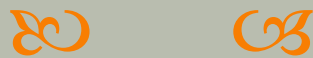


Hadron Structure: Lattice QCD and Effective Field Theory



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Plan of Lectures

- ∞ **Lecture 1: QCD and Hadron Physics Phenomenology**
- ∞ **Lecture 2: Chiral Perturbation Theory**
- ∞ **Lecture 3: Partonic Structure of Hadrons**
- ∞ **Lecture 4: Lattice QCD**

Lecture 1: QCD and Hadron Physics Phenomenology



QCD

∞ The fundamental theory of strong interactions, with quarks and gluons as basic degrees of freedom (quarks and gluons have no known sub-structure)

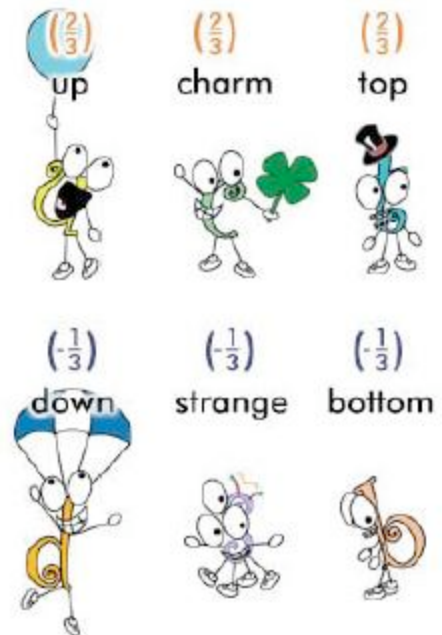
Quarks: spin-1/2, up, down, strange, ...

Dirac field $\psi(x)$

Gluons: spin-1, massless, 8 colors,

Lorentz vector fields A^μ

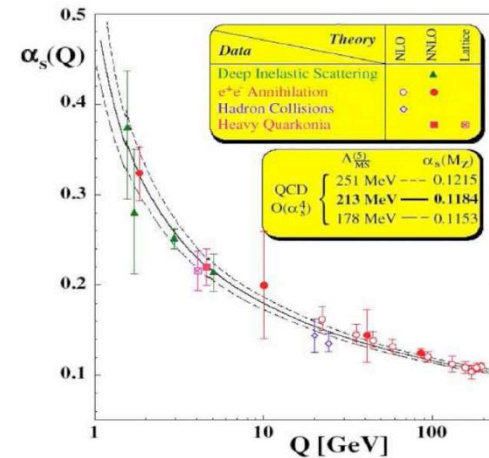
The interaction among these particles are determined by coupling constant, α_s ,



Asymptotic Freedom

- One of the striking features of QCD is **asymptotic freedom**: the interaction strength becomes logarithmically small at short distance.

$$\alpha_s(\mu) = \frac{2\pi}{\beta_0 \ln \mu / \Lambda_{\text{QCD}}}$$

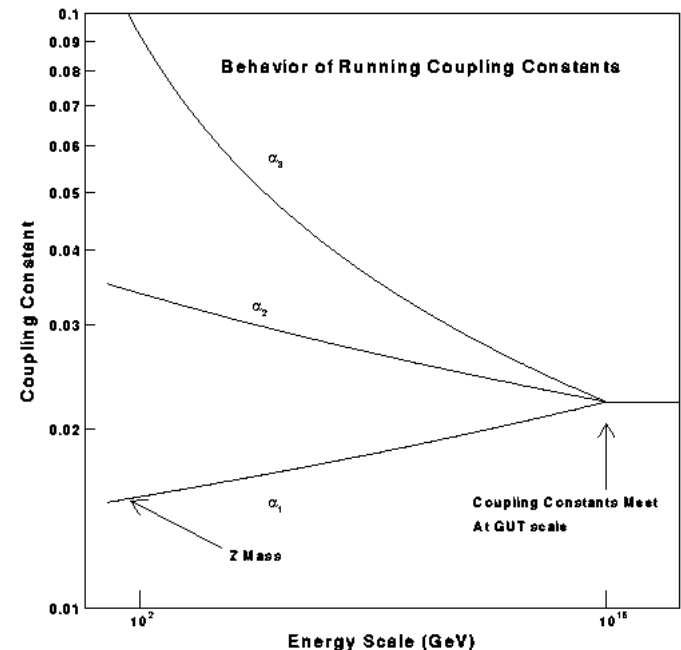


- The interaction strength becoming small does not mean the interaction between particles vanishes at short distances. To the contrary, the forces between particles do become increasingly large as they become small!

$$F = \frac{e_{\text{eff}}^2}{4\pi r^2} = \frac{\alpha_{\text{em}}(r)}{r^2}$$

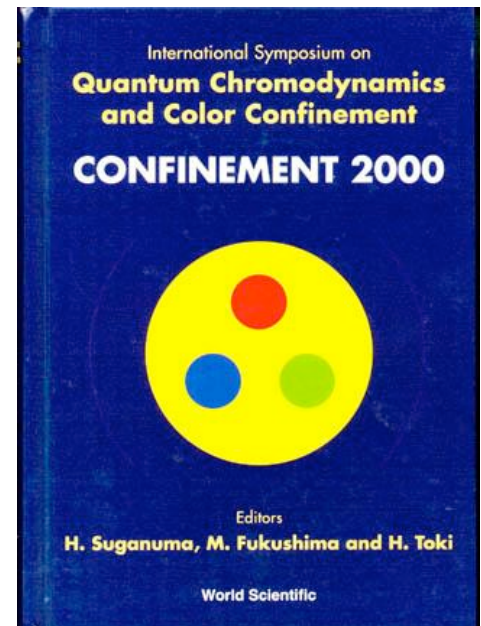
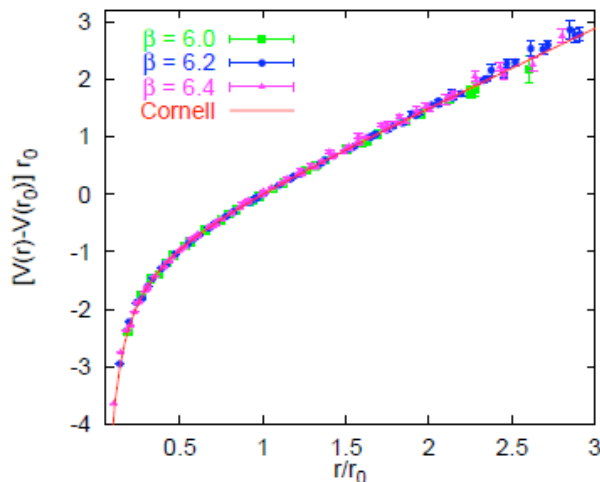
QCD scale Λ_{QCD}

- ✧ The unknown QCD coupling is determined by the QCD scale parameter Λ_{QCD} .
- ✧ Λ_{QCD} determines when the QCD interactions become indeed strong. It cannot be fixed by the theory itself, but determined by experimental data.
- ✧ Λ_{QCD} in theory is perhaps fixed by grand unification theory: all coupling must become the same at some small distance scale.



Color Confinement

- At zero-temperature, the asymptotic states are color-less hadrons instead of colored quarks and gluons
- This statement is a summary of experimental situation and has no rigorous theoretical proof yet, worth a \$1M (Witten: QCD has a mass gap?)
- Theoretical evidence:



Quark Masses

- ✎ Since the free quarks do not exist, one cannot measure the masses of the quarks using standard approaches.
- ✎ Masses appear as free parameters in the QCD lagrangian, to which the physical observables depend on. One must fix the masses through computing physical observables.
- ✎ Masses are defined in some schemes, $\overline{\text{MS}}$ scheme in Dim. Reg. is quite standard. Moreover the mass depends on the scale as well.
- ✎ Light quarks have small masses ($\overline{\text{MS}}$, 2 GeV) compared to Λ_{QCD}
 - Up quark 1.5-3 MeV:
 - Down quark 3-7 MeV
 - Strange quark 70-120 MeV

Hadron Family: Colorless Quark-Gluon States

- ∞ **Protons and neutrons:** main constituents of atomic nuclei
- ∞ **Mesons:** pion and eta
- ∞ **Adding strange quark:**
 - Hyperons (Σ , Λ Ξ)
 - Strange mesons: K , η'
 - more hadrons with charm and bottom quarks.
- ∞ All the hadrons above are stable if weak interactions are turned off and flavors become good quantum numbers.
- ∞ **Resonances of the above hadrons:** excited hadron states that live a relatively short time.

Proton and Its Mass

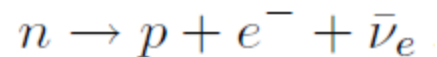
- ∞ Stable, the dominant form of baryonic matter in the universe (4%).
- ∞ Has the quantum numbers of $u+u+d$, at least 3-quark to make a color singlet!
Due to strong interactions, the Fock components of the proton wave function can be much more complicated.
- ∞ **Mass $M=938.3 \text{ MeV}/c^2$, consistent with the QCD scale.**
- ∞ **A smaller part of the mass comes from the quark mass. Most of the proton mass comes from non-perturbative QCD dynamics.**

Proton Spin

- ∞ **Spin -1/2**
- ∞ **With a well-measured magnetic moment $2.793 \mu_N$, the first evidence that proton is not an elementary particle.**
- ∞ **Proton spin:**
 - Nuclear Spin
 - Nuclear Magnetic Resonance
 - 21 cm radiation in the universe
 - Much interest in recent years in understanding the fundamental spin structure of the proton.

Neutron

- ∞ Has the quantum numbers of u+d+d quarks,
- ∞ Has a mass slightly heavier than that of the proton: **1.3 MeV**, (down quark is a bit heavier, but there is a bit more electromagnetic energy in the proton).
- ∞ It is not stable, has a lifetime about **15 min**, it decays through weak interaction



- ∞ Has a negative magnetic momentum: **-1.913 μ_N**
- ∞ The charge distribution in the neutron can be measured in elastic electron scattering, showing overall neutral-ness does not mean that it is neutral everywhere.

Isospin Symmetry

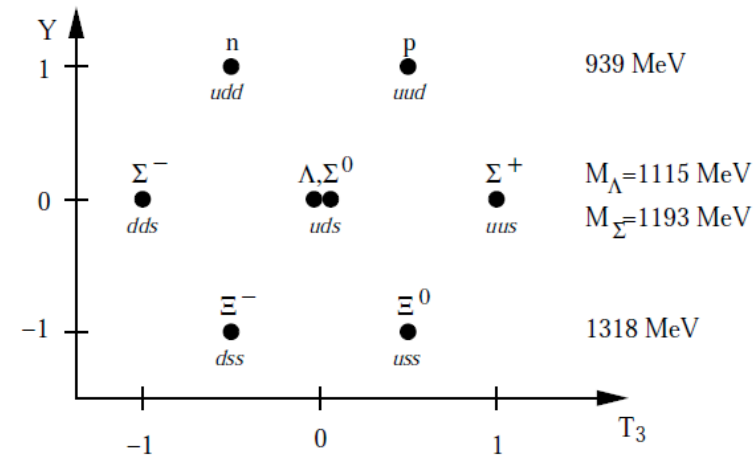
- ∞ The original observation of isospin symmetry by W. Heisenberg comes from the near degeneracy of proton and neutron masses.
- ∞ In nuclear physics, isospin symmetry of the nuclear force and nuclear structure has been well-established.
- ∞ In QCD, this symmetry is due to the flavor blindness of the QCD interactions: all quark flavors have the exact same interactions except for the differences of their masses. For up and down quarks, their mass effects are small compared to the QCD interactions. The isospin symmetry is a part of the larger symmetry when the quark masses are neglected: **Chiral Symmetry.**

Pions

- ✧ **First predicted by Yukawa as the mediator of the strong forces between nucleons.**
- ✧ **It was first discovered through the cosmic ray, comes with three different charge states (+, -, 0), with mass around 140 MeV, much lighter than the masses of the nucleons.**
- ✧ **Pions have finite lifetime due to electromagnetic and weak interactions.**
 - Neutral pion decays into 2γ
 - Charged pion decays into lepton + ν
- ✧ **Pions were recognized in 1960's as the Goldstone bosons resulting from spontaneous breaking of Chiral Symmetry.**

SU(3) Flavor Symmetry

- Hadrons exhibit SU(3) flavor symmetry in the sense that the hadrons with different quark flavors tend to be degenerate in mass.
- Eight-fold way leading to the suggestion of quarks
- Flavor symmetry is a result that QCD scale is much larger than the scale of quark masses.
- SU(3) flavor symmetry help to understand hadron phenomenology.



QCD and hadronic physics

- ∞ **Hadronic physics has been around for more than half of the century. Much of the studies were done in the context of phenomenology and models. The modern goal is to understand hadronic physics from the fundamental theory. However, this is non-trivial.**
- ∞ **QCD is quite difficult to solve!**
 - Models (QCD-inspired)
 - Chiral effective theory
 - Lattice QCD calculations

Simple Quark Models

- ⌘ **Hadronic structure is relativistic! However, QFT is hard. Let's consider non-relativistic “approximation”**
- ⌘ **Non-relativistic quantum mechanics is a lot of easier to understand. For one thing, one deals with finite number of degrees of freedoms: particles dot not get created or annihilated.**
- ⌘ **This approximation works much better for heavy quarks than lighter ones, such as charm, bottom and top quarks. One can create an effective theory of QCD called NonRelativistic QCD, or simply NRQCD.**

NR Quark Model

- ∞ In NR quark model, the proton and neutron are made of three point-like non-relativistic quarks, moving in a harmonic oscillator type of potential,

$$H_0 = \frac{1}{2m}(\vec{p}_1^2 + \vec{p}_2^2) + \frac{1}{2m'}\vec{p}_3^2 + \frac{1}{2}k \sum_{i<j} (\vec{r}_i - \vec{r}_j)^2$$

- ∞ The spin-flavor wave function obeys the so-called SU(6) wave function,

$$\begin{aligned} |p \uparrow\rangle &= \sqrt{\frac{1}{18}} [|u \uparrow u \downarrow d \uparrow\rangle + |u \downarrow u \uparrow d \uparrow\rangle - 2|u \uparrow u \uparrow d \downarrow\rangle \\ &\quad + |u \uparrow d \uparrow u \downarrow\rangle + |u \downarrow d \uparrow u \uparrow\rangle - 2|u \uparrow d \downarrow u \uparrow\rangle \\ &\quad + |d \uparrow u \uparrow u \downarrow\rangle + |d \uparrow u \downarrow u \uparrow\rangle - 2|d \downarrow u \uparrow u \uparrow\rangle] \times |\psi_{\text{color}}\rangle \\ &= \frac{1}{\sqrt{3}} [|u \uparrow u \downarrow d \uparrow\rangle_S - 2|u \uparrow u \uparrow d \downarrow\rangle_S] \times |\psi_{\text{color}}\rangle , \end{aligned}$$

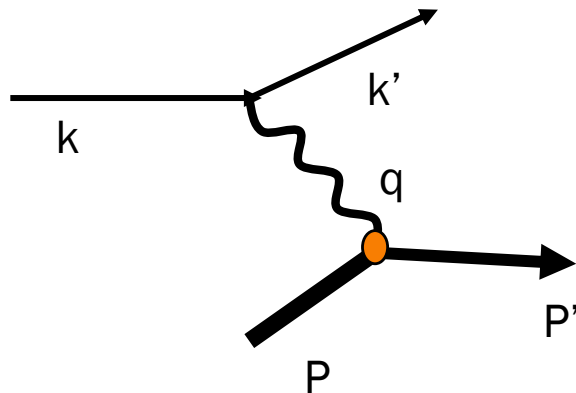
Successes of quark models

∞ Magnetic moment of the proton

- Since the NR quark mass is about $1/3$ of the mass of the proton, the magnetic moment of a quark is about 3 times the nucleon magneton.
- Ratio of the proton to neutron magnetic moment is close $3/2$, consistent with the prediction.
- **Δ is an excited state of the proton with spin $J=3/2$. When it makes a transition to the proton, it emits electric quadrupole (E2) & magnetic dipole (M1) radiations.**
 - Experimentally, the E2/M1 transition ratio is found to be 2~3%.
 - In NR quark model, E2 transition is close to zero.

The Proton Elastic Form Factors

- ∞ First measured by Hofstadter *et. al.* in the mid 1950's
Elastic electron scattering



$$\langle p' | j^\mu | p \rangle = U(p') \left[F_1(q^2) \gamma^\mu + F_2(q^2) i \sigma^{\mu\nu} q_\nu / 2M \right] U(p)$$

What does $F_{1,2}$ tell us about the structure of the nucleon?

Sachs Interpretation of Form Factors

- ∞ According to Sachs, the FT of $\mathbf{G}_E = \mathbf{F}_1 - \tau \mathbf{F}_2$ and $\mathbf{G}_M = \mathbf{F}_1 + \mathbf{F}_2$ are related to charge and magnetization distributions.
- ∞ This is obtained by first constructing a wave packet of the proton (*a spatially-fixed proton*)

$$|R\rangle = \int \frac{d^3 p}{(2\pi)^3} e^{iRp} \phi(p) |p\rangle$$

then measure the charge density relative to the center

$$\rho(r) = \langle R=0 | j_0(r) | R=0 \rangle$$

Sachs' Interpretation (Continued)

- ∞ Calculate the FT of the charge density, which now depends on the wave-packet profile

$$F(q) \propto \int dP \phi^* \left(P - \frac{q}{2} \right) \phi \left(P + \frac{q}{2} \right) \langle P + \frac{q}{2} | j_0 | P - \frac{q}{2} \rangle$$

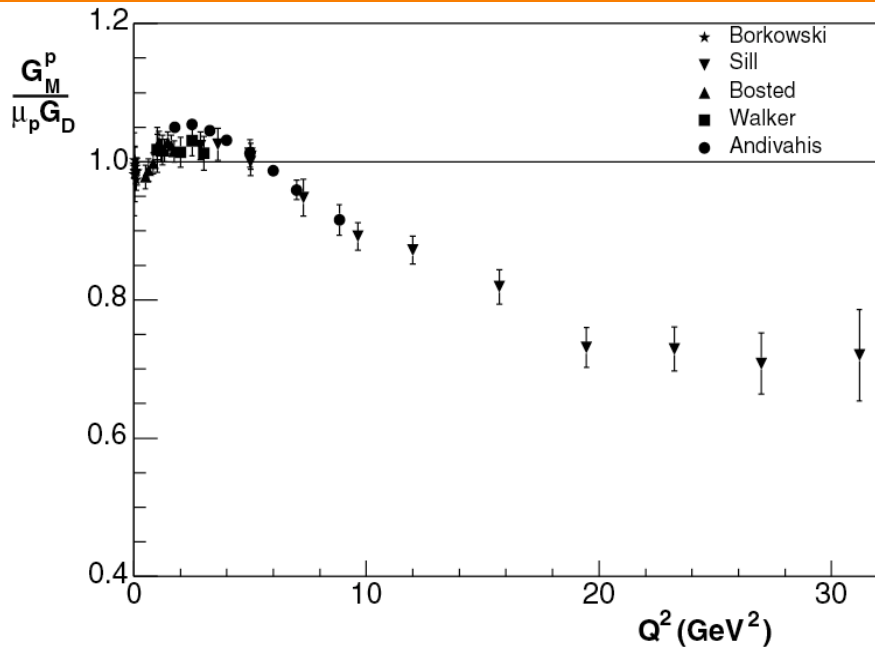
- ∞ Additional assumptions

- The wave packet has no dependence on the relative momentum q
- $|\phi(P)|^2 \sim \delta(P)$

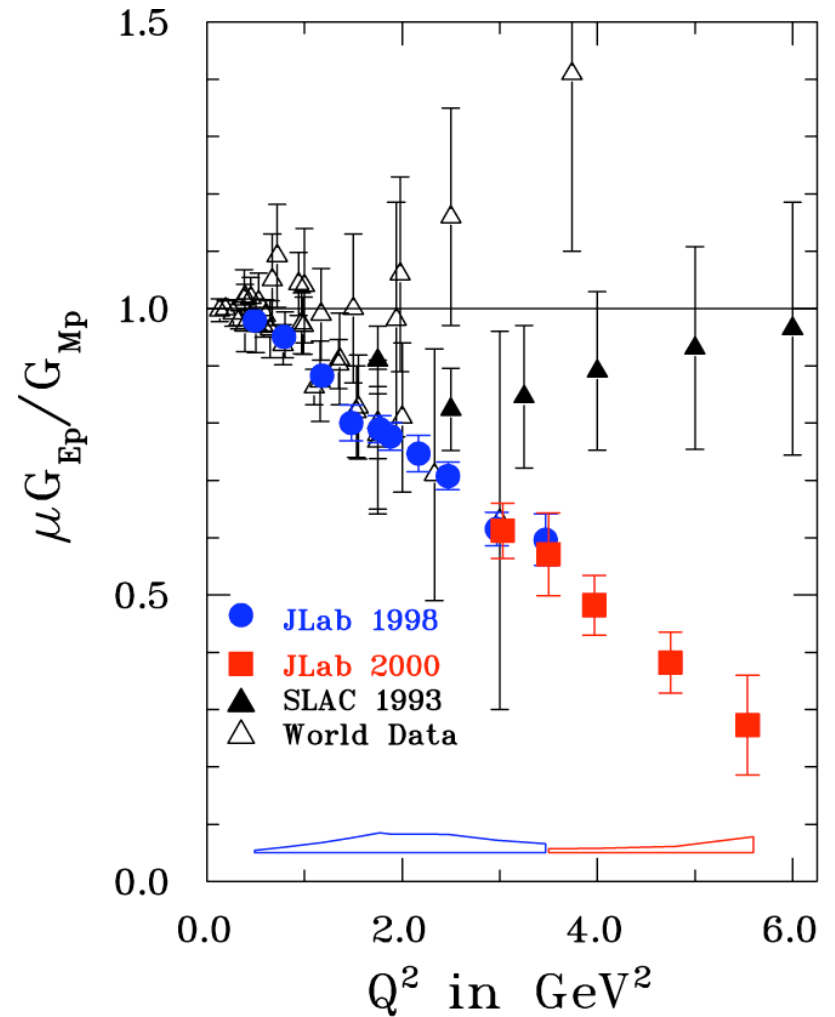
$$F(q) \propto \langle q/2 | j_0 | -q/2 \rangle$$

Matrix element
In the Breit frame

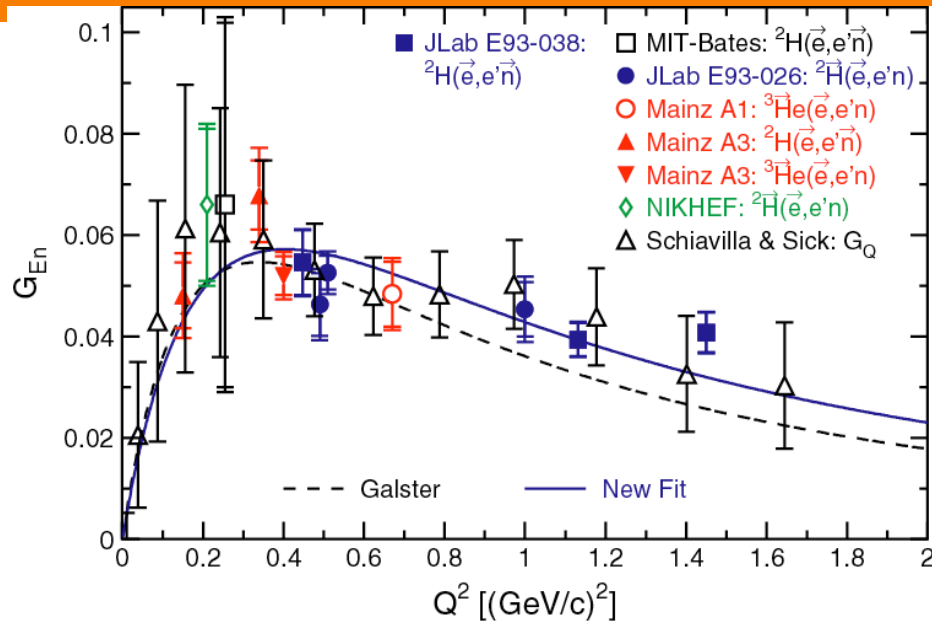
Proton's Elastic Form Factor



$$G_D = 1/(1+Q^2/m^2)^2$$



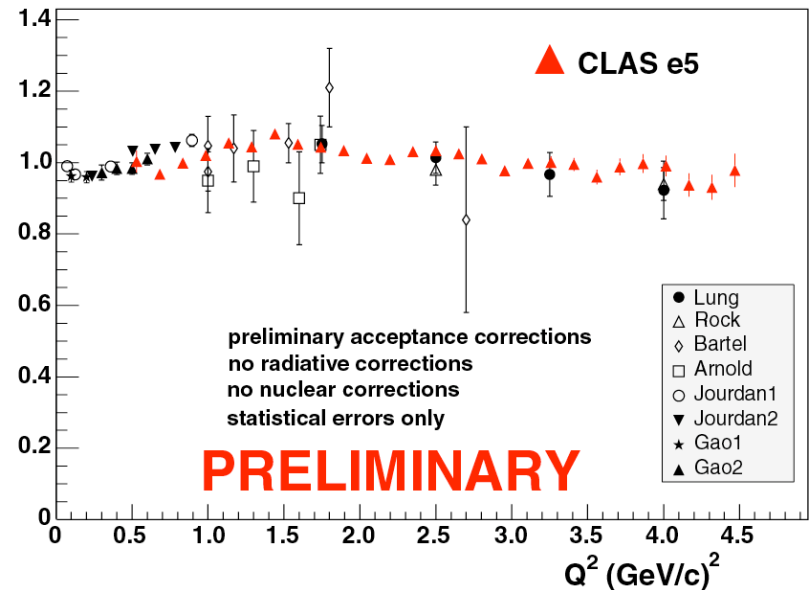
Neutron Form Factors



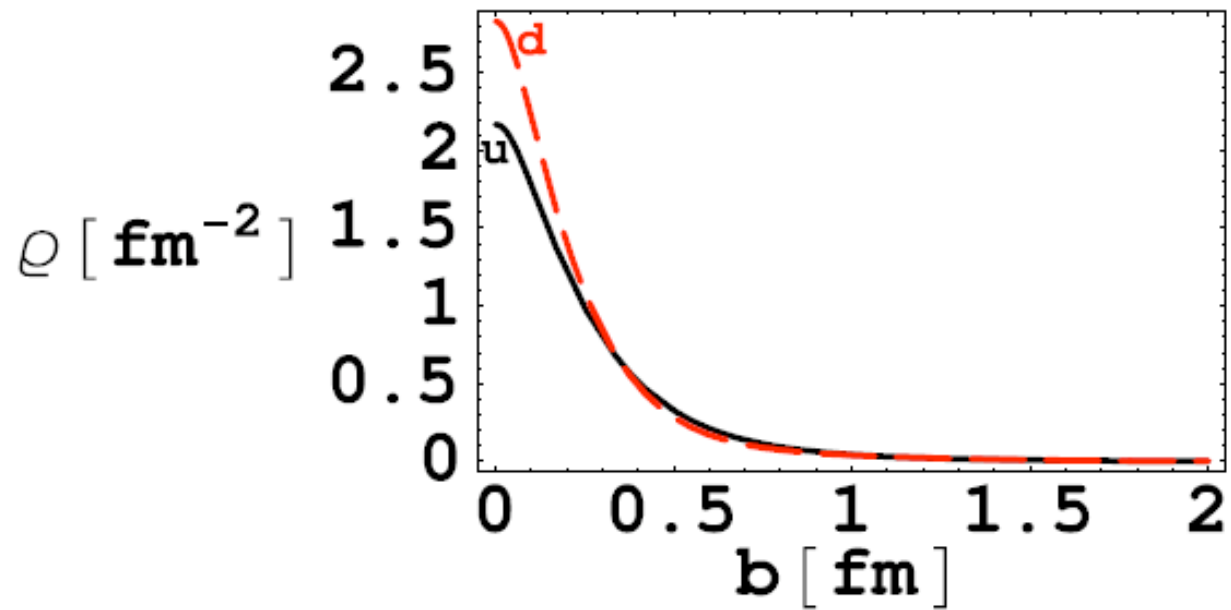
Very close to a dipole form

Negative charge radius!
Like H-atom

$$\frac{G_M^n}{\mu_n G_D}$$



Charge distributions



Resonances

- ☞ Recall hydrogen atoms, there are many excited states.
- ☞ However, when considering the effects of the spontaneous emission, the excited states have finite life-time.
- ☞ Finite life-time leads to uncertainty in their energy (width)
- ☞ The width is usually much smaller than the energy spacing. However, this is not the case in the strong interaction theory.
- ☞ Interesting resonances
 - Spin-1/2 negative parity states
 - Spin-3/2 positive parity states

Resonances in Quark Model

