Introduction to High Energy Nuclear Collisions I (QCD at high gluon density)

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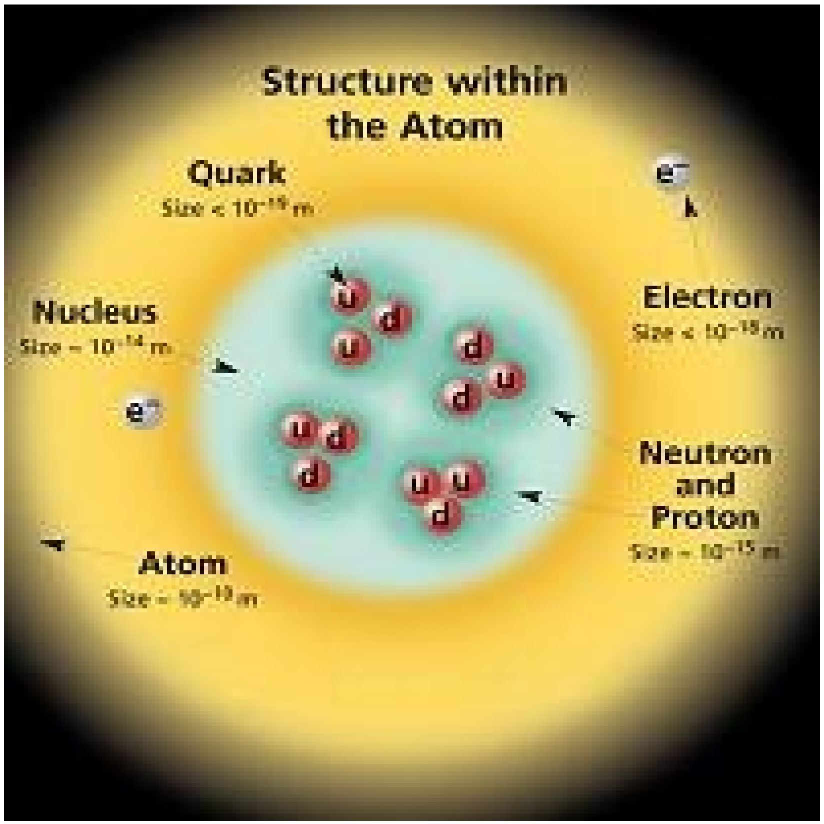
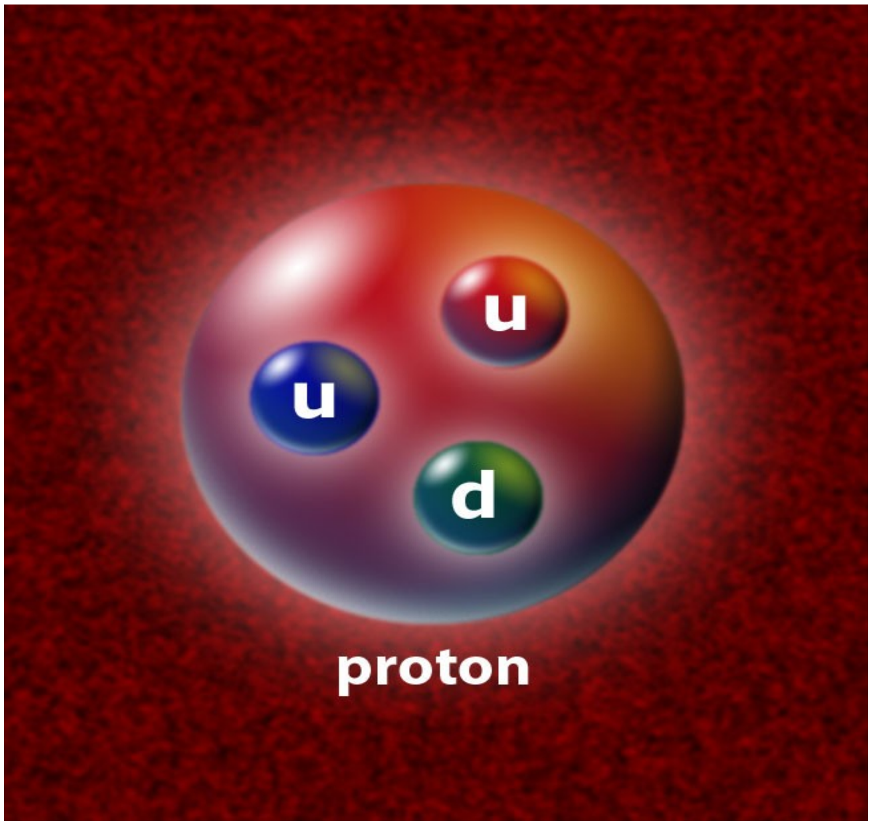
Many thanks to my colleagues, A. Deshpande, F. Gelis, B. Surrow and specially M. Stratmann for providing me with their slides

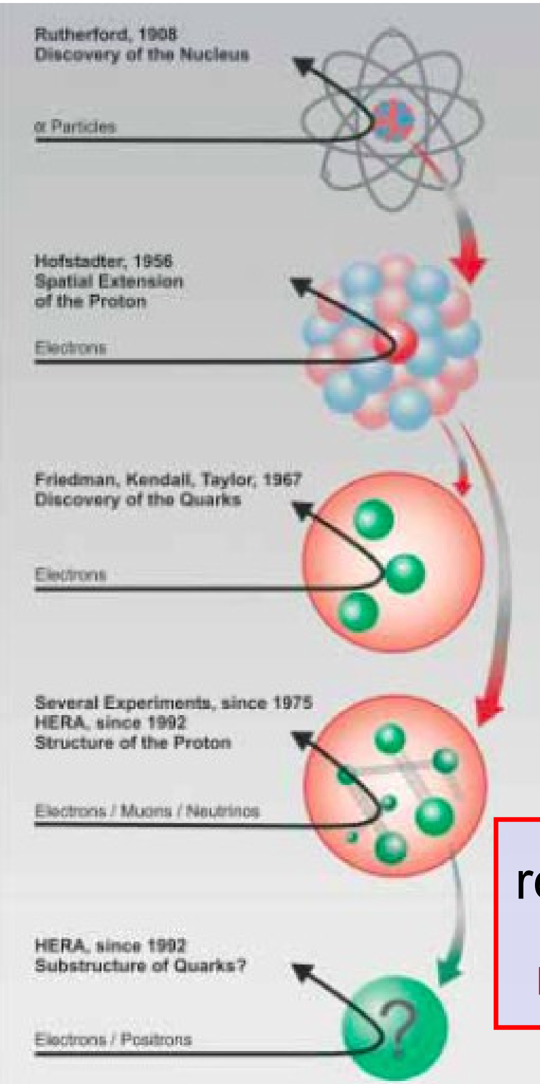
Strong Interactions: Quantum ChromoDynamics (QCD)



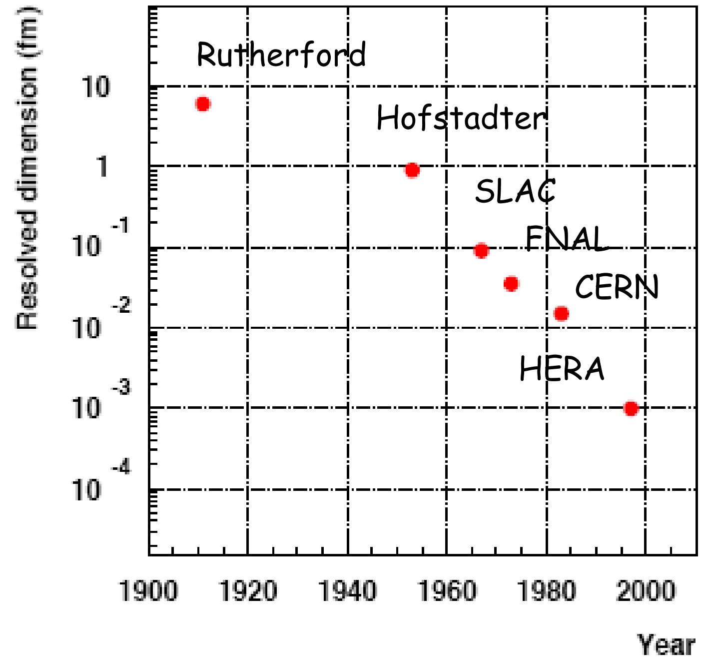


Confinement *(no free quarks/gluons)*

Deeply Inelastic Scattering (DIS)

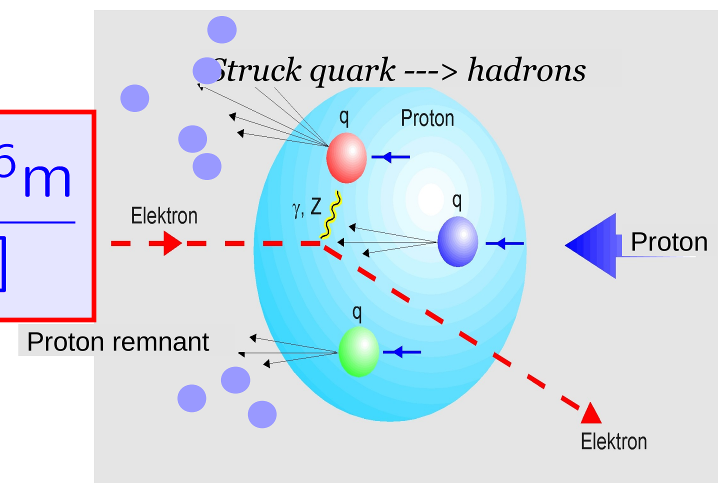


Probing
smaller distances
requires
larger momentum
transfer q
(small wavelength)

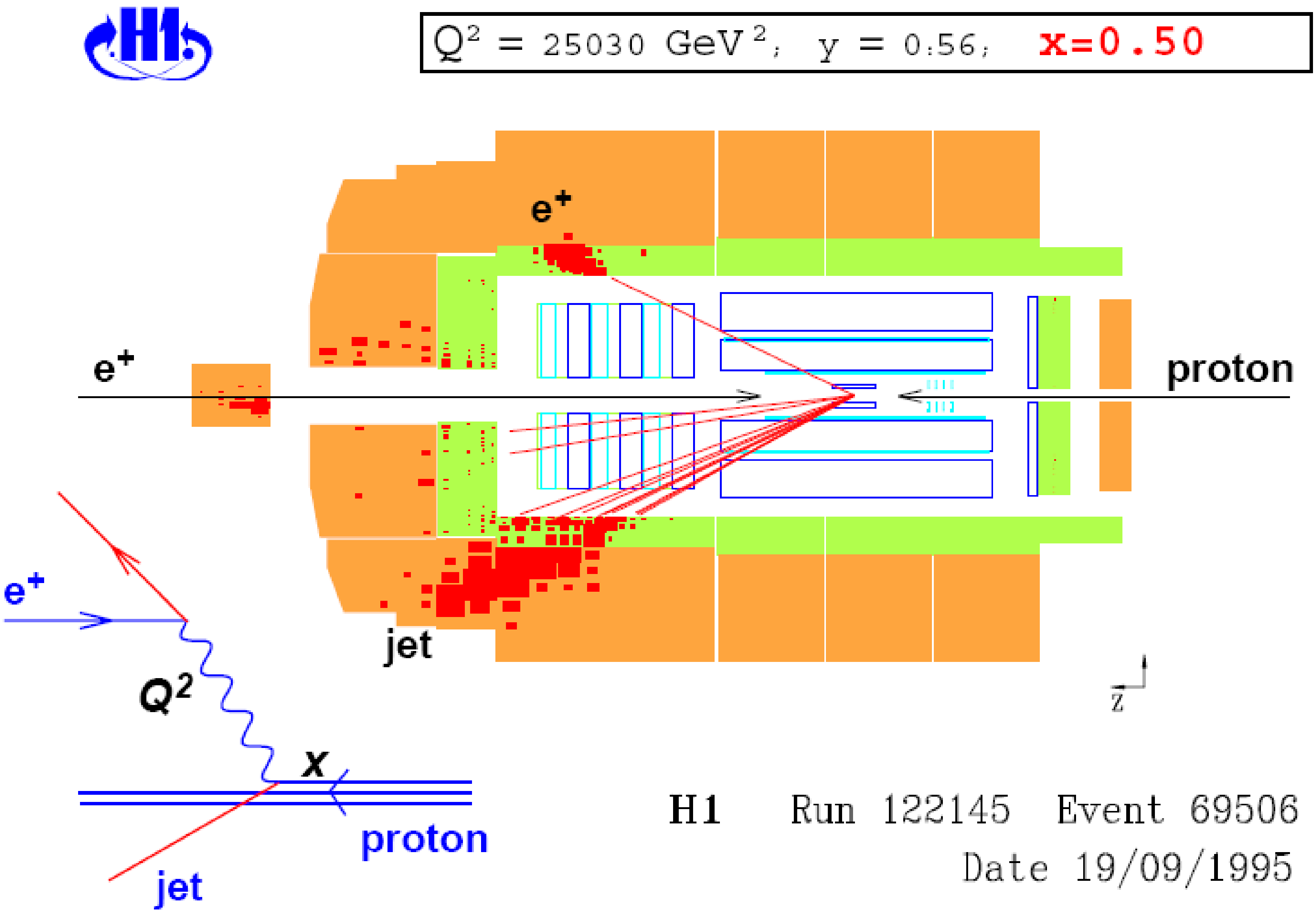


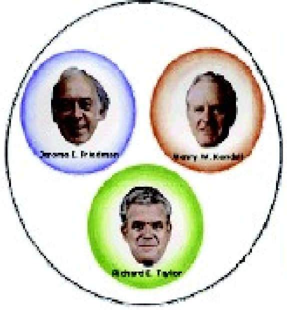
resolution: $\frac{\hbar}{Q} \approx \frac{2 \times 10^{-16}\mathrm{m}}{Q[\mathrm{GeV}]}$

r » 1/Q



A typical DIS event

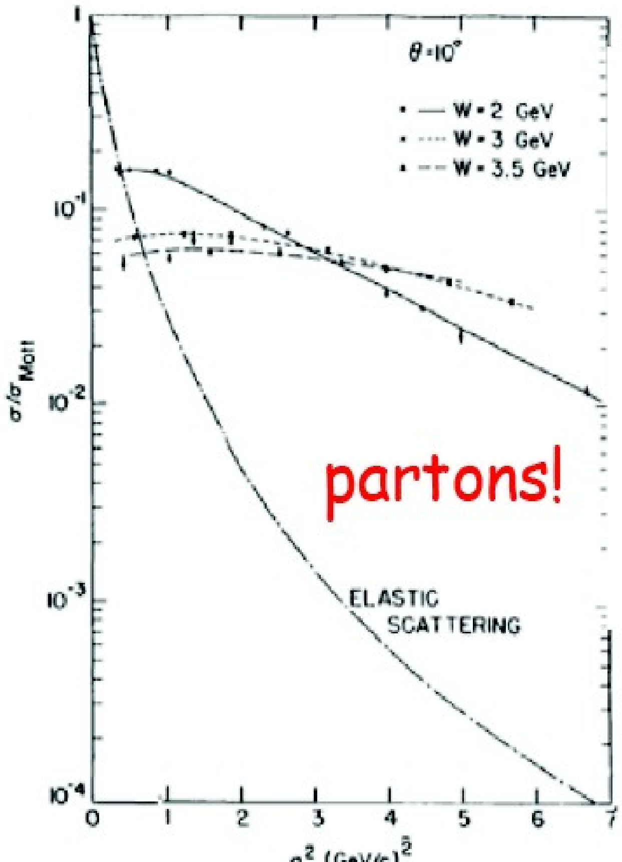


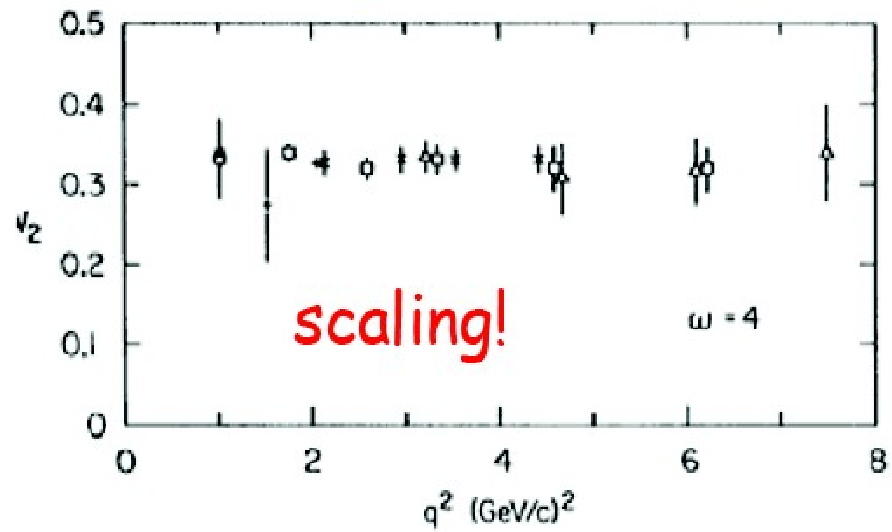


Deep-inelastic scattering (DIS) SLAC-MIT experiment of 1969

1990

two unexpected results:



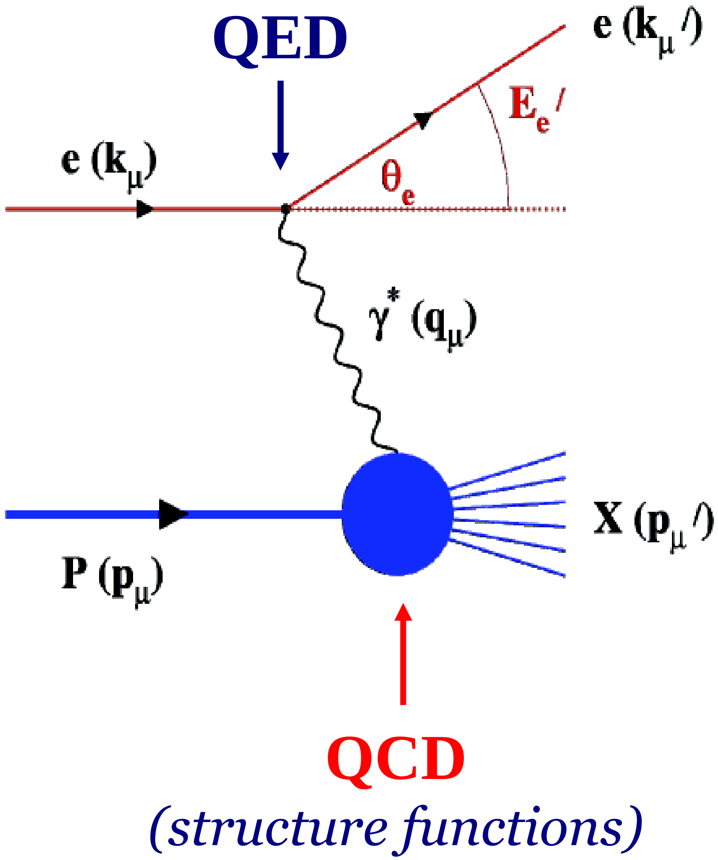


[recall Rutherford experiments]()

birth of the pre-QCD parton model

Deeply Inelastic Scattering (DIS)

probing hadron structure



Kinematic Invariants

$$Q^2 = -q^2 = -(k_\mu - k_\mu')^2$$

$$Q^2 = 4E_e E_e' \sin^2\left(\frac{\theta_e'}{2}\right)$$

Measure of resolution power

$$y = \frac{pq}{pk} = 1 - \frac{E_e'}{E_e}\cos^2\left(\frac{\theta_e'}{2}\right)$$

Measure of inelasticity

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{sy}$$

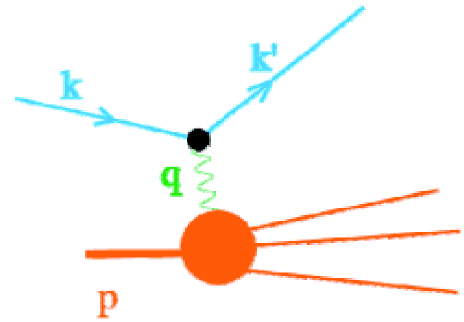
Measure of momentum fraction of struck quark

$$\mathbf{s \equiv (p + k)^2}$$

Deeply Inelastic Scattering

first analysis of DIS does not require any knowledge about QCD

electroweak theory tells us how
the virtual vector boson couples:
(let's assume only photon exchange)



$$d\sigma = \frac{4\alpha^2}{s}\frac{d^3\vec{k}'}{2|\vec{k}'|}\frac{1}{Q^4}L^{\mu\nu}(k,q)W_{\mu\nu}(p,q)$$

phase space scat. lepton

photon propagator^2

leptonic tensor

hadronic tensor contains information about hadronic structure

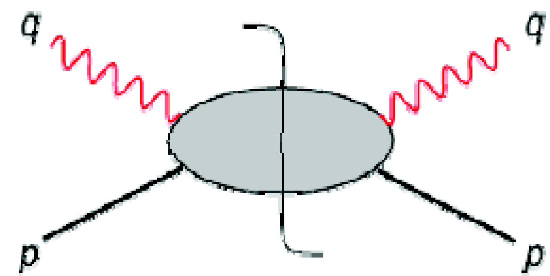
(can be easily generalized to W/Z-boson exchange)

with $$L_{\mu\nu} = 2(k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu} k\cdot k')$$

Deeply Inelastic Scattering

Strong interactions: contained in the hadronic tensor $\mathbf{W}_{\mu\nu}(\mathbf{p},\mathbf{q})$

to all orders in the strong interaction $W_{\mu\nu}$ is given by the square of $\gamma^*(q)\, h(p) \rightarrow X$



symmetries (parity, Lorentz), hermiticity & current conservation tell us that $W^{\nu\mu} = W^{\mu\nu *}$ $\quad q_\mu W^{\mu\nu} = 0$

$$
\begin{aligned}
W_{\mu\nu}(p,q) &= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1(x,Q^2) \\
&+ \left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right)\frac{1}{p\cdot q} F_2(x,Q^2)
\end{aligned}
$$

structure functions

DIS cross section- So far this is totally general

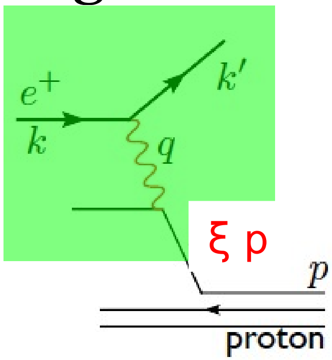
$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[[1+(1-y)^2]F_1(x,Q^2) + \frac{(1-y)}{x}[F_2(x,Q^2) - 2xF_1(x,Q^2)]\right]$$

different y-dep. can differentiate between F_1 and F_2-$2xF_1$

"naive" *parton model*: consider electron-parton scattering

find $\overline{\sum}|\mathcal{M}|^2 = 2e_q^2 e^4 \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$ with the usual Mandelstam's

$$\hat{s} = (k + p_q)^2$$
$$\hat{t} = (k - k')^2$$
$$\hat{u} = (p_q - k')^2$$



and use the massless 2->2 cross section

$$\frac{d\sigma}{dt} = \frac{1}{16\pi\hat{s}^2}\overline{\sum}|\mathcal{M}|^2$$ to obtain $$\frac{d\sigma}{dQ^2} = \frac{2\pi\alpha^2 e_q^2}{Q^4}[1+(1-y)^2]$$

next: use on-mass shell constraint

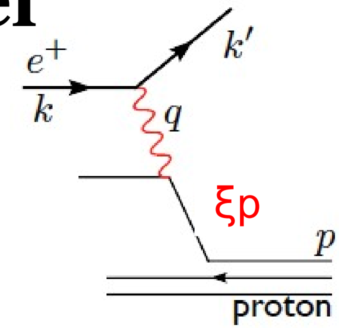
$$p_q'^2 = (p_q + q)^2 = q^2 + 2p_q \cdot q = -2p\cdot q = -2p\cdot q\,(x - \xi) = 0$$

this implies that ξ is equal to Bjorken x

$$\hat{s} = \xi Q^2/(xy) = \xi\, s$$
$$\hat{t} = q^2 = -Q^2$$
$$\hat{u} = \hat{s}\,(y-1)$$

to obtain $$\frac{d\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}[1+(1-y)^2]\frac{1}{2}e_q^2\delta(x-\xi)$$

DIS in the “naive” parton model



compare our result

$$\frac{d\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}[1+(1-y)^2]\frac{1}{2}e_q^2\delta(x-\xi)$$

to what one obtains with the hadronic tensor (parton level)

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[[1+(1-y)^2]F_1(x) + \frac{(1-y)}{x}(F_2(x) - 2xF_1(x))\right]$$

and read off

$$F_2 = 2xF_1 = xe_q^2\,\delta(x-\xi)$$

Callan-Gross relation

proton structure functions then obtained by weighting the parton str. fct.

with the **parton distribution functions** (probability to find a parton with momentum ξ)

$$F_2 = 2xF_1 = \sum_{q,q'}\int_0^1 d\xi\, q(\xi)\, xe_q^2\,\delta(x-\xi)$$

$$= \sum_{q,q'} e_q^2\, x\, q(x)$$

DIS measures the sum of quarks and anti-quarks

space-time picture of DIS

light cone variables

advantages: boosting is easy

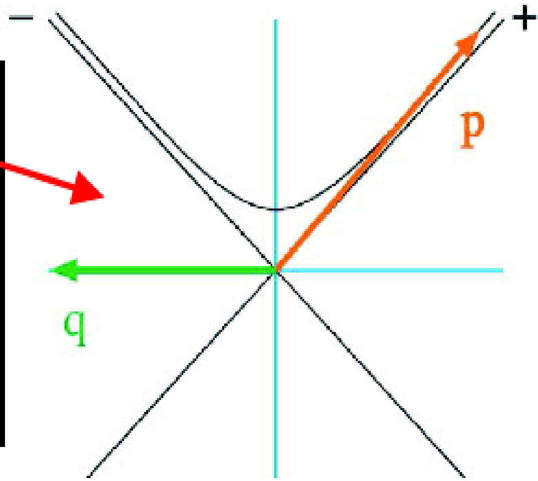
separation of large and small components of vectors

$$P^+ \equiv \frac{E + P_z}{\sqrt{2}}$$

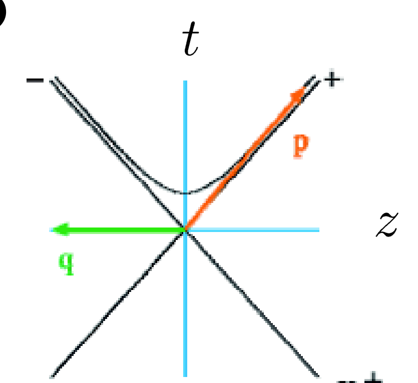
$$P^- \equiv \frac{E - P_z}{\sqrt{2}} \qquad (\mathbf{V^+}, \mathbf{V^-}, \mathbf{V_t}) \to (\mathbf{e^{\omega} V^+}, \mathbf{e^{-\omega} V^-}, \mathbf{V_t}) \text{ with } \mathbf{e^{\omega}} = \frac{\mathbf{Q}}{\mathbf{x\, m_h}}$$

$$P_t = P_t$$

4-vector	hadron rest frame	Breit frame
$(p^+, p^-, \vec{p}_T)$	$\frac{1}{\sqrt{2}}(m_h, m_h, \vec{0})$	$\frac{1}{\sqrt{2}}(\frac{Q}{x}, \frac{x m_h^2}{Q}, \vec{0})$
$(q^+, q^-, \vec{q}_T)$	$\frac{1}{\sqrt{2}}(-m_h x, \frac{Q^2}{m_h x}, \vec{0})$	$\frac{1}{\sqrt{2}}(-Q, Q, \vec{0})$



space-time picture of DIS

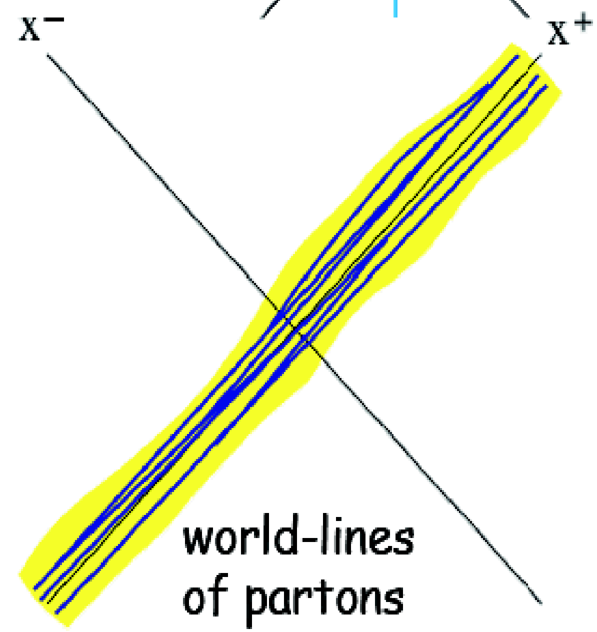


simple estimate for typical time-scale of interactions among the partons inside a fast-moving hadron:

rest frame: $\Delta x^+ \sim \Delta x^- \sim \frac{1}{m}$

Breit frame: $\Delta x^+ \sim \frac{1}{m}\frac{Q}{m} = \frac{Q}{m^2}$ large

$$\Delta x^- \sim \frac{1}{m}\frac{m}{Q} = \frac{1}{Q} \quad \text{small}$$

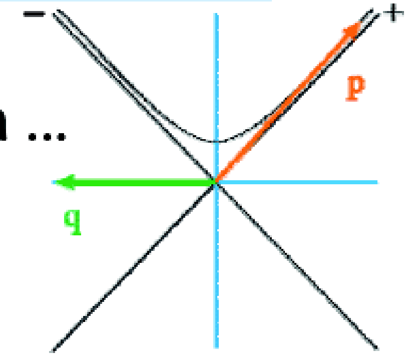


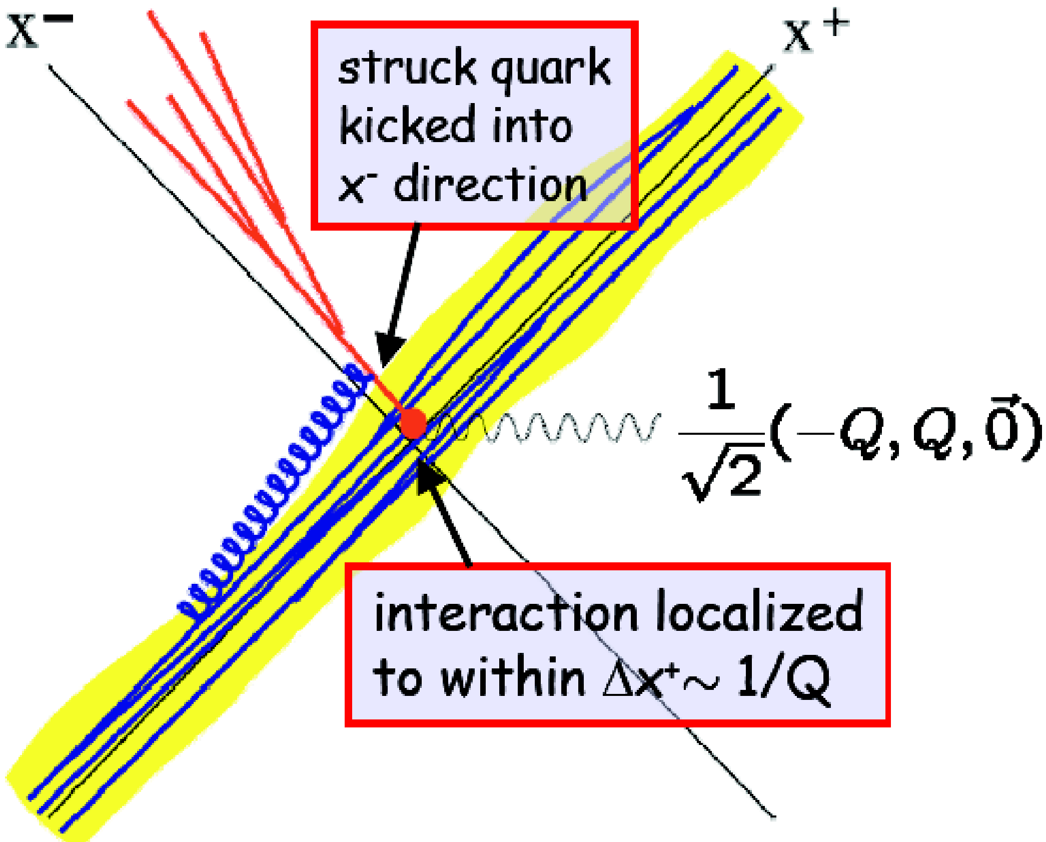
> interactions between partons are spread out inside a fast moving hadron

How does this compare with the time-scale of the hard scattering?

space-time picture of DIS

now let the virtual photon meet our fast moving hadron ...





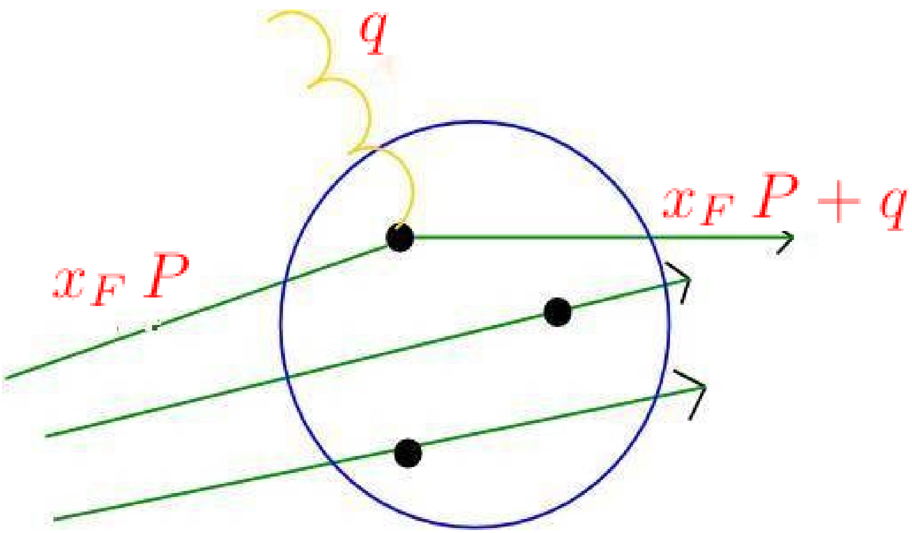
upshot:

- partons are free during the hard interaction
- hadron effectively consists of partons that have momenta $(p_i^+, p_i^-, \vec{p}_i)$
- convenient to introduce momentum fractions
  $0 < \xi_i \equiv p_i^+/p^+ < 1$

What is inside a hadron: Parton model

Bjorken: $\mathbf{Q^2}$, $\mathbf{S} \to \infty$ but keep $x_{Bj} = \frac{Q^2}{S}$ fixed (scaling limit)

Structure functions depend only on x_{Bj}

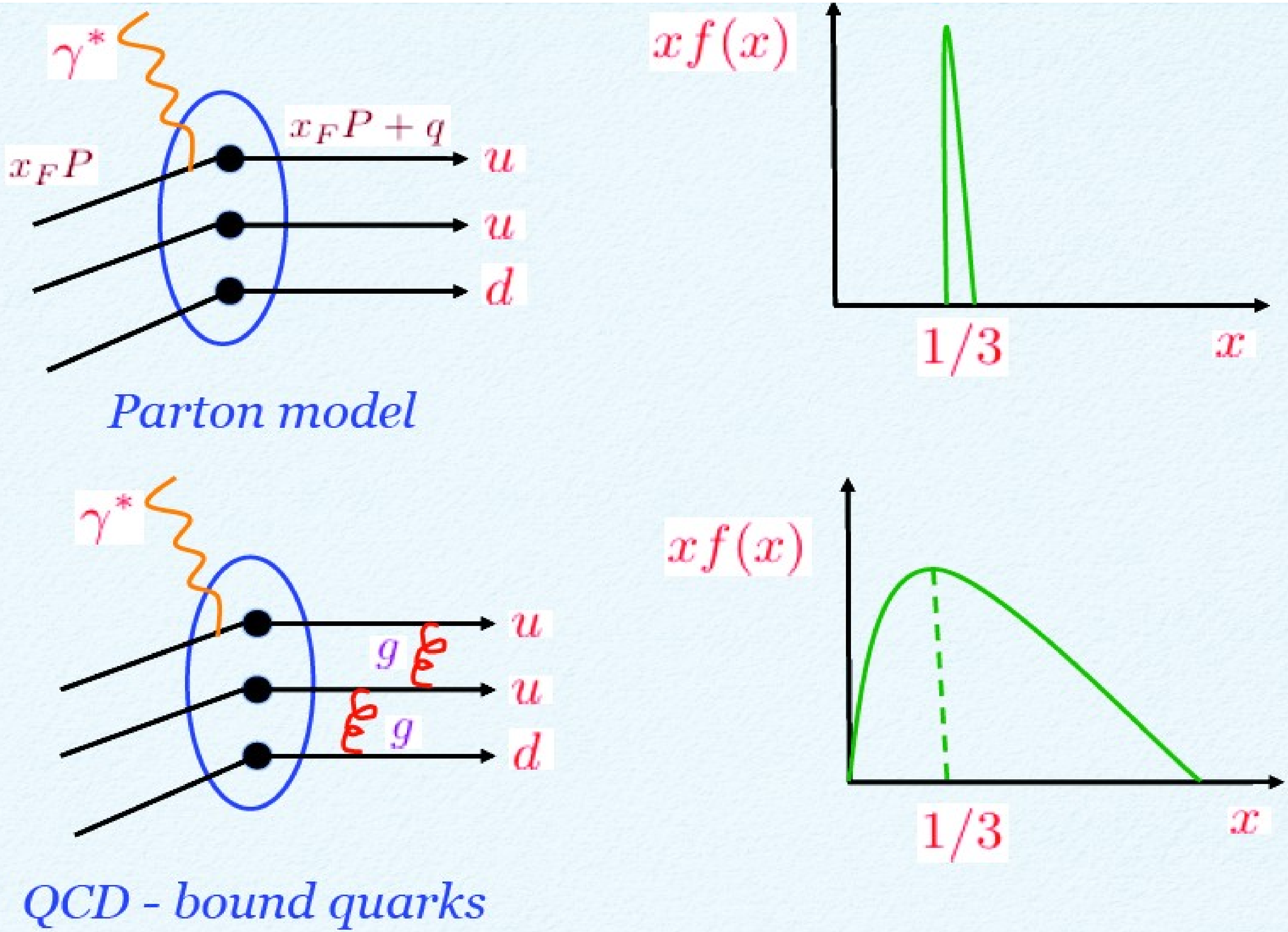


Feynman:

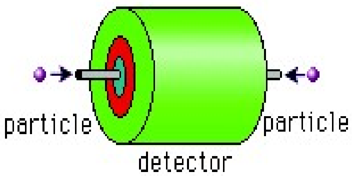
Parton constituents of proton are "free" on time scale $1/Q << 1/\Lambda$ (interaction time scale between partons)

X_{Bj} = ***fraction of hadron momentum carried by a parton*** = X_F

A hadron according to parton model

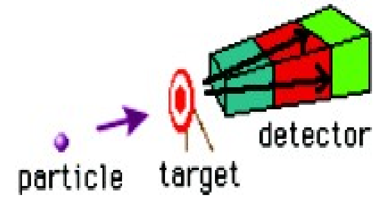


- Collider experiment: Electron-Proton collisions at HERA (DESY, Hamburg, Germany)

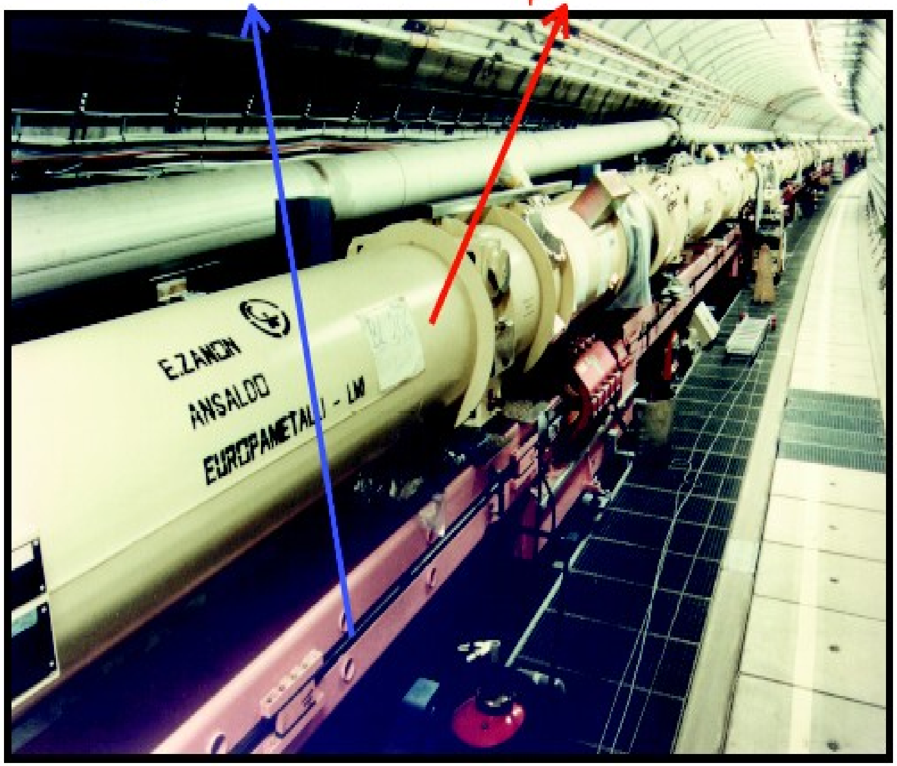


Equivalent to fixed target of

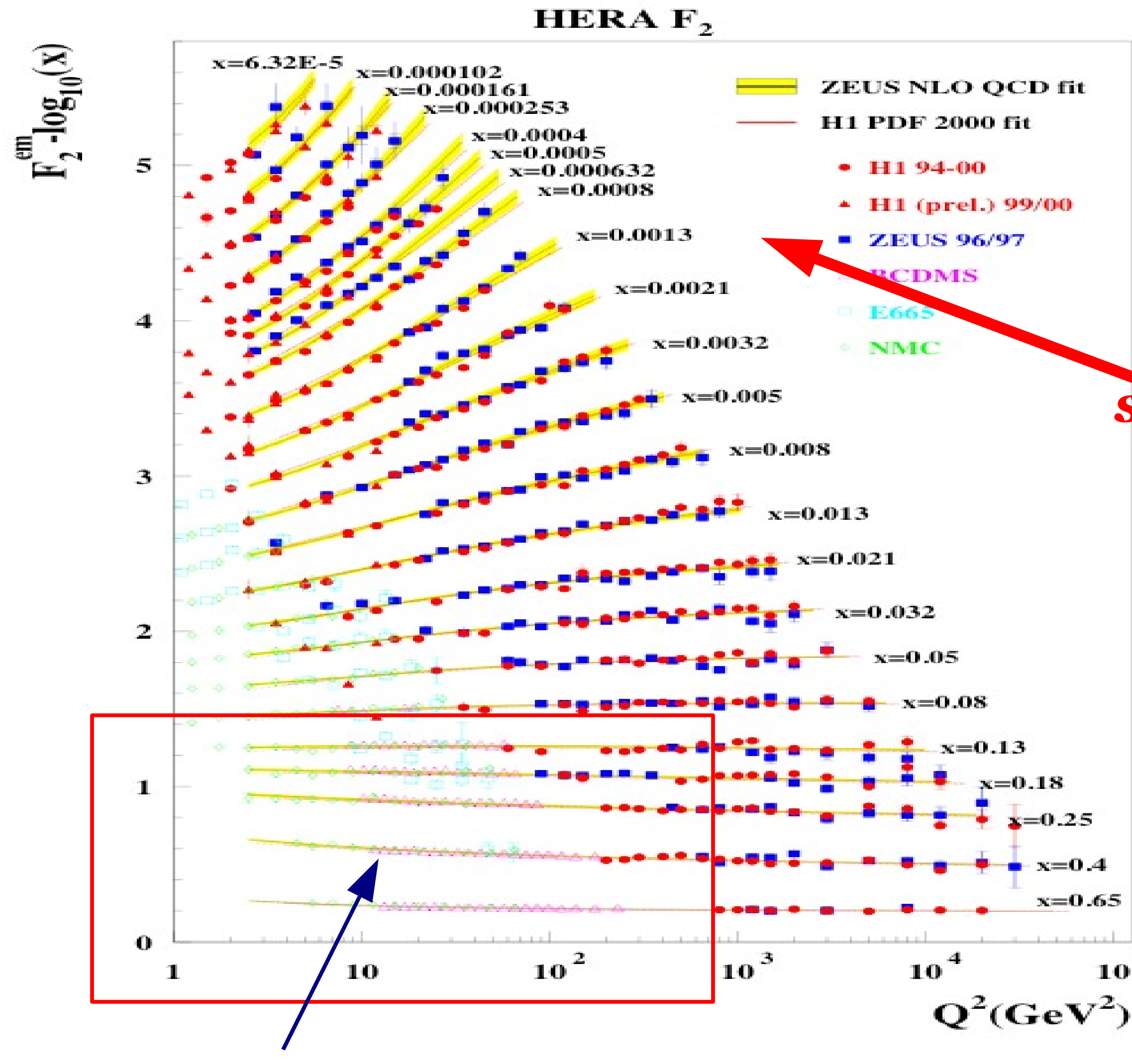
E_e = 50600 GeV:



E_e = 27.5 GeV E_p = 920 GeV



Deeply Inelastic Scattering $\mathbf{e(k)\,p(p) \to e(k')\,X}$



$$\mathbf{F_2} \equiv \frac{\mathbf{Q^2}}{4\,\pi^2\,\alpha_{\mathbf{em}}}\sigma^{\gamma^\star \mathbf{p}}$$

strong scaling violations

Parton model of a hadron leads to scale invariance

Can we understand scaling violations from QCD ?

early experiments (SLAC,...): scale invariance of hadron structure

Quantum ChromoDynamics (QCD)

Theory of strong interactions between quarks and gluons $\mathbf{SU(N_c)}$

$\mathbf{N_c = 3}$

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \sum^{\mathrm{f}} \bar{\Psi}^\alpha_i \left[i \not{D} - m_f \right]^{ij}_{\alpha\beta} \Psi^\beta_j$$

$$G^a_{\mu\nu}(x) \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$

color index:

$$a, b, c = 1, \cdots, 8$$

$$\alpha, \beta = 1, 2, 3$$

f^{abc} *Group structure constant*

Lorentz index: $\mu, \nu = 0, 1, 2, 3$

$\not{D} \equiv D_\mu \gamma^\mu$ *with* $\{\gamma^\mu, \gamma^\nu\} = 2 g^{\mu\mu}$

spinor index: $i, j = 1, 2, 3, 4$

$D_\mu \equiv \partial_\mu + i g A_\mu$ *Covariant derivative*

Quarks:
Fermions, spin 1/2
4x1 spinor, come in N_c *colors*
6 flavors (up, down, ..., top)
carry electric charge

Gluons:
Bosons, spin 1
come in $N_c^2 - 1$ *colors*
flavor blind
have no electric charge

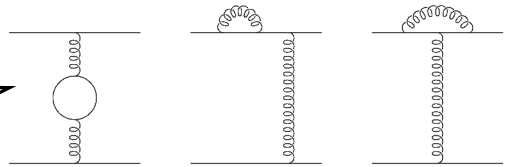
running of the coupling constant

asymptotic freedom

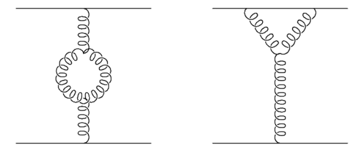
Gross, Wilczek; Politzer ('73/'74)
Nobel prize 2004

value of strong coupling $\alpha_s = g^2/4\pi$ depends on distance r (i.e. energy Q)

like QED

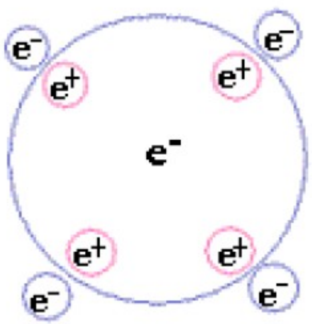


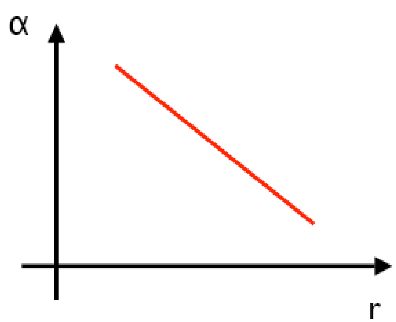
"screening" of the charge

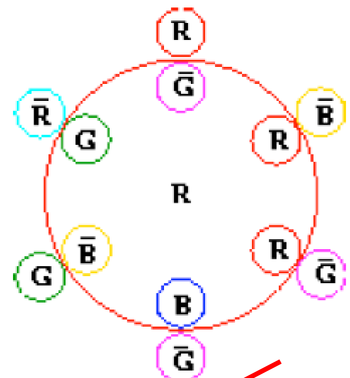


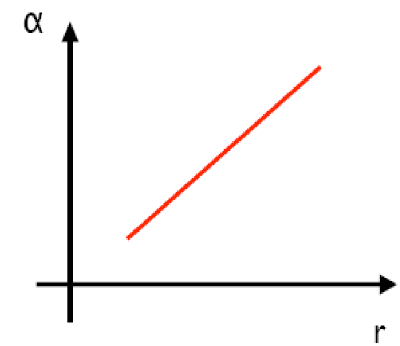
"anti-screening"

non Abelian









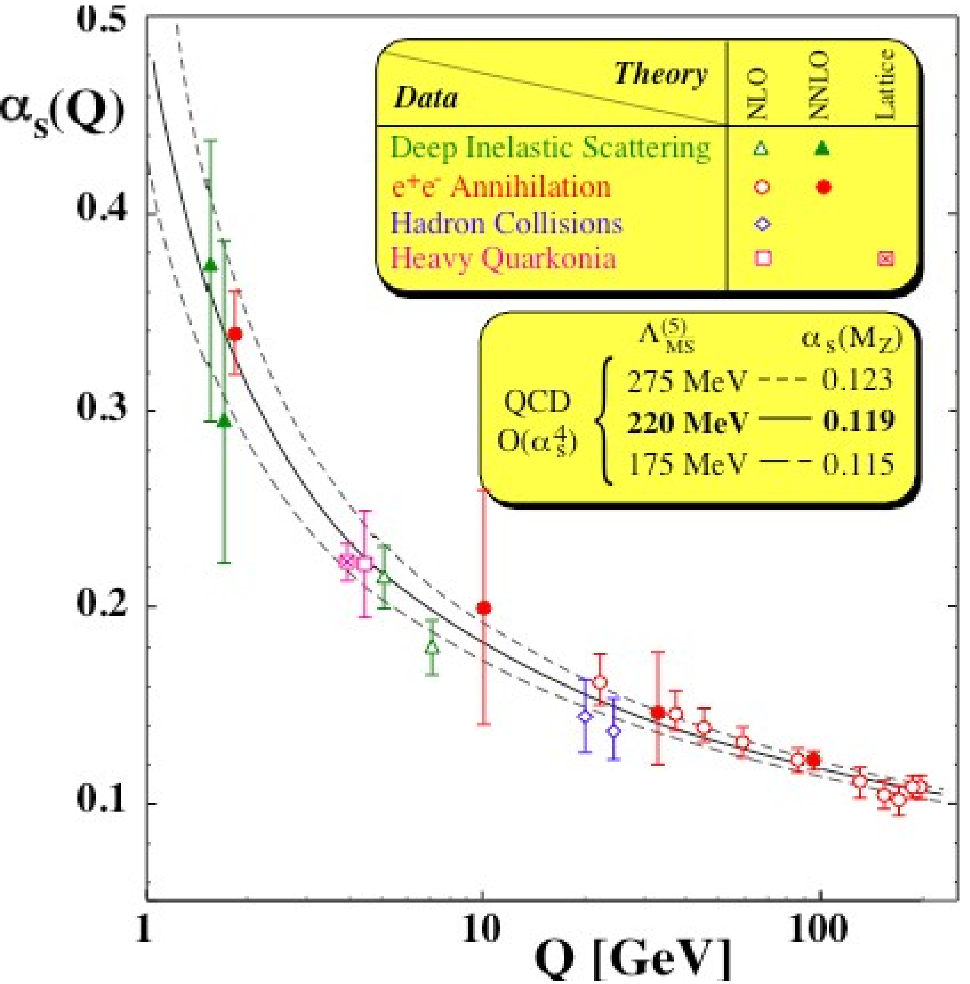
who wins?

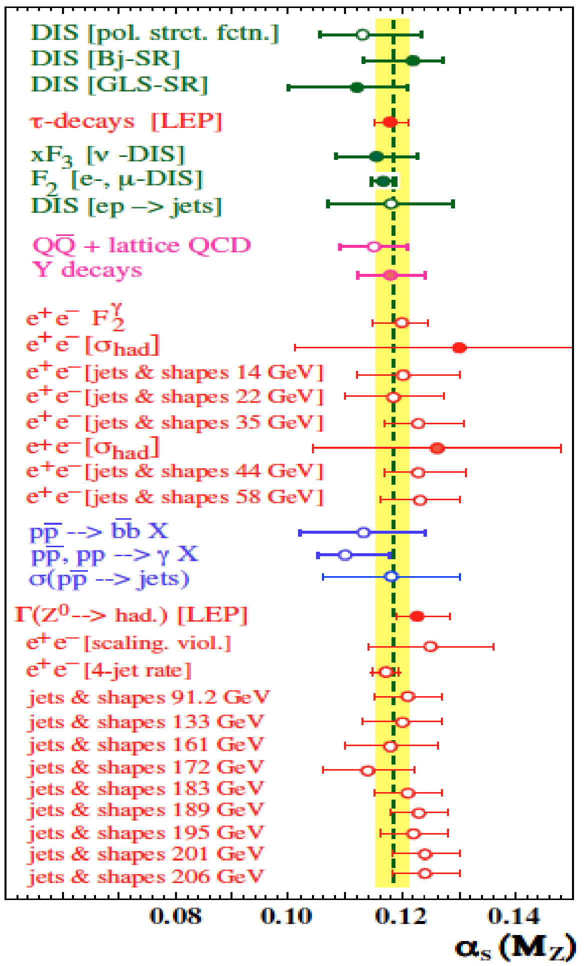
$$\alpha_s(Q^2) \approx \frac{4\pi}{(\frac{11}{3}C_A - \frac{4}{3}T_F N_f)\ln(Q^2/\Lambda^2)} \qquad Q \sim 1/r$$

typical hadronic scale O(200 MeV)

Λ depends on N_f, pert. order and scheme

running of the coupling constant





Perturbative QCD:

expansion in powers of the coupling constant

Tests of QCD:

running of the coupling constant
scaling violations in DIS
vector boson production
jet cross sections
....

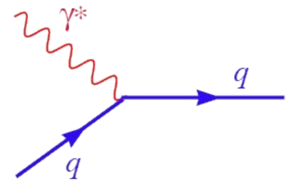
QCD is firmly established

we need to explore it

and

learn how to use it!

DIS in the QCD-improved parton model

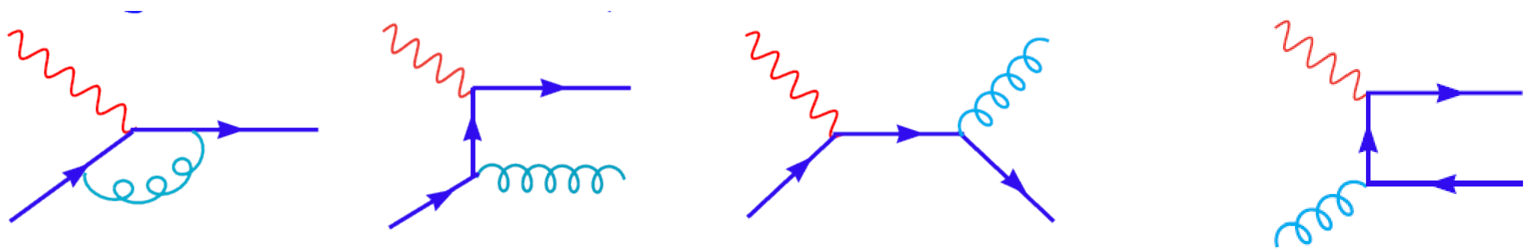


we got a long way (parton model) *without* invoking QCD

now we have to study **QCD dynamics in DIS**

– this leads to similar problems already encountered in e^+e^-

let's try to compute the **$O(\alpha_s)$ QCD corrections** to the naive picture



α_S corrections to the LO process photon-gluon fusion

caveat: ***expect divergencies***

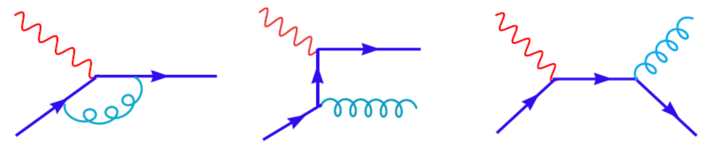
related to soft/collinear emission or from loops

what to do with infinities?

introduce "**regulator**" in the intermediate stages, remove it at the end

general structure of the O(α_s) corrections

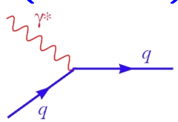
using small quark/gluon masses we obtain:



$$\left.\frac{d^2\hat{\sigma}}{dx\,dQ^2}\right|_{F_2} \equiv \hat{F}_2^q$$

LO

$$= e_q^2 x \left[\delta(1-x) + \frac{\alpha_s(\mu_r)}{4\pi}\left[P_{qq}(x)\ln\frac{Q^2}{m_g^2} + C_2^q(x)\right]\right]$$

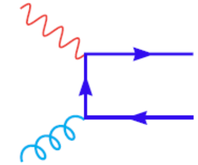


large logarithms (collinear emission)

finite coefficients

$$\left.\frac{d^2\hat{\sigma}}{dx\,dQ^2}\right|_{F_2} \equiv \hat{F}_2^g$$

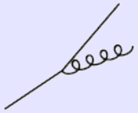
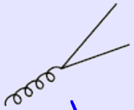
$$= \sum_q e_q^2 x \left[0 + \frac{\alpha_s(\mu_r)}{4\pi}\left[P_{qg}(x)\ln\frac{Q^2}{m_q^2} + C_2^g(x)\right]\right]$$



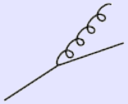
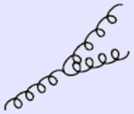
convolute with the PDFs $\mathbf{F_2(x, Q^2) \equiv \sum^{f} e_f^2\, x[q_f(x, Q^2) + \bar{q}_f(x, Q^2)]}$

DGLAP "evolution" equation:

scale dependence of parton distribution functions

$$\frac{d}{d \ln \mu} \begin{pmatrix} q(x,\mu) \\ g(x,\mu) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix}_{(z,\alpha_s)} \cdot \begin{pmatrix} q(x/z,\mu) \\ g(x/z),\mu) \end{pmatrix}$$

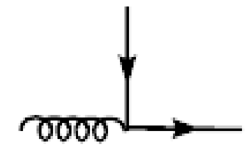
best solved in Mellin moment space: set of ordinary differential eqs.;
no closed solution in exp. form beyond LO (commutators of P matrices!)

properties of LO splitting functions

soft gluon divergence (z=1)
regulated by plus distribution

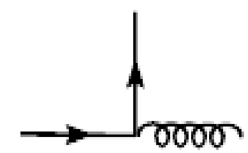
$$P_{qq}^{(0)} = P_{\bar{q}\bar{q}}^{(0)} = C_F \left[\left(\frac{1+z^2}{1-z} \right)_+ + \frac{3}{2}\delta(1-z) \right]$$

$$P_{qg}^{(0)} = P_{\bar{q}g}^{(0)} = T_R \left(z^2 + (1-z) \right)$$



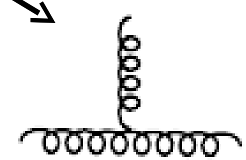
symmetric under
z -> (1-z)
except virtuals

$$P_{gq}^{(0)} = P_{g\bar{q}}^{(0)} = C_F \frac{1+(1-z)^2}{z}$$



soft gluon divergence (z=0)
not reached, always z > 0

$$P_{gg}^{(0)} = 2C_A \left[z \left(\frac{1}{1-z} \right)_+ + \frac{1-z}{z} + z(1-z) + b_0 \delta(1-z) \right]$$



soft gluon divergence (z=1)
regulated by plus distribution

involves "**plus distribution**" $$\int_0^1 dz\, [g(z)]_+ f(z) \equiv \int_0^1 dz\, g(z) \left[f(z) - f(1) \right]$$

condition: f(z) sufficiently smooth for z → 1

PDFs as bi-local operators

more physical formulation in Bjorken-x space:

matrix elements of *bi-local operators* on the light-cone

for quarks: (similar for gluons)

$$f_a(\xi,\mu_f) = \frac{1}{2}\int \frac{dy^-}{2\pi} e^{-i\xi p^+ y^-} \langle p|\overline{\Psi}_a(0,y^-,\vec{0})\gamma^+ \mathcal{F}\Psi_a(0)|p\rangle_{\overline{\mathrm{MS}}}$$

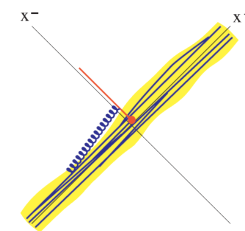
Fourier transform with momentum $\xi\, p^+$

recreates quark at $x^+=0$ and $x^-=y^-$

annihilates quark at $x^\mu=0$

in general we need a "**gauge link**" for a gauge invariant definition:

$$\mathcal{F} = \mathcal{P}\exp\left(-ig\int_0^{y^-} dz^- A_c^+(0,z^-,\vec{0})T_c\right)$$



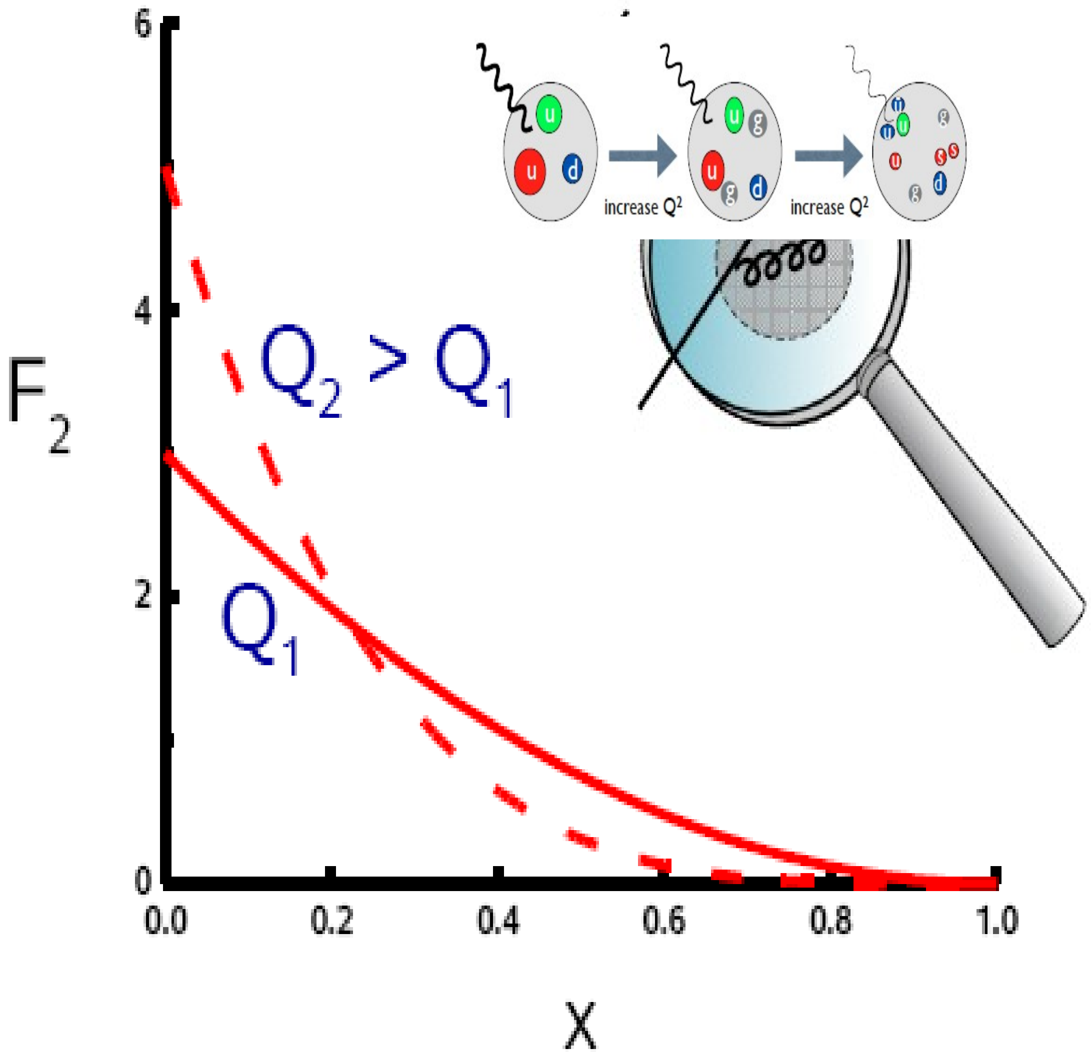
crucial role for a special class of **"transverse-momentum dep. PDFs"**
describing phenomena with transverse polarization **("Sivers function", …)**

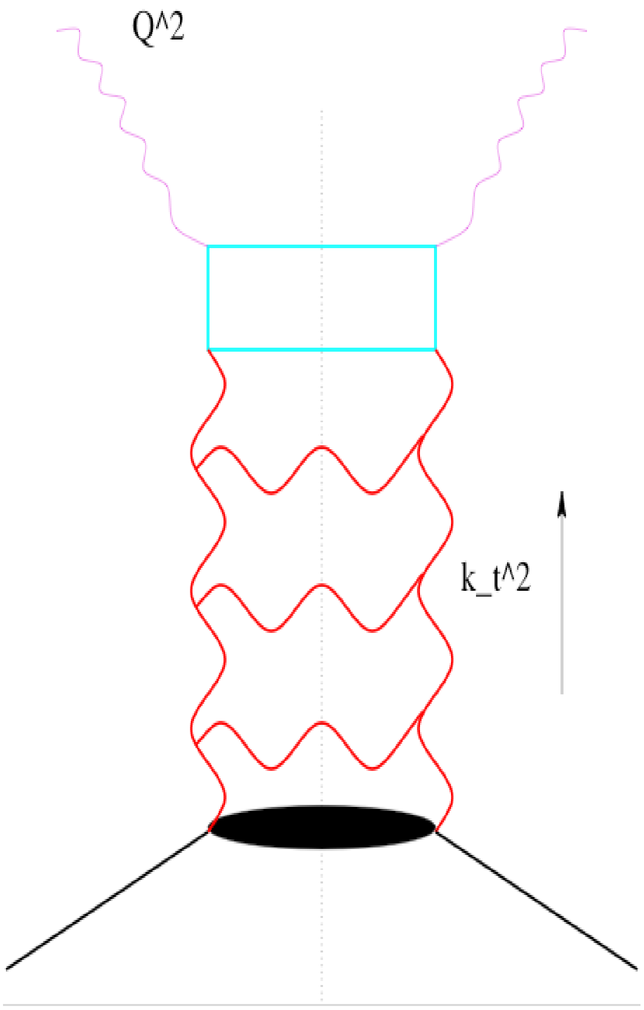
interpretation as ***number operator*** only in "**$A^+ = 0$ gauge**"

DGLAP "evolution" equation:

scale dependence of parton distribution functions

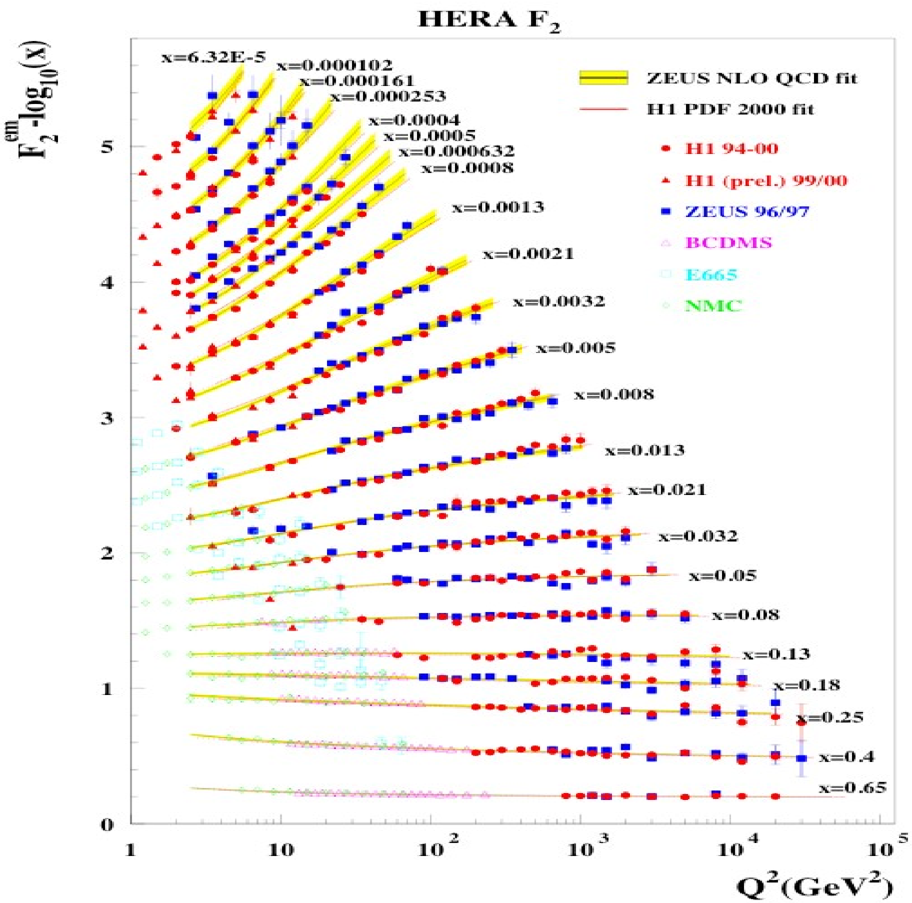
Dokshitzer-**G**ribov-**L**ipatov-**A**ltarelli-**P**arisi

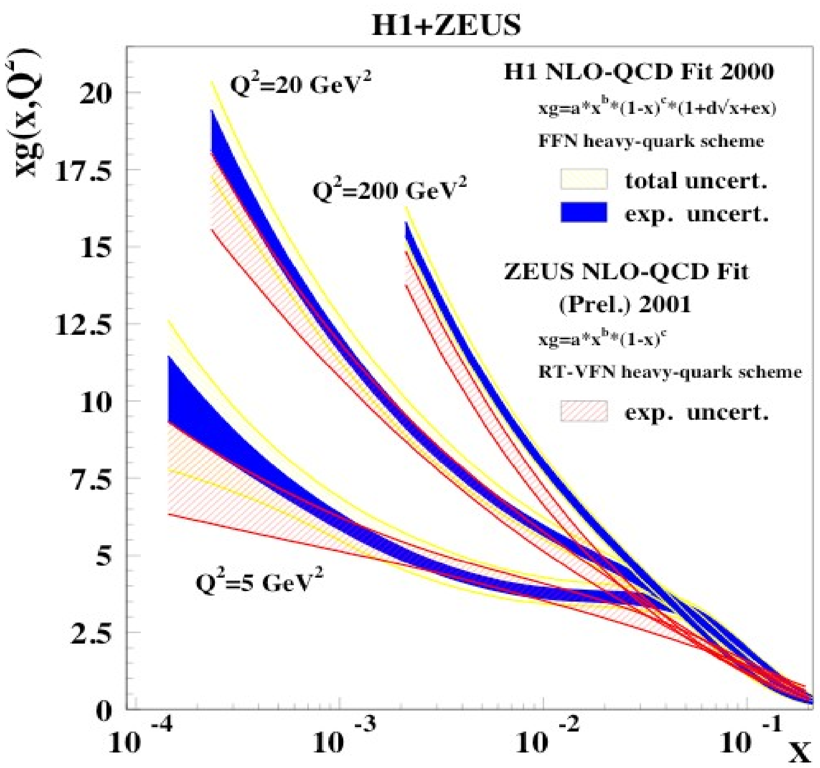




DGLAP "evolution" equation:

scale dependence of parton distribution functions





What drives the growth of parton distributions?

Splitting functions at leading order $O(\alpha_S{}^0)$ $(x \neq 1)$

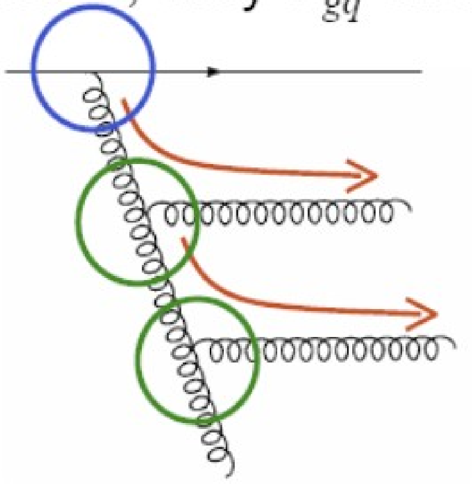
$$P_{qq}^{(0)}(x) = C_F \frac{1+x^2}{1-x}$$

$$P_{qg}^{(0)}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right]$$

$$P_{gq}^{(0)}(x) = C_F \frac{1+(1-x)^2}{x}$$

$$P_{gg}^{(0)}(x) = 2C_A\left[\frac{x}{1-x} + \frac{1-x}{x} + x(1-x)\right]$$

At small x, only P_{gq} and P_{gg} are relevant.



→ Gluon dominant at small *x*!

The double log approximation (DLA) of DGLAP is easily solved.

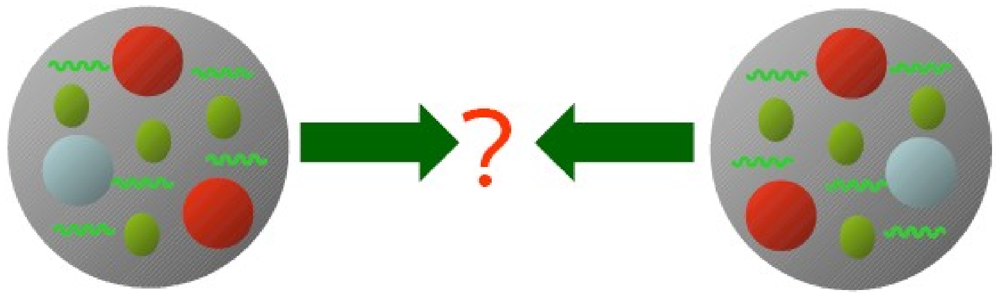
-- increase of gluon distribution at small x

$$\mathbf{xg(x, Q^2) \sim e^{\sqrt{\alpha_s\,(\log 1/x)\,(\log Q^2)}}}$$

Keep this in mind: will be the <u>focus</u> of next part

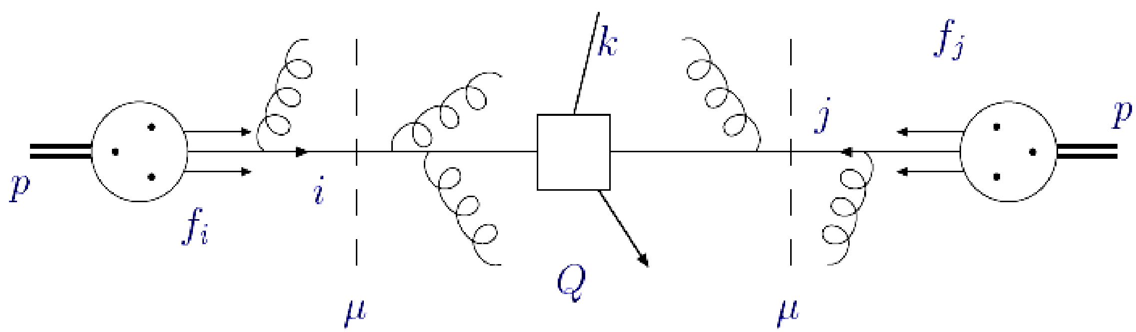
pQCD at work:hadron-hadron collisions

What happens when two hadrons collide ?



concepts discussed so far fall in the category of collinear factorization in pQCD

Production of jets, hadrons, heavy quarks, ...

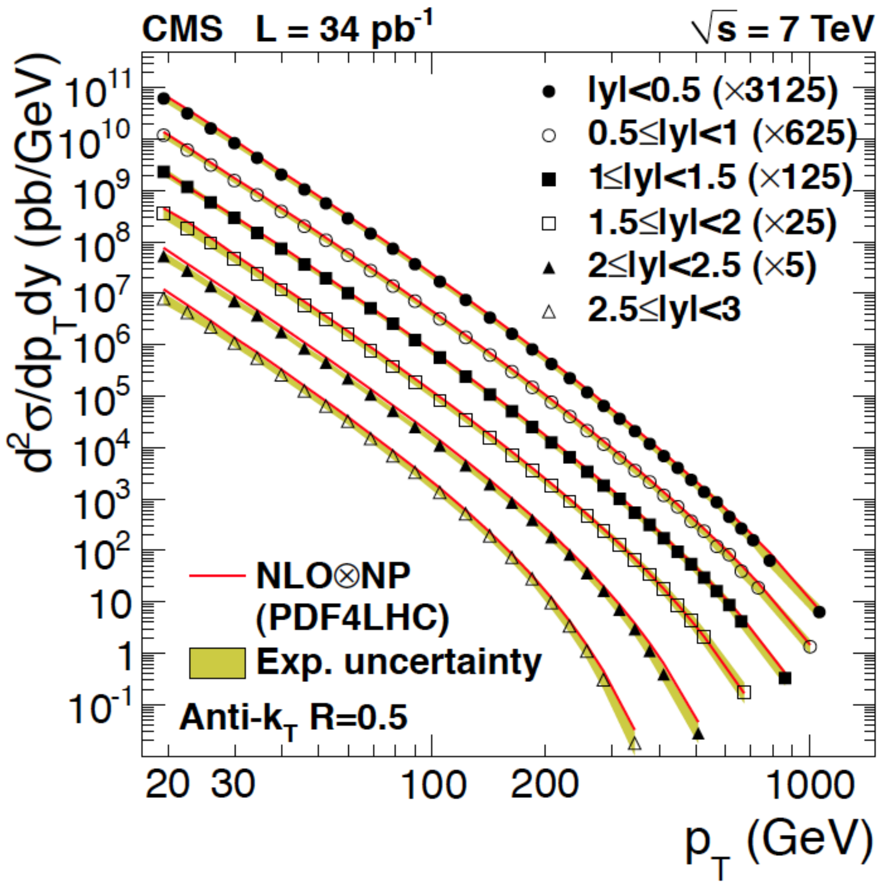


$$d\sigma = \sum_{ij} \int dx_i dx_j \, f_i(x_i, \mu^2) \, f_j(x_j, \mu^2) \, d\hat{\sigma}_{ij}(\alpha_s(\mu_r), Q^2, \mu^2, x_i, x_j)$$

non-perturbative but universal PDFs ← **linked by μ** → **hard scattering of two partons: pQCD**

Large Hadron Collider (LHC)

pQCD: a success story



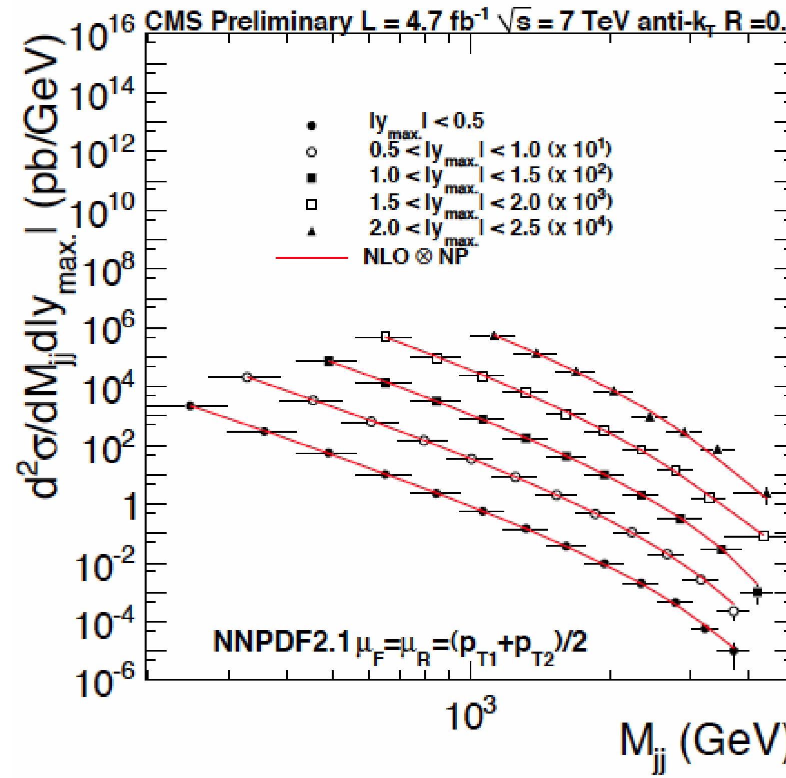
two recent examples from the LHC:

1-jet and di-jet cross sections

many other final-states available

$$\mathbf{y} = \ln\tan\frac{\theta}{2} \sim \frac{1}{2}\ln\frac{\mathbf{x_1}}{\mathbf{x_2}} \qquad \mathbf{M} = \sqrt{\mathbf{x_1 x_2 s}}$$

$$\mathbf{x_1} = \frac{\mathbf{M}}{\sqrt{\mathbf{s}}}\mathbf{e}^{+\mathbf{y}} \qquad \mathbf{x_2} = \frac{\mathbf{M}}{\sqrt{\mathbf{s}}}\mathbf{e}^{-\mathbf{y}}$$



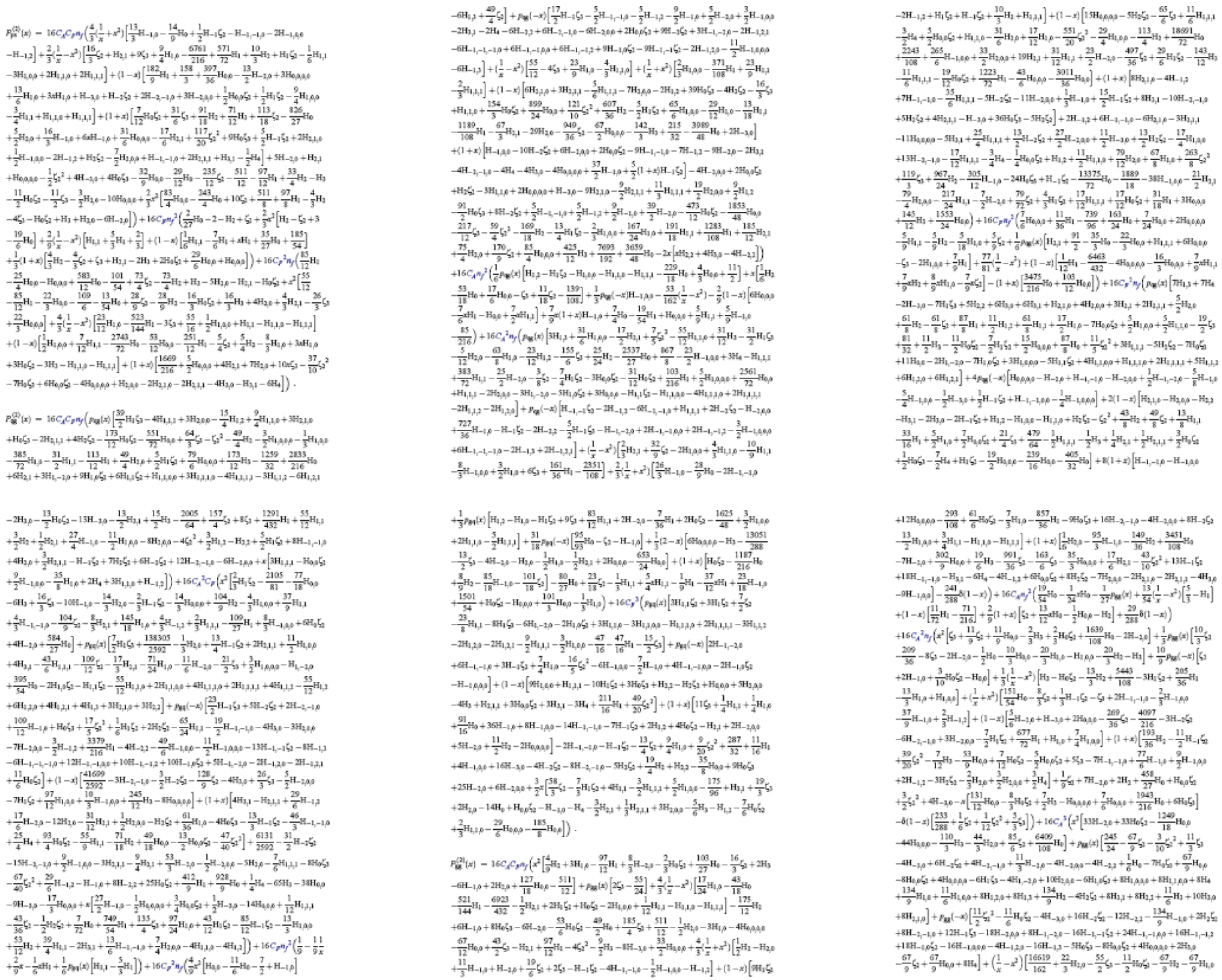
results now start to being used in global fits to constrain PDFs

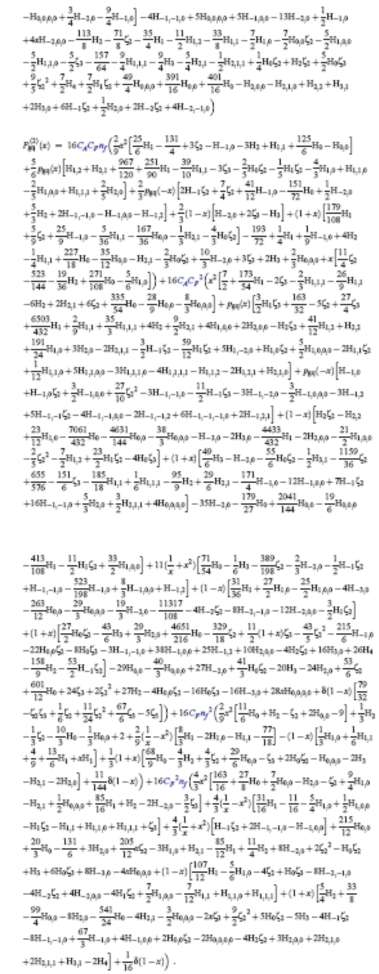
particularly sensitive to gluons

$$\mathbf{gg \to gg} \qquad \mathbf{gq \to gq}$$

Precision QCD : P_{ij} @ NNLO

10000 diagrams, 10^5 integrals, 10 man years, and several CPU years later:





Moch, Vermaseren, Vogt

2004

NNLO the new emerging standard in QCD – essential for precision physics

You may be wondering,

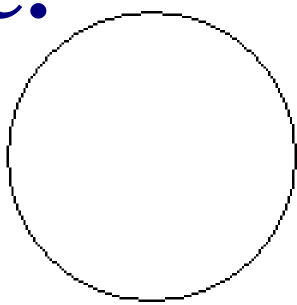
“Am I in the right place?”

What does this have to do with NUNCLEAR PHYSICS?

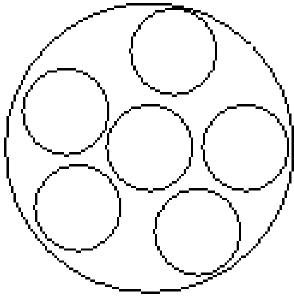
What is a nucleus?

depends on the resolution$_T$ scale!

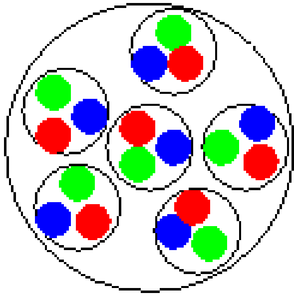
A point particle: λ_t >> 10 fm



A collection of nucleons: $\lambda_t \sim 1$ fm

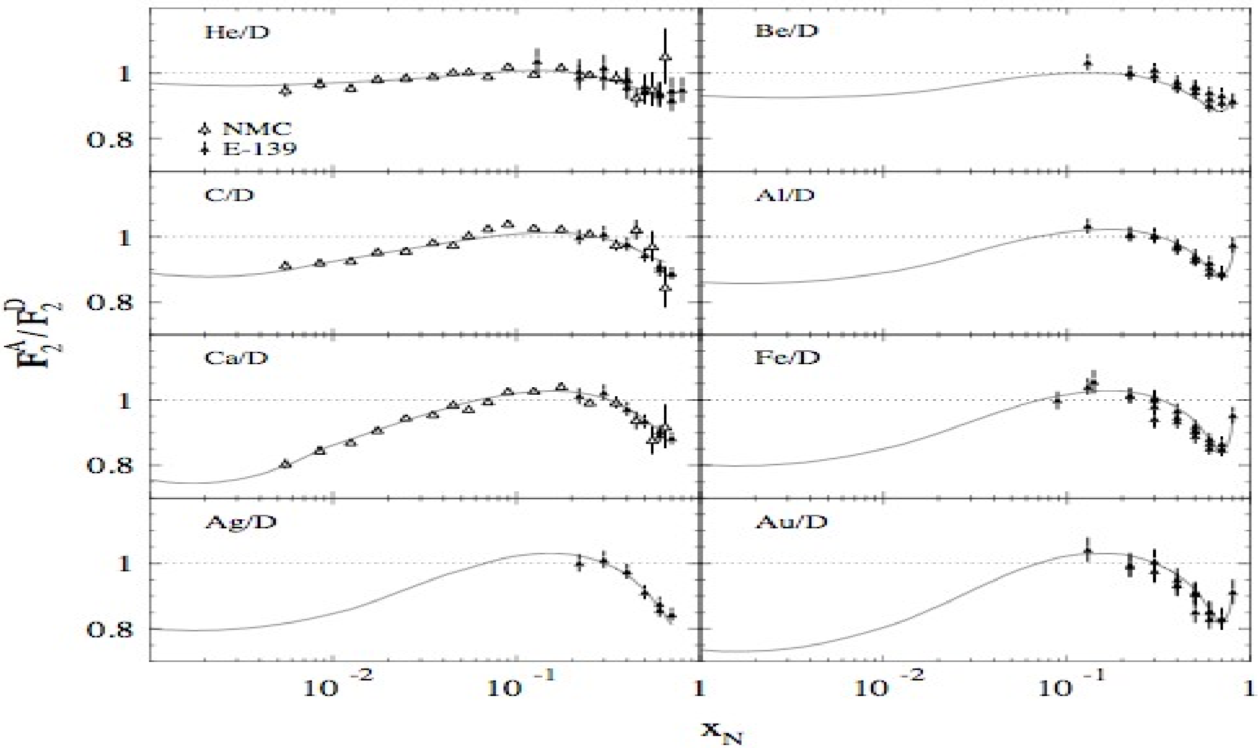


A system of <u>quarks and gluons</u>: λ_t << 1 fm



we should be able to use pQCD

modification of the nuclear structure functions



$(\propto 1/c.m.\,energy)$

How about scattering of nuclei?

RHIC, LHC

I) modification of initial state: "nuclear shadowing"

II) modification of hard scattering: multiple scattering

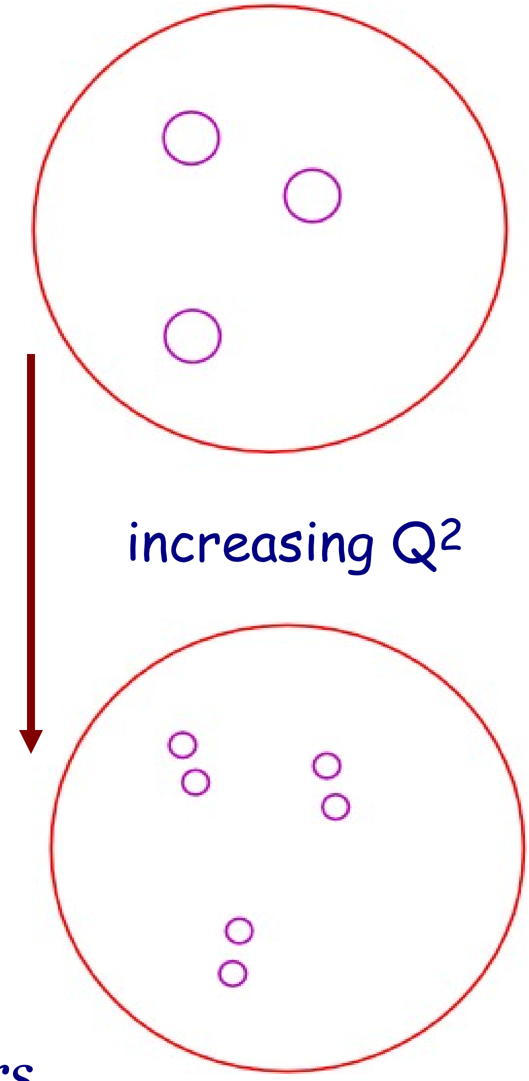
III) modification of fragmentation functions

how do partons hadronize?

so far we have considered PQCD in the Bjorken limit $Q^2, \ S \to \infty \quad x_{Bj} \equiv \frac{Q^2}{S}$ *fixed*

DGLAP evolution of partons

- *number of partons increases with Q²*
- *but parton number density decreases*
- ***hadron becomes more dilute***



Excellent tool for high Q² inclusive observables

- *higher twists become important at low Q²*

Not designed to treat collective phenomena:

- *shadowing*
- *multiple scattering*
- *diffraction*
- *impact parameter dependence*
- *............*

Extension beyond leading twist is very difficult

- *many-body dynamics hidden in parameters*