### Neutrino interactions and cross sections

- $\cdot v$  scattering on a free nucleon
- $\cdot v$  electron scattering
- $\cdot$  v scattering on light nuclei at low energies
- v quasielastic scattering
- v pion production
- v deep inelastic scattering

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In this talk I use a number of slides of Sam Zeller (LANL) from her excellent talk on this topic at the INSS Summer School at Fermilab, July 2009

# Number of v Events

• neutrino interaction cross section plays a critical role in determining number of  $\nu$  interactions expect to collect

$$N_{v}(E) \sim \Phi_{v}(E) \times \sigma_{v}(E) \times \text{target}$$

$$v \text{ flux}$$
(# neutrinos)  
depends on your v source
$$v \text{ cross section}$$

$$\frac{G_{F}}{\sqrt{2}} = \frac{g^{2}}{8M_{W}^{2}}$$

$$tiny (\sim 10^{-38} \text{ cm}^{2})$$

$$\sigma_{v}^{\text{tot}} \sim E_{v}$$

Fundamental couplings. There is until now no indication that neutrinos interact by any other nonstandard way.



$$J_W^\mu = \bar{u}_{f'} \tau_+ \gamma^\mu (1 - \gamma^5) u_f$$

Charged current

$$J_Z^\mu = \bar{u}_f \gamma^\mu (g_V^f - g_A^f \gamma^5) u_f$$

$$g_V^f = T_3^f - 2Q^f \sin^2 \theta_W$$
  
$$g_A^f = T_3^f$$

Neutral current



## Charged Current (CC)

- neutrino in
- charged lepton out

$$\begin{array}{ccc} \nu_{e} \rightarrow e^{-} & \overline{\nu_{e}} \rightarrow e^{+} \\ \nu_{\mu} \rightarrow \mu^{-} & \overline{\nu_{\mu}} \rightarrow \mu^{+} \\ \nu_{\tau} \rightarrow \tau^{-} & \overline{\nu_{\tau}} \rightarrow \tau^{+} \end{array}$$

this is how we detected neutrinos in the first place

- flavor of outgoing lepton "tags" flavor of incoming neutrino
- charge of outgoing lepton determines whether  ${\bf v}$  or anti- ${\bf v}$



### Neutral Current (NC)

- neutrino in
- neutrino out

#### 1st observed in 1972



$$\nu_{\mu} \ e^{-} \rightarrow \nu_{\mu} \ e^{-}$$

Different neutrino sources determine the range of energies Description of the nuclear and hadronic effects is also energy dependent



• also, treatment of nuclear effects is energy dependent ...



### The simplest hadronic system - a single nucleon at low (a few MeV) energies

The neutron decay and the antineutrino capture on proton are governed by the same hadronic matrix element:  $n \rightarrow p + e^{-} + v_{e}$  (neutron decay)  $v_{e} + p \rightarrow n + e^{+}$  (inverse neutron decay)

Knowing the neutron lifetime,  $\tau_n = 885.7(0.8)$ , f = 1.715, fixes the cross section for the relevant energies:  $\sigma_{tot} = (2\pi^2/m_e^5)/f\tau_n \times E_e p_e$  or  $[2\pi^2(hc)^3]/[(m_ec^2)^5 f\tau_n c] p_e c E_e$ 

( $E_e, p_e$  are the energy and momentum of the positron,  $E_e = E_{e} - (M_n - M_p + m_e)$ )

Note, however, that life is not simple even in this "classical" case. The measurement of Serebrov et al. (2005) gives  $\tau_n = 878.5 \pm 0.7 \pm 0.3$ , which differs from the official 885.7±0.8 by ~9 $\sigma$ ; it is not yet clear which is correct.

The neutron lifetime fixes cross section of all processes involving a single nucleon, e.g.,  $v_e + n \rightarrow p + e^-$  or  $v + p(n) \rightarrow v + p(n)$  (neutral current)

If one wants something really accurate (no matter the lifetime controversy) one should consider corrections:

The (relatively) small corrections of order  $E_{\nu}/M_{p}$  and  $\alpha/\pi$  can be accurately evaluated: (see Vogel & Beacom, Phys. Rev. D60,053003 (1999) and Kurylov, Ramsey-Musolf & Vogel, Phys.Rev.C67,035502(2003))

In this way the cross section of the inverse neutron decay (and any low energy weak process involving only free nucleons) can be evaluated with the accuracy of ~0.2%, even though only few reactions were actually observed, and the experimental errors are much larger, ~ 2%. (Also, at higher energies the uncertainties in the nucleon form factors must be included.)

### Cross section for the inverse beta decay has been checked to a few % accuracy.

•  $\sigma_{\text{IBD}}$  has been checked in reactor experiments (a short distance from the reactor where possible oscillation effects are negligible)

$$\overline{v_e} p \rightarrow e^+ n$$

• measurements at few-% level, consistent with prediction

	Goesgen	Krasnoyarsk	Bugey
	PRD <b>34</b> , 2621 (1986)	JETP Lett <b>54</b> , 2225 (1991)	PLB <b>338</b> , 383 (1994)
$\sigma_{exp}$	3.0%	2.8%	1.4%

- theory is ahead here,  $\sigma_{\!_{\! \rm V}}$  measurements limited by how well know reactor neutrino flux

# Before discussing v-nucleus scattering lets briefly describe the scattering on electrons $v + e^- \rightarrow v + e^-$

- process in which we 1<sup>st</sup> discovered NC's!
- purely-leptonic process, so σ calculation is very straightforward (no form factors!)



 $\sigma = \frac{2G_F^2 m_{\rm e}}{\pi} \begin{bmatrix} \left(g_L^2 + \frac{g_R^2}{3}\right) E_\nu - g_L g_R \frac{m_{\rm e}}{2} \end{bmatrix} \qquad \begin{array}{c} g_L = \sin^2 \theta_W \stackrel{\rm e}{\pm} \frac{1}{2} \\ g_R = \sin^2 \theta_W \end{array} \qquad \begin{array}{c} e^- & e^- \\ Both \ {\it CC} \ {\it and} \ {\it NC} \ {\it are} \\ present \ {\it for} \ v_e \ {\it but not} \end{array}$  $g_{L} \Leftrightarrow g_{R} \text{ for anti } v$   $\sigma \text{ is } \sim \text{ linear with } E_{v} \text{ (generic feature of point-like scattering)}$   $\sigma (v_{e} e^{-}) > \sigma(v_{\mu,\tau} e^{-}) \text{ (v}_{e} \text{ can scatter both by NC \& CC)}$   $\sigma \text{ is small:}$   $\sigma \sim s = (E_{CM})^{2} = 2m_{target}E_{v}$   $\sigma \text{ for anti } v$ 4 orders of magnitude less likely than scattering off nucleons at 1 GeV! The differential cross section in terms of the lab. electron recoil kinetic energy T is  $d\sigma/dT = 2G_{F}^{2}m_{e}/\pi [g_{1}^{2} + g_{R}^{2}(1 - T/E_{y})^{2} - g_{1}g_{R}m_{e}T/E_{y}]$   appealing to use for SN and solar v detection because it is directional! (e- emitted at a very small angle wrt incoming v direction)

$$E_e \, \theta_e^2 < 2 \, m_e$$

can derive from simple E, mom conservation

 recoiling e<sup>-</sup> preserves knowledge of incident v direction (compared to e<sup>+</sup> from IBD which is essentially isotropic for low E<sub>v</sub>)



SuperKamiokande solar neutrino data (see Cravens et al. Phys. Rev. D**78**, 032002(2008) **)**.

#### What about v interaction with complex nuclei?

At lowest energies we must consider **exclusive** scattering to specific bound (or resonance) nuclear states. At somewhat higher energies we are typically interested in the **inclusive** scattering, summing over all possible nuclear final states.

The initial state is usually the nuclear ground state. However, in various astrophysics applications the temperature might be high enough that excited states are populated as well.

Essentially absent is the truly elastic (NC) scattering, never observed as yet. Note that at low energies ( $E_v < 50$  MeV) such scattering is coherent with the maximum nuclear recoil energy of only  $\sim E_v^2/(A \times m_p)$ , thus very difficult to observe, even though the cross section is enhanced  $\sigma_{tot} \sim G_F^2 E_v^2 N^2/4\pi$ .

Neutrino interaction with the simplest nucleus, deuteron, at low energy:

There are no bound states in d, the only open channel is the deuteron disintegration. Consider the CC scattering  $v_e + d = p + p + e^{-1}$ 

The tree level cross section at low energies is

### $(d\sigma/dE)_{tree} = 2G_F^2/\pi V_{ud}^2 g_A^2 M_p p_e E_e p |I(p^2)|^2$

where p is the relative momentum of the outgoing protons and the overlap integral is  $I(p^2) = \int u_{cont}^*(pr) u_d(r) dr$ ,

This integral depends on the pp scattering length, effective radius, and on the deuteron binding energy. It is peaked at low values of  $p^2/M_p$  and is about 1 MeV wide in that variable

With deuterons there are many possible reactions now:

 $v_e+d \rightarrow p+p+e^-(CC)$   $v+d \rightarrow v+p+n(NC)$  (for any v) and the corresponding reactions with antineutrinos. In addition, the reactions powering Sun involve the same physics:  $p+p \quad d+v_e+e^+$  (pp in the Sun, endpoint 420 keV)  $p+p+e^- \quad d+v_e$  (pep in the Sun, monoenergetic  $E_v = 1.44$  MeV)

For all these reactions we should also consider the two-body currents (pion exchange currents in the traditional language). In the effective field theory all corresponding unknown effects can be lumped together in one unknown parameter  $L_{1A}$  (isovector two-body axial current) that must be fixed **experimentally**. The cross section is then of the form  $\sigma(E) = a(E) + b(E)L_{1A}$ , where the functions a(E), b(E) are known, and  $b(E)L_{1A}$  contributes ~`a few' %.

### How can one fix the parameter $L_{1A}$ ?

There are several ways to do this. One can use reactordata ( $*_eCC$  and NC), solar luminosity + helioseismology,SNO data, and tritum beta decay. Here is what you get:reactors:3.6(5.5) fm³ (Butler,Chen,Vogel)Helioseismology:4.8(6.7) fm³ (Brown,Butler,Guenther)SNO:4.0(6.3) fm³ (Chen,Heeger,Robertson)tritium b decay:6.5(2.4) fm³ (Schiavilla et al.)

All these values are consistent, but have rather large uncertainties. To reduce them substantially, one would have to measure one of these cross sections to ~1%. This is <u>very difficult</u>. Thus the considered neutrino-deuteron reaction cross sections remain ~ 2% uncertain. We will not consider the two-body currents for heavier nuclei. Experimental data on neutrino-nucleus cross sections at low energies are rare or nonexistent, here is the **full** list:

v-d: for reactor  $v_e$  (E<sub>v</sub> < 8 MeV) and solar  $v_{e,}$ , E<sub>v</sub> < 14 MeV. v- <sup>12</sup>C: for v from  $\pi^+$  and  $\mu^+$  decay at rest, E<sub>v</sub> < 52 MeV; exclusive transition to the <sup>12</sup>N<sub>g.s</sub>. and to the 15.11 MeV state and inclusive transition to the continuum in <sup>12</sup>N  $v_e$  - <sup>56</sup>Fe: inclusive transition to <sup>56</sup>Co, error 50%  $v_e$ -<sup>37</sup>Cl and <sup>71</sup>Ga: radiochemical measurements with solar neutrinos, inclusive cross section for states below neutron emission  $v_e$ -<sup>71</sup>Ga: radiochemical calibration with the <sup>51</sup>Cr source  $v_e$ -<sup>127</sup>I: radiochemical measurement with the  $\mu^+$  decay at rest spectrum

Thus we need to rely on theory. However, since this problem belongs to the ``neutrino engineering" category, it is not among the high priority and high visibility programs.

# Reaction $v_e + {}^{12}C \rightarrow {}^{12}N_{g.s.} + e^{-}$

This is an example of a process where the cross section can be evaluated with little uncertainty. We can use the known <sup>12</sup>N and <sup>12</sup>B  $\beta$  decay rate, as well as the exclusive  $\mu$  capture on <sup>12</sup>B and the *M1* formfactor for the excitation of the analog 1<sup>+</sup>, T=1 state at 15.11 MeV in  $^{12}$ C. This fixes the cross section value for (almost) all energies, for both  $v_e$  and  $v_u$ .

A=12 triad



# Experiment and theory agree quite well

 intensive program of beam dump v experiments at Los Alamos & Rutherford lab (10-20% measurements)

flux-averaged $\sigma$ in units of cm <sup>2</sup>	${ m ^{12}C}( u_e,e^-){ m ^{12}N}_{gs}$ decay at rest	${ m ^{12}C}( u_{\mu},\mu^{-})^{12}{ m N}_{gs}$ decay in flight	$^{12}C(\nu,\nu')^{12}C(15.11)$ decay at rest
KARMEN	9.1 ± 0.5 ± 0.8	-	$10.4 \pm 1.0 \pm 0.9$
LSND	8.9 ± 0.3 ± 0.9	$66{\pm}10{\pm}10$	-
E225	10.5 ± 1.0 ± 1.0	-	-
Shell model <sup>10</sup>	9.1	63.5	9.8
CRPA <sup>4,5</sup>	8.9	63.0	10.5
EPT <sup>11</sup>	9.2	59	9.9

(DAR means `decay at rest' E < 52 MeV, DIF means `decay in flight', E ~ 180 MeV )

# Cross section for ${}^{12}C(v_{\mu},\mu){}^{12}N_{gs}$ in 10<sup>-42</sup>cm<sup>2</sup>, see Engel et al, Phys. Rev. C54, 2740 (1996)



### $v_e + {}^{12}C \rightarrow {}^{12}N^* + e^-$ (inclusive reaction)

Here the final state is not fixed and not known, one cannot use (at least not simply as before) the known weak processes to fix the parameters of the nuclear models. This is dominated by

negative parity multipoles, calculation becomes more difficult. The measurement is also more difficult since the experimental signature is less specific (units as before 10<sup>-42</sup>cm<sup>2</sup>) (DAR spectrum) experiment

4.3	±	0.4	±	0.6	(LSND, 01)	
5.7	±	0.6	±	0.6	(LSND, 97)	
5.1	±	0.6	±	0.5	(KARMEN, 98	3
3.6	±	2.0			( <i>E225</i> , 92)	

calculatior	เร	
Kolbe 95	5.9-6.3	CRPA
Singh 98	6.5	local density app.
Kolbe 99	5.4-5.6	CRPA, frac. filling
Hayes 00	3.8-4.1	SM, 3hw,
Volpe 00	8.3	SM, 3hw
Volpe 00	9.1	QRPA

The agreement between different calculations, and with the experiment, is less than perfect.

# True challenge, inclusive ${}^{12}C(\nu_{\mu},\mu^{-})N^{*}$ with DIF

Exp: LSND 02, (10.6± 0.3± 1.8)x10<sup>-40</sup>cm<sup>2</sup> Calc: 17.5 - 17.8 (Kolbe, CRPA, 99) 16.6± 1.4 (Singh, loc.den.app.,98) 15.2 (Volpe, SM, 00) 20.3 (Volpe, QRPA, 00) 13.8 (Hayes, SM, 00)

Thus all calculations overestimate the cros section, with SM results noticeably smaller than CRPA or QRPA. The reason for that remains a mystery.

Note: Meucci et al, nucl-th/0311081 claim 11.15 in agreement with exp. using Green's function approach

### Related process: $\mu$ capture $\mu^- + {}^{Z}A \rightarrow \nu_{\mu} + {}^{(Z-1)}A^*$

In this process  $|q| \sim 100$  MeV (muon mass). Total capture rate is thus analogous to the inclusive neutrino neutrino scattering with similar q.





What happens if the incoming neutrino beam has is broad, has no well defined energy? Can one deduce the incoming neutrino energy from the observation of the outgoing muon (or electron)? Yes, provided you can neglect the nucleon binding energy:

$$E_{\nu}^{QE} = \frac{M_{N}E_{\mu} - \frac{m_{\mu}^{2}}{2}}{M_{N} - E_{\mu} + p_{\mu}\cos\theta_{\mu}}$$

# QE scattering at ~ 1 GeV, need to take into account the nucleon structure characterized by form factors



Discrepancy between past higher energy determination of  $M_A$  and the more recent ones for  $E_v \sim 1$  GeV.

# Modern M<sub>A</sub>



past world average:  $M_A = 1.03 \pm 0.02 \text{ GeV}$ 

- K2K SciFi (<sup>16</sup>O, Q<sup>2</sup>>0.2) Phys. Rev. D74, 052002 (2006) M<sub>A</sub>=1.20 ± 0.12 GeV
- K2K SciBar (<sup>12</sup>C, Q<sup>2</sup>>0.2) AIP Conf. Proc. 967, 117 (2007) M<sub>A</sub>=1.14 ± 0.11 GeV
- MiniBooNE (<sup>12</sup>C, Q<sup>2</sup>>0) paper in preparation  $M_A$ =1.35 ± 0.17 GeV
- **MINOS** (Fe, Q<sup>2</sup>>0.3) NuInt09, preliminary M<sub>A</sub>=1.26 ± 0.17 GeV

 $(E_v 3-100 \text{ GeV})$ 

• NOMAD (<sup>12</sup>C, Q<sup>2</sup>>0) arXiv:0812.4543 [hep-ex] M<sub>A</sub>=1.07 ± 0.07 GeV The discrepancy in  $M_A$  is a consequence of apparent difference in the trend of the cross sections. So far unexplained.



 ~ 30% difference between QE σ measured at low & high E both on <sup>12</sup>C ?!
 ? who ordered this? Summary of older CC QE data with curves for  $M_A = 1$  GeV, nuclear effects represent only about ~10% reduction



#### How can one take into account the effects of nuclear structure?

From general considerations one can identify three different energy ranges with different demands on the details with which the nuclear structure should be treated:

- i) For relatively low neutrino energies, comparable with the nuclear excitation energy, the model of choice is the nuclear shell model. The shell model calculations are indeed able to reproduce the allowed (Fermi, Gamow-Teller) response.
- ii) The Random Phase Approximation (RPA) has been developed to describe the collective excitation of a nucleus. The RPA is the methods of choice at intermediate energies where the reaction rate is sensitive dominantly to the total strength and the energy of the giant resonances. A variant of RPA, so-called CRPA takes into account that the final nucleon is in the continuum.
- iii) At high incoming energies neutrinos scatter `quasi-freely' on individual nucleons. The remaining nucleons can be treated as (non-interacting) spectators. This situation is most simply realized in the relativistic Fermi gas model.

### Comparison of shell model and RPA



Comparison of Fermi gas model (full lines) and the CRPA (dashed lines)



General formula for cross section contains a product of the leptonic  $d\sigma = \frac{G_F^2 \cos^2 \vartheta_c}{2} 2\pi L^{\mu\nu} W_{\mu\nu} \frac{d^3 k}{(2\pi)^3}$  and hadronic tensors

 in a nucleus, target nucleon has some initial momentum which modifies the observed scattering



hadronic tensor now an integral over initial nucleon states

$$W_{A}^{\mu\nu} = \frac{1}{2} \int d^{3}p \, dE(\mathbf{P}(\mathbf{p}, E) \underbrace{1}_{4 E_{|\mathbf{p}|} E_{|\mathbf{p}+\mathbf{q}|}} W^{\mu\nu}(\tilde{p}, \tilde{q})$$
  
• simplest: Fermi Gas model  
(2 free parameters)  

$$p_{F}=220 \text{ MeV/c (}^{12}\text{C}) P_{RFGM}(\mathbf{p}, E) = \left(\frac{6 \pi^{2} A}{p_{F}^{3}}\right) \theta(p_{F} - \mathbf{p}) \, \delta(E_{\mathbf{p}} - E_{B} + E)$$
  

$$P_{B}=25 \text{ MeV} E_{B}=25 \text{ MeV} E_{B}$$

The RFGM is simple but crude. A better approximation uses spectral functions (tested in electron scattering).



Nuclear effects cause a significant suppression of  $\sigma$ at low E<sub>v</sub> and low Q<sup>2</sup> compared to scattering on free nucleons

#### Opening a new channel, resonance production



Note:  $\sigma$  scaled with 1/E

- main contribution is from  $\Delta(1232) \rightarrow N\pi$
- most widely used model (Rein, Sehgal, Annals Phys 133, 179 (1981))
- experiments typically simulate ~18 different resonances ( $\Delta$ ,N\*) including their single- $\pi$  & multi- $\pi$  decay modes, also  $\Delta \rightarrow N\gamma!$

#### Available data:

 variety of σ measurements, mostly bubble chamber experiments (1970's-80's), 25-40% level uncertainties



Sam Zeller, INSS, 07/08/09

Pions produced inside the nucleus can be absorbed or charge exchanged before getting out, this are Final State Effects

> • nuclear effects further complicate this description (once produce  $\pi^0$ , has to get out of nucleus, FSI alter  $\pi^0$  kinematics!)





example, at E<sub>v</sub>=1 GeV
 ~20% of π<sup>0</sup> get absorbed
 ~10% charge exchange (π<sup>0</sup> →π<sup>+,-</sup>)

(T. Leitner,  $E_v$  beam ~ 1 GeV)

- need to predict initial interaction  $\sigma \, \underline{\text{and}}$  final state effects

Deep inelastic scattering,  $E_v \sim 100 \text{ GeV}$ 



#### Quantities used in the description of DIS



$$\nu_{\mu} \mathsf{N} \rightarrow \mu^{-} \mathsf{X}$$

 in the quark parton model, these reactions are described as the scattering of v's from q (and q) constituents in nucleon

$$Q^{2} = 4(E_{\mu} + E_{had})E_{\mu}sin^{2} \frac{\theta_{\mu}}{2}$$
 (4-momentum  
transfer squared)  
$$v = E_{had}$$
 (energy transfer)  
$$y = E_{had}/E_{\nu}$$
 (inelasticity)  
$$x = \frac{Q^{2}}{2M\nu}$$
 (fraction of the nucleon momentum  
carried by struck quark, i.e. Bjorken x)

• at LO, neglecting lepton mass terms, the DIS  $\sigma$  reduces to:

 $\frac{d^2 \sigma^{\nu(\overline{\nu})}}{dxdy} = \frac{G_F^2 M E_{\nu}}{\pi (1 + \frac{Q^2}{M_W^2})^2} \left[ \left( 1 - y - \frac{Mxy}{2E_{\nu}} \right) F_2^{\nu(\overline{\nu})} + \frac{y^2}{2} \frac{2x F_1^{\nu(\overline{\nu})}}{1} \pm y(1 - \frac{y}{2}) x F_3^{\nu(\overline{\nu})} \right]$  $F_1$ ,  $F_2$ ,  $F_3$  contain direct information on nucleon structure; they are functions of x,  $Q^2$  $F_3$  is unique to neutrino  $F_2$  measures the scattering since it is density distribution parity violating. Changes of all guarks and sign for ¥. Measures the antiquarks in the valence quark distribution nucleon.



PDG, 2009

### Conclusions

- Cross sections of neutrinos on nucleons and nuclei are tiny and many processes contribute simultaneously. That makes the analysis of data and theoretical predictions challenging.
- But it is critically important to know the  $\sigma$ . You need them in order to estimate how many events you should expect and what kind of signals, that is final states, you will observe.
- The  $\sigma$  are reasonably well known at low and at very large energies. However, in the intermediate energy range ~1 GeV, they are known only crudely. Yet it is this energy range that is crucially important in neutrino oscillation studies.
- Study of  $\sigma$ , both experimental and theoretical, is not as glamorous as other problems of neutrinos (so-called neutrino engineering). Yet, it is vital part of the whole field.