The 2010 US National Nuclear Physics Summer School and the TRIUMF Summer Institute, NNPSS-TSI
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1st Lecture:	Some Aspects of Collective Oscillations and Superfluidity in Atomic Nuclei
2nd Lecture:	Giant Resonances – Wavelets, Scales and Level Densities
3rd Lecture:	Nuclear Structure in Astrophysics Studied with Electromagnetic Probes – Some Examples
4th Lecture:	Quantum Manifestation of Classical Chaos – Universal Features of Billiards and Nuclei

Some Aspects of Collective Oscillations and Superfluidity in Atomic Nuclei

- Examples of modes of nuclear sound
- Remarks on nuclear superfluidity (pairing) and its experimental manifestation
- The magnetic dipole Scissors Mode in nuclei revisited
- Magnetic quadrupole resonances – the nuclear Twist Mode

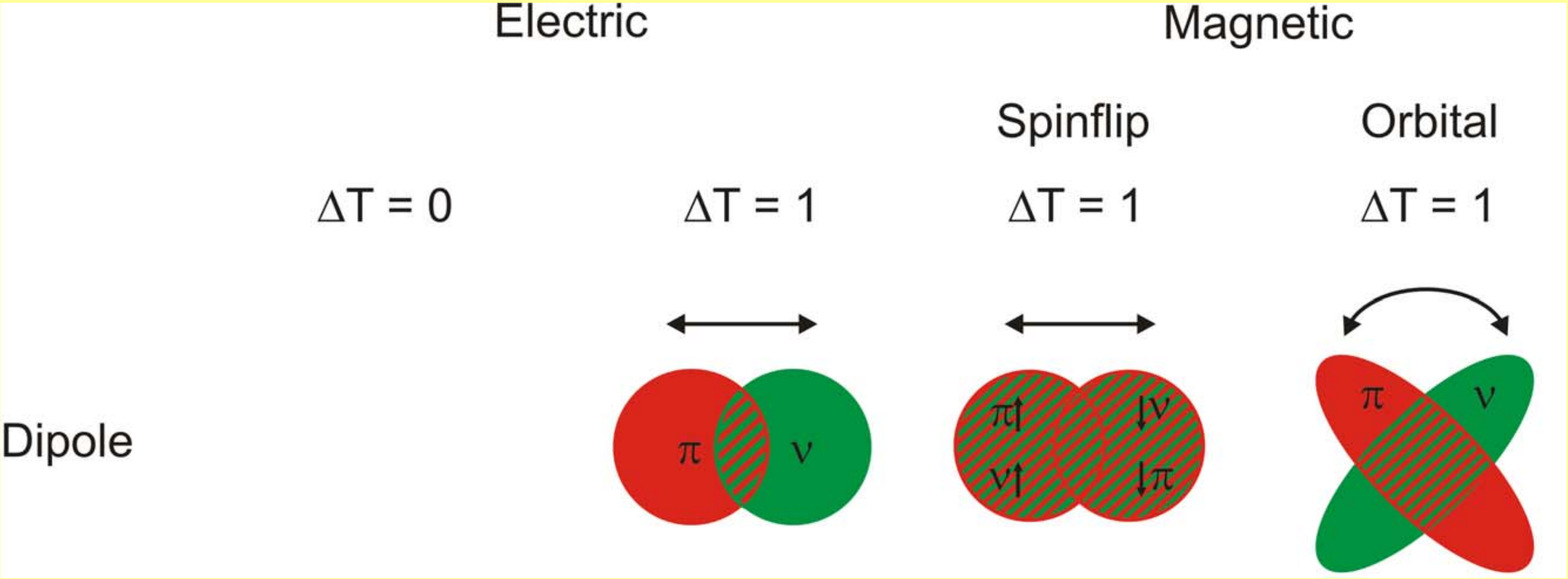
Supported by DFG within SFB 634

Key Reference for 1st Lecture

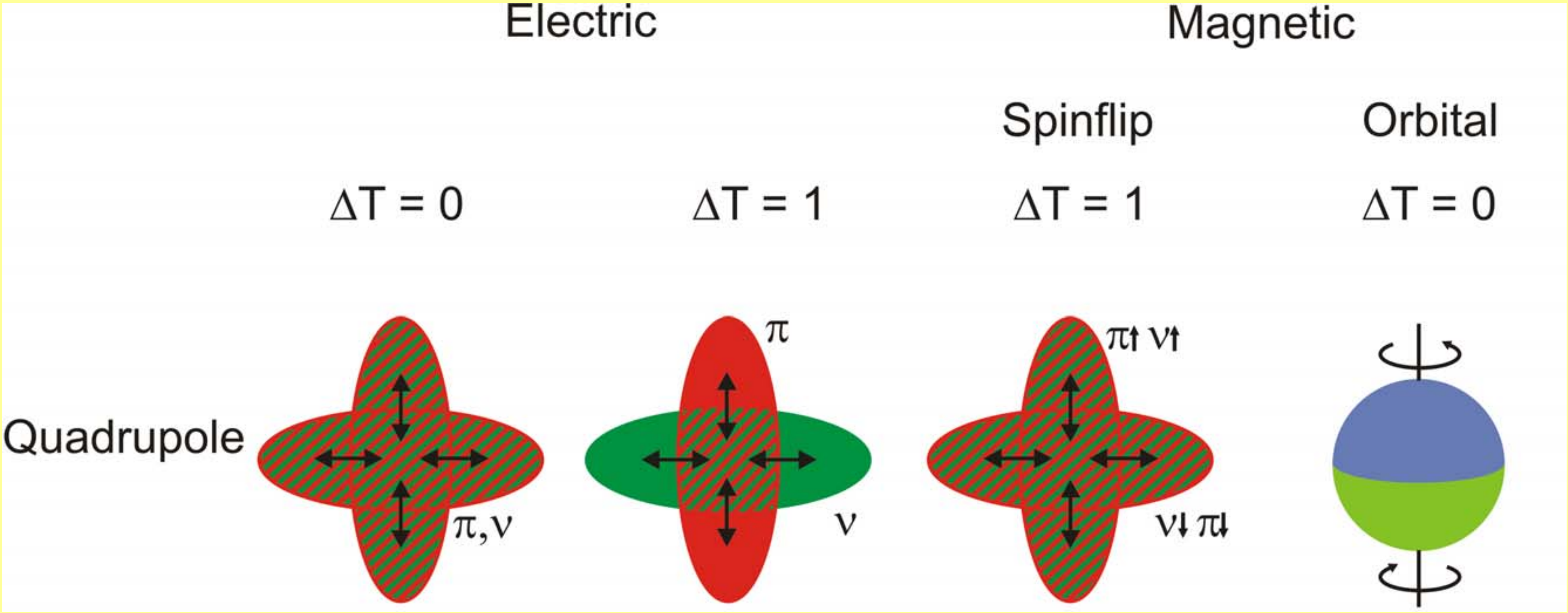
Magnetic dipole excitations in nuclei: elementary modes of nucleonic motion

K. Heyde, A. Richter and P. von Neumann-Cosel
Rev. Mod. Phys., in press (arXiv:1004.3429)

Examples of Modes of Nuclear Sound



Examples of Modes of Nuclear Sound



Some Remarks on Nuclear Superfluidity and its Experimental Manifestation

- Nuclei are build up from protons (π) and neutrons (ν), i.e. fermions, interacting with essentially charge independent NN forces.

- The independent particle shell model (Mayer/Jensen) explains a large fraction of experimental data (g.s. spins, shell gaps as seen at given numbers of abundances of elements, excitation energies of first excited states, magnetic moments of s.p. states, …).

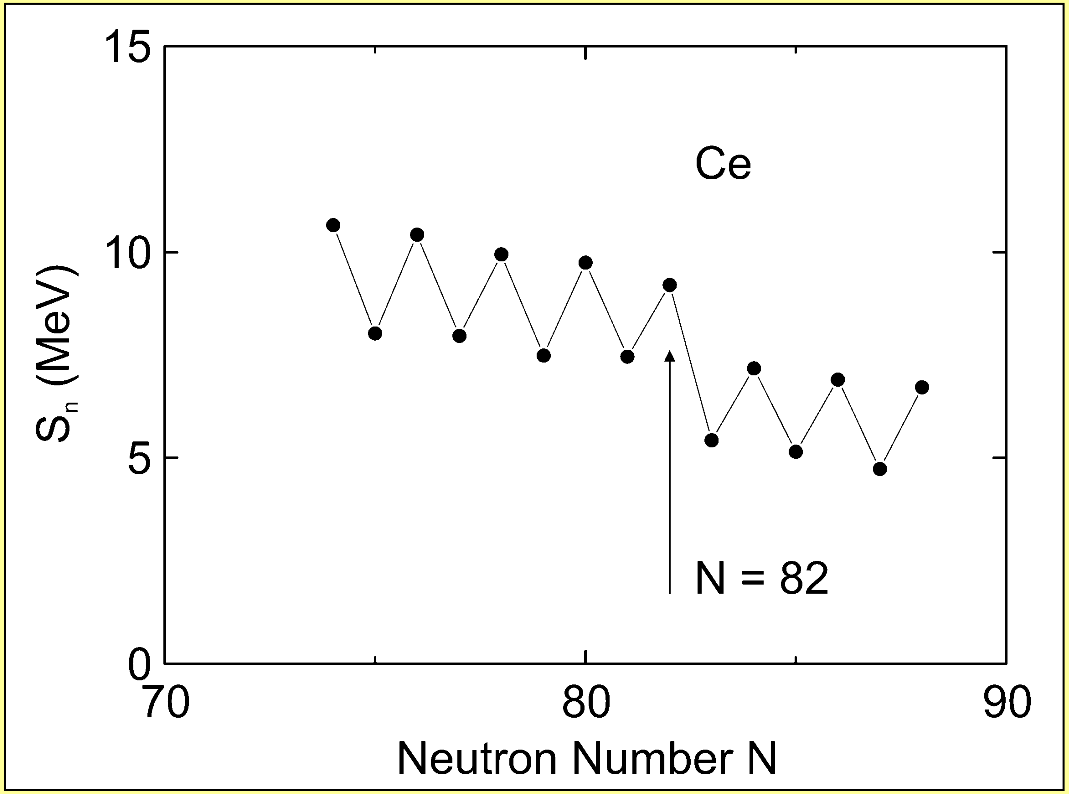
- Later Hartree-Fock theory has shown how this mean field (1-body field) could be derived from an effective 2-body interaction acting inside the nucleus: $U(\vec{r}) = \int \rho(\vec{r}\,')\, V(\vec{r},\vec{r}\,')\, d\vec{r}\,'$ (Hartree term).

- But important facts could not be understood at all on the basis of the independent particle motion of π's and ν's.
  Let's summarize them:

Evidence for Pairing Correlations in Nuclei

(i) Odd-even effect: mass of an odd-even nucleus is larger than the mean of adjacent two even-even nuclear masses → shows up in S_n and S_p for all nuclei.

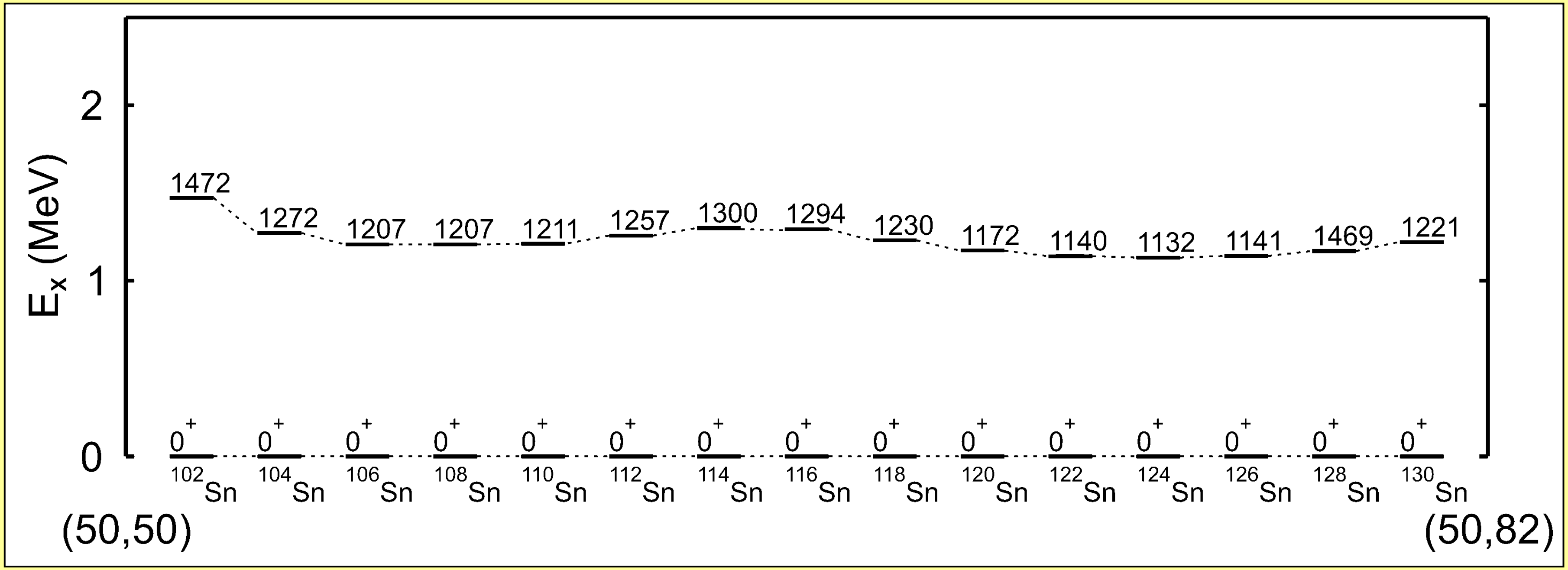
Example: $S_n = BE(A,Z) - BE(A-1,Z)$ of Ce nuclei



- Behavior points towards pair formation of nucleons.

Evidence for Pairing Correlations in Nuclei

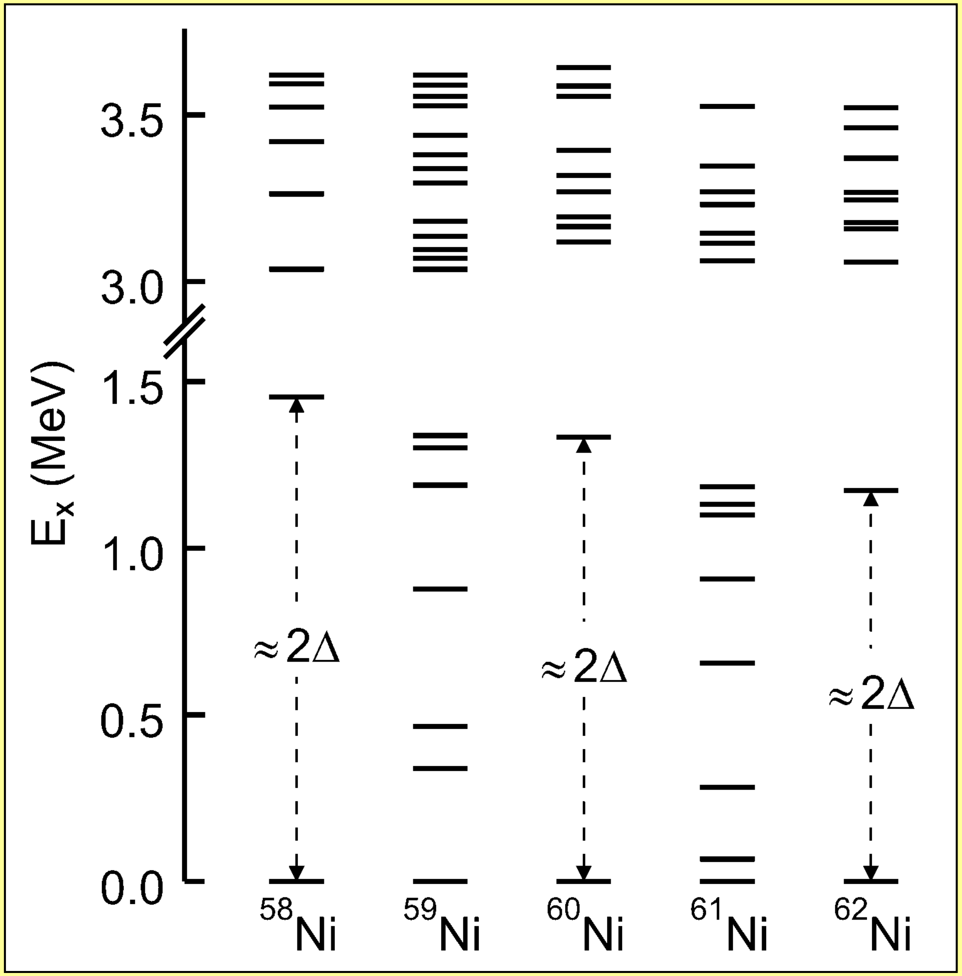
(ii) The excitation energy of the first excited 2^+ state in nuclei remains remarkably constant over large intervals of neutron (proton) numbers.
Example: 2^+_1 excitation energy in Sn nuclei



- These 2^+ states are not rotational states but are connected to a coherent pairing condensate.
- Pair breaking energy: $2\Delta \approx 2$ MeV

Evidence for Pairing Correlations in Nuclei

(iii) Energy gap: odd-even and even-odd nuclei (especially deformed nuclei) have energy spectra different from even-even nuclei.
Example: Ni isotopes



- e-e nuclei: only a few states at most (vibrations, rotations) appear below the pairing gap 2Δ.
- But in o-e and e-o nuclei (where the last nucleon is unpaired) many s.p. and collective states appear.
- Note: above the pair breaking energy 2Δ many excited states are possible → level density $\rho = \rho(\Delta)$
  → **2nd Lecture**

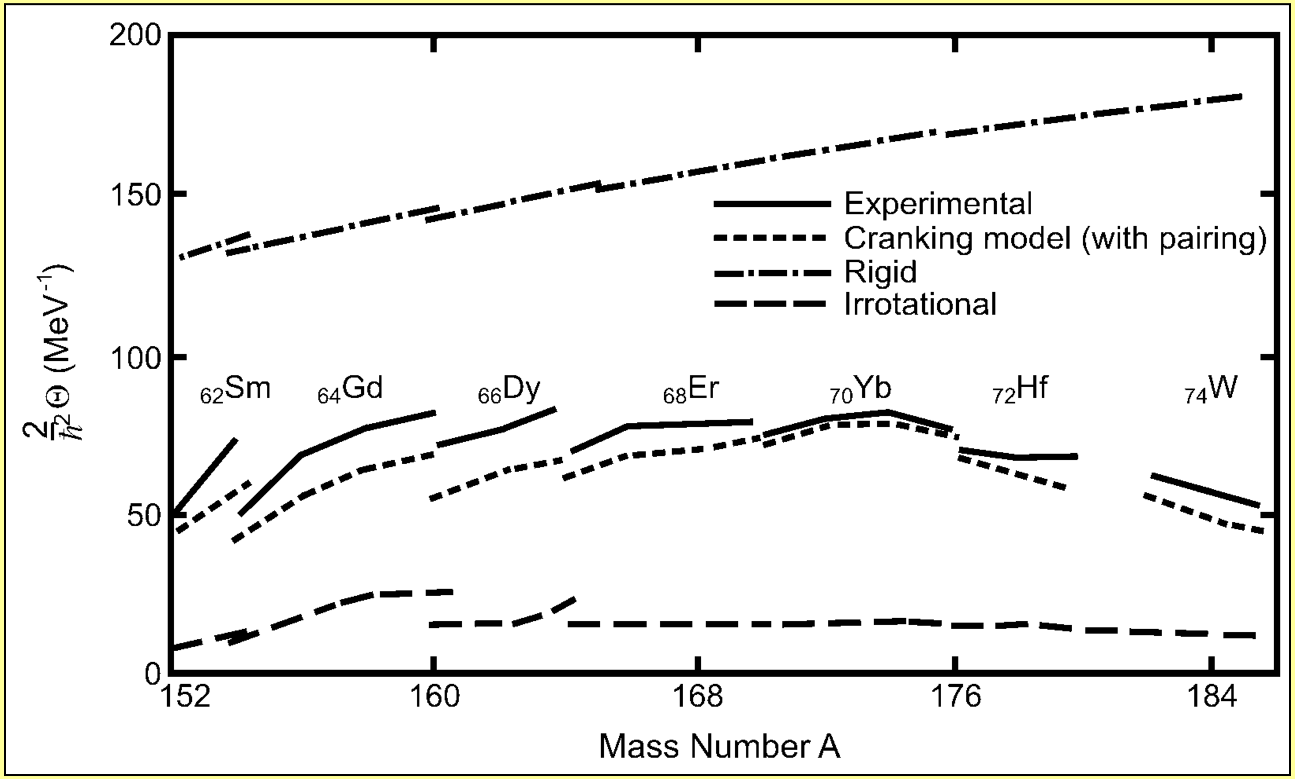
Evidence for Pairing Correlations in Nuclei

(iv) Moment of inertia: extracted from level spacing in rotational bands

$$E = \frac{\hbar^2}{2\Theta} J(J+1)$$

deviates about a factor of two from the rigid rotor values.

Example: Moments of inertia of even-nuclei in the rare-earth region



- $\Theta_{irrot} < \Theta_{exp} < \Theta_{rigid}$
- Pairing correlations have a dramatic influence on collective modes.

BCS Theory and Pairing in Nuclei

- Soon after BCS theory (1957) for electrons in metals had been formulated it was adopted for nuclei (Bohr, Mottelson, Pines (1958), Belyaev (1959), Nilsson, Prior (1960), …).

- Although in finite microscopic systems like nuclei direct evidence of superfluid flow cannot be obtained, the experimental evidence (i) – (iv) for the existing of pairing of nucleons (and hence of a short-range pairing force) points naturally to correlations of two electrons in a superconductor.
  Example: BCS ground state and concept of quasiparticles

$$|\Phi_0\rangle^{BCS} = \prod_{j,m>0} (\mathrm{u}_j + \mathrm{v}_j\, a^{+}_{jm} a^{+}_{j-m})|0\rangle$$

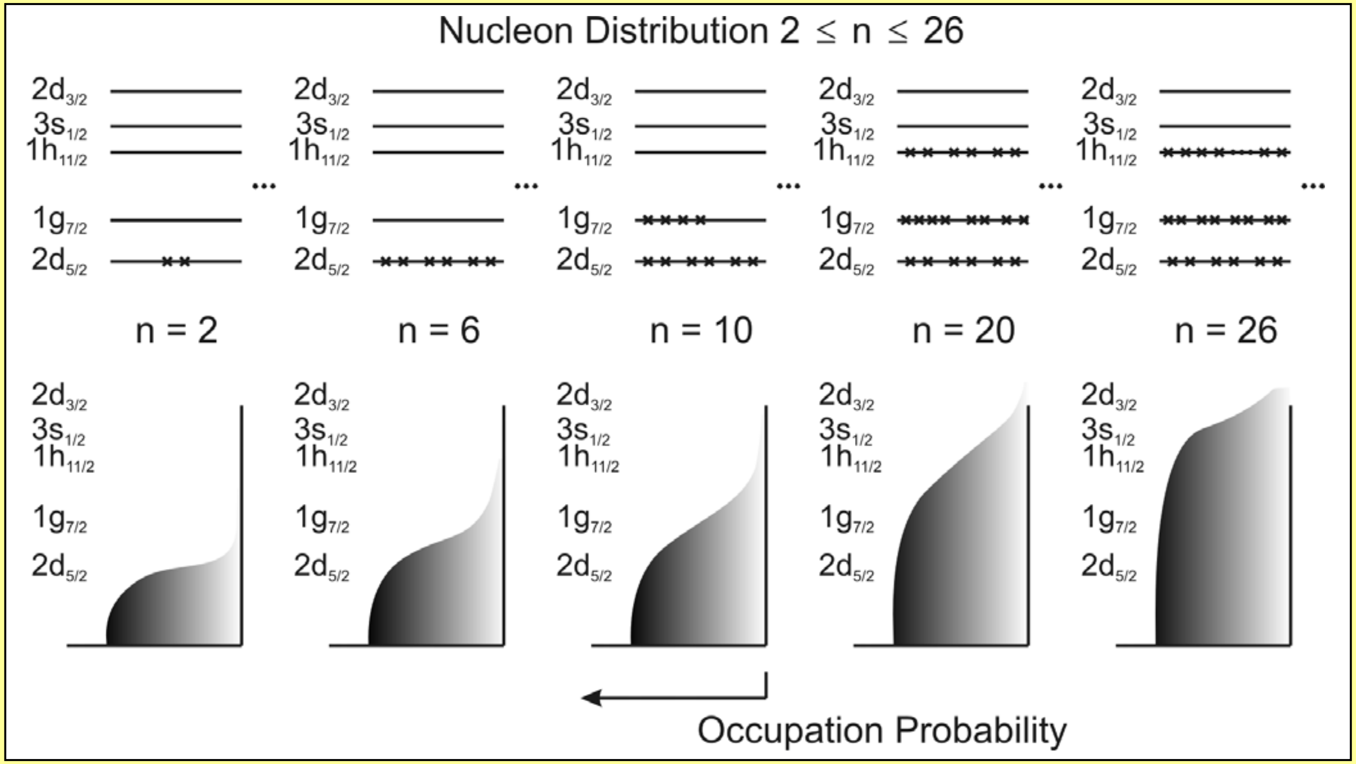
prob. amplitude of a state *j* not occupied by a pair

… a state being occupied

vacuum state of real particle $a_{jm}|0\rangle = 0$

Discrete vs. BCS-Pair Distribution

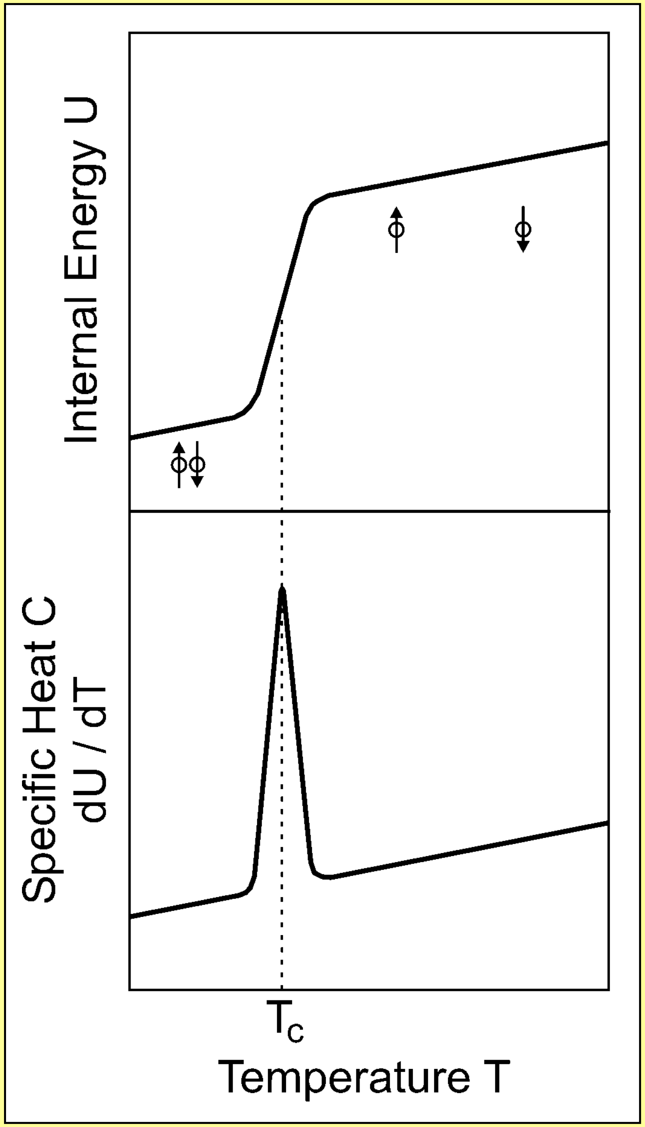
Example: distribution of a number of nucleons over five orbitals



- Short-range pairing force scatters pairs of particles across the sharp Fermi level leading to 2p – 2h, 4p – 4h, … correlations in the g.s.
- BCS pairing correlations modify the nuclear g.s. nucleon distribution.
- Electromagnetic properties (Eλ, Mλ, Q, μ, …) are influenced by various combinations of pairing factors $[(u_i u_j \pm v_i v_j),(u_i^2 - v_i^2),...]$.

Breaking the Pairs by Heating the Nucleus

Schematically: what happens in a finite system ?



- Quantum fluctuations cause broadening.

Thermodynamics in Hot Nuclei

$$\langle A \rangle_C = \frac{1}{Z_\beta} \hat{T}_{r_N}(A\, e^{-\beta \hat{H}})$$

arbitrary observable | partition function | microscopic propagator

$$\hat{H} = \hat{H}_{Mean\ field} + \hat{H}_{Pairing} + \hat{H}_{QQ}$$

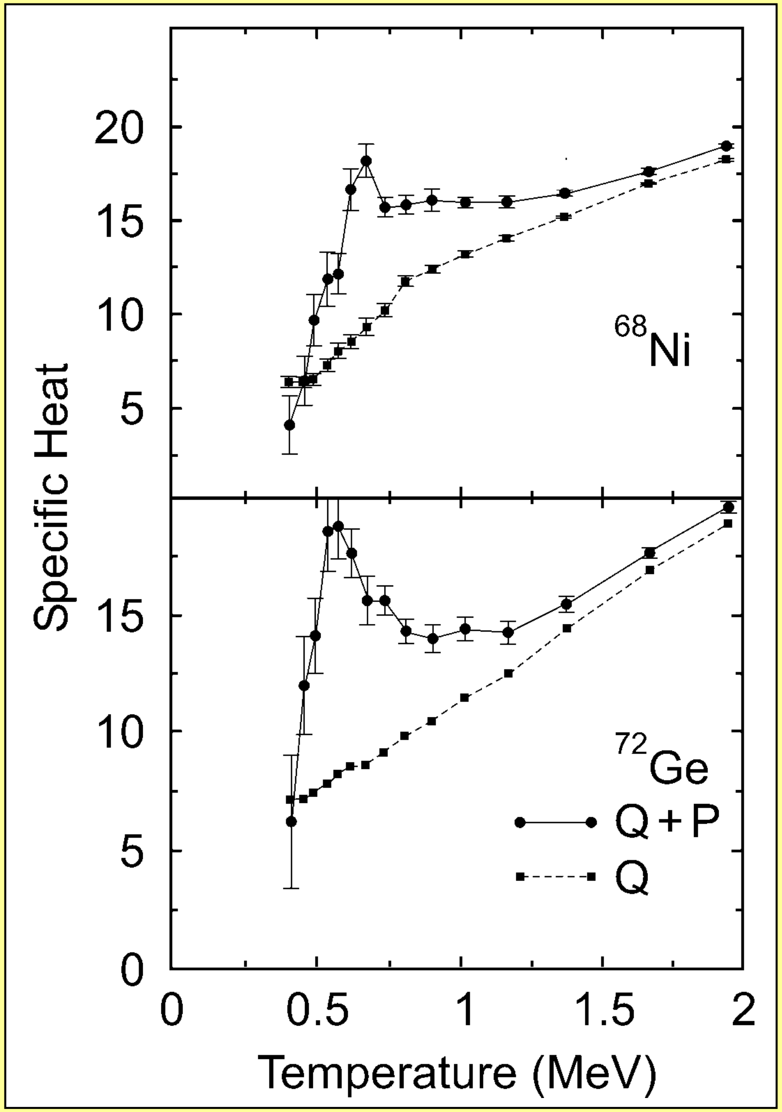
Model space: 50 orbitals for π's and ν's each

Technique: shell model Monte-Carlo

Koonin et al. (1992)

Realistic Examples

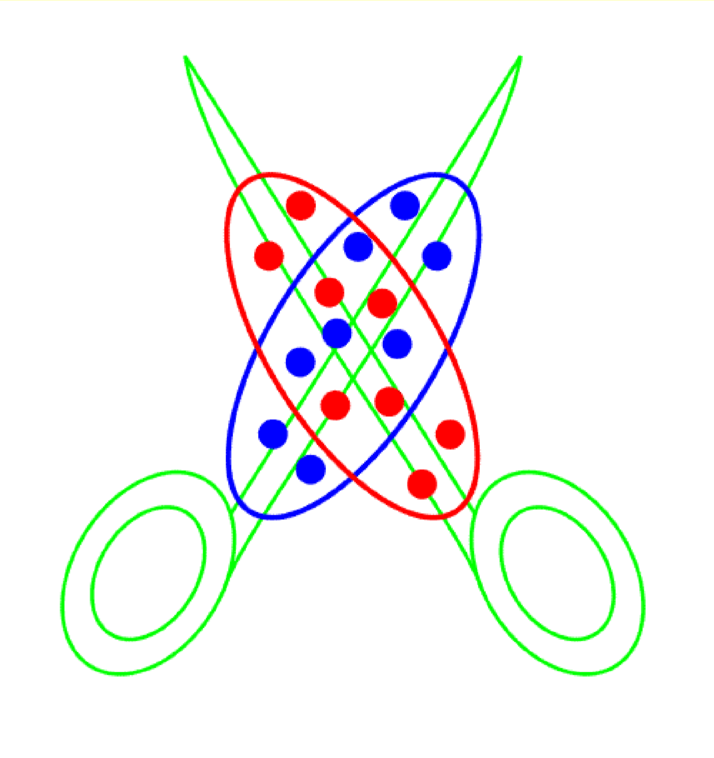
Langanke, Dean, Nazarewicz (2005)



- Pairing correlations in a finite system vanish if the nucleus is excited to about 6 – 10 MeV internal (excitation) energy.

- Specific heat is related to level density ρ
  $\rightarrow$ experimental test possible?
  $\rightarrow$ **2nd Lecture**

The Magnetic Dipole Scissors Mode in Nuclei Revisited



Overview

- Qualitative nature of the nuclear M1 response
- Scissors Mode
- Excitation energy and strength
- Collectivity: sum-rule approach
- Collectivity and fine structure: level spacing statistics
- The Scissors Mode in nuclei and BECs

Qualitative Nature of the M1 Response in Nuclei

(i) Structure of the M1 operator

(ii) Properties of the <u>known</u> p – h interactions

$$T(M1) = \sum_{i=1}^{A} \{g_l(i)\,\vec{l}_i + g_s(i)\,\vec{s}_i\} = T(M1)_{IV} + T(M1)_{IS} \qquad [\mu_N]$$

$$T(M1)_{IS} = \frac{1}{2}\,\vec{J} + \frac{1}{2}\,(g_p + g_n)\,\vec{S} = \frac{1}{2}\,\vec{J} + 0.38\,\vec{S} \qquad [\mu_N]$$

small “spin“

Qualitative Nature of the M1 Response in Nuclei

$$T(M1)_{IV} = \sum_i t_z(i)\,\vec{l}_i + (g_p - g_n)\,\sum_i t_z(i)\,\vec{s}_i \qquad [\mu_N]$$

$$= \frac{1}{2}\,(\vec{L}_p - \vec{L}_n) + \frac{1}{2}\,(g_p - g_n)\,T(M1)_{\Delta T_z=0} \qquad [\mu_N]$$

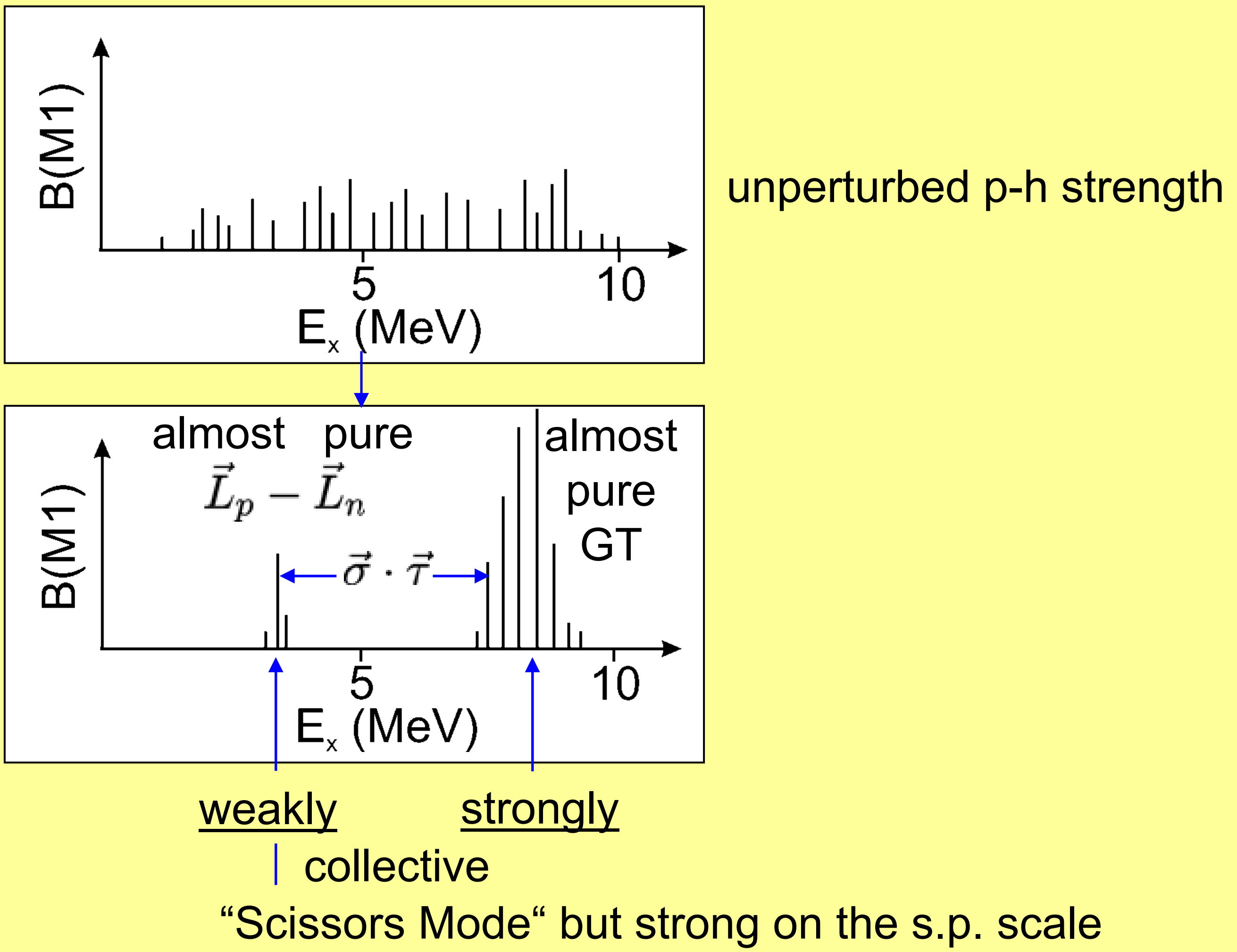
$$= \frac{1}{2}\,(\vec{L}_p - \vec{L}_n) + 4.71\,T(M1)_{\Delta T_z=0} \qquad [\mu_N]$$

rotation generator
“scissors motion“

enhanced “spin-flip”
(Gamow-Teller)

Qualitative Nature of the M1 Response in Nuclei

(iii) Schematically: RPA calculation



- Ideal candidate for the test of models !

Discovery of the Scissors Mode

Volume 137B, number 1,2 PHYSICS LETTERS 22 March 1984

NEW MAGNETIC DIPOLE EXCITATION MODE STUDIED IN THE HEAVY DEFORMED NUCLEUS ^{156}Gd BY INELASTIC ELECTRON SCATTERING ☆

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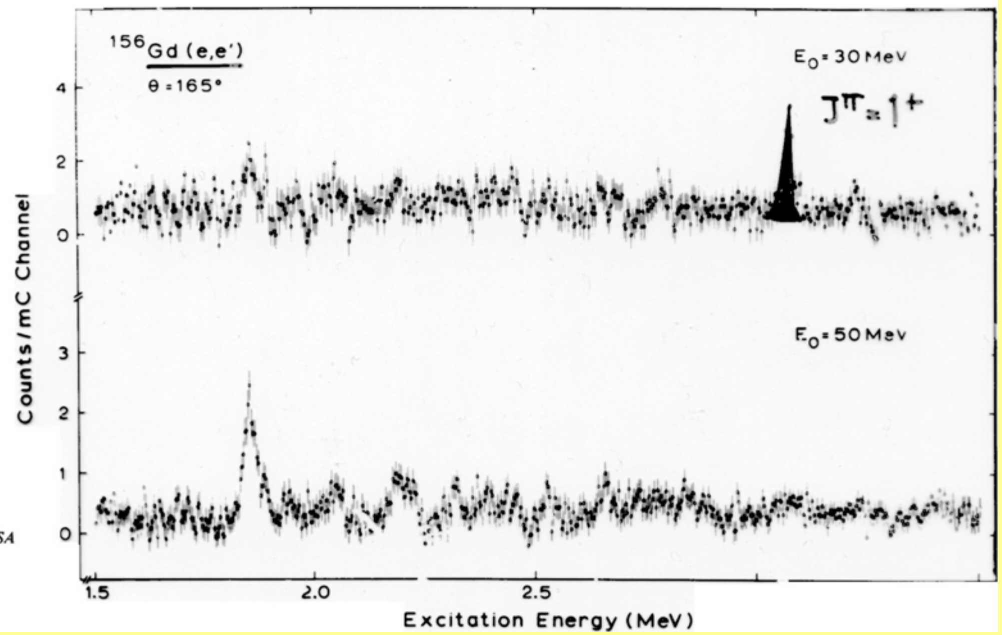
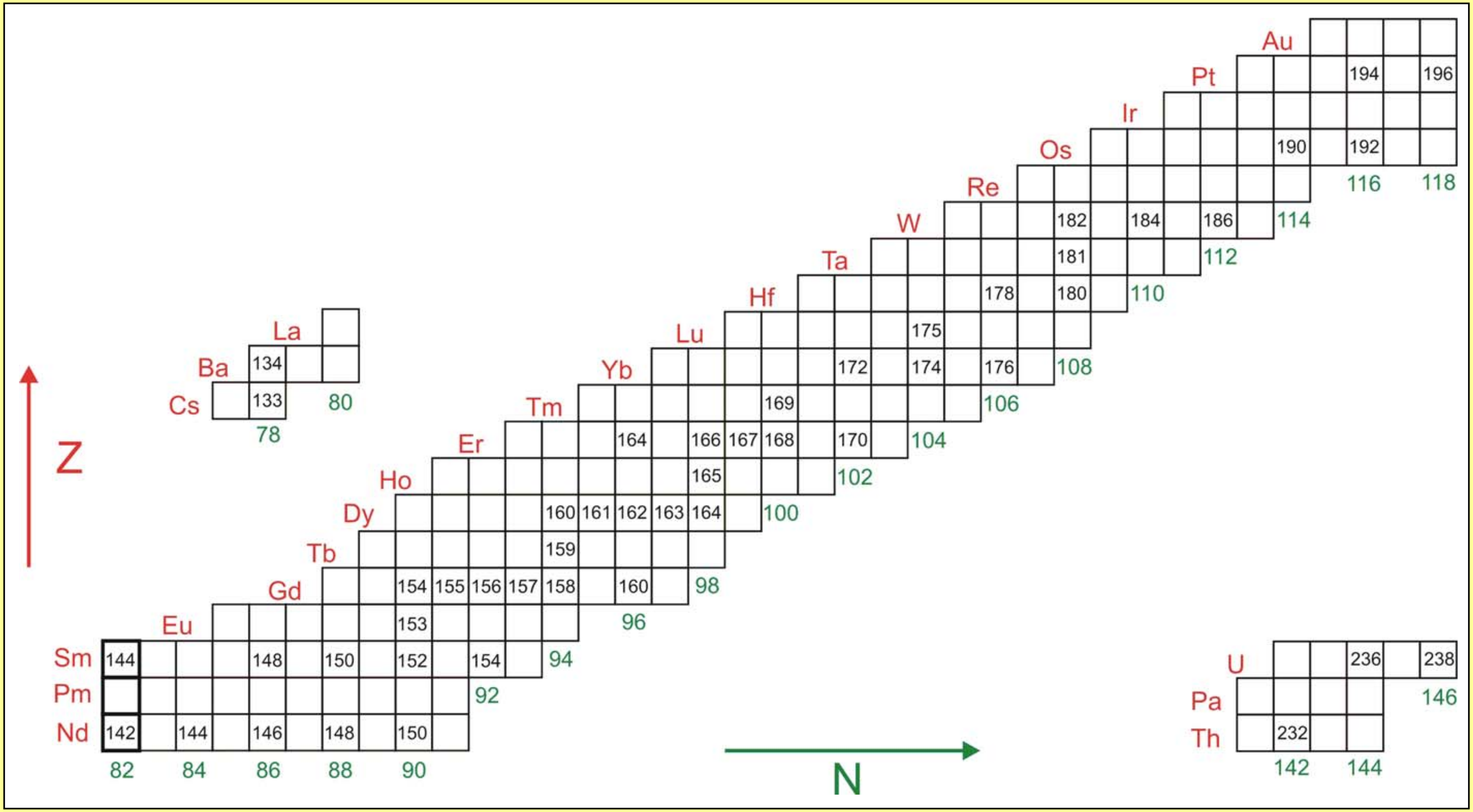
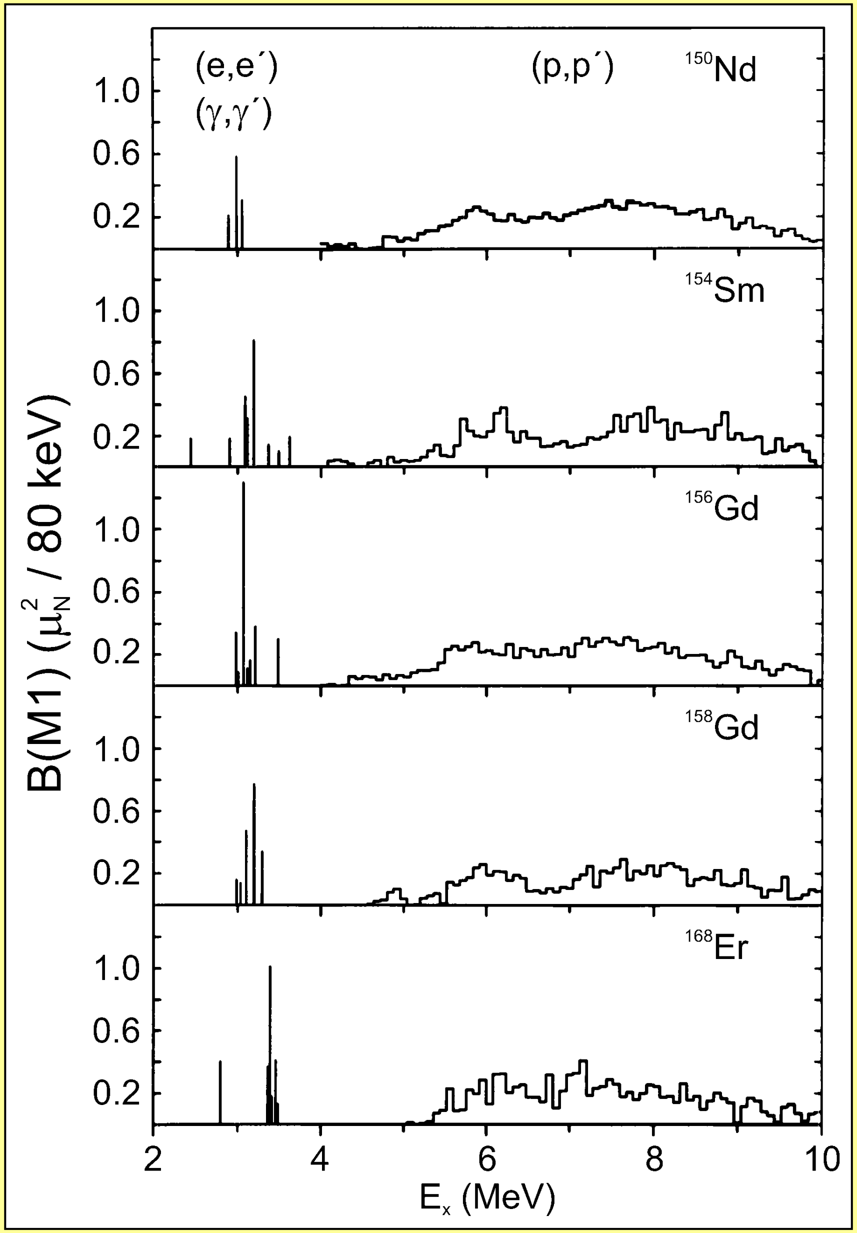


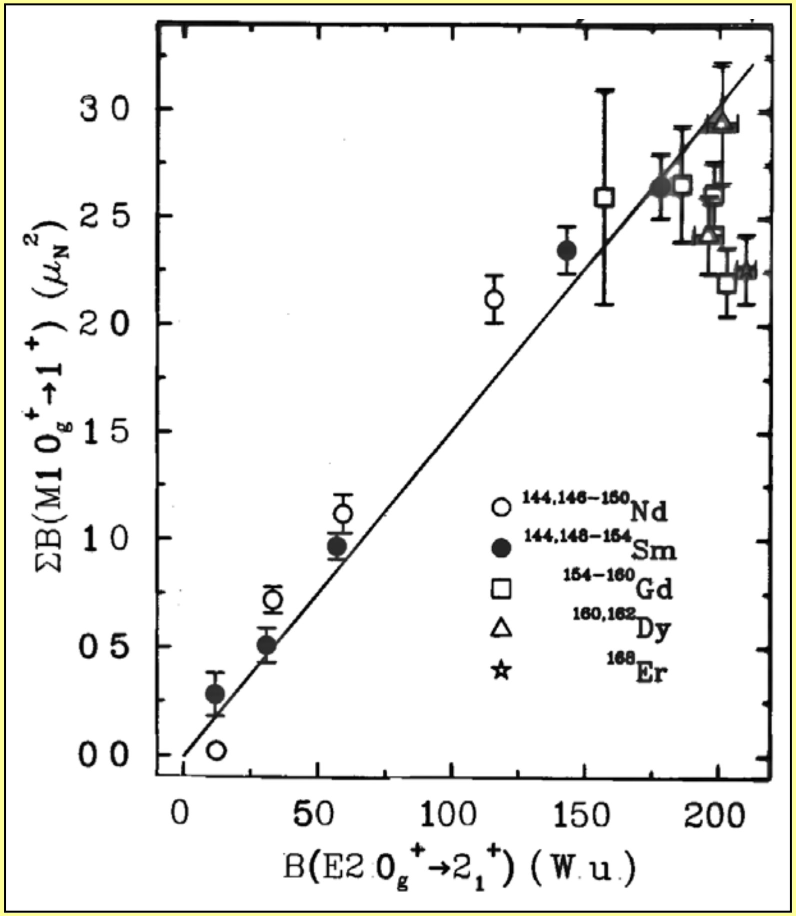
Systematics of the Scissors Mode in Heavy Deformed Nuclei

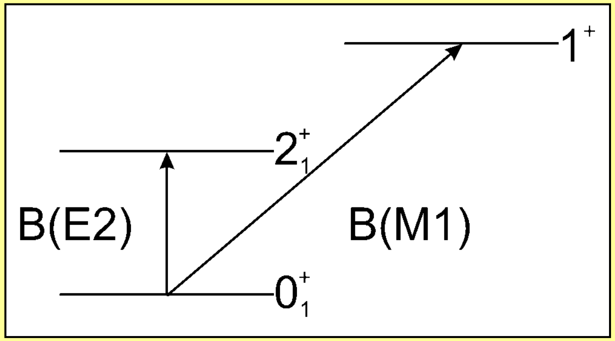


Magnetic Dipole Response in Heavy Deformed Nuclei



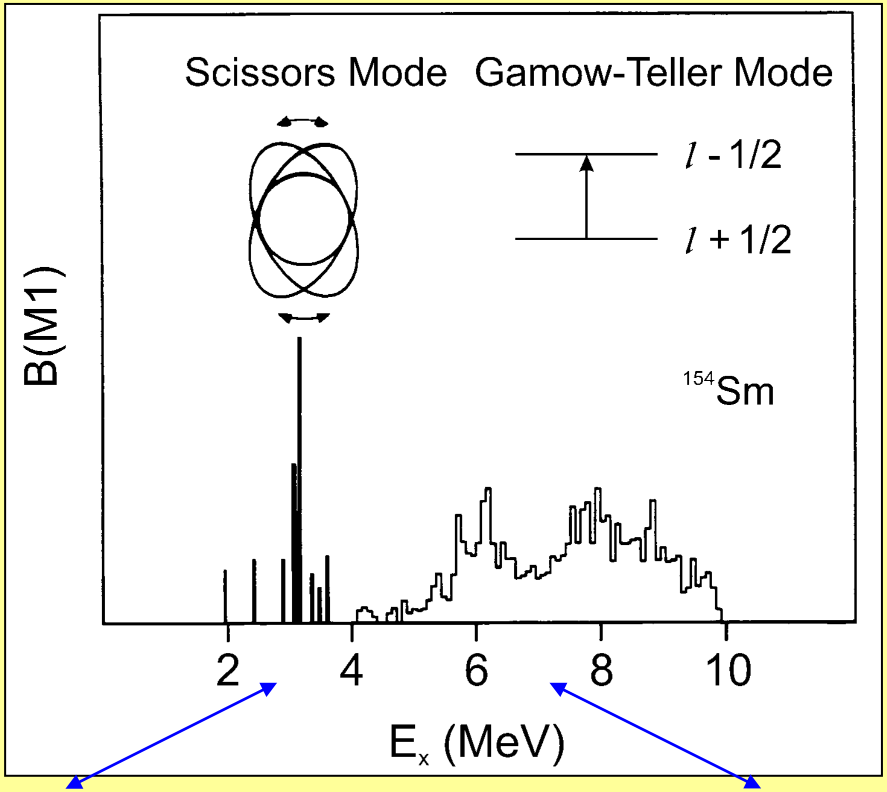
Correlation between B(M1) and B(E2) Strengths





- Experimentally: B(M1) ~ δ^2 → strong dependence on deformation
- Since also B(E2) ~ δ^2 and pairing plays a dominant role in explaining this deformation dependence → B(M1) must also depend strongly on pairing.

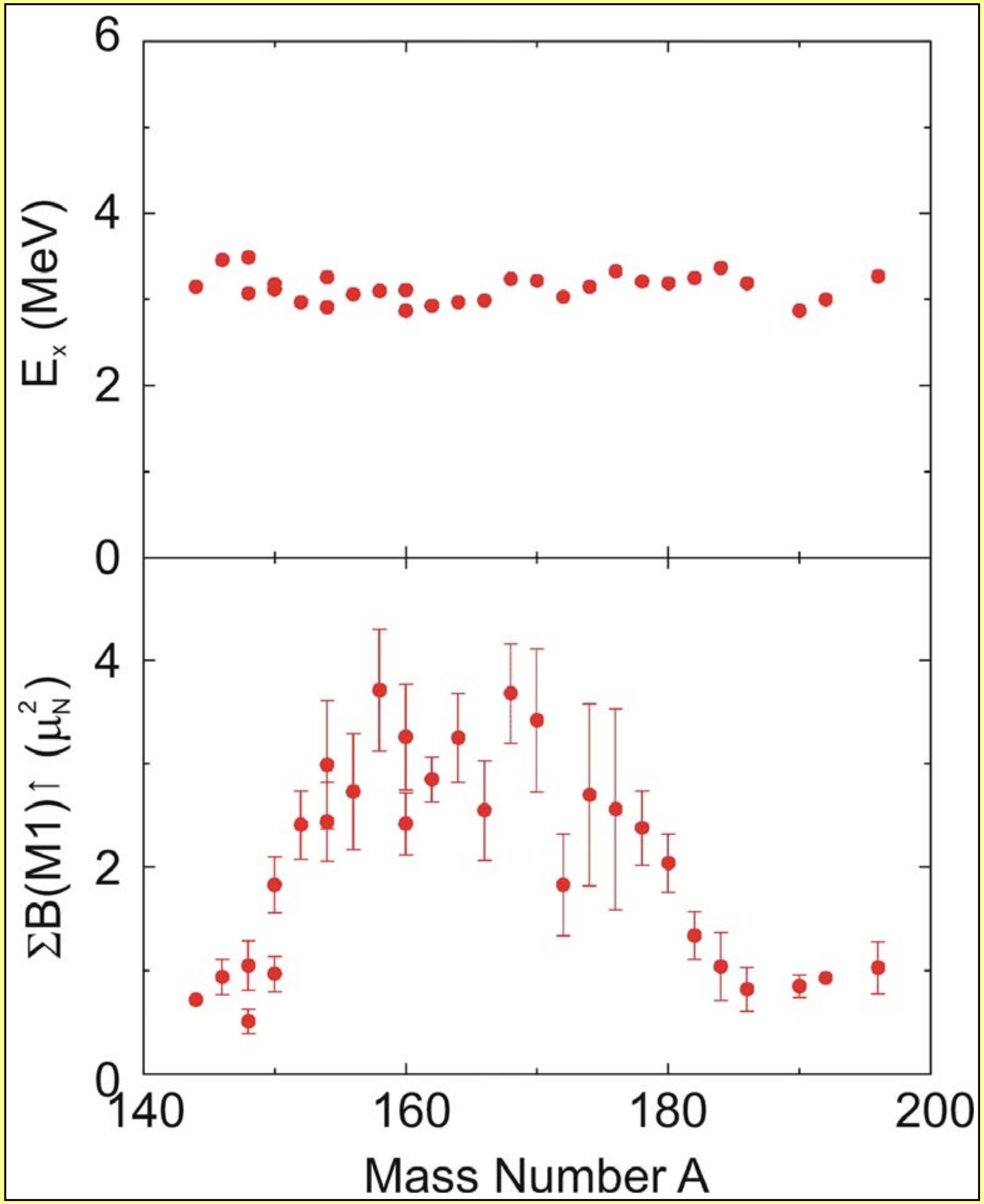
M1 Response in a Heavy Deformed Nucleus



- $(e, e'), (\gamma, \gamma')$
- $E_x \approx 3\,\mathrm{MeV}$
- $\sum B(M1) \approx 3\,\mu_N^2$
- $B_l/B_\sigma \approx 10$
- $\sum B(M1) \approx \delta^2$
- Spreading

- (p, p')
- $E_x \approx 5 - 10\,\mathrm{MeV}$
- $\sum B_\sigma(M1) \approx 11\,\mu_N^2$
- $B_l/B_\sigma \approx ?$
- Two maxima: $p - n$, $IS - IV$
- Spreading

Energy and Strength of the Scissors Mode



- Excitation energy approximately constant, independent of deformation
- Strength depends strongly on deformation: $\sum B(M1) \sim \delta^2 \sim B(E2)$
- Collectivity
- Sum-rule approach

Sum-rule Approach

Lipparini and Stringari (1983)

- Sum rules $$S_j(\mathcal{M}) = \sum_i B_i(\mathcal{M})\, E_{xi}^j$$
- Structure of the field operator $\longrightarrow$ commutation relations

$$S_{+1}(\mathcal{M}) = \frac{1}{2}\,\langle 0|\,[\mathcal{M},[\mathcal{H},\mathcal{M}]]\,|0\rangle$$

$$S_{-1}(\mathcal{M}) = \frac{1}{2}\,\langle 0|\,[[\mathcal{X}^+,\mathcal{H}],\mathcal{X}]\,|0\rangle$$

$$\text{with}\quad \mathcal{X} \quad \text{from} \quad [\mathcal{H},\mathcal{X}] = \mathcal{M}$$

Physics Parameters

- Mean excitation energy and summed excitation strength

$$E_x = \sqrt{S_{+1}/S_{-1}}$$

$$B(M1) = \sqrt{S_{+1} \cdot S_{-1}}$$

- Sum rules depend on two parameters:

$$g_{IV} \approx g_{IS} = g(2_1^+)$$

$$\Theta_{IV} = \Theta_{IS} = 3\hbar^2/E_{2_1^+}$$

Transition Operator

Enders et al. (2005)

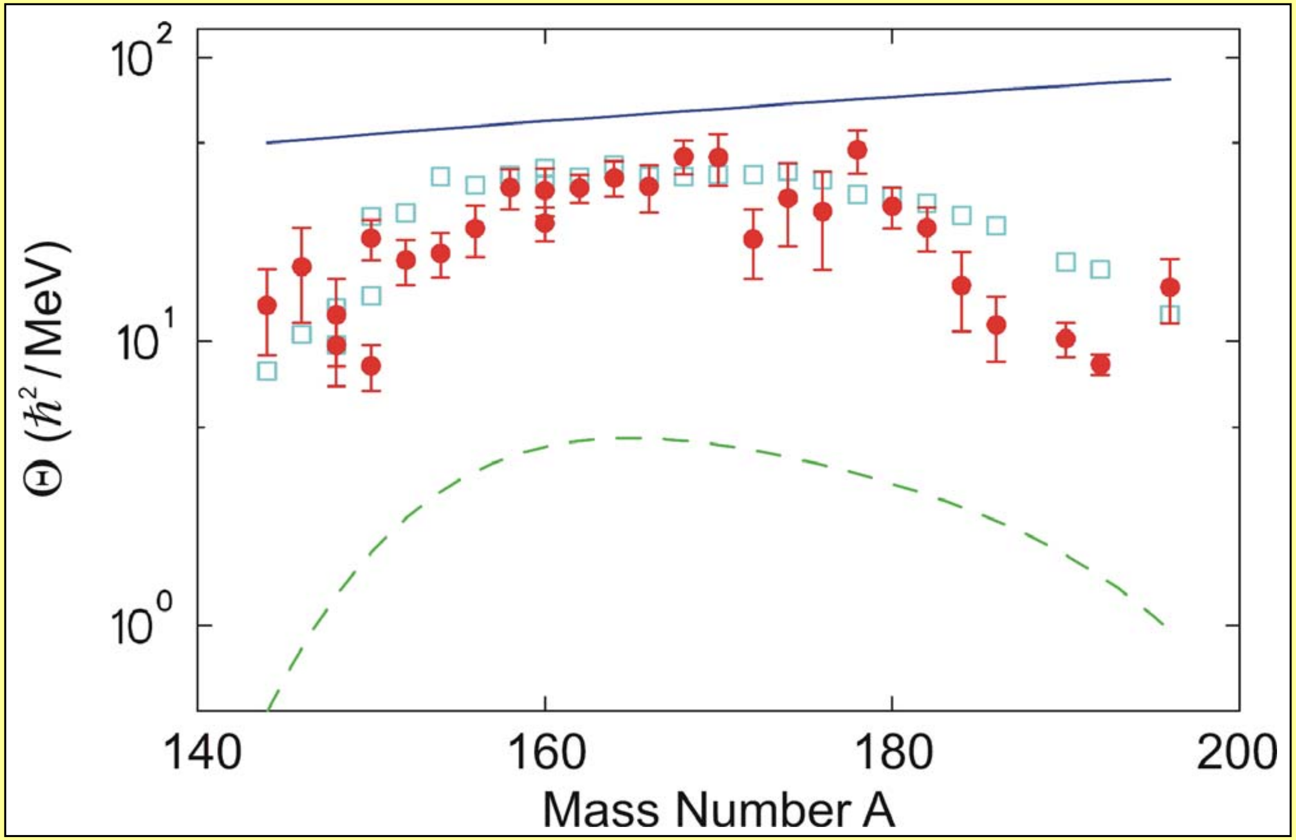
- Isovector rotation (TRM of Lo Iudice and Palumbo) $\mathcal{M}(M1) \propto \sum_i \hat{J}_i^x \, \hat{\tau}_i^3$

- $S_{+1}(M1) = \sum B(M1) \cdot E_x = \frac{3}{5\pi} \, r_0^2 \, A^{5/3} \, \delta^2 \, E_{GDR}^2 \, m_N \, g_{IV}^3$

- $S_{-1}(M1) = \sum \frac{B(M1)}{E_x} = \frac{3}{16\pi} \, \Theta_{M1} \, g_{IV}^2$

with: $g_{IV} = \frac{1}{2} \, (g_p^l - g_n^l) \approx \frac{1}{2} \, g_p^l \, (\approx g(2_1^+))$

Moments of Inertia



$$\Theta_{M1} = \frac{4\pi\, S_{-1}}{3g_{IV}^2}$$

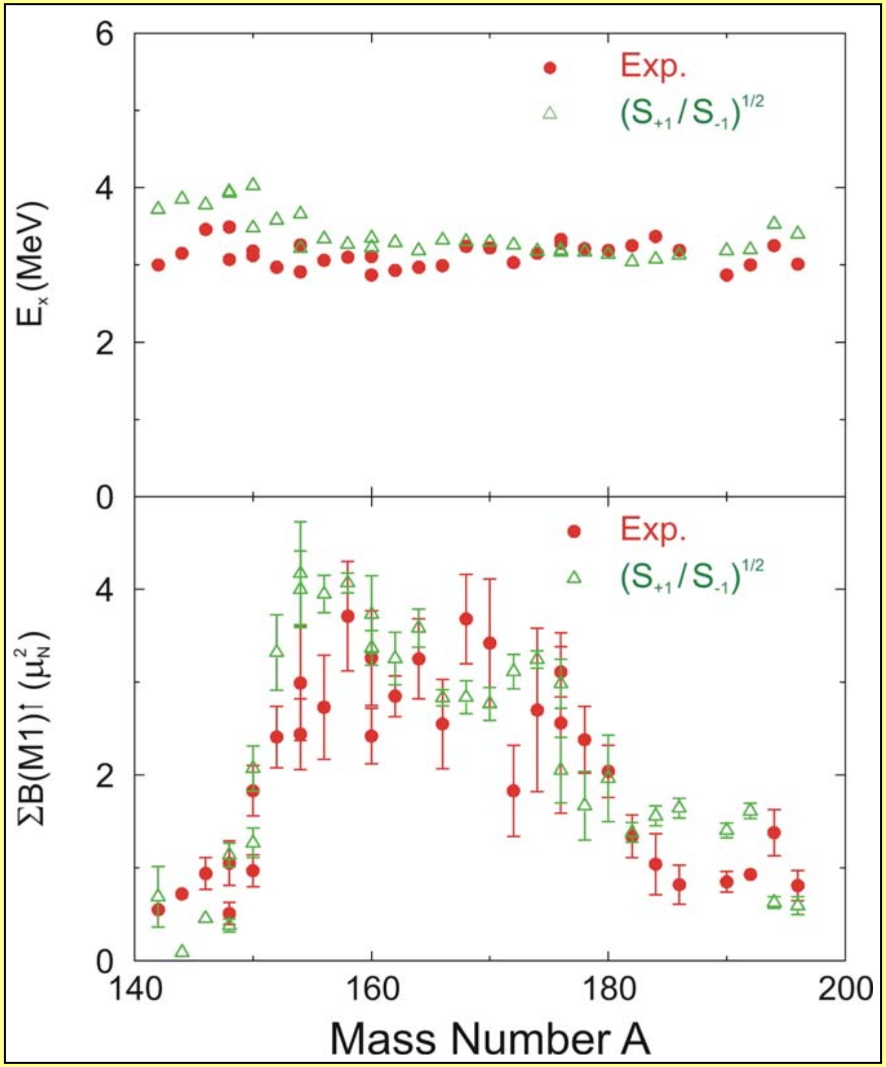
$$\Theta_{gsb} = \frac{3\hbar^2}{E_{2_1^+}}$$

$$\Theta_{rig} = \frac{2}{5}\, MR^2$$

$$\Theta_{irrot} = \Theta_{rig} \cdot \delta^2$$

- Strong effect of pairing (nuclear superfluidity) is evident.

Parameter-free Sum Rule Description



- $E_x \sim \sqrt{E_{2_1^+}} \cdot \delta \approx const.$,

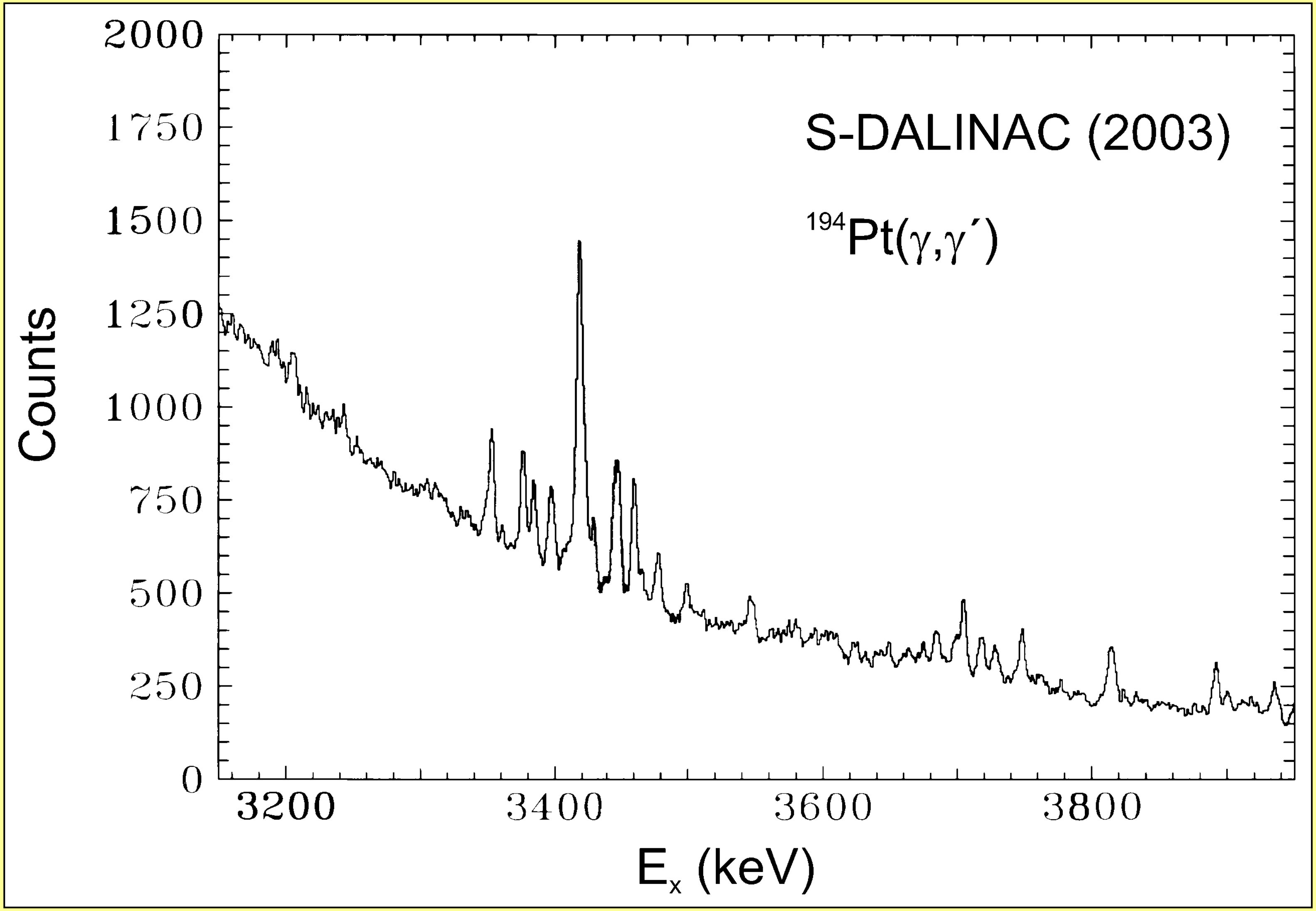
  i.e. contributions from deformation and moment of inertia cancel each other.

- $B(M1) \sim \frac{\delta}{\sqrt{E_{2_1^+}}} \sim \delta^2$,

  i.e. "δ^2 law" results from an interplay of deformation and the moment of inertia.

- Deformed mean field $\rightarrow$ B(M1) depends on occupation probabilities, i.e. the pairing factor $(u_1 v_2 - u_2 v_1)^2$ vanishes for $\delta \approx 0$ but becomes sizable for a large δ (De Coster and Heyde, 1989).

Example for Fragmentation and Spreading of Scissors Mode Strength

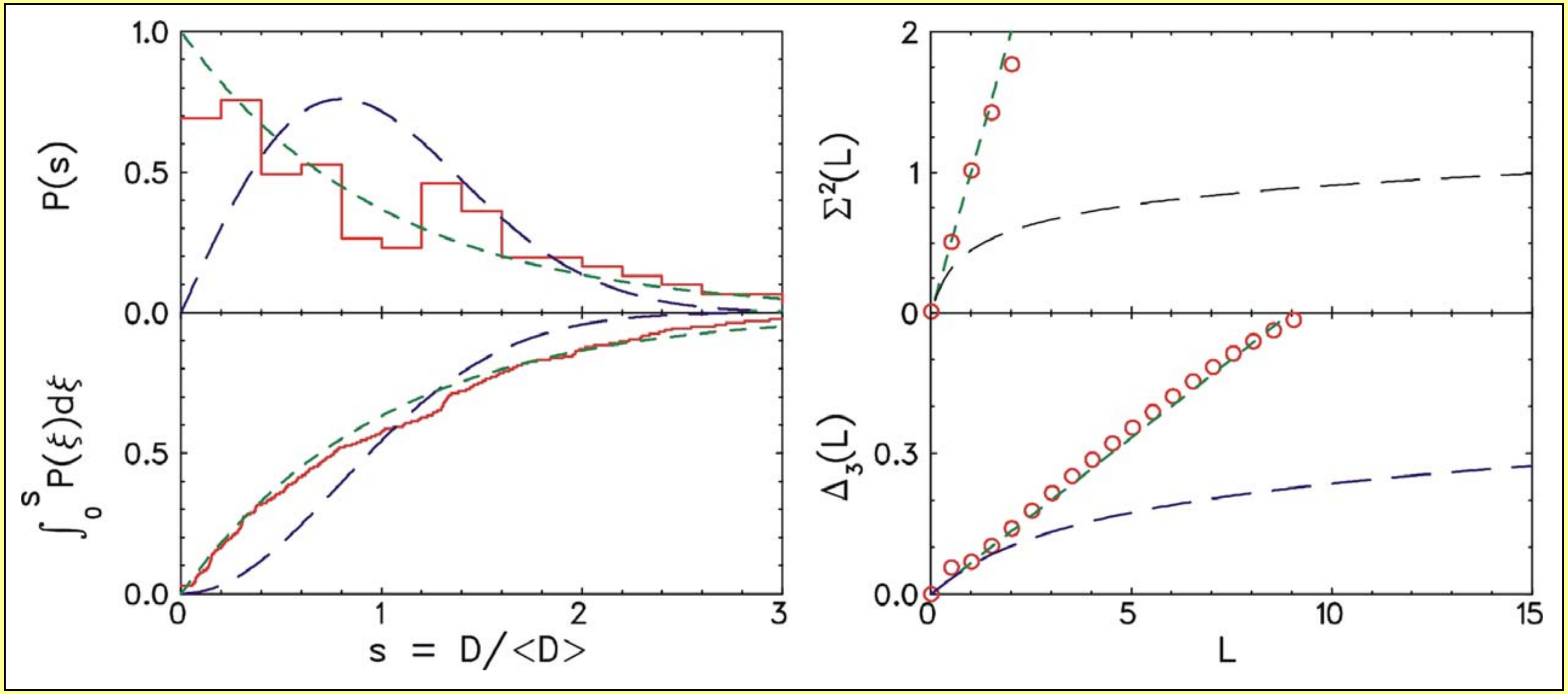


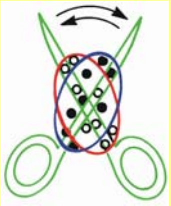
→ **2nd Lecture**

Level Spacing Distribution of Scissors Mode Strength

Enders et al. (2000)

Nuclear data ensemble: 152 $J^{\pi} = 1^{+}$ states from 13 nuclei





- Non-generic behavior according to Poisson
- Proof of a simple collective excitation

→ **4th Lecture**

Magnetic Dipole Scissors Mode

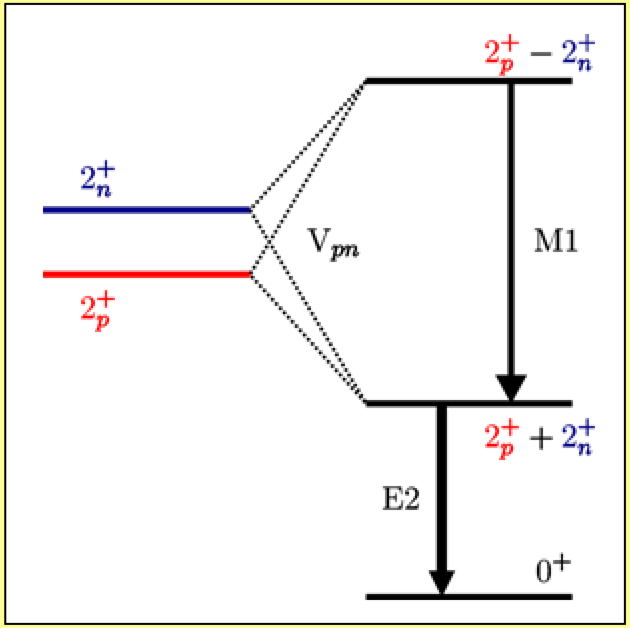
- Sum-rule approach

- All physics parameters are fixed by known nuclear properties:
  - deformation
  - magnetic *g* factor of collective states
  - giant dipole (and isoscalar giant quadrupole resonance)

- Properties of the Scissors Mode result from an interplay of
  - moment of inertia
  - deformation

  } → pairing (superfluidity)

- Number of microscopic models (IBM, QRPA, …) for the Scissors Mode which is a paradigm of mixed symmetry states → pn-symmetry in valence shell, e.g.



Temperature Dependence of Damping and Frequency Shifts of the Scissors Mode of a Trapped Bose-Einstein Condensate

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The experimental discovery of the scissors mode [1], first predicted in a geometrical model [2], has been one of the most exciting findings in nuclear physics during the past two decades (see [3] for a review). According to the geometrical picture, such a mode arises from a counter-rotational oscillation of the deformed proton and neutron fluids. Extensive studies of this mode in the past two decades investigated the dependence of its strength on the nuclear deformation and the relationship with the quadrupole collective mode and with the superfluid moment of inertia of the nucleus [3].

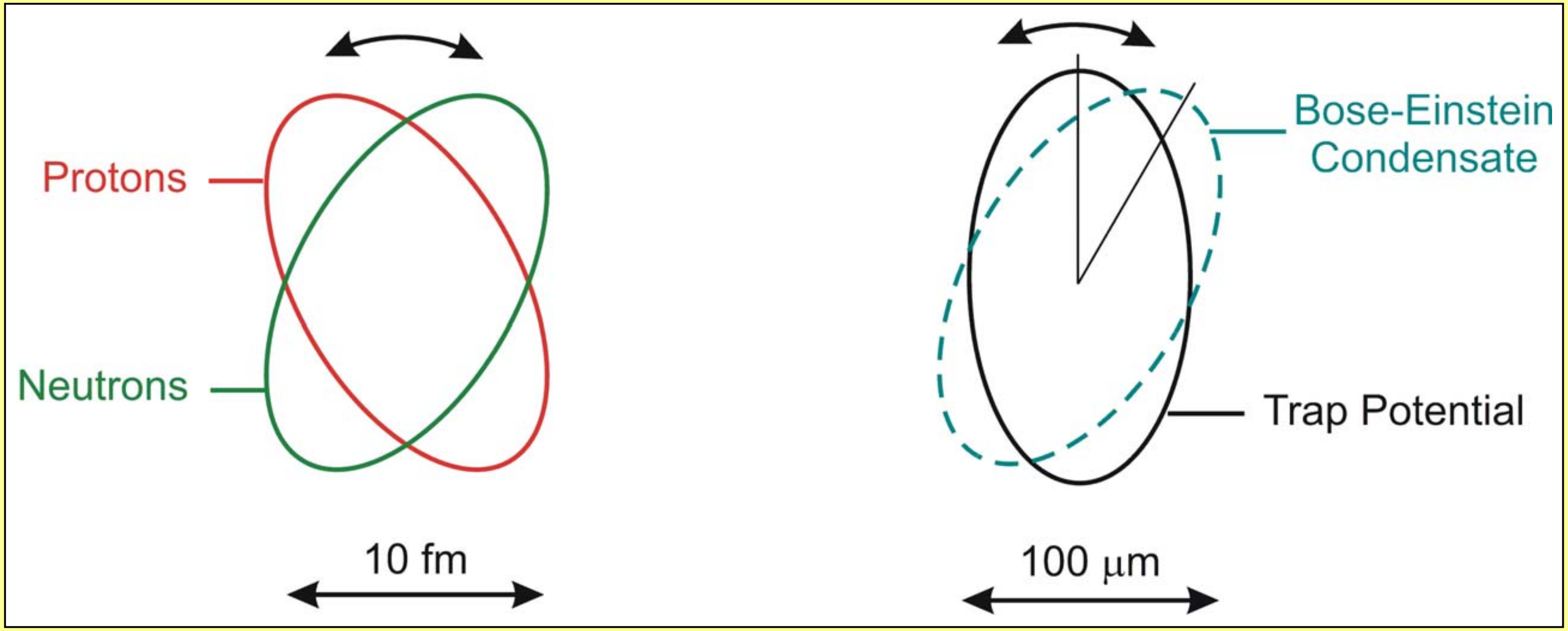
Recently it has been possible to study the scissors mode in Bose-Einstein condensates (BEC) of dilute gases [4]. This transverse mode of excitation was used to demonstrate that the condensate can flow only in an irrotational

In this paper we report a systematic study of the temperature dependent damping and frequency shifts of the scissors mode excitation in a trapped condensed gas. After a brief description of the experimental procedure, we present the scissors mode data for the frequency shifts and damping rates. We then compare our results with the available theoretical calculations for other collective modes. Finally, we show how the scissors mode frequency shifts are related to quenching of the moment of inertia of the boson gas.

We excite the scissors mode by using the technique described in our previous paper [6]. In summary we prepare atoms at the desired temperature T in an untilted time-averaged orbiting potential (TOP) trap ($\omega_x = \omega_y = 126$ Hz, $\omega_z = \sqrt{8}\,\omega_x$). We then adiabatically tilt the con-

Scissors Mode

Deformed Nuclei | **Trapped BEC**

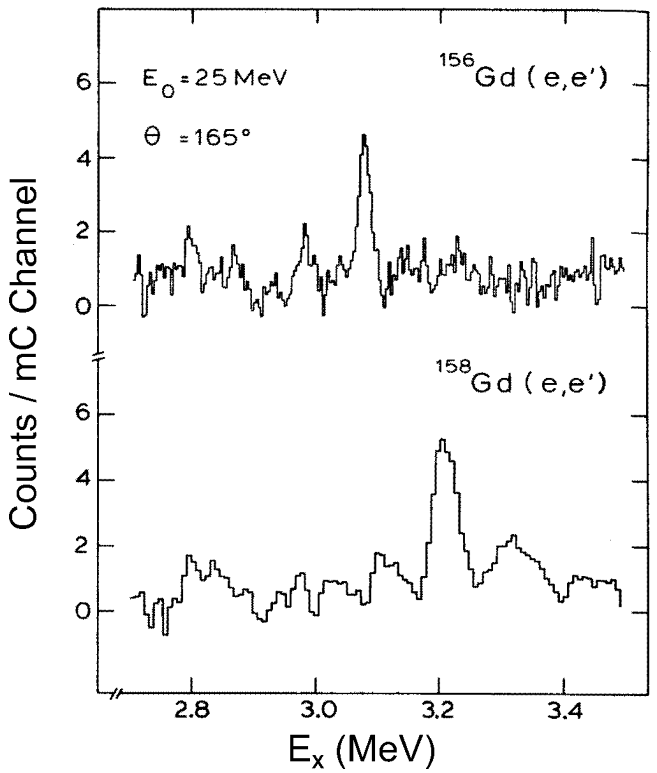


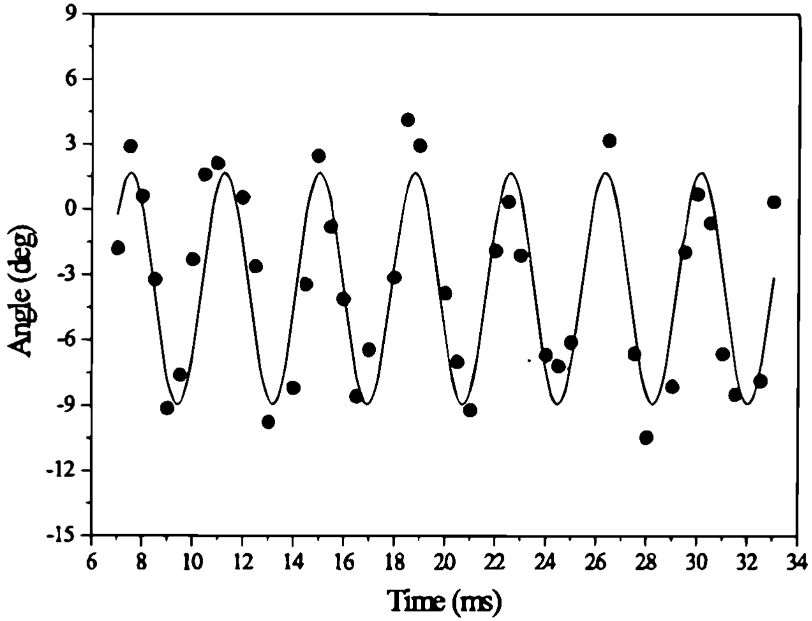
- Superfluidity ?
- Moment of inertia ?

Guéry-Odelin and Stringari (1999)

Experimental Studies

Deformed Nuclei / Trapped BEC





$\nu \approx 7\cdot10^{20}$ s^{-1}

$\alpha \approx 6.0°$ (collective model)

Bohle et al. (1984)

$\nu \approx 3\cdot10^{2}$ s^{-1}

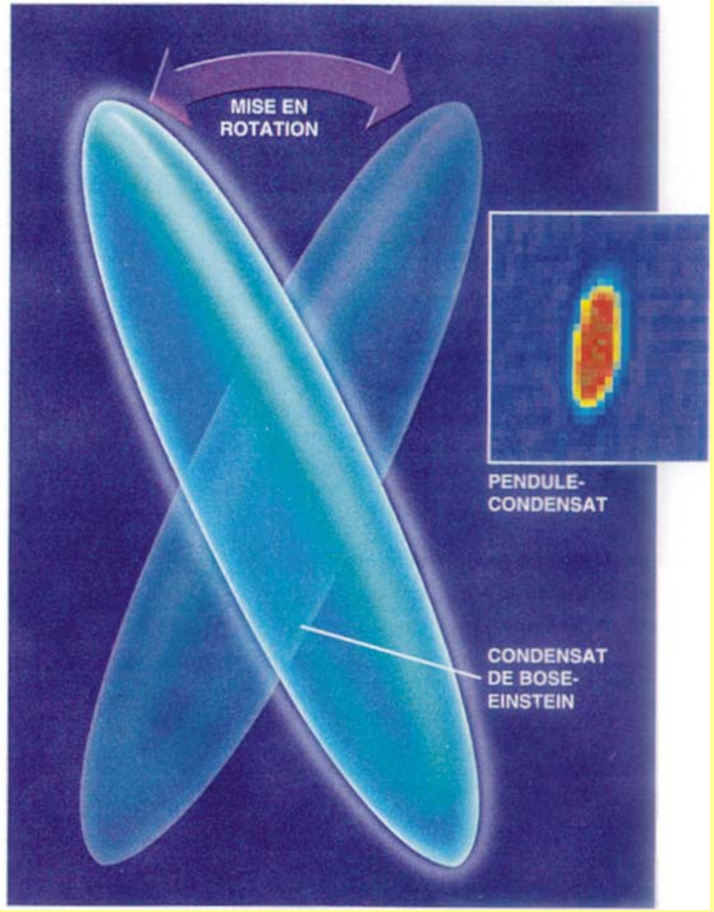
$\alpha \approx 5.5°$ (measured)

Marago et al. (2000)

La superfluidité des condensats

DAVID GUÉRY-ODELIN

POUR LA SCIENCE - N° 296 JUIN 2002



Magnetic Quadrupole Resonance – the Nuclear Twist Mode

Magnetic Quadrupole Response in Nuclei

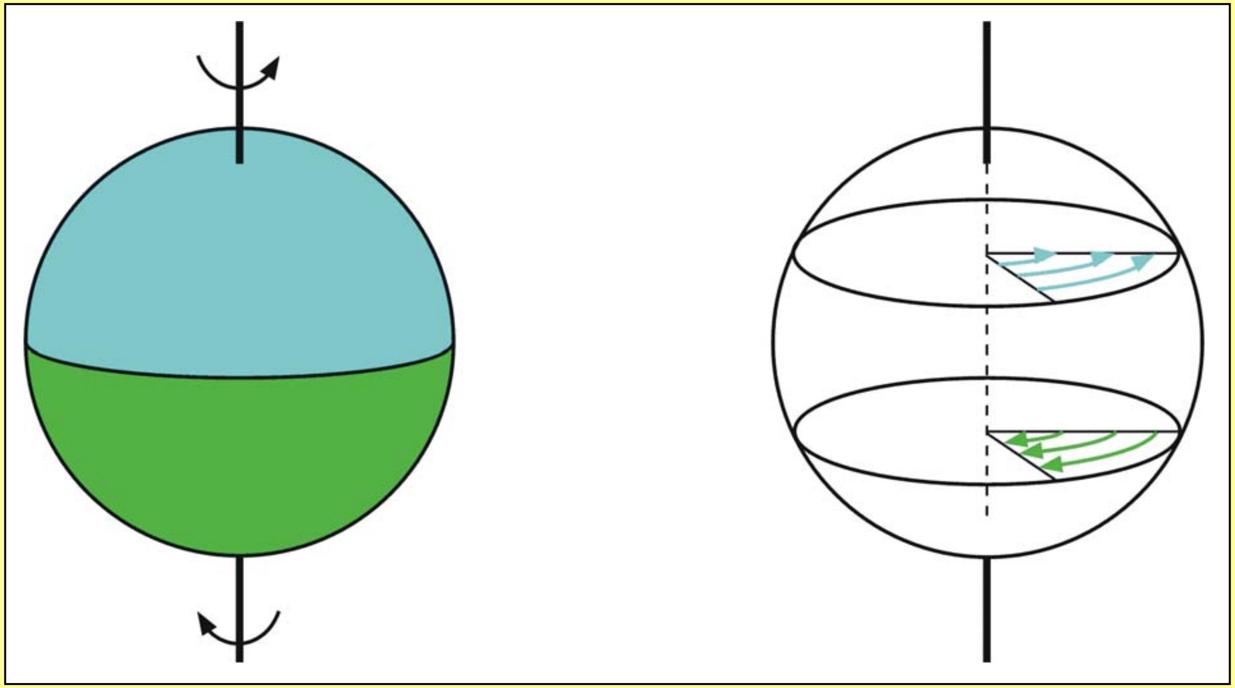
- Scarcely studied as compared to the M1 response
  - ^{90}Zr, ^{58}Ni, ^{48}Ca

- Spin and orbital parts are about of equal magnitude

- Orbital M2 mode (“Nuclear Twist Mode”)
  - existence ?
  - nature ?
  - general feature of finite Fermi systems ?

The $J^{\pi} = 2^{-}$ Twist Mode

Holzwarth and Eckart (1977)



- No restoring force in an ideal fluid → its observation would be a direct proof of the zero-sound nature of magnetic giant resonances in nuclei.

- Shear module $\mu/\rho = \begin{cases} 15.3 \text{ MeV nuclear matter} \\ 6.3 \text{ MeV } {}^{48}\text{Ca} \\ 7.2 \text{ MeV } {}^{90}\text{Zr} \end{cases}$

The Nuclear "Twist"

- Operator $\hat{T} = e^{-i\alpha\, z l_z} = e^{\alpha\, \vec{u}\cdot\vec{\nabla}}$ with $\vec{u} = (yz, -xz, 0)$

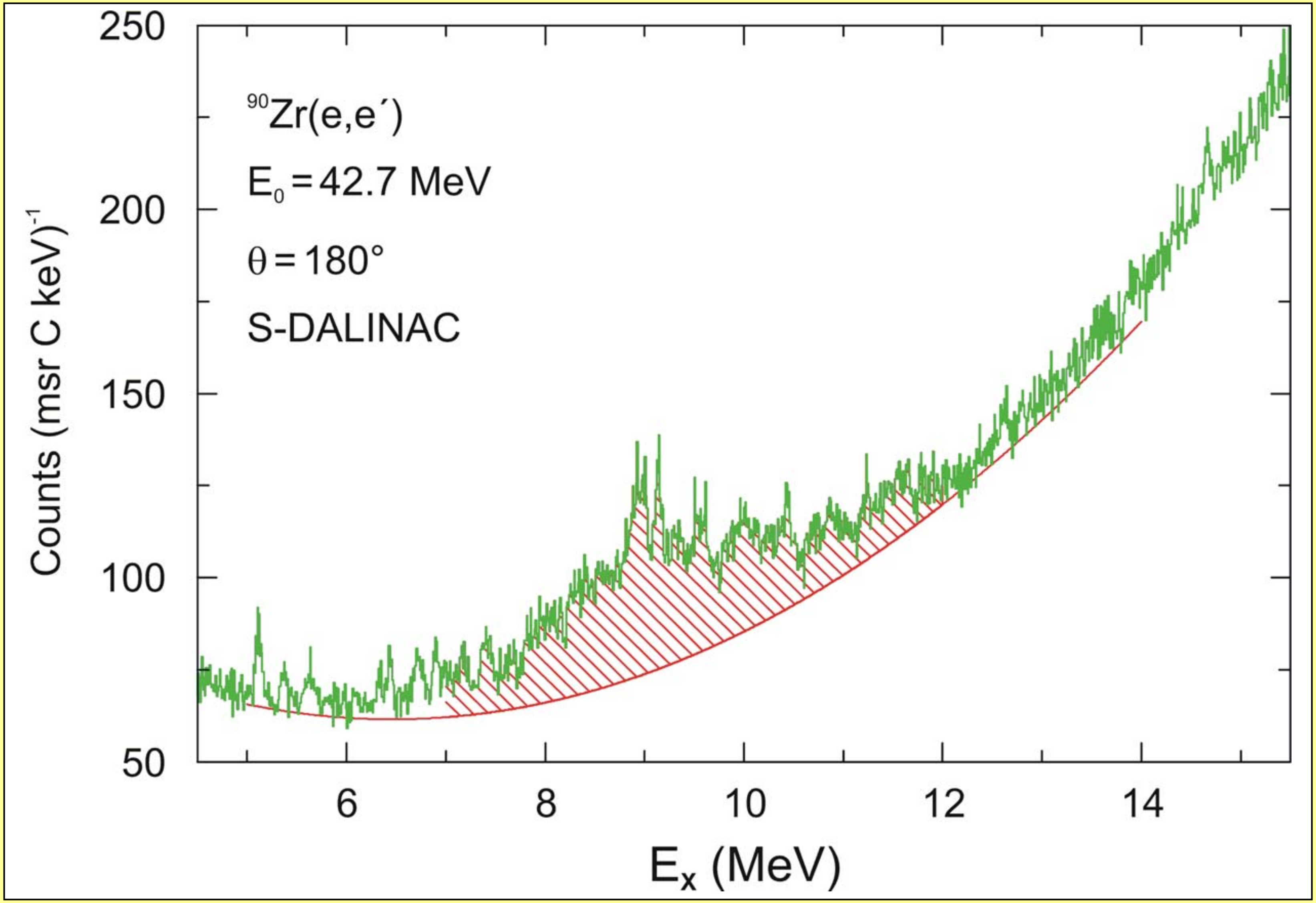
  i.e. rotation around the body-fixed z-axis with a rotation angle proportional to z (clockwise for $z > 0$ and counterclockwise for $z < 0$)

- Operator $z l_z$

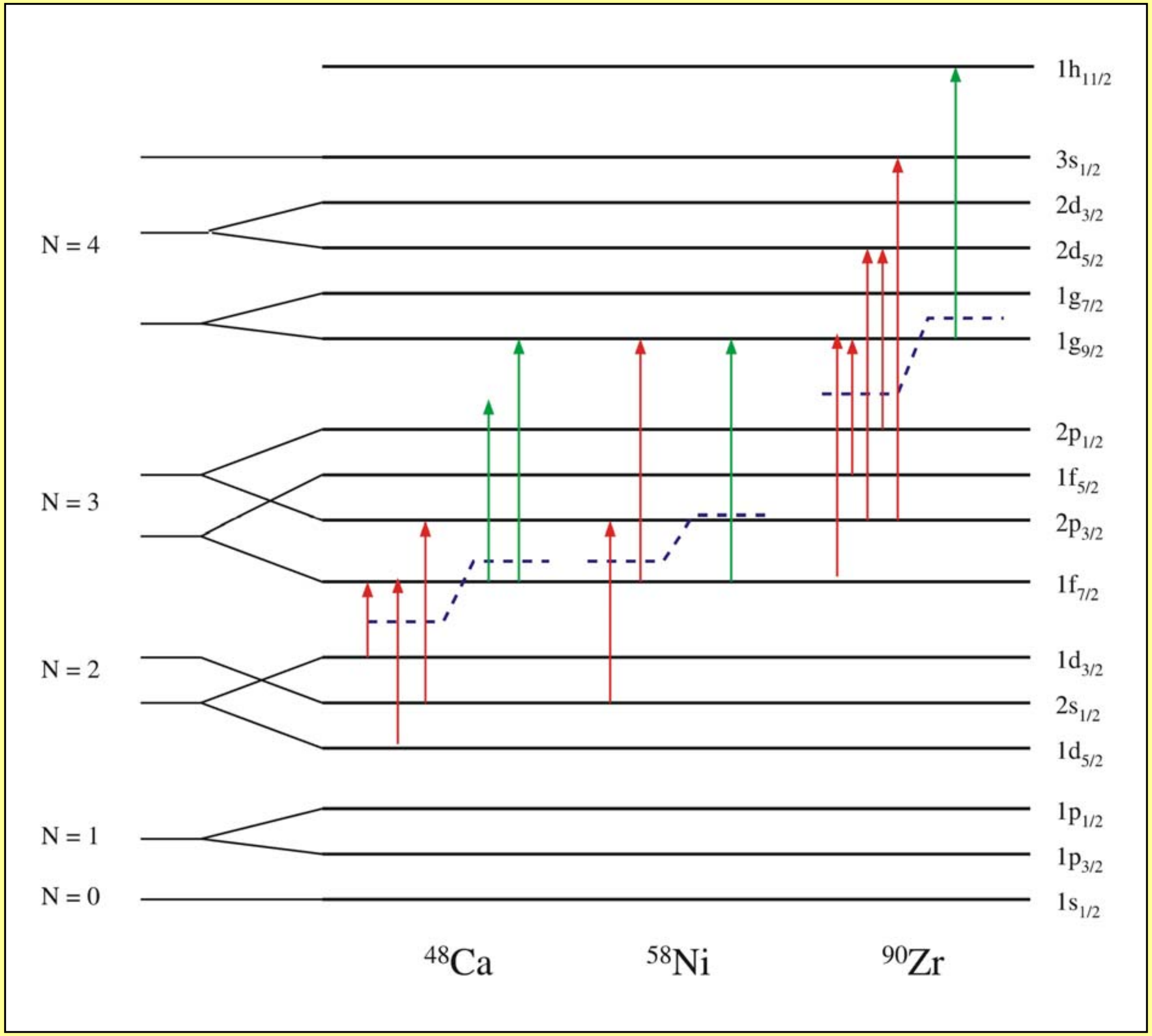
  has spin-parity $J^{\pi} = 2^{-}$ (because the scalar part of the tensor product $\vec{r} \otimes \vec{l}$, i.e. $\vec{r} \cdot \vec{l}$, vanishes identically).

- Although for axially symmetric nuclei there is evidently no change in the local density, the "twist" still creates a distortion of the local Fermi surface characterized by α.

180° Electron Scattering: Selectivity

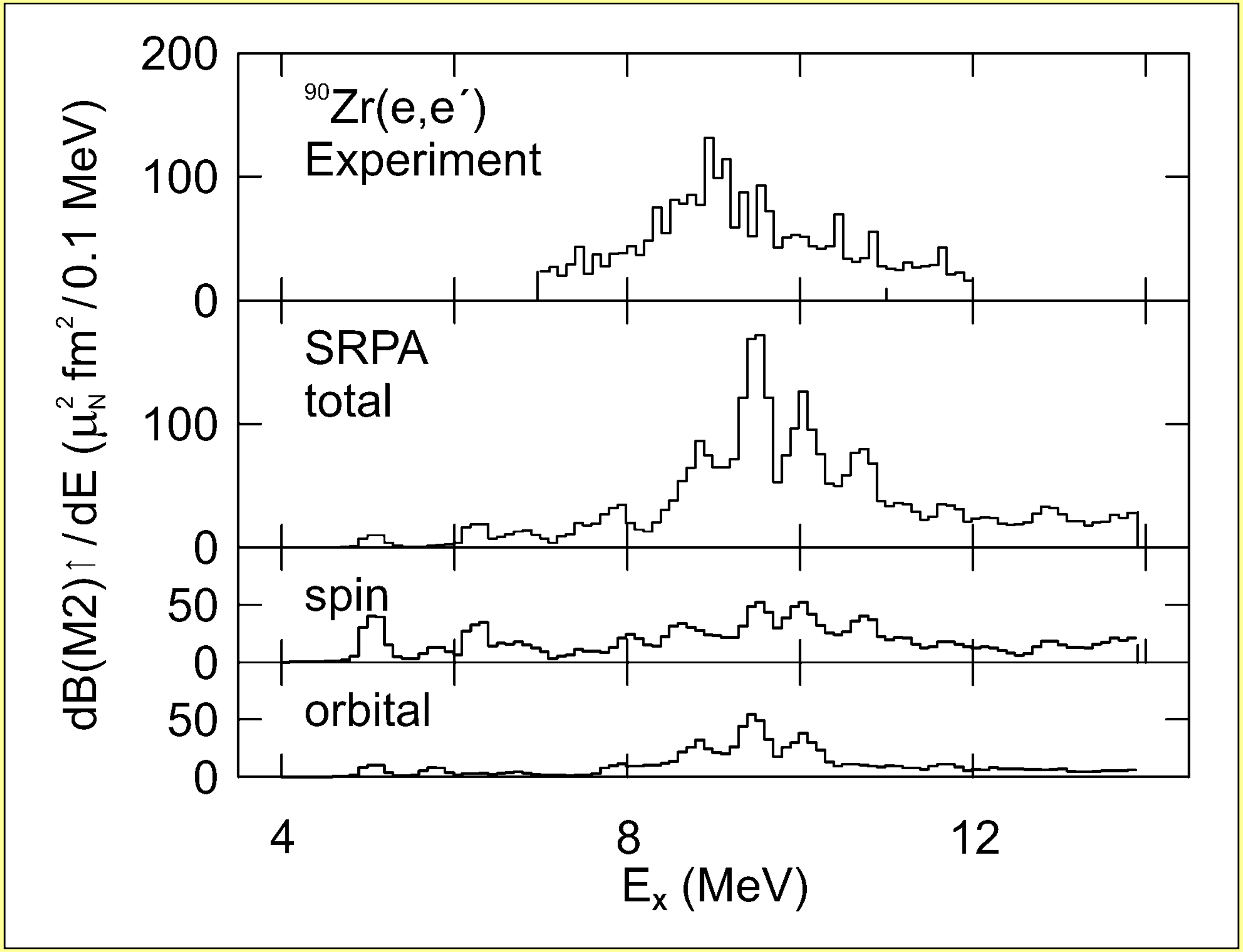


M2 Transitions in Medium Heavy Nuclei



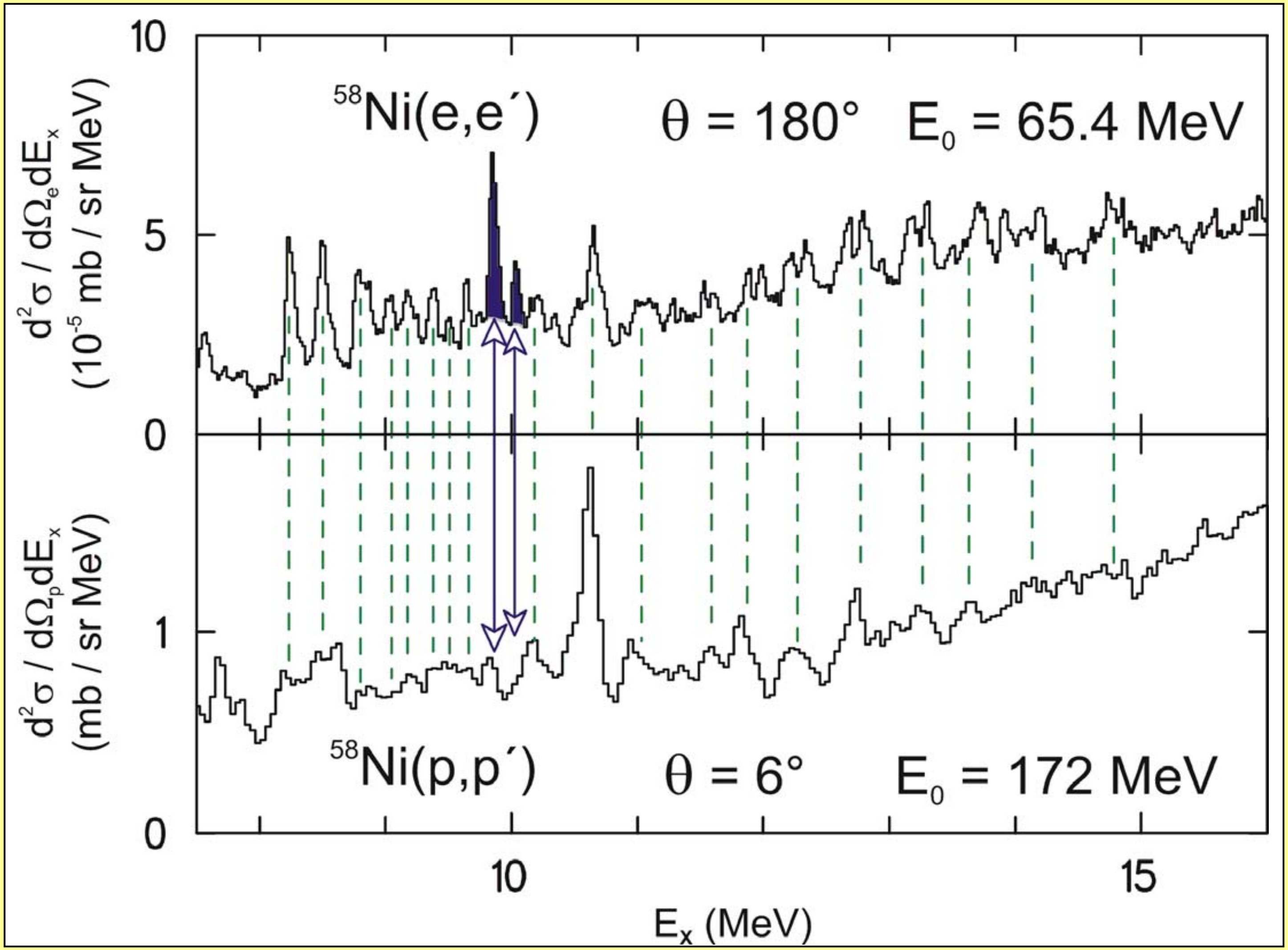
$$M(M2) = \frac{e\hbar}{2Mc} \sum_i \left(g_s(i)\, s_i + \frac{2}{3}\, g_l(i)\, l_i \right) \nabla_i\, r_i^2\, Y_2$$

B(M2) Strength in ^{90}Zr: Spin and Orbital Parts



Direct Evidence for Orbital M2 Excitations

Reitz et al. (2002)

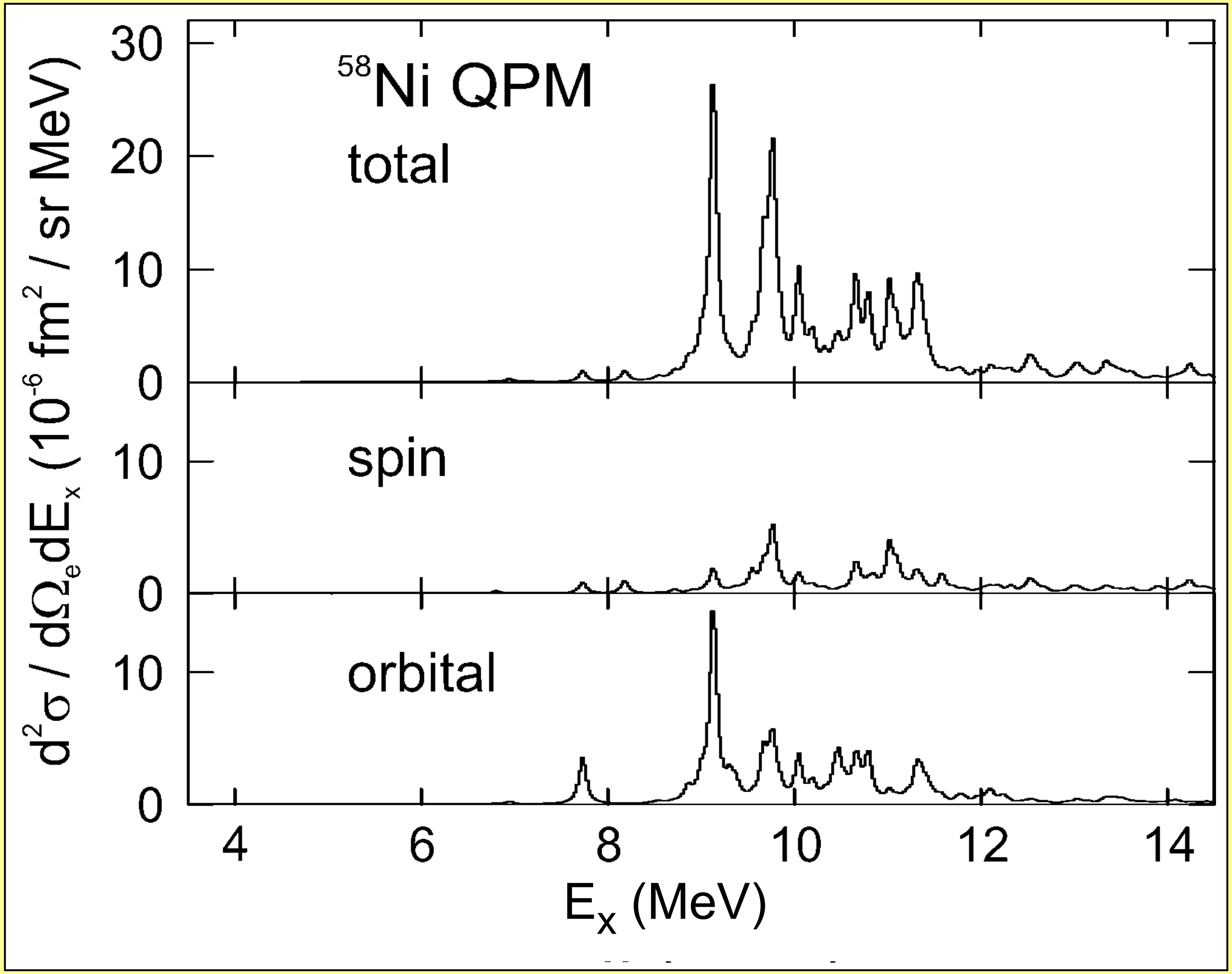


orbital

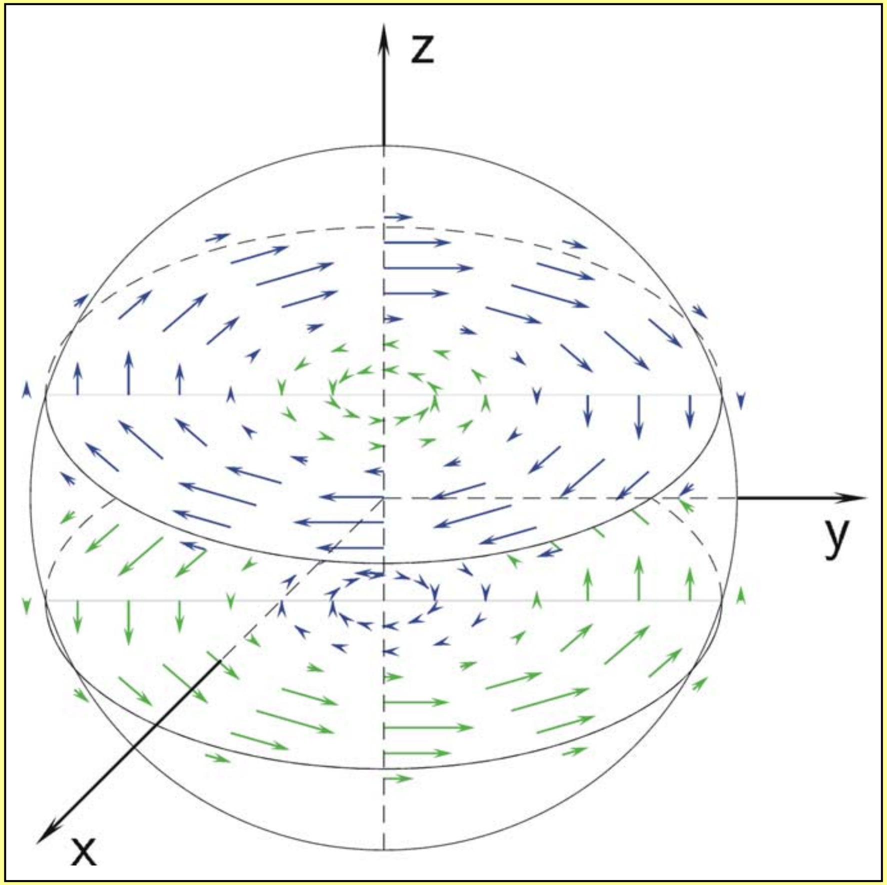
+ spin

spin

Decomposition: Spin vs. Orbital M2 Excitations



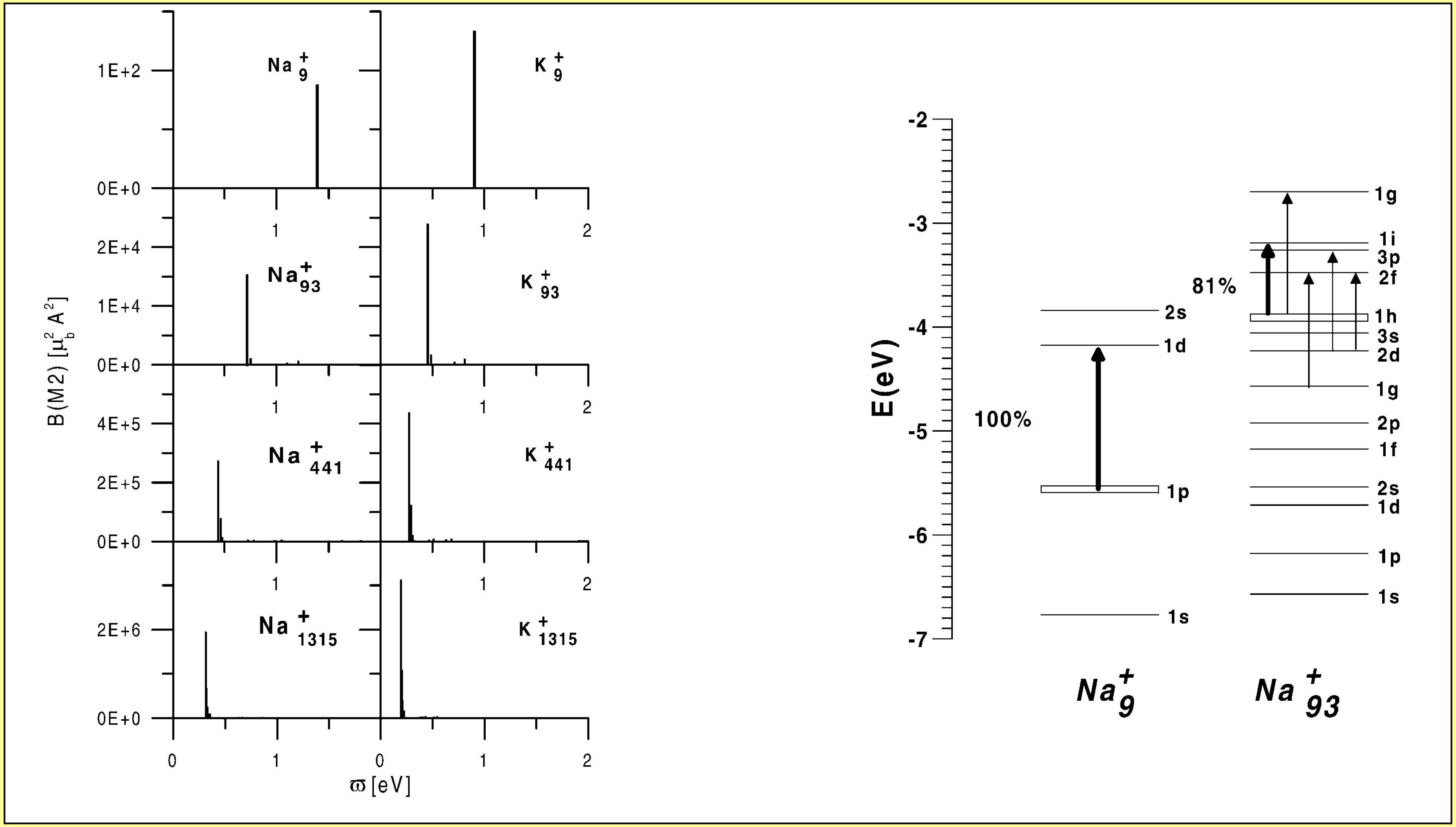
Orbital Transition Currents of the Twist Mode



- Clockwise respective counterclockwise flow in the two hemispheres
- Note the reversal of direction of flow in the interior → node of the transition current.
- Semiclassical picture of the Twist Mode is confirmed.

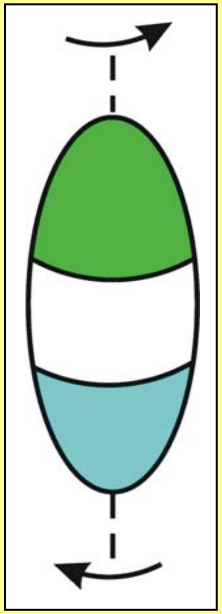
Twist Mode in Metallic Clusters

Nestorenko et al. (2000)



Twist Mode in Ultracold Atomic Fermi Gases

Viñas et al. (2001)



Trapped Fermi gas within

$$T \ll \frac{k_F^2}{2m} = \varepsilon_F$$

(e.g. ^{6}Li, ^{40}K at $\cong$ 600 nK)

- Unique mode for degenerate Fermi gases
- Landau theory predicts quadrupole deformation of the Fermi sphere:
  - anisotropic pressure tensor
  - transverse zero sound
  - "R-Modes" in neutron stars
- But excitation of the Twist Mode in traps difficult