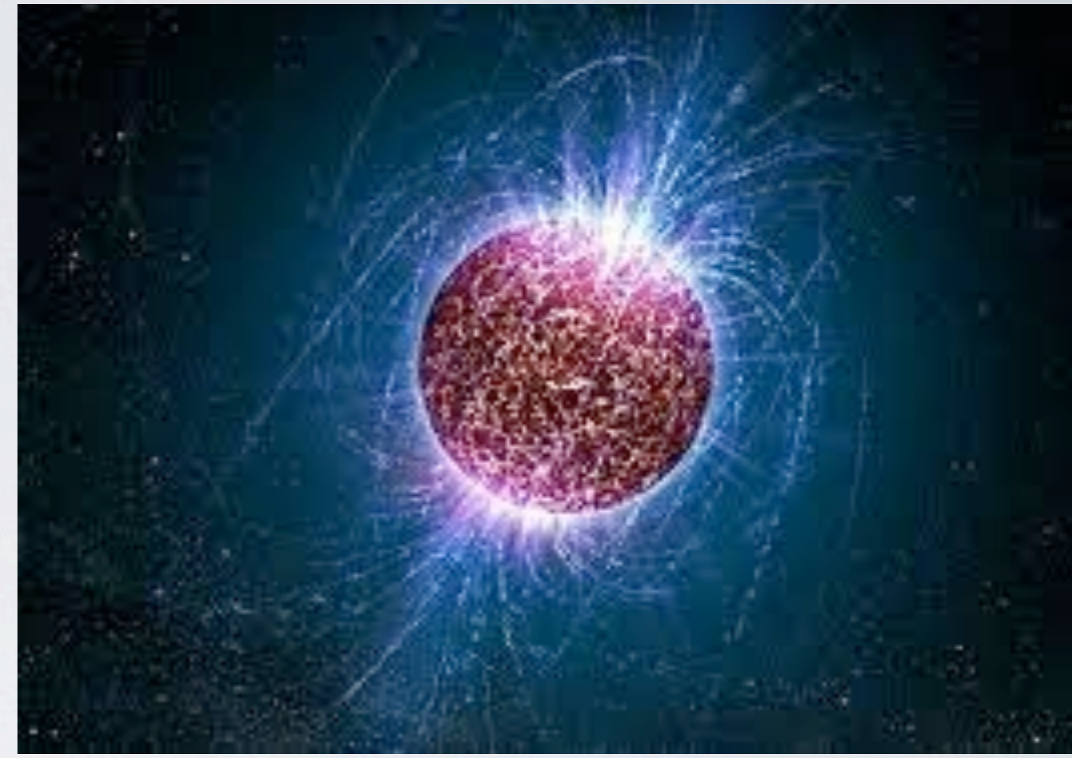

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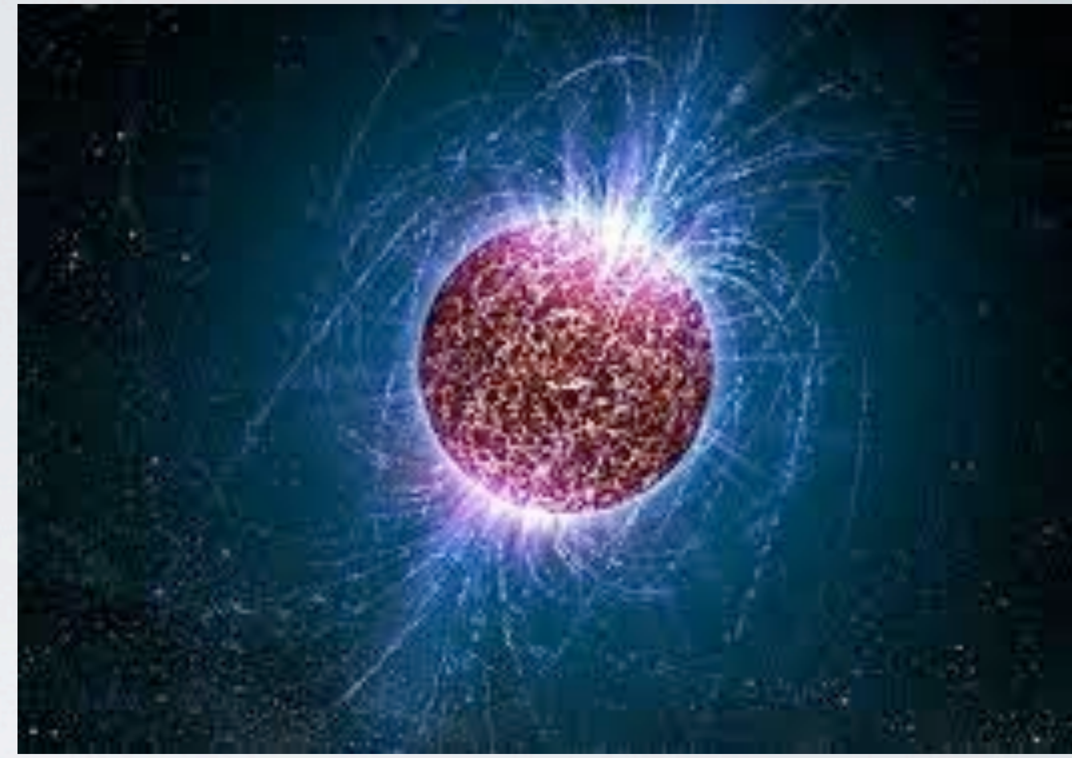
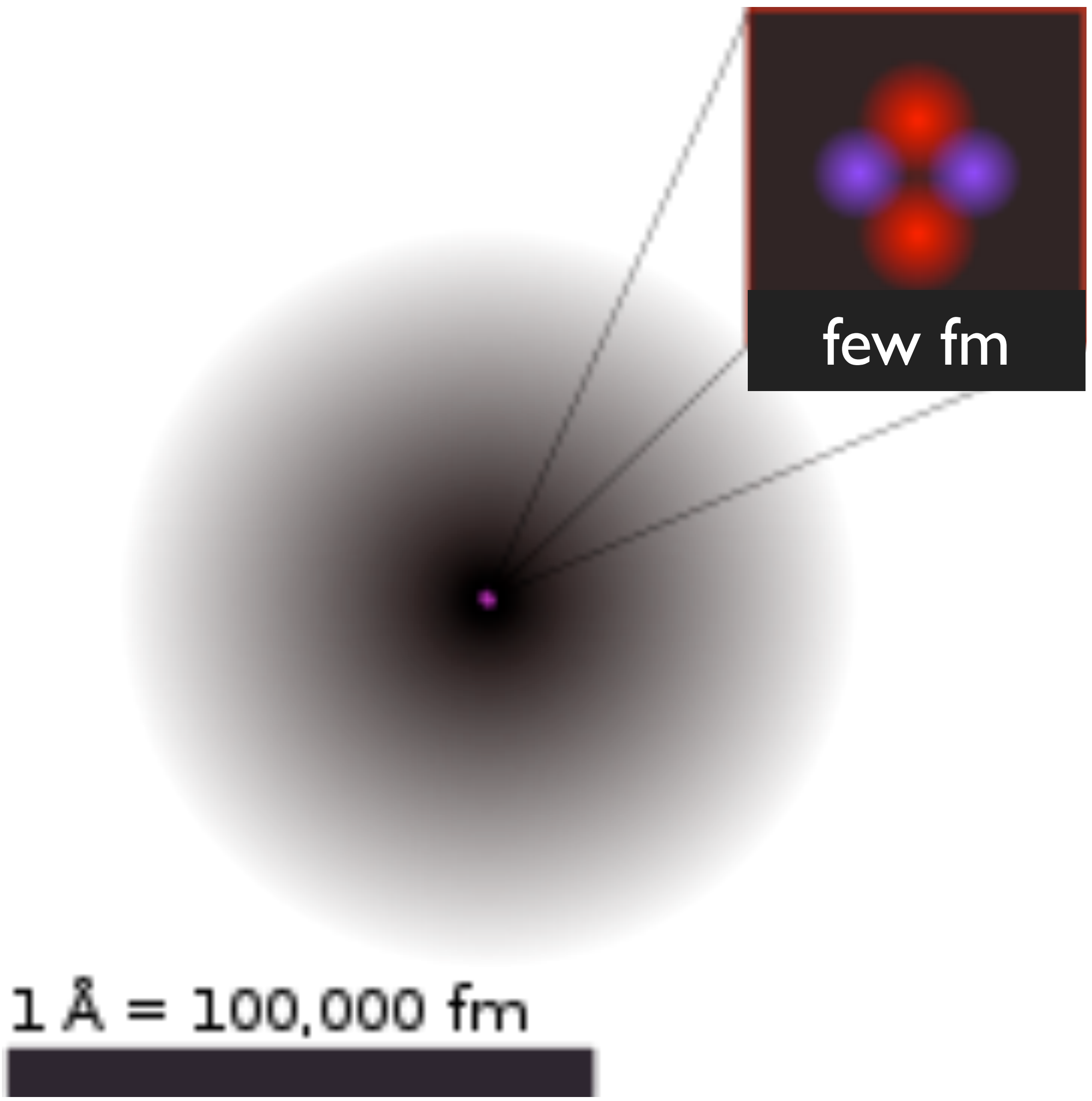


$x \Rightarrow (1-\varepsilon)\, x$

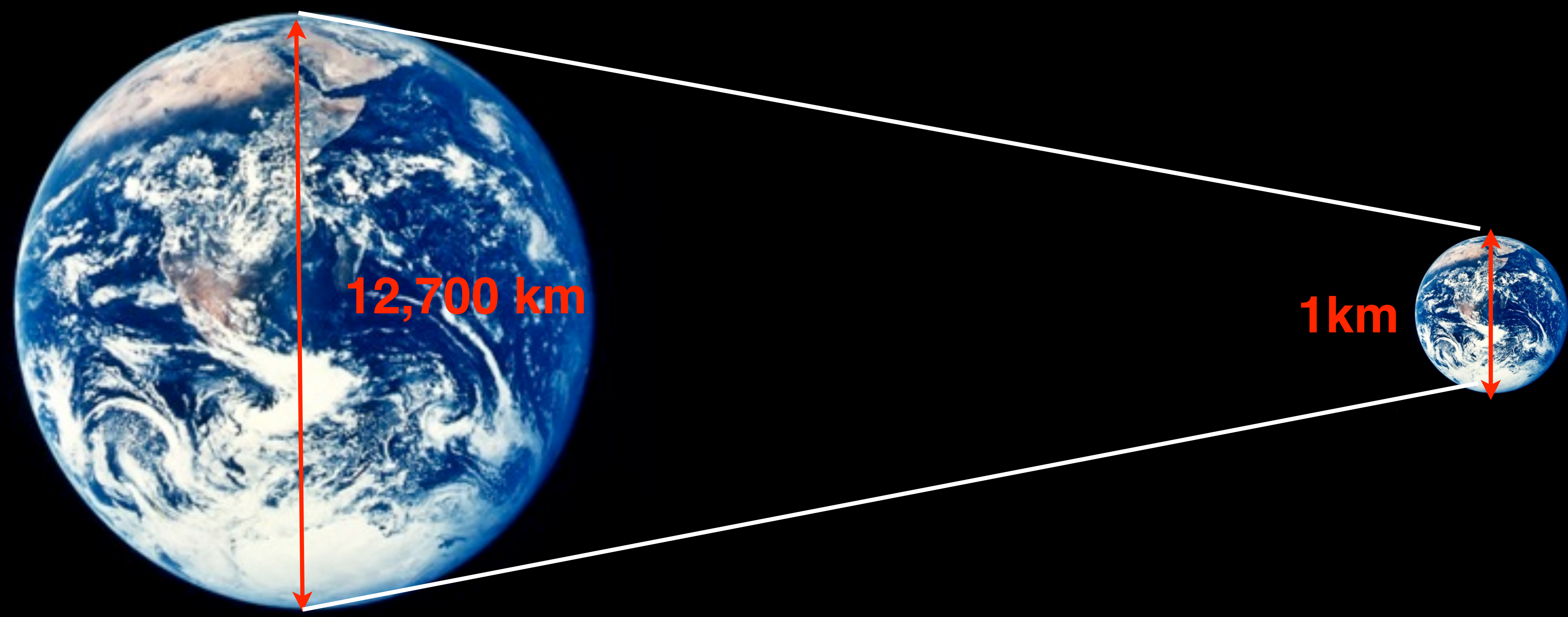
$x \Rightarrow 10^{-10}\, x$

Getting to ~ 2.5x10^{14} g/cm^3: Nuclear Physics



Properties of nuclei - almost get us there !

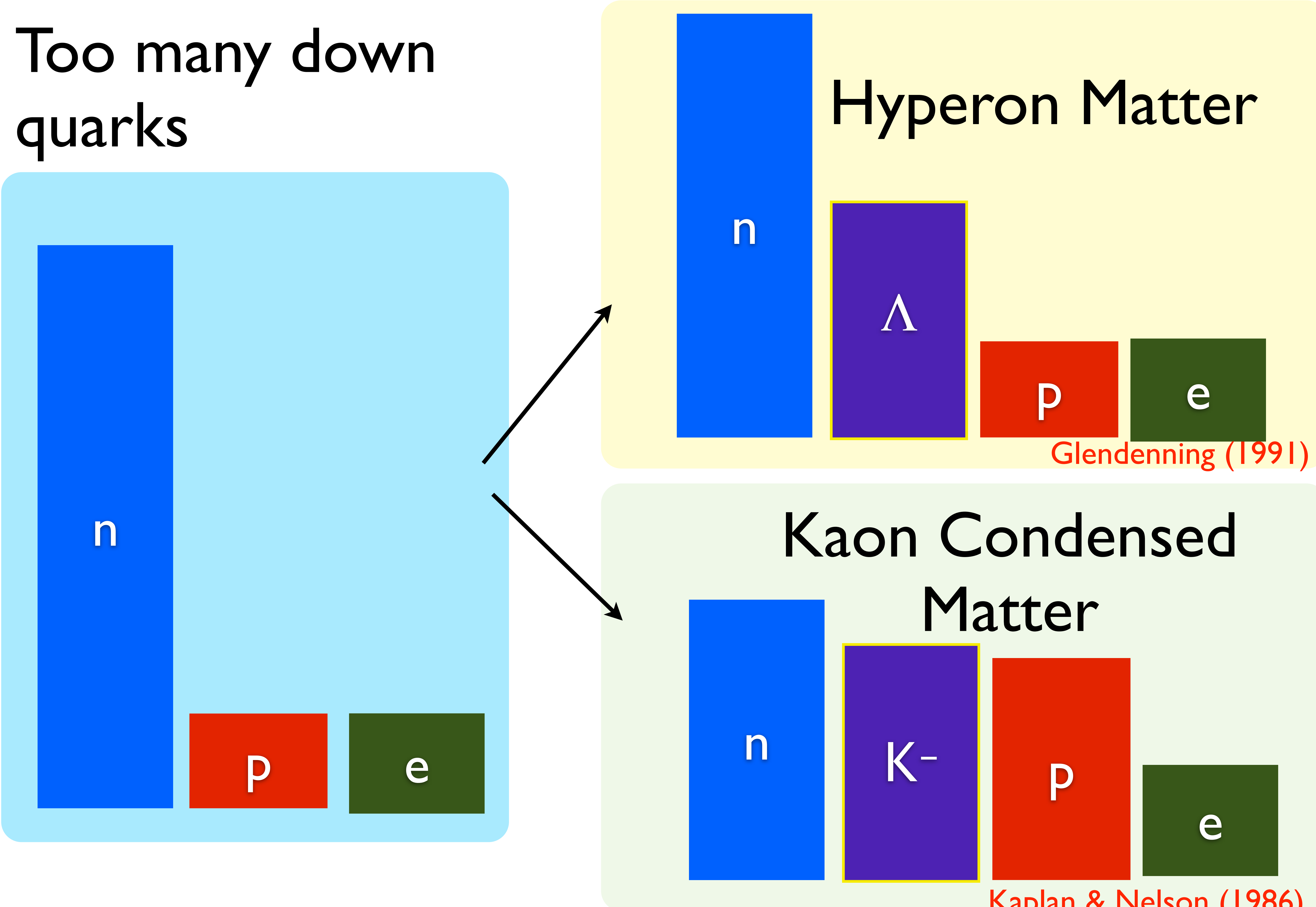
Compression: Frustration and Liberation



Density	Energy	Phenomena
10^3 - 10^6 g/cm^3	Electron Chemical Pot. μ_e= 10 keV– MeV	Ionization
10^6 - 10^{11} g/cm^3	Electron Chemical Pot. μ_e= 1−25 MeV	Neutron-rich Nuclei
10^{11} - 10^{14} g/cm^3	Neutron Chemical Pot. μ_n= 1−30 MeV	Neutron-drip
10^{14} - 10^{15} g/cm^3	Neutron Chemical Pot. μ_n=30−1000 MeV	Nuclear matter Hyperons or Quarks ?

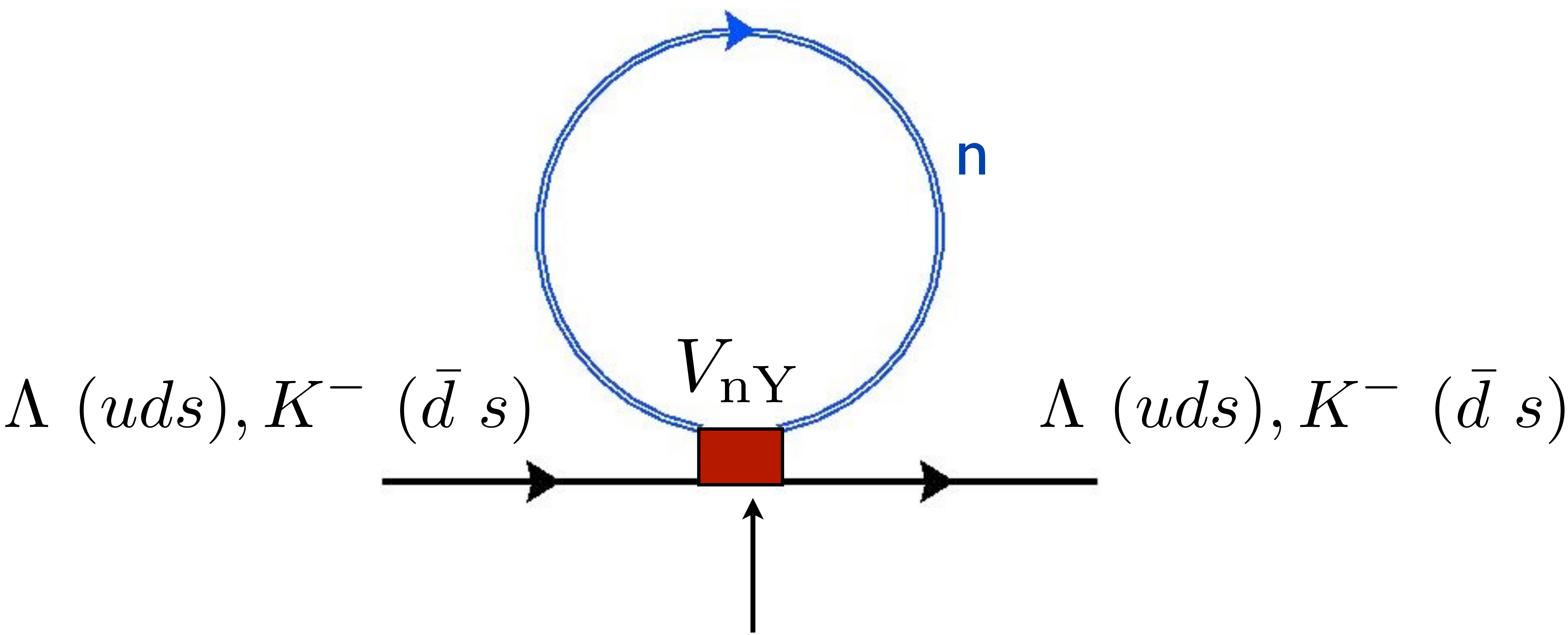
Frustration in Neutron Matter

Too many down quarks



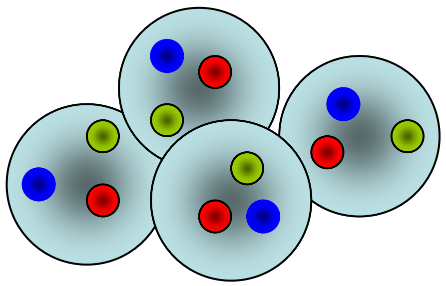
Strangeness in Dense Matter: Theory very uncertain.

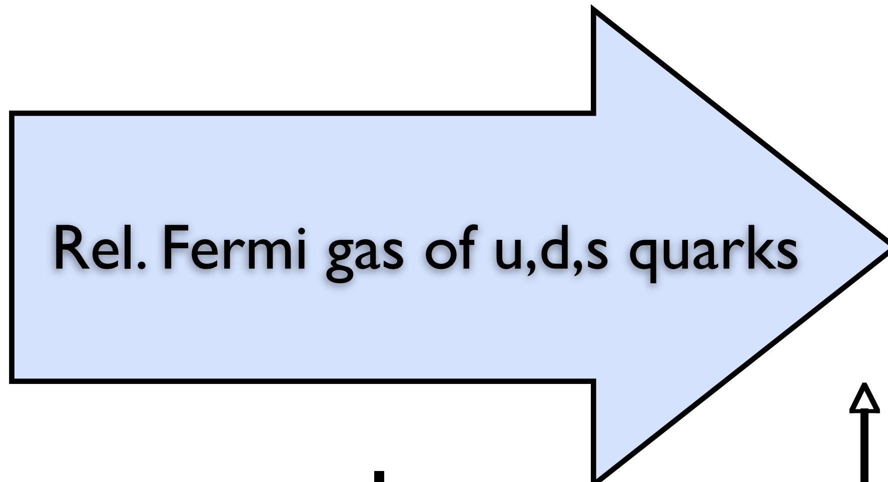
$$
\begin{aligned}
E_\Lambda(p=0) &= M_\Lambda + V_{\mathrm{n}\Lambda}(\rho) \le \mu_B \\
E_{\mathrm{K}^-}(p=0) &= M_{\mathrm{K}^-} + V_{\mathrm{nK}^-}(\rho) \le \mu_e
\end{aligned}
$$

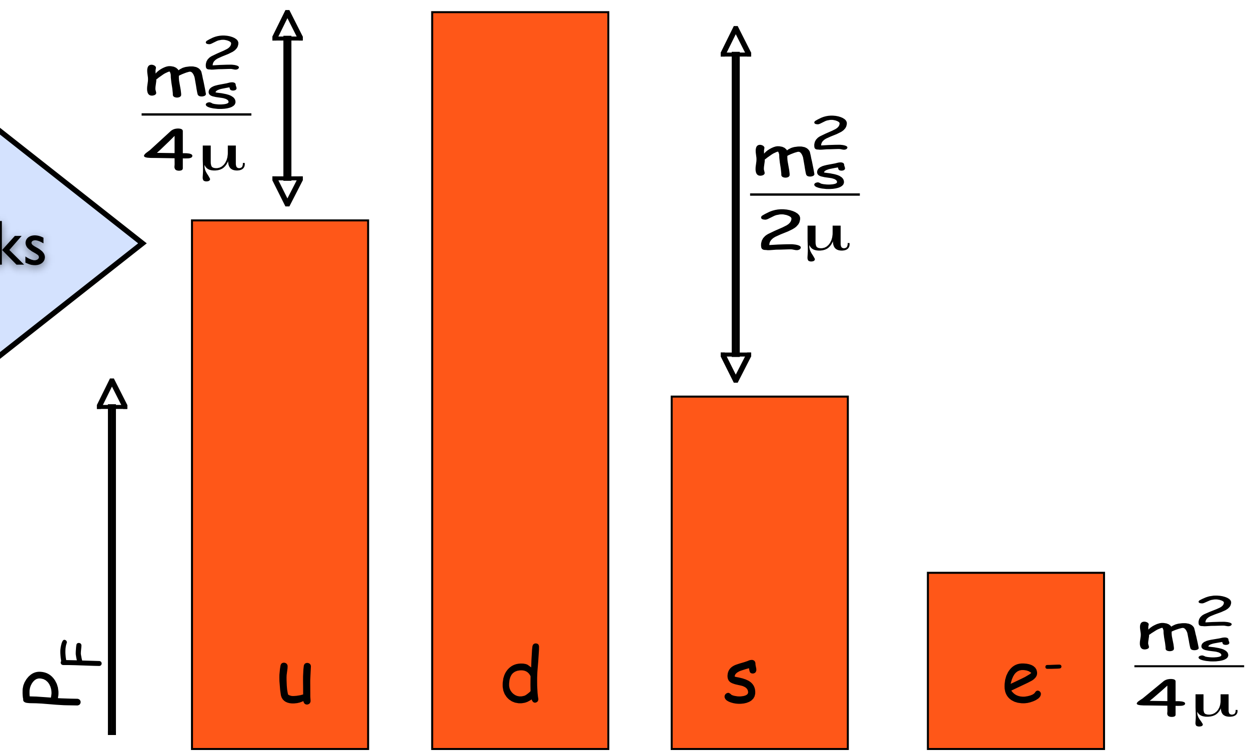


Interactions are poorly known

Asymptotic Density





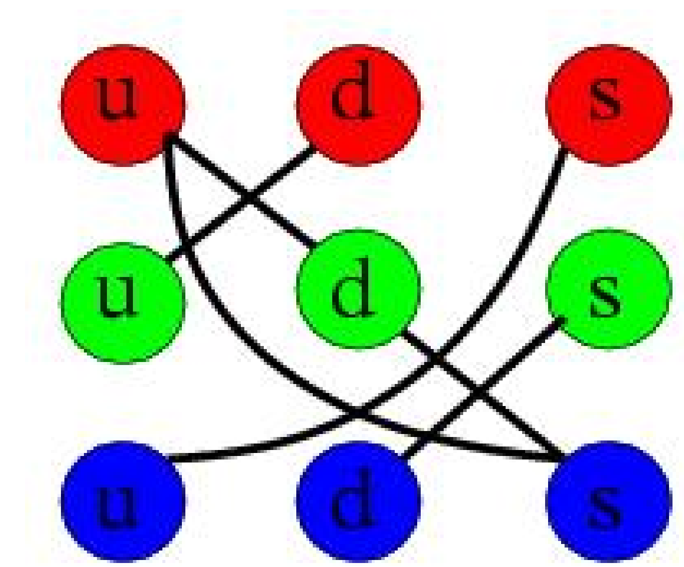


Interactions are nearly perturbative - calculable.

Interactions lead to pairing and color superconductivity

Strongest attraction in color-antisymmetric channel:
Color-Flavor-Locking

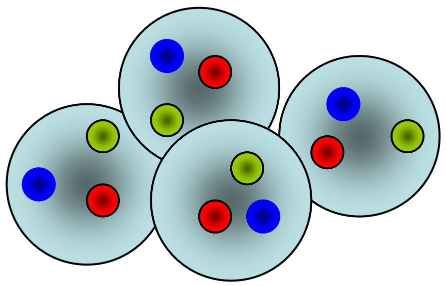
$$\Delta \gg \frac{m_s^2}{4\mu}$$

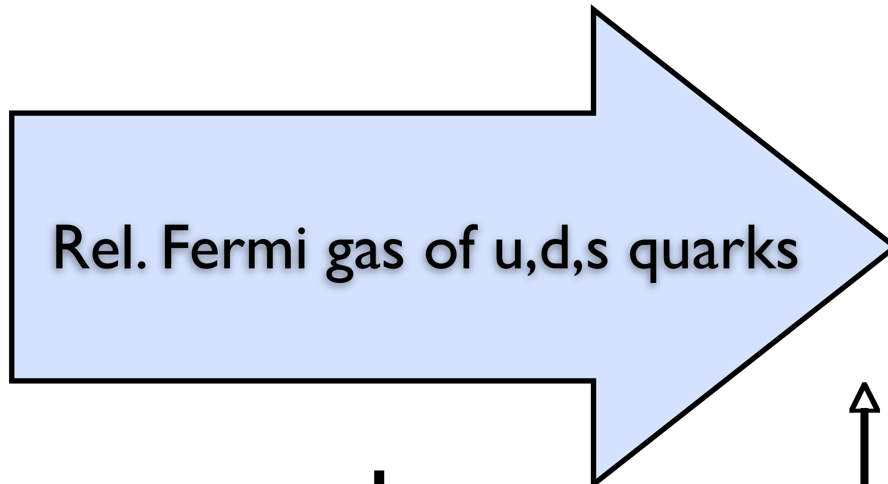


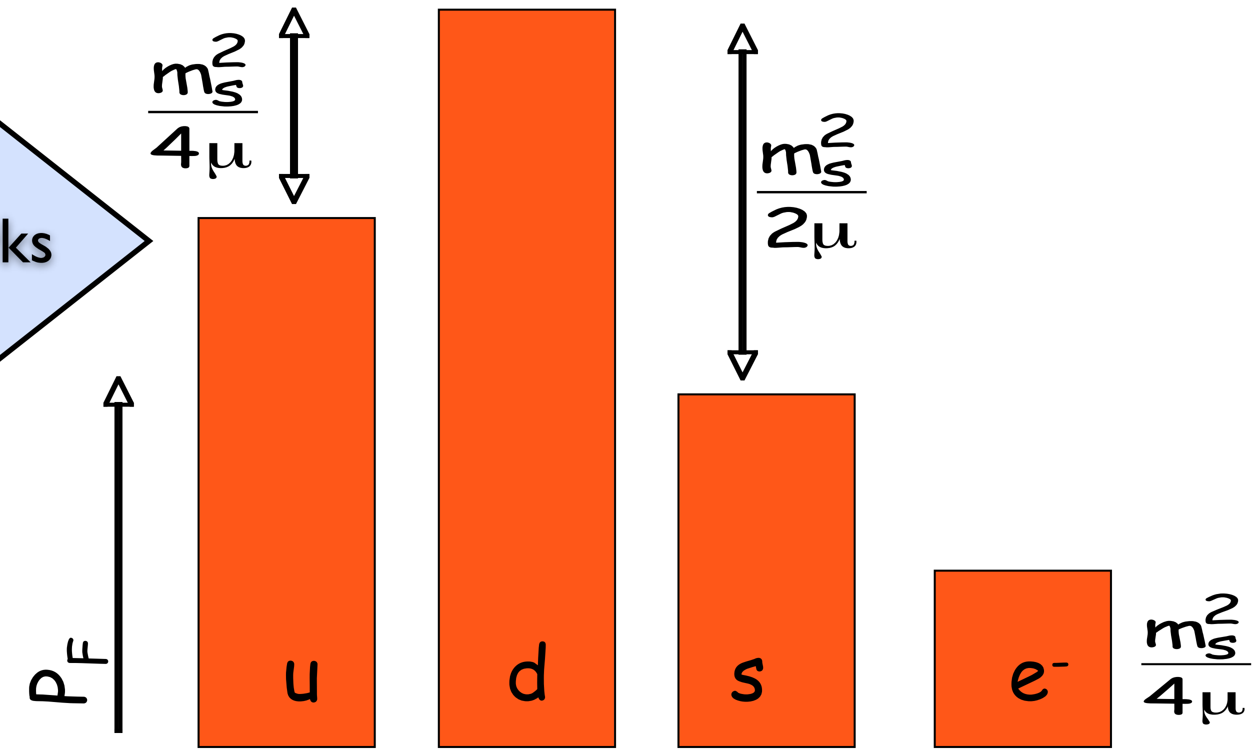
$$n_u = n_d = n_s$$

Alford, Rajagopal, Wilczek (1999)

Asymptotic Density







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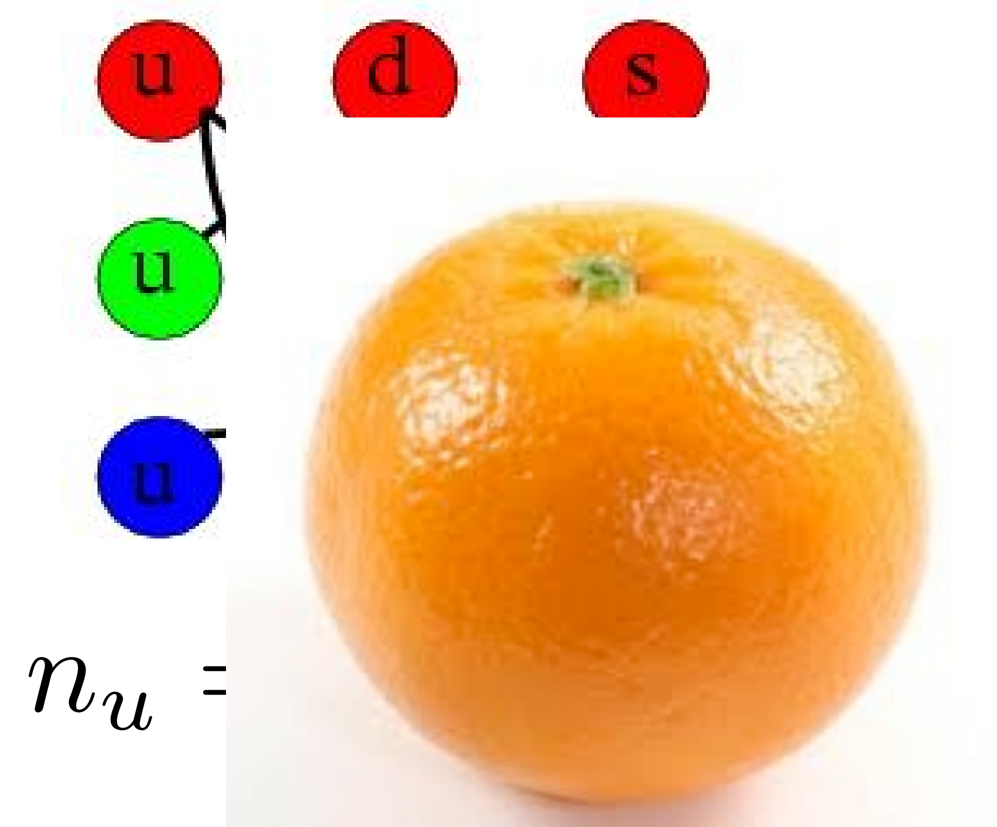

Alford, Rajagopal, Wilczek (1999)

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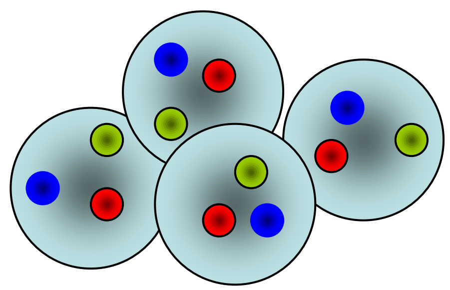

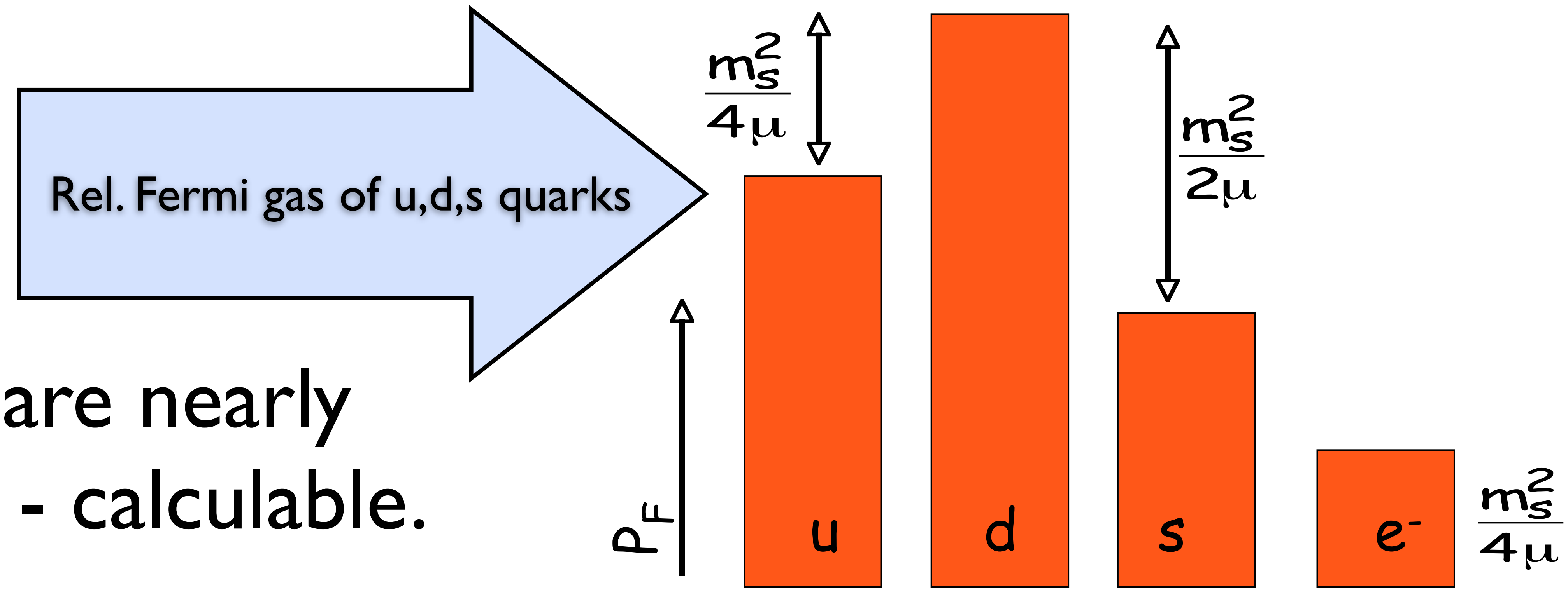

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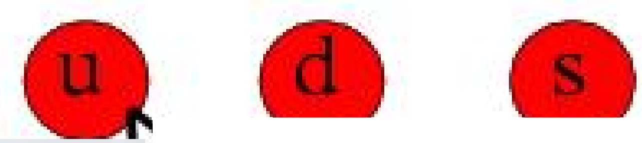
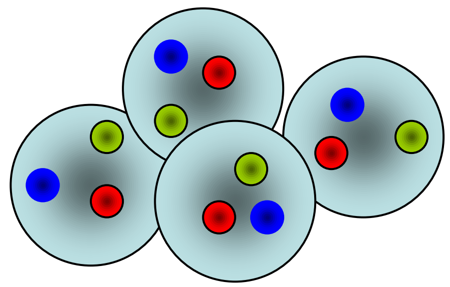


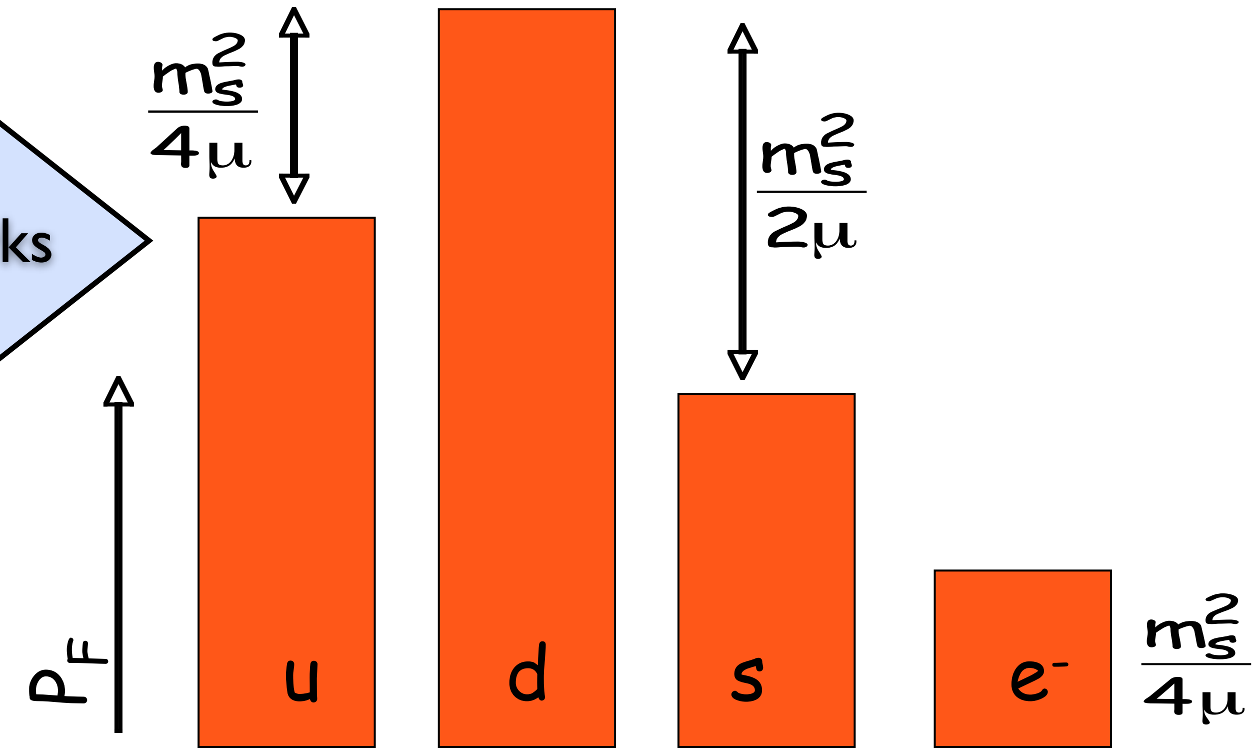
$$\Delta \gg \frac{m_s^2}{4\mu}$$

Alford, Rajagopal, Wilczek (1999)

Quark Matter in Neutron Stars



Rel. Fermi gas of u,d,s quarks



Interactions are non- perturbative. Difficult to predict critical density.

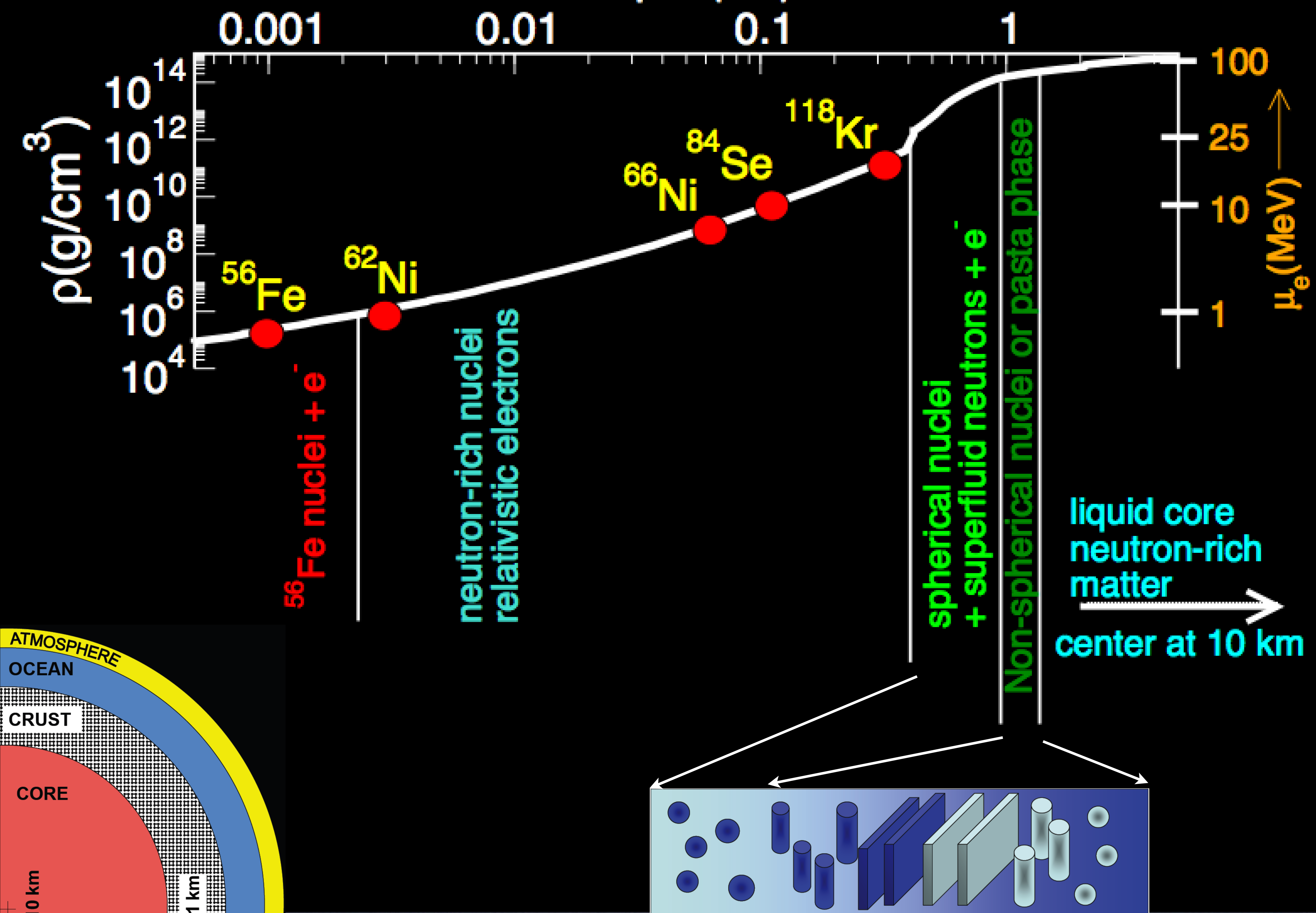
$$\Delta \simeq \frac{m_s^2}{4\mu}$$

- Difficult to predict ground state.
- Complicated spectrum of excitations (Strongly coupled quasi-particles)

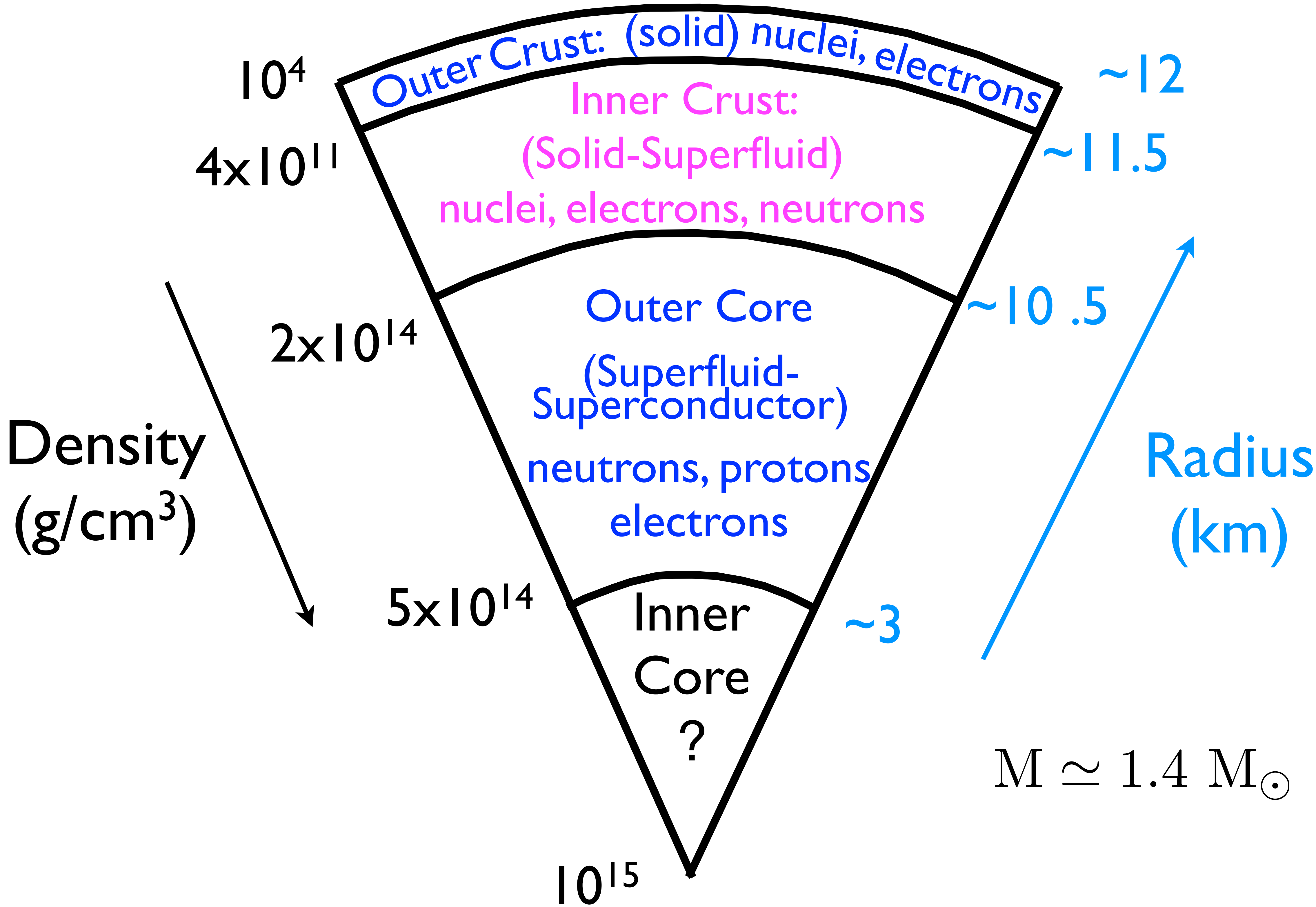
$$\Delta \geq \frac{m_s^2}{4\mu}$$

- Ground state is CFL.
- Low energy spectrum is simple (Goldstone modes - weakly coupled)

Neutron Star in Depth



Phase & Composition



What can we observe?

- Orbital Characteristics in Binaries
- Surface Luminosity
- Spin
- Explosions & Flares
- Neutrinos (Supernova)
- Gravity Waves (likely within 5 yrs!)

What can we infer ?

Hard Physics

- Mass
- Radius
- Crust thickness
- Oscillations frequencies

Ground state EoS

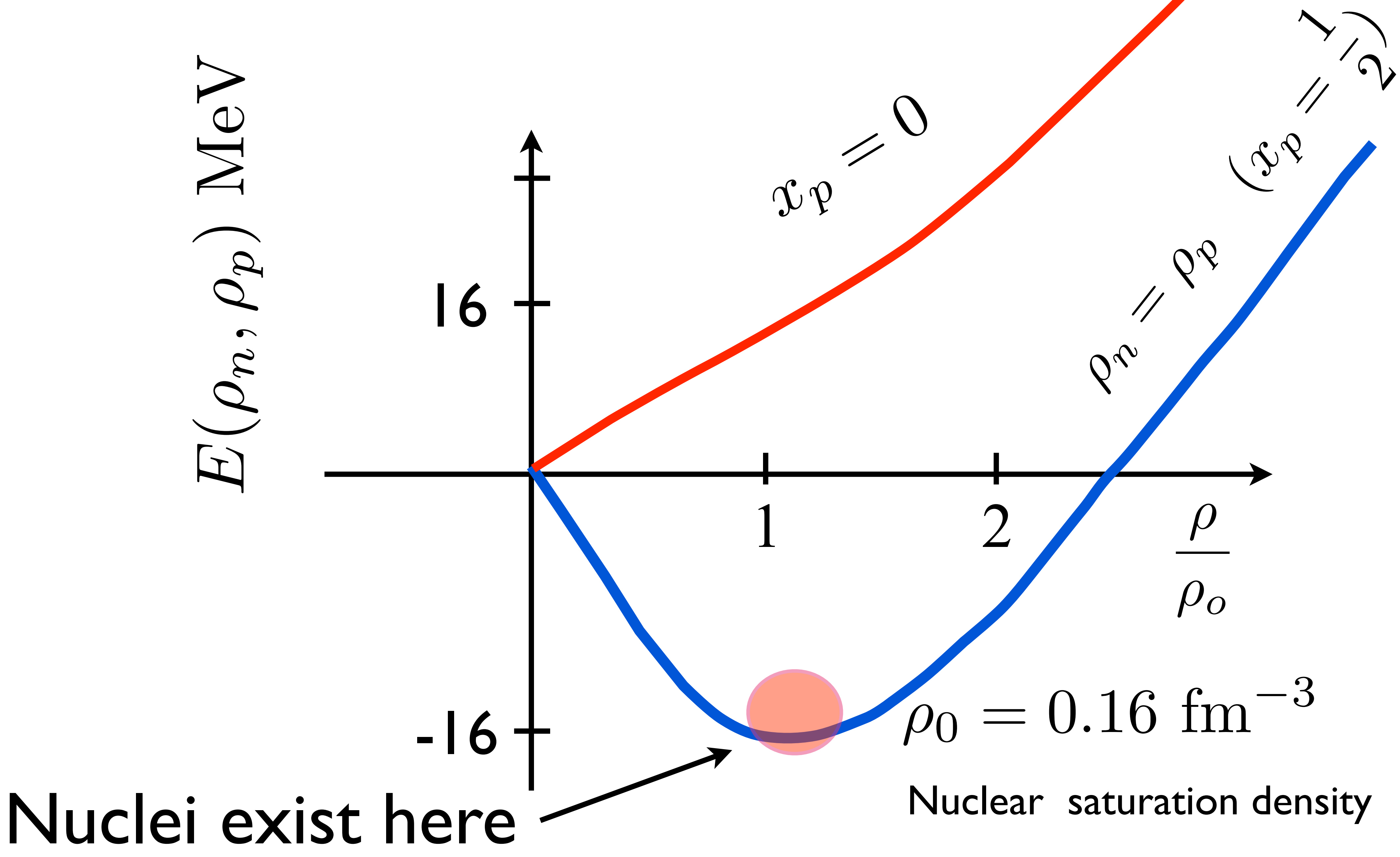
Soft Physics

- Surface and interior temperature
- Neutrino cooling and scattering rates
- Electrical & Thermal Conductivities
- Damping rates

Low energy fluctuations

The Nuclear Equation of State

$$\begin{cases} \epsilon = \rho\, E(\rho) \\ P(\epsilon) = \rho^2 \dfrac{\partial E(\rho)}{\partial \rho} \end{cases}$$



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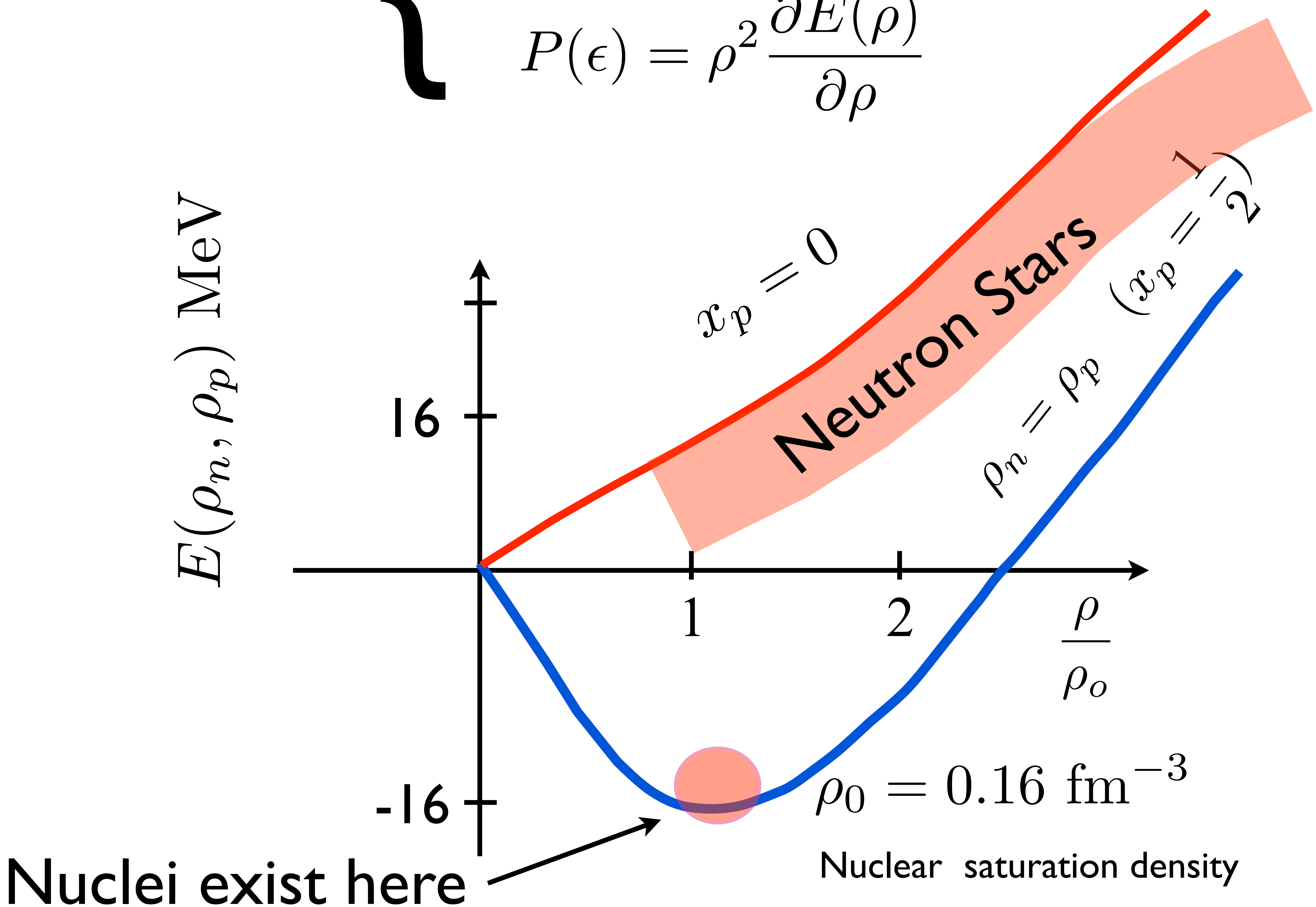

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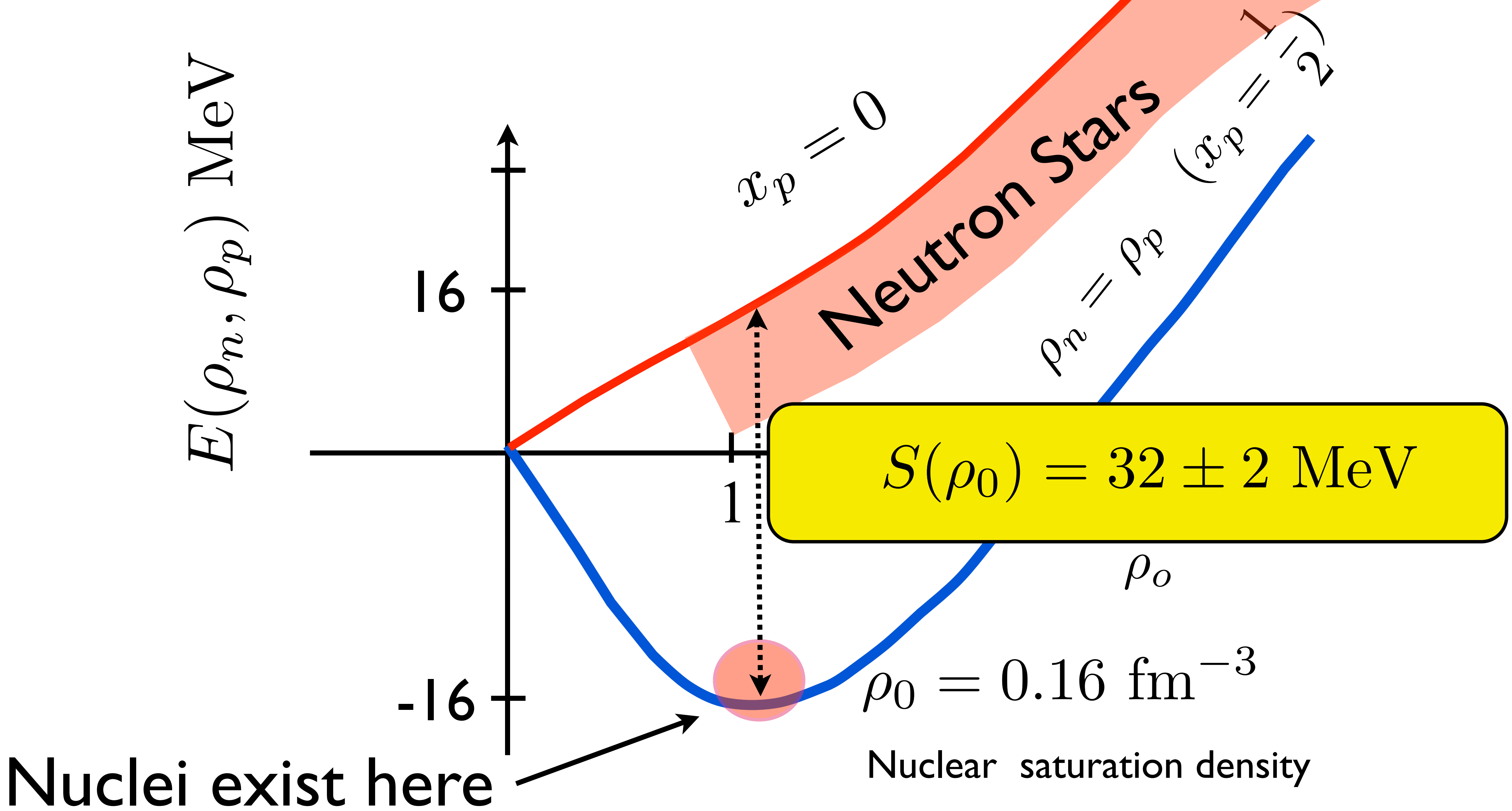
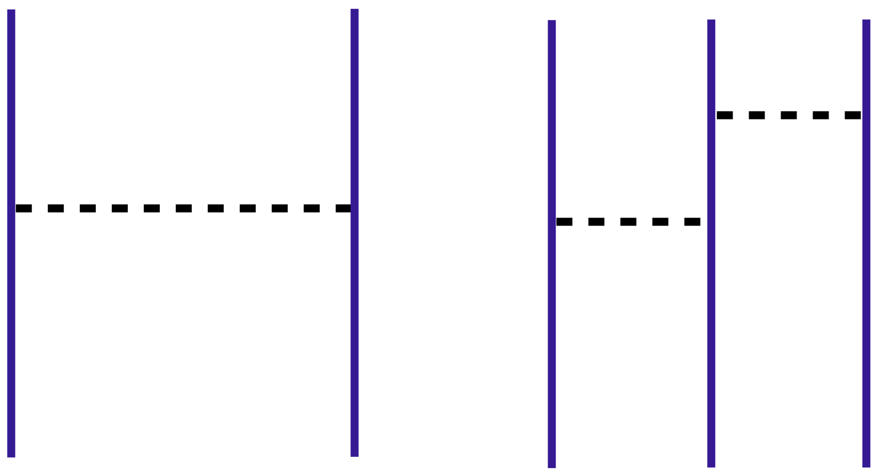
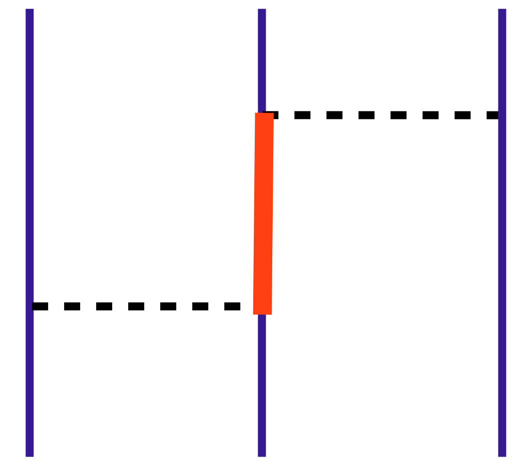


Nuclear Many Body Theory

$$H_{\text{nuclear}} = \frac{\nabla^2}{2M} + V_{\text{NN}} + V_{\text{NNN}} + \cdots$$





Phenomenological potentials (Argonne etc) tuned to fit scattering and light nuclei.

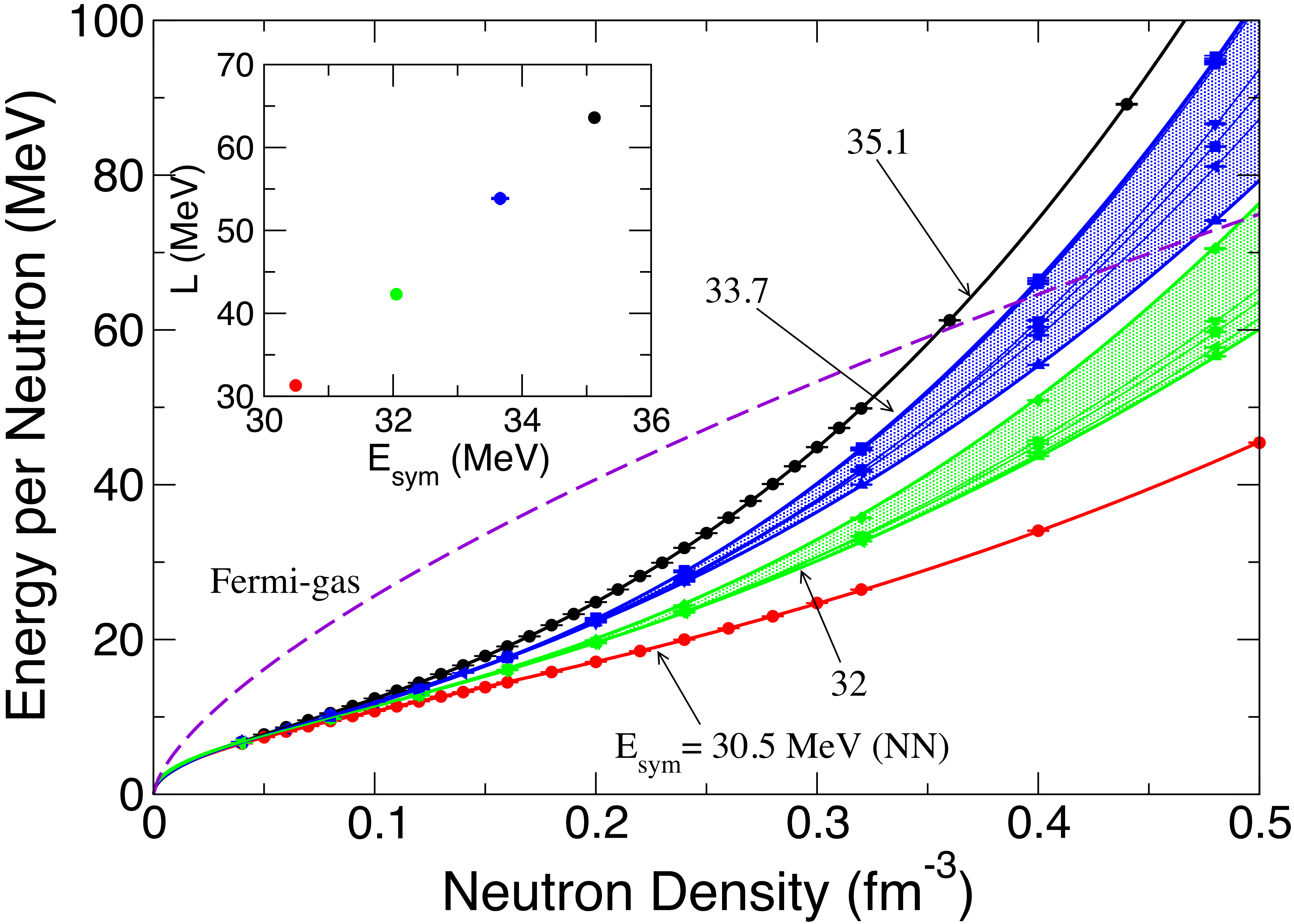
Chiral potentials and softer low energy potentials obtained using RG.

Computational Methods: Quantum Monte Carlo

Diagrammatic Methods Perturbation Theory

$E(\rho_n, \rho_p)$: Energy per particle

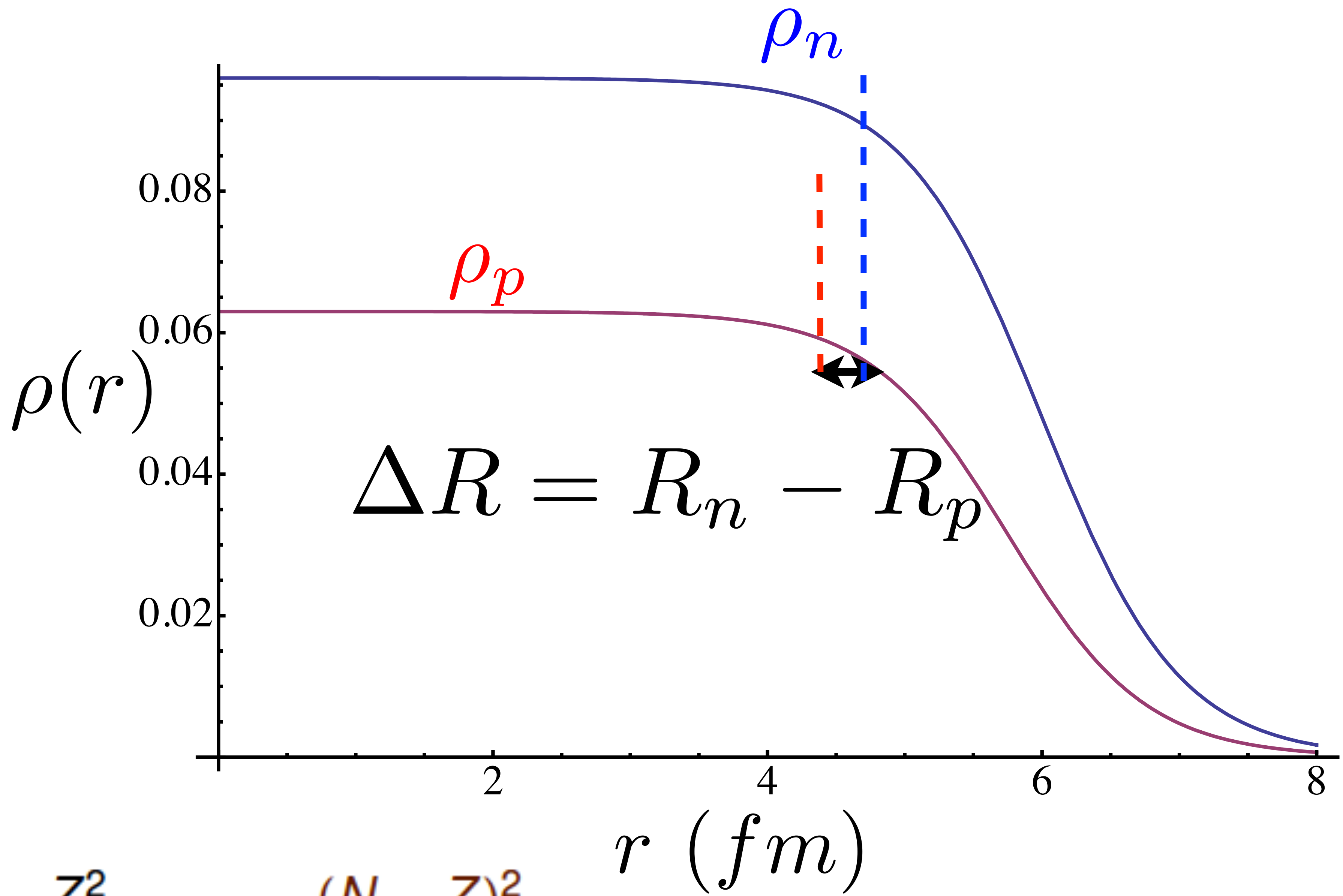
Neutron Matter & 3N Forces



Gandolfi, Carlson, Reddy (2010)

Neutron-rich Nuclei

- Nuclear masses are sensitive to the symmetry energy.
- Neutron distribution at the surface is sensitive to its density dependence.



Bethe-Weizsäcker formula:

$$E = -a_V A + a_S A^{2/3} + a_C \frac{Z^2}{A^{1/3}} + a_a(A)\frac{(N-Z)^2}{A} + E_{\text{mic}}$$

$$\frac{1}{a_a(A)} = \frac{1}{a_a^V} + \frac{1}{a_a^S} A^{1/3}$$

$$\left\{ \begin{array}{l} a_a^V \Rightarrow E_{\text{sym}}\ (S) \\ a_a^S \Rightarrow \dfrac{\partial E_{\text{sym}}}{\partial \rho}\ (L/3) \end{array} \right.$$

Horowitz (2007) Danielewicz (2008)

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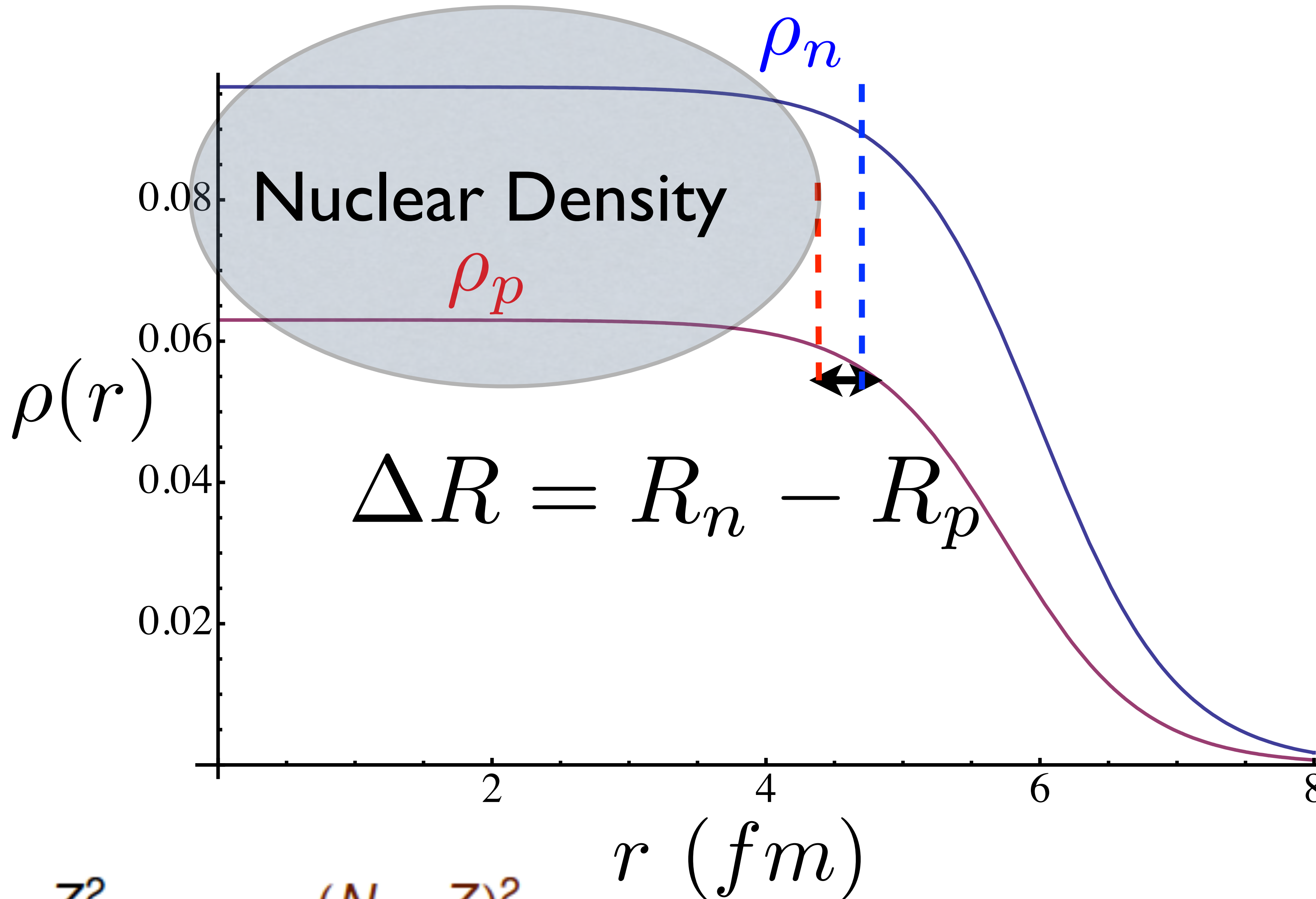

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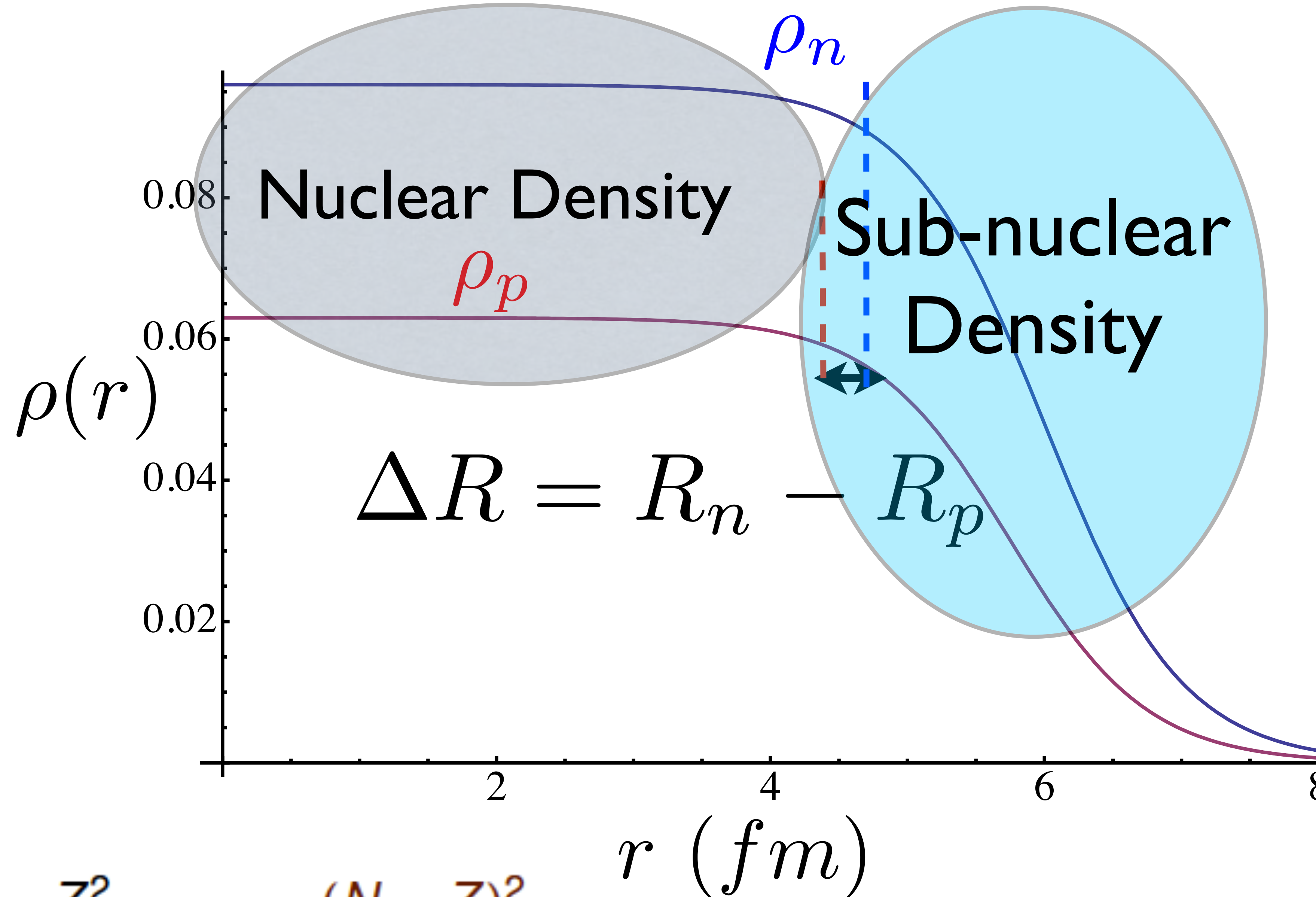

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Horowitz (2007) Danielewicz (2008)

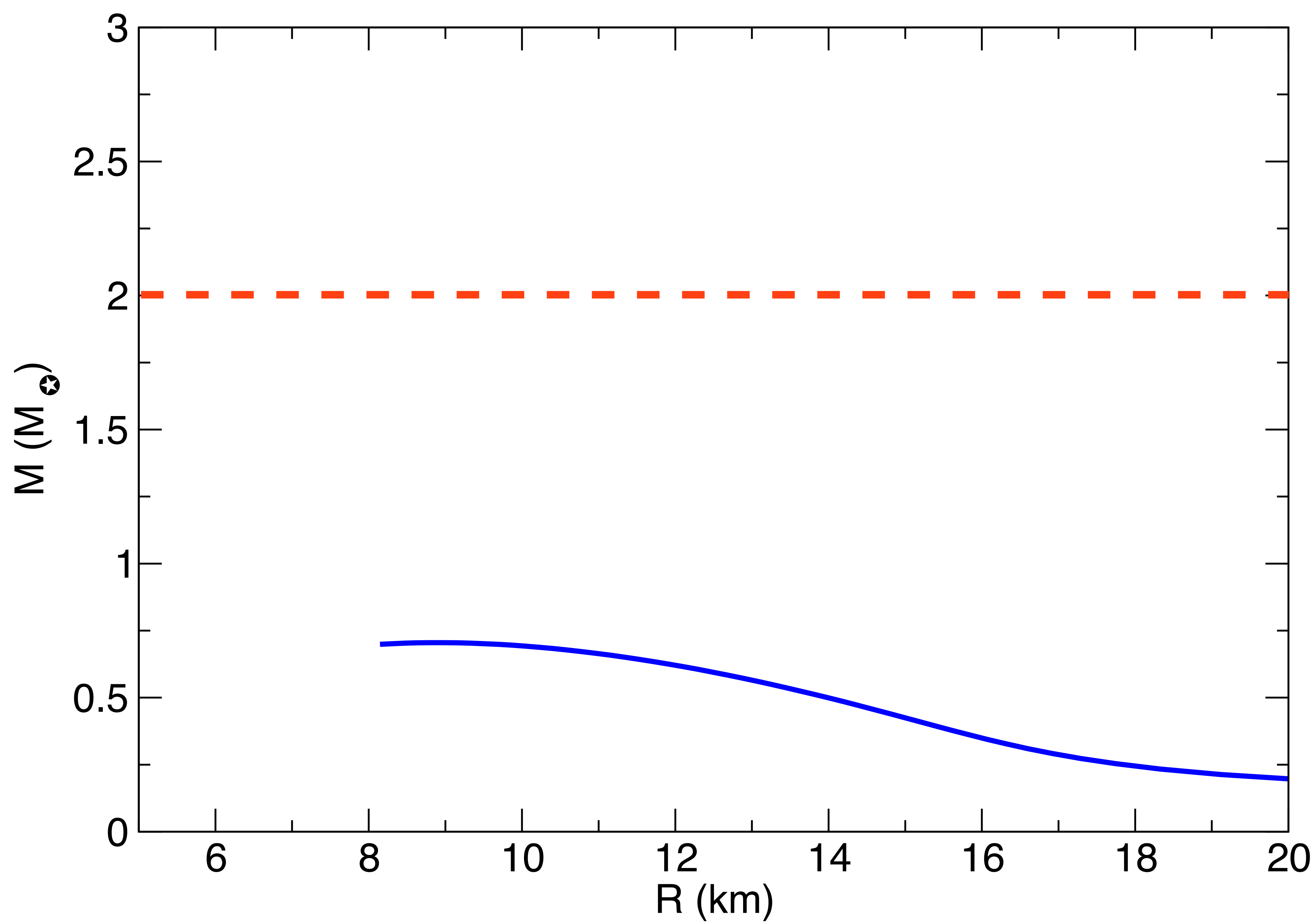
Nuclear experiments to measure S and L:

- Masses of very neutron-rich nuclei near the neutron-drip will reduce systematic errors in extracting S from model fits.
(Facilities such as FRIB, FAIR, JPARC)
- Distribution of neutrons in the surface region of neutron-rich nuclei can measure L indirectly.
Eg. PREX at Jefferson lab.

To extrapolate to high density we need a theory that can predict S & L.

Mass-Radius

$$\mathrm{M(R)} \leftrightarrow \mathrm{P}(\epsilon)$$



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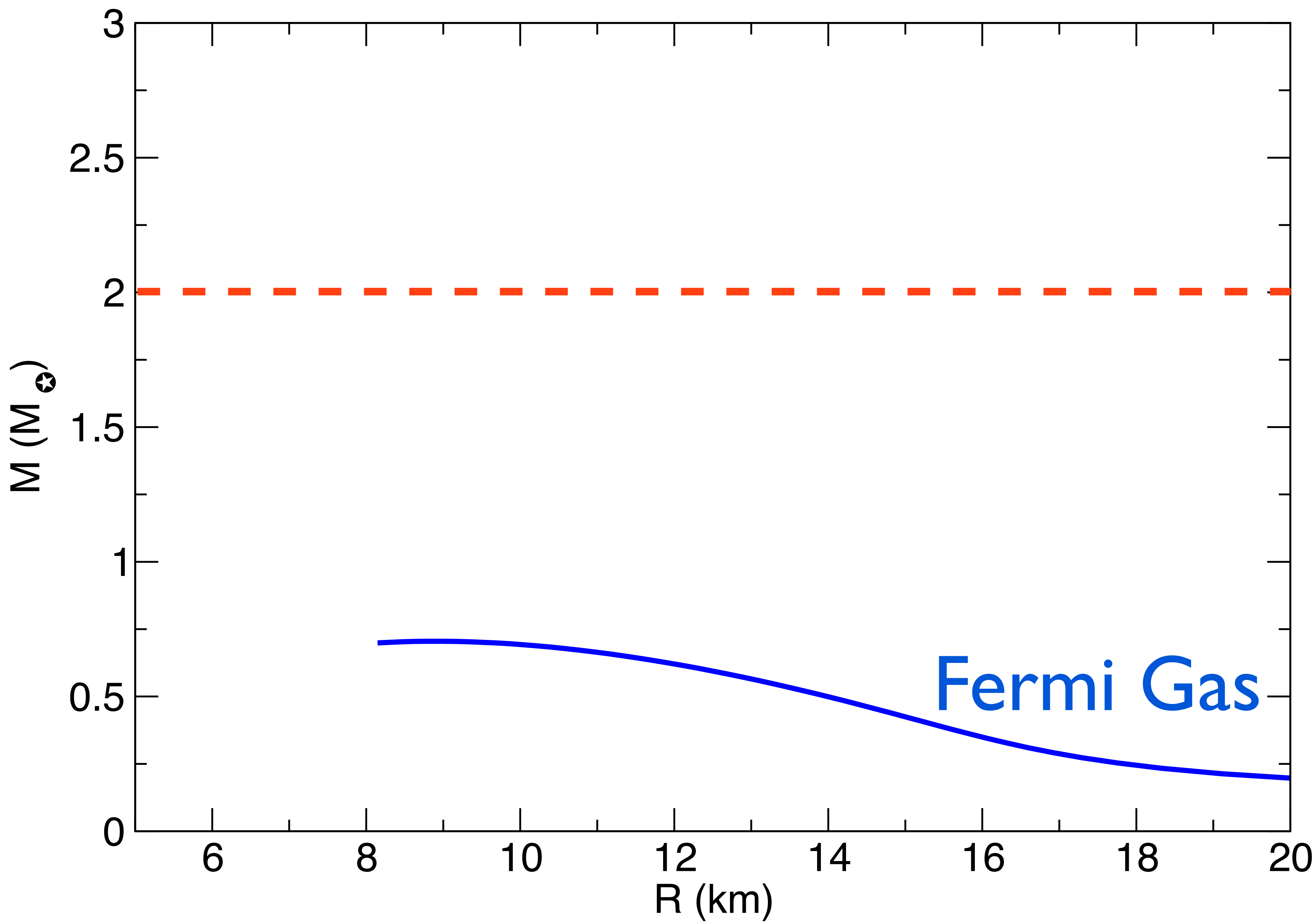

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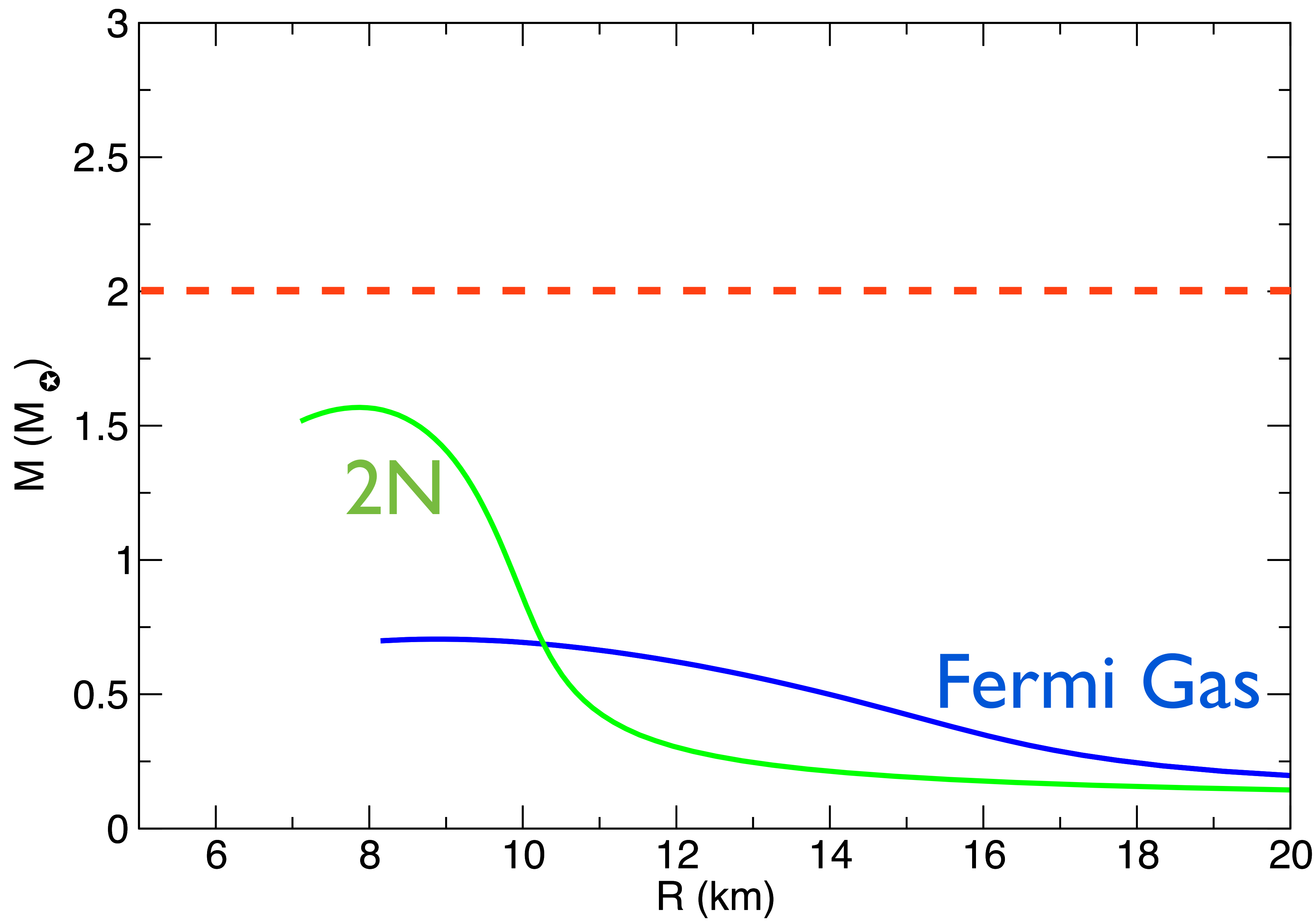

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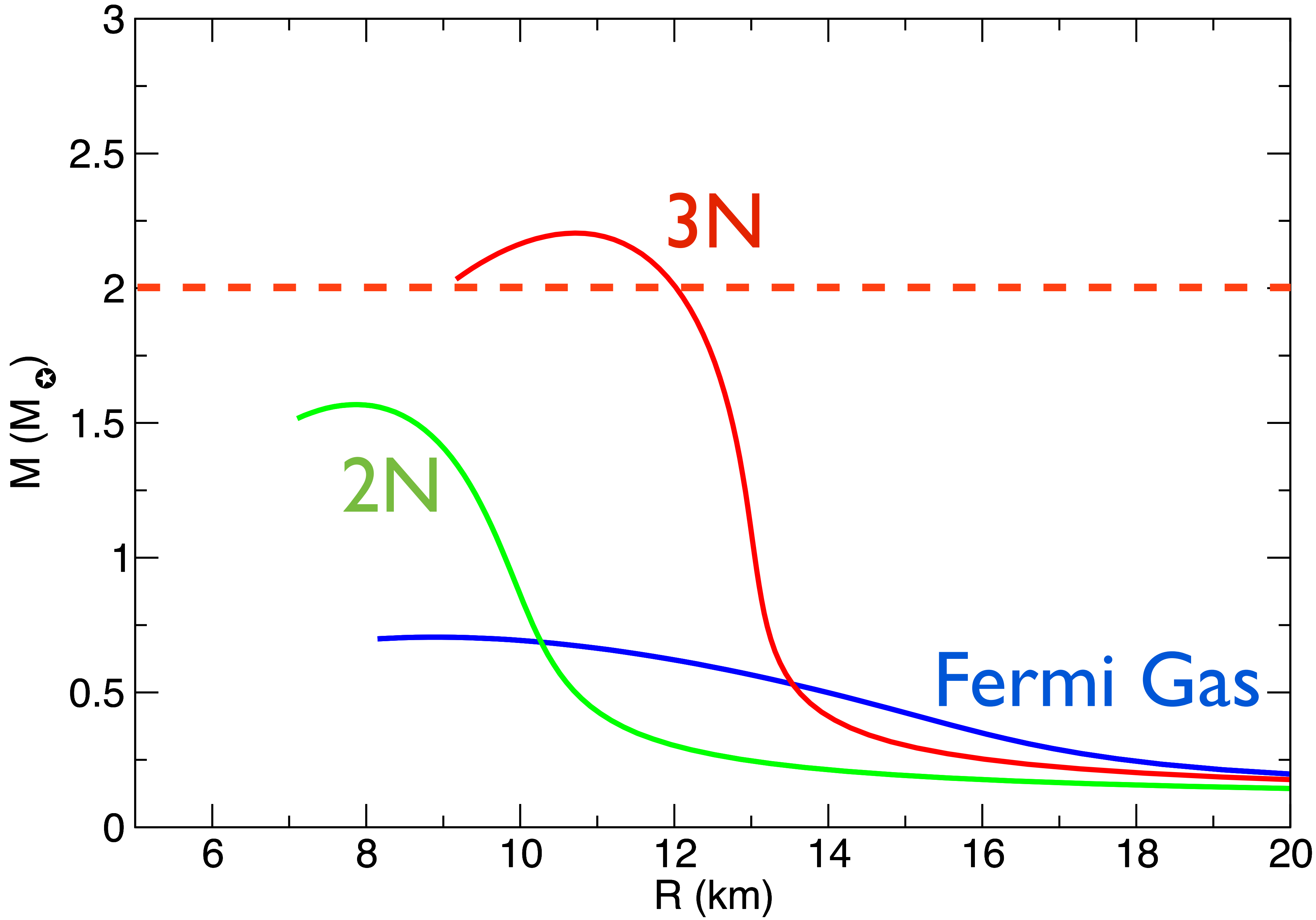
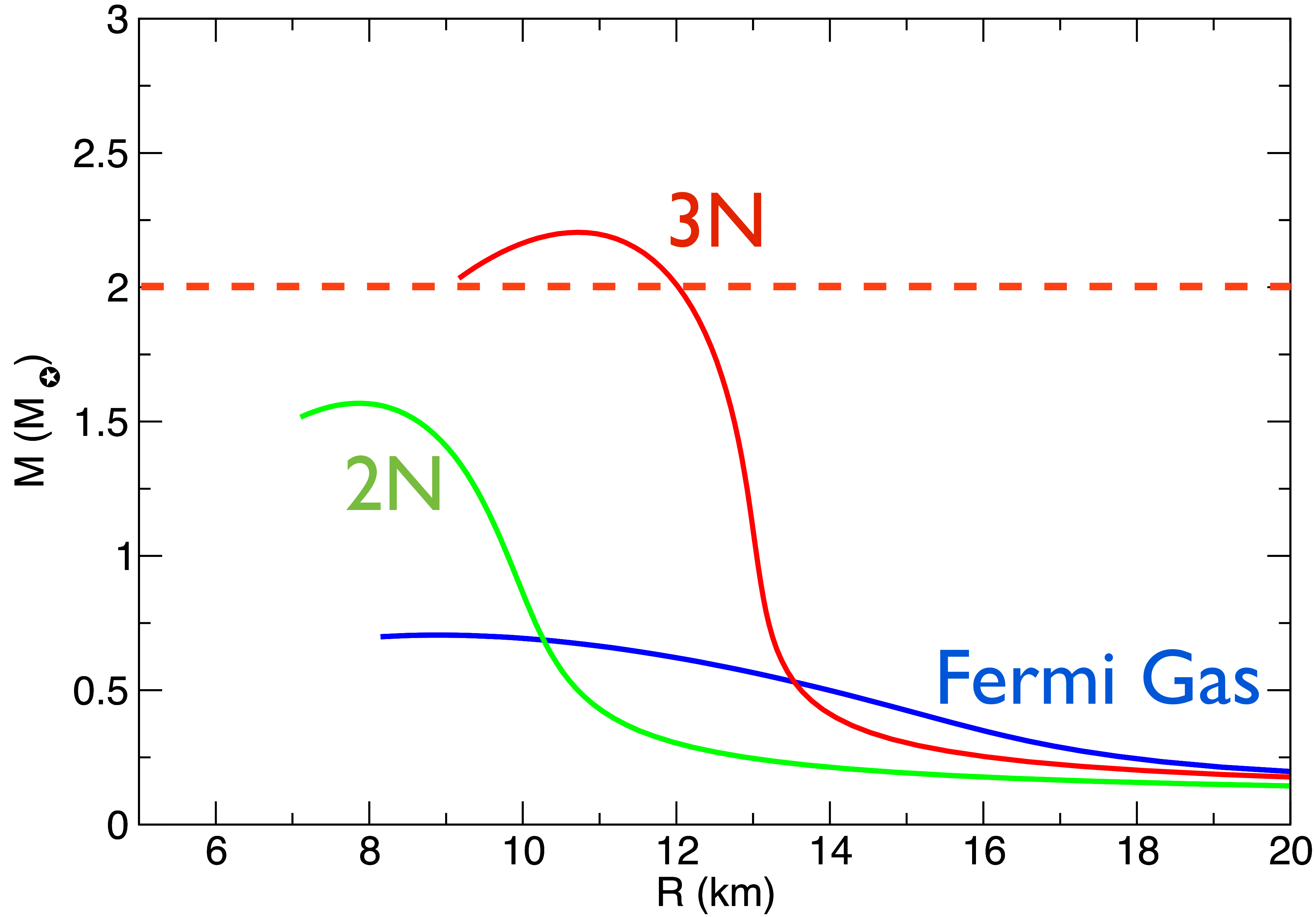
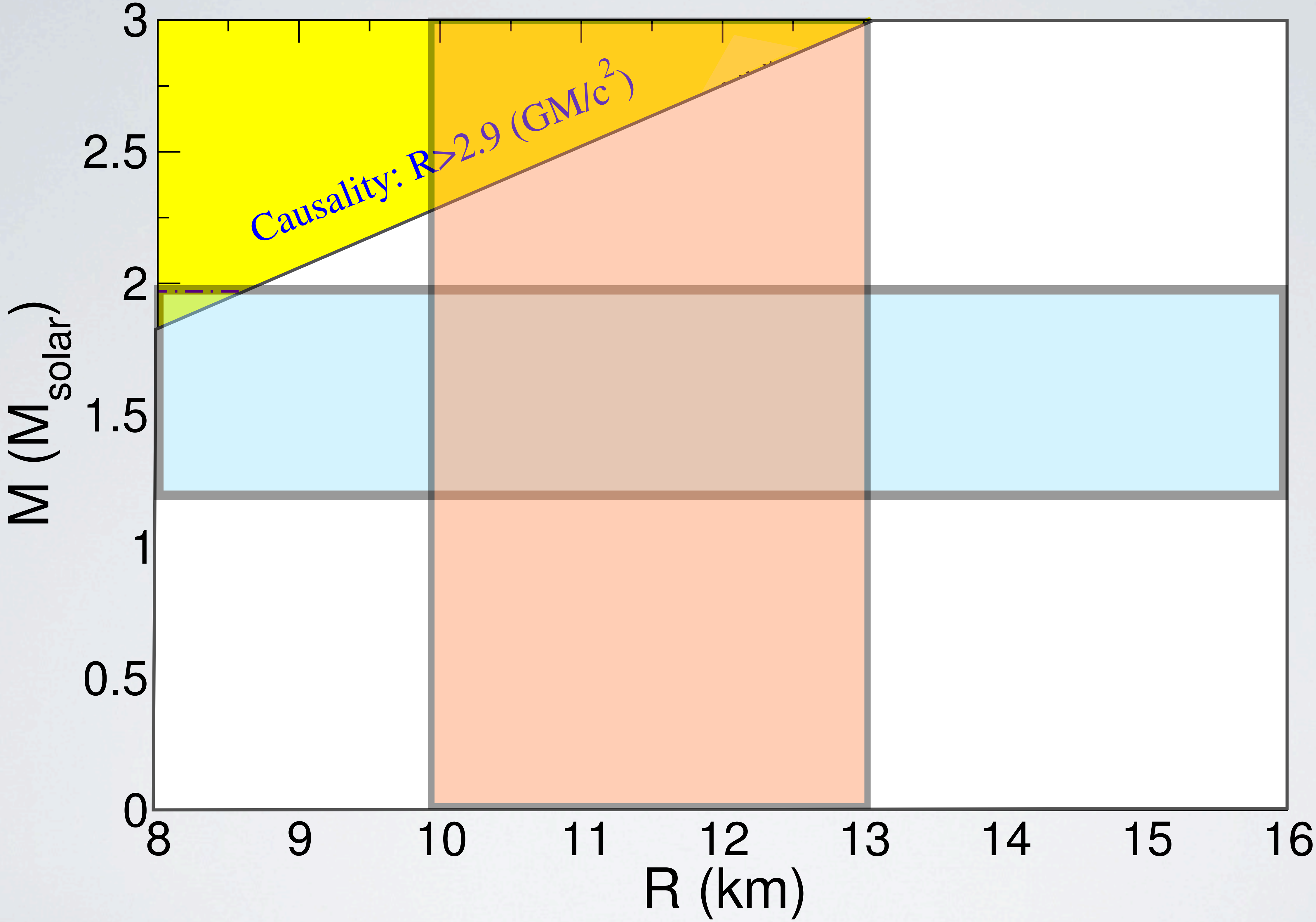


Mass-Radius

- Soft EoS: low maximum mass and small radii
- Stiff EoS: high maximum mass and large radii



Mass and Radius



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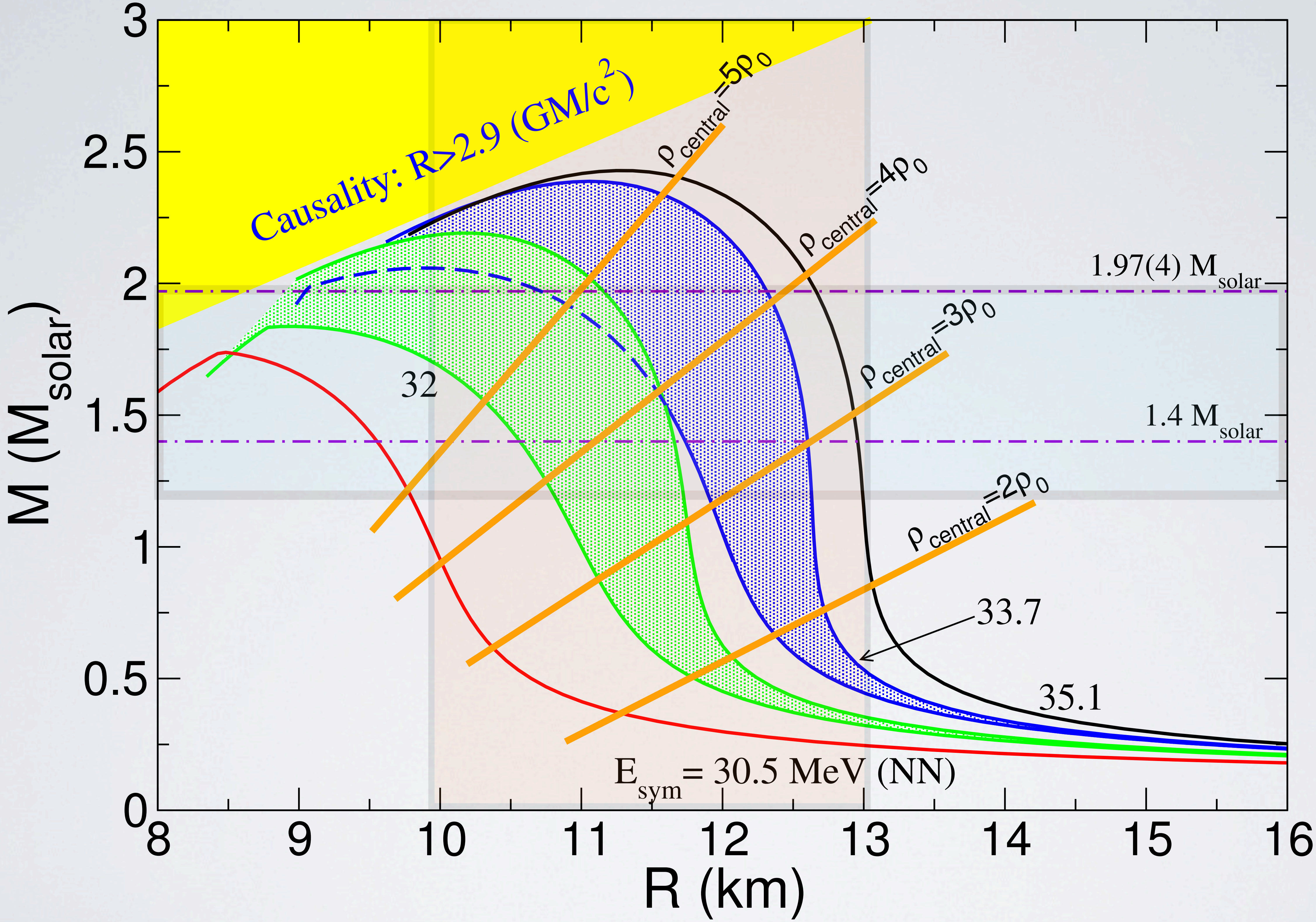
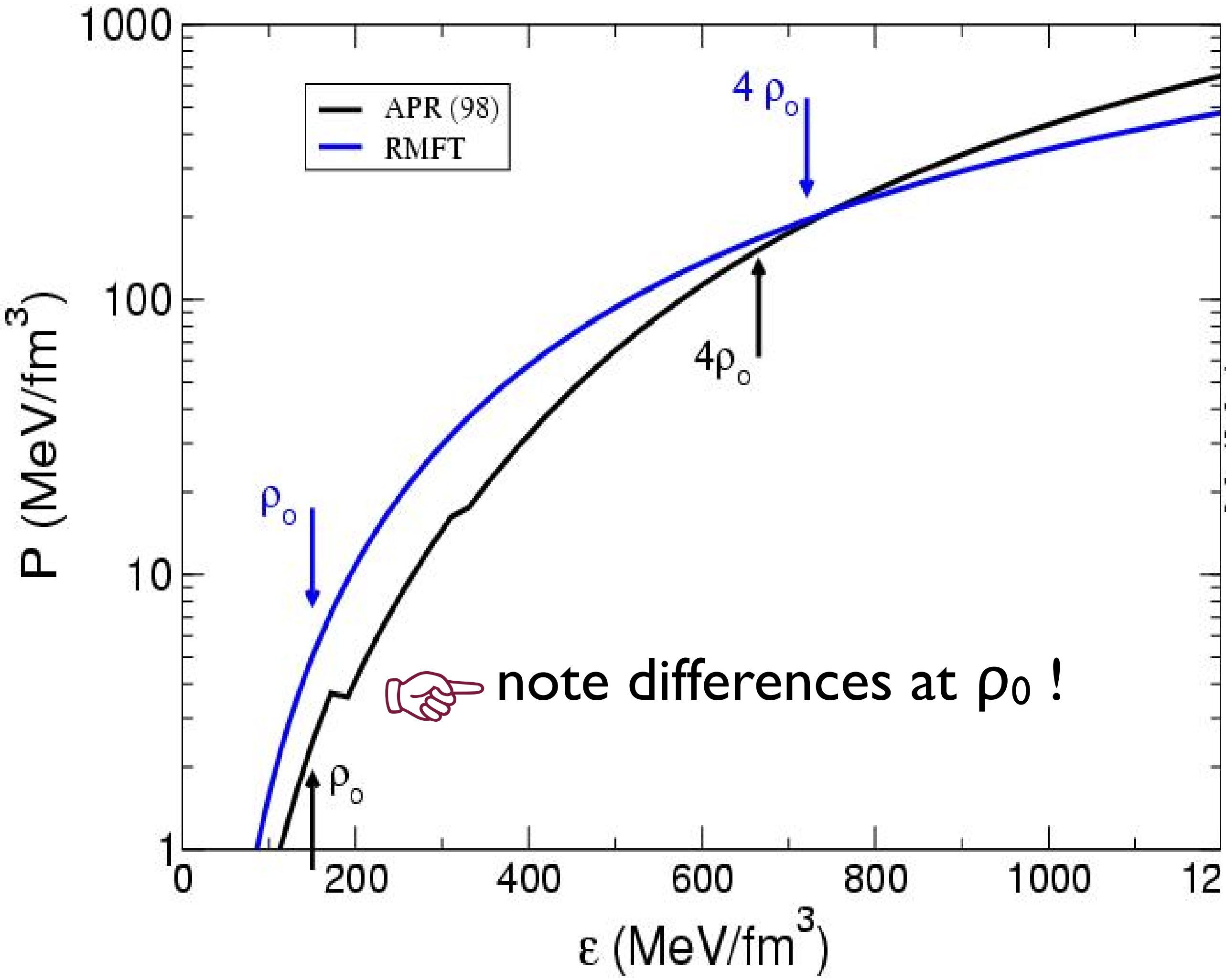
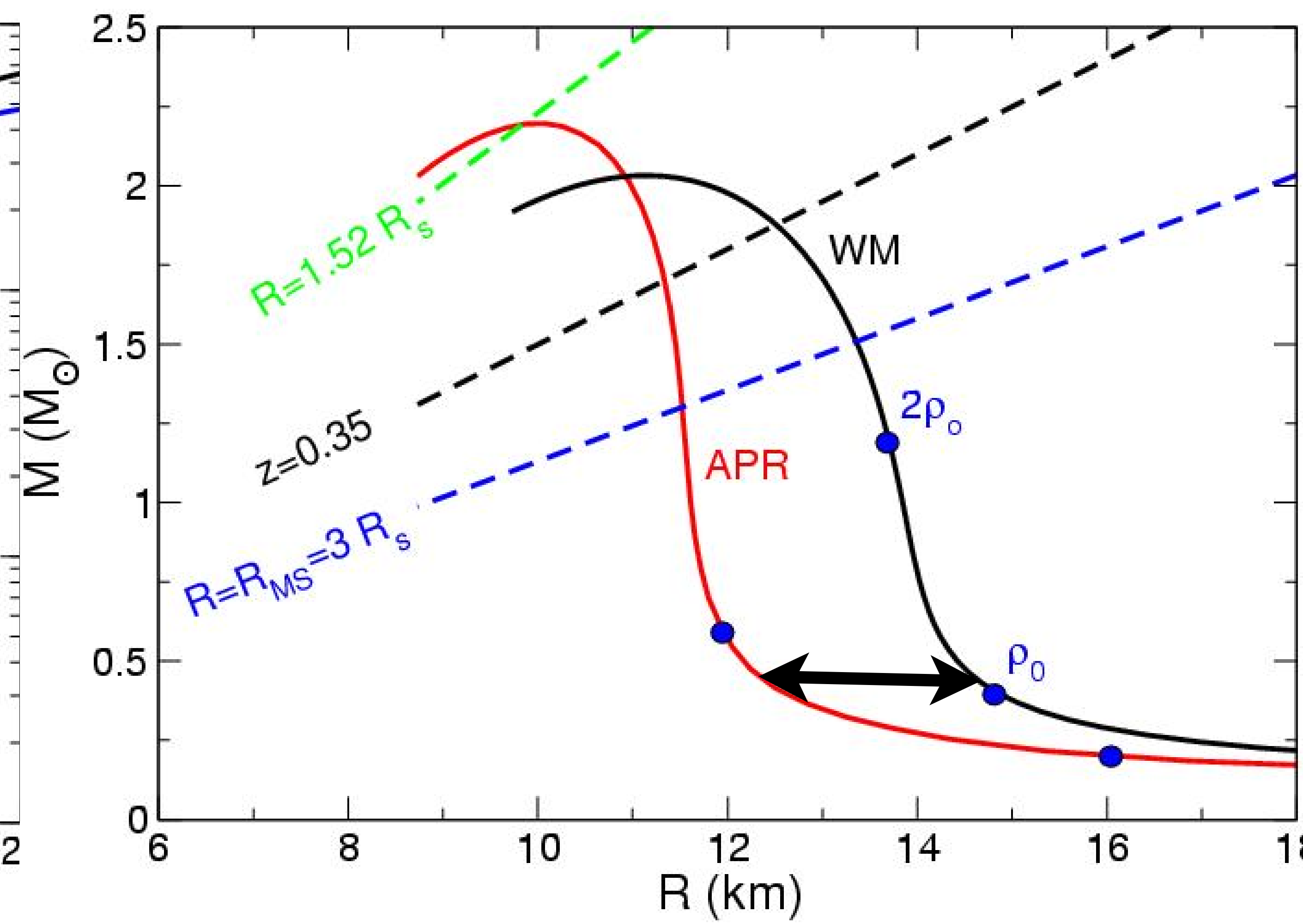


Maximum Mass & Phase Transitions





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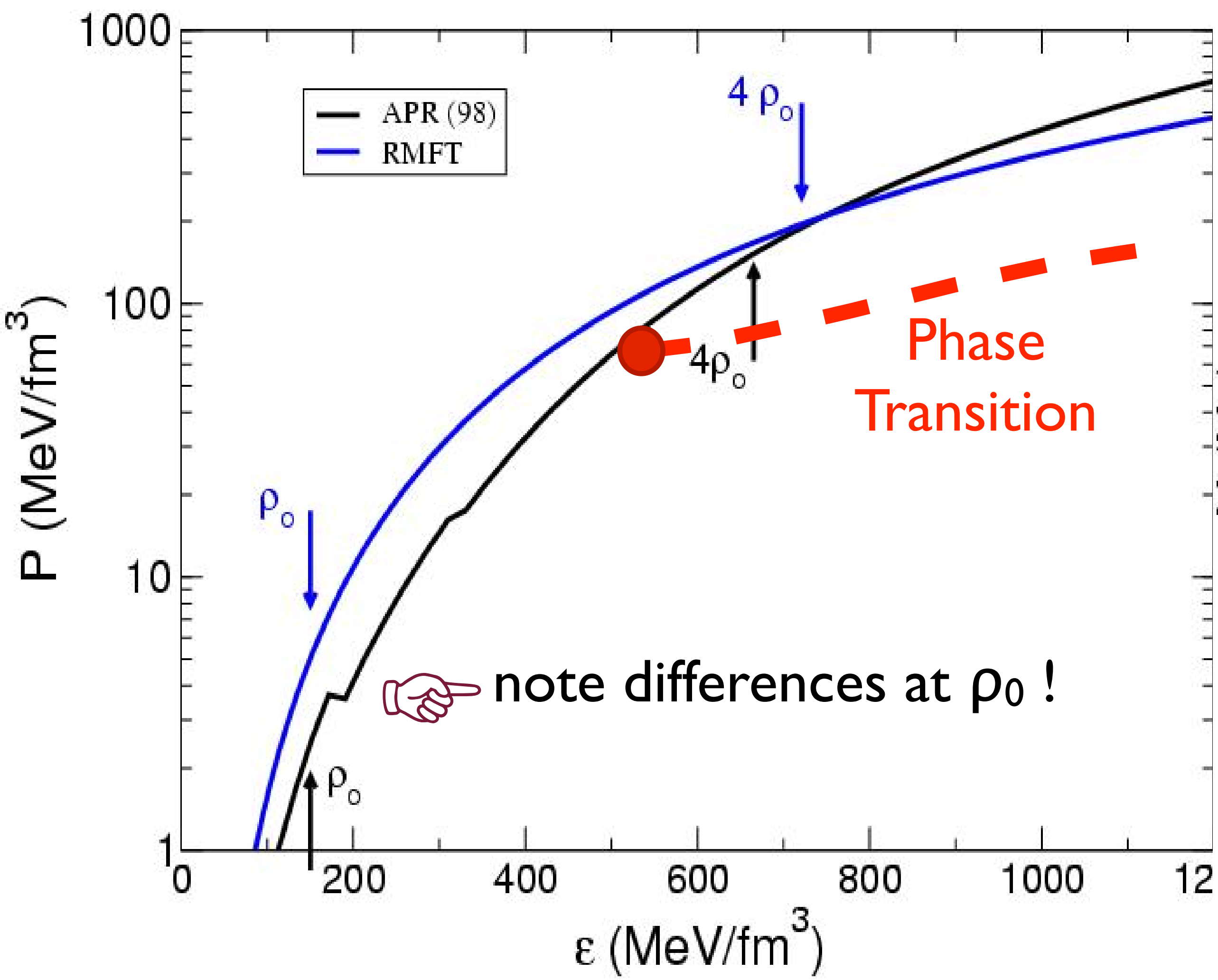

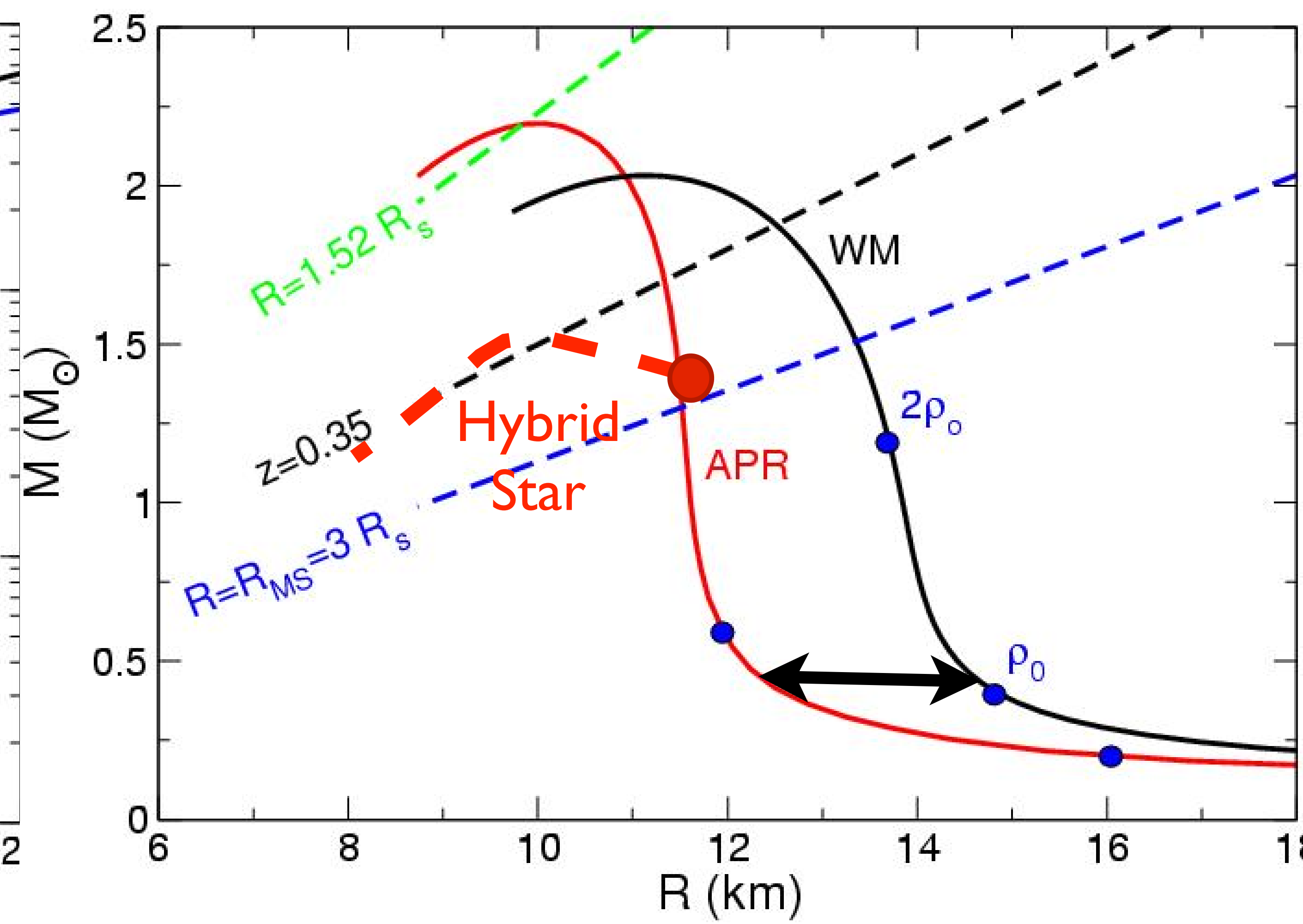

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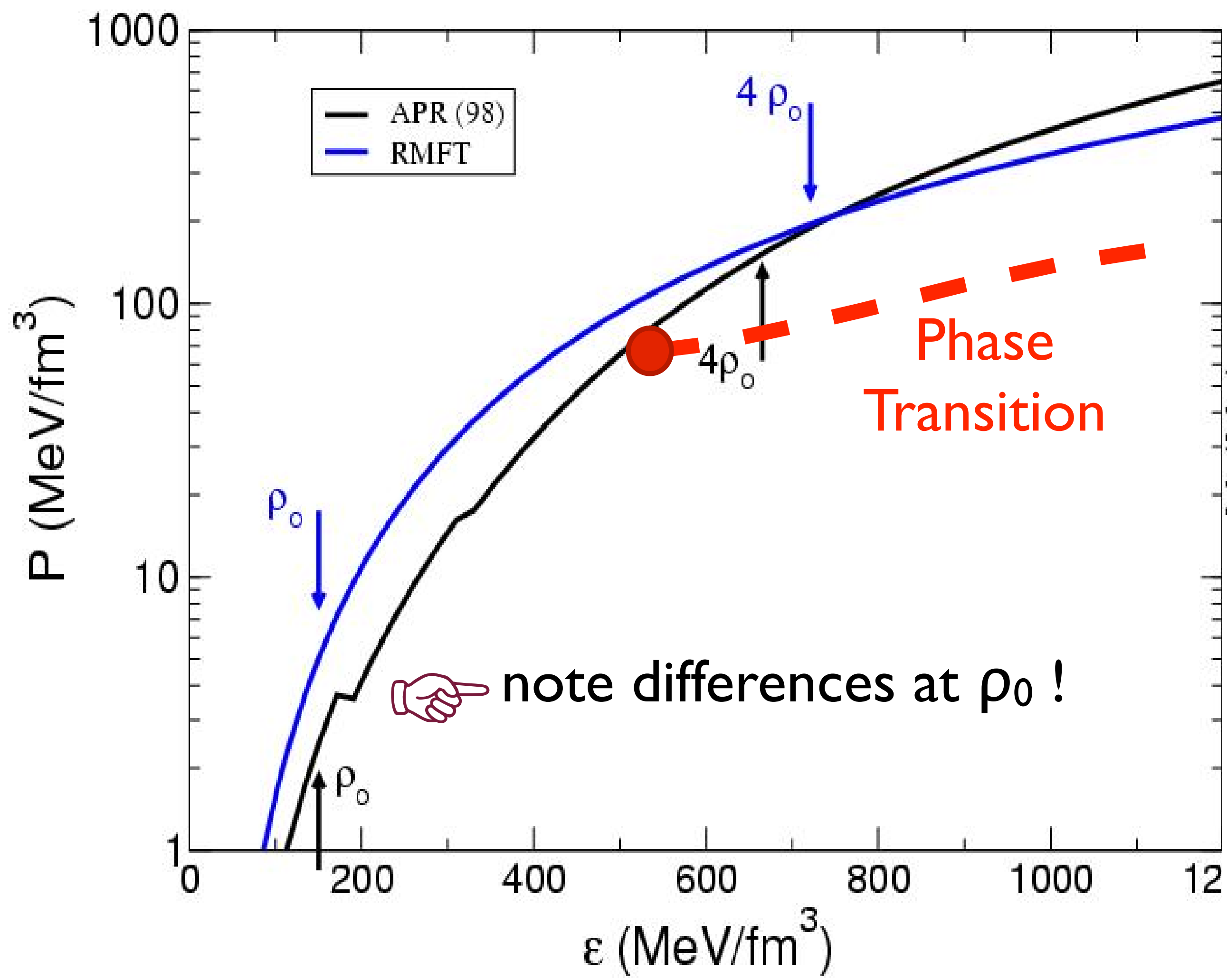

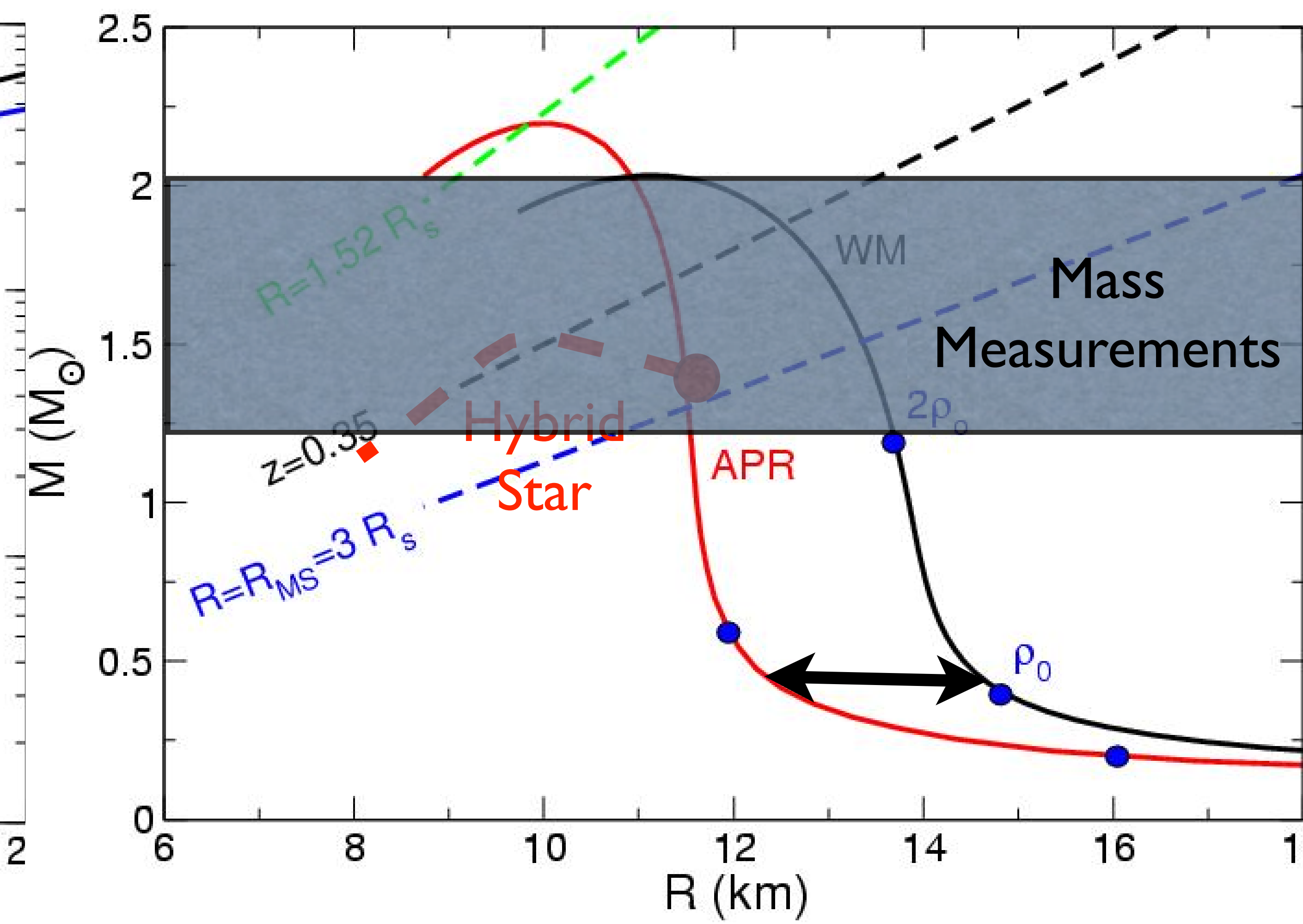

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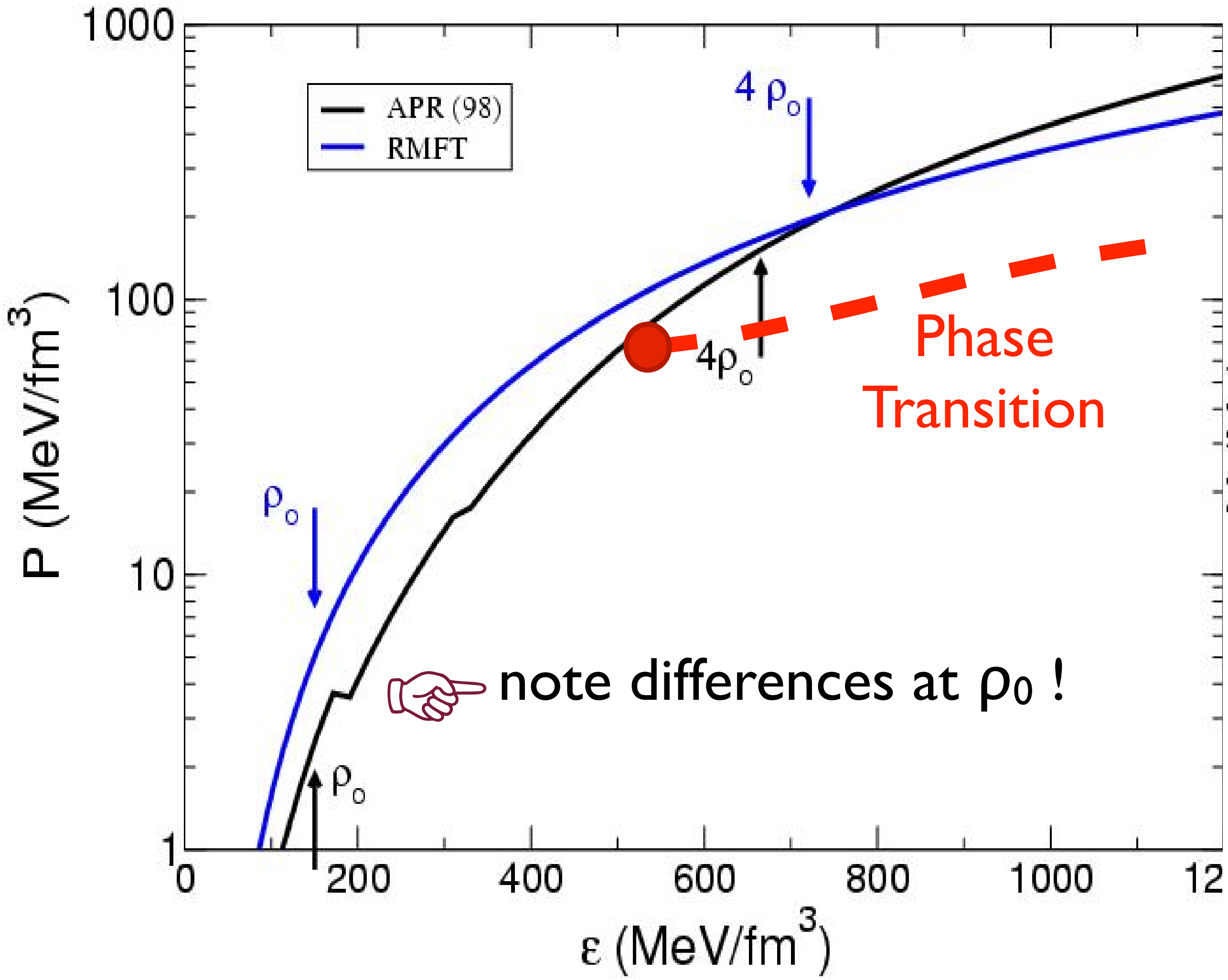
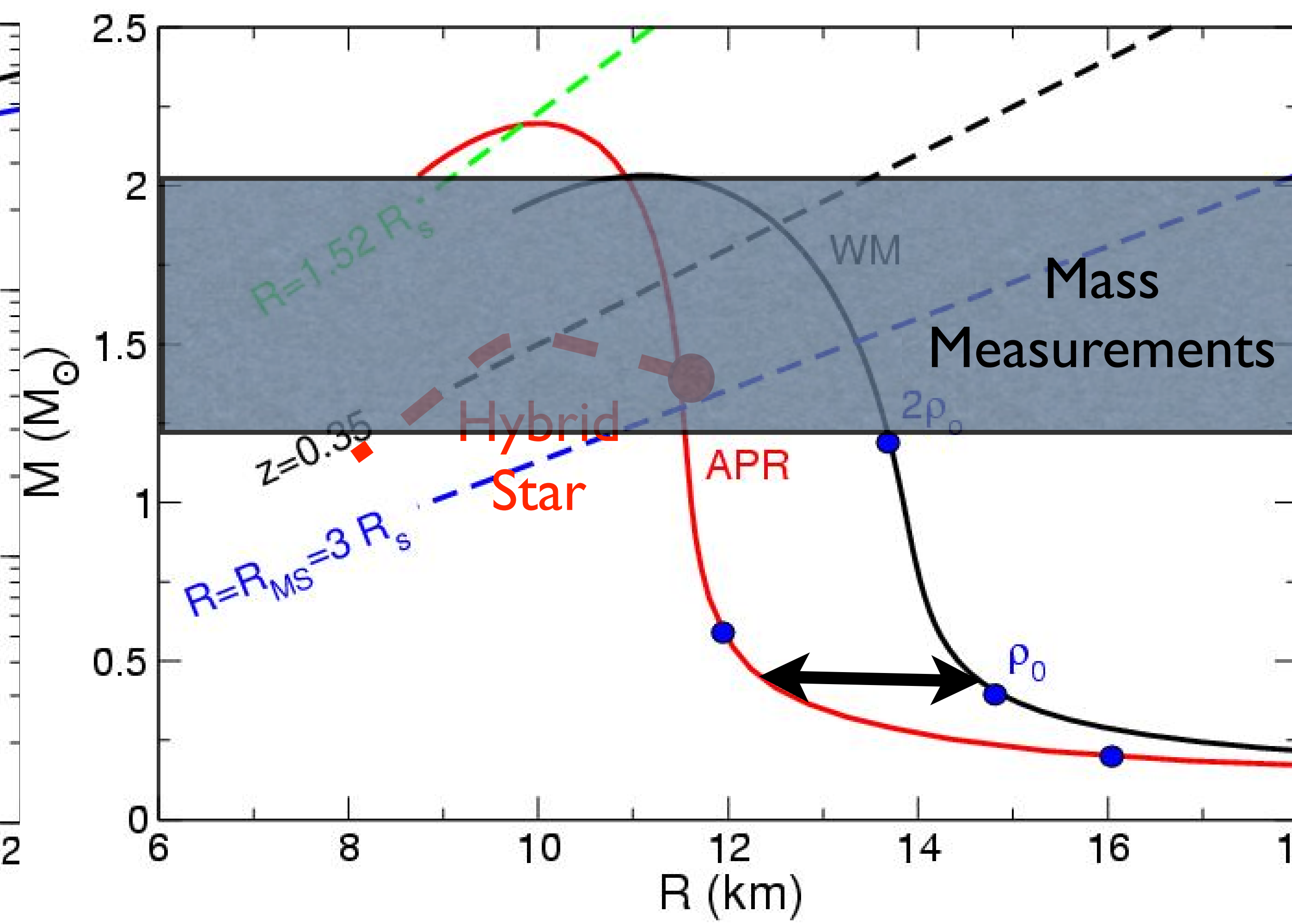




The 2 solar mass neutron star rules out a strong first-order transitions at supra-nuclear density

BEYOND MASS & RADIUS: TRANSPORT PHENOMENA

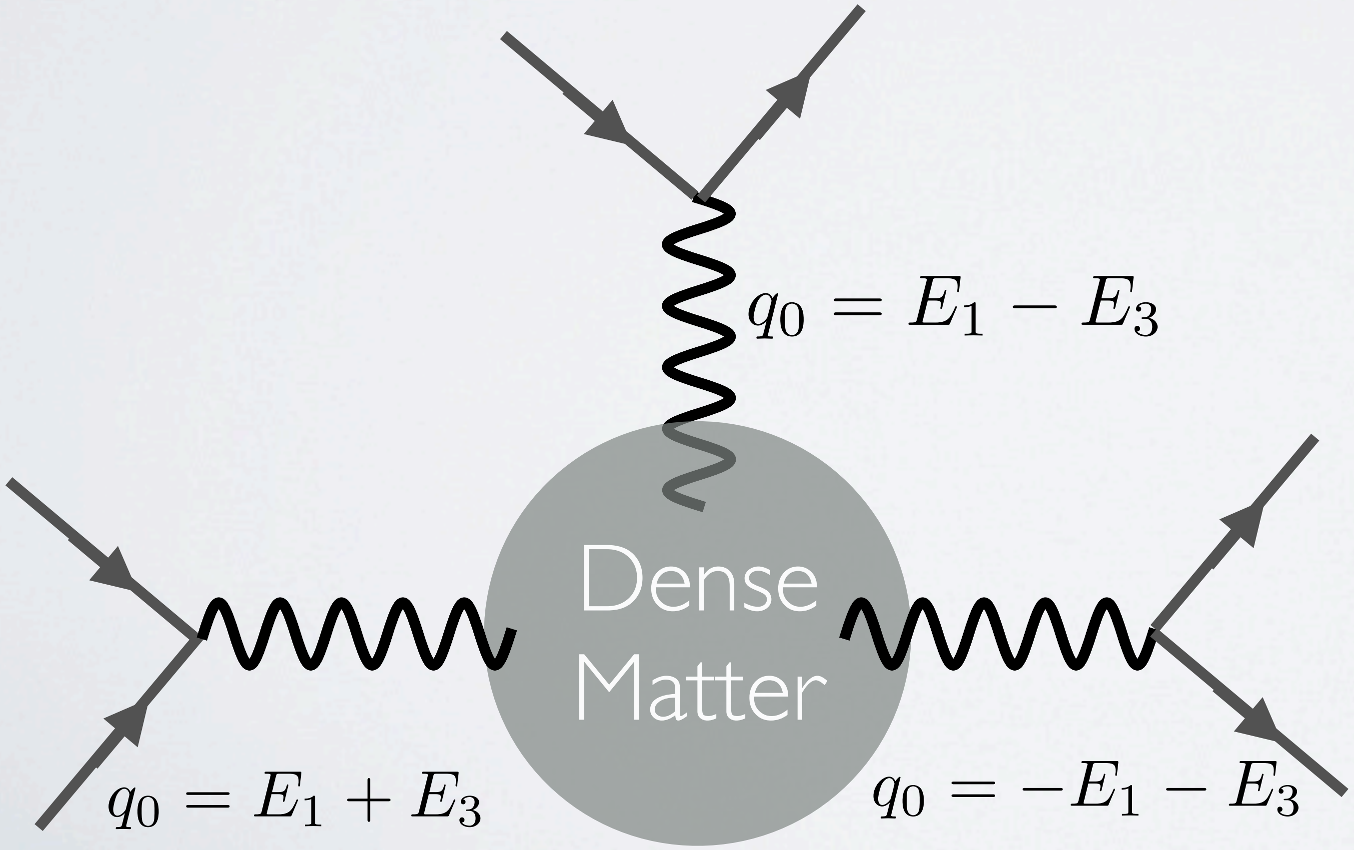
How does dense matter:

- Cool
- Conduct heat and electric currents
- Respond to angular momentum
- Oscillate when its perturbed   ?

FLUCTUATIONS

- The rate of production and scattering of neutrinos (neutron star cooling, supernova), and scattering of electrons (thermal relaxation) are related to the thermal fluctuation spectrum.

Rate = Coupling **x** Kinematics **x** Response Function



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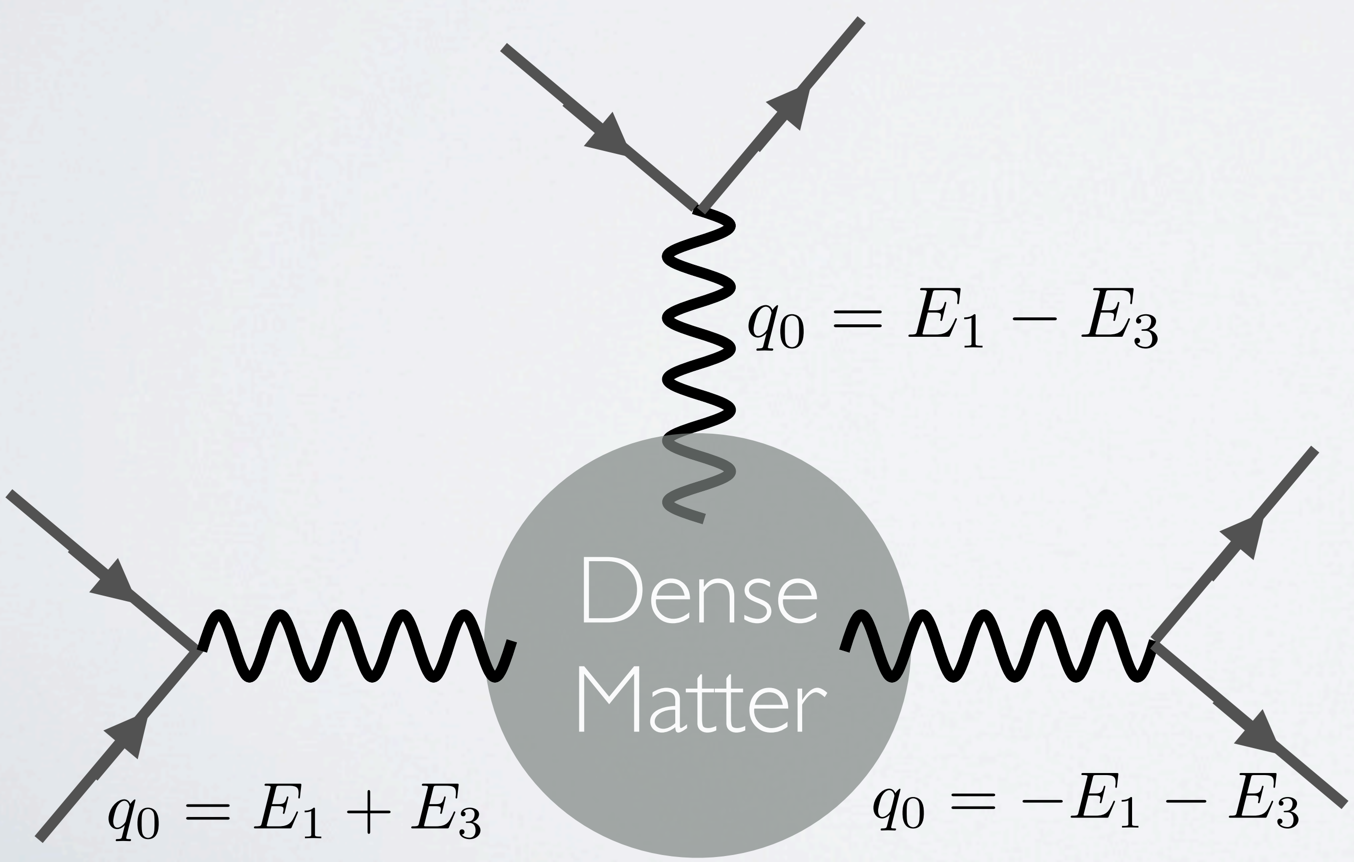
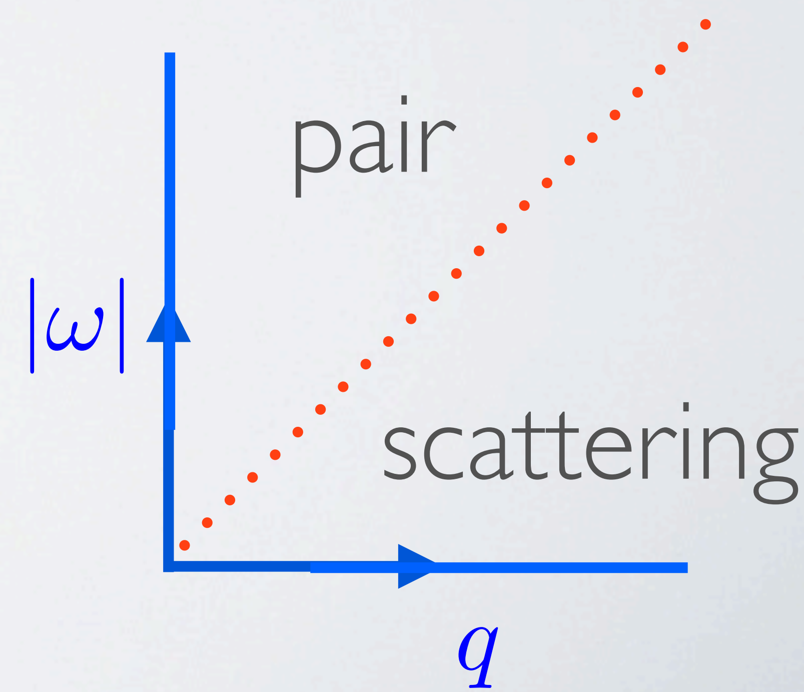
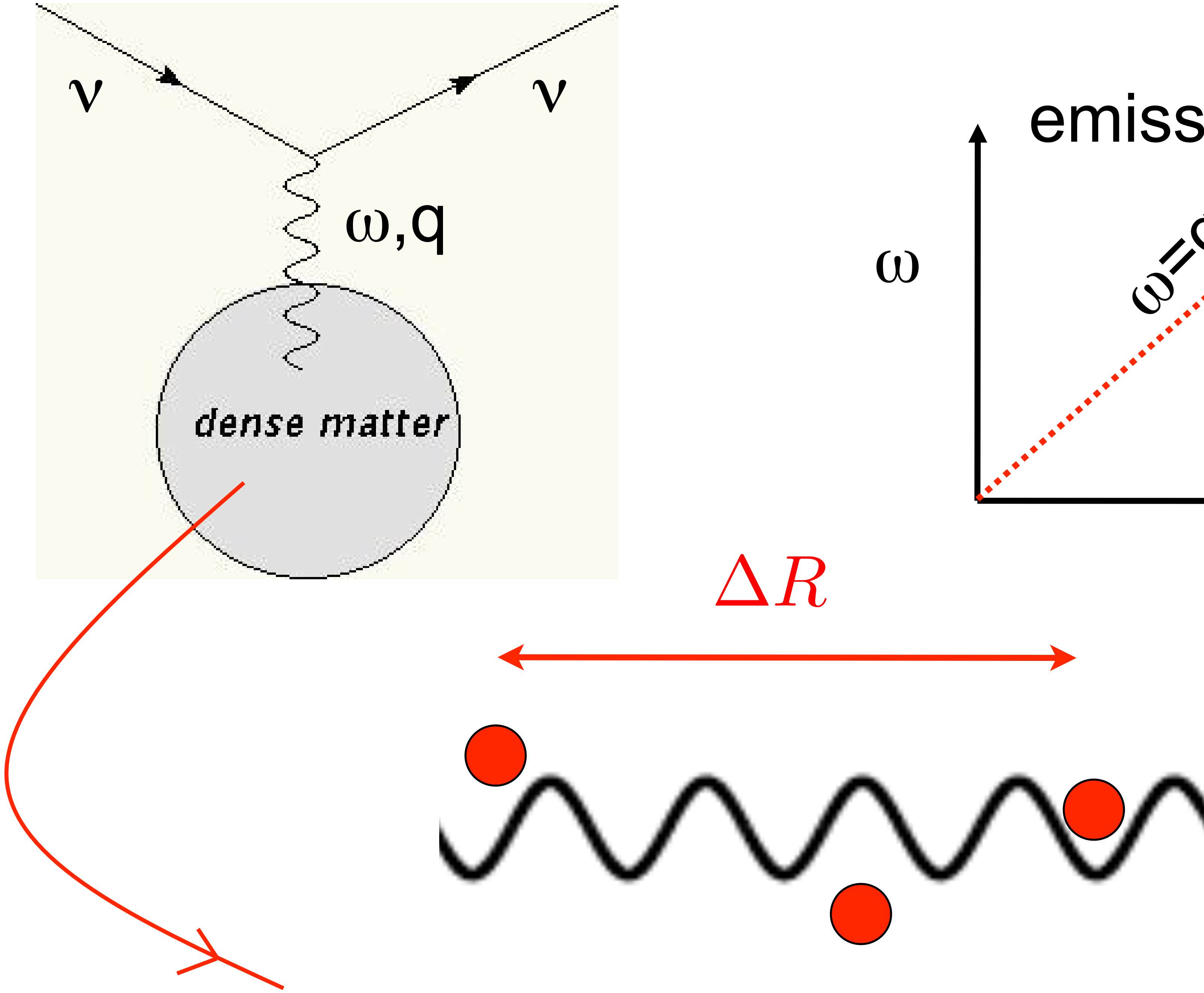
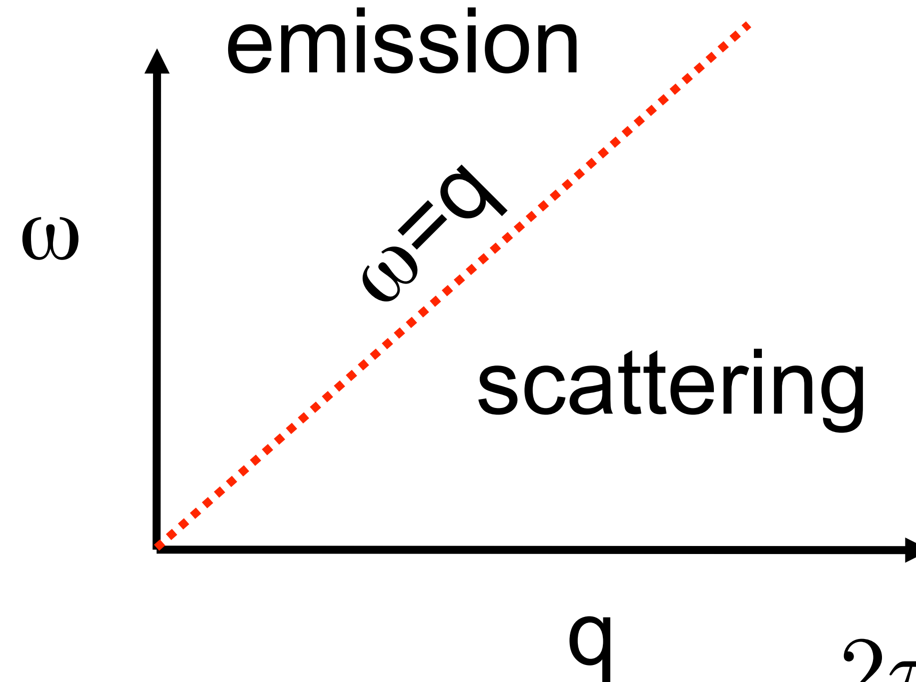




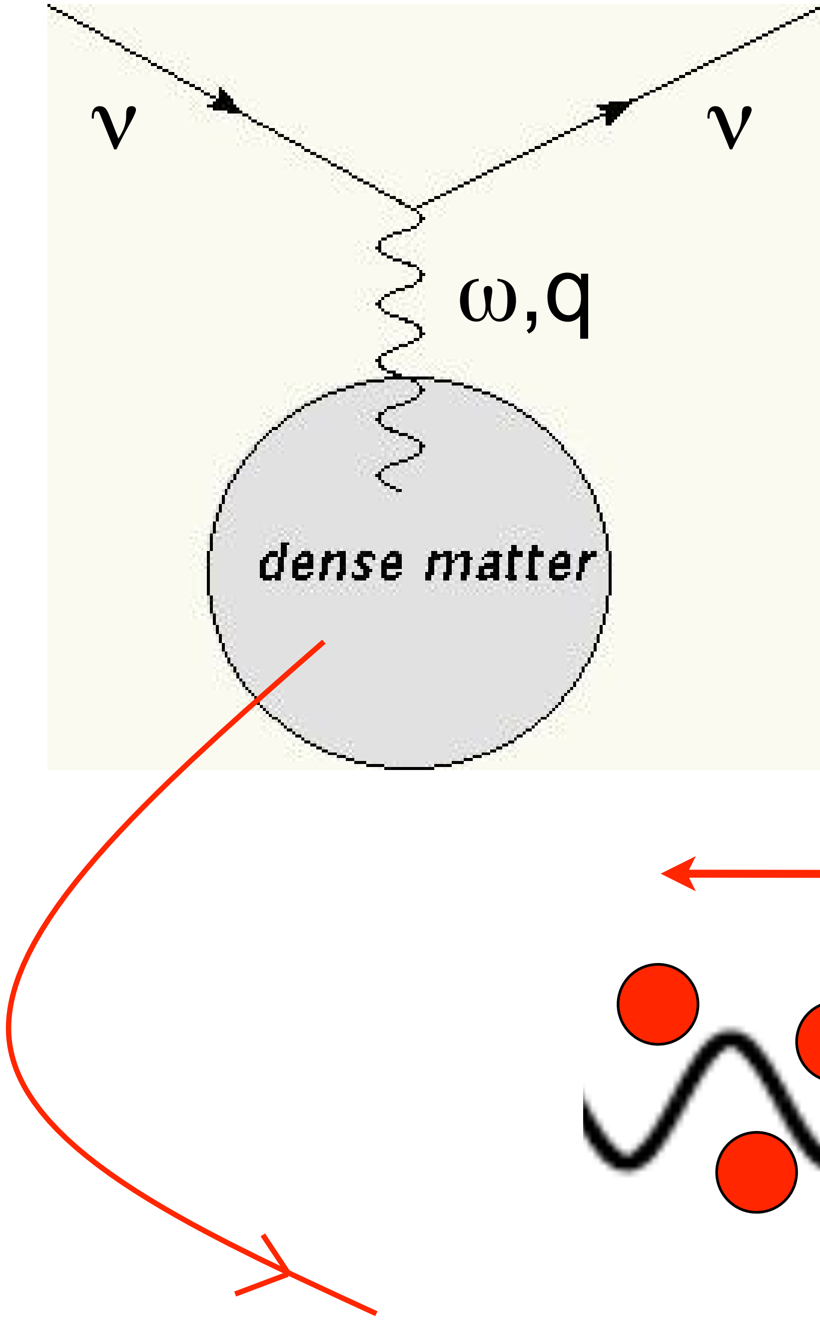
Response of Interacting System

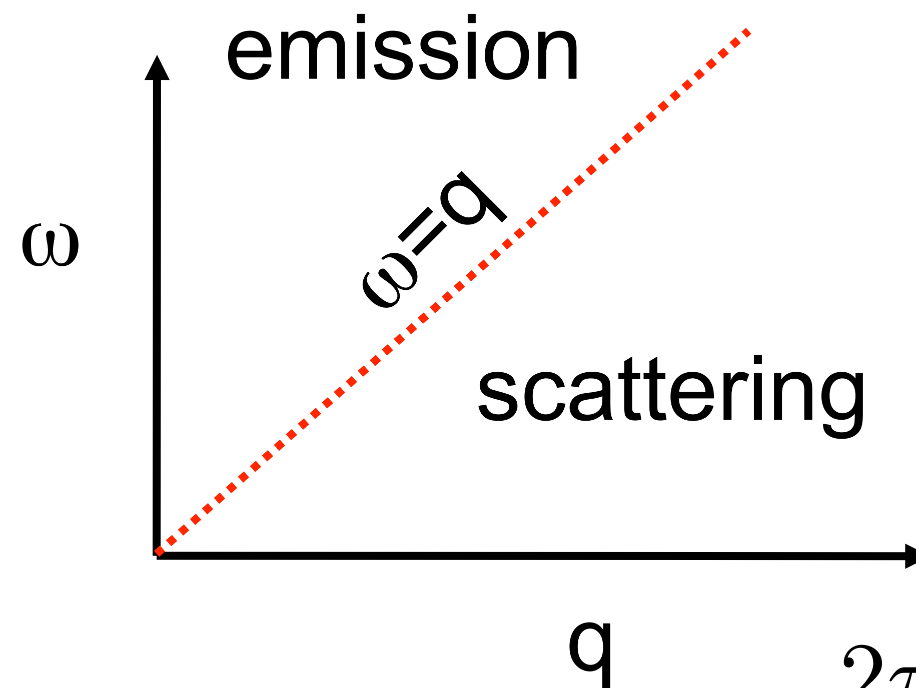


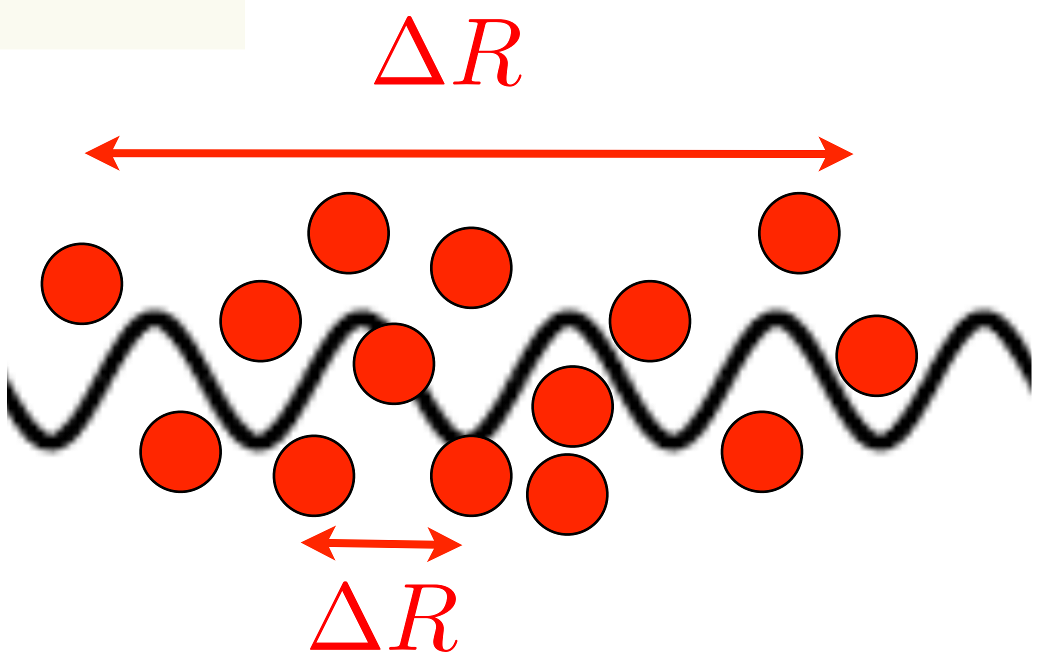


$$q = \frac{2\pi}{\lambda} > \frac{1}{\Delta R}$$

Response of Interacting System



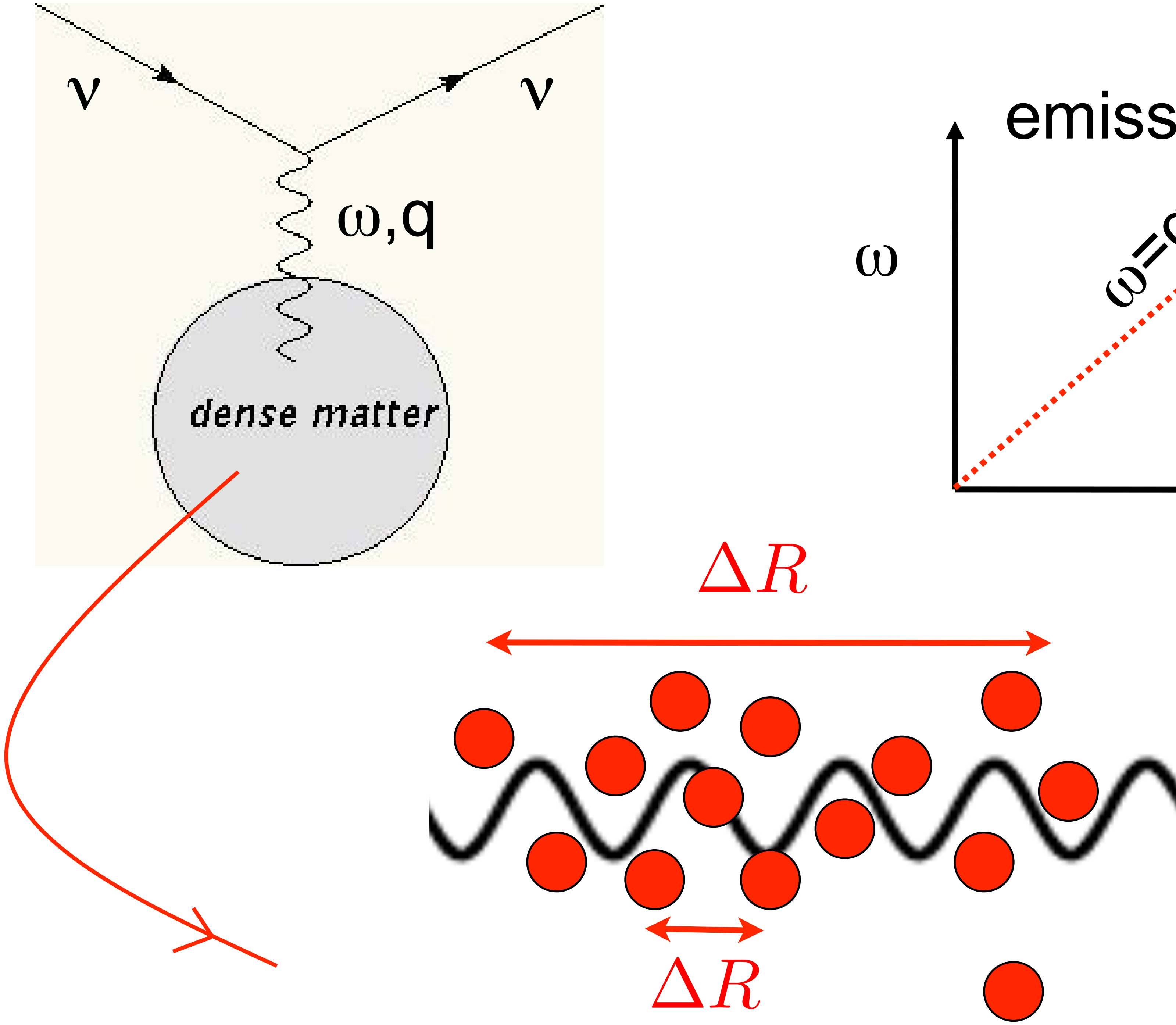


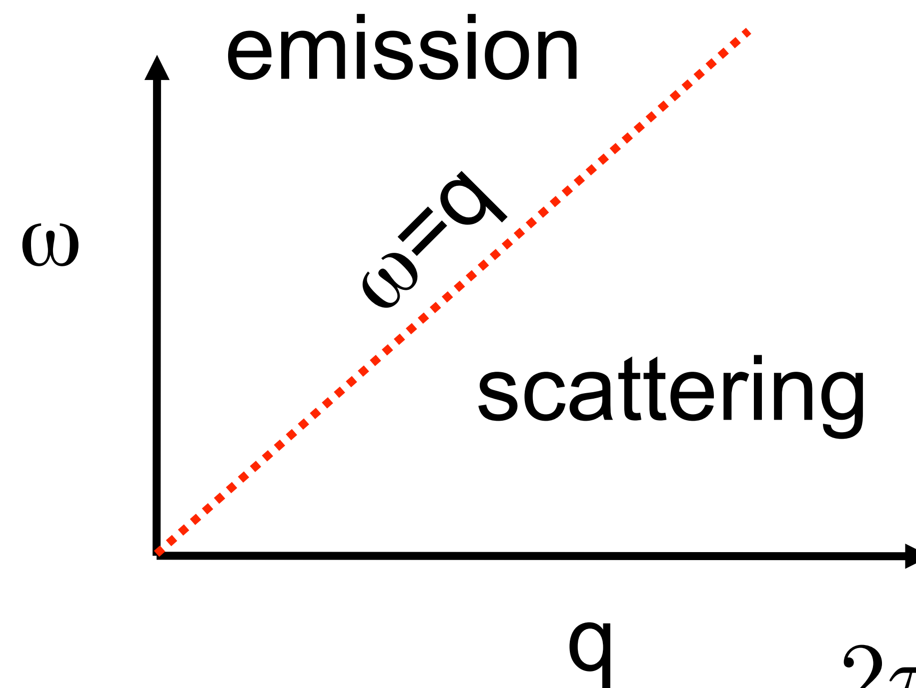


$$q = \frac{2\pi}{\lambda} > \frac{1}{\Delta R}$$

$$q = \frac{2\pi}{\lambda} < \frac{1}{\Delta R}$$

Response of Interacting System





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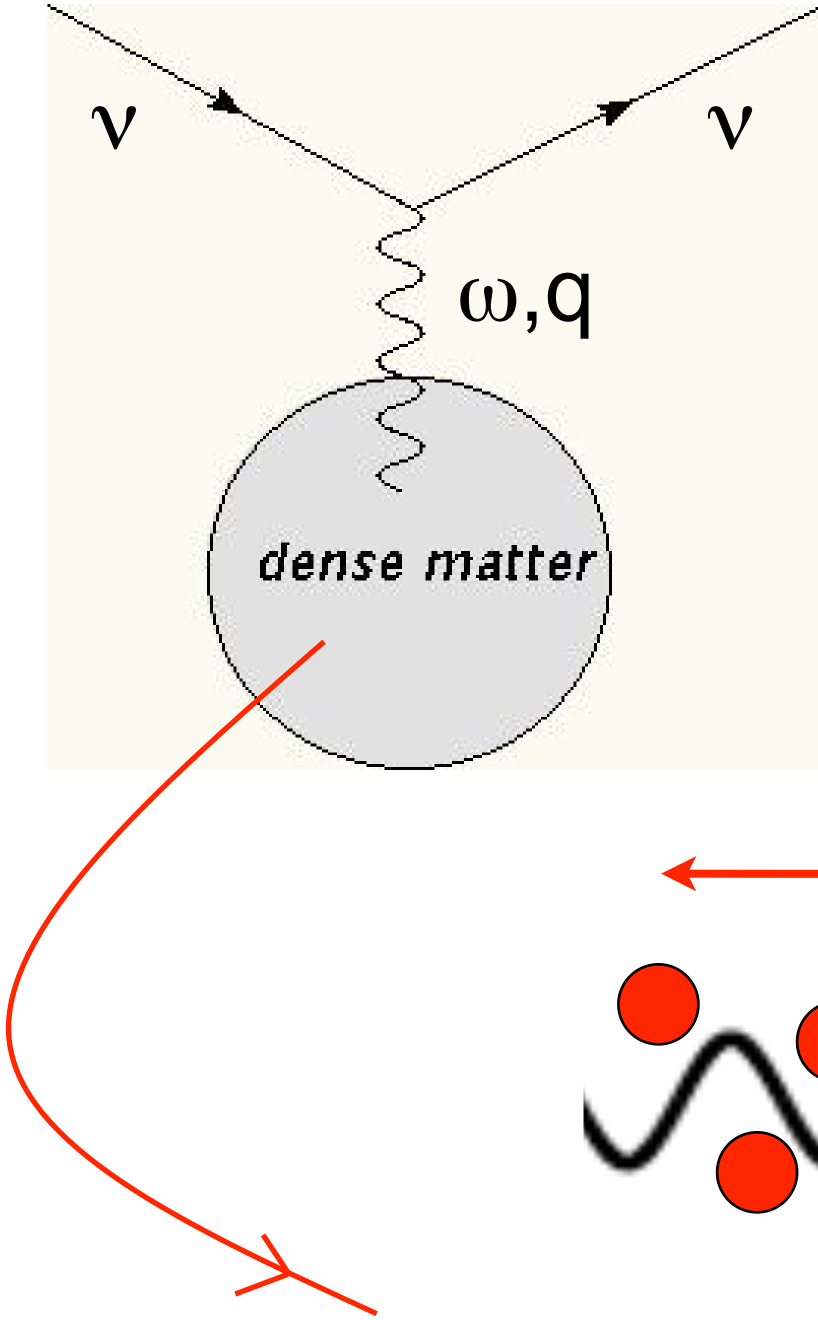
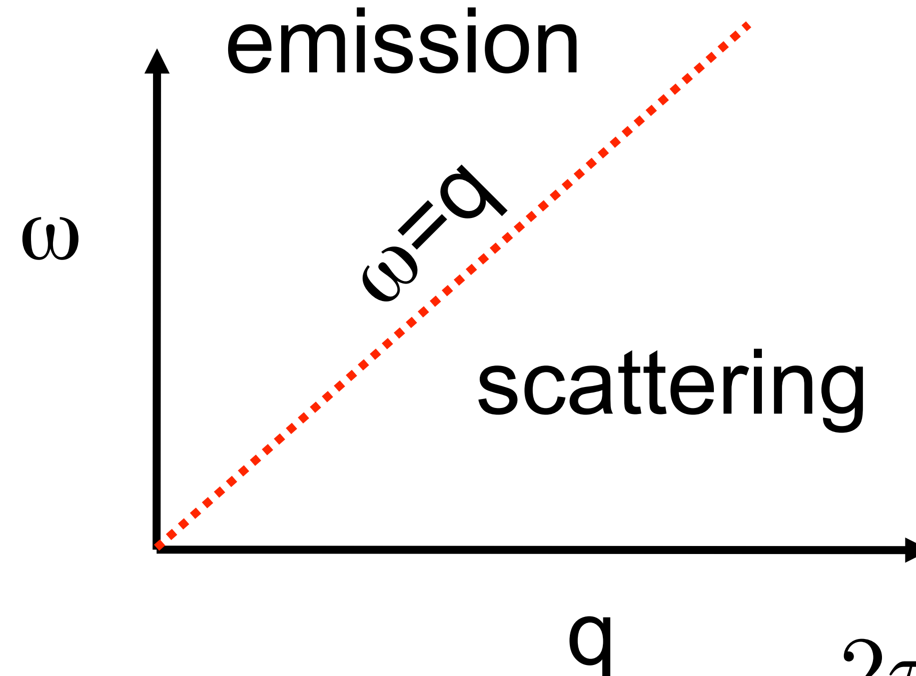
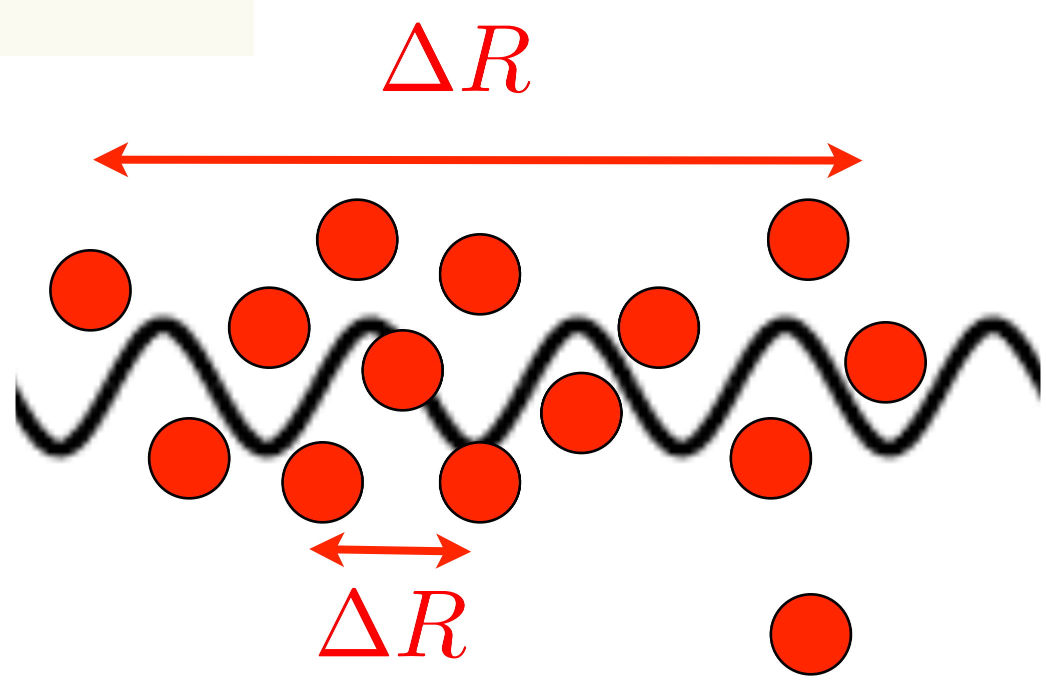
Response of Interacting System







$$q = \frac{2\pi}{\lambda} > \frac{1}{\Delta R}$$

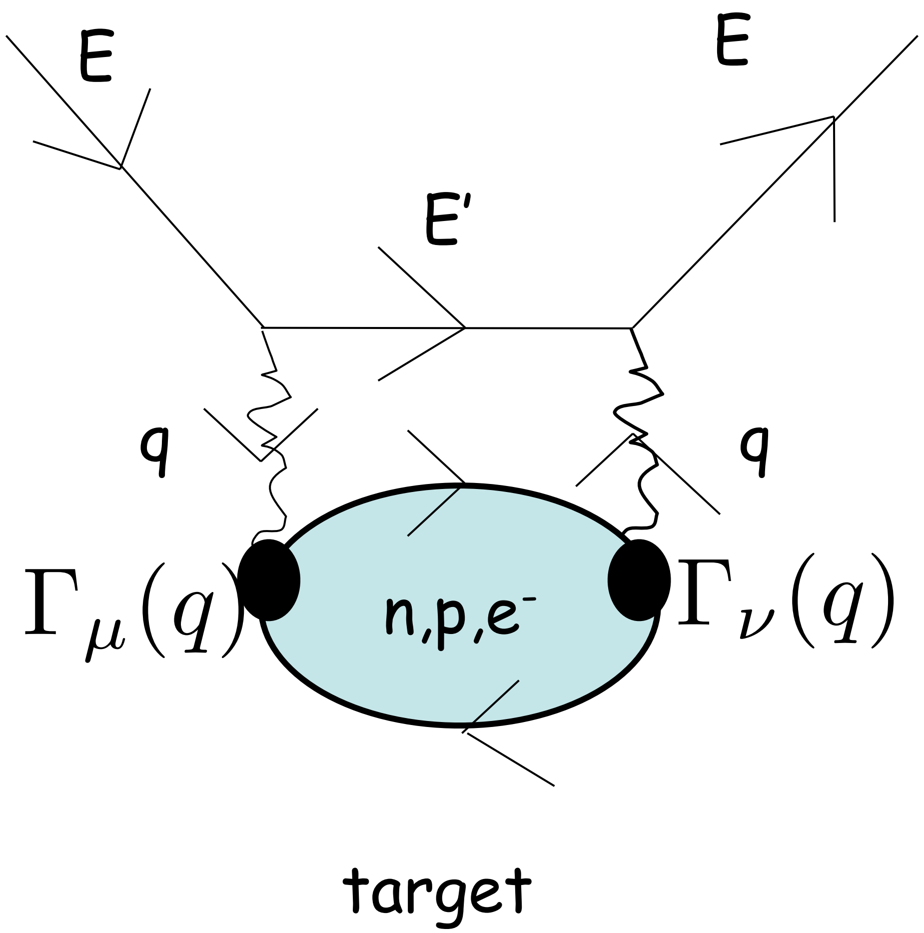
$$q = \frac{2\pi}{\lambda} < \frac{1}{\Delta R}$$

$$\omega = \frac{2\pi}{\tau} > \frac{2\pi}{\tau_{\text{collision}}}$$

$$\omega = \frac{2\pi}{\tau} < \frac{2\pi}{\tau_{\text{collision}}}$$

$\tau_{\text{collision}}$ = Collision Time

Weak Interaction Rates



$$L = \frac{G_F}{2\sqrt{2}}\, l_\nu(x)\; j^\mu(x)$$

$$l_\nu = \bar{\nu}(x)\gamma_\nu(1-\gamma_5)\nu(x)$$

$$j^\mu = \bar{\psi}(x)(c_V\gamma^\mu - c_A\gamma^\mu\gamma_5 + iF_2\,\sigma^{\mu\nu}\frac{q_\nu}{2M})\,\psi(x)$$

$$\frac{d^2\sigma}{V\, d\cos\theta\, dE'} \approx G_F^2\,\frac{E}{E'}\,\mathrm{Im}\left[L_{\mu\nu}(k,k+q)\,\Pi^{\mu\nu}(q)\right]$$

$$L_{\mu\nu} = \mathrm{Tr}[\, l_\mu(k)\, l_\nu(k+q)\,]$$

$$\Pi^{\mu\nu} = \int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}[\, j^\mu(p)\;\; j^\nu(p+q)\,]$$

Neutrino-Nucleon Scattering

Neutrinos couple to density and spin

$$j^\mu(x) = \overline{\psi}(x)\,\gamma^\mu (c_V - c_A \gamma_5)\,\psi(x)$$

$$\xrightarrow{NR} c_V\ \psi^+\psi\,\delta^{\mu 0} - c_A\ \ \psi^+\,\sigma^i\,\psi\,\delta^{\mu i}$$

$$\frac{d\Gamma}{d\cos\theta dE'_\nu} = \frac{G_F^2}{4\pi^2}\,(1 - f_\nu(E'_\nu))\ {E'_\nu}^2$$
$$\times\ \left(c_V{}^2\ (1+\cos\theta)\ S(|\vec{q}|,\omega)\ +\ c_A{}^2\ (3-\cos\theta)\ S^A(|\vec{q}|,\omega)\right)$$

$$S(|\vec{q}|,\omega) = \int_{-\infty}^{\infty} dt\ \ \exp(i\omega t)\ \langle\ \rho(\vec{q},t)\rho(-\vec{q},0)\ \rangle$$

$$S^A(|\vec{q}|,\omega) = \int_{-\infty}^{\infty} dt\ \ \exp(i\omega t)\ \delta_{ij}\ \langle\ \sigma_i(\vec{q},t)\sigma(-\vec{q},0)\ \rangle$$

Iwamoto & Pethick (1982)

Response of a classical liquid

The density-density correlation for N particles is

$$\langle \rho(\mathbf{q},\mathbf{0})\rho(\mathbf{q},\mathbf{t})\rangle = \langle \boldsymbol{\Sigma}_{\mathbf{i}}\ \mathbf{e}^{-\mathbf{iq}.\mathbf{r}_{\mathbf{i}}} \boldsymbol{\Sigma}_{\mathbf{j}}\ \mathbf{e}^{-\mathbf{iq}.\mathbf{r}_{\mathbf{j}}(\mathbf{t})}\rangle$$

Ensemble average

Positions at t

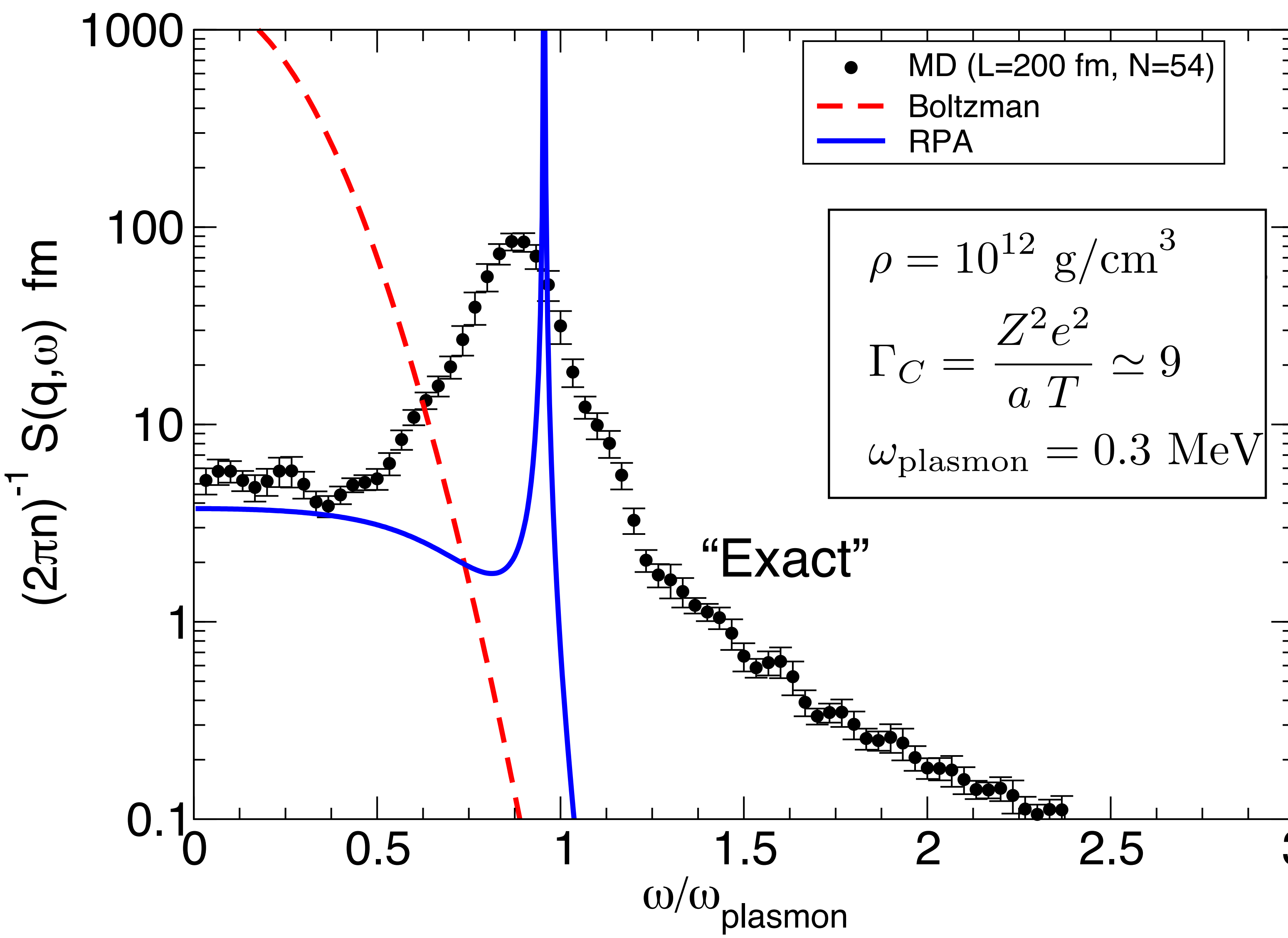
Positions at t=0

Need to specify equations of motion or $\mathbf{r}_j(t)$.
Classical limit:

$$\mathbf{r}_j(\Delta t) = \mathbf{r}_j(0) + \mathbf{v}_j\ \Delta t + \frac{1}{2\ m}\Sigma_{i\neq j}\mathbf{F}_{ij}\ t^2$$

Screening, Damping & Collective Modes

- Strong repulsive Coulomb forces affect the spatial distribution.
- A collective mode exists in the system.
- Response is pushed to high energy.
- Multi-particle excitations smears the response.

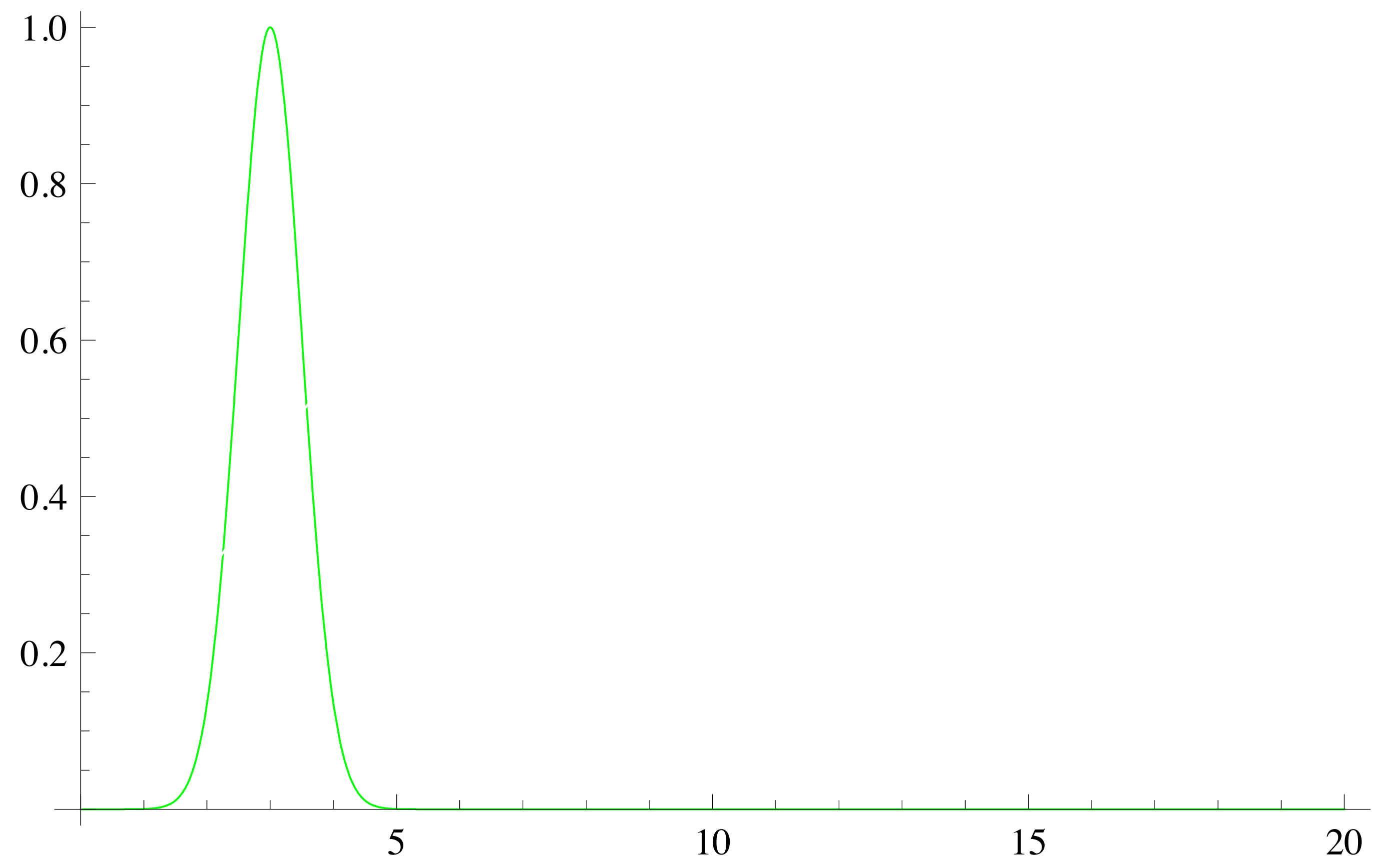


Response Functions: In Quantum Fluids

$$S_q(\omega) = \sum_{\lambda,\lambda'} f_\lambda \, |\langle\lambda|A_q|\lambda'\rangle|^2 \, \delta(E_\lambda - E'_\lambda - \omega)$$

$$= \int dt \, e^{i\omega t} \langle\langle\lambda|A_q(t) \, A_q^\dagger(0)|\lambda\rangle\rangle$$

$S_q(\omega)$



$$\omega = \frac{q^2}{2M} + \frac{\vec{p}.\vec{q}}{M}$$

ω

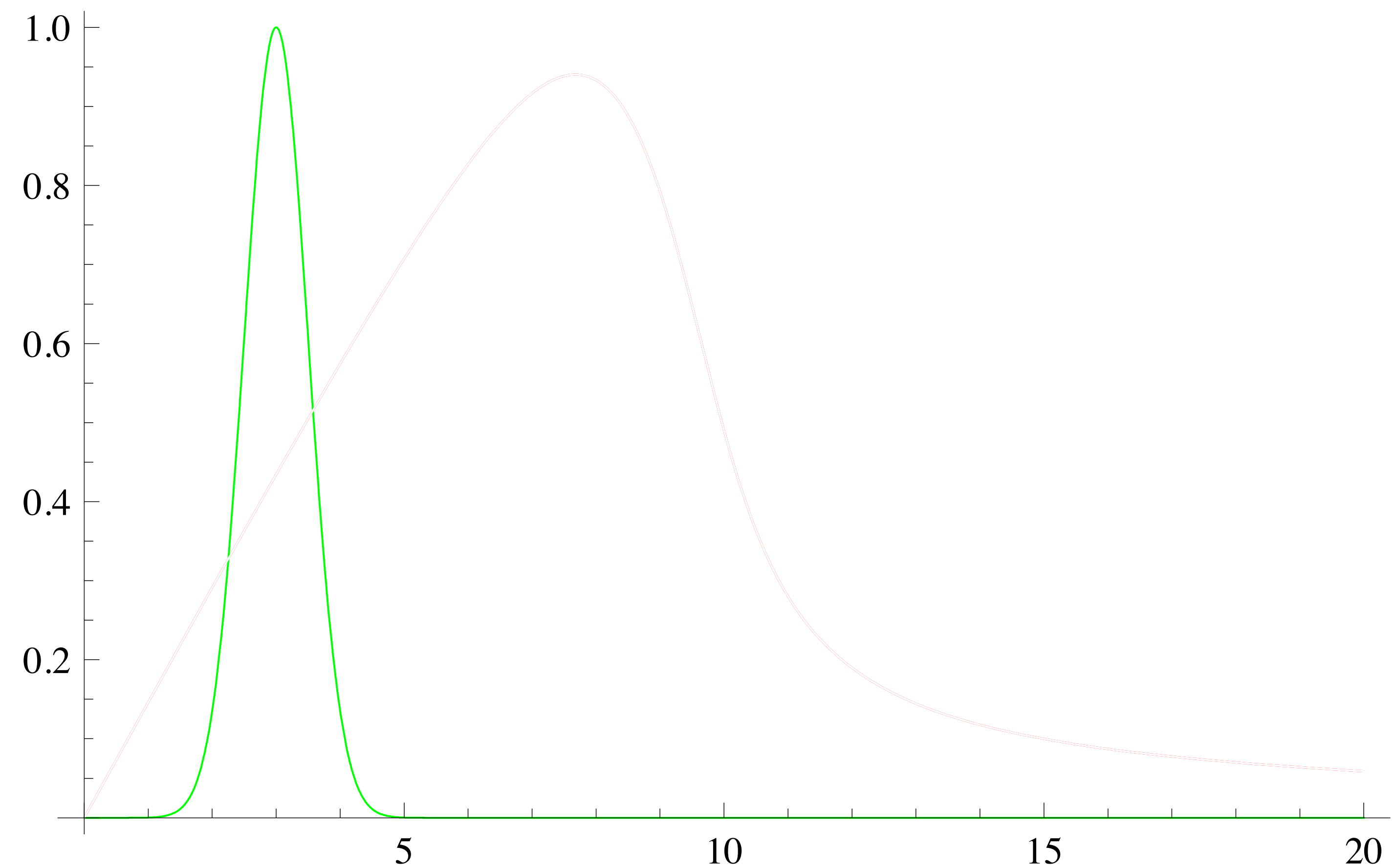
Response Functions: In Quantum Fluids

$$S_q(\omega) = \sum_{\lambda,\lambda'} f_\lambda \, |\langle \lambda | A_q | \lambda' \rangle|^2 \, \delta(E_\lambda - E'_\lambda - \omega)$$

$$= \int dt \, e^{i\omega t} \langle \langle \lambda | A_q(t) \, A_q^\dagger(0) | \lambda \rangle \rangle$$

Fermi Motion and
Pauli Blocking.

$S_q(\omega)$



$$\omega = \frac{q^2}{2M} + \frac{\vec{p}.\vec{q}}{M}$$

ω

Response Functions: In Quantum Fluids

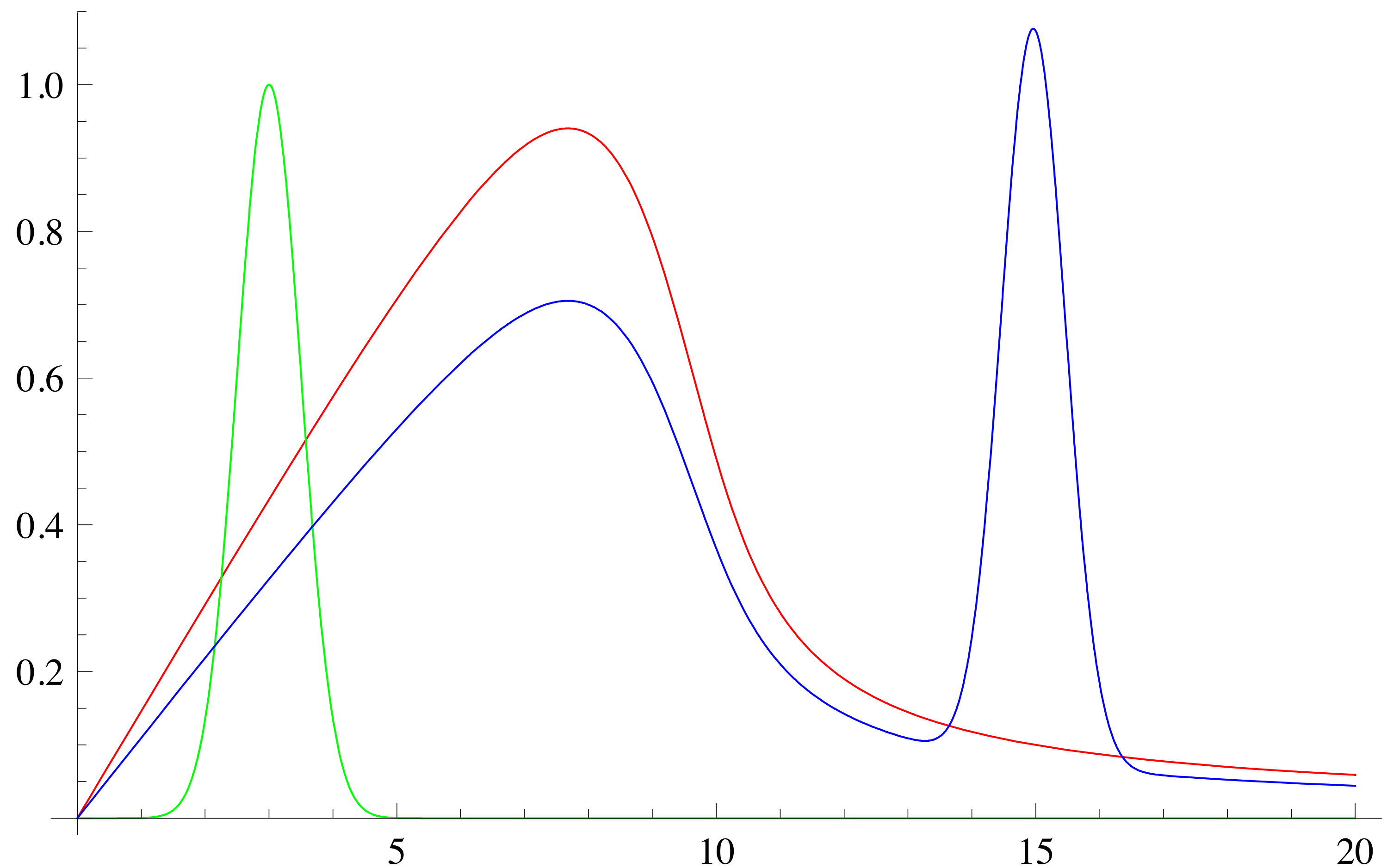
$$S_q(\omega) = \sum_{\lambda,\lambda'} f_\lambda \, |\langle \lambda | A_q | \lambda' \rangle|^2 \, \delta(E_\lambda - E'_\lambda - \omega)$$

$$= \int dt \, e^{i\omega t} \langle\langle \lambda | A_q(t) \, A_q^\dagger(0) | \lambda \rangle\rangle$$

Fermi Motion and Pauli Blocking.

Correlations and collisions.

$$S_q(\omega)$$



$$\omega = \frac{q^2}{2M} + \frac{\vec{p}.\vec{q}}{M} \qquad \omega$$

Response Functions: In Quantum Fluids

$$S_q(\omega) = \sum_{\lambda,\lambda'} f_\lambda \, |\langle\lambda|A_q|\lambda'\rangle|^2 \, \delta(E_\lambda - E'_\lambda - \omega)$$

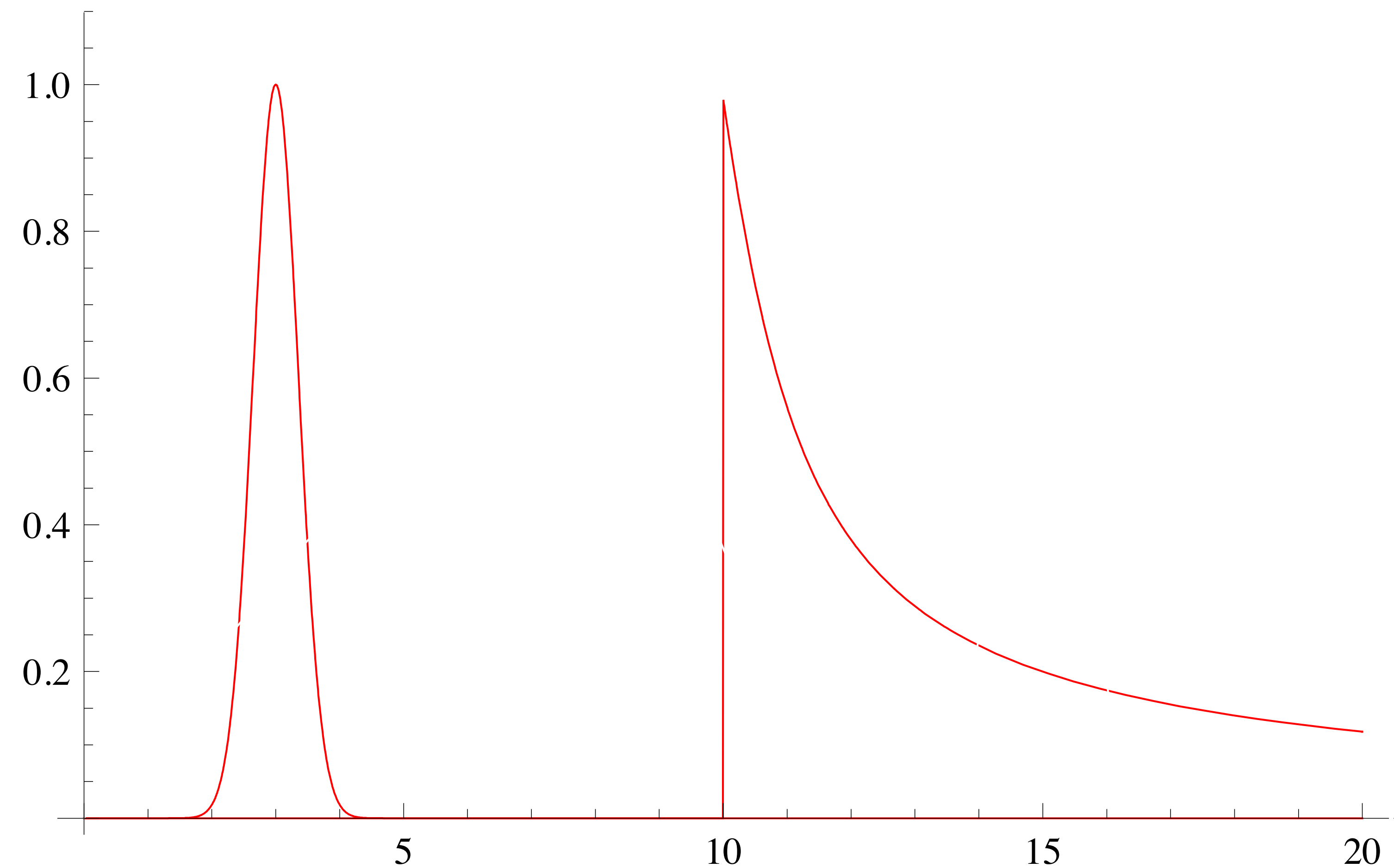
$$= \int dt \, e^{i\omega t} \langle\langle\lambda|A_q(t) \, A_q^\dagger(0)|\lambda\rangle\rangle$$

Fermi Motion and Pauli Blocking.

Correlations and collisions.

Cooper Pairing & Superfluidity.

$S_q(\omega)$



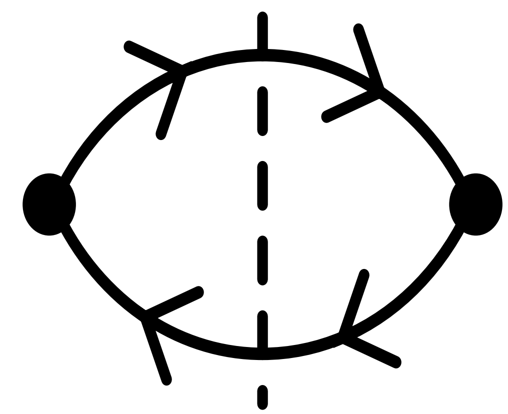
$$\omega = \frac{q^2}{2M} + \frac{\vec{p}.\vec{q}}{M}$$

ω

Computing Correlation Functions

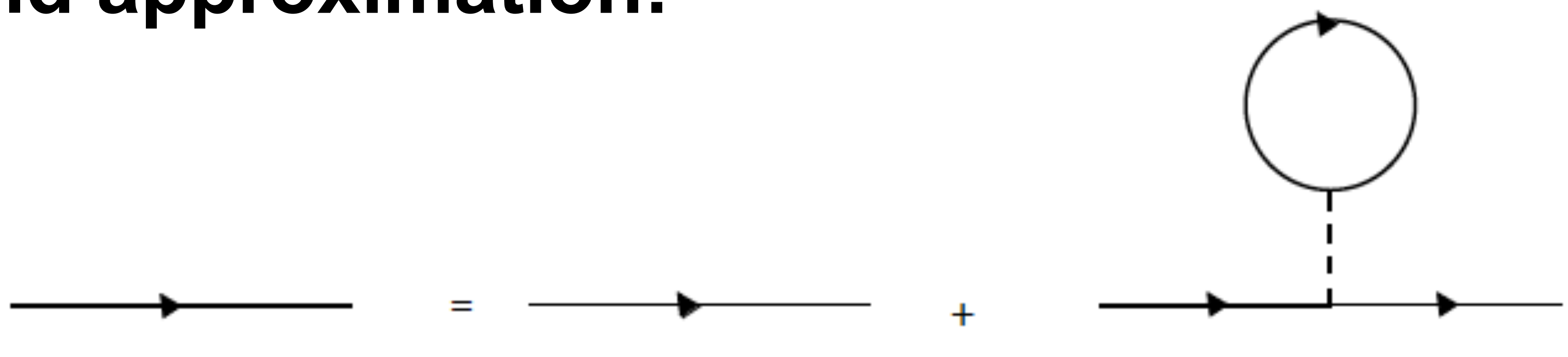
No exact methods exist in strongly coupled quantum systems.

In the Fermi gas:



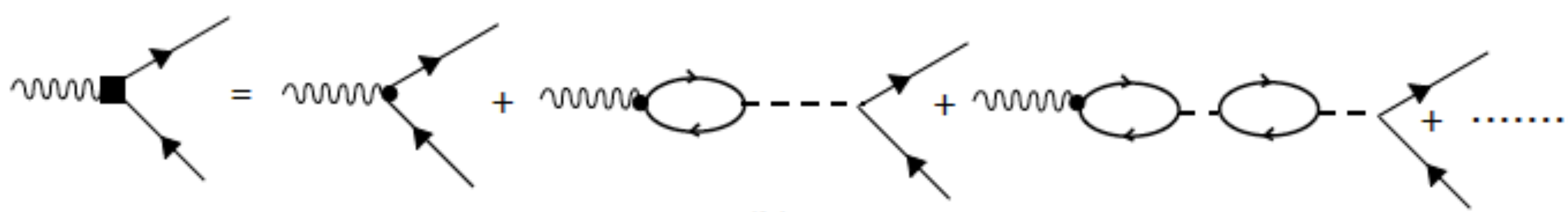
In the mean field approximation:

Corrections to single particle energy

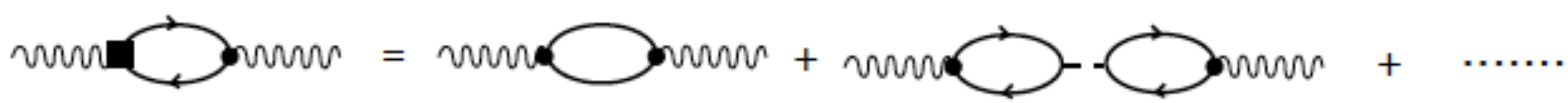


(a)

Vertex Corrections



(b)

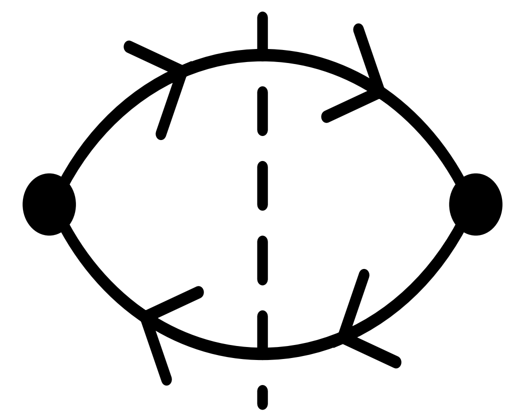


(c)

Computing Correlation Functions

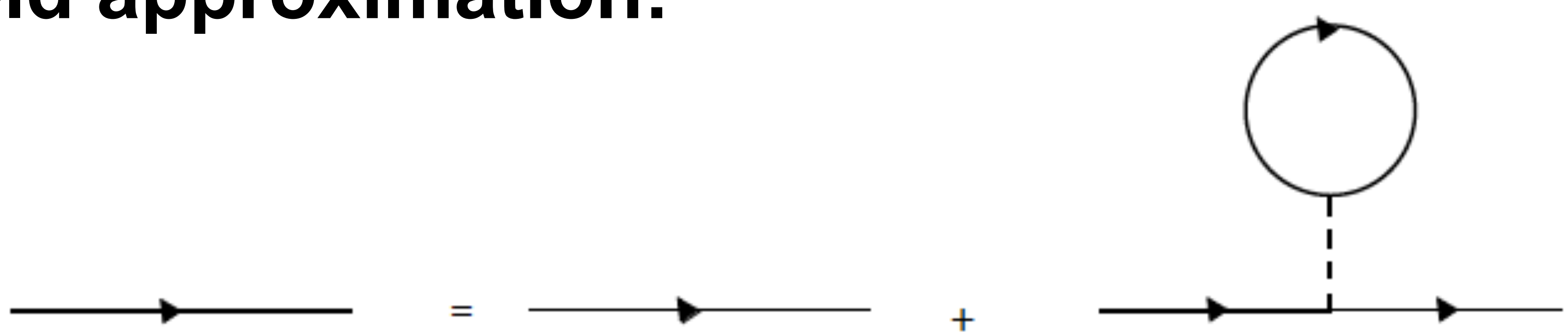
No exact methods exist in strongly coupled quantum systems.

In the Fermi gas:



In the mean field approximation:

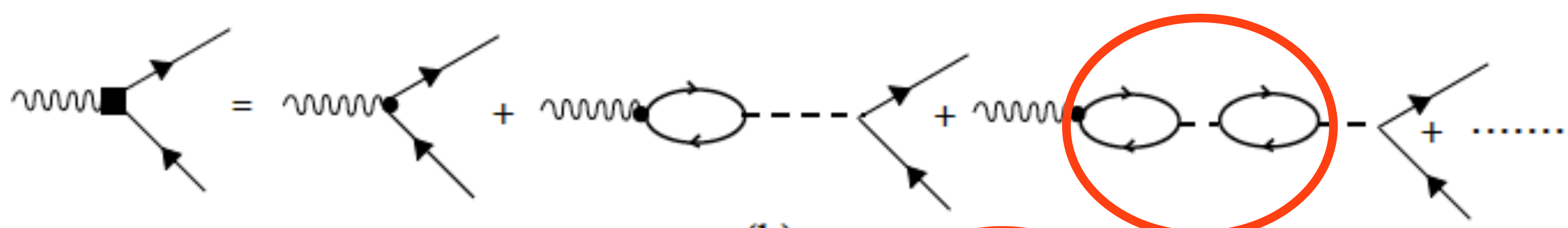
Corrections to single particle energy



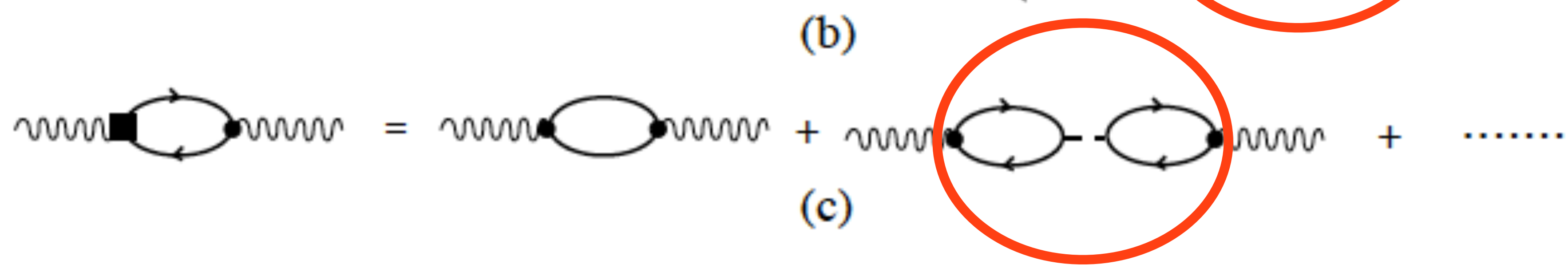
(a)

RPA

Vertex Corrections



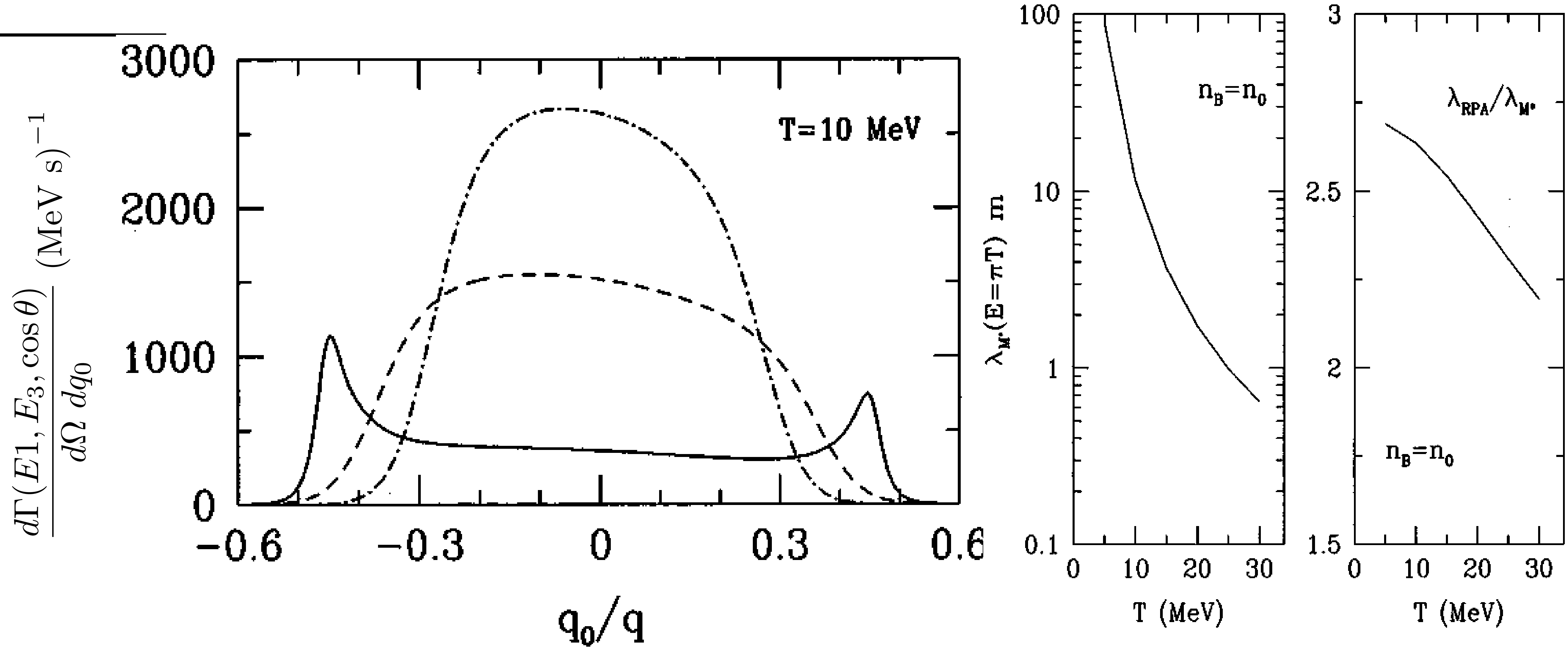
(b)



(c)

Correlations in a nuclear liquid

$$\frac{d\Gamma(E_1)}{d\cos\theta\ dq_0} = \frac{G_F^2}{4\pi^2}\ (E_1 - q_0)^2\ \left[(1+\cos\theta)\ S_{\rm V}^{\rm RPA}(q_0,q) + (3-\cos\theta)\ S_{\rm A}^{\rm RPA}(q_0,q)\right]$$



Neutrino scattering can be significantly reduced

OBSERVING TRANSPORT PHENOMENA

- Can we measure correlation functions in neutron star matter ?

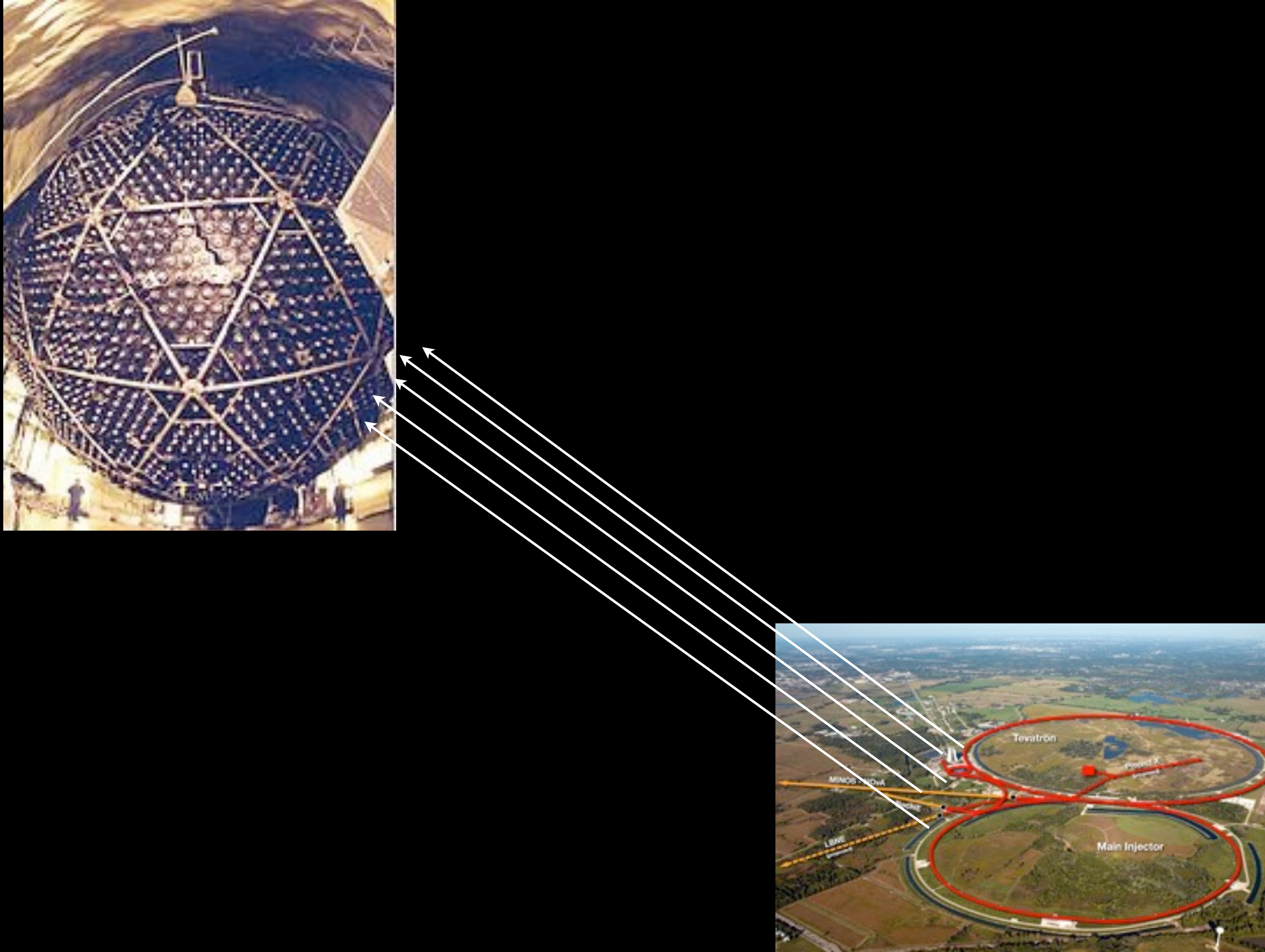
OBSERVING TRANSPORT PHENOMENA

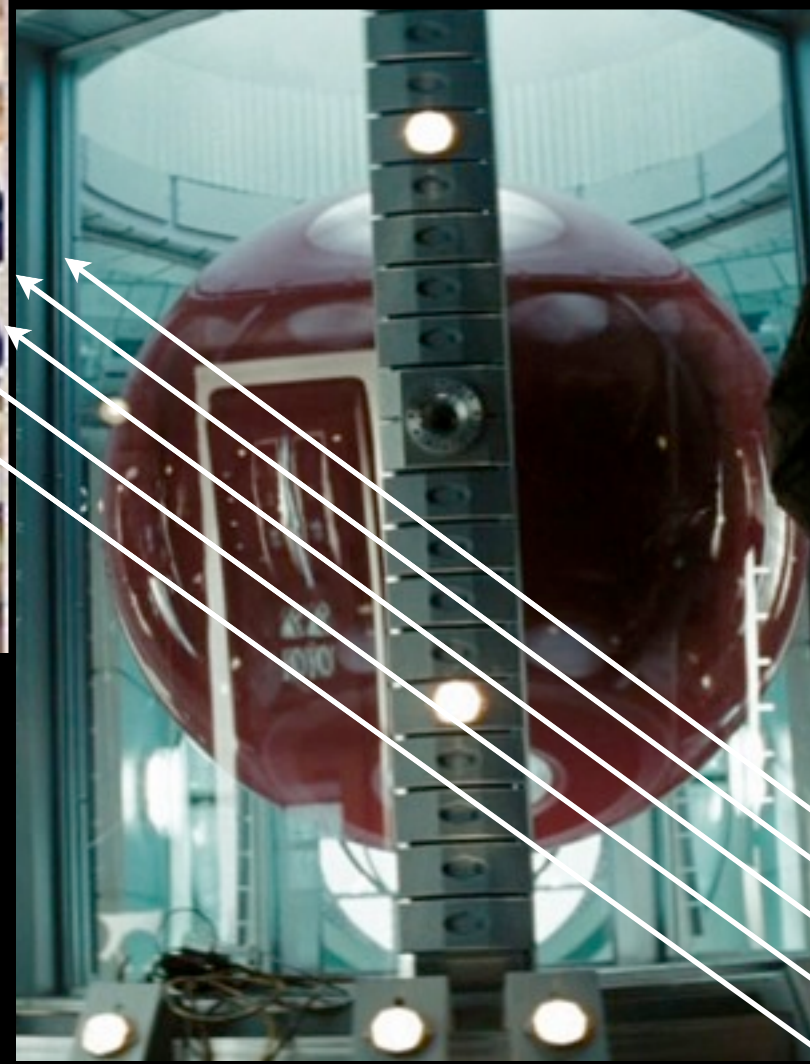
- Can we measure correlation functions in neutron star matter ?

Yes.

This will require at least two of the following:

- New experiments with exotic targets.
- Temporal phenomena in neutron stars.
- Theoretical understanding of transport properties.


Tevatron
Main Injector



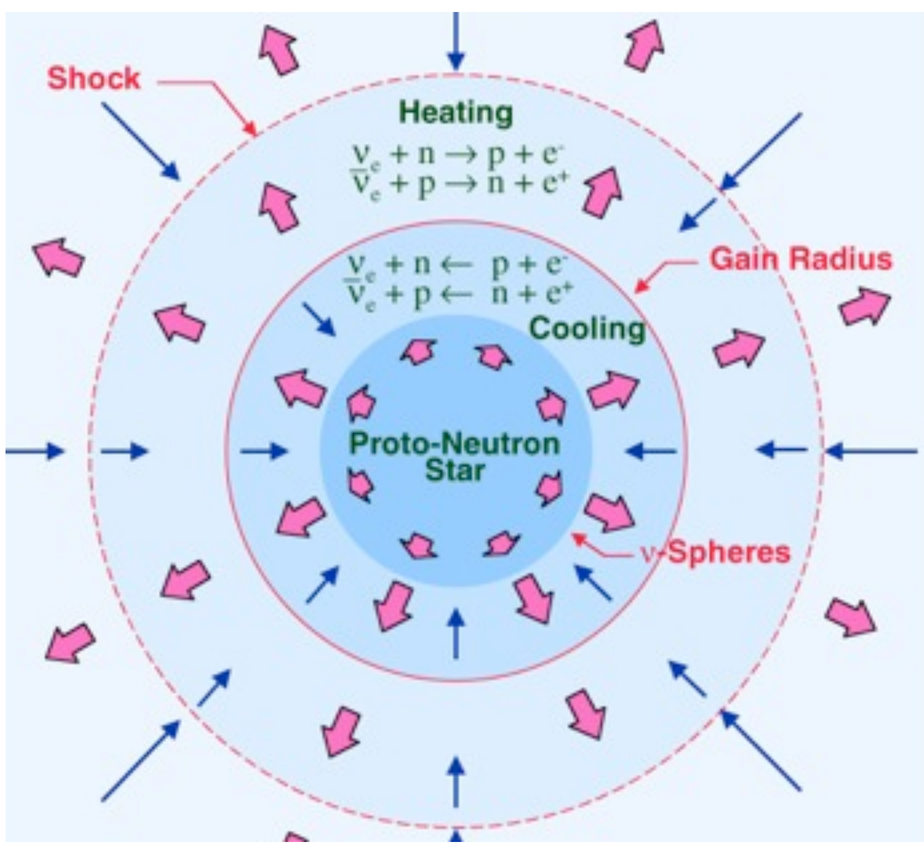
Missing target. Awaiting new collaborators.

Time Dependent Phenomena

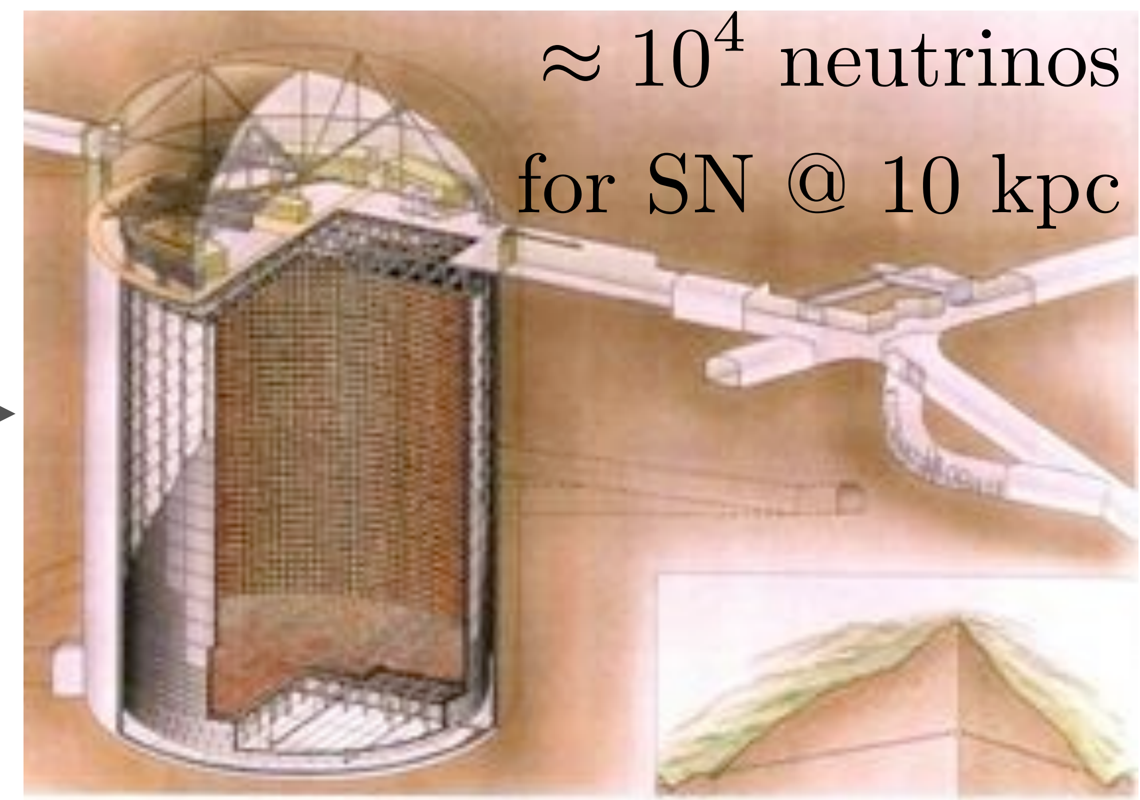
- Thermal relaxation of the core.
- Neutron star cooling.
- Thermal relaxation of the crust.

Thermal Relaxation of the Core

Once in a lifetime we may detect a neutrino burst from a galactic supernova.



$$\nu_e, \bar{\nu}_e, \nu_X, \bar{\nu}_X \longrightarrow$$



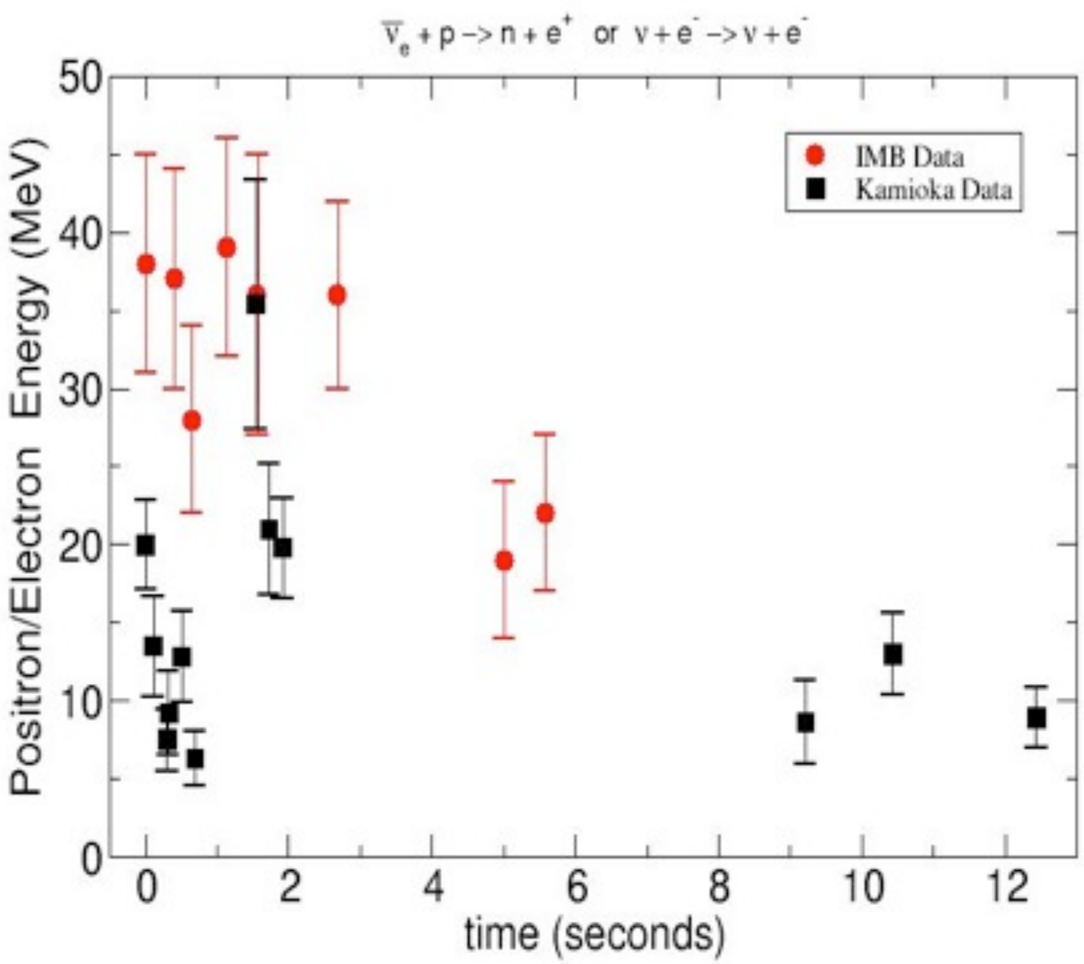
$\approx 10^4$ neutrinos for SN @ 10 kpc

Supernova Neutrinos

3×10^{53} ergs = $10^{58} \times$ 20 MeV Neutrinos

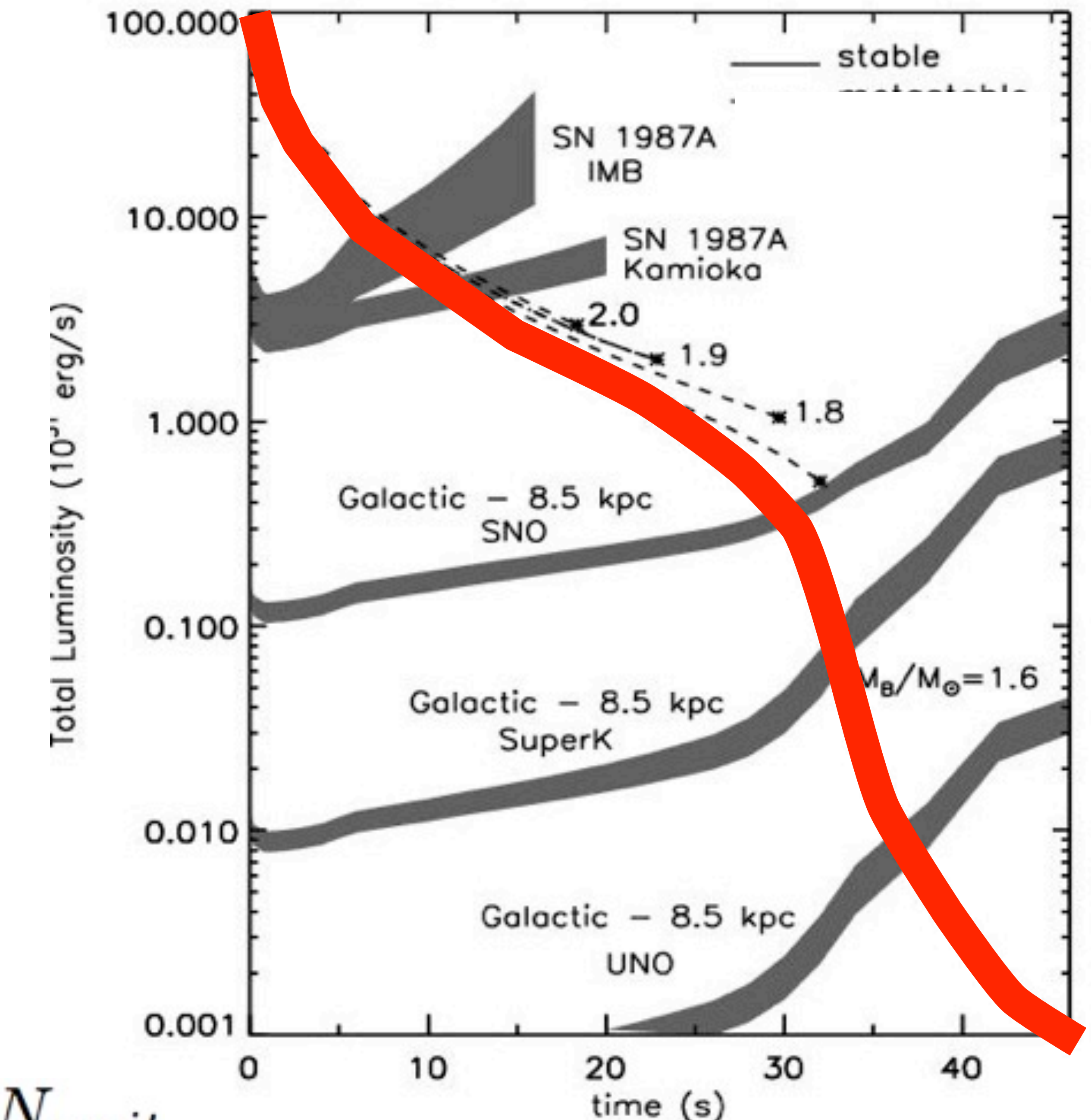
Past:

SN 1987a: ~ 20 neutrinos ..in support of supernova theory



Future:

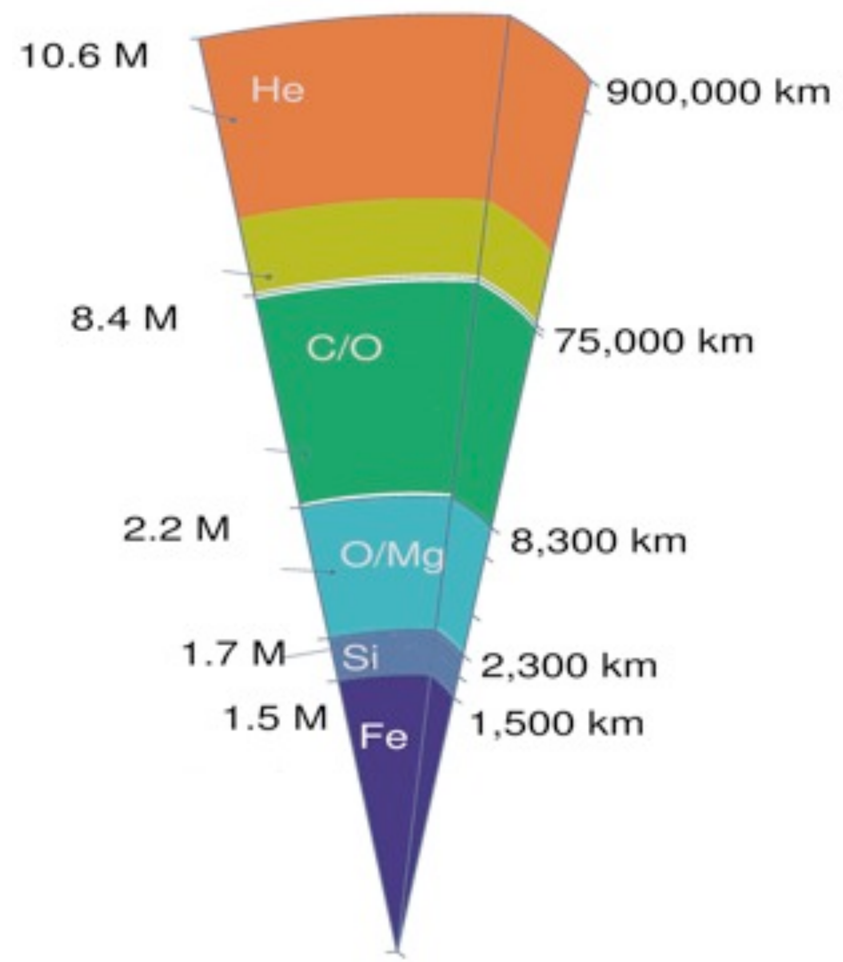
Can detect ~10,000 neutrinos from galactic supernova



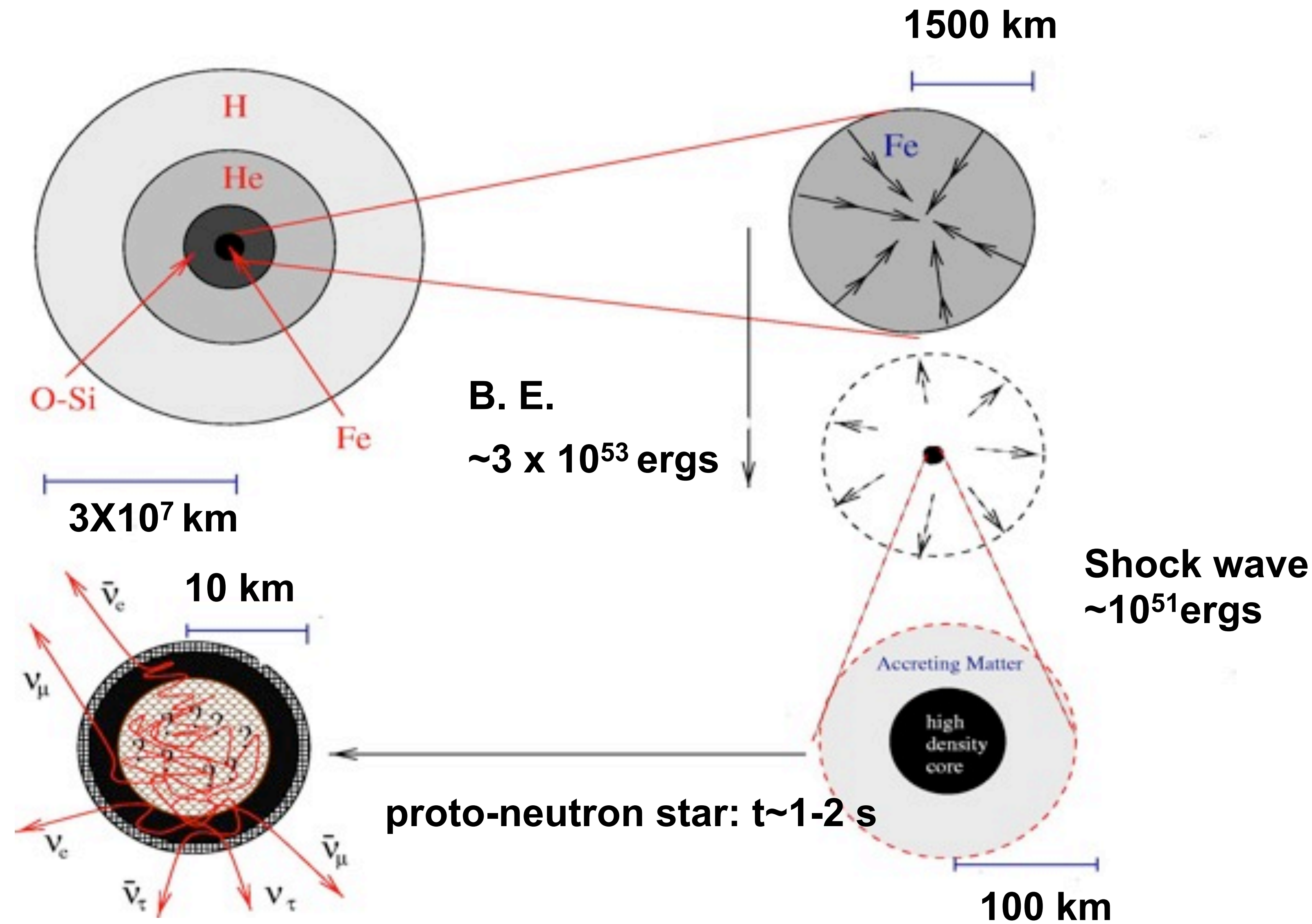
$$\frac{dN_{\rm detect}}{dt} \simeq \frac{\sigma_{\rm ref} \times n_p \times M_{\rm tons}}{4\pi D^2} \frac{E_\nu^2}{m_e^2} \frac{dN_{emit}}{dt}$$

Pons et al. (2002)

Core Collapse Supernova

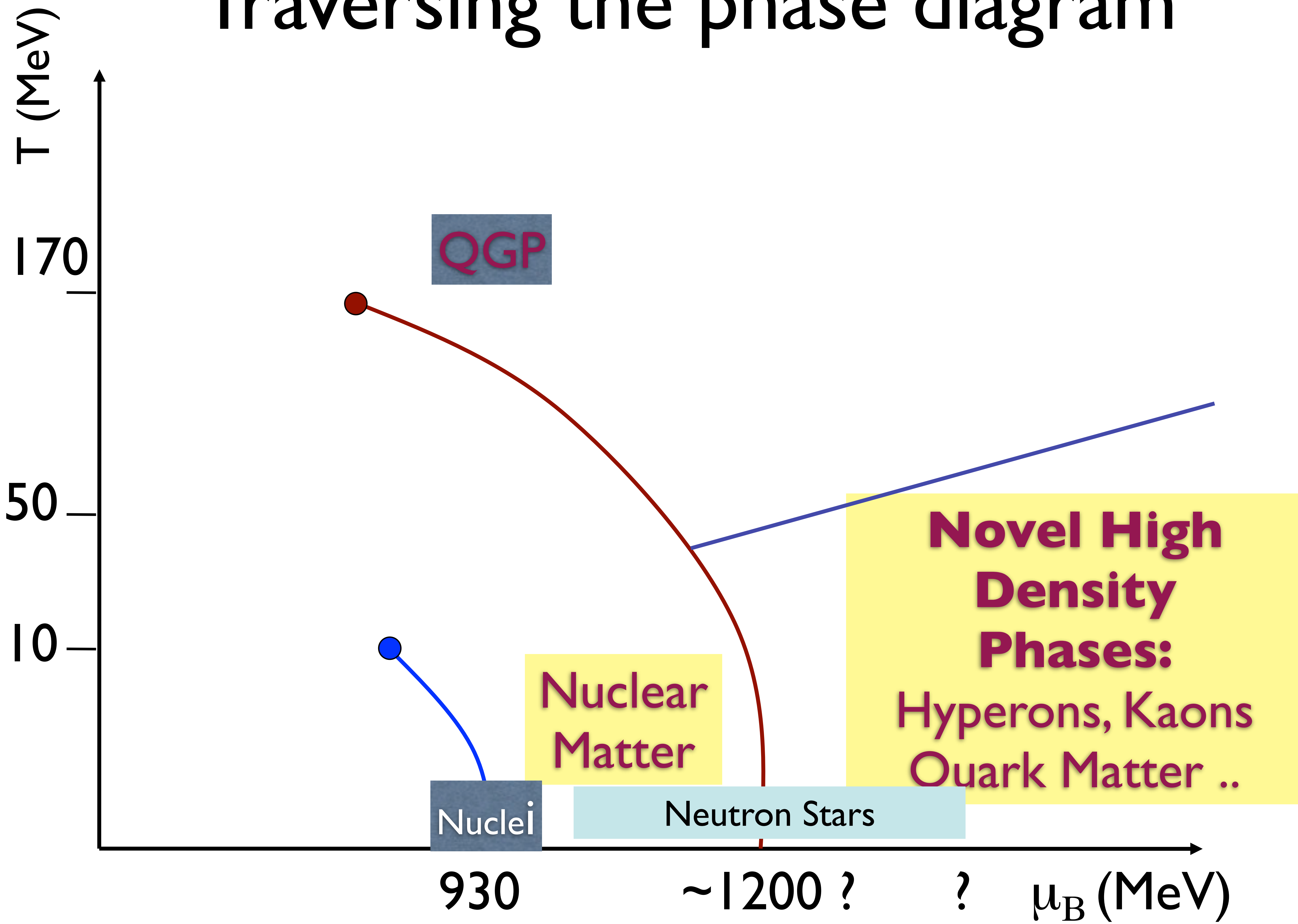


Based on illustration by Francis Reddy and computations by Alexander Heger, LANL

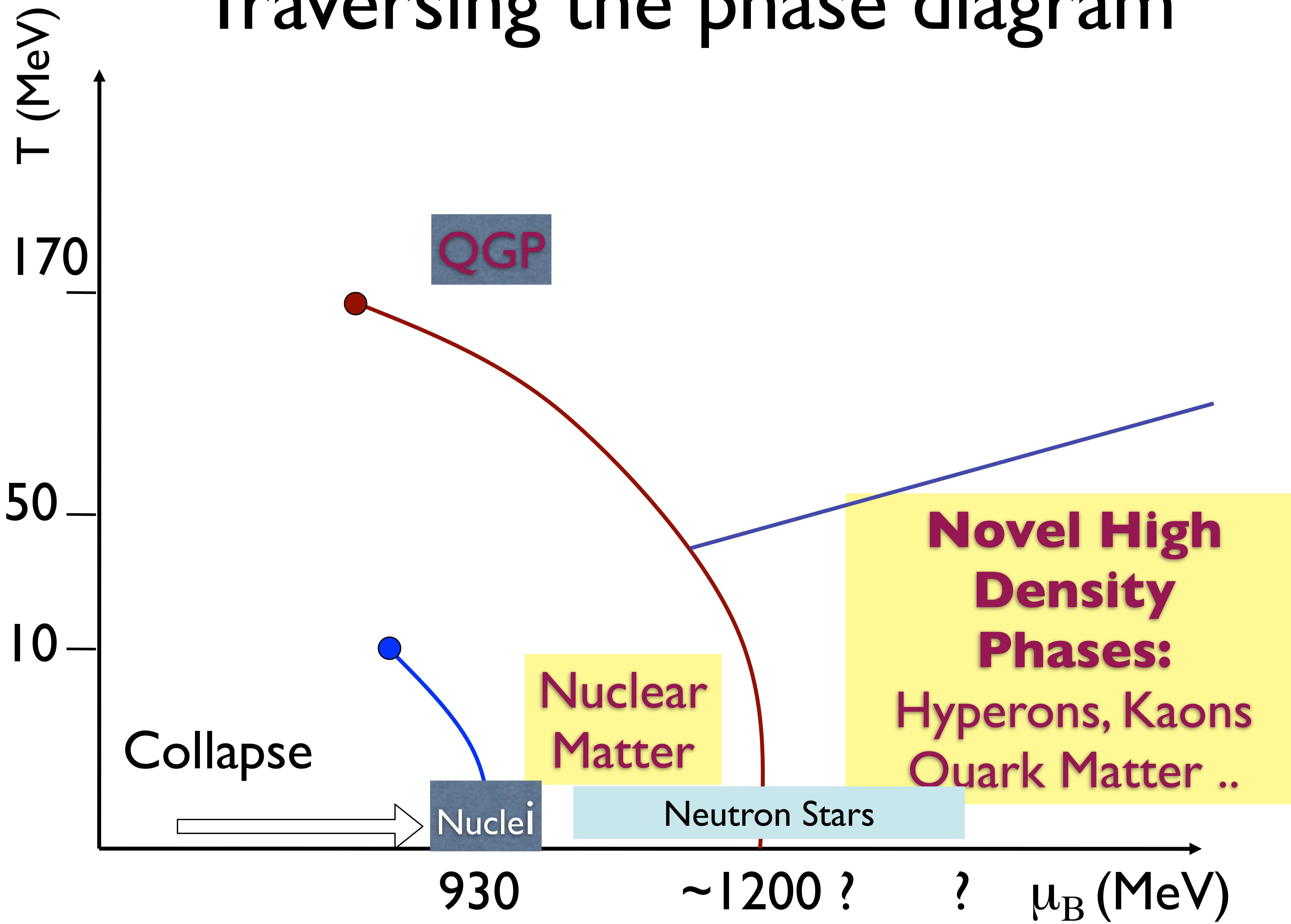


- Neutrinos are trapped during core collapse. Collapse is nearly adiabatic.
- Gravitational binding energy is stored as thermal energy and lepton degeneracy energy.

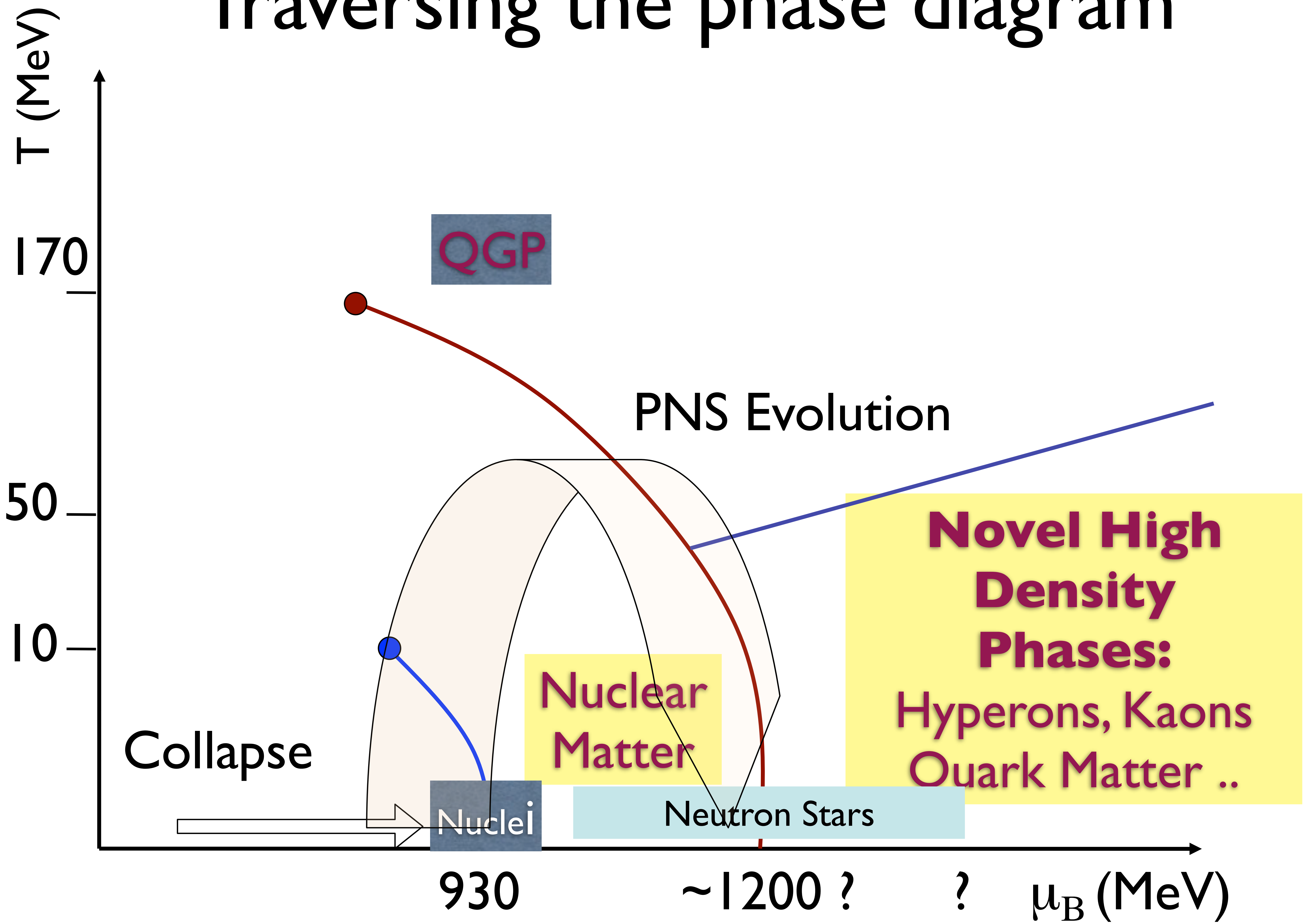
Traversing the phase diagram



Traversing the phase diagram

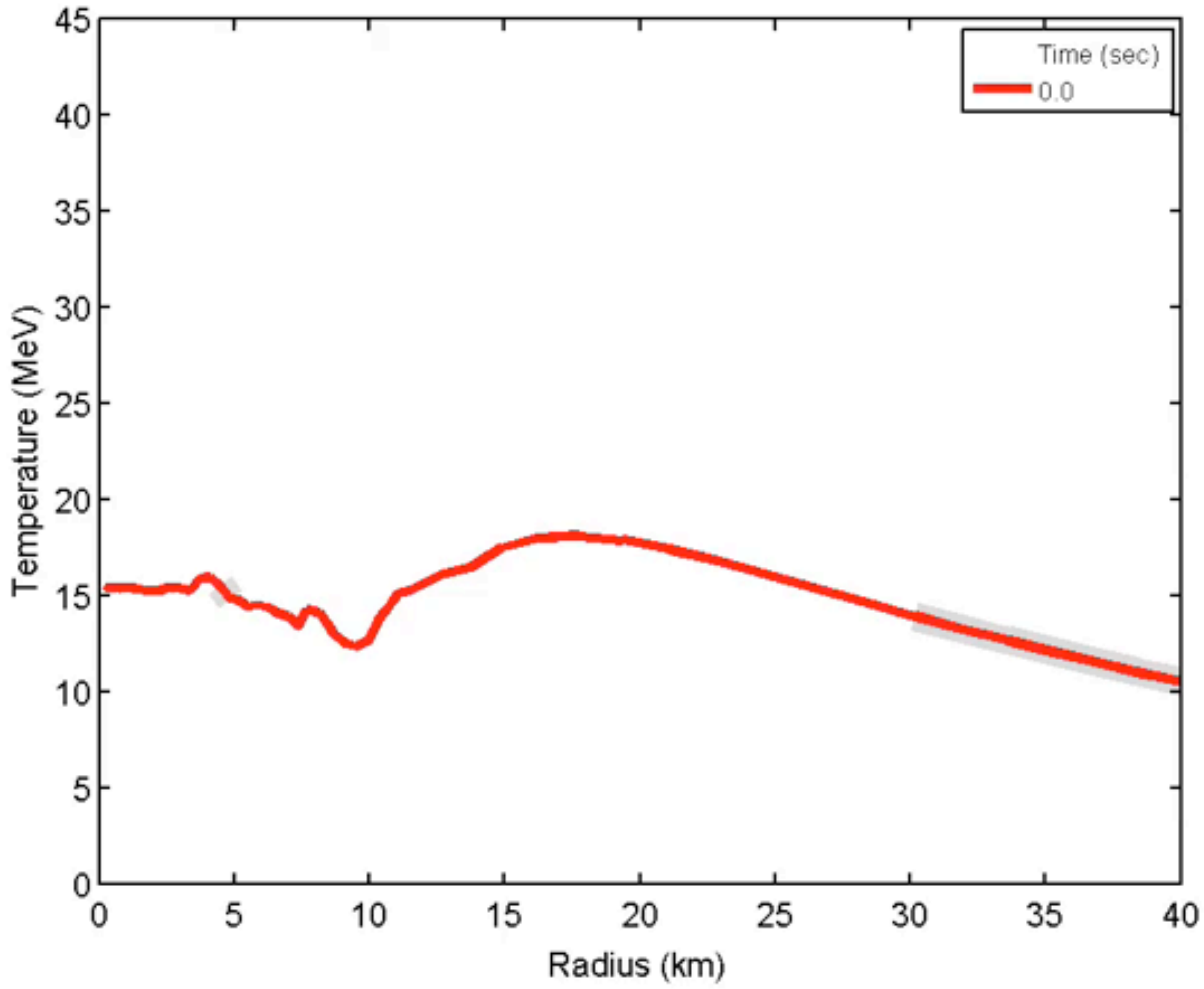


Traversing the phase diagram



Protoneutron Star Evolution

Neutrino diffusion cools the PNS.



Pons, Reddy, Lattimer & Prakash (1999)

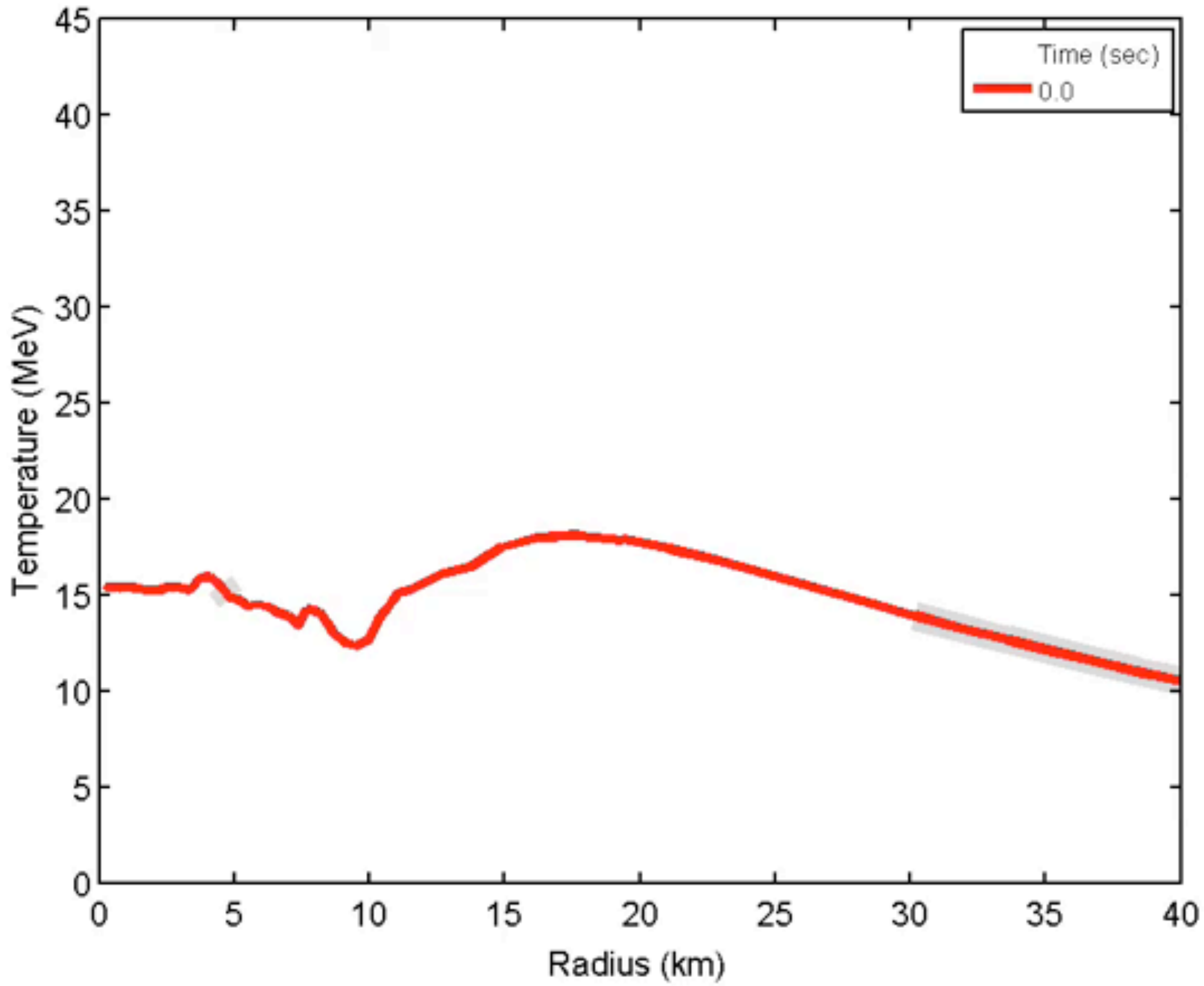
Protoneutron Star Evolution

Neutrino diffusion cools the PNS.

Typical time-scales:

$$T(t) \approx T(t=0)\left(1-\frac{t}{\tau_C}\right)$$

$$\tau_C \approx C_V \frac{R^2}{c\langle\lambda_\nu\rangle}$$



Pons, Reddy, Lattimer & Prakash (1999)

Neutron Star Tomography

- Time structure of the neutrino signal maps the neutrino opacity as a function of depth.
- Opacity is directly related to spectrum of density and spin fluctuations in dense matter.
- Important to note that several other astrophysical effects can complicate this simple interpretation.

Late Time Cooling in X-Rays

- Cooling of isolated neutron stars.

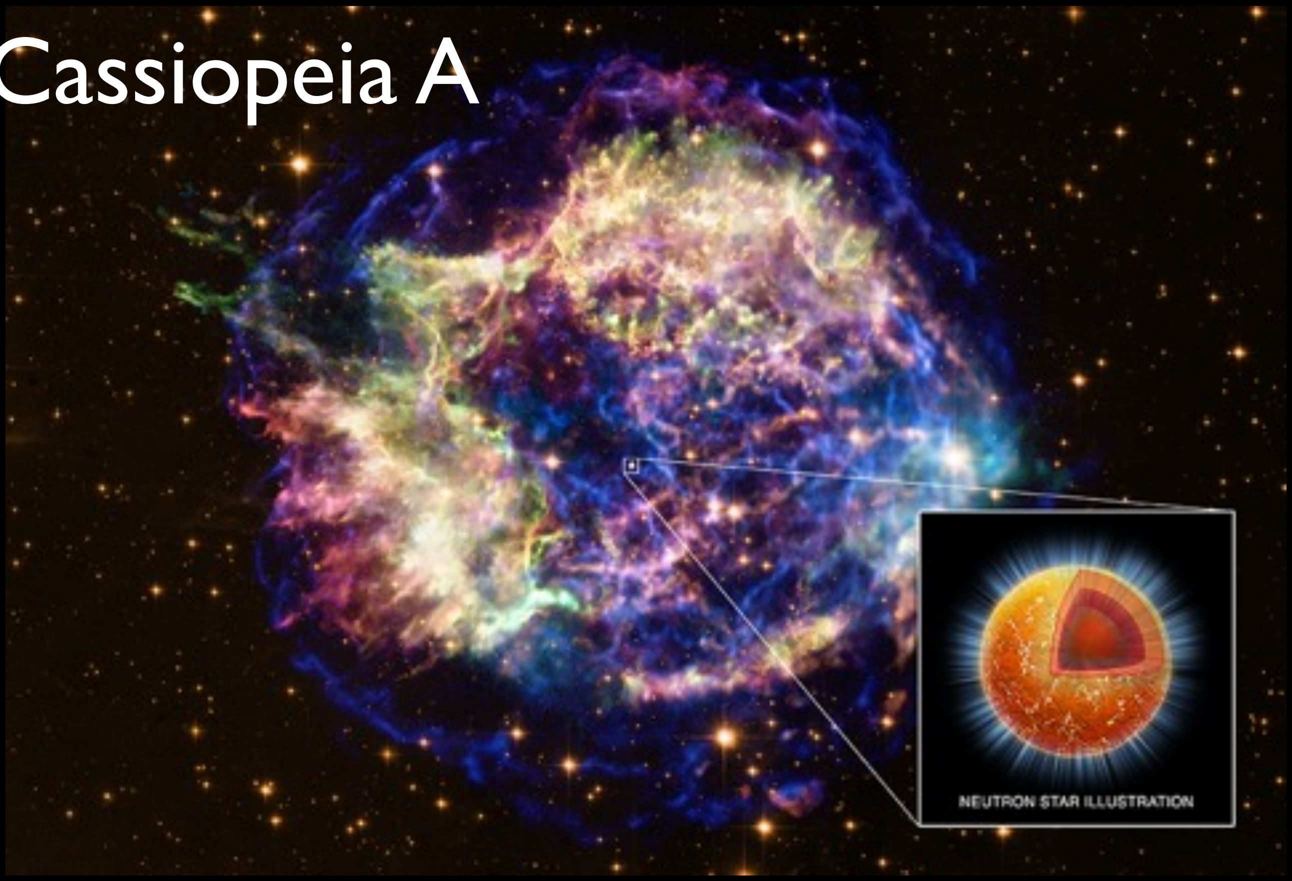
Shternin et al (2011) Page et al. (2011)

- Thermal relaxation of accreting neutron stars.

Brown & Cumming (2007) Shternin et al (2007)

Neutron Star in a Supernova Remnant

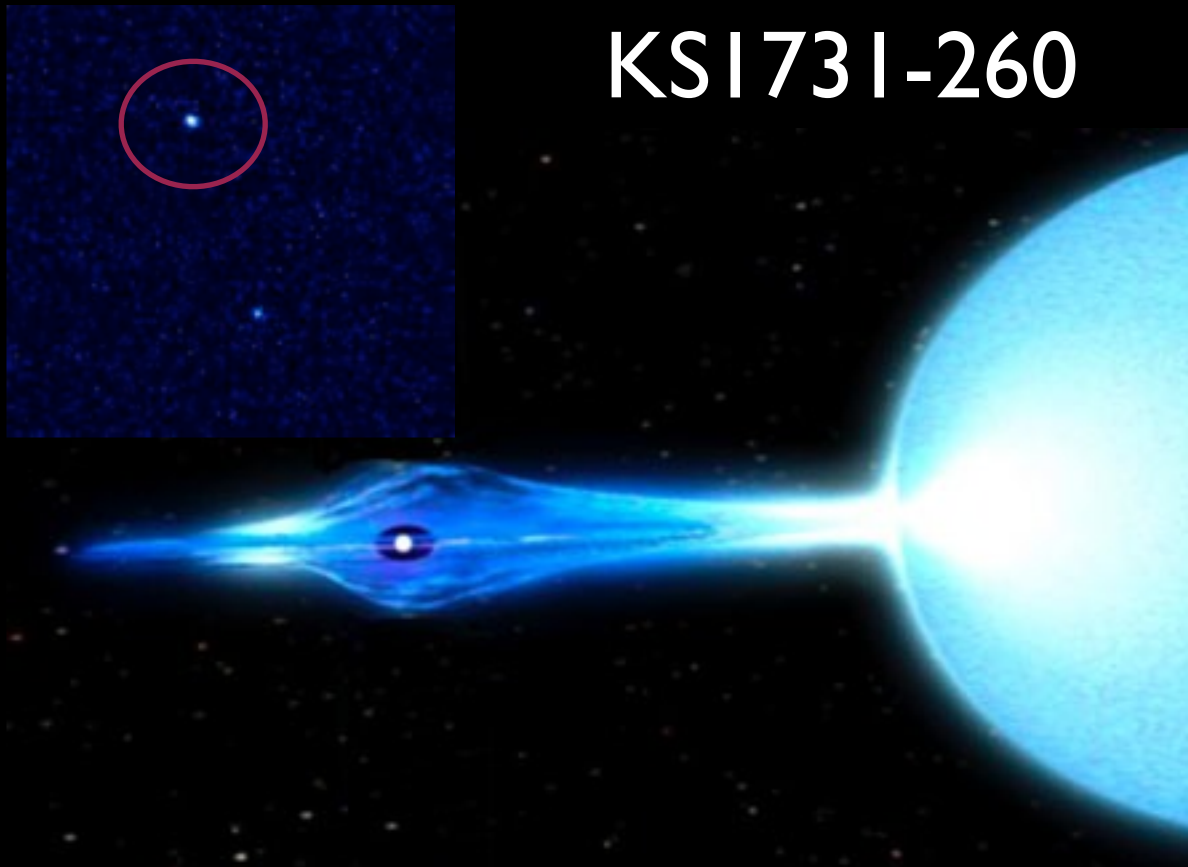
Cassiopeia A



chandra.harvard.edu/photo/2011/casa/

Neutron star in X-ray Binary

KS1731-260

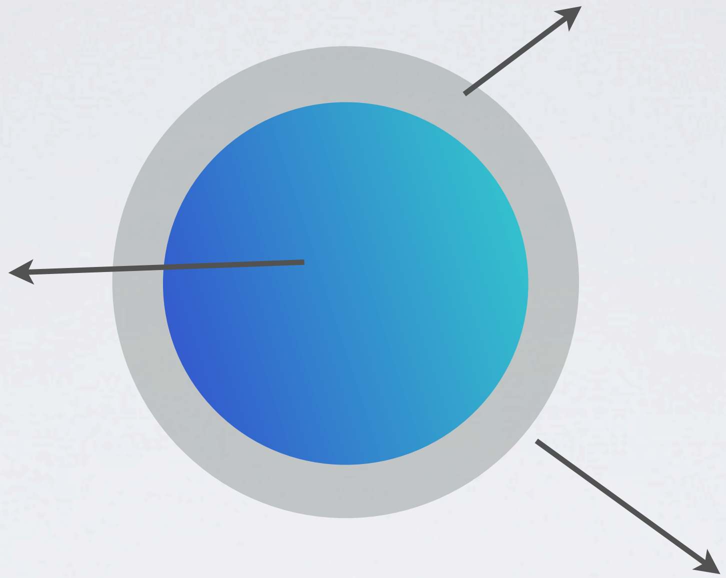


chandra.harvard.edu/photo/2001/ks1731/

NEUTRON STAR COOLING

Crust cools by conduction

Isothermal core cools by neutrino emission



Surface photon emission dominates at late time t > 10^6 yrs

Basic neutrino reactions:

$$\begin{aligned} n &\rightarrow p + e^- + \bar{\nu}_e \\ e^- + p &\rightarrow n + \nu_e \end{aligned}$$

Fast: Direct URCA

$$\begin{aligned} n + n &\rightarrow n + p + e^- + \bar{\nu}_e \\ e^- + p + n &\rightarrow n + n + \nu_e \end{aligned}$$

Slow: Modified URCA

Table 1. Dominant neutrino emission processes.

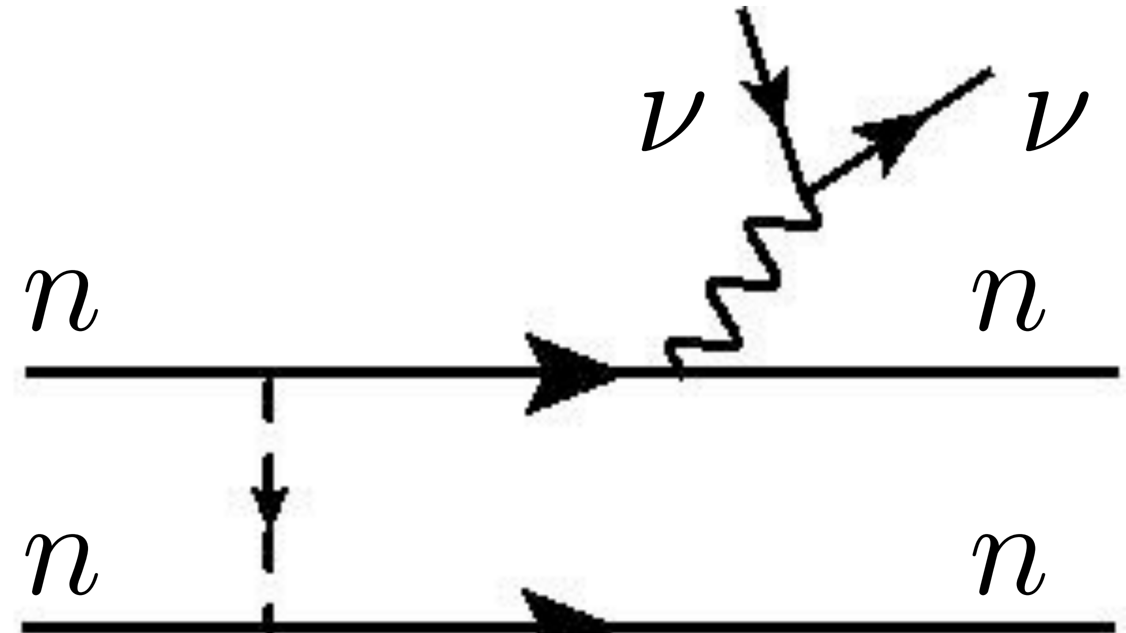
Name	Process	Emissivity[†] (erg cm^{-3} s^{-1})	Efficiency
Modified Urca cycle (neutron branch)	$n + n \to n + p + e^- + \bar{\nu}_e$ $n + p + e^- \to n + n + \nu_e$	$\sim 2{\times}10^{21}\ R\, T_9^8$	Slow
Modified Urca cycle (proton branch)	$p + n \to p + p + e^- + \bar{\nu}_e$ $p + p + e^- \to p + n + \nu_e$	$\sim 10^{21}\ R\, T_9^8$	Slow
Bremsstrahlung	$n + n \to n + n + \nu + \bar{\nu}$ $n + p \to n + p + \nu + \bar{\nu}$ $p + p \to p + p + \nu + \bar{\nu}$	$\sim 10^{19}\ R\, T_9^8$	Slow
Cooper pair formations	$n + n \to [nn] + \nu + \bar{\nu}$ $p + p \to [pp] + \nu + \bar{\nu}$	$\sim 5{\times}10^{21}\ R\, T_9^7$ $\sim 5{\times}10^{19}\ R\, T_9^7$	Medium
Direct Urca cycle (nucleons)	$n \to p + e^- + \bar{\nu}_e$ $p + e^- \to n + \nu_e$	$\sim 10^{27}\ R\, T_9^6$	Fast
Direct Urca cycle (Λ hyperons)	$\Lambda \to p + e^- + \bar{\nu}_e$ $p + e^- \to \Lambda + \nu_e$	$\sim 10^{27}\ R\, T_9^6$	Fast
Direct Urca cycle (Σ^- hyperons)	$\Sigma^- \to n + e^- + \bar{\nu}_e$ $n + e^- \to \Sigma^- + \nu_e$	$\sim 10^{27}\ R\, T_9^6$	Fast
π^- condensate	$n + < \pi^- > \to n + e^- + \bar{\nu}_e$	$\sim 10^{26}\ R\, T_9^6$	Fast
K^- condensate	$n + < K^- > \to n + e^- + \bar{\nu}_e$	$\sim 10^{25}\ R\, T_9^6$	Fast
Direct Urca cycle (u-d quarks)	$d \to u + e^- + \bar{\nu}_e$ $u + e^- \to d + \nu_e$	$\sim 10^{27}\ R\, T_9^6$	Fast
Direct Urca cycle (u-s quarks)	$s \to u + e^- + \bar{\nu}_e$ $u + e^- \to s + \nu_e$	$\sim 10^{27}\ R\, T_9^6$	Fast

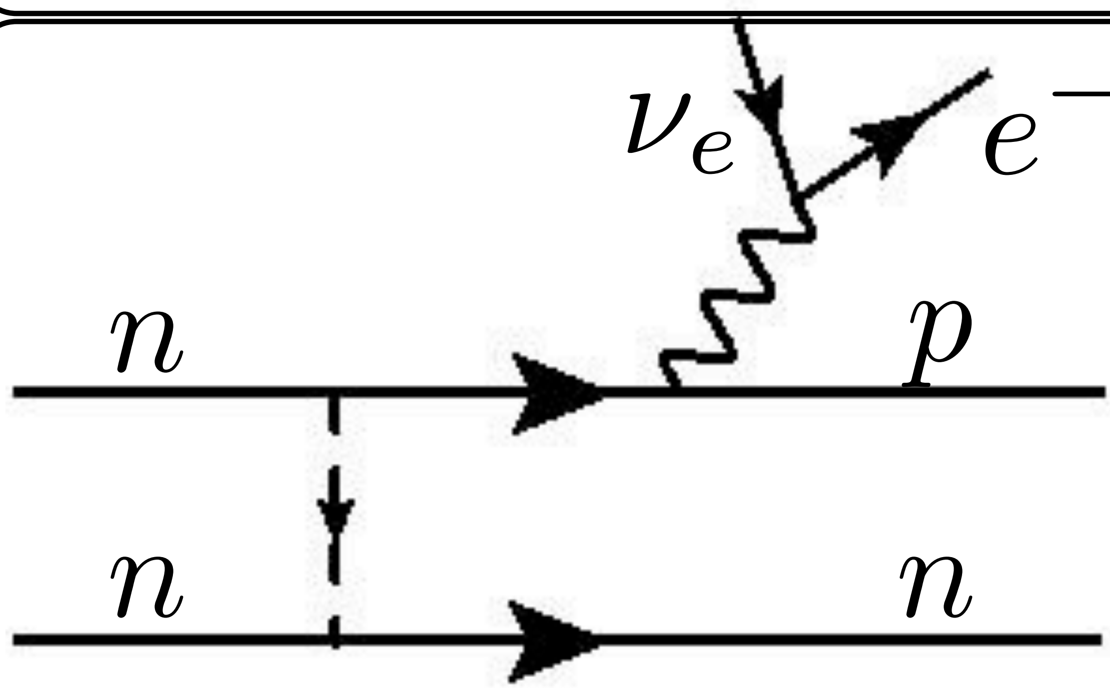
Page (2012)

Standard Cooling

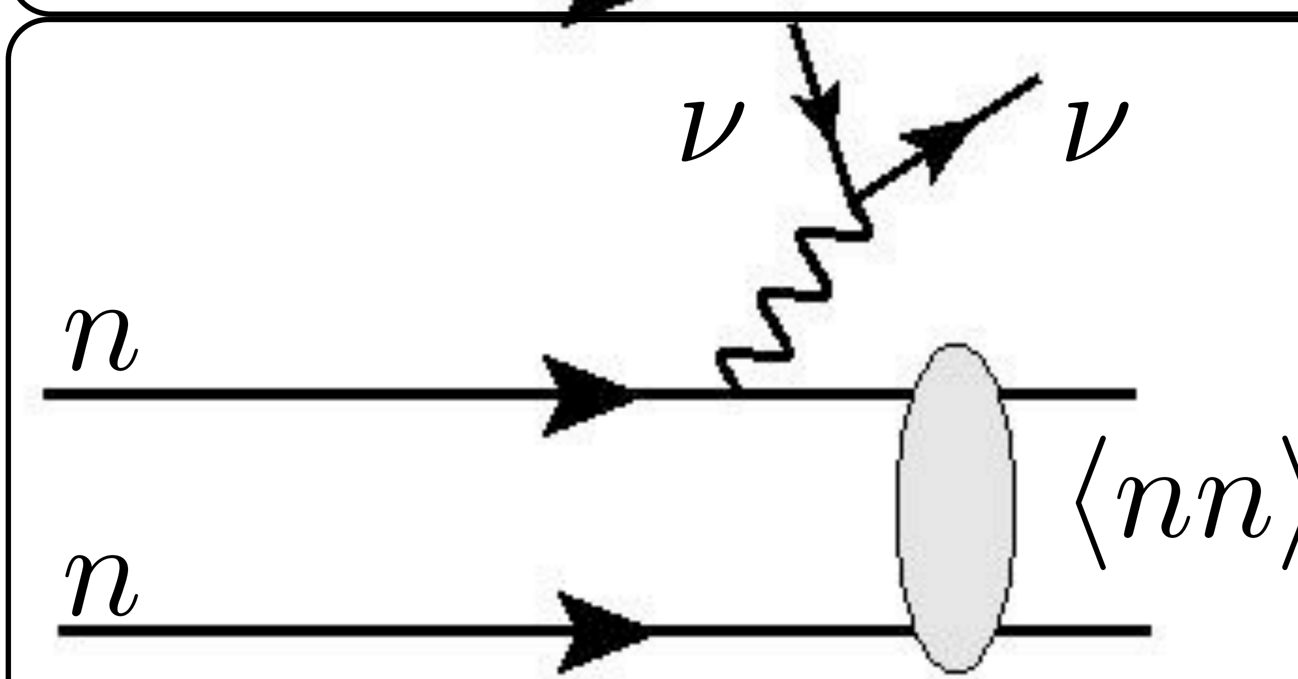
$\dot{\epsilon} \approx e^{-2\Delta/T}\ 10^{27}\ T_9^6\ \text{erg cm}^{-3}\text{s}^{-1}$

$\dot{\epsilon} \approx e^{-2\Delta/T}\ 10^{19}\ T_9^8\ \text{erg cm}^{-3}\text{s}^{-1}$





$\dot{\epsilon} \approx e^{-2\Delta/T}\ 10^{21}\ T_9^8\ \text{erg cm}^{-3}\text{s}^{-1}$



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Standard Cooling

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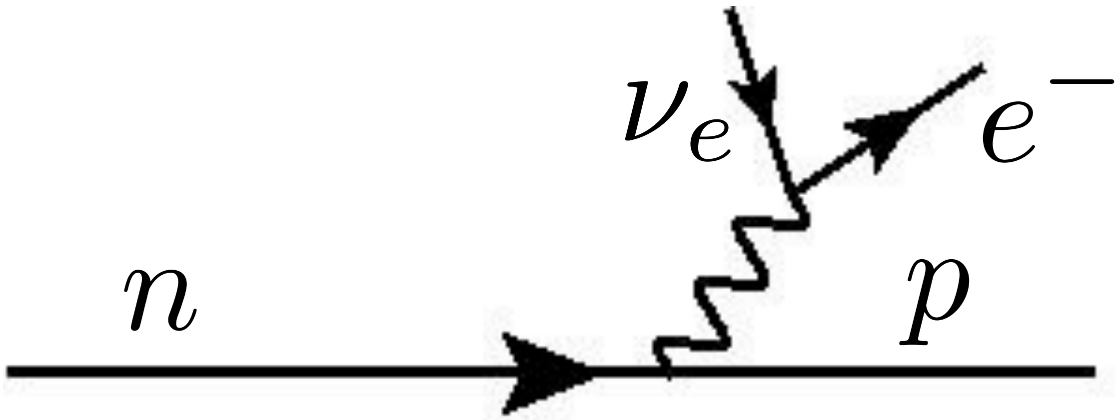

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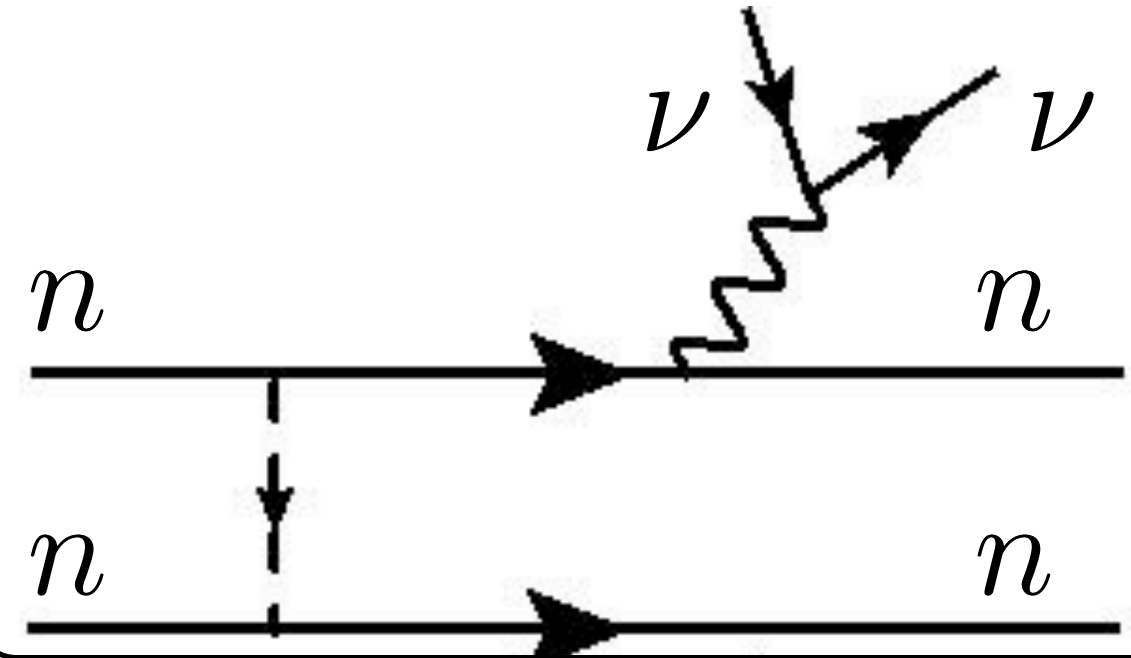

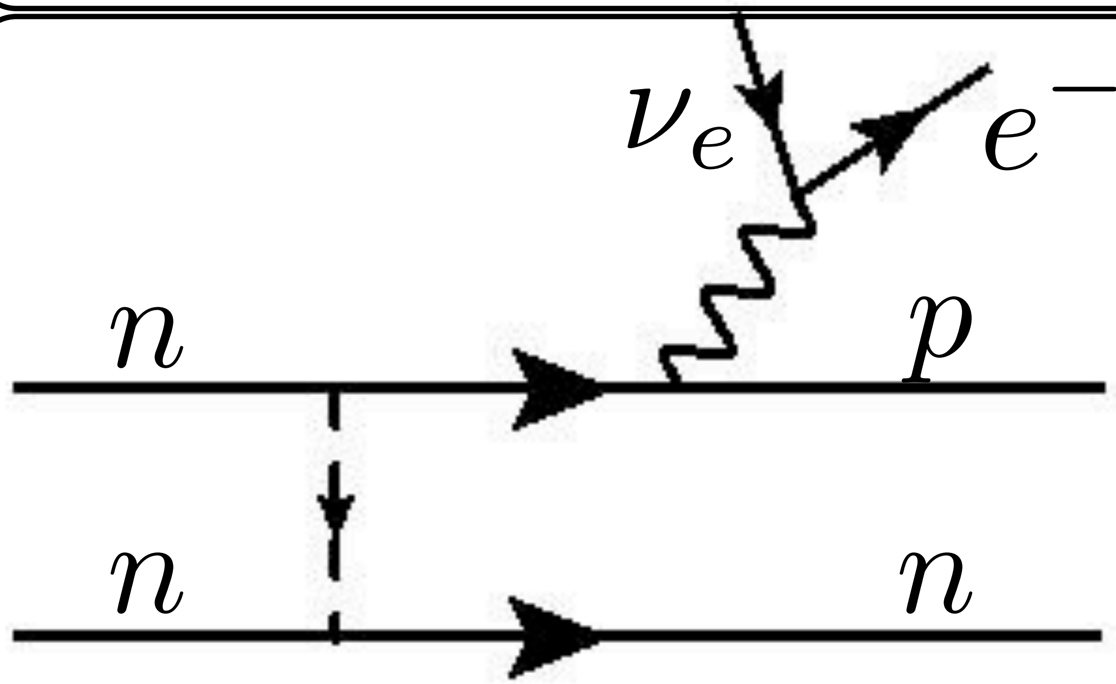

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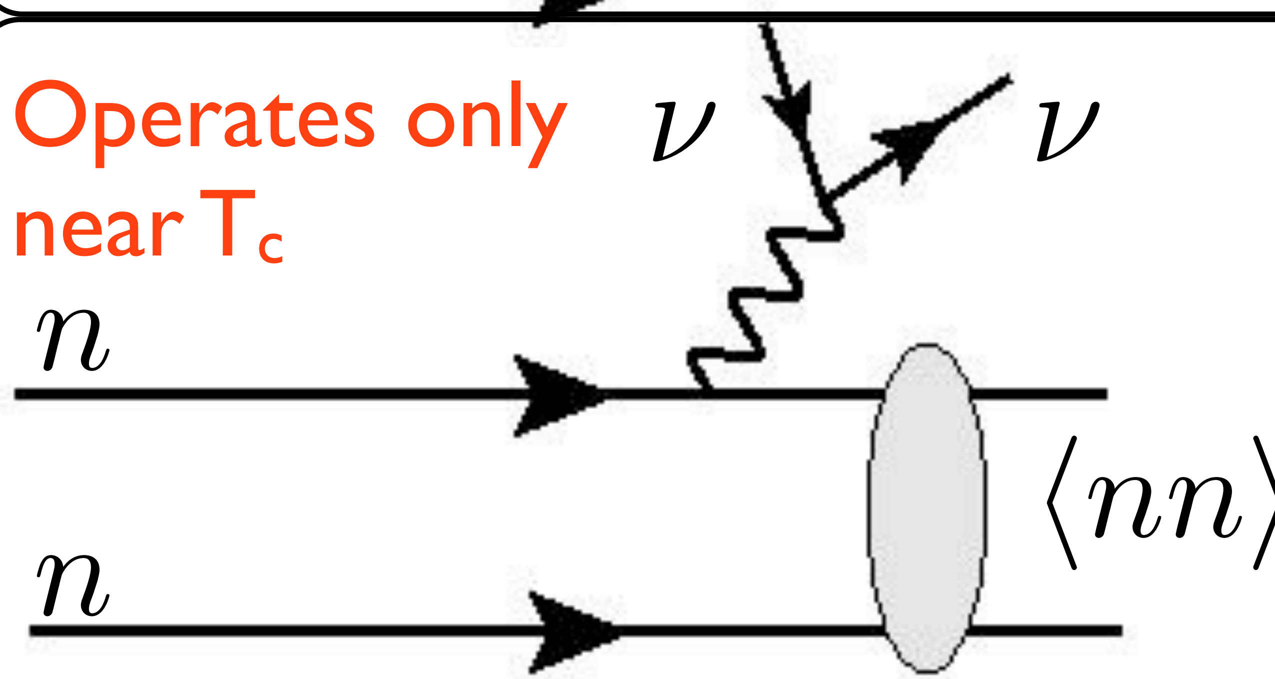
Operates only near T_c



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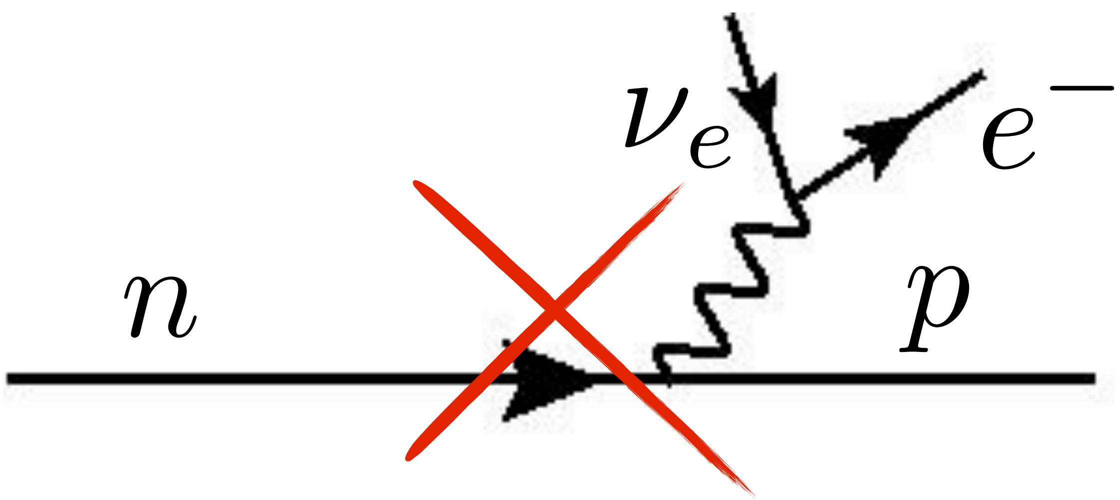

Kinematically Forbidden

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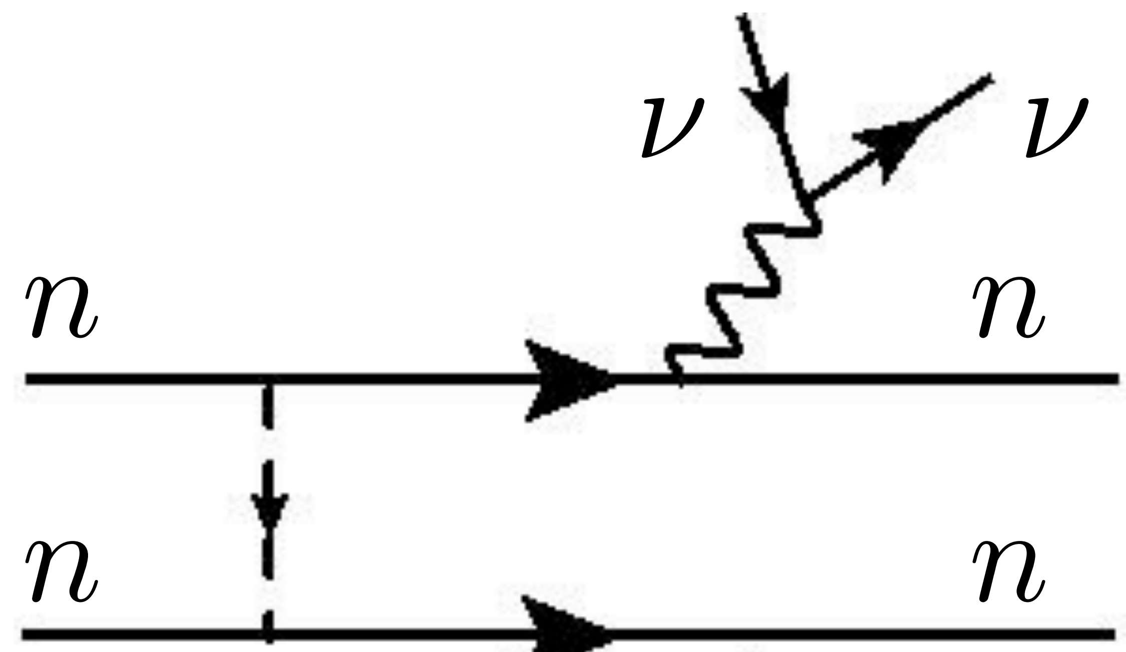

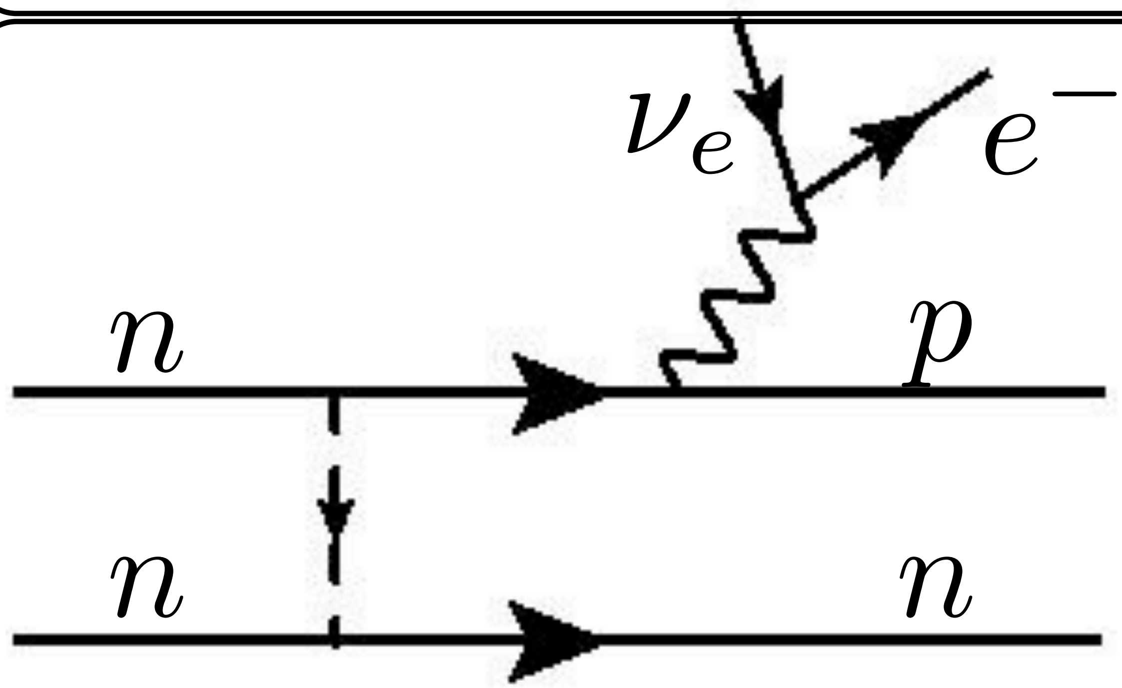

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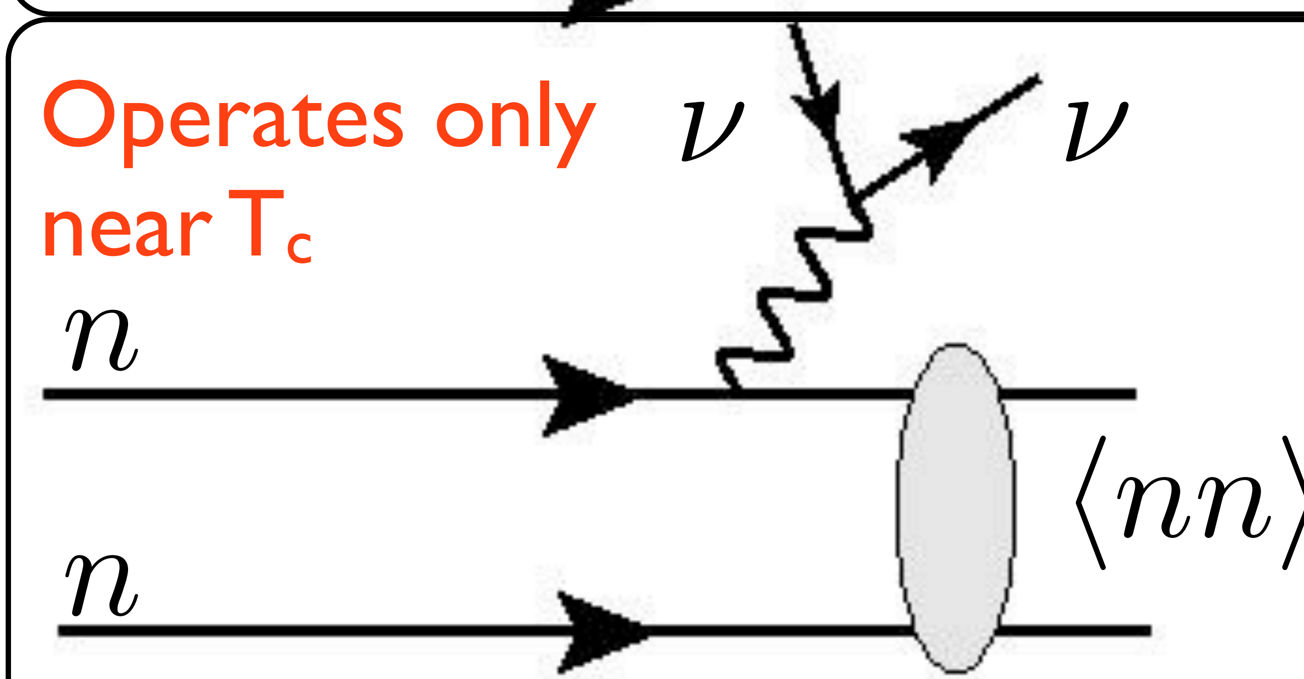

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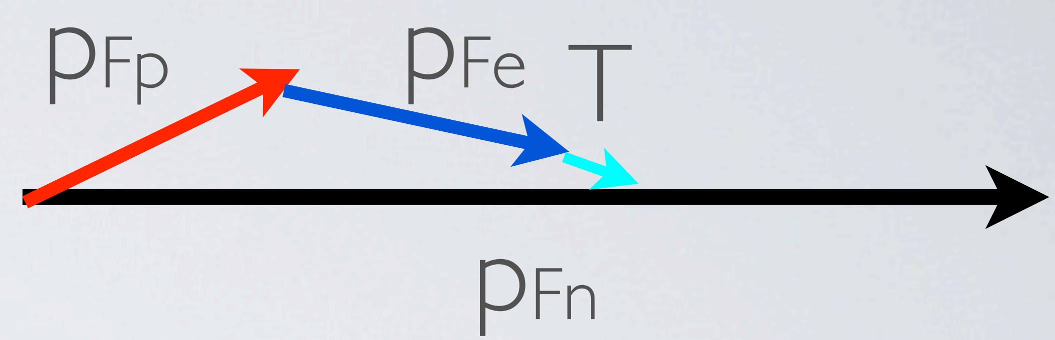
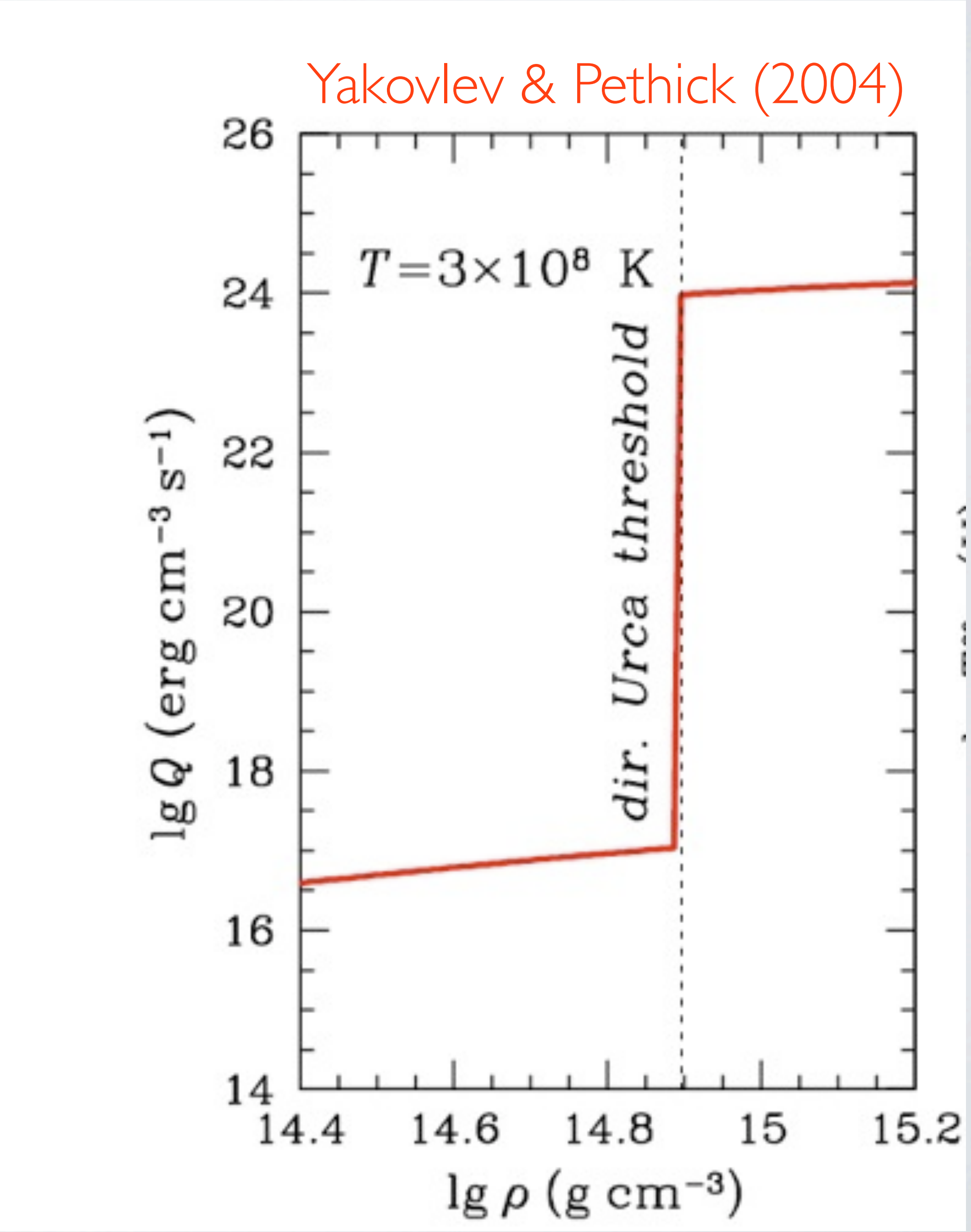
COOLING AND EOS

Neutron decay at the Fermi surface cannot conserve momentum if

$x_p \sim (p_{Fp}/p_{Fn})^3 < 0.12\text{-}14$



- In the standard scenario only massive stars ($M \sim 2\,M_{\odot}$) cool rapidly.



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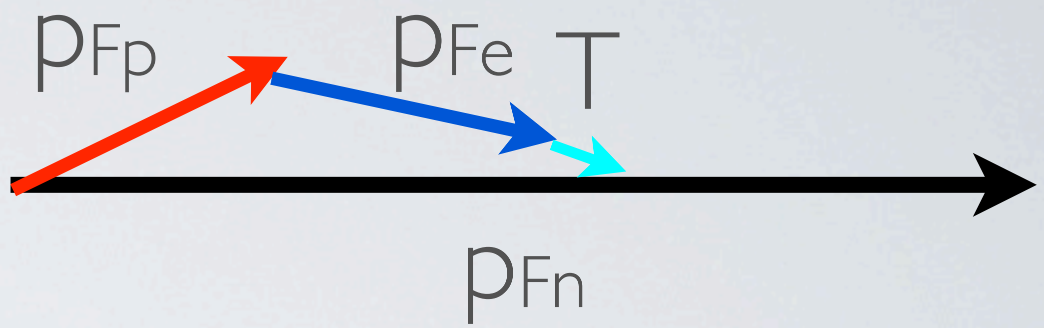
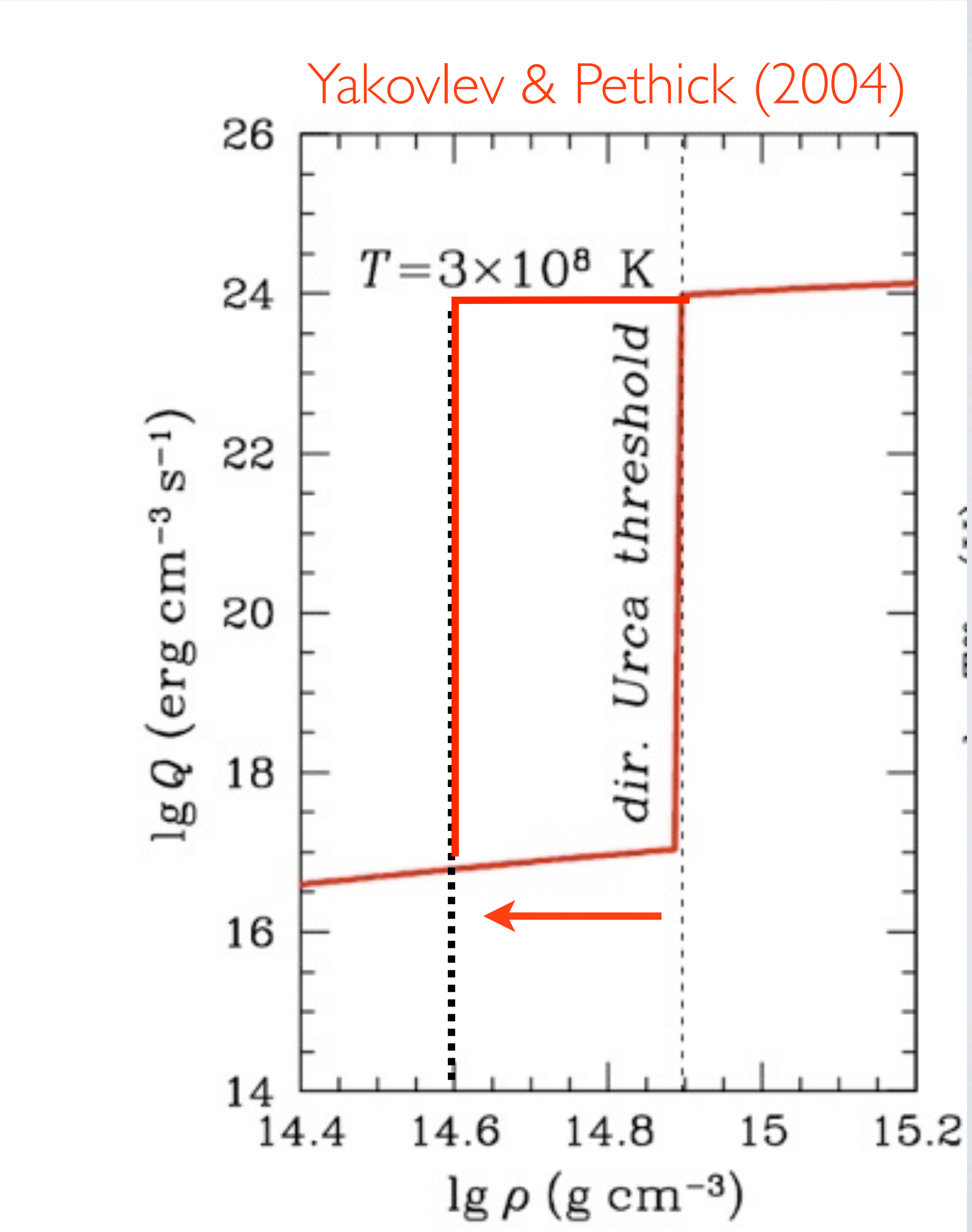


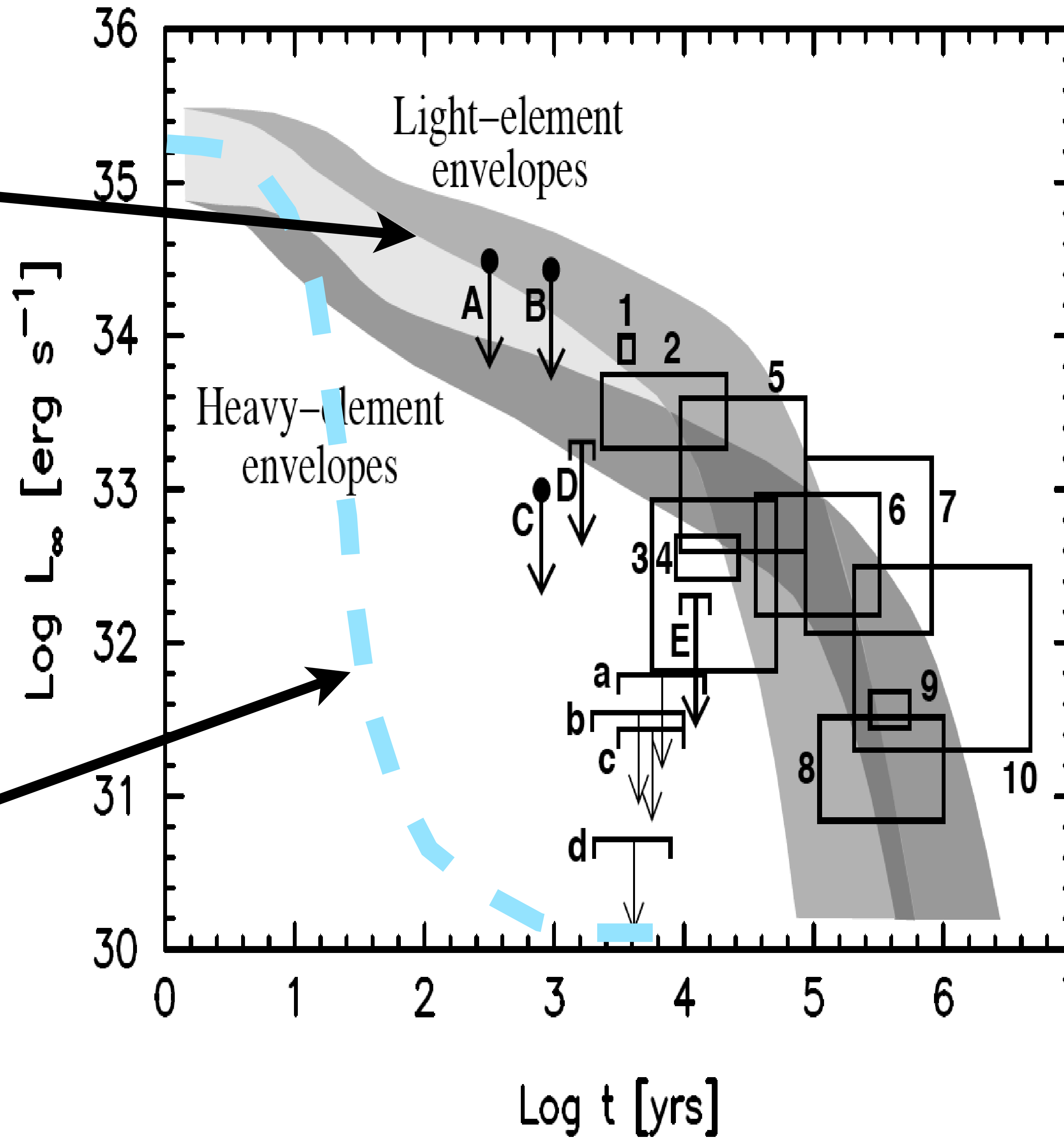
- In the standard scenario only massive stars (M ~ 2 M☉) cool rapidly.
- A large symmetry energy will allow direct URCA for typical NS (M ~ 1.4 M☉) .
- Recall a large symmetry energy also favors large radii.



$$\frac{dE_{\rm th}}{dt} \equiv C_V \frac{dT}{dt} = -L_\nu - L_\gamma$$

Standard or Slow Cooling

Rapid Cooling



Page et al (2009,2010)

Pairing

I. Too hot for electron pairing:

$$T_c \approx \omega_p^{\text{ion}} \ \exp\left(-\frac{v_{Fe}}{\alpha_{\text{em}}}\right)$$

Ginzburg (1969)

Relativistic electrons move too quickly to feel the phonon induced attraction.

II. Pairing between nucleons is inevitable.

$$T_c \approx E_{Fn} \ \exp\left(-\frac{\pi}{2k_{Fn} \ a_{\text{nn}}}\right)$$

Bohr, Mottelson, Pines (1958)
Migdal (1959)

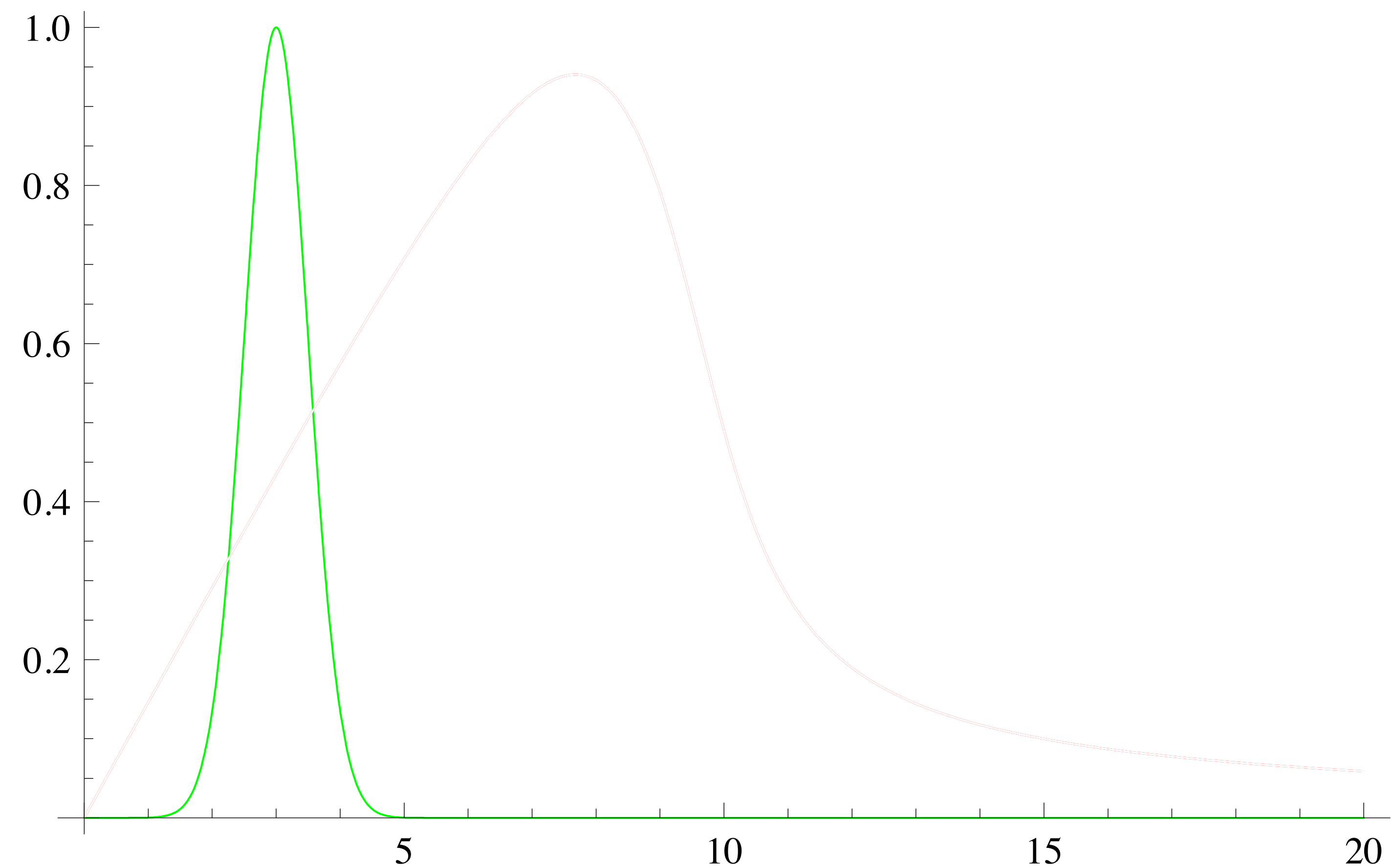
Typical energy scale is MeV (~10^{10} K)

Recall Response Function in Superfluid !

$$S_q(\omega) = \sum_{\lambda,\lambda'} f_\lambda \, |\langle \lambda | A_q | \lambda' \rangle|^2 \, \delta(E_\lambda - E'_\lambda - \omega)$$

$$= \int dt \, e^{i\omega t} \langle \langle \lambda | A_q(t) \; A_q^\dagger(0) | \lambda \rangle \rangle$$

$S_q(\omega)$



$$\omega = \frac{q^2}{2M} + \frac{\vec{p}.\vec{q}}{M}$$

ω

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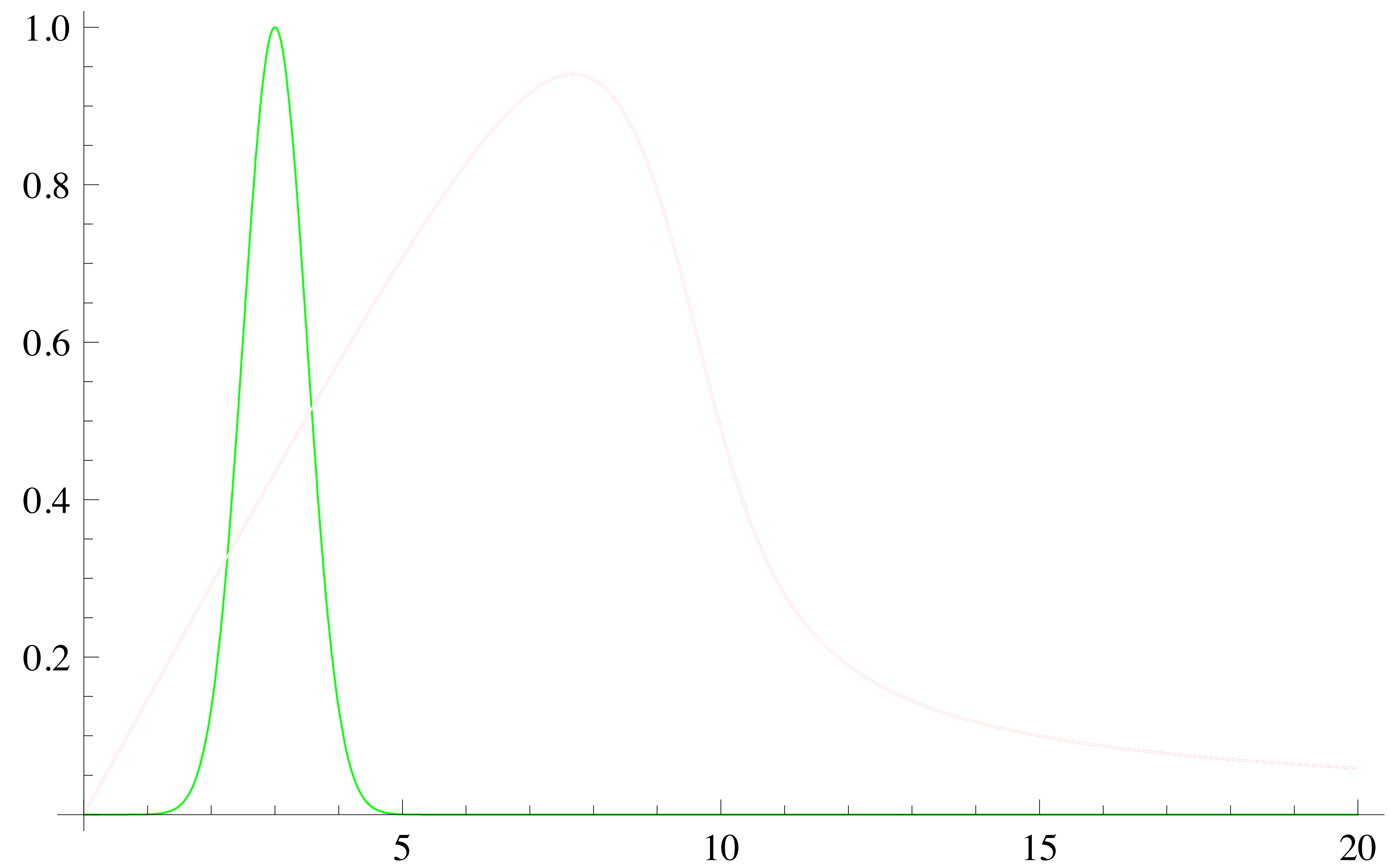
Fermi Motion and
Pauli Blocking.

$S_q(\omega)$



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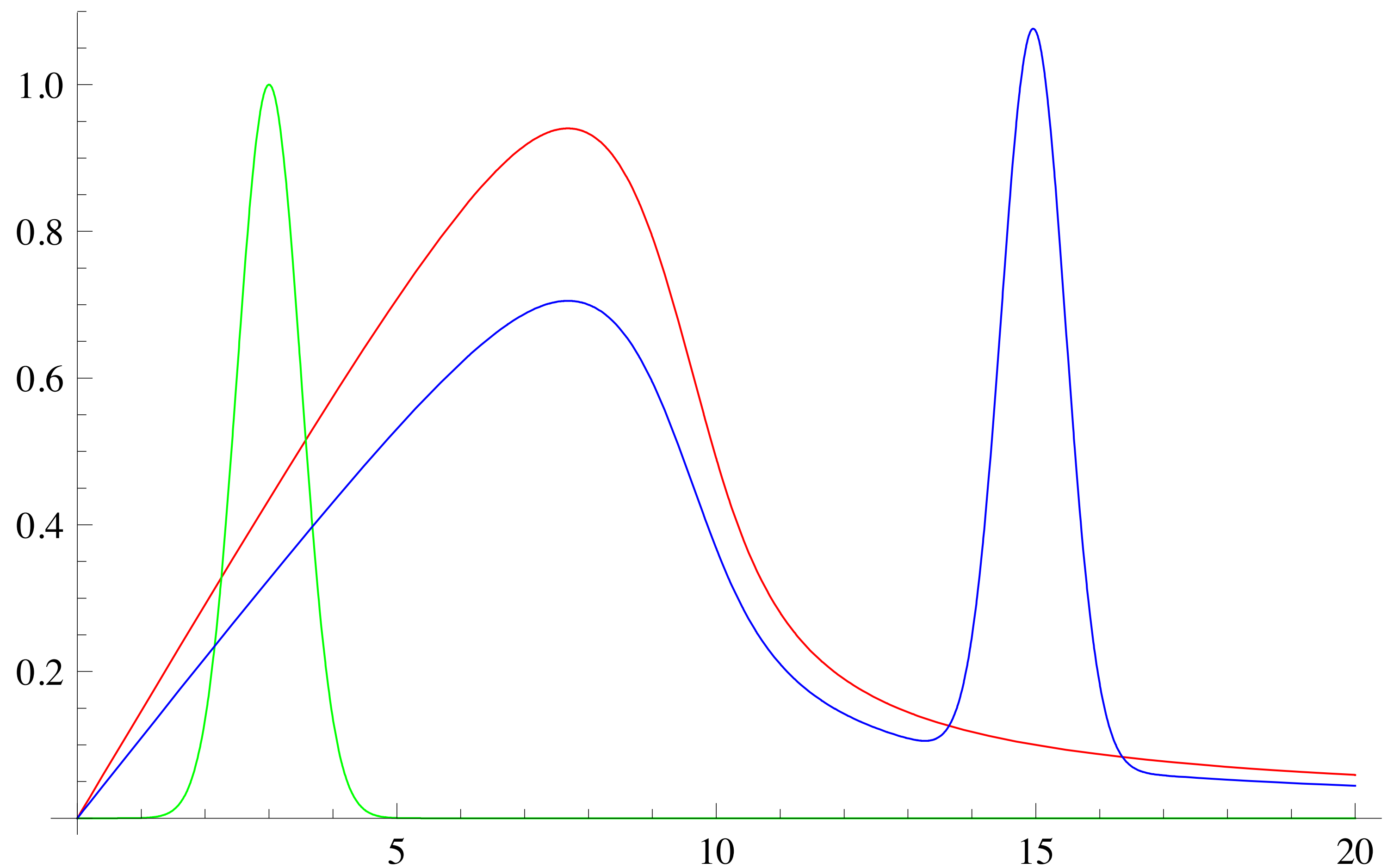
Fermi Motion and Pauli Blocking.

Correlations and collisions.

$S_q(\omega)$



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Fermi Motion and Pauli Blocking.

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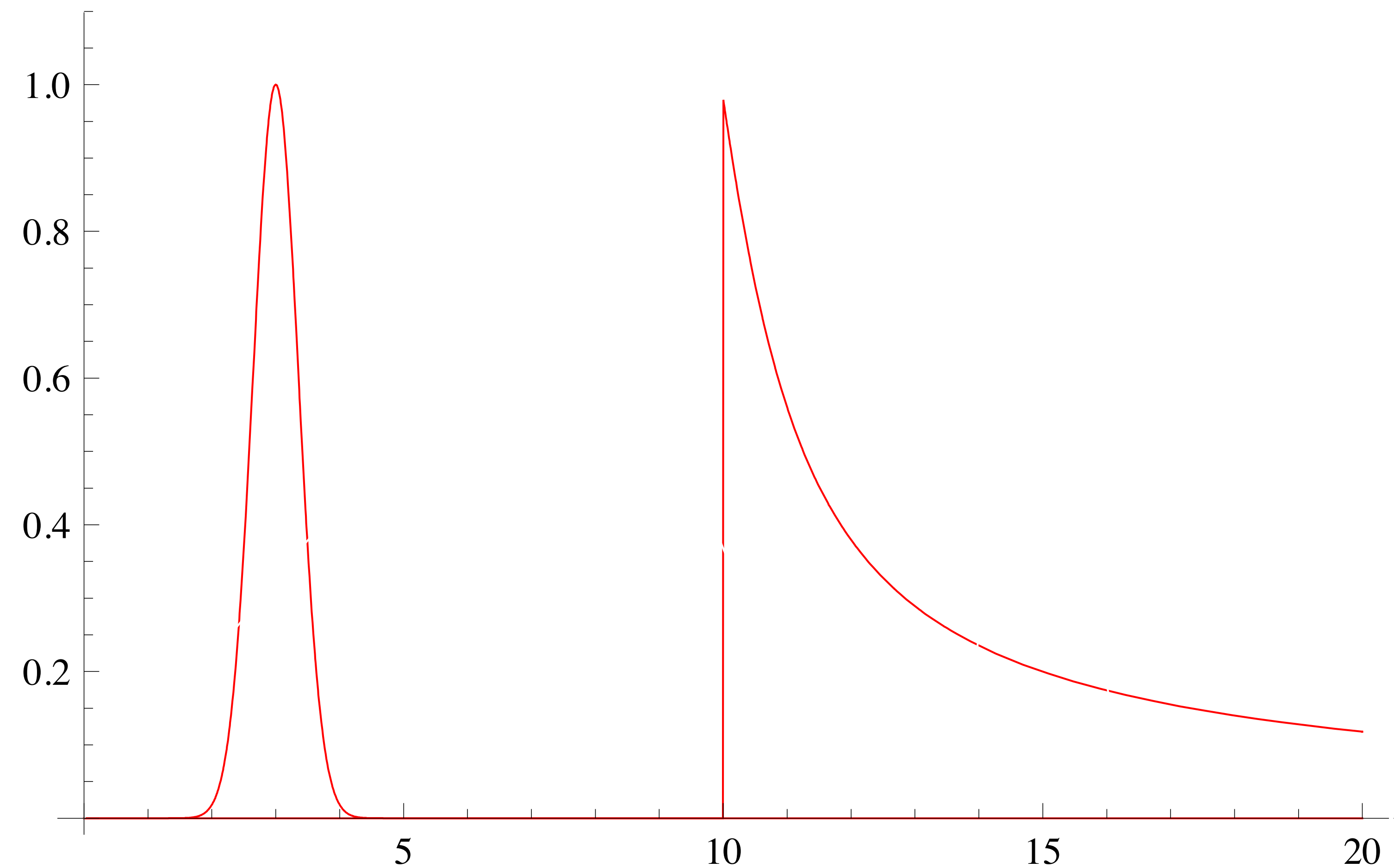
Cooper Pairing & Superfluidity.

$S_q(\omega)$



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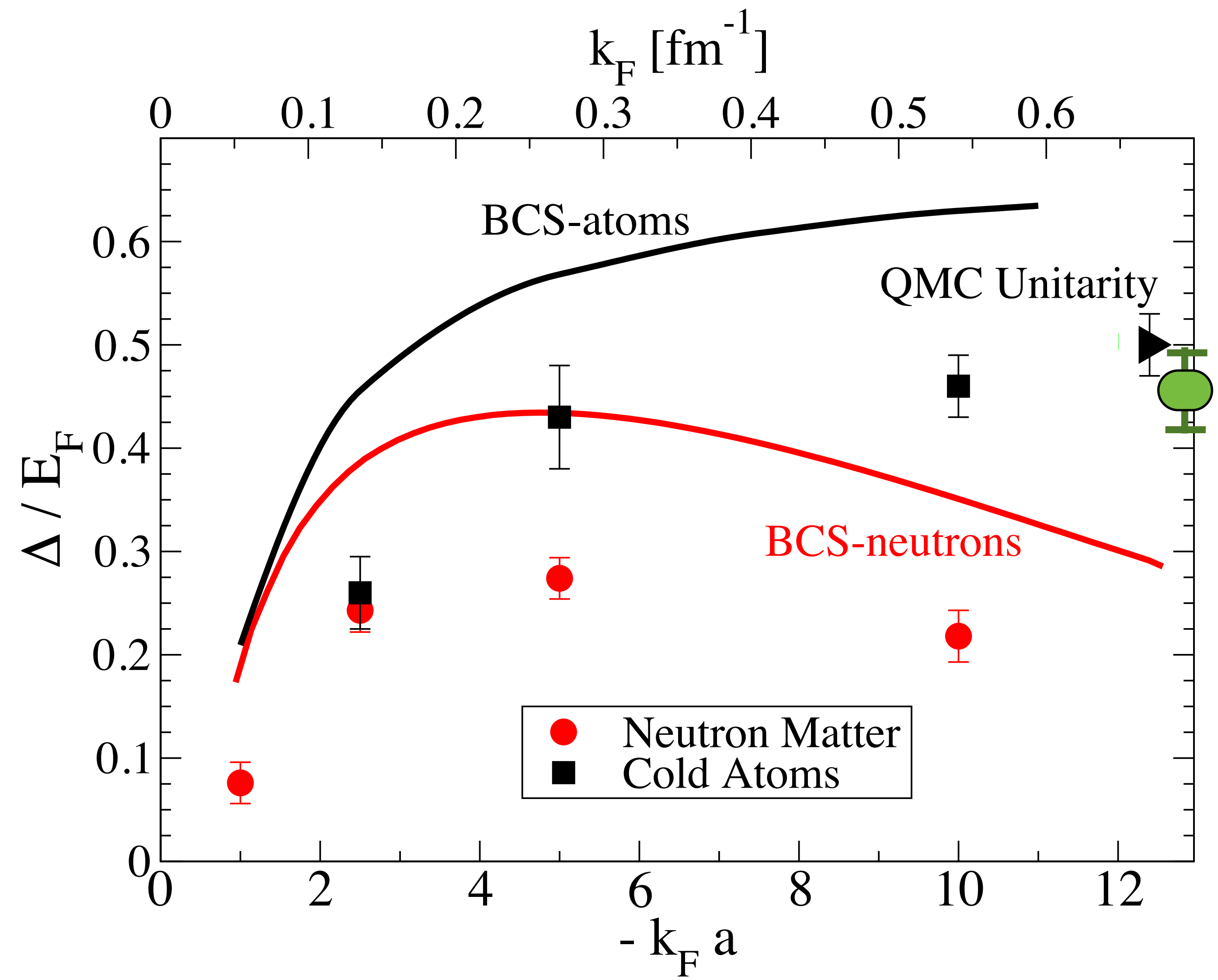
S-wave pairing

Gezerlis & Carlson (2008)

- The nucleon-nucleon interaction is known up to relative momenta ~ 350 MeV.
- Perturbation theory fails, but Quantum Monte Carlo and lattice methods may be reliable.
- Best estimates for the gap indicate that it reaches a maximum value ~ 1 MeV in the crust.



Cold atom experiments help validate many-body theory of strong short-range interactions.

Bulgac, Carlson, Drut, Gandolfi,

Forbes, Kaplan ...

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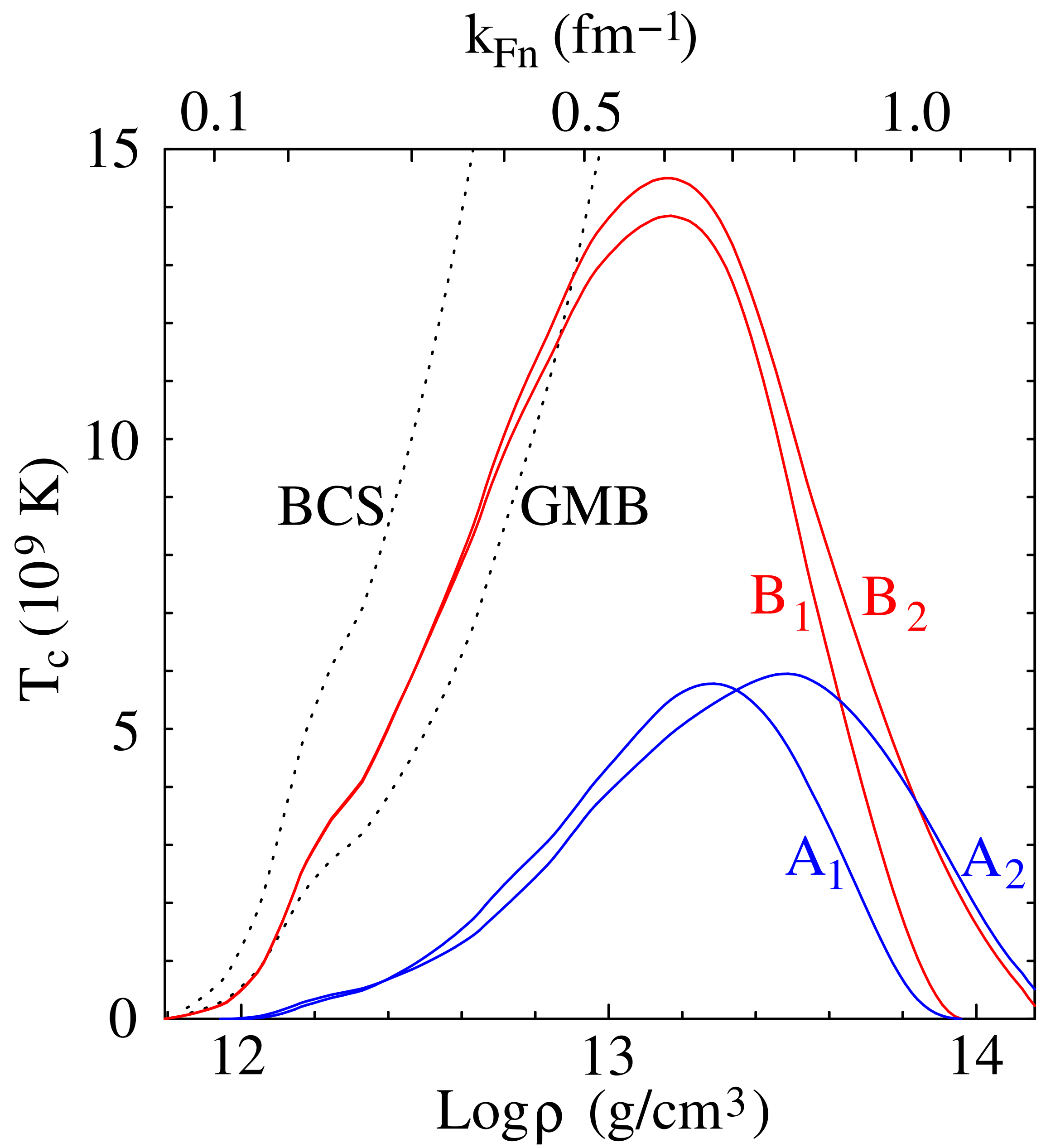
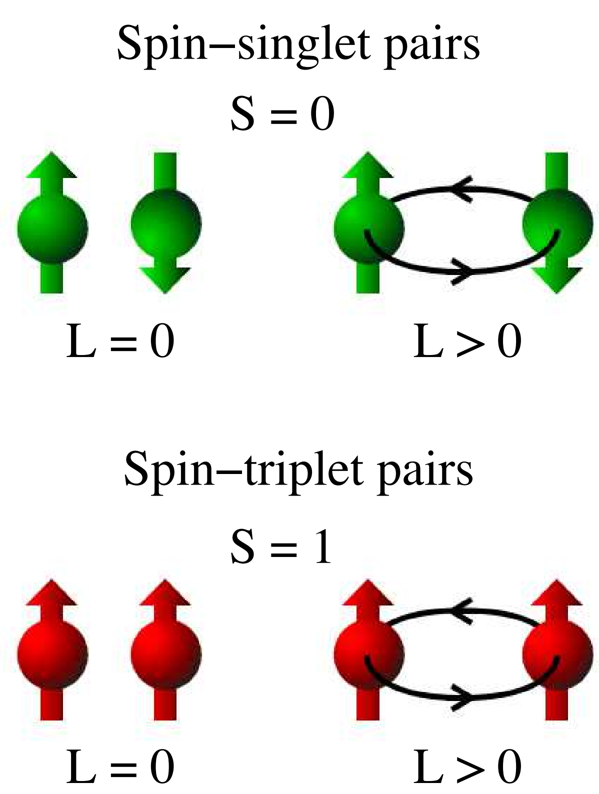
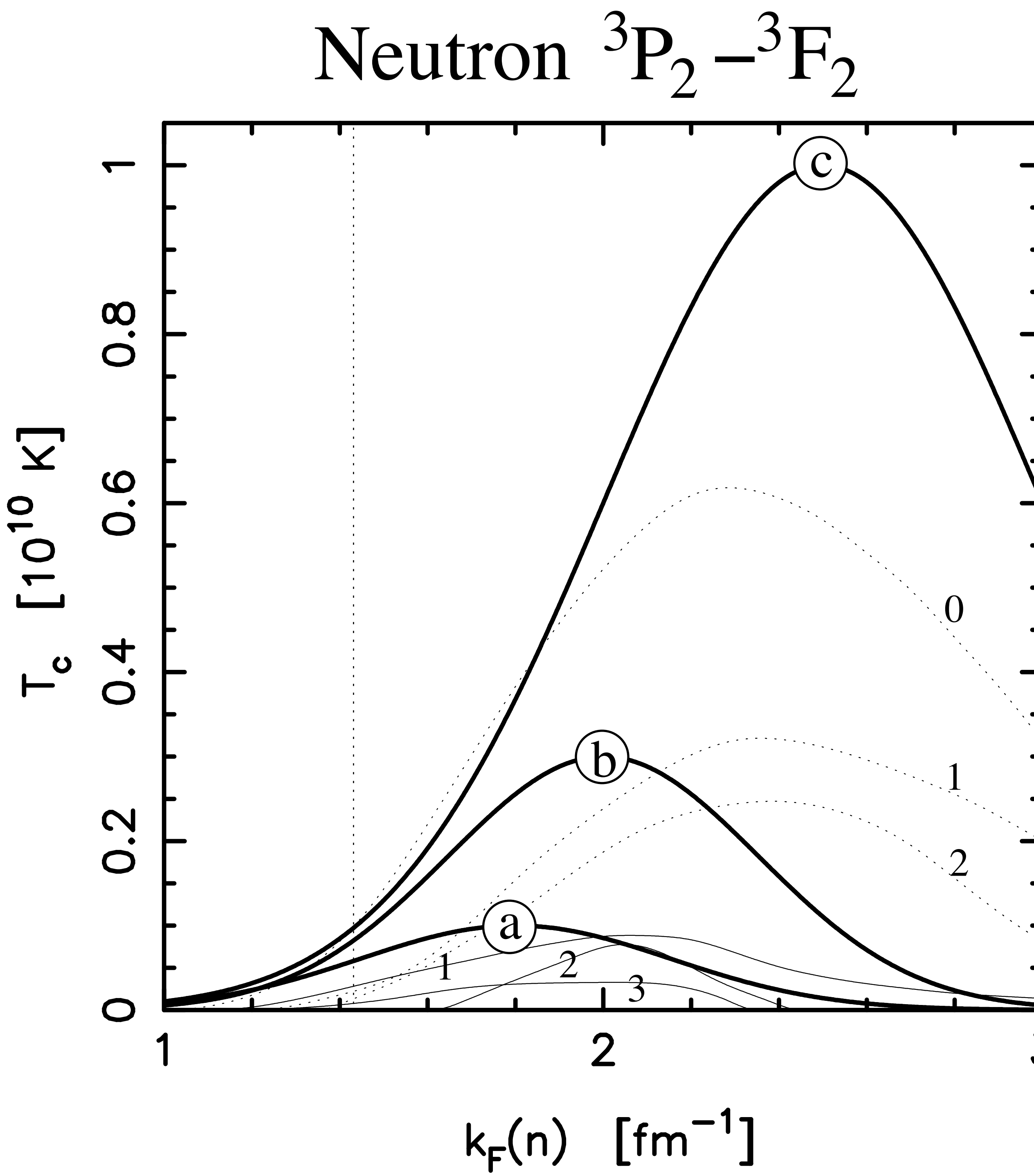


P-wave Triplet Pairing



S-wave interaction is repulsive at high density. Attraction in spin 1 channel due to P-wave interaction

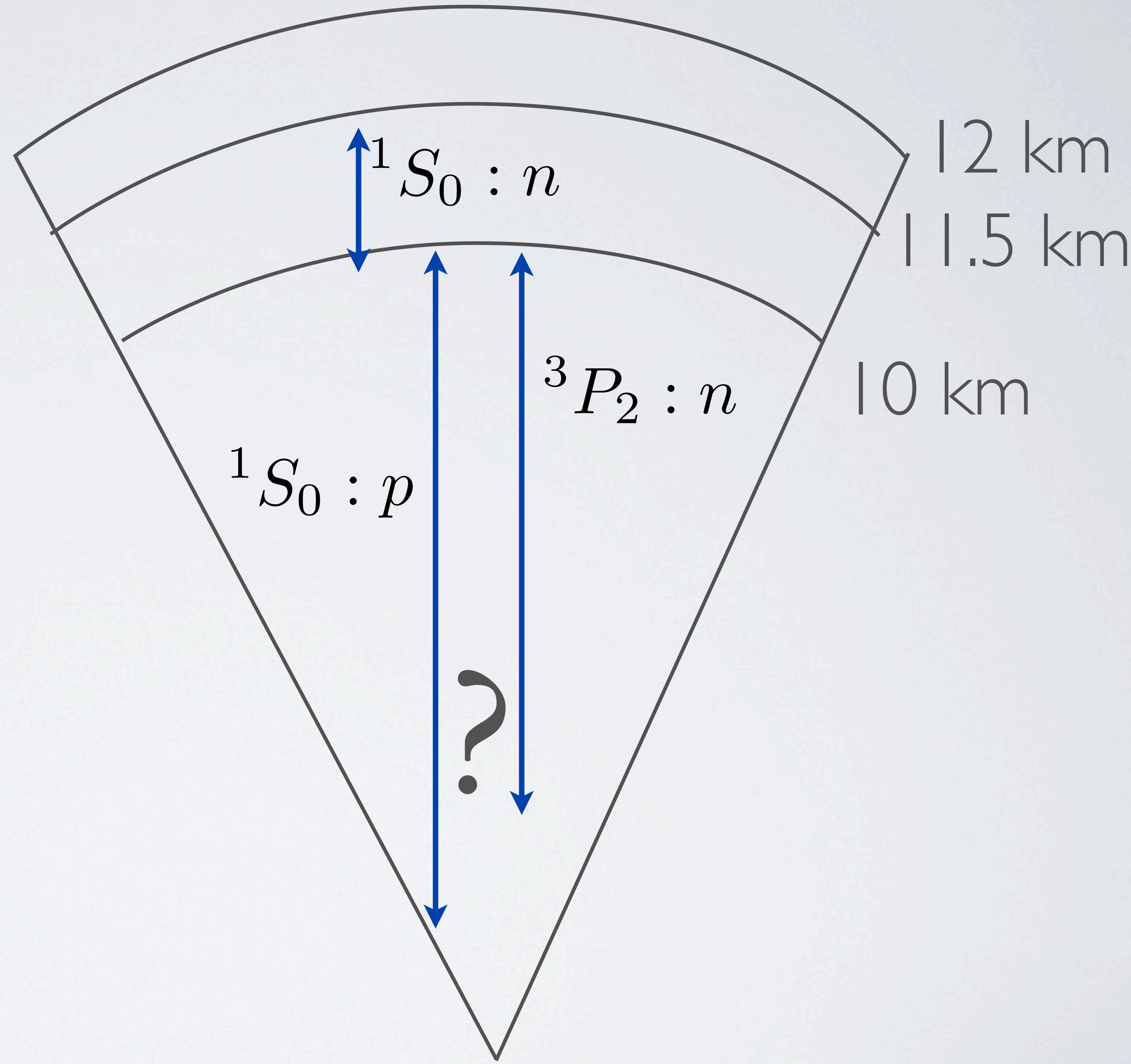


Page et al. (2011)

PAIRING PROFILE

Predictions based on the sign and magnitude of the interaction.

Screening, the multi-component nature, and other many-body effects can be important at high density.



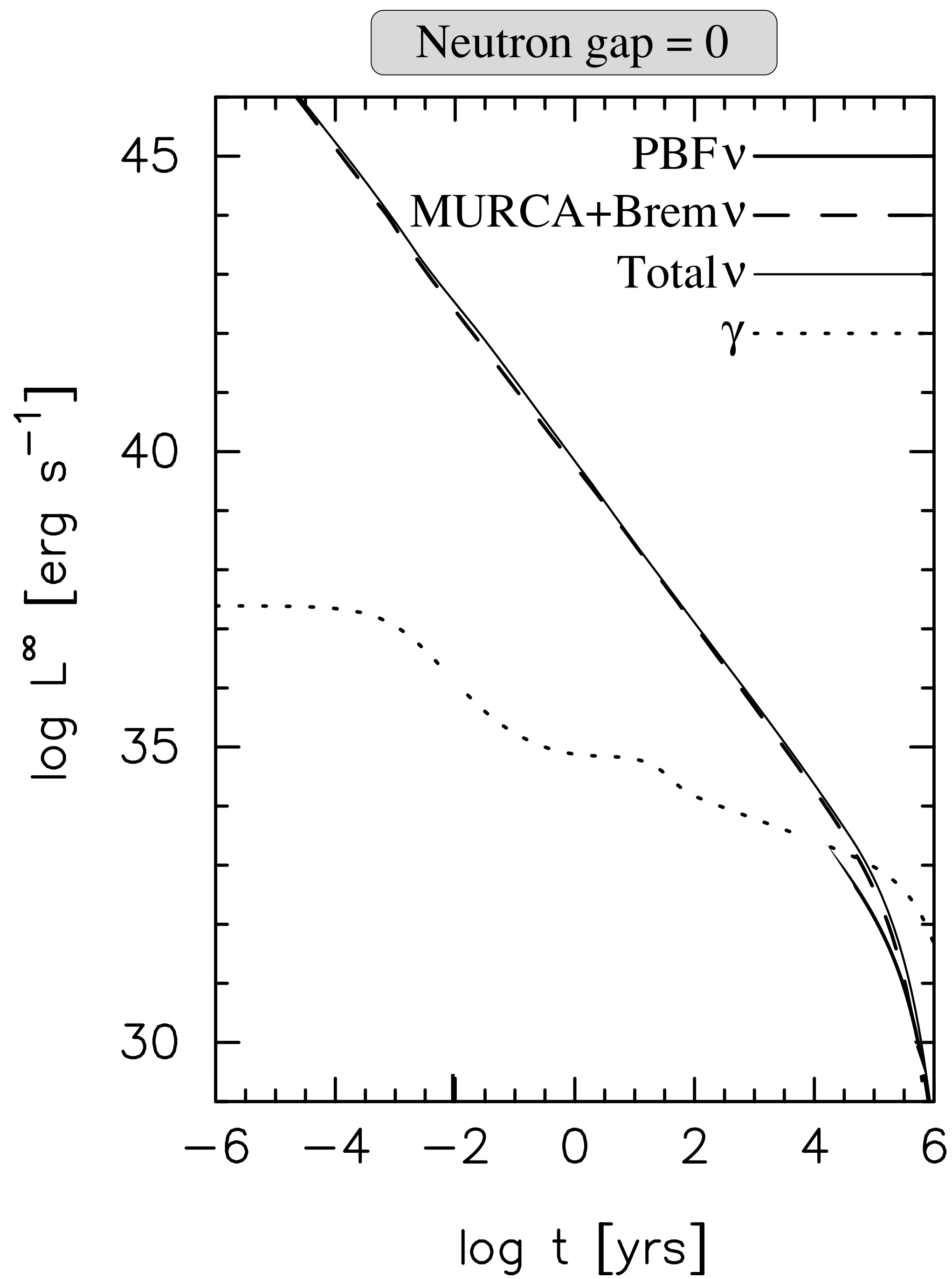
Pair Breaking & Formation

- Fluctuations near T_c are efficient at producing neutrinos.

Flowers, Ruderman, Sutherland (1976)

- Fluctuations in the $^3P_2$ superfluid are most efficient: (i) because neutrino's couple to spin, (ii) conservation laws suppress the emission in the $^1S_0$ channel.

Leinson (2008)
Steiner & Reddy (2009)



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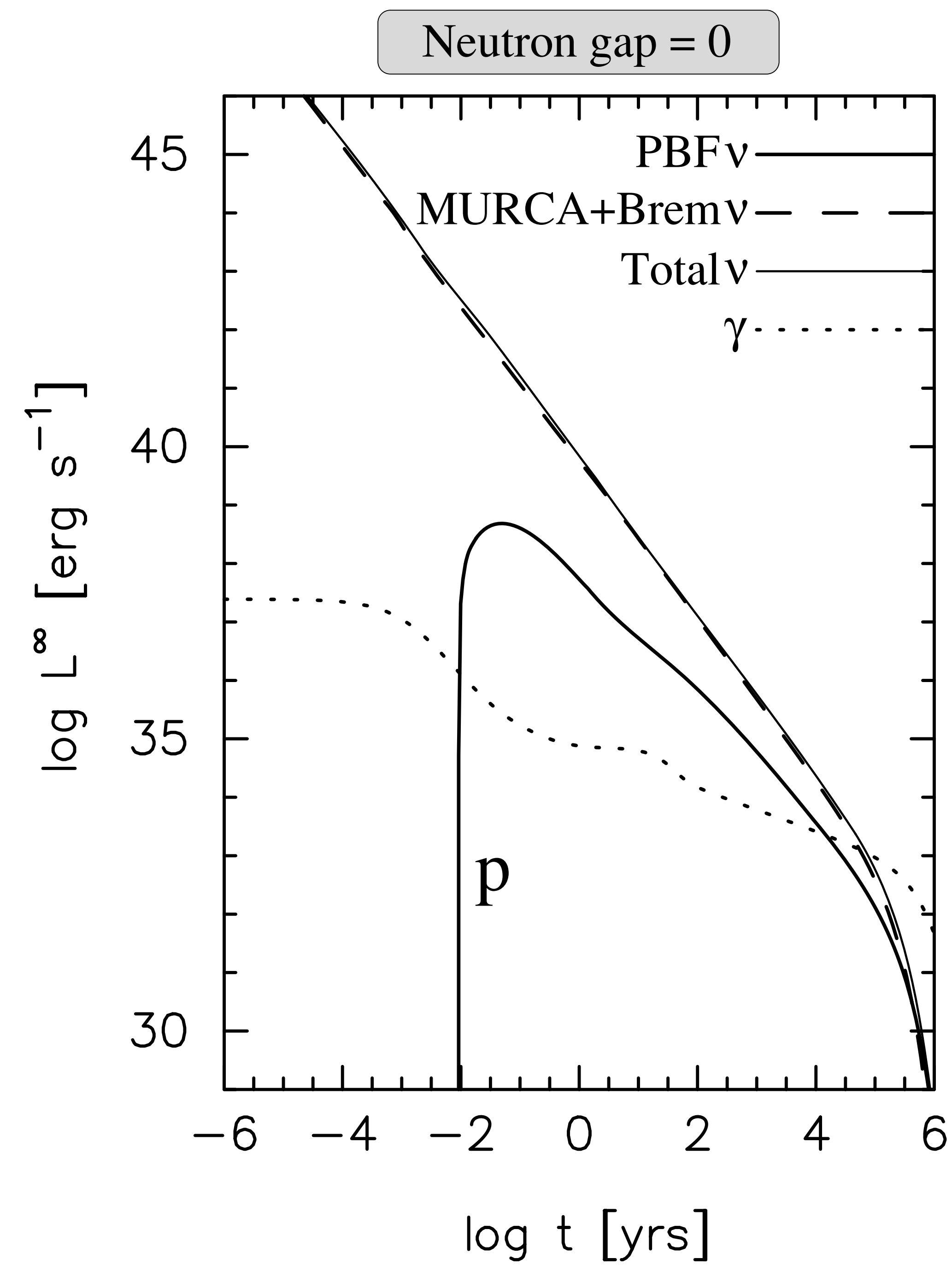

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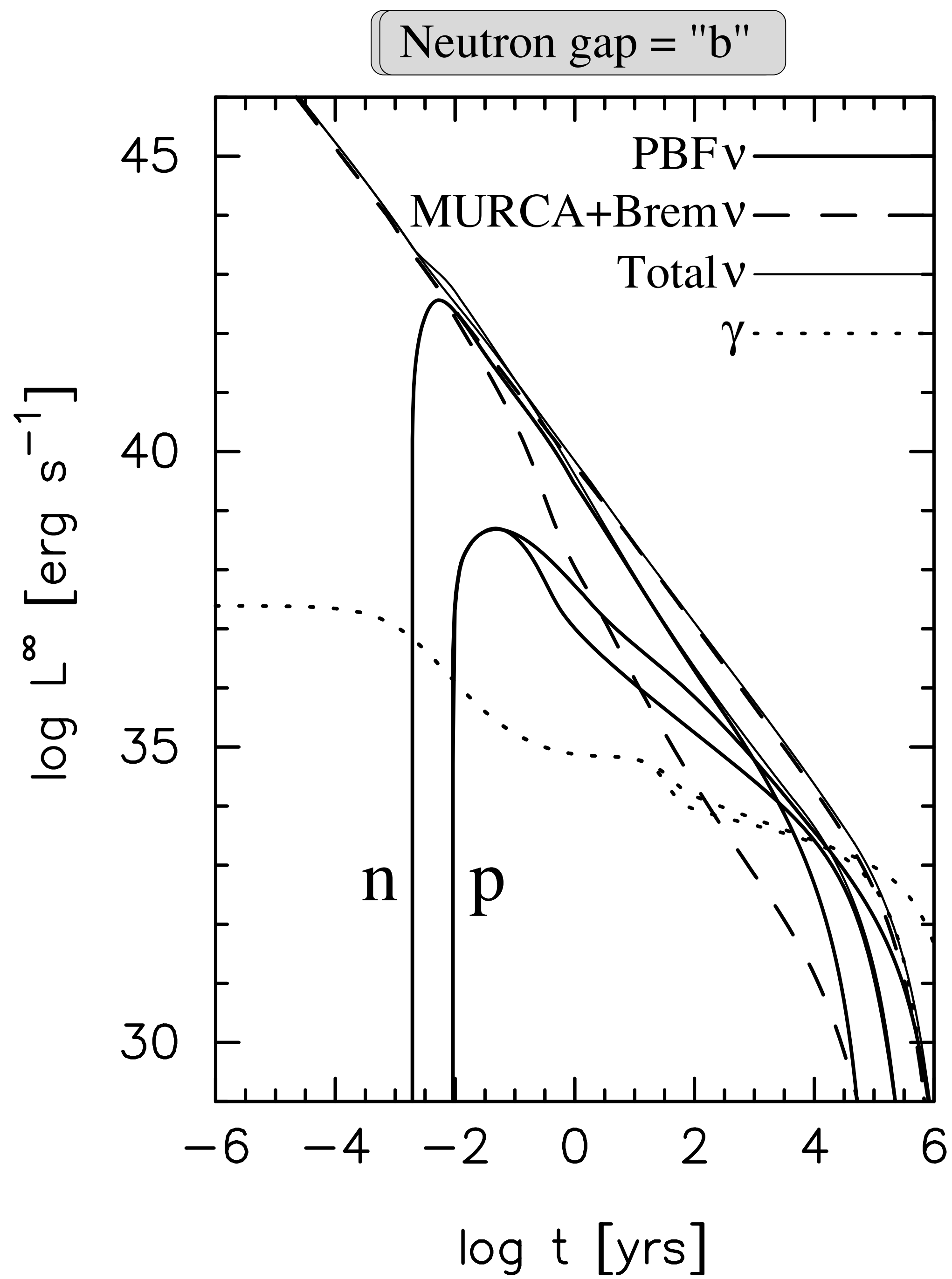


Leinson (2008)
Steiner & Reddy (2009)

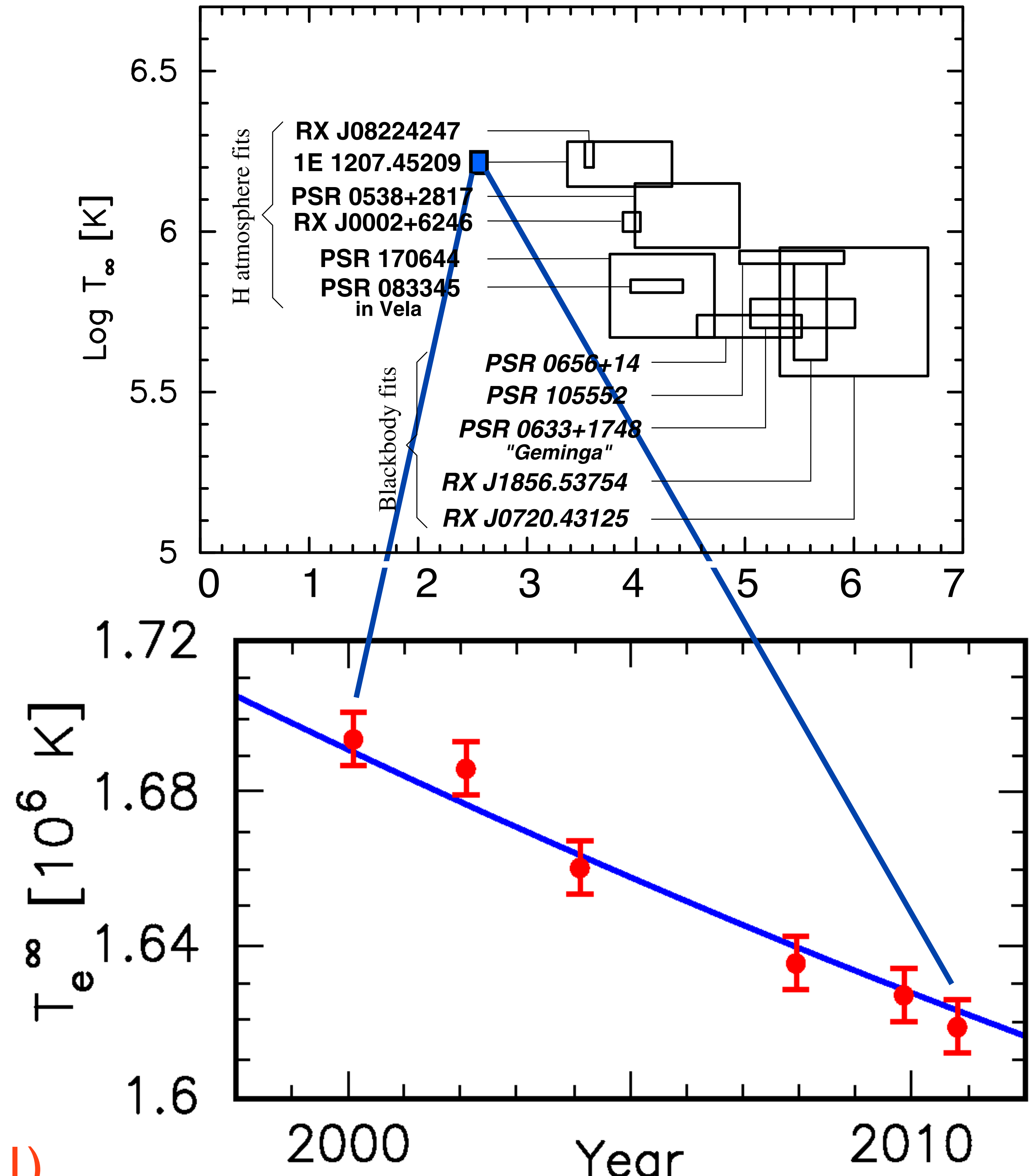
Page et al. (2011)

Cooling of the Neutron Star in Cas A

Cooling on a 10 year time scale requires very rapid cooling.

Is a large volume inside the neutron star undergoing a superfluid transition to produce enhanced cooling ?

Cooling behavior over the next decade will tell.



Heinke, & Ho (2010)
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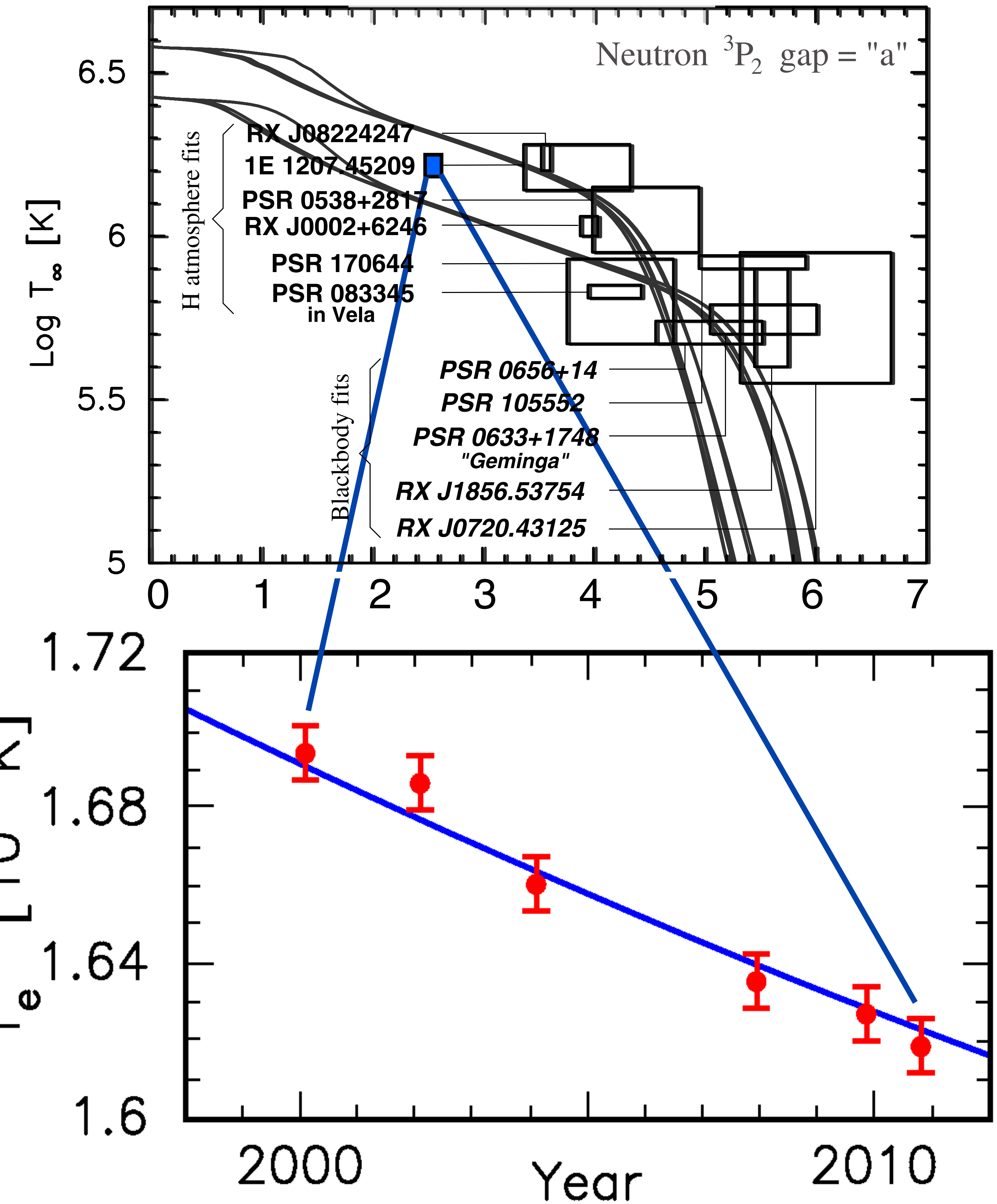

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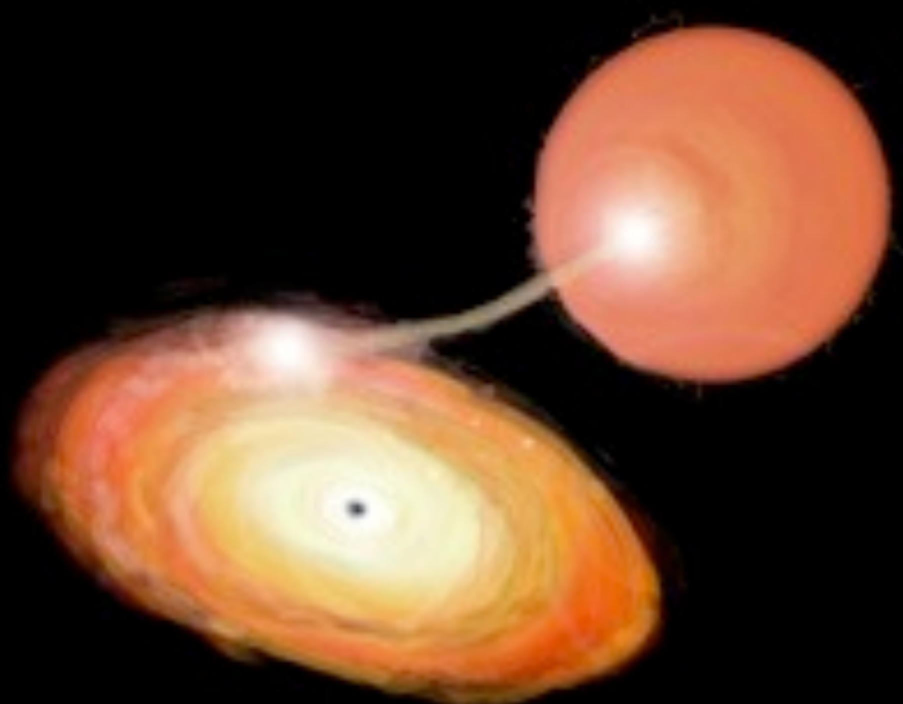
Page et al. (2011), Shternin et al. (2011)

Transient Accretion

- Nuclear reactions heat the crust during accretion.
  Haensel & Zdunik 1990, Brown, Bildsten, Rutledge (1998)
- Crust relaxes during quiescence.
  Shternin & Yakovlev (2007), Brown & Cumming (2009)

crust



Simulations by Dany Page

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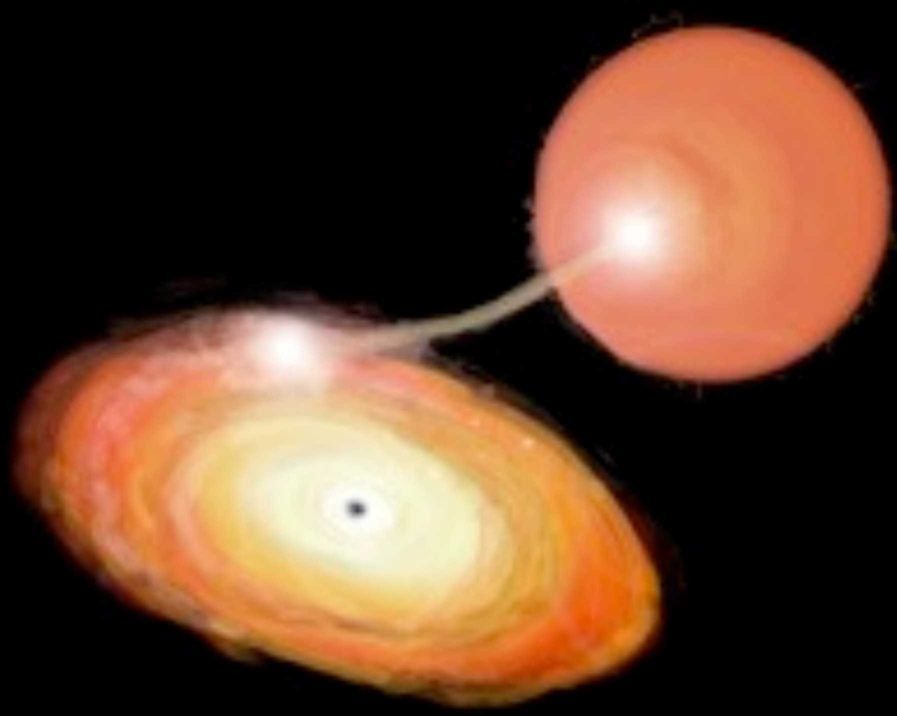

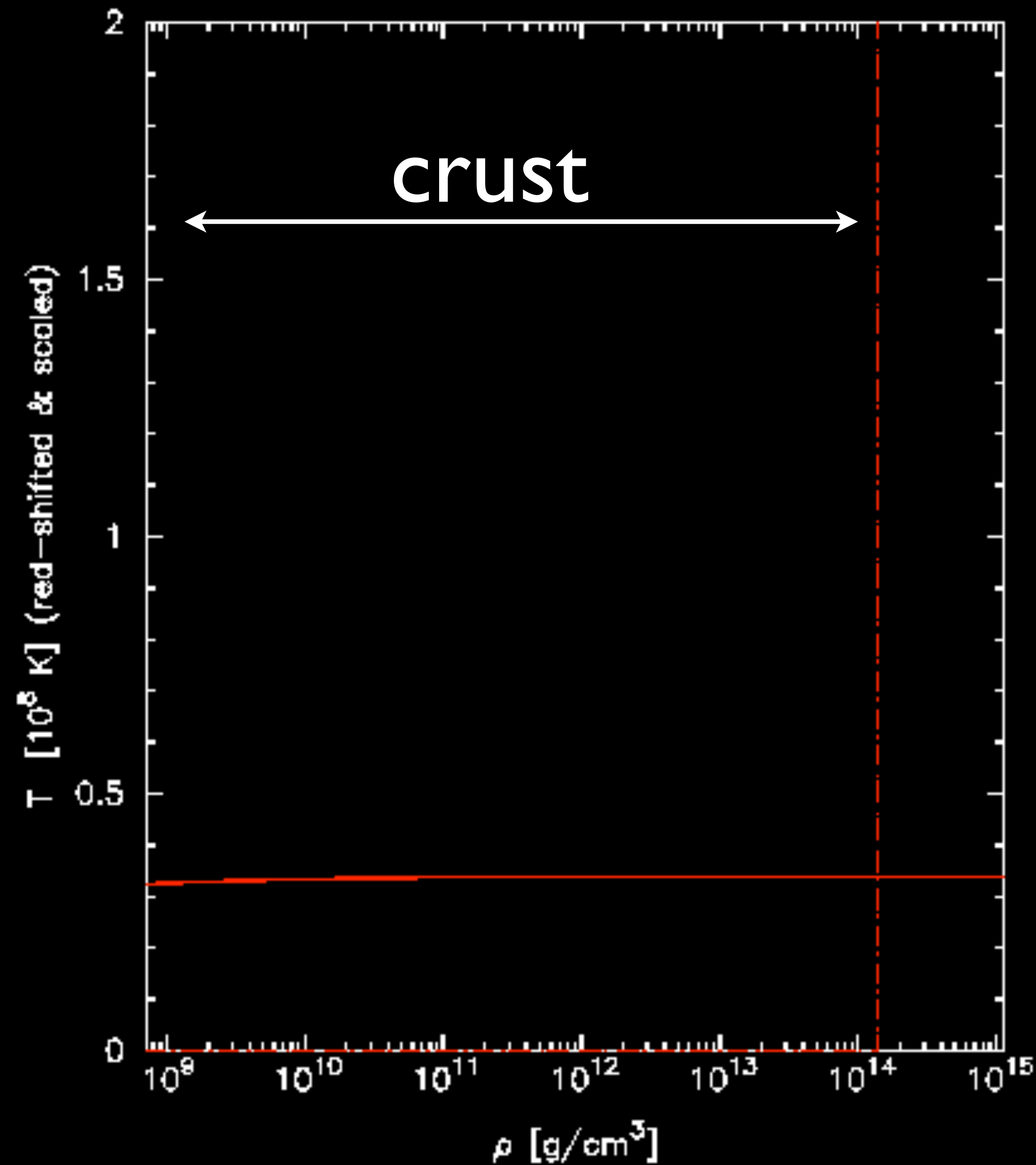

Simulations by Dany Page

Crust Relaxation

- Crust relaxes during quiescence.

Shternin & Yakovlev (2007), Brown & Cumming (2009)

data from
MXB 1659

$T_e^\infty\,[\mathrm{eV}]$

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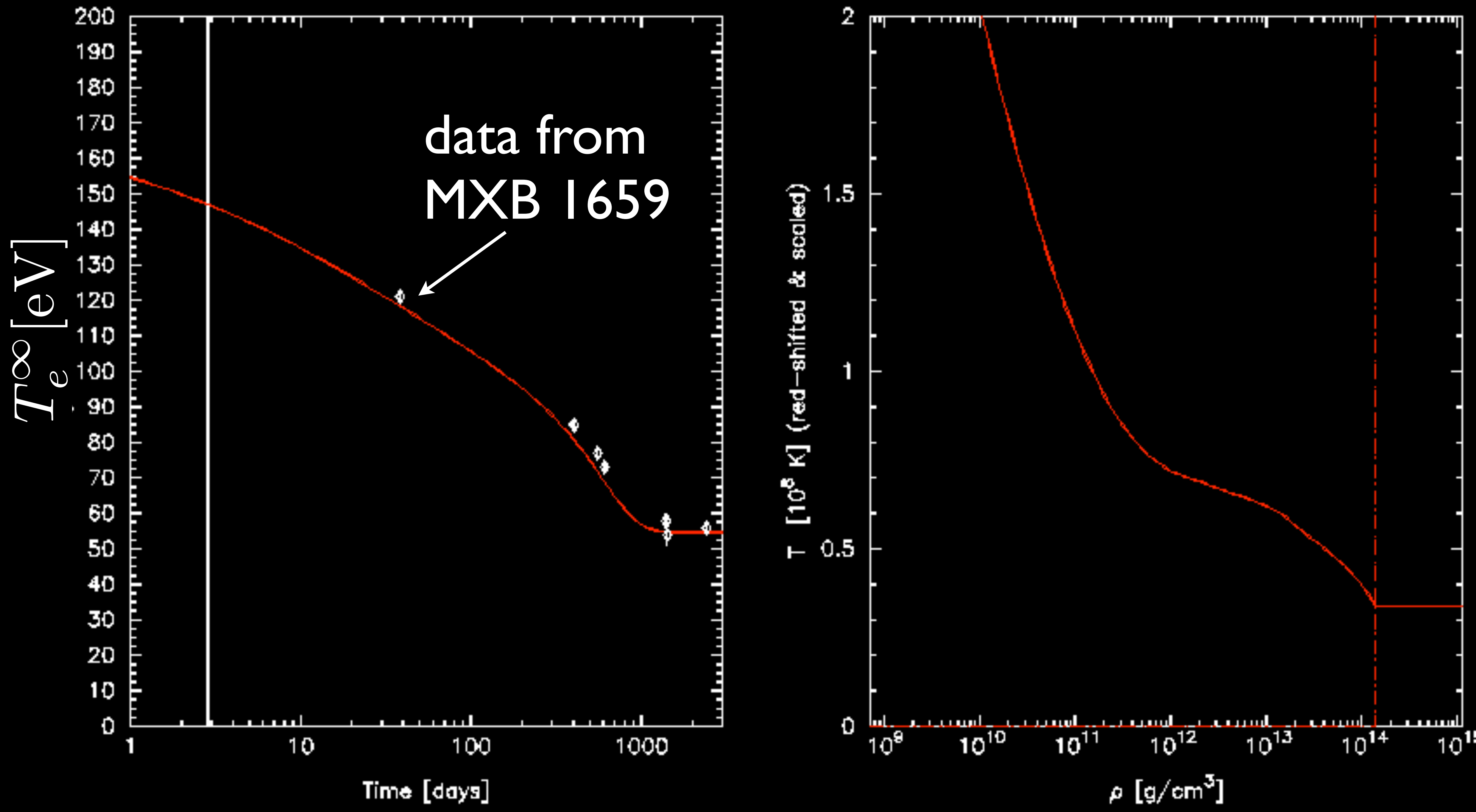
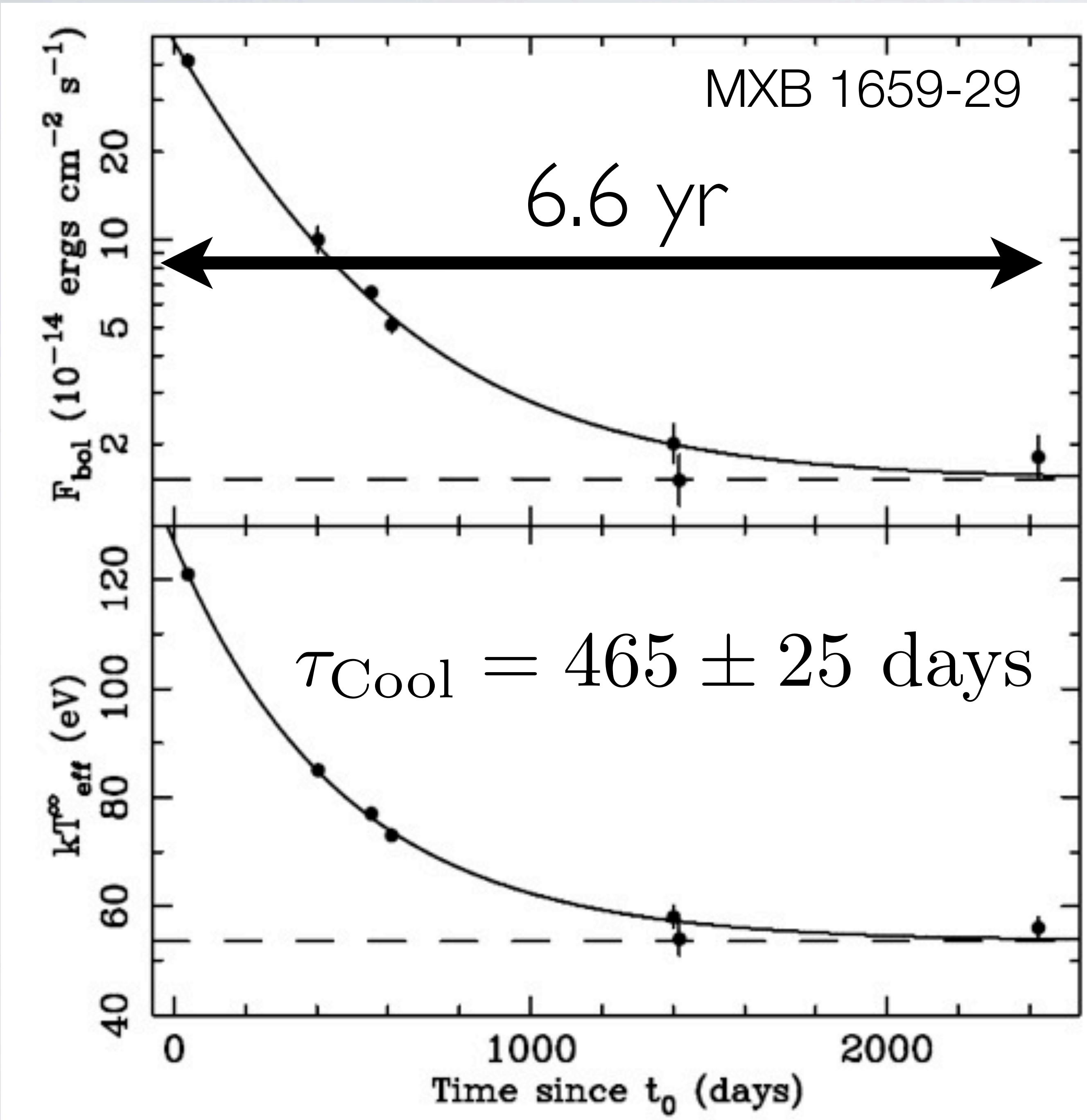
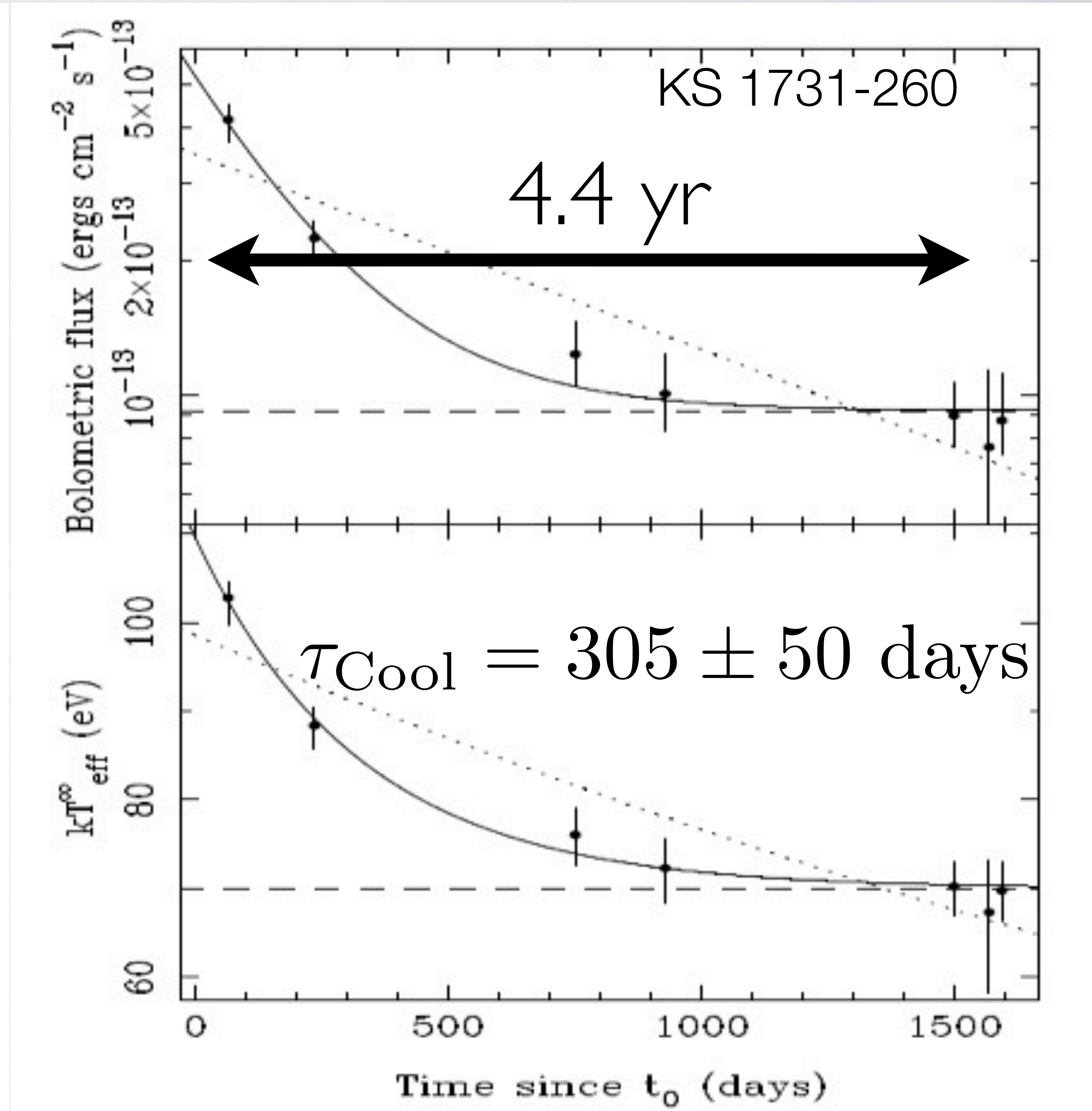


MORE THAN ONE SOURCE !

Cackett et al. 2006



Cackett et al. 2008



CONNECTING TO CRUST MICROPHYSICS

$$\tau_{\mathrm{Cool}} \simeq \frac{C_V}{\kappa} (\Delta R)^2$$

CONNECTING TO CRUST MICROPHYSICS

Crustal Specific Heat

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Thermal Conductivity

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Crustal Specific Heat

Crust Thickness

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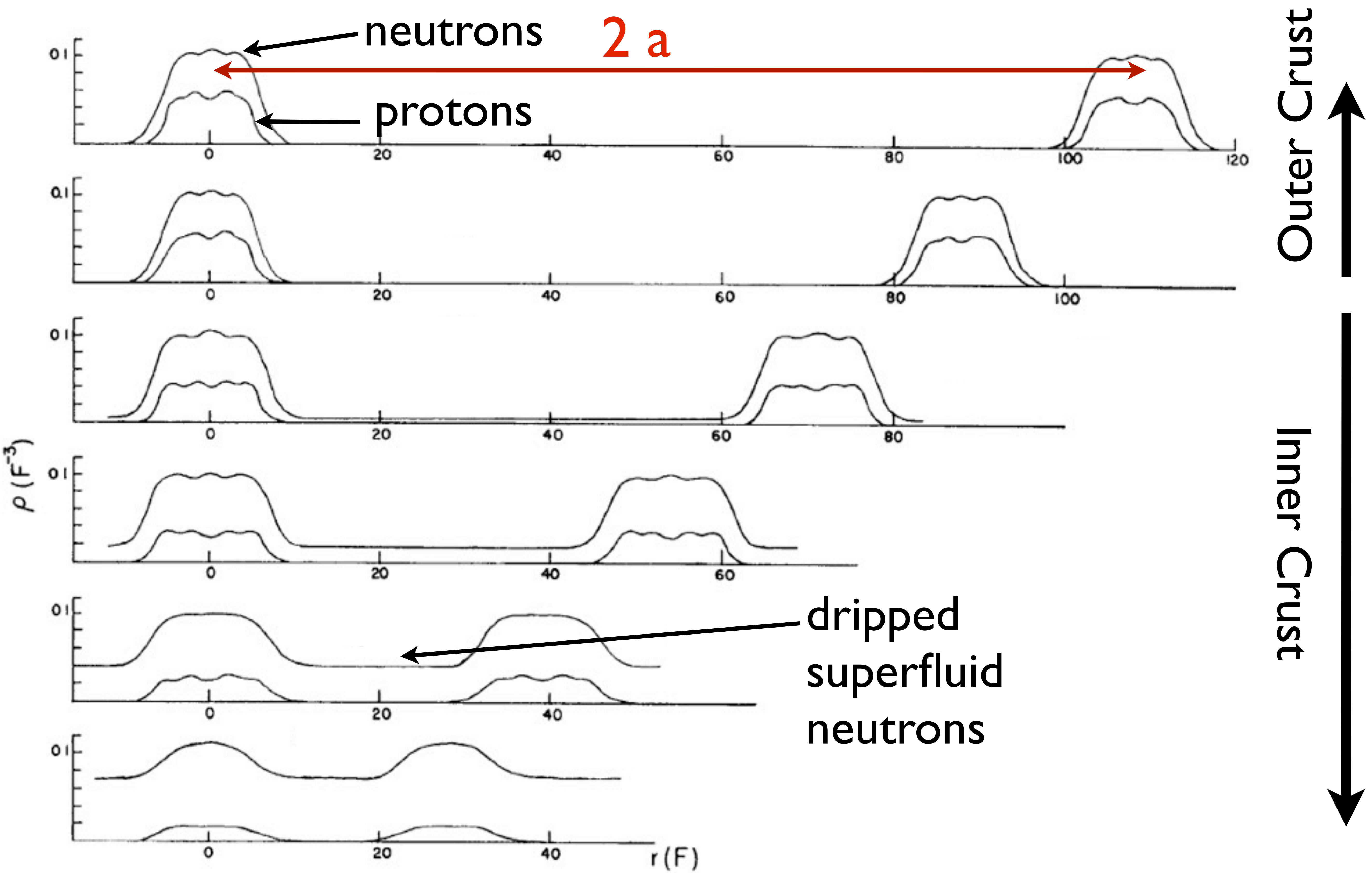
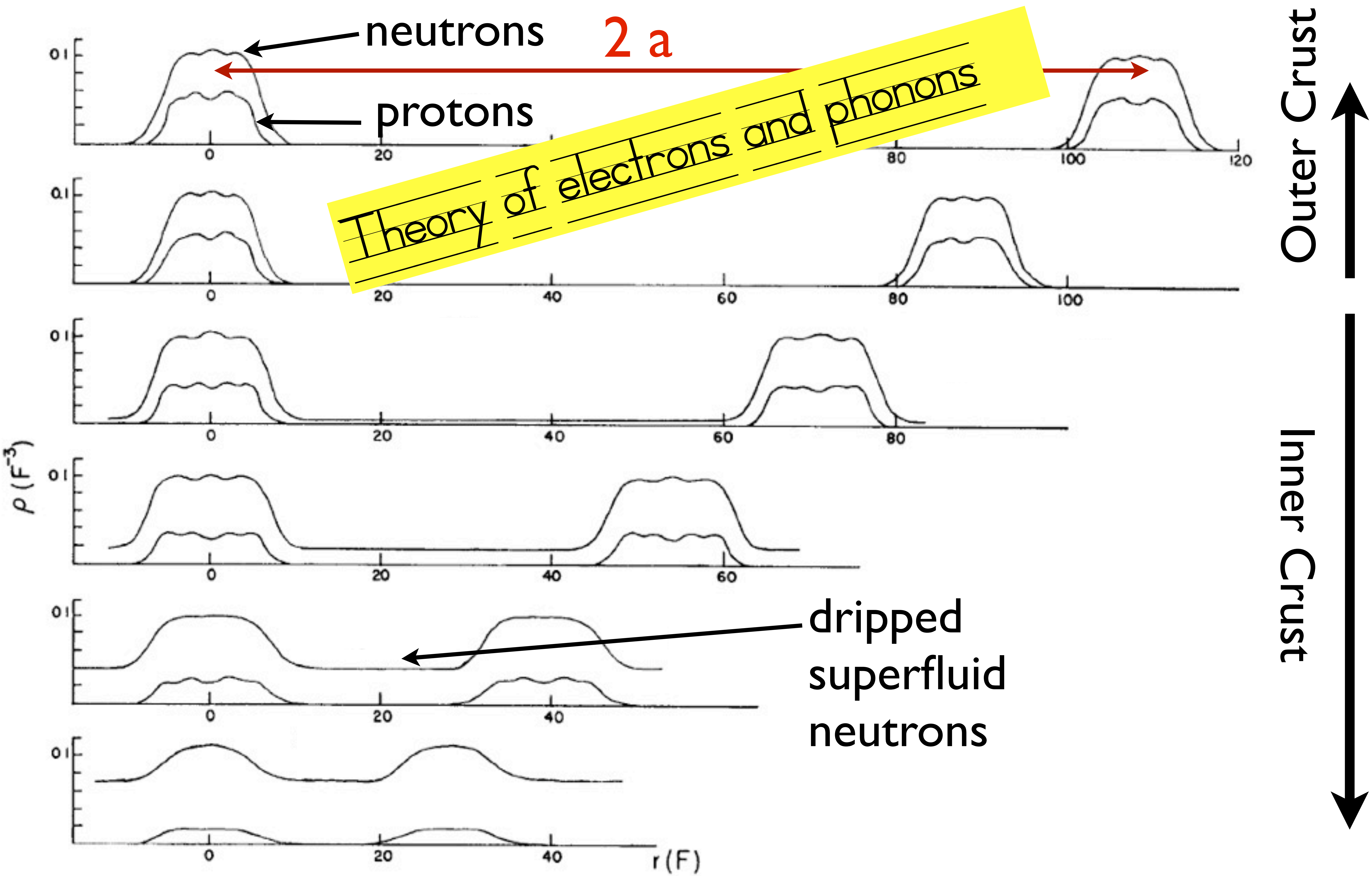
Thermal Conductivity

Microscopic Structure of the Crust



Baym Pethick & Sutherland (1971)   Negele & Vautherin (1973)

Microscopic Structure of the Crust



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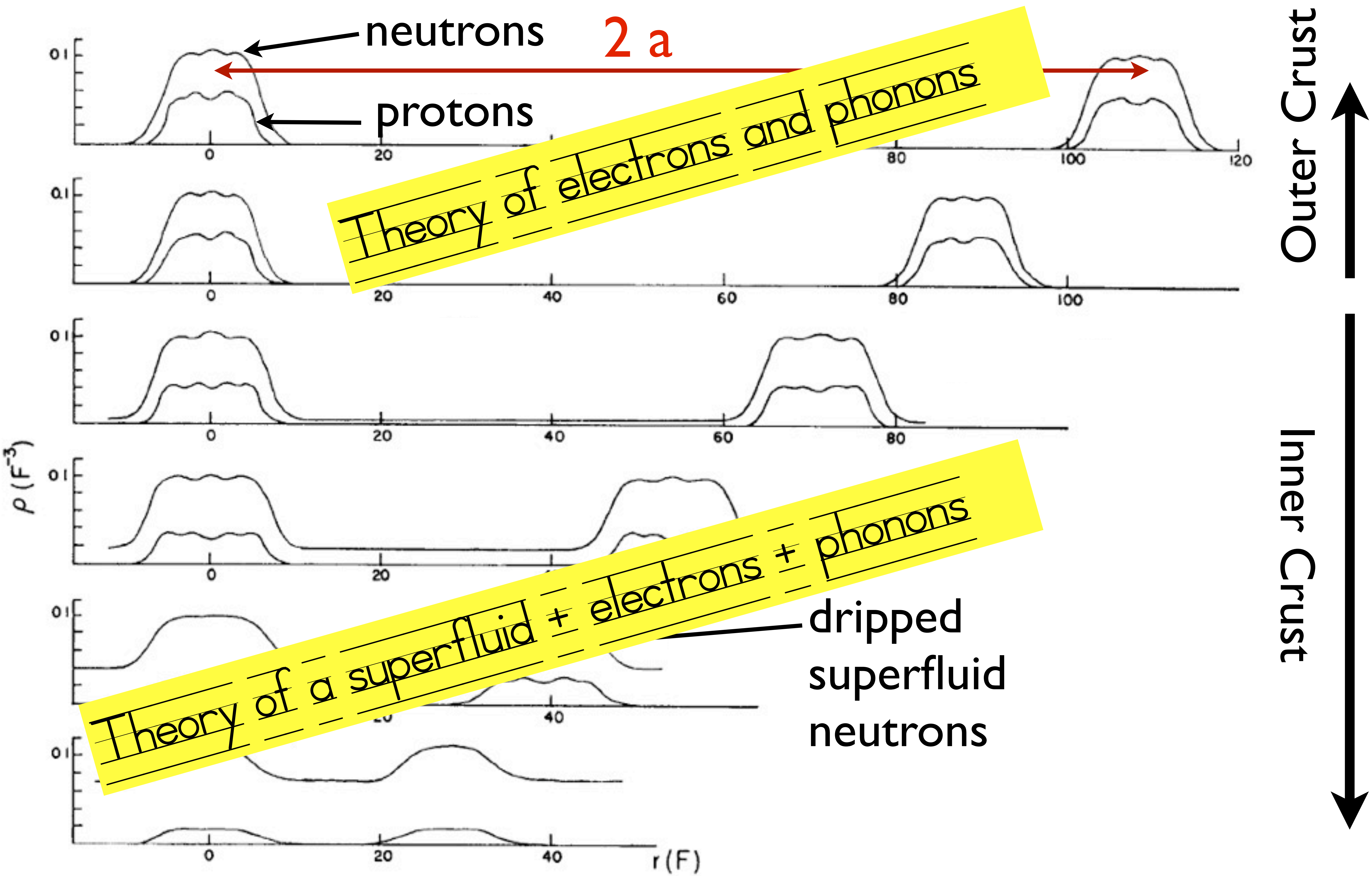

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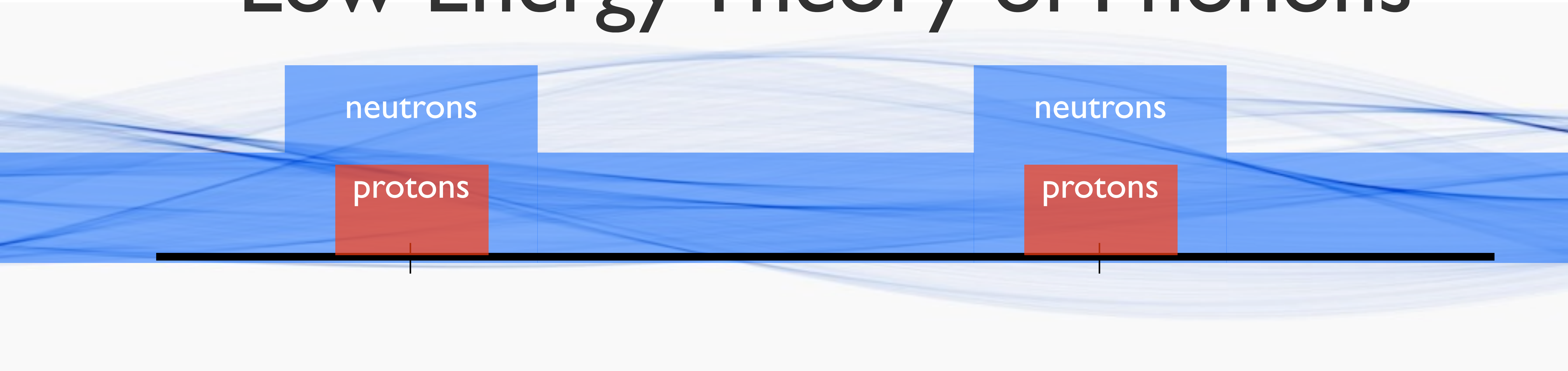
Low Energy Theory of Phonons



Proton (clusters) move collectively on lattice sites. Displacement is a good coordinate.

Neutron superfluid: Goldstone excitation is the phase of the condensate.

Cirigliano, Reddy & Sharma (2011)

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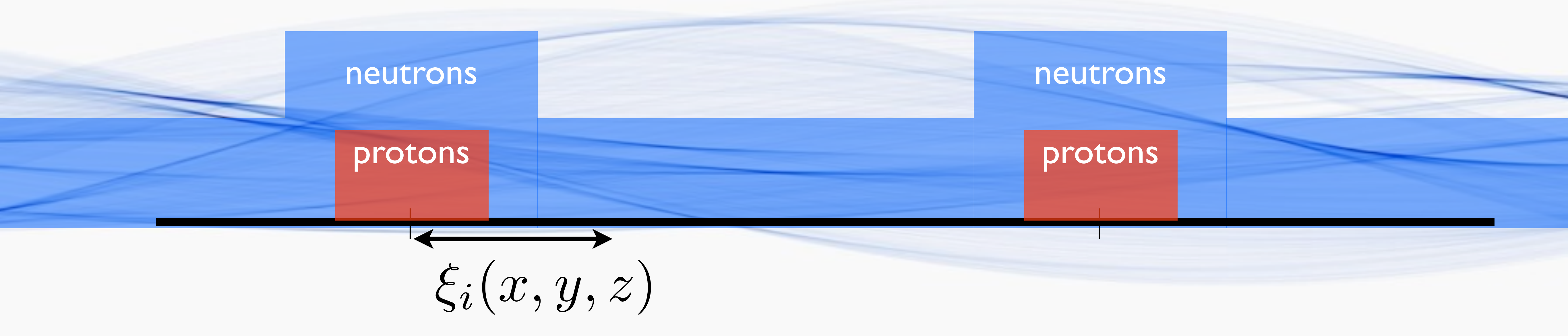

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Neutron superfluid: Goldstone excitation is the phase of the condensate.

“coarse-grain”

$$\langle \psi_\uparrow(r)\psi_\downarrow(r)\rangle = |\Delta| \ \exp\left(-2i \ \theta\right)$$

Collective coordinates:

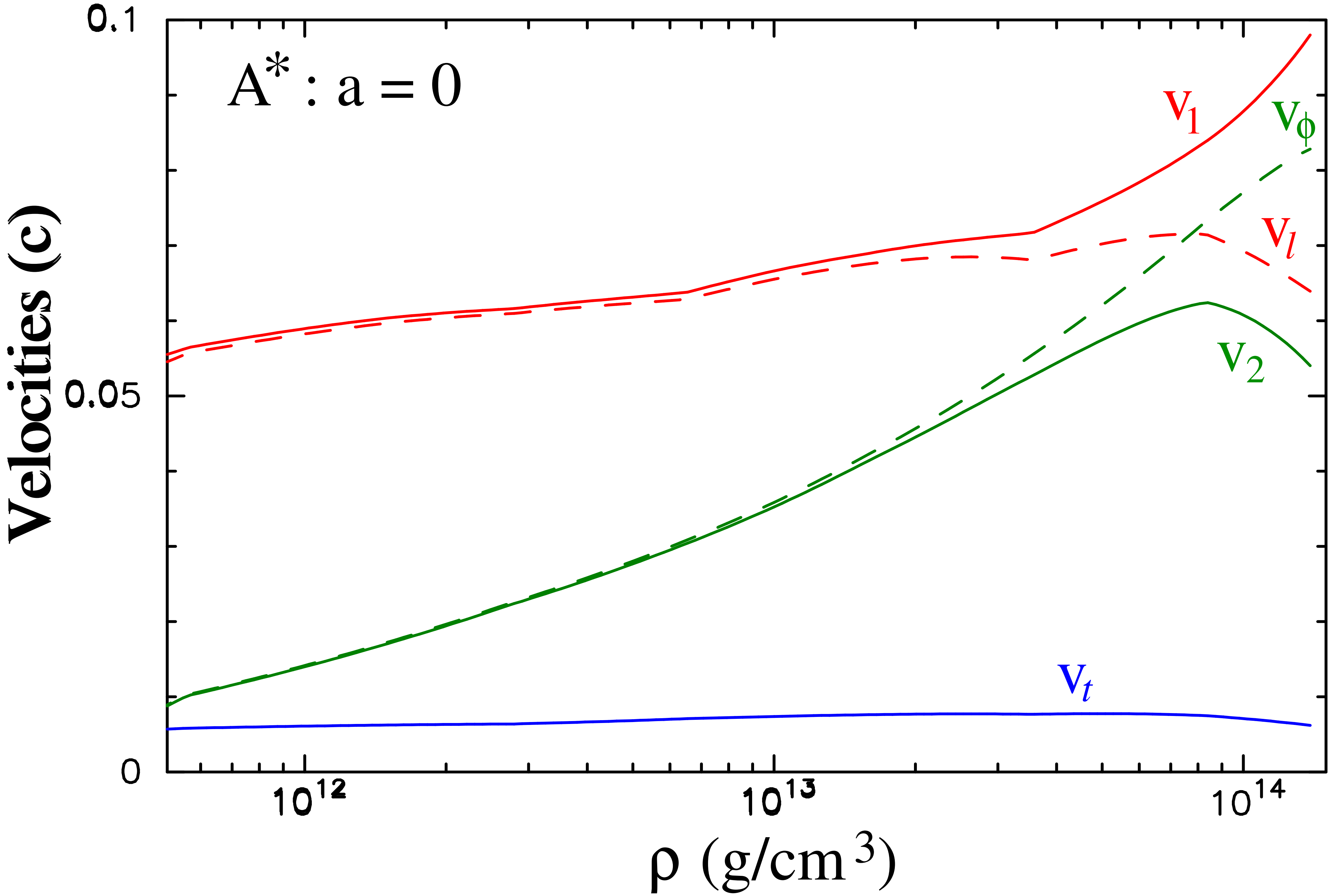
Vector Field: $\xi_i(r,t)$
Scalar Field: $\phi(r,t)$

Cirigliano, Reddy & Sharma (2011)

Low Energy Excitations

$$\omega_{\rm lPh}(q) = c_l \; q \qquad \omega_{\rm sPh}(q) = v_s \; q$$

$$\omega_{\rm tPh}(q) = c_t \; q \qquad \omega_{\rm electron} = q$$



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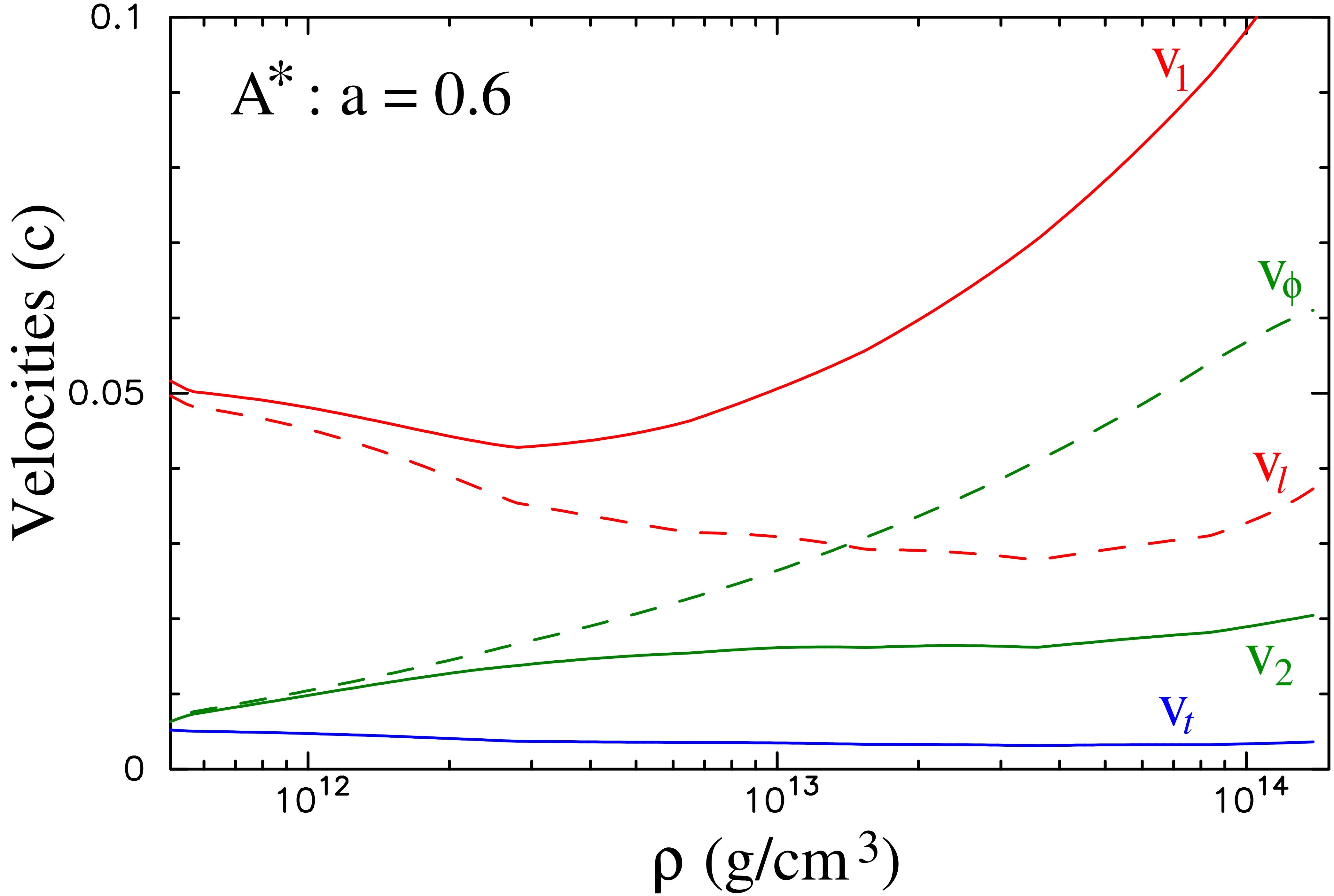
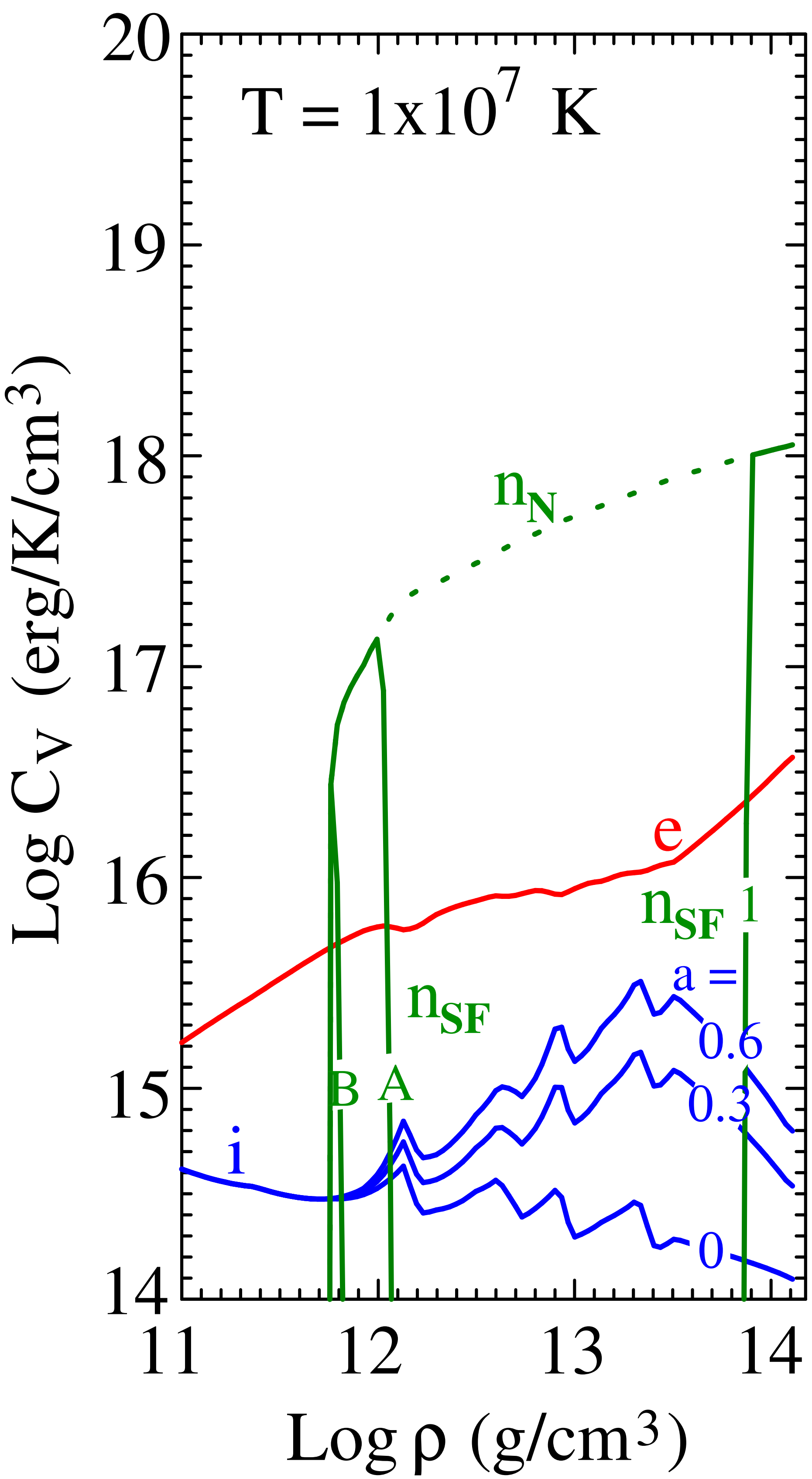


Crustal Specific Heat



Electrons: $C_v^e = \frac{1}{3}\mu_e^2\, T$

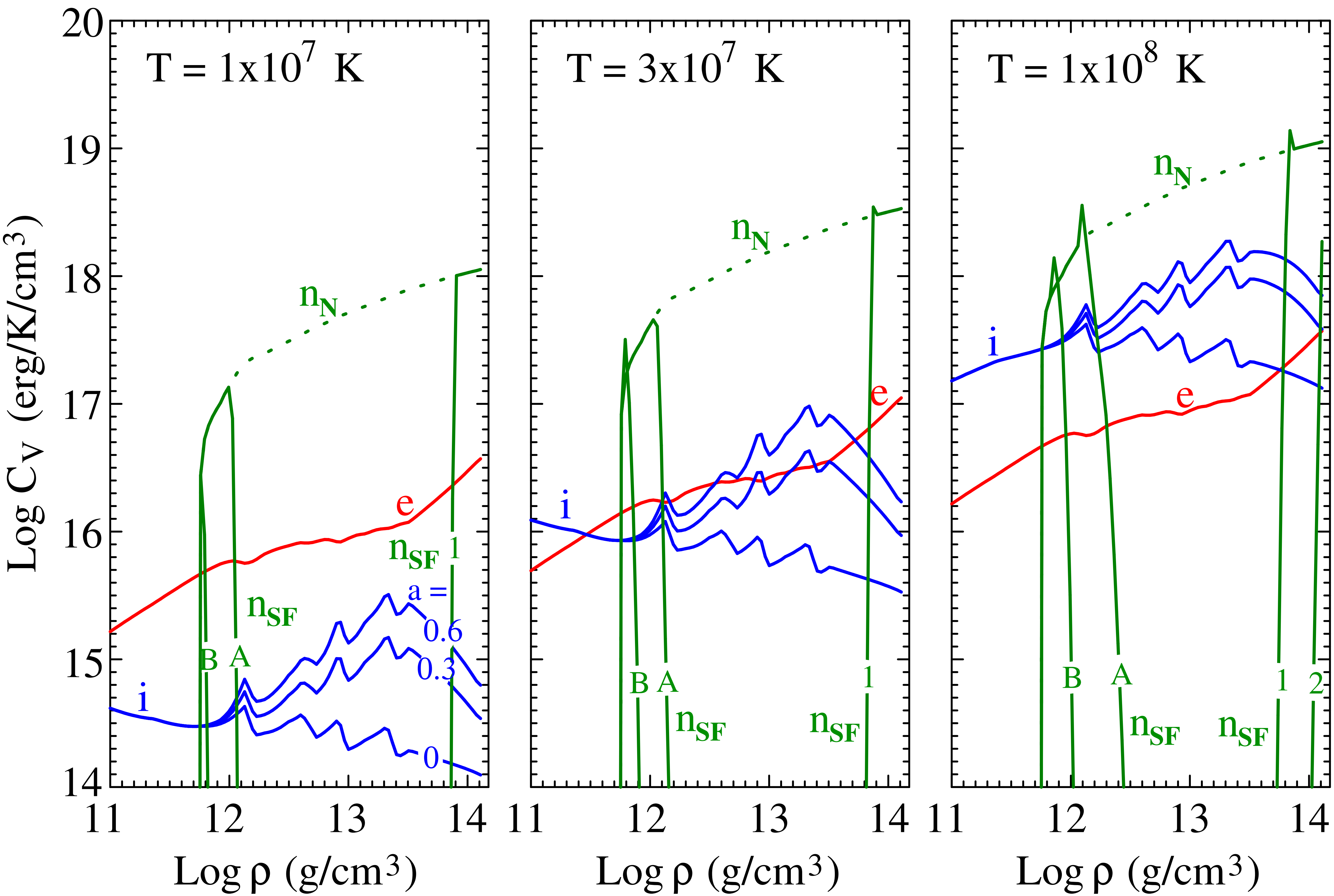
Ions: $C_v^{\mathrm{lph}} = \frac{2\pi^2}{15}\left(\frac{T^3}{v_l^3} + \frac{2\,T^3}{v_t^3}\right)$

Neutrons:

$$
\begin{cases}
C_v^{\mathrm{sph}} = \frac{2\pi^2}{15}\frac{T^3}{v_\phi^3} & (T \ll T_c) \\
C_v^{\mathrm{neutron}} = \frac{1}{3}\, m_n\, k_{\mathrm{Fn}}\, T & (T > T_c)
\end{cases}
$$

Page & Reddy (2012)

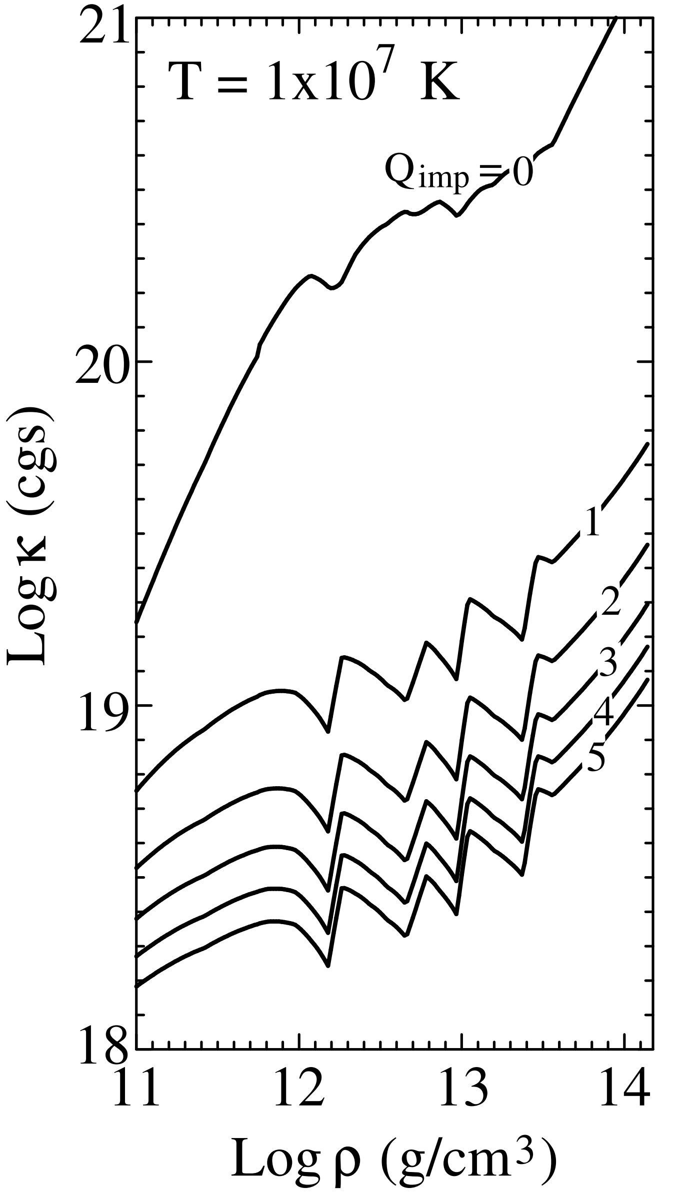
Crustal Specific Heat



Page & Reddy (2012)

Electron Conduction

$$\kappa_e = \frac{1}{9}\mu_e^2\, T\, \lambda_e$$



Electron-phonon:

$$\begin{cases} \lambda_e^{\rm ph} \propto v_t^3/T^2 & T \geq T_{\rm um} \\ \lambda_e^{\rm ph} \propto v_l^4/T^3 & T \ll T_{\rm um} \end{cases}$$

$$T_{\rm um} = (4e^3/9\pi)\, v_t\, k_{\rm Fe}$$

Electron-impurity:

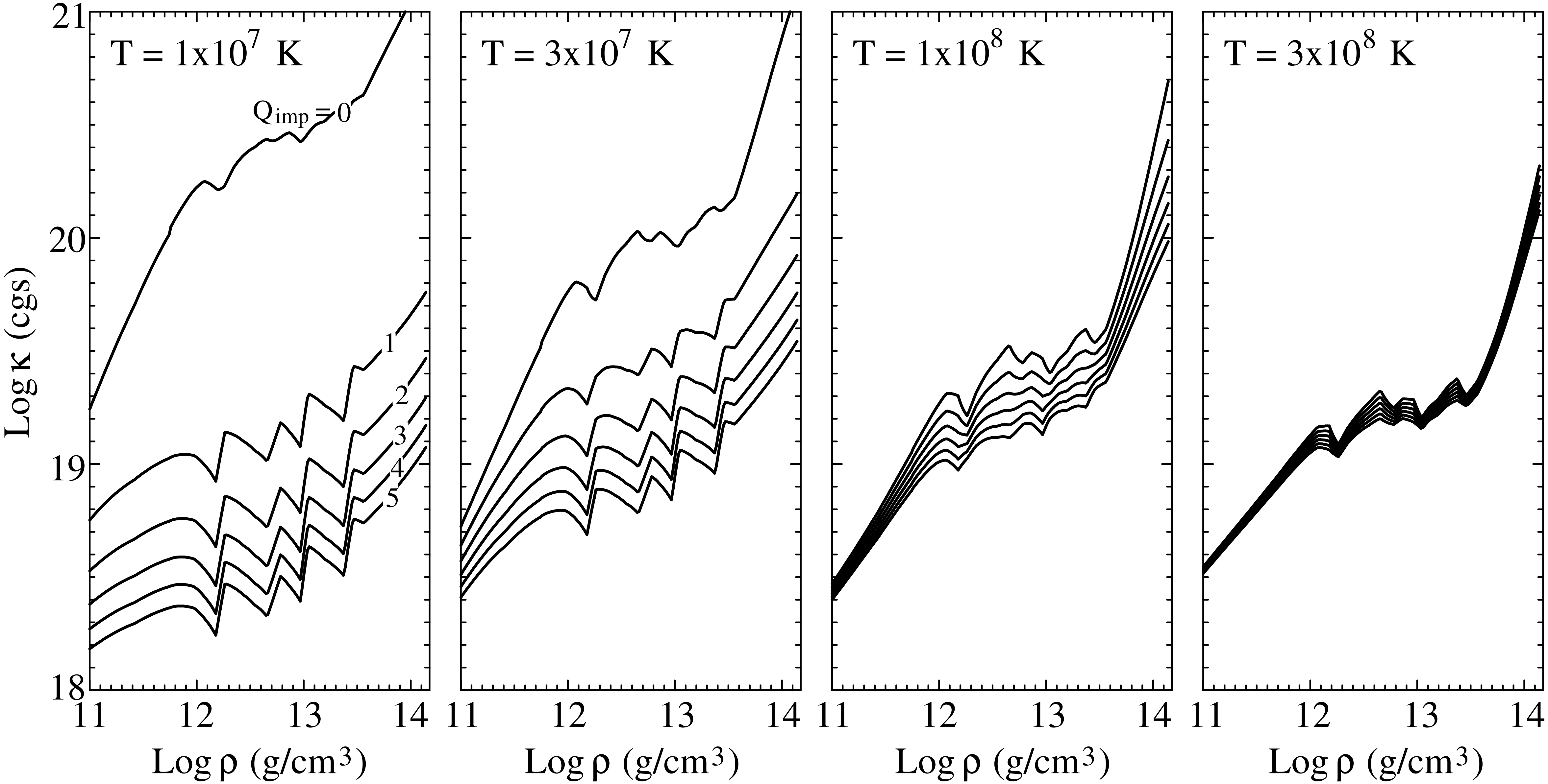
$$\lambda_e^{\rm imp} = \frac{3\pi\langle Z\rangle}{4e^4 Q_{\rm imp}\, k_{\rm Fe}}\,\Lambda^{-1} \qquad Q_{\rm imp} = \frac{1}{n_{\rm ion}}\sum_i n_i\,(Z_i - \langle Z\rangle)^2$$

Impurity scattering is important at low temperature.

Flowers & Itoh (1976)

Electron Conduction

$$\kappa_e = \frac{1}{9}\mu_e^2 \, T \, \lambda_e$$



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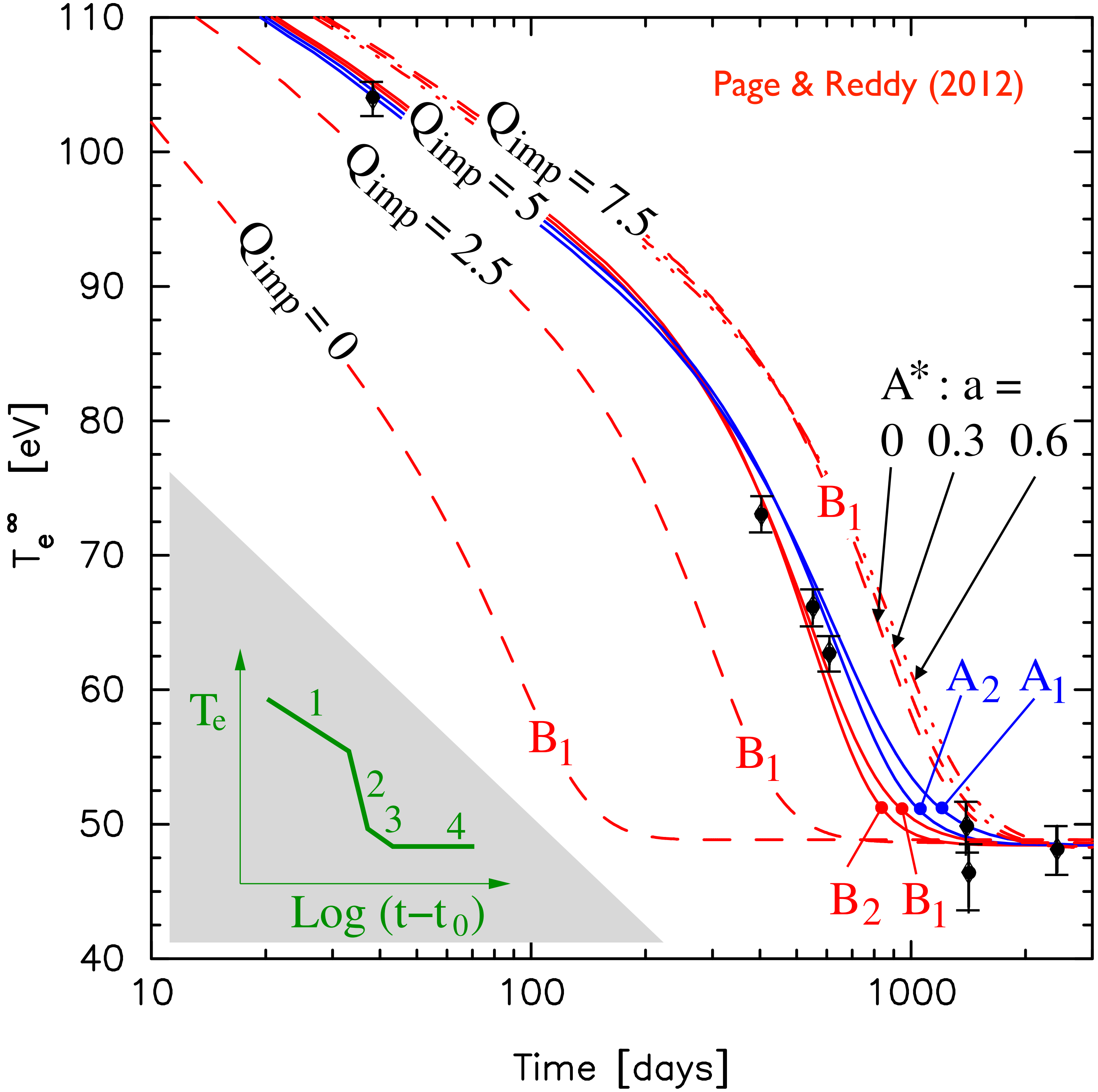
Unraveling thermal relaxation

- Late time signal is sensitive to inner crust thermal and transport properties.
- Impurity parameter can be fixed at earlier times. Shternin & Yakovlev (2007) Brown & Cumming (2009)
- Variations in the pairing gap (changes the fraction of normal neutrons) are discernible !

Page & Reddy (2012)



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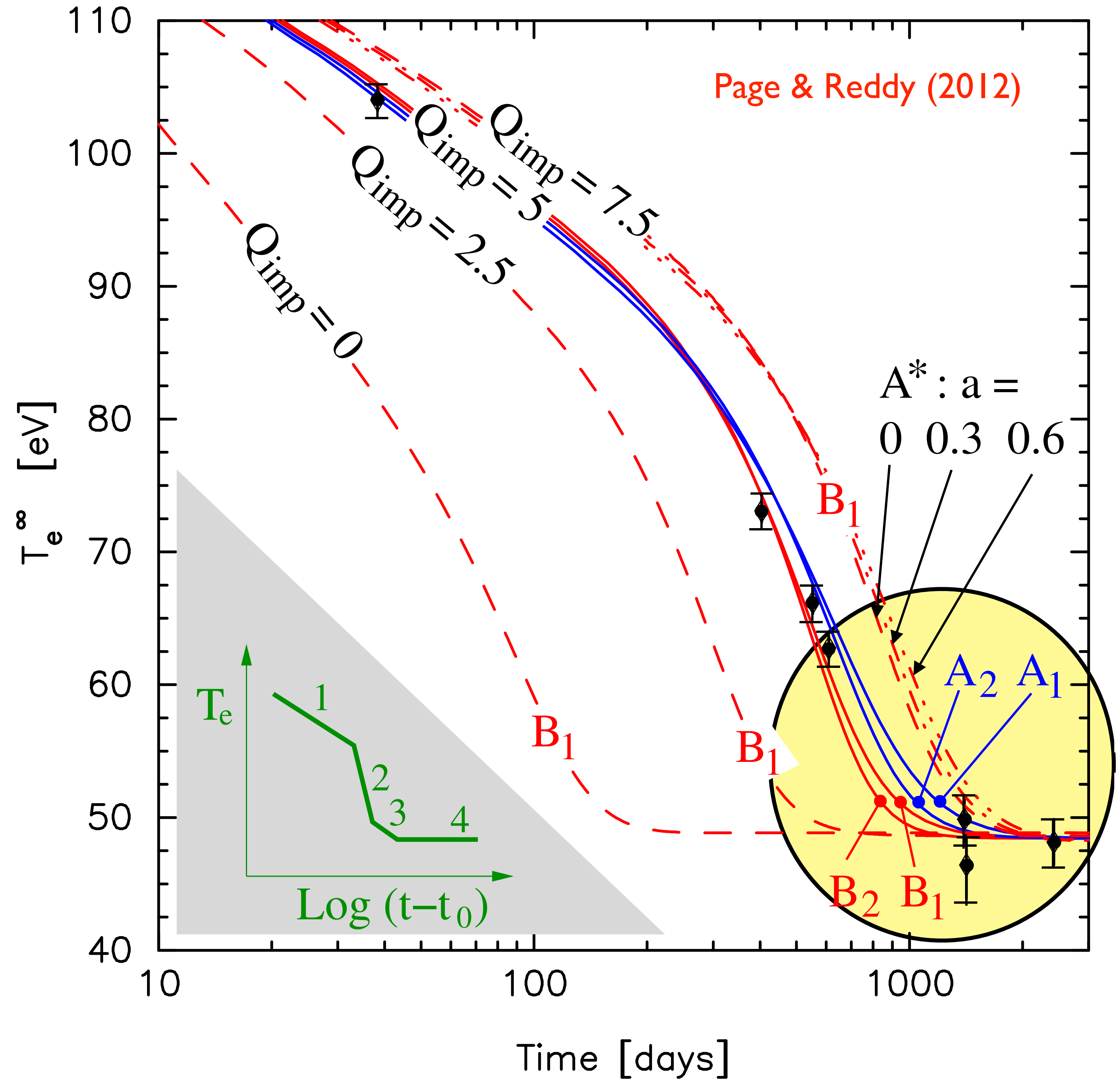
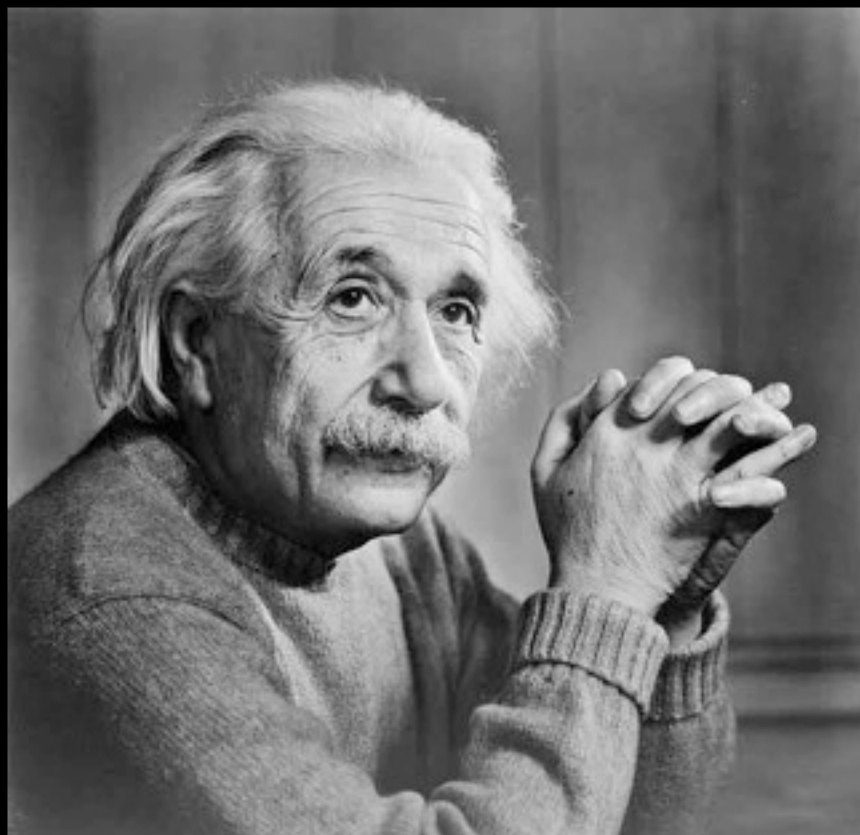
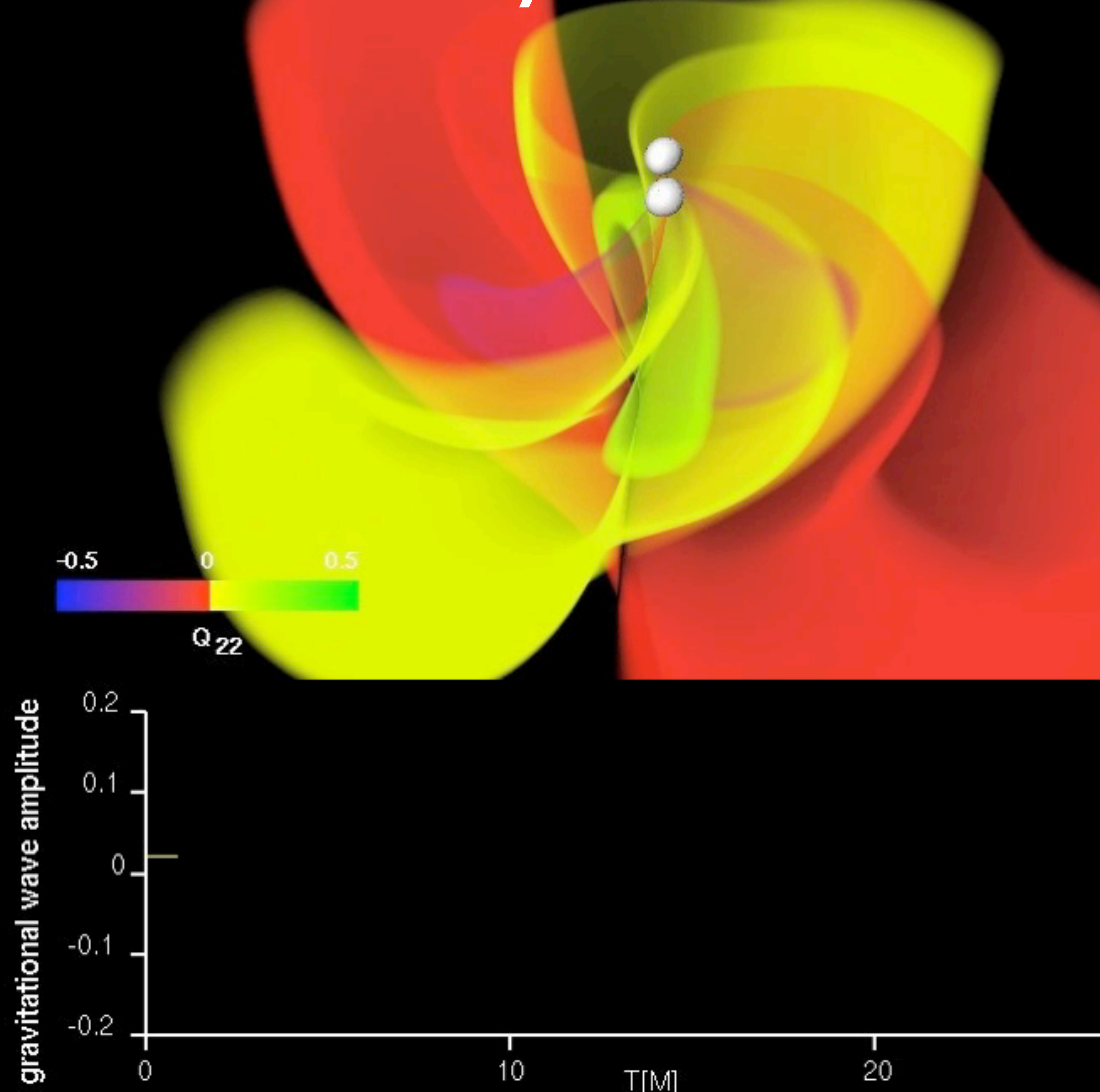


A: Low T_c - large normal fraction

B: High T_c- small normal fraction

NS Collisions - Gravity Waves

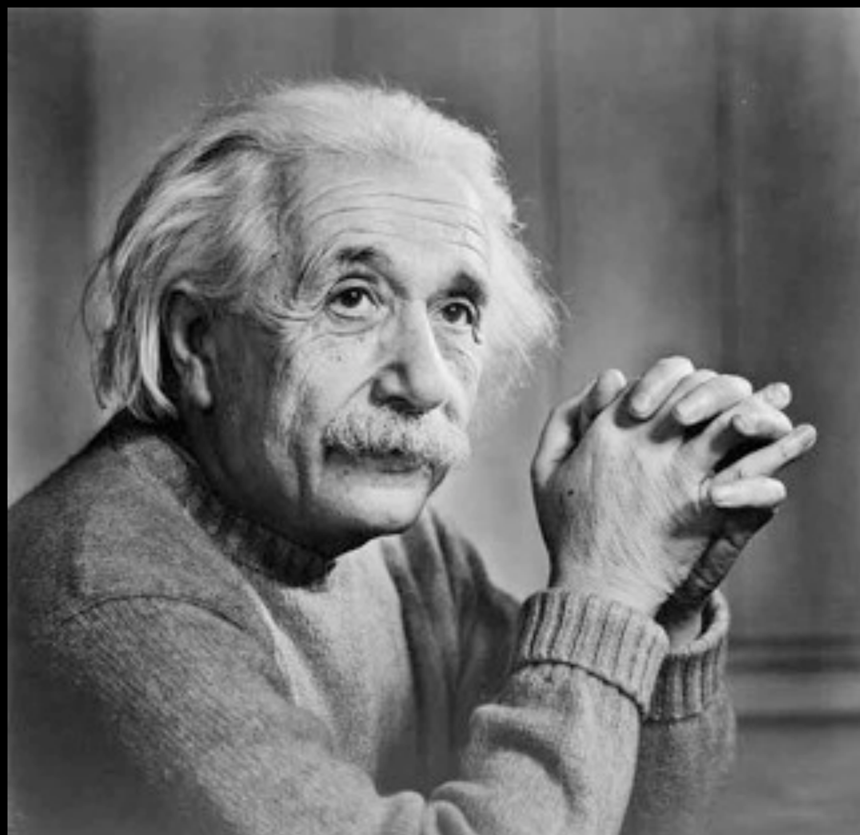




The gravity wave and EM counterparts are also likely to be sensitive to both the EoS and response functions.

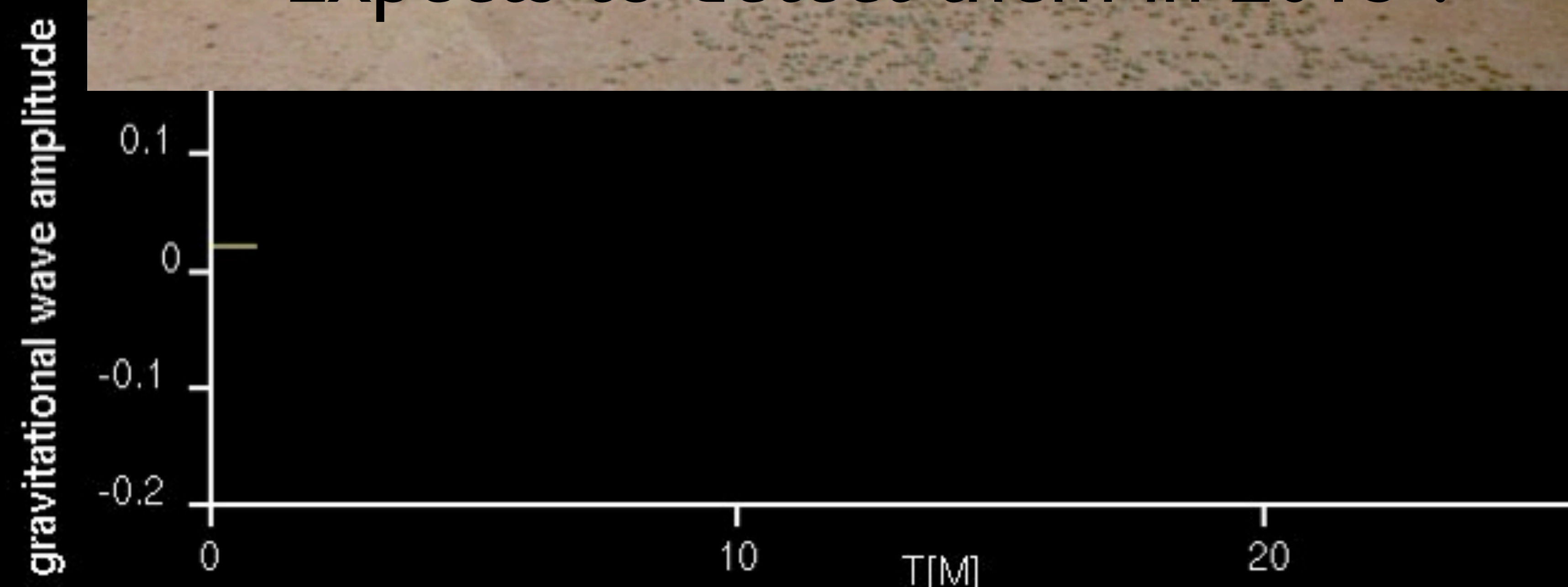
Simulations by Rezolla et al.

NS Collisions - Gravity Waves



Advanced LIGO
- Expects to detect them in 2015 !

The gravity wave and EM counterparts are also likely to be sensitive to both the EoS and response functions.



Simulations by Rezolla et al.

Summary & Outlook

- A qualitative understanding of connections between dense matter properties and neutron star observations have emerged in the past decade.
- There is much to do. Pursuing theoretical work to provide a quantitative description of the equation of state and correlations functions of interest will be both challenging and rewarding.
- Multi-messenger probes of the neutron star interior (x-rays, neutrinos, and GWs) contain a wealth of information .. extracting it will require good ideas, theory, and large-scale simulations.