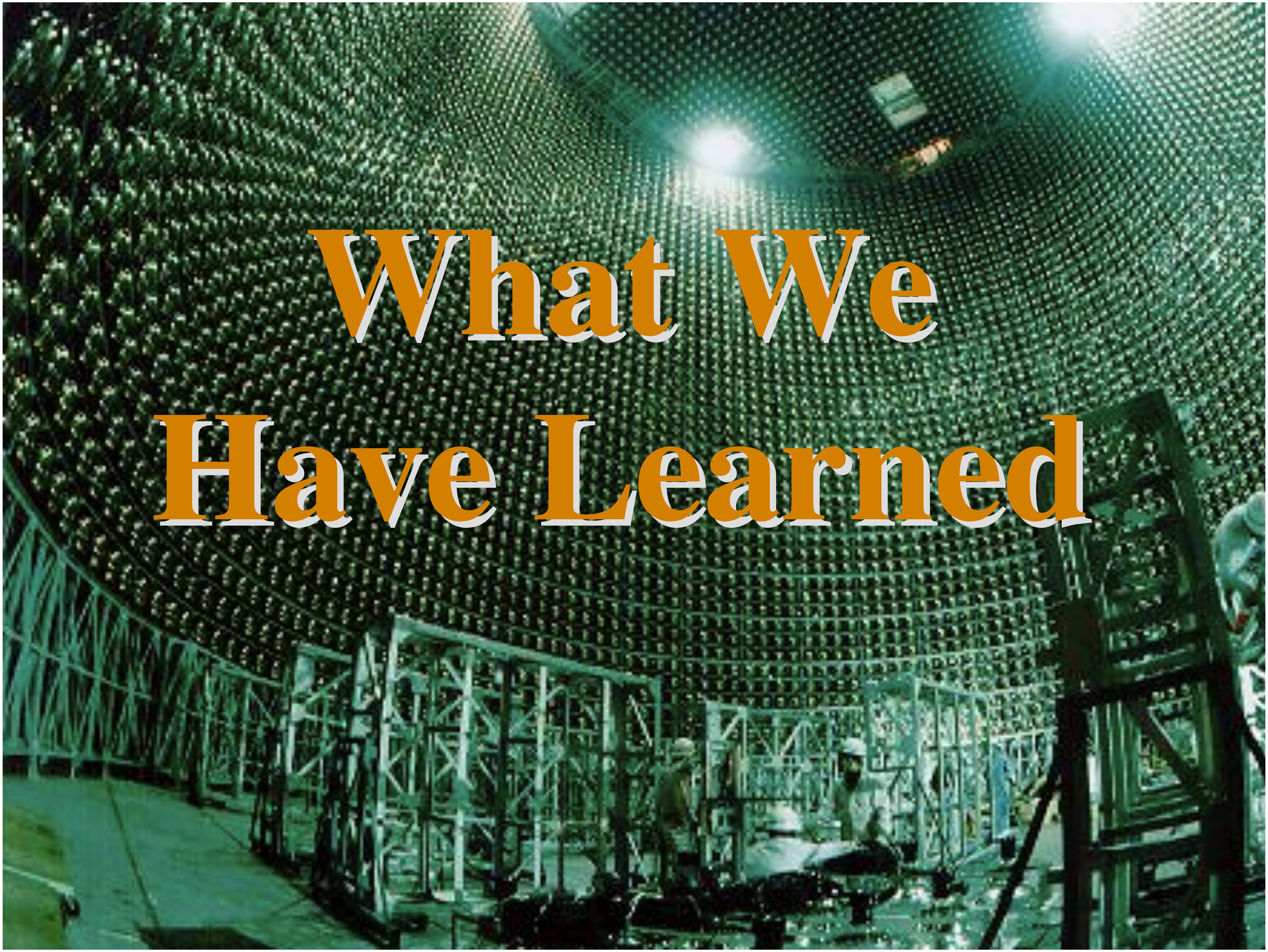
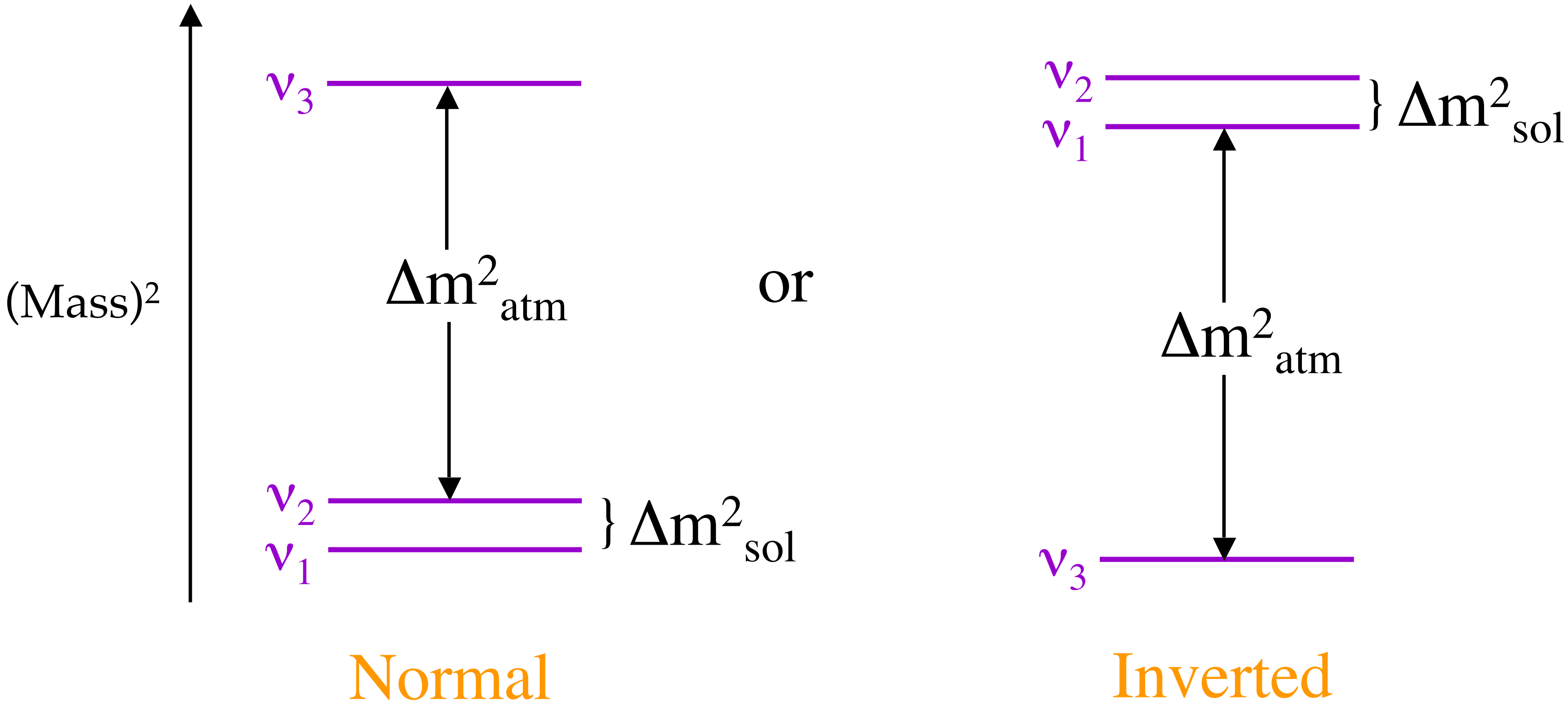


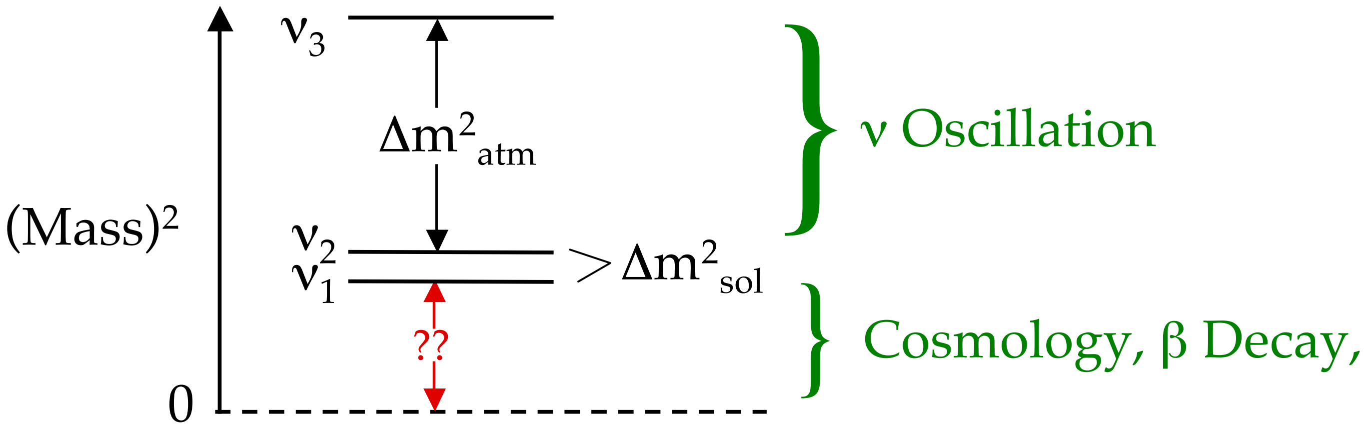
What We Have Learned

The $(\text{Mass})^2$ Spectrum



$\Delta m^2_{\text{sol}} \cong 7.5 \text{ x } 10^{-5}\ \text{eV}^2$, $\Delta m^2_{\text{atm}} \cong 2.3 \text{ x } 10^{-3}\ \text{eV}^2$

The Absolute Scale of Neutrino Mass



How far above zero is the whole pattern?

Oscillation Data $\Rightarrow \sqrt{\Delta m^2_{atm}} <$ Mass[Heaviest ν_i]

The Upper Bound From Cosmology

Neutrino mass affects large scale structure.

Cosmological Data + Cosmological Assumptions ⇒

$$\Sigma\, m_i < (0.17 - 1.0)\ \text{eV}\ .$$

Mass(ν_i)

(Seljak, Slosar, McDonald
Hannestad; Pastor)

If there are only 3 neutrinos,

$$0.04\ \text{eV} \lesssim \text{Mass}[\text{Heaviest } \nu_i] < (0.07 - 0.4)\ \text{eV}$$

$\sqrt{\Delta m^2_{\text{atm}}}$

Cosmology

The Upper Bound From Tritium

Cosmology is wonderful, but there are known loopholes in its argument concerning neutrino mass.

The absolute neutrino mass can in principle also be measured by the kinematics of β decay.

Tritium decay: $^3H \rightarrow {}^3He + e^- + \overline{\nu_i}$; $i = 1, 2,$ or 3

$$BR\left({}^3H \rightarrow {}^3He + e^- + \overline{\nu_i}\right) \propto \left|U_{ei}\right|^2$$

In $^3H \rightarrow {}^3He + e^- + \overline{\nu_i}$, the bigger m_i is, the smaller the maximum electron energy is.

There are 3 separate thresholds in the β energy spectrum.

The β energy spectrum is modified according to —

$$(E_0 - E)^2 \Theta[E_0 - E] \Rightarrow \sum_i |U_{ei}|^2 (E_0 - E)\sqrt{(E_0 - E)^2 - m_i^2}\,\Theta[(E_0 - m_i) - E]$$

Maximum β energy when
there is no neutrino mass

β energy

Present experimental energy resolution is insufficient to separate the thresholds.

Measurements of the spectrum bound the average neutrino mass —

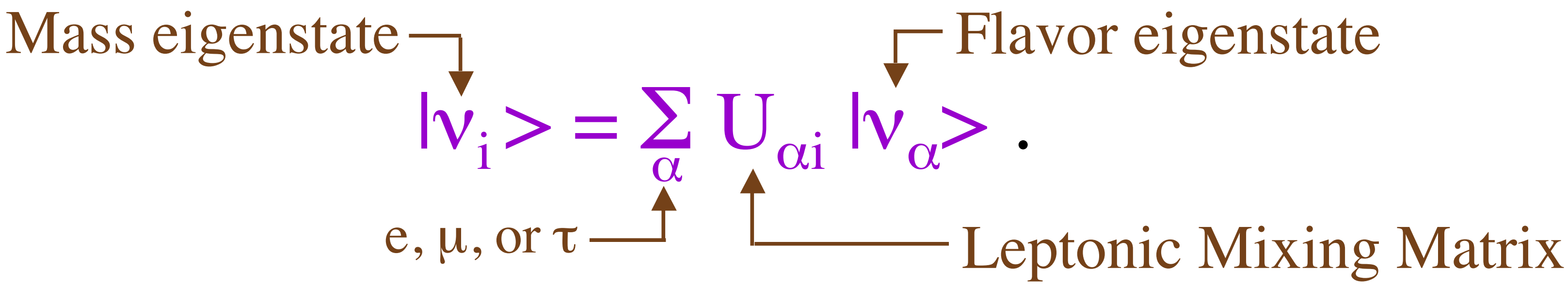
$$\langle m_\beta \rangle = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$$

$$\boxed{\text{Presently: } \langle m_\beta \rangle < 2 \text{ eV}}$$

Mainz &
Troitzk

Leptonic Mixing

This has the consequence that —



Mass eigenstate

Flavor eigenstate

$$|\nu_i> = \sum_\alpha U_{\alpha i}\, |\nu_\alpha> .$$

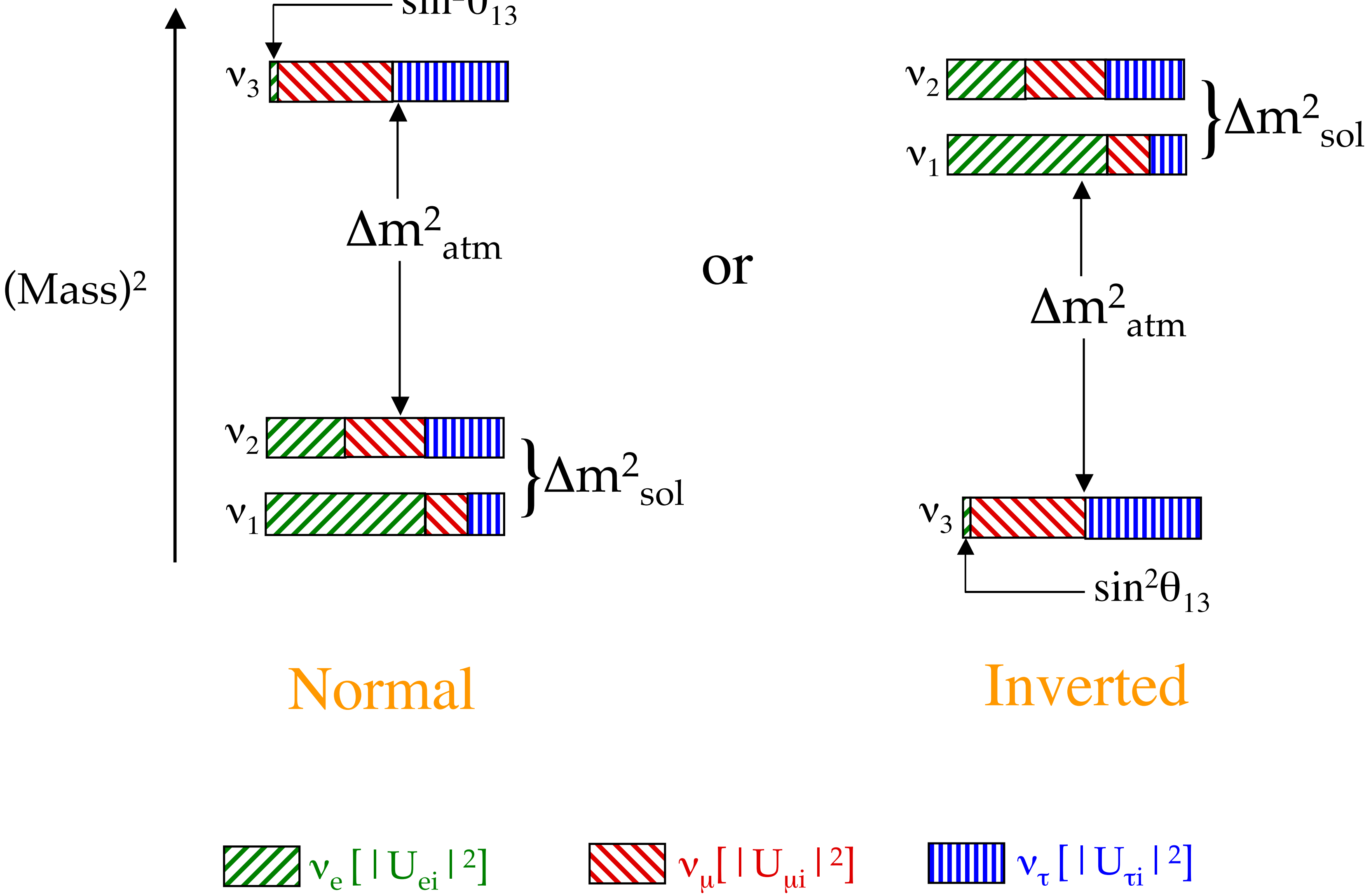
e, μ, or τ

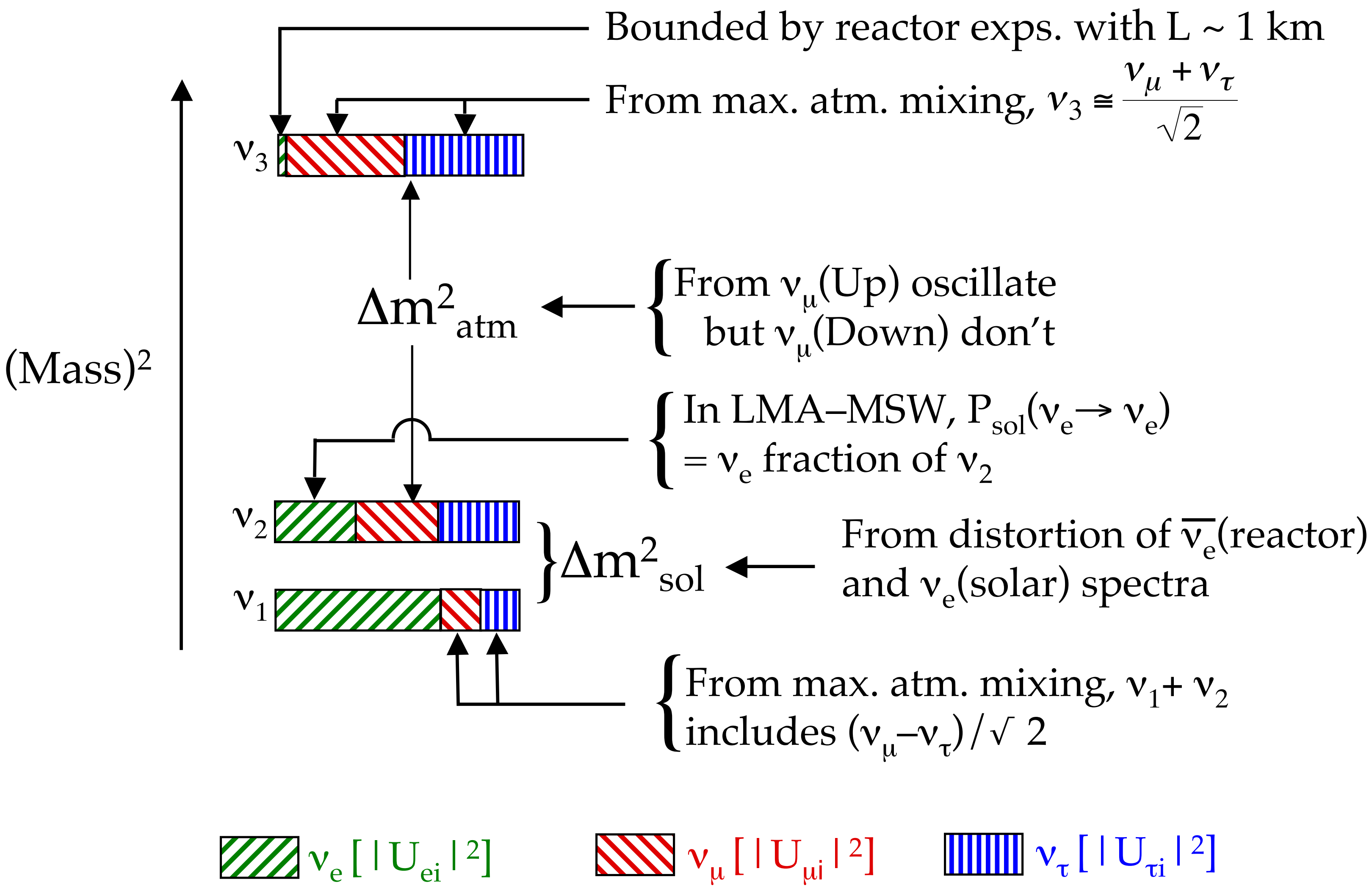
Leptonic Mixing Matrix

Flavor-α fraction of $\nu_i = |U_{\alpha i}|^2$.

When a ν_i interacts and produces a charged lepton, the probability that this charged lepton will be of flavor α is $|U_{\alpha i}|^2$.

The spectrum, showing its approximate flavor content, is





Bounded by reactor exps. with L ~ 1 km

From max. atm. mixing, $\nu_3 \cong \frac{\nu_\mu + \nu_\tau}{\sqrt{2}}$

From ν_μ(Up) oscillate but ν_μ(Down) don't

In LMA–MSW, $P_{sol}(\nu_e \rightarrow \nu_e)$ = ν_e fraction of ν_2

From distortion of $\overline{\nu_e}$(reactor) and ν_e(solar) spectra

From max. atm. mixing, $\nu_1 + \nu_2$ includes $(\nu_\mu - \nu_\tau)/\sqrt{2}$

▨ ν_e [$|U_{ei}|^2$] ▨ ν_μ [$|U_{\mu i}|^2$] ▥ ν_τ [$|U_{\tau i}|^2$]

The Mixing Matrix

$$U = \overset{\text{Atmospheric}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}} \times \overset{\text{Cross-Mixing}}{\begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix}} \times \overset{\text{Solar}}{\begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$\times \underset{\text{Majorana } \cancel{CP} \text{ phases}}{\begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$c_{ij} \equiv \cos\theta_{ij}$
$s_{ij} \equiv \sin\theta_{ij}$

Hints??

$\theta_{12} \approx \theta_{sol} \approx 34°$, $\theta_{23} \approx \theta_{atm} \approx 39\text{-}51°$, $\theta_{13} \lesssim 12°$

δ would lead to $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$. $\cancel{CP}$

But note the crucial role of $s_{13} \equiv \sin\theta_{13}$.

Recent Evidence For Non-Zero θ_{13}

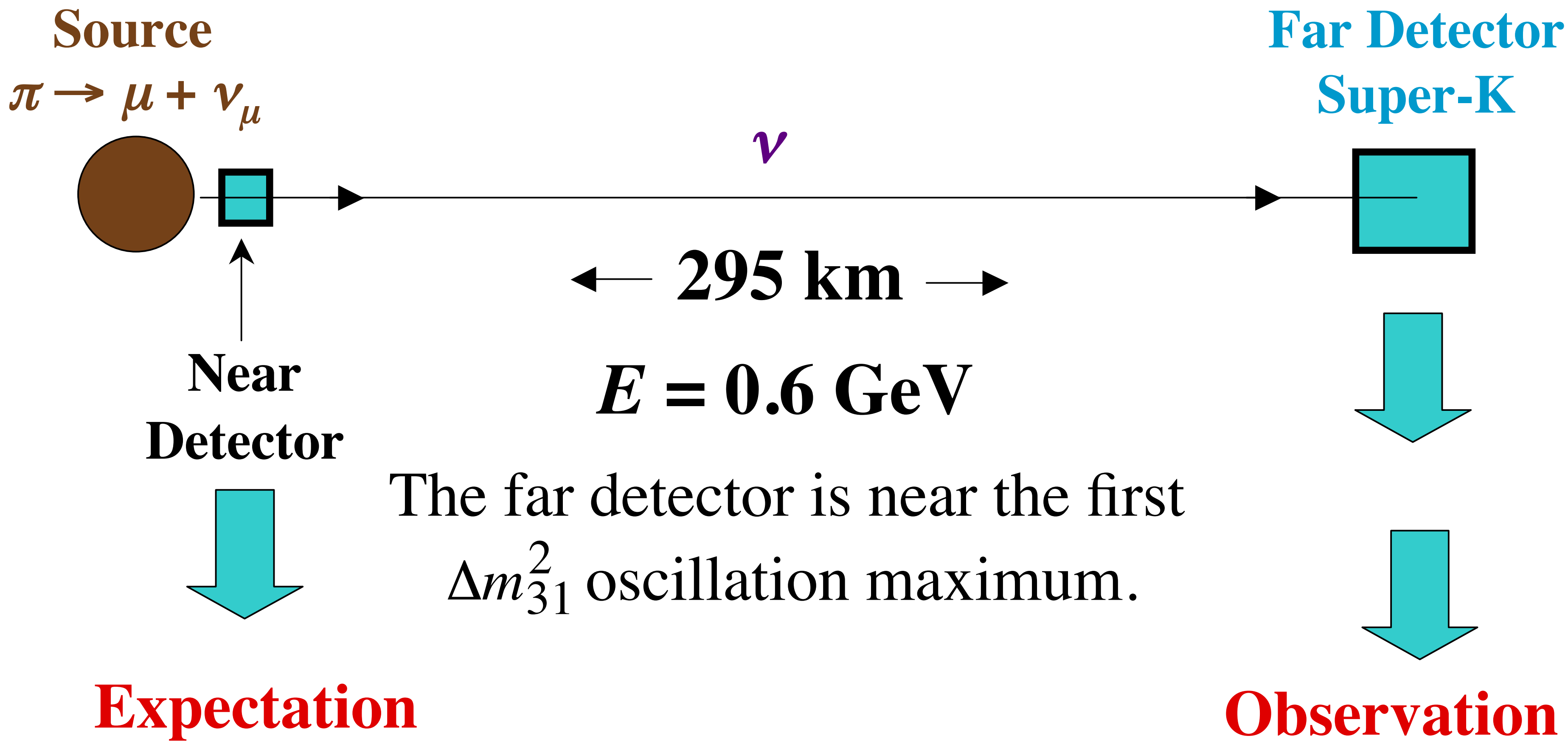
In an experiment where *L*/*E* is too small for the small splitting $\Delta m_{21}^2 \equiv m_2^2 - m_1^2$ to be seen,

$$P\left(\nu_\mu \to \nu_e\right) \cong 4\left|U_{\mu 3}U_{e3}\right|^2 \sin^2\left(\Delta m_{31}^2 \frac{L}{4E}\right)$$

$$= \boxed{\sin^2 2\theta_{13}} \sin^2 \theta_{23} \sin^2\left(\Delta m_{31}^2 \frac{L}{4E}\right)$$

T2K has looked for $\nu_\mu \to \nu_e$ in a long-baseline experiment:

The T2K experiment



Source

$\pi \rightarrow \mu + \nu_\mu$

Far Detector
Super-K

ν

← **295 km** →

Near Detector

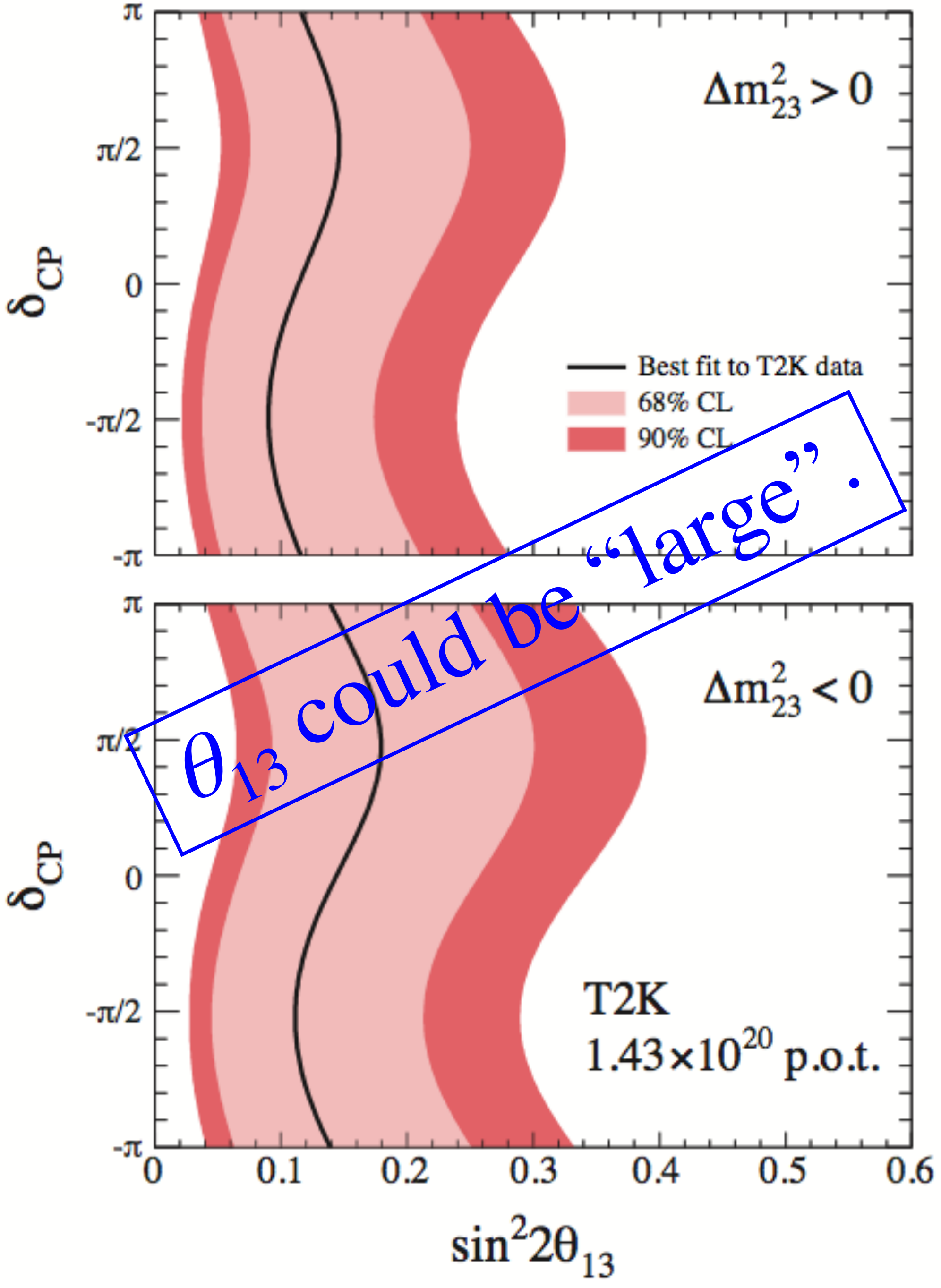
$E = 0.6$ **GeV**

The far detector is near the first Δm^2_{31} oscillation maximum.

Expectation

Observation

T2K sees 6 ν_e candidate events in the far detector, whereas 1.5 are expected if $\theta_{13} = 0$.



θ_{13} could be “large”.

These take the Δm^2_{21} contributions and matter effects into account.

MINOS, not designed to look for $\nu_\mu \rightarrow \nu_e$, sees 62 candidate events where 50 are expected if $\theta_{13} = 0$.

While not highly significant by itself, this result is consistent with that from T2K.

There Is Nothing Special About θ_{13}

All mixing angles must be nonzero for $\not{CP}$ in oscillation.

For example —

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) - P(\nu_\mu \rightarrow \nu_e) = 2\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin\delta$$
$$\times \sin\left(\Delta m^2{}_{31}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{32}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{21}\frac{L}{4E}\right)$$

In the factored form of U, one can put δ next to θ_{12} instead of θ_{13}.

The Majorana ~~CP~~ Phases

The phase α_i is associated with
neutrino mass eigenstate ν_i:

$$U_{\alpha i} = U^0_{\alpha i} \exp(i\alpha_i/2) \text{ for all flavors } \alpha.$$

$$\text{Amp}(\nu_\alpha \rightarrow \nu_\beta) = \sum_i U_{\alpha i}^* \exp(-\, i m_i^2 L/2E)\, U_{\beta i}$$

is insensitive to the Majorana phases α_i.

Only the phase δ can cause CP violation in
neutrino oscillation.