Neutrinoless double beta decay. What its observation would prove, and its nuclear matrix elements.

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Outline:

- Introduction: What is $\beta\beta$ decay, candidate nuclei, the allowed $2\nu\beta\beta$ mode, significance of the $0\nu\beta\beta$ mode, other tests of the lepton number conservation.

- Mechanism of the $0\nu\beta\beta$ decay. Exchange of a light Majorana neutrino or a TeV scale physics?

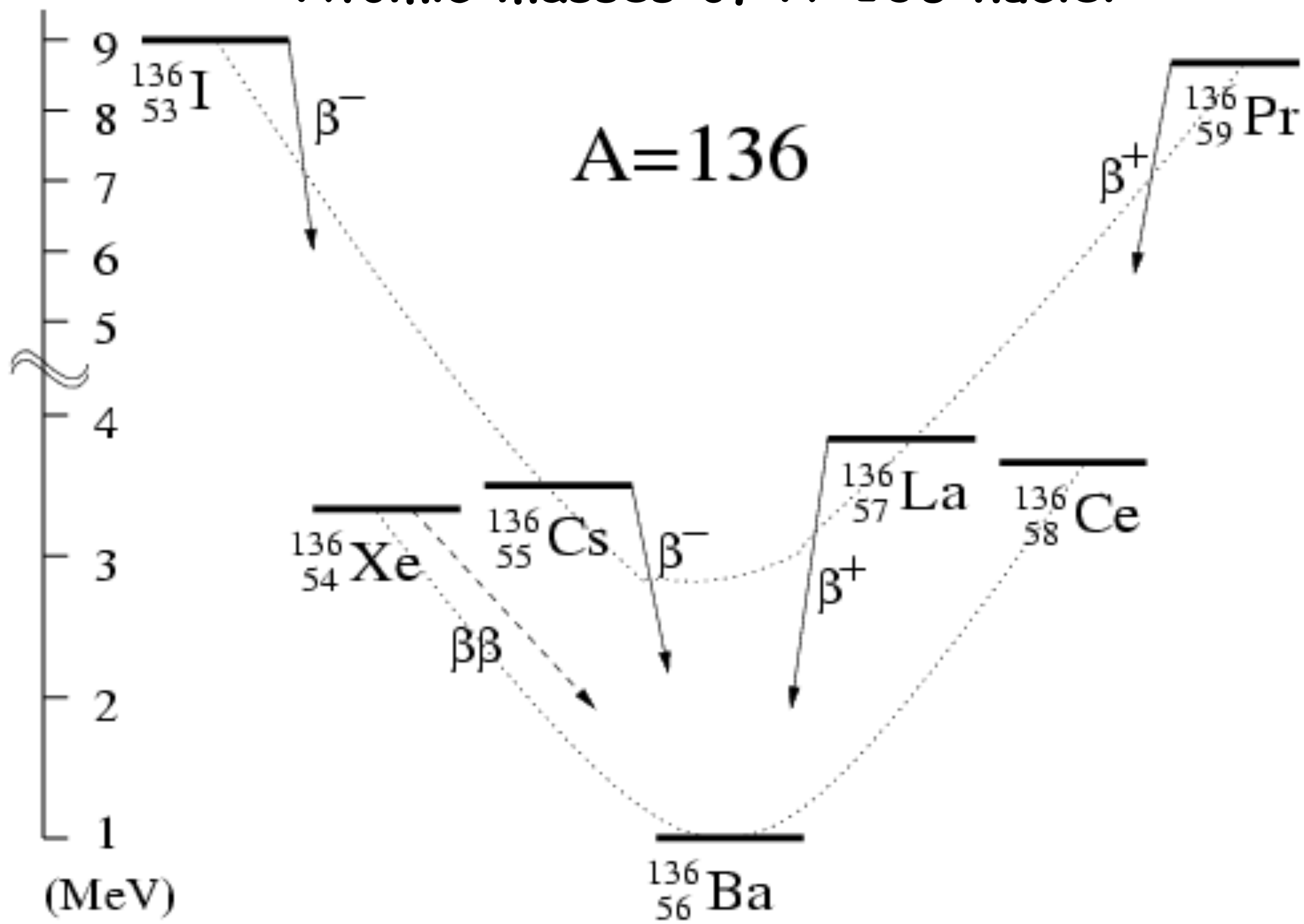
- Effective Majorana mass $\langle m_{\beta\beta} \rangle$ and its relation to other ways of neutrino mass determination.

- Nuclear matrix elements. Issues related to nucleon structure. How important these things are and how to include them in the evaluation of nuclear matrix elements?

- Nuclear matrix elements and nuclear structure. Can we estimate the relevant uncertainty? What the comparison of different methods tells us?

Double ββ decay is observable because even-even nuclei are more bound than the odd-odd ones (due to the pairing interaction)

Atomic masses of A=136 nuclei



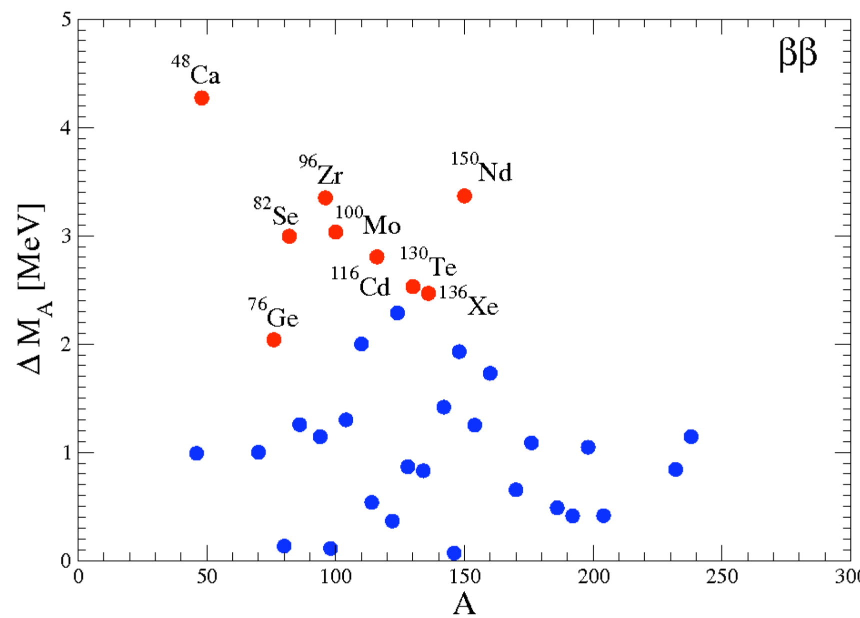
^{136}Xe and ^{136}Ce are stable against β decay (they exist in nature), but unstable against ββ decay ($\beta^-\beta^-$ for ^{136}Xe and $\beta^+\beta^+$ for ^{136}Ce)

Candidate nuclei for double beta decay with Q>2 MeV

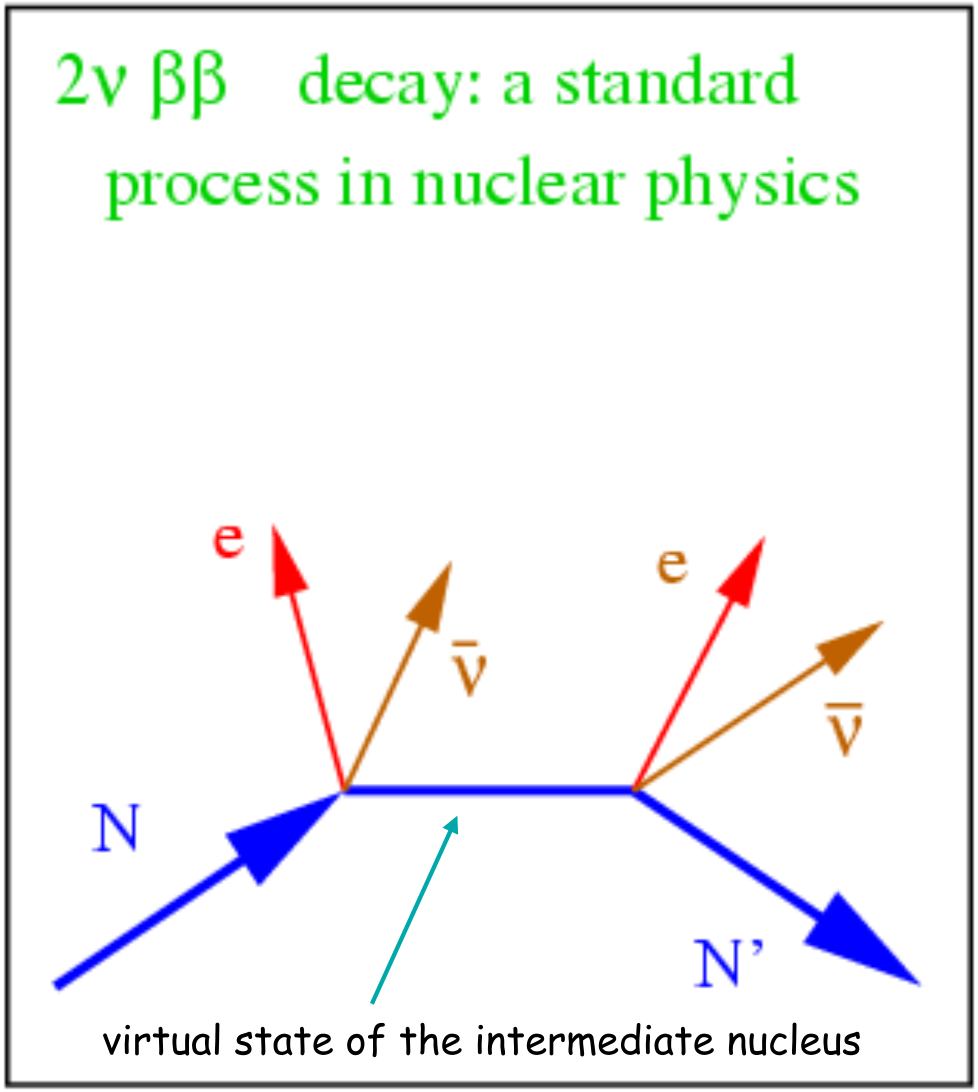
		Q (MeV)	Abund.(%)
	$^{48}Ca \rightarrow {}^{48}Ti$	4.271	0.187
→	$^{76}Ge \rightarrow {}^{76}Se$	2.040	7.8
→	$^{82}Se \rightarrow {}^{82}Kr$	2.995	9.2
	$^{96}Zr \rightarrow {}^{96}Mo$	3.350	2.8
→	$^{100}Mo \rightarrow {}^{100}Ru$	3.034	9.6
	$^{110}Pd \rightarrow {}^{110}Cd$	2.013	11.8
	$^{116}Cd \rightarrow {}^{116}Sn$	2.802	7.5
	$^{124}Sn \rightarrow {}^{124}Te$	2.228	5.64
	$^{130}Te \rightarrow {}^{130}Xe$	2.533	34.5
→	$^{136}Xe \rightarrow {}^{136}Ba$	2.479	8.9
→	$^{150}Nd \rightarrow {}^{150}Sm$	3.367	5.6

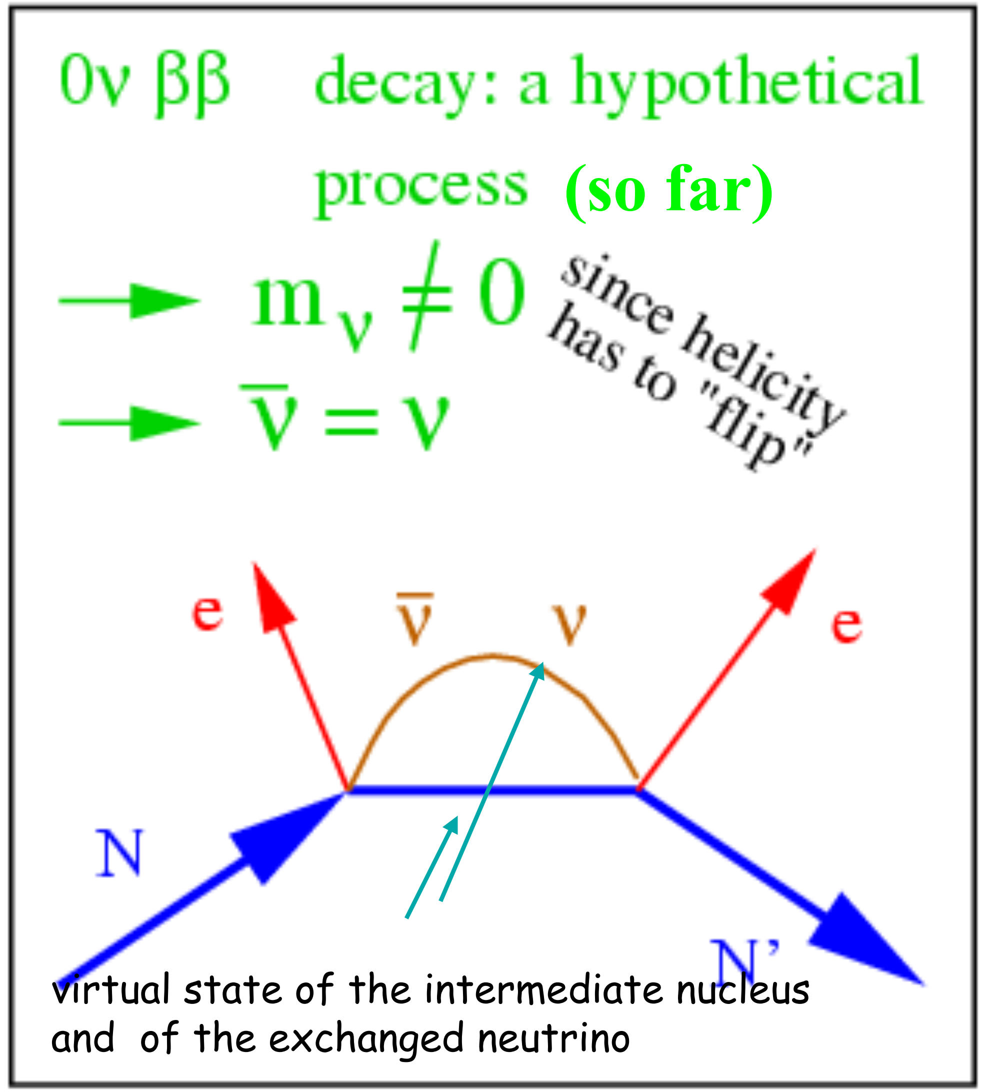
The nuclei with an arrow are used in the present or planned large experiments. For most of the nuclei in this list the $2\nu\beta\beta$ decay has been observed

The Q values of all candidate nuclei.
Those with Q > 2 MeV are in red.

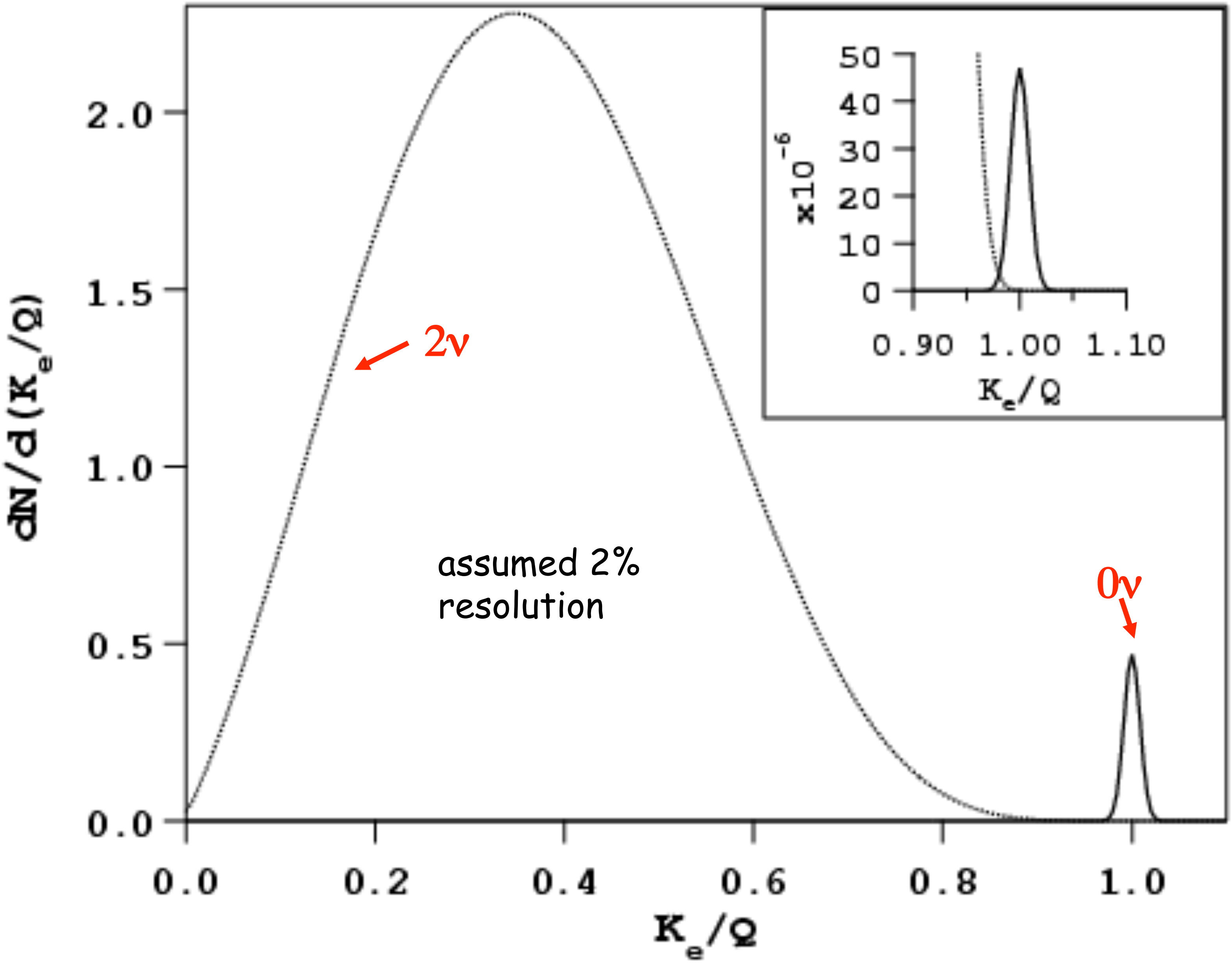


ββ decay can exist in two modes. The two-neutrino (2νββ) decay is an allowed but slow process, while the neutrinoless (0νββ) mode would violate the total lepton number conservation law and thus would be a sign of **new physics**





One can distinguish the two modes by measuring the sum electron energy. Ultimately, though, the 2ν decay is an unavoidable background to the 0νββ.



assumed rate ratio 1:10^6

assumed rate ratio 1:10^2

How can we tell whether the total lepton number is conserved?

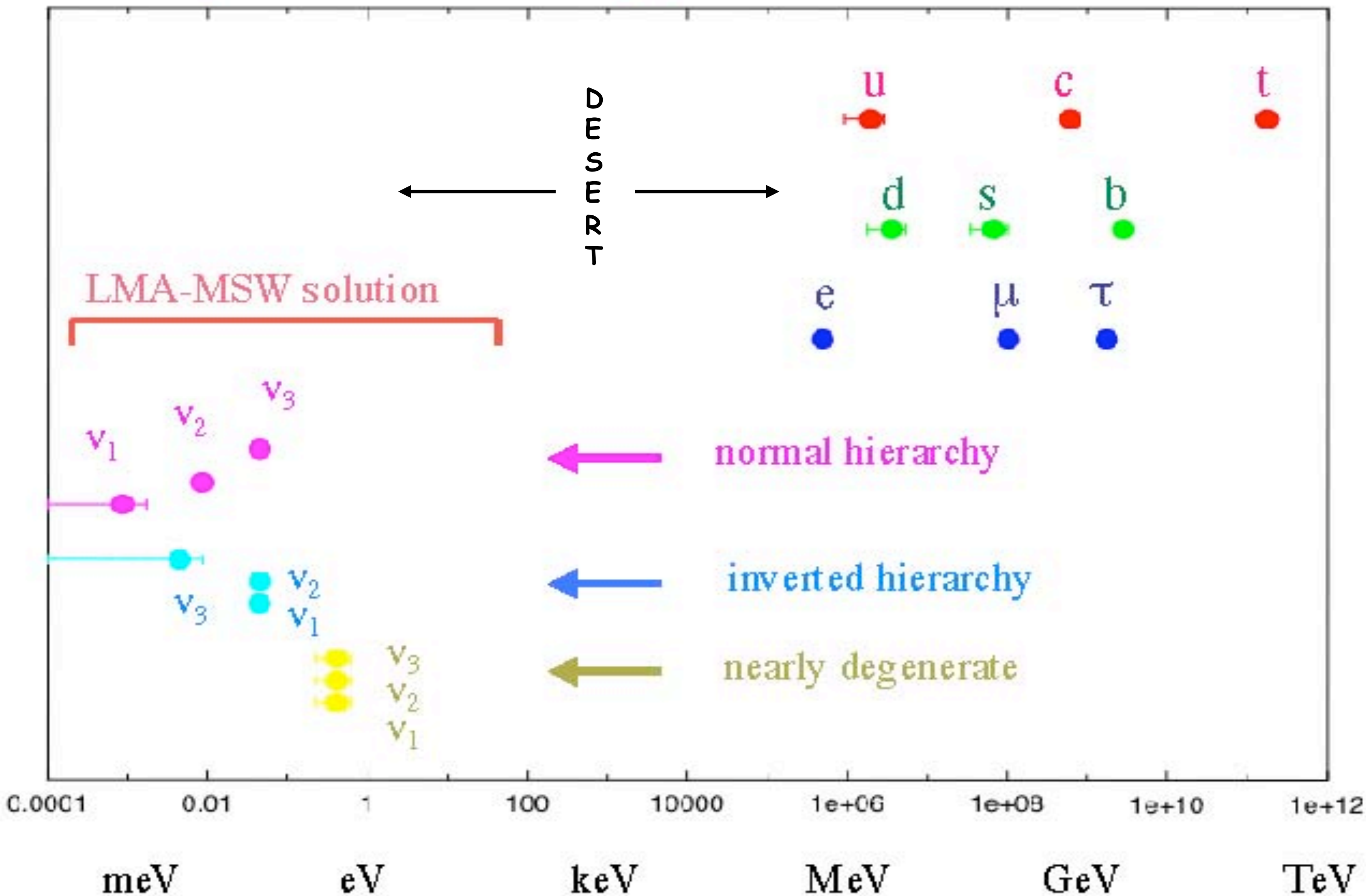
A partial list of processes where the lepton number would be violated:

Neutrinoless $\beta\beta$ decay: (Z,A) -> (Z±2,A) + $2e^{(\pm)}$, $T_{1/2}$ > ~10^{25} y
Muon conversion: μ^- + (Z,A) -> e^+ + (Z-2,A), BR < 10^{-12}
Anomalous kaon decays: $K^+ \rightarrow \pi^-\mu^+\mu^+$, BR < 10^{-9}
Flux of ν_e from the Sun: BR < 10^{-4}
Flux of $\bar{\nu}_e$ from a nuclear reactor: BR < ?
Production at LHC of pair of the same charge leptons with no missing energy through production of doubly charged scalar that decays that way?

Observing any of these processes would mean that the lepton number is not conserved, and that neutrinos are massive Majorana particles.

It turns out that the study of the $0\nu\beta\beta$ decay is by far the most sensitive test of the total lepton number conservation, so we restrict further discussion to this process.

We know that ν masses are much much smaller than the masses of other fermions



Is that a possible “Hint of” a new mass-generating mechanism?

To solve the dilemma of `unnaturally' small neutrino mass we can give up on renormalizability and add operators of dimension d > 4 that are suppressed by inverse powers of some scale Λ but are consistent with the SM symmetries.

Weinberg already in 1979 (PLR **43**, 1566) showed that there is **only one** dimension d=5 gauge-invariant operator given the particle content of the standard model:

$$L^{(5)} = C^{(5)}/\Lambda\ (\overline{L^c}\varepsilon H)(H^T\varepsilon L) + h.c.$$

Here $\overline{L^c} = L^T C$, where C is charge conjugation and $\varepsilon = -i\tau_2$. This operator clearly violates the lepton number by two units and represents neutrino Majorana mass

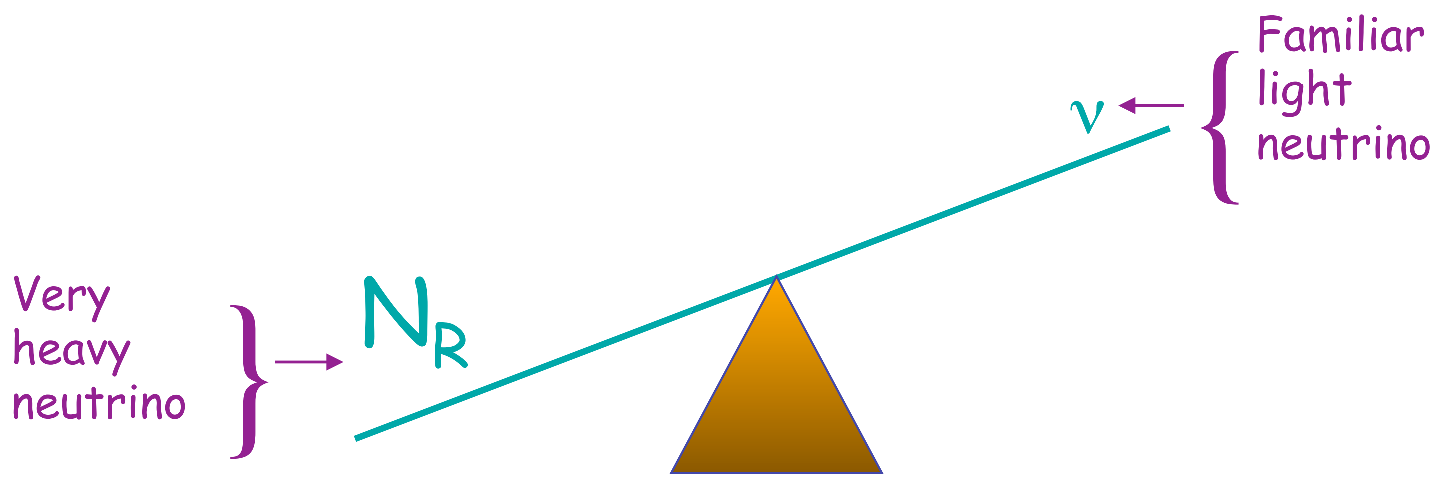
$$L^{(M)} = C^{(5)}/\Lambda\ v^2/2\ (\overline{\nu_L{}^c}\,\nu_L) + h.c.$$

If Λ is larger than v, the Higgs vacuum expectation value, the neutrinos will be `naturally' lighter than the charged fermions.

The most popular theory of why neutrinos are so light is the —See-Saw Mechanism

(Minkowski (1977), Gell-Mann, Ramond, Slansky (1979), Yanagida(1979), Mohapatra, Senjanovic(1980))

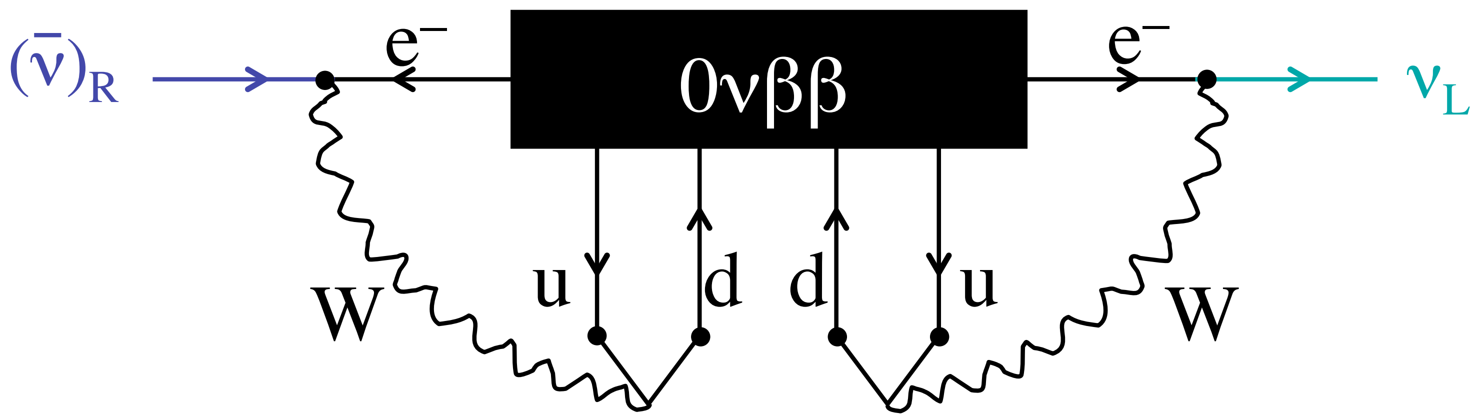
It assumes that the very heavy neutrinos N_R exist. Their mass plays an analogous role as the scale Λ of Weinberg, i.e., $m_\nu \sim v^2/M_N$. Both the light and heavy neutrinos are Majorana fermions.



For formalism of Majorana fermions and the difference between them and the usual Dirac fermions, see the nice pedagogical review of P.B.Pal, arXiv:1006.1718

Whatever processes cause $0\nu\beta\beta$, its observation would imply the existence of a Majorana mass term and thus would represent ``New Physics":

Schechter and Valle,82

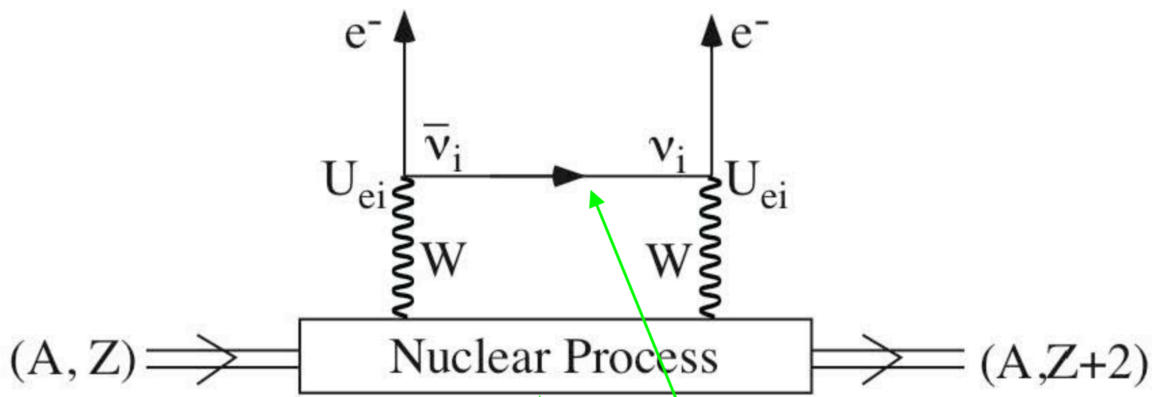


By adding only Standard model interactions we obtain

$(\bar{\nu})_R \rightarrow (\nu)_L$ ***Majorana mass term***

Hence observing the $0\nu\beta\beta$ decay guaranties that ν are massive Majorana particles. But the relation between the decay rate and neutrino mass might be complicated.

If (or when) the $0\nu\beta\beta$ decay is observed two problems must be resolved:



a) What is the mechan
   i.e., what kind of virtual particle is exchanged between the affected nucleons (or quarks)? (what is ν_1?)

b) How to relate the observed decay rate to the fundamental parameters, that is what is the value of the corresponding nuclear matrix elements? (how to describe NP above?)

Two basic categories are **long-range** and **short-range** contributions to the $0\nu\beta\beta$ decay.

The **long-range** category involves two pointlike vertices and the exchange of a light Majorana neutrino between them. The standard (plain vanilla) type of that category is when

$1/T_{1/2}^{0\nu} = G^{0\nu}(Q,Z)\ |M^{0\nu}|^2\ |\langle m_{\beta\beta}\rangle|^2$, $\langle m_{\beta\beta}\rangle = \Sigma_i U_{ei}^2\ m_i$,

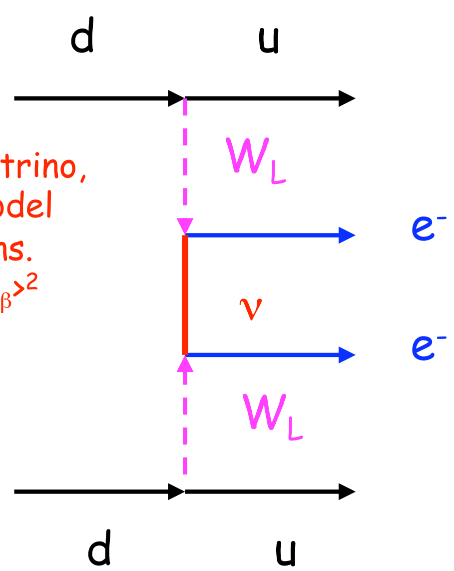
which represents simple relation between the decay rate and the parameters of the neutrino mass matrix.

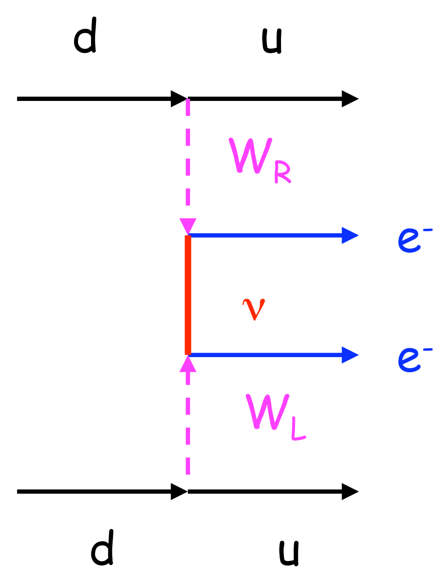
The **short-range** category involves only a single pointlike vertex (six fermions, four hadrons and two leptons), i.e. a dimension 9 operator. The relation between the decay rate and neutrino mass is not simple in that case.

What is the nature of the `black box'? In other words, what kinds of effects can contribute to both categories?

All these diagrams can in principle contribute to the $0\nu\beta\beta$ decay amplitude

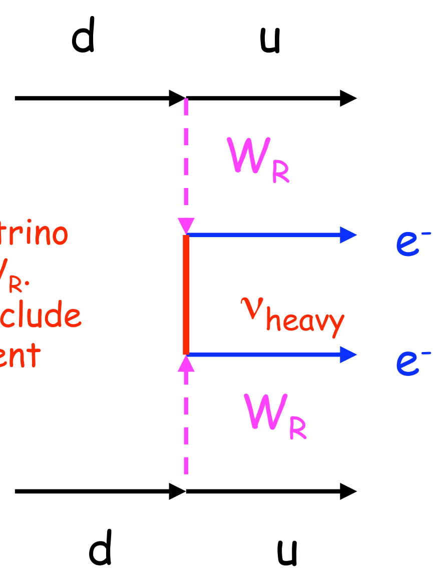
Light Majorana neutrino, only Standard Model weak interactions. Decay rate $\sim\langle m_{\beta\beta}\rangle^2$

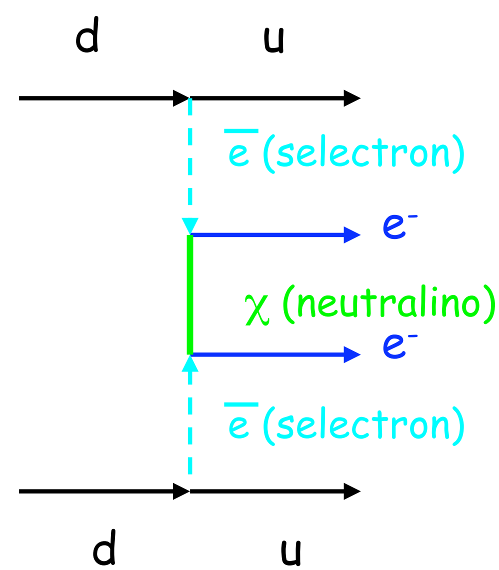




Light Majorana neutrinos. Model extended to include right-handed W_R. Mixing extended between the left and right-handed neutrinos. This is the mode where the rate $\sim\lambda^2$ or η^2

Heavy Majorana neutrino interacting with W_R. Model extended to include right-handed current interactions.





Supersymmetry with R-parity violation. Many new particles invoked. Light Majorana neutrinos exist also.

It is well known that the amplitude for the light neutrino exchange scales as $\langle m_{\beta\beta} \rangle$. On the other hand, if heavy particles of scale Λ are involved the amplitude scales as $1/\Lambda^5$ (dimension 9 operator)

.

The relative size of the heavy (A_H) vs. light particle (A_L) exchange to the decay amplitude is (a crude estimate, due originaly to Mohapatra)

$A_L \sim G_F^2 \, m_{\beta\beta}/\langle q^2 \rangle$, $\qquad A_H \sim G_F^2 \, M_W^4/\Lambda^5$,

where Λ is the heavy scale and $q \sim 100$ MeV is the virtual neutrino momentum.

For $\Lambda \sim 1$ TeV and $m_{\beta\beta} \sim 0.1 - 0.5$ eV $A_L/A_H \sim 1$, hence both mechanisms contribute equally.

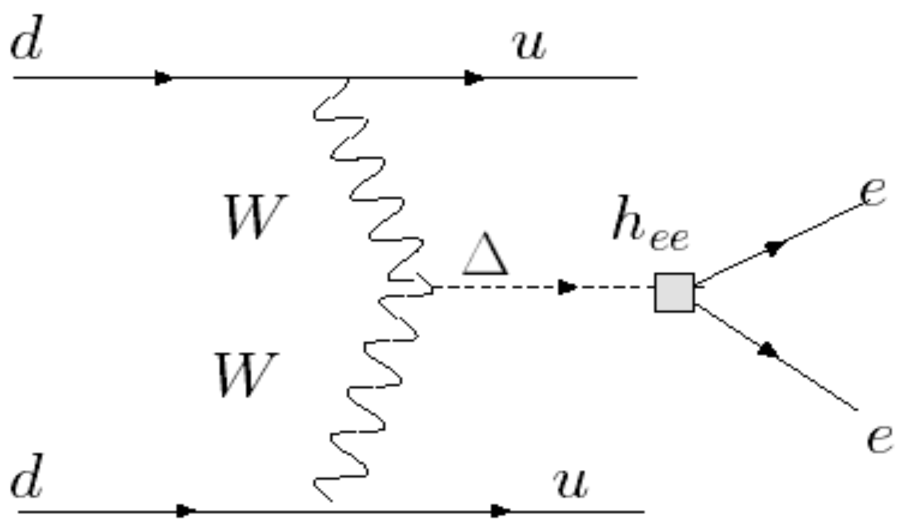
As long as only a limit on the $0\nu\beta\beta$ decay rate exists, we can constrain all parameters entering the decay amplitudes (light and heavy neutrino masses, strength of the right-handed current, SUSY R-parity violating amplitudes, etc.).

However, once the decay rate is convincingly measured, we will need to determine which of the possible mechanism is responsible for the observation.

Lets consider the particle physics models in which $0\nu\beta\beta$-decay of the short-range category might exist. In them LNV violation is associated with low-scale (~TeV) physics, unlike see-saw with LNV at very high scale.

Low scale LNV: Left-Right Symmetric Model (LRSM)

$$\mathcal{L}_{\delta^{\pm\pm}_{L,R}} = \frac{g}{2}\left[\delta^{++}_{L,R}\,\overline{l^c}\,(h_{L,R}\,P_{L,R})\,l + \delta^{--}_{L,R}\,\bar{l}\left(h^{\dagger}_{L,R}\,P_{R,L}\right) l^c\right]$$



The model includes a doubly charged Higgs that couples to leptons as shown

This is an example of 0νββ decay mediated by this coupling. The amplitude scales like

$$\frac{g_2^3\ \ h_{ee}}{M^3_{W_R}\,M^2_{\Delta}}$$

Another example is the exchange of heavy right-handed ν_R and two W_R that scales like

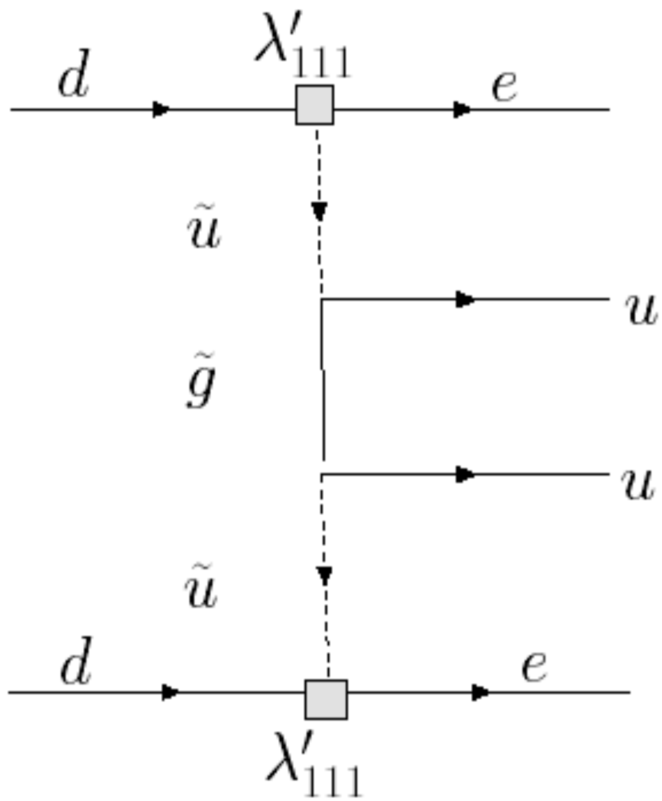
$$\frac{g_2^4}{M^4_{W_R}\,M_{\nu_R}}$$

In both cases the amplitude scales like $1/\Lambda^5$ with $\Lambda \sim M_{W(R)} \sim M_{\Delta} \sim M_{\nu\,(R)}$

Illustration II: RPV SUSY [R = $(-1)^{3(B-L) + 2s}$]

$$W_{RPV} = \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i^c D_j^c D_k^c + \mu'_i L_i H_u$$

0νββ



The 0νββ amplitude scales as

$$\frac{\pi\alpha_s \;\; \lambda'^2_{111}}{m_{\bar{g}} \;\; m^4_{\bar{f}}}$$

or in another example as

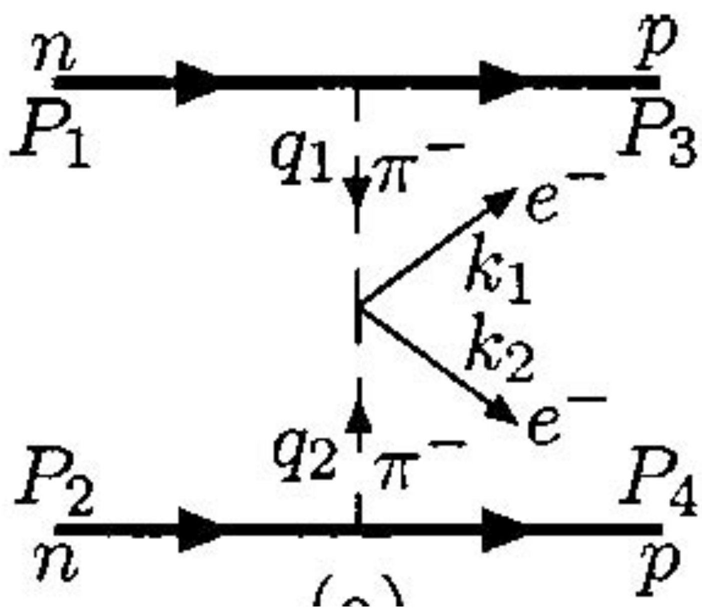
$$\frac{\pi\alpha_2 \;\; \lambda'^2_{111}}{m_\chi \;\; m^4_{\bar{f}}}$$

Again with the characteristic $1/\Lambda^5$ scaling

Note in passing that less attention has been devoted in the past to the evaluation of the nuclear matrix elements for the case of heavy particle exchange (short-range contribution to $0\nu\beta\beta$ decay). Proper treatment of the nucleon-nucleon repulsion in that case is obviously crucial; it is traditionally treated crudely using nucleon form factors.

Including pion exchange avoids this problem and seems to lead to larger and more consistently evaluated matrix elements.

(Vergados 82, Faessler *et al.* 97, Prezeau *et al.* 03)



$0\nu\beta\beta$ amplitude is contained in the $\pi\pi ee$ vertex

The study of lepton flavor violation (LFV) can help to decide what mechanism is responsible for the $0\nu\beta\beta$ decay if it is observed in a foreseeable future.

This is based on "Lepton number violation without supersymmetry" Phys.Rev.D 70 (2004) 075007
V. Cirigliano, A. Kurylov, M.J.Ramsey-Musolf, and P.V.

and on "Neutrinoless double beta decay and lepton flavor violation" Phys. Rev. Lett. 93 (2004) 231802
V. Cirigliano, A. Kurylov, M.J.Ramsey-Musolf, and P.V.

Lepton flavor violation (LFV) involving charged leptons has not been observed as yet. The most sensitive limits are for the decay

$$B_{\mu\to e\gamma} = \Gamma(\mu\to e\gamma)/\Gamma(\mu\to e\nu_\mu\nu_e) < 1.2\times10^{-11}$$

New experiment, MEG at PSI, started data taking in 2008 and should reach sensitivity ~ 2 orders of magnitude better.

The "muon conversion" is constrained by

$$B_{\mu\to e} = \frac{\Gamma(\mu^- + (Z,A) \to e^- + (Z,A))}{\Gamma(\mu^- + (Z,A) \to \nu_\mu + (Z-1,A))} < 8\times10^{-13}$$

Several proposals extending the sensitivity to $\sim10^{-17}$ have been proposed.

The fact that neutrinos have finite mass and that they mix will not make these LFV processes observable, they are suppressed by $(\Delta m^2/M_W^2)^2 \leq 10^{-50}$. Hence observation of them would imply "new physics" unrelated (or only indirectly related) to neutrino mass.

Summary so far:

1) Short-range contributions to the $0\nu\beta\beta$ decay with ~TeV mass scale can lead to the decay rate similar to that of light Majorana neutrino exchange with $\langle m_{\beta\beta} \rangle \sim 0.1 - 1$ eV.
2) In order to correctly interpret the experimental results and plan new experiments, it is important to determine the mechanism of the decay. Relation to LFV can help in that respect.
3) Next generation of experiments on LFV will extend the sensitivity considerably. In parallel, running of LHC will shed light on the existence of particles with ~TeV masses.

Lets restrict our further considerations to the simplest long-range mechanism involving the virtual exchange of a light Majorana neutrino.

As long as the mass eigenstates ν_i that are the components of the flavor neutrinos ν_e, ν_μ, and ν_τ are really Majorana neutrinos, the $0\nu\beta\beta$ decay **will occur,** with the rate

$$1/T_{1/2} = G(E_{tot},Z)\,(M^{0\nu})^2 \langle m_{\beta\beta}\rangle^2,$$

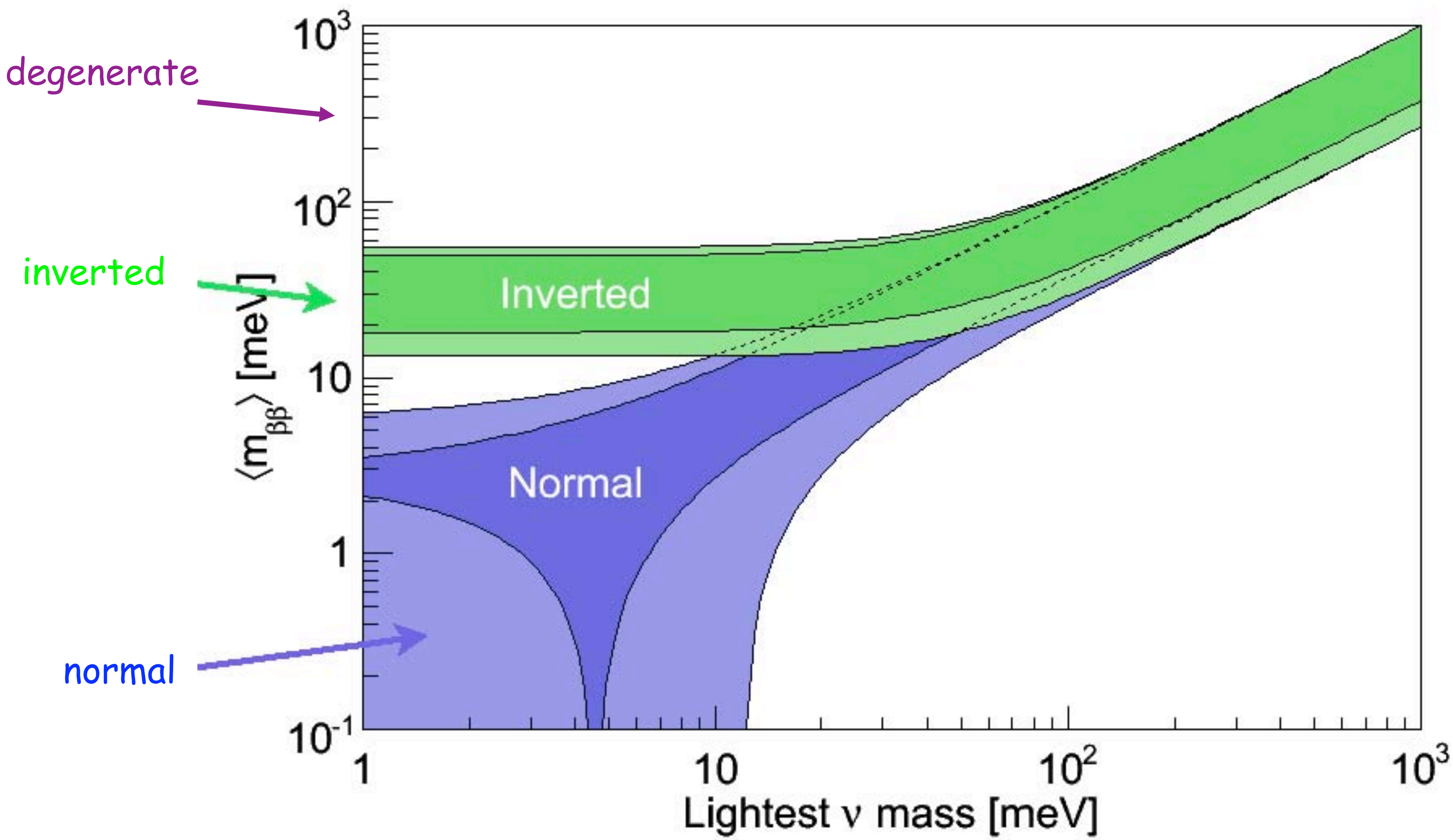
where $G(E_{tot},Z)$ is easily calculable phase space factor, $M^{0\nu}$ is the nuclear matrix element, calculable with difficulties (and discussed later), and

$$\langle m_{\beta\beta}\rangle = | \Sigma_i |U_{ei}|^2 \exp(i\alpha_i)\, m_i |,$$

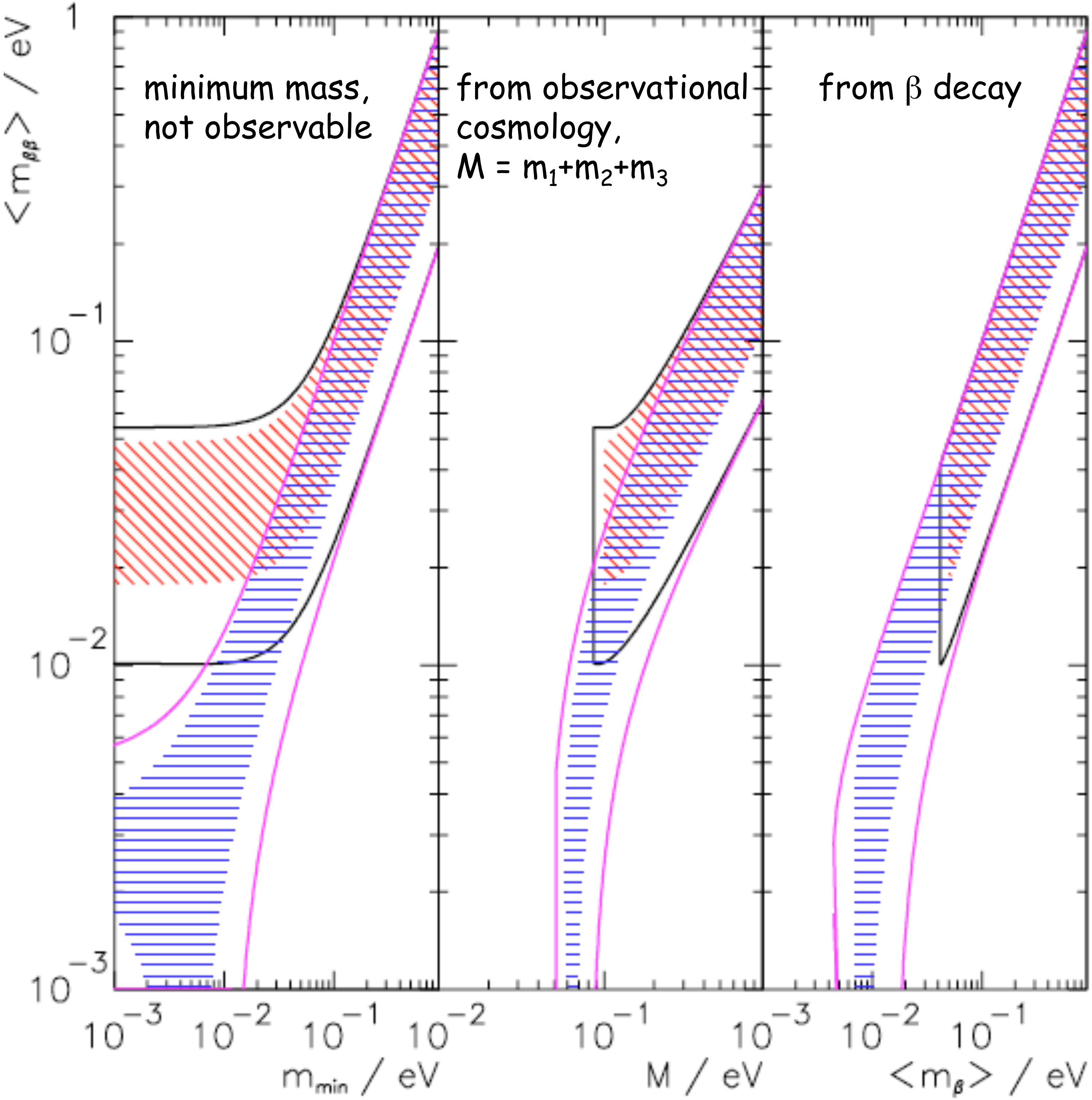
where α_i are unknown Majorana phases (only two of them are relevant).

Using the formula above we can relate $\langle m_{\beta\beta}\rangle$ to other observables related to the absolute neutrino mass.

Usual representation of that relation. It shows that the $\langle m_{\beta\beta}\rangle$ axis can be divided into three distinct regions as indicated. However, it creates the impression (**false**) that determining $\langle m_{\beta\beta}\rangle$ would decide between the two competing hierarchies.



$\langle m_{\beta\beta} \rangle$ vs. the absolute mass scales



blue shading: normal hierarchy, $\Delta m^2_{31} > 0$.

red shading: inverted hierarchy $\Delta m^2_{31} < 0$

shading:best fit parameters, lines 95% CL errors.

Thanks to A. Piepke

Note as a curiosity:
$\langle m_{\beta\beta}\rangle$ may vanish even though all m_i are nonvanishing and all ν_i are Majorana neutrinos.

What can we do in that case?

In principle, although probably not in practice, we can look for the lepton number violation involving muons.

Numerical example: take $\theta_{13} = 0$, and Majorana phase $\alpha_2 - \alpha_1 = \pi$ (only for this choice of phases can $\langle m_{\beta\beta}\rangle$ vanish when $\theta_{13} = 0$).
$\langle m_{\beta\beta}\rangle = 0$ if $m_1/m_2 = \tan^2\theta_{12}$, with $m_2 = (m_1^2 + \Delta m_{sol}^2)^{1/2}$.
That happens for $m_1 = 4.58$ meV and $m_2 = 10$ meV
(this is, therefore, fine tuning).
But then $\langle m_{\mu e}\rangle = \sin 2\theta_{12}\cos\theta_{23}/2\times(m_1 + m_2) = 4.78$ meV,
Which is, at least in principle, observable using
$\mu^- + (Z,A) \rightarrow e^+ + (Z-2,A)$.

Nuclear Matrix Elements:

In double beta decay two neutrons bound in the ground state of an initial even-even nucleus are simultaneously transformed into two protons that again are bound in the ground state of the final nucleus.

It is therefore necessary to evaluate, with a sufficient accuracy, **the ground state wave functions of both nuclei**, and evaluate the matrix element of the $0\nu\beta\beta$-decay operator connecting them.

This cannot be done exactly; some approximation and/or truncation is always needed. Moreover, unfortunately, there is no other analogous observable that can be used to judge the quality of the result.

Can one use the $2\nu\beta\beta$-decay matris elements for that?
What are the similarities and differences?

Both $2\nu\beta\beta$ and $0\nu\beta\beta$ operators connect the same states.
Both change two neutrons into two protons.

However, in $2\nu\beta\beta$ the momentum transfer $q <$ few MeV; thus $e^{iqr} \sim 1$, **long wavelength approximation** is valid, only the GT operator $\sigma\tau$ need to be considered.

In $0\nu\beta\beta$ $q \sim 100$-200 MeV, $e^{iqr} = 1 +$ **many terms**, there is no natural cutoff in that expansion.

Explaining $2\nu\beta\beta$-decay rate is necessary but not sufficient

To obtain the $0\nu\beta\beta$ operator, we need to integrate over dq^0 (the energy of the virtual neutrino), and Fourier transform the corresponding second order perturbation expression (this is now the integral over the three-momentum of the virtual neutrino):

$$H(r, E_m) = \frac{R}{2\pi^2 g_A^2} \int \frac{d\vec{q}}{\omega} \frac{1}{\omega + A_m} e^{i\vec{q}\cdot\vec{r}} = \frac{2R}{\pi r g_A^2} \int_0^\infty dq \frac{q \sin(qr)}{\omega(\omega + A_m)}$$

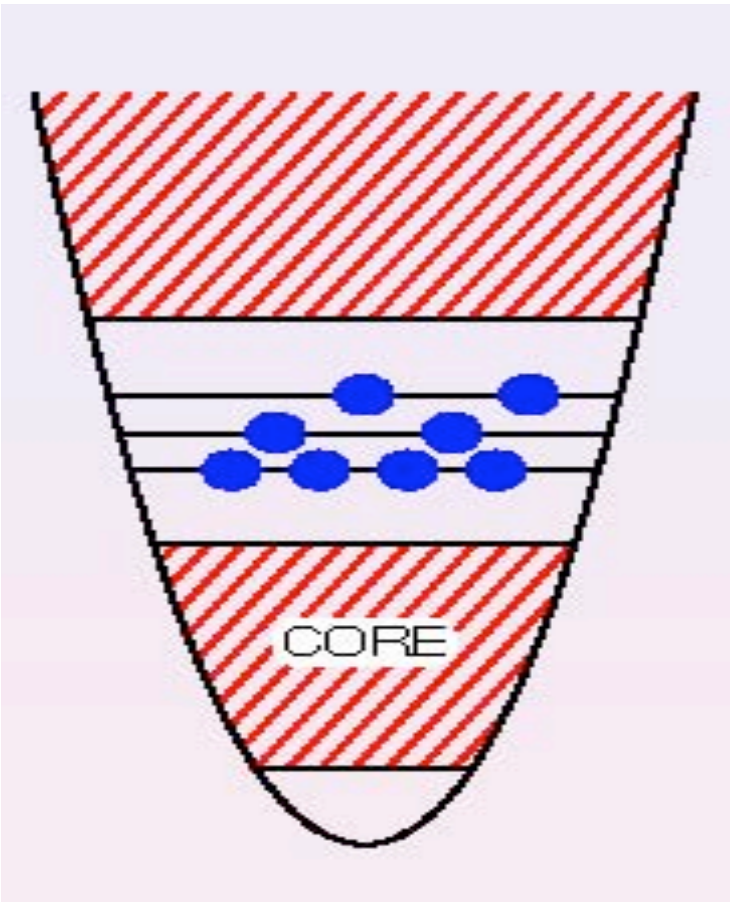
This is a `neutrino potential' (A_m is the total energy in the virtual nuclear state with respect to $(M_i + M_f)/2$). The constants are added for future convenience. r is the distance between the two neutrons that are transformed into protons.

This H(r, E), together with spin and isospin operators will appear in the nuclear matrix elements, In compact form

$H(r) = R/r\ \Phi(Er)$

where $\Phi(Er) \le 1$ is a slowly varying function. Since $r < R$ the potential is ≥ 1 (but less than 5-10).

Basic procedures: Treat the nucleus as a collection of protons and neutrons bound in a potential well, and interacting through an effective interaction. The procedure consists of several steps:



1) Define the valence space
2) Derive the effective hamiltonian H_{eff} using the nucleon-nucleon interaction plus some empirical nuclear data.
3) Solve the equations of motion to obtain the ground state wave functions.

Note: Completely full or completely empty subshells in both the initial and final nuclei will not participate in the $\beta\beta$ decay.

Two complementary procedures are commonly used:

a) Nuclear shell model (NSM)

b) Quasiparticle random phase approximation (QRPA)

In NSM a **limited** valence space is used but **all** configurations of valence nucleons are included. Describes well properties of low-lying nuclear states. Technically difficult, thus only few $0\nu\beta\beta$ calculations.

In QRPA a **large** valence space is used, but **only a class** of configurations is included. Describes collective states, but not details of dominantly few-particle states. Relatively simple, thus many $0\nu\beta\beta$ calculations.

QRPA proceeds in two steps.

1) First pairing between like nucleons is included in a simple fashion:

$$\begin{pmatrix} a^\dagger_{jm} \\ \tilde{a}_{jm} \end{pmatrix} = \begin{pmatrix} u_j c^\dagger_{jm} & + & v_j \tilde{c}_{jm} \\ -v_j c^\dagger_{jm} & + & u_j \tilde{c}_{jm} \end{pmatrix}$$

particles — quasiparticles

Bogoliubov transformation, proton and neutron Fermi levels are smeared. However, particle numbers are conserved only in average.

2) Then the proton-neutron interaction is included

$$|J^\pi M; m\rangle = \Sigma_{pn} \left[X^m_{pn,J\pi} A^\dagger(pn; J^\pi M) + Y^m_{pn,J\pi} \tilde{A}(pn; J^\pi M) \right] |0^+_{QRPA}\rangle$$

two quasiparticle creation operator

two quasiparticle annihilation operator

correlated ground state, includes zero-point motion

Evaluation of $M^{0\nu}$ involves transformation to the relative coordinates of the nucleons (the operators O_K depend on r_{ij})

$$M_K = \sum_{J^\pi, k_i, k_f, \mathcal{J}} \sum_{pnp'n'} (-1)^{j_n + j_{p'} + J + \mathcal{J}} \times$$

$$\sqrt{2\mathcal{J}+1} \left\{ \begin{array}{ccc} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{array} \right\} \times$$

$$\langle p(1), p'(2); \mathcal{J} \parallel \bar{f}(r_{12}) \tau_1^+ \tau_2^+ O_K \bar{f}(r_{12}) \parallel n(1), n'(2); \mathcal{J} \rangle \times$$

$$\langle 0_f^+ ||| [\widetilde{c_{p'}^+ \tilde{c}_{n'}}]_J ||J^\pi k_f \rangle \langle J^\pi k_f | J^\pi k_i \rangle \langle J^\pi k_f i ||[c_p^+ \tilde{c}_n]_J || 0_i^+ \rangle \,.$$

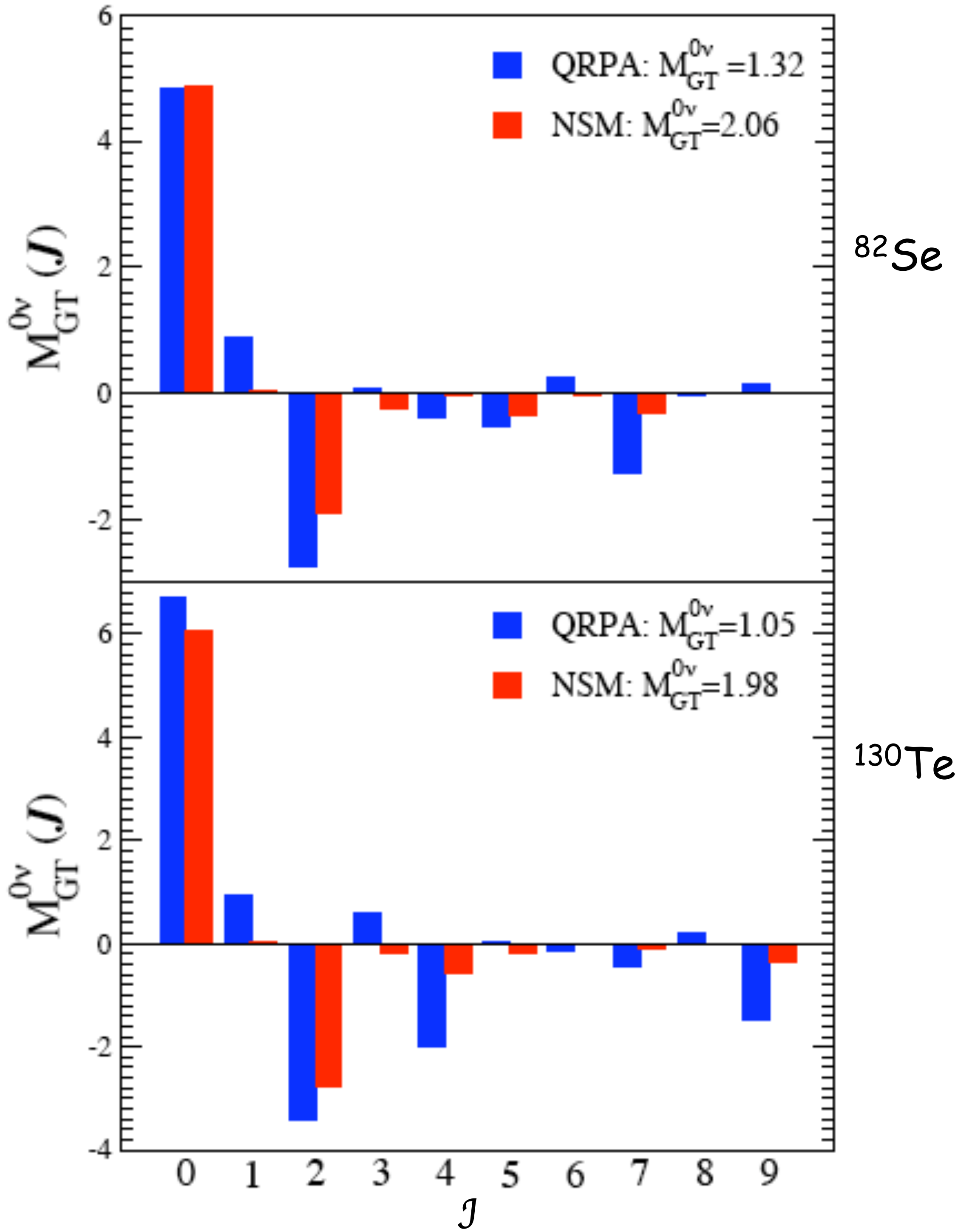
unsymmetrized two-body radial integral involves `neutrino potentials'

From QRPA for final nucleus

overlap

From QRPA for initial nucleus

Note the two separate multipole decompositions. J^π refers to the virtual state in odd-odd nucleus, while $\mathcal{J}$ refers to the angular momentum of the neutron pair transformed into proton pair.



Why it is difficult to calculate the matrix elements accurately?

Contributions of different angular momenta *J* of the neutron pair that is transformed in the decay into the proton pair with the same *J*.

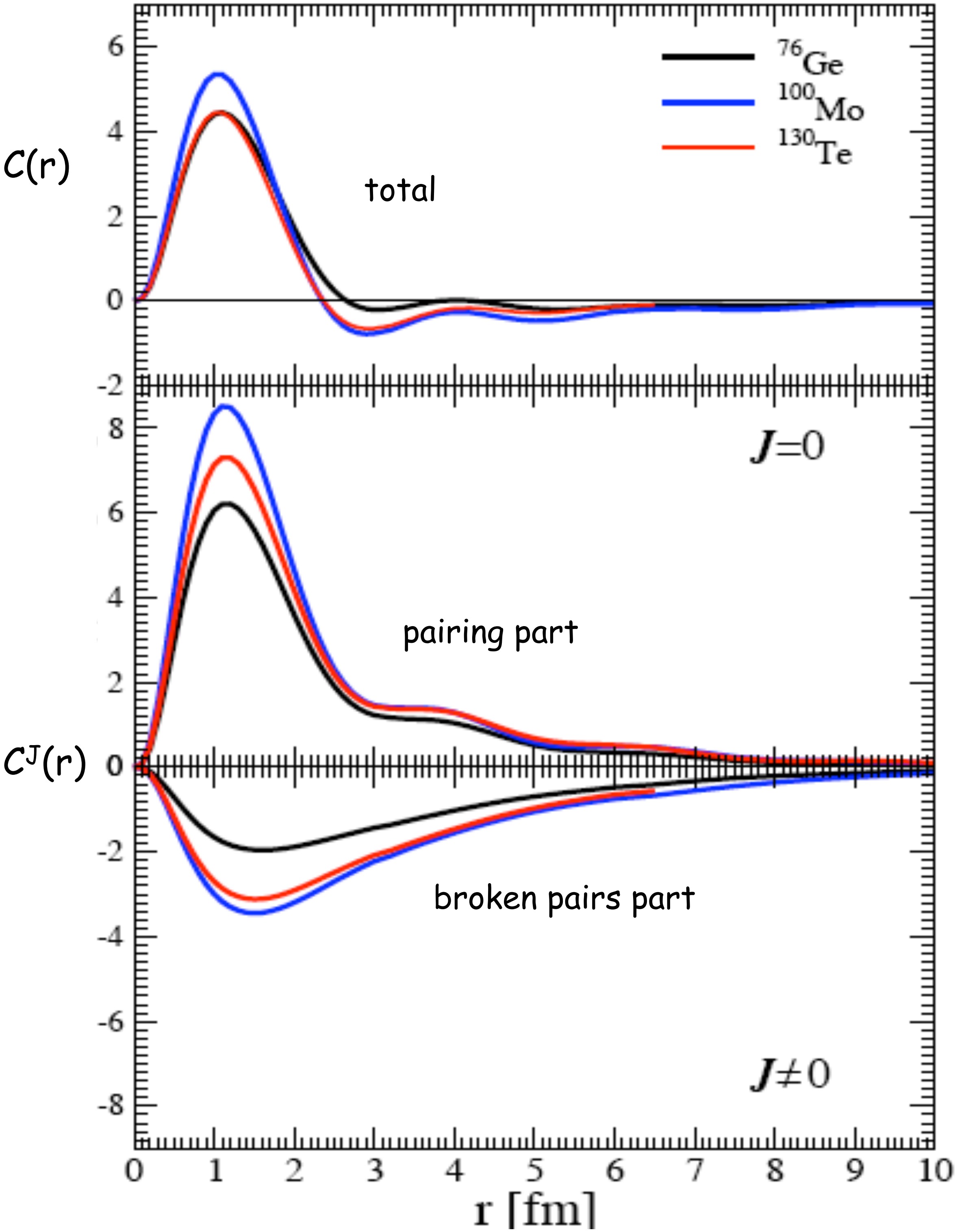
Note the opposite signs, and thus tendency to cancel, between the *J* = 0 (pairing) and the *J* ≠ 0 (ground state correlations) parts.

The same restricted s.p. space is used for QRPA and NSM. There is a reasonable qualitative agreement between the two methods

Dependence on the relative distance, nucleon structure, short range repulsion, higher order currents, etc.

The neutrino propagator connecting the two participating nucleons introduces dependence on the relative distance ***r*** (or equivalently momentum transfer ***q***) between them.
Recall that the ``neutrino potential" is **H(r) = R/r** $\Phi(\omega r)$, where $\Phi(\omega r)$ is slowly varying function. Thus, **naively**, one expects that the typical distance is r ~ R.

If small values of ***r*** (or large values of ***q***) are important, we have to worry about induced weak currents (terms q/M_p), nucleon finite size, and the short range nucleon-nucleon repulsion.

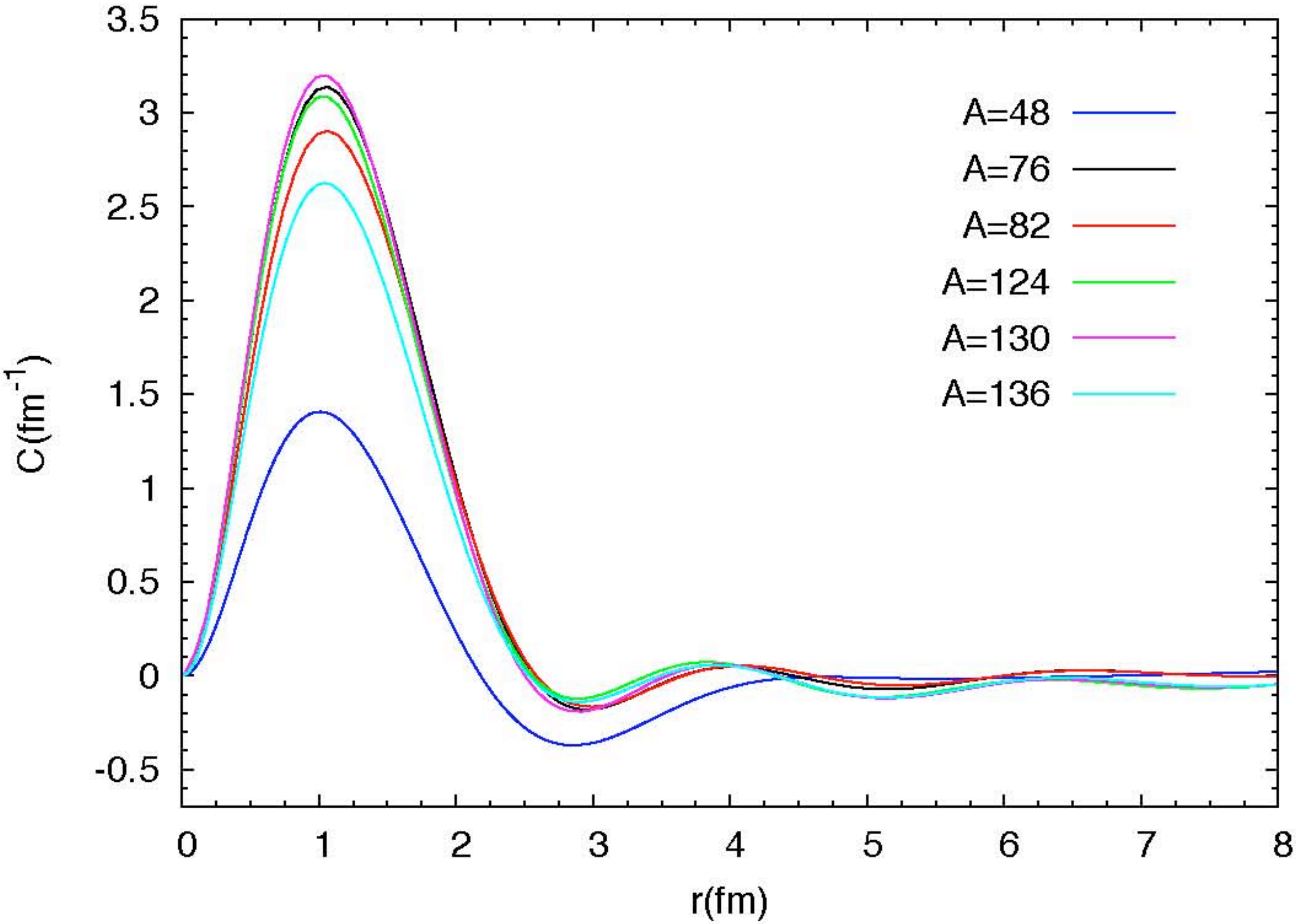


The radial dependence of $M^{0\nu}$ for the three indicated nuclei. The contributions summed over all components ss shown in the upper panel. The `pairing' $J = 0$ and `broken pairs' $J \neq 0$ parts are shown separately below.

Note that these two parts essentially cancel each other for r > 2-3 fm. This is a generic behavior. Hence the treatment of small values of *r* and large values of *q* are quite important.

$$M^{0\nu} = \int C(r)dr$$

The radial dependence of $M^{0\nu}$ for the indicated nuclei, evaluated in the **nuclear shell model**. (Menendes et al, arXiv:0801.3760).
Note the similarity to the QRPA evaluation of the same function.



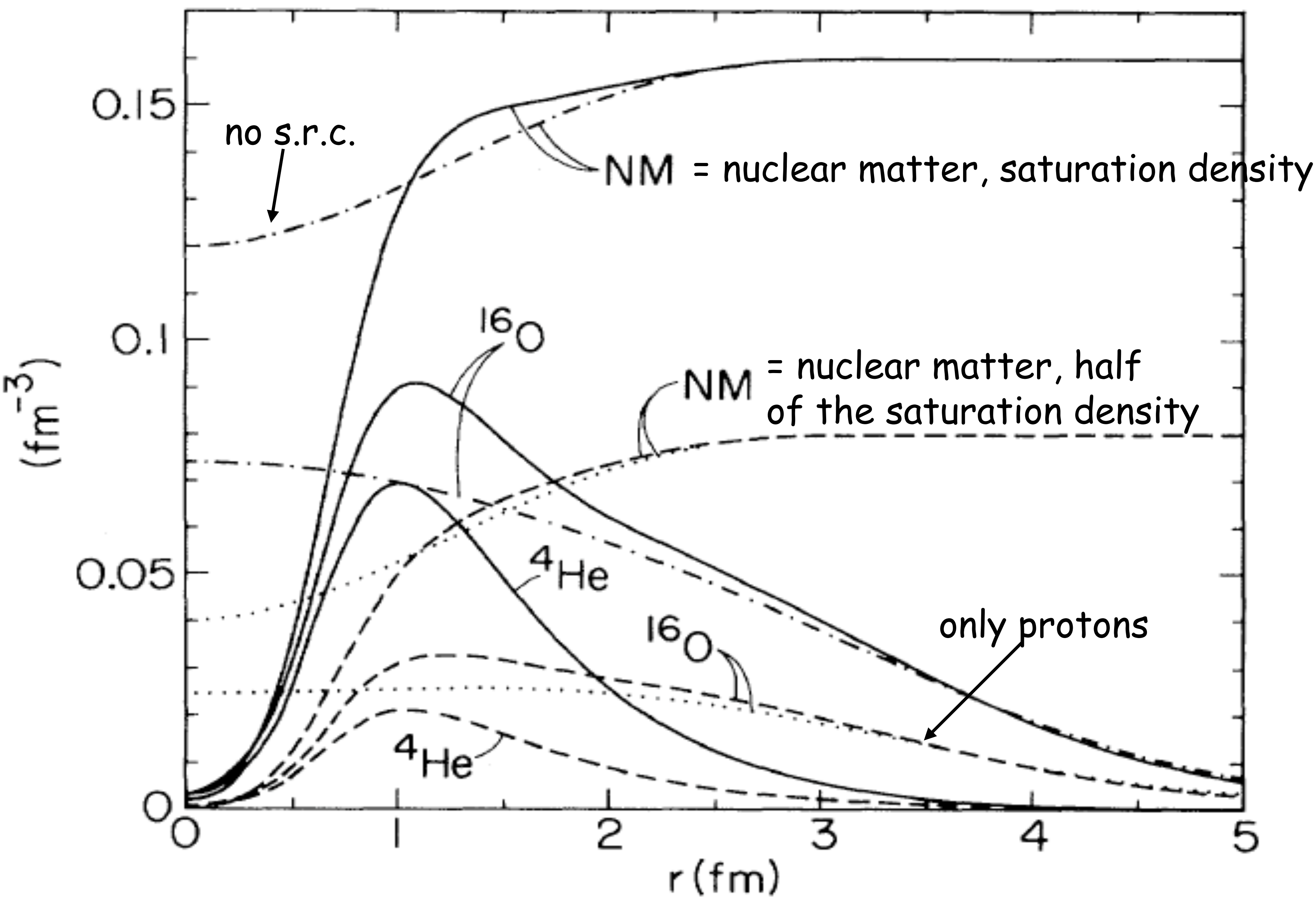
The finding that the relative distances $r < 2\text{- }3$ fm, and correspondingly that the momentum transfer $q > \sim 100$ MeV means that one needs to consider a number of effects that typically play a minor role in the structure of nuclear ground states:

a) Short range repulsion
c) Nucleon finite size
d) Induced weak currents (Pseudoscalar and weak magnetism)

Each of these, with the present treatment, causes correction (or uncertainty) of ~20% in the $0\nu\beta\beta$ matrix element.

There is a consensus now that these effects must be included but only emerging consensus how to treat them, in particular a).

Two-nucleon probability distribution, with and without correlations, MC with realistic interaction. O. Benhar et al. RMP**65**,817(1993)



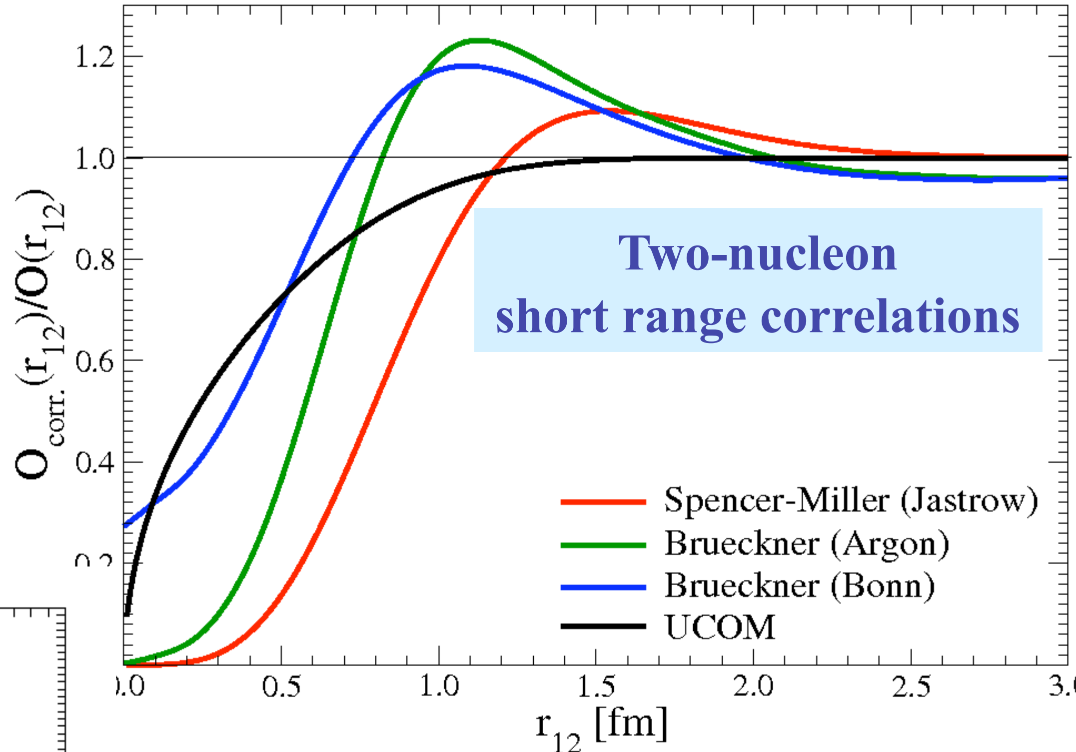
See also Bisconti et al., Phys. Rev. C73, 054304(2006) for the more modern version of this

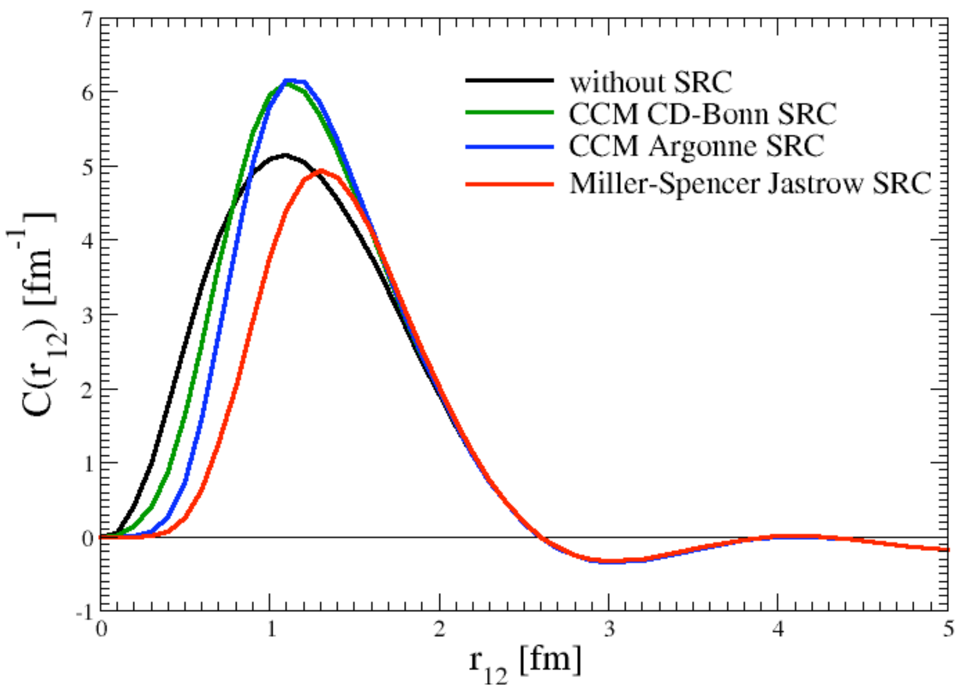
Dependence on the distance between the two transformed nucleons and the effect of different treatments of short range correlations. This causes changes of $M^{0\nu}$ by up to ~ 20%.

$$|\Psi\rangle_{corr.} = f(r_{12})\,|\Psi\rangle$$

$$O_{corr.}(r_{12}) = f(r_{12})O(r_{12})f(r_{12})$$

ββ decay operator





Effect of including src on the radial function c(r) for the $0\nu\beta\beta$ nuclear matrix element, $M^{0\nu} = \int C(r)dr$

Hadronic current expressed in terms of nucleon fields Ψ:

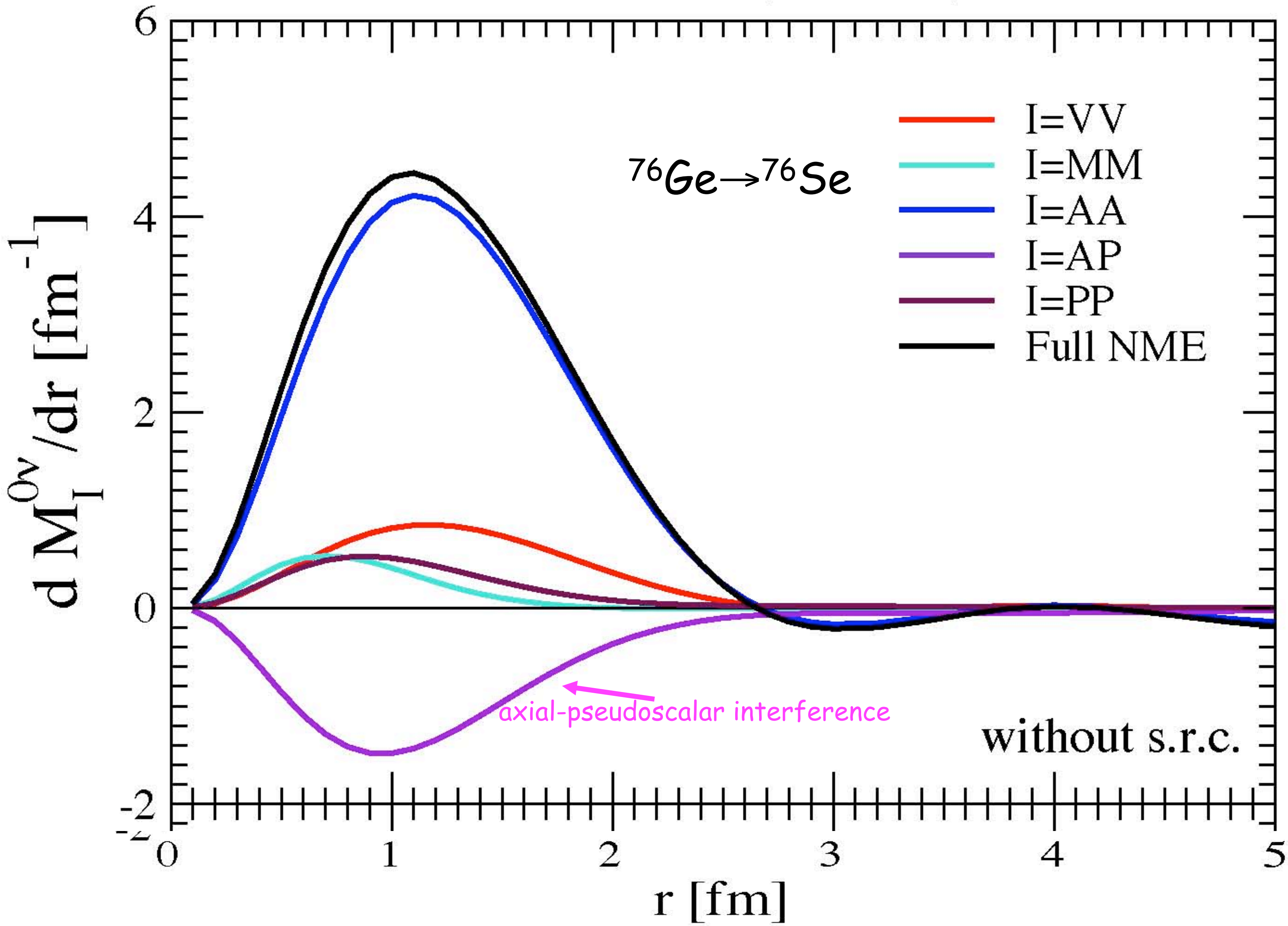
$$j^{\rho\dagger} = \overline{\Psi}\tau^{+}\left[g_V(q^2)\gamma^{\rho} + i g_M(q^2)\frac{\sigma^{\rho\nu}}{2m_p}q_{\nu} - g_A(q^2)\gamma^{\rho}\gamma_5 - g_P(q^2)q^{\rho}\gamma_5 \right]\Psi,$$

Vector $g_V(q^2) = g_V/(1 + q^2/M_V^2)^2$, $g_V = 1$, $M_V = 0.85$ GeV
Axial vector $g_A(q^2) = g_A/(1 + q^2/M_A^2)^2$, $g_A = 1.25$, $M_A = 1.09$ GeV
Weak Magnetism $g_M(q^2) = (\mu_p - \mu_n)\, g_V(q^2)$
Induced pseudoscalar $g_P(q^2) = 2m_p g_A(q^2)/(q^2 + m_\pi^2)$

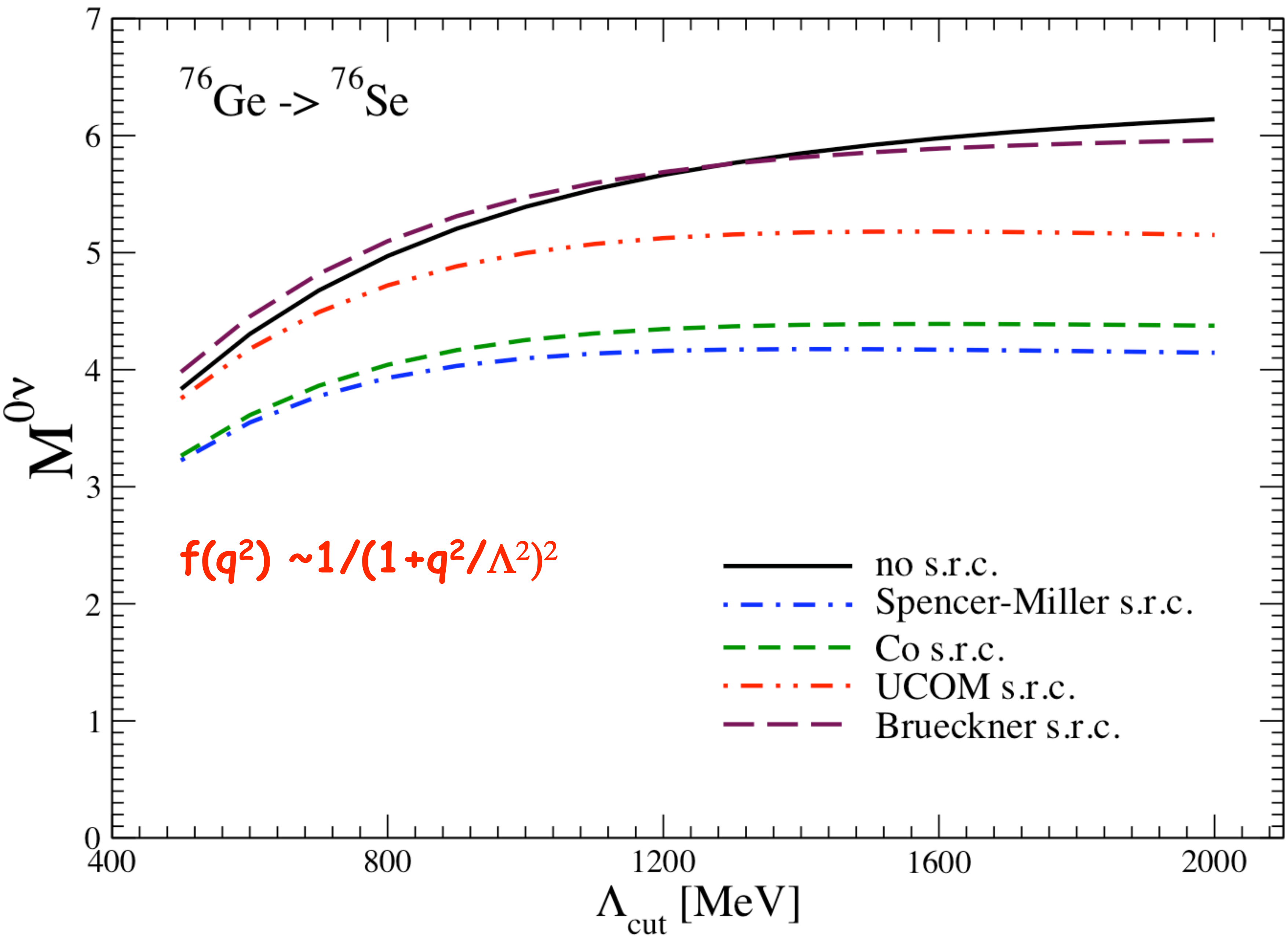
After the nonrelativistic reduction the space part of the current is

$$\vec{J}_n(\vec{q}^2) = g_M(\vec{q}^2) i \frac{\vec{\sigma}_n \times \vec{q}}{2m_p} + g_A(\vec{q}^2)\vec{\sigma} - g_P(\vec{q}^2)\frac{\vec{q}\,\vec{\sigma}_n \cdot \vec{q}}{2m_p}$$

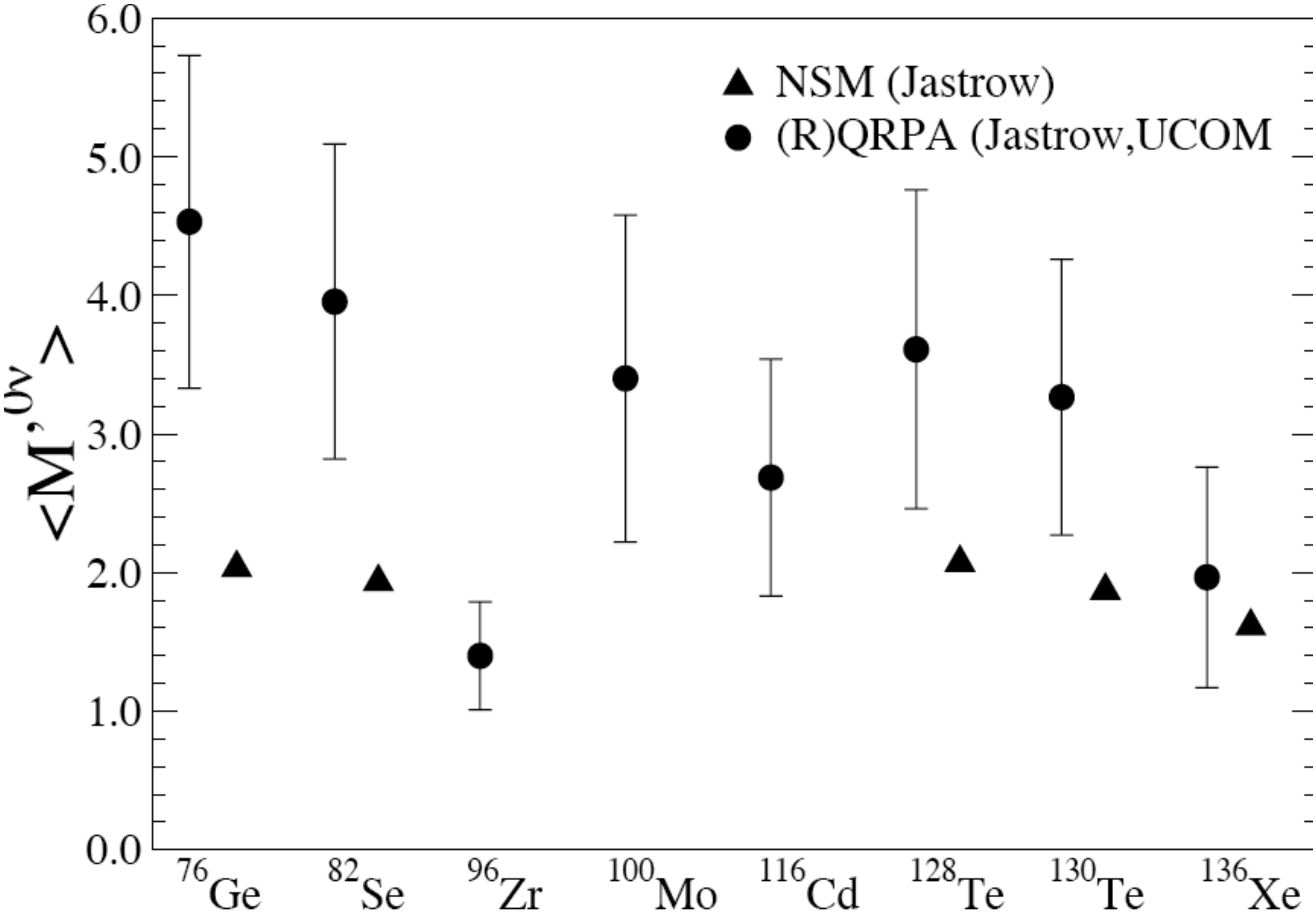
Contributions of different parts of the nucleon current. Note that the AP (axial-pseudoscalar interference) contains $q^2/(q^2 + m_\pi^2)$, and MM contains $q^2/4M_p^2$.



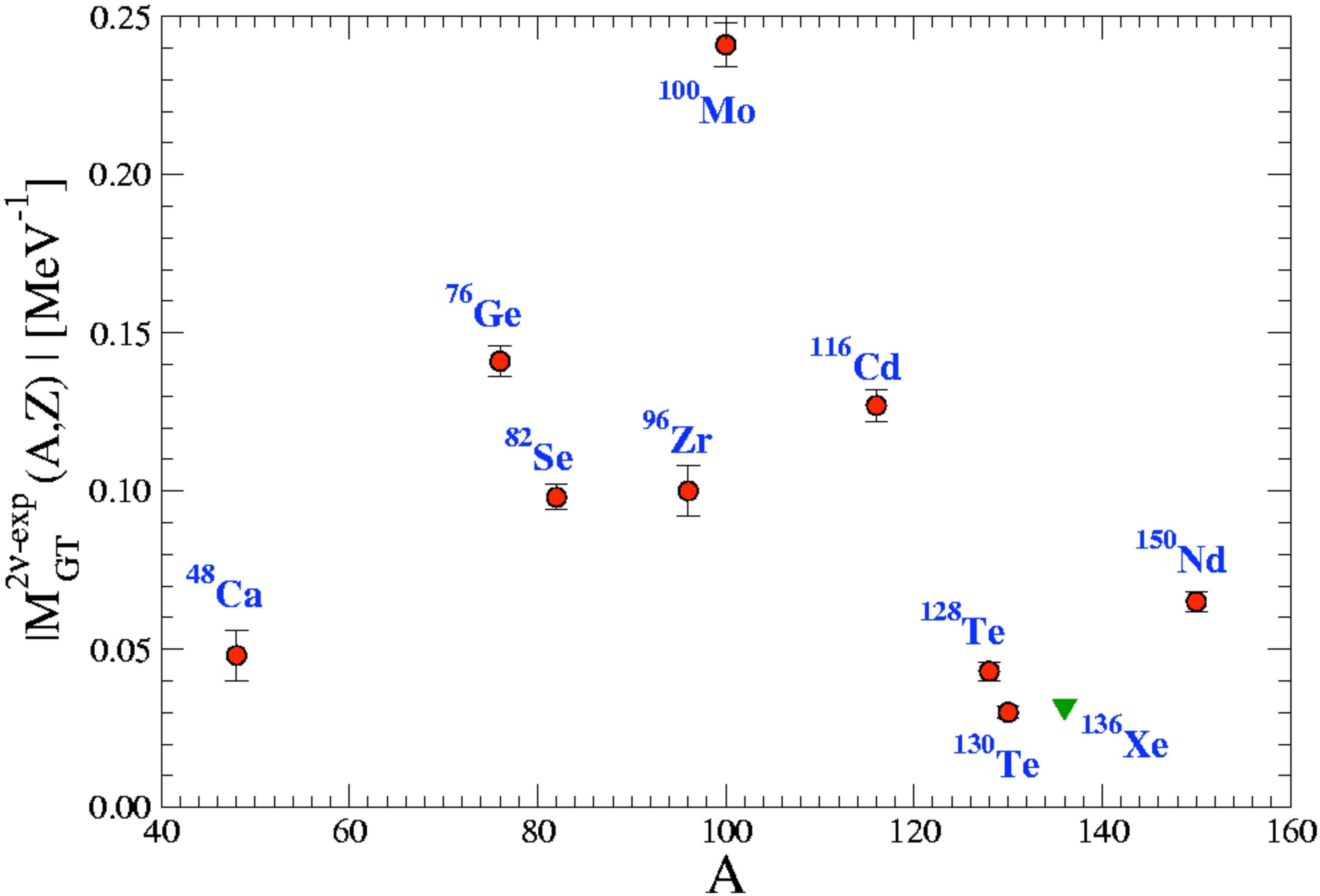
Dependence of the $0\nu\beta\beta$ matrix element on the $M_A = M_V = \Lambda_{cut}$ parameter in the usual dipole nucleon form factor . When correction for short range correlations is included the $M^{0\nu}$ changes little for $\Lambda_{cut} \geq 1000$ MeV.



Full estimated range of $M^{0\nu}$ within QRPA framework and comparison with NSM (higher order currents now included in NSM, status as of 2008)



The 2ν matrix elements, unlike the 0ν ones, exhibit pronounced shell effects. They vary fast as a function of Z or A.



$0\nu\beta\beta$ nuclear matrix elements calculated very recently with the **Interacting Boson Model-2**, see Barea and Iachello, Phys. Rev. **C79**, 044301(2009).

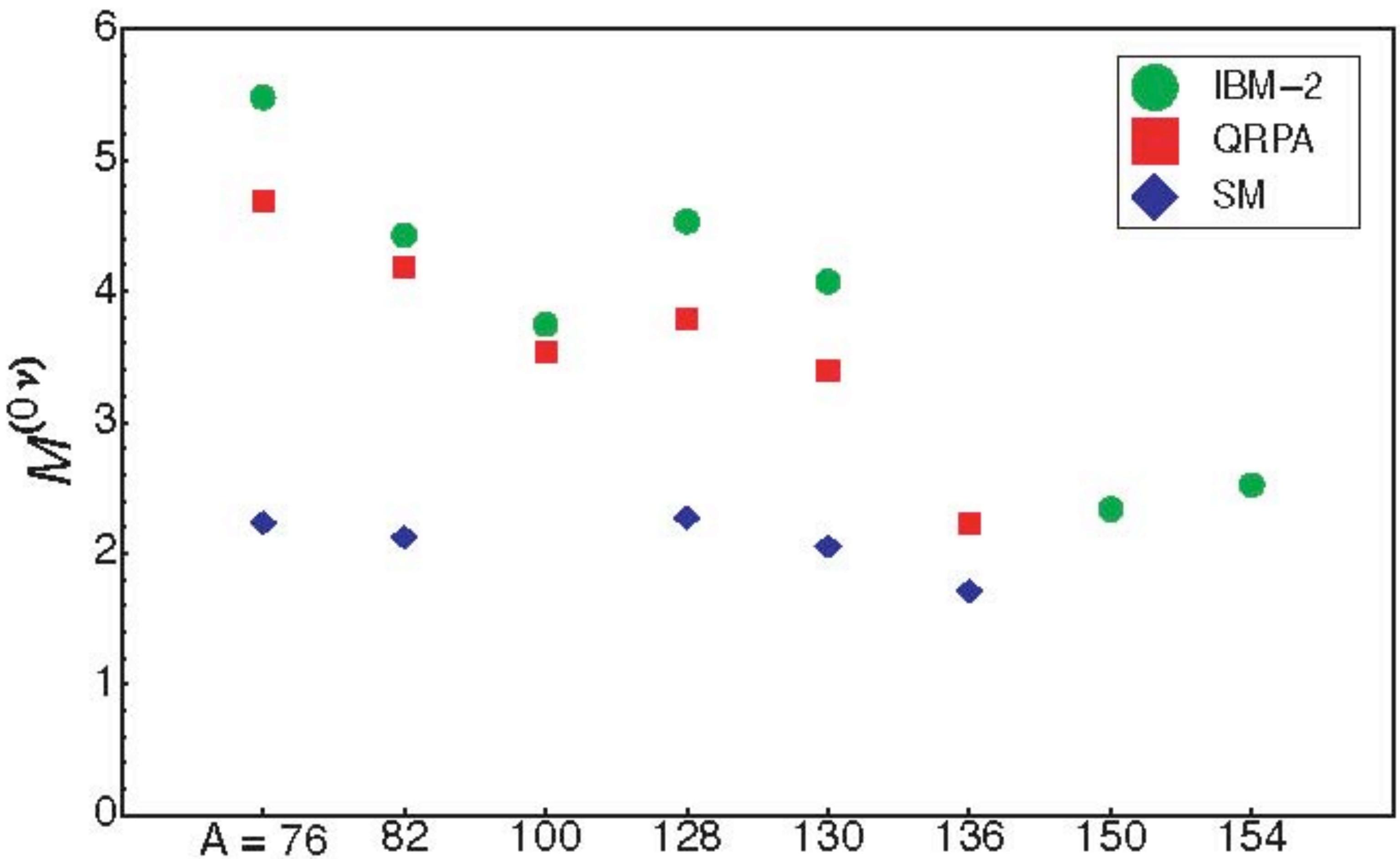
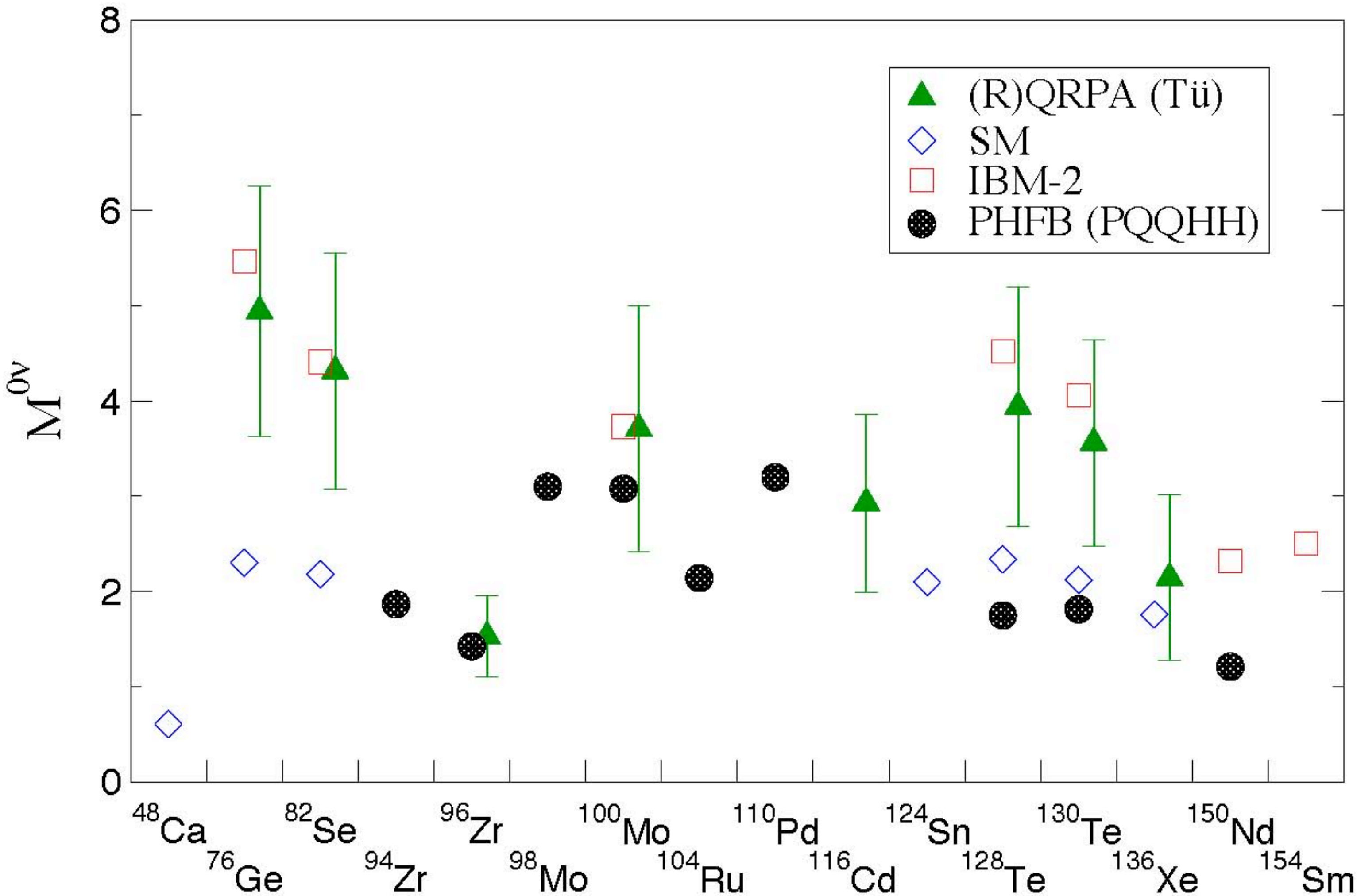


FIG. 2. (Color online) Neutrinoless double-β decay matrix elements in the formulation of Šimkovic *et al.* [10] for IBM-2, Set I (this work), QRPA with $g_A = 1.25$ and Jastrow SRC [20], and SM [8].

Same as before but the results of Projected HFB method added.
Note that all agree on a rather smooth dependence on A and Z.
However, the results of different methods can differ by ~2.



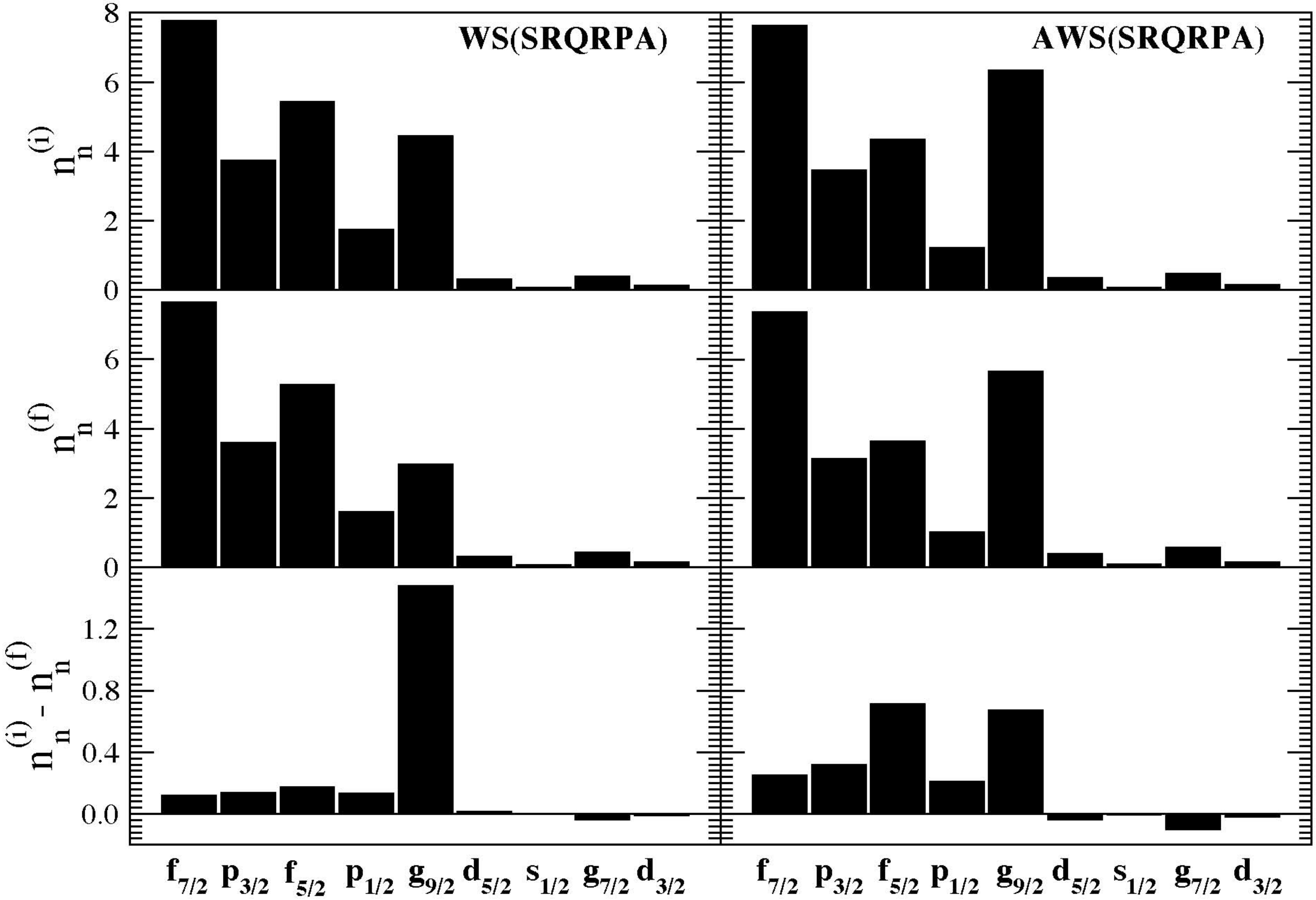
Why are the QRPA and NSM matrix elements different?

Various possible explanations:

a) Assumed occupancies of individual valence orbits might be different
b) In QRPA more single particle states are included
c) In NSM all configurations (seniorities) are included
d) In NSM the deformation effects are included
e) All of the above

Assumed occupancies of individual valence orbits might be different

Neutron orbit occupancies, original Woods-Saxon vs. adjusted effective mean field. For $^{76}Ge \rightarrow {}^{76}Se$



experiment

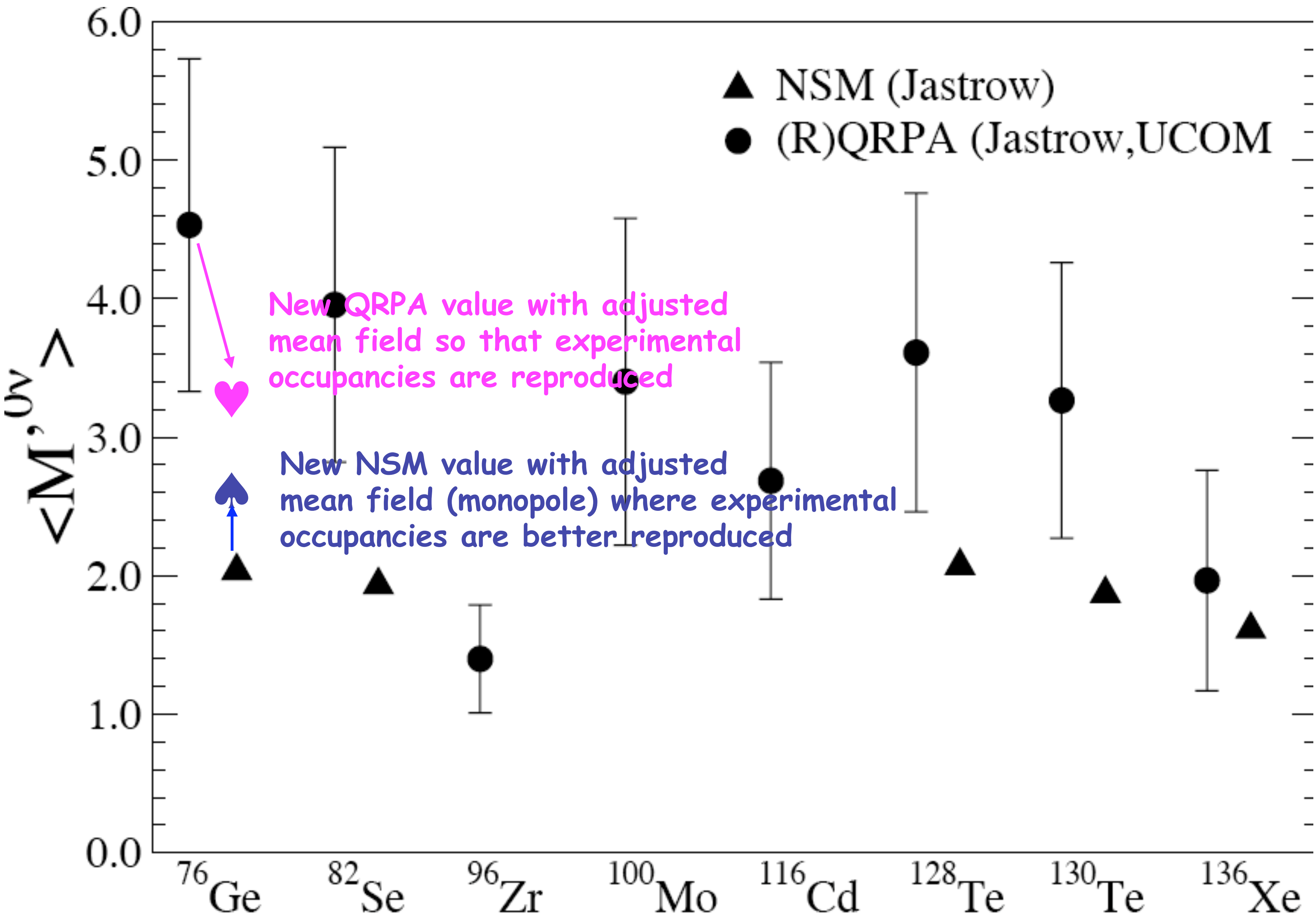
p 4.9 ± 0.2
$f_{5/2}$ 4.6 ± 0.4
$g_{9/2}$ 6.5 ± 0.3

p 4.4 ± 0.2
$f_{5/2}$ 3.8 ± 0.4
$g_{9/2}$ 5.8 ± 0.3

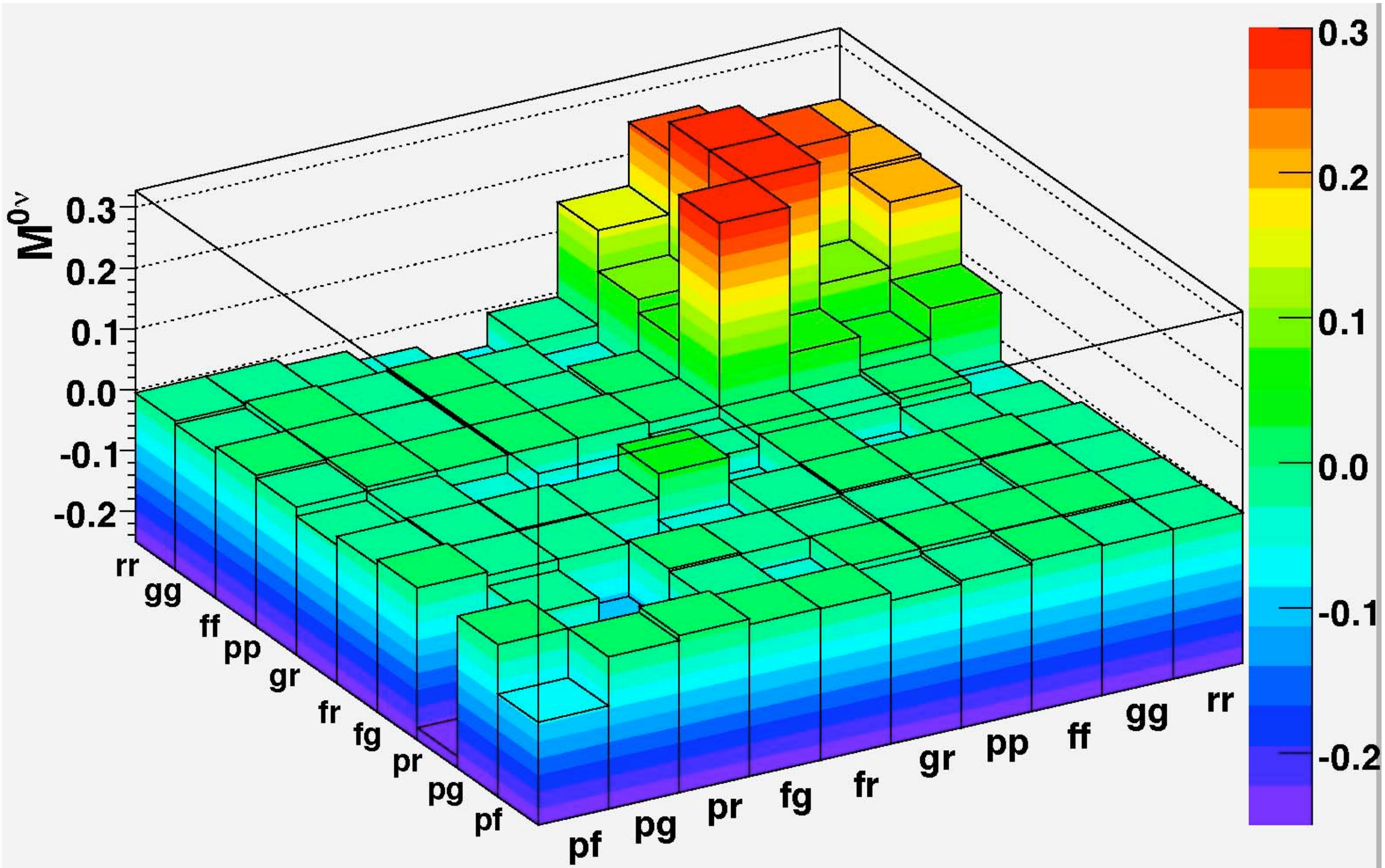
p 0.5
$f_{5/2}$ 0.8
$g_{9/2}$ 0.7

Experiment from J.P.Schiffer et al, Phys.Rev.Lett. **100**, 1120501(2008), used (d,p),(p,d),($^3He,\alpha$),($\alpha,{}^3He$) to derive occupancies of neutron orbits

Full estimated range of $M^{0\nu}$ within QRPA framework and comparison with NSM (higher order currents now included in NSM)



In QRPA more single particle states are included



Contribution of initial neutron orbit pairs against the final proton pairs.
The nonvalence orbits are labeled as **r**. Adding all parts with r-type orbits gives +2.83 - 3.22 = -0.39 which is only ~12% of the total matrix element 3.27 (The total matrix element is made of 9.74 - 6.46 = 3.27.)
In the figure all entries are, however, normalized so that their sum is unity..

It appears, therefore, that all of these effects, possible differences in the assumed occupancies of valence orbits, additional single particle states included in QRPA but not in NSM, inclusion of complicated configurations (higher seniority and/or deformation) in NSM but only crudely in QRPA, can, and probably do, affect the resulting nuclear matrix elements, and might explain the different outcomes of the two methods.

In particular, the difference in deformation of the initial and final nuclei makes the evaluation of the matrix element for ^{150}Nd -> ^{150}Sm very difficult.

Summary

1) There is, as of now, agreement of all practitioners on what needs to be included in the evaluation of the $0\nu\beta\beta$ nuclear matrix elements, even though there is no complete agreement how to do it (e.g. for the short range correlations).
2) The NSM and QRPA have both many basic features in common, in particular the (sometimes severe) cancellation between the effect of pairing and `broken pairs' configurations and in the radial distance dependence.
3) There are still noticeable differences between the two methods, and several possible causes have been identified.
4) Both methods predict that the $0\nu\beta\beta$ nuclear matrix elements should vary slowly and rather smoothly with A and Z, unlike the $2\nu\beta\beta$ matrix elements. That makes the comparison of experiments with different sources easier.