# Fundamental Symmetries – I
# The Standard Model

R. Tribble

**Texas A&M University**

# Outline of Lectures

- Introduction – a bit of history (I)
- The electroweak Standard Model - basics
- QCD and the SM
- SM parameters – the SM today
- NP and HEP tests of the electroweak SM
- Parity violating electron scattering (I)
- Beta decay – from the neutron to nuclei (II)
- Matter-antimatter asymmetry and CP (II)
- Muon decay  and muon properties(III)

# Questions to consider

- What symmetries can you identify in physics? Which of them do you consider ‘fundamental’? Can you identify a way to test the symmetries?
- What ‘fundamental forces’ do we use?
- How do hadrons interact via these forces? How do we test the interactions?
- What are some of the ways that symmetries and forces affect the evolution of the universe?

# Historical Context – 1950’s

(HEP and NP)

- Many ‘elementary’ particles discovered
- QED well established as a renormalizable Quantum Field Theory
- Prevailing view – four distinct forces
- Symmetries associated with forces: charge conjugation (C), parity (P) and time-reversal (T) transformation – forces invariant under these symmetries?

# Symmetries

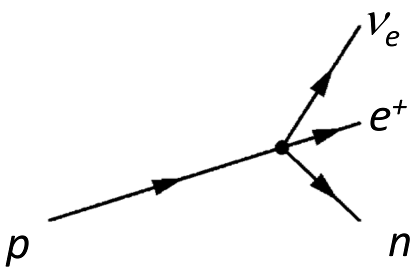
- Parity inversion $\Rightarrow \vec{r} \rightarrow -\vec{r}$
- Charge conjugation $\Rightarrow q \rightarrow -q$

  $|\psi\rangle \rightarrow |\bar{\psi}\rangle$ (no change in handedness)
- Time reversal $\Rightarrow t \rightarrow -t$
- CP symmetry
- CPT

"Parity Violation"

# Historical Context – β decay

- Weak interaction characterized by β decay
- Fermi theory – decay described via point interaction



- General form for nuclear β decay allows for Scalar, Vector, Tensor, Pseudo Scalar, and Axial Vector interactions

# **General Form** for Nuclear $\beta$ Decay

$$n \to p + e^- + \nu_e$$

- Hamiltonian given by

$$H_\beta = \sum_{i=1}^{5} G_i \{(\bar{\psi}_p O_i \psi_n)(\bar{\psi}_e O_i \psi_{\nu e}) + h.c.\}$$

- Mid 1950's ⇒ nucleon decay favored *ST*, pion decay favored *A* or *P* and muon decay favored *VA*
- Rather than develop operators for nucleons in above, let's consider a simpler process

# Muon decay – I

$$\mu^{+} \rightarrow e^{+} + \nu_{e} + \bar{\nu}_{\mu}$$

- Purely leptonic decay – spin 1/2 particles
- W.F. satisfy Dirac Equation - $(\gamma_{\mu} + m)\psi(x) = 0$
- Five covariant interactions:
  - scalar $\bar{\psi}_1(x)\psi_2(x)$
  - vector $\bar{\psi}_1(x)\gamma_{\mu}\psi_2(x)$
  - tensor $\bar{\psi}_1(x)\sigma_{\mu v}\psi_2(x)$
  - axial vector $\bar{\psi}_1(x)i\gamma_{\mu}\gamma_5\psi_2(x)$
  - pseudo scalar $\bar{\psi}_1(x)\gamma_5\psi_2(x)$
- Note $x$ is a four vector here

# **Muon decay** – II

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$

- Use momentum dependent free Dirac spinors
- $u(p), v(p)$ satisfy

$$(i\gamma \cdot p + m)u_r(p) = 0$$

$$(-i\gamma \cdot p + m)v_r(p) = 0$$

- Reduced T matrix is then

$$\begin{aligned}
\tilde{T}_{fi} = \frac{1}{(2\pi)^6}\sqrt{\frac{m_\mu m_e m_1 m_2}{p_{10}p_{20}k_{10}k_{20}}}\,&[\bar{u}(p_2)u(p_1)\bar{u}_2(k_2)(C_S + C'_S\gamma_5)v_1(k_1)\\
&+ \bar{u}(p_2)\gamma_\lambda u(p_1)\bar{u}_2(k_2)\gamma_\lambda(C_V + C'_V\gamma_5)v_1(k_1)\\
&+ \tfrac{1}{2}\bar{u}(p_2)\sigma_{\lambda\nu}u(p_1)\bar{u}_2(k_2)\sigma_{\lambda\nu}(C_T + C'_T\gamma_5)v_1(k_1)\\
&+ \bar{u}(p_2)i\gamma_\lambda\gamma_5 u(p_1)\bar{u}_2(k_2)i\gamma_\lambda\gamma_5(C_A + C'_A\gamma_5)v_1(k_1)\\
&+ \bar{u}(p_2)\gamma_5 u(p_1)\bar{u}_2(k_2)\gamma_5(C_P + C'_P\gamma_5)v_1(k_1)]
\end{aligned}$$

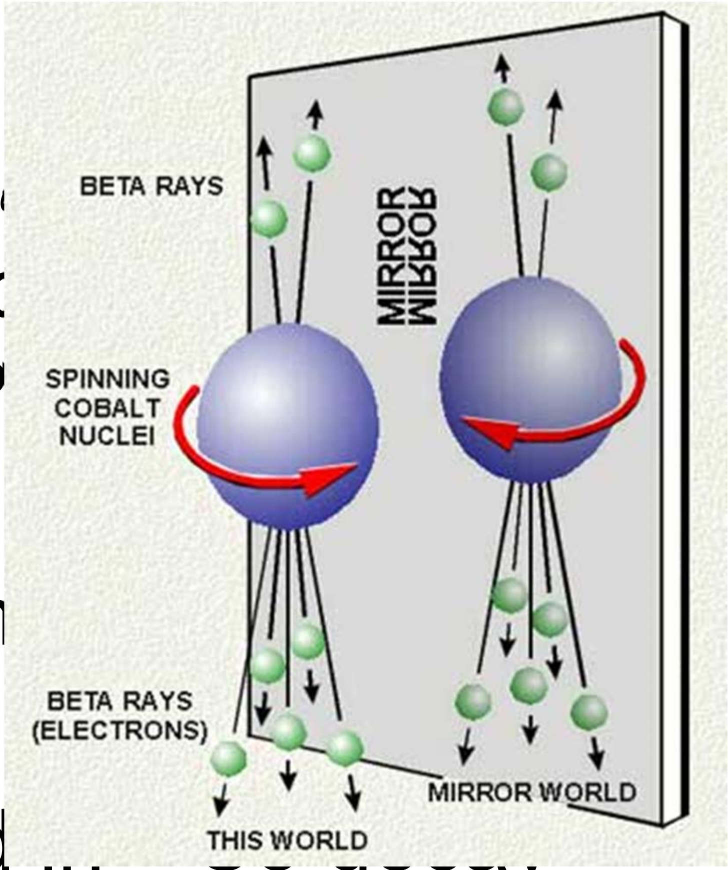
# **General Form** for Nuclear β Decay

## $n \rightarrow p + e^- + \nu_e$

- Hamiltonian given by

$$H_\beta = \sum_{i=1}^{5} G_i \{(\bar{\psi}_p O_i \psi_n)(\bar{\psi}$$

- Mid 1950's ⇒ nucleon dec
  pion decay favored *A* or *P*
  decay favored *VA*
- Lee and Yang predicted th
  did not conserve parity
- Parity violation discovered



Turn upside down

# Ingredients leading to Electroweak Standard Model

- QED was very successful model built around concept of local symmetry and was satisfied by U(1) gauge symmetry
- Yang-Mills developed SU(2) gauge theory of strong interactions (1954), but symmetry was approximate
- Broken symmetry concept introduced – Hamiltonian and commutation relations could posses exact symmetry but physical states might differ from it (1960) due to spontaneous symmetry breaking $\Rightarrow$ vector boson
- Following QED, renormalizable gauge theory sought that included intermediate vector boson to mediate weak interaction
- New model built on quark and lepton families, not nucleons

# Electroweak Standard Model - I

- Weinberg first to write down the basics
- With gauge fields A(x), spin-1/2 fields ψ(x), spin 0 fields ϕ(x), θ and t Lie algebra matrices, gauge-covariant Yukawa coupling G, the Lagrangian for SU(2)$_L$×U(1) renormalizable gauge theory is:

$$\mathfrak{L} = -\tfrac{1}{4}F_{\alpha\mu\nu}F_{\alpha}^{\mu\nu} - \tfrac{1}{2}(D_\mu\phi)_i(D^\mu\phi)_i - \bar{\psi}\gamma^\mu D_\mu\psi - \bar{\psi}\Gamma_i\psi\phi_i$$
$$- P(\phi) - \bar{\psi}m_o\psi,$$

where $F_{\alpha\mu\nu}$ is the gauge-covariant curl

$$F_{\alpha\mu\nu} \equiv \partial_\mu A_{\alpha\nu} - \partial_\nu A_{\alpha\mu} - C_{\alpha\beta\gamma}A_{\beta\mu}A_{\gamma\nu},$$

and $D_\mu\phi$ and $D_\mu\psi$ are gauge-covariant derivatives

$$(D_\mu\phi)_i \equiv \partial_\mu\phi_i - i(\theta_\alpha)_{ij}\phi_j A_{\alpha\mu},$$
$$(D_\mu\psi)_n \equiv \partial_\mu\psi_n - i(t_\alpha)_{nm}\psi_m A_{\alpha\mu}.$$

# Electroweak Standard Model - II

- Observed leptonic multiplets (1970) assumed as irreducible representations, note left handed doublets and right handed singlets

$$\left(\frac{1+\gamma_5}{2}\right)\begin{pmatrix}\nu_e\\ e^-\end{pmatrix} \qquad \left(\frac{1-\gamma_5}{2}\right)e^-$$

$$\left(\frac{1+\gamma_5}{2}\right)\begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix} \qquad \left(\frac{1-\nu_5}{2}\right)\mu^-$$

- Define a ‘hypercharge’ as Y ≡ $\mathbf{T}_3$ - *Q*
- Group algebra *Q* and leptonic ‘isospin’ **T** acting on left-handed doublets ⇒ $SU(2)_L \times U(1)$ symmetry with gauge fields $A_\mu$ and $B_\mu$
- Two gauge coupling constants, *g* and *g*’, and the weak mixing angle θ introduced, then $Q=(1/g)(g\mathbf{T}_3)-(1/g')(g'Y)$ is charge operator ⇒ a massless vector particle – photon associated with field $A_\mu$

# Electroweak Standard Model - III

- The field $A_\mu$ formed by orthogonal linear combination, called $Z_\mu$, which required *massive* neutral vector boson (Z)
- Remaining gauge fields used to generate charged massive vector bosons $W_\mu$
- Couplings imply both charged and neutral current interactions
- With this formulation, get:

$$tan\theta_W \equiv \frac{g'}{g}$$

$$e = gsin\theta_W$$

$$\frac{G_F}{\sqrt{2}} = \frac{1}{{m_W}^2}\left(\frac{g}{2\sqrt{2}}\right)^2$$

$$m_W = \frac{(2^{-5/4})(\sqrt{G_F})e}{sin\theta_W} = \frac{37.3\ GeV}{sin\theta_W}$$

$$m_Z = \frac{74.6\ GeV}{sin2\theta_W}$$

# Additions to SM

- T'Hooft showed how to choose gauge so that Feynman rules would be renormalizable; proofs that original model was renormalizable followed
- Specific gauge models based on early work developed
- Two quark doublets added to account for semi-leptonic decay; c quark was discovered
- Mixing in quark sector added (early CKM)
- Neutral currents were verified
- QCD developed
- b, t quarks and $\tau$ and $\nu_\tau$ leptons discovered
- CKM matrix for three generations of quarks

# The Standard Model today

- Gauge symmetry – SU(3)×SU(2)$_L$×U(1)
- Some QCD attributes:
  - – 'Color' charge used to describe force
  - – Quarks come in 3 colors (antiquarks – 3 anticolors)
  - – Strong force mediated by exchange of massless gluons
  - – Physical states have no net color charge – confinement

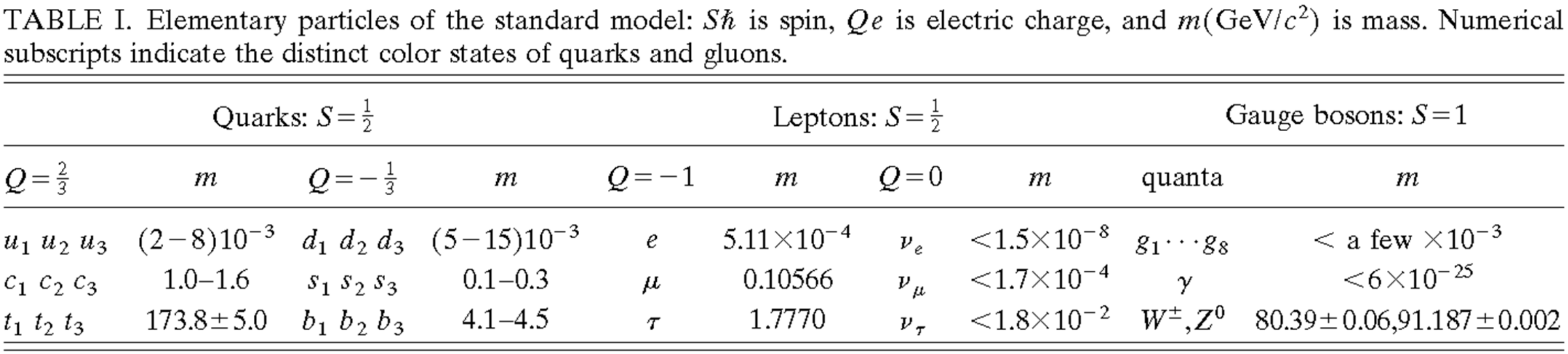


TABLE I. Elementary particles of the standard model: $S\hbar$ is spin, $Qe$ is electric charge, and $m(\mathrm{GeV}/c^2)$ is mass. Numerical subscripts indicate the distinct color states of quarks and gluons.

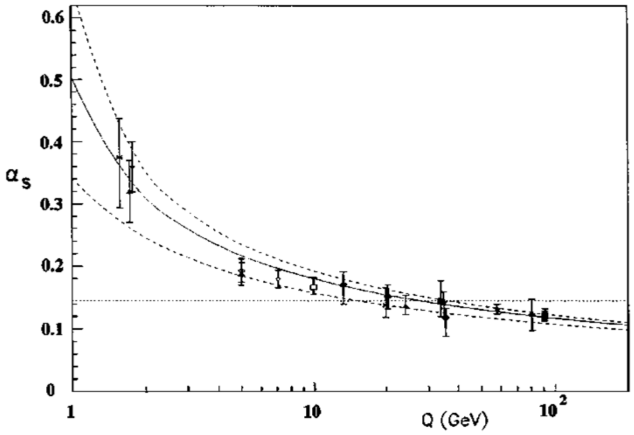
| Quarks: $S=\frac{1}{2}$ | | | | Leptons: $S=\frac{1}{2}$ | | | | Gauge bosons: $S=1$ | |
|---|---|---|---|---|---|---|---|---|---|
| $Q=\frac{2}{3}$ | $m$ | $Q=-\frac{1}{3}$ | $m$ | $Q=-1$ | $m$ | $Q=0$ | $m$ | quanta | $m$ |
| $u_1\ u_2\ u_3$ | $(2-8)10^{-3}$ | $d_1\ d_2\ d_3$ | $(5-15)10^{-3}$ | $e$ | $5.11\times10^{-4}$ | $\nu_e$ | $<1.5\times10^{-8}$ | $g_1\cdots g_8$ | $<$ a few $\times10^{-3}$ |
| $c_1\ c_2\ c_3$ | 1.0–1.6 | $s_1\ s_2\ s_3$ | 0.1–0.3 | $\mu$ | 0.10566 | $\nu_\mu$ | $<1.7\times10^{-4}$ | $\gamma$ | $<6\times10^{-25}$ |
| $t_1\ t_2\ t_3$ | $173.8\pm5.0$ | $b_1\ b_2\ b_3$ | 4.1–4.5 | $\tau$ | 1.7770 | $\nu_\tau$ | $<1.8\times10^{-2}$ | $W^\pm, Z^0$ | $80.39\pm0.06, 91.187\pm0.002$ |

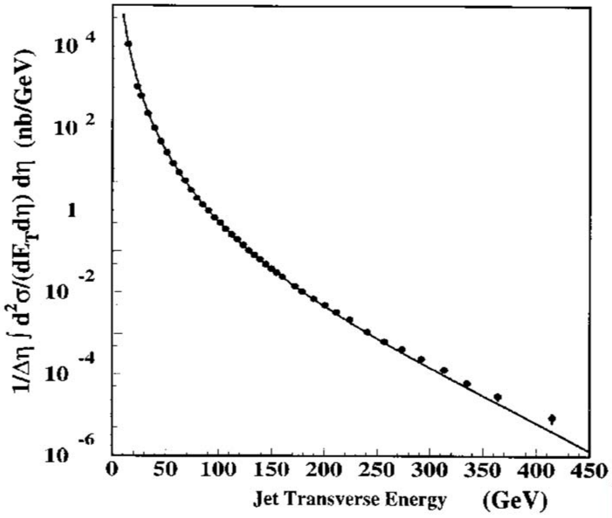
# Standard Model Symmetries

- P violation is maximal
- C violation also maximal
- CP violation small – two sources, one from electroweak and one from QCD ($\theta_{QCD}$)
  - CP violation from electroweak sector observed in neutral K decays
- Universality of interaction
- Conserved Vector Current relates $\beta$ and $\gamma$ decay parameters

# Some Key Experiments for **QCD**

- Deep-inelastic scattering with electrons
- Follow up studies with electrons, muons, and neutrinos followed QCD expectations for scaling
- Observation of ‘three-pronged’ jets in $e^+e^-$ scattering confirmed gluon as force carrier
- ‘Running’ of strong coupling constant verified as expected by confinement
- QCD prediction for inclusive jet cross section (NLO)





TEXAS A&M UNIVERSITY

# Some Key Experiments for **E-W Int**.

- Neutral currents found in neutrino scattering
- Discovery of W and Z bosons
- Photon – Z interference via polarized electron scattering of left and right handed beams
- Z properties measured, including width, which fixes number of light neutrinos
- Apparent unitarity of CKM matrix (more on this later!)
- Observation of top quark and constraints on Higgs mass from W and top masses
- Determination that $\sin^2\theta_W$ has small energy dependence
  - New measurements at JLab will provide precision test of the energy dependence – come back to this soon!

# Testing the SM

- About 40 years of experiments probing the SM
- Only neutrino mass (thus mixing) found that is not included in minimal SM
- No other tests of SM predictions vary by more than 3$\sigma$ and only muon (g-2) at this level
- So why test it?
  - Gravity not included
  - Many arbitrary parameters – quark and lepton masses, coupling strengths, CKM parameters (including phase), electroweak symmetry breaking – Higgs mass, scalar field mass, neutrino mixing angles, ...
  - Expect that it is part of a larger description of nature
  - Popular extensions include Supersymmetry

# SM current-current interaction

- Weak interaction Hamiltonian

$$H_W = \frac{G_F}{\sqrt{2}} J_\mu{}^\dagger J_\mu + H.C.$$

$$J_\mu = J_\mu{}^{had} + J_\mu{}^{lep}$$

$$J_\mu{}^{lep} = \bar{e}\gamma_\mu(1-\gamma_5)\nu_e + \bar{\mu}\gamma_\mu(1-\gamma_5)\nu_\mu + \bar{\tau}\gamma_\mu(1-\gamma_5)\nu_\tau$$

$$J_\mu{}^{had} = \overline{d_c}\gamma_\mu(1-\gamma_5)u + \overline{s_c}\gamma_\mu(1-\gamma_5)c + \overline{b_c}\gamma_\mu(1-\gamma_5)t$$

- *(V-A)* interaction that includes all weak processes
- Charge changing interactions mediated by $W^\pm$
- Neutral current interactions mediated by Z
- Electromagnetic interactions $\Rightarrow \gamma$

# Quark Mixing

- Cabibbo introduced the idea of quark mixing to account for slightly different values of vector coupling in muon and nuclear decays
- Extended about a decade later by Kobayashi and Maskawa
- CKM matrix in SM should be unitary

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

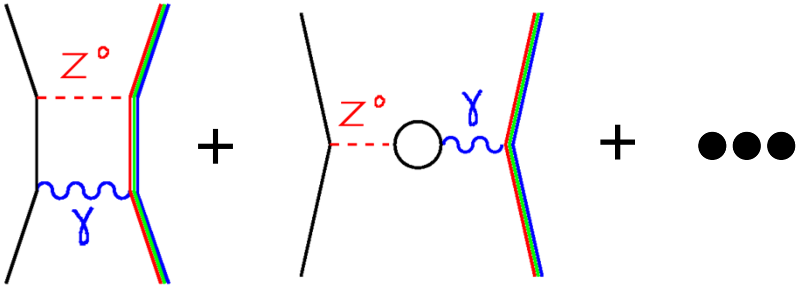
- A sensitive SM test is to verify unitarity

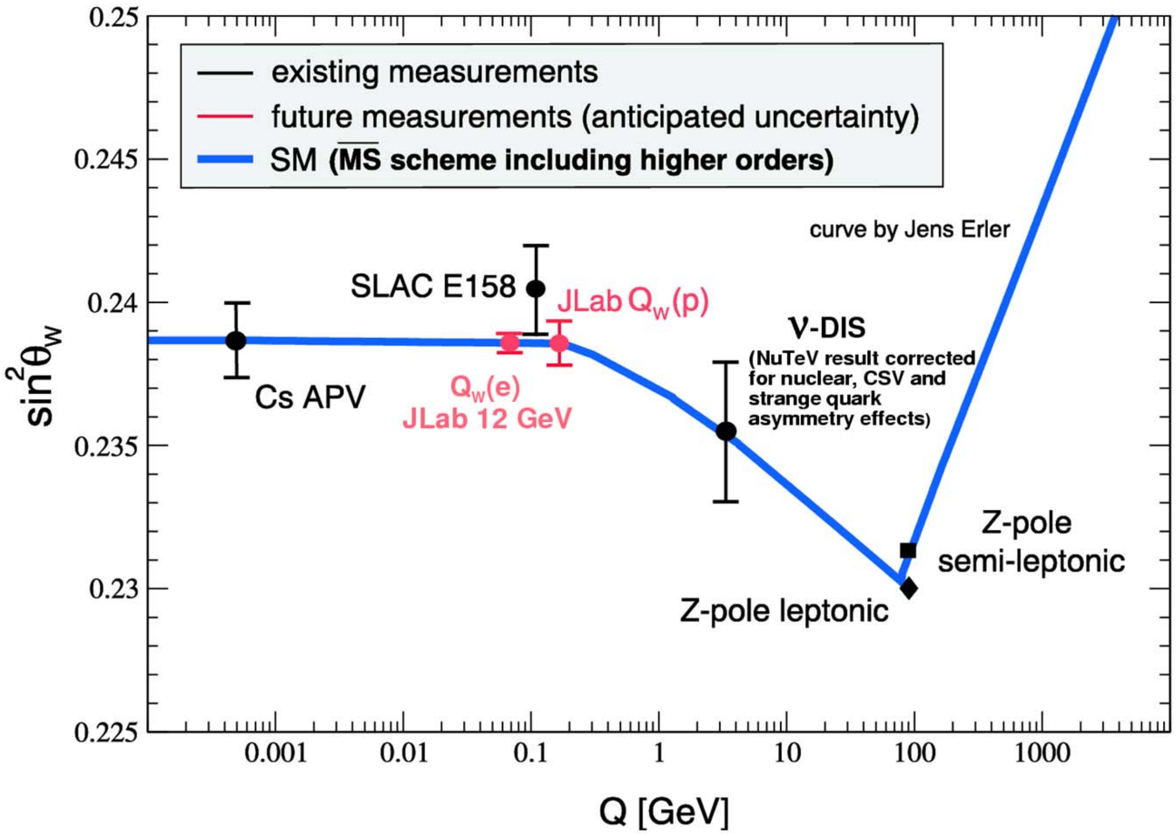
# A more detailed look at $\theta_W$

- Running of $\sin^2\theta_W$ established
- New experiments at JLab will yield much improved test of the SM prediction
- $Q_{Weak}$ experiment now underway
- Moller Scattering experiment being planned for 12 GeV upgraded facility
- Look at these two experiments now

# "Running of $\sin^2\theta_W$" in the Electroweak Standard Model

- Electroweak radiative corrections
  $\rightarrow \sin^2\theta_W$ varies with Q

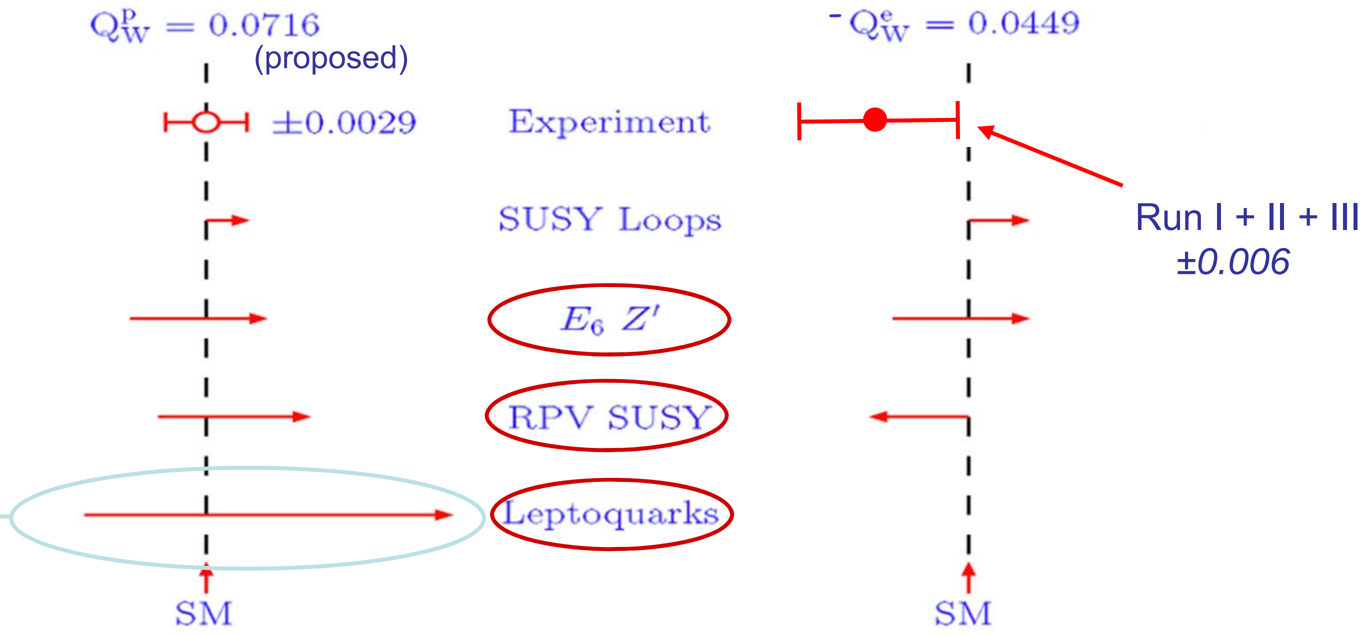




- All "extracted" values of $\sin^2\theta_W$ <u>must</u> agree with the Standard Model prediction or <u>new</u> physics is indicated.

# $Q^{p}_{weak}$ & $Q^{e}_{weak}$ - Complementary Diagnostics for New Physics

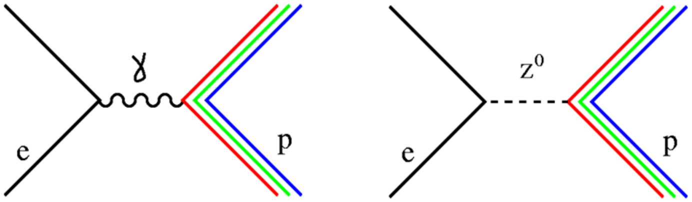
## JLab Qweak          SLAC E158



Erler, Kurylov, Ramsey-Musolf, PRD 68, 016006 (2003)

- Qweak measurement will provide a stringent stand alone constraint on lepto-quark based extensions to the SM.
- $Q^{p}_{weak}$ (semi-leptonic) and E158 (pure leptonic) together make a powerful program to search for and identify new physics.
- MOLLER (pure leptonic) is intended to do considerably better.
- Shifts are based on errors of +/- 0.0029 and +/- 0.0040, respectively

# Weak Charge Phenomenology



| | EM Charge | Weak Charge |
|---|---|---|
| $q^{up}$ | $+2/3$ | $1 - \frac{8}{3}\sin^2\theta_W \approx 1/3$ |
| $q^{down}$ | $-1/3$ | $-1 + \frac{4}{3}\sin^2\theta_W \approx -2/3$ |
| $Q^p = 2q^{up} + 1q^{down}$ | $+1$ | $1 - 4\sin^2\theta_W = .048$ |
| $Q^n = 1q^{up} + 2q^{down}$ | $0$ | $-1$ |

**Note how the roles of the proton and neutron have become almost reversed**

**(ie, neutron weak charge is dominant, proton weak charge is almost zero!)**

**$Q^e$ -1 $-(1 - 4\sin^2\theta_W) = -.048$**

**This accidental suppression of the proton weak charge in the SM makes it more sensitive to new physics (all other things being equal). Similarly for the electron weak charge.**

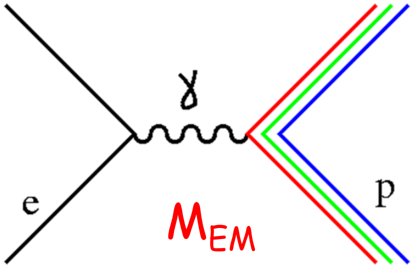
# The $Q^{p}_{weak}$ Experiment: A Search for New TeV Scale Physics via a Measurement of the Proton's Weak Charge

**Measure:** Parity-violating asymmetry in
$\vec{e}$ + p elastic scattering at $Q^2 \sim 0.03$ GeV$^2$
to ~4% relative accuracy at JLab

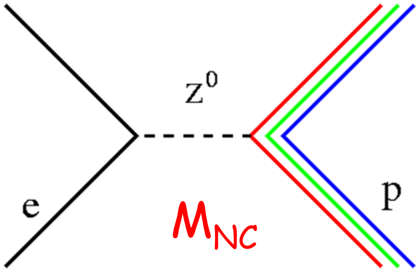
**Extract:** Proton's weak charge $Q^{p}_{weak} \sim 1 - 4 \sin^2\theta_W$
to get ~0.3% on $\sin^2\theta_W$ at $Q^2 \sim 0.03$ GeV$^2$

➡ tests "running of $\sin^2\theta_W$" from $M^2_Z$ to low $Q^2$
➡ sensitive to new TeV scale physics

# $Q^p_{weak}$: Extract from Parity-Violating Electron-Proton Scattering



As $Q^2 \to 0$



measures $Q^p$ – proton's electric charge

measures $Q^p_{weak}$ – proton's weak charge

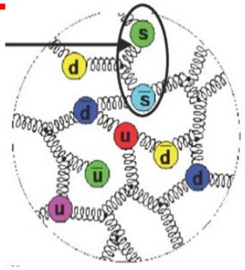
$$A = \frac{2M_{NC}}{M_{EM}} = \left[\frac{-G_F}{4\pi\alpha\sqrt{2}}\right]\left[Q^2\, Q^p_{weak} + F^p\left(Q^2,\theta\right)\right]$$

$$\xrightarrow[\theta \to 0]{Q^2 \to 0} \left[\frac{-G_F}{4\pi\alpha\sqrt{2}}\right]\left[Q^2\, Q^p_{weak} + Q^4 B\left(Q^2\right)\right]$$

contains hadronic structure information – strange form factors

$$Q^p_{weak} = 1 - 4\sin^2\theta_W \sim 0.072$$ (at tree level)

- $Q^p_{weak}$ is a well-defined experimental observable
- $Q^p_{weak}$ has a definite prediction in the electroweak Standard Model



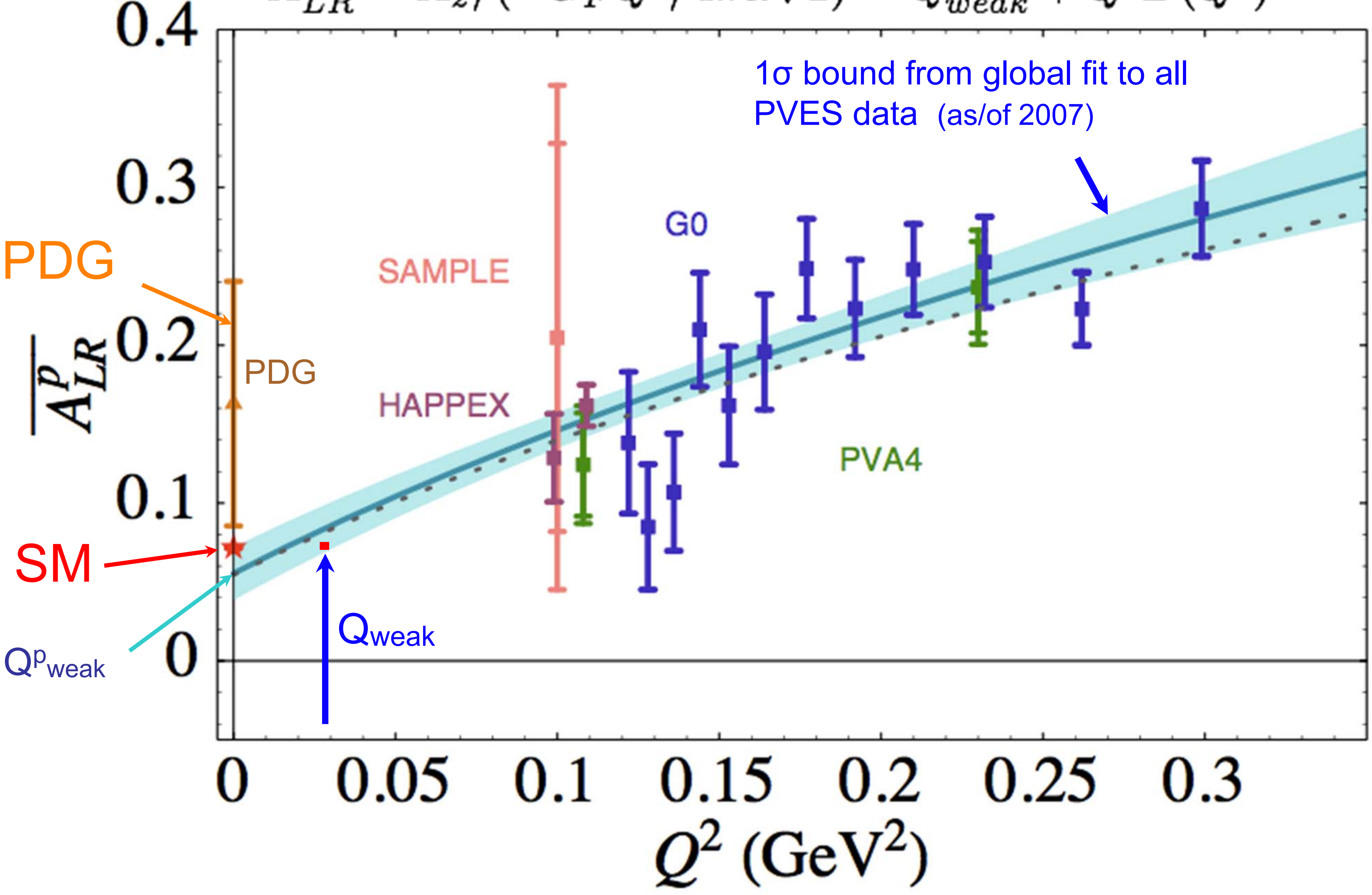
$$G^s_E\,(Q^2)$$
$$G^s_M\,(Q^2)$$

Strange electric and magnetic form factors
-measure contribution of strange quark sea
to nucleon structure

# Parity-Violating Asymmetry Extrapolated to $Q^2 = 0$

(Young, Carlini, Thomas & Roche, PRL 99, 122003 (2007) )

$$\overline{A^p_{LR}} = A_z/(-G_F Q^2/4\pi\alpha\sqrt{2}) = Q^p_{weak} + Q^2 B(Q^2)$$



1σ bound from global fit to all PVES data (as/of 2007)

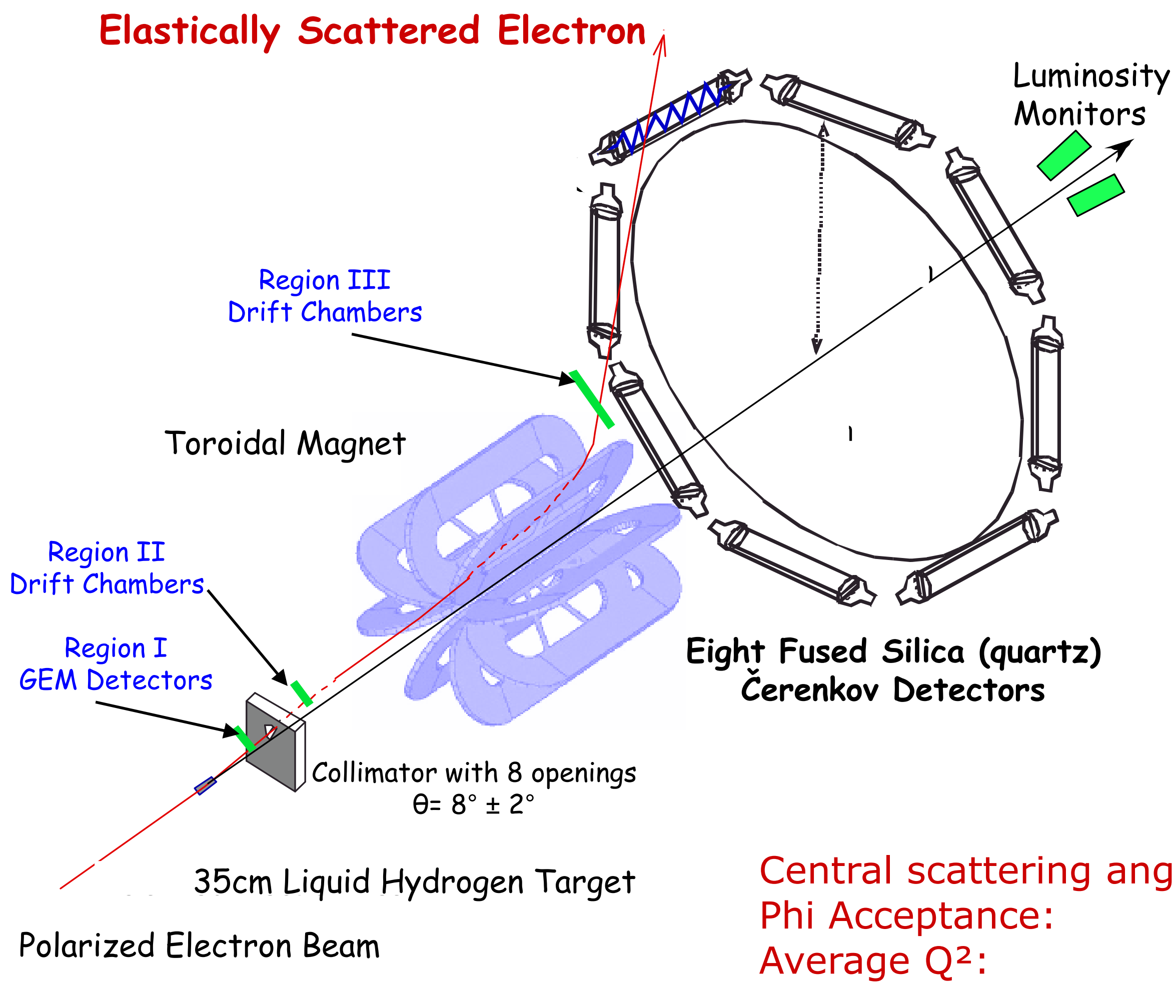
PDG

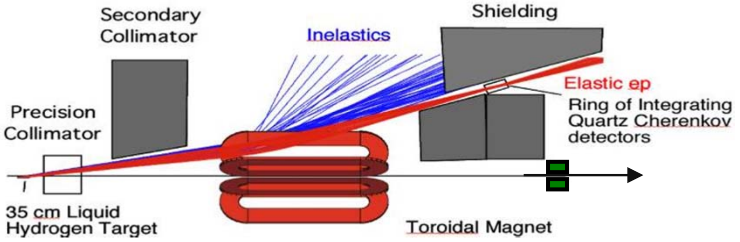
SM

$Q^p_{weak}$

Qweak

# Overview of the $Q^p_{Weak}$ Experiment





## Experiment Parameters (integration mode)

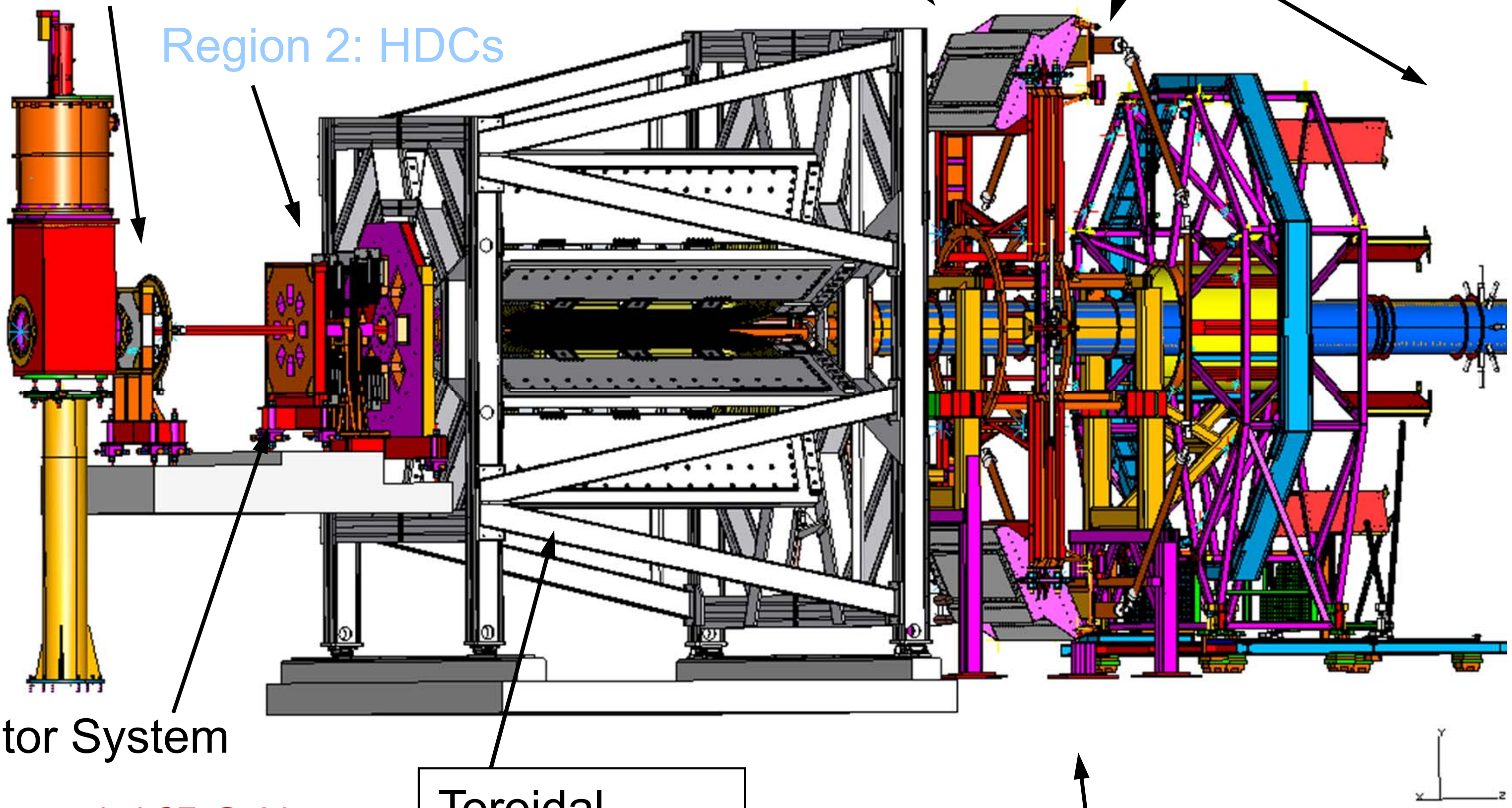
| | |
|---|---|
| Incident beam energy: | 1.165 GeV |
| Beam Current: | 170 μA |
| Beam Polarization: | 85% |
| $LH_2$ target power: | 2.5 KW |

| | |
|---|---|
| Central scattering angle: | 8.4° ± 3° |
| Phi Acceptance: | 53% of $2\pi$ |
| Average $Q^2$: | 0.026 $(GeV/c)^2$ |
| Acceptance averaged asymmetry: | –0.27 ppm |
| Integrated Rate (all sectors): | 6.4 GHz |
| Integrated Rate (per detector): | 800 MHz |

Polarimetry: Moller and Compton Polarimeters

# Principal Parts of the $Q^p_{weak}$ Experiment

light blue = counting mode
black = current mode



Region 3: VDCs

Synthetic Quartz Scintillator Bars

Lumi Monitors

Region 1: GEMs

Liquid Hydrogen Target (2.5 kW)

Region 2: HDCs

Electron Beam
1.165 GeV
170 μA (0.2 nA)
P ~ 85%

Collimator System

Toroidal Spectrometer Magnet

Trigger Scintillator

| | |
|---|---|
| Incident beam energy: | 1.165 GeV |
| Beam Current: | 170 μA |
| Beam Polarization: | 85% |
| $LH_2$ target power: | 2.5 KW |

# Parity-Violating Electron-Electron Scattering at 11 GeV

- $Q^e_{weak}$ would tightly constrain RPV SUSY (ie tree-level

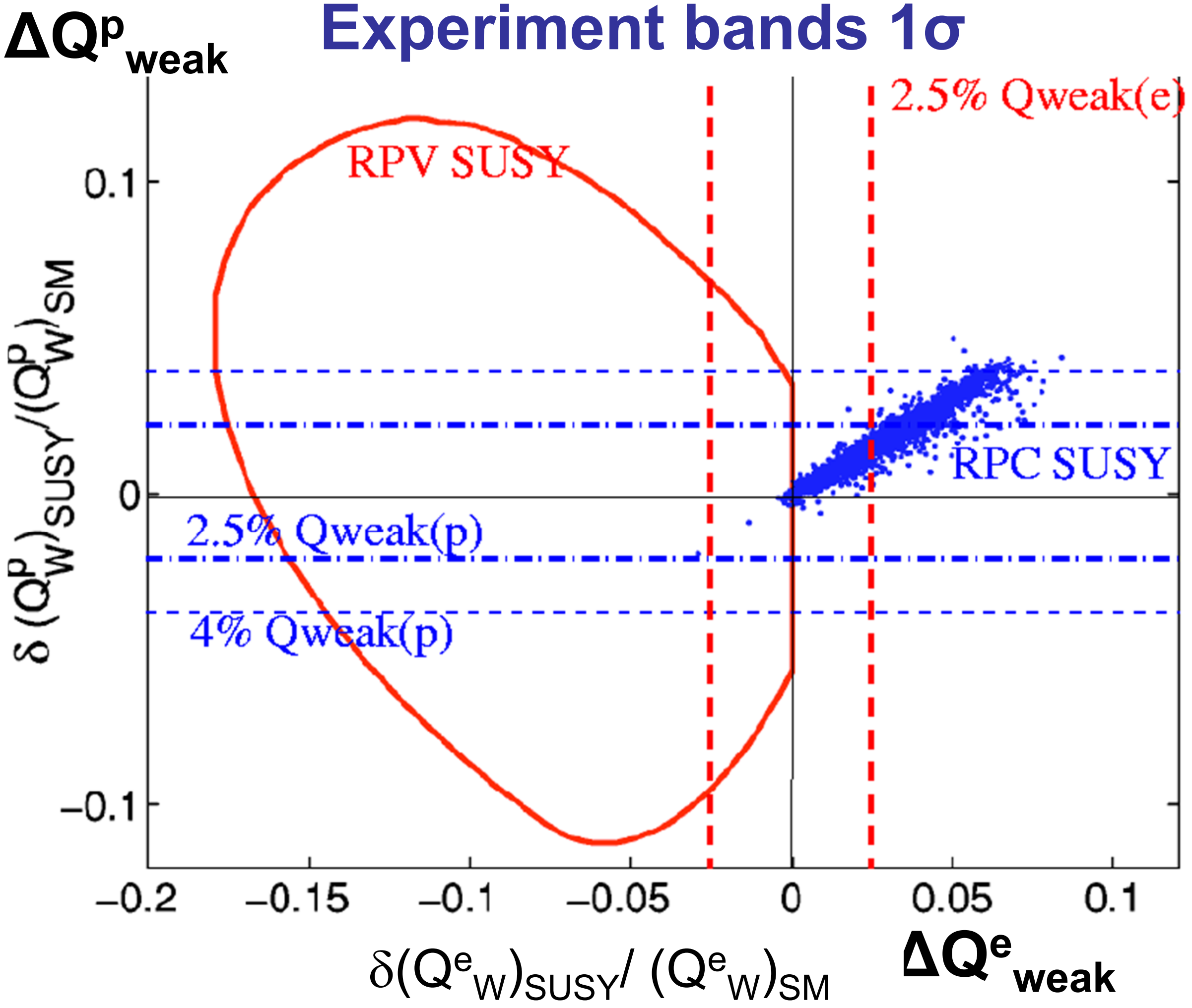
  One of few ways to constrain RPC SUSY if it happens to conserve CP (hence SUSY EDM = 0).

  Direct associated-production of a pair of RPC SUSY particles might not be possible even at the LHC.

  A.Kurylov, M.J. Ramsey-Musolf, S.Su, Phys.Rev. D68, 035008 (2003).

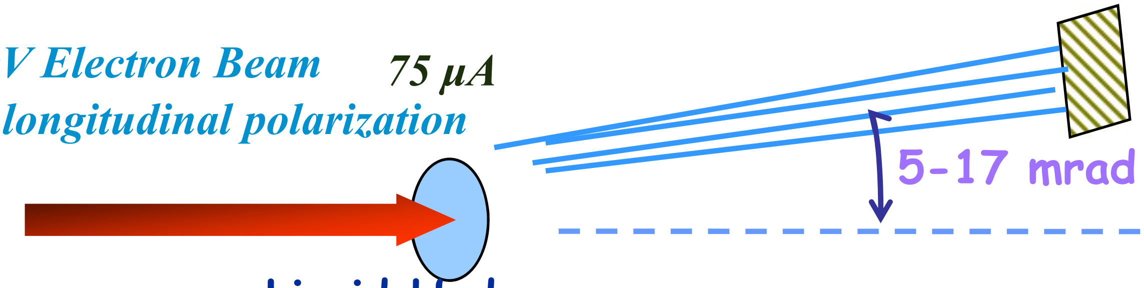
**Theory contours 95% CL**

**Experiment bands 1σ**



# MOLLER at Jefferson Laboratory

*Measurement of Lepton-Lepton Electroweak Reaction*



*11 GeV Electron Beam* *75 µA*

*80% longitudinal polarization*

150 GHz

5-17 mrad

**Liquid Hydrogen Target**

$$A_{PV} = \frac{\sigma_{\uparrow} - \sigma_{\downarrow}}{\sigma_{\uparrow} + \sigma_{\downarrow}} = -A_{LR}$$

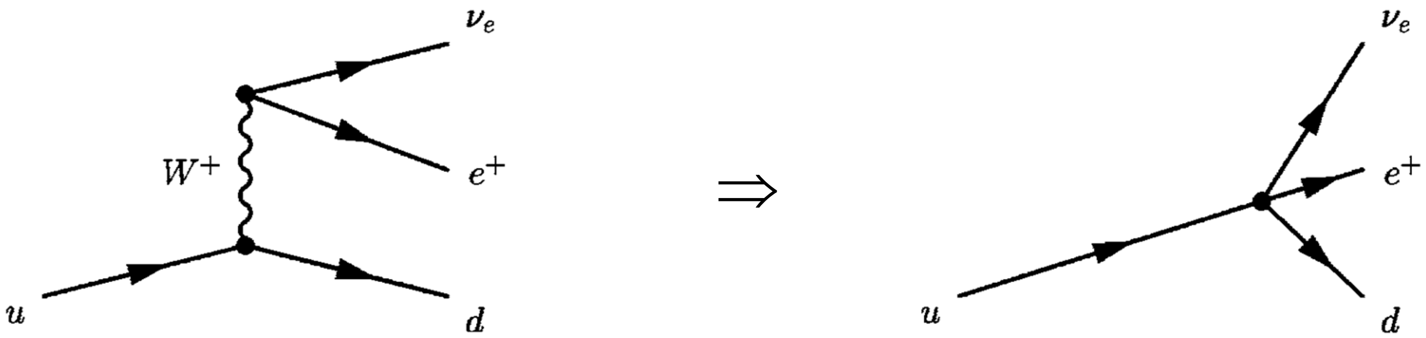
$A_{PV} = 35.6$ ppb → *38 weeks* → $\delta(A_{PV}) = 0.73$ ppb

*Compositeness length scale probed:* $4 \times 10^{-21}$ m

*Equivalent to the reach of a several TeV electron linear collider*

# **SM** Interaction for **low energy processes**

- Since W is very massive, can treat nuclear, pion, and muon beta decay as a point interaction



- This reverts to early formulation of nuclear beta decay by Fermi but with only (V-A) in the interaction

Starting point for next lecture!