

Renormalization group methods in nuclear few- and many-body problems

Lecture 1

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2011 National Nuclear Physics Summer School
University of North Carolina at Chapel Hill

Useful readings for these lectures

“From low-momentum interactions to nuclear structure”

S. K. Bogner, R. J. Furnstahl and A. Schwenk

Prog. Part. Nucl. Phys. **65**, 94 (2010) [arXiv:0912.3688 [nucl-th]] [SPIRES entry](#)

“How to renormalize the Schrodinger equation”

G. P. Lepage

arXiv:nucl-th/9706029 [SPIRES entry](#)

Lectures given at 8th Jorge Andre Swieca Summer School on Nuclear Physics, Sao Paulo, Brazil, 26 Jan - 7 Feb 1997

“Modern Theory of Nuclear Forces”

E. Epelbaum, H. W. Hammer and U. G. Meissner

Rev. Mod. Phys. **81**, 1773 (2009) [arXiv:0811.1338 [nucl-th]] [SPIRES entry](#)

“Toward ab initio density functional theory for nuclei”

J. E. Drut, R. J. Furnstahl and L. Platter

Prog. Part. Nucl. Phys. **64**, 120 (2010) [arXiv:0906.1463 [nucl-th]] [SPIRES entry](#)

Lecture 1 outline

Objective: Give an overview of how renormalization group methods can be used to simplify microscopic few- and many-body calculations in low energy nuclear structure and reactions.

Technical details and selected results for nuclei and nuclear matter will be revisited in lectures 2 and 3.

- 1) Overview
- 2) Nuclear interactions
- 3) Motivation for RG in nuclear physics
- 4) Simplifications at low resolution
- 5) Take-away points and preview of lectures 2,3



Questions that drive low-energy nuclear physics

- How do protons and neutrons make stable nuclei and rare isotopes? Where are the limits?
- What are the heaviest nuclei that can exist?
- What is the equation of state of nucleonic matter?
- What is the origin of simple patterns in complex nuclei?
- How do we describe fission, fusion, reactions, ... ?
- How did the elements from iron to uranium originate?
- How do stars explode?
- What is the nature of neutron star matter?
- How can our knowledge of nuclei and our ability to produce them benefit humankind? Life Sciences, Material Sciences, Nuclear Energy, Security

Physics of
Nuclei

Nuclear
Astrophysics

Applications
of Nuclei

Frontiers in low E nuclear theory

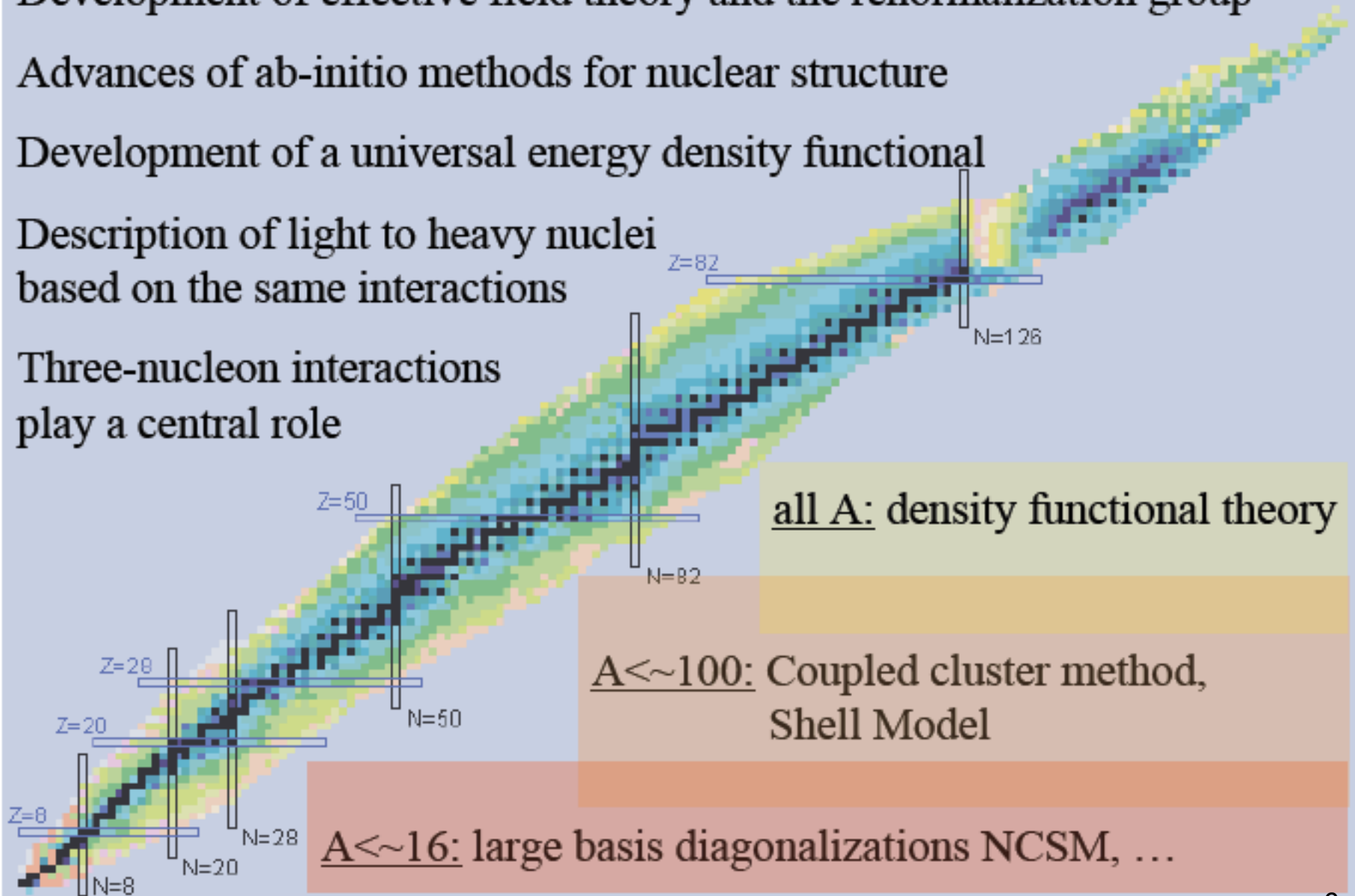
Development of effective field theory and the renormalization group

Advances of ab-initio methods for nuclear structure

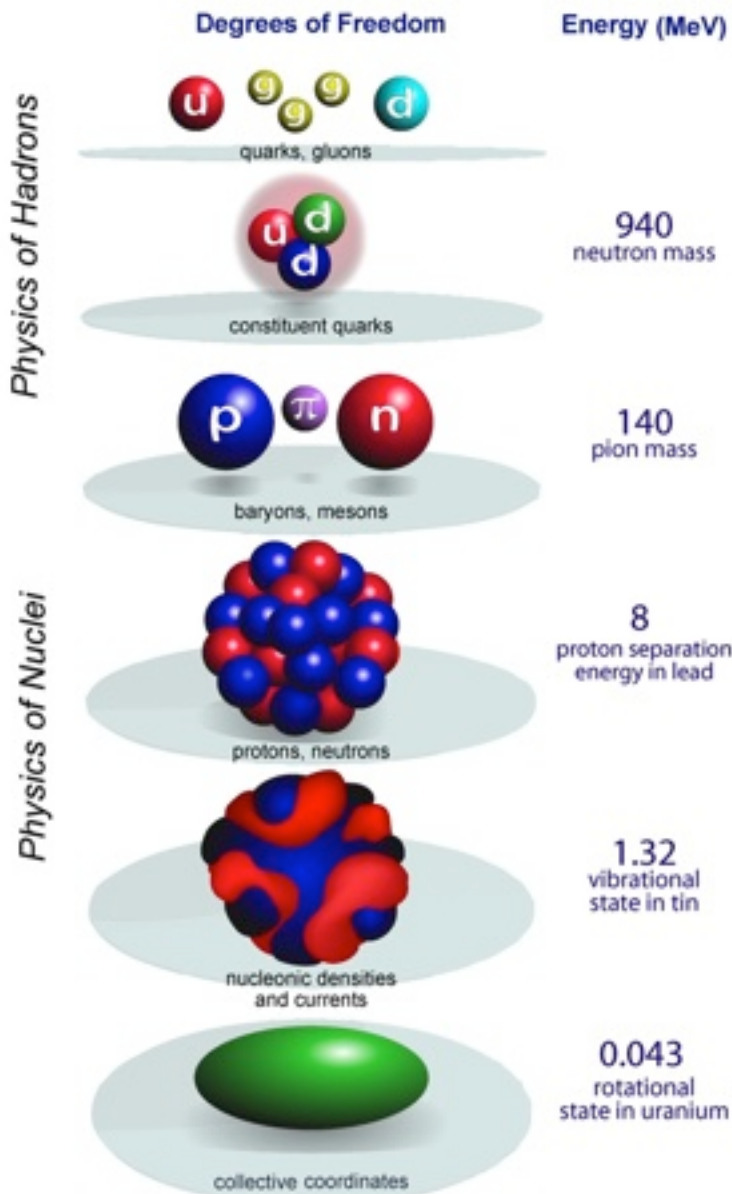
Development of a universal energy density functional

Description of light to heavy nuclei based on the same interactions

Three-nucleon interactions play a central role



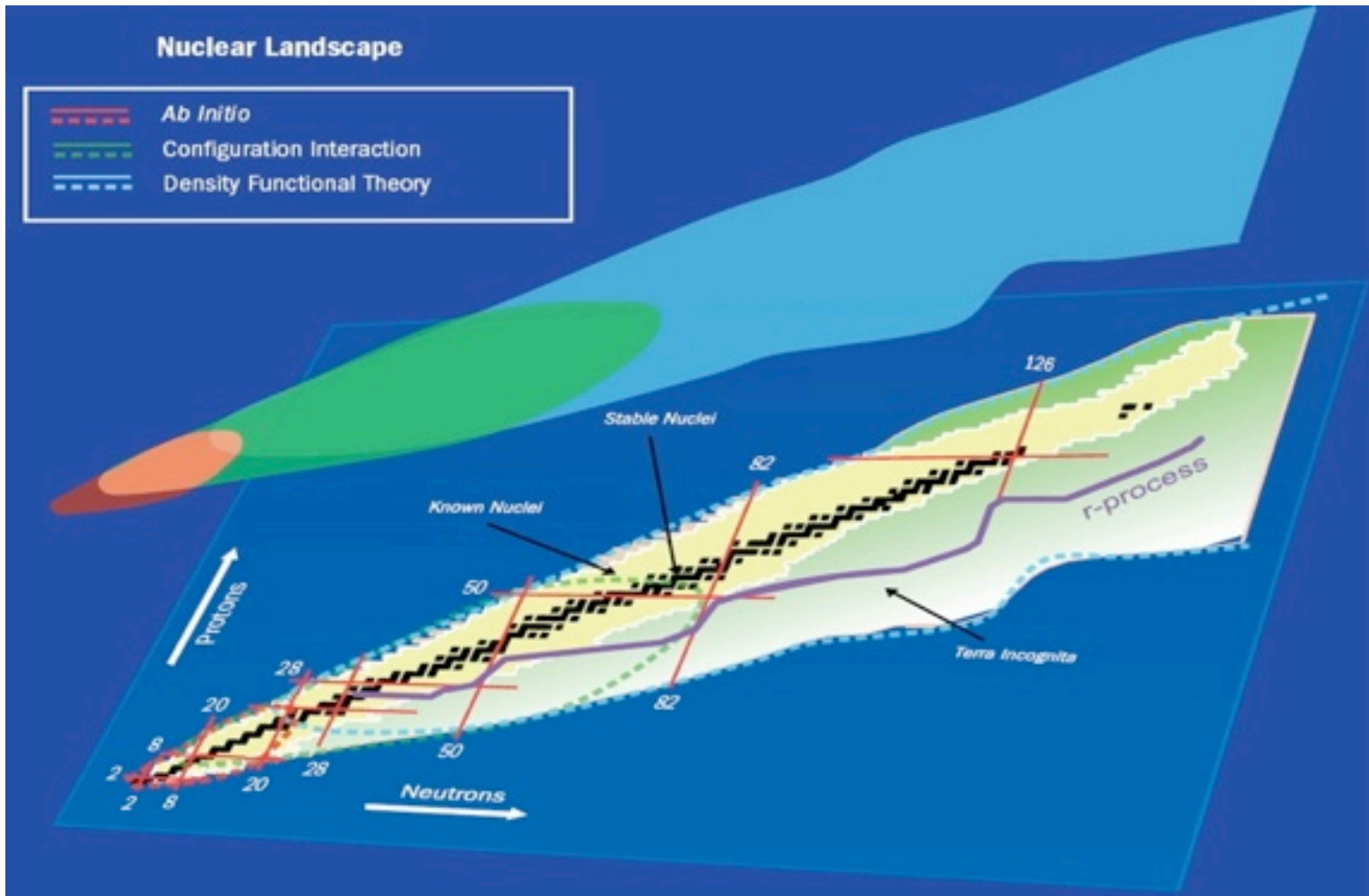
What are the relevant degrees of freedom?



RG/EFT methods tailor-made to develop systematic effective theories that focus on a limited range of scales/DOF at a time.

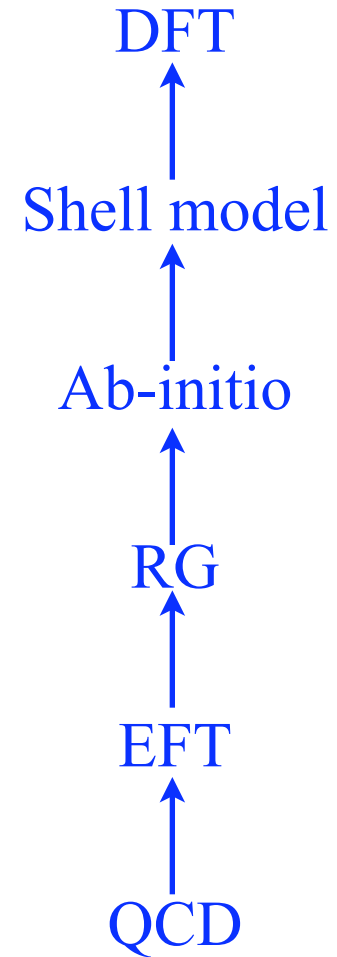
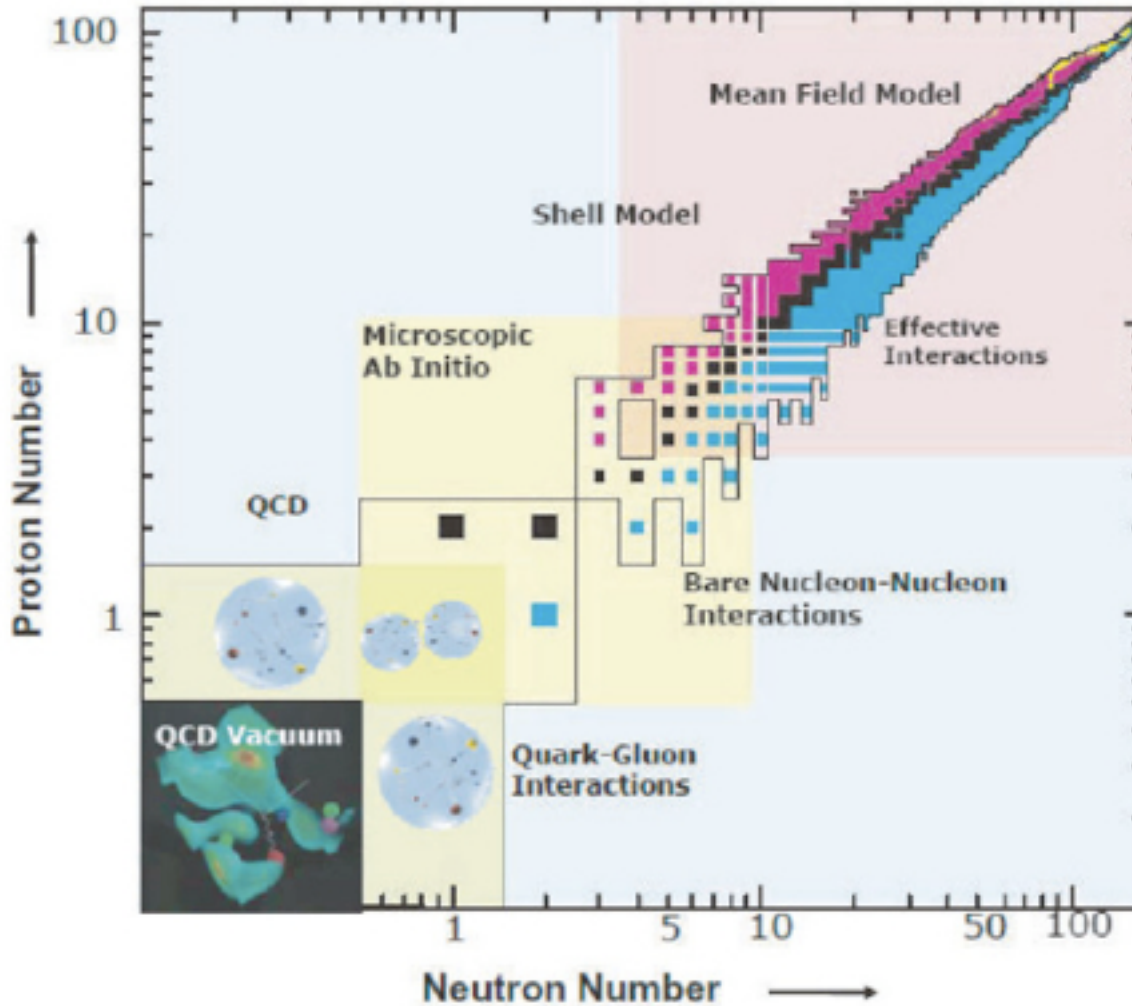
Nucleonic matter
(our domain in
nuclear structure)

The nuclear landscape



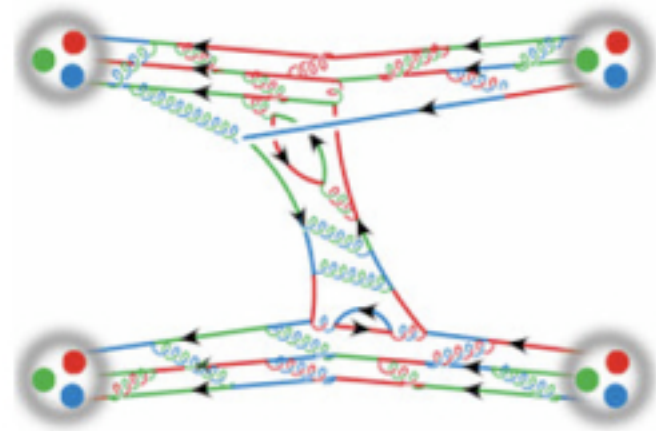
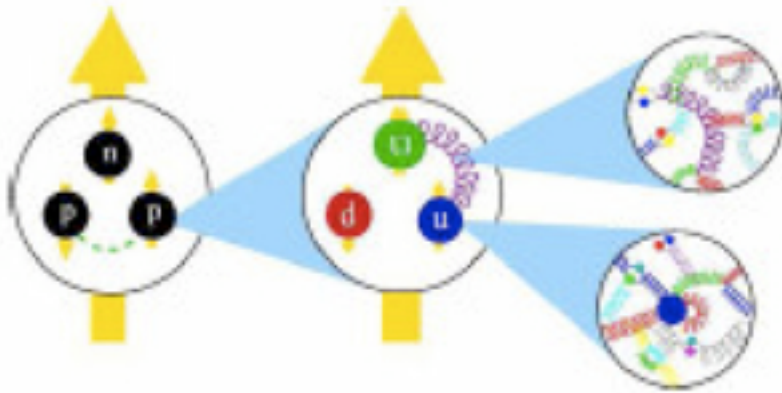
- no 1-size fits all method
- Density functional theory covers the most ground, but is the most phenomenological.

Ultimate goal: Bottom-up approach to nuclear structure



Nuclear Interactions

Choosing the right DOF: The effective NN interaction

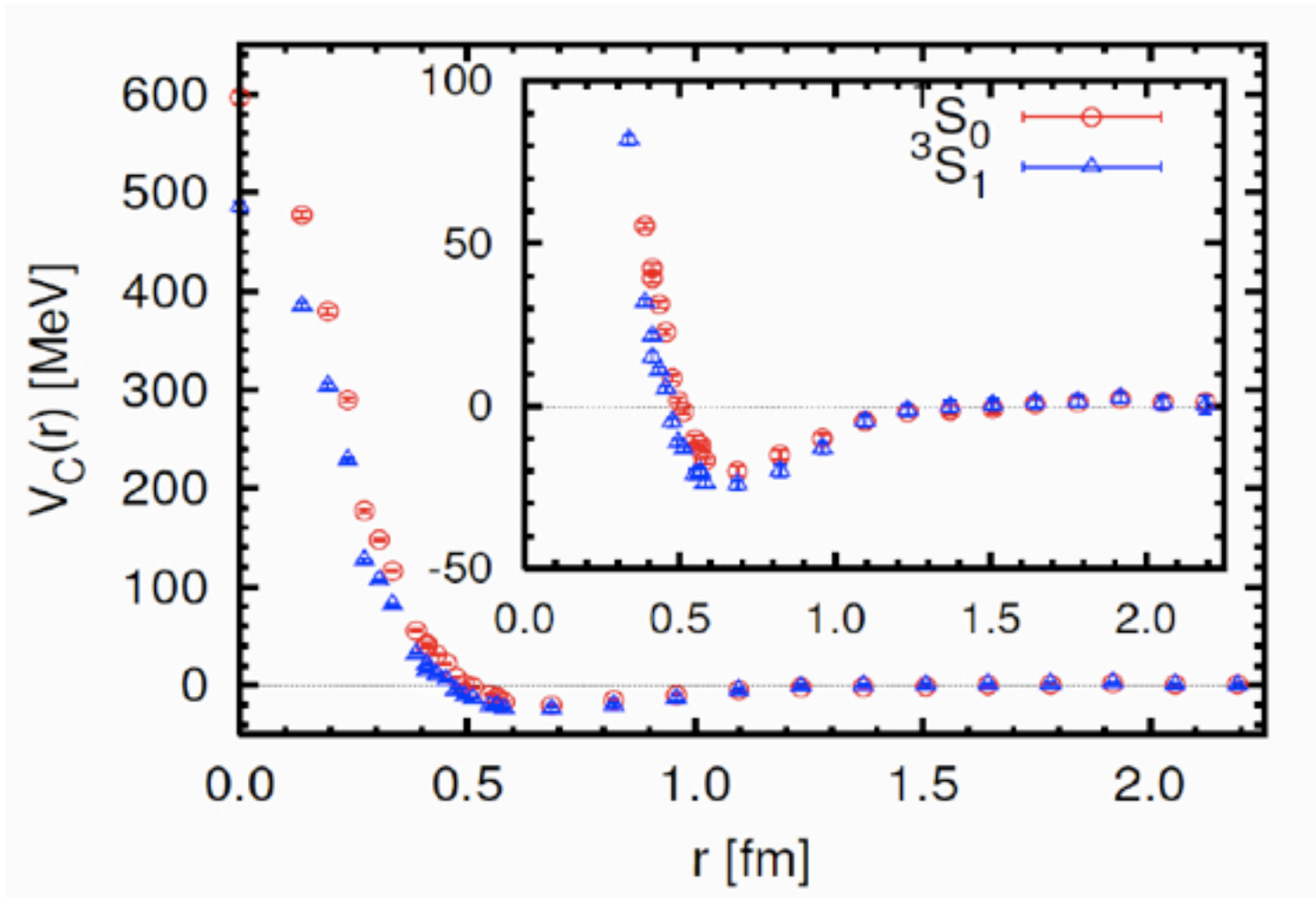


How to get it?

- Ideally, from lattice QCD
- effective field theory + phase shifts

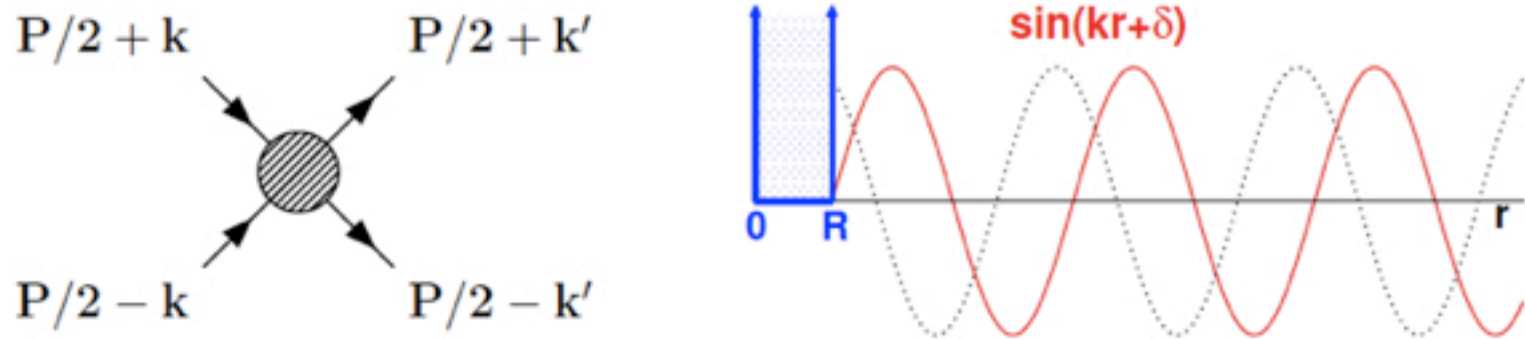
(or phenomenological
meson-exchange models)

NN central potential $V_C(r)$ for $m_\pi = 530$ MeV from lattice QCD



- Ishii, Aoki & T.Hatsuda., Phys. Rev. Lett. 99, 022001 (2007).
- Nemura, Ishii, Aoki & T.Hatsuda, arXiv:0710.3622 [hep-lat]
- Aoki, Ishii & T.Hatsuda, arXiv:0805.2462 [hep-ph]

NN Scattering review



- Relative motion with total $P = 0$: $\psi(r) \xrightarrow{r \rightarrow \infty} e^{i\mathbf{k} \cdot \mathbf{r}} + f(k, \theta) \frac{e^{ikr}}{r}$
 where $k^2 = k'^2 = ME_k$ and $\cos \theta = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}}'$
- Differential cross section is $d\sigma/d\Omega = |f(k, \theta)|^2$
- Central $V \implies$ partial waves:

$$f(k, \theta) = \sum_l (2l + 1) f_l(k) P_l(\cos \theta)$$

$$\text{where } f_l(k) = \frac{e^{i\delta_l(k)} \sin \delta_l(k)}{k} = \frac{1}{k \cot \delta_l(k) - ik}$$

and the S -wave phase shift is defined by

$$u_0(r) \xrightarrow{r \rightarrow \infty} \sin[kr + \delta_0(k)] \implies \delta_0(k) = -kR \text{ for hard sphere}$$

Low energy limit: Effective range expansion

- As first shown by Schwinger, $k^{l+1} \cot \delta_l(k)$ has a power series expansion. For $l = 0$:

$$k \cot \delta_0(k) = -\frac{1}{a_0} + \frac{1}{2}r_0 k^2 - Pr_0^3 k^4 + \dots$$

defines the *scattering length* a_0 and the *effective range* r_0

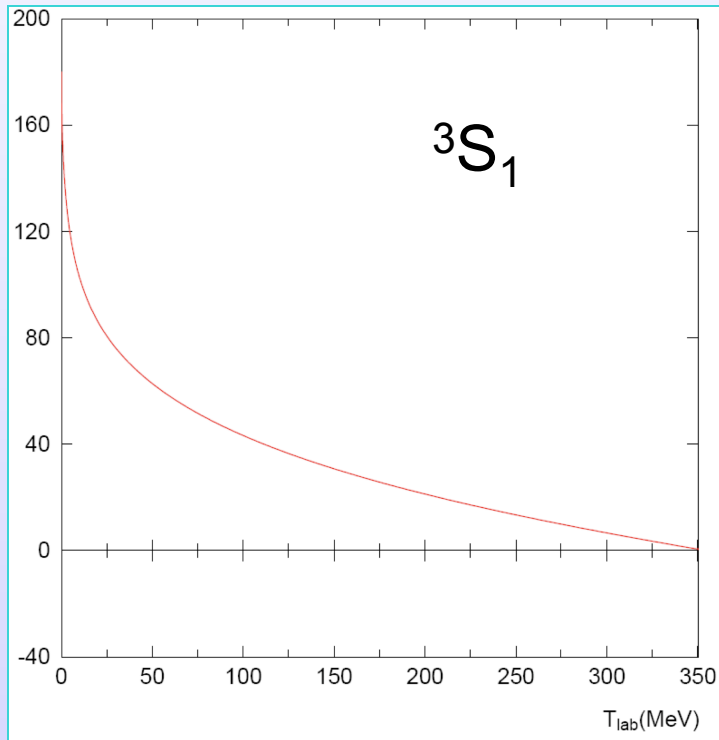
- While $r_0 \sim R$, the range of the potential, a_0 can be anything
 - if $a_0 \sim R$, it is called “natural”
 - $|a_0| \gg R$ (unnatural) is particularly interesting \implies cold atoms
- The effective range expansion for hard sphere scattering is:

$$k \cot(-kR) = -\frac{1}{R} + \frac{1}{3}Rk^2 + \dots \implies a_0 = R \quad r_0 = 2R/3$$

so the low-energy effective theory is natural

Nuclear s-wave phase shifts

<http://nn-online.org/>

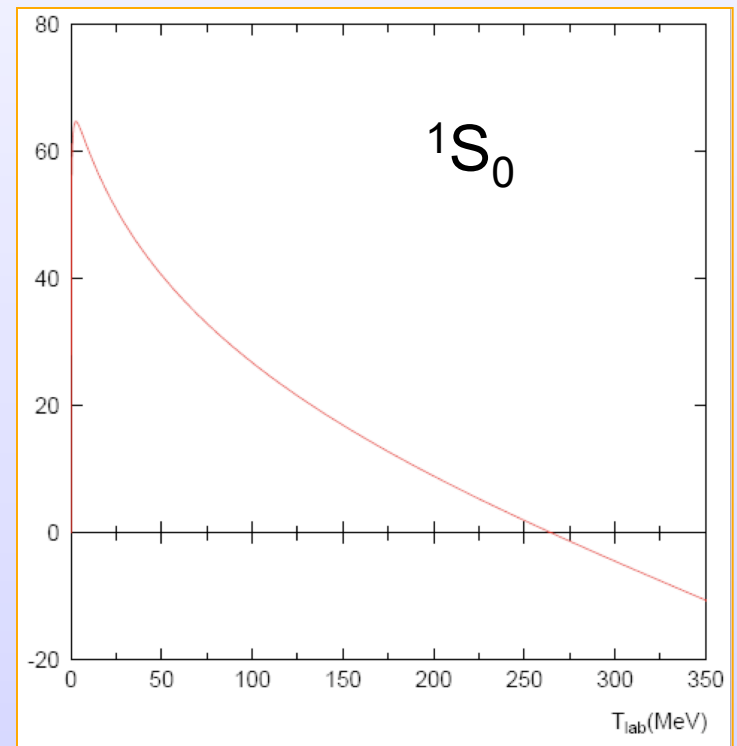


Deuteron is a very weakly bound system!

System has one bound state.

Steep decrease from 180 degrees due to large scattering length $a = 5.5$ fm.

Acts repulsive due to large (positive) scattering length.

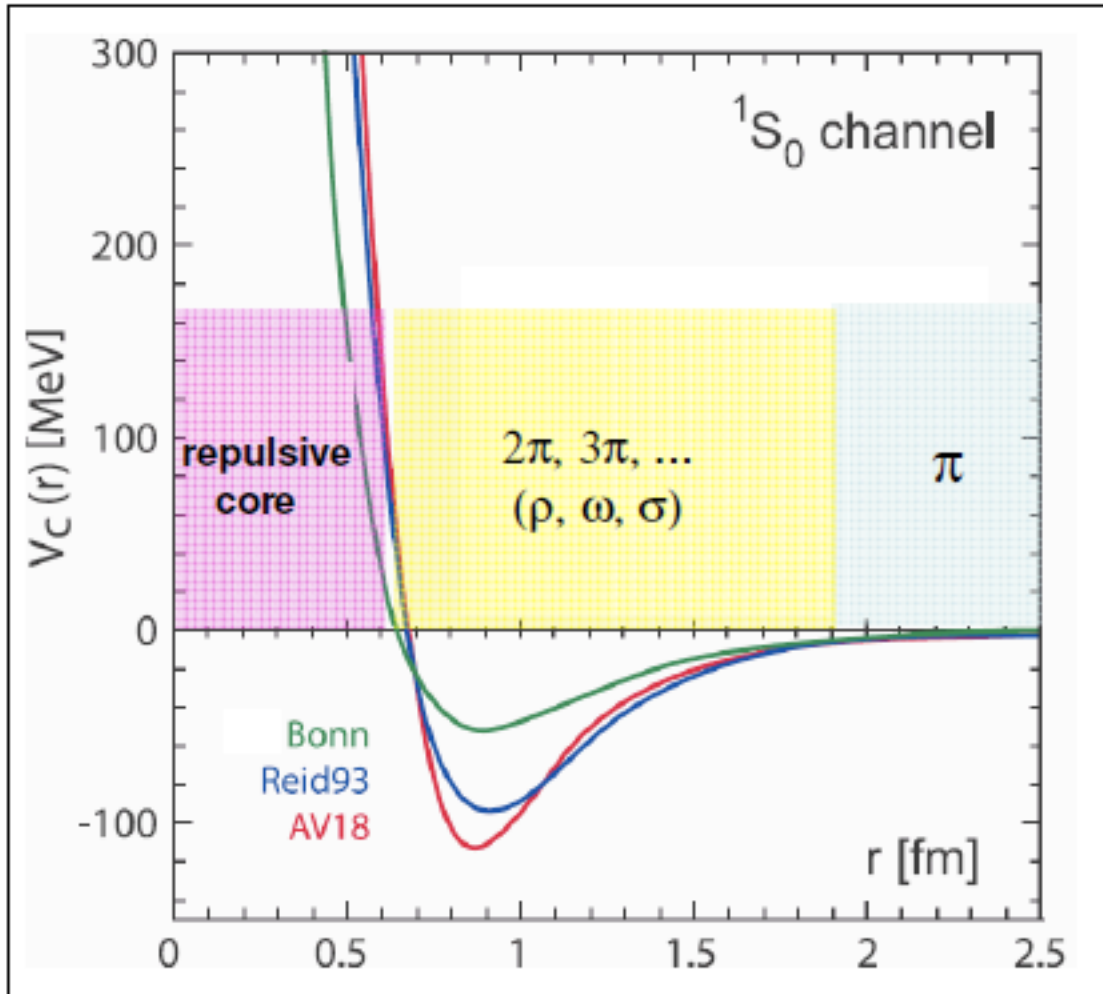


System (barely) fails to exhibit bound state.

Steep rise at 0 due to large scattering length $a = -18$ fm.

Monotonous decrease due to “hard core”.

Phenomenological NN Models

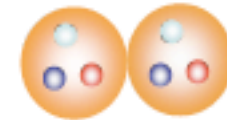


From T. Hatsuda (Oslo 2008)

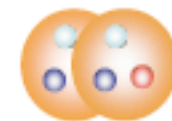
One-pion exchange
by Yukawa (1935)



Multi-pions
by Taketani (1951)



Repulsive core
by Jastrow (1951)



short- and mid-range tuned (~ 20 parameters) to phase shifts and deuteron pole

Phenomenological NN Potential Models

(CD-Bonn, Argonne v18, Reid93, Nijmegen I and II,...)

- all share one pion exchange (OPE) at long distances
- model-dependent mid-range attraction and short-distance repulsion
- fit ~ 6000 NN data with $\chi^2/\text{dof} \sim 1$ ☺
- many ab-initio successes in light nuclei ☺

but

- difficult to estimate theoretical errors and range of applicability ☹
- no obvious connection to QCD ☹
- not obvious how to define fully *consistent* 3NF's and operators (e.g., meson-exchange currents) ☹
- hard to work with in most many-body methods ☹

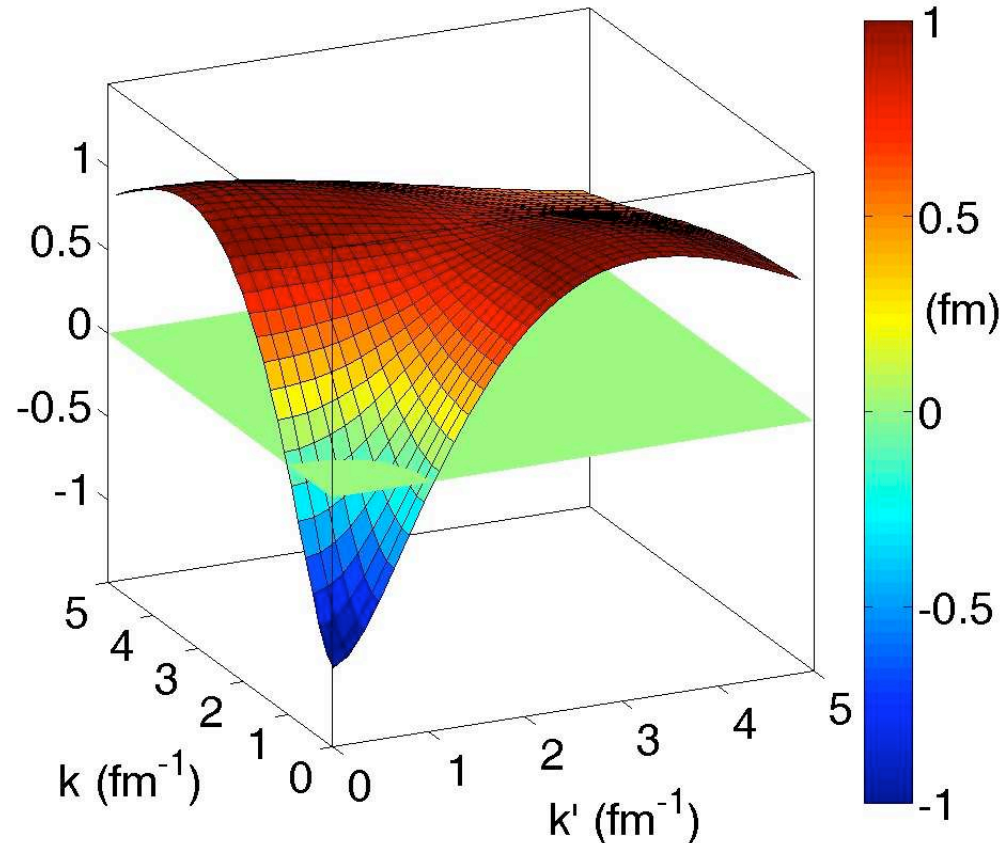
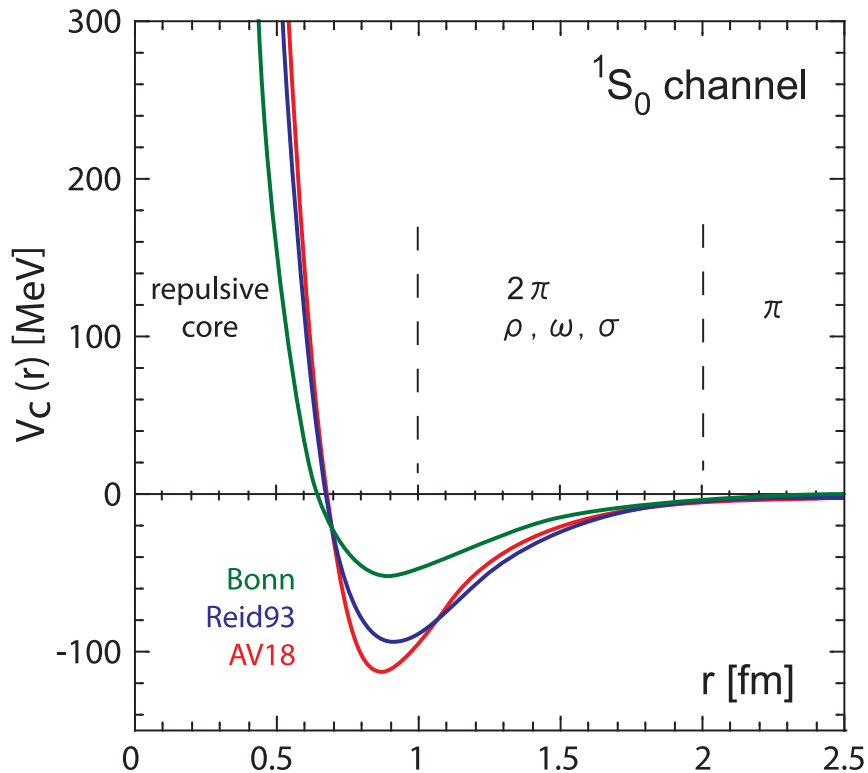
chiral EFT (lecture 2) addresses these shortcomings

Renormalization Group Methods

“The method in its most general form can I think be understood as a way to arrange in various theories that the degrees of freedom that you’re talking about are the relevant ones for the problem at hand.”

-S. Weinberg

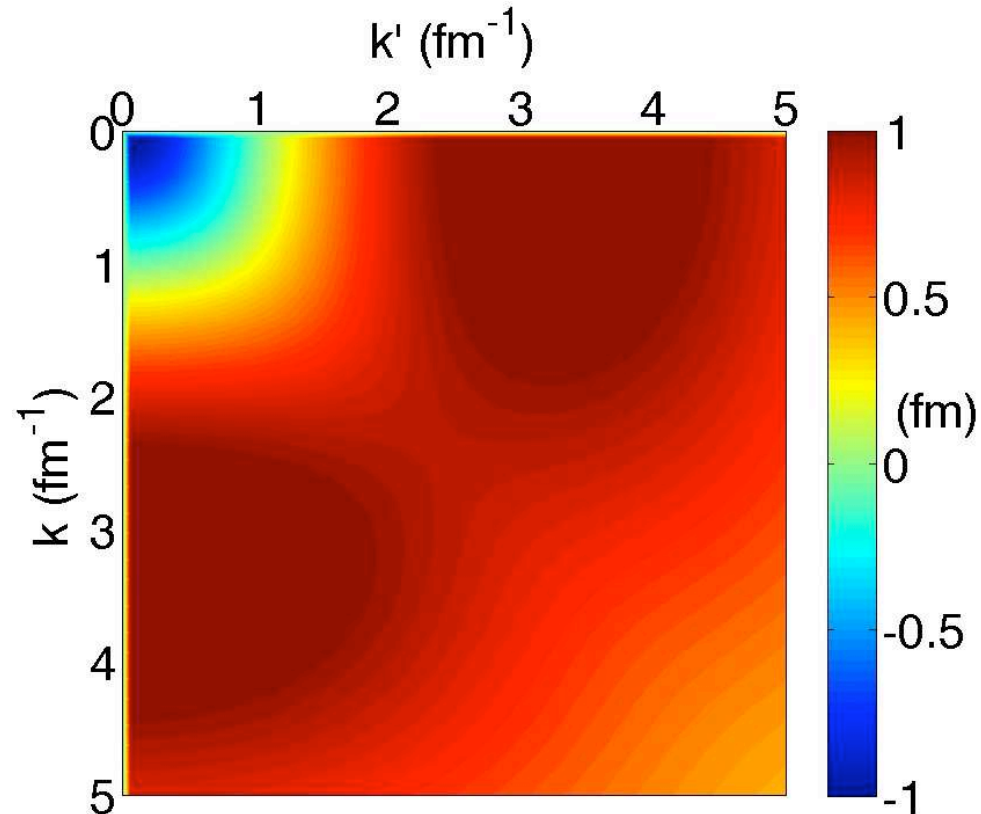
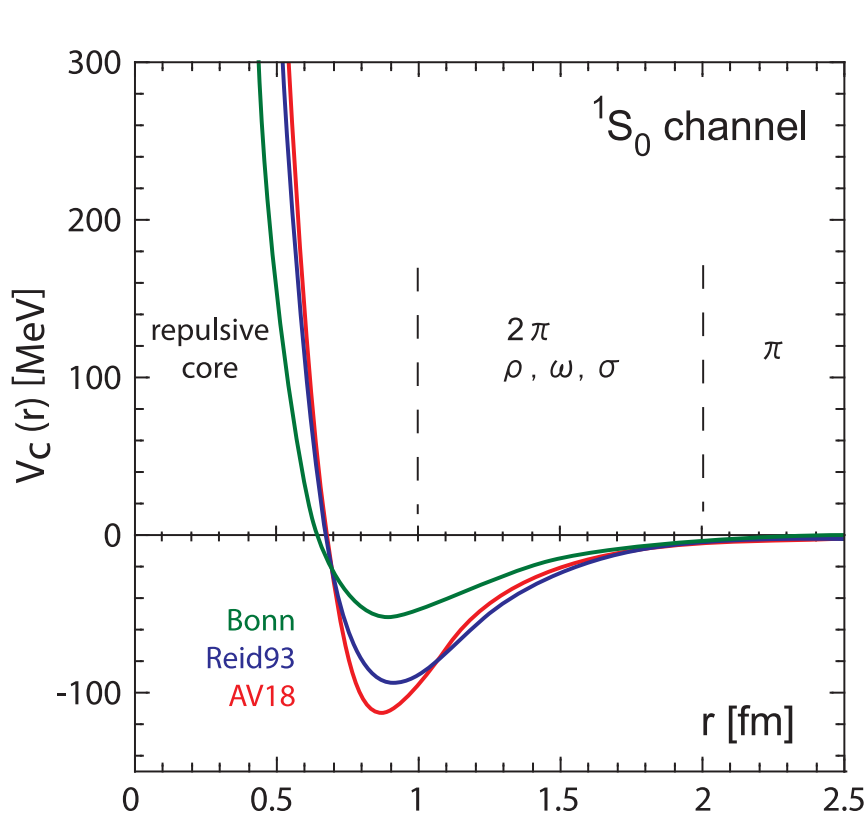
Why is “textbook” nuclear physics is so hard?



Repulsive core & strong tensor force \Rightarrow low and high k modes strongly coupled by the interaction (reminder: typical $k \sim 1 \text{ fm}^{-1}$ in nuclei)

$$V_{l=0}(k, k') = \int d^3r j_0(kr) V(r) j_0(k'r)$$

Why is “textbook” nuclear physics is so hard?

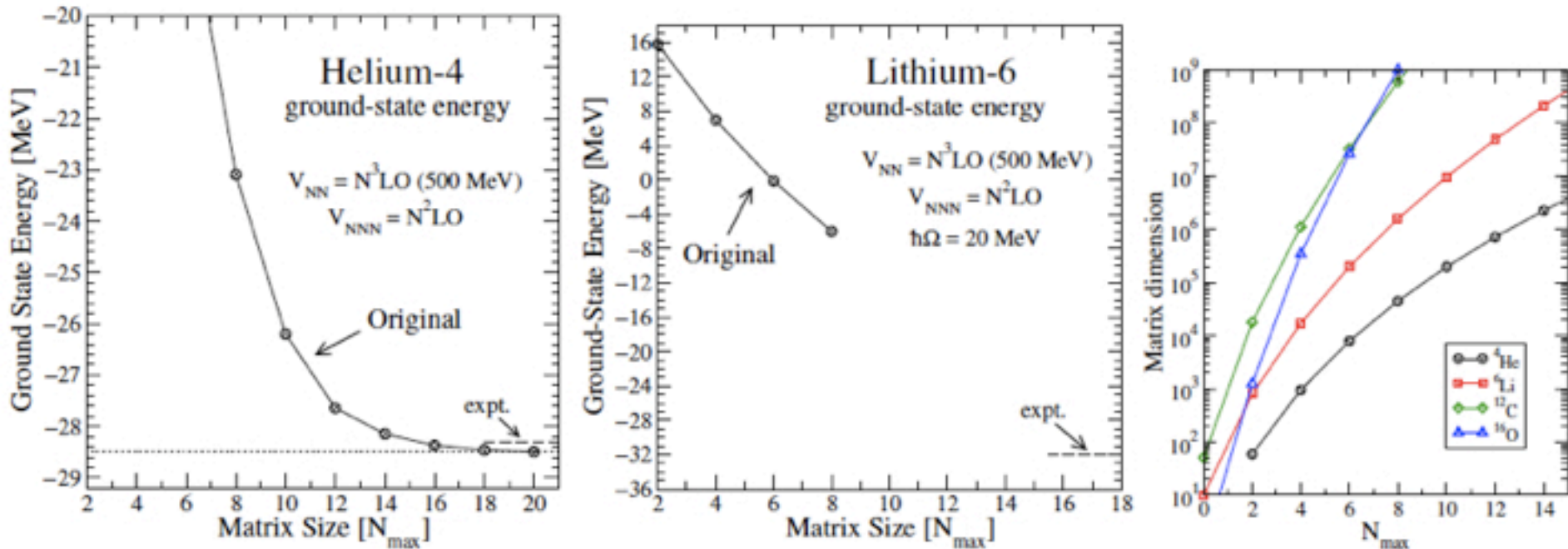


Repulsive core & strong tensor force \Rightarrow low and high k modes strongly coupled by the interaction (reminder: typical $k \sim 1 \text{ fm}^{-1}$ in nuclei)

Complications: strong correlations, non-perturbative, poorly convergent basis expansions, ...

Many short wavelengths => Large matrices to diagonalize

- Harmonic oscillator basis with N_{\max} shells for excitations
- Graphs show convergence for *soft* chiral EFT potential (although not at optimal $\hbar\Omega$ for ${}^6\text{Li}$)



- Factorial growth of basis with $A \implies$ limits calculations
- Too much resolution from potential \implies mismatch of scales

Why large Λ 's are painful

Suppose we want to compute the ground state E of a nucleus with mass number A by brute force diagonalization. Assume the interaction has a cutoff Λ .

Exercise:

Estimate how the size of the s.p. basis scales with Λ . Given this, estimate the size of the Hamiltonian matrix for ^{16}O .

Why large Λ 's are painful

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Exercise:

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Hints: 1) The basis must be sufficiently extended in **space** to capture the size of the nucleus ($R \sim 1.2A^{1/3}$ fm).

2) The basis must be sufficiently extended in **momentum** to capture the size of the cutoff Λ in the Hamiltonian.

3) Use a phase space argument to get # of sp states

Why large Λ 's are painful

Suppose we want to compute the ground state E of a nucleus with mass number A by brute force diagonalization. Assume the interaction has a cutoff Λ .

Exercise:

Estimate how the size of the s.p. basis scales with Λ . Given this, estimate the size of the Hamiltonian matrix for ^{16}O .

Answer: # of s.p. states $D \sim \Lambda^3 A$

$$\begin{aligned}\text{Dim}(H) &= \# \text{ of } A\text{-body Slater determinants} \\ &= D! / (D-A)! / A!\end{aligned}$$

e.g., for $\Lambda = 4.0 \text{ fm}^{-1}$ $\text{Dim}(H) \sim 10^{14}$

Why large Λ 's are painful

Suppose we want to compute the ground state E of a nucleus with mass number A by brute force diagonalization. Assume the interaction has a cutoff Λ .

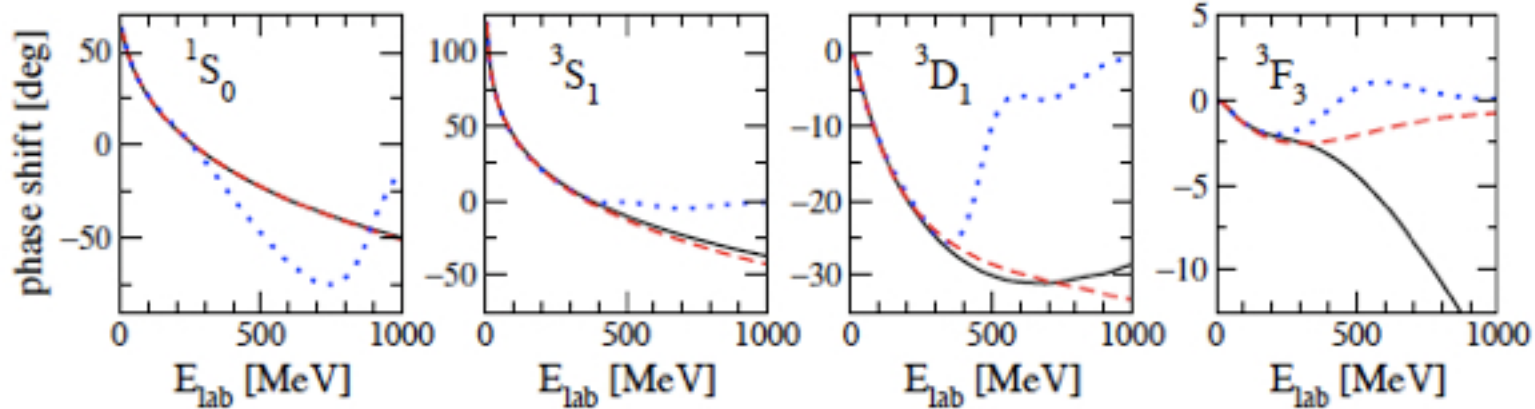
Moral:

Easiest way to extend the reach of ab-initio to heavier nuclei is to use lower resolutions (Λ)

“physical” scales k_F and $m_\pi \sim 1 \text{ fm}^{-1} \dots$

Arguments for using “low-resolution” interactions

SKB et al., arXiv:0912.3688

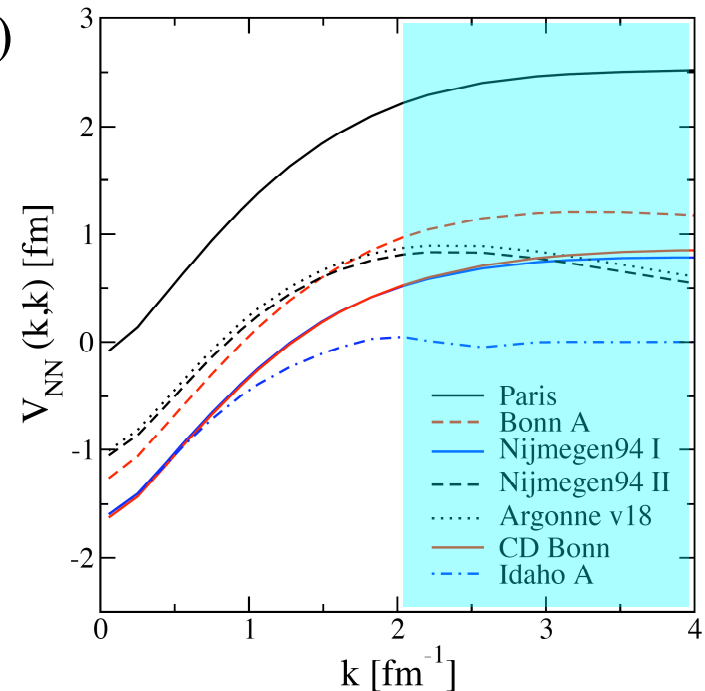


NN models share same long-distance physics (V_π)
 Phase shifts to $E_{\text{lab}} \sim 350$ MeV ($k_{\text{rel}} \sim 2.1$ fm $^{-1}$);
 beyond this, totally model-dependent

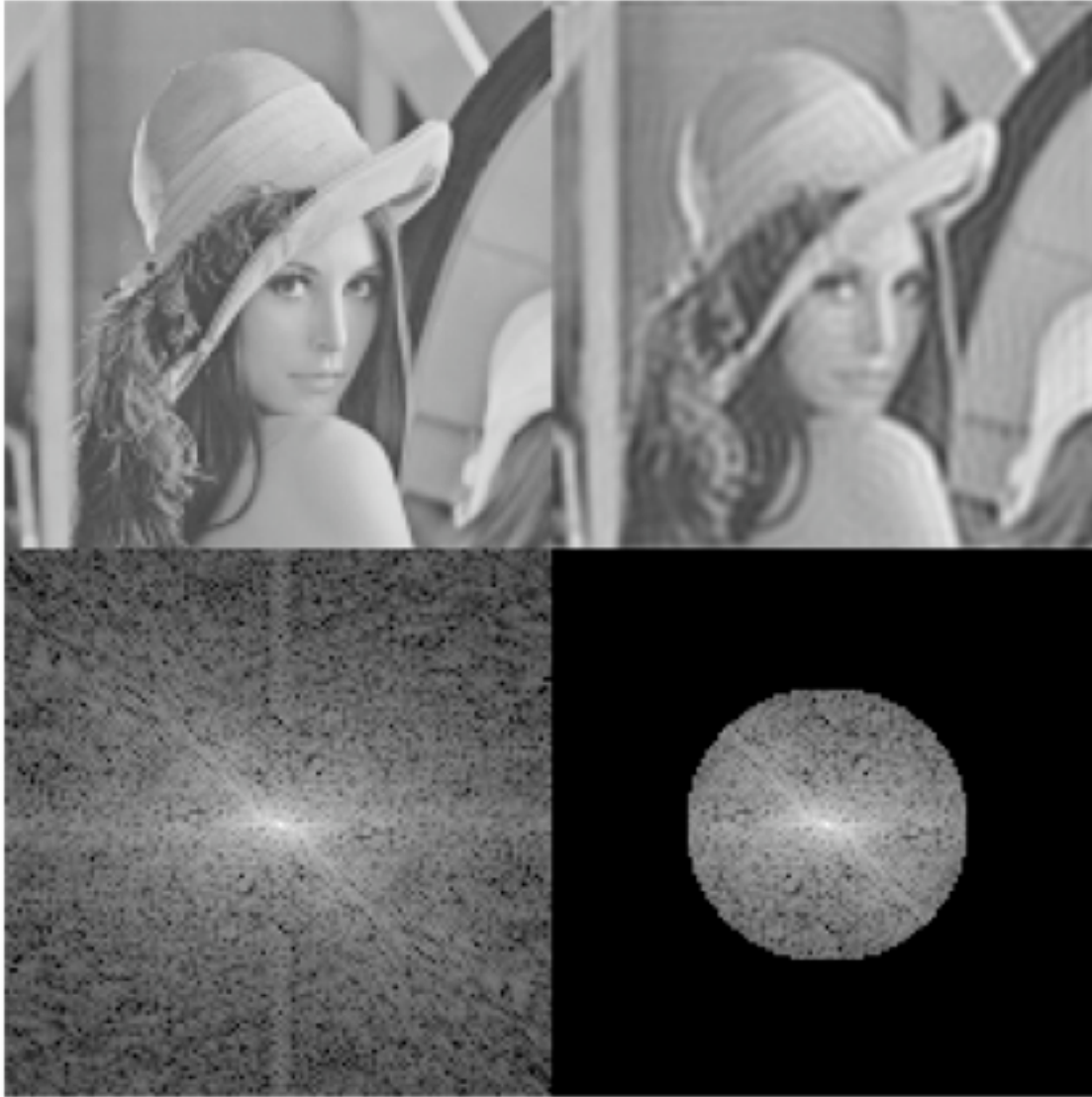
Most $H(\Lambda)$ have $\Lambda \gg \Lambda_{\text{data}} \sim 2.1$ fm $^{-1}$

$k_F \sim 1.35$ fm $^{-1}$, $m_\pi \sim 0.7$ fm $^{-1}$

Why work so hard to treat high k modes that are unconstrained by NN data?



Low-pass filter on fourier transform of a 2d-image

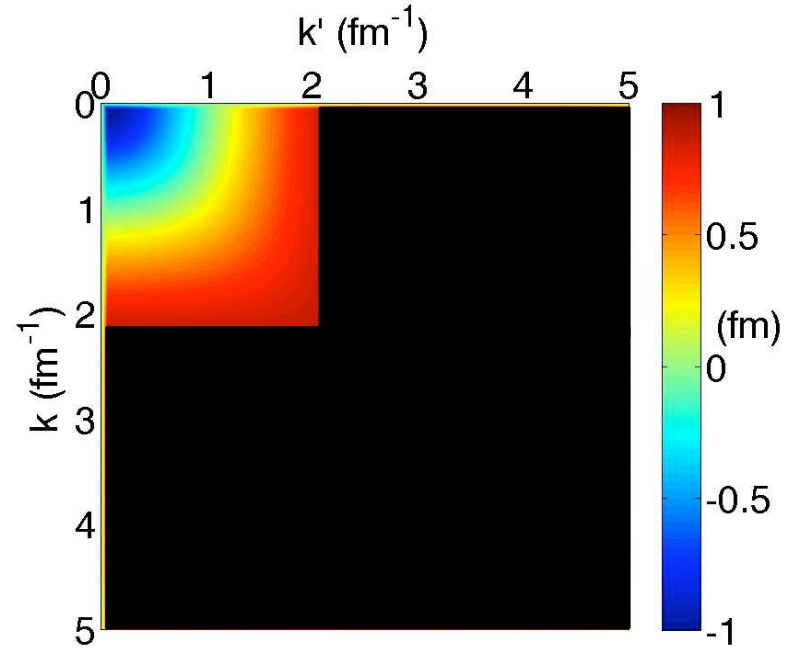
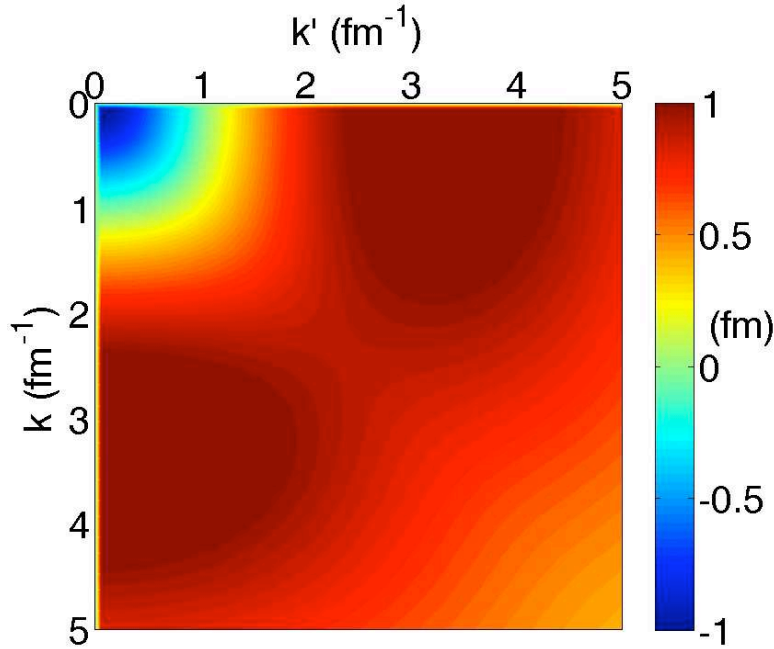


Much less information
needed

BUT

Long-wavelength info
preserved

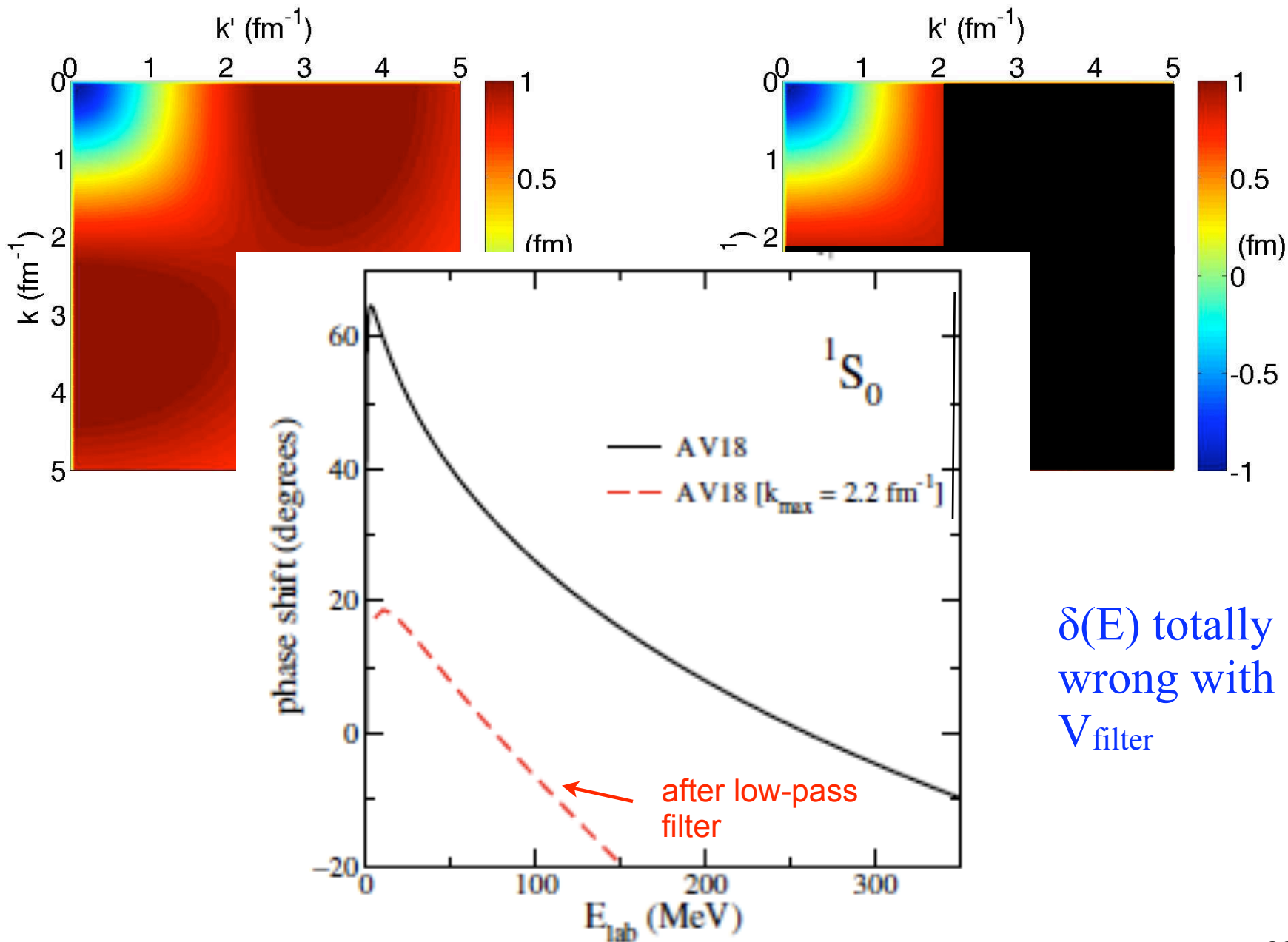
Try a naive “low-pass” filter on V :



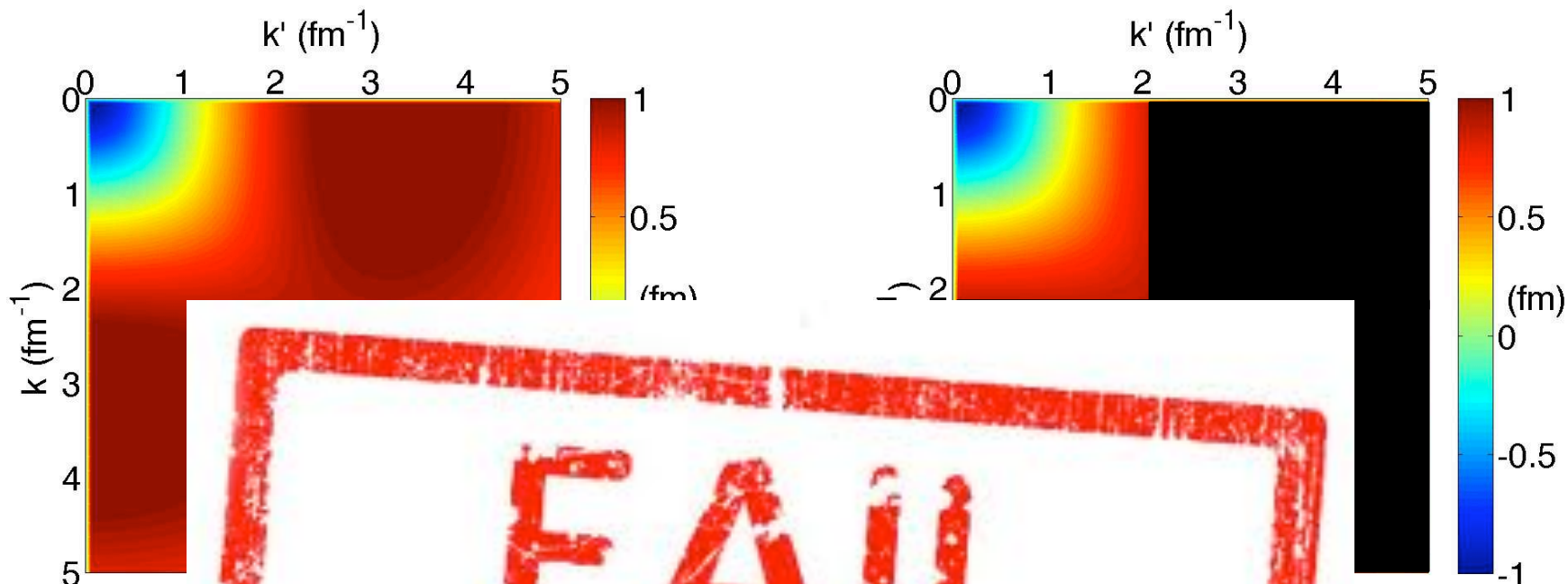
$$V_{filter}(k', k) \equiv 0 \quad k, k' > 2.2 \text{ fm}^{-1}$$

Now calculate low E observables (e.g., NN scattering) and see what happens...

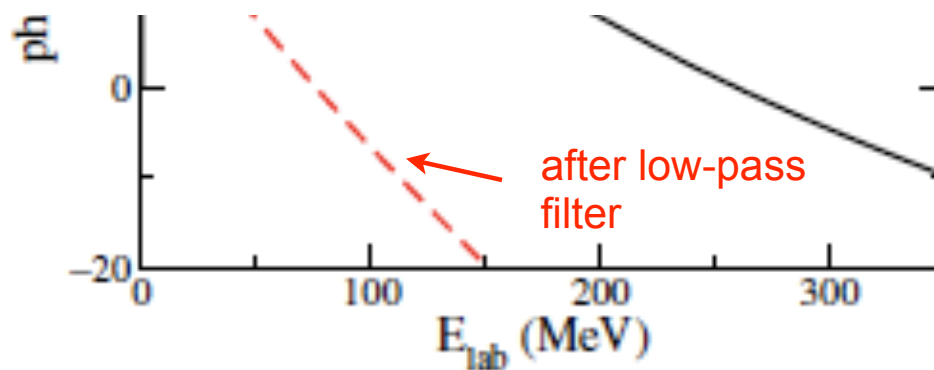
Try a naive “low-pass” filter on V :



Try a naive “low-pass” filter on V :



3) totally wrong with V filter



Why did the low-pass filter fail?

Low and high k are coupled by quantum fluctuations (virtual states)

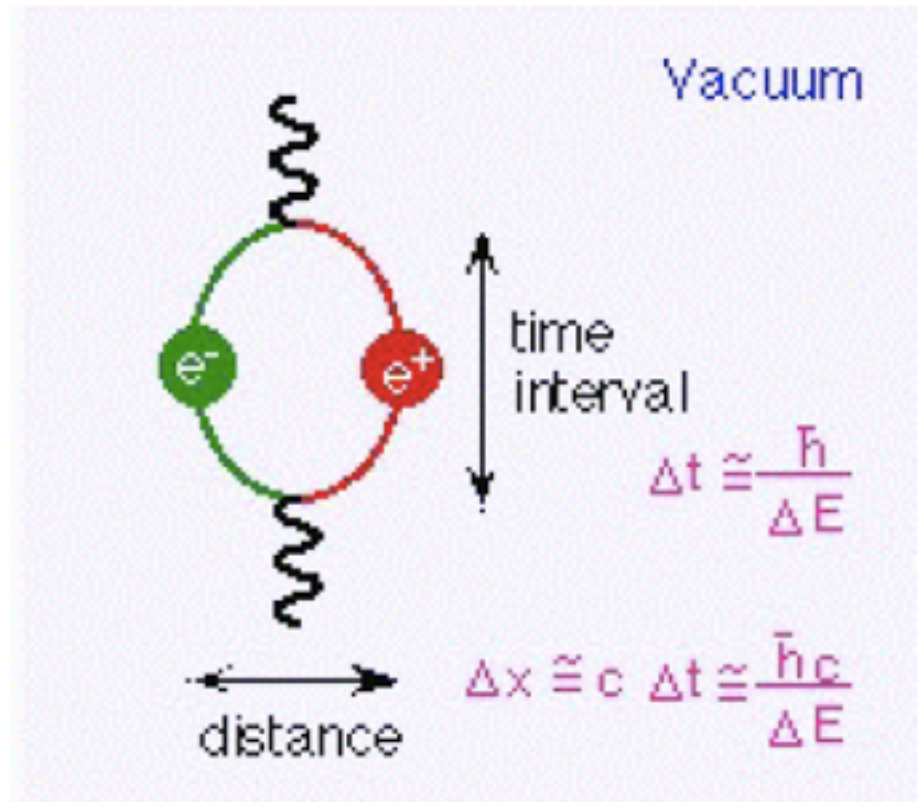
$$\langle k|V|k'\rangle + \sum_{q=0}^{\Lambda} \frac{\langle k|V|q\rangle\langle q|V|k'\rangle}{\epsilon_{k'} - \epsilon_q} + \sum_{q=\Lambda}^{\infty} \frac{\langle k|V|q\rangle\langle q|V|k'\rangle}{\epsilon_{k'} - \epsilon_q}$$

Can't simply drop **high q** without changing low k observables.

But the effect of short-distance physics on low-energy physics can be absorbed by adjustments in the basic forces

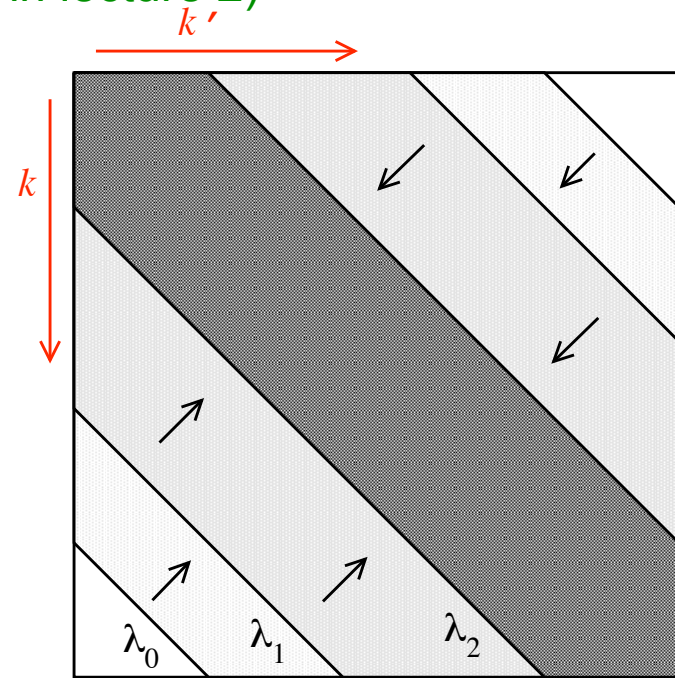
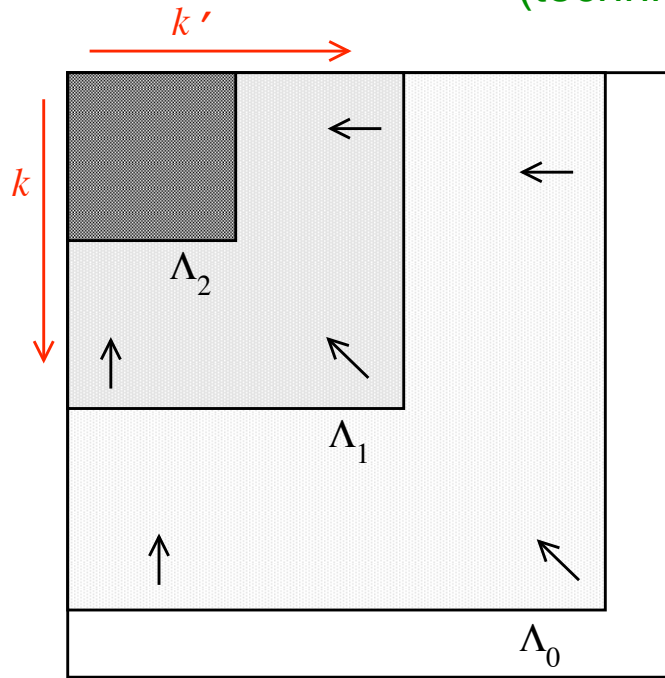
\implies “Renormalization Group”

$$\alpha(0) \approx \frac{1}{137}; \quad \alpha(M_W) \approx \frac{1}{128}$$



2 Types of Renormalization Group Transformations

(technical details in lecture 2)



“ $V_{\text{low } k}$ ”

integrate-out high k states

preserves observables for $k < \Lambda$

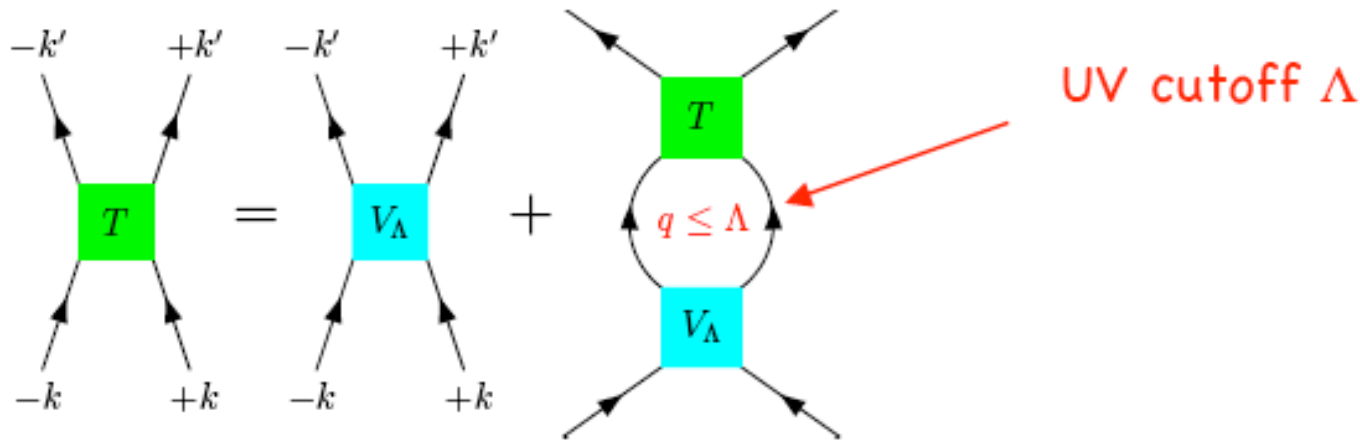
“Similarity RG”

eliminate far off-diagonal coupling

preserves “all” observables

Very similar consequences despite differences in appearance!

Integrating out high-momentum modes ("V_{low k}")



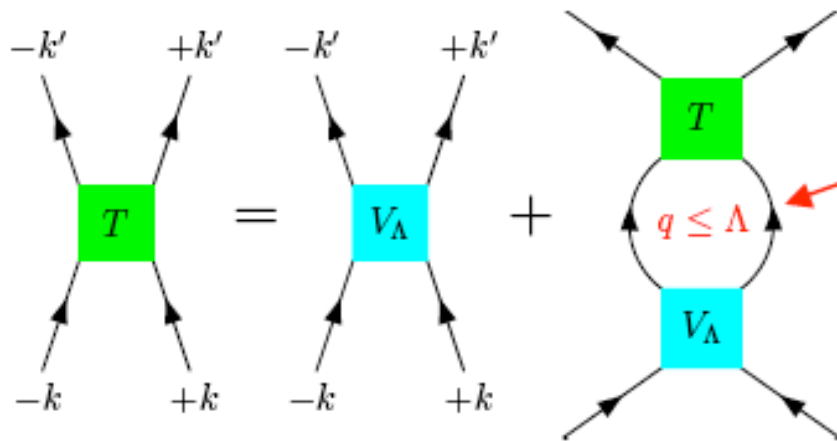
- Demand $\frac{d}{d\Lambda} T = 0$

\Rightarrow RGE's for "running" of V_Λ w/ Λ

$$\frac{d}{d\Lambda} V_{\text{low } k}^\Lambda(k', k) = \frac{2}{\pi} \frac{V_{\text{low } k}^\Lambda(k', \Lambda) T^\Lambda(\Lambda, k; \Lambda^2)}{1 - (k/\Lambda)^2}$$

Solve coupled RGE's given input V_{NN} as large Λ initial condition

Integrating out high-momentum modes (“ $V_{\text{low } k}$ ”)



- Demand $\frac{d}{d\Lambda} T = 0$

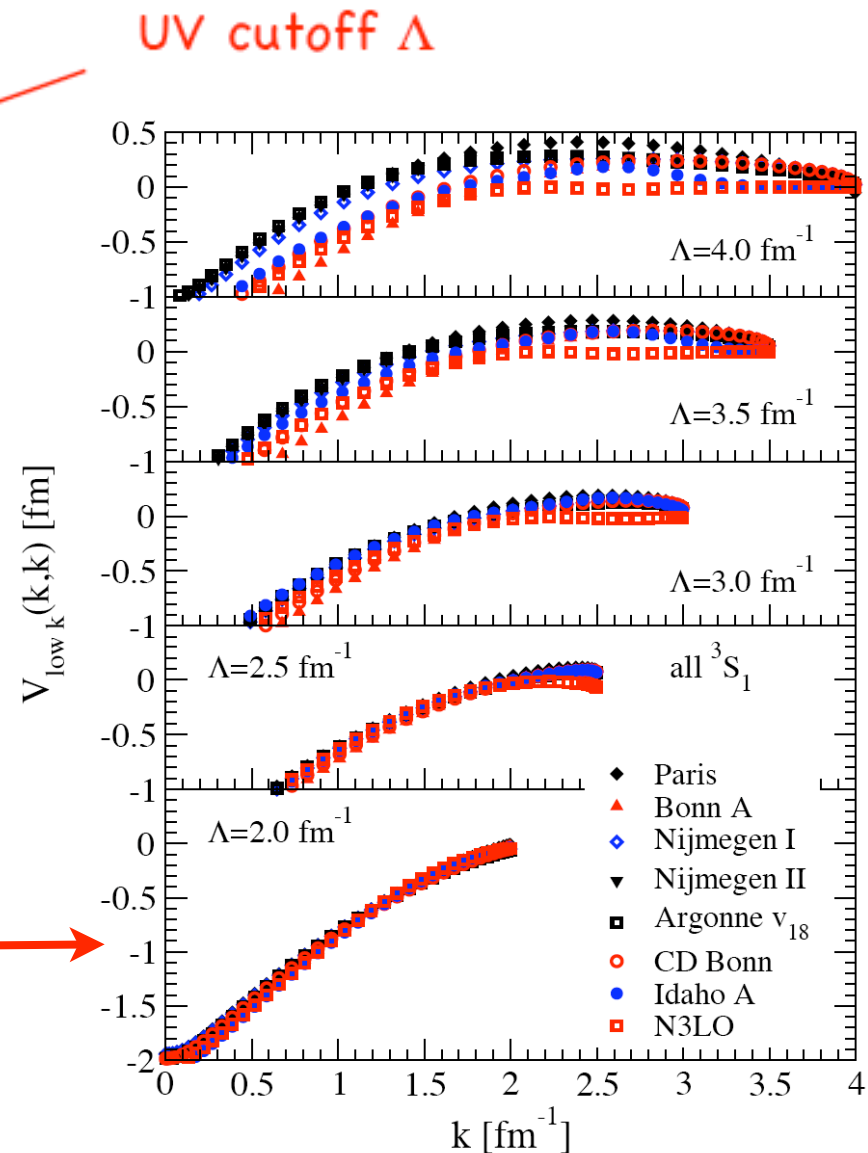
\Rightarrow RGE's for “running” of V_Λ w/ Λ

- Integrate RGE's to smaller Λ
 \Rightarrow decouples high k modes

- Low momentum universality

$$\Lambda \sim \Lambda_{\text{data}}$$

$$2 \text{ fm}^{-1} \Leftrightarrow 330 \text{ MeV lab}$$



The Similarity Renormalization Group

Wegner, Glazek and Wilson

Unitary transformation on an initial $H = T + V$

$$H_\lambda = U(\lambda) H U^\dagger(\lambda) \equiv T + V_\lambda \quad \lambda = \text{continuous flow parameter}$$

Differentiating with respect to λ :

$$\frac{dH_\lambda}{d\lambda} = [\eta(\lambda), H_\lambda] \quad \text{with} \quad \eta(\lambda) \equiv \frac{dU(\lambda)}{d\lambda} U^\dagger(\lambda)$$

Engineer η to do different things as $\lambda \Rightarrow 0$

$$\eta(\lambda) = [\mathcal{G}_\lambda, H_\lambda]$$

$\mathcal{G}_\lambda = T \Rightarrow H_\lambda$ driven towards diagonal in k – space

$\mathcal{G}_\lambda = P H_\lambda P + Q H_\lambda Q \Rightarrow H_\lambda$ driven to block – diagonal

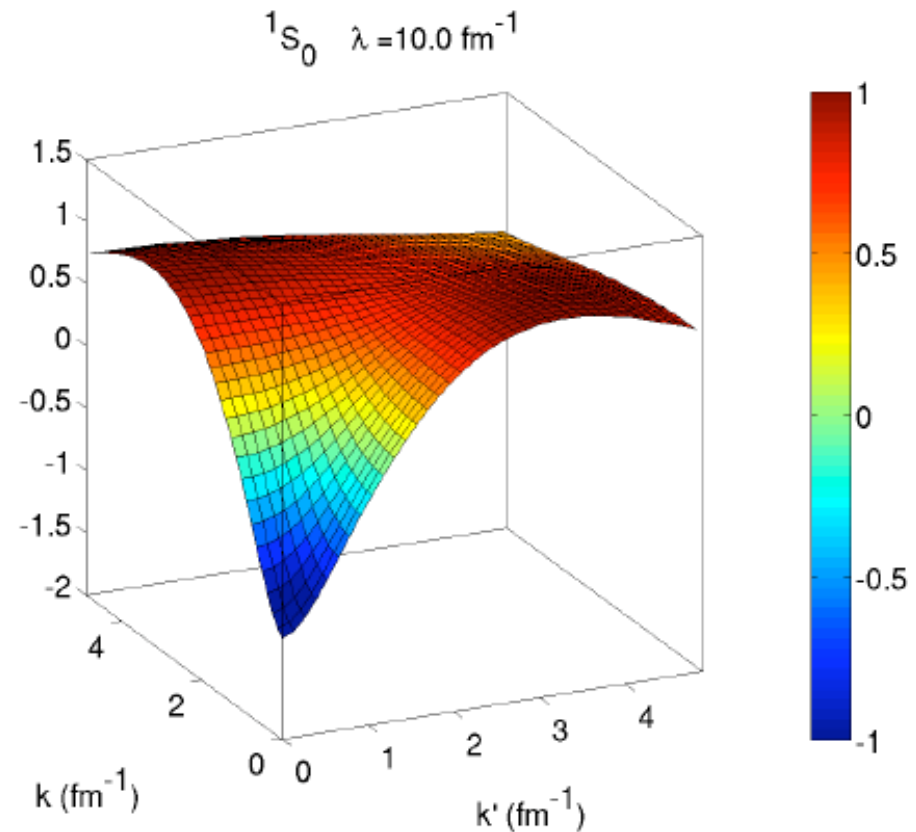
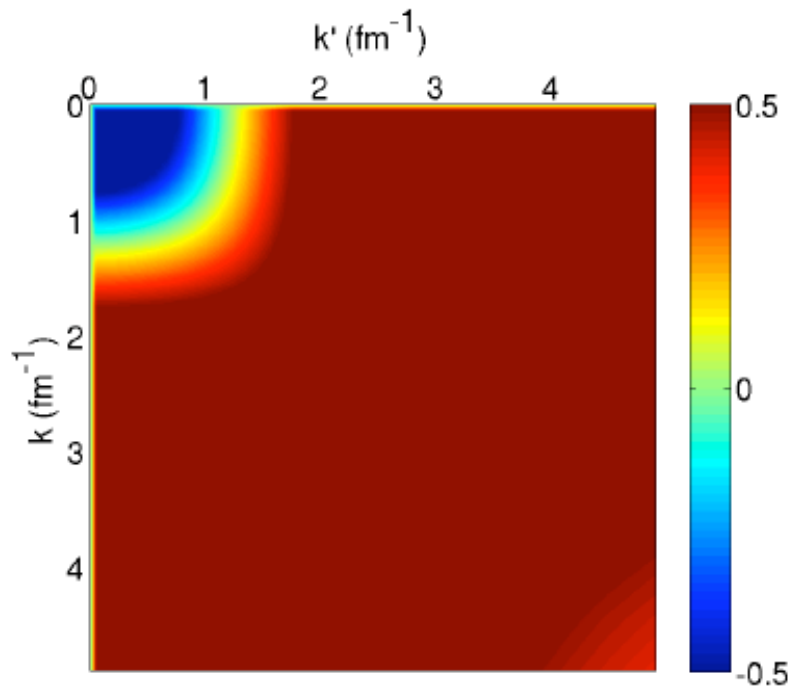
⋮

SRG evolved NN interactions with $\eta = [T,H]$

- In each partial wave with $\epsilon_k = \hbar^2 k^2 / M$ and $\lambda^2 = 1 / \sqrt{s}$

$$\frac{dV_\lambda}{d\lambda}(k, k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k, k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k, q) V_\lambda(q, k')$$

${}^1S_0 \quad \lambda = 10.0 \text{ fm}^{-1}$



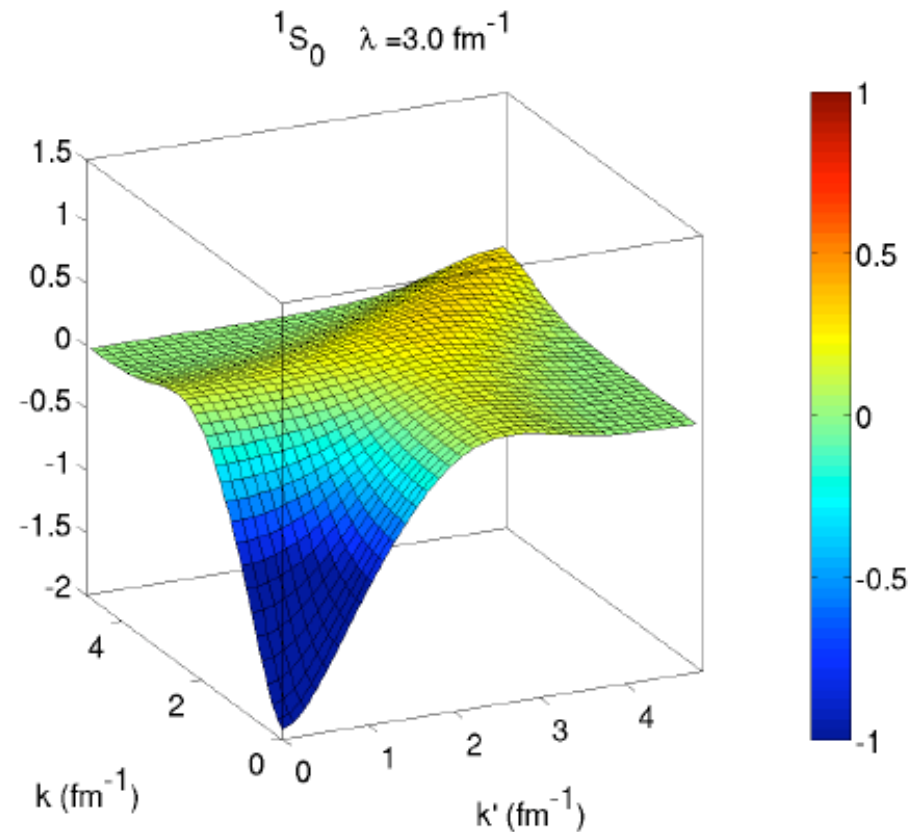
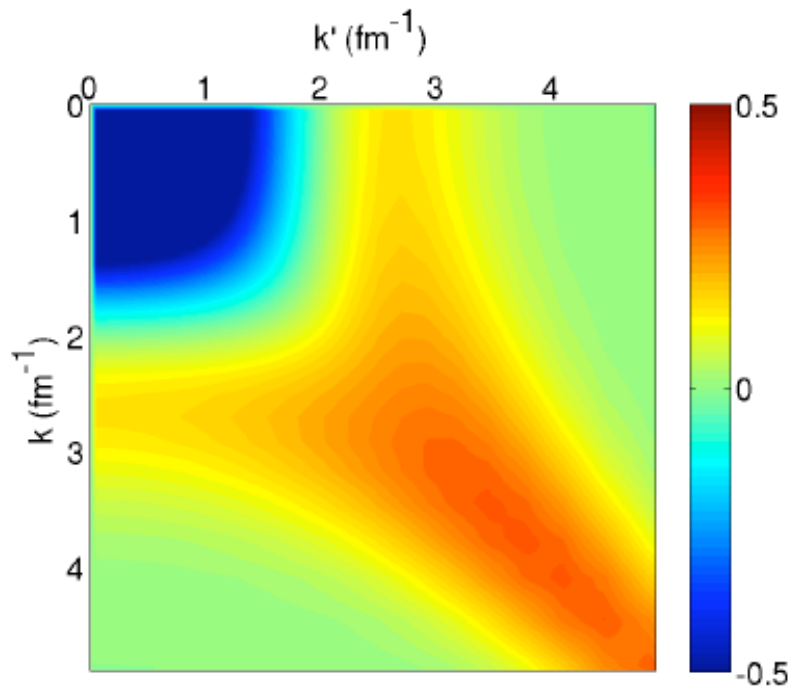
$$\lambda = 10.0 \text{ fm}^{-1}$$

SRG evolved NN interactions with $\eta = [T,H]$

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${}^1S_0 \quad \lambda = 3.0 \text{ fm}^{-1}$



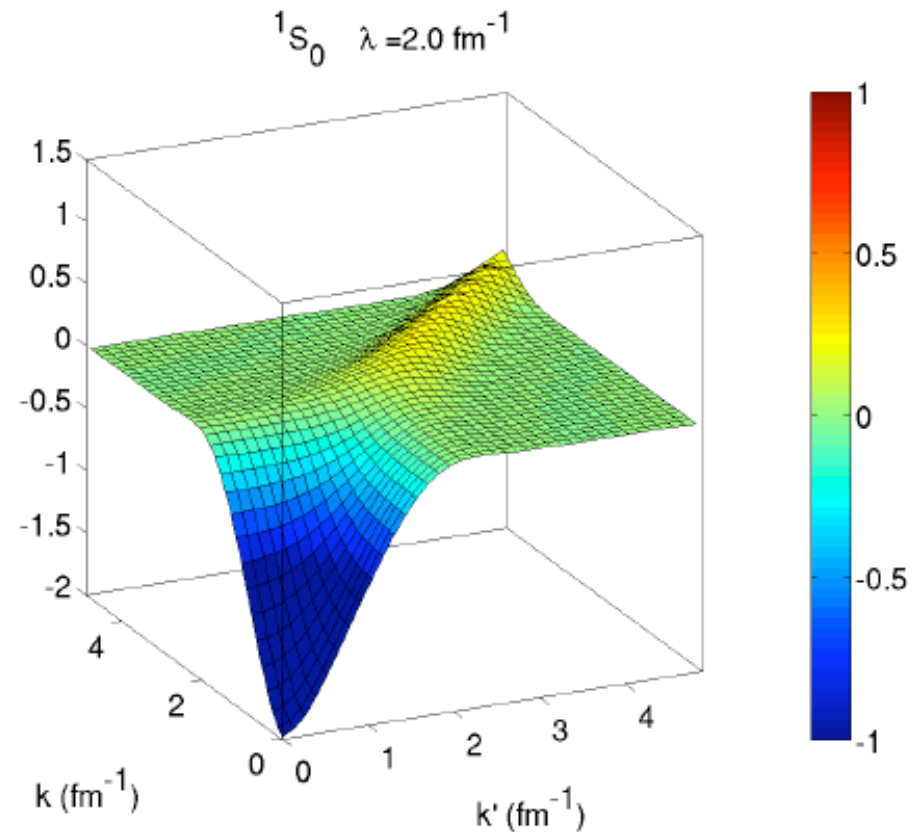
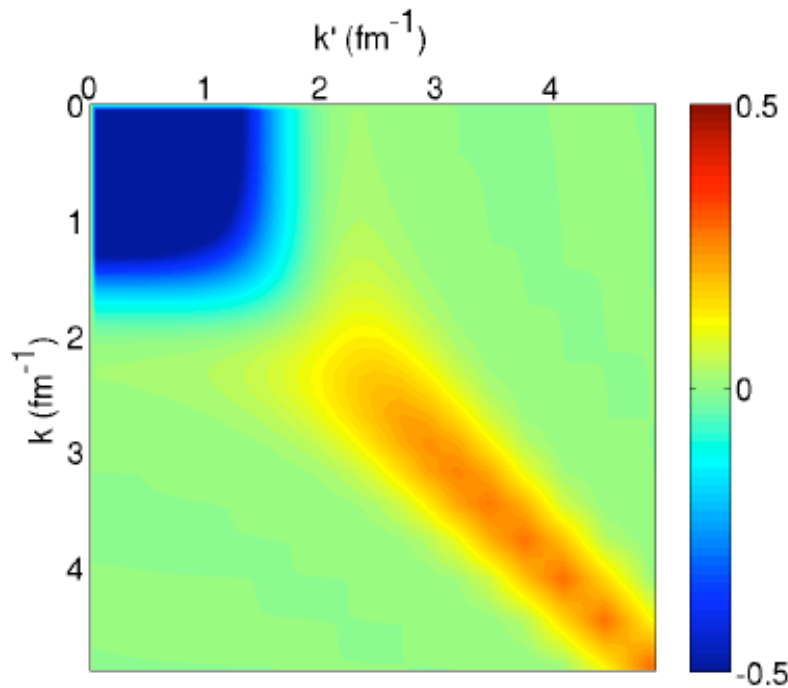
$$\lambda = 3.0 \text{ fm}^{-1}$$

SRG evolved NN interactions with $\eta = [T,H]$

- In each partial wave with $\epsilon_k = \hbar^2 k^2 / M$ and $\lambda^2 = 1 / \sqrt{s}$

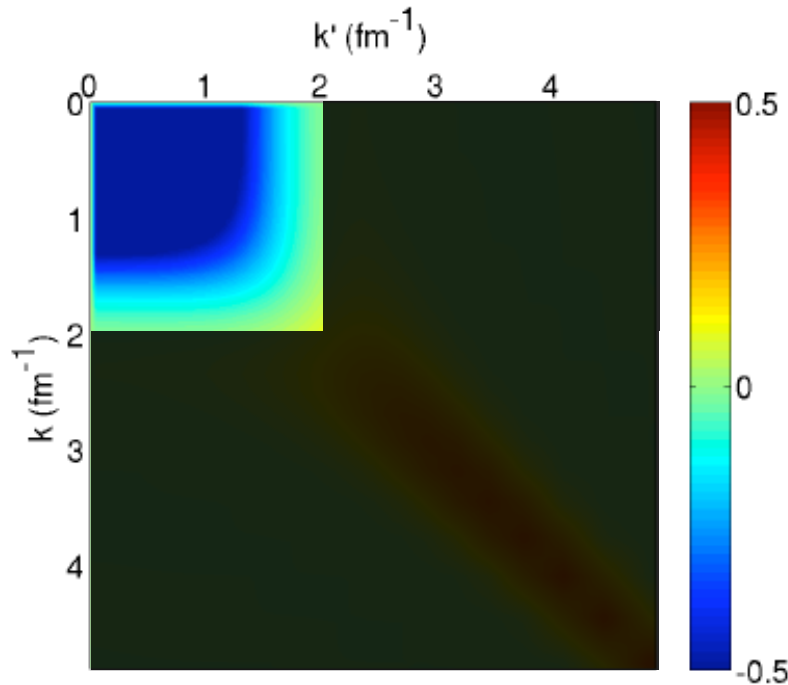
$$\frac{dV_\lambda}{d\lambda}(k, k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k, k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k, q) V_\lambda(q, k')$$

${}^1S_0 \quad \lambda = 2.0 \text{ fm}^{-1}$

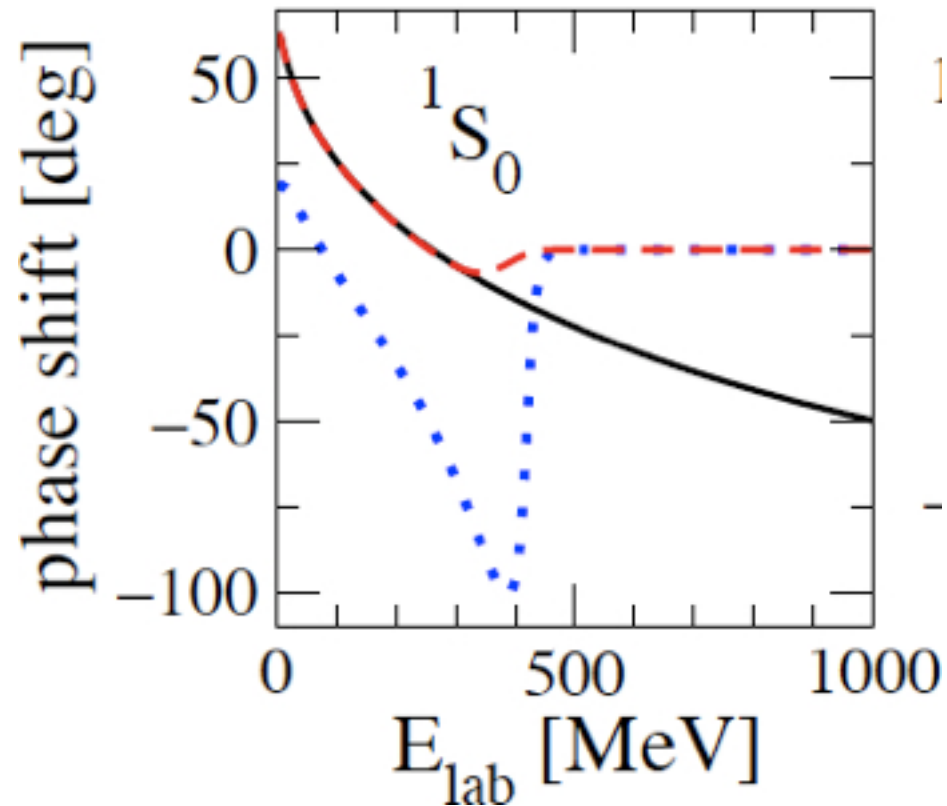


$\lambda = 2.0 \text{ fm}^{-1}$

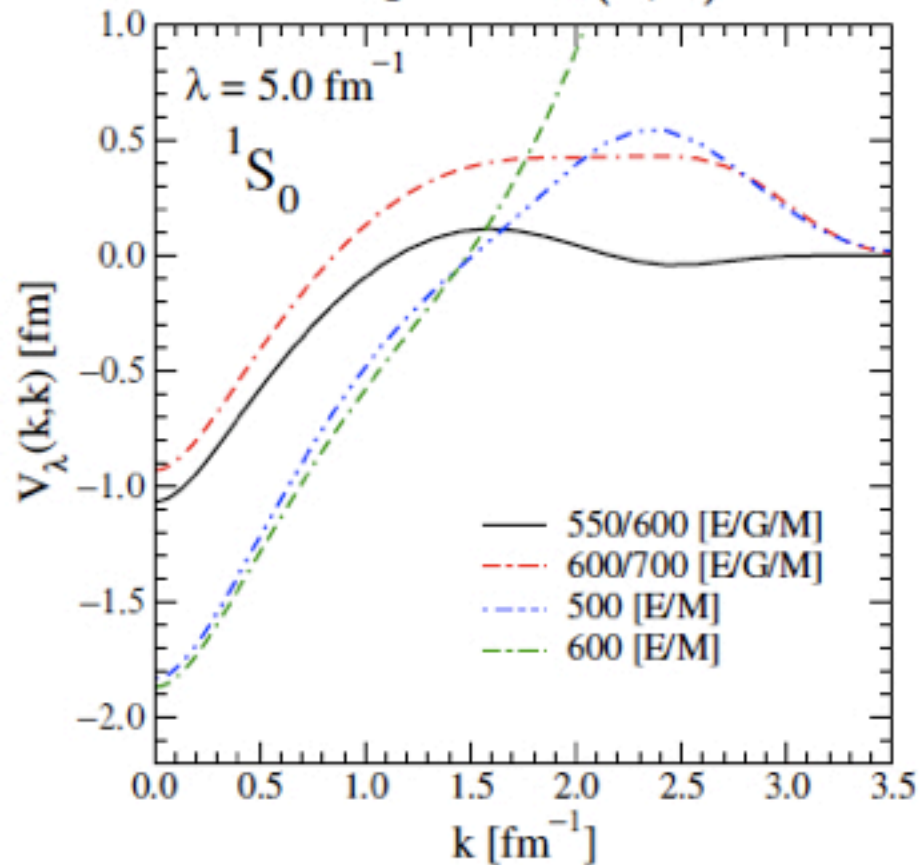
Our low-pass filter now works. If you do things right, (i.e., RG/SRG transformations) problematic high-k modes can be eliminated! **High momentum modes decouple.**



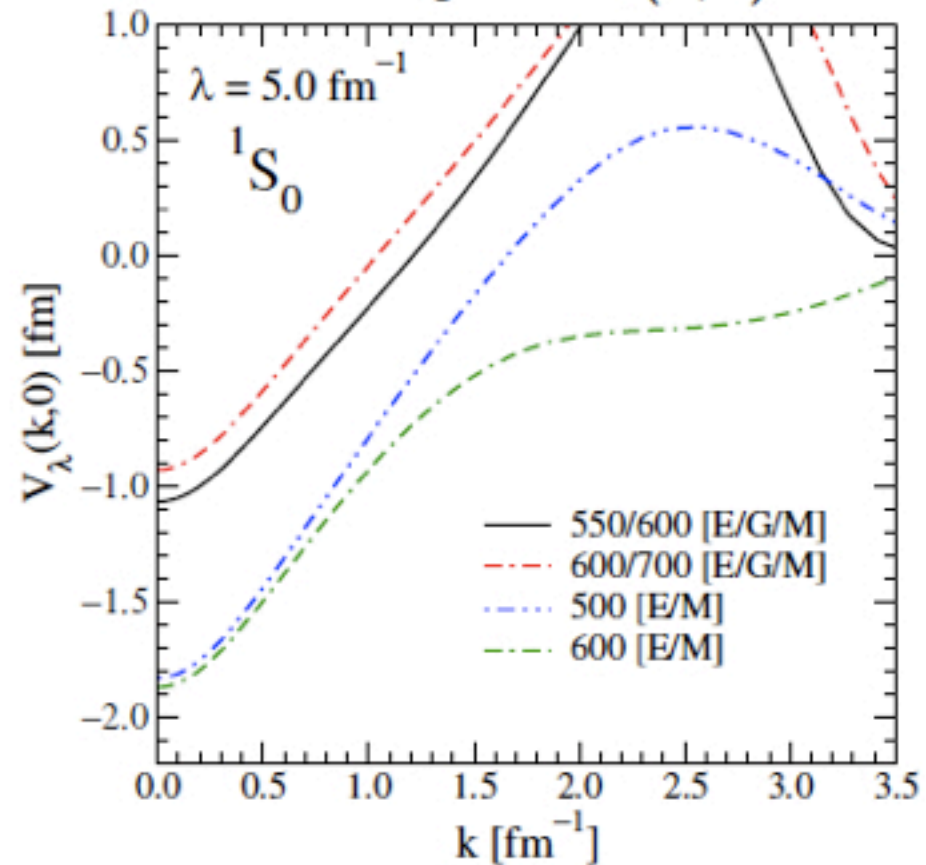
$$\lambda = 2.0 \text{ fm}^{-1}$$



Diagonal $V_\lambda(k, k)$

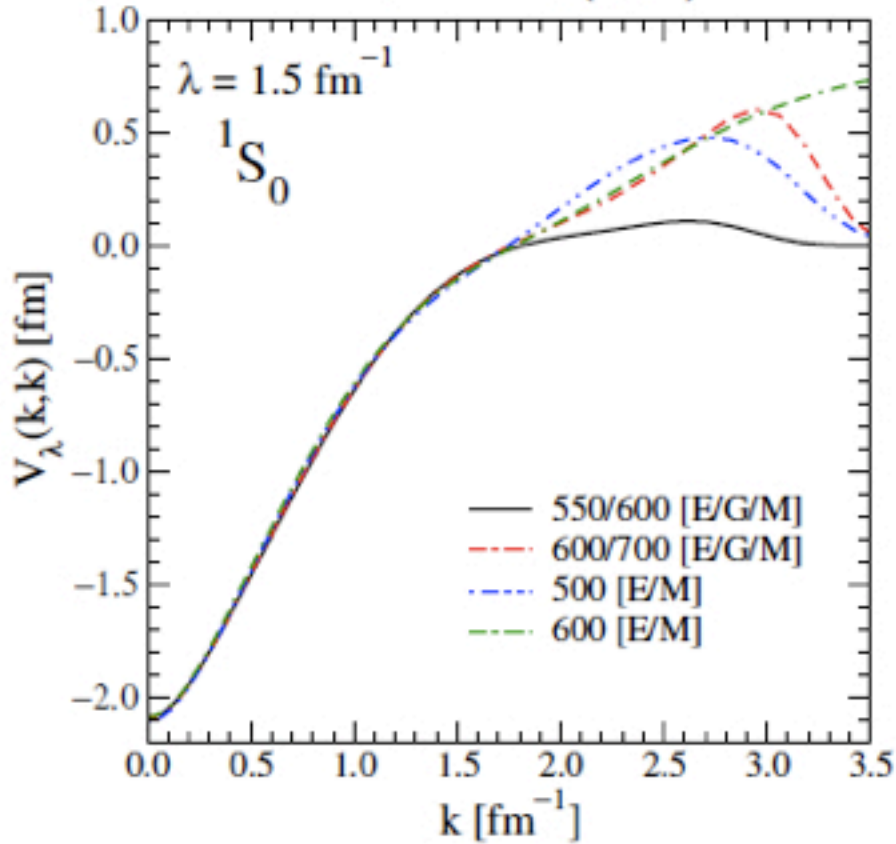


Off-Diagonal $V_\lambda(k, 0)$

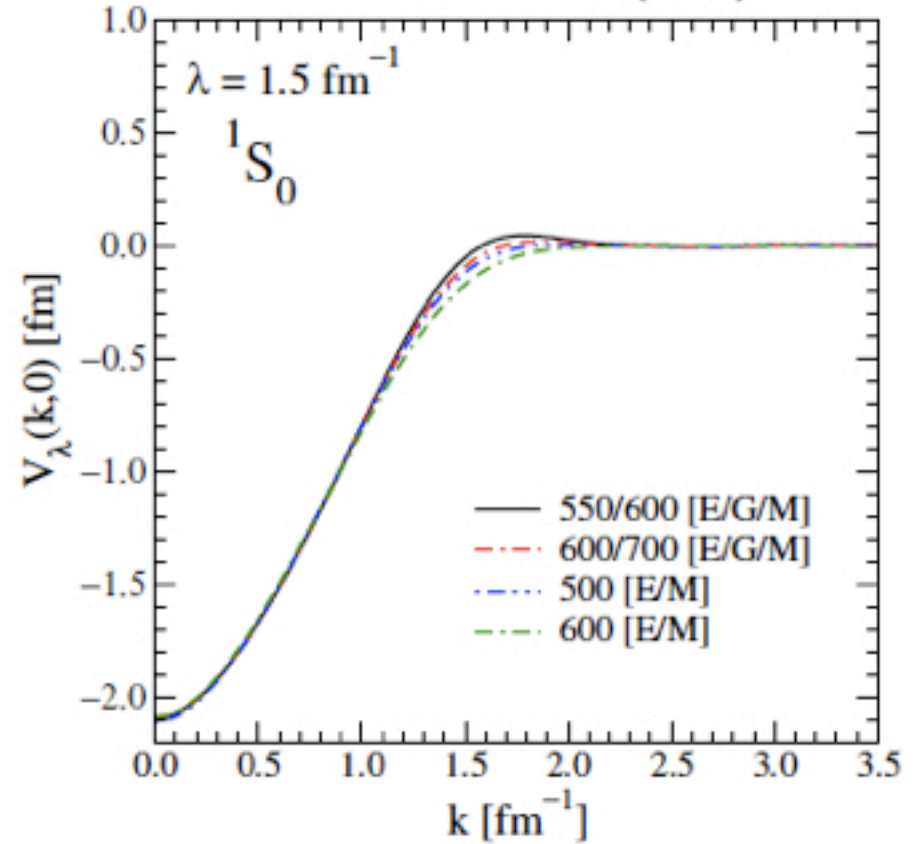


Initially very different-looking chiral EFT potentials at N³LO ...

Diagonal $V_\lambda(k, k)$



Off-Diagonal $V_\lambda(k, 0)$

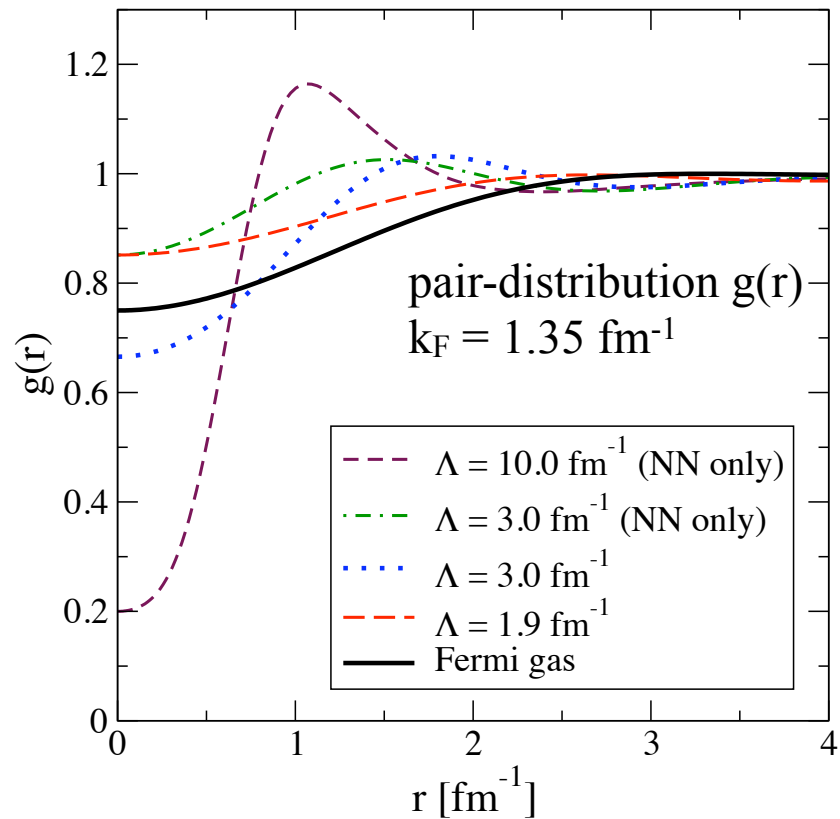
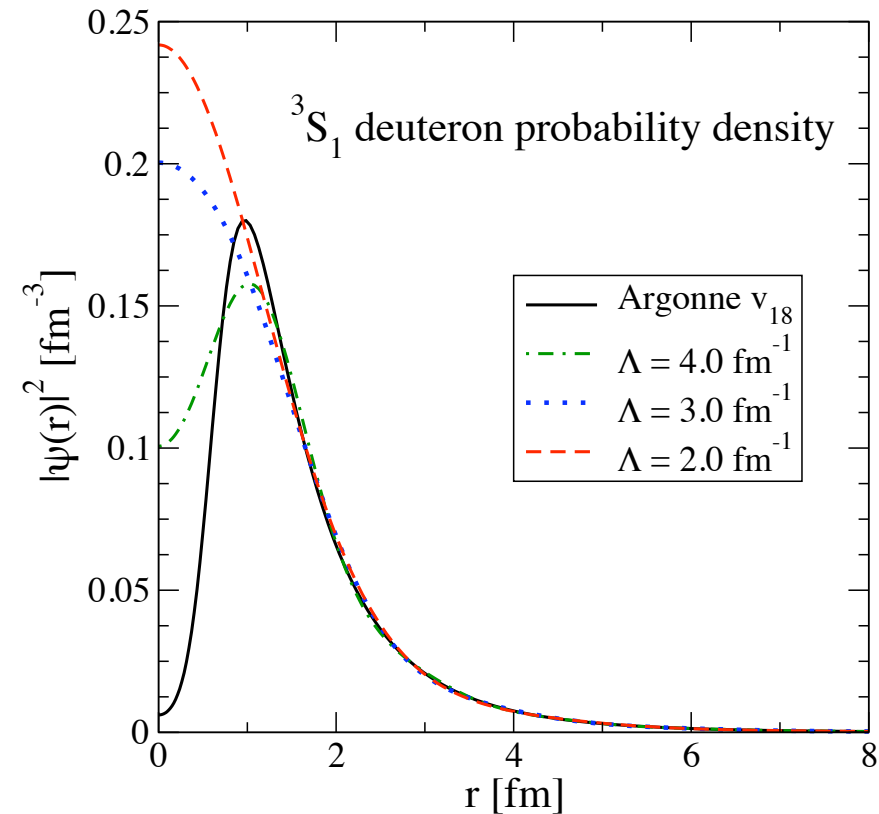


Low-momentum universality like for $V_{\text{low}k}$

Note, however, the model-dependent modes at high k along the diagonal

Simplifications at low resolution

Simplifications from lowering Λ



weaker short-range correlations,
more effective variational calcs.,
efficient basis expansions
(SM, coupled cluster, etc.),
more perturbative

Weinberg Eigenvalue Analysis of Convergence

Born series:
$$T(E) = V + V \frac{1}{E - H_0} V + V \frac{1}{E - H_0} V \frac{1}{E - H_0} V + \dots = V + V \frac{1}{E - H} V$$

- If bound state E_b , series must diverge at $E = E_b$ where

$$(H_0 + V)|b\rangle = E_b|b\rangle \quad \Longrightarrow \quad V|b\rangle = (E_b - H_0)|b\rangle$$

- For *any* E , generalize to find the eigenvalue of the kernel (Weinberg, 1962)

$$\frac{1}{E_b - H_0} V|b\rangle = |b\rangle \quad \Longrightarrow \quad \frac{1}{E - H_0} V|\Gamma_\nu\rangle = \eta_\nu|\Gamma_\nu\rangle$$

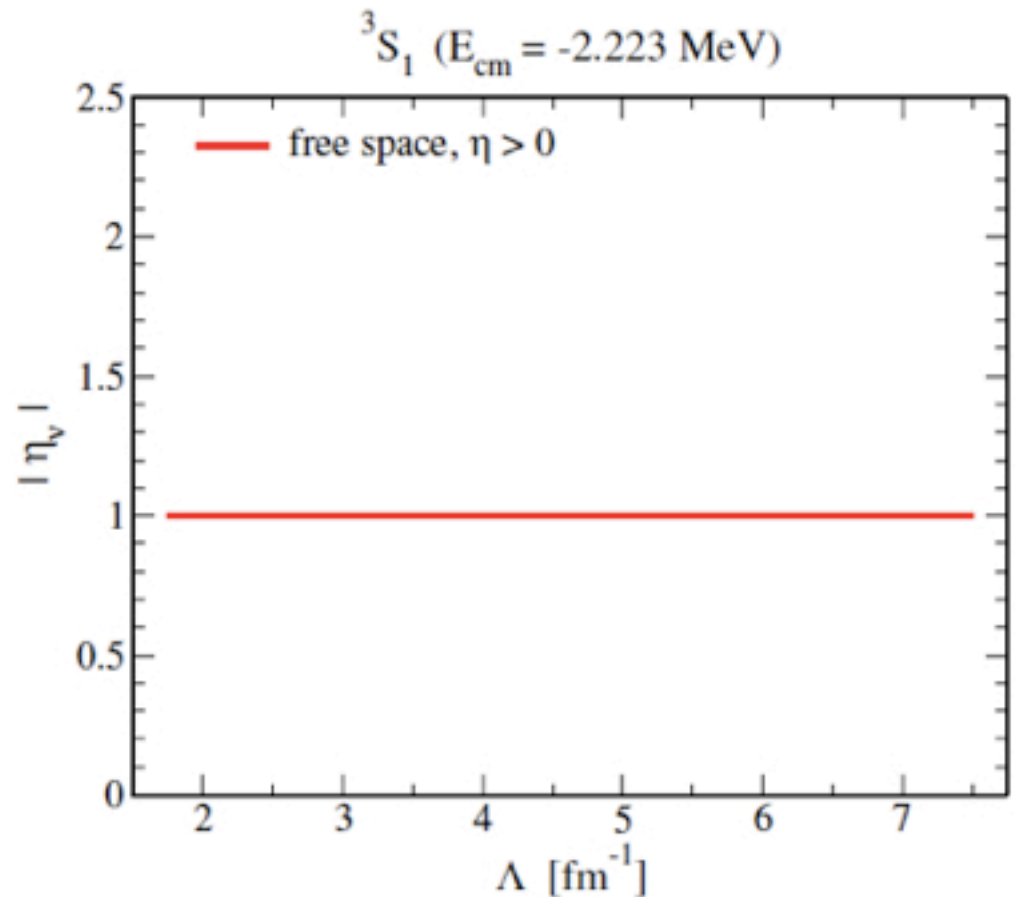
- Acting with $T(E)$ on *any* Γ_ν gives

$$T(E)|\Gamma_\nu\rangle = V|\Gamma_\nu\rangle (1 + \eta_\nu + \eta_\nu^2 + \dots)$$

\Longrightarrow series diverges at E if *any* $|\eta_\nu(E)| \geq 1$

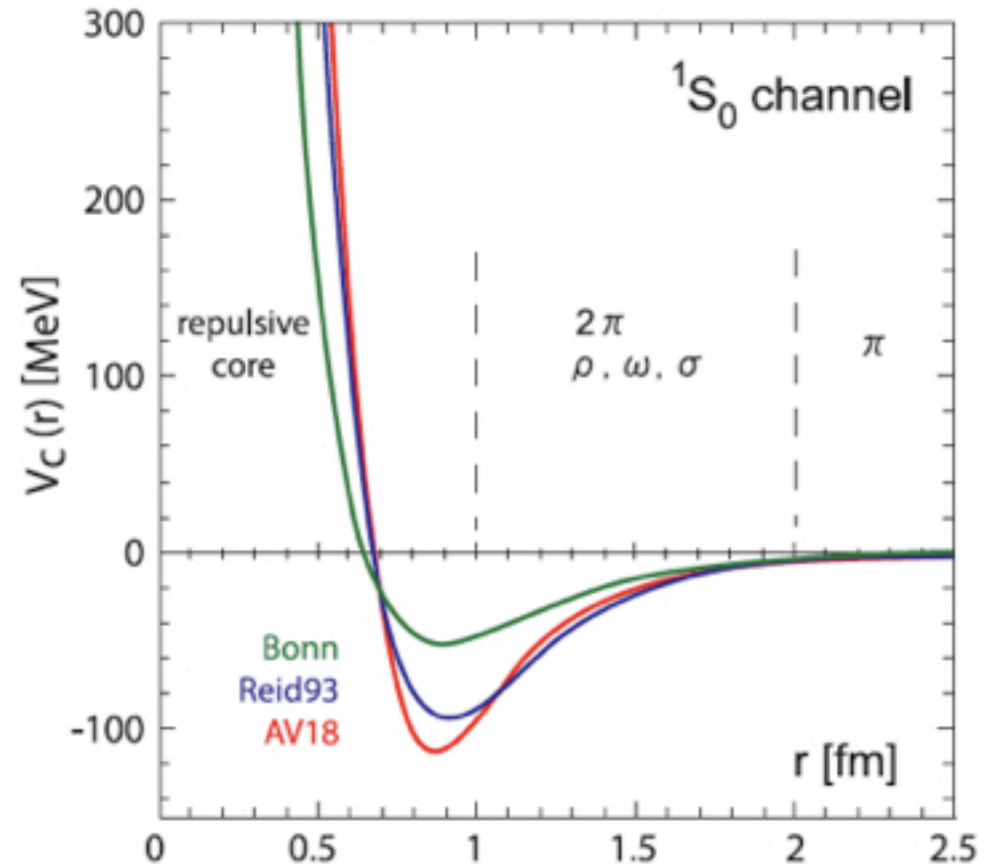
Weinberg eigenvalues as function of cutoff Λ/λ

- Consider $\eta_\nu(E = -2.22 \text{ MeV})$
- Deuteron \implies attractive eigenvalue $\eta_\nu = 1$
 - $\Lambda \downarrow \implies$ unchanged



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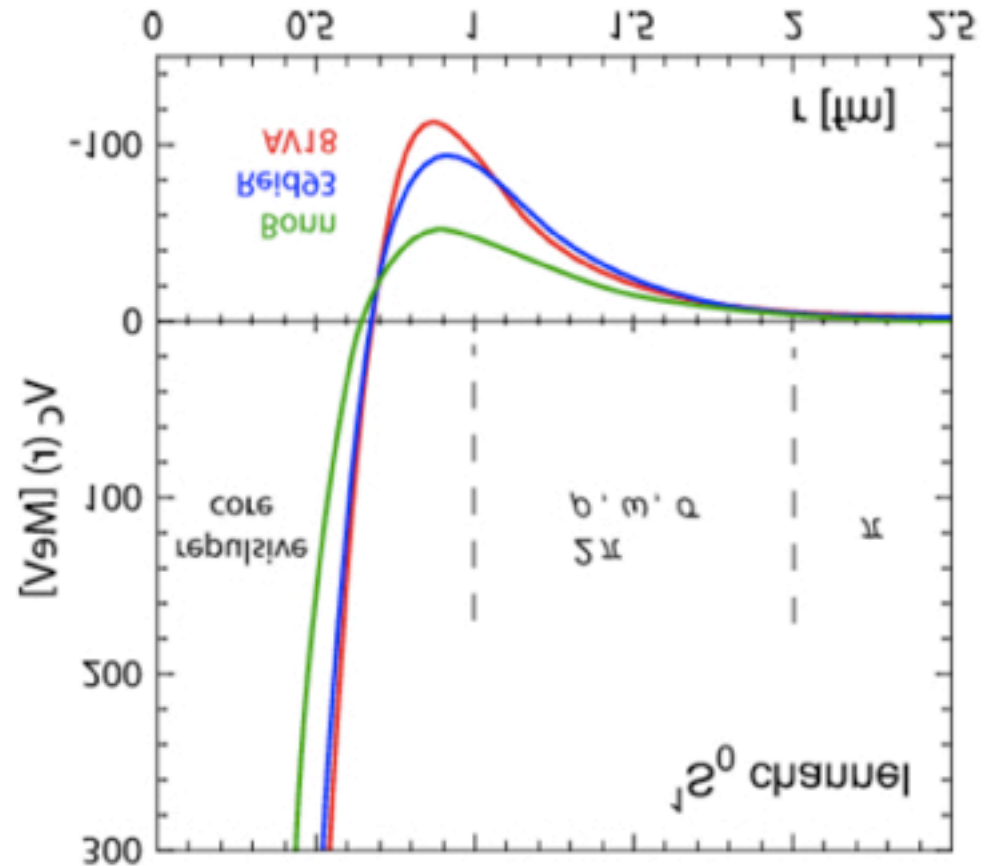


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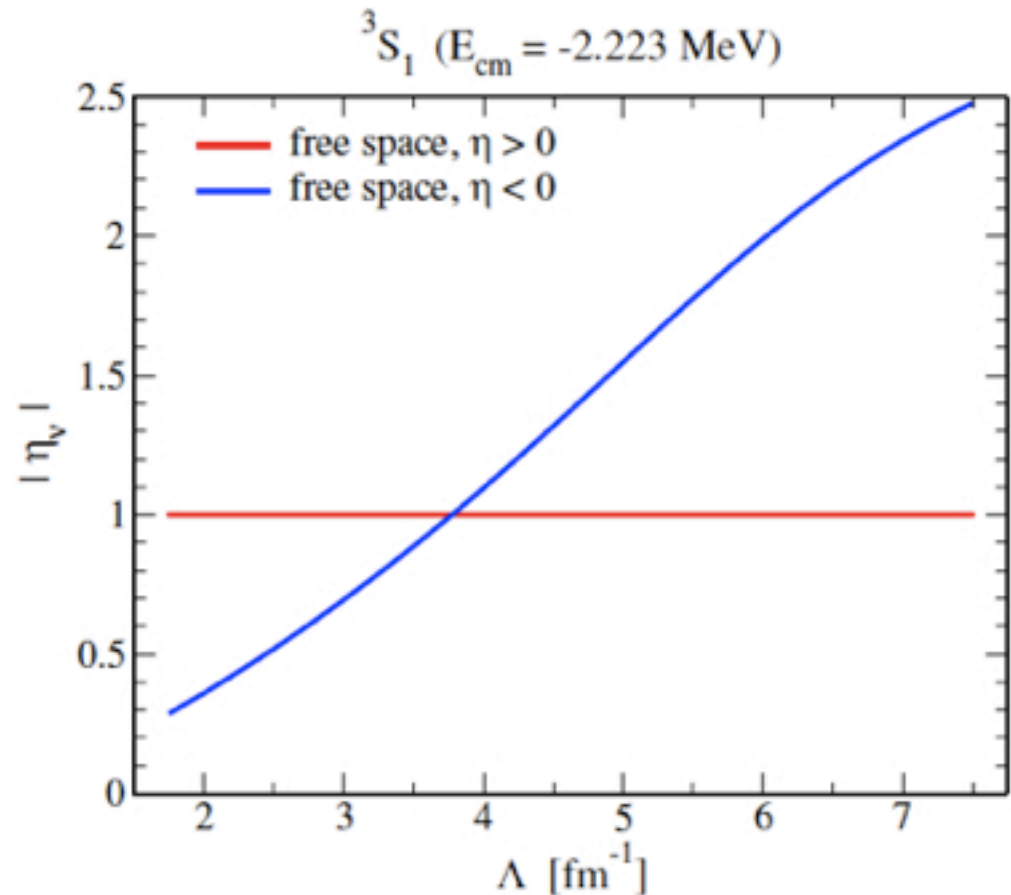
$$\frac{1}{E - H_0} V |\Gamma_\nu\rangle = \eta_\nu |\Gamma_\nu\rangle$$

$$\implies \left(H_0 + \frac{V}{\eta_\nu} \right) |\Gamma_\nu\rangle = E |\Gamma_\nu\rangle$$

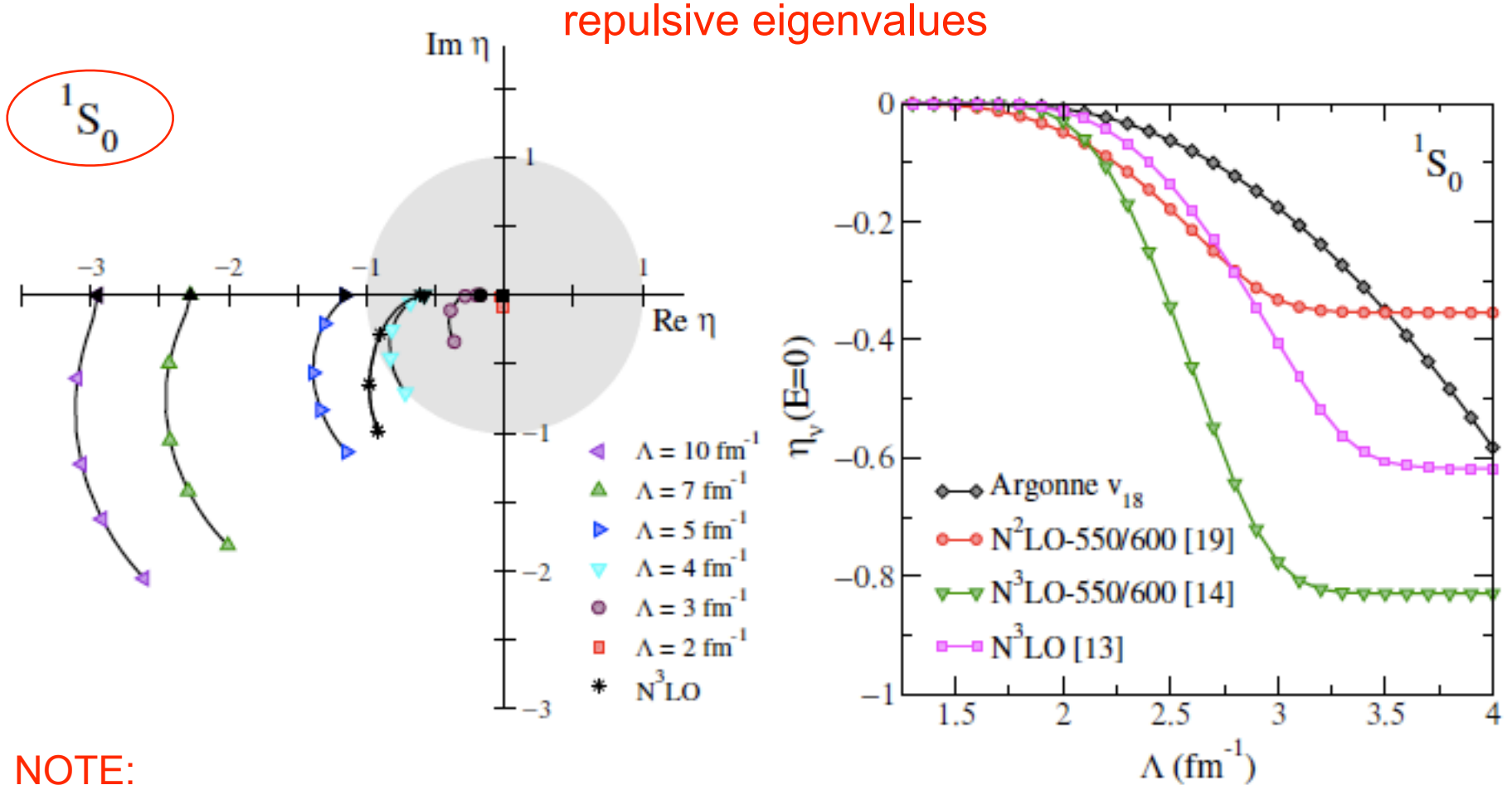


Weinberg eigenvalues as function of cutoff Λ/λ

- Consider $\eta_\nu(E = -2.22 \text{ MeV})$
- Deuteron \implies **attractive** eigenvalue $\eta_\nu = 1$
 - $\Lambda \downarrow \implies$ unchanged
- But η_ν can be negative, so $V/\eta_\nu \implies$ flip potential
- Hard core \implies **repulsive** eigenvalue η_ν
 - $\Lambda \downarrow \implies$ reduced



Evolving to low resolution increases “perturbativeness”

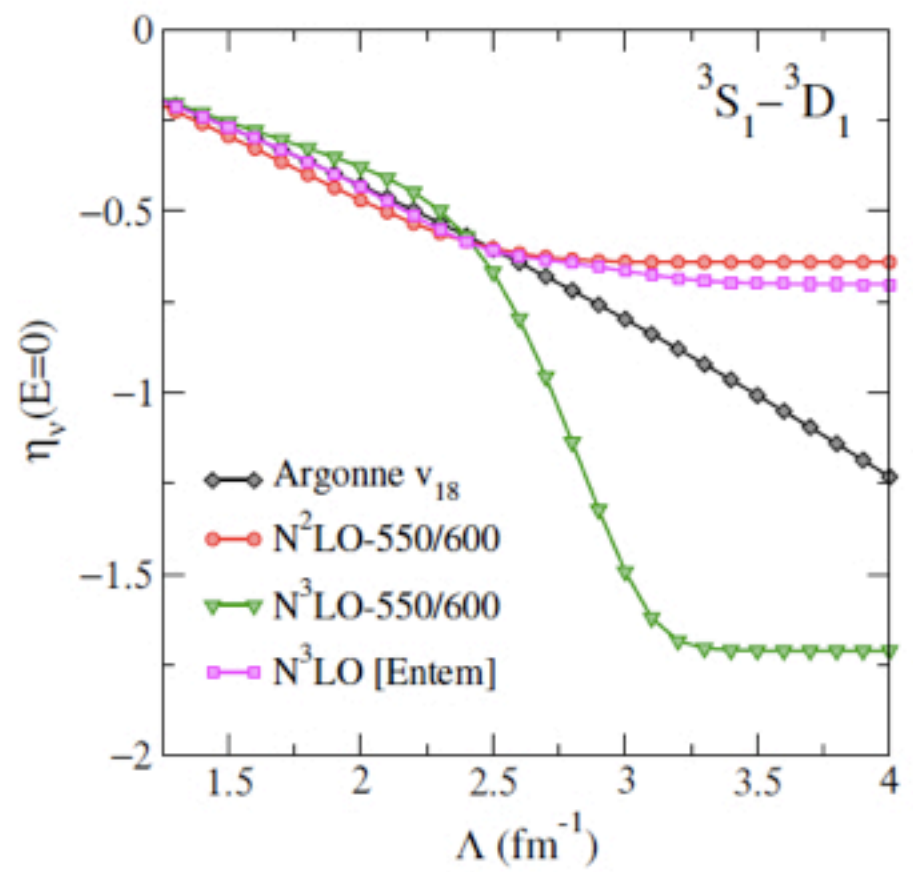
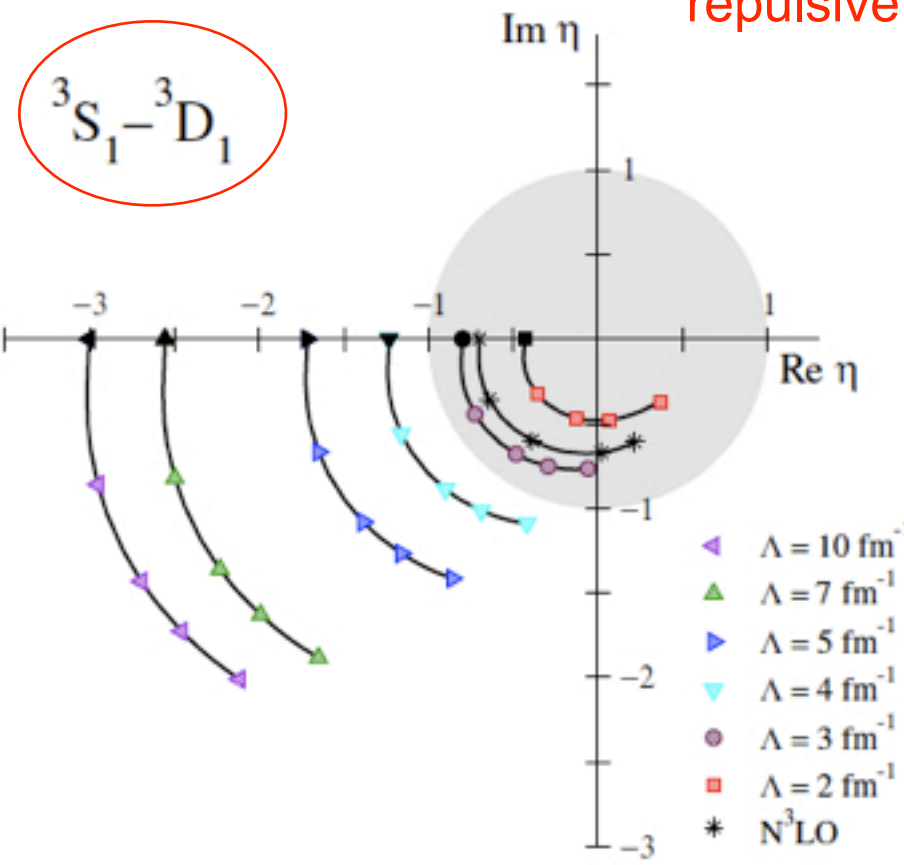


- 1) For real $E \leq 0$, $\text{Im}(\eta) = 0$
- 2) “repulsive” eigenvalue for $\eta \leq 0$ and visa versa (why?)

Evolving to low resolution increases “perturbativeness”

repulsive eigenvalues

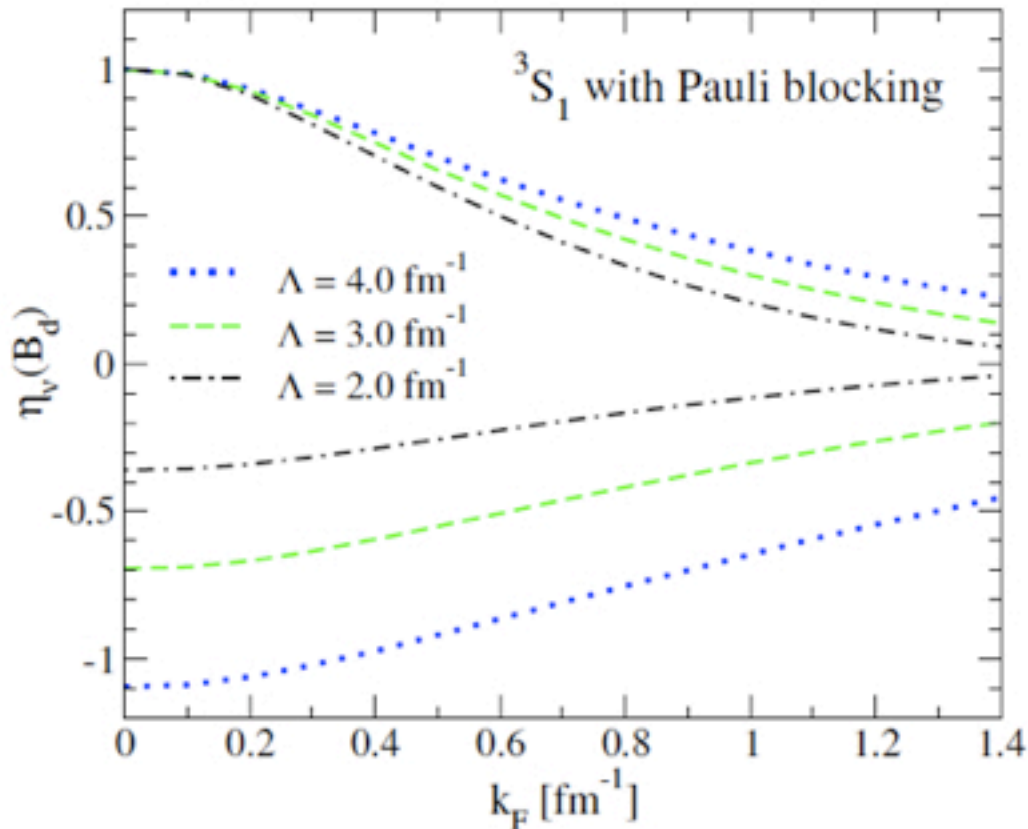
$^3S_1-^3D_1$



Any idea why in the $3S_1-3D_1$ channel η doesn't decay as dramatically as $1S_0$?
 (HINT: typical q scale of the “hard core” $\sim 5-7 \text{ fm}^{-1}$ in both channels.)

Weinberg eigenvalues at finite density

- Weinberg eigenvalue analysis (η_ν at -2.22 MeV vs. density)

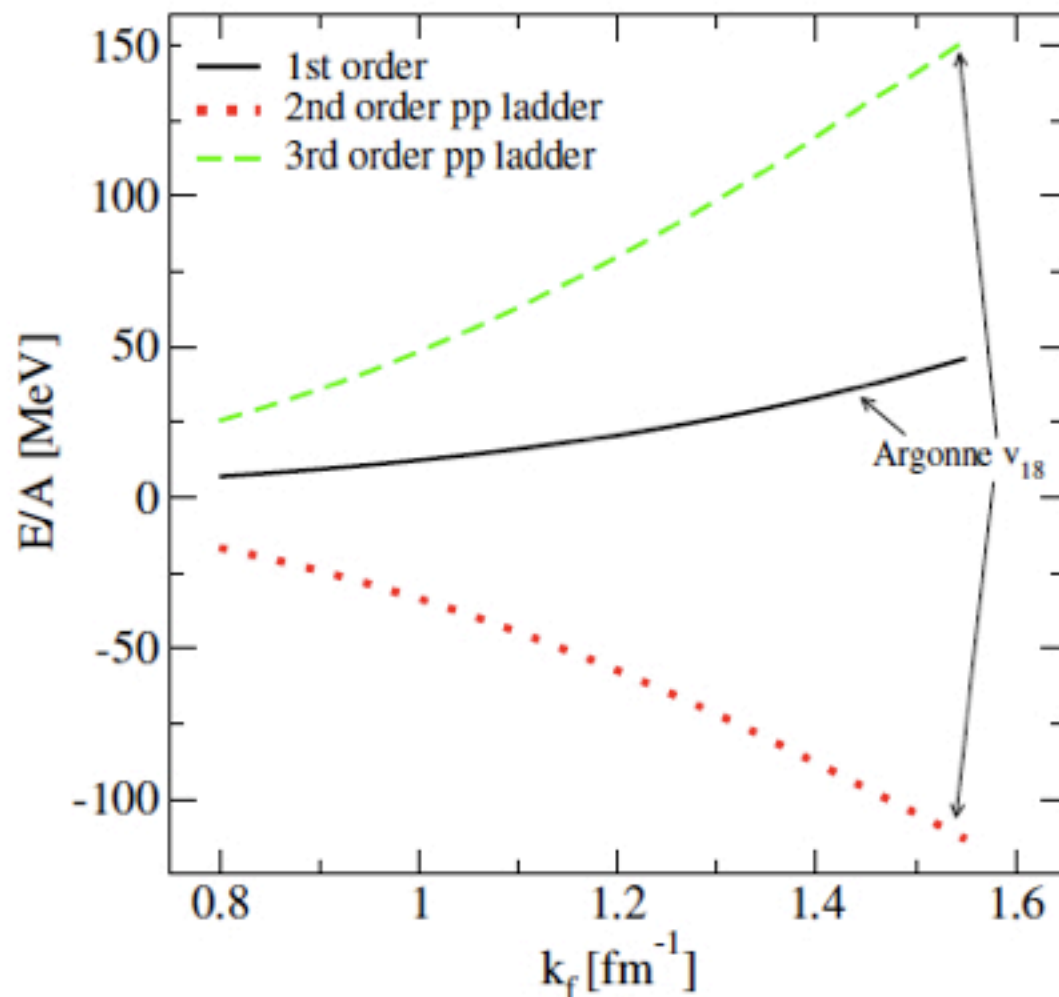
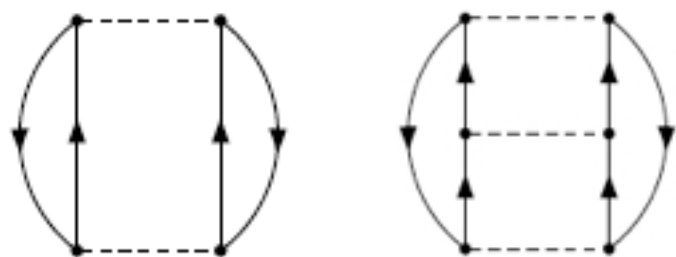


- Pauli blocking in nuclear matter increases it even more!
 - at Fermi surface, pairing revealed by $|\eta_\nu| > 1$

Low resolution \implies MBPT is feasible!

- MBPT \equiv Many-Body Perturbation Theory

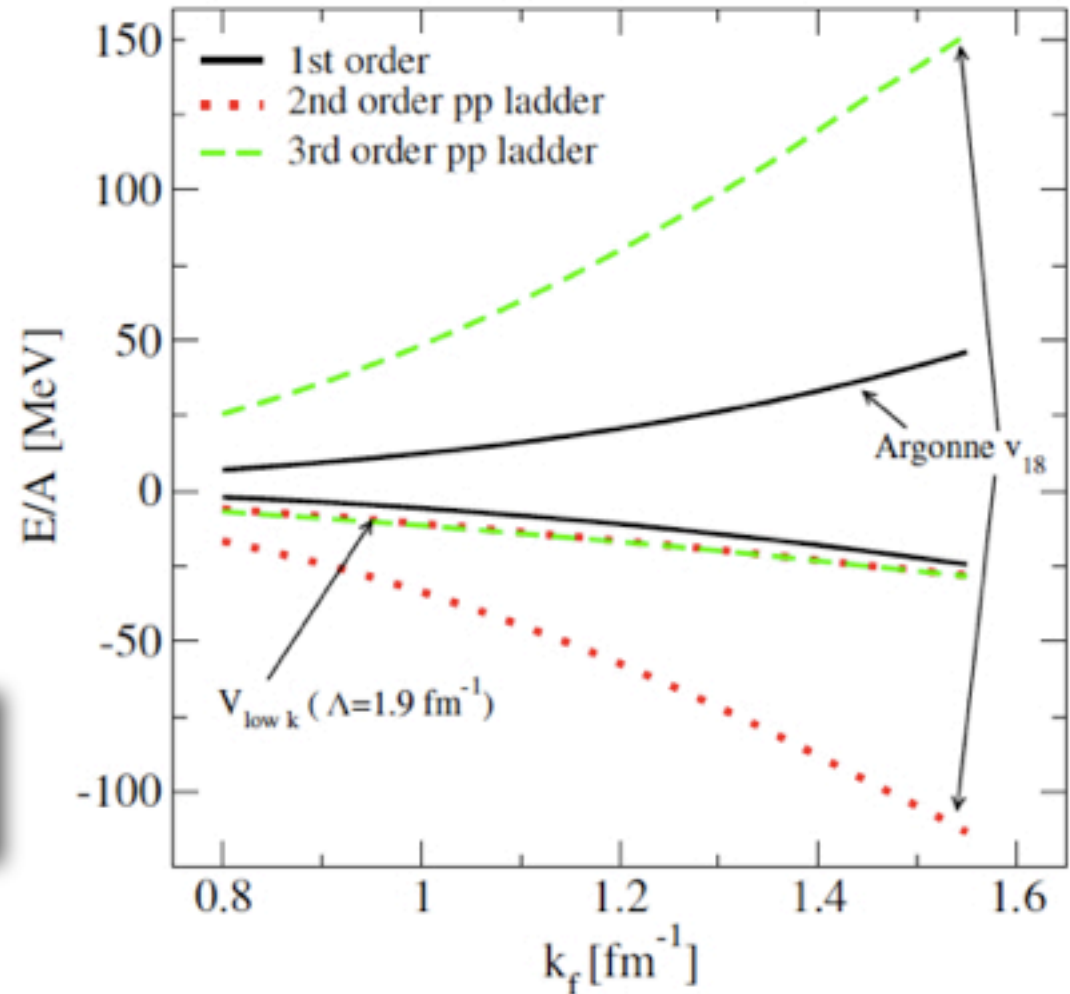
- Compare high resolution (AV18) to low resolution $V_{\text{low } k}$ or SRG



Low resolution \implies MBPT is feasible!

- MBPT \equiv Many-Body Perturbation Theory

- Compare high resolution (AV18) to low resolution $V_{\text{low } k}$ or SRG
- MBPT converges!
- Need evolved 3-body force for saturation
- More on this in lectures 2 and 3!



Why does MBPT work better at low cutoffs?

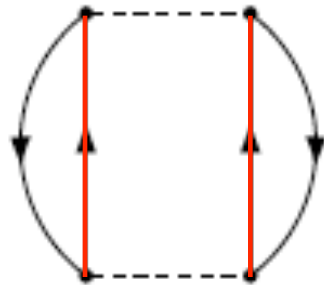
- Phase space in pp-channel strongly suppressed:

$$\int_{k_F}^{\infty} q^2 dq \frac{V_{NN}(k', q) V_{NN}(q, k)}{k^2 - q^2}$$

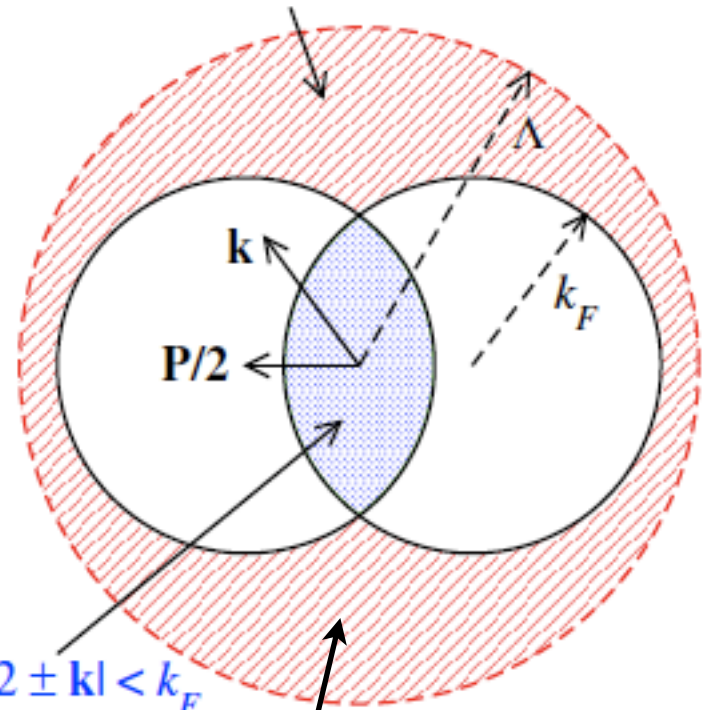
vs.

$$\int_{k_F}^{\Lambda} q^2 dq \frac{V_{\text{low } k}(k', q) V_{\text{low } k}(q, k)}{k^2 - q^2}$$

- Tames hard core, tensor, and bound state



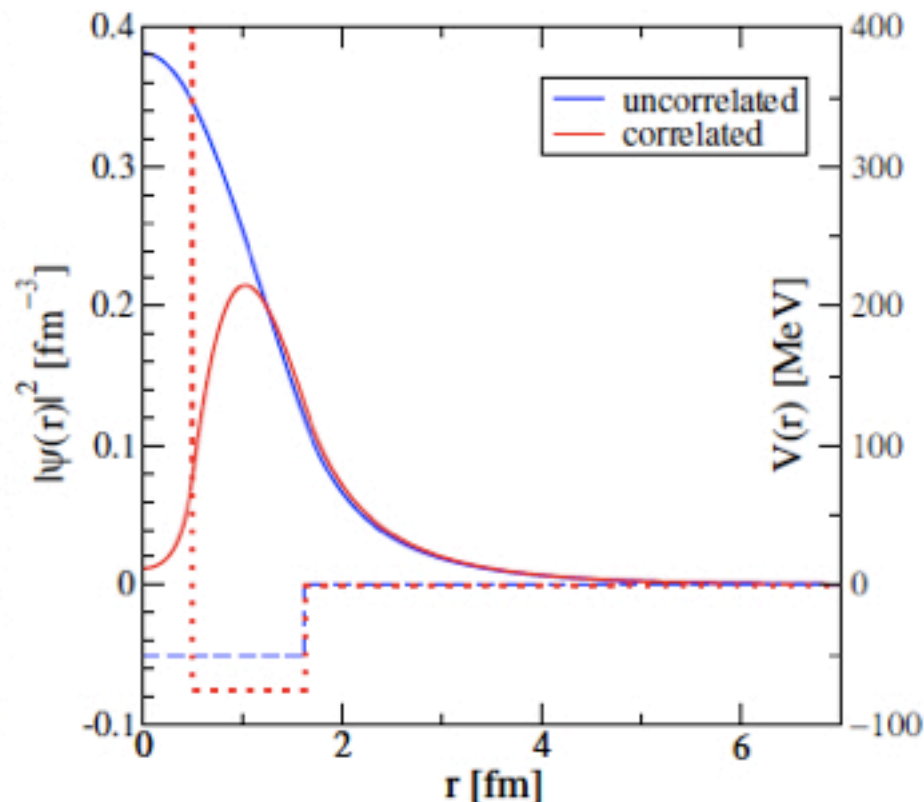
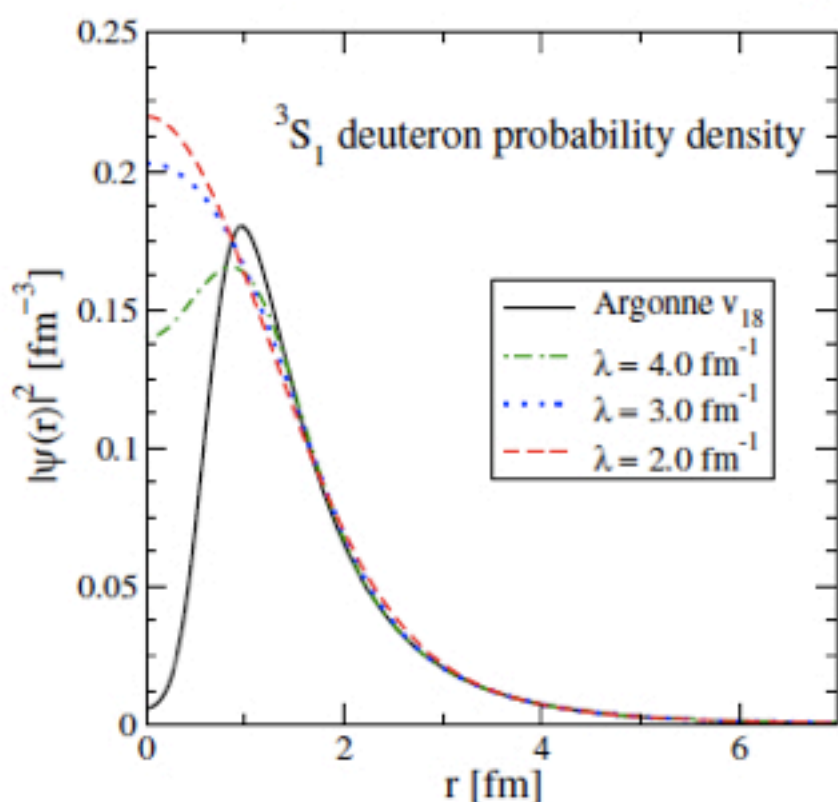
$\Lambda: |P/2 \pm k| > k_F$ and $|k| < \Lambda$



$F: |P/2 \pm k| < k_F$

phase space for 2-particles to scatter out of fermi sea

Consequences of a repulsive core revisited



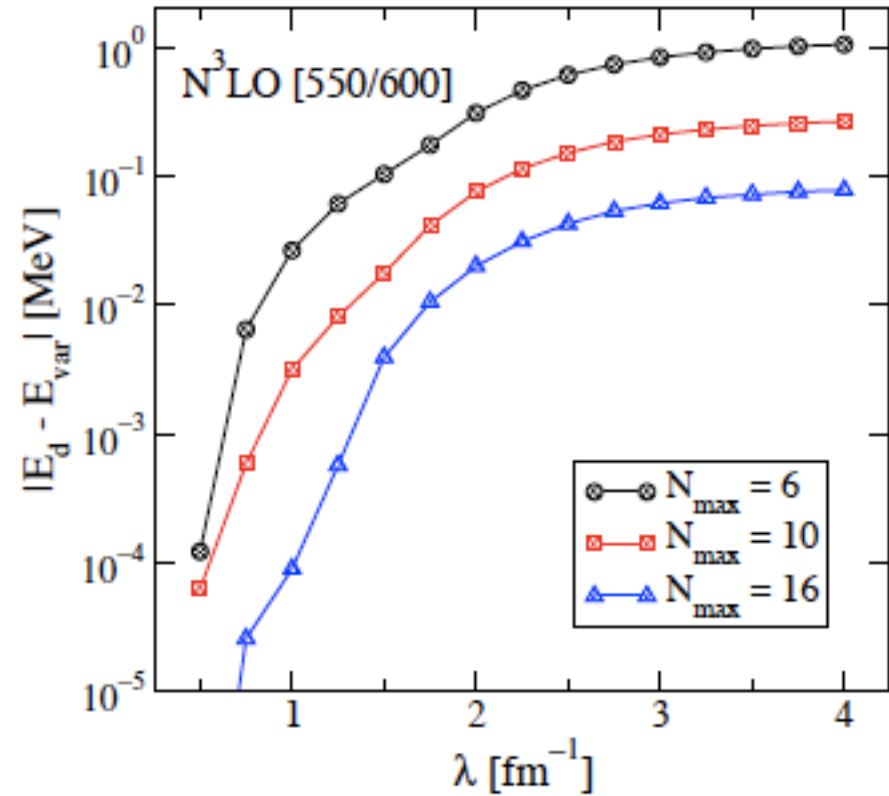
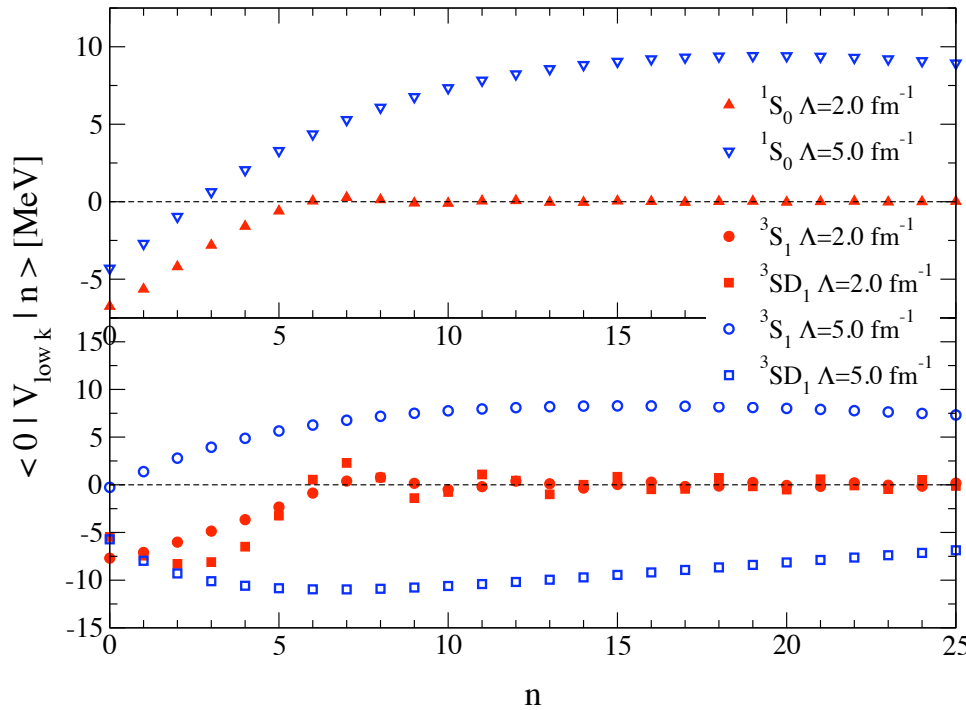
- Transformed potential \implies no short-range correlations in wf!
- Potential is now **non-local**: $V(\mathbf{r})\psi(\mathbf{r}) \longrightarrow \int d^3\mathbf{r}' V(\mathbf{r}, \mathbf{r}')\psi(\mathbf{r}')$
 - A problem for Green's Function Monte Carlo approach
 - Not a problem for many-body methods using HO matrix elements

Evolution of potential in coordinate space

AV18 1S_0 Potential

- Non-diagonal (“nonlocal”) potential, so how to plot?
- Calculate integral over off-diagonal $\int d\mathbf{r}' V(\mathbf{r}, \mathbf{r}')$
- If local, then equal to $V(r)$
- Core meltdown!

Faster convergence in HO basis expansions

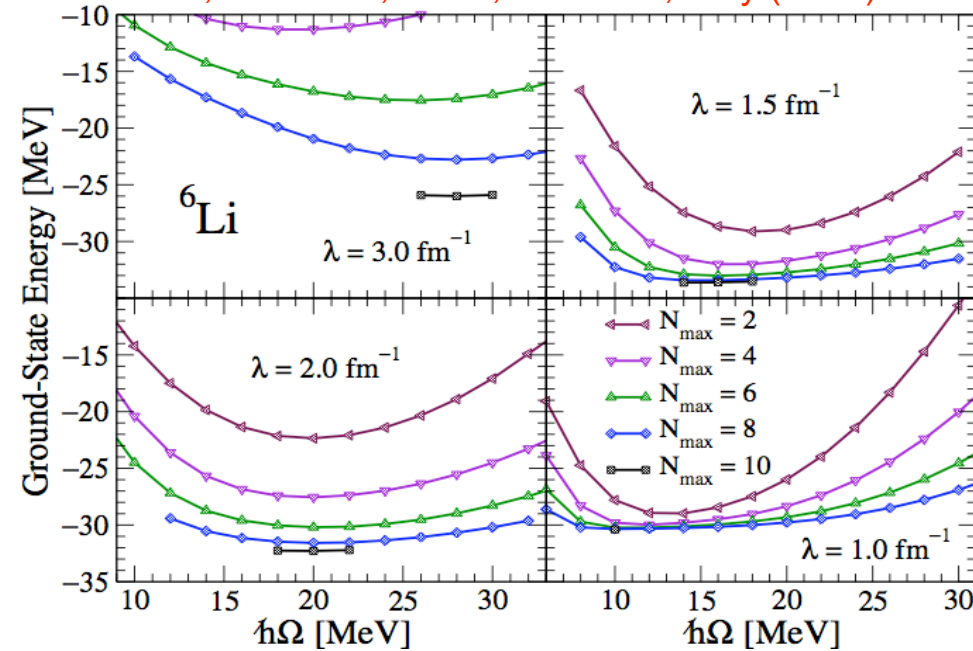


many-body methods that expand on finite HO basis converge much faster (weaker coupling to high momentum)

variational calculations improve (weaker correlations)

RG-Improved Convergence in ab-initio calculations

SKB, Furnstahl, Maris, Schwenk, Vary (2008)



Li-6 diagonalization
in HO basis

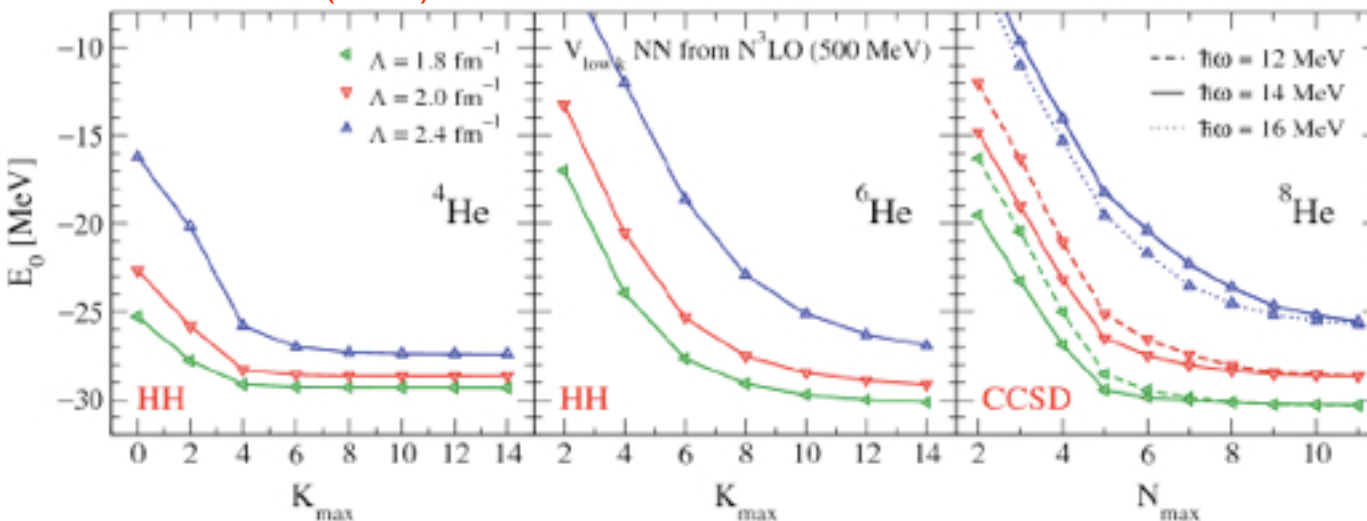
10^3 states for $N_{\text{max}} = 2$

versus

10^7 states for $N_{\text{max}} = 10$

Helium Halo Nuclei

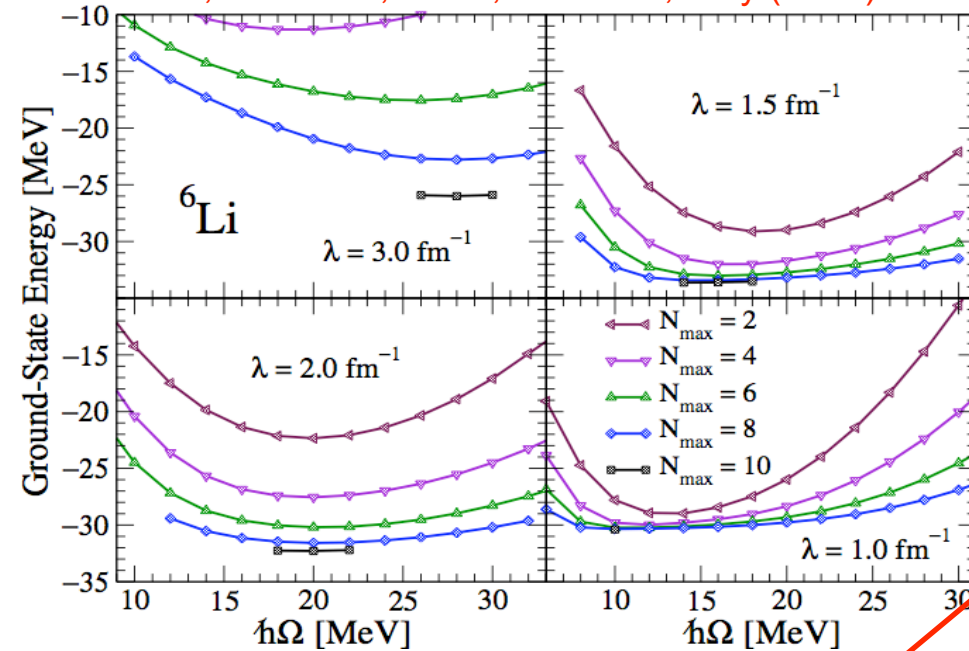
Bacca et al. (2009)



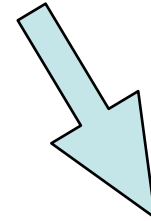
Ab-initio calculations
of heavier nuclei
accessible...

RG-Improved Convergence in ab-initio calculations

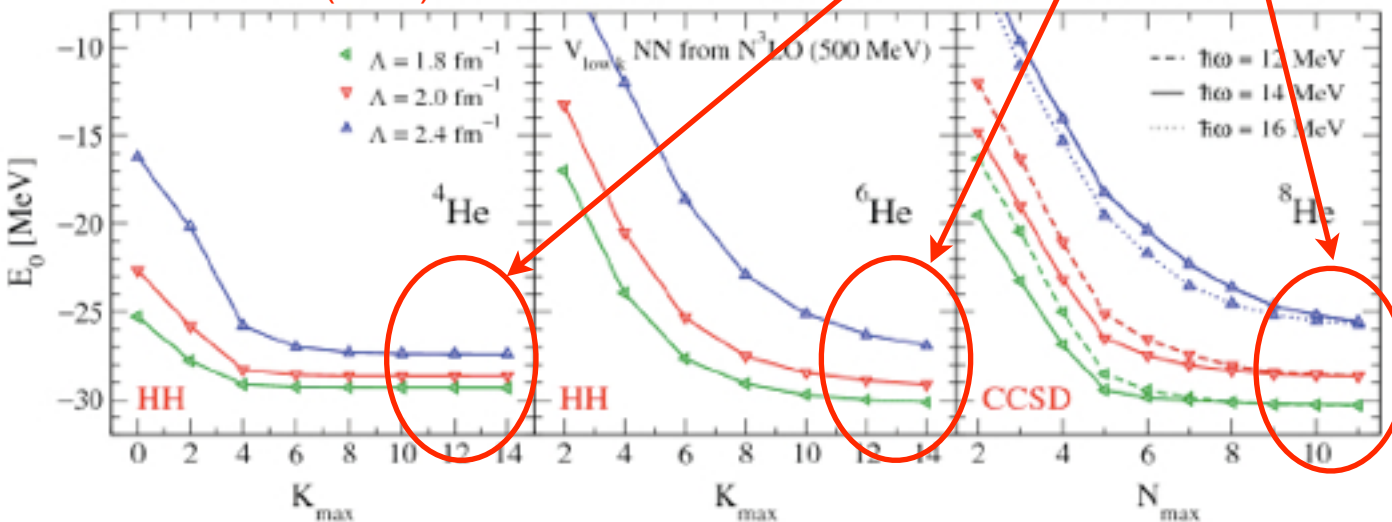
SKB, Furnstahl, Maris, Schwenk, Vary (2008)



Faster convergence, but λ -dependent answers!



Bacca et al. (2009)



We've neglected 3N interactions induced by RG.
(lecture 2)

Take-away points from lecture 1

- Nuclear forces are “resolution-dependent” quantities
- High resolution scales $\Lambda \gg k_{\text{data}}$ in most interaction models
 - strong coupling of low- and high- k states
 - highly non-perturbative with strong correlations (hard!)
- Strategy: Use RG to evolve to lower resolutions
 - exploits **decoupling**
 - various implementations available ($V_{\text{low } k}$, SRG flow eqns.)
 - faster convergence of many-body problems
 - correlations in wf's reduced dramatically
 - use cutoff dependence as a tool (theoretical uncertainties)
- Lectures 2 and 3 preview
 - EFT ideology, chiral EFT for nuclear potentials
 - details of RG alternatives, 3N (and higher) interactions
 - effective operators and factorization
 - in-medium SRG
 - more results in finite nuclei and nuclear matter
 - towards ab-initio energy density functionals