

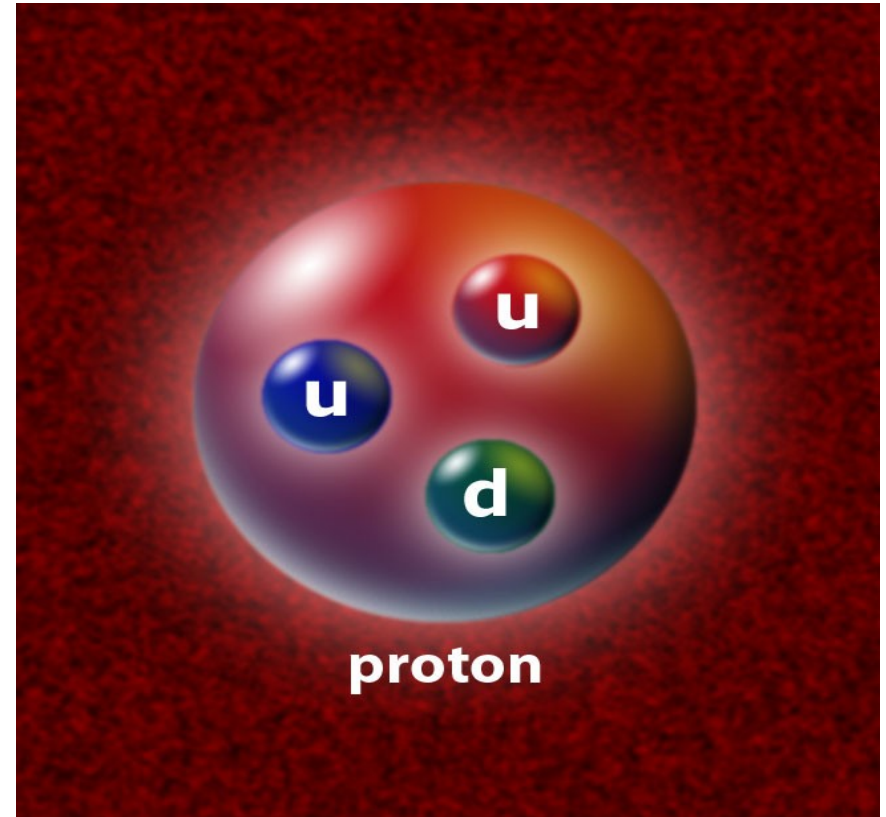
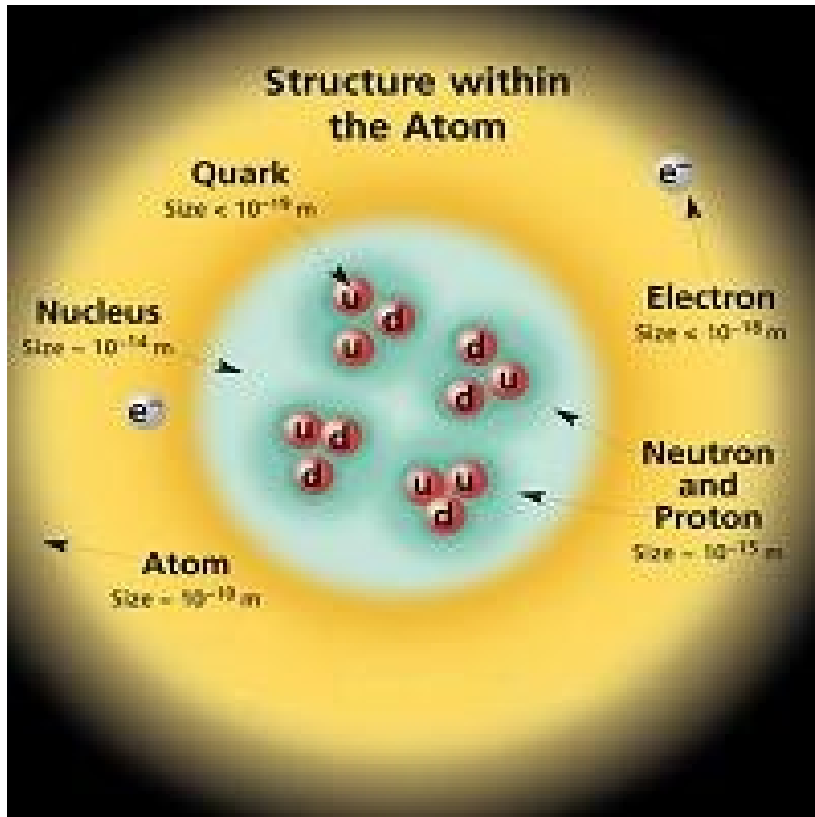
Introduction to High Energy Nuclear Collisions I (QCD at high gluon density)

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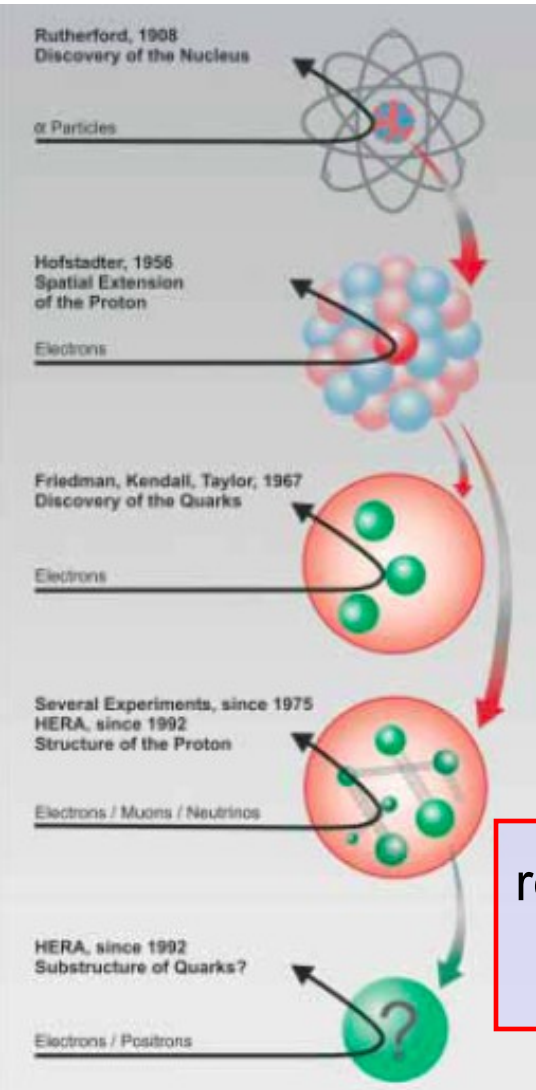
**Many thanks to my
colleagues, A. Deshpande, F.
Gelis, B. Surrow and
specially M. Stratmann for
providing me with their
slides**

Strong Interactions: Quantum ChromoDynamics (QCD)

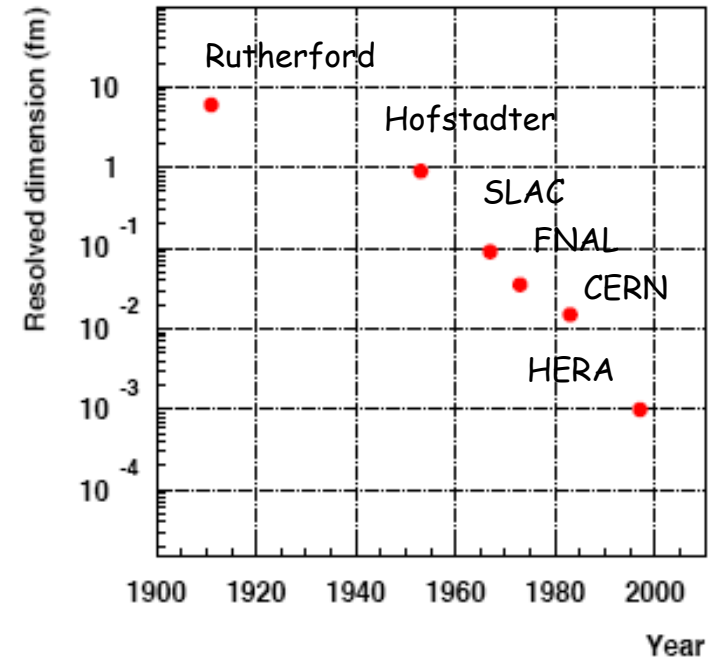


Confinement (*no free quarks/gluons*)

Deeply Inelastic Scattering (DIS)

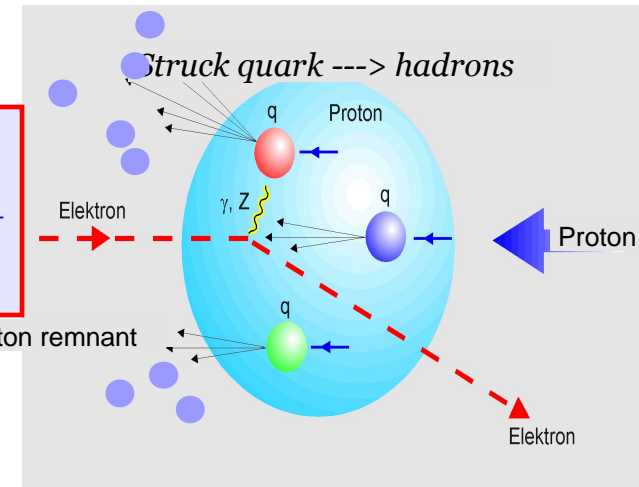


Probing
smaller distances
requires
larger momentum
transfer q
(small wavelength)



resolution: $\frac{\hbar}{Q} \approx \frac{2 \times 10^{-16} \text{m}}{Q[\text{GeV}]}$

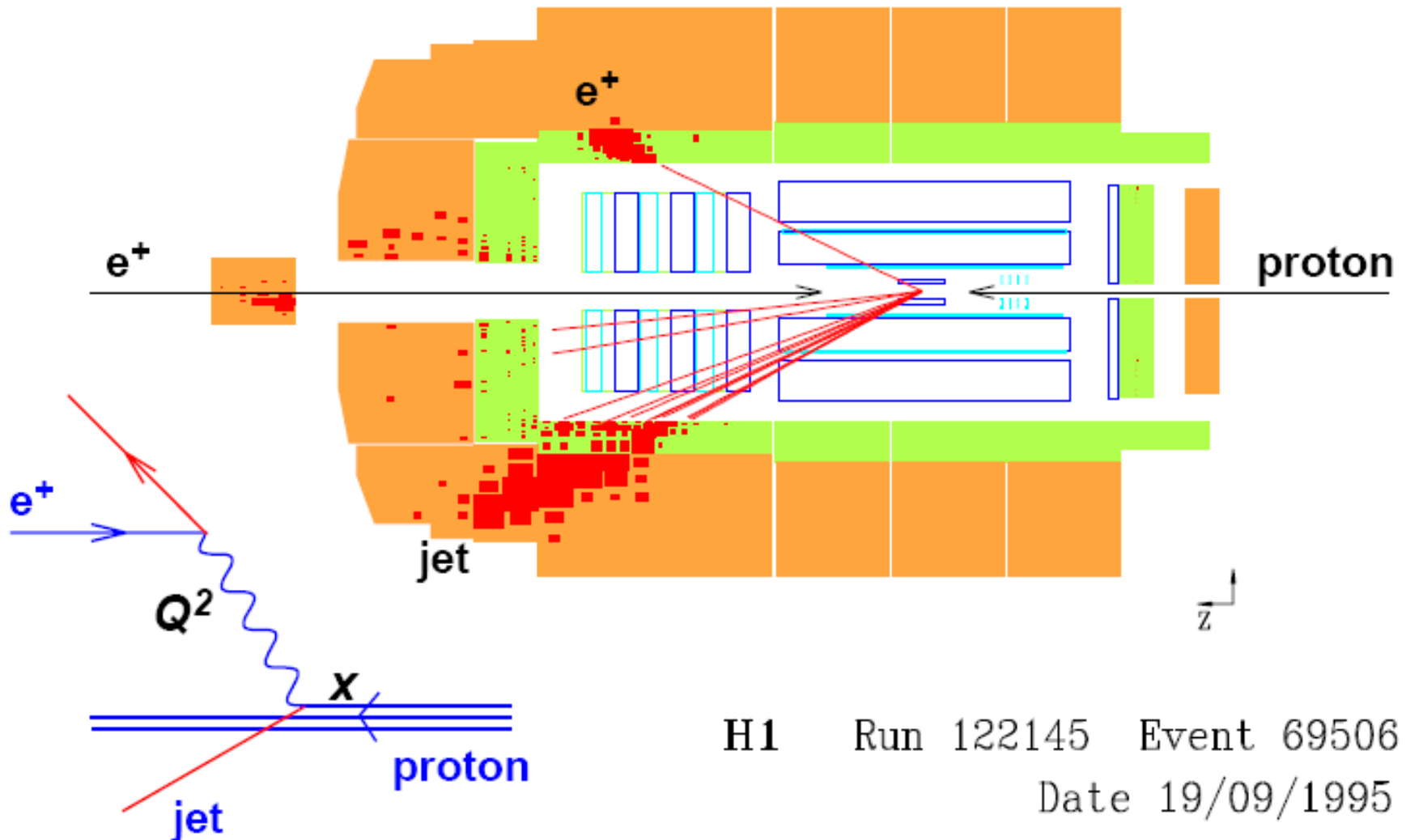
$r \gg 1/Q$



A typical DIS event



$Q^2 = 25030 \text{ GeV}^2$, $y = 0.56$, $x = 0.50$

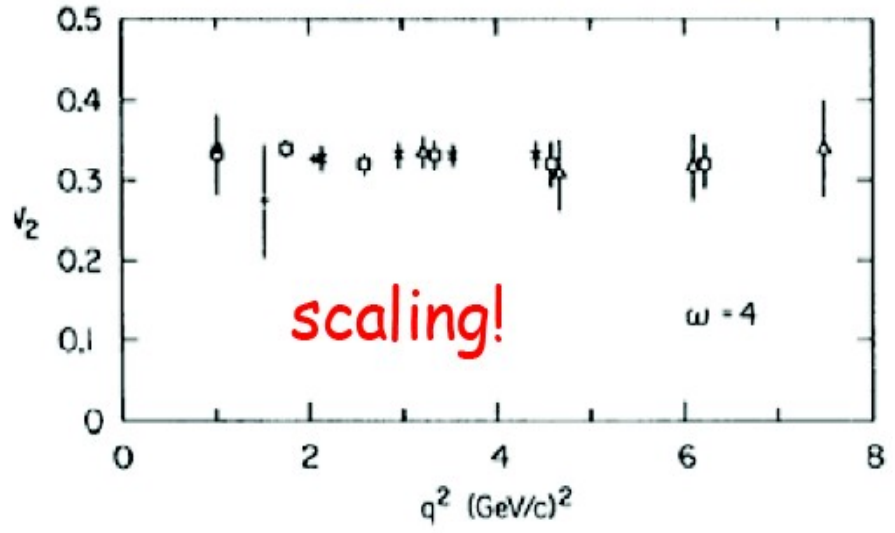
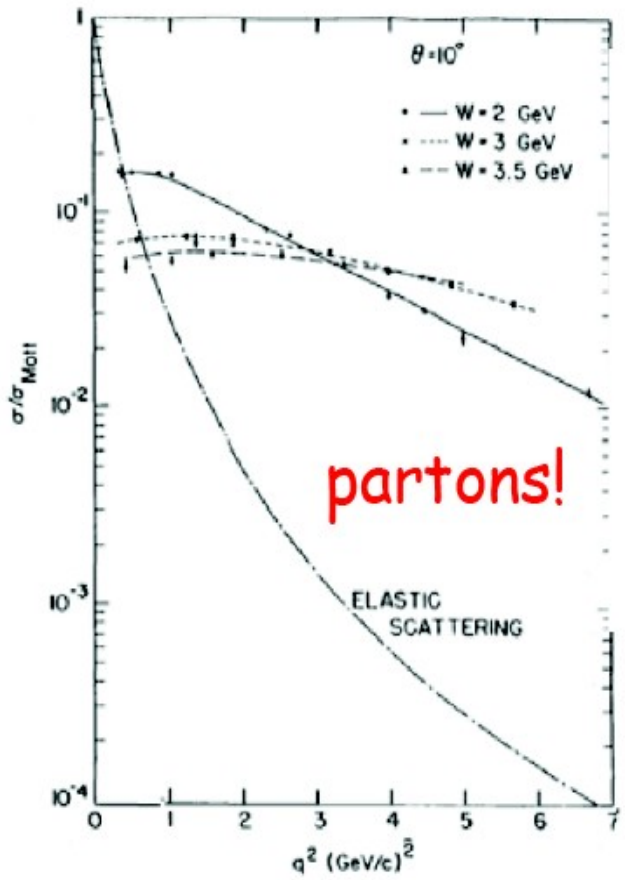




Deep-inelastic scattering (DIS) SLAC-MIT experiment of 1969



two unexpected results:



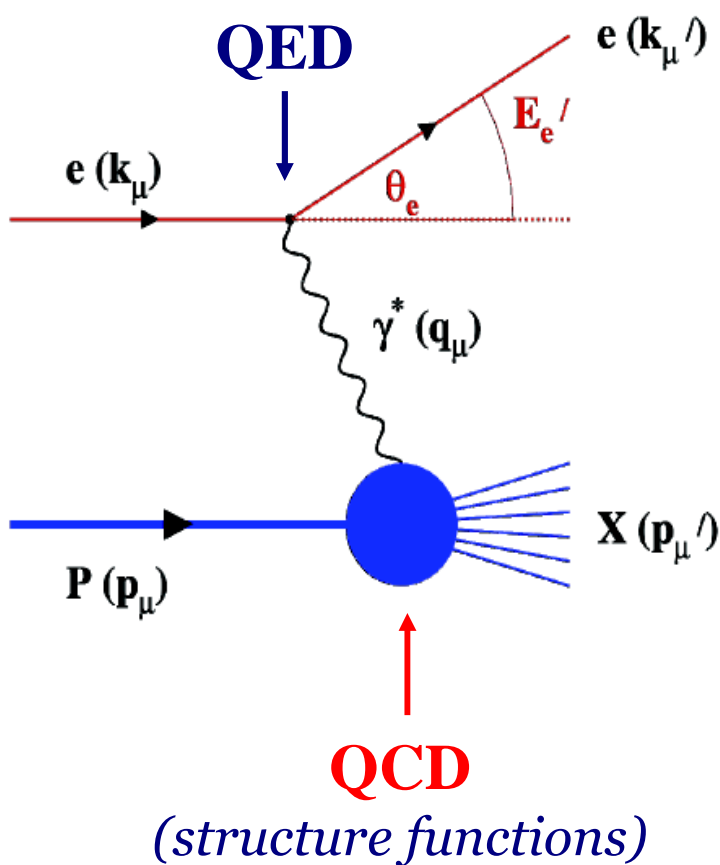
recall Rutherford experiments

birth of the pre-QCD parton model

Deeply Inelastic Scattering (DIS)

probing hadron structure

Kinematic Invariants



$$Q^2 = -q^2 = -(\mathbf{k}_\mu - \mathbf{k}'_\mu)^2$$

$$Q^2 = 4E_e E'_e \sin^2\left(\frac{\theta'_e}{2}\right)$$

$$y = \frac{pq}{pk} = 1 - \frac{E'_e}{E_e} \cos^2\left(\frac{\theta'_e}{2}\right)$$

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{sy}$$

$$s \equiv (\mathbf{p} + \mathbf{k})^2$$

Measure of
resolution
power

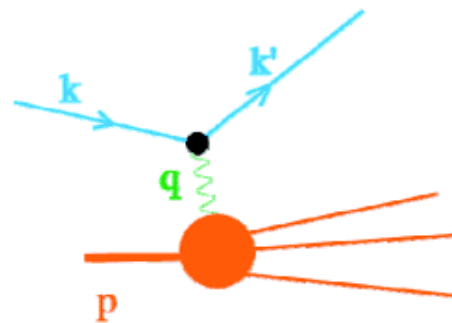
Measure of
inelasticity

Measure of
momentum
fraction of
struck quark

Deeply Inelastic Scattering

first analysis of DIS does not require any knowledge about QCD

electroweak theory tells us how the virtual vector boson couples:
(let's assume only photon exchange)



$$d\sigma = \frac{4\alpha^2}{s} \frac{d^3\vec{k}'}{2|\vec{k}'|} \frac{1}{Q^4} L^{\mu\nu}(k, q) W_{\mu\nu}(p, q)$$

phase space
scat. lepton

photon
propagator²

leptonic
tensor

hadronic tensor
contains information
about hadronic structure

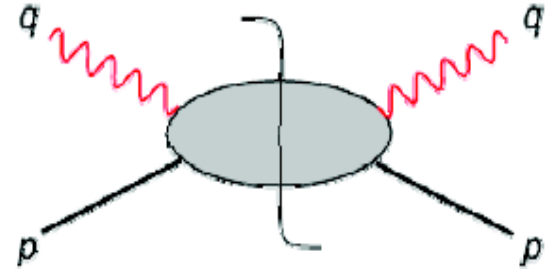
(can be easily generalized to W/Z-boson exchange)

with $L_{\mu\nu} = 2(k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu} k \cdot k')$

Deeply Inelastic Scattering

Strong interactions: contained in the hadronic tensor $W_{\mu\nu}(\mathbf{p}, \mathbf{q})$

to all orders in the strong interaction $W_{\mu\nu}$ is given by the square of $\gamma^*(q) h(p) \rightarrow X$



symmetries (parity, Lorentz), hermiticity & current conservation tell us that

$$W_{\nu\mu} = W_{\mu\nu}^*$$

$$q_\mu W^{\mu\nu} = 0$$

$$W_{\mu\nu}(p, q) = - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) F_1(x, Q^2) + \left(p_\mu - q_\mu \frac{p \cdot q}{q^2} \right) \left(p_\nu - q_\nu \frac{p \cdot q}{q^2} \right) \frac{1}{p \cdot q} F_2(x, Q^2)$$

structure functions

DIS cross section- So far this is totally general

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[[1 + (1-y)^2] F_1(x, Q^2) + \frac{(1-y)}{x} [F_2(x, Q^2) - 2xF_1(x, Q^2)] \right]$$

different y -dep. can differentiate between F_1 and $F_2 - 2xF_1$

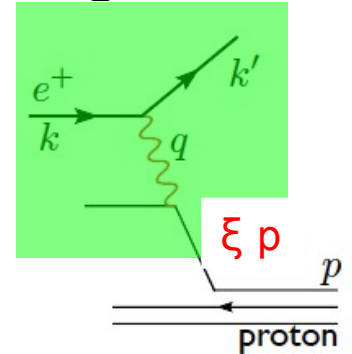
“naive” *parton model*: consider electron-parton scattering

find $\sum \overline{|\mathcal{M}|^2} = 2e_q^2 e^4 \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$ with the usual Mandelstam's

$$\hat{s} = (k + p_q)^2$$

$$\hat{t} = (k - k')^2$$

$$\hat{u} = (p_q - k')^2$$



and use the massless 2->2 cross section

$$\frac{d\sigma}{dt} = \frac{1}{16\pi\hat{s}^2} \sum \overline{|\mathcal{M}|^2} \quad \text{to obtain} \quad \frac{d\sigma}{dQ^2} = \frac{2\pi\alpha^2 e_q^2}{Q^4} [1 + (1-y)^2]$$

next: use on-mass shell constraint

$$p_q'^2 = (p_q + q)^2 = q^2 + 2p_q \cdot q = -2p \cdot q = -2p \cdot q (x - \xi) = 0$$

this implies that ξ is equal to Bjorken x

$$\hat{s} = \xi Q^2 / (xy) = \xi s$$

$$\hat{t} = q^2 = -Q^2$$

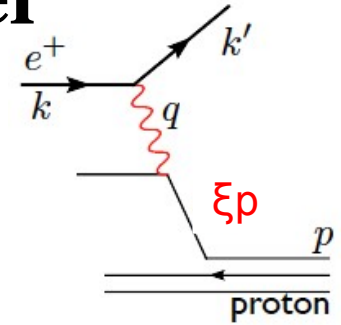
$$\hat{u} = \hat{s}(y - 1)$$

to obtain $\frac{d\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} [1 + (1-y)^2] \frac{1}{2} e_q^2 \delta(x - \xi)$

DIS in the “naive” parton model

compare our result

$$\frac{d\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} [1 + (1-y)^2] \frac{1}{2} e_q^2 \delta(x - \xi)$$



to what one obtains with the hadronic tensor (parton level)

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[[1 + (1-y)^2] F_1(x) + \frac{(1-y)}{x} (F_2(x) - 2xF_1(x)) \right]$$

and read off

$$F_2 = 2xF_1 = xe_q^2 \delta(x - \xi)$$

Callan-Gross relation

proton structure functions then obtained by weighting the parton str. fct.

with the **parton distribution functions** (probability to find a parton with momentum ξ)

$$\begin{aligned} F_2 = 2xF_1 &= \sum_{q,q'} \int_0^1 d\xi \overset{\downarrow}{q(\xi)} xe_q^2 \delta(x - \xi) \\ &= \sum_{q,q'} e_q^2 x q(x) \end{aligned}$$

DIS measures the sum of quarks and anti-quarks

space-time picture of DIS

light cone variables

advantages: boosting is easy

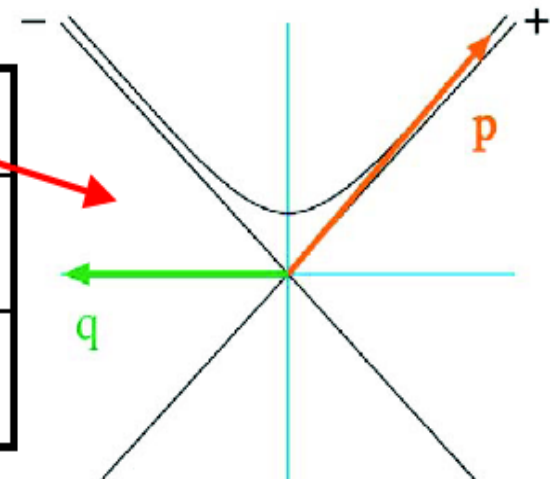
separation of large and small components of vectors

$$P^+ \equiv \frac{E + P_z}{\sqrt{2}}$$

$$P^- \equiv \frac{E - P_z}{\sqrt{2}} \quad (\mathbf{V}^+, \mathbf{V}^-, \mathbf{V}_t) \rightarrow (e^\omega \mathbf{V}^+, e^{-\omega} \mathbf{V}^-, \mathbf{V}_t) \quad \text{with} \quad e^\omega = \frac{Q}{x m_h}$$

$$P_t = P_t$$

4-vector	hadron rest frame	Breit frame
(p^+, p^-, \vec{p}_T)	$\frac{1}{\sqrt{2}}(m_h, m_h, \vec{0})$	$\frac{1}{\sqrt{2}}(\frac{Q}{x}, \frac{xm_h^2}{Q}, \vec{0})$
(q^+, q^-, \vec{q}_T)	$\frac{1}{\sqrt{2}}(-m_h x, \frac{Q^2}{m_h x}, \vec{0})$	$\frac{1}{\sqrt{2}}(-Q, Q, \vec{0})$



space-time picture of DIS

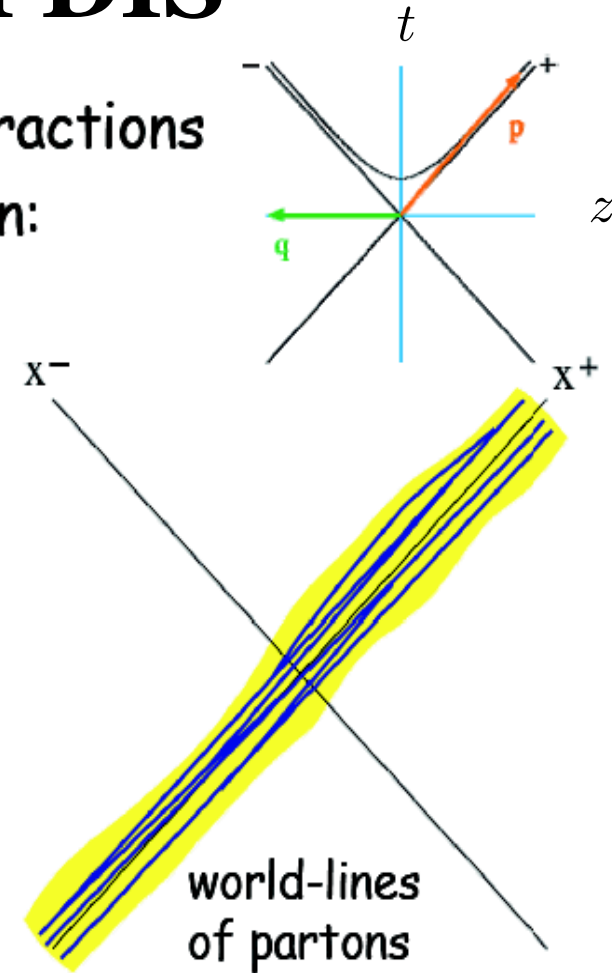
simple estimate for typical time-scale of interactions among the partons inside a fast-moving hadron:

$$\text{rest frame: } \Delta x^+ \sim \Delta x^- \sim \frac{1}{m}$$

$$\text{Breit frame: } \Delta x^+ \sim \frac{1}{m} \frac{Q}{m} = \frac{Q}{m^2} \quad \text{large}$$

$$\Delta x^- \sim \frac{1}{m} \frac{m}{Q} = \frac{1}{Q} \quad \text{small}$$

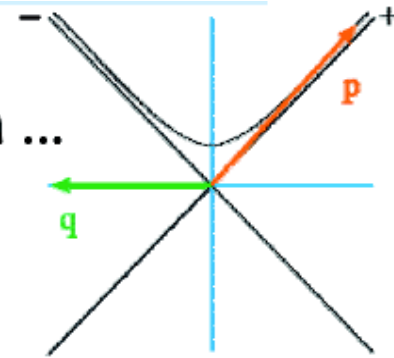
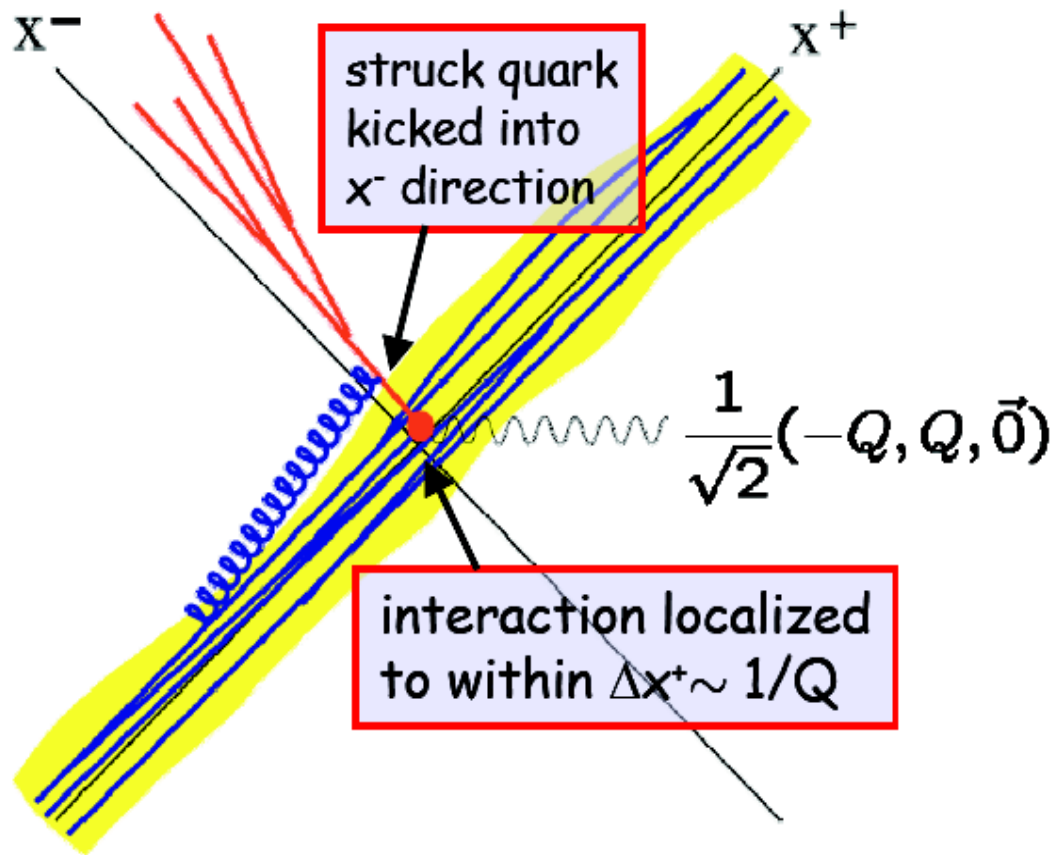
interactions between partons are spread out inside a fast moving hadron



How does this compare with the time-scale of the hard scattering?

space-time picture of DIS

now let the virtual photon meet our fast moving hadron ...



upshot:

- partons are free during the hard interaction
- hadron effectively consists of partons that have momenta $(p_i^+, p_i^-, \vec{p}_i)$
- convenient to introduce **momentum fractions**
 $0 < \xi_i \equiv p_i^+ / p^+ < 1$

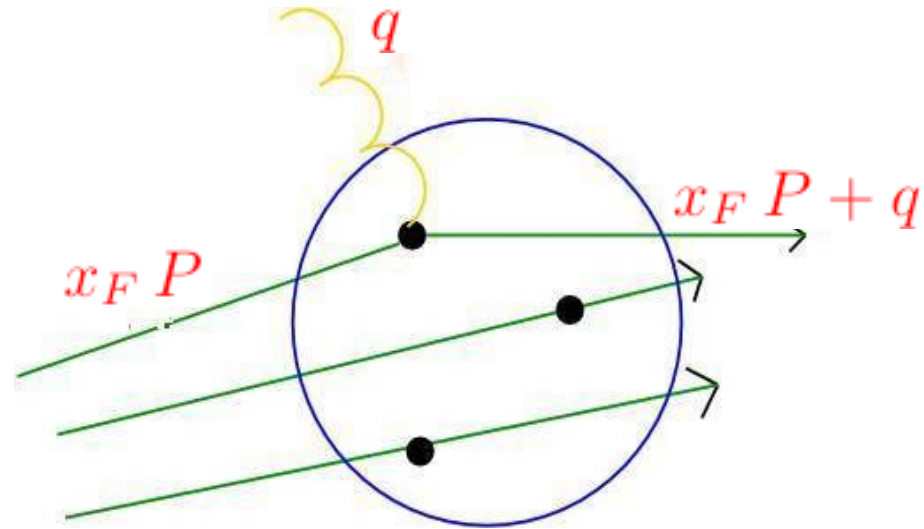
What is inside a hadron: Parton model

Bjorken: $Q^2, S \rightarrow \infty$ but keep $x_{Bj} = \frac{Q^2}{S}$ fixed (scaling limit)

Structure functions
depend only on x_{Bj}

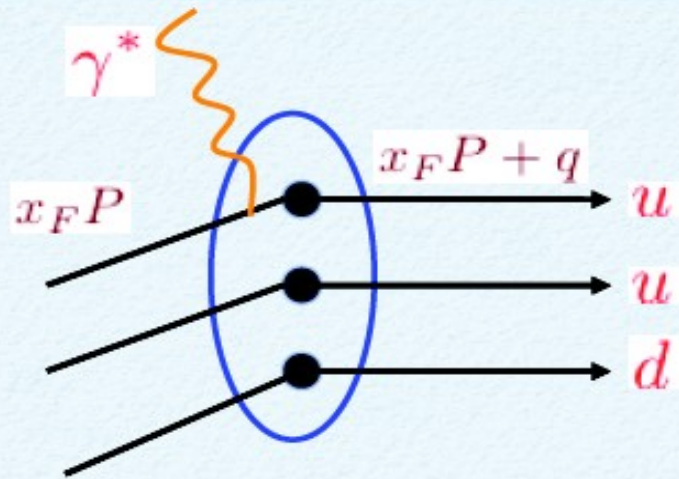
Feynman:

Parton constituents of
proton are “free” on time scale
 $1/Q \ll 1/\Lambda$ (interaction
time scale between partons)

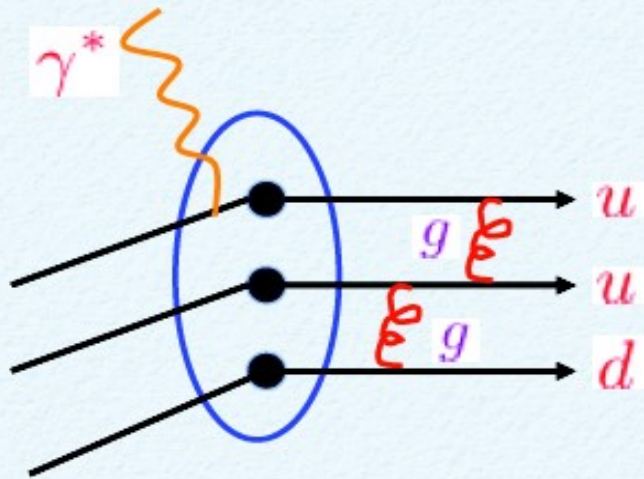


$x_{Bj} = \text{fraction of hadron momentum carried by a parton} = x_F$

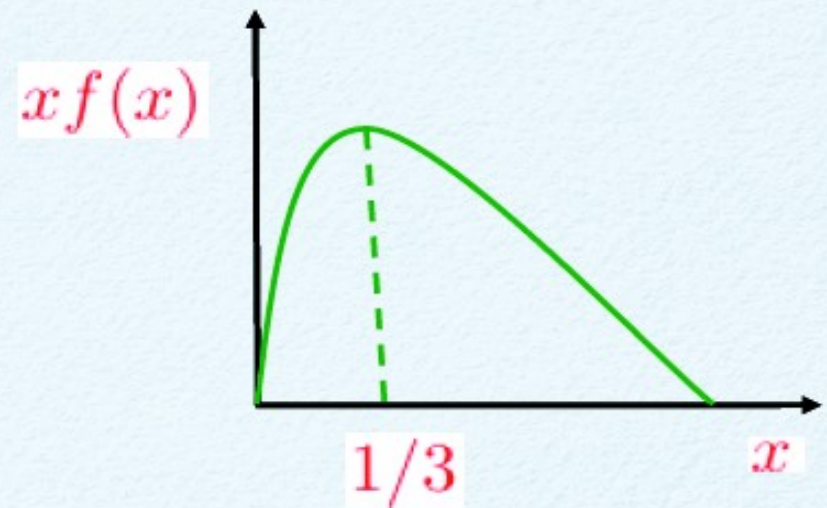
A hadron according to parton model



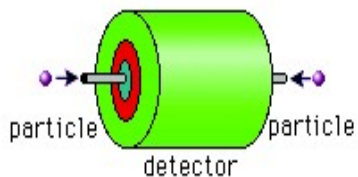
Parton model



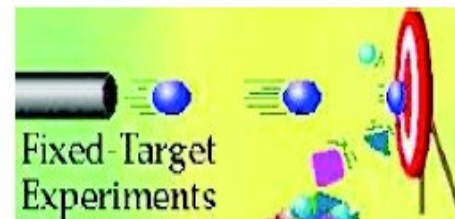
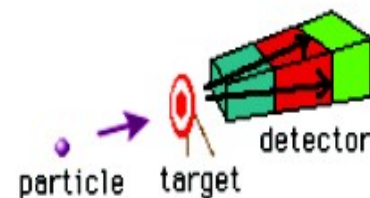
QCD - bound quarks



Collider experiment: Electron-Proton collisions at HERA (DESY, Hamburg, Germany)

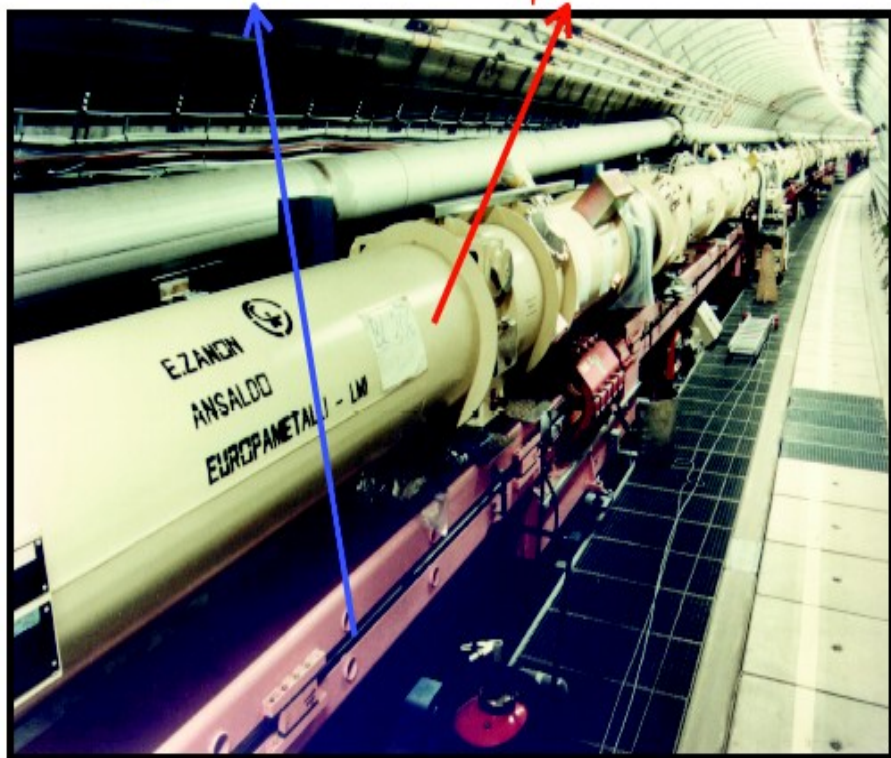


Equivalent to fixed target of
 $E_e = 50600 \text{ GeV}$:

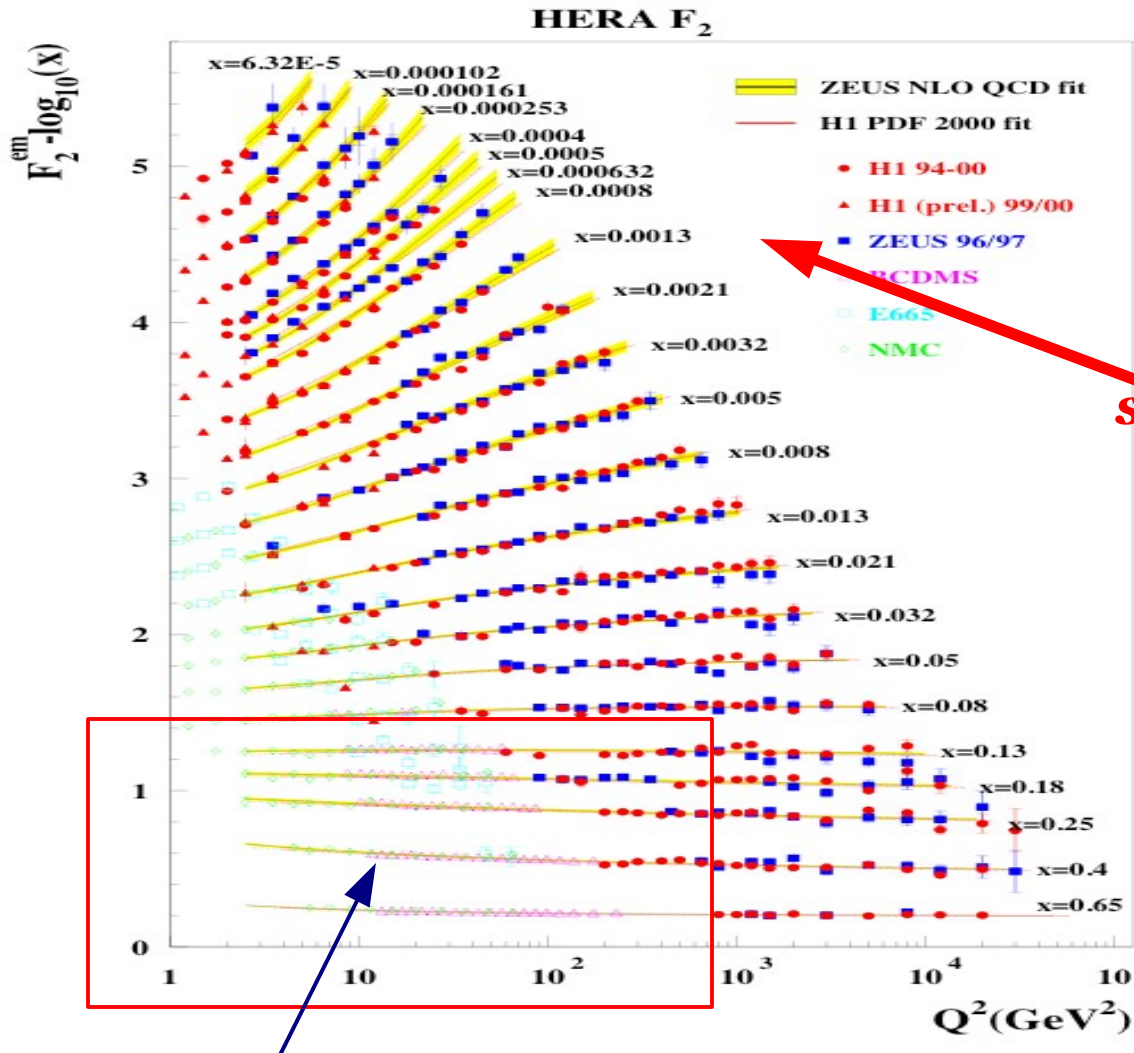


$E_e = 27.5 \text{ GeV}$

$E_p = 920 \text{ GeV}$



Deeply Inelastic Scattering $e(k) p(p) \rightarrow e(k') X$



$$F_2 \equiv \frac{Q^2}{4\pi^2 \alpha_{\text{em}}} \sigma^{\gamma^* p}$$

Parton model of a hadron leads to scale invariance

Can we understand scaling violations from QCD ?

early experiments (SLAC,...): *scale invariance* of hadron structure

Quantum Chromodynamics (QCD)

Theory of strong interactions between quarks and gluons $SU(N_c)$
 $N_c = 3$

$$\mathcal{L} = -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} + \sum_f \bar{\Psi}_i^\alpha [i \not{D} - m_f]_{\alpha\beta}^{ij} \Psi_j^\beta$$

$$G_{\mu\nu}^a(x) \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$$

$$a, b, c = 1, \dots, 8$$

color index: $\alpha, \beta = 1, 2, 3$

f^{abc} Group structure constant

Lorentz index: $\mu, \nu = 0, 1, 2, 3$

$$\not{D} \equiv D_\mu \gamma^\mu \quad \text{with} \quad \{\gamma^\mu, \gamma^\nu\} = 2 g^{\mu\nu}$$

spinor index: $i, j = 1, 2, 3, 4$

$$D_\mu \equiv \partial_\mu + ig A_\mu \quad \text{Covariant derivative}$$

Quarks:

Fermions, spin 1/2

4x1 spinor, come in N_c colors

6 flavors (up, down,, top)

carry electric charge

Gluons:

Bosons, spin 1

come in $N_c^2 - 1$ colors

flavor blind

have no electric charge

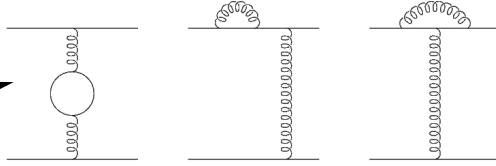
running of the coupling constant

asymptotic freedom

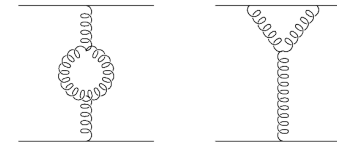


Gross, Wilczek;
Politzer ('73/'74)
Nobel prize 2004

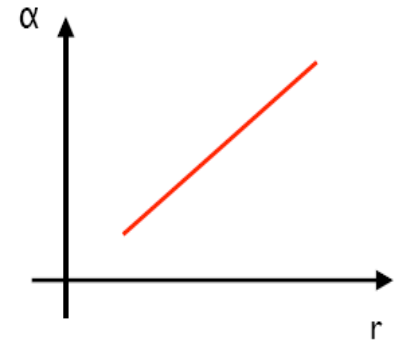
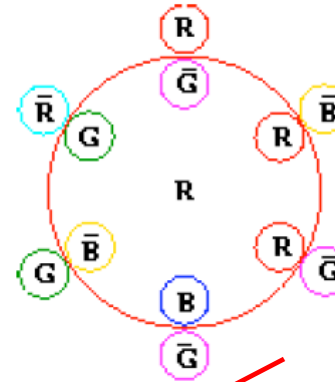
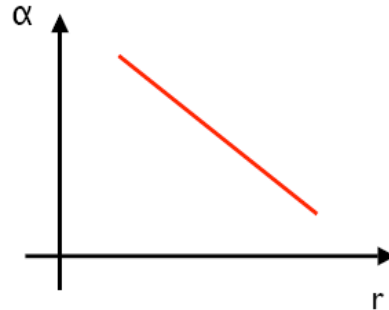
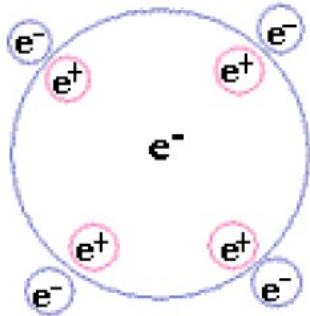
value of strong coupling $\alpha_s = g^2/4\pi$ depends on distance r (i.e. energy Q)



“screening“ of the charge



“anti-screening“



who wins?

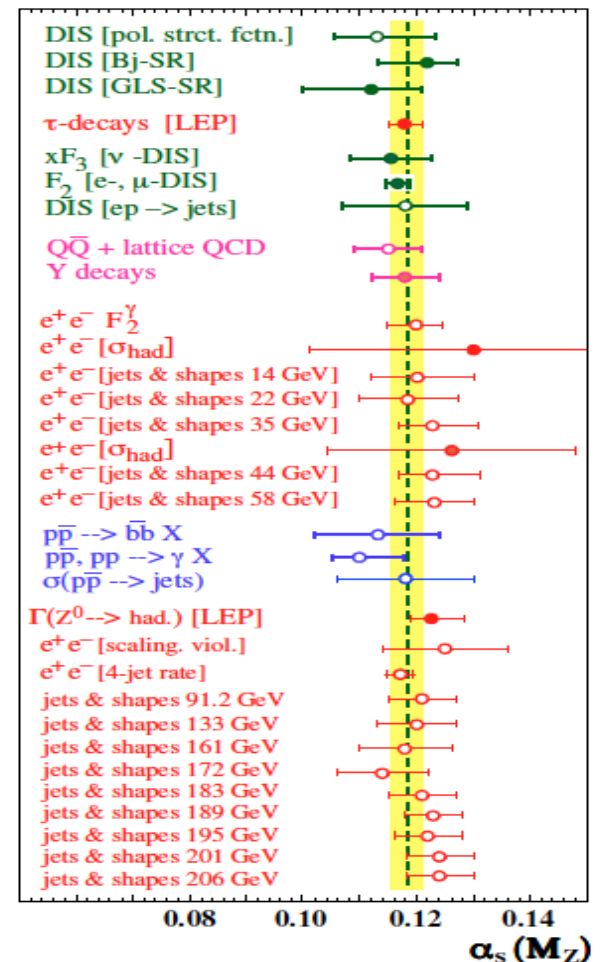
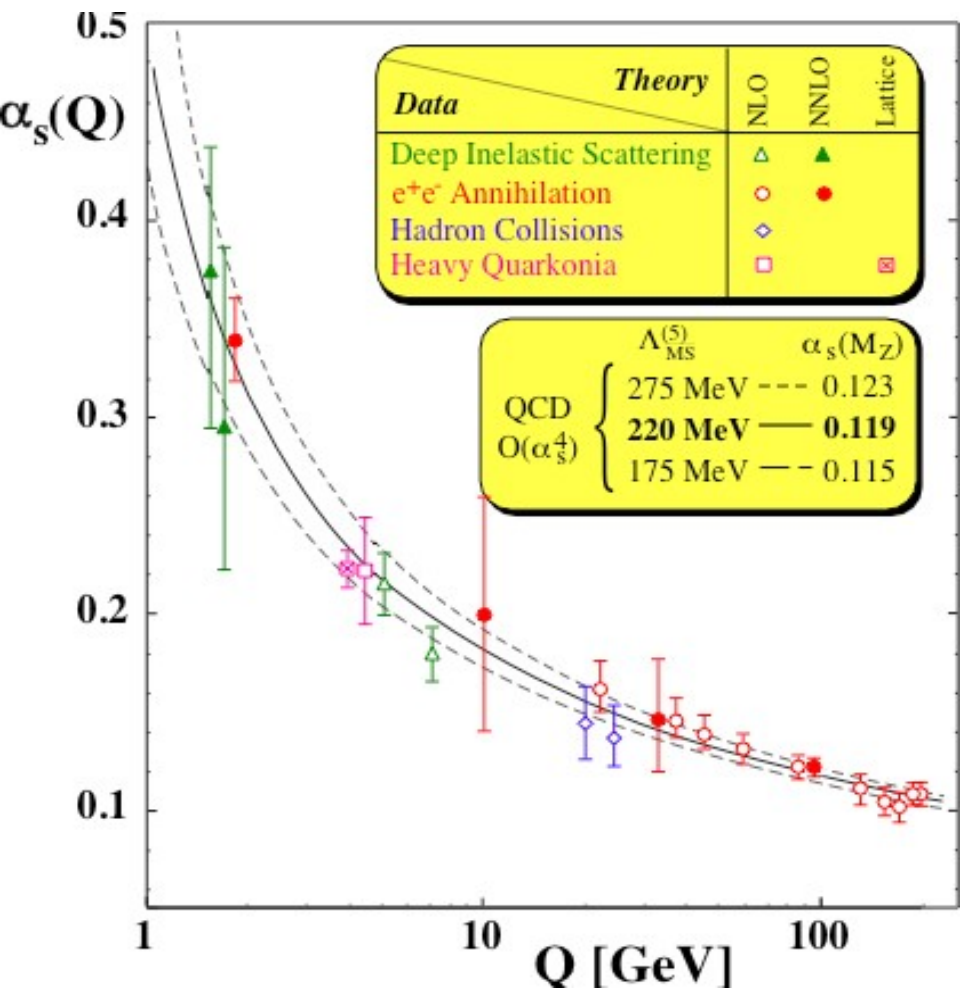
$$\alpha_s(Q^2) \approx \frac{4\pi}{\left(\frac{11}{3}C_A - \frac{4}{3}T_F N_f\right) \ln(Q^2/\Lambda^2)}$$

$$Q \sim 1/r$$

typical hadronic scale $O(200 \text{ MeV})$

Λ depends on N_f , pert. order and scheme

running of the coupling constant



Perturbative QCD:

expansion in powers of the coupling constant

Tests of QCD:

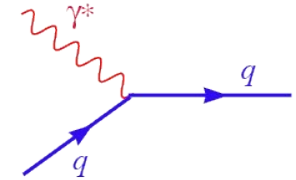
running of the coupling constant
scaling violations in DIS
vector boson production
jet cross sections

.....

QCD is firmly established
we need to explore it
and
learn how to use it!

DIS in the QCD-improved parton model

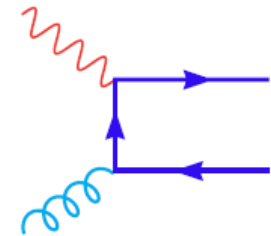
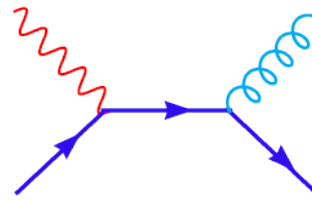
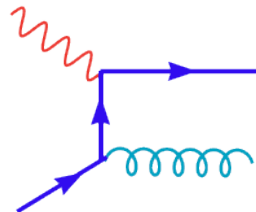
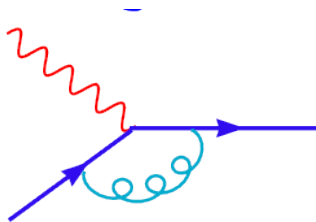
we got a long way (parton model) *without* invoking QCD



now we have to study **QCD dynamics in DIS**

– this leads to similar problems already encountered in e^+e^-

let's try to compute the **$O(\alpha_s)$ QCD corrections** to the naive picture



α_s corrections to the LO process

photon-gluon fusion

caveat: *expect divergencies*

related to soft/collinear emission or from loops

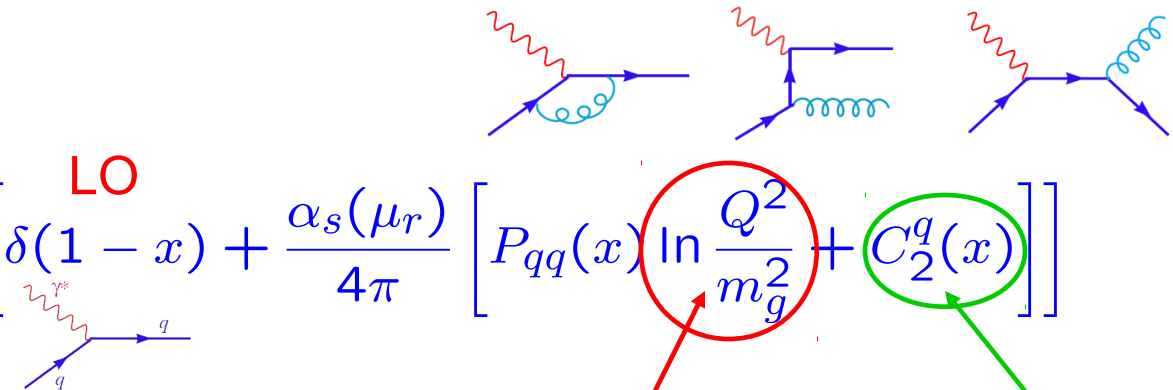
what to do with infinities?

introduce “**regulator**” in the intermediate stages, remove it at the end

general structure of the $O(\alpha_s)$ corrections

using small quark/gluon masses we obtain:

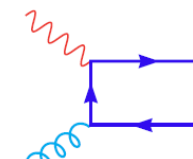
$$\frac{d^2 \hat{\sigma}}{dx dQ^2} \Big|_{F_2} \equiv \hat{F}_2^q$$

$$= e_q^2 x \left[\overset{\text{LO}}{\delta(1-x)} + \frac{\alpha_s(\mu_r)}{4\pi} \left[P_{qq}(x) \ln \frac{Q^2}{m_g^2} + C_2^q(x) \right] \right]$$


large logarithms
(collinear emission)

finite coefficients

$$\frac{d^2 \hat{\sigma}}{dx dQ^2} \Big|_{F_2} \equiv \hat{F}_2^g$$

$$= \sum_q e_q^2 x \left[0 + \frac{\alpha_s(\mu_r)}{4\pi} \left[P_{qg}(x) \ln \frac{Q^2}{m_q^2} + C_2^g(x) \right] \right]$$


convolute with the PDFs $\mathbf{F}_2(\mathbf{x}, \mathbf{Q}^2) \equiv \sum_f^f e_f^2 \mathbf{x} [\mathbf{q}_f(\mathbf{x}, \mathbf{Q}^2) + \bar{\mathbf{q}}_f(\mathbf{x}, \mathbf{Q}^2)]$

DGLAP “evolution” equation:

scale dependence of parton distribution functions

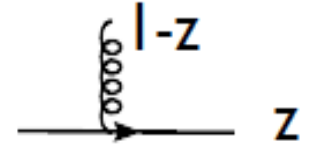
$$\frac{d}{d \ln \mu} \begin{pmatrix} q(x, \mu) \\ g(x, \mu) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} \mathcal{P}_{qq} & \mathcal{P}_{qg} \\ \mathcal{P}_{gq} & \mathcal{P}_{gg} \end{pmatrix} (z, \alpha_s) \cdot \begin{pmatrix} q(x/z, \mu) \\ g(x/z, \mu) \end{pmatrix}$$

best solved in Mellin moment space: set of ordinary differential eqs.;
no closed solution in exp. form beyond LO (commutators of P matrices!)

properties of LO splitting functions

$$P_{qq}^{(0)} = P_{\bar{q}\bar{q}}^{(0)} = C_F \left[\left(\frac{1+z^2}{1-z} \right)_+ + \frac{3}{2} \delta(1-z) \right]$$

soft gluon divergence (z=1)
regulated by plus distribution



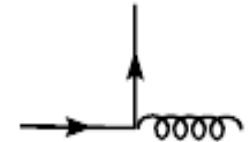
$$P_{qg}^{(0)} = P_{\bar{q}g}^{(0)} = T_R (z^2 + (1-z))$$

symmetric under
 $z \rightarrow (1-z)$
except virtuals



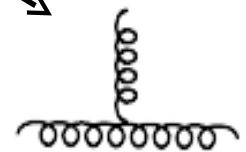
$$P_{gq}^{(0)} = P_{g\bar{q}}^{(0)} = C_F \frac{1+(1-z)^2}{z}$$

soft gluon divergence (z=0)
not reached, always $z > 0$



$$P_{gg}^{(0)} = 2C_A \left[z \left(\frac{1}{1-z} \right)_+ + \frac{1-z}{z} + z(1-z) + b_0 \delta(1-z) \right]$$

soft gluon divergence (z=1)
regulated by plus distribution



involves **“plus distribution”**

$$\int_0^1 dz [g(z)]_+ f(z) \equiv \int_0^1 dz g(z) [f(z) - f(1)]$$

condition: $f(z)$ sufficiently smooth for $z \rightarrow 1$

PDFs as bi-local operators

more physical formulation in Bjorken-x space:

matrix elements of *bi-local operators* on the light-cone

for quarks: (similar for gluons)

$$f_a(\xi, \mu_f) = \frac{1}{2} \int \frac{dy^-}{2\pi} e^{-i\xi p^+ y^-} \langle p | \bar{\Psi}_a(0, y^-, \vec{0}) \gamma^+ \mathcal{F} \Psi_a(0) | p \rangle_{\overline{MS}}$$

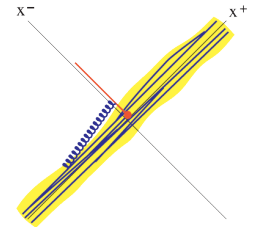
Fourier transform
with momentum ξp^+

recreates quark
at $x^+=0$ and $x^-=y^-$

annihilates
quark at $x^\mu=0$

in general we need a “**gauge link**“ for a gauge invariant definition:

$$\mathcal{F} = \mathcal{P} \exp \left(-ig \int_0^{y^-} dz^- A_c^+(0, z^-, \vec{0}) T_c \right)$$



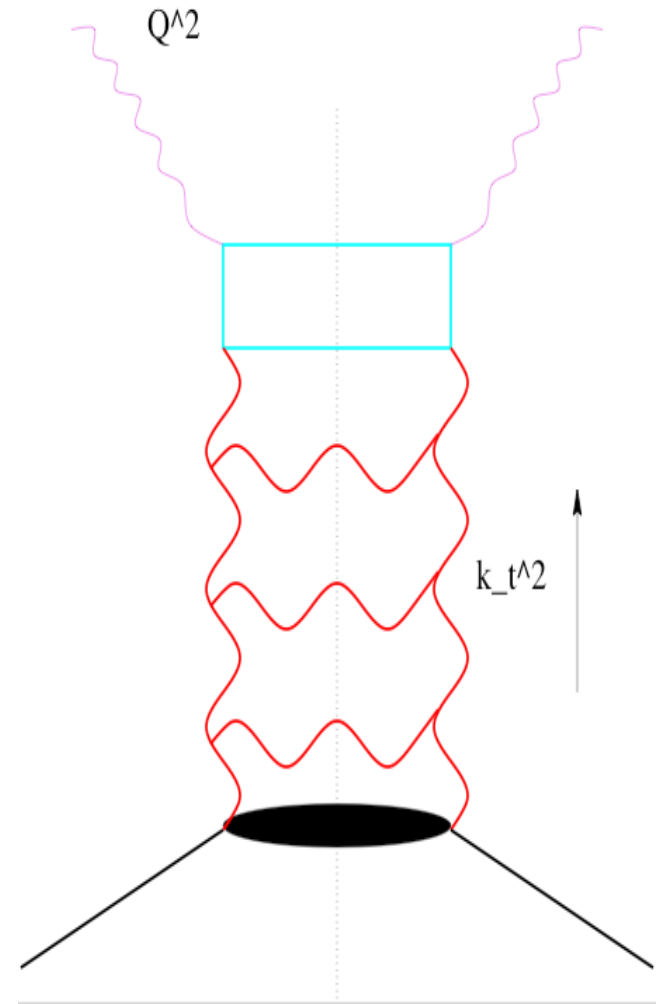
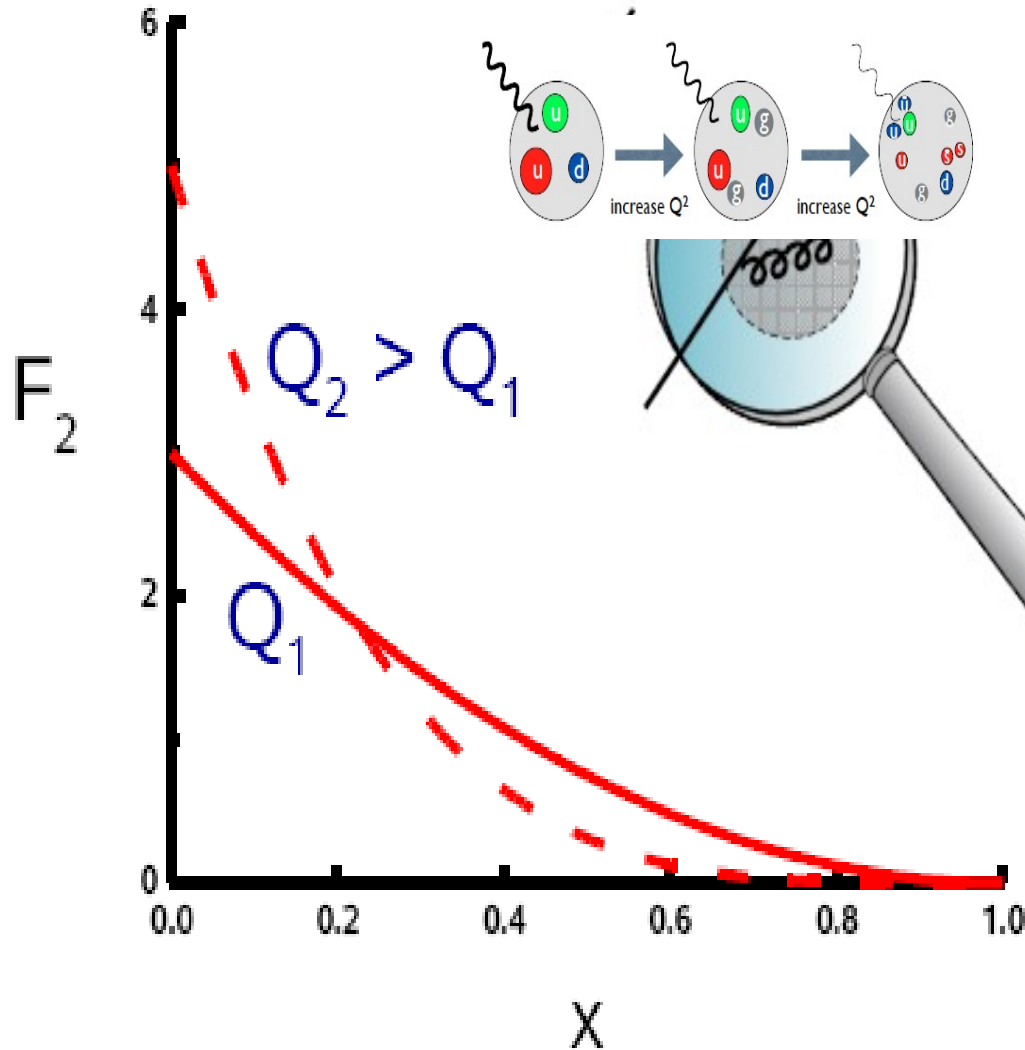
crucial role for a special class of “**transverse-momentum dep. PDFs**”
describing phenomena with transverse polarization (“**Sivers function**”, ...)

interpretation as ***number operator*** only in “ **$A^+ = 0$ gauge**“

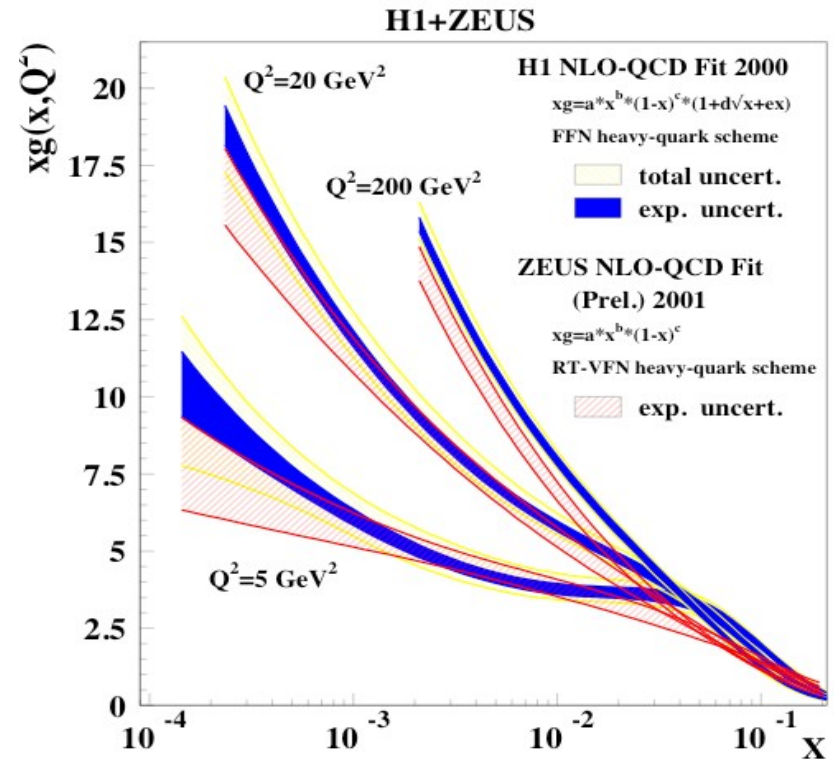
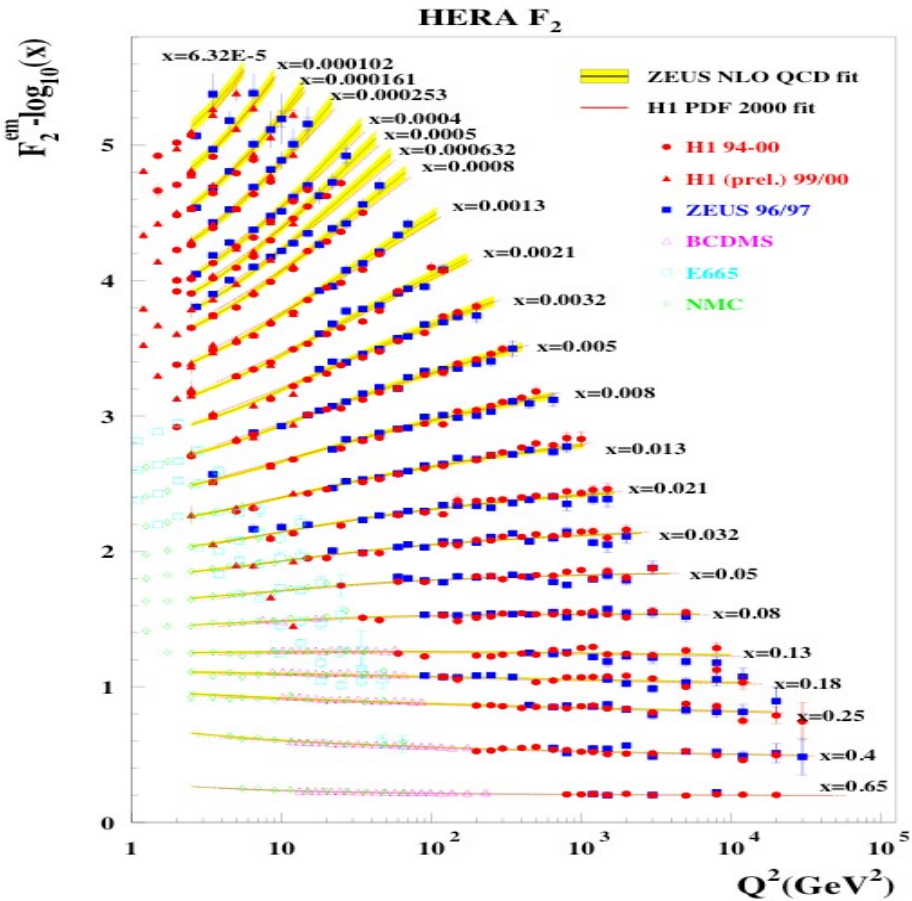
DGLAP “evolution” equation:

scale dependence of parton distribution functions

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi



DGLAP “evolution” equation: scale dependence of parton distribution functions



What drives the growth of parton distributions?

Splitting functions at leading order $O(\alpha_s^0)$ ($x \neq 1$)

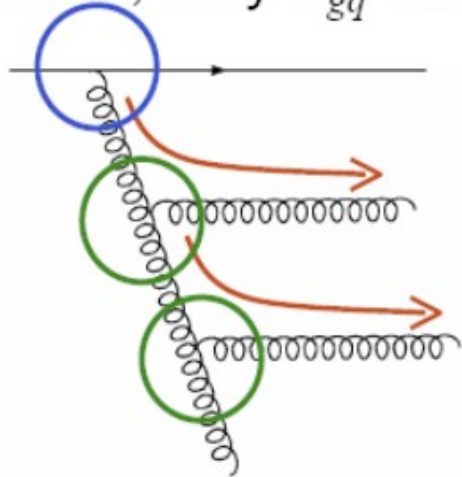
$$P_{qq}^{(0)}(x) = C_F \frac{1+x^2}{1-x}$$

$$P_{qg}^{(0)}(x) = \frac{1}{2} [x^2 + (1-x)^2]$$

$$P_{gq}^{(0)}(x) = C_F \frac{1+(1-x)^2}{x}$$

$$P_{gg}^{(0)}(x) = 2C_A \left[\frac{x}{1-x} + \frac{1-x}{x} + x(1-x) \right]$$

At small x , only P_{gq} and P_{gg} are relevant.



→ Gluon dominant at small x !

The double log approximation (DLA) of DGLAP is easily solved.

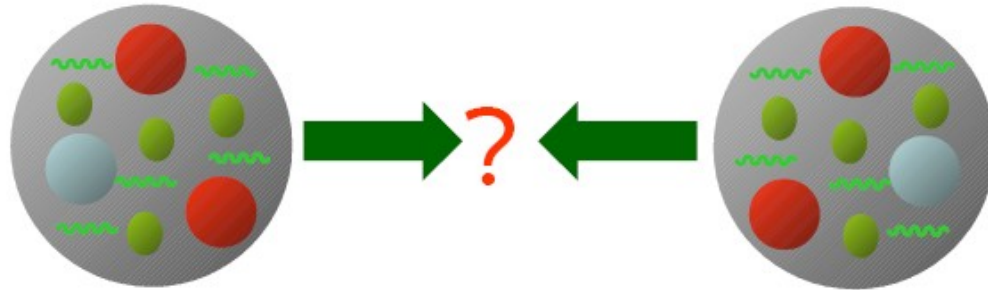
-- increase of gluon distribution at small x

$$xg(x, Q^2) \sim e^{\sqrt{\alpha_s (\log 1/x) (\log Q^2)}}$$

Keep this in mind: will be the focus of next part

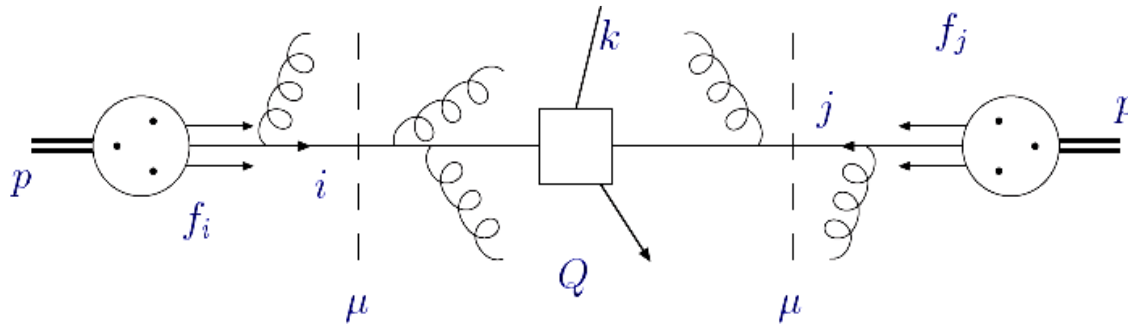
pQCD at work: hadron-hadron collisions

What happens when two hadrons collide ?



concepts discussed so far fall in the category of collinear factorization in pQCD

Production of jets, hadrons, heavy quarks, ...



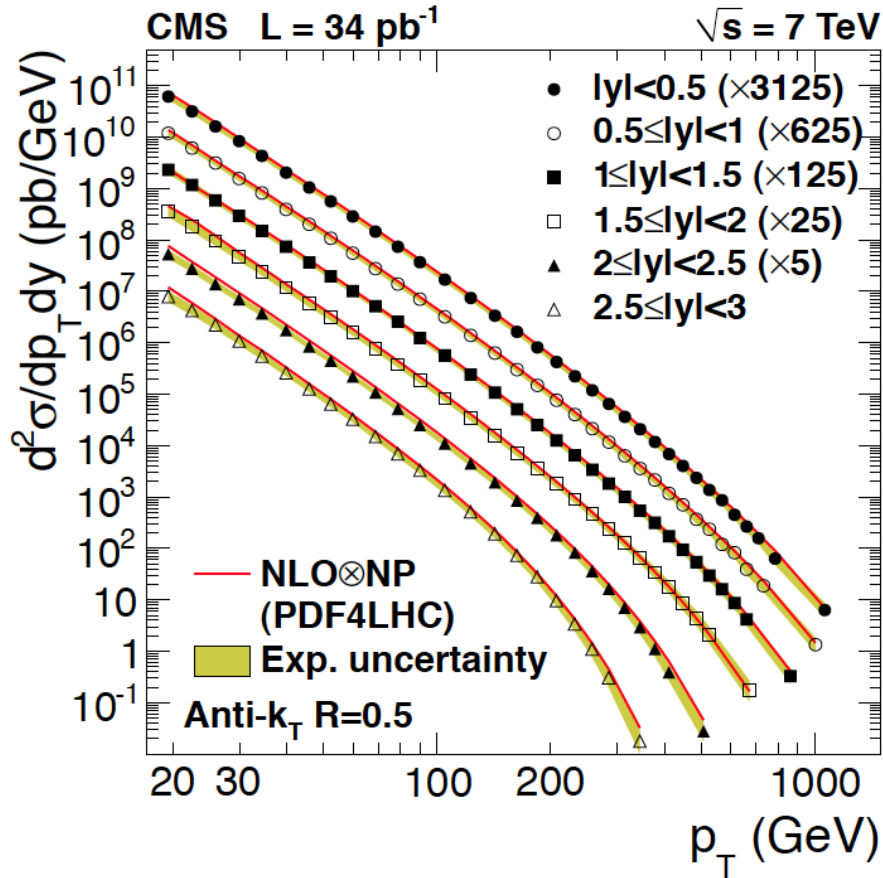
$$d\sigma = \sum_{ij} \int dx_i dx_j f_i(x_i, \mu^2) f_j(x_j, \mu^2) d\tilde{\sigma}_{ij}(\alpha_s(\mu_r), Q^2, \mu^2, x_i, x_j)$$

non-perturbative but universal PDFs $\xleftrightarrow[\text{by } \mu]{\text{linked}}$ hard scattering of two partons: pQCD

Large Hadron Collider (LHC)



pQCD: a success story



results now start to being used
 in global fits to constrain PDFs
 particularly sensitive to gluons

$$gg \rightarrow gg \quad gq \rightarrow gq$$

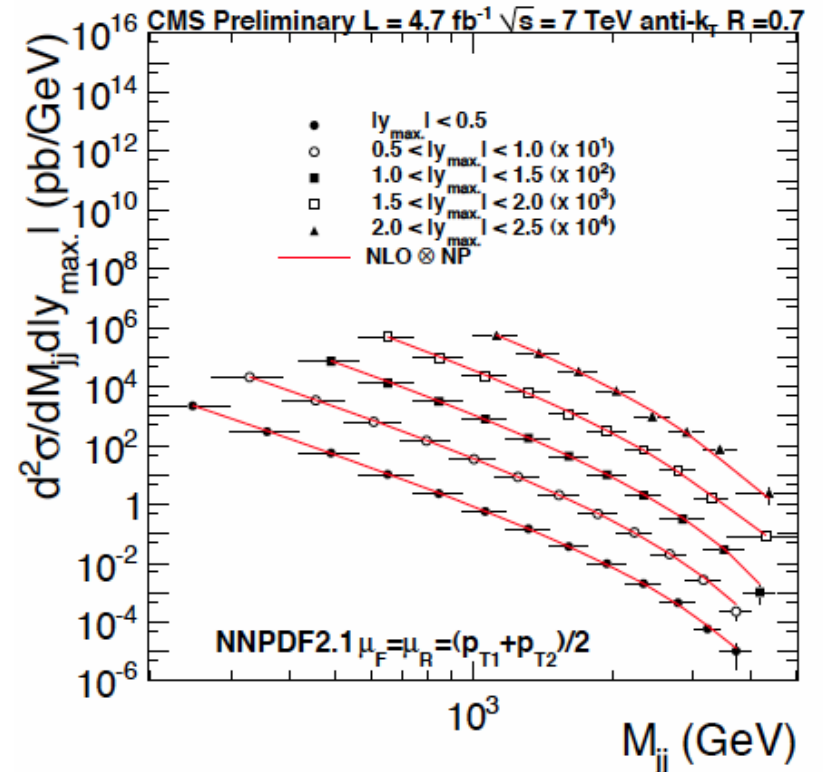
two recent examples from the LHC:

1-jet and di-jet cross sections

many other final-states available

$$y = \ln \tan \frac{\theta}{2} \sim \frac{1}{2} \ln \frac{x_1}{x_2} \quad M = \sqrt{x_1 x_2 s}$$

$$x_1 = \frac{M}{\sqrt{s}} e^{+y} \quad x_2 = \frac{M}{\sqrt{s}} e^{-y}$$



Precision QCD: P_{ij}@NNLO

10000 diagrams, 10⁵ integrals, 10 man years, and several CPU years later:

$$P_{ij}^{(2)}(x) = 16C_F C_F \gamma \left(\frac{2}{3} x^2 + x^3 \right) \left[\frac{13}{3} H_{-1,0} - \frac{14}{3} H_{-1,1} + \frac{1}{3} H_{-1,2} - H_{-1,3} - 2H_{-1,4,0} \right. \\ \left. - H_{-1,2} \right] + \frac{2}{3} x^2 \left[\frac{16}{3} \zeta_2 + H_{-1,1} + 9\zeta_3 + \frac{9}{2} H_{-1,0} - \frac{6701}{18} H_{-1,1} + \frac{10}{3} H_{-1,2} + H_{-1,3} - \frac{1}{2} H_{-1,4} \right. \\ \left. - 3H_{-1,4,0} + 2H_{-1,4,1} + 2H_{-1,4,2} \right] + (1-x) \left[\frac{2}{3} H_{-1,0} - \frac{1}{3} H_{-1,1} - \frac{1}{3} H_{-1,2} + \frac{1}{3} H_{-1,3} + \frac{1}{3} H_{-1,4} \right. \\ \left. + \frac{1}{3} H_{-1,4,0} + 34H_{-1,4,1} + H_{-1,4,2} + H_{-1,4,3} + 2H_{-1,4,4} - 3H_{-1,4,4,0} + 2H_{-1,4,4,1} + \frac{9}{2} H_{-1,4,4,2} \right. \\ \left. + \frac{1}{2} H_{-1,4,4,3} + H_{-1,4,4,4} \right] + (1+x) \left[\frac{7}{12} H_{-1,0} + \frac{31}{6} \zeta_2 + \frac{91}{12} H_{-1,1} + \frac{71}{18} H_{-1,2} + \frac{113}{18} \zeta_3 + \frac{826}{27} H_{-1,3} \right. \\ \left. + \frac{5}{27} H_{-1,4} - \frac{154}{27} \zeta_2 + \frac{899}{27} \zeta_3 + \frac{121}{27} \zeta_4 + \frac{607}{27} H_{-1,0} - 2H_{-1,1} + 399H_{-1,2} - \frac{29}{18} H_{-1,3} + \frac{13}{18} H_{-1,4} \right. \\ \left. + \frac{119}{108} H_{-1,4,0} - \frac{10H_{-1,4,1} - 20H_{-1,4,2} - 36\zeta_2 H_{-1,4,3} - 42\zeta_3 H_{-1,4,4} - 2H_{-1,4,4,0}}{36} \right. \\ \left. + (1+x) \left[H_{-1,4,0} - 10H_{-1,4,1} + 6H_{-1,4,2} - 9H_{-1,4,3} - 7H_{-1,4,4} - 2H_{-1,4,4,0} \right. \right. \\ \left. \left. - 4H_{-1,4,4,1} - 4H_{-1,4,4,2} - 4H_{-1,4,4,3} + \frac{17}{18} H_{-1,4,4,4} + \frac{5}{18} (1+x) H_{-1,4,4,4} \right] - 4H_{-1,4,4,0} + 2H_{-1,4,4,1} \right. \\ \left. + H_{-1,4,4,2} - 3H_{-1,4,4,3} + 2H_{-1,4,4,4} + H_{-1,4,4,4,0} - 9H_{-1,4,4,4,1} + \frac{11}{18} H_{-1,4,4,4,2} + \frac{19}{18} H_{-1,4,4,4,3} \right. \\ \left. + \frac{9}{18} H_{-1,4,4,4,4} + 8H_{-1,4,4,4,4,0} + 2H_{-1,4,4,4,4,1} + 2H_{-1,4,4,4,4,2} - \frac{473}{18} H_{-1,4,4,4,4,3} - \frac{183}{48} H_{-1,4,4,4,4,4} \right. \\ \left. + \frac{217}{54} \zeta_2 - \frac{59}{54} \zeta_3 - \frac{169}{54} \zeta_4 + 4H_{-1,0} - \frac{2}{3} H_{-1,1} - \frac{2}{3} H_{-1,2} + \frac{1}{3} H_{-1,3} + \frac{191}{18} H_{-1,4} + \frac{108}{108} H_{-1,4,0} + \frac{185}{108} H_{-1,4,1} \right. \\ \left. + \frac{25}{108} H_{-1,4,2} + \frac{170}{108} H_{-1,4,3} + \frac{85}{108} H_{-1,4,4} + \frac{425}{108} H_{-1,4,4,0} + \frac{3629}{108} H_{-1,4,4,1} - 2\zeta_2 H_{-1,4,4,2} + 4H_{-1,4,4,3} - 4H_{-1,4,4,4} \right] \\ \left. + 16C_F C_F \gamma \left(\frac{2}{3} H_{-1,0} - H_{-1,1} - 2H_{-1,2} + 2H_{-1,3} + H_{-1,4} \right) + 16C_F C_F \gamma \left(\frac{85}{18} H_{-1,0} \right. \right. \\ \left. \left. + \frac{2}{3} H_{-1,1} - H_{-1,2} + H_{-1,3} + H_{-1,4} \right) + 16C_F C_F \gamma \left(\frac{1}{3} H_{-1,0} - H_{-1,1} - H_{-1,2} + H_{-1,3} + H_{-1,4} \right) \right. \\ \left. + \frac{2}{3} H_{-1,4,0} + \frac{98}{3} H_{-1,4,1} + \frac{100}{3} H_{-1,4,2} + \frac{28}{3} H_{-1,4,3} + \frac{16}{3} H_{-1,4,4} + \frac{16}{3} H_{-1,4,4,0} + 4H_{-1,4,4,1} + \frac{16}{3} H_{-1,4,4,2} \right. \\ \left. + \frac{2}{3} H_{-1,4,4,3} + \frac{4}{3} \zeta_2 \left(x^2 - x^3 \right) \left[\frac{23}{12} H_{-1,0} - \frac{23}{12} H_{-1,1} - \frac{5}{6} \zeta_2 H_{-1,2} + H_{-1,3} + H_{-1,4} \right. \right. \\ \left. \left. + (1-x) \left[\frac{1}{2} H_{-1,0} + \frac{7}{12} H_{-1,1} - \frac{2743}{72} H_{-1,2} + \frac{251}{72} H_{-1,3} + \frac{5}{24} H_{-1,4} + \frac{341}{72} H_{-1,4,0} \right. \right. \right. \\ \left. \left. + 3H_{-1,4,1} - 3H_{-1,4,2} - H_{-1,4,3} - H_{-1,4,4} \right] + (1+x) \left[\frac{1669}{216} H_{-1,0} + \frac{5}{216} H_{-1,1} + \frac{1703}{108} H_{-1,2} + \frac{37}{108} \zeta_2 \right. \right. \\ \left. \left. - 7H_{-1,3} + 6H_{-1,4,0} - 4H_{-1,4,1} + H_{-1,4,2} - 4H_{-1,4,3} + H_{-1,4,4} \right] \right)$$

$$P_{ij}^{(2)}(x) = 16C_F C_F \gamma \left(\frac{2}{3} H_{-1,0} - 4H_{-1,1} + 3H_{-1,2} - \frac{14}{3} H_{-1,3} + \frac{2}{3} H_{-1,4} - 3H_{-1,4,0} \right. \\ \left. + H_{-1,4,1} + 4H_{-1,4,2} - \frac{17}{12} H_{-1,4,3} + \frac{51}{12} H_{-1,4,4} - \frac{64}{3} \zeta_2 - \zeta_3 - \frac{49}{3} \zeta_4 - \frac{3}{2} H_{-1,4,0,0} - \frac{1}{2} H_{-1,4,0,1} \right. \\ \left. + \frac{385}{12} H_{-1,4,1,0} + \frac{113}{12} H_{-1,4,1,1} + \frac{49}{12} H_{-1,4,1,2} + \frac{5}{12} H_{-1,4,1,3} + \frac{79}{12} H_{-1,4,1,4} + \frac{173}{12} H_{-1,4,1,5} + \frac{2833}{12} H_{-1,4,1,6} \right. \\ \left. - 6H_{-1,4,2,0} + 3H_{-1,4,2,1} + 6H_{-1,4,2,2} + 4H_{-1,4,2,3} + 3H_{-1,4,2,4} - 4H_{-1,4,2,5} - 3H_{-1,4,2,6} - 6H_{-1,4,2,7} \right) \\ \left. + \frac{1}{2} P_{ij}^{(2)}(x) \left[H_{-1,0} - H_{-1,1} + \zeta_2 + \frac{81}{12} H_{-1,2} + 2H_{-1,3} + \frac{7}{36} H_{-1,4} + 2H_{-1,4,0} - \frac{1625}{48} H_{-1,4,1} \right. \right. \\ \left. \left. + 2H_{-1,4,2} - \frac{5}{24} H_{-1,4,3} \right] + \frac{31}{12} P_{ij}^{(2)}(x) \left[\zeta_2 - \zeta_3 - H_{-1,0} \right] + \frac{1}{2} (1-x) \left[6H_{-1,0,0} - H_{-1,0,1} - \frac{13051}{288} \right. \right. \\ \left. \left. - \frac{13}{24} H_{-1,0,2} - H_{-1,0,3} - \frac{2}{3} H_{-1,0,4} + H_{-1,0,5} + 2H_{-1,0,6} - \frac{655}{24} H_{-1,0,7} + (1+x) H_{-1,0,8} \right] + \frac{1167}{144} H_{-1,0,9} \right. \\ \left. + 1H_{-1,0,10} - H_{-1,0,11} - 2H_{-1,0,12} - \frac{5}{24} H_{-1,0,13} + \frac{1}{24} H_{-1,0,14} - \frac{5}{24} H_{-1,0,15} + \frac{1}{24} H_{-1,0,16} \right. \\ \left. + \frac{85}{18} H_{-1,0,17} - \frac{101}{18} \zeta_2 + \frac{80}{18} \zeta_3 + \frac{23}{18} \zeta_4 + \frac{1}{3} H_{-1,1} + \frac{5}{3} H_{-1,2} + \frac{37}{18} H_{-1,3} + \frac{23}{18} H_{-1,4} \right. \\ \left. + \frac{1501}{144} H_{-1,4,0} - \frac{1}{144} H_{-1,4,1} + 16C_F C_F \gamma \left(\frac{19}{144} \zeta_2 + \frac{1}{324} \zeta_3 + \frac{13}{72} \zeta_4 + \frac{1}{3} H_{-1,0} - \frac{1}{3} H_{-1,1} \right) \right. \\ \left. + \frac{22}{144} H_{-1,4,0} - 8H_{-1,4,1} - 2H_{-1,4,2} + 3H_{-1,4,3} - 3H_{-1,4,4} - H_{-1,4,4,0} + 2H_{-1,4,4,1} - 3H_{-1,4,4,2} \right. \\ \left. - 2H_{-1,4,4,3} - 2H_{-1,4,4,4} - \frac{9}{2} H_{-1,4,4,5} - \frac{3}{2} H_{-1,4,4,6} + \frac{47}{18} \zeta_2 + \frac{17}{18} \zeta_3 + P_{ij}^{(2)}(x) \left[2H_{-1,0} - 1,0 \right. \right. \\ \left. \left. + 6H_{-1,1} - 1,0 + 3H_{-1,2} + \frac{1}{2} H_{-1,3} - \frac{16}{3} \zeta_2 - 2H_{-1,4} + 4H_{-1,4,0} - 2H_{-1,4,1,0} - 2H_{-1,4,1,1} \right. \right. \\ \left. \left. - H_{-1,4,1,2} \right] + (1-x) \left[9H_{-1,0,0} + H_{-1,1,0} - 10H_{-1,2,0} + 3H_{-1,3,0} + 4H_{-1,4,0} + H_{-1,4,0,0} \right. \right. \\ \left. \left. + H_{-1,4,0,1} + 3H_{-1,4,0,2} + 3H_{-1,4,0,3} - 3H_{-1,4,0,4} - \frac{211}{18} H_{-1,4,0,5} - \frac{49}{18} H_{-1,4,0,6} \right] + (1+x) \left[\zeta_2 + \frac{1}{2} H_{-1,1} + \frac{1}{2} H_{-1,2} \right. \right. \\ \left. \left. + \frac{21}{16} H_{-1,3} + 3H_{-1,4,0} + 8H_{-1,4,1,0} - 14H_{-1,4,1,1} - 7H_{-1,4,2,0} + 2H_{-1,4,2,1} + 4H_{-1,4,2,2} + 2H_{-1,4,2,3} \right. \right. \\ \left. \left. + \frac{21}{16} H_{-1,4,2,4} + 3H_{-1,4,2,5} + 3H_{-1,4,2,6} - 14H_{-1,4,2,7} - 7H_{-1,4,3,0} + 2H_{-1,4,3,1} + 4H_{-1,4,3,2} + 2H_{-1,4,3,3} \right. \right. \\ \left. \left. + 3H_{-1,4,3,4} + \frac{11}{2} H_{-1,4,3,5} - 2H_{-1,4,3,6} - 1H_{-1,4,3,7} + \frac{13}{4} \zeta_2 + \frac{9}{16} H_{-1,4,4} + \frac{287}{32} H_{-1,4,4,0} \right. \right. \\ \left. \left. + 6H_{-1,4,4,1} - 16H_{-1,4,4,2} - 4H_{-1,4,4,3} - 8H_{-1,4,4,4} - 5H_{-1,4,4,5} + \frac{13}{4} \zeta_3 + \frac{1}{16} \zeta_4 + \frac{287}{32} H_{-1,4,4,6} \right. \right. \\ \left. \left. + 2H_{-1,4,4,7} + 3H_{-1,4,4,8} + 3 \left(\frac{189}{4} \zeta_2 - 2H_{-1,4,4,9} \right) + \frac{3}{2} H_{-1,4,4,10} - \frac{5}{2} H_{-1,4,4,11} - \frac{175}{36} H_{-1,4,4,12} + \frac{19}{3} \zeta_3 \right. \right. \\ \left. \left. + 2H_{-1,4,4,13} + 4H_{-1,4,4,14} + \frac{3}{2} H_{-1,4,4,15} - \frac{3}{2} H_{-1,4,4,16} + \frac{5}{2} H_{-1,4,4,17} - \frac{175}{36} H_{-1,4,4,18} + \frac{19}{3} \zeta_3 \right. \right. \\ \left. \left. + 2H_{-1,4,4,19} + 4H_{-1,4,4,20} - H_{-1,4,4,21} - H_{-1,4,4,22} + \frac{1}{2} H_{-1,4,4,23} + 3H_{-1,4,4,24} - \frac{1}{2} H_{-1,4,4,25} - \frac{7}{2} H_{-1,4,4,26} \right. \right. \\ \left. \left. + \frac{5}{2} H_{-1,4,4,27} - \frac{29}{8} H_{-1,4,4,28} + \frac{185}{8} H_{-1,4,4,29} \right) \right)$$

$$-2H_{-1,2} + H_{-1,3} + H_{-1,4} + (1-x) \left[9H_{-1,0,0} - 5H_{-1,0,1} - \frac{65}{36} \zeta_2 + \frac{11}{36} H_{-1,1} \right. \\ \left. - \frac{2}{3} H_{-1,2} + \frac{2}{3} H_{-1,3} + H_{-1,4} - \frac{31}{36} H_{-1,4,0} - \frac{551}{36} \zeta_2 + \frac{29}{36} H_{-1,4,1} - \frac{113}{36} H_{-1,4,2} - \frac{18691}{72} H_{-1,4,3} \right. \\ \left. + \frac{2043}{72} H_{-1,4,4} - \frac{33}{72} H_{-1,4,5} + 19H_{-1,4,6} + \frac{31}{72} H_{-1,4,7} - \frac{407}{36} \zeta_2 + \frac{29}{36} H_{-1,4,8} - \frac{143}{72} H_{-1,4,9} \right. \\ \left. - \frac{5}{72} H_{-1,4,10} - \frac{5}{72} \zeta_3 + \frac{157}{72} H_{-1,4,11} - \frac{9}{72} \zeta_4 - \frac{5}{72} H_{-1,4,12} - \frac{1}{72} H_{-1,4,13} + H_{-1,4,14} + \frac{5}{72} H_{-1,4,15} \right. \\ \left. + \frac{9}{72} \zeta_2 + \frac{241}{72} H_{-1,4,16} + \frac{49}{72} H_{-1,4,17} + \frac{291}{72} H_{-1,4,18} - \frac{407}{72} H_{-1,4,19} - H_{-1,4,20} + H_{-1,4,21} \right. \\ \left. + 2H_{-1,4,22} + 2H_{-1,4,23} + 2H_{-1,4,24} + 2H_{-1,4,25} + 4H_{-1,4,26} \right) \\ \left. - 113H_{-1,4,0} - 5H_{-1,4,1} + 2H_{-1,4,2} + \frac{27}{24} H_{-1,4,3} + \frac{11}{24} H_{-1,4,4} - \frac{67}{24} H_{-1,4,5} + \frac{12}{24} H_{-1,4,6} \right. \\ \left. + 13H_{-1,4,7} - \frac{17}{24} H_{-1,4,8} - \frac{3}{24} H_{-1,4,9} + H_{-1,4,10} + \frac{79}{24} H_{-1,4,11} + \frac{67}{24} H_{-1,4,12} + \frac{263}{8} \zeta_2 \right. \\ \left. + \frac{119}{24} \zeta_3 + \frac{303}{24} H_{-1,4,13} - 24H_{-1,4,14} + H_{-1,4,15} - \frac{13375}{72} H_{-1,4,16} - \frac{1889}{72} H_{-1,4,17} - \frac{107}{72} H_{-1,4,18} \right. \\ \left. + \frac{145}{72} H_{-1,4,19} + \frac{217}{72} H_{-1,4,20} + \frac{29}{72} \zeta_2 + \frac{17}{72} H_{-1,4,21} + \frac{17}{72} H_{-1,4,22} + \frac{19}{72} H_{-1,4,23} + \frac{2}{72} H_{-1,4,24} \right. \\ \left. + \frac{145}{72} H_{-1,4,25} + 16C_F C_F \gamma \left(\frac{4}{3} H_{-1,0} + \frac{11}{36} H_{-1,1} + \frac{17}{36} H_{-1,2} + \frac{1}{18} H_{-1,3} + 2H_{-1,4,0} \right. \right. \\ \left. \left. - \frac{1}{12} H_{-1,4,1} - \frac{1}{12} H_{-1,4,2} + \frac{1}{2} P_{ij}^{(2)}(x) \left[\frac{1}{2} H_{-1,0} - H_{-1,1} - H_{-1,2} + H_{-1,3} + H_{-1,4} \right] + 16C_F C_F \gamma \left(\frac{85}{18} H_{-1,0} \right. \right. \right. \\ \left. \left. + \frac{2}{3} H_{-1,1} - H_{-1,2} + H_{-1,3} + H_{-1,4} \right) + 16C_F C_F \gamma \left(\frac{1}{3} H_{-1,0} - H_{-1,1} - H_{-1,2} + H_{-1,3} + H_{-1,4} \right) \right. \\ \left. + \frac{2}{3} H_{-1,4,0} + \frac{98}{3} H_{-1,4,1} + \frac{100}{3} H_{-1,4,2} + \frac{28}{3} H_{-1,4,3} + \frac{16}{3} H_{-1,4,4} + \frac{16}{3} H_{-1,4,4,0} + 4H_{-1,4,4,1} + \frac{16}{3} H_{-1,4,4,2} \right. \\ \left. + \frac{2}{3} H_{-1,4,4,3} + \frac{4}{3} \zeta_2 \left(x^2 - x^3 \right) \left[\frac{23}{12} H_{-1,0} - \frac{23}{12} H_{-1,1} - \frac{5}{6} \zeta_2 H_{-1,2} + H_{-1,3} + H_{-1,4} \right. \right. \\ \left. \left. + (1-x) \left[\frac{1}{2} H_{-1,0} + \frac{7}{12} H_{-1,1} - \frac{2743}{72} H_{-1,2} + \frac{251}{72} H_{-1,3} + \frac{5}{24} H_{-1,4} + \frac{341}{72} H_{-1,4,0} \right. \right. \right. \\ \left. \left. + 3H_{-1,4,1} - 3H_{-1,4,2} - H_{-1,4,3} - H_{-1,4,4} \right] + (1+x) \left[\frac{1669}{216} H_{-1,0} + \frac{5}{216} H_{-1,1} + \frac{1703}{108} H_{-1,2} + \frac{37}{108} \zeta_2 \right. \right. \\ \left. \left. - 7H_{-1,3} + 6H_{-1,4,0} - 4H_{-1,4,1} + H_{-1,4,2} - 4H_{-1,4,3} + H_{-1,4,4} \right] \right)$$

$$-2H_{-1,2} + H_{-1,3} + H_{-1,4} + (1-x) \left[9H_{-1,0,0} - 5H_{-1,0,1} - \frac{65}{36} \zeta_2 + \frac{11}{36} H_{-1,1} \right. \\ \left. - \frac{2}{3} H_{-1,2} + \frac{2}{3} H_{-1,3} + H_{-1,4} - \frac{31}{36} H_{-1,4,0} - \frac{551}{36} \zeta_2 + \frac{29}{36} H_{-1,4,1} - \frac{113}{36} H_{-1,4,2} - \frac{18691}{72} H_{-1,4,3} \right. \\ \left. + \frac{2043}{72} H_{-1,4,4} - \frac{33}{72} H_{-1,4,5} + 19H_{-1,4,6} + \frac{31}{72} H_{-1,4,7} - \frac{407}{36} \zeta_2 + \frac{29}{36} H_{-1,4,8} - \frac{143}{72} H_{-1,4,9} \right. \\ \left. - \frac{5}{72} H_{-1,4,10} - \frac{5}{72} \zeta_3 + \frac{157}{72} H_{-1,4,11} - \frac{9}{72} \zeta_4 - \frac{5}{72} H_{-1,4,12} - \frac{1}{72} H_{-1,4,13} + H_{-1,4,14} + \frac{5}{72} H_{-1,4,15} \right. \\ \left. + \frac{9}{72} \zeta_2 + \frac{241}{72} H_{-1,4,16} + \frac{49}{72} H_{-1,4,17} + \frac{291}{72} H_{-1,4,18} - \frac{407}{72} H_{-1,4,19} - H_{-1,4,20} + H_{-1,4,21} \right. \\ \left. + 2H_{-1,4,22} + 2H_{-1,4,23} + 2H_{-1,4,24} + 2H_{-1,4,25} + 4H_{-1,4,26} \right) \\ \left. - 113H_{-1,4,0} - 5H_{-1,4,1} + 2H_{-1,4,2} + \frac{27}{24} H_{-1,4,3} + \frac{11}{24} H_{-1,4,4} - \frac{67}{24} H_{-1,4,5} + \frac{12}{24} H_{-1,4,6} \right. \\ \left. + 13H_{-1,4,7} - \frac{17}{24} H_{-1,4,8} - \frac{3}{24} H_{-1,4,9} + H_{-1,4,10} + \frac{79}{24} H_{-1,4,11} + \frac{67}{24} H_{-1,4,12} + \frac{263}{8} \zeta_2 \right. \\ \left. + \frac{119}{24} \zeta_3 + \frac{303}{24} H_{-1,4,13} - 24H_{-1,4,14} + H_{-1,4,15} - \frac{13375}{72} H_{-1,4,16} - \frac{1889}{72} H_{-1,4,17} - \frac{107}{72} H_{-1,4,18} \right. \\ \left. + \frac{145}{72} H_{-1,4,19} + \frac{217}{72} H_{-1,4,20} + \frac{29}{72} \zeta_2 + \frac{17}{72} H_{-1,4,21} + \frac{17}{72} H_{-1,4,22} + \frac{19}{72} H_{-1,4,23} + \frac{2}{72} H_{-1,4,24} \right. \\ \left. + \frac{145}{72} H_{-1,4,25} + 16C_F C_F \gamma \left(\frac{4}{3} H_{-1,0} + \frac{11}{36} H_{-1,1} + \frac{17}{36} H_{-1,2} + \frac{1}{18} H_{-1,3} + 2H_{-1,4,0} \right. \right. \\ \left. \left. - \frac{1}{12} H_{-1,4,1} - \frac{1}{12} H_{-1,4,2} + \frac{1}{2} P_{ij}^{(2)}(x) \left[\frac{1}{2} H_{-1,0} - H_{-1,1} - H_{-1,2} + H_{-1,3} + H_{-1,4} \right] + 16C_F C_F \gamma \left(\frac{85}{18} H_{-1,0} \right. \right. \right. \\ \left. \left. + \frac{2}{3} H_{-1,1} - H_{-1,2} + H_{-1,3} + H_{-1,4} \right) + 16C_F C_F \gamma \left(\frac{1}{3} H_{-1,0} - H_{-1,1} - H_{-1,2} + H_{-1,3} + H_{-1,4} \right) \right. \\ \left. + \frac{2}{3} H_{-1,4,0} + \frac{98}{3} H_{-1,4,1} + \frac{100}{3} H_{-1,4,2} + \frac{28}{3} H_{-1,4,3} + \frac{16}{3} H_{-1,4,4} + \frac{16}{3} H_{-1,4,4,0} + 4H_{-1,4,4,1} + \frac{16}{3} H_{-1,4,4,2} \right. \\ \left. + \frac{2}{3} H_{-1,4,4,3} + \frac{4}{3} \zeta_2 \left(x^2 - x^3 \right) \left[\frac{23}{12} H_{-1,0} - \frac{23}{12} H_{-1,1} - \frac{5}{6} \zeta_2 H_{-1,2} + H_{-1,3} + H_{-1,4} \right. \right. \\ \left. \left. + (1-x) \left[\frac{1}{2} H_{-1,0} + \frac{7}{12} H_{-1,1} - \frac{2743}{72} H_{-1,2} + \frac{251}{72} H_{-1,3} + \frac{5}{24} H_{-1,4} + \frac{341}{72} H_{-1,4,0} \right. \right. \right. \\ \left. \left. + 3H_{-1,4,1} - 3H_{-1,4,2} - H_{-1,4,3} - H_{-1,4,4} \right] + (1+x) \left[\frac{1669}{216} H_{-1,0} + \frac{5}{216} H_{-1,1} + \frac{1703}{108} H_{-1,2} + \frac{37}{108} \zeta_2 \right. \right. \\ \left. \left. - 7H_{-1,3} + 6H_{-1,4,0} - 4H_{-1,4,1} + H_{-1,4,2} - 4H_{-1,4,3} + H_{-1,4,4} \right] \right)$$

$$-2H_{-1,2} + H_{-1,3} + H_{-1,4} + (1-x) \left[9H_{-1,0,0} - 5H_{-1,0,1} - \frac{65}{36} \zeta_2 + \frac{11}{36} H_{-1,1} \right. \\ \left. - \frac{2}{3} H_{-1,2} + \frac{2}{3} H_{-1,3} + H_{-1,4} - \frac{31}{36} H_{-1,4,0} - \frac{551}{36} \zeta_2 + \frac{29}{36} H_{-1,4,1} - \frac{113}{36} H_{-1,4,2} - \frac{18691}{72} H_{-1,4,3} \right. \\ \left. + \frac{2043}{72} H_{-1,4,4} - \frac{33}{72} H_{-1,4,5} + 19H_{-1,4,6} + \frac{31}{72} H_{-1,4,7} - \frac{407}{36} \zeta_2 + \frac{29}{36} H_{-1,4,8} - \frac{143}{72} H_{-1,4,9} \right. \\ \left. - \frac{5}{72} H_{-1,4,10} - \frac{5}{72} \zeta_3 + \frac{157}{72} H_{-1,4,11} - \frac{9}{72} \zeta_4 - \frac{5}{72} H_{-1,4,12} - \frac{1}{72} H_{-1,4,13} + H_{-1,4,14} + \frac{5}{72} H_{-1,4,15} \right. \\ \left. + \frac{9}{72} \zeta_2 + \frac{241}{72} H_{-1,4,16} + \frac{49}{72} H_{-1,4,17} + \frac{291}{72} H_{-1,4,18} - \frac{407}{72} H_{-1,4,19} - H_{-1,4,20} + H_{-1,4,21} \right. \\ \left. + 2H_{-1,4,22} + 2H_{-1,4,23} + 2H_{-1,4,24} + 2H_{-1,4,25} + 4H_{-1,4,26} \right) \\ \left. - 113H_{-1,4,0} - 5H_{-1,4,1} + 2H_{-1,4,2} + \frac{27}{24} H_{-1,4,3} + \frac{11}{24} H_{-1,4,4} - \frac{67}{24} H_{-1,4,5} + \frac{12}{24} H_{-1,4,6} \right. \\ \left. + 13H_{-1,4,7} - \frac{17}{24} H_{-1,4,8} - \frac{3}{24} H_{-1,4,9} + H_{-1,4,10} + \frac{79}{24} H_{-1,4,11} + \frac{67}{24} H_{-1,4,12} + \frac{263}{8} \zeta_2 \right. \\ \left. + \frac{119}{24} \zeta_3 + \frac{303}{24} H_{-1,4,13} - 24H_{-1,4,14} + H_{-1,4,15} - \frac{13375}{72} H_{-1,4,16} - \frac{1889}{72} H_{-1,4,17} - \frac{107}{72} H_{-1,4,18} \right. \\ \left. + \frac{145}{72} H_{-1,4,19} + \frac{217}{72} H_{-1,4,20} + \frac{29}{72} \zeta_2 + \frac{17}{72} H_{-1,4,21} + \frac{17}{72} H_{-1,4,22} + \frac{19}{72} H_{-1,4,23} + \frac{2}{72} H_{-1,4,24} \right. \\ \left. + \frac{145}{72} H_{-1,4,25} + 16C_F C_F \gamma \left(\frac{4}{3} H_{-1,0} + \frac{11}{36} H_{-1,1} + \frac{17}{36} H_{-1,2} + \frac{1}{18} H_{-1,3} + 2H_{-1,4,0} \right. \right. \\ \left. \left. - \frac{1}{12} H_{-1,4,1} - \frac{1}{12} H_{-1,4,2} + \frac{1}{2} P_{ij}^{(2)}(x) \left[\frac{1}{2} H_{-1,0} - H_{-1,1} - H_{-1,2} + H_{-1,3} + H_{-1,4} \right] + 16C_F C_F \gamma \left(\frac{85}{18} H_{-1,0} \right. \right. \right. \\ \left. \left. + \frac{2}{3} H_{-1,1} - H_{-1,2} + H_{-1,3} + H_{-1,4} \right) + 16C_F C_F \gamma \left(\frac{1}{3} H_{-1,0} - H_{-1,1} - H_{-1,2} + H_{-1,3} + H_{-1,4} \right) \right. \\ \left. + \frac{2}{3} H_{-1,4,0} + \frac{98}{3} H_{-1,4,1} + \frac{100}{3} H_{-1,4,2} + \frac{28}{3} H_{-1,4,3} + \frac{16}{3} H_{-1,4,4} + \frac{16}{3} H_{-1,4,4,0} + 4H_{-1,4,4,1} + \frac{16}{3} H_{-1,4,4,2} \right. \\ \left. + \frac{2}{3} H_{-1,4,4,3} + \frac{4}{3} \zeta_2 \left(x^2 - x^3 \right) \left[\frac{23}{12} H_{-1,0} - \frac{23}{12} H_{-1,1} - \frac{5}{6} \zeta_2 H_{-1,2} + H_{-1,3} + H_{-1,4} \right. \right. \\ \left. \left. + (1-x) \left[\frac{1}{2} H_{-1,0} + \frac{7}{12} H_{-1,1} - \frac{2743}{72} H_{-1,2} + \frac{251}{72} H_{-1,3} + \frac{5}{24} H_{-1,4} + \frac{341}{72} H_{-1,4,0} \right. \right. \right. \\ \left. \left. + 3H_{-1,4,1} - 3H_{-1,4,2} - H_{-1,4,3} - H_{-1,4,4} \$$

You may be wondering,

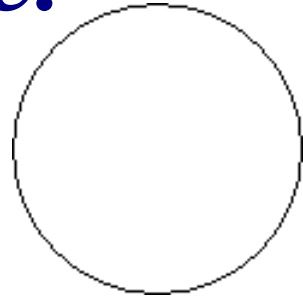
“Am I in the right place?”

**What does this have to do with
NUNCLEAR PHYSICS?**

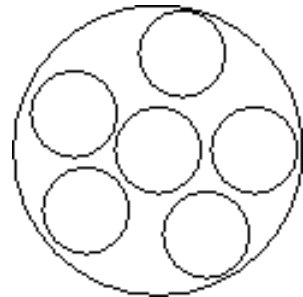
What is a nucleus?

depends on the resolution_T scale!

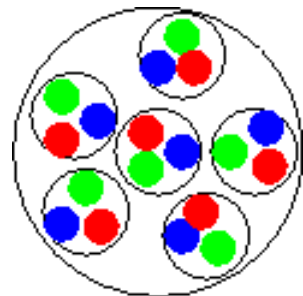
A point particle: $\lambda_t \gg 10 \text{ fm}$



A collection of nucleons: $\lambda_t \sim 1 \text{ fm}$

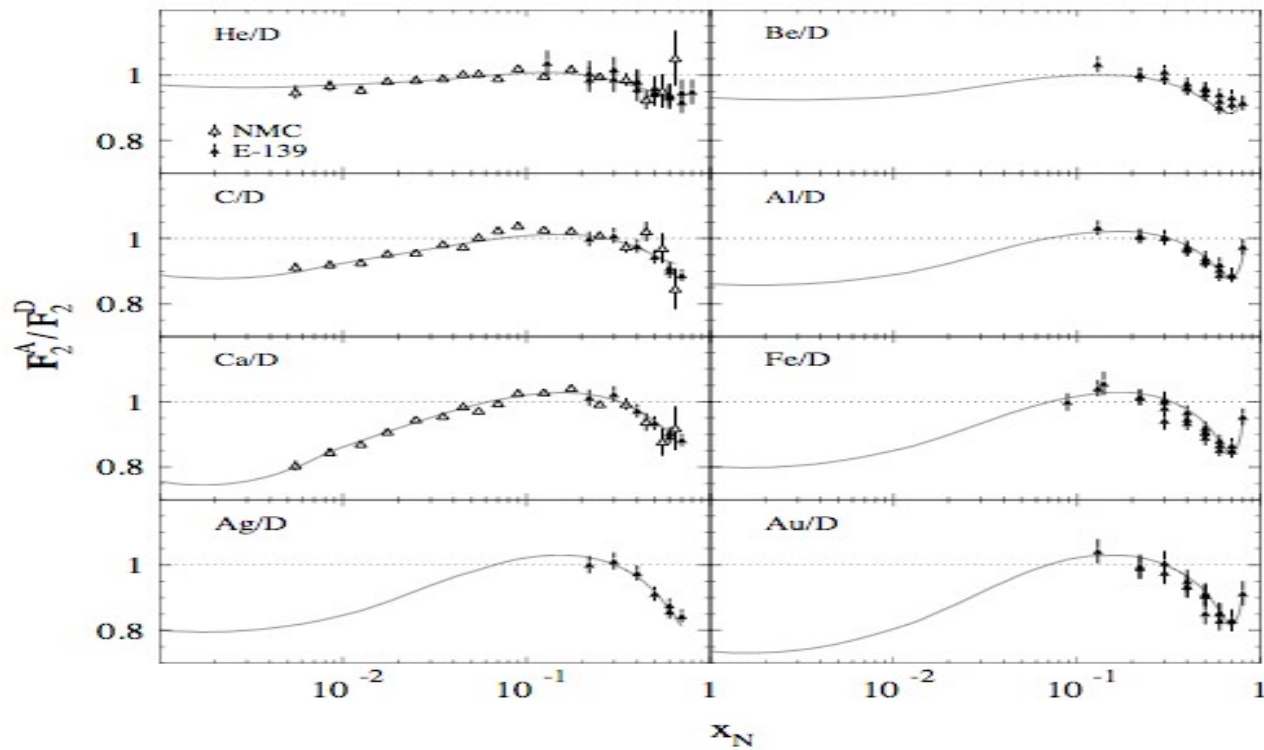


A system of quarks and gluons: $\lambda_t \ll 1 \text{ fm}$



we should be able to use pQCD

modification of the nuclear structure functions



($\propto 1/c.m. \text{ energy}$)

How about scattering of nuclei?

RHIC, LHC

I) modification of initial state: "nuclear shadowing"

II) modification of hard scattering: multiple scattering

III) modification of fragmentation functions

how do partons hadronize?



so far we have considered PQCD in the Bjorken limit

$$Q^2, S \rightarrow \infty \quad x_{Bj} \equiv \frac{Q^2}{S} \text{ fixed}$$

DGLAP evolution of partons

*number of partons increases with Q^2
but parton number density decreases
hadron becomes more dilute*

*Excellent tool for high Q^2 inclusive observables
higher twists become important at low Q^2*

Not designed to treat collective phenomena:

*shadowing
multiple scattering
diffraction
impact parameter dependence
.....*

*Extension beyond leading twist is very difficult
many-body dynamics hidden in parameters*

