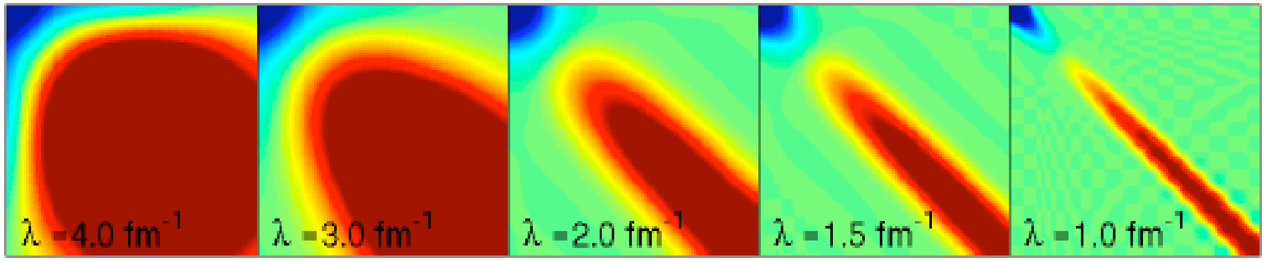

Renormalization group methods in nuclear few- and many-body problems

Lecture 2

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2011 National Nuclear Physics Summer School
University of North Carolina at Chapel Hill

Lecture 2 outline

1) Recap/questions from Lecture 1

2) low-energy effective theories (general)

3) chiral EFT for NN and NNN interactions

4) many-body interactions in flow equation

Questions to think about

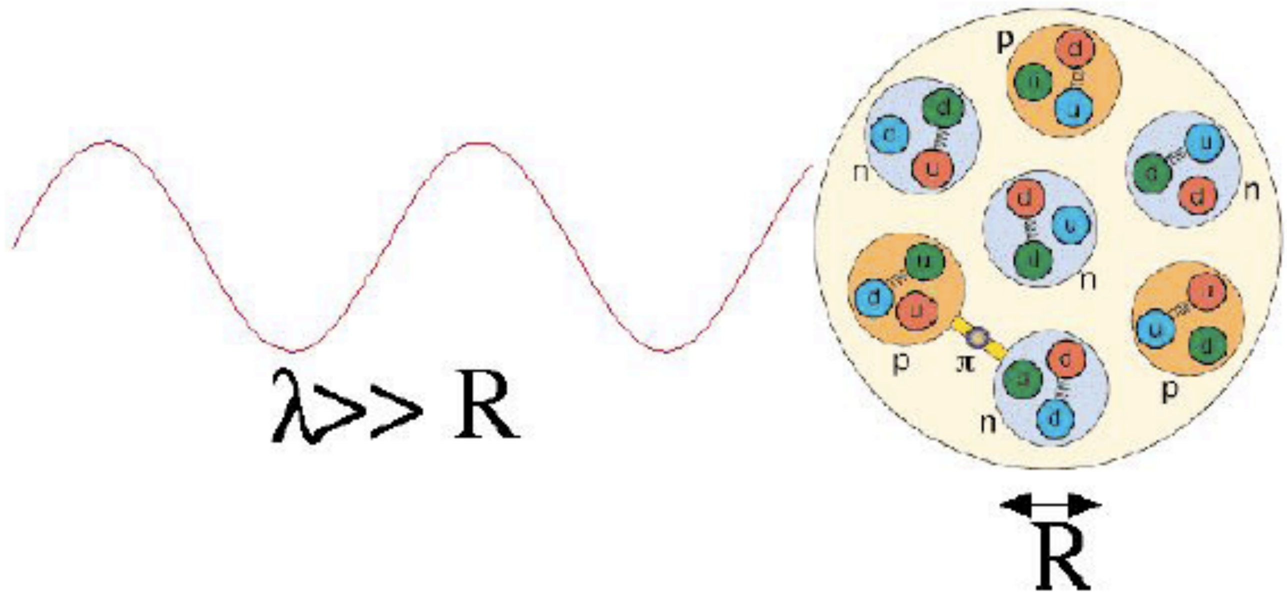
1) What do I mean when I say that the form of low energy effective theories are “universal” or model-independent ? (Think of the multipole expansion example)

2) Why do RG transformations only affect short-distance pieces of the Hamiltonian? Would you be alarmed if it modified long-distance pieces?

3) Last time we saw that using the RG to lower the cutoff in the nuclear Hamiltonian gives a much “softer” problem that is amenable to perturbative treatments.

However, we learned that this transformation “induces” 3-body (and higher) forces. What then, have we gained? Have we really simplified anything at all?

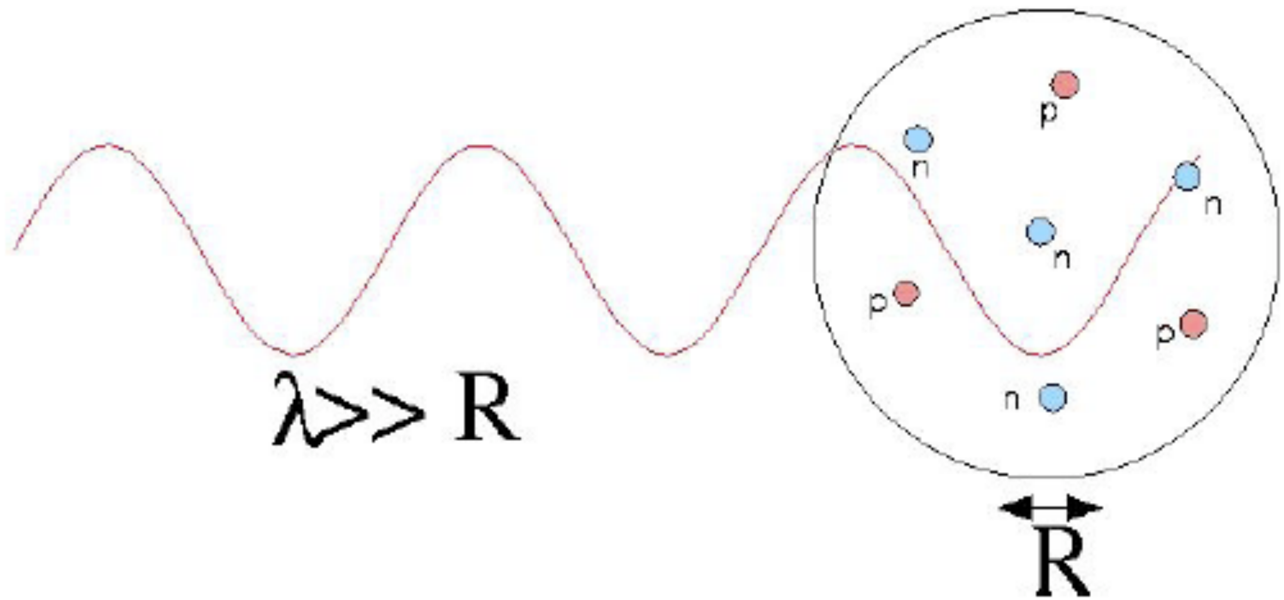
Principle of Low-Energy Effective Theories



- If a system is probed at low energies, fine details not resolved

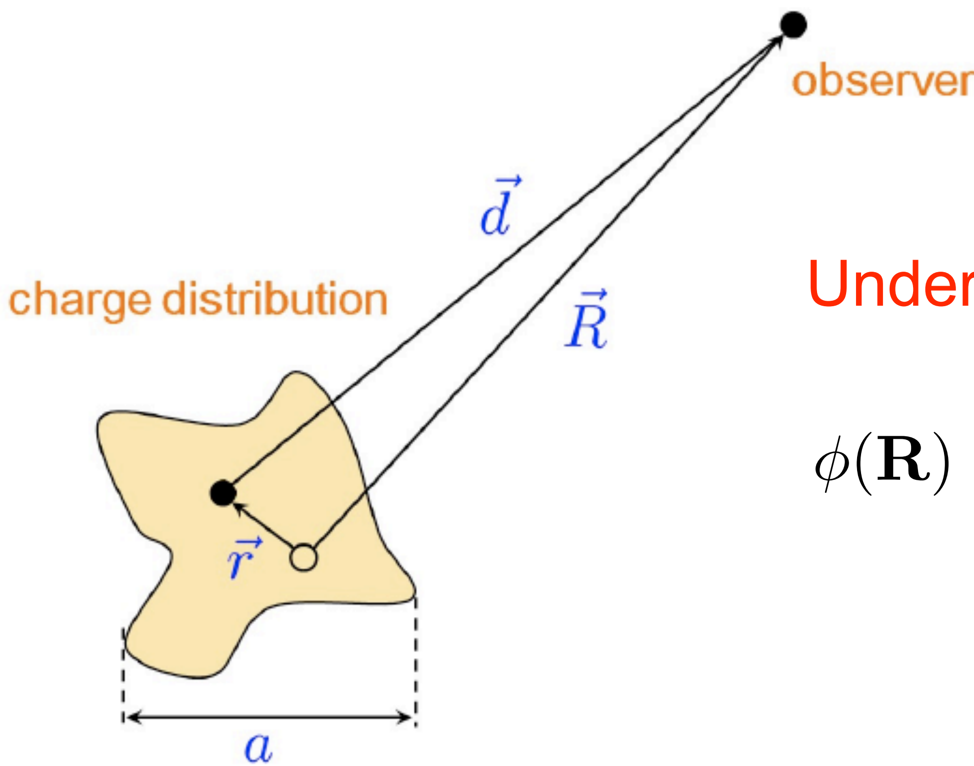
e.g., proton EM form factor $F(Q) \sim 1$ for $Q << 800$ MeV

Principle of Low-Energy Effective Theories



- If a system is probed at low energies, fine details not resolved
  - Use convenient dof to describe low-energy processes
  - Complicated short-distance structure replaced by something simpler without distorting low-E observables
  - Systematically achieved by effective field theories (EFT)

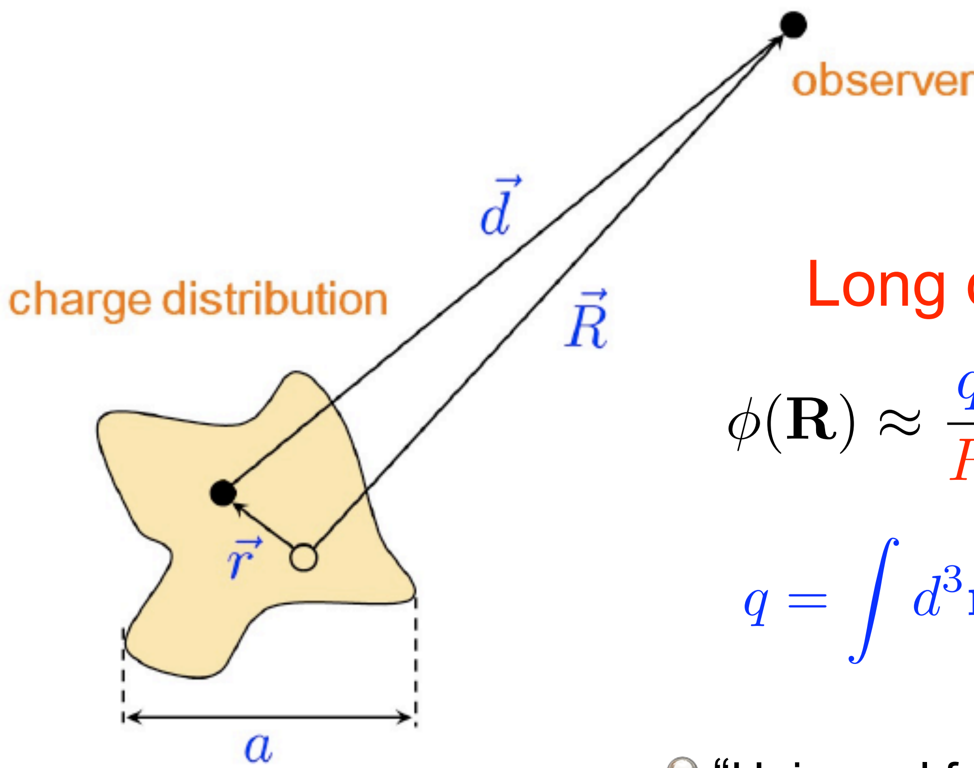
Example: Multipole expansion in E&M



Underlying theory:

$$\phi(\mathbf{R}) = \int d^3\mathbf{r} \, \frac{\rho(\mathbf{r})}{|\mathbf{R} - \mathbf{r}|}$$

Example: Multipole expansion in E&M



Underlying ρ replaced by pointlike multipoles

Long distance effective theory:

$$\phi(\mathbf{R}) \approx \frac{q}{R} + \frac{\mathbf{p}\cdot\mathbf{R}}{R^3} + \frac{Q_{ij}R_iR_j}{R^5} + \cdots$$

$$q = \int d^3\mathbf{r}\,\rho(\mathbf{r}) \quad \mathbf{p} = \int d^3r\,\mathbf{r}\,\rho(\mathbf{r})\,, \text{etc.}$$

- "Universal form" (same for **all** localized charge distributions) given by symmetry
- Details of ρ(r) encoded a few numbers (q,**p**, Q_{ij}) that can be calculated from "underlying" theory or extracted from experiment if ρ(r) unknown.

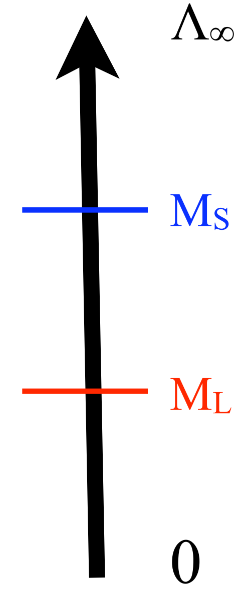
Low energy effective theories (QM)

Underlying theory with cutoff Λ_∞

$$V = V_L + V_S$$

known long-distance physics (e.g. 1π-exchange) with some scale M_L

short-distance physics with some scale M_s (e.g., ρ,ω-exchange)



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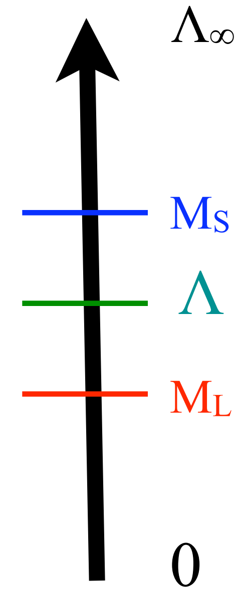


Now suppose we want an low E effective theory that describes physics up to some $M_L < \Lambda < M_S$.

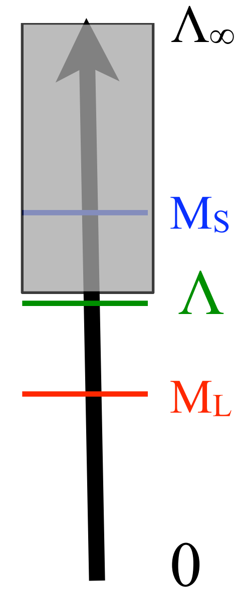
Low energy effective theories (QM)

Underlying theory with cutoff Λ_∞

$$V = V_L + V_S$$

known long-distance physics (e.g. 1π-exchange) with some scale M_L

short-distance physics with some scale M_s (e.g., ρ,ω-exchange)



Our task is to "integrate out" states above Λ using the RG

Generic form of the effective theory

$$V_{eff} = V_L + \delta V_{c.t.}(\Lambda)$$

Low energy effective theories (QM)

Underlying theory
with cutoff Λ_∞

$$V = V_L + V_S$$

known
long-distance
physics (e.g.
1π-exchange)
with some scale
M_L

short-distance
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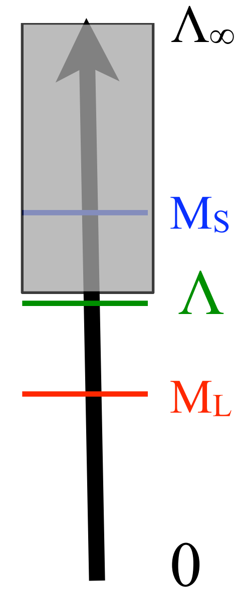


Our task is to "integrate out" states above Λ using the RG

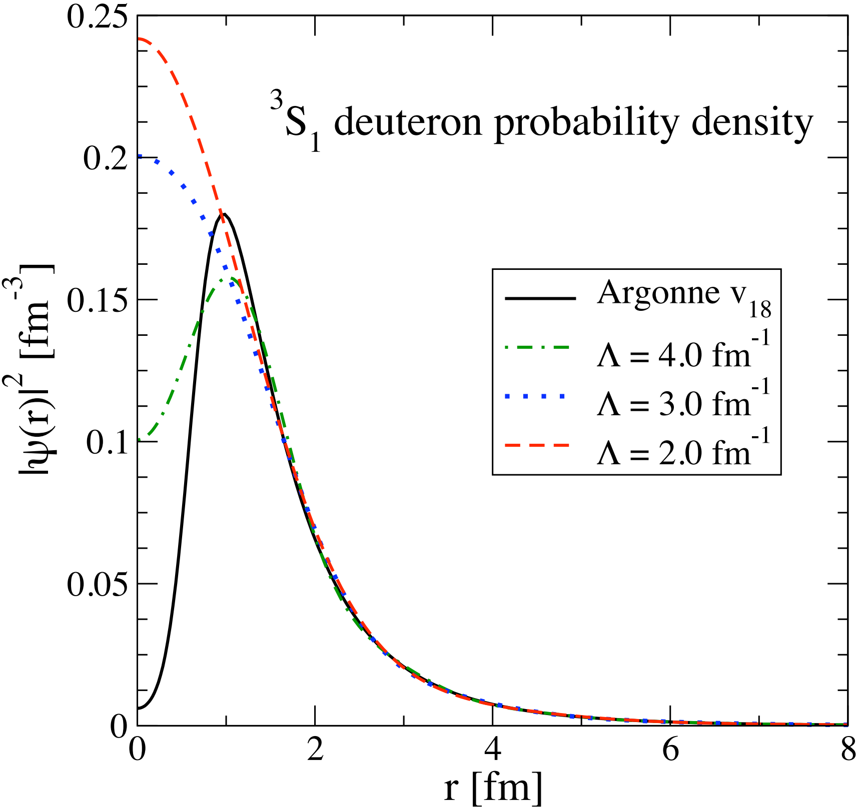
Generic form of
the effective theory

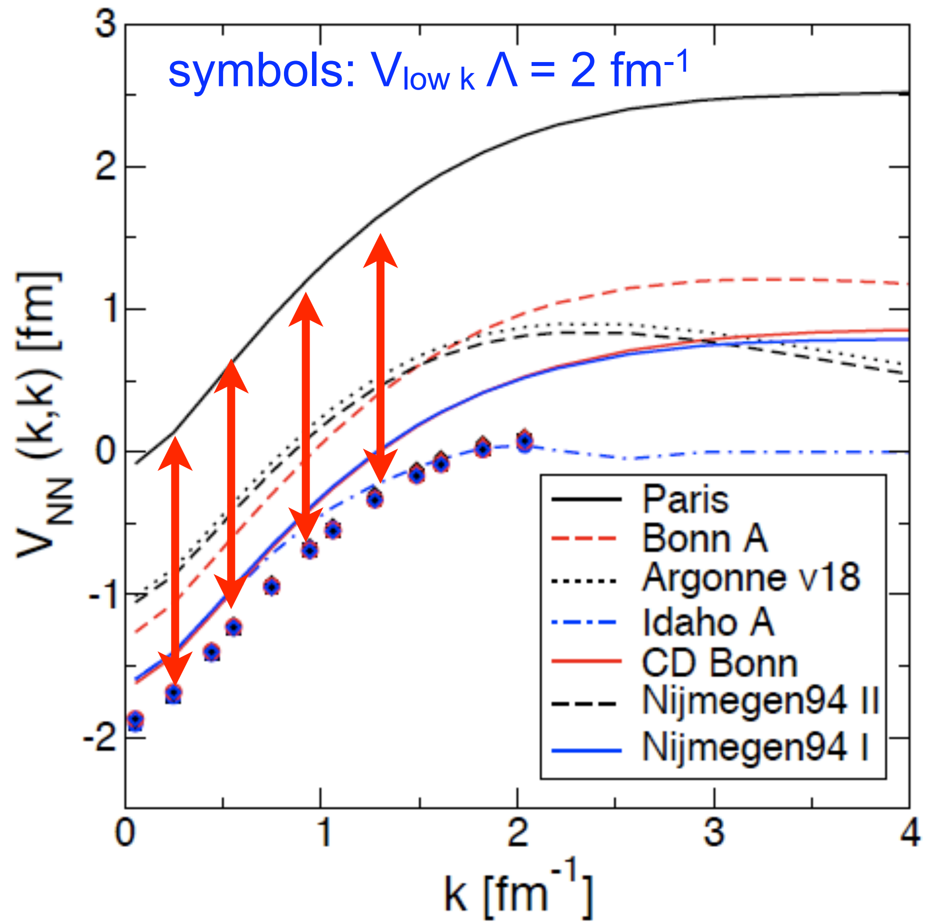
$$V_{eff} = V_L + \delta V_{c.t.}(\Lambda)$$

$$\delta V_{ct} = C_0(\Lambda)\delta^3(\mathbf{r}) + C_2(\Lambda)\nabla^2\delta^3(\mathbf{r}) + \cdots$$

encodes the
effects of integrated
dof on low-E physics

universal form; depends
only on symmetries





Evidence that $V_{eff} = V_L + \delta V_{c.t.}(\Lambda)$

- main effect of RG evolution is a constant shift (delta function!)
- tail of deuteron wf doesn't change
- consistent with collapse to "universal" interaction

Low energy effective theories (QM)

Generic form of the effective theory

$$V_{eff} = V_L + \delta V_{c.t.}(\Lambda)$$

$$\delta V_{ct} = C_0(\Lambda)\delta^3(\mathbf{r}) + C_2(\Lambda)\nabla^2\delta^3(\mathbf{r}) + \cdots$$

encodes the effects of integrated dof on low-E physics

universal form; depends only on symmetries

Like the multipole example, the complicated short-distance structure of the "true" theory is encoded in a few numbers that can be calculated from the the underlying theory

OR

in cases where the short-distance structure is unknown or too complicated, can be extracted from low E data

Effective Field Theory (EFT) is based on these ideas (see Lepage reference)

Construction of nuclear potentials via chiral EFT

Weinberg, van Kolck, Epelbaum, Machleidt, ...

1) Identify the **relevant degrees of freedom** for the resolution scale of nuclei (nucleons and pions)

2) Identify **relevant symmetries** of low-E QCD (spontaneously broken chiral symmetry)

3) Write the **most general** Lagrangian consistent with the symmetries (infinite number of interactions; non-normalizeable)

4) Design an **organizational scheme** that can distinguish between more or less important contributions. (low-momentum expansion; **power counting**)

5) Calculate finite # of Feynman diagrams to the desired accuracy dictated by the power counting.

Reviews:

Bedaque and van Kolck, Ann. Rev. Nucl. Part. Sci. 52 (2002) 339, nucl-th/0205058.

Machleidt, arxiv:0704.0807.

Epelbaum, Hammer, Meißner, Rev. Mod. Phys. 81, 1773 (2009); arXiv:0811.1338.

1. Identify relevant dof/separation of scales

Kinetic energy at Fermi surface: $T \approx 80$ MeV (Fermi momentum $k_F \sim 1.4$ / fm)

Pion mass: $m_\pi \approx 140$ MeV

Nucleon resonance (Delta excitation): $E_\Delta \approx 300$ MeV

Rho meson: $m_\rho \approx 770$ MeV

Omega meson: $m_\Omega \approx 780$ MeV

Nucleon mass: $m_N \approx 940$ MeV

EFTs:

1. Pion-less EFT (low-density nuclear/neutron matter, few-body systems)
2. Chiral EFT (includes pions): High-momentum cutoff $\Lambda \sim 500 \dots 600$ MeV/c
3. Chiral EFT (includes pions and delta resonance) $\Lambda \sim 500 \dots 600$ MeV/c

2. Identify low E symmetries of QCD

- Besides space-time symmetries and parity, what else?
- Is SU(3) color gauge symmetry encoded in the EFT?
- Consider chiral symmetry:

$$\mathcal{L}_{\rm QCD} = \overline{q}_L i \not{D} q_L + \overline{q}_R i \not{D} q_R - \frac{1}{2} \mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu} - \overline{q}_R \mathcal{M} q_L - \overline{q}_L \mathcal{M} q_R$$

$$\not{D} \equiv \not{\partial} - i g_s \not{G}^a T^a \;; \qquad T^a = SU(3) \text{ Gell-Mann matrices}$$

$$\mathcal{M} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \qquad SU(2) \text{ quark mass matrix}$$

$$q_{L,R} = \frac{1}{2}(1 \pm \gamma_5) q \,, \qquad \text{projection on left,right-handed quarks}$$

- m_u and m_d are small compared to typical Hadrons

  (~ 5 and 9 MeV at 1 GeV renormalization scale versus about 1 GeV)

$$\mathcal{M} \approx 0 \Longrightarrow \text{approximate } SU(2)_L \otimes SU(2)_R \text{ chiral symmetry}$$

2. Identify low E symmetries of QCD

- What happens if we have a symmetry of the Hamiltonian?
  - Could have a multiplet of ~ degenerate states (masses)
  - Could be a spontaneously broken (hidden) symmetry

- Experimentally we notice:
  - Isospin multiplets like (p,n) or (Σ^+, Σ^-, Σ^0), etc.
  - But we **don't** find opposite parity partners for these states with close to the same mass. Axial part spontaneously broken.

- Pions are "pseudo Goldstone bosons." Explicit symmetry breaking of quark masses ($u \neq d \neq 0$) implies $m_\pi << M_{QCD}$ but non-zero.

> Chiral symmetry relates states with different numbers of pions and dictates that pion interactions get weak at low energy $\Longrightarrow$ pion as long-distance dof in χEFT!

3) Write down the most general Lagrangian consistent with symmetries; hierarchy of terms => Power counting

$$\mathcal{L}_{\text{eft}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN}$$

$$\begin{aligned}
\mathcal{L}^{(0)} &= \frac{1}{2}\partial_\mu \boldsymbol{\pi}\cdot\partial^\mu \boldsymbol{\pi} - \frac{1}{2}m_\pi^2 \boldsymbol{\pi}^2 + N^\dagger\left[i\partial_0 + \frac{g_A}{2f_\pi}\boldsymbol{\tau}\boldsymbol{\sigma}\cdot\nabla\boldsymbol{\pi} - \frac{1}{4f_\pi^2}\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}})\right]N \\
&\quad - \frac{1}{2}C_S(N^\dagger N)(N^\dagger N) - \frac{1}{2}C_T(N^\dagger\boldsymbol{\sigma}N)(N^\dagger\boldsymbol{\sigma}N) + \ldots, \\
\mathcal{L}^{(1)} &= N^\dagger\left[4c_1 m_\pi^2 - \frac{2c_1}{f_\pi^2}m_\pi^2\boldsymbol{\pi}^2 + \frac{c_2}{f_\pi^2}\dot{\boldsymbol{\pi}}^2 + \frac{c_3}{f_\pi^2}(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}) \right.\\
&\quad\left. - \frac{c_4}{2f_\pi^2}\epsilon_{ijk}\,\epsilon_{abc}\,\sigma_i\tau_a(\nabla_j\,\pi_b)(\nabla_k\,\pi_c)\right]N \\
&\quad - \frac{D}{4f_\pi}(N^\dagger N)(N^\dagger\boldsymbol{\sigma}\boldsymbol{\tau}N)\cdot\nabla\boldsymbol{\pi} - \frac{1}{2}E\,(N^\dagger N)(N^\dagger\boldsymbol{\tau}N)\cdot(N^\dagger\boldsymbol{\tau}N) + \ldots
\end{aligned}$$

(Weinberg counting)

Infinite # of unknown parameters (LEC's), but leads to hierarchy of diagrams: $\nu = -4 + 2N + 2L + \sum_i (d_i + n_i/2 - 2) \geq 0$

N = # external nucleons

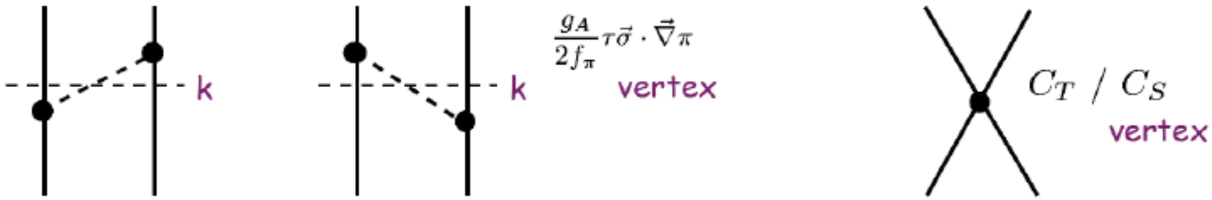
L = # loops

d_i = # derivatives or m_π at i^{th} vertex

n_i = # nucleons at i^{th} vertex

5) Calculate to the desired order

- LO time-ordered diagrams



$$V_{1\pi}(\mathbf{q}) = \left(\frac{g_A}{2f_\pi}\right)^2 \frac{\tau_1\cdot\tau_2}{2\omega_q}\left\{\frac{\sigma_1\cdot\mathbf{q}\,\sigma_2\cdot\mathbf{q}}{\frac{\mathbf{p}_1^2}{2M}+\frac{\mathbf{p}_2^2}{2M}-\frac{\mathbf{p}_1'^2}{2M}-\frac{\mathbf{p}_2^2}{2M}-\omega_q} + \text{2nd diagram}\right\}$$
$$= -\left(\frac{g_A}{2f_\pi}\right)^2 \tau_1\cdot\tau_2 \frac{\sigma_1\cdot\mathbf{q}\,\sigma_2\cdot\mathbf{q}}{\mathbf{q}^2+m_\pi^2}$$

one pion exchange

- zero-range contact term at LO

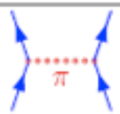
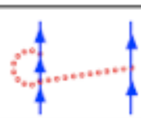
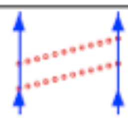
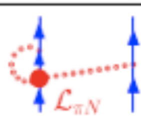
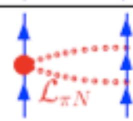
$$V_C = C_S + C_T\sigma_1\cdot\sigma_2$$

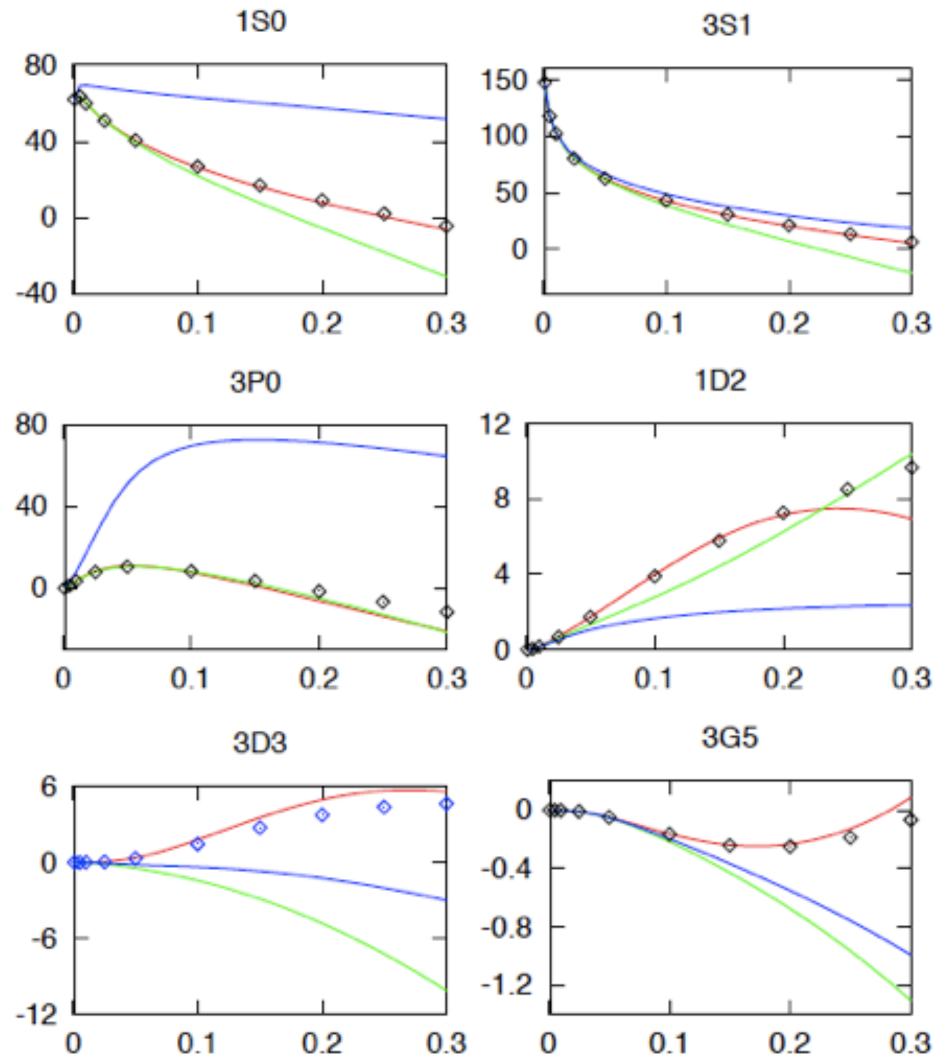
- regularize (WHY?)

$$V(\mathbf{p}',\mathbf{p}) \rightarrow e^{-(p'/\Lambda)^{2n}} V(\mathbf{p}',\mathbf{p}) e^{-(p/\Lambda)^{2n}}$$

Chiral EFT for two-nucleon potential

- Epelbaum, Meißner, et al.
- Also Entem, Machleidt
- $\mathcal{L}_{\pi N}$ + match at low energy

Q^ν	1π	2π	$4N$
Q^0		—	 (2)
Q^1			
Q^2			 (7)
Q^3			
Q^4	many	many	 (15)



Approaches level of accuracy (and fit parameters via the LECs) of "conventional" models at N3LO

Why the cutoff Λ?

- Need to match unknown LECs to data (e.g., phaseshifts). Solve LS eqn:

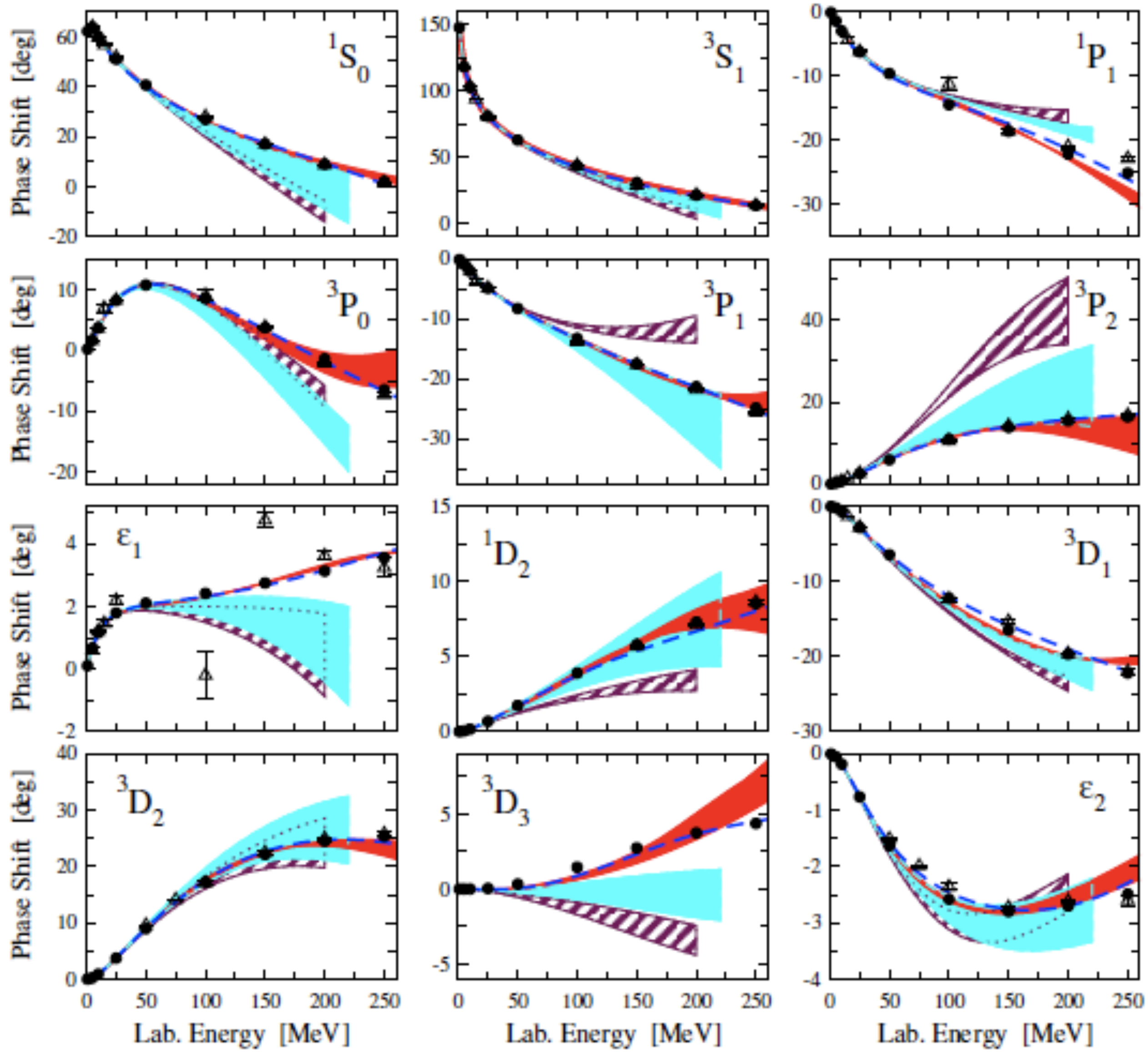
$$T(k,k) = V(k,k) + \frac{2}{\ } \int q^2 dq \frac{V(k,q)T(q,k)}{k^2 - q^2} \quad \text{where} \quad \tan \quad (k) = -kT(k,k)$$

- NN loop integral UV divergent => regularization and renormalization
  - details of cutoff (sharp, smooth, etc.) don't matter to low E physics
  - LECs now "run" with Λ

- No such thing as "the" chiral potential of a given order. Infinitely many regularization/renormalization schemes => any differences should be higher order effects.

- **Truncation errors** of observables go as $\mathcal{O}(\frac{Q^\nu}{\Lambda^\nu})$
- **"theoretical error bars" from varying Λ**

Error bands thru N^{3}LO (Epelbaum et al., nucl-th/0509032)



Question: Consider two high-precision NN potentials from chiral EFT with different cutoffs. How will the solutions of the nuclear many-body problem depend on the cutoff?

1. There will be (almost) no cutoff dependence in the two-body system.
2. There will be (almost) no cutoff dependence in many-body systems as Nature must be cutoff independent.
3. The cutoff dependence measures missing contributions from higher orders.
4. The cutoff dependence measures missing short-range contributions from higher orders.

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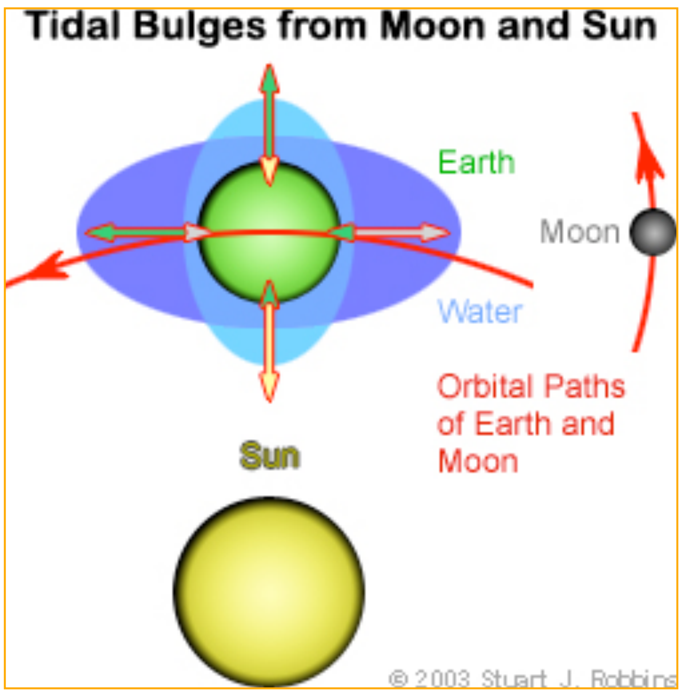
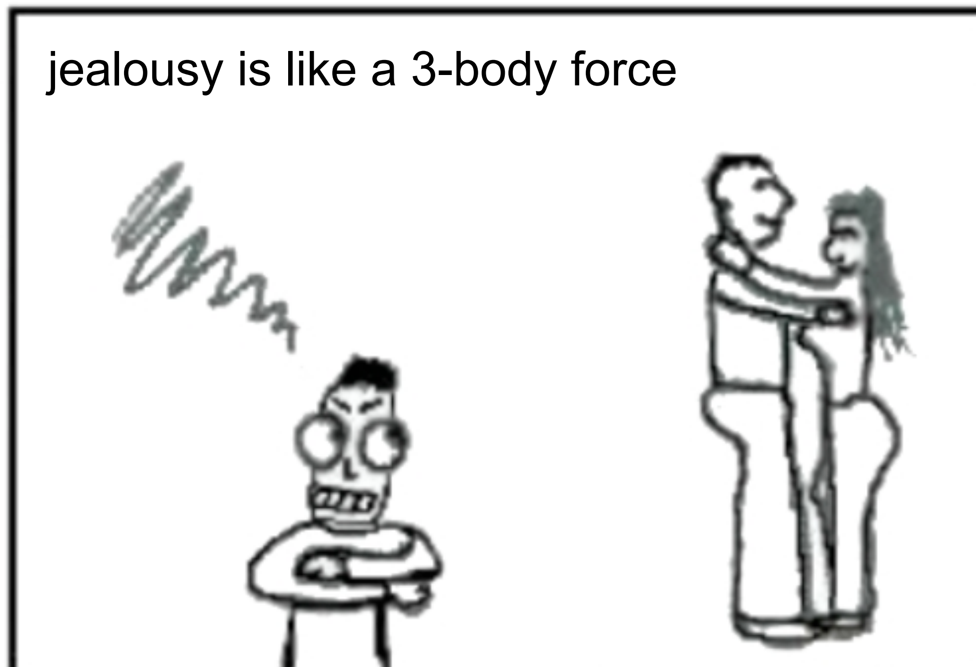
3. The cutoff dependence measures missing contributions from higher orders.
4. The cutoff dependence measures missing short-range contributions from higher orders.
5. Answers 1 and 4 are correct.

Three-body force

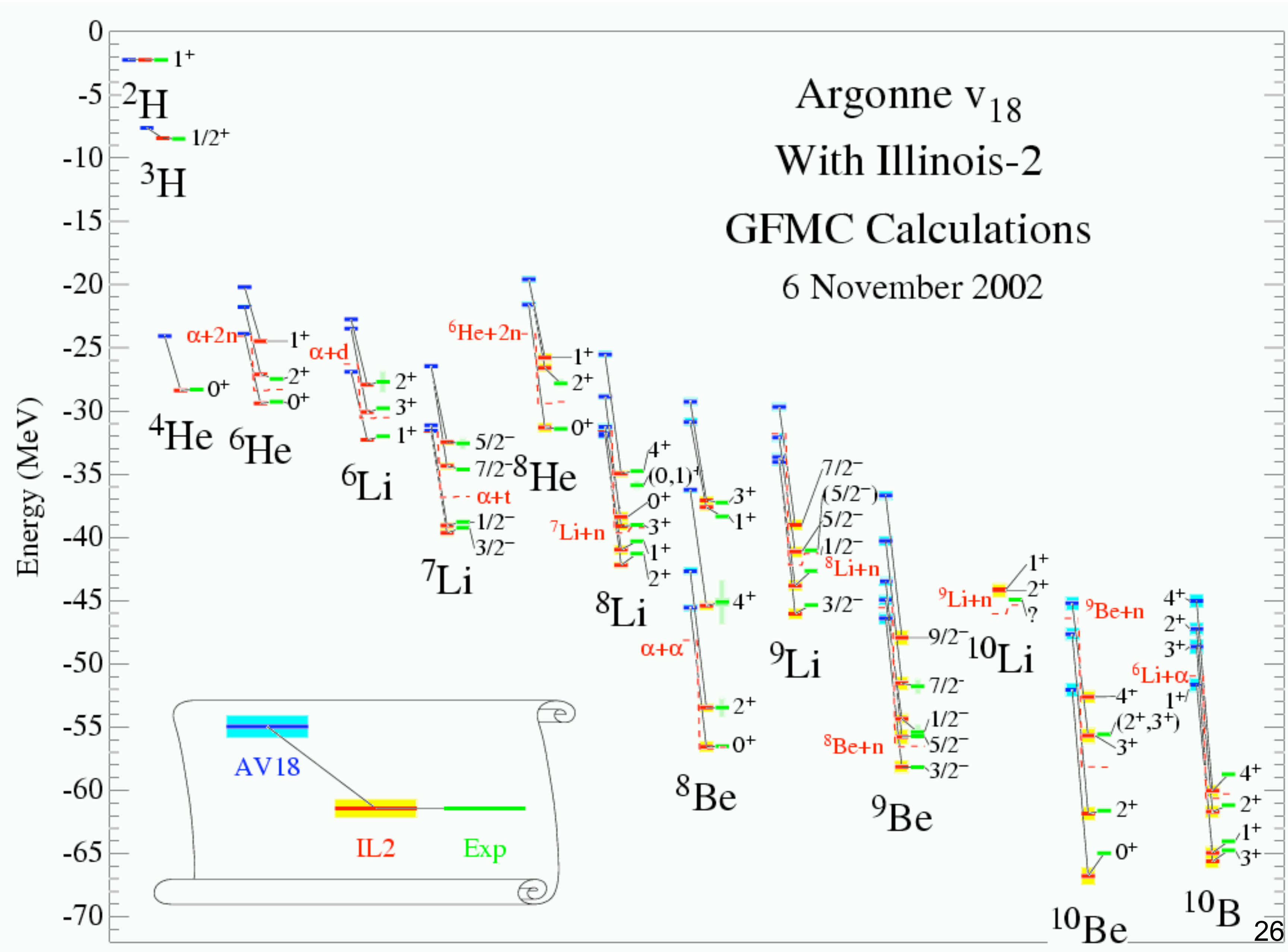
From Wikipedia, the free encyclopedia

A **three-body force** is a force that does not exist in a system of two objects but appears in a three-body system. In general, if the behaviour of a system of more than two objects cannot be described by the two-body interactions between all possible pairs, as a first approximation, the deviation is mainly due to a three-body force.

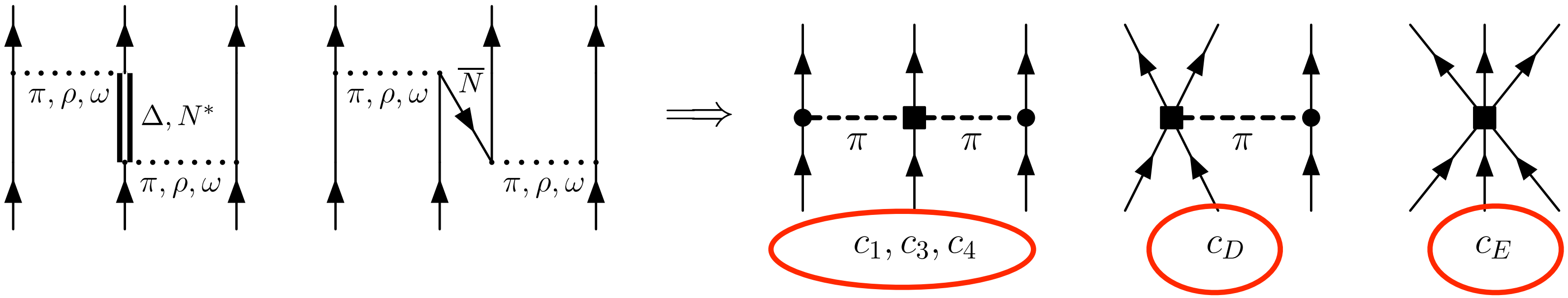




Evidence for 3N in light nuclei: overall binding & level ordering



Eliminating DOF leads to 3-body forces



- integrating out non-nucleonic DOF and/or high-momentum states renormalizes the strength of 3N (and higher) interactions.

- artificial to speak of "true" 3N and "induced" 3N forces!

Leading three-nucleon force

1. Long-ranged two-pion term (Fujita & Miza …)
2. Intermediate-ranged one-poin term
3. Short-ranged three-nucleon contact

The question is not: Do three-body forces enter the description?

The (only) question is: How large are three-body forces?

And at what resolution scale?

A theorem for three-body Hamiltonians

Polyzou and Glöckle, Few Body Systems 9, 97 (1990)

Different two-body Hamiltonians can be made to fit two-body and three-body data by including a 3NF into one of the Hamiltonians.

Theorem. *Let*

$$H_{ij} = H_i + H_j + V_{ij} \quad \textit{and} \quad \bar{H}_{ij} = H_i + H_j + \bar{V}_{ij} \tag{1.1}$$

be two-body Hamiltonians with the same binding energies and scattering matrices for each pair of particles i and j. Assume that the two-body Hamiltonians are asymptotically complete and that the unitary transformations relating these two-body Hamiltonians, which necessarily exist, have bounded Cayley transforms. Then there exists a three-body interaction, W, such that the two three-body Hamiltonians

$$H = H_1 + H_2 + H_3 + V_{12} + V_{23} + V_{31} \tag{1.2}$$

and

$$\bar{H}' = \bar{H} + W \tag{1.3}$$

with

$$\bar{H} = H_1 + H_2 + H_3 + \bar{V}_{12} + \bar{V}_{23} + \bar{V}_{31} \tag{1.4}$$

have the same binding energies and scattering matrix.

Corollary. *Under the assumptions of the theorem, if $V_{(123)}$ is a three-body interaction then there exists another three-body interaction $\bar{V}_{(123)}$ such that*

$$H = H_1 + H_2 + H_3 + V_{12} + V_{23} + V_{31} + V_{(123)}$$

and

$$\bar{H} = H_1 + H_2 + H_3 + \bar{V}_{12} + \bar{V}_{23} + \bar{V}_{31} + \bar{V}_{(123)}$$

have the same binding energies and scattering matrix.

Implications:

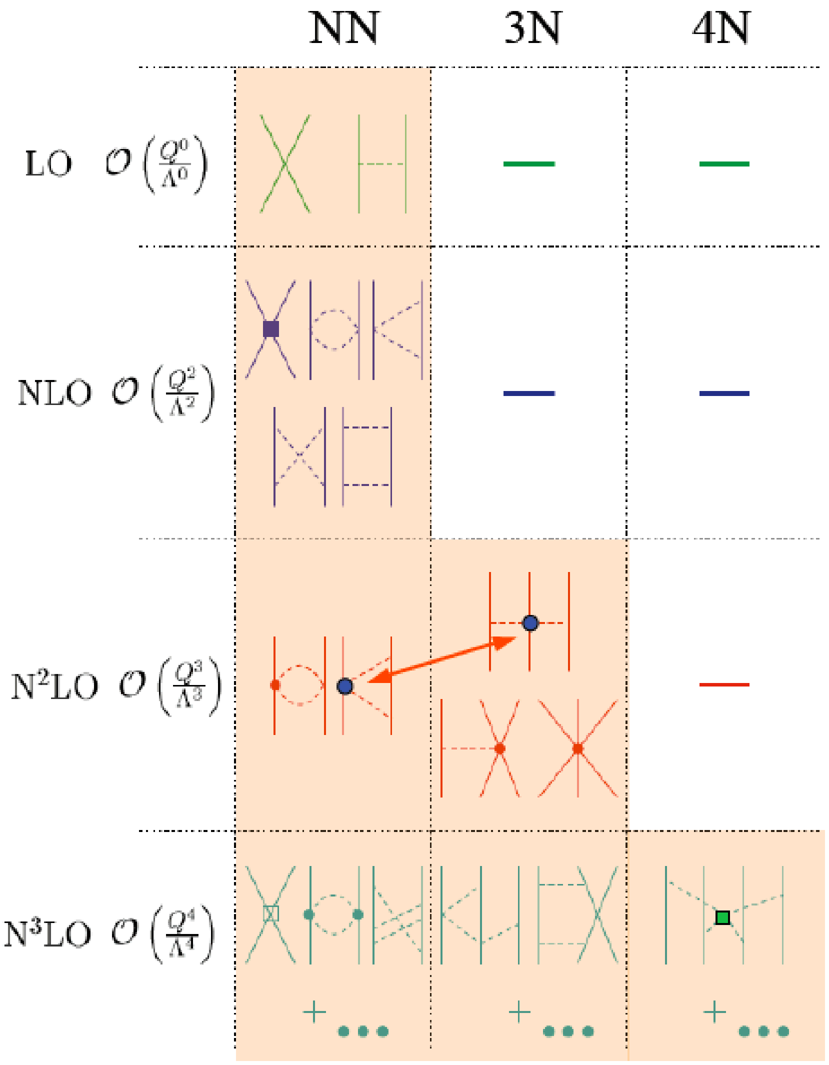
(1) There are no experiments measuring only three-body binding energies and phase shifts that can determine if there are no three-body forces in a three-body system. The question makes no sense. The correct statement is that there may be some systems for which it is possible to find a representation in which three-body forces are not needed.

(2) Different off-shell extensions of two-body forces can be equivalently realized as three-body interactions.

(4) Three-body forces cannot be determined in a manner that is independent of the two-body interaction.

Few-body forces from Chiral EFT

Separation of scales: low momenta Q << Λ_b breakdown scale

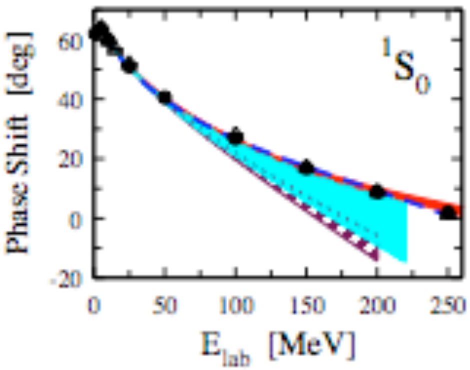


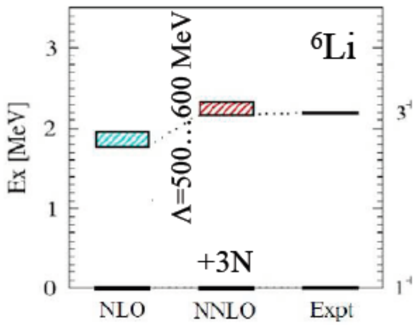
Explains 2N > 3N > 4N

Formal Consistency

- NN and NNN from same Lagrangian
- ππ and πN, electroweak
- Broken chiral symmetry of QCD

Error estimates from Λ variation





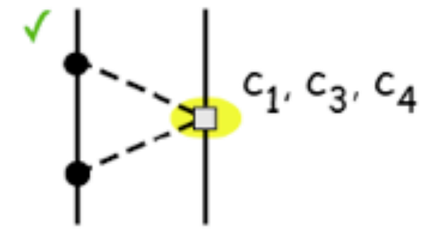
from A. Nogga

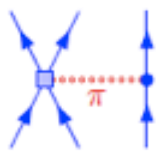
Weinberg, van Kolck, Epelbaum, Meissner, Machleidt, …

Fitting 3NF LEC's at N^{2}LO [A. Nogga]

- Fitting c_1, c_3, and c_4

	c_1	c_3	c_4
NN phase shift analysis	-0.76	-4.78	3.96
πN scattering (dispersion rel.)	-0.81	-4.70	3.40
πN scattering (directly)	-1.23	-5.94	3.47
NN pert. 3F4	-0.81	-3.40	3.40
NN potential fit to data	-0.81	-3.20	5.40

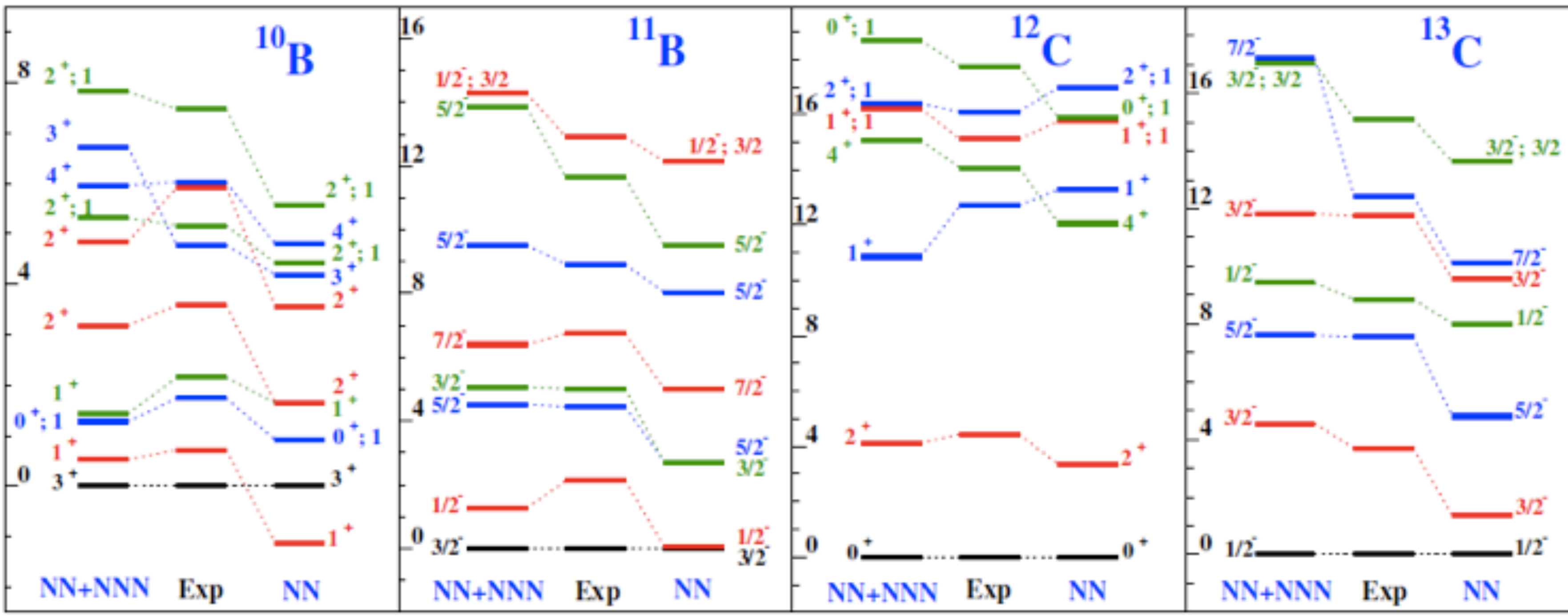


- Significant uncertainties!
- Fitting D 

and E 
  - D appears in pion production from NN, but not analyzed
  - E requires a 3N observable
  - Typically D and E fit together to triton binding energy and ^{4}He binding energy *or* radius; or sometimes to 3-body energy and scattering length

No-Core Shell Model (NCSM) with 3NF

- Nuclear structure results point to importance of 3NF
  - Note ^{10}B ground state
  - Note spin-orbit splittings



[Navratil et al., (2007)]

Extras

Green's Function Monte Carlo

Idea:

1. Determine accurate approximate wave function via variation of the energy (The high-dimensional integrals are done via Monte Carlo integration).

$$E = \frac{\langle \Psi_{\text{trial}} | \hat{H} | \Psi_{\text{trial}} \rangle}{\langle \Psi_{\text{trial}} | \Psi_{\text{trial}} \rangle}$$

2. Refine wave function and energy via projection with Green's function

$$|\Psi\rangle = \lim_{\tau \to \infty} e^{-\tau(\hat{H}-E)} |\Psi_{\text{trial}}\rangle$$

☺ Virtually exact method.

☹ Limited to certain forms of Hamiltonians; computationally expensive method. (local potentials only)

Working in a finite model space

NCSM and Coupled-cluster theory solve the Schroedinger equation in a model space with a *finite* (albeit large) number of configurations or basis states.

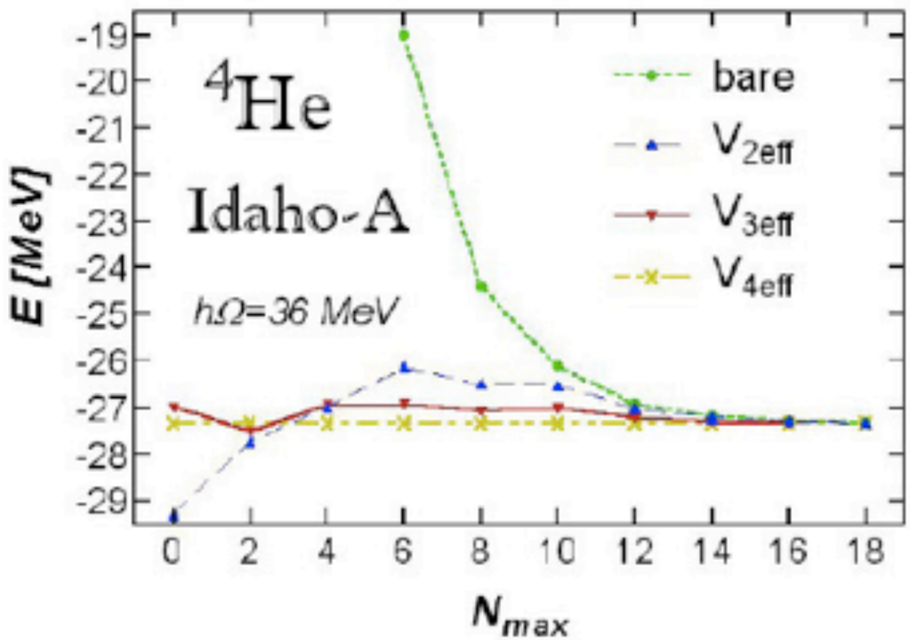
Problem: High-momentum components of high-precision NN interactions require enormously large spaces.

Solution: Get rid of the high-momentum modes via a renormalization procedure. (Vlow-k is an example)

Price tag:

Generation of 3, 4, …, A-body forces unavoidable.

Observables other than the energy also need to be transformed.



E. Ormand
http://www.phy.ornl.gov/npss03/ormand2.ppt

No core shell model

Idea: Solve the A-body problem in a harmonic oscillator basis.

1. Take K single particle orbitals
2. Construct a basis of Slater determinants
3. Express Hamiltonian in this basis
4. Find low-lying states via diagonalization

☺ Get eigenstates and energies

☺ No restrictions regarding Hamiltonian

☹ Number of configurations and resulting matrix very large: There are

$$\binom{K}{A} = \frac{K!}{(K-A)!A!}$$

ways to distribute A nucleons over K single-particle orbitals.

Coupled Cluster (CC) Calculations (figures from G. Hagen)

- Size extensive, based on the Linked Cluster theorem
- Softer polynomial scaling with # of orbitals
- Extended to include 3NF's (Dean, Hagen, Papenbrock...)

$$|\Psi\rangle = e^{T^{(A)}}|\Phi\rangle, \quad T^{(A)} = \sum_{k=1}^{m_A} T_k$$

$$T_1 = \sum_{\substack{i\\a}} t_i^a |\Phi_i^a\rangle, \quad T_2 = \sum_{\substack{i>j\\a>b}} t_{ij}^{ab}|\Phi_{ij}^{ab}\rangle, \quad T_3 = \sum_{\substack{i>j>k\\a>b>c}} t_{ijk}^{abc}|\Phi_{ijk}^{abc}\rangle$$