The 2010 NNPSS-TSI

A. Richter 4^{th} Lecture

Quantum Manifestations of Classical Chaos – Some Universal Features of Billiards and Nuclei

- Classical billiards and quantum billiards
- Random Matrix Theory (Wigner 1951 – Dyson 1962)
- Spectral properties of billiards and mesoscopic systems
- Microwave resonator as a model for the compound nucleus

  S-Matrix fluctuations in the regime of overlapping resonances

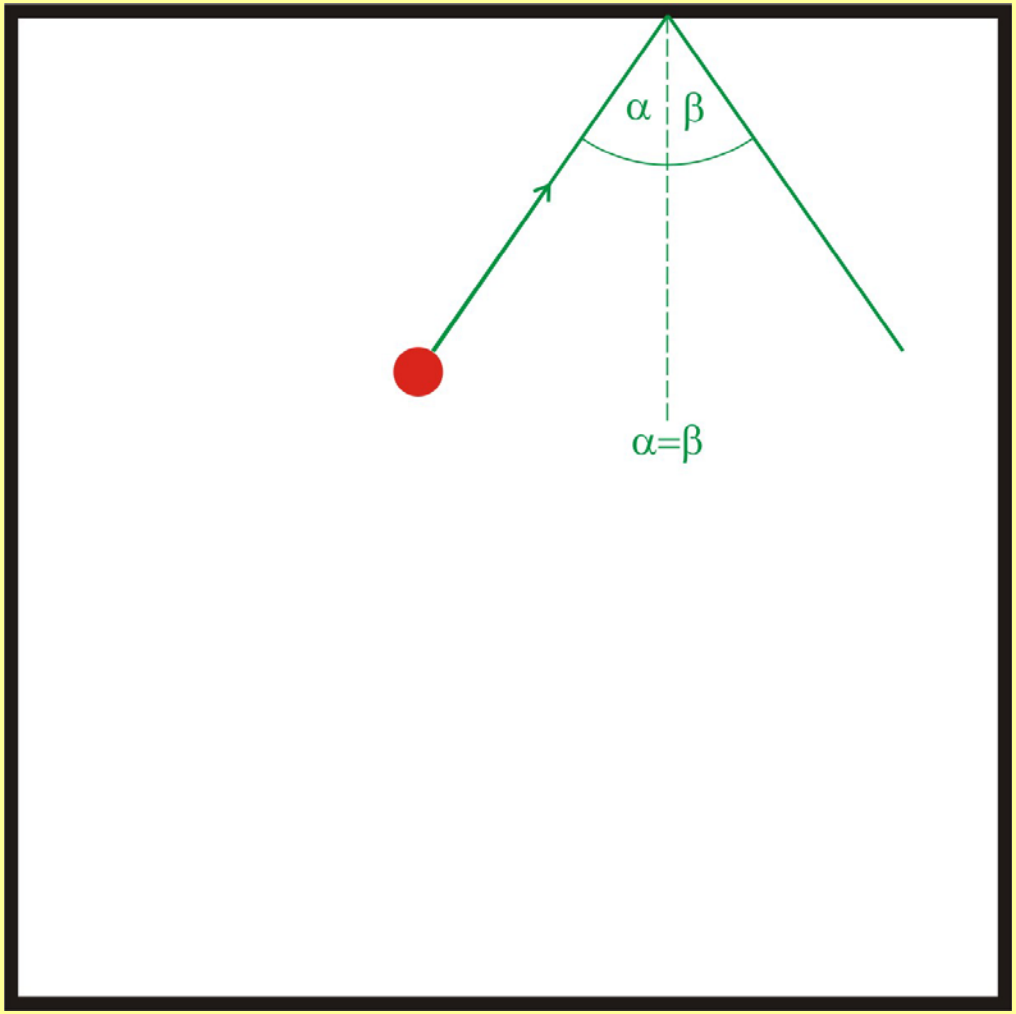
  Induced time-reversal symmetry breaking

Supported by DFG under SFB 634

Key References for 4th Lecture

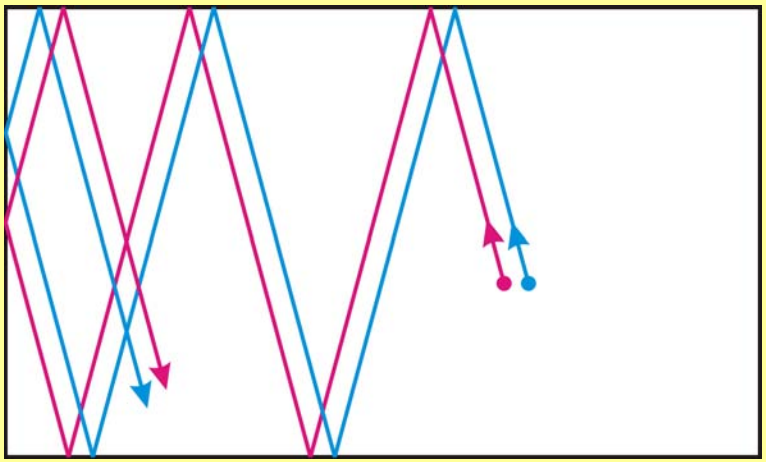
- H. A. Weidenmüller and G. E. Mitchell, Rev. Mod. Phys. **81**, 539 (2009)
- G. E. Mitchell, A. Richter and H. A. Weidenmüller, arXiv:1001.2422v1 (2010)
- B. Dietz et al., Phys. Rev. Lett. **98**, 074103 (2007)
- B. Dietz et al., Phys. Rev. Lett. **103**, 064101 (2009)
- B. Dietz et al., Phys. Lett. B **685**, 263 (2010)
- B. Dietz et al., Phys. Rev. E **81**, 036205 (2010)

Classical Billiard

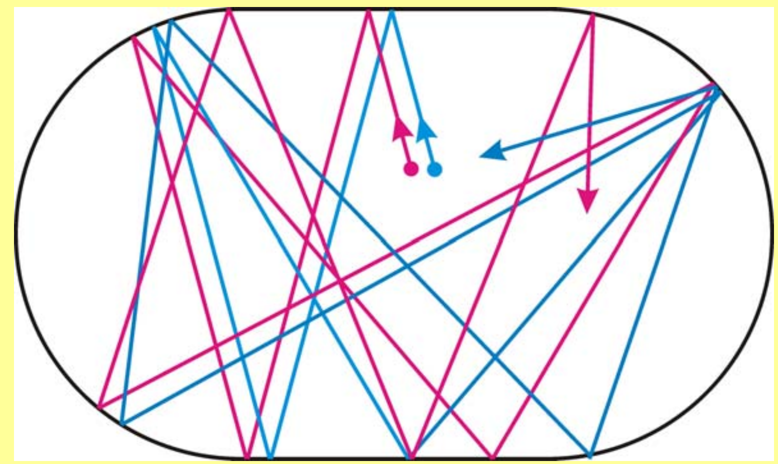


Regular and Chaotic Dynamics

Regular



Bunimovich stadium (chaotic)



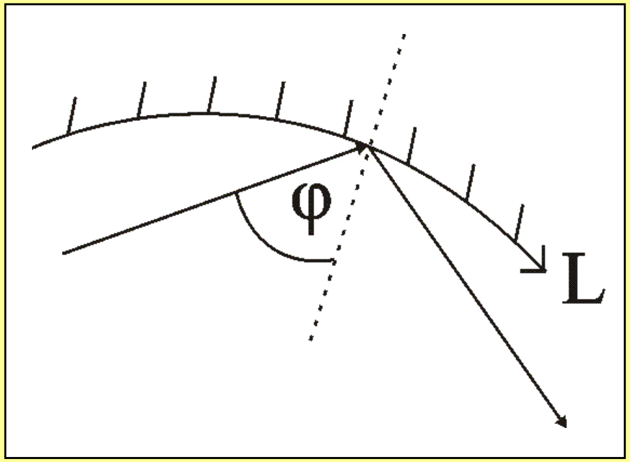
- Energy and p_x^2 are conserved
- Equations of motion are integrable
- Predictable for infinite long times

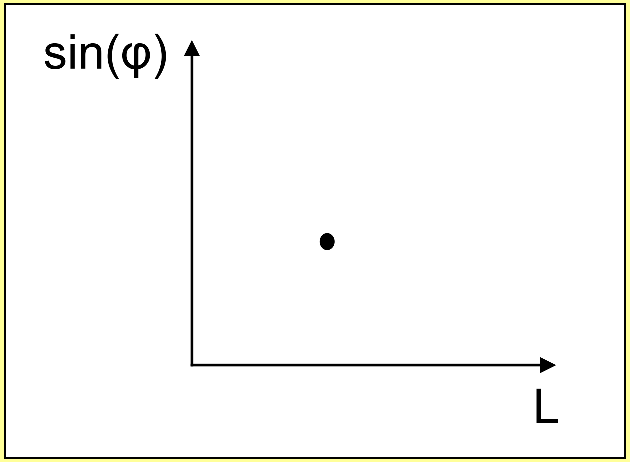
- Only energy is conserved
- Equations of motion are not integrable
- Predictable for a finite time only

Tool: Poincaré Sections of Phase Space

- Parametertization of billiard boindary: L
- Momentum component along the boundary: $\sin(\varphi)$

conjugate variables





Small Changes $\rightarrow$ Large Actions

- Sensitivity of the solutions of a deterministic problem with respect to small changes in the initial conditions is called Deterministic Chaos

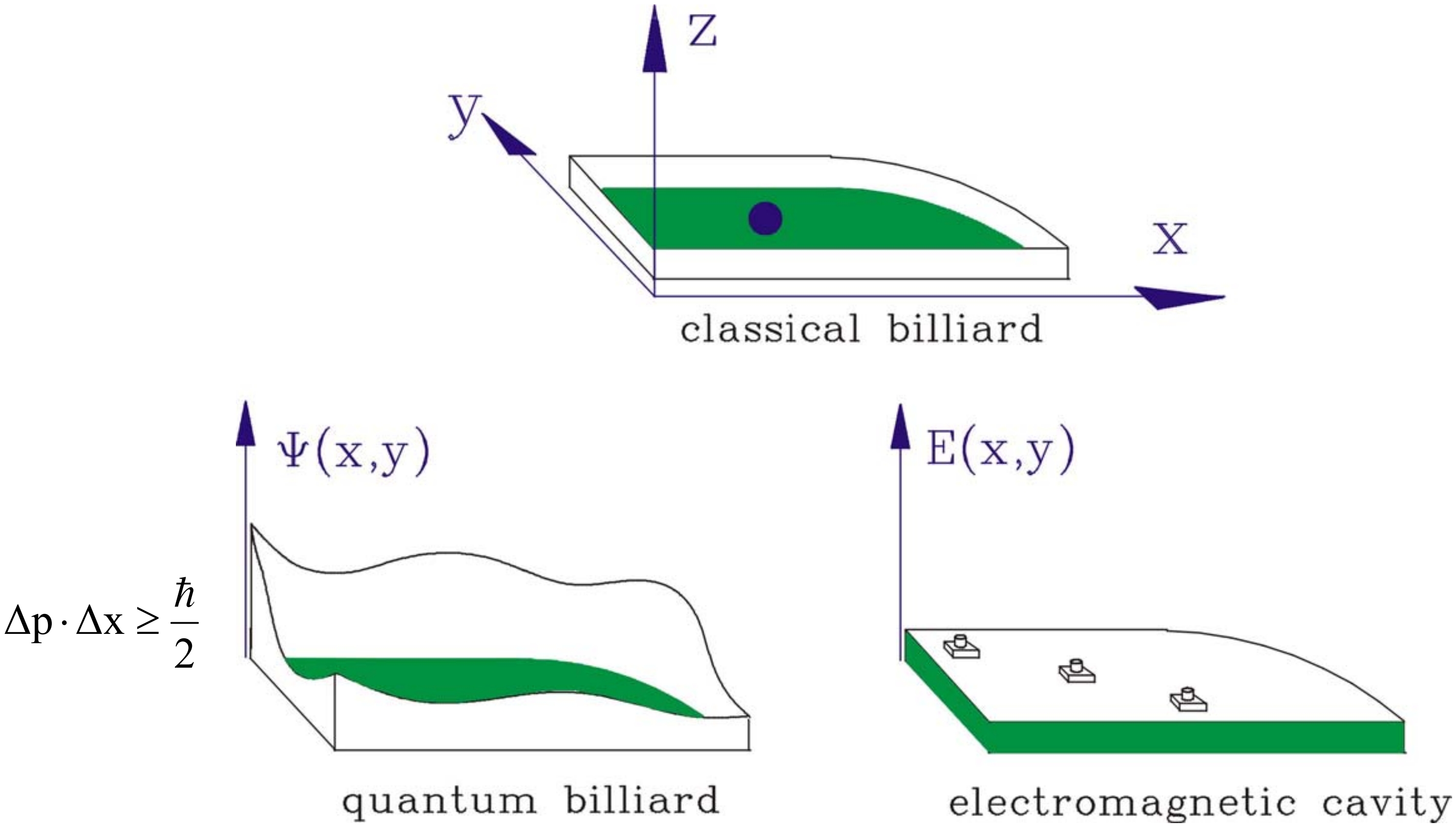
- Beyond a fixed, for the system characteristic time becomes every prediction impossible. The system behaves in such a way as if not determined by physical laws but randomness

Our Main Interest

- How are these properties of classical systems transformed into corresponding quantum-mechanical systems ?
  $\rightarrow$ Quantum chaos ?

- What might we learn from generic features of billiards and mesoscopic systems (hadrons, nuclei, atoms, molecules, metal clusters, quantum dots) ?

The Quantum Billiard and its Simulation



Schrödinger $\leftrightarrow$ Helmholtz

quantum billiard

2D microwave cavity: $h_z < \lambda_{min}/2$

$$\left(\Delta + k^2\right)\Psi = 0$$

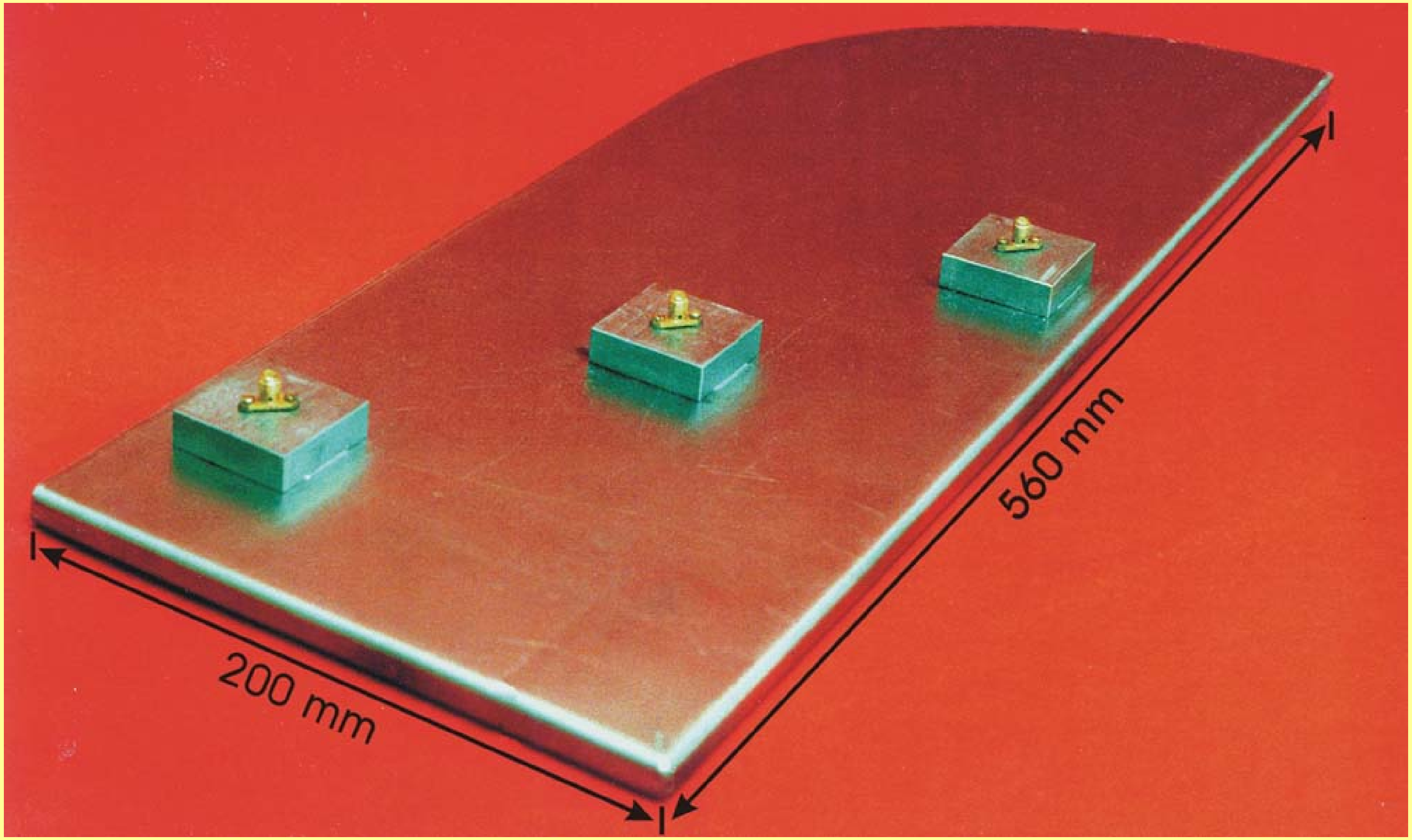
$$\left(\Delta + k^2\right)E_z = 0$$

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

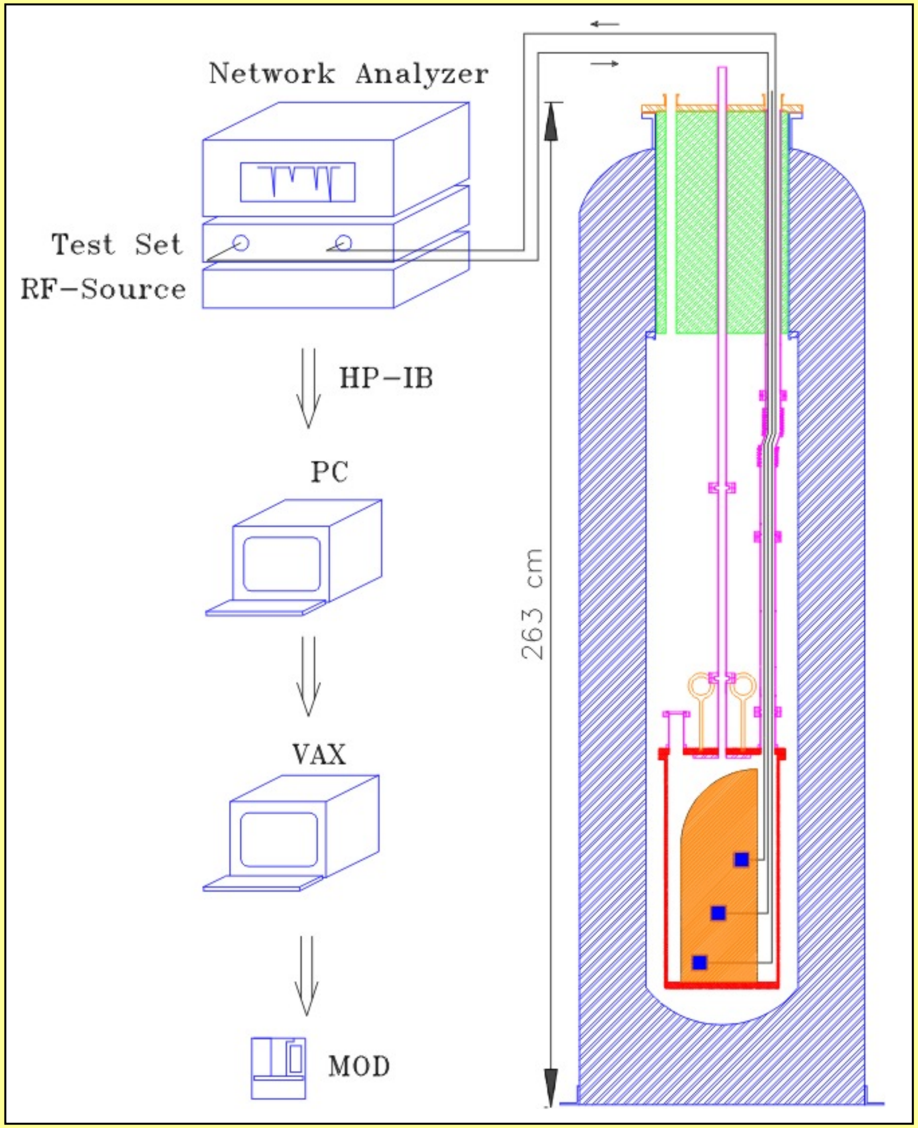
$$k = \frac{2\pi f}{c}$$

Helmholtz equation and Schrödinger equation are equivalent in 2D. The motion of the quantum particle in its potential can be simulated by electromagnetic waves inside a two-dimensional microwave resonator.

Superconducting Niobium Microwave Resonator

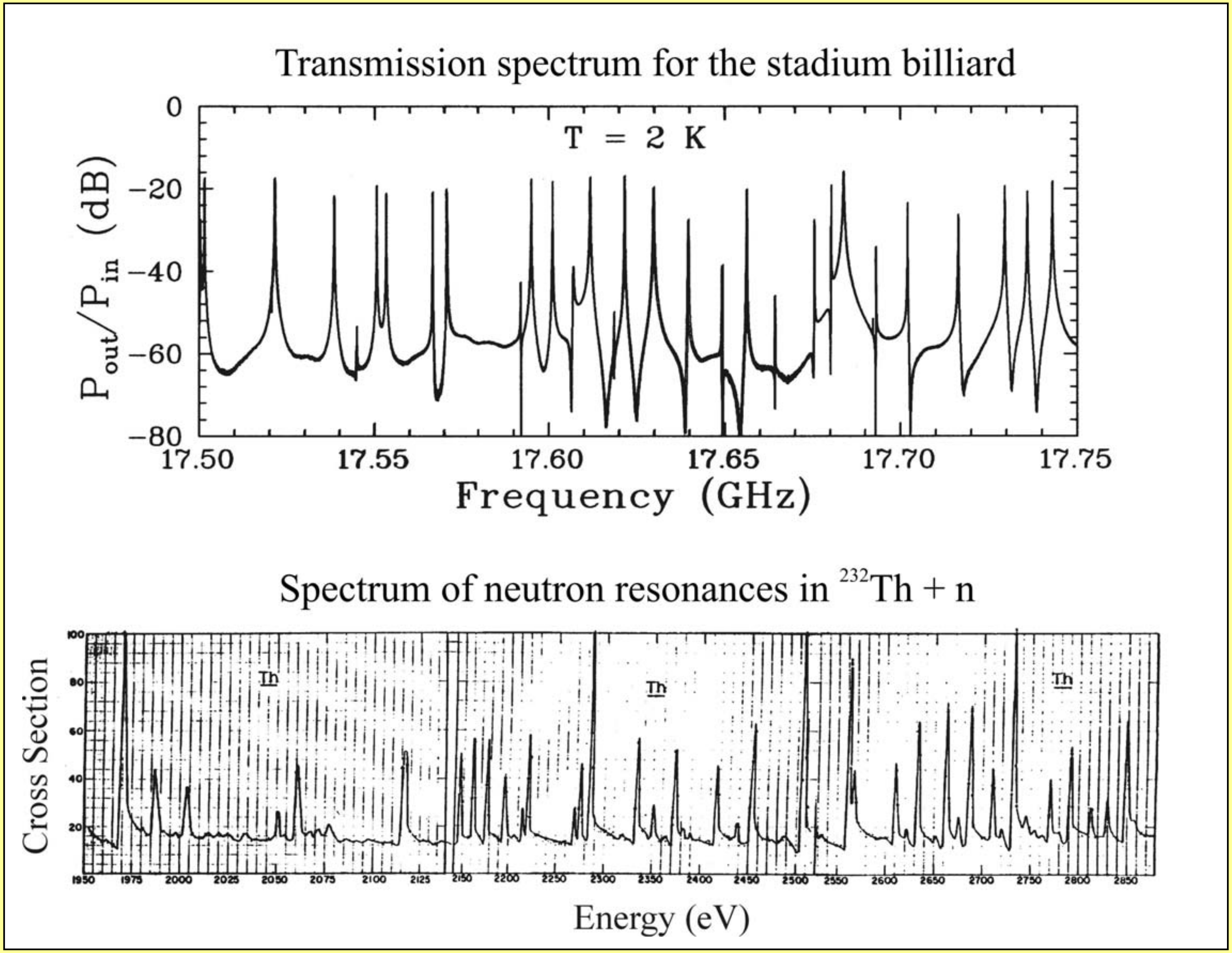


Experimental Setup



- Superconducting cavities
- LHe (T = 4.2 K)
- f = 45 MHz ... 50 GHz
- $10^3 \ldots 10^4$ eigenfrequencies
- $Q = f/\Delta f \approx 10^6$

Stadium Billiard ↔ n + ^{232}Th



Niels Bohr's Model of the Compound Nucleus

The first of these is intended to convey an idea of events arising out of a collision between a neutron and the nucleus. Imagine a shallow basin with a number of billiard balls in it as shown in the accompanying figure. If the basin were empty, then upon striking a ball from the outside, it would go down one slope and pass out on the opposite side with its original velocity. But with other balls in the basin, there would not be a free passage of this kind. The struck ball would divide its energy first with one of the balls in the basin, these two would similarly share their energies with others, and so on until the original kinetic energy was divided among all the balls. If the basin and the balls are regarded as perfectly smooth and elastic, the collisions would continue until the kinetic energy happens again to be concentrated upon a ball close to the edge. This ball would then escape from the basin and the remainder of the balls would be left with insufficient total energy for any of them to climb the slope.

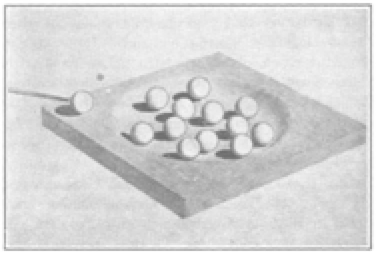


Fig. 1.

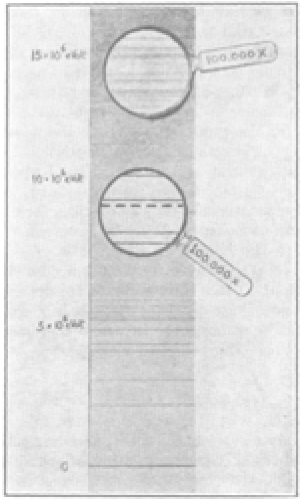


Fig. 2.

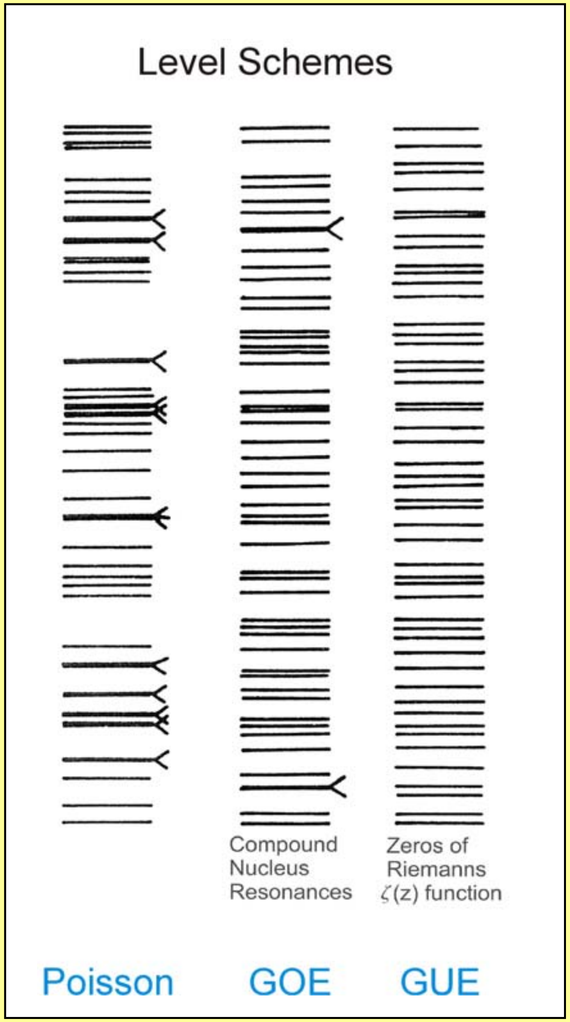
Random Matrices ↔ Level Schemes

Random Matrix

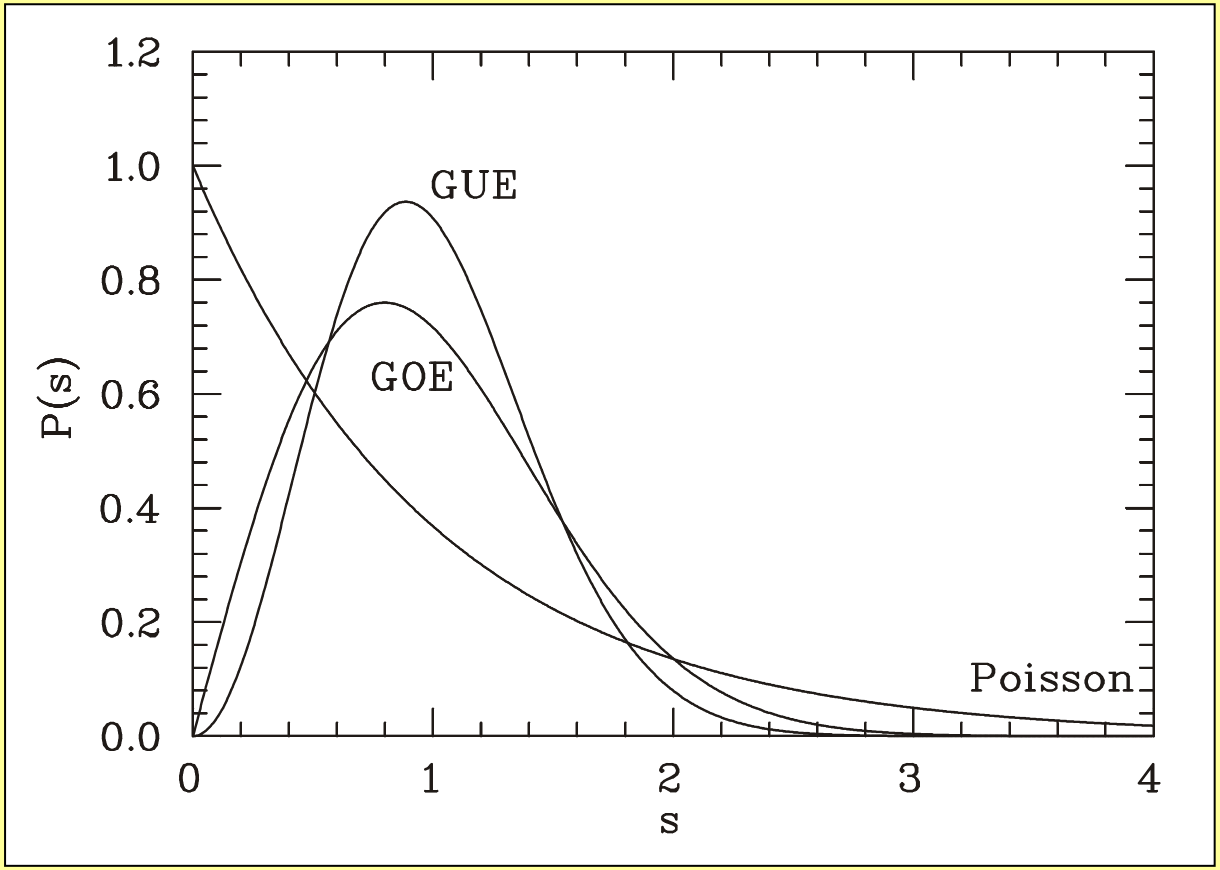
$$H = \begin{pmatrix} H_{11} & \cdots & H_{1N} \\ \vdots & \ddots & \vdots \\ H_{N1} & \cdots & H_{NN} \end{pmatrix}$$

Eigenvalues

$$H\phi_n = E_n\phi_n$$



Nearest Neighbor Spacings Distribution

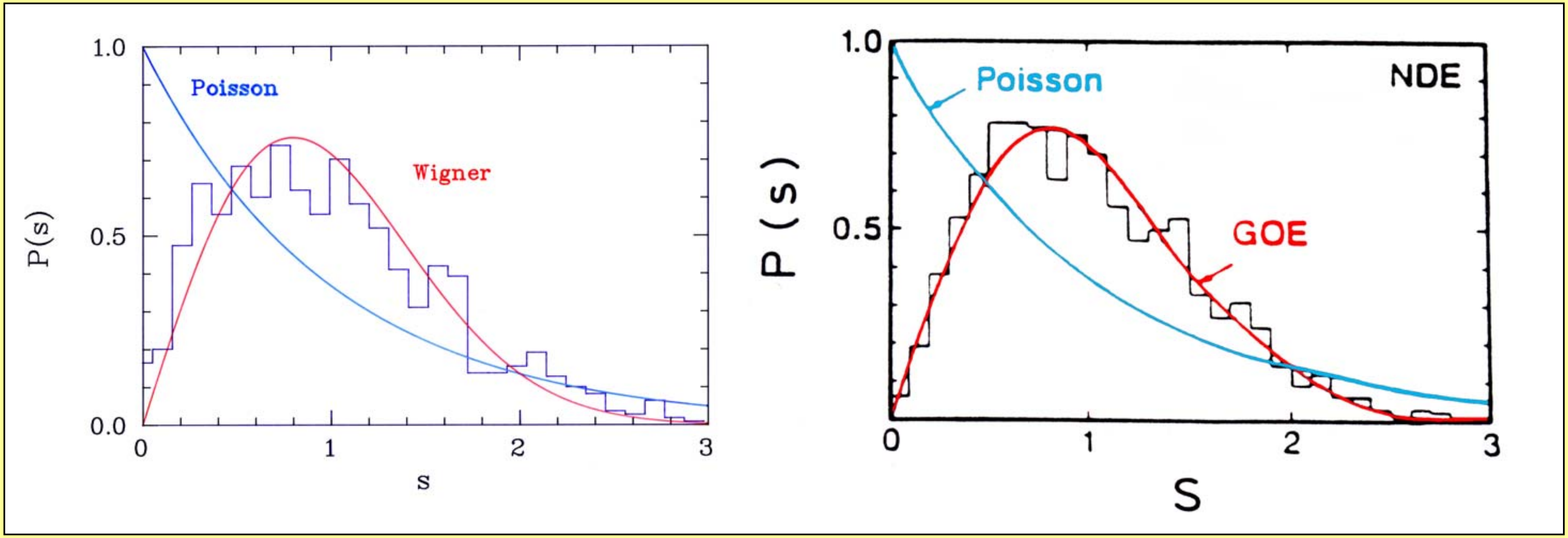


- GOE and GUE ↔ "Level Repulsion"
- Poissonian Random Numbers ↔ "Level Clustering"

Nearest Neighbor Spacings Distribution

stadium billiard

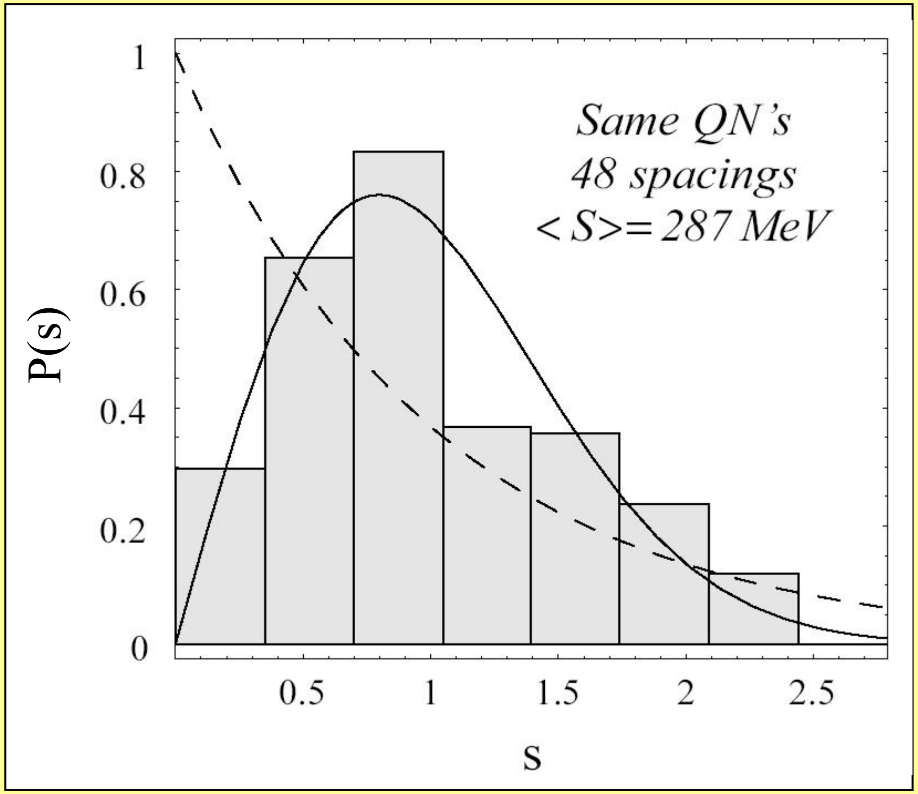
nuclear data ensemble



- Universal (generic) behaviour of the two systems

Universality in Mesoscopic Systems: Quantum Chaos in Hadrons

- Combined data from measured baryon and meson mass spectra up to 2.5 GeV (from PDG)
- Spectra can be organized into multiplets characterized by a set of definite quantum numbers: isospin, spin, parity, strangeness, baryon number, ...

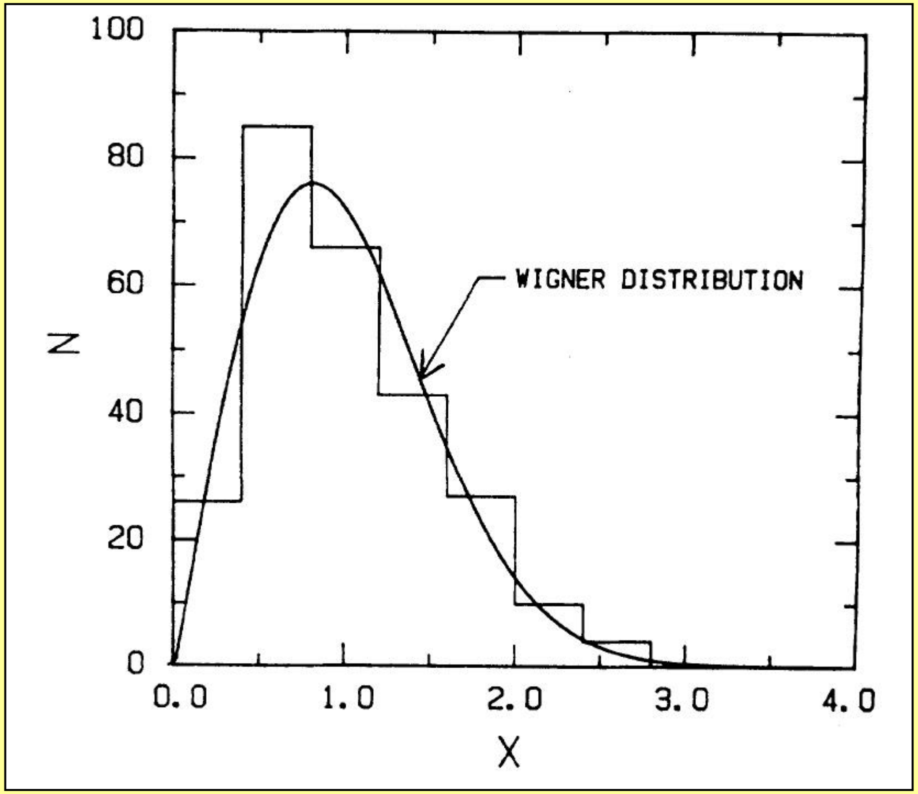


- Scale: 10^{-16} m

Pascalutsa (2003)

Universality in Mesoscopic Systems: Quantum Chaos in Atoms

- 8 sets of atomic spectra of highly excited neutral and ionized rare earth atoms combined into a data ensemble
- States of same total angular momentum and parity

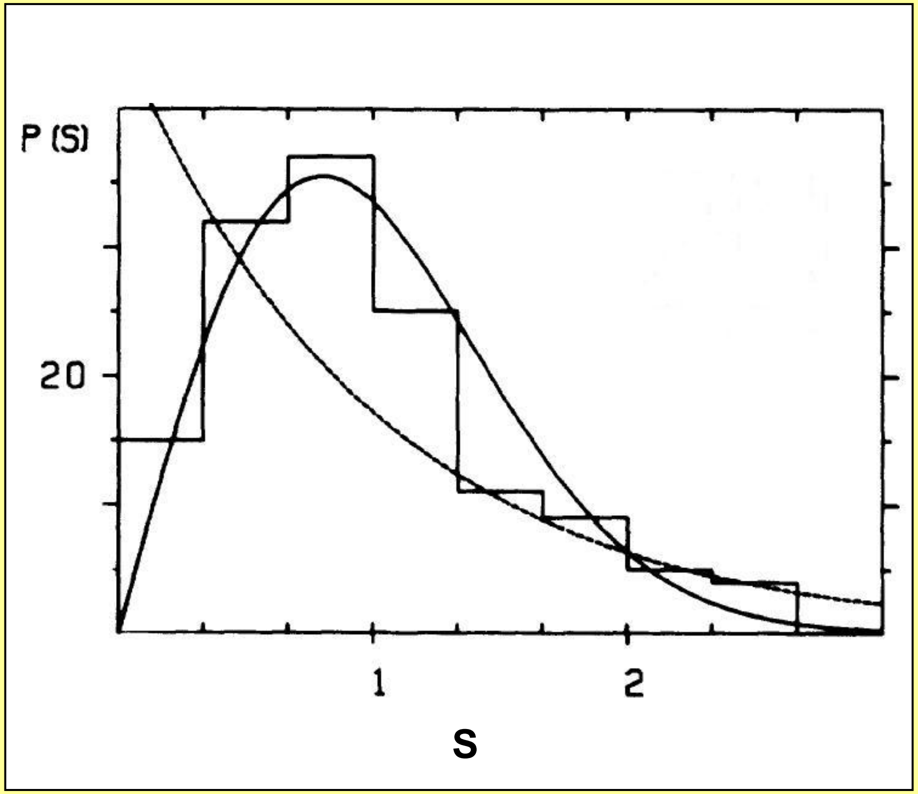


- Scale: 10^{-10} m

Camarda + Georgopulos (1983)

Universality in Mesoscopic Systems: Quantum Chaos in Molecules

- Vibronic levels of NO_2
- States of same quantum numbers



- Scale: 10^{-9} m

Zimmermann et al. (1988)

Conjecture of Bohigas, Giannoni + Schmit (1984)

- How is the behaviour of the classical system transferred to the quantum system ?

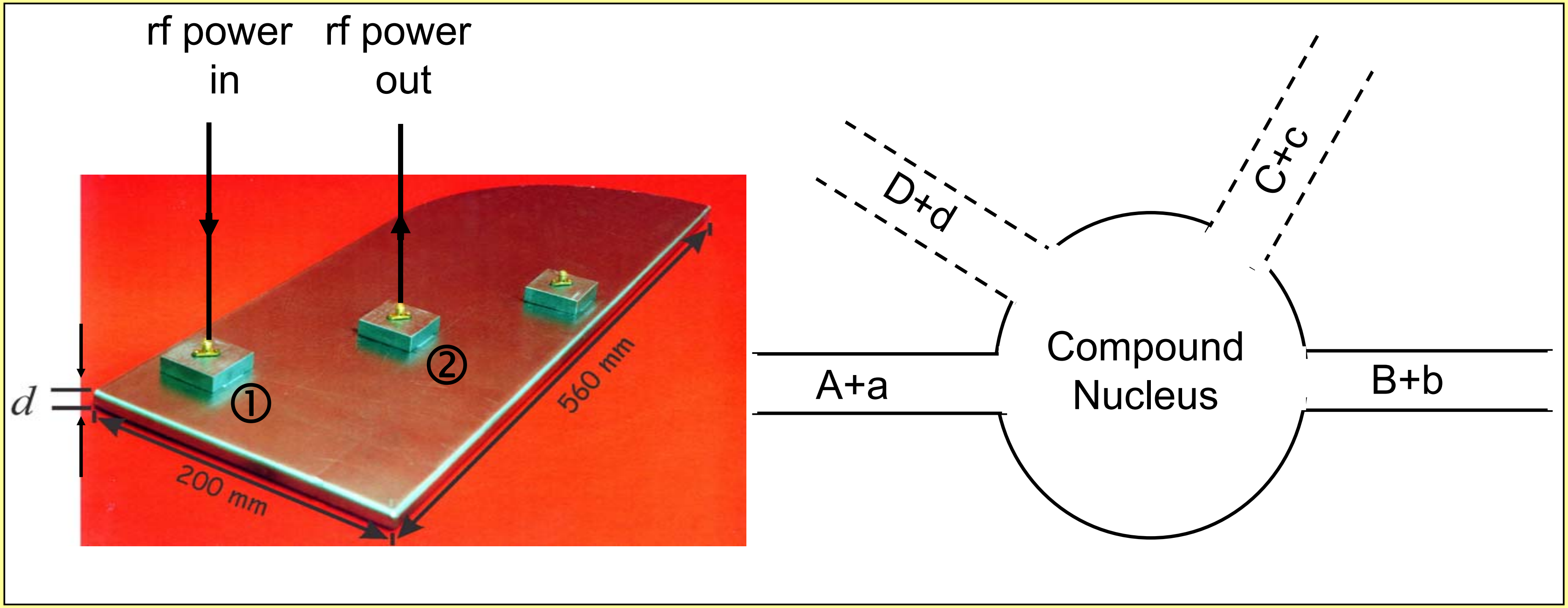
- Answer: There is a one-to-one correspondence between billiards and mesoscopic systems on all scales

- For chaotic systems, the spectral fluctuation properties of eigenvalues coincide with the predictions of random-matrix theory (RMT) for matrices of the same symmetry class

- Numerous tests of various spectral properties (NNSD, Σ^2, Δ_3, …) and wave functions exist

- Our aim: to test this conjecture in scattering systems, i.e. in open chaotic microwave billiards particularly in the regime of weakly overlapping resonances

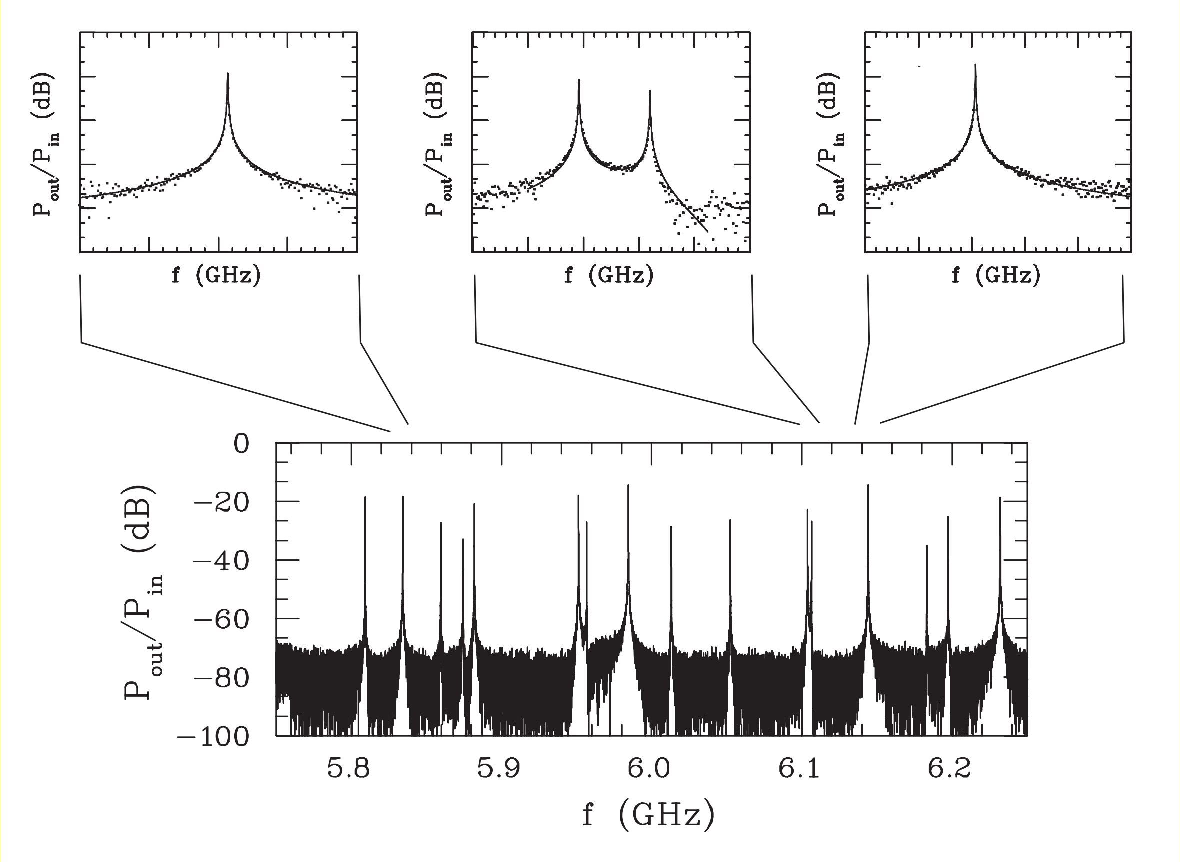
Microwave Resonator as a Model for the Compound Nucleus



- Microwave power is **emitted** into the resonator by antenna ① and the output signal is **received** by antenna ② → **Open scattering system**
- The antennas act as **single scattering channels**
- Absorption into the walls is modelled by **additive channels**

G. E. Mitchell, A. Richter and H. A. Weidenmüller, arXiv:1001.2422v1 (2010)

Typical Transmission Spectrum



- Transmission measurements: relative power from antenna a → b

$$P_{out,b} / P_{in,a} = \left|S_{ba}\right|^2$$

Scattering Matrix Description

- Scattering matrix for both scattering processes

$$\hat{S}(E) = \mathbb{1} - 2\pi i\, \hat{W}^T (E\mathbb{1} - \hat{H} + i\pi\, \hat{W}\hat{W}^T)^{-1} \hat{W}$$

Compound-nucleus reactions		**Microwave billiard**
nuclear Hamiltonian	$\leftarrow \hat{H} \rightarrow$	resonator Hamiltonian
coupling of quasi-bound states to channel states	$\leftarrow \hat{W} \rightarrow$	coupling of resonator states to antenna states and to the walls

- Experiment: complex S-matrix elements
- **RMT description**: replace $\hat{H}$ by a GOE / GUE matrix for T-inv / T-noninv systems

Resonance Parameters

- Use eigenrepresentation of

$$\hat{H}_{eff} = \hat{H} - i\pi\hat{W}\hat{W}^T$$

and obtain for a scattering system with isolated resonances

a → resonator → b

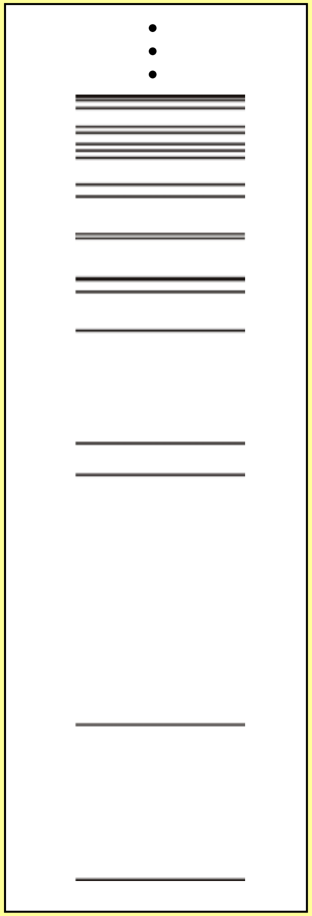
$$S_{ba} = \delta_{ba} - i\sum_{\mu}\frac{\sqrt{\Gamma_{\mu a}\Gamma_{\mu b}}}{f - f_{\mu} + (i/2)\Gamma_{\mu}}$$

- Here: $f_{\mu} =$ real part, $\Gamma_{\mu} =$ imaginary part } of eigenvalues of $\hat{H}_{eff}$

- Partial widths $\Gamma_{\mu a}, \Gamma_{\mu b}$ fluctuate and total widths Γ_{μ} also

Excitation Spectra

atomic nucleus

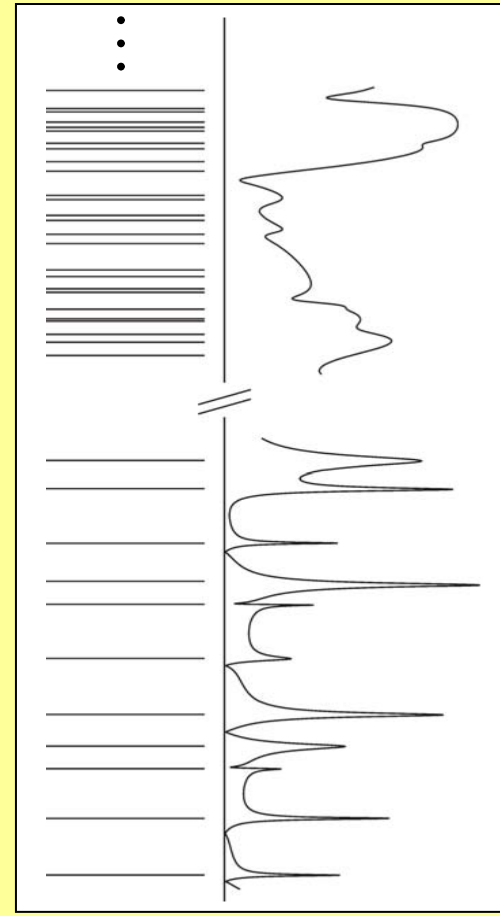


$\rho \sim \exp(E^{1/2})$

overlapping resonances
for $\Gamma/D > 1$
Ericson fluctuations

isolated resonances
for $\Gamma/D << 1$

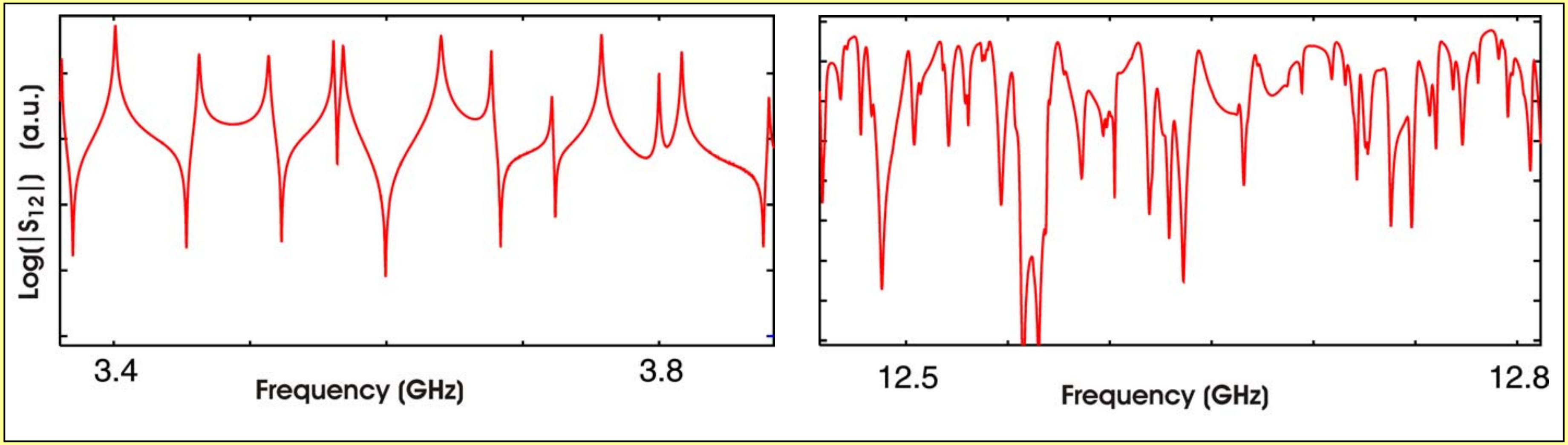
microwave cavity



$\rho \sim f$

- Universal description of spectra and fluctuations: Verbaarschot, Weidenmüller + Zirnbauer (1984)

Spectra and Correlation of S-Matrix Elements



- Regime of isolated resonances
- Γ/D small
- Resonances: eigenvalues

- Overlapping resonances
- Γ/D ~ 1
- Fluctuations: Γ_{coh}

Correlation function: $$C(\varepsilon) = \langle S(f)S^*(f+\varepsilon)\rangle - \langle S(f)\rangle\langle S^*(f+\varepsilon)\rangle$$

Ericson's Prediction for Γ > D

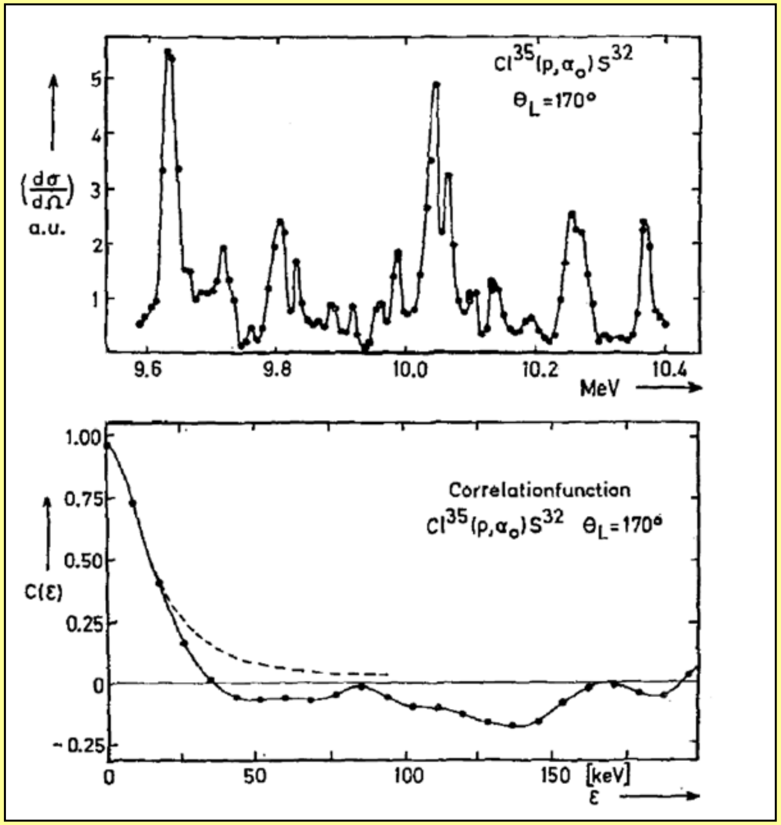
- Ericson fluctuations (1960):

$$|C(\varepsilon)|^2 \propto \frac{\Gamma_{coh}^2}{\Gamma_{coh}^2 + \varepsilon^2}$$

- Correlation function is Lorentzian

- Measured 1964 for overlapping compound nuclear resonances

- Now observed in lots of different systems: molecules, quantum dots, laser cavities, microwave cavities, …



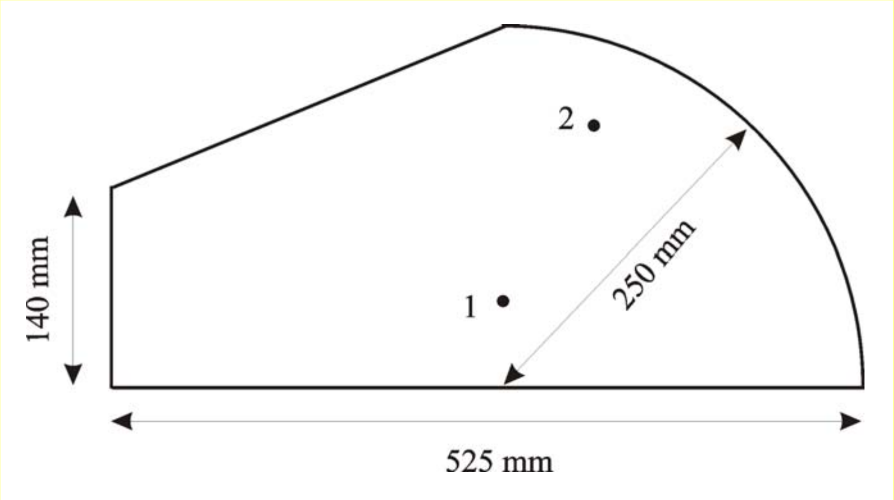
P. v. Brentano et al., PL 9 (1964) 48

- Different theoretical approaches: Ericson → energy and time domain
  VWZ → RMT
  Blümel & Smilansky → semiclassical approach

- Applicable for Γ/*D* >> 1 and for many open channels only

Fluctuations in a Fully Chaotic Cavity with T-Invariance

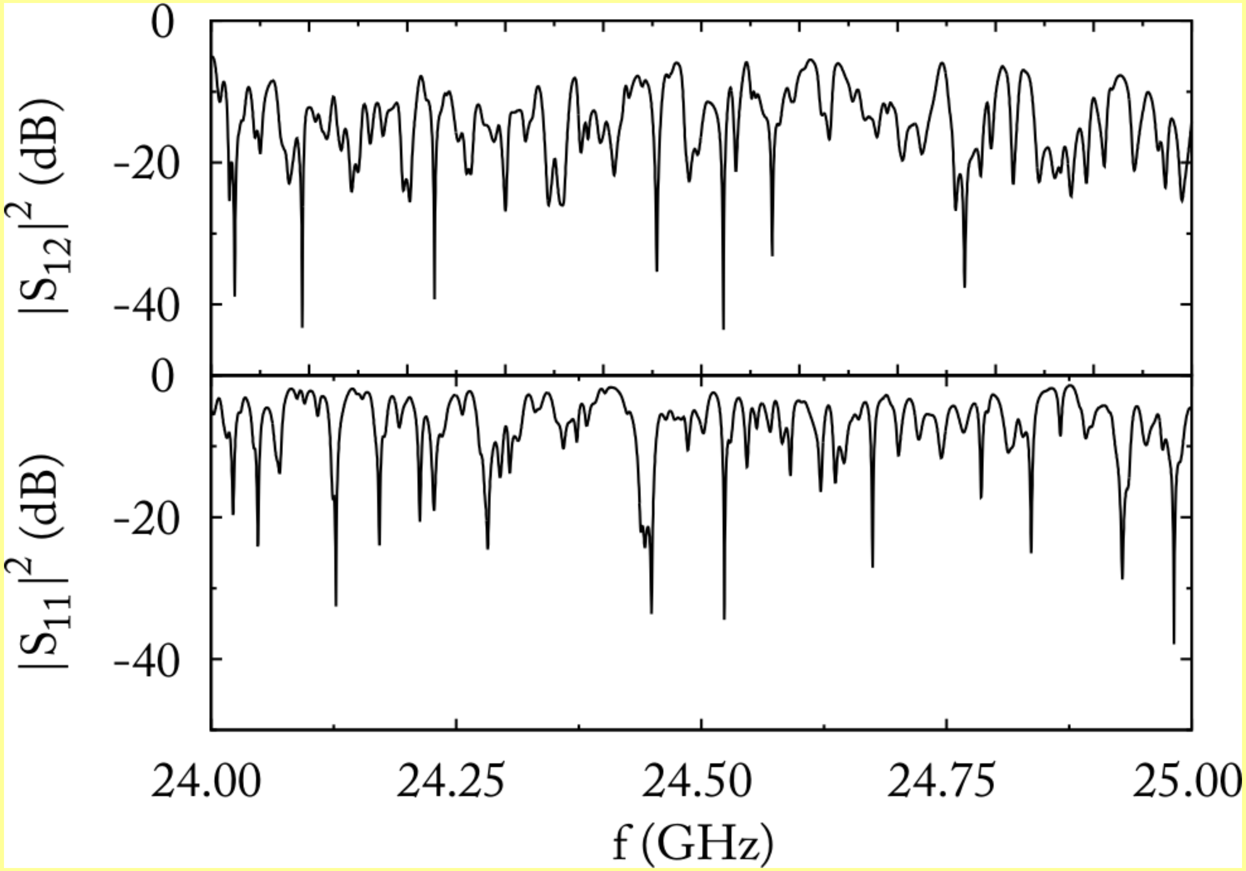
- Tilted stadium (Primack + Smilansky, 1994)



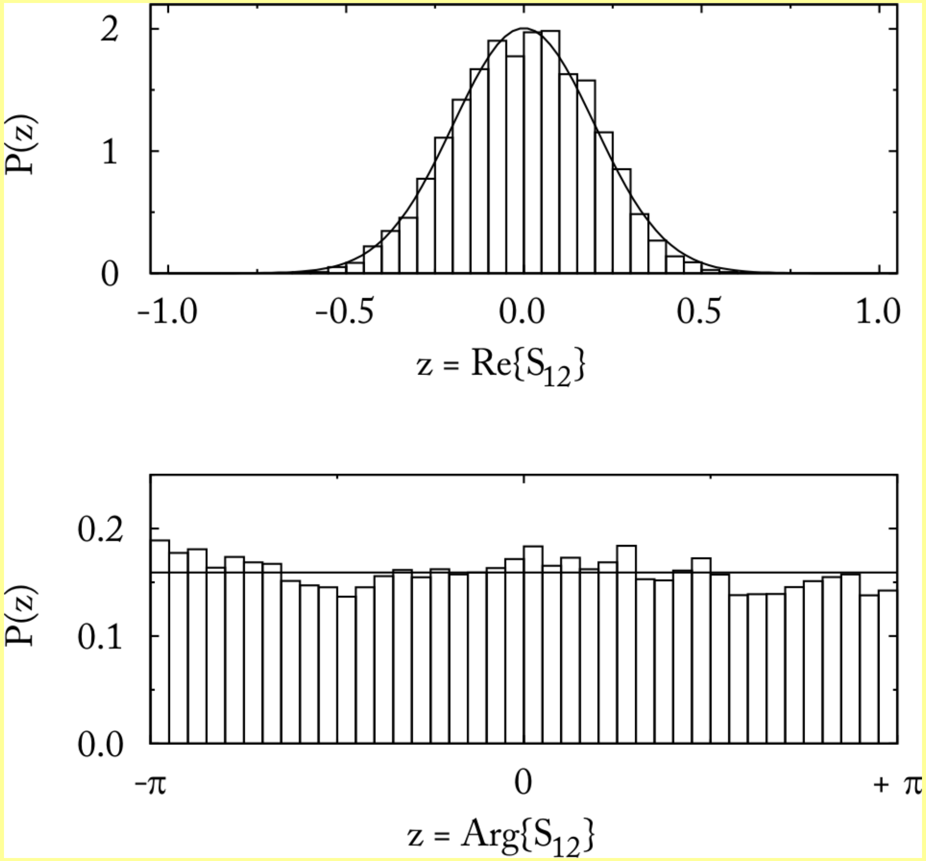
- GOE behaviour checked

- Measure full complex S-matrix for two antennas: S_{11}, S_{22}, S_{12}

Spectra of S-Matrix Elements in the Ericson Regime



Distributions of S-Matrix Elements the Ericson Regime



Road to Analysis of the Measured Fluctuations

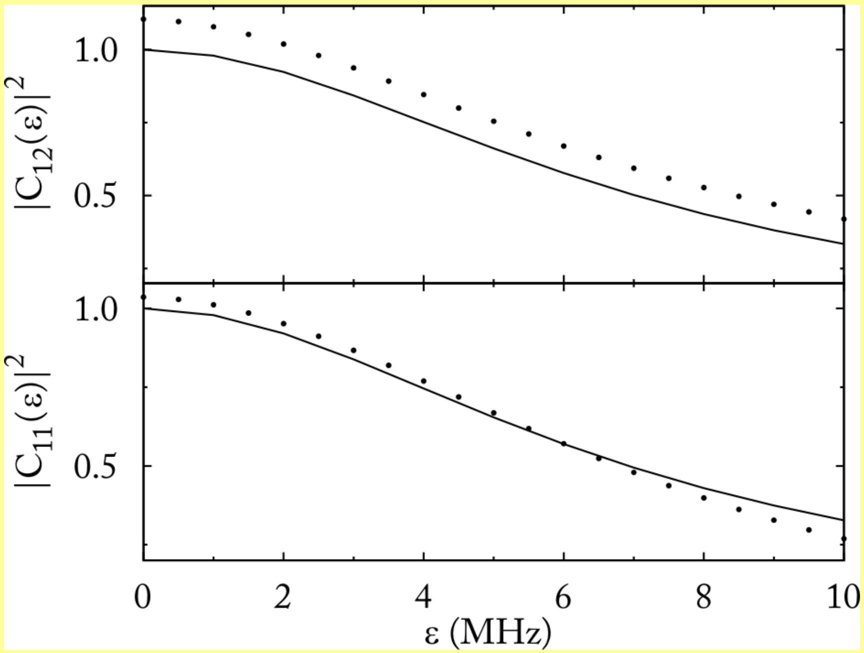
- Problem: adjacent points in $C(\varepsilon)$ are correlated

- Solution: FT of $C(\varepsilon) \rightarrow$ uncorrelated Fourier coefficients $\tilde{C}(t)$
  Ericson (1965)

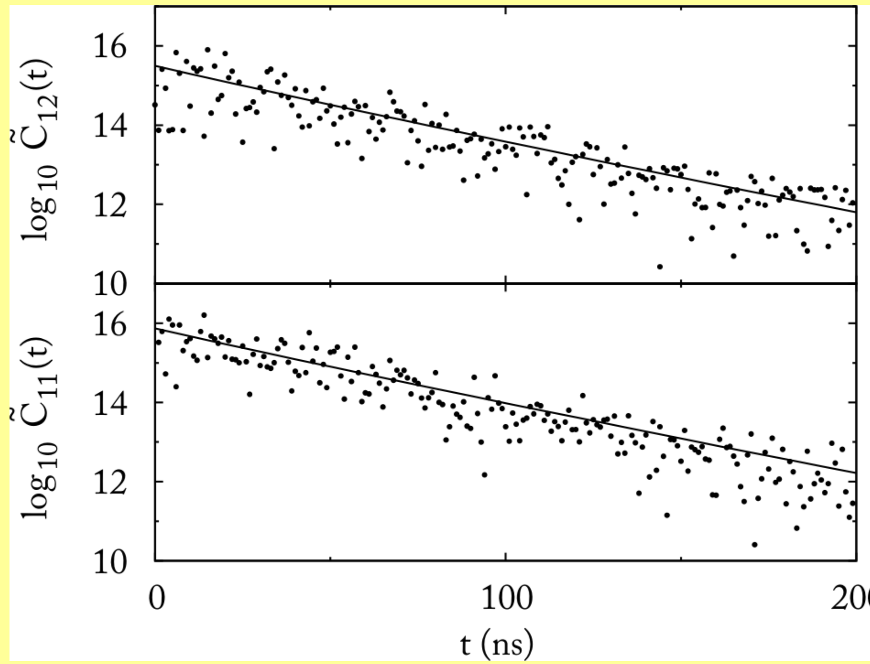
- Development: Non Gaussian fit and test procedure

Autocorrelation Function and Fourier Coefficients in the Ericson Regime

Frequency domain



Time domain



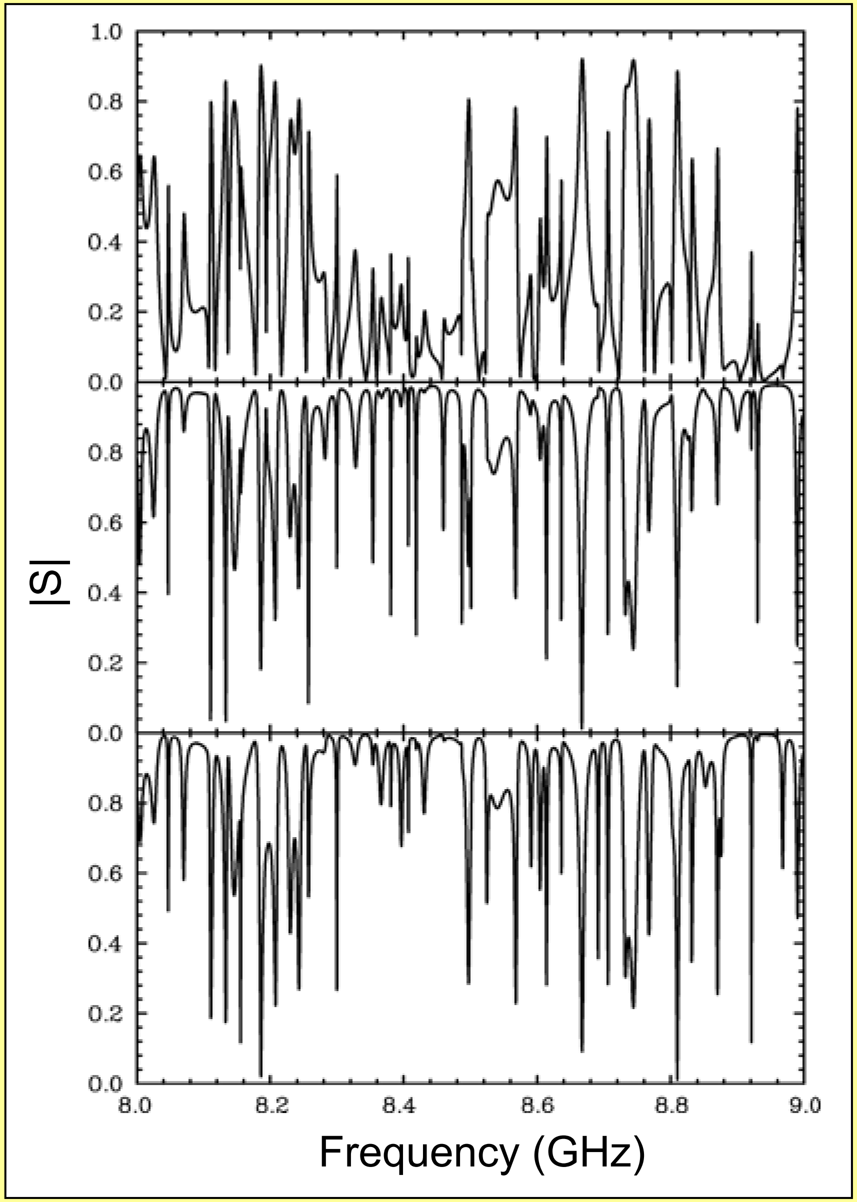
Spectra of S-Matrix Elements in the Regime $\Gamma/D \lesssim 1$

Example: 8-9 GHz

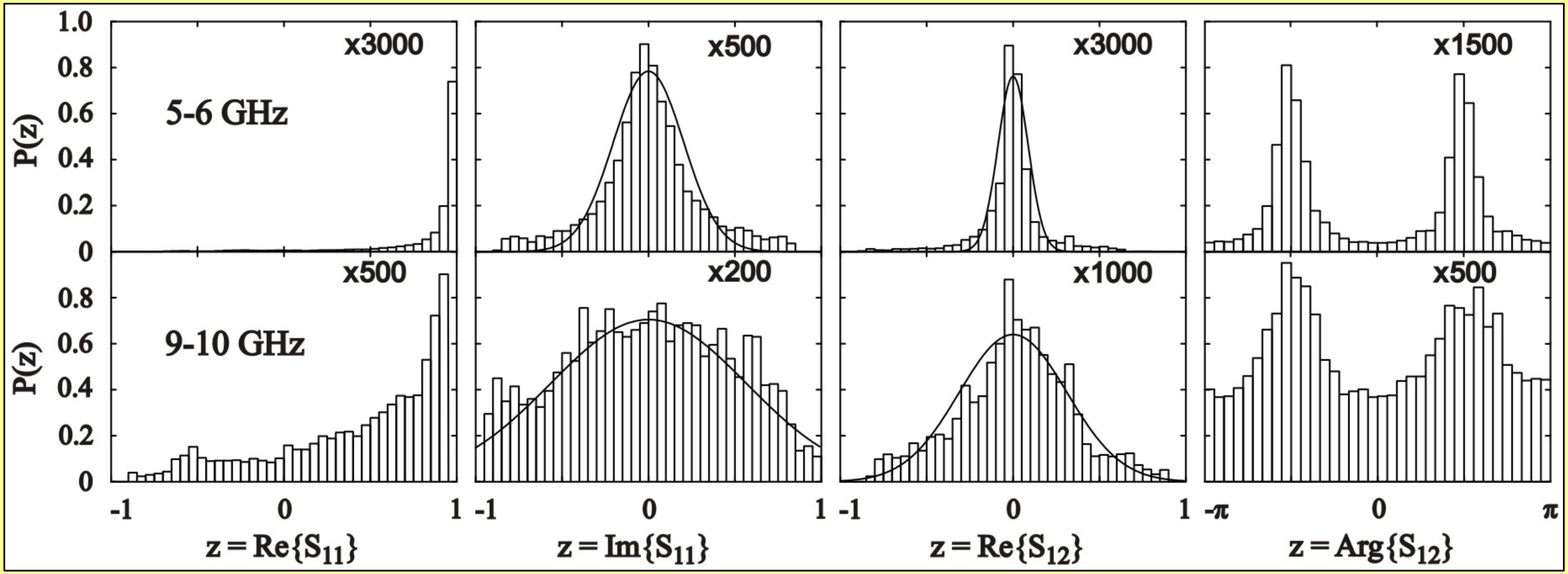
$S_{12} \rightarrow$

$S_{11} \rightarrow$

$S_{22} \rightarrow$



Distributions of S-Matrix Elements in the Regime Γ/D ≲ 1



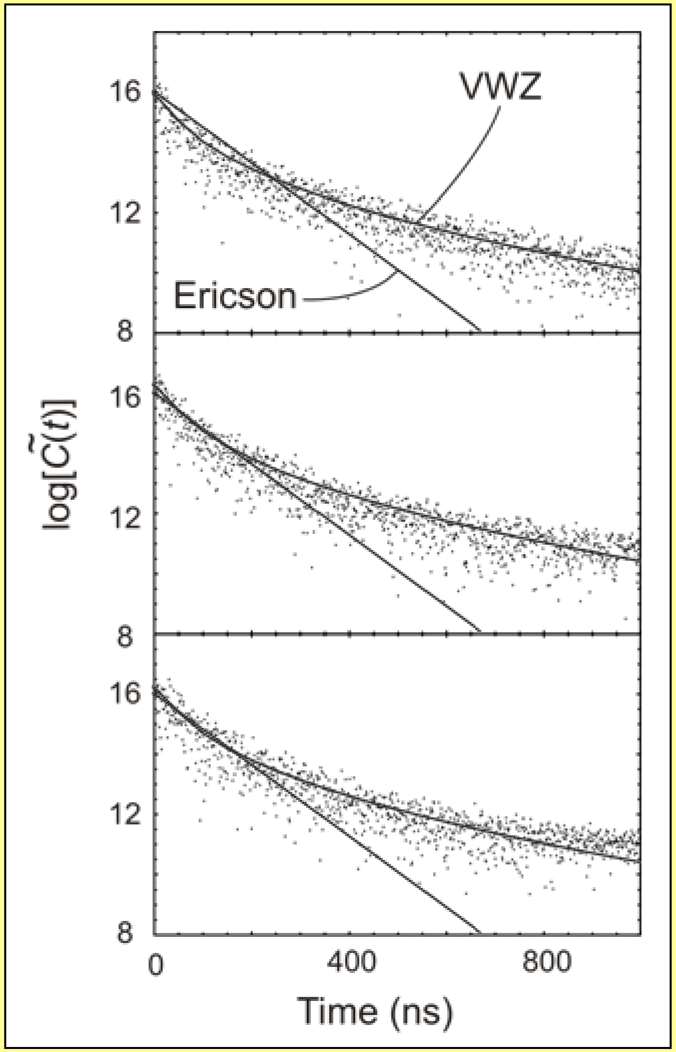
- Ericson regime: Re{S} and Im{S} should be Gaussian and phases uniformly distributed
- Clear deviations for Γ/D ≲ 1 which still need to be modeled theoretically

Fourier Transform vs. Autocorrelation Function

Time domain

Example 8-9 GHz

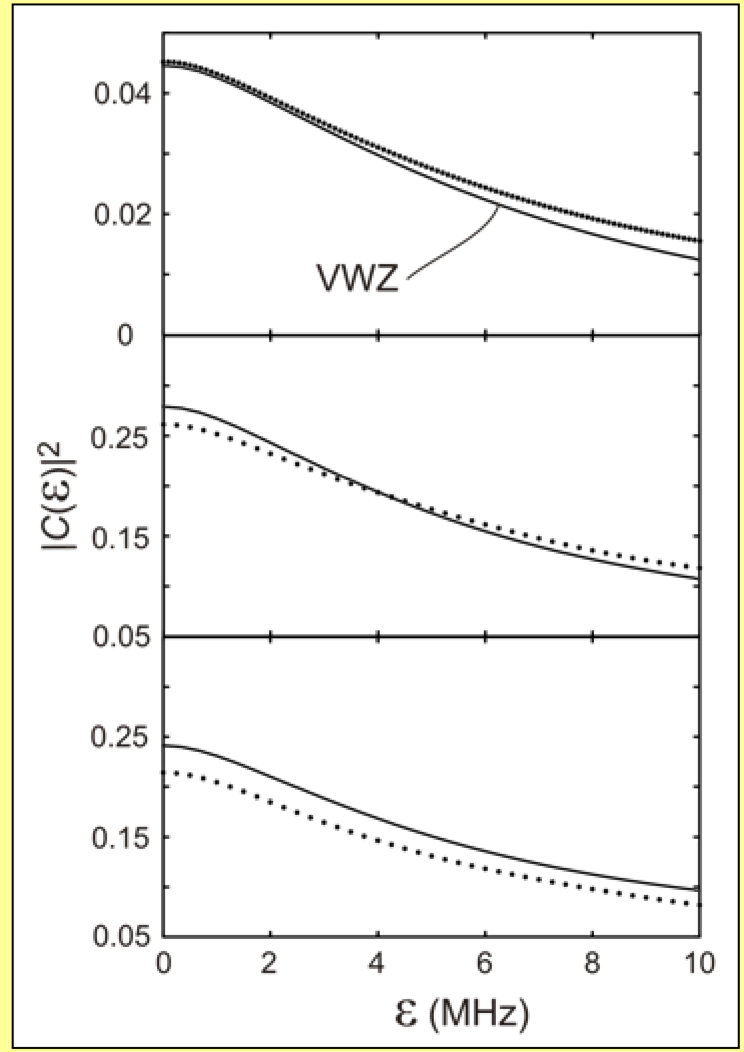
Frequency domain



$\leftarrow S_{12} \rightarrow$

$\leftarrow S_{11} \rightarrow$

$\leftarrow S_{22} \rightarrow$



Exact RMT Result for GOE Systems

- Verbaarschot, Weidenmüller and Zirnbauer (VWZ) 1984 for arbitrary Γ/D :
- VWZ-integral: $C = C(T_i, D\,; \varepsilon)$

$$C_{ab}(\epsilon) = \frac{1}{8}\int_0^\infty d\lambda_1 \int_0^\infty d\lambda_2 \int_0^1 d\lambda\, \mu(\lambda,\lambda_1,\lambda_2) \times \exp\left(-i\pi\epsilon(\lambda_1+\lambda_2+2\lambda)/D\right) \times J_{ab}(\lambda,\lambda_1,\lambda_2) \times \prod_e \frac{(1-T_e\lambda)}{((1+T_e\lambda_1)(1+T_e\lambda_2))^{1/2}}$$

$$\mu(\lambda,\lambda_1,\lambda_2) = \frac{\lambda(1-\lambda)|\lambda_1-\lambda_2|}{(\lambda+\lambda_1)^2(\lambda+\lambda_2)^2(\lambda_1\lambda_2(1+\lambda_1)(1+\lambda_2))^{1/2}}$$

$$J_{ab}(\lambda,\lambda_1,\lambda_2) = \delta_{ab}\,T_a^2\,(1-T_a) \times \left(\frac{\lambda_1}{1+T_a\lambda_1} + \frac{\lambda_2}{1+T_a\lambda_2} + \frac{2\lambda}{1-T_a\lambda}\right) + (1+\delta_{ab})\,T_a\,T_b + \left(\frac{\lambda_1(1+\lambda_1)}{(1+T_a\lambda_1)(1+T_b\lambda_1)} + \frac{\lambda_2(1+\lambda_2)}{(1+T_a\lambda_2)(1+T_b\lambda_2)} + \frac{2\lambda(1-\lambda)}{(1-T_a\lambda)(1-T_b\lambda)}\right)$$

- Rigorous test of VWZ: isolated resonances, i.e. $\Gamma << D$
- First test of VWZ in the intermediate regime, i.e. $\Gamma/D \approx 1$, with high statistical significance only achievable with microwave billiards
- Note: nuclear cross section fluctuation experiments yield only $|S|^2$

Corollary

- **Present work:**
  S-matrix → Fourier transform → decay time (indirectly measured)

- **Future work at NIF:**
  Direct measurement of the decay time of an excited nucleus might become possible by exciting all nuclear resonances (or a subset of them) simultaneously by a short laser pulse.

Search for TRSB in Nuclei: Ericson Regime

VOLUME 51, NUMBER 5 PHYSICAL REVIEW LETTERS 1 AUGUST 1983

Improved Experimental Test of Detailed Balance and Time Reversibility in the Reactions $^{27}Al+p \rightleftharpoons {}^{24}Mg+\alpha$

E. Blanke,[a] H. Driller,[b] and W. Glöckle
Abteilung für Physik und Astronomie, Ruhr Universität Bochum, D-4630 Bochum, Germany

and

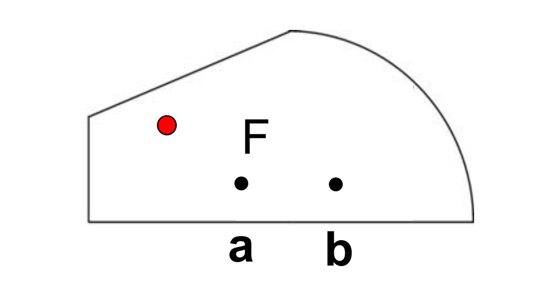
H. Genz, A. Richter, and G. Schrieder
Institut für Kernphysik, Technische Hochschule Darmstadt, D-6100 Darmstadt, Germany

(Received 25 April 1983)

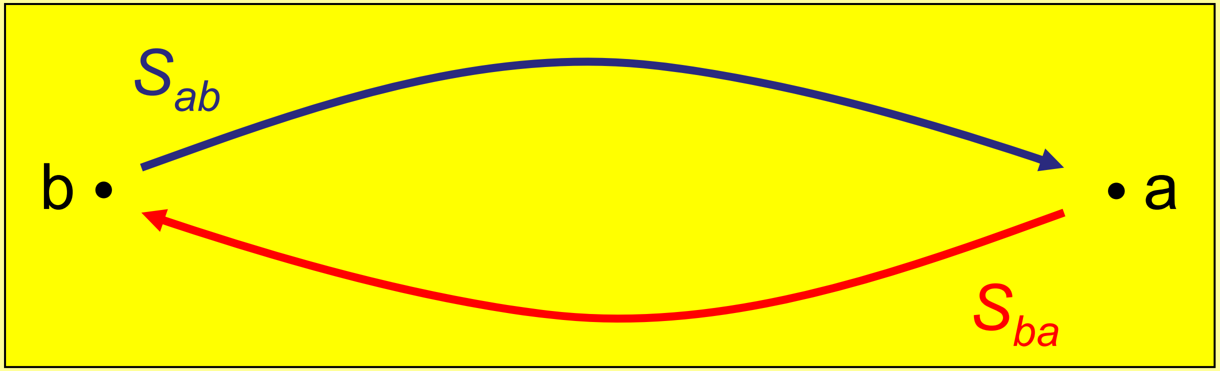
A new test of the principle of detailed balance in the nuclear reactions $^{27}Al(p,\alpha_0)^{24}Mg$ and $^{24}Mg(\alpha,p_0)^{27}Al$ at bombarding energies 7.3 MeV $\leq E_p \leq$ 7.7 MeV and 10.1 MeV $\leq E_\alpha \leq$ 10.5 MeV, respectively, is reported. Measured relative differential cross sections agree within the experimental uncertainty $\Delta = \pm 0.51\%$ and hence are consistent with time-reversal invariance. From this result an upper limit $\xi \leq 5\times 10^{-4}$ (80% confidence) is derived for a possible time-reversal–noninvariant amplitude in the reaction.

Induced Time-Reversal Symmetry Breaking (TRSB) in Billiards

- T-symmetry breaking caused by a magnetized ferrite

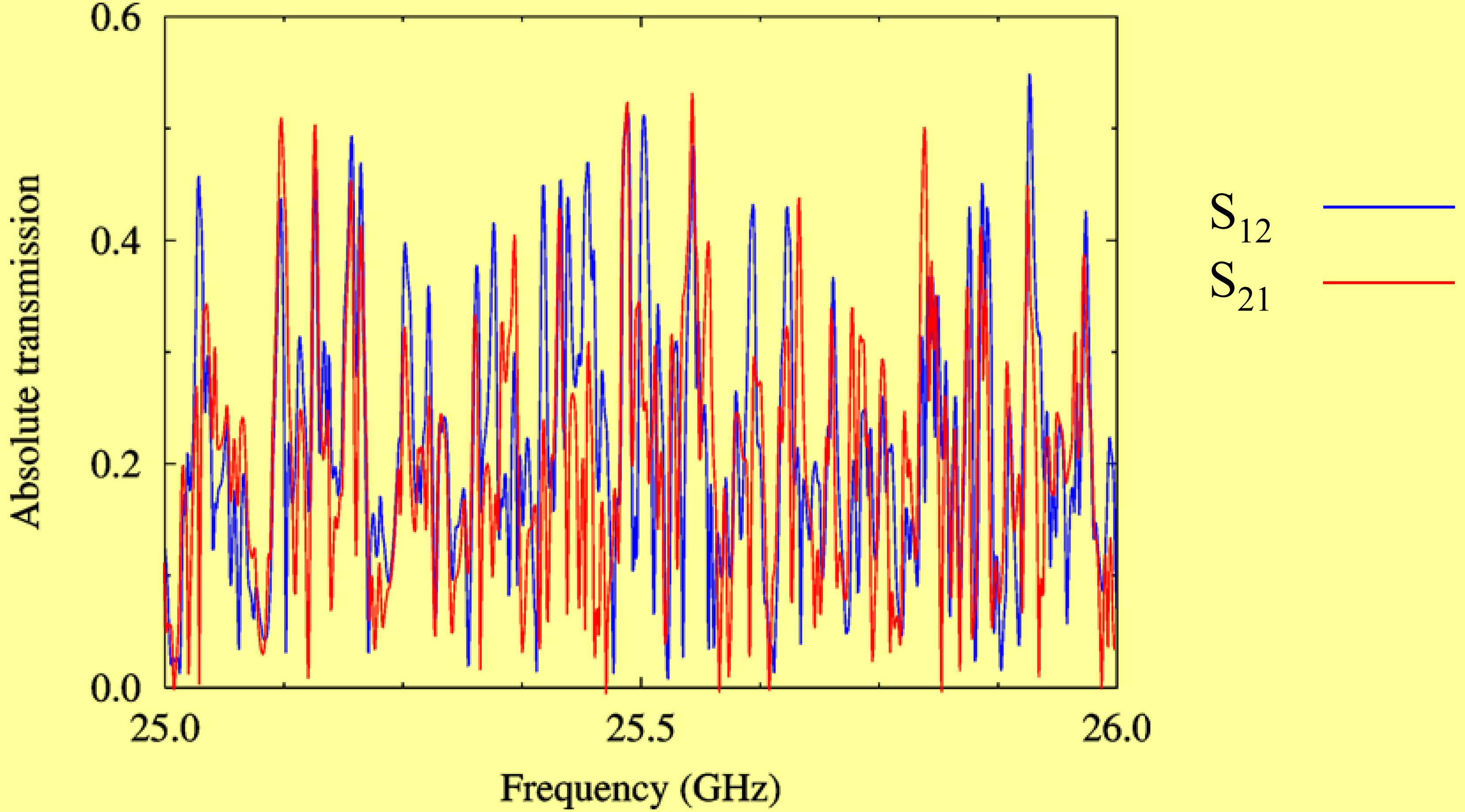


- Ferrite features ferromagnetic resonance (FMR)
- Coupling of microwaves to the FMR depends on the direction a $\leftrightarrows$ b



- Principle of detailed balance: $|S_{ab}|^2 = |S_{ba}|^2$
- Principle of reciprocity: $S_{ab} = S_{ba}$

Violation of Reciprocity



- Clear violation of reciprocity in the regime of $\Gamma / D \approx 1$

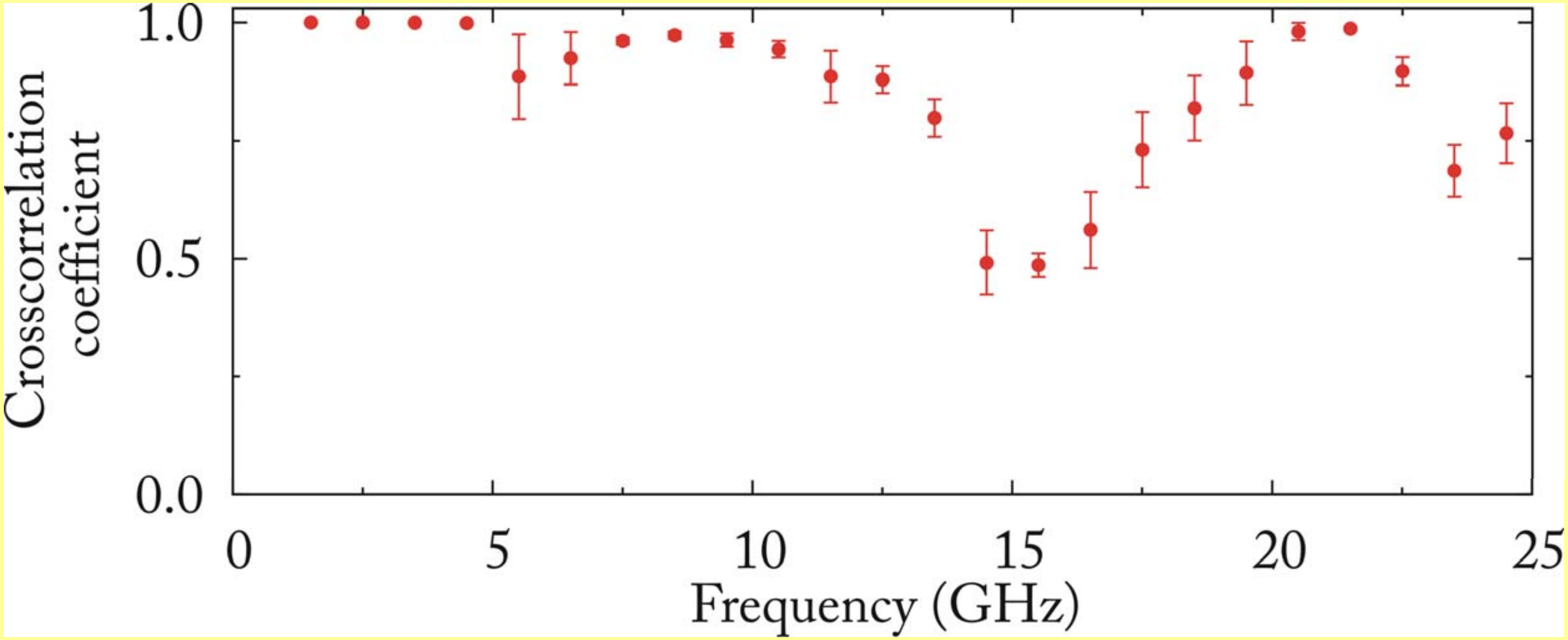
Analysis of Fluctuations with Crosscorrelation Function

Crosscorrelation function:

$$C(S_{12}, S_{21}^*, \varepsilon) = \left\langle S_{12}(f)\, S_{21}^*(f+\varepsilon) \right\rangle - \left\langle S_{12}(f) \right\rangle \left\langle S_{21}^*(f) \right\rangle$$

- Determination of T-breaking strength from the data
- Special interest in first coefficient ($\varepsilon = 0$)

Experimental Crosscorrelation Coefficients



- $C(S_{12}, S_{21}^{*}) = \begin{cases} 1 & \text{for GOE} \\ 0 & \text{for GUE} \end{cases}$
- Data: TRSB is incomplete → mixed GOE/GUE system

Exact RMT Result for Partial T Breaking

- RMT analysis based on Pluhař, Weidenmüller, Zuk, Lewenkopf and Wegner, 1995

$$C(T_1, T_2, \tau_{abs}; D, \varepsilon, \xi)$$

T-symmetry breaking parameter

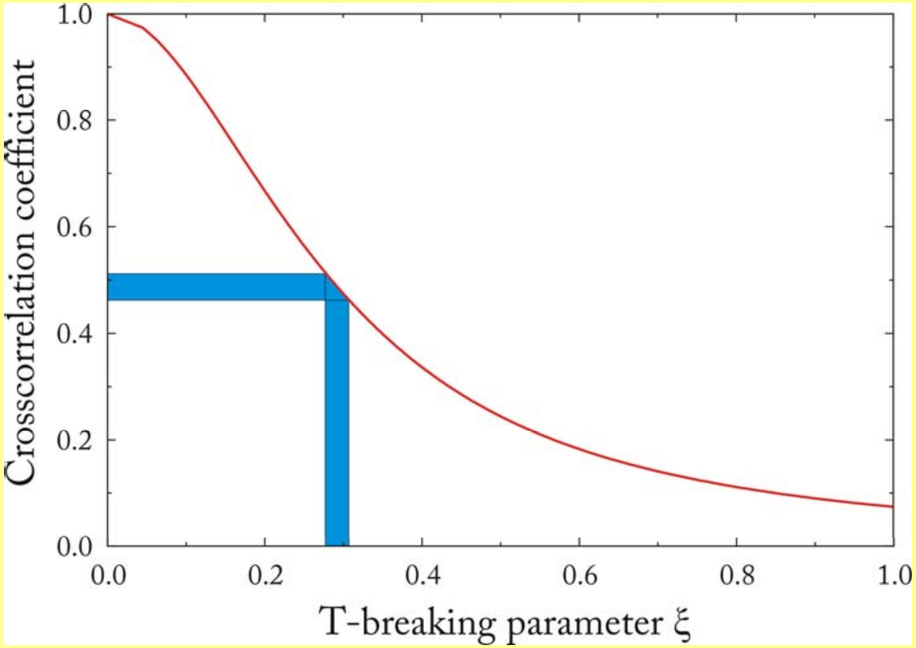
$$
\begin{aligned}
C_{ab}(\epsilon) &= \frac{T_a T_b}{16} \int_0^\infty \mathrm{d}\mu_1 \int_0^\infty \mathrm{d}\mu_2 \int_0^1 \mathrm{d}\mu \, \frac{|\mu_1 - \mu_2|}{\mathcal{U}} \\
&\times \frac{1}{(\mu+\mu_1)^2} \frac{1}{(\mu+\mu_2)^2} \exp\left(-\frac{i\pi\epsilon}{D}(\mu_1+\mu_2+2\mu)\right) \\
&\times J_{ab} \cdot \prod_c \frac{1 - T_c \mu}{\sqrt{(1+T_c\mu_1)(1+T_c\mu_2)}} \exp(-2\,\mathfrak{t}\,\mathcal{H}) .
\end{aligned}
$$

$$
\begin{aligned}
J_{ab} &= \Bigg\{ \Bigg[\Bigg(\frac{1}{2} \frac{\mu_1(1+\mu_1)}{(1+T_a\mu_1)(1+T_b\mu_1)} + \frac{1}{2} \frac{\mu_2(1+\mu_2)}{(1+T_a\mu_2)(1+T_b\mu_2)} \\
&+ \frac{\mu(1-\mu)}{(1-T_a\mu)(1-T_b\mu)} \Bigg) (1+\delta_{ab}) \\
&+ 2\,\delta_{ab}\,\overline{S_{aa}}^2 \left(\frac{\mu_1}{2(1+T_a\mu_1)} + \frac{\mu_2}{2(1+T_a\mu_2)} + \frac{\mu}{1-T_a\mu} \right)^2 \Bigg] \\
&\times \left[\mathcal{F}\,\varepsilon_+ + (\lambda_2^2 - \lambda_1^2)\,\varepsilon_- + 4\,\mathfrak{t}\,\mathcal{R}\,(\lambda_2^2 + \mathcal{F}\,(\varepsilon_+ - 1)) \right] \\
&\pm (1-\delta_{ab})\,K_{ab} \Bigg\} + \{\lambda_1 \rightleftharpoons \lambda_2\}
\end{aligned}
$$

$$
\begin{aligned}
K_{ab} &= \varepsilon_- \Big[2\,\mathcal{F} \left\{ (\tilde{A}_a\tilde{C}_b + \tilde{A}_b\tilde{C}_a)\,\mathcal{G}\lambda_2 + (\tilde{B}_a\tilde{C}_b + \tilde{B}_b\tilde{C}_a)\,\mathcal{H}\lambda_1 \right\} \\
&+ 3\,C_3\,\mathcal{F} - C_2\,(\lambda_2^2 - \lambda_1^2) + C_2\,\mathfrak{t}\,\mathcal{R}\,(4\,\lambda_2^2 - 2\,\mathcal{F}) \\
&+ 2\,\mathfrak{t}\,\mathcal{R}\,C_3\,\mathcal{F} \Big] \\
&+ \left(\varepsilon_+ - \frac{\varepsilon_-}{\mathfrak{t}\,\mathcal{F}} \right) \Big[3\,C_3\,(\lambda_2^2 - \lambda_1^2) + \mathfrak{t}\,\mathcal{R}\,C_3\,(4\,\lambda_2^2 - 2\,\mathcal{F}) \\
&+ 2\,\mathcal{F} \left\{ (\tilde{A}_a\tilde{C}_b + \tilde{A}_b\tilde{C}_a)\,\mathcal{G}\,\lambda_2 - (\tilde{B}_a\tilde{C}_b + \tilde{B}_b\tilde{C}_a)\,\mathcal{H}\,\lambda_1 \right\} \\
&+ (2\,\mathfrak{t}\,\mathcal{R} - 1)\,C_2\,\mathcal{F} \Big] .
\end{aligned}
$$

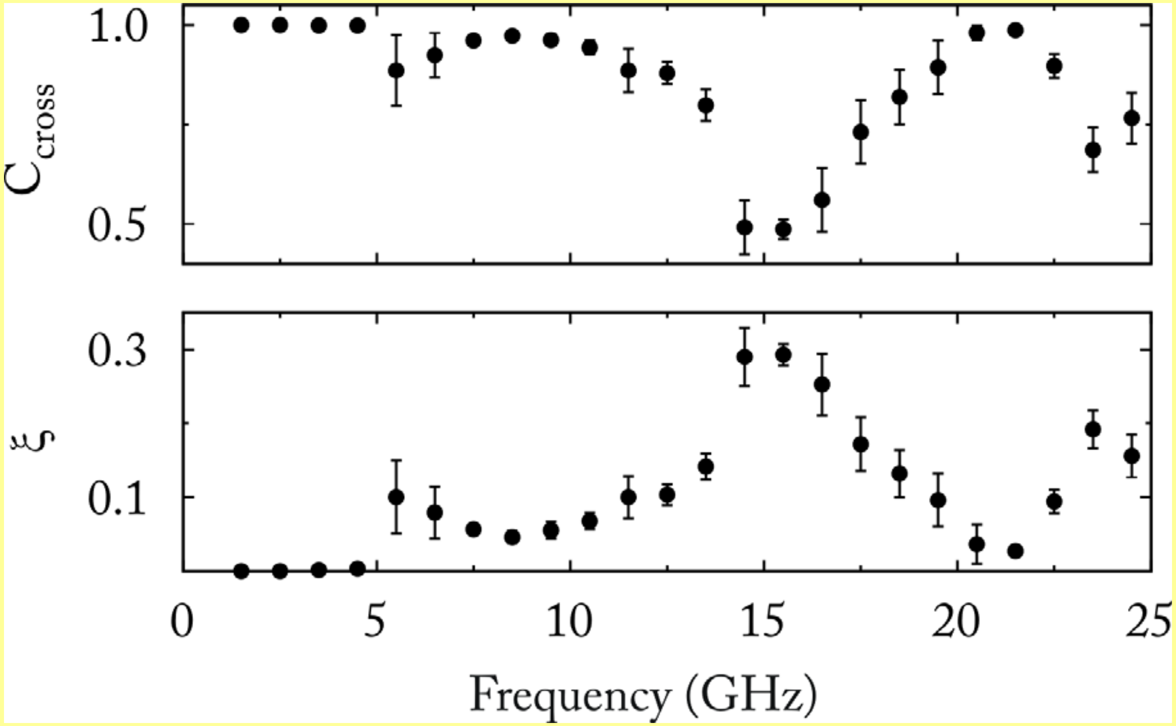
Exact RMT Result for Partial T Breaking

- RMT analysis based on Pluhař, Weidenmüller, Zuk, Lewenkopf and Wegner, 1995



- RMT $\rightarrow$ $H = H^s + i\left(\pi\xi/\sqrt{N}\right)H^a, \quad \xi = \begin{cases} 0 & \text{for GOE} \\ 1 & \text{for GUE} \end{cases}$

Determination of T-Breaking Strength



- B. Dietz *et al.*, Phys. Rev. Lett. **103**, 064101 (2009).

Summary

- Spectra of wave-dynamical systems show universal behaviour
- Test of RMT predictions with microwave billiards
- Investigated a chaotic T-invariant microwave resonator (i.e. a GOE system) both in the regime of weakly overlapping resonances ($\Gamma \lesssim D$) and in the Ericson regime ($\Gamma \gg D$)
- Exponential decay and gaussian distribution of S-matrix elements found in the Ericson regime
- Non-exponential decay and deviations from gaussian distribution of S-matrix elements found in the regime of weakly overlapping resonances
- Data are limited by rather small FRD errors, not by noise
- Stringent test of the VWZ theory of chaotic scattering using this large number of data points and a goodness-of-fit test

Summary ctd.

- Investigated furthermore a chaotic T-noninvariant microwave resonator (i.e. a GUE system) in the regime of weakly overlapping resonances
- Principle of reciprocity is strongly violated ($S_{ab} \neq S_{ba}$)
- Data show, however, that TRSB is incomplete → mixed GOE / GUE system
- RMT approach shows that full TRSB sets already in when the symmetry breaking matrix element is of the order of the mean level spacing