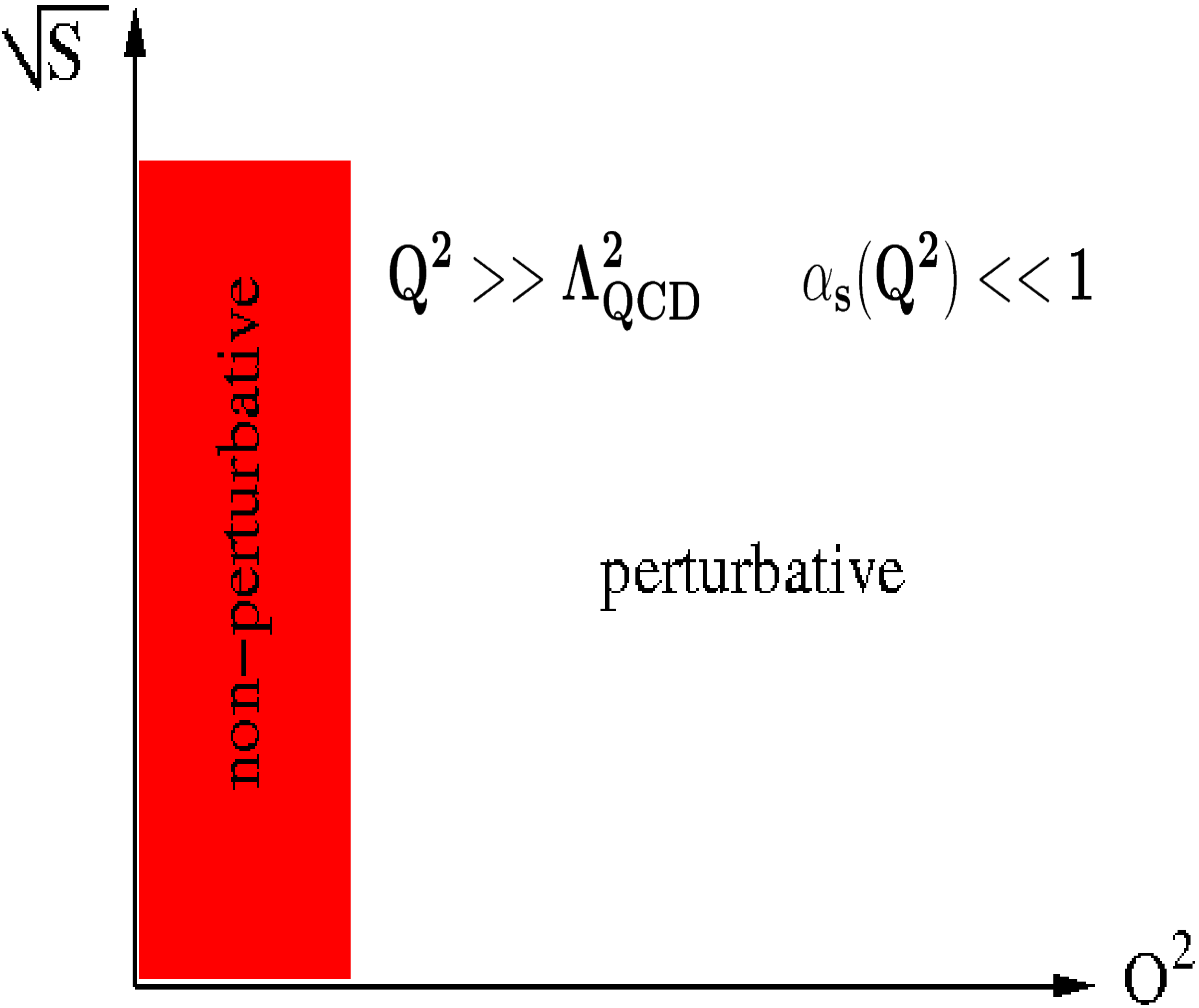
Introduction to High Energy Nuclear Collisions II (QCD at high gluon density)

Jamal Jalilian-Marian
Baruch College, City University of New York

Perturbative QCD

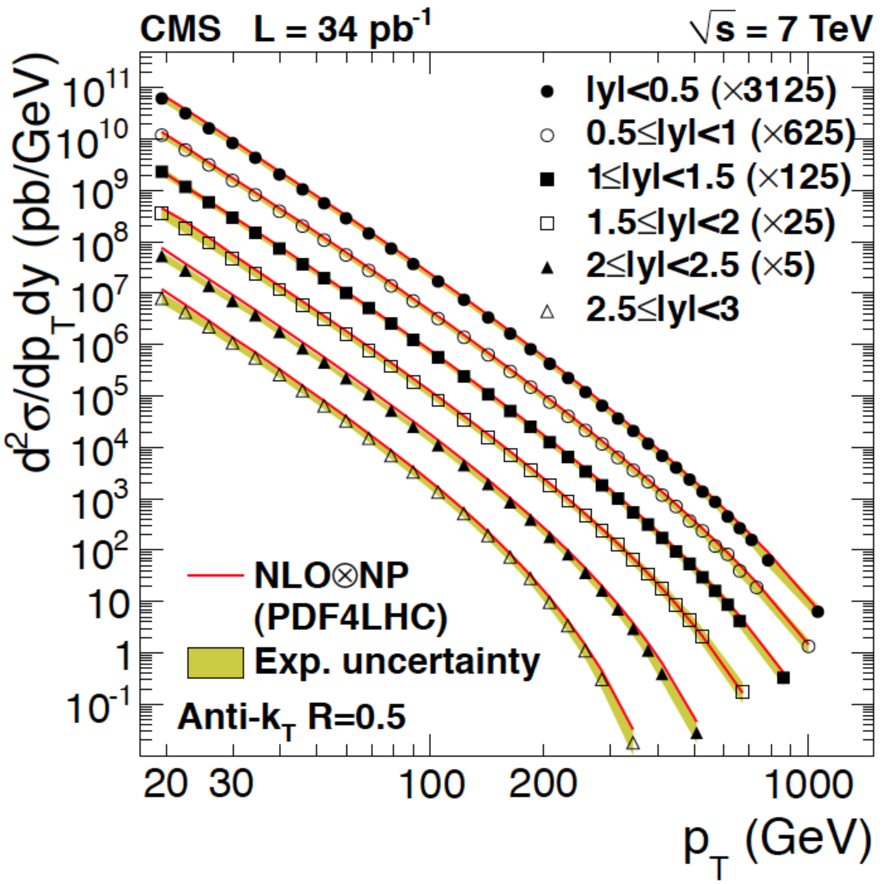


pQCD tools:
twist expansion,
collinear factorization

$$\sqrt{\mathbf{S}} \to \infty$$

$$\mathbf{Q^2} \to \infty$$

pQCD: a success story



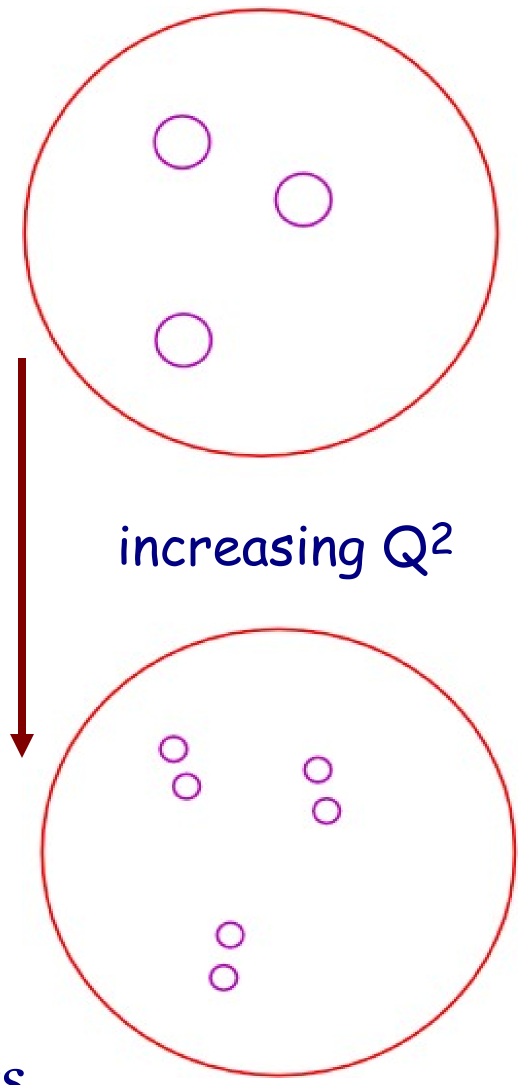
but bulk of QCD phenomena happens at low Q

so far we have considered PQCD in the Bjorken limit

$$\mathrm{Q}^2,\ \mathrm{S} \to \infty \quad x_{Bj} \equiv \frac{\mathrm{Q}^2}{\mathrm{S}} \ \textit{fixed}$$

DGLAP evolution of partons

number of partons increases with Q^2
but parton number density decreases
hadron becomes more dilute



Excellent tool for high Q^2 inclusive observables

higher twists become important at low Q^2

Not designed to treat collective phenomena:

shadowing
multiple scattering
diffraction
impact parameter dependence
....

Extension beyond leading twist is very difficult

many-body dynamics hidden in parameters

QCD in the Regge-Gribov limit

recall $x_{Bj} \equiv \frac{Q^2}{S}$

$S \to \infty \,,\; Q^2 \text{ fixed} : x_{Bj} \to 0$

Regge

Gribov

why QCD at high energy/small x?

why QCD at small x?

many formal as well as practical reasons

QCD dynamics at high energy

beyond partons: collective dynamics in a hadron/nucleus wave function

initial state/early stage of high energy nuclear collisions

cosmic rays/ultra high energy neutrinos

....

Remember the light cone variables

$$\mathbf{P^+ \equiv \frac{E + P_z}{\sqrt{2}}\ , P^- \equiv \frac{E - P_z}{\sqrt{2}}\ , P_t = P_t}$$

Define rapidity

$$\mathbf{y \equiv \frac{1}{2}\ln\frac{P^+}{P^-} = \frac{1}{2}\ln\frac{E + P_z}{E - P_z} \rightarrow v_z}$$

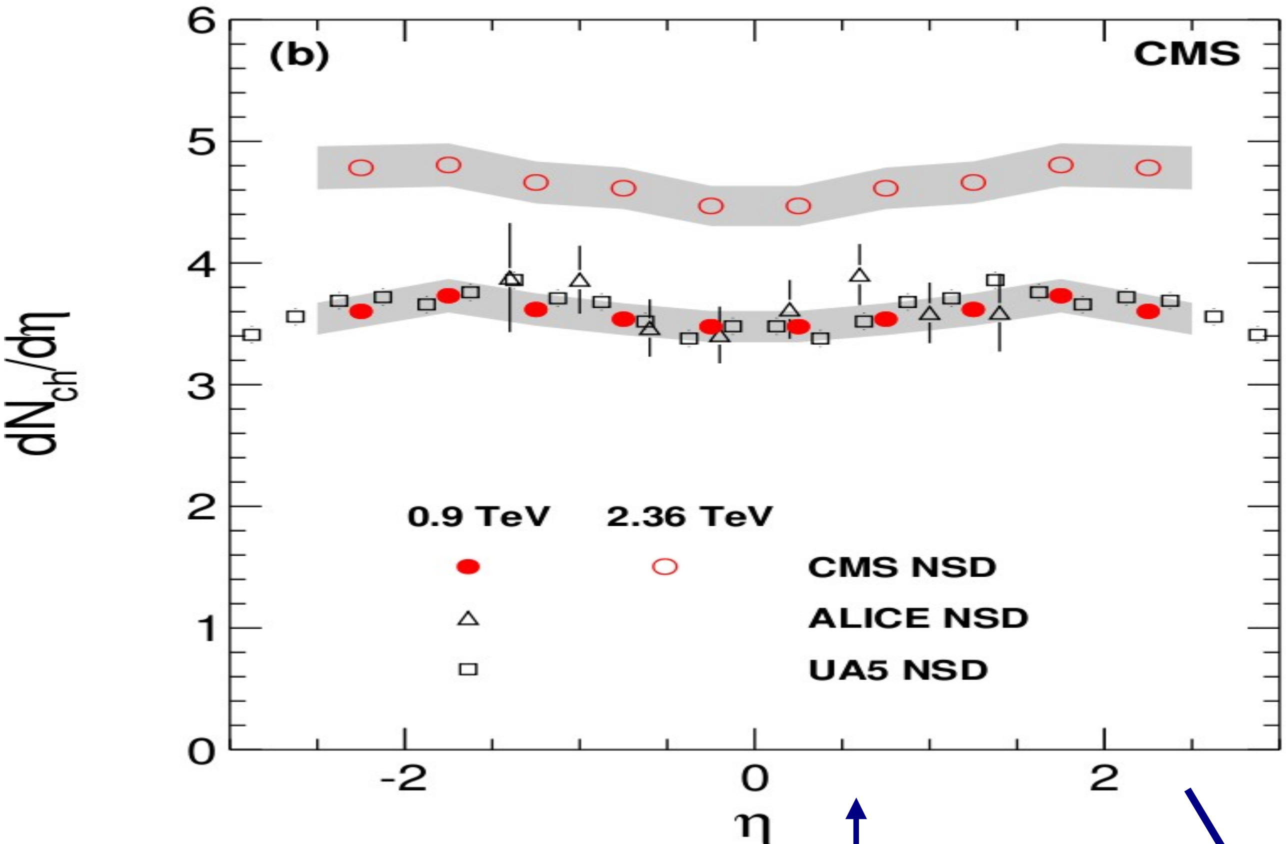
Under boosts: $\mathbf{y \rightarrow y + \Psi}$

Define pseudo-rapidity

$$\eta \equiv \mathbf{-\ln\tan\frac{\theta}{2}}$$

$$\mathbf{y = \ln\frac{\sqrt{m^2 + p_t^2\cosh^2\eta} + p_t\sinh\eta}{\sqrt{p_t^2 + m^2}}}$$

pp collisions at LHC

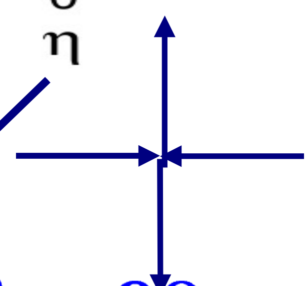


7 TeV data now available

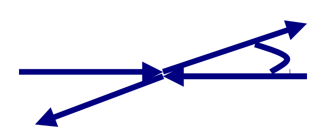
most particles produced with p_t < 1 GeV

$$\frac{p_t}{\sqrt{S}} e^{-\eta} \rightarrow 0$$

$$y \sim \ln \frac{\sqrt{S}}{M_N}$$

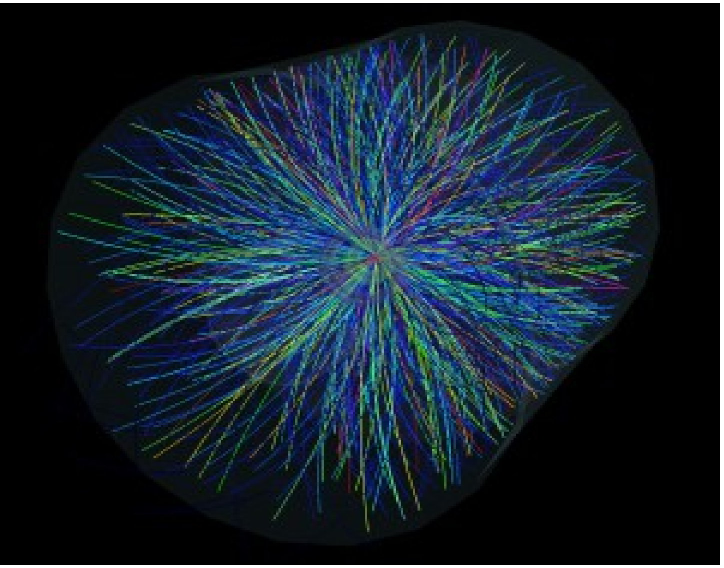


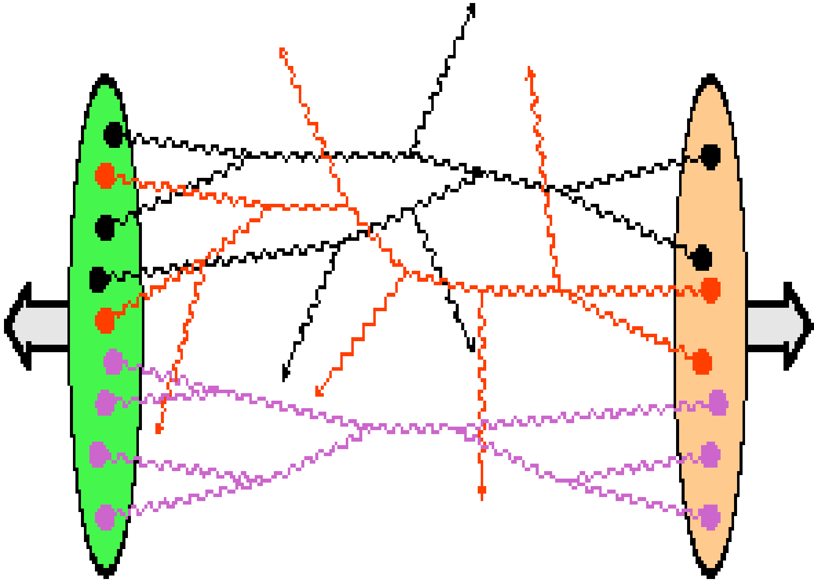
mid-rapidity $\theta = 90$



forward-rapidity $\theta \rightarrow 0$

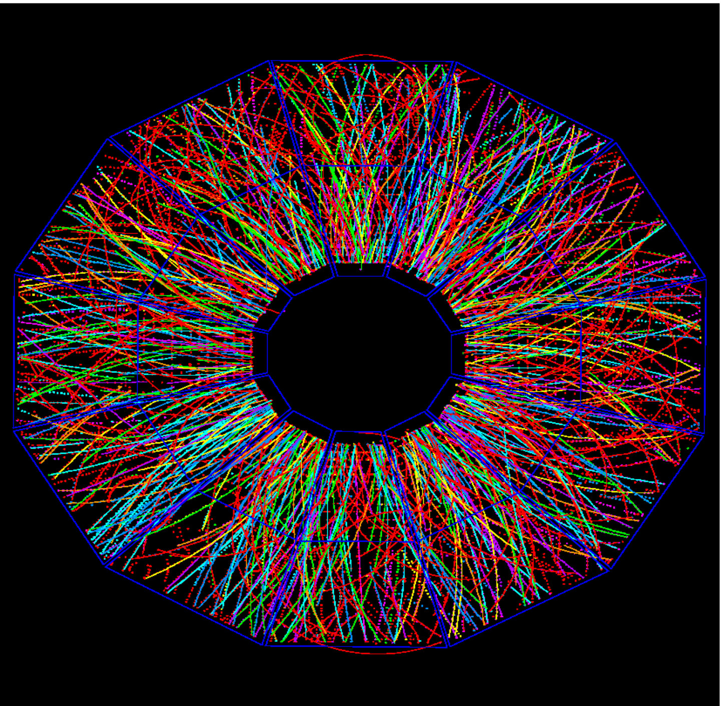
Nucleus-nucleus collisions at LHC





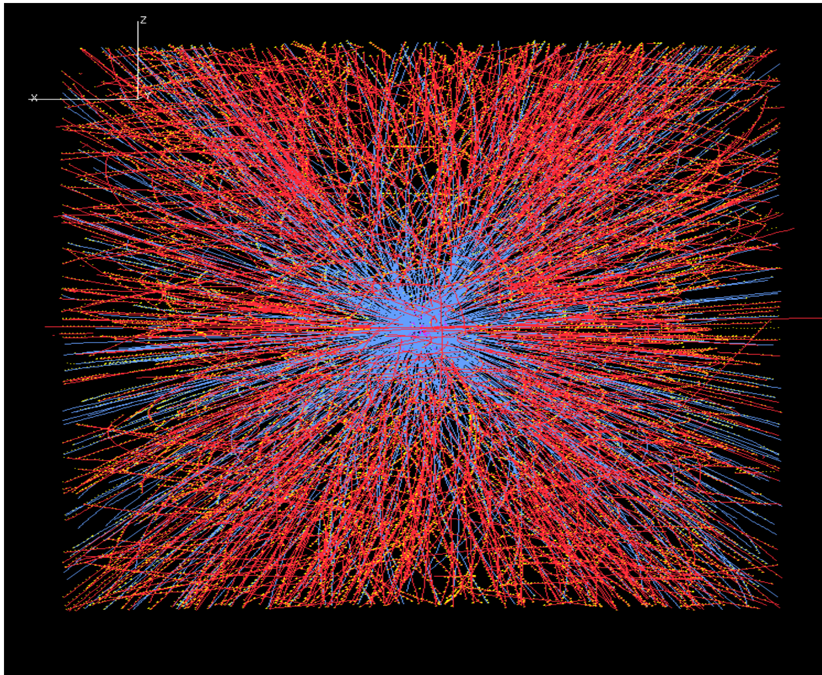
$$\frac{dN_{ch}}{d\eta} \sim 1600$$

at LHC

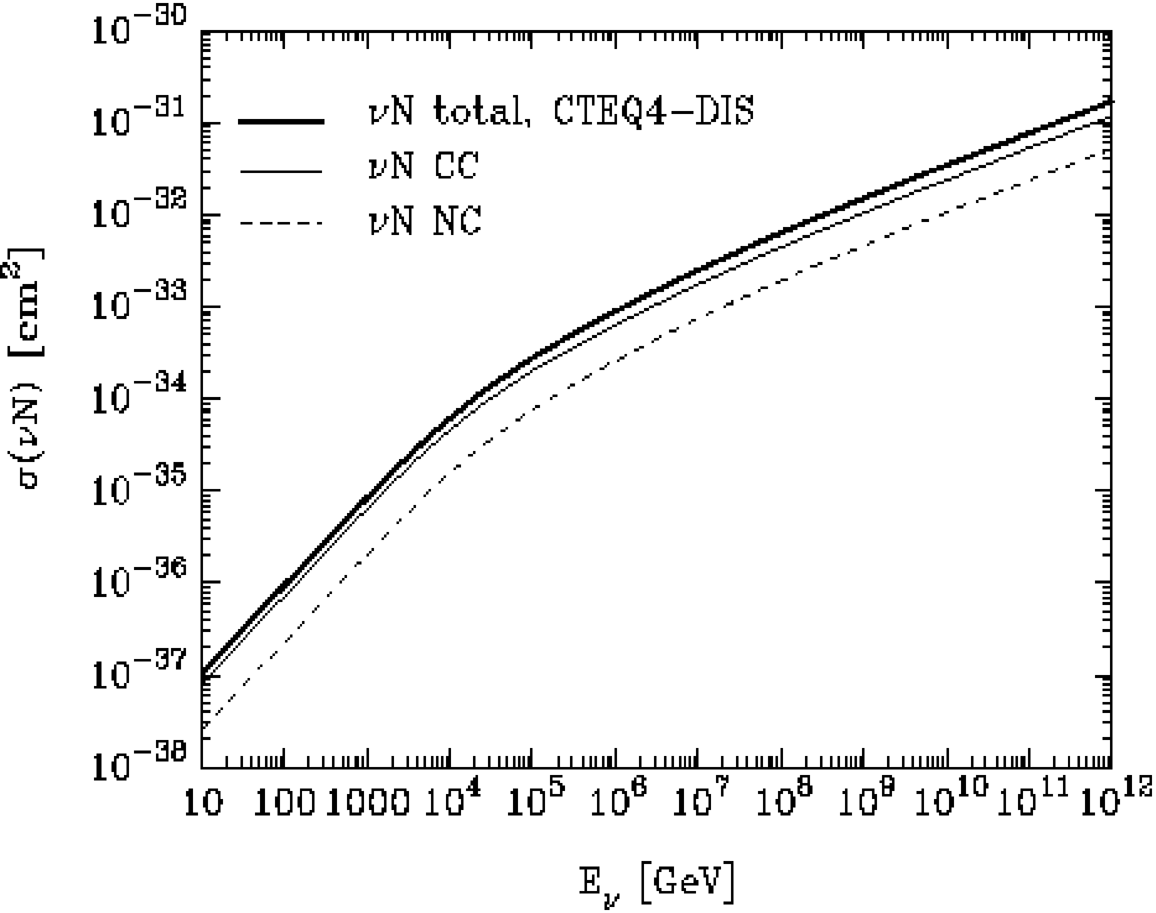


$$\frac{dN_{ch}}{d\eta} \sim 700$$

at RHIC



Ultra-High Energy Neutrinos



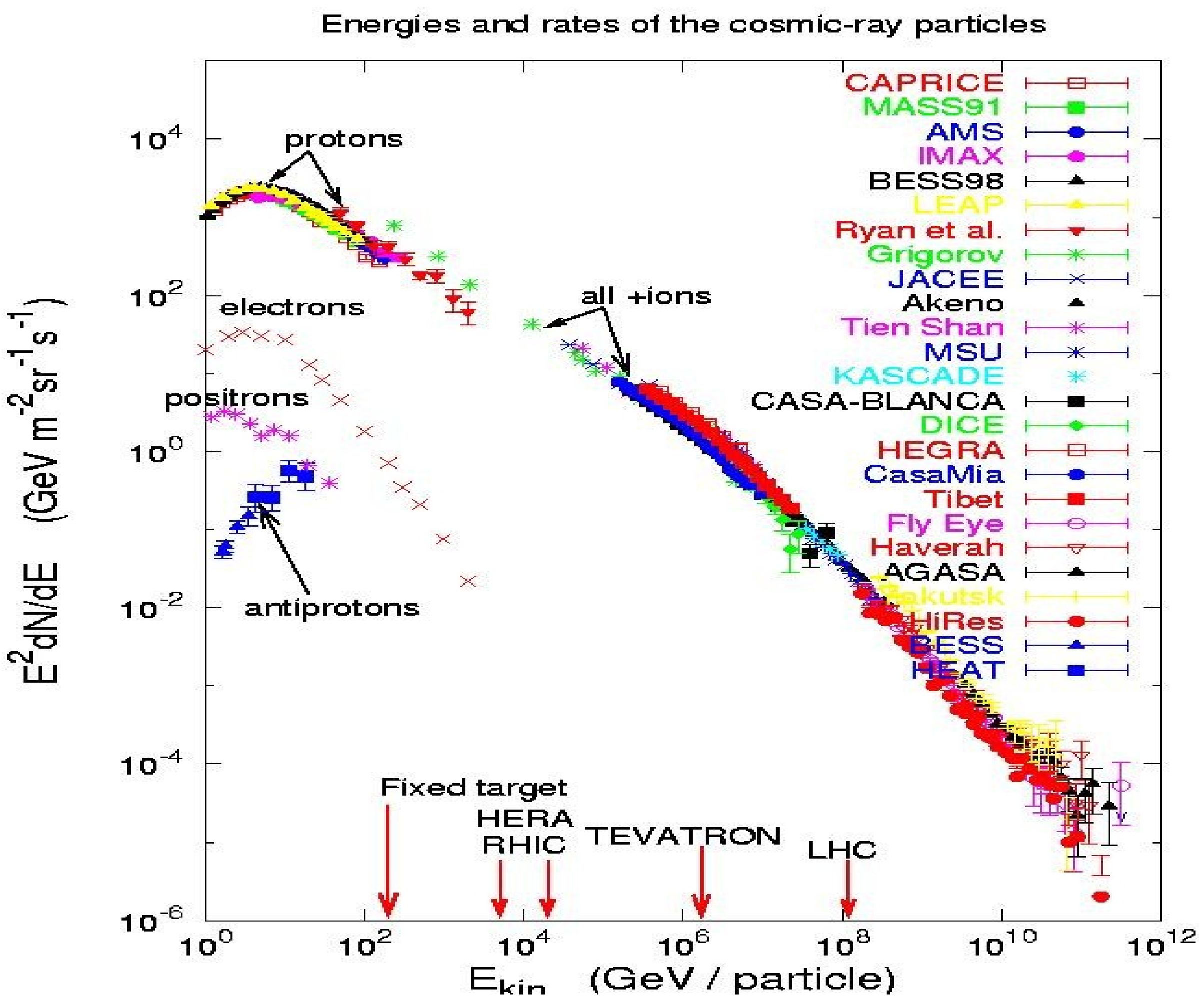
$$\nu\,\mathbf{N} \to \nu\,\mathbf{X}$$

$$\sqrt{\mathbf{S}} \sim \mathbf{10^{2-3}TeV}$$

total cross section dominated by Q ~ M_z

$$\frac{\mathbf{M_z}}{\sqrt{\mathbf{S}}} \to \mathbf{0}$$

High Energy Cosmic Rays



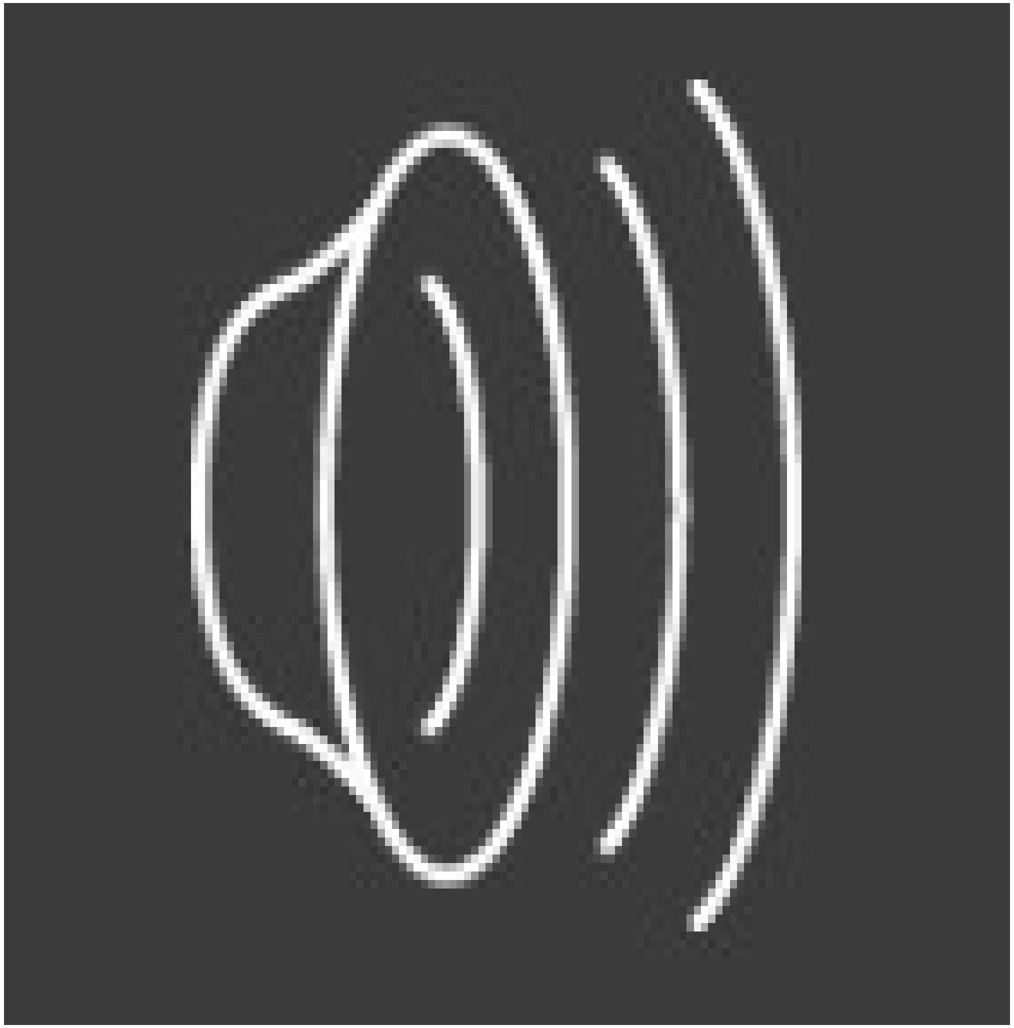
$$\mathbf{p\ A \rightarrow X}$$

$$\sqrt{S} \sim 10^{2-3}\,\mathrm{TeV}$$

most particles are produced with low p_t

$$\frac{p_t}{\sqrt{S}} \rightarrow 0$$

High Energy Cosmic Rays



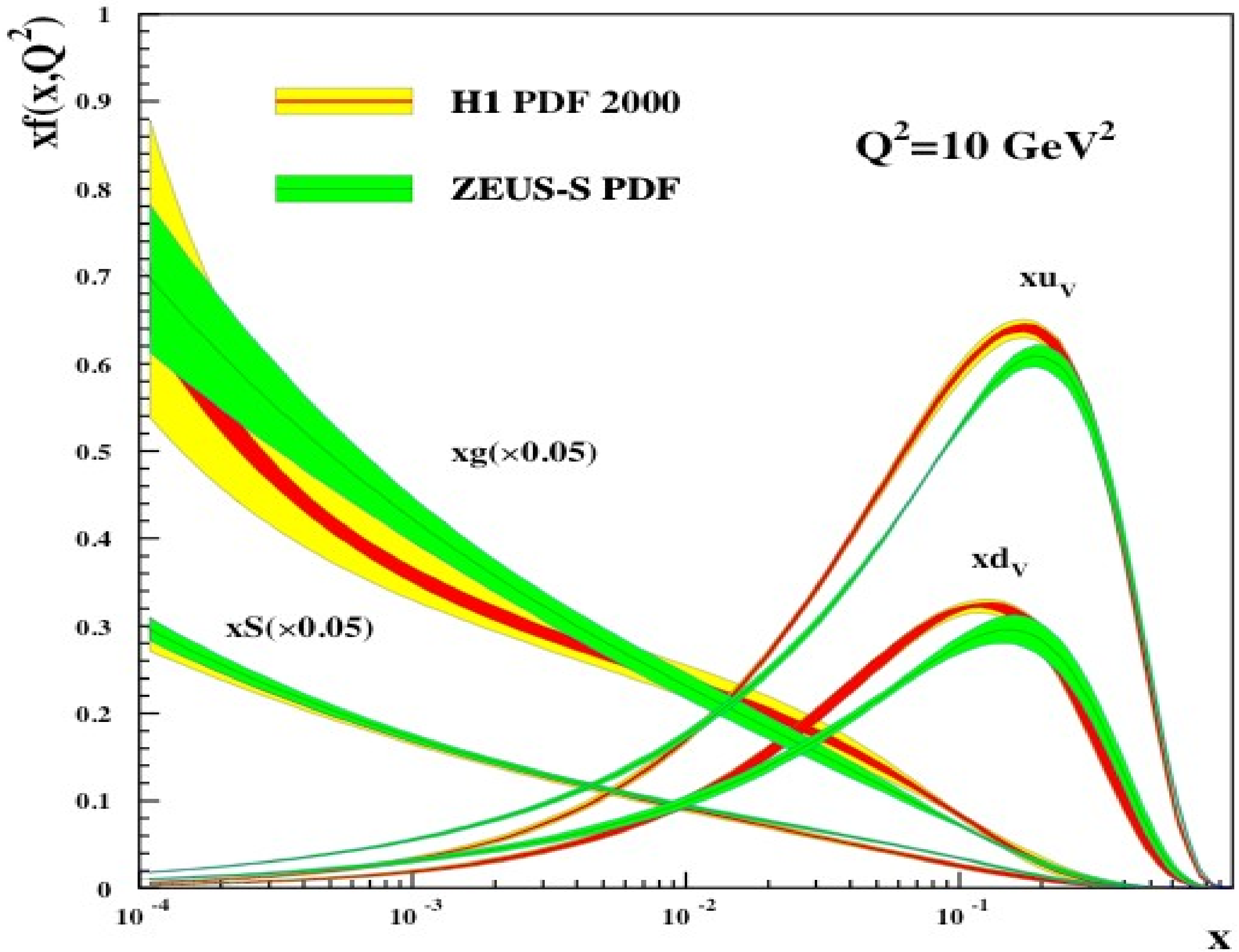
$$\mathbf{p\,A \rightarrow X}$$

$$\sqrt{\mathbf{S}} \sim \mathbf{10^{2-3}TeV}$$

most particles/energy are in the <u>forward rapidity</u> region

$$\frac{\mathbf{p_t}}{\sqrt{\mathbf{S}}}\mathbf{e^{-y}} \rightarrow \mathbf{0}$$

parton distribution functions at small x

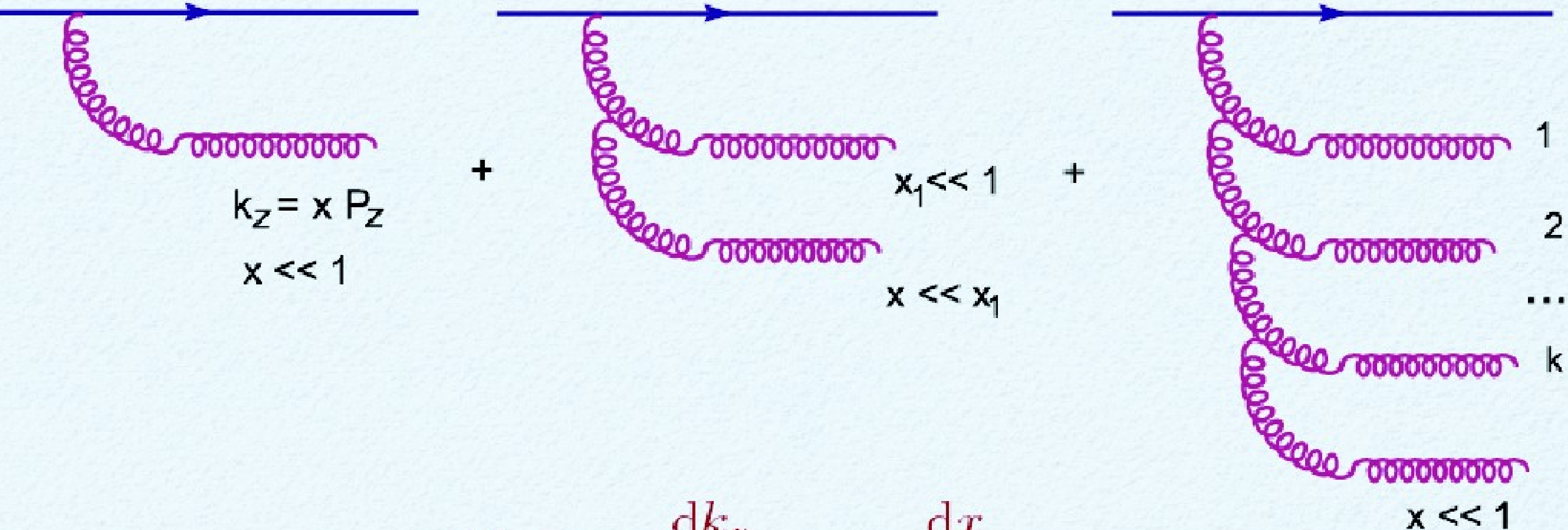


QCD at high energy is dominated by <u>gluons</u>

gluon radiation at small x :pQCD

The infrared sensitivity of bremsstrahlung favors the emission of 'soft' (= small–x) gluons

$$\mathbf{P_{gg}(x) \sim \frac{1}{x} \; for \; x \to 0}$$



$$\mathrm{d}\mathcal{P} \propto \alpha_s \frac{\mathrm{d}k_z}{k_z} = \alpha_s \frac{\mathrm{d}x}{x}$$

The 'price' of an additional gluon:

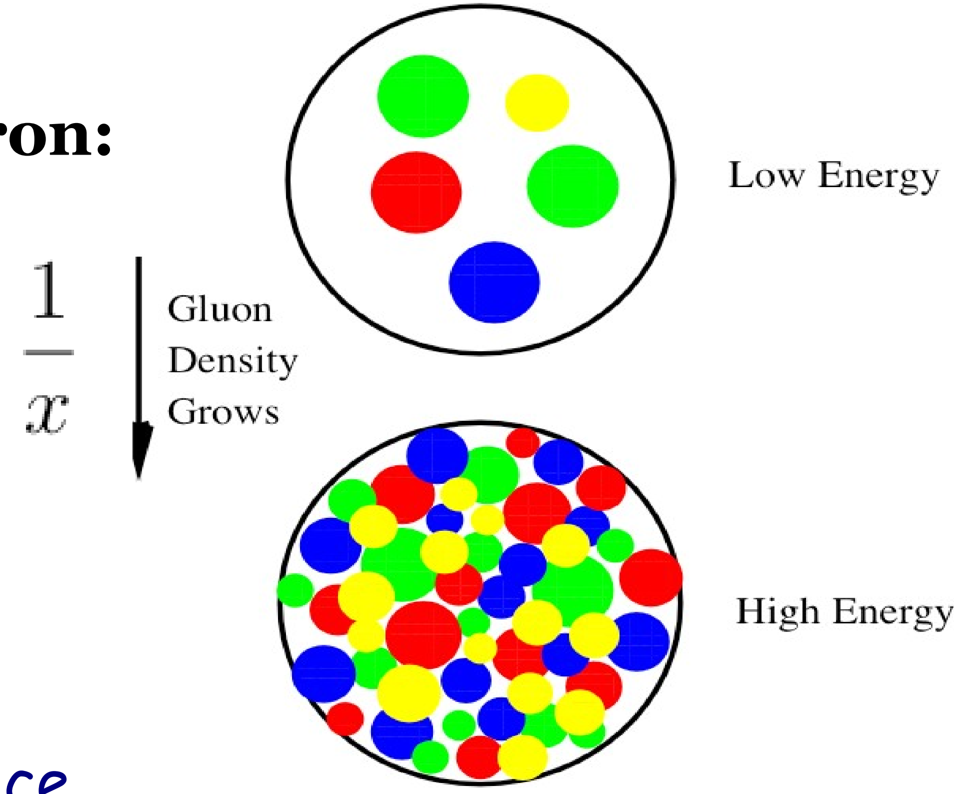
$$\mathcal{P}(1) \propto \alpha_s \int_x^1 \frac{\mathrm{d}x_1}{x_1} = \alpha_s \ln \frac{1}{x}$$

number of gluons grows fast

$$n \sim e^{\alpha_s \, ln 1/x}$$

Resolving the nucleus/hadron:

Regge-Gribov limit



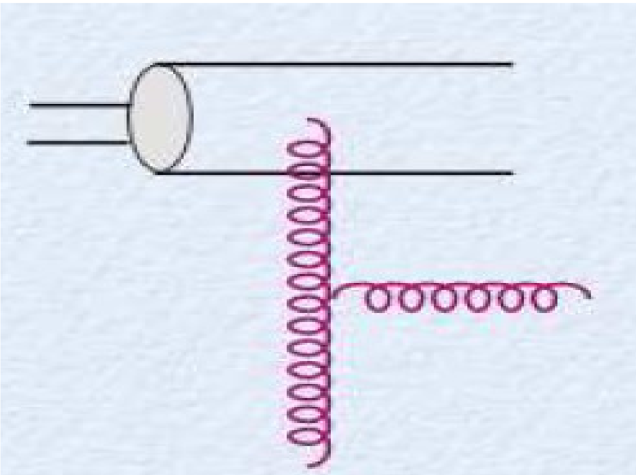
radiated gluons have the same size ($1/Q^2$) - the number of partons increase due to the increased longitudinal phase space

hadron/nucleus becomes a dense system of gluons: concept of a quasi-free parton is not useful

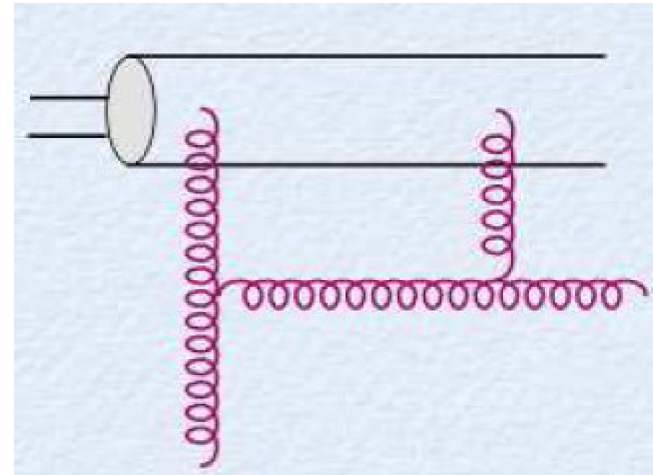
Physics of strong color fields in QCD, multi-particle production- possibly discover novel universal properties of theory in this limit

Why does pQCD break down at small x?

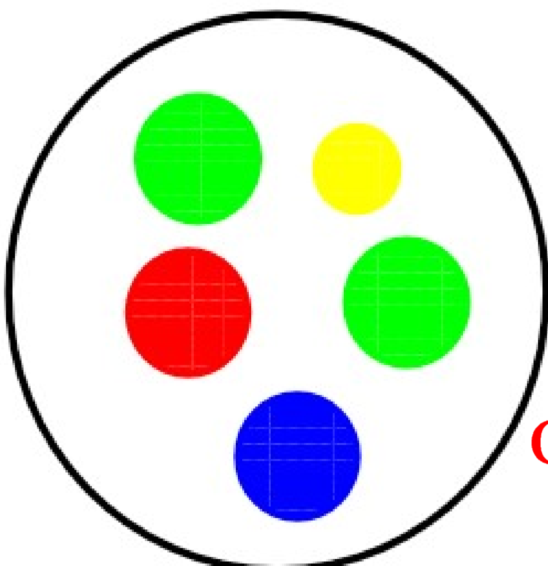
"attractive" bremsstrahlung *vs.* *"repulsive" recombination*



included in pQCD

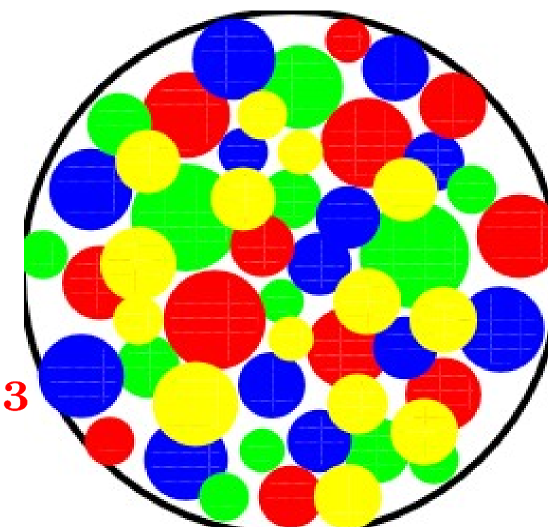


not included in pQCD
(collinear factorization)



$$\frac{\alpha_s}{Q^2}\,\frac{xG(x,Q^2)}{\pi r^2} \sim 1$$

$$Q_s^2(x, b_t, A) \sim A^{1/3}\,\left(\frac{1}{x}\right)^{0.3}$$



The need to go beyond pQCD

High gluon density: multiple scattering

pQCD (col. fact.) includes single scattering only

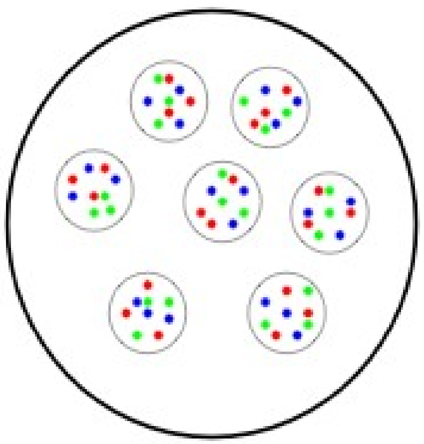
High energy: include ln 1/x corrections

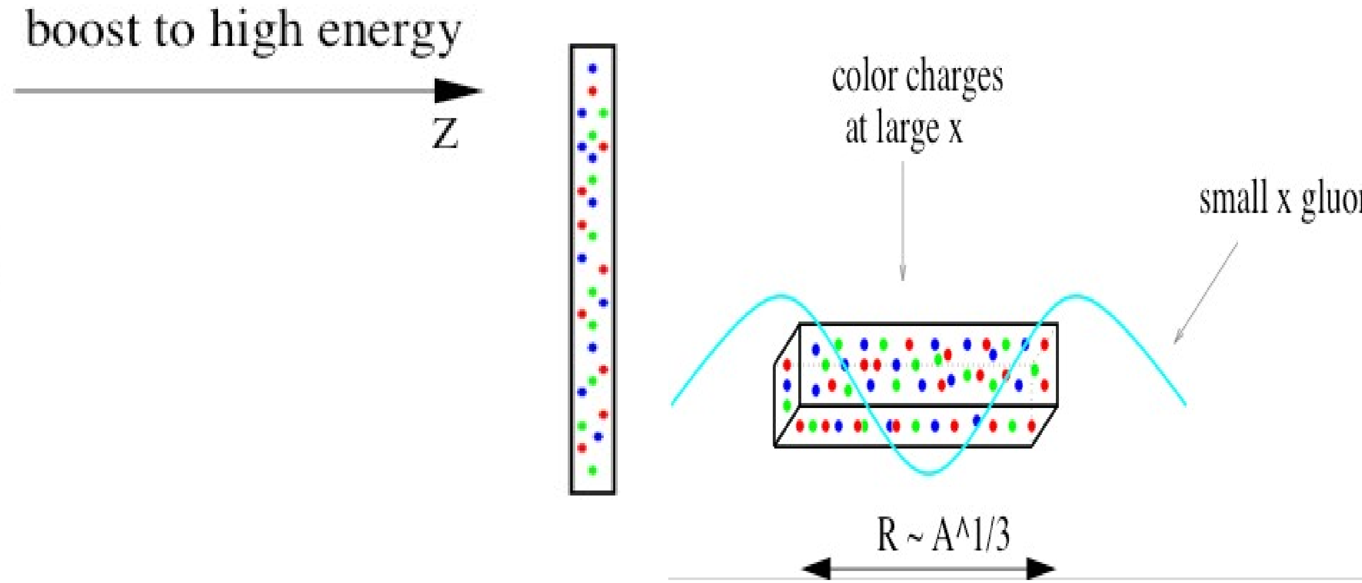
pQCD (DGLAP) includes ln Q2 corrections

we will use an effective action approach

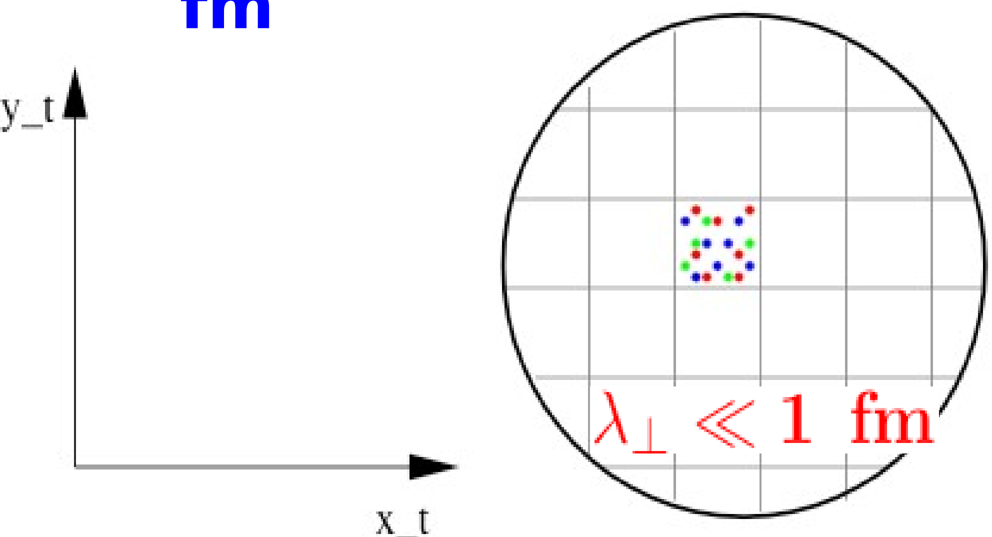
A model of nuclei at high energy

(a system of color charges)





R ~ 6 fm

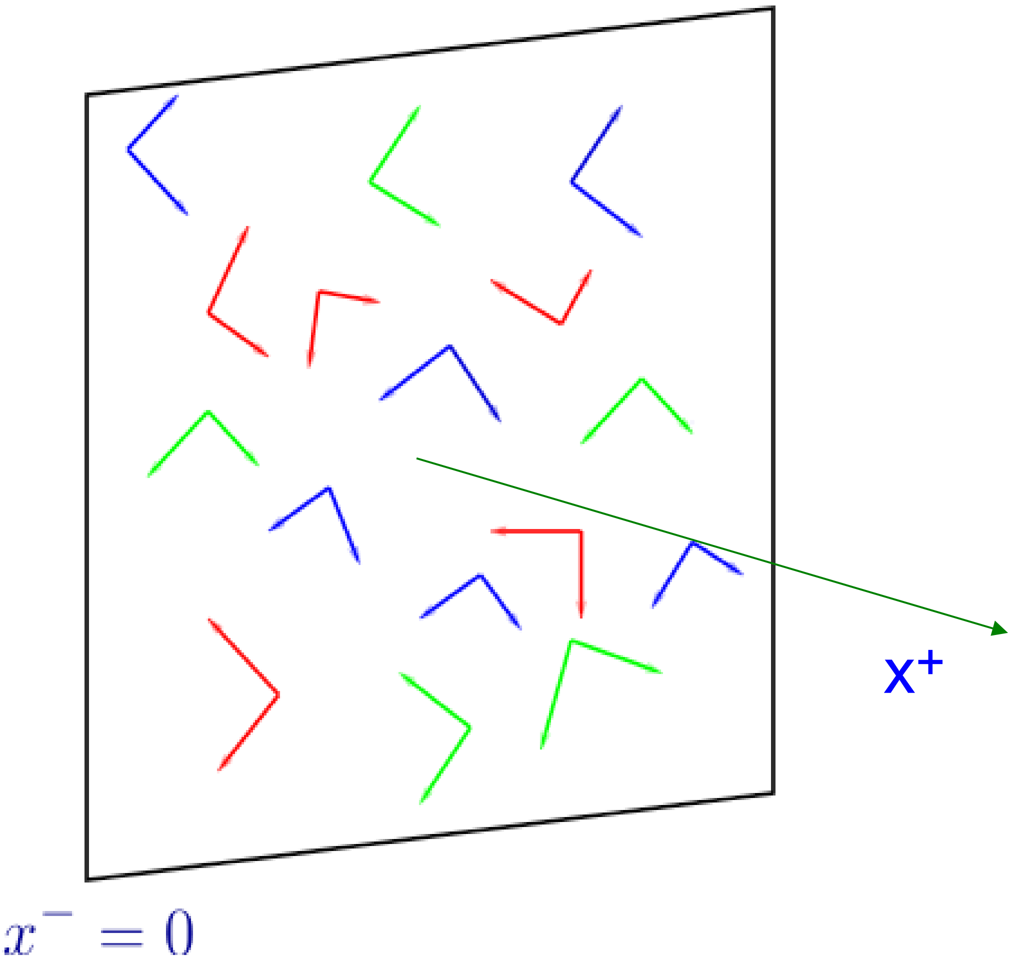


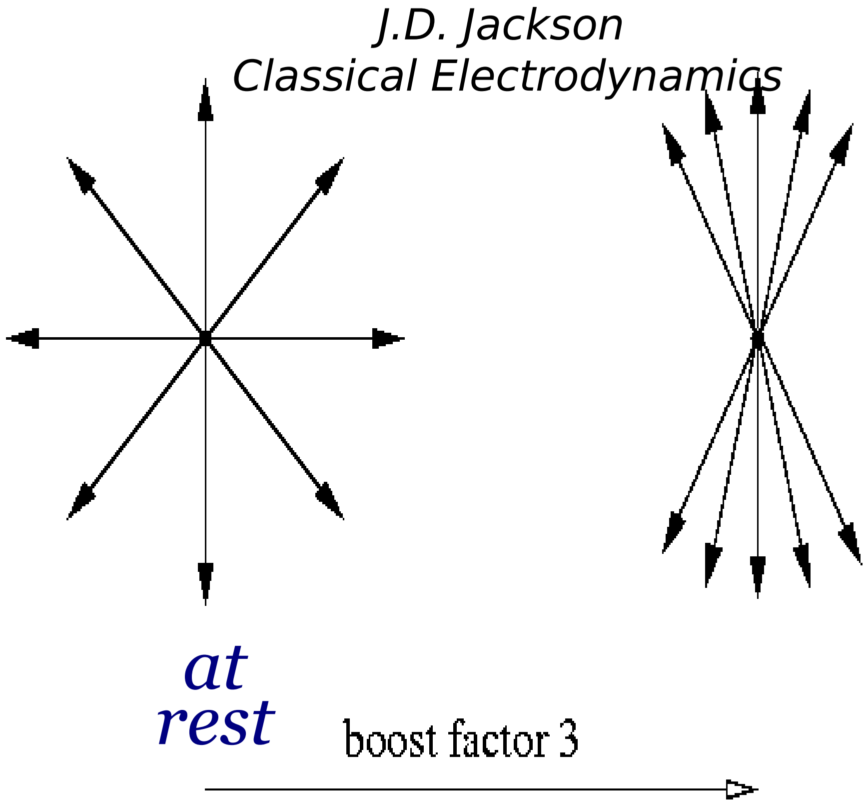
$$R \rightarrow \frac{R}{\gamma}$$

$$\gamma = 100 \quad RHIC$$

$$\gamma = 5500 \quad LHC$$

A model of nuclei at high energy

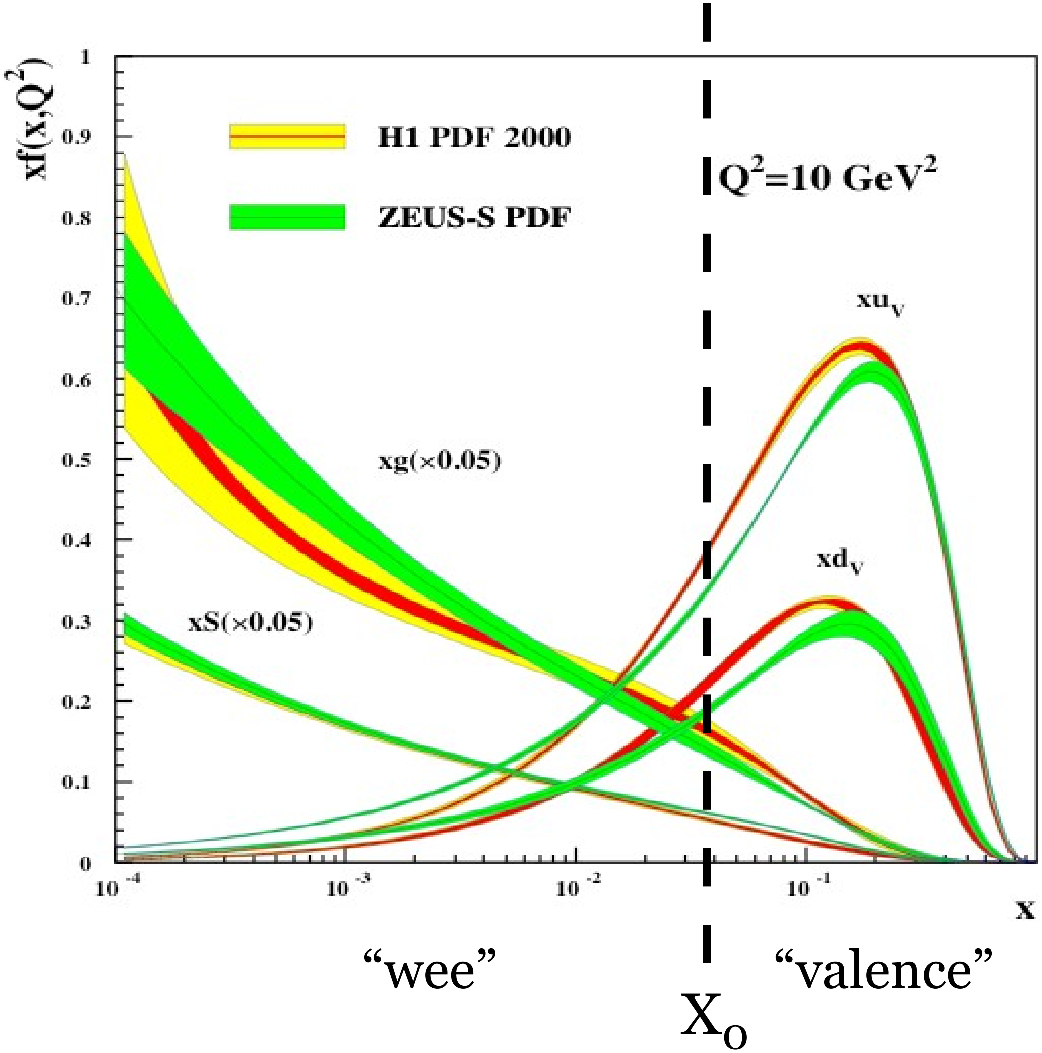




***Electric field** of a*
point charge at high energy

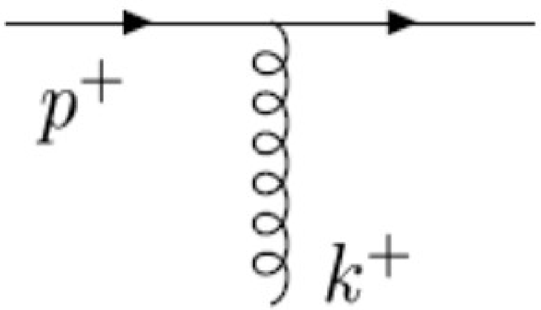
*random **color Electric & Magnetic fields***
in the plane of the fast moving nucleus

high x partons as static color charges ρ



recall for any 4-momentum

$$\mathbf{p^2 = 2p^+p^- - p_t^2}$$

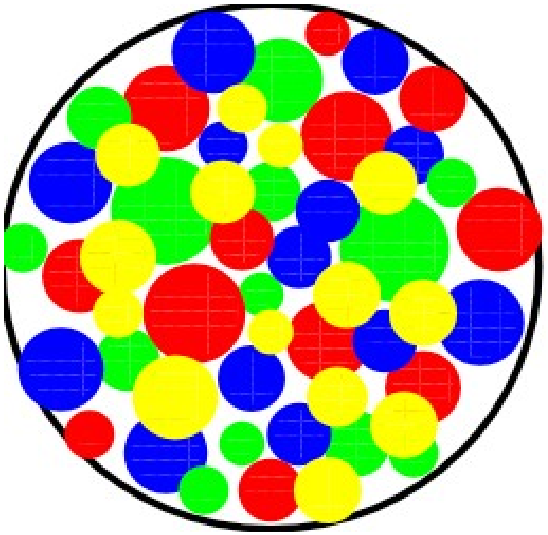


small x: $\mathbf{k^+ << p^+}$

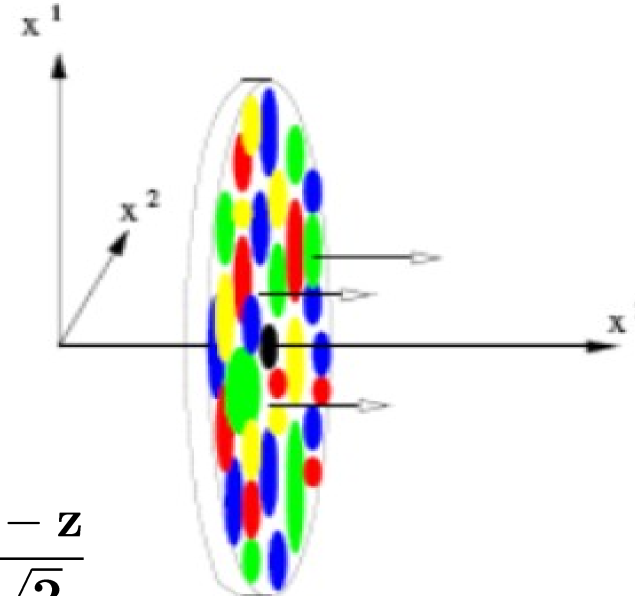
$$\tau_{\mathrm{val}} \sim \frac{1}{p^-} = \frac{2p^+}{k_t^2} \gg \tau_{\mathrm{wee}} \sim \frac{1}{k^-} = \frac{2k^+}{k_t^2}$$

natural time scale for “valence” partons is much larger than that of “wee” partons

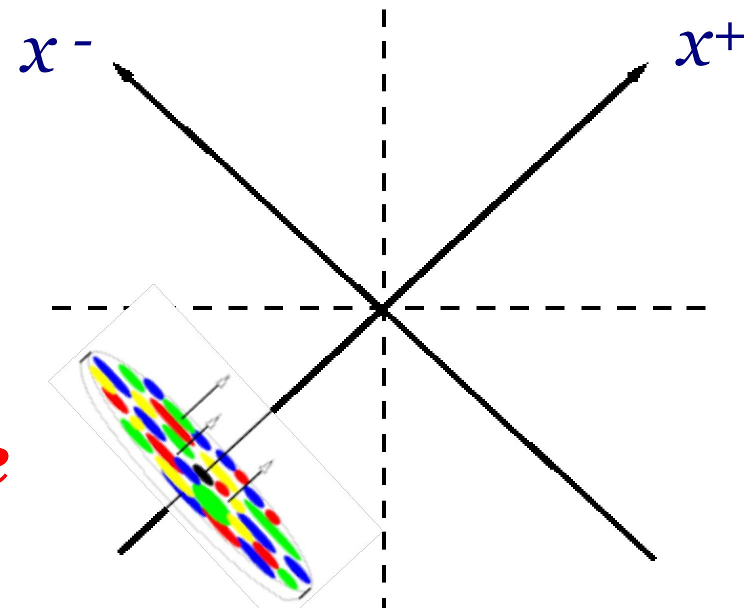
A model of nuclei at high energy



boost →



$$\mathbf{x^+} \equiv \frac{\mathbf{t + z}}{\sqrt{\mathbf{2}}} \qquad \mathbf{x^-} \equiv \frac{\mathbf{t - z}}{\sqrt{\mathbf{2}}}$$



sheet of color charge moving along x^+ and sitting at $x^- = 0$

$$\mathbf{J^\mu_a(x)} \equiv \delta^{\mu +}\, \delta(\mathbf{x^-})\, \rho_{\mathbf{a}}(\mathbf{x_t})$$

color current *color charge*

current has only one large component

static color charges ρ

Let's see what kind of color fields they generate

Solve the classical equations of motion

$$\mathbf{D}_\mu \, \mathbf{F}^{\mu\nu}_{\mathbf{a}} = \mathbf{g}\, \mathbf{J}^\nu_{\mathbf{a}}$$

solution *(in light cone gauge $A^+ = 0$) :*

can not be inverted

$$A^-_a = 0 \qquad A^a_i = \theta(x^-)\, \alpha^a_i(x_t)$$

with

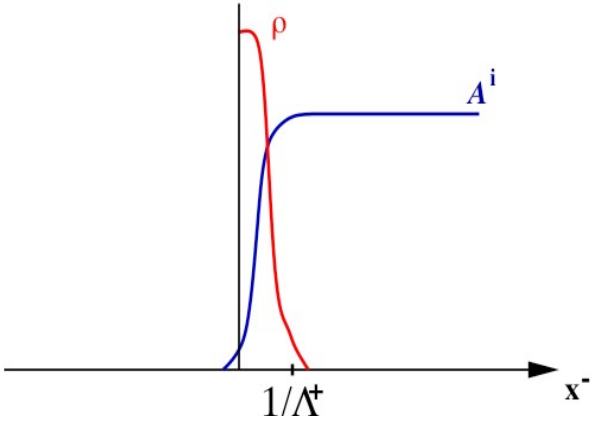
$$\partial_i \alpha^a_i(x_t) = g\, \rho^a(x_t) \qquad \alpha_i = \frac{i}{g} U(x_t) \partial_i U^\dagger(x_t)$$

solution is a 2-d pure gauge

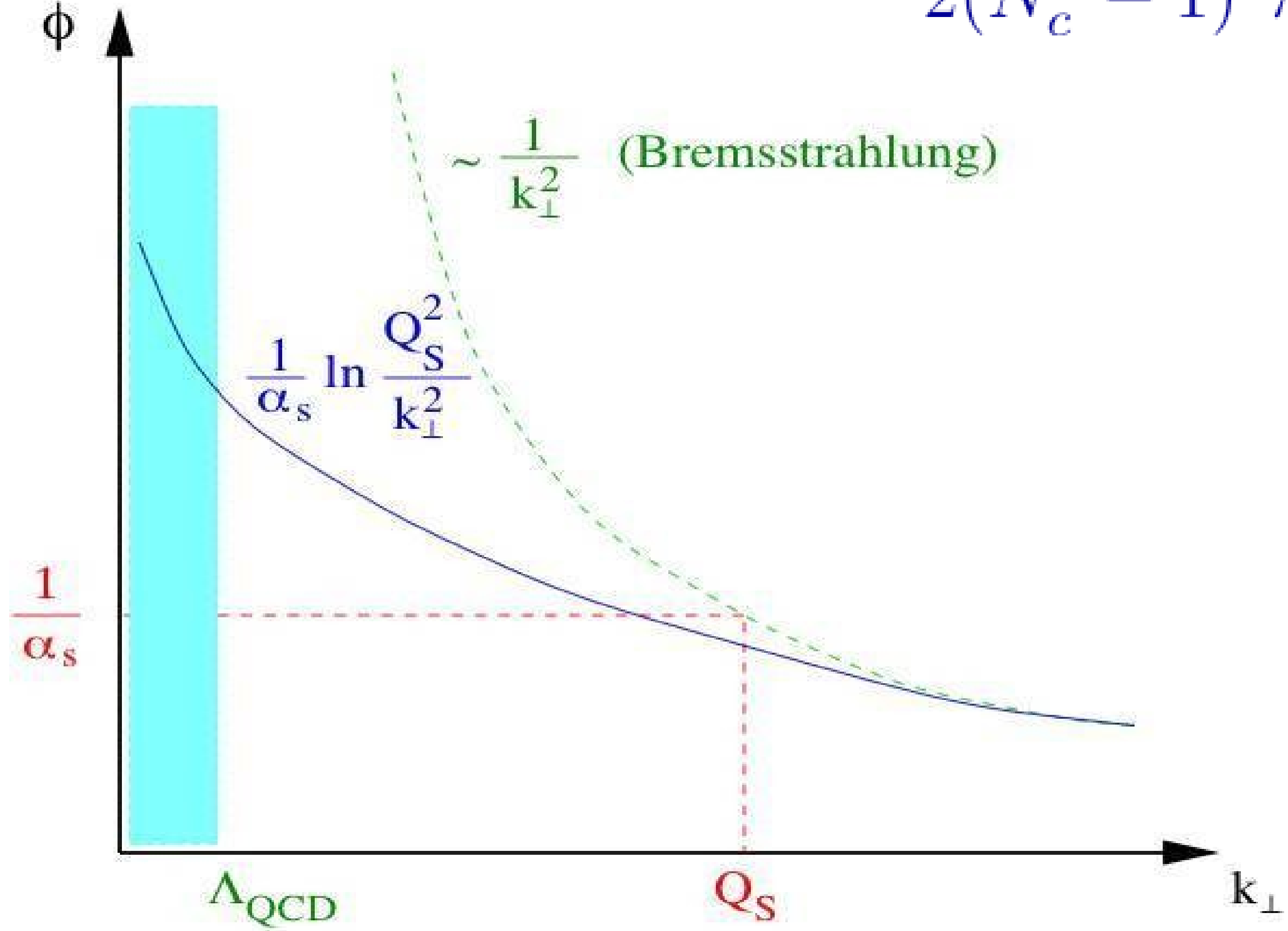
it is (LC) time-independent

the only "physical" color field is

$$\mathbf{F}^{+\,\mathbf{i}} \sim \delta(\mathbf{x}^-)\alpha^{\mathbf{i}} \neq \mathbf{0} \quad (\mathbf{F}^{\mathbf{i}\,\mathbf{j}} = \mathbf{0})$$

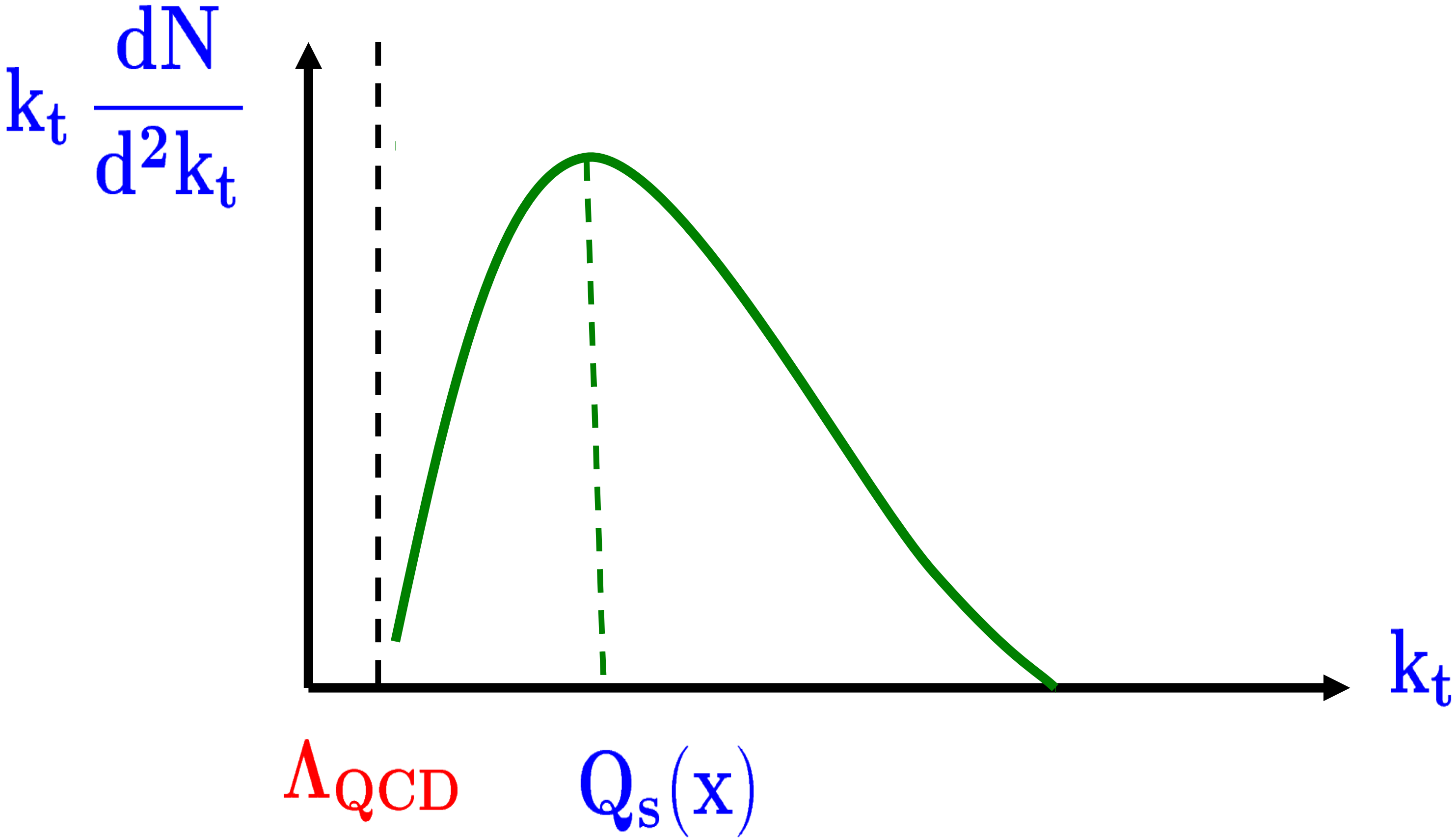


compute the gluon distribution function <Ai Ai> $\frac{(2\pi)^3}{2(N_c^2-1)}\frac{dN}{\pi R^2 d^2k_\perp dy}$



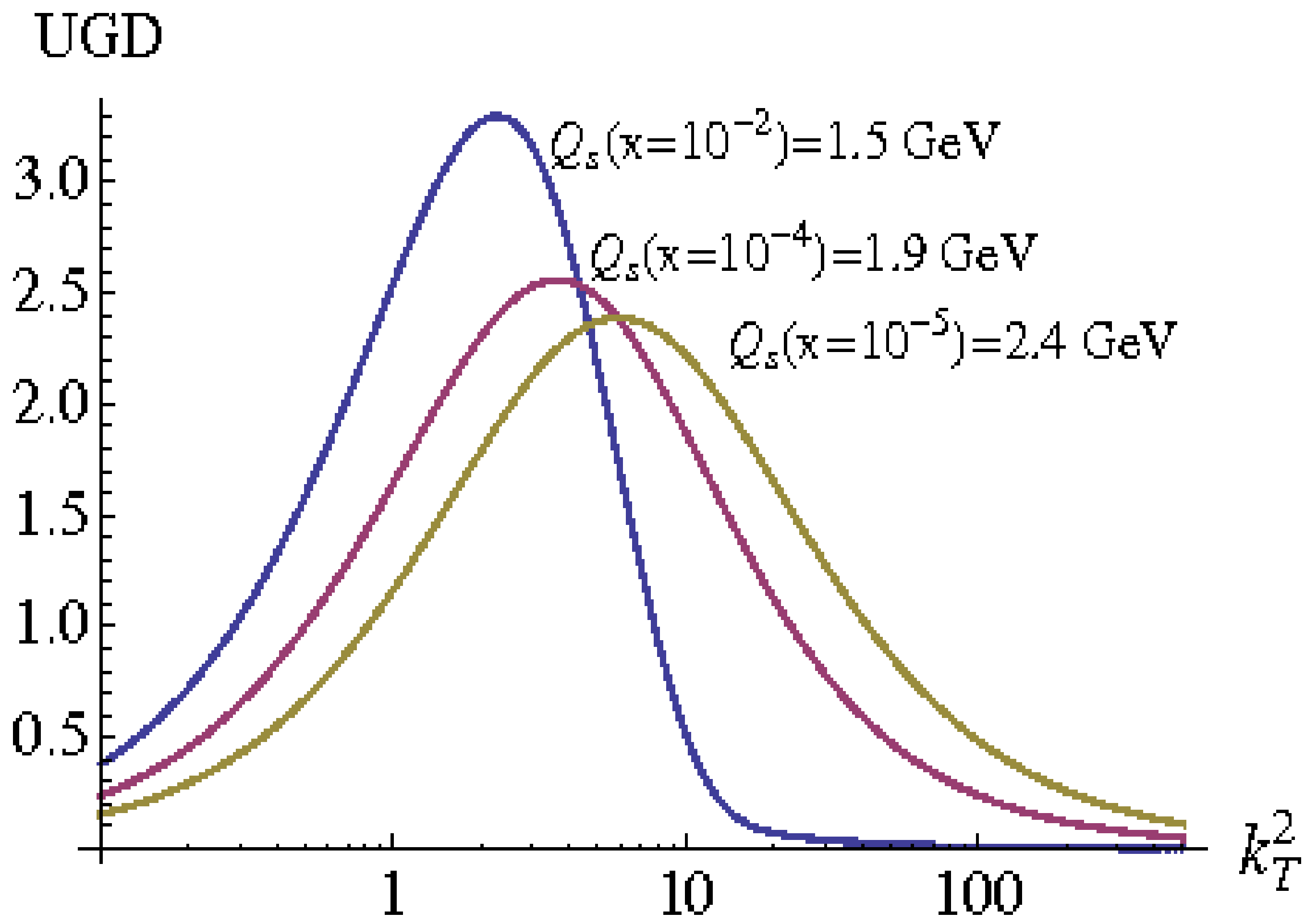
$$xG(x,Q^2) \sim \int d^2k_t \frac{dN}{d^2k_t}$$

Small x gluons in a hadron

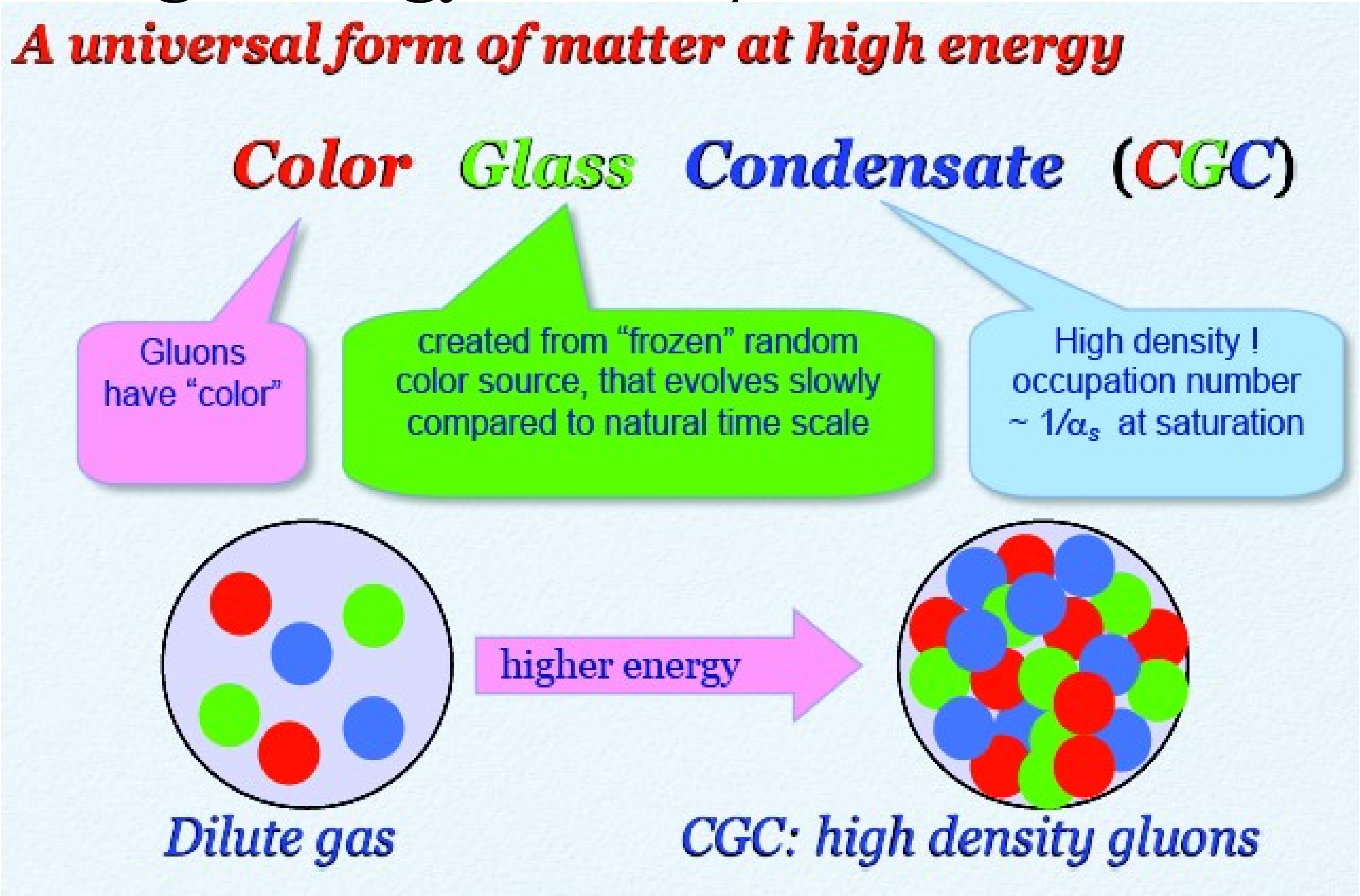


$Q_s(x, b_t, A)$ ***can provide a <u>hard</u> infrared cutoff***

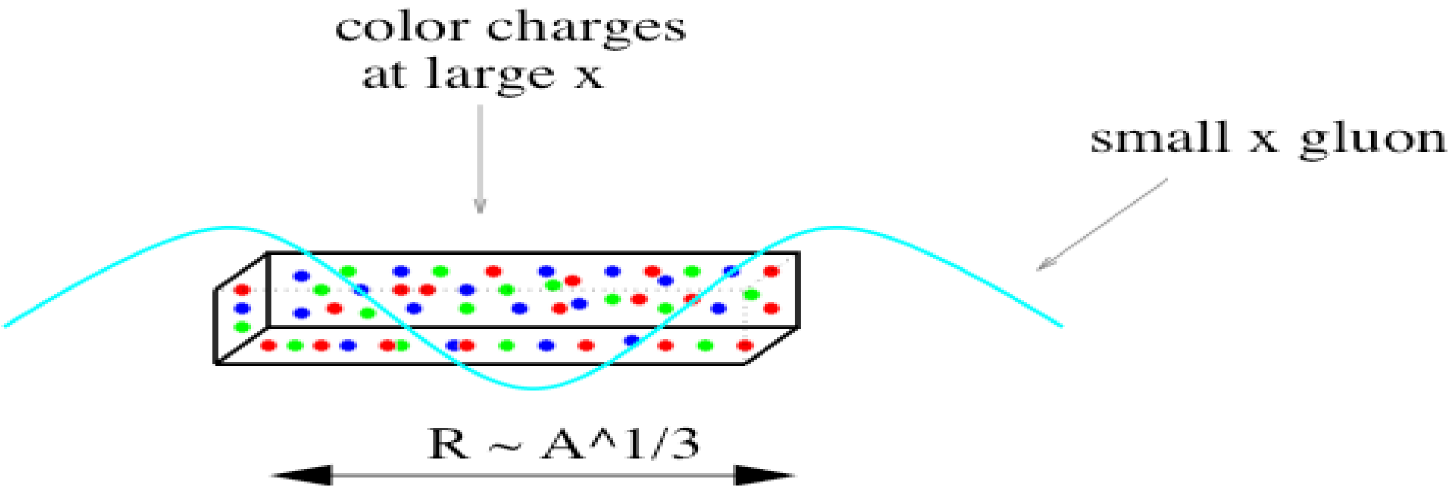
what happens if you try to put more gluons in?



A high energy hadron/nucleus as a CGC



The nuclear "oomph" factor

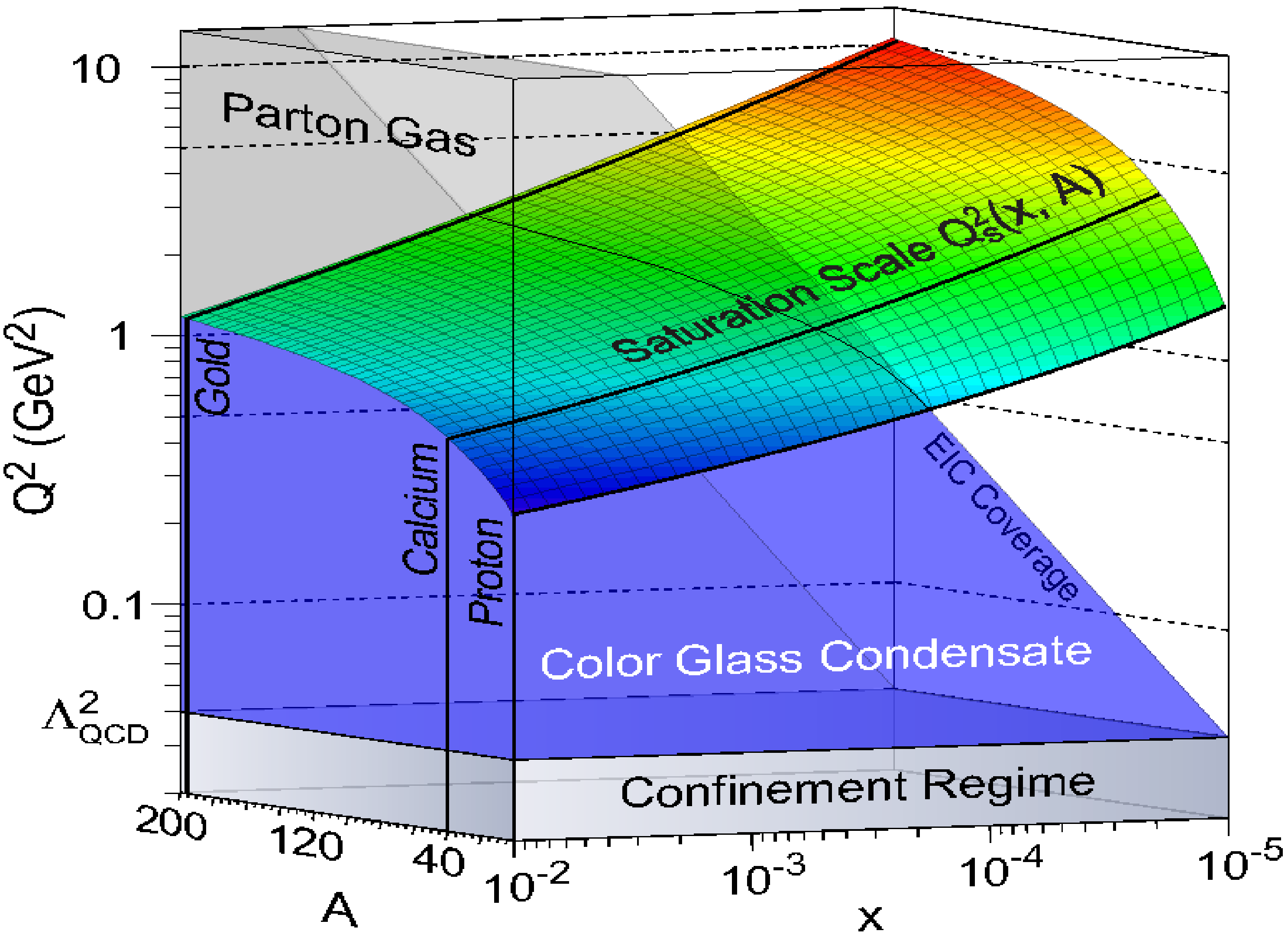


$$Q_s^2(x, A) \sim \frac{A^{1/3}}{x^\delta}$$

$\delta \sim 0.3$

$$\alpha_s(Q_s^2) \ll 1$$

The Saturation Scale Q_s



X 9/4 for gluons

The classical field

It looks like (in LC gauge) a shock wave

its strength is O(1/g) (dense system of gluons: state with maximum occupation number)

keep in mind gauge fields are gauge dependent

how does the solution look like in another gauge?

***solution** (in covariant gauge $\partial_\mu \tilde{\mathbf{A}}^\mu = \mathbf{0}$) :*

$$\tilde{\mathbf{A}}^\mu_{\mathbf{a}} = \delta^{\mu +}\tilde{\alpha}_{\mathbf{a}} \quad \textit{with} \quad \partial^{\mathbf{2}}_{\mathbf{t}}\tilde{\alpha}_{\mathbf{a}}(\mathbf{x_t}) = -\mathbf{g}\,\tilde{\rho}_{\mathbf{a}}(\mathbf{x_t})$$

this can be inverted to give

$$\tilde{\alpha}_{\mathbf{a}}(\mathbf{x_t}) = -\frac{\mathbf{1}}{\partial^{\mathbf{2}}_{\mathbf{t}}}\mathbf{g}\,\tilde{\rho}_{\mathbf{a}}(\mathbf{x_t})$$

$$\mathbf{A}^\mu = \mathbf{U}[\tilde{\mathbf{A}}^\mu + \frac{\mathbf{i}}{\mathbf{g}}\partial^\mu]\mathbf{U}^\dagger$$

$$\textit{with} \quad \mathbf{U}^\dagger \equiv \hat{\mathbf{P}}\exp\left\{\mathbf{ig}\int_{-\infty}^{\mathbf{x}^-}\mathbf{dz}^-\,\tilde{\alpha}_{\mathbf{a}}\,\mathbf{T_a}\right\}$$

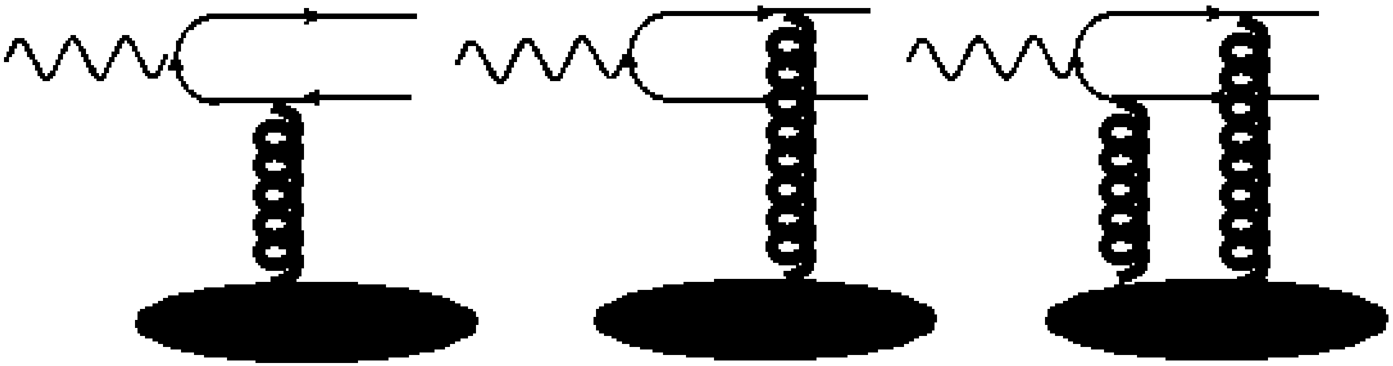
DIS total cross section

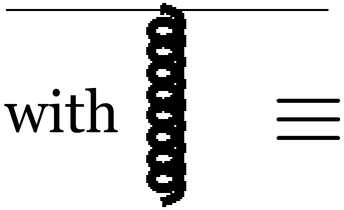
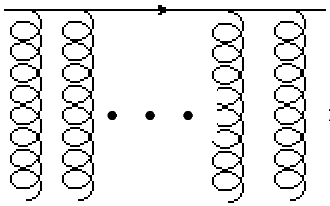
need to generalize the parton model to include high gluon density effects

"tree level" first (no quantum corrections yet)

let's consider the amplitude for $\gamma^\star \mathbf{T} \rightarrow \mathbf{q}\,\bar{\mathbf{q}}\,\mathbf{X}$

F. Gelis, J. Jalilian-Marian
PRD67 (2003) 074019



with  $\equiv$  $= \mathbf{2}\pi\delta(\mathbf{p}^- - \mathbf{q}^-)\gamma^- \int \mathbf{d^2 x_t e^{-i(p_t - q_t)\cdot x_t}}[\mathbf{V}(\mathbf{x_t}) - \mathbf{1}]$

$U(x_t)$ re-sums multiple scattering of a quark from the target represented by the classical field

DIS total cross section

$$\gamma^\star \mathbf{T} \to \mathbf{q}\,\bar{\mathbf{q}}\,\mathbf{X}$$

$$\mathcal{M}^\mu(k|q,p) = \frac{i}{2}\int \frac{d^2 l_t}{(2\pi)^2}\int d^2x_t d^2y_t e^{il_t\cdot x_t} e^{i(p_t+q_t-k_t-l_t)\cdot y_t}$$

$$\left[V(x_t)V^\dagger(y_t) - 1\right]\overline{u}(q)\,\Gamma^\mu(k^\pm, k_t|q^-, p^-, q_t - l_t)\,v(p)$$

with $$\Gamma^\mu \equiv \frac{\gamma^-(\not{Q} - \not{L} + \mathbf{m})\gamma^\mu(\not{Q} - \not{K} - \not{L} + \mathbf{m})\gamma^-}{\mathbf{p}^-[(\mathbf{q_t}-\mathbf{l_t})^2 + \mathbf{m}^2 - 2\mathbf{q}^-\mathbf{k}^+] + \mathbf{q}^-[(\mathbf{q_t}-\mathbf{k_t}-\mathbf{l_t})^2 + \mathbf{m}^2]}$$

$$d\sigma = \frac{d^3q}{(2\pi)^2 2q_0}\frac{d^3p}{(2\pi)^3 2p_0}\frac{1}{2k^-}2\pi\delta(k^- - p^- - q^-)$$
$$\langle \mathcal{M}^\mu(k|q,p)\mathcal{M}^{\nu *}(k|q,p)\rangle_\rho\,\epsilon_\mu(K)\epsilon^*_\nu(K)$$

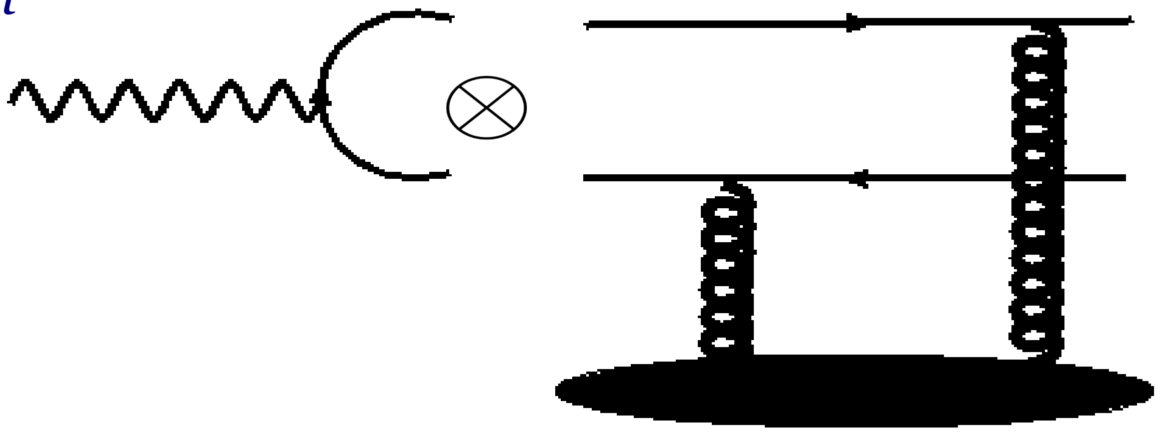
integrate over the quark and anti-quark momenta to get the total cross section

DIS total cross section

$$\sigma^{\mathrm{total}}_{\mathrm{DIS}} = 2\int_0^1 dz \int d^2x_t\, d^2y_t \left|\Psi(k^\pm, k_t | z, x_t, y_t)\right|^2 \sigma_{\mathrm{dipole}}(x_t, y_t)$$

can be written in closed form in terms of Bessel functions K_0, K_1

$$\sigma_{\mathrm{dipole}}(x_t, y_t) \equiv \frac{1}{N_c} \mathrm{Tr} \left\langle 1 - U(x_t) U^\dagger(y_t) \right\rangle$$



total cross section = *probability of photon decaying into a quark anti-quark pair* $\otimes$ *probability of the quark anti-quark "dipole" scattering on the target*

QED *QCD*

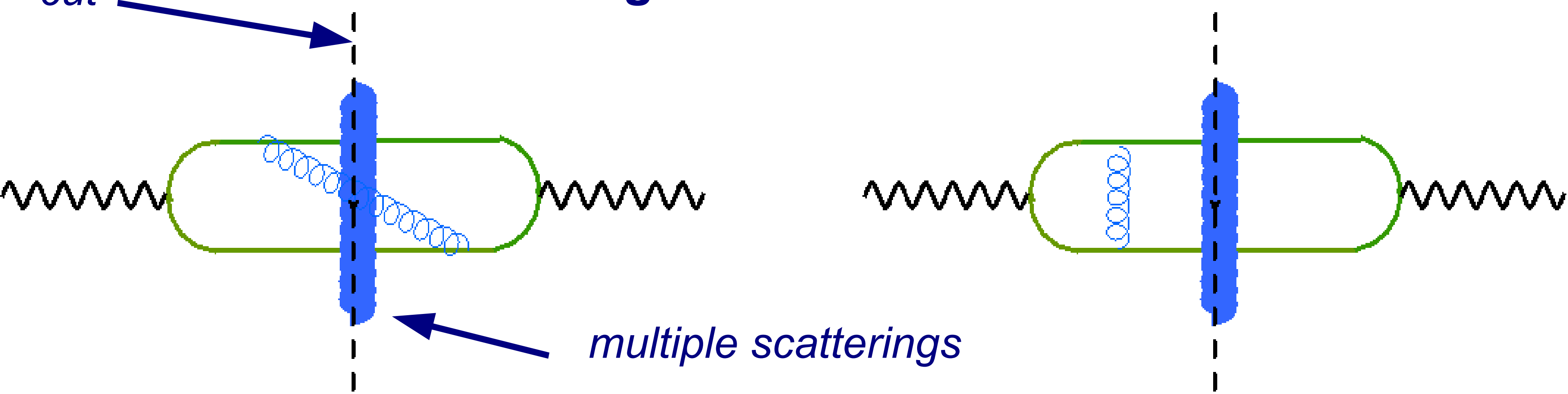
DIS total cross section: energy dependence

recall the parton model was scale invariant, scaling violation (dependence on Q^2) came after quantum corrections - $O(\alpha_s)$

what we have done so far is to include high gluon density effects but no energy dependence yet

to include the energy dependence, need quantum corrections - $O(\alpha_s)$

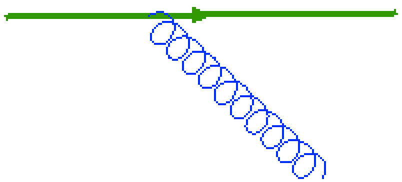
diagrams like



real corrections *virtual corrections*

Energy dependence

radiation vertex (LC gauge)



a, k_t

$$= 2\,g\,t^a\,\frac{\epsilon_\lambda\cdot\mathbf{k_t}}{\mathbf{k_t^2}}$$

which looks like

$$\int\frac{d^2k_t}{(2\pi)^2}e^{ik_t\cdot(x_t-z_t)}\,2\,g\,t^a\,\frac{\epsilon_\lambda\cdot k_t}{k_t^2}=\frac{2\,i\,g}{2\pi}\,t^a\,\frac{\epsilon_\lambda\cdot(x_t-z_t)}{(x_t-z_t)^2}$$

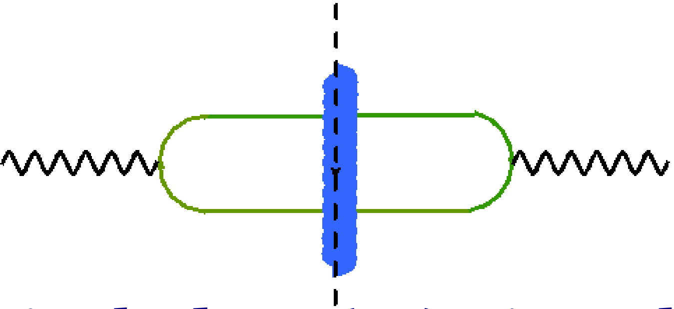
in coordinate space (x_t and z_t are 2-d transverse coordinates of the quark and gluon)

sum over physical gluon polarizations

$$\sum^{\lambda}\epsilon^i_\lambda\,\epsilon^j_\lambda=-g^{ij}$$

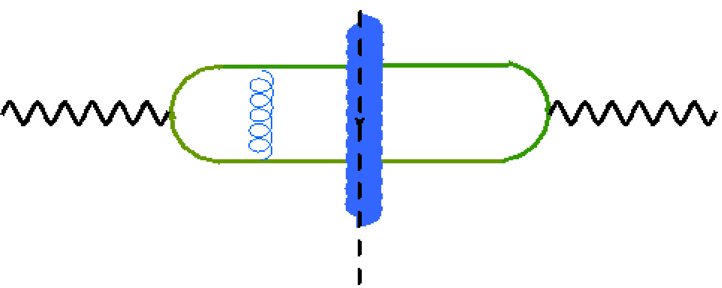
Energy dependence

we started with ("tree level")



$$\sim |\Psi(\mathbf{x_t}, \mathbf{y_t})|^2 \, \mathbf{Tr}\, \mathbf{V}(\mathbf{x_t})\, \mathbf{V}^\dagger(\mathbf{y_t})$$

include $O(\alpha_s)$ virtual corrections – subleading N_c terms will cancel between real and virtual contributions, not included



+ ....

rapidity

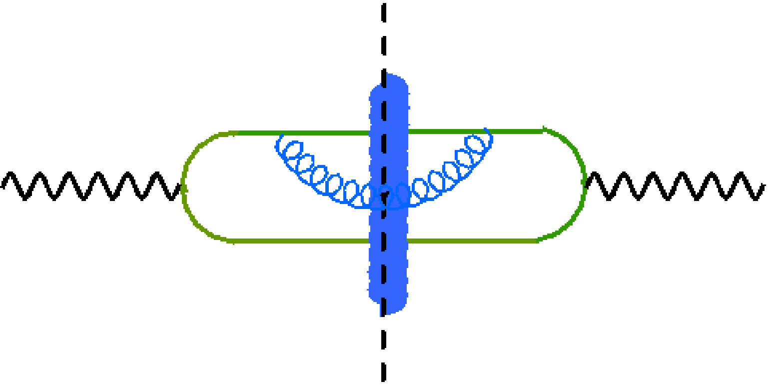
divergent ~ ln P^+ ~ Y

$$= \quad -\frac{N_c \alpha_s}{2\pi^2} \int^{P^+} \frac{dk^+}{k^+} \int d^2 z_t \frac{(x_t - y_t)^2}{(x_t - z_t)^2 (y_t - z_t)^2}$$

$$|\Psi(x_t, y_t)|^2 \, Tr\, V(x_t)\, V^\dagger(y_t)$$

Energy dependence

include real corrections - O (α_s)



+ ....

divergent ~ ln P^+

$$= |\Psi(x_t, y_t)|^2 \frac{\alpha_s}{2\pi^2} \int d^2 z_t \frac{(x_t - y_t)^2}{(x_t - z_t)^2 (y_t - z_t)^2} \int^{P^+} \frac{dk^+}{k^+}$$

$$Tr\,\left[V^\dagger(x_t)\, V(z_t)\right]\, Tr\,\left[V(y_t)\, V^\dagger(z_t)\right]$$

have used $t^a\, U_{ab}(z_t) = V(z_t)\, t^b\, V^\dagger(z_t)$

and $[t^a]_{ij}\, [t^a]_{kl} = \frac{1}{2}[\delta_{il}\delta_{jk} - \frac{1}{N_c}\delta_{ij}\delta_{kl}]$ *Fierz identity*

Energy dependence

sum of all $O(\alpha_s)$ corrections gives

$$-\frac{N_c^2 \alpha_s \, Y}{2\pi^2} \, |\Psi(x_t, y_t)|^2 \int d^2 z_t \frac{(x_t - y_t)^2}{(x_t - z_t)^2 (y_t - z_t)^2}$$

$$\left\{ S(x_t - y_t) - S(x_t, z_t) \, S(z_t, y_t) \right\}$$

where the S(cattering) matrix is defined as

$$\mathbf{S(x_t, y_t) \equiv \frac{1}{N_c} \, Tr \left[V(x_t) \, V^\dagger(y_t) \right]}$$

recall we started with

$$|\mathbf{\Psi(x_t, y_t)}|^2 \, \mathbf{N_c \, S(x_t, y_t)}$$

then the change (evolution) after one gluon radiation is

(BK) Evolution equation

$$\frac{\partial S(x_t, y_t)}{\partial Y} = -\frac{N_c \alpha_s}{2\pi^2} \int d^2 z_t \frac{(x_t - y_t)^2}{(x_t - z_t)^2 (y_t - z_t)^2}$$
$$\times \left\{ S(x_t - y_t) - S(x_t, z_t)\, S(z_t, y_t) \right\}$$

non-linear

this equation describes evolution of the cross section with <u>energy (rapidity) and includes multiple scatterings</u>

remember DGLAP gives Q^2 evolution of structure functions

(BK) Evolution equation

let's consider the limit when gluon density is not high,
define the T matrix

$$\mathbf{T}(\mathbf{x_t}, \mathbf{y_t}) \equiv \mathbf{1} - \mathbf{S}(\mathbf{x_t}, \mathbf{y_t})$$

then our non-linear equations becomes

$$\frac{\partial T(x_t, y_t)}{\partial Y} = \frac{N_c \alpha_s}{2\pi^2} \int d^2 z_t \frac{(x_t - y_t)^2}{(x_t - z_t)^2 (y_t - z_t)^2}$$

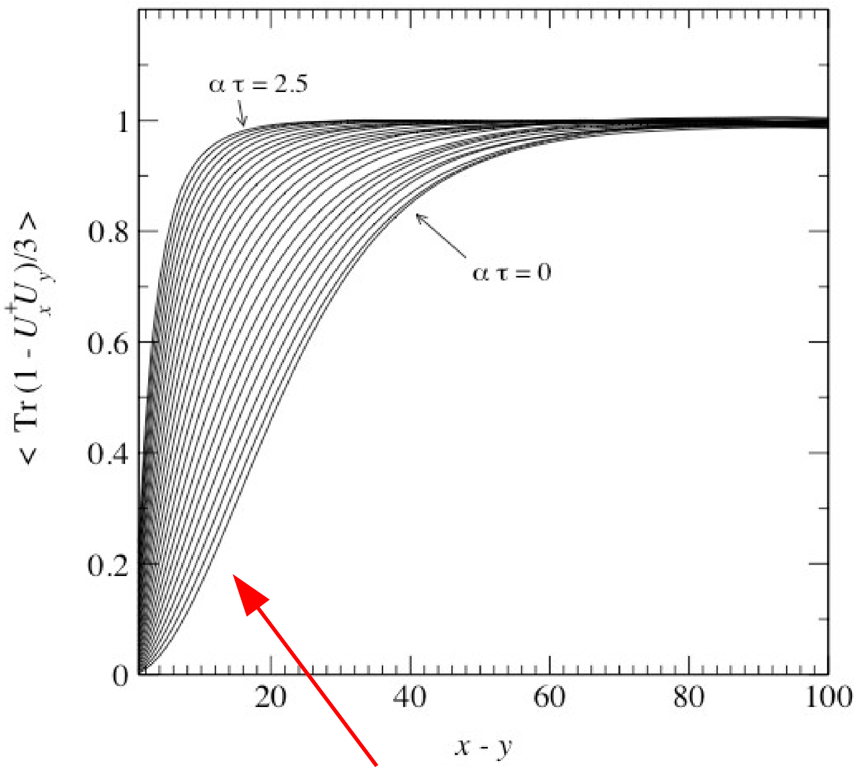
$$\{T(x_t, z_t) + T(z_t, y_t) - T(x_t, y_t) - T(x_t, z_t)\, T(z_t, y_t)\}$$

linear (BFKL) non-linear

both T = 0 and T = 1 are fixed points

unstable stable

Solution of (BK) evolution equation



$$\sim \mathbf{r_t^2\, xG(x, 1/r_t^2)}$$

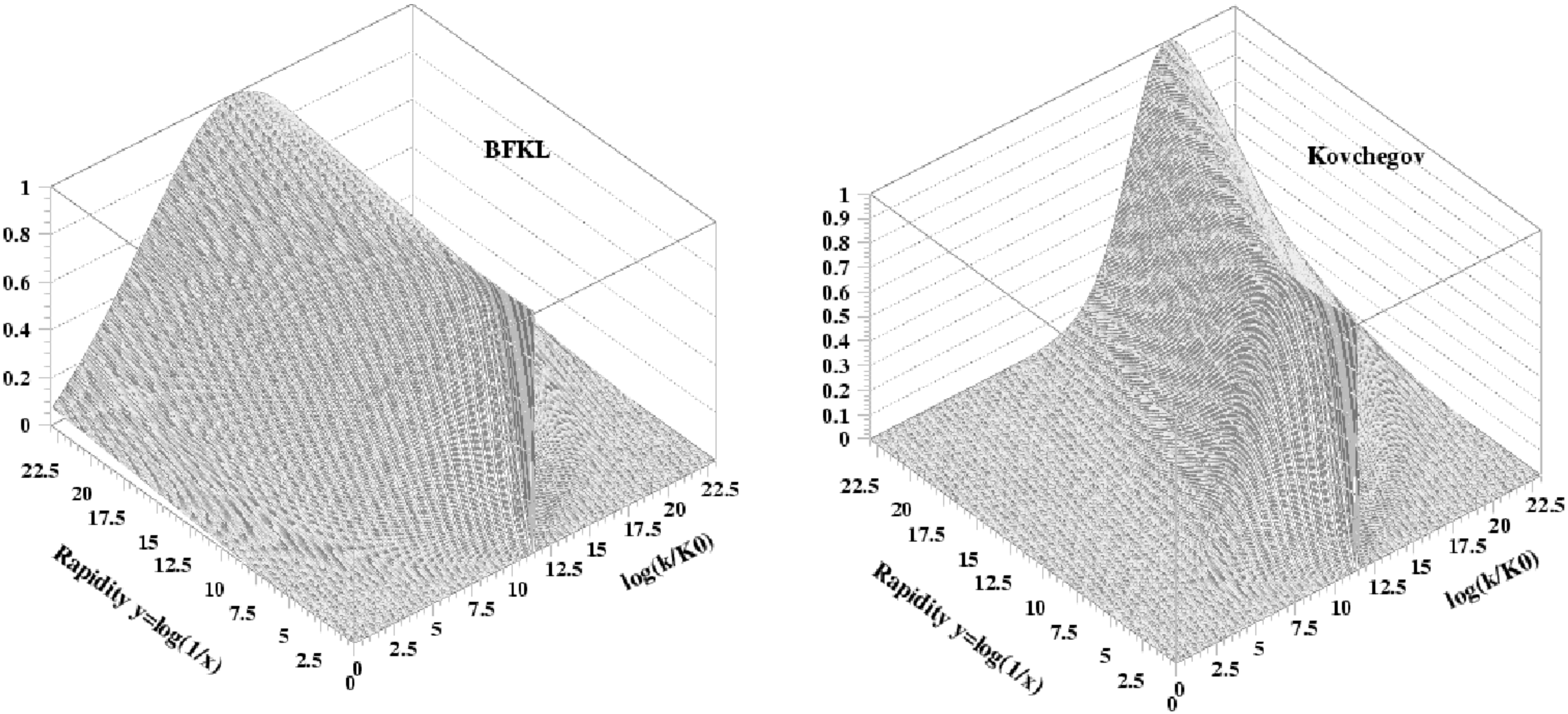
$$\tilde{N}_F(p_t) \rightarrow \log\left[\frac{Q_s^2}{p_t^2}\right]$$ **saturation region**

$$\tilde{N}_F(p_t) \rightarrow \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right]^\gamma$$ **extended scaling region**

$$\tilde{N}_F(p_t) \rightarrow \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right]$$ **pQCD region**

RW, NPA739 (2004) 183

Solution of (BK) evolution equation



G-BMS, PRD65 (2002) 074037

Effective Action + RGE

$$\mathbf{S[A,\rho] = -\frac{1}{4}\int d^4x\, F^2_{\mu\nu} \;+ \frac{i}{N_c}\int d^2x_t dx^-\;\delta(x^-) Tr[\rho(x_t) U(A^-)]}$$

$$\mathbf{U(A^-) = \hat{P}\, Exp\left[ig\int dx^+\; A_a^-\; T_a\right]}$$

eikonal coupling of fast and slow modes

$$\mathbf{Z[j] = \int [D\rho]\, W_{\Lambda^+}[\rho] \left[\frac{\int^{\Lambda^+}[DA]\delta(A^+) e^{iS[A,\rho]-\int j\cdot A}}{\int^{\Lambda^+}[DA]\delta(A^+) e^{iS[A,\rho]}}\right]}$$

$\mathbf{W_{\Lambda^+}[\rho]}$

weight functional:
probability distribution of color charges ρ
at longitudinal scale $\mathbf{\Lambda^+}$

invariance under change of $\mathbf{\Lambda^+}$ $\longrightarrow$ RGE for $\mathbf{W_{\Lambda^+}[\rho]}$

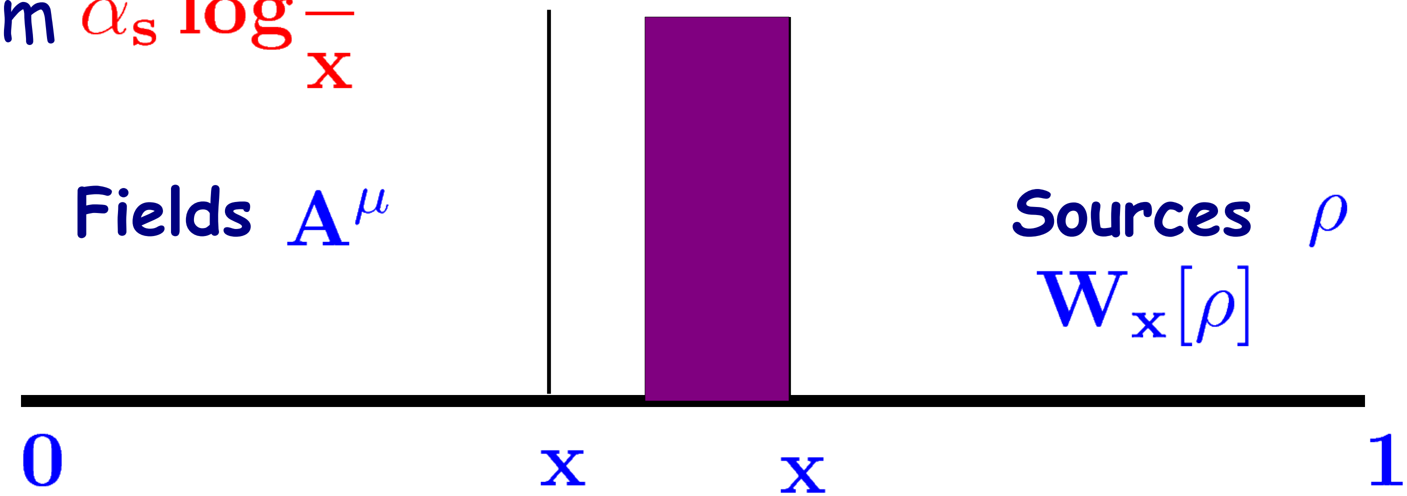
QCD at High Energy: Wilsonian RG

resum $\alpha_s \log \frac{1}{x}$

Fields A^μ

Sources ρ

$W_x[\rho]$



$$A^\mu = A^\mu_{class} + \delta\, A^\mu$$

integrate out field fluctuations quadratically

$$\rho \rightarrow \rho' = \rho + \delta\rho$$

$$\frac{\partial W[\rho]}{\partial \ln 1/x} = \frac{1}{2} \int_{x_t, y_t} \frac{\delta}{\delta \rho^a(x_t)} \chi^{ab}(x_t, y_t)[\rho] \frac{\delta}{\delta \rho^a(y_t)} W[\rho]$$

JIMWLK eq. describes x evolution of observables

JIMWLK evolution equation

$$\frac{d}{d\,ln 1/x}\langle O\rangle = \frac{1}{2}\left\langle \int d^2x\, d^2y\, \frac{\delta}{\delta\alpha_x^b}\,\eta_{xy}^{bd}\,\frac{\delta}{\delta\alpha_y^d}\,O\right\rangle$$

$$\eta_{xy}^{bd} = \frac{1}{\pi}\int \frac{d^2z}{(2\pi)^2}\,\frac{(x-z)\cdot(y-z)}{(x-z)^2(y-z)^2}\,\Big[\underbrace{1+U_x^\dagger U_y}_{\text{virtual}} - \underbrace{U_x^\dagger U_z - U_z^\dagger U_y}_{\text{real}}\Big]^{bd}$$

U is a Wilson line in adjoint representation

QCD at low x: CGC

(a high gluon density environment)

two main effects:
- *"multiple scatterings" via classical field*
- *evolution with ln (1/x) via JIMWLK/BK*

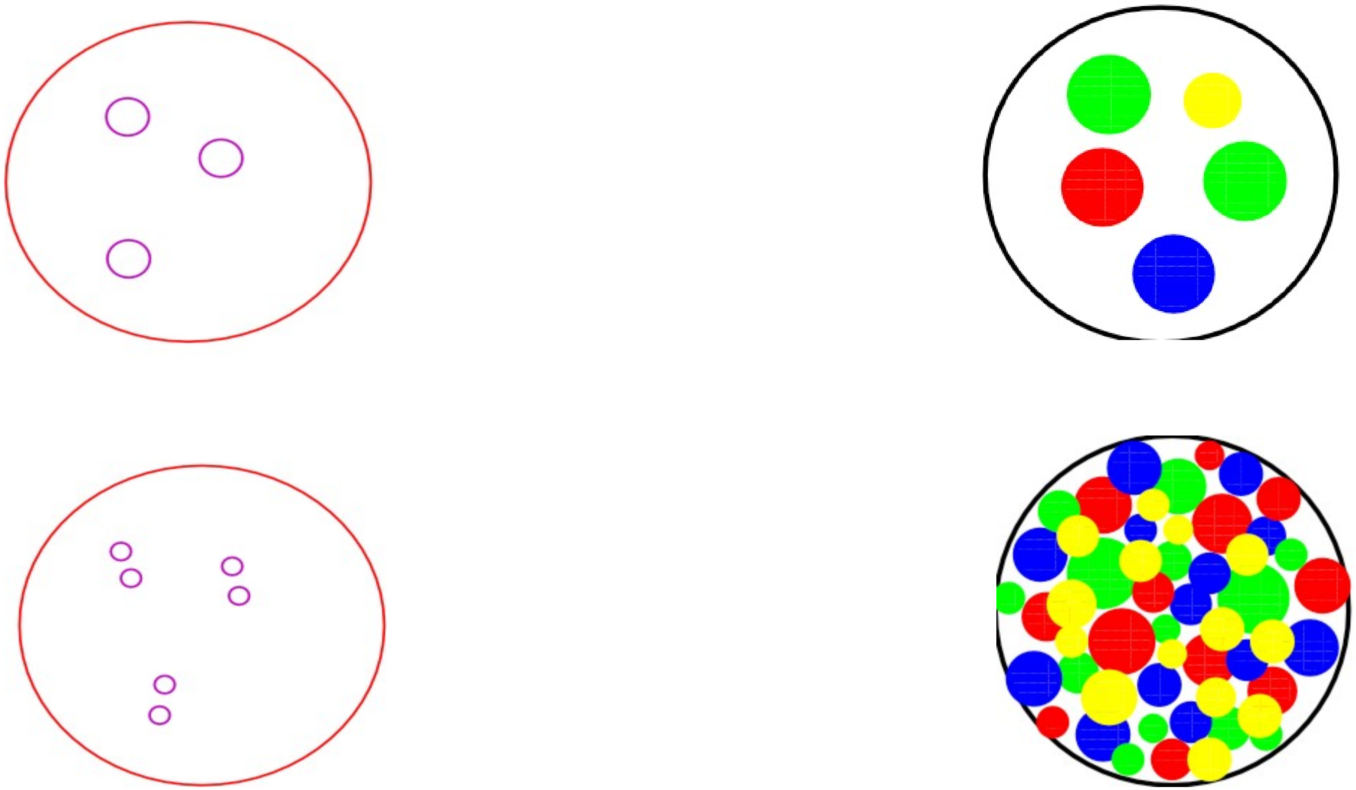
CGC observables: $< \mathrm{Tr}\, V \cdots\cdots V^{\dagger} >$ with $\mathbf{V(x_t) = \hat{P} e^{ig \int dx^- A_a^+ t_a}}$

$$\mathbf{A_a^\mu (x_t, x^-) \sim \delta^{\mu +} \delta(x^-) \alpha_a (x_t)} \qquad \mathbf{\alpha^a (k_t) = g\, \rho^a (k_t)/k_t^2}$$

gluon distribution: $\mathbf{xG(x, Q^2) \sim \int^{Q^2} \frac{d^2 k_t}{k_t^2} \phi(x, k_t)}$ with $\mathbf{\phi(x, k_t^2) \sim < \rho_a^\star (k_t)\, \rho_a (k_t) >}$

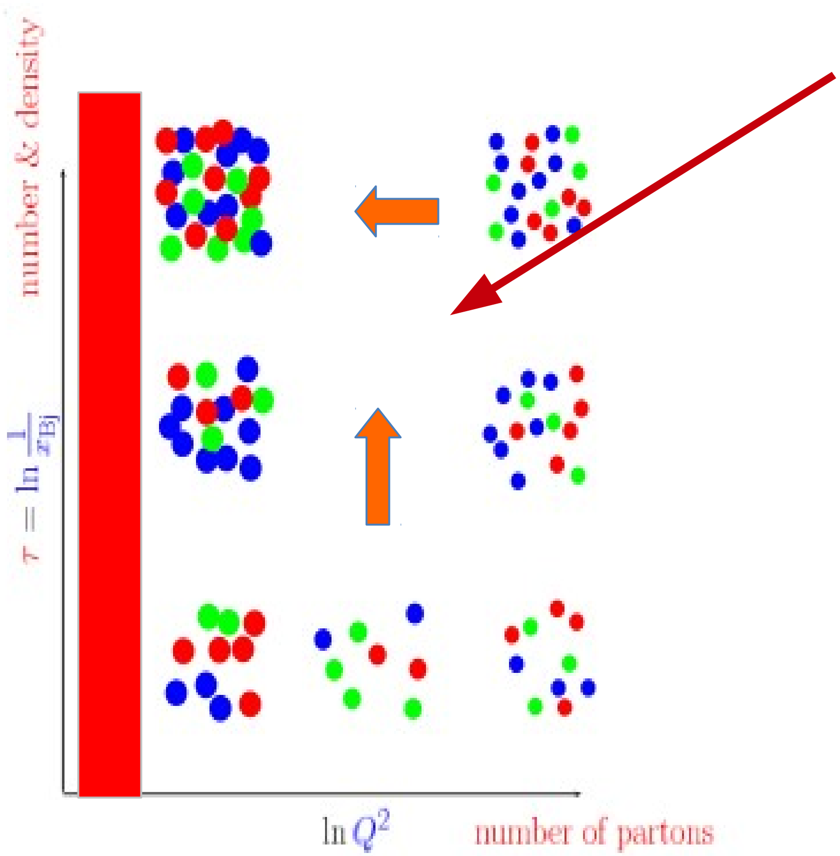
pQCD with collinear factorization:
- *single scattering*
- *evolution with ln Q^2*

Bjorken/Feynman or Regge/Gribov?



depends on kinematics!

Many-body dynamics of universal gluonic matter



How does this happen ?

How do correlation functions of these evolve ?

Is there a universal fixed point for the RG evolution of d.o.f

How does the coupling run ?

How does saturation transition to chiral symmetry breaking and confinement

Color Glass Condensate

Advantages:

A systematic, first-principle approach to high energy scattering in QCD

Controlled approximations

Same formalism can describe a wide range of phenomena

Disadvantages:

Applicable at low x (high x, Q2 missing)

Initial conditions

Observables

DIS:

structure functions
particle production

dilute-dense (pA, forward pp) collisions:

multiplicities
p_t spectra
di-hadron angular correlations

dense-dense (AA, pp) collisions:

multiplicities, spectra
long range rapidity correlations

Spin

DIS total cross section

Recall we can write the DIS cross section as

$$\sigma^{tot}_{DIS}(Y, Q^2) = \int d^2x_t\, d^2y_t \int_0^1 dz\, \left|\Psi(z, x_t, y_t, Q^2)\right|^2 < T(x_t, y_t) >_Y$$

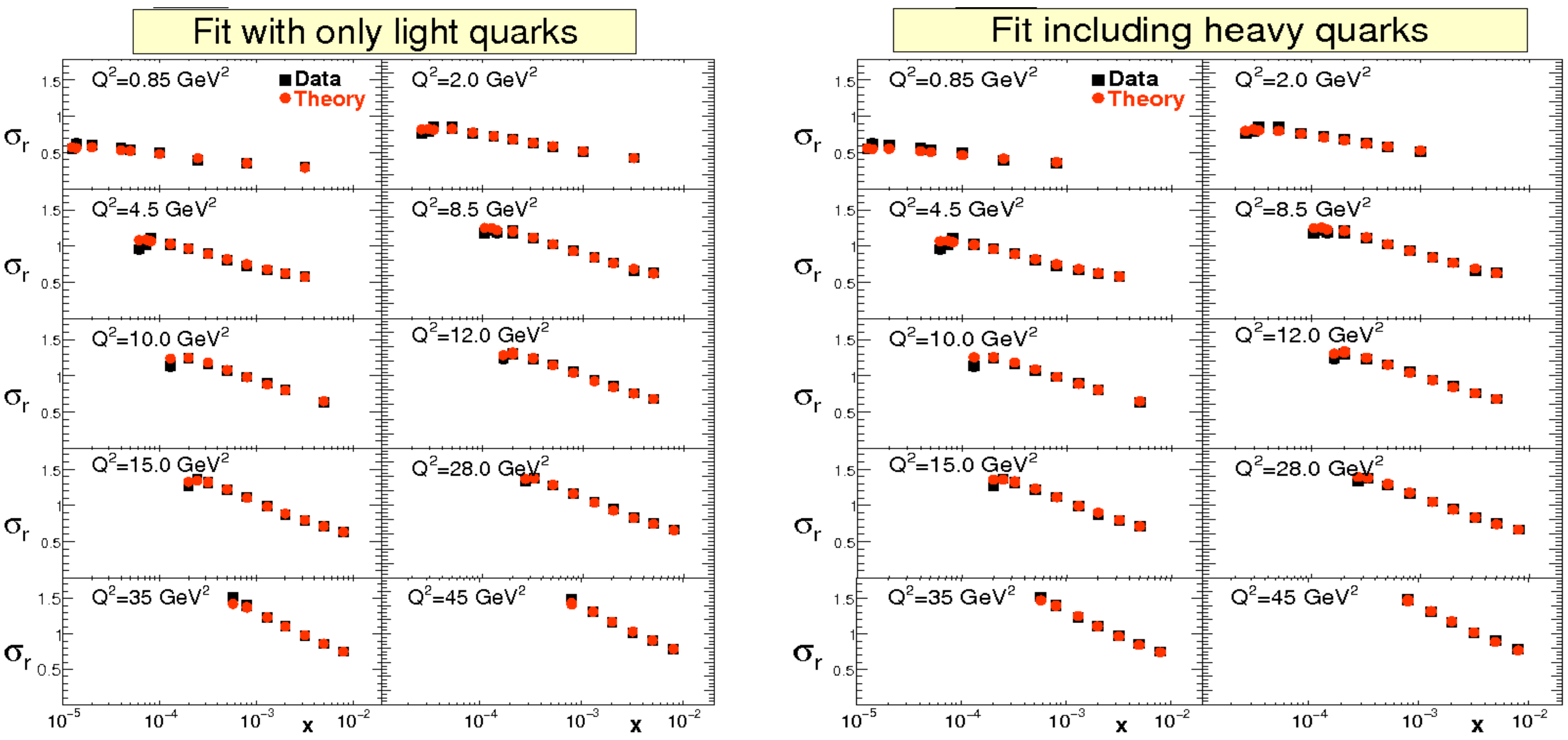
Photon wave function is known

Can use the solution to BK equation to find T

need an initial condition (parameterize)

Compare the results with data

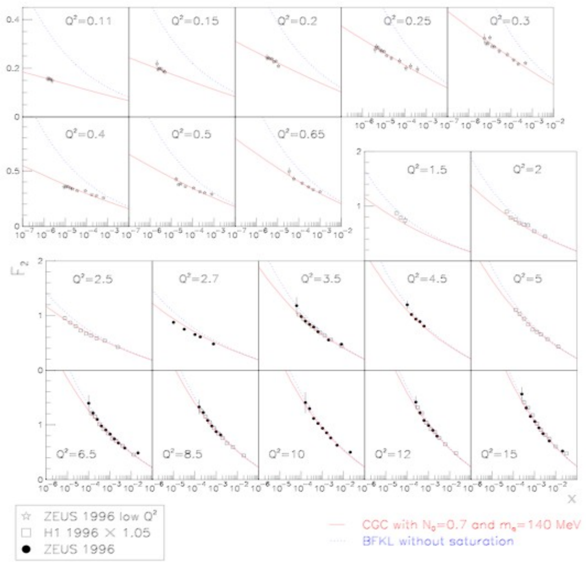
Structure functions at HERA



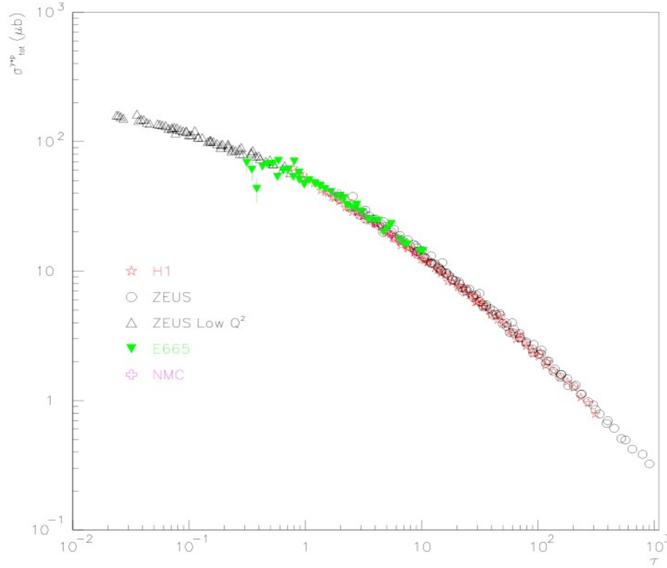
AAMQS(2010)

PQCD: DGLAP-based approaches also "work" : need more discriminatory observables

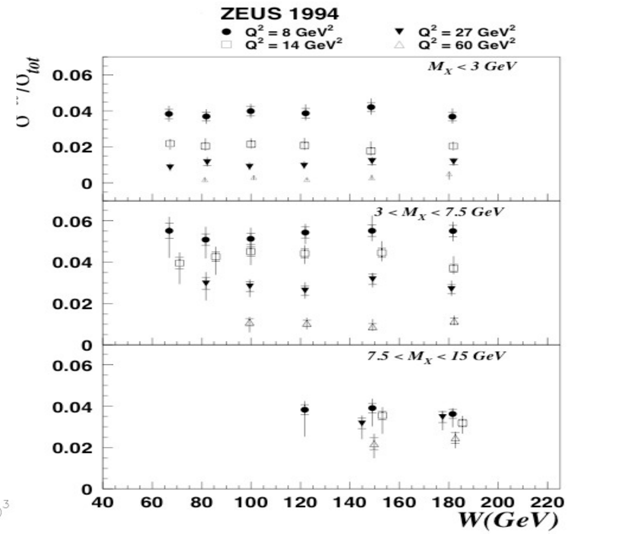
CGC at HERA ?



Structure Functions



Extended Scaling



$\sigma^{diff}/\sigma^{tot}$

CGC applicable at HERA for $x < 10^{-2}$ $Q < 20\,\mathrm{GeV}$

Saturation scale of a proton $Q_s^2(x) = 1\,\mathrm{GeV}^2 \left[\frac{x_0}{x}\right]^{0.28}$

with $x_0 = 3 \times 10^{-4}$

RHIC is an extreme QCD machine

Large Hadron Collider (LHC)

Proton-Nucleus (CGC X CGC) Collisions

Classical (before evolution with x):

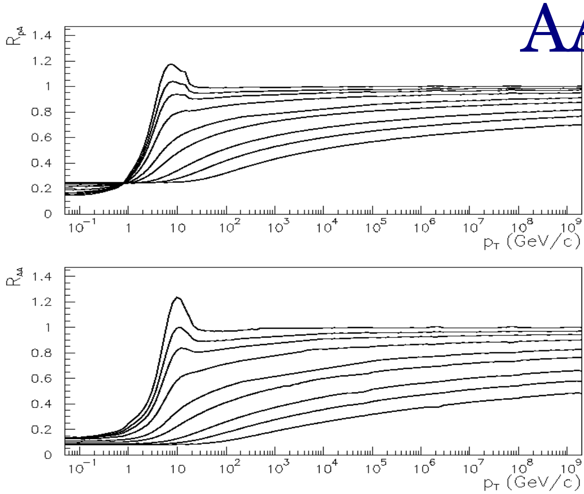
$R_{pA} = 1 + ...$ for $P_t >> Q_s$ GJM

$R_{pA} < 1$ for $P_t << Q_s$

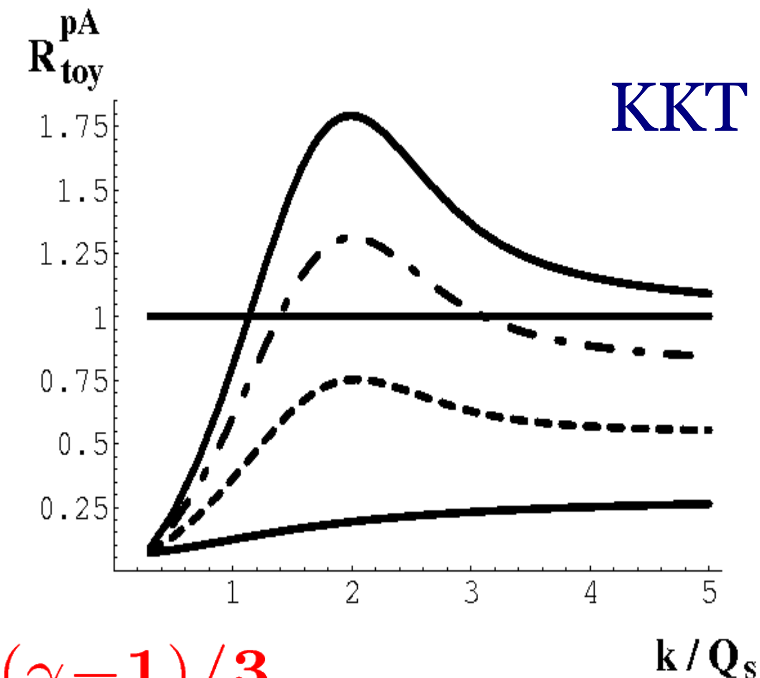
With x evolution :

$R_{pA} < 1$ for all P_t

AAKSW

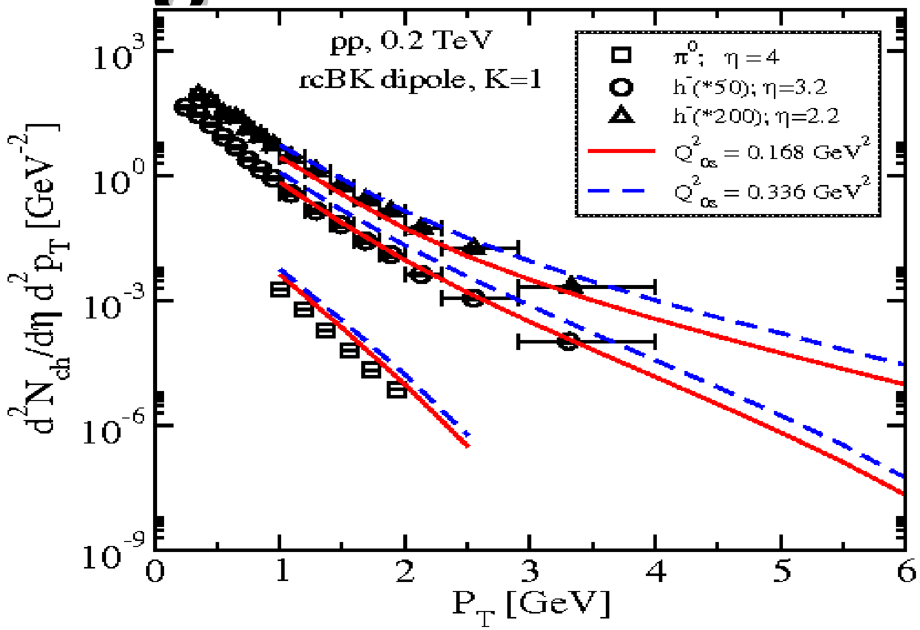


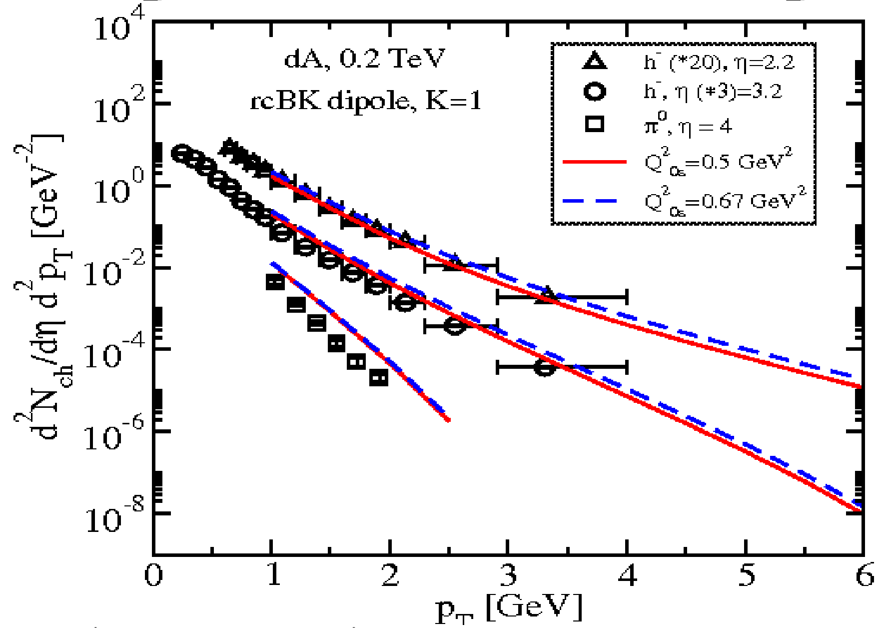
KKT



$$R_{pA} \to A^{(\gamma-1)/3}$$

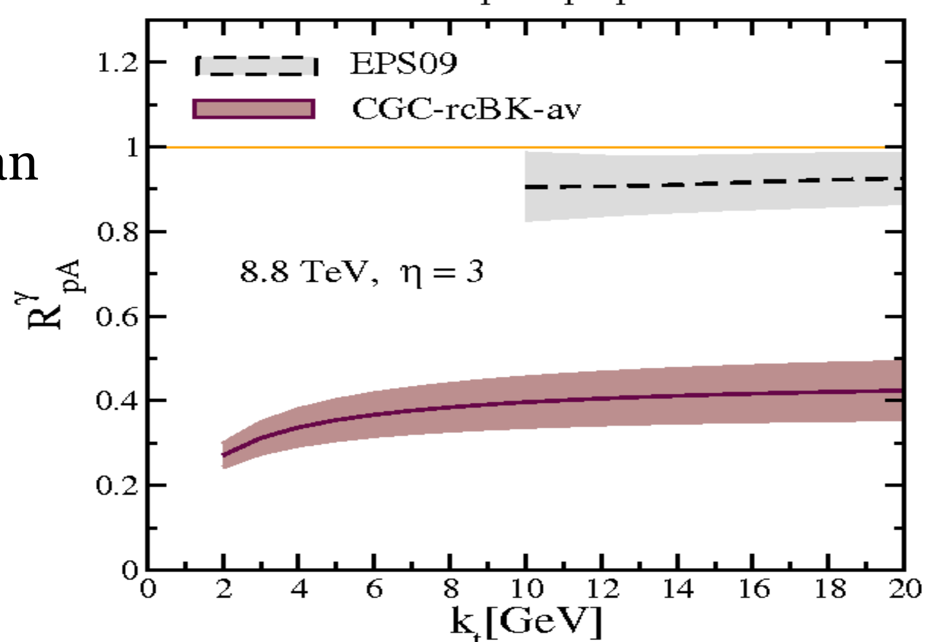
Single inclusive hadron production in pA





J. Jalilian-Marian, A. Rezaeian
PRD85 (2012) 014017,
arXiv:1204.1319

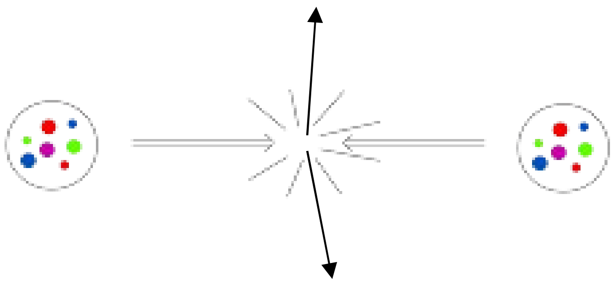
Inclusive prompt photon



Di-hadron production in pA: CGC

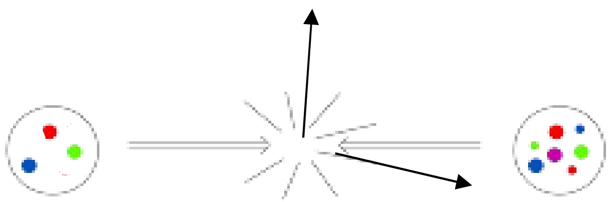
produced partons: k_1, y_1 $\quad k_2, y_2$ $\quad x_p = \frac{k_1\, e^{y_1} + k_2\, e^{y_2}}{\sqrt{s}}$ $\quad x_A = \frac{k_1\, e^{-y_1} + k_2\, e^{-y_2}}{\sqrt{s}}$

scanning the wave-functions $\quad k_1 \sim k_2 \sim k \ \sim 2\, GeV$



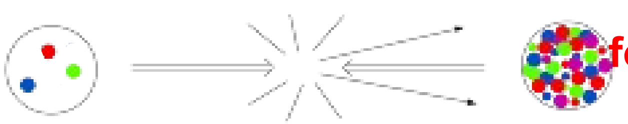
mid-mid correlations
probe moderate x

$$x_p \sim x_A \sim \frac{k}{\sqrt{S}} \sim 10^{-2}$$



mid-forward

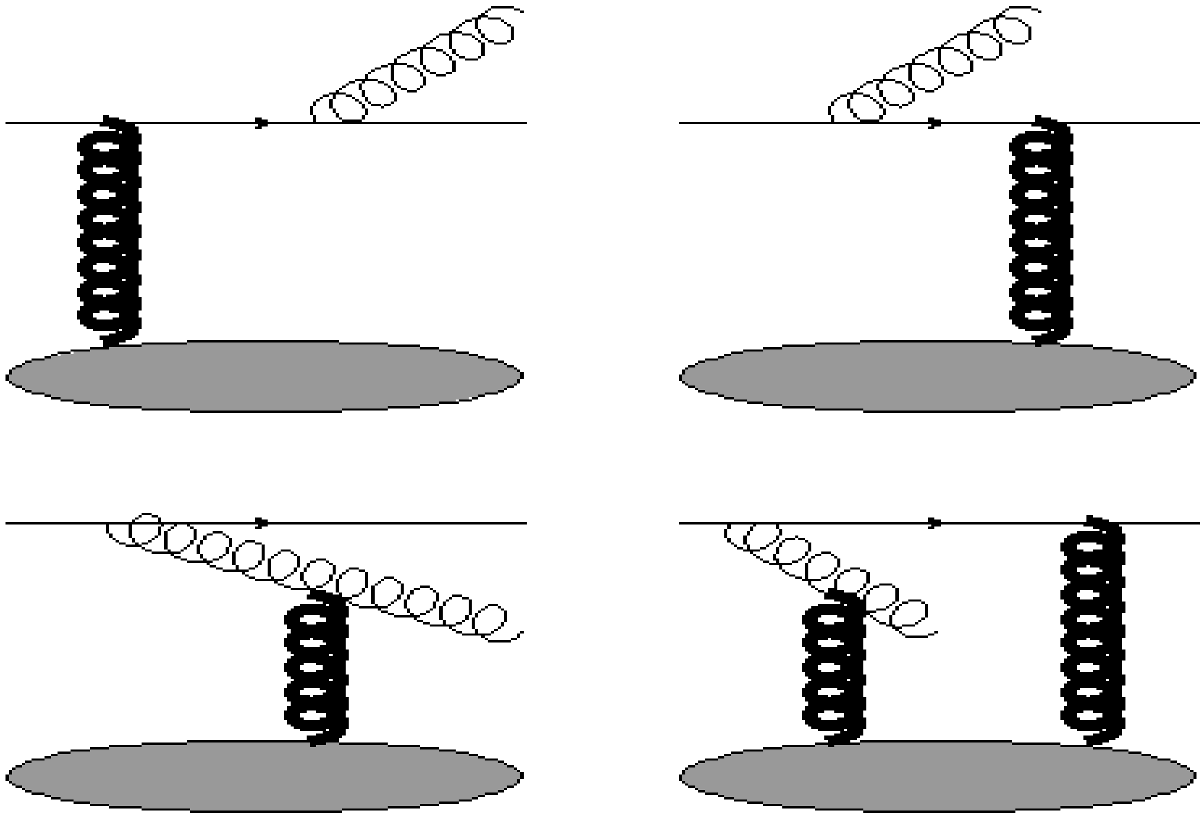
$$x_p \sim \frac{k}{\sqrt{S}}\, e^{y_1} \rightarrow 1 \quad x_A \sim \frac{k}{\sqrt{S}} \sim 10^{-2}$$



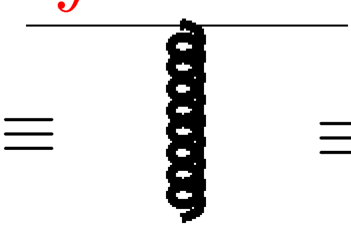
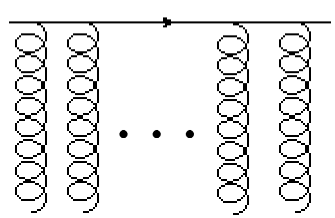
forward-forward $\quad x_p \sim \frac{k}{\sqrt{S}}\, e^{y_1} \rightarrow 1 \quad x_A \sim \frac{k}{\sqrt{S}}\, e^{-y_2} \sim 10^{-4}$

Di-jet production: pA

$$\mathbf{q(p)\,T \rightarrow q(q)\,g(k)}$$

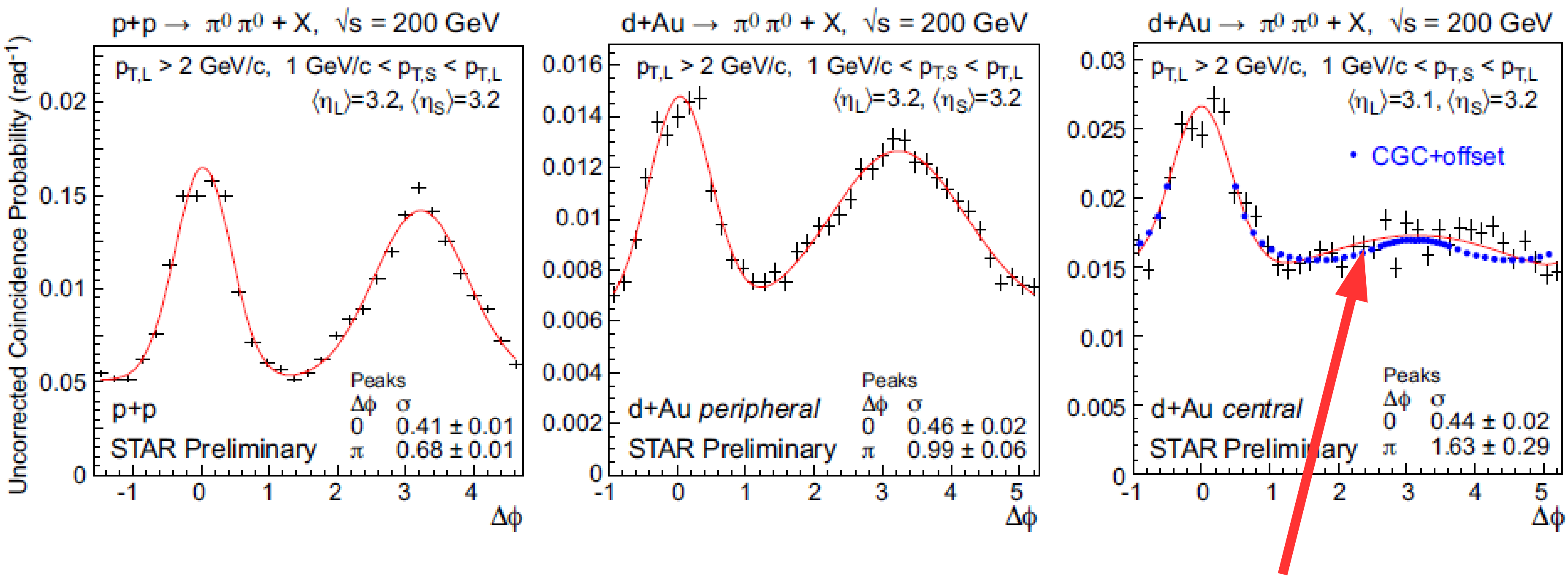


$$\mathbf{d}\sigma \sim \int \mathbf{K} \otimes \left[< \mathbf{Tr V\, V^\dagger} > + < \mathbf{Tr V\, V^\dagger\, V\, V^\dagger} > + \cdots \right]$$

$\mathbf{V} \equiv$  $\equiv$  $\sim \mathbf{1} + \mathbf{O}(\mathbf{g}\,\rho) + \mathbf{O}(\mathbf{g^2}\,\rho^2)$

disappearance of back to back jets

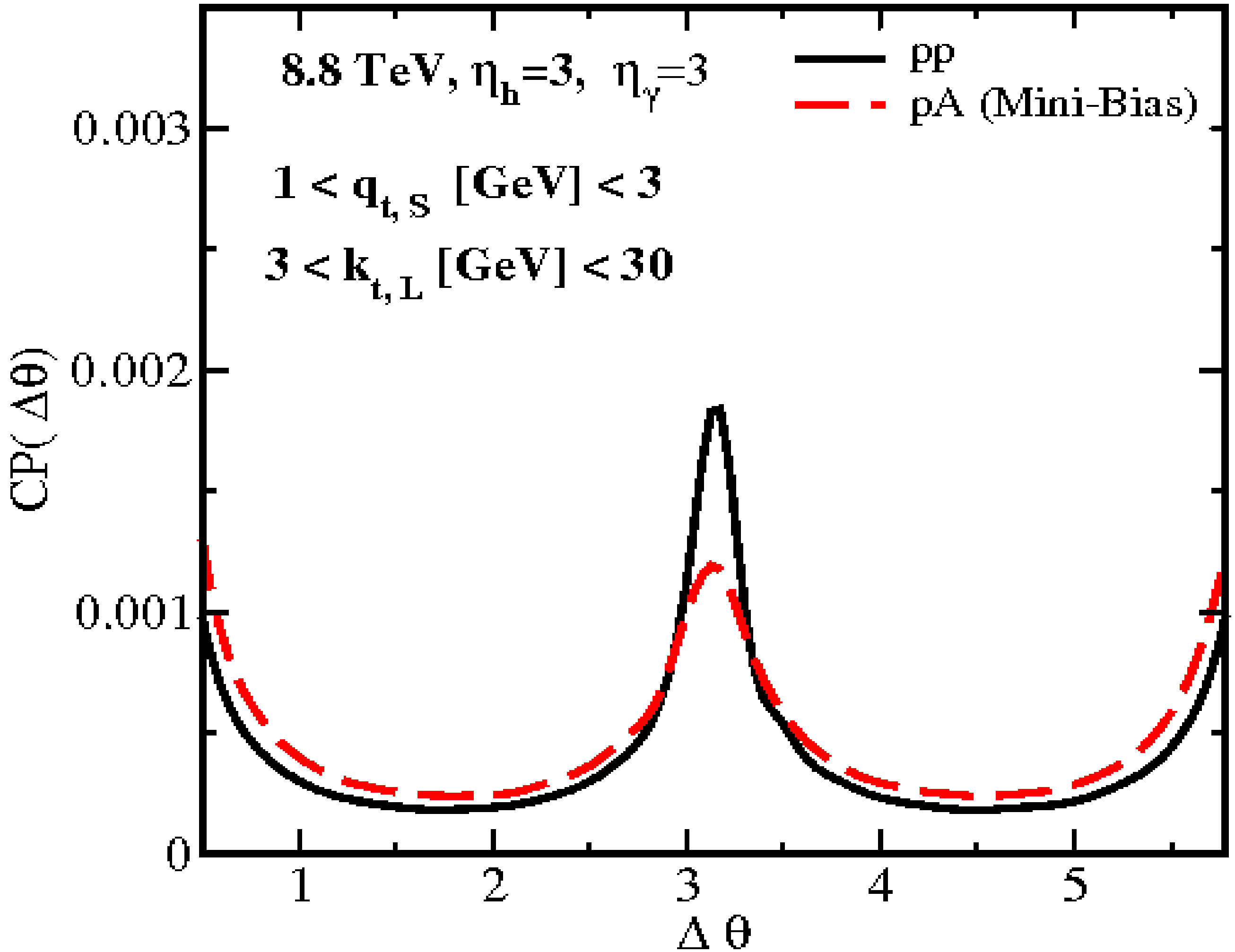
Recent STAR measurement (arXiv:1008.3989v1):



CGC fit from Albacete + Marquet, PRL (2010)
Also by Tuchin, NPA846 (2010) and
A. Stasto, B-W. Xiao, F. Yuan, arXiv:1109.1817

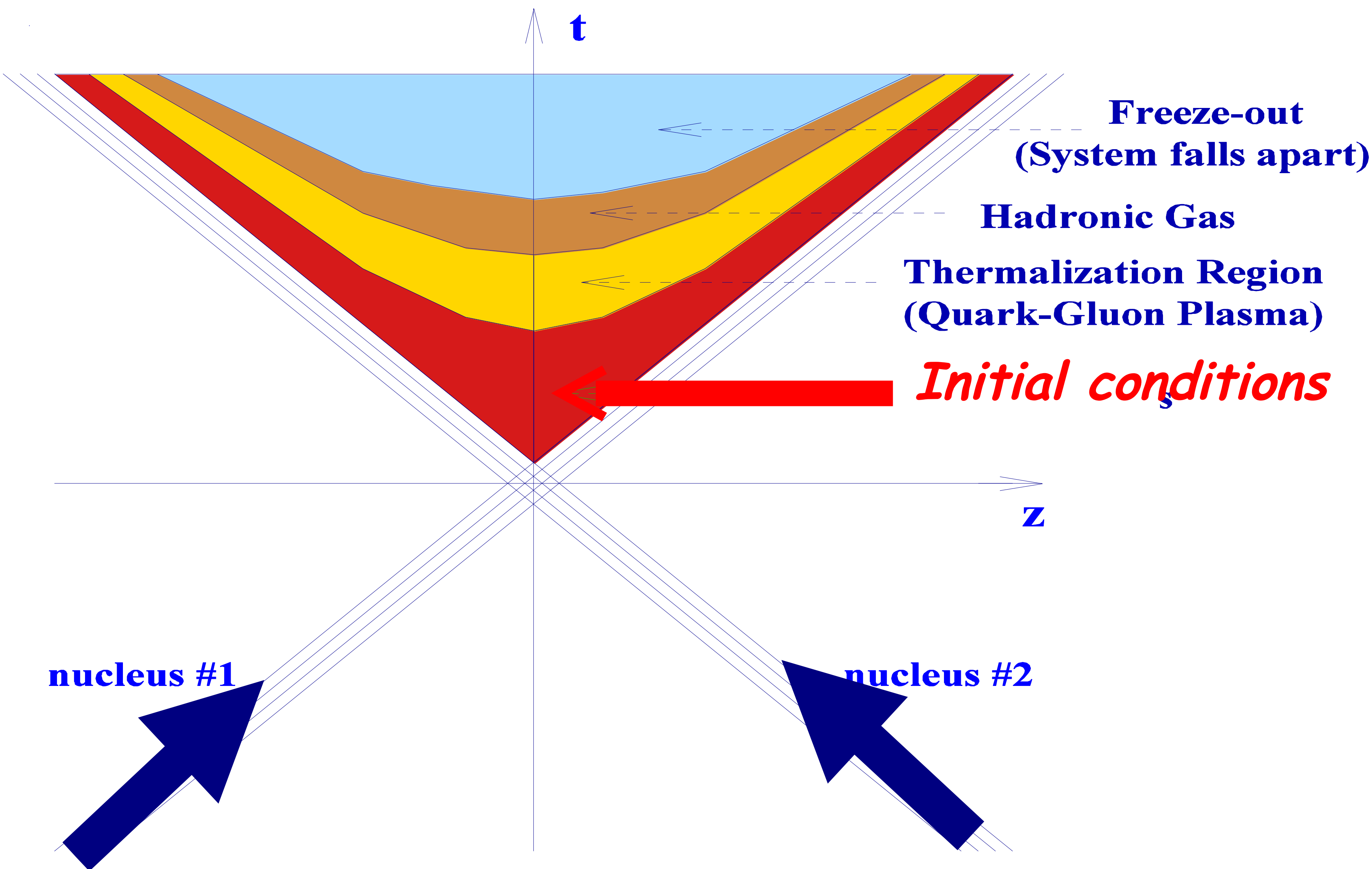
multiple scatterings de-correlate the hadrons

Photon-hadron angular correlation



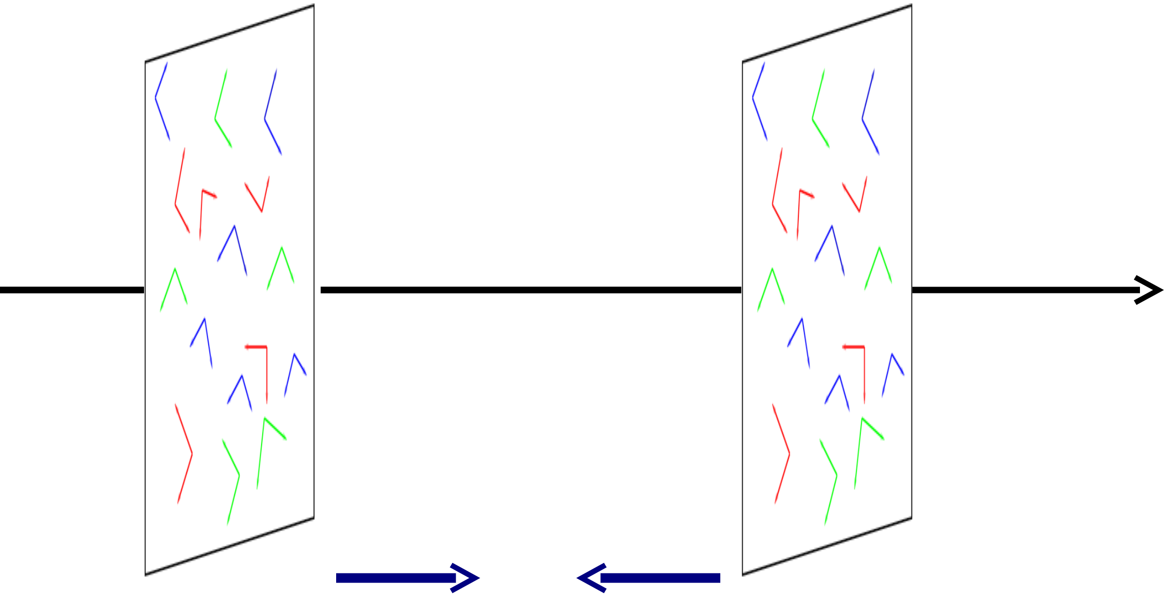
J. Jalilian-Marian, A. Rezaeian
arXiv:1204.1319

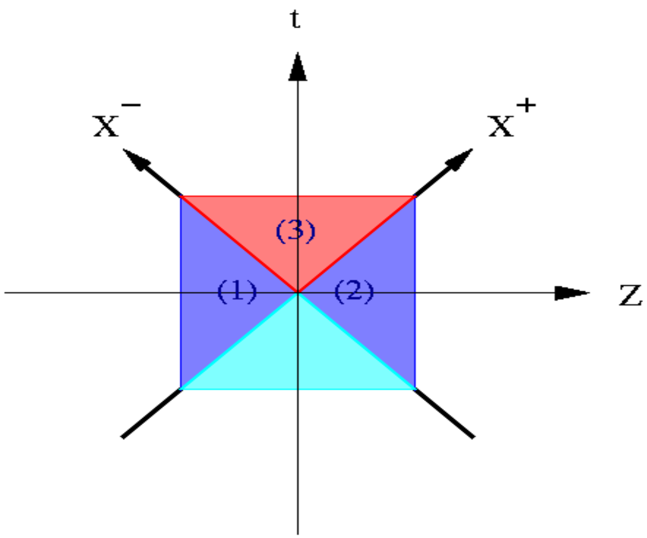
Space-Time History of a Heavy Ion Collision



Heavy Ion Collisions at High Energy:

Colliding Sheets of Color Glass





before the collision:

$$\mathbf{A}^+ = \mathbf{A}^- = \mathbf{0}$$
$$\mathbf{A}^{\mathbf{i}} = \mathbf{A}^{\mathbf{i}}_{\mathbf{1}} + \mathbf{A}^{\mathbf{i}}_{\mathbf{2}}$$
$$\mathbf{A}^{\mathbf{i}}_{\mathbf{1}} = \theta(\mathbf{x}^-)\theta(-\mathbf{x}^+)\alpha^{\mathbf{i}}_{\mathbf{1}}$$
$$\mathbf{A}^{\mathbf{i}}_{\mathbf{2}} = \theta(-\mathbf{x}^-)\theta(\mathbf{x}^+)\alpha^{\mathbf{i}}_{\mathbf{2}}$$

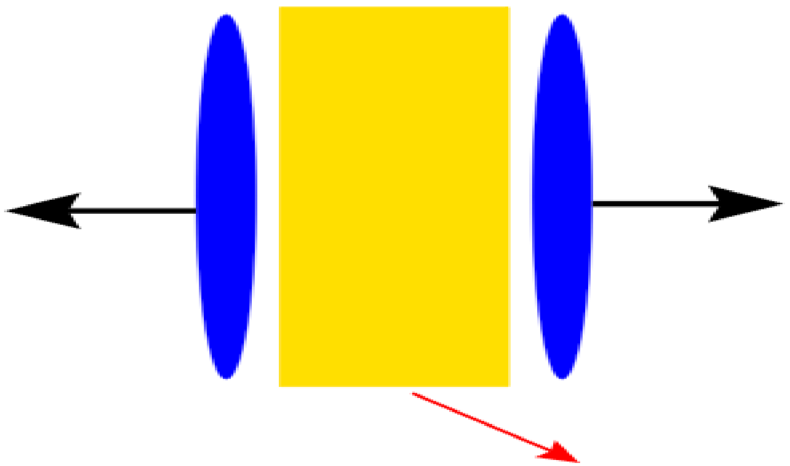
after the collision:

solve for $\mathbf{A}_\mu$

in the forward LC

Colliding Sheets of Color Glass at High Energies

solve the classical eqs. of motion in the forward light cone: subject to initial conditions given by one nucleus solution



GLASMA:strong color fields with occupation number $\sim \frac{1}{\alpha_s}$

initial energy and multiplicity of produced gluons depend on Q_s

$$\frac{1}{A_\perp}\frac{dE_\perp}{d\eta} = \frac{0.25}{g^2} Q_s^3 \qquad \frac{1}{A_\perp}\frac{dN}{d\eta} = \frac{0.3}{g^2} Q_s^2$$

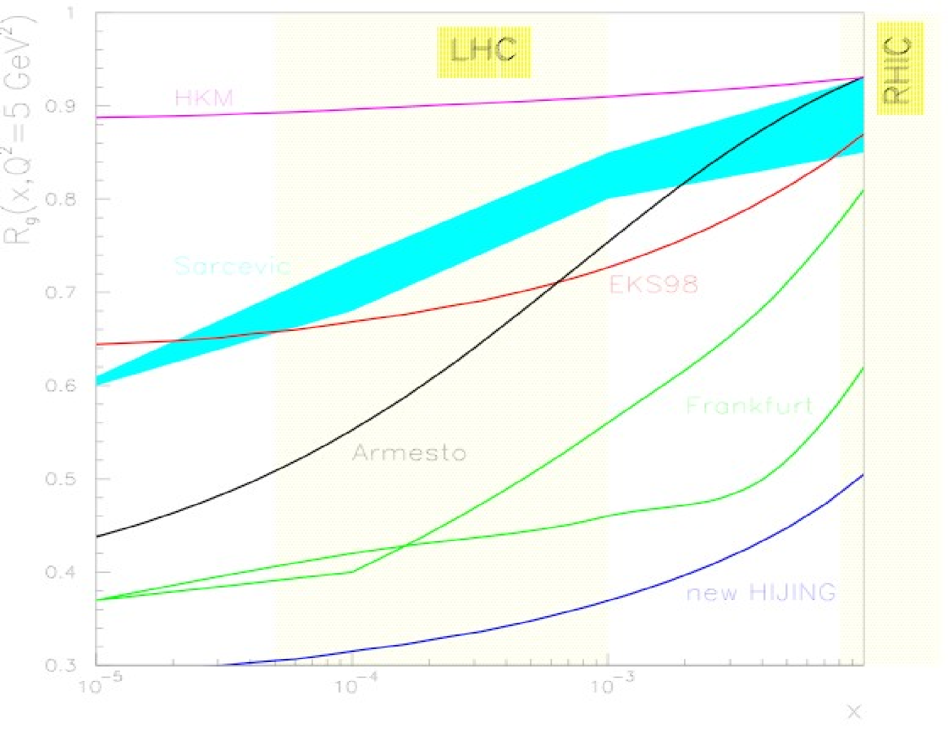
Aspects of low X physics with ep/A at an EIC

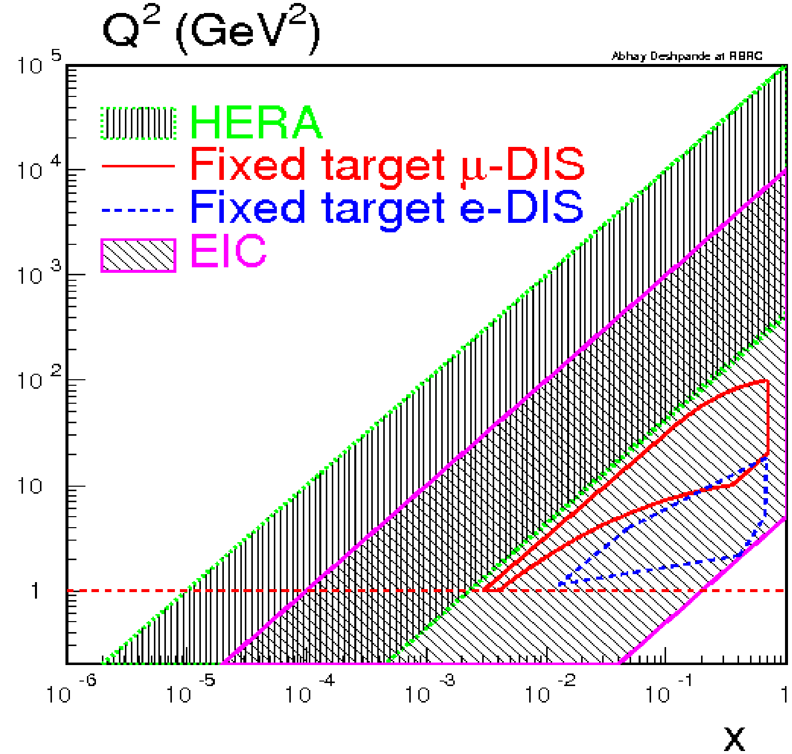
Probing extreme QCD:

unitarity, universality, strong color fields

Connection to heavy ion physics at RHIC/LHC

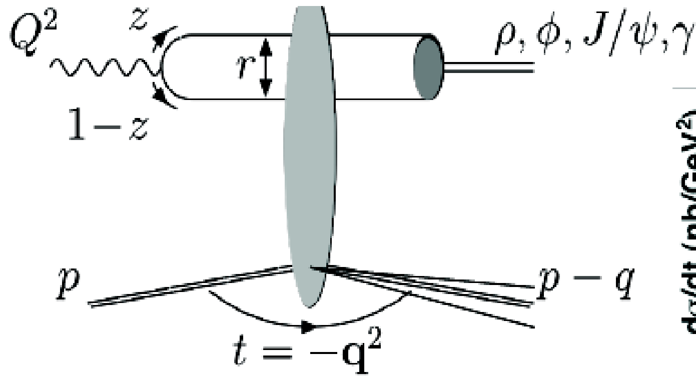
Nuclear structure functions at EIC



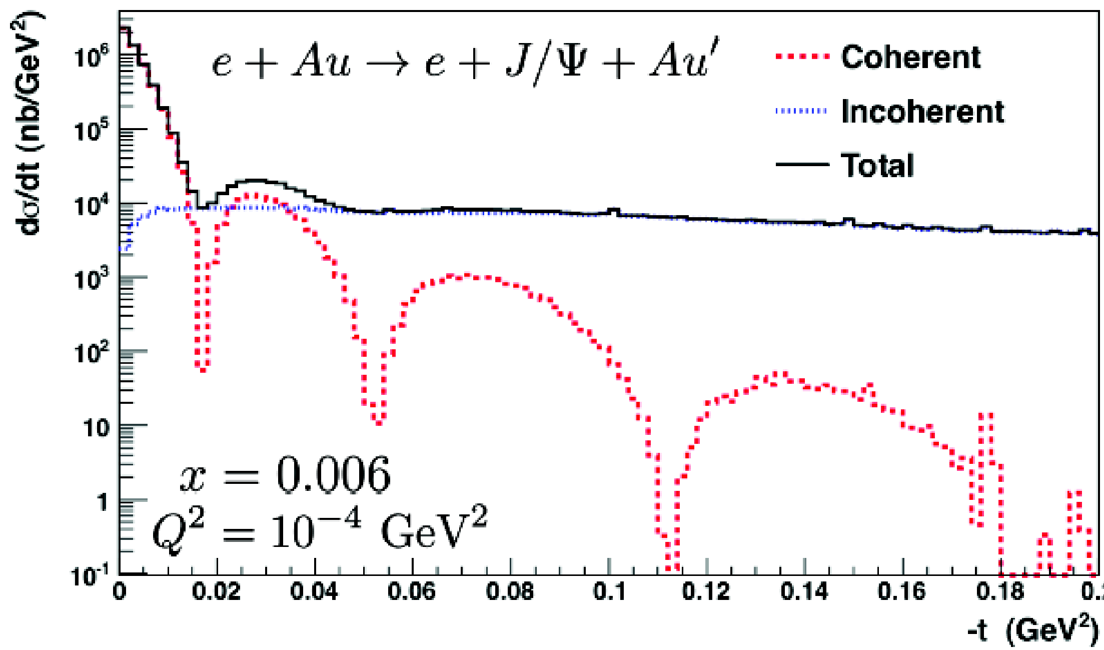


Nuclear parton (gluon) distributions are poorly known:
severe consequences for RHIC/LHC

Diffractive vector meson production



Toll and Ullrich (2011)



- as a function of t

exclusive production (coherent): the target undergoes elastic scattering, dominates at small |t|

→ steep exp. fall at small |t|

target dissociation (incoherent): the target undergoes inelastic scattering, dominates at large |t|

breakup into the nucleons
→ slower exp. fall at $0.02 < -t < 0.7$ GeV^2

breakup of the nucleons
→ power-law tail at large |t|

Fourier transform of t dependence gives the <u>impact parameter</u> profile of target

The role of initial conditions

McLerran-Venugopalan (93)

$$< \mathbf{O}(\rho) > \equiv \int \mathbf{D}[\rho]\, \mathbf{O}(\rho)\, \mathbf{W}[\rho]$$

$$\mathbf{W}[\rho] \simeq \mathbf{e}^{-\int \mathbf{d^2 x_t} \frac{\rho^a(x_t)\rho^a(x_t)}{2\,\mu^2}} \qquad \mu^2 \equiv \frac{\mathbf{g^2 A}}{\mathbf{S}_\perp}$$

$$\mathbf{T(r_t)} \equiv \frac{1}{\mathbf{N_c}} < \mathbf{Tr}\,[1 - \mathbf{V(r_t)}^\dagger\, \mathbf{V(0)}] > \sim \mathbf{1 - e}^{-[\mathbf{r_t^2\, Q_s^2}]^\gamma \log(\mathbf{e} + \frac{1}{\mathbf{r_t}\,\Lambda_{\mathbf{QCD}}})}$$

with $\gamma = 1.119$

how about higher order terms in ρ *?*

$$\mathbf{W}[\rho] \simeq \mathbf{e}^{-\int \mathbf{d^2 x_t} \left[\frac{\rho^a(x_t)\rho^a(x_t)}{2\,\mu^2} - \frac{d^{abc}\,\rho^a(x_t)\rho^b(x_t)\rho^c(x_t)}{\kappa_3} + \frac{F^{abcd}\,\rho^a(x_t)\rho^b(x_t)\rho^c(x_t)\rho^d(x_t)}{\kappa_4}\right]}$$

these higher order terms make the single inclusive spectra steeper

Dumitru-Jalilian-Marian-Petreska, PRD84 (2011) 014018
Dumitru-Petreska,, NPA879 (2012) 59

Summary

CGC is an effective theory of QCD at high energy limited to low-intermediate P_t region

No smoking gun but that may be unrealistic
- **Need to measure as many different processes as possible**

CGC describes a wide range of phenomena

Large uncertainties at LHC at y =0
- **Much more robust in forward rapidity**

pA@LHC run will constrain CGC significantly
Need an EIC for precision studies of CGC