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Effective Field Theories Beyond the Standard Model

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Plan of the lecture

- Introduction: Search for physics BSM
  - ★ direct vs indirect probes (energy vs precision frontiers) and the role of EFT
- BSM EFT prototype: Fermi theory of β decay
- General BSM EFT (dim 5 & 6 operators)

- Applications:
  - ★ β decays: weak universality, non V-A, etc
  - ★ Lepton Flavor Violation (μ→eγ and μ→e conversion)

↓ discussion sessions

Energy vs precision frontiers
and
the role of EFT

Energy and Precision frontiers

- While the SM successfully describes phenomena from atomic to collider energy scales, a number of open questions (both empirical and theoretical) points to the existence of new degrees of freedom & interactions active at scales $d < 10^{-16}$ cm ($E > 100$ GeV)

- Two *complementary* strategies to probe BSM physics:

Energy Frontier
(direct access to new d.o.f)

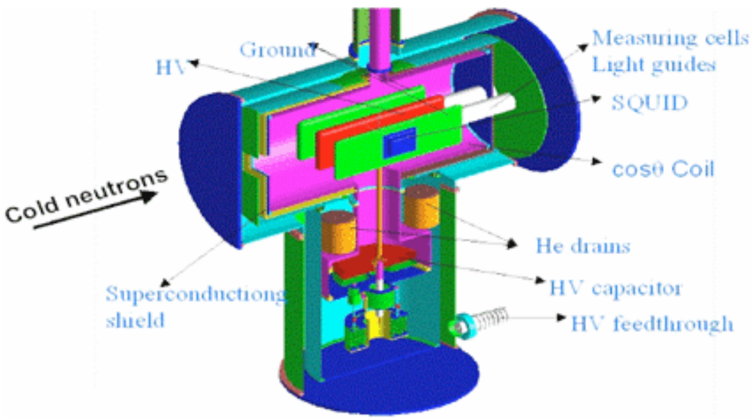
- EWSB mechanism
- Discover new particles
- …

Precision Frontier
(indirect access to new d.o.f through virtual effects)

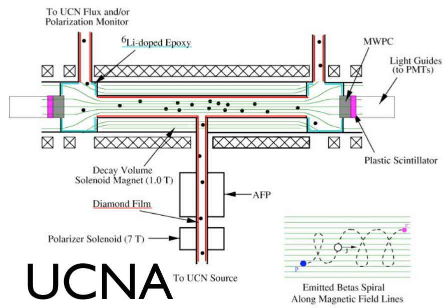
- CP violation (w/o flavor)
- Flavor symmetries (quarks, leptons)
- L and B violation
- Gauge universality
- ....

Both needed to fully describe the "New Standard Model" at the TeV scale and address the outstanding open questions!

- Nuclear Physics plays a central role at the Precision Frontier



nEDM



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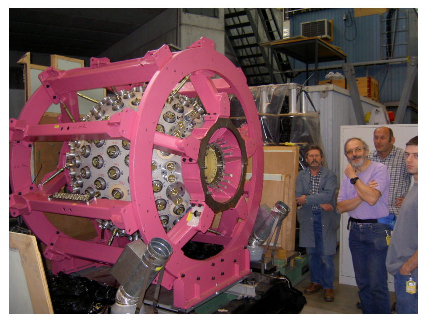
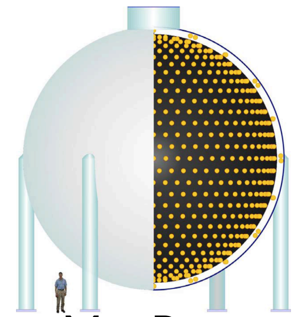
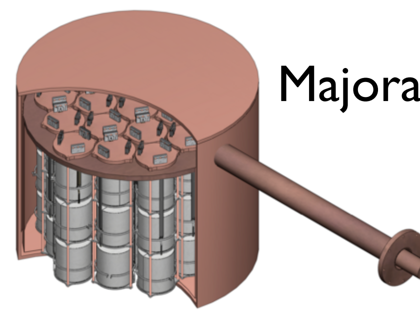
PEN





MiniBoone



Majorana

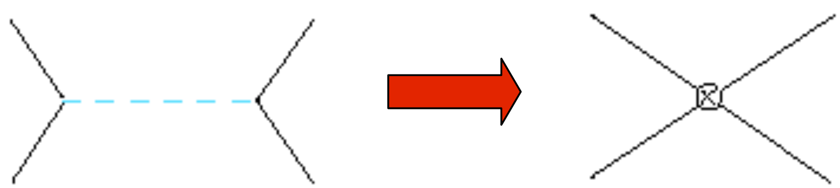
- The whole field of "indirect probes" is based on EFT ideas
- EFT provides a *model-independent* (= that applies to classes of models) framework to analyze and interpret experimental results

How does the precision frontier work?

- Key observation: at low E, the presence of heavy particles induces either a renormalization of the coupling constants or new local operators suppressed by powers of the heavy scale

Appelquist-Carazzone '75

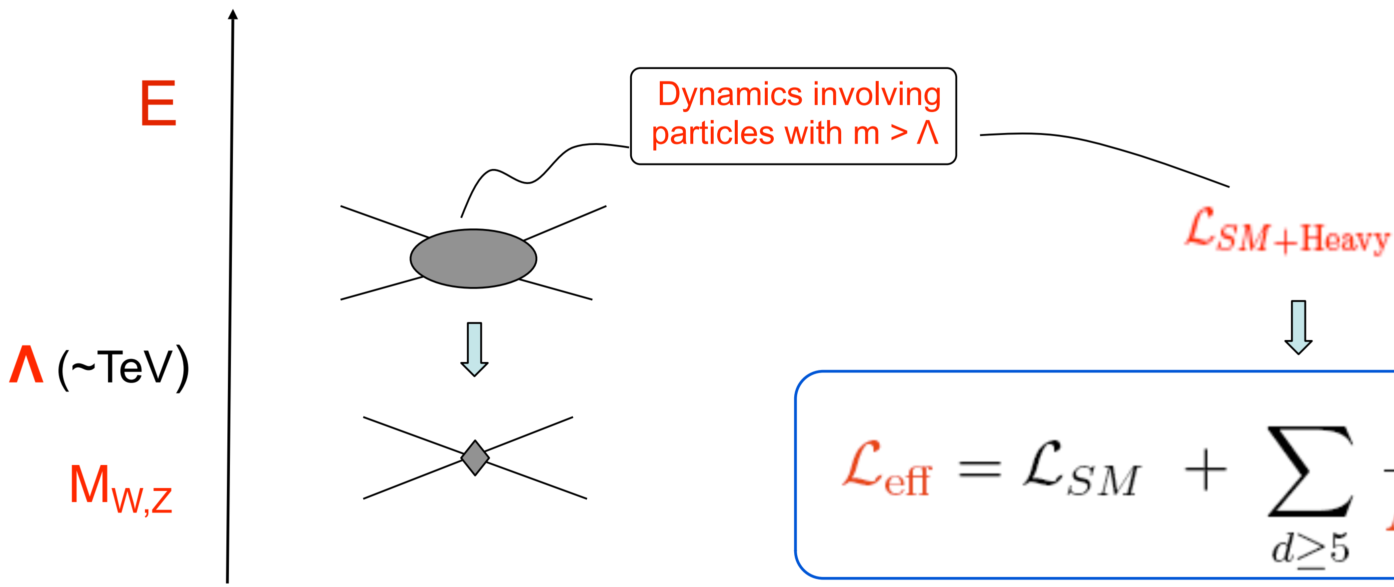
Example: heavy particle exchange generates new local interaction



$$\frac{g^2}{M_W^2 - q^2}$$

$$\frac{g^2}{M_W^2} \sim G_{\mathrm{Fermi}}$$

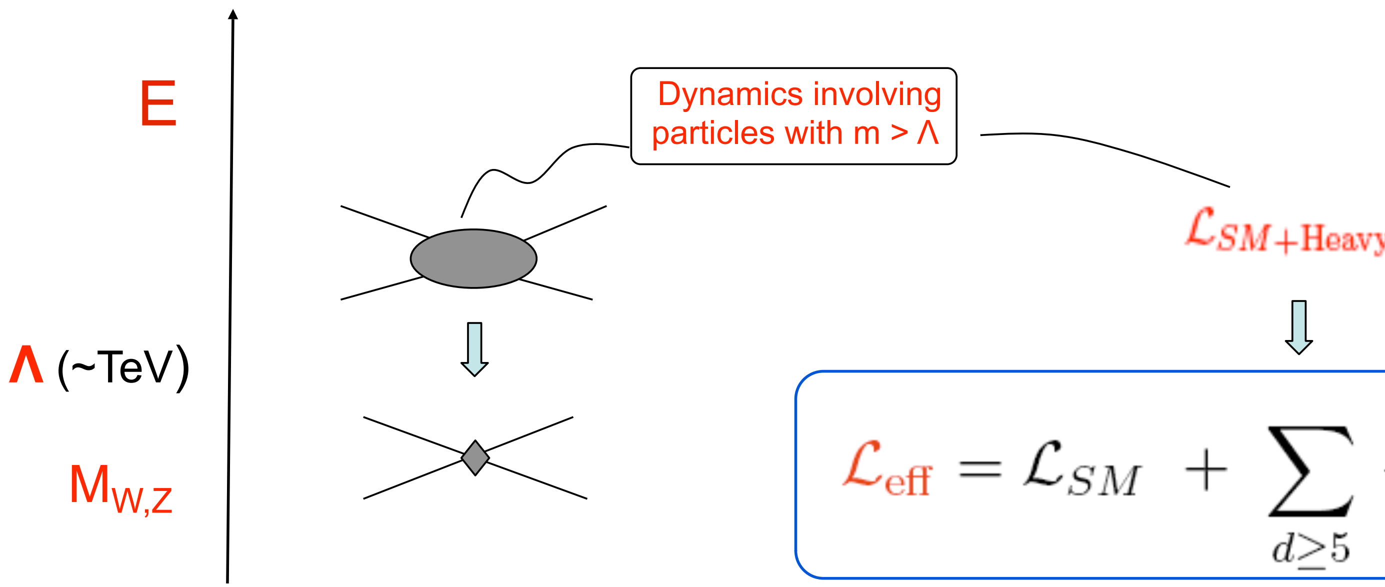
- Dynamics below scale Λ [~ mass of new particles] described by L_{eff}



$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{SM} + \sum_{d \geq 5} \frac{c_n^{(d)}}{\Lambda^{d-4}} \hat{O}_n^{(d)}[\phi_{SM}]$$

- L_{eff} is built out of relevant low-E degrees of freedom (SM fields)
- L_{eff} reflects symmetries of underlying theory (but not necessarily of SM)
- L_{eff} is organized in inverse powers of heavy scale (amplitudes suppressed by powers of (E/Λ))

- Dynamics below scale Λ [~ mass of new particles] described by L_{eff}



$$\mathcal{L}_{\rm eff} = \mathcal{L}_{SM} + \sum_{d\geq 5} \frac{c_n^{(d)}}{\Lambda^{d-4}} \hat{O}_n^{(d)}[\phi_{SM}]$$

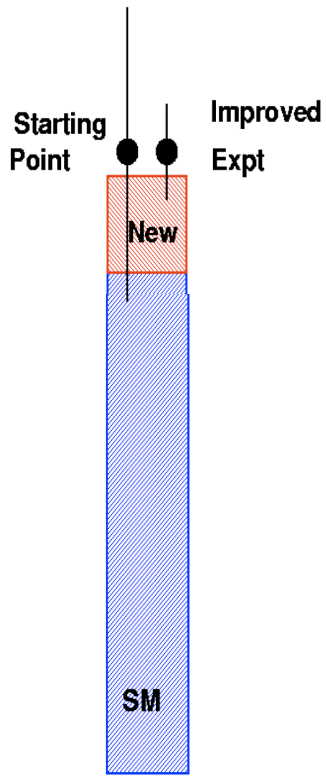
- Experiments at the precision frontier probe energy scale Λ and symmetries of the new interactions (⇔ coeff. & structure of $\hat{O}_n^{(d)}$)

- $\hat{O}_n^{(d)}$ can be roughly divided in two classes:

(i) Those that generate corrections to SM "allowed" processes: probe them with precision measurements (β-decays, muon g-2, Q_W, ...).

(ii) Those that violate (approximate) SM symmetries and hence mediate rare/forbidden processes (quark and lepton FCNC, LNV, BNV, EDMs).

(i)



(ii)

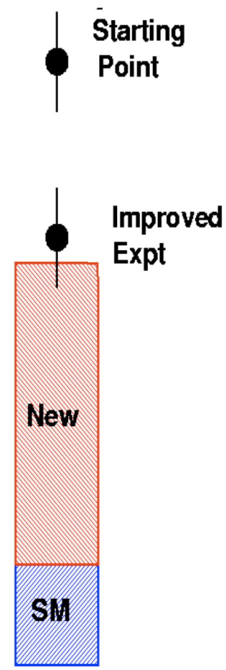
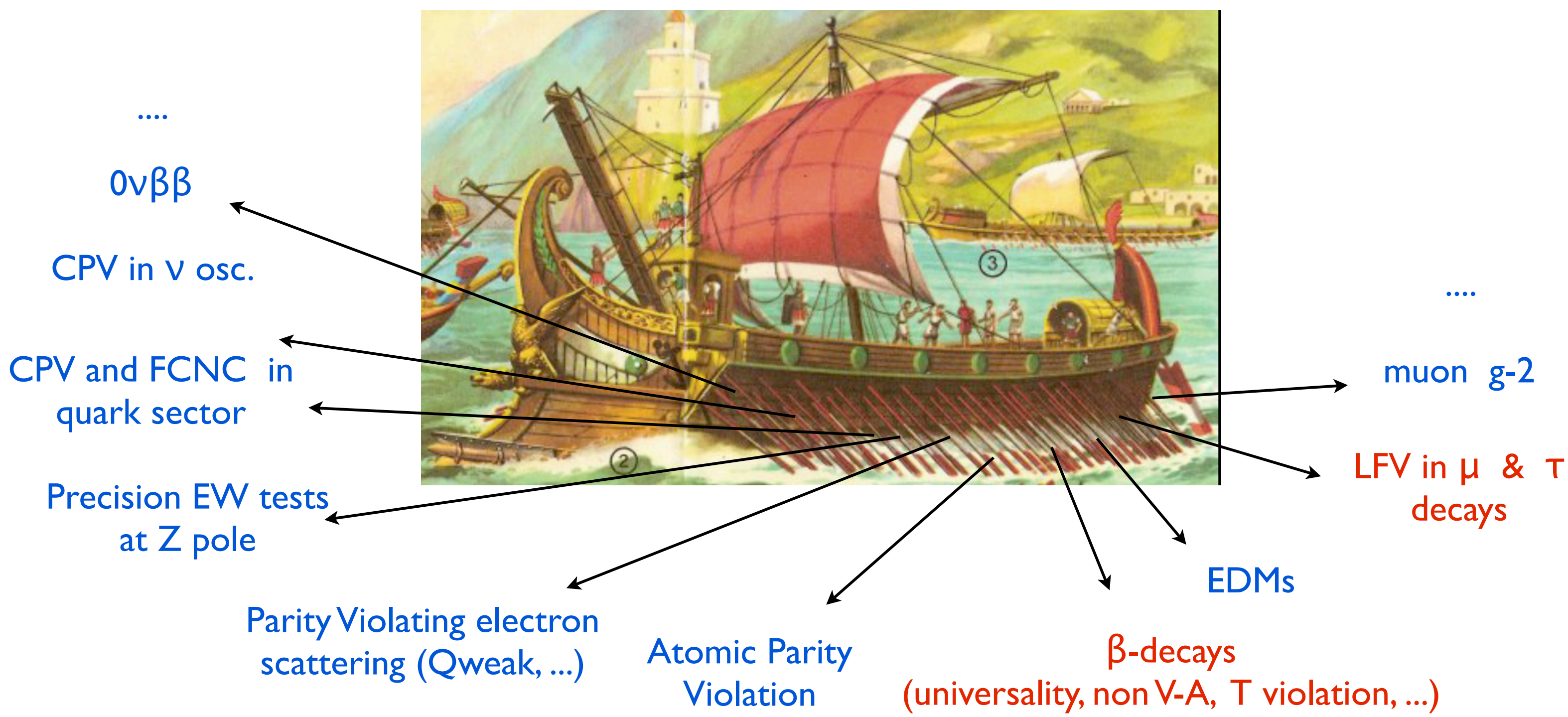


Figure copyright: David Mack

- No single low-energy probe by itself will uncover the fundamental TeV scale dynamics
- It is the combination of all these efforts (and collider searches) that will ultimately help discriminate among BSM scenarios

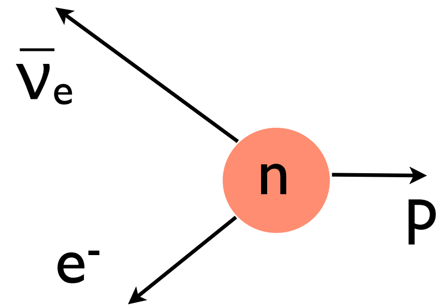


BSM EFT prototype: Fermi theory of β decay

- Write down O(GeV) scale EFT with given assumptions on symmetries: phenomenology ("bottom-up")
- *Match* electroweak theory (SM) onto GeV-scale EFT ("top-down")

EFT approach to β decay

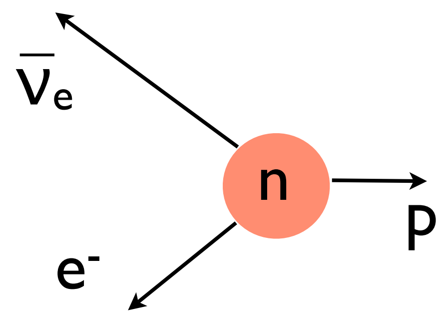
- Neutron beta decay: $n \rightarrow p\, e\, \bar{\nu}_e$



$m_{n,p} \sim 1\,\mathrm{GeV}$
$m_e \sim 0.5\,\mathrm{MeV}$
$m_\nu \sim 0$

- Simplified picture:
  - ★ "Standard Model" (E~GeV) ↔ QED + strong interactions (Yukawa): β decay is forbidden
  - ★ "New physics" mediating weak decay originates at Λ_W >> 1 GeV
  - ★ Want to describe the new physics that induces β decay through L_{eff}, using a systematic expansion in E/Λ_W

EFT approach to β decay



$m_{n,p}$ ~ 1GeV
m_e ~ 0.5 MeV
m_ν ~ 0

- Neutron beta decay: $n \to p\, e\, \bar{\nu}_e$
- Low energy theory (E~GeV): QED + strong interaction (Yukawa) + "new physics" mediating weak decay (originating at Λ_W >> 1 GeV)
- Identify ingredients for EFT description:

massless spin 1/2 with in principle both helicity states

★ Degrees of freedom (field content): n, p, e, $(\nu_e)_{L/R} = (1 \pm \gamma_5)/2\ \nu_e$

★ Symmetries: Lorentz, $U(1)_{EM}$ gauge invariance, possibly P,C,T ?

★ Power counting in E/Λ_W: non-derivative 4-fermion interactions

- Most general non-derivative effective interaction (Lee-Yang ’57) involves product of fermion bilinears

Dimensionless coefficients

$$\mathcal{L}_{\rm eff} \supset \frac{c_{12}}{\Lambda_W^2} \; \bar{p}\,\Gamma_1\, n \;\; \bar{e}\,\Gamma_2\,\nu_e$$

Scale of weak interactions

Operators of mass dimension 6
(recall $[\Psi] = m^{3/2}$)
invariant under $U(1)_{EM}$

Dirac structures:

$$\Gamma_i \;=\; I,\; \gamma_5,\; \gamma_\mu,\; \gamma_\mu\gamma_5,\; \sigma_{\mu\nu} = i/2[\gamma_\mu, \gamma_\nu]$$

S P V A T

- Problem (discussion): Make sense of dimensional factors in $L_{\rm eff}$

(1) mass dimension of lagrangian density; (2) mass dimension of fields & operators

- Most general non-derivative effective interaction (Lee-Yang '57) involves product of fermion bilinears

- Impose Lorentz invariance: $\mathcal{L}_{\rm eff} = \mathcal{L}_{V,A} + \mathcal{L}_{S,P} + \mathcal{L}_T$

$$-\mathcal{L}_{V,A} = \bar{p}\gamma_\mu n \;\; \bar{e}\gamma^\mu \, (C_V + C_V' \, \gamma_5)\nu_e \;+\; \bar{p}\gamma_\mu\gamma_5 n \;\; \bar{e}\gamma^\mu\gamma_5 \, (C_A + C_A' \, \gamma_5)\nu_e$$

$$-\mathcal{L}_{S,P} = \bar{p}n \;\; \bar{e}(C_S + C_S' \, \gamma_5)\nu_e \;+\; \bar{p}\gamma_5 n \;\; \bar{e}\gamma_5 \, (C_P + C_P' \, \gamma_5)\nu_e + \text{h.c.}$$

$$-\mathcal{L}_T = \frac{1}{2} \, \bar{p}\sigma_{\mu\nu} n \;\; \bar{e}\sigma^{\mu\nu} \, (C_T + C_T' \, \gamma_5)\nu_e \;+\; \text{h.c.}$$

- Problem (homework): what happened to the extra tensor term? Hint: use the identity

$$\sigma_{\mu\nu}\gamma_5 = \frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}$$

- Most general non-derivative effective interaction (Lee-Yang '57) involves product of fermion bilinears

- Impose Lorentz invariance: $\mathcal{L}_{\rm eff} = \mathcal{L}_{V,A} + \mathcal{L}_{S,P} + \mathcal{L}_T$

$$-\mathcal{L}_{V,A} = \bar{p}\gamma_\mu n \;\; \bar{e}\gamma^\mu \, (C_V + C_V' \, \gamma_5)\nu_e \; + \; \bar{p}\gamma_\mu\gamma_5 n \;\; \bar{e}\gamma^\mu\gamma_5 \, (C_A + C_A' \, \gamma_5)\nu_e$$

$$-\mathcal{L}_{S,P} = \bar{p}n \;\; \bar{e}(C_S + C_S' \, \gamma_5)\nu_e \; + \; \bar{p}\gamma_5 n \;\; \bar{e}\gamma_5 \, (C_P + C_P' \, \gamma_5)\nu_e + \text{h.c.}$$

$$-\mathcal{L}_T = \frac{1}{2} \, \bar{p}\sigma_{\mu\nu} n \;\; \bar{e}\sigma^{\mu\nu} \, (C_T + C_T' \, \gamma_5)\nu_e \; + \; \text{h.c.}$$

- P-invariance ⇔ $C'_{V,A,S,P,T} = 0$

- T-invariance ⇔ C_i, C_i' relatively real

Phenomenology with L_{eff}

- Experimental information on β-decays (rates, angular distributions) ⇒

$$C_V \equiv \frac{1}{\Lambda_W^2} \qquad \Lambda_W \sim 350\,\mathrm{GeV}$$

$$C_A \sim 1.25\, C_V$$

$$C_V \sim C'_V \qquad C_A \sim C'_A$$

$$C_{S,P,T}/C_V,\; C'_{S,P,T}/C_V \;\leq\; \%$$

• Weak decays probe scales of O(100 GeV) >> $m_{n,p}$!!

• Parity maximally violated; chiral nature of the weak couplings

• Information on nature of underlying force mediators ($\Lambda_{S,T} \geq 2$ TeV)

- Important input in developing what we now call Standard Model

Phenomenology with L_{eff}

- Experimental information on β-decays (rates, angular distributions) ⇒

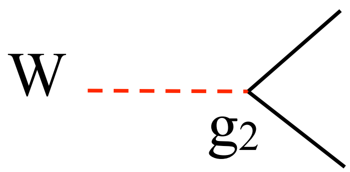
- In applications to BSM physics, one mostly uses the model-independent phenomenological approach described here
- However, if we know the underlying high-energy theory we can calculate the effective couplings in L_{eff} via a so-called matching calculation
  - ★ Constraints on the C_{eff} can be converted into constraints on the parameters of any underlying theory
- Next, work out a simple example of matching calculation

($\Lambda_{S,T} \geq 2$ TeV)

- Important input in developing what we now call Standard Model

Matching SM onto L_{eff}

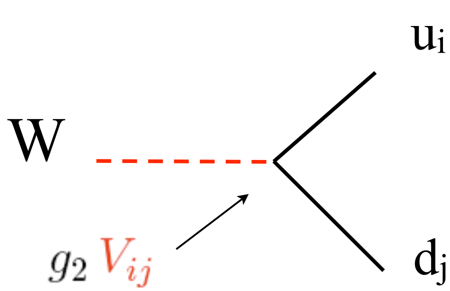
- In full underlying theory (=SM), charged current weak processes are mediated by exchange of the W boson
- W couples to up-down states of weak isospin doublets with strength g_2



$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \begin{pmatrix} c \\ s \end{pmatrix}_L \begin{pmatrix} t \\ b \end{pmatrix}_L$$

$$\Psi_L = (1 - \gamma_5)/2 \; \Psi$$

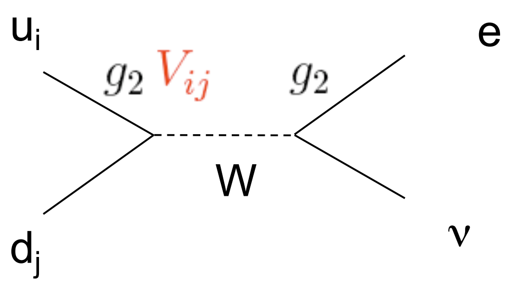
- When expressed in terms of quark *mass eigenstates,* the u-d-W vertex involves unitary matrix V_{ij} (Cabibbo-Kobayashi-Maskawa) describing misalignment of "u" and "d" mass matrices



$$\mathbf{V} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

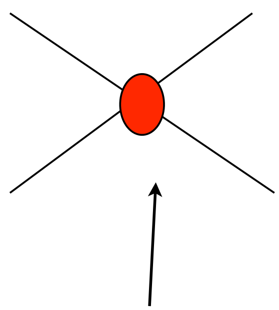
- Calculate $d \to u\, e\, \nu$ amplitude within the SM

- Exploit hierarchy of scales: $m_{had} \ll M_{W,Z,t}$



$$A = \frac{g^2}{8} V_{ud} \frac{i}{k^2 - M_W^2}\, \bar{u}\gamma_\mu (1-\gamma_5) d\; \bar{e}\gamma^\mu (1-\gamma_5)\nu_e$$

- To lowest order in k^2/M_W^2, same answer is obtained in a theory with no W and a new local 4-quark operator



$$\mathcal{L}_{\rm eff}^{SL} = -\frac{G_F}{\sqrt{2}} V_{ud}\, \hat{O} \;+\; h.c. \qquad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$

$$\hat{O} = \bar{u}\gamma_\mu (1-\gamma_5) d\; \bar{e}\gamma^\mu (1-\gamma_5)\nu_e$$

$$A = -i \frac{G_F}{\sqrt{2}} V_{ud}\, \langle \hat{O} \rangle \;+\; O\left(\frac{k^2}{M_W^2}\right)$$

- Next step: go from quark-level to nucleon level description

$$\langle p|\bar{u}\gamma_\mu d|n\rangle = g_V\ \bar{u}_p\gamma_\mu u_n + O(q)$$

$$q = p_n - p_p$$

$$\langle p|\bar{u}\gamma_\mu\gamma_5 d|n\rangle = g_A\ \bar{u}_p\gamma_\mu\gamma_5 u_n + O(q)$$

$$g_V = 1 \qquad g_A \simeq 1.27$$

- Final results of matching calculation:

$$C_V = C'_V = \frac{g^2}{8M_W^2}V_{ud} \ \equiv \frac{1}{\Lambda_W^2}$$

$$C_A = C'_A = -g_A\ \frac{g^2}{8M_W^2}V_{ud}$$

$$C_{S,P,T} = C'_{S,P,T} = 0$$

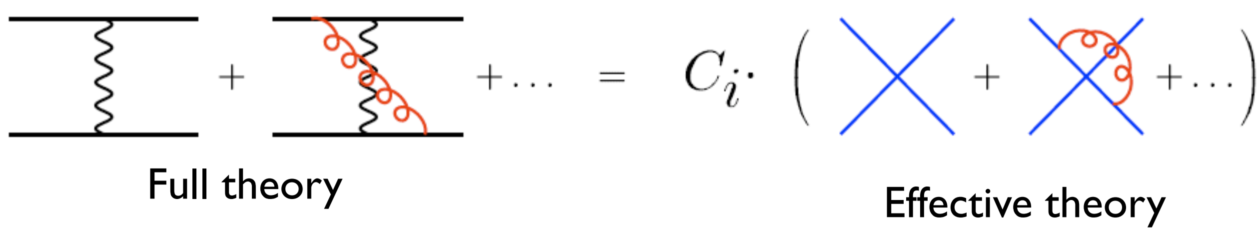
- Effective couplings know about masses and coupling constants of the underlying theory

- Effective scale Λ_W does not coincide in general with mass of new particle (factors of couplings, possibly loops....)

- This was a simple example of matching calculation in EFT:

$$A_{\text{full}} = \sum_i C_i \langle O_i \rangle \equiv A_{EFT}$$

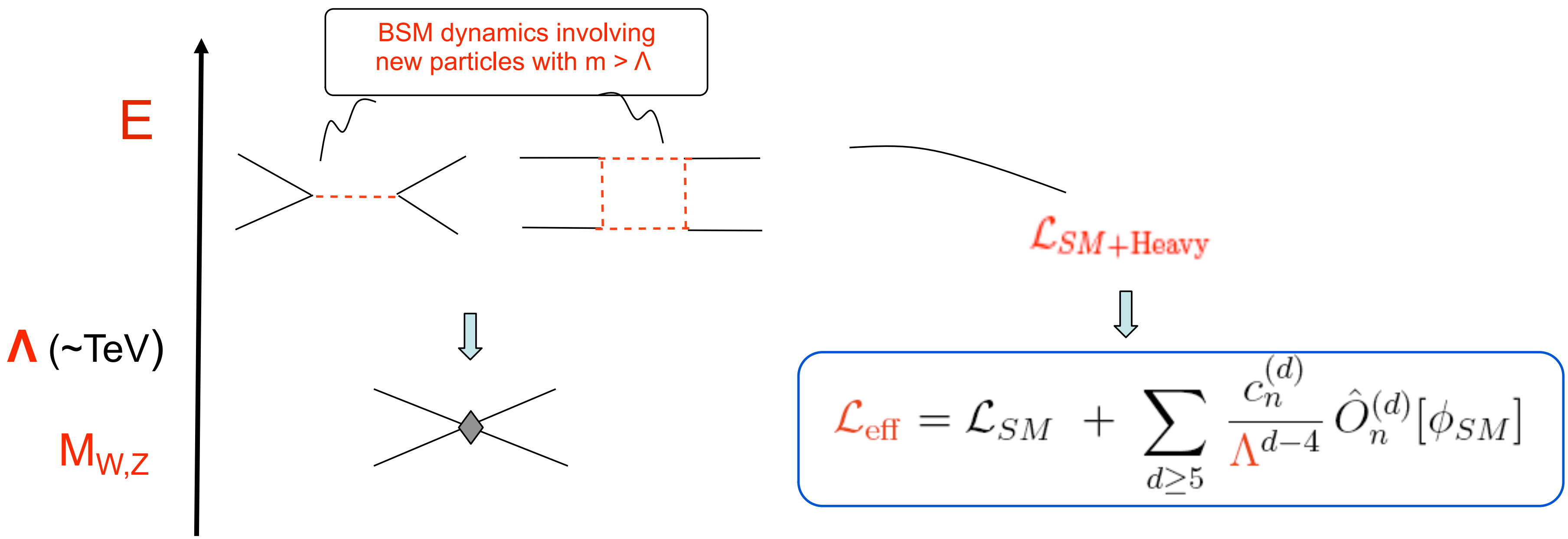
- ★ "Integrate out" heavy d.o.f (W,Z,t); write L_{eff} in terms of local operators built from low-energy d.o.f.
- ★ To a given order in E/M_W, determine effective couplings (Wilson coefficients) from the matching condition $A_{full} = A_{EFT}$ with amplitudes involving "light" states
- ★ We did matching at tree-level, but strong and electroweak higher order corrections can be included



General BSM EFT

Big picture

- Assume existence of new particles with M >> $E_{accessible}$ ~ $G_F^{-1/2}$
- "Integrate out" these particles: describe dynamics below scale Λ [~ mass of new particles] via L_{eff}



$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{SM} + \sum_{d\geq 5} \frac{c_n^{(d)}}{\Lambda^{d-4}} \hat{O}_n^{(d)}[\phi_{SM}]$$

- Building L_{eff} requires specifying:

  - ★ Degrees of freedom*: SM field content

    - ★ One Higgs doublet, no light ν_R and no other light fields

  - ★ Symmetries*: SM gauge group $SU(3)_c \times SU(2)_W \times U(1)_Y$

    - ★ Underlying theory respects SM gauge group

  - ★ Power counting in E/Λ: organize analysis in terms of operators of increasing dimension (5,6,...)

Lightning review of the SM

- Gauge group: $SU(3)_c \times SU(2)_W \times U(1)_Y$

$$\psi'(x) = e^{ig_s \alpha_A(x) \frac{\lambda_A}{2}} \psi(x)$$

$$\psi'(x) = e^{ig \beta_a(x) \frac{\sigma_a}{2}} \psi(x)$$

$$\psi'(x) = e^{ig' \gamma(x) Y} \psi(x)$$

Fundamental representation

- Notation for gauge group representations:

$$(\dim[SU(3)_c],\ \ \dim[SU(2)_W],\ \ Y)$$

- Building blocks I: gauge fields

		$SU(3)_c \times SU(2)_W \times U(1)_Y$ representation
gluons:	$G^A_\mu\,, \quad A = 1 \cdots 8\,,$ $G^A_{\mu\nu} = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + g_s f_{ABC} G^B_\mu G^C_\nu\,.$	(8,1,0)
W bosons:	$W^I_\mu\,, \quad I = 1 \cdots 3\,,$ $W^I_{\mu\nu} = \partial_\mu W^I_\nu - \partial_\nu W^I_\mu + g \varepsilon_{IJK} W^J_\mu W^K_\nu$	(1,3,0)
B boson:	$B_\mu\,,$ $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu\,.$	(1,1,0)

Gauge transformation:

$$W^I_{\mu\nu}\frac{\sigma^I}{2} \longrightarrow V(x)\left[W^I_{\mu\nu}\frac{\sigma^I}{2}\right] V^\dagger(x)$$

$$V(x) = e^{ig\beta_a(x)\frac{\sigma_a}{2}}$$

- Building blocks 2: fermions and Higgs

	SU(3)$_c$ x SU(2)$_W$ x U(1)$_Y$ representation	SU(2)$_W$ transformation
$l = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	(1,2,-1/2)	$l \to V_{SU(2)}\, l$
$e = e_R$	(1,1,-1)	
$q^i = \begin{pmatrix} u^i_L \\ d^i_L \end{pmatrix}$	(3,2,1/6)	$q \to V_{SU(2)}\, q$
$u^i = u^i_R$	(3,1,2/3)	
$d^i = d^i_R$	(3,1,-1/3)	
$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$	(1,2,1/2)	$\varphi \to V_{SU(2)}\, \varphi$
$\tilde{\varphi} = \epsilon\, \varphi^* = \begin{pmatrix} \varphi^{0*} \\ -\varphi^- \end{pmatrix}$	(1,2,-1/2)	$\tilde{\varphi} \to V_{SU(2)}\, \tilde{\varphi}$

$\epsilon = i\sigma_2$

- SM Lagrangian: $$\mathcal{L}_{SM} = \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}$$

$$\mathcal{L}_{\text{Gauge}} = -\frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^I_{\mu\nu} W^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$+\, i\bar{\ell}\not{D}\ell \;+\; i\bar{e}\not{D}e \;+\; i\bar{q}\not{D}q \;+\; i\bar{u}\not{D}u \;+\; i\bar{d}\not{D}d$$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu\varphi)^\dagger (D^\mu\varphi) - \lambda(\varphi^\dagger\varphi - v^2)^2$$

$$\mathcal{L}_{\text{Yukawa}} = Y_e\,\bar{\ell}e\varphi \;+\; Y_d\,\bar{q}d\varphi \;+\; Y_u\,\bar{q}u\tilde{\varphi} \;+\; \text{h.c.}$$

EWSB →

$$\langle\varphi\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\langle\tilde{\varphi}\rangle = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$v = 174\,\text{GeV}$$

- Covariant derivative

$$D_\mu = I\,\partial_\mu \;-\; ig_s\frac{\lambda^A}{2}G^A_\mu \;-\; ig\frac{\sigma^a}{2}W^a_\mu \;-\; ig'Y B_\mu$$

BSM: dimension 5

- Construct all possible dim=5 effective operators in detail: this illustrates the method and leads to a physically interesting result

- Fermions only (and derivatives)? No
  - Ψ's are chiral fermions and [Ψ]=3/2, so e.g. $\bar{\psi}\, \not{D}\not{D}\psi = 0$

    $\left(\not{D}\not{D} = D_\mu D_\nu g^{\mu\nu} - i D_\mu D_\nu \sigma^{\mu\nu} \right)$

- Scalars only, vectors only? No: use [φ] = [V] =1 and gauge invariance

- Vectors + Fermions & Vectors + scalars? No

- So, we are lead to consider operators with fermions (2) and scalars (2) and no derivatives

- If scalars are φ and φ^* $\Rightarrow$
  - total hypercharge Y of fermions Ψ_1 and Ψ_2 is 0
  - need a multiplet and its charge-conjugate
  - but cannot make non-vanishing Lorentz scalar of dim3 ($\bar{\psi}\psi = 0$)

- We are left with building blocks φ, φ, Ψ_1, Ψ_2
  - Forming $SU(2)_W$ invariants: $\varphi^T \varepsilon \varphi = 0 \Rightarrow \Psi_1, \Psi_2$ must be doublets (so we are left with ℓ or q)

Recall: $\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

- Problem (discussion): prove that $d_1^T \varepsilon\, d_2$ is SU(2) invariant ($d_{1,2}$ doublets)

- If scalars are φ and φ^* $\Rightarrow$
    - total hypercharge Y of fermions Ψ_1 and Ψ_2 is 0
    - need a multiplet and its charge-conjugate
    - but cannot make non-vanishing Lorentz scalar of dim3 ($\bar{\psi}\psi = 0$)

- We are left with building blocks φ, φ, Ψ_1, Ψ_2
    - Forming $SU(2)_W$ invariants: $\varphi^T \varepsilon \varphi = 0 \Rightarrow \Psi_1, \Psi_2$ must be doublets (so we are left with ℓ or q)
    - $\ell^T \varepsilon \varphi$ and $\varphi^T \varepsilon \ell$ are $SU(2)_W$ and $U(1)_Y$ invariant
    - Connect them to make Lorentz scalar:

$$\hat{O}_{\text{dim}=5} = \ell^T C \epsilon \varphi \; \varphi^T \epsilon \ell \qquad C = i\gamma_2\gamma_0$$

- Could one replace ℓ with q? No: invariance under $SU(3)_c$ and $U(1)_Y$

- Conclusion: there is only one dim=5 operator (Weinberg '79)

$$\hat{O}_{\rm dim=5} = \ell^T C \epsilon \varphi \; \varphi^T \epsilon \ell \qquad\qquad C = i\gamma_2\gamma_0$$

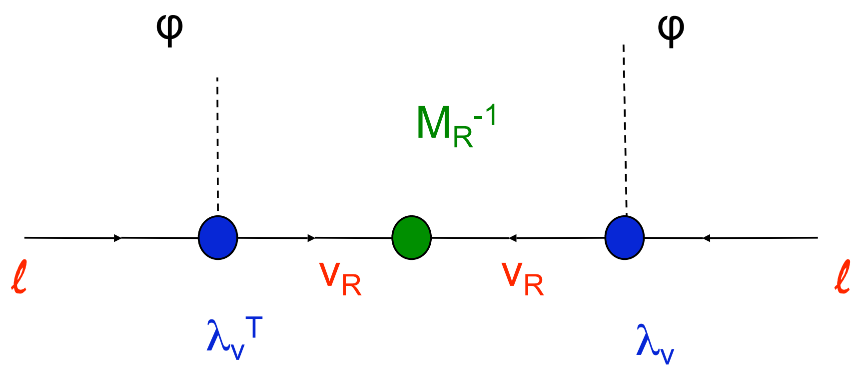
  - it violates total lepton number ($\ell \to e^{i\alpha}\ell$, $e \to e^{i\alpha} e$)
  - it generates Majorana mass for L-handed neutrinos (after EWSB)

$$\frac{1}{\Lambda}\hat{O}_{\rm dim=5} \xrightarrow{\langle\varphi\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}} \frac{v^2}{\Lambda}\nu_L^T C \nu_L$$

  - light neutrino mass scale (≤ eV) points to high scale of lepton number breaking

$$m_\nu \sim 1\,{\rm eV} \;\; \to \;\; \Lambda \sim 10^{13}\,{\rm GeV}$$

- Explicit realization of this operator in models with heavy R-handed Majorana neutrinos

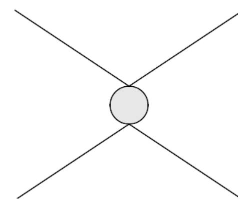


$$g \sim \lambda_\nu^T M_R^{-1} \lambda_\nu$$

$$\mathcal{L}_5 = g_{\alpha\beta}\, \ell_\alpha^T C \epsilon \varphi \, \varphi^T \epsilon \ell_\beta$$

BSM: dimension 6

- Many possible structures, but methodology is the same
- B violating operators: Weinberg'79 & Wilczek-Zee '79
- B & L conserving operators: first systematic analysis by Buchmuller-Wyler '86 (~ 80 operators)
- Here give just a few examples:
  - 4-fermion operators



  - operators involving vectors-fermions-scalars

After EWSB these generate corrections to fermion - gauge boson vertex (vector and dipole)

- Examples of 4-fermion operators (relevant for β decay discussion and Lepton Flavor Violation) [Homework: check gauge invariance]

$$O_{ll}^{(1)} = \frac{1}{2}(\bar{l}\gamma^\mu l)(\bar{l}\gamma_\mu l), \qquad O_{ll}^{(3)} = \frac{1}{2}(\bar{l}\gamma^\mu \sigma^a l)(\bar{l}\gamma_\mu \sigma^a l)$$

$$O_{lq}^{(1)} = (\bar{l}\gamma^\mu l)(\bar{q}\gamma_\mu q), \qquad O_{lq}^{(3)} = (\bar{l}\gamma^\mu \sigma^a l)(\bar{q}\gamma_\mu \sigma^a q)$$

$$O_{qde} = (\bar{\ell}e)(\bar{d}q) + \mathrm{h.c.}$$

$$O_{lq} = (\bar{l}_a e)\epsilon^{ab}(\bar{q}_b u) + \mathrm{h.c.}$$

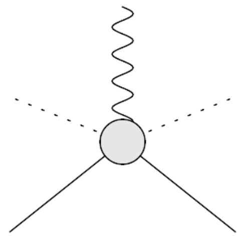
$$O_{lq}^t = (\bar{l}_a \sigma^{\mu\nu} e)\epsilon^{ab}(\bar{q}_b \sigma_{\mu\nu} u) + \mathrm{h.c.}$$

- Recall:

$$(t^a)_{ij}\,(t^a)_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{kl}\right)$$

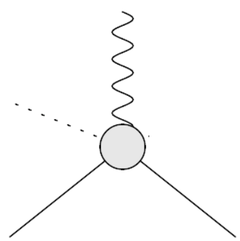
- These operators contribute to both charged-current and neutral current transitions

- Examples of vectors-fermions-scalars operators (relevant for β decay and Lepton Flavor Violation) [Homework: check gauge invariance]



$$O_{\varphi l}^{(1)} = i(\varphi^\dagger D^\mu \varphi)(\bar{l}\gamma_\mu l) + \text{h.c.}, \quad O_{\varphi l}^{(3)} = i(\varphi^\dagger \sigma^a D^\mu \varphi)(\bar{l}\gamma_\mu \sigma^a l) + \text{h.c.}$$

$$O_{\varphi q}^{(1)} = i(\varphi^\dagger D^\mu \varphi)(\overline{q}\gamma_\mu q) + \text{h.c.}, \quad O_{\varphi q}^{(3)} = i(\varphi^\dagger \sigma^a D^\mu \varphi)(\overline{q}\gamma_\mu \sigma^a q) + \text{h.c.}$$



$$O_{eW} = (\bar{l}\sigma^{\mu\nu}\sigma^a e)\varphi\, W^a_{\mu\nu} \qquad O_{eB} = (\bar{l}\sigma^{\mu\nu} e)\varphi\, B_{\mu\nu}$$

Applications

- ⋆ β decays: weak universality, non V-A, etc
- ⋆ Lepton Flavor Violation: discriminating power of $\mu \to e\gamma$ and $\mu \to e$ conversion

“β-decays”

Semi-leptonic CC decays

- Mediated by W exchange in the SM

★ Only V-A structure

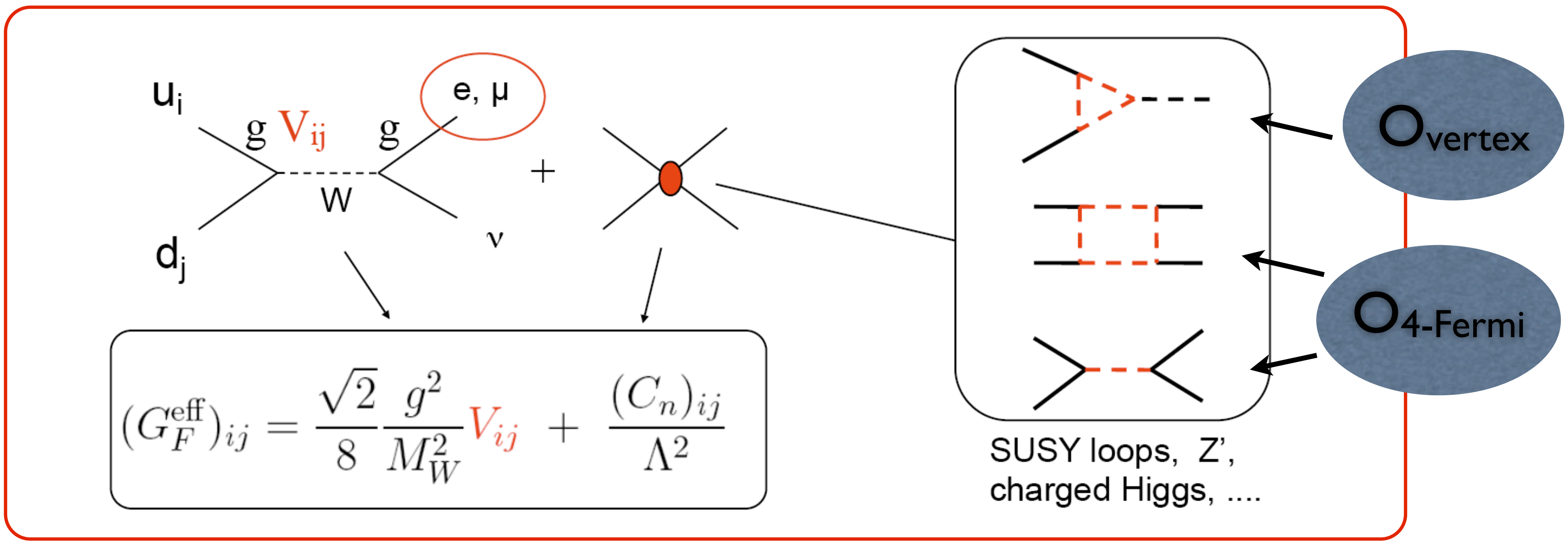
★ Universality relations

$$[G_F]_e/[G_F]_\mu = 1 + \Delta_{e/\mu}$$

Lepton universality

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{\mathrm{CKM}}$$

Cabibbo universality

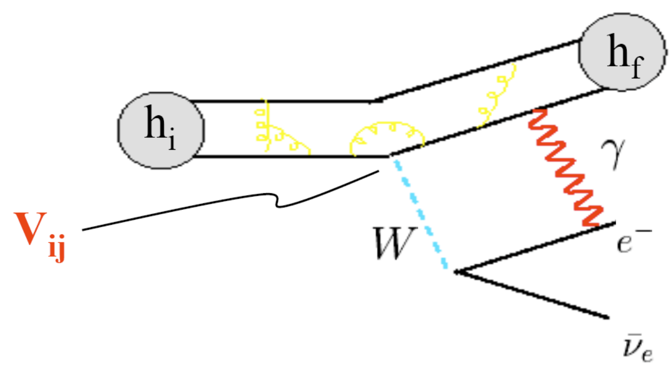


$$(G_F^{\mathrm{eff}})_{ij} = \frac{\sqrt{2}}{8}\frac{g^2}{M_W^2} V_{ij} + \frac{(C_n)_{ij}}{\Lambda^2}$$

★ Sensitivity to BSM scale: $\Delta \sim \frac{c_n}{g^2}\frac{M_W^2}{\Lambda^2} \leq 10^{-2} - 10^{-3} \longleftrightarrow \Lambda \sim 1\text{-}10$ TeV

Paths to V_{ud} and V_{us}

V_{ud}	$0^+ \to 0^+$ $(\pi^\pm \to \pi^0 e\nu)$	$n \to pe\bar{\nu}$	$\pi \to \mu\nu$	$\tau \to h_{NS}\,\nu_\tau$
V_{us}	$K \to \pi\ \ell\nu$	$\Lambda \to pe\bar{\nu},...$	$K \to \mu\nu$	$\tau \to h_S \nu_\tau$ (*inclusive*)



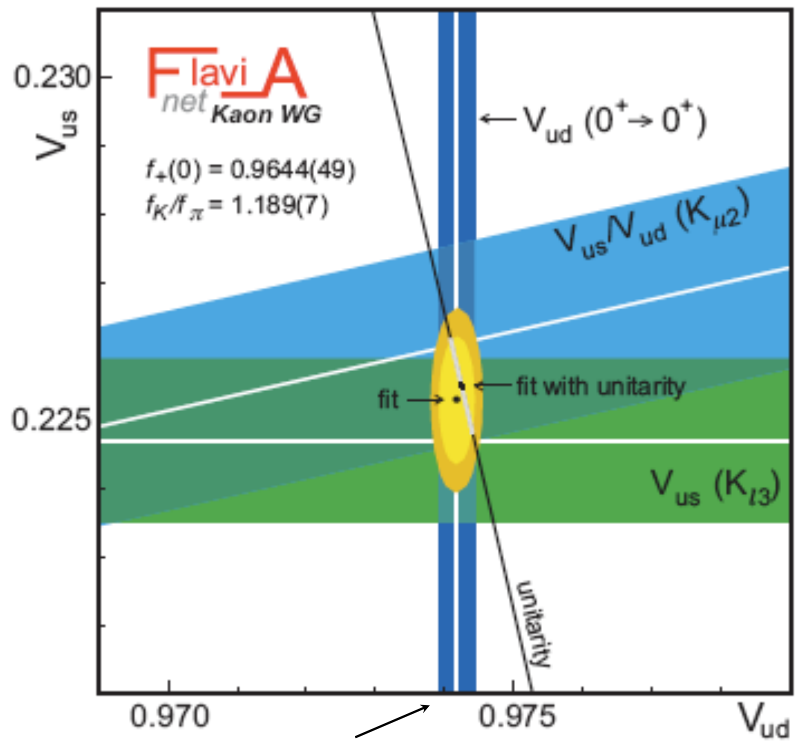
$$\Gamma_{ij} = \left[G_F^{(\mu)} V_{ij}\right]^2 \times |M_{\rm had}|^2 \times (1 + \delta_{\rm em}) \times F_{\rm kin}$$

Hadronic matrix elements

Radiative corrections

Global Fit to V_{ud} and V_{us}

arXiv:0907.5386



V_{ud} from $0^+ \rightarrow 0^+$

Fit result

$$V_{ud} = 0.97425\ (22)$$
$$V_{us} = 0.2252\ (9)$$

$\chi^2/\text{dof} = 0.65/1$

↓

$$|V_{ud}|^2 + |V_{us}|^2 = 0.9999(6)$$

Error equally shared between V_{ud} and V_{us}

- Remarkable agreement with Cabibbo universality: $\Delta_{CKM} = -(1 \pm 6) * 10^{-4}$
- Confirms large EW rad. corr. $(2\ \alpha/\pi\ \log(M_Z/M_p) = +3.6\%)$ Marciano-Sirlin
- It would naively fit $M_Z = (90 \pm 7)$ GeV

Implications for BSM physics

- Extraction of V_{ij} uses Fermi constant from muon decay

$$\Gamma_{ij} \;=\; \left[G_F^{(\mu)}\, V_{ij}\right]^2 \times |M_{\rm had}|^2 \times (1 + \delta_{\rm em}) \times F_{\rm kin}$$

- In SM extensions, Fermi constant in muon decay and semi-leptonic transitions may differ (vertex corrections and boxes)
- $\Delta_{\rm CKM}$ is sensitive to these apparent violations of weak universality from TeV extensions of the SM:

$$\frac{[G_F^{(\beta)}]^2}{[G_F^{(\mu)}]^2} = 1 + \Delta_{\rm CKM}$$

EFT analysis

- Explore in a model-independent way:

  (1) significance of Δ_{CKM} constraint vs other precision measurements.
  (2) correlations between potential universality deviations and other low- and high-energy observables

- Setup: parameterize BSM interactions via SU(2)xU(1) gauge-invariant higher-dim operators built out of SM fields

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_X \frac{1}{\Lambda_X^2} O_X$$

stop at dim=6

Buchmuller-Wyler 1986, … … Han-Skiba 2004

- Flavor properties: include only U(3)^5-invariant operators ⇒ no problems with FCNC.

$$\psi^i = [Q_L^i, u_R^i, d_R^i \,;\, L_L^i, e_R^i]$$

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

V. Cirigliano, M. Gonzalez, J. Jenkins: arXiv: 0908.1754

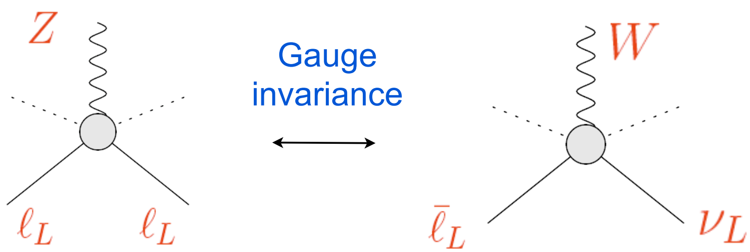
- Δ_{CKM} is sensitive to four operators:

$$\Delta_{\rm CKM} = 4\left(\hat{\alpha}^{(3)}_{ll} - \hat{\alpha}^{(3)}_{lq} - \hat{\alpha}^{(3)}_{\varphi l} + \hat{\alpha}^{(3)}_{\varphi q}\right) \qquad \hat{\alpha}_X = \frac{v^2}{\Lambda_X^2}$$

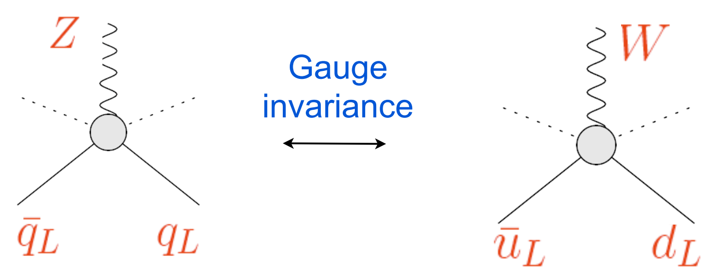
v~200 GeV

Vertex corrections

$$O^{(3)}_{\varphi l} = i(\varphi^\dagger D^\mu \sigma^a \varphi))(\bar{l}\gamma_\mu \sigma^a l)$$



$$O^{(3)}_{\varphi q} = i(\varphi^\dagger D^\mu \sigma^a \varphi)(\bar{q}\gamma_\mu \sigma^a q)$$



$$l^i = \begin{pmatrix} \nu^i_L \\ e^i_L \end{pmatrix} \qquad \varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \qquad q^i = \begin{pmatrix} u^i_L \\ d^i_L \end{pmatrix}$$

V. Cirigliano, M. Gonzalez, J. Jenkins: arXiv: 0908.1754

- Δ_{CKM} is sensitive to four operators:

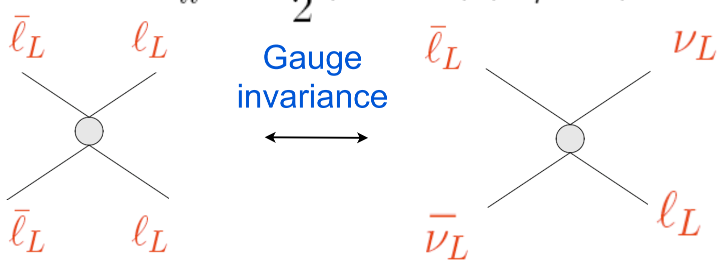
$$\Delta_{\rm CKM} = 4\left(\hat{\alpha}_{ll}^{(3)} - \hat{\alpha}_{lq}^{(3)} - \hat{\alpha}_{\varphi l}^{(3)} + \hat{\alpha}_{\varphi q}^{(3)}\right)$$

$$\hat{\alpha}_X = \frac{v^2}{\Lambda_X^2}$$

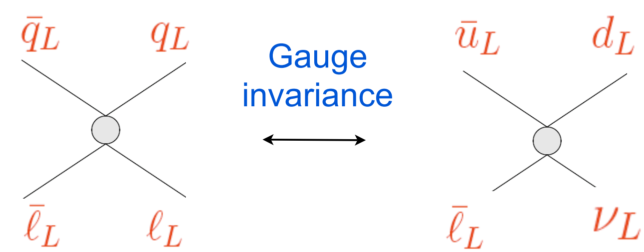
v~200 GeV

4-fermion operators

$$O_{ll}^{(3)} = \frac{1}{2}(\bar{l}\gamma^\mu\sigma^a l)(\bar{l}\gamma_\mu\sigma^a l)$$



$$O_{lq}^{(3)} = (\bar{l}\gamma^\mu\sigma^a l)(\bar{q}\gamma_\mu\sigma^a q)$$



$$l^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix} \qquad \varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \qquad q^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}$$

- Relevant operators affect other precision EW observables! Assess significance of Δ_{CKM} vs other EWPT

Question (1): What is the range of Δ_{CKM} allowed by precision EW tests?

- Global fit and covariance matrix from Han-Skiba 04

↓

$$-9.5 \times 10^{-3} \leq \Delta_{\rm CKM} \leq 0.1 \times 10^{-3}$$ 90% C.L.

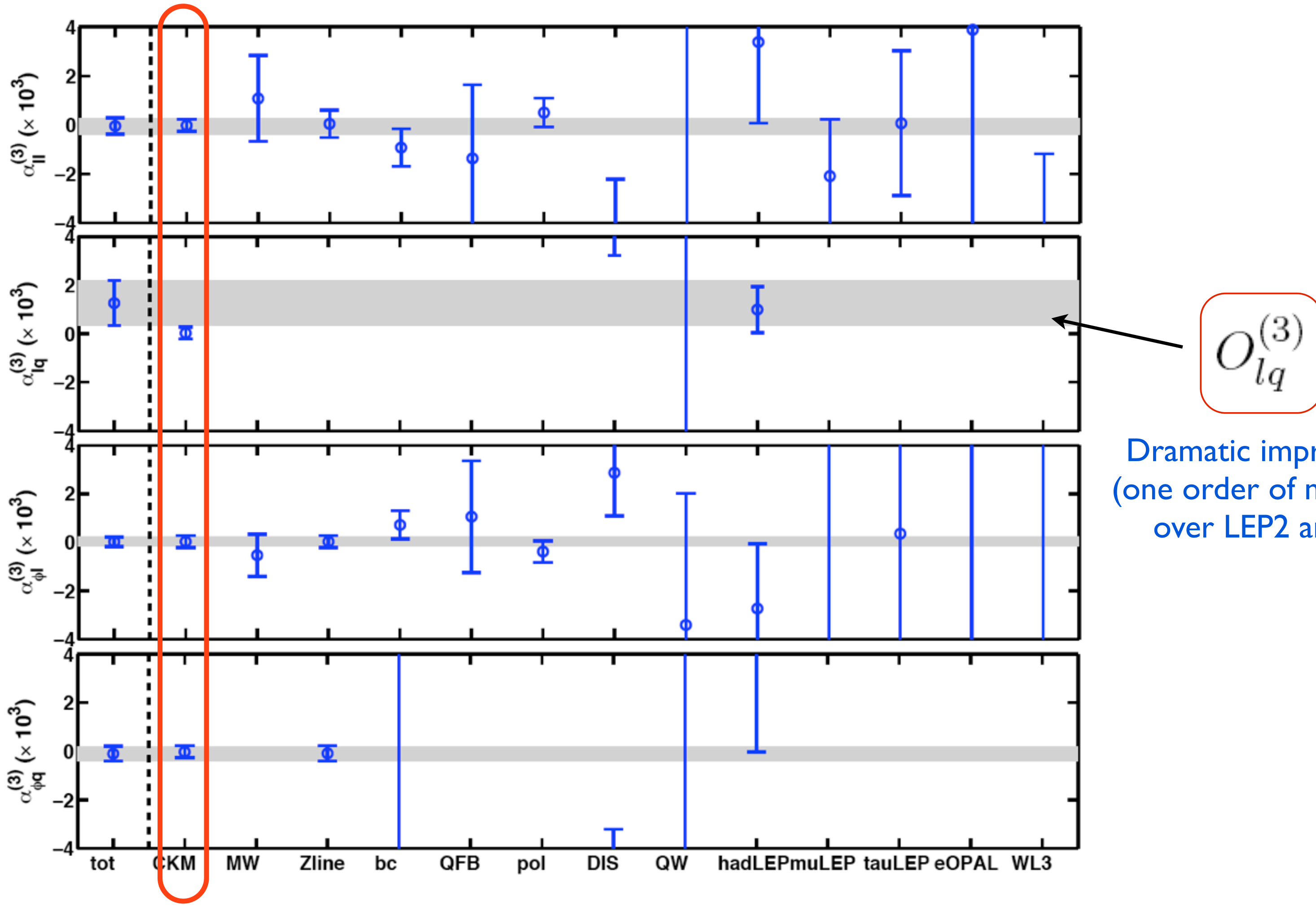
- Direct constraint implies $|\Delta_{CKM}| \leq 1.x10^{-3}$ @ 90% CL

EW precision data alone would leave room for large Δ_{CKM}!

Question (2): What is the strength of Δ_{CKM} constraint?

Same level or better than Z-pole obs.: $\Lambda > 11$ TeV @ 90% CL

$$\hat{\alpha}_X = \frac{v^2}{\Lambda_X^2}$$



Dramatic improvement (one order of magnitude) over LEP2 and APV

Question (2): What is the strength of Δ_{CKM} constraint?

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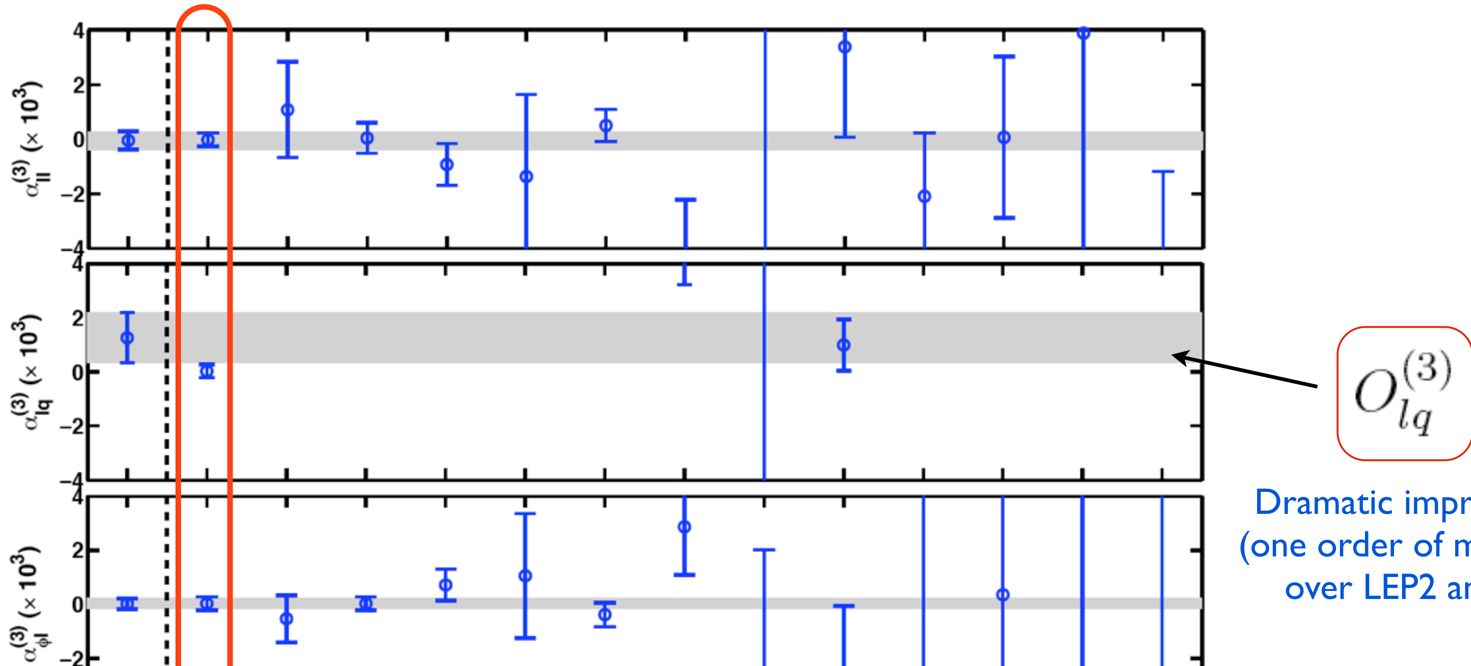


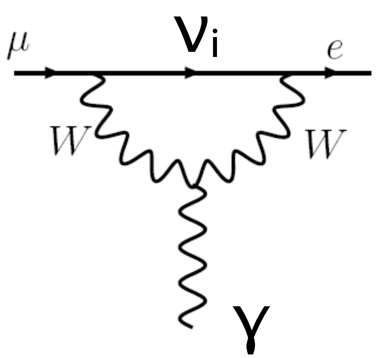
Dramatic improvement (one order of magnitude) over LEP2 and APV

Deviations as large as Δ_{CKM} ~ -0.01 at 90% CL could be blamed on $O_{lq}^{(3)}$ without conflicting with LEP2 data on hadronic cross section

Muons and Lepton Flavor Violation: an EFT perspective

Charged LFV: general considerations

- Evidence of ν oscillations implies that individual lepton family numbers ($L_{e,\mu,\tau}$) are not conserved
- In SM + massive ν, charged LFV rates are negligible (GIM-suppression)



$$Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M^2_W}\right|^2 < 10^{-54}$$

Great discovery channels. Extremely clean probe of BSM physics

- Experimental status (90% CL): muons

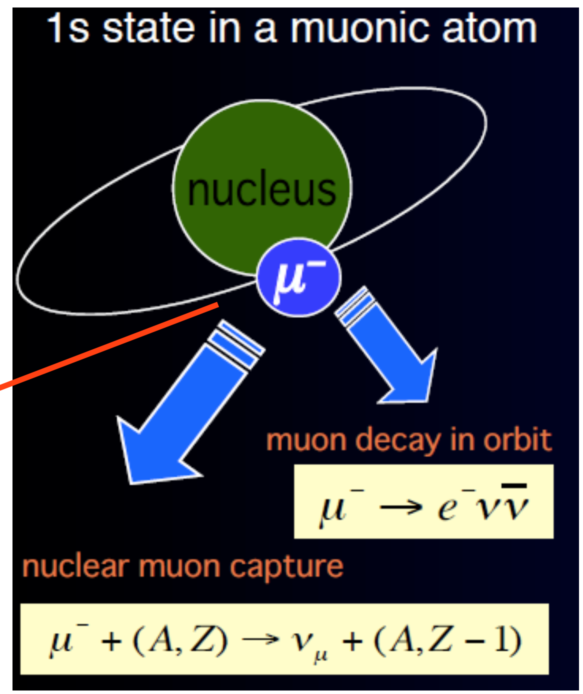
$B_{\mu\to e\gamma}$	$<$	1.2×10^{-11}
$B_{\mu\to 3e}$	$<$	1.0×10^{-12}
$B^{Ti}_{\mu-e}$	$<$	4.3×10^{-12}
$B^{Au}_{\mu-e}$	$<$	8×10^{-13}
$B^{Pb}_{\mu-e}$	$<$	4.6×10^{-11}

⟶ $10^{-13/14}$ (MEG at PSI, *now running*)

⟶ $10^{-16/17}$ (Mu2e, COMET)

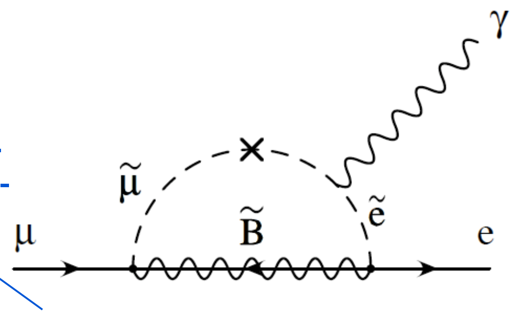
- μ-to-e conversion rate is normalized to total muon capture rate

$$B_{\mu\to e} = \frac{\Gamma(\mu^- + (Z,A) \to e^- + (Z,A))}{\Gamma(\mu^- + (Z,A) \to \nu_\mu + (Z-1,A))}$$

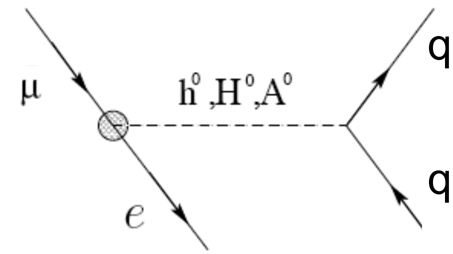


- BSM, several dim-6 operators contribute to LFV processes

Dominant in SUSY-GUT and SUSY see-saw scenarios



Dominant in RPV SUSY and RPC SUSY for large tan(β) and low m_A

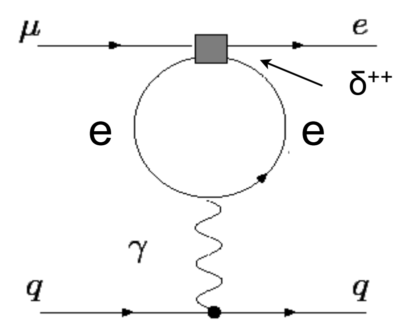


$$\mathcal{L}_{eff} \supset \frac{[\alpha_D]^{ij}}{\Lambda^2} \varphi^\dagger \, \bar{e}^i_R \sigma_{\mu\nu} \ell^j_L \, F^{\mu\nu} + \frac{[\alpha_S]^{ij}}{\Lambda^2} \bar{e}^i_R \ell^j_L \, \bar{q}_L d_R$$

$$+ \frac{[\alpha_{V(z)}]^{ij}}{\Lambda^2} \bar{\ell}^i_L \gamma_\mu \ell^j_L \, \varphi^\dagger D^\mu \varphi + \frac{[\alpha_{V(\gamma)}]^{ij} e_q}{\Lambda^2} \bar{\ell}^i_L \gamma_\mu \ell^j_L \, \bar{q}_L \gamma^\mu q_L + \ldots$$

Generated by Z-penguin

Enhanced in Left-Right symmetric models



- BSM, several dim-6 operators contribute to LFV processes

Key Questions for LFV dynamics in LHC era

0 - What is the overall size of LFV effects?

Current limit from $\mu \to e\gamma$ implies $\Lambda/\sqrt{[\alpha_D]^{e\mu}} > 2 \times 10^4\ \mathrm{TeV}$

- In TeV extensions of the SM, flavor symmetry is broken in a non-generic way (small mixing!)
- New physics at TeV (and reasonable mixing pattern) $\Leftrightarrow$ LFV signals are within reach of planned searches

↓

Be optimistic: assume that BSM physics produces observable rates. Ask questions that probe more deeply LFV dynamics and help discriminating underlying SM extensions

- BSM, several dim-6 operators contribute to LFV processes

Key Questions for LFV dynamics in LHC era

1 - What is the relative strength of various operators (α_D vs α_S ...) ?

- Can be addressed experimentally through analysis of $\mu \to e\gamma$ and $\mu \to e$ conversion in different target nuclei

VC, R. Kitano, Y. Okada, P. Tuzon PRD 80 013002 (2009)

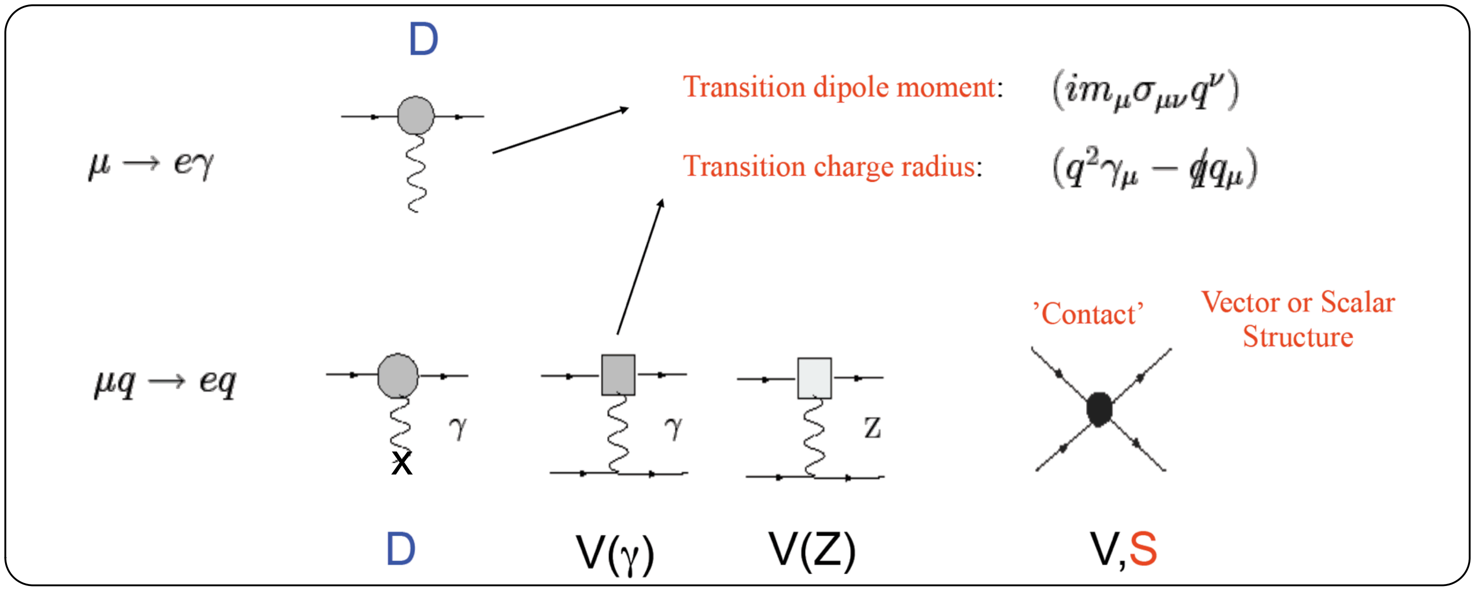
2 - What is the flavor structure of the couplings ($[\alpha_D]^{e\mu}$ vs $[\alpha_D]^{\tau\mu}$ ...) ?

- Many possible scenarios
- Question can in part be addressed experimentally, by testing the predicted pattern of $\mu \to e\gamma$ vs $\tau \to \mu\gamma$ rates
- For a simple and predictive scheme (Minimal Flavor Violation) see references below

VC, B. Grinstein, G. Isidori, M. Wise NPB 728, 121 (2005)
VC, B. Grinstein, G. Isidori, M. Wise NPB 763, 35 (2006)

Discriminating power of μ →eγ and μ →e conversion

- μ →eγ and μ →e conversion probe different combinations of operators



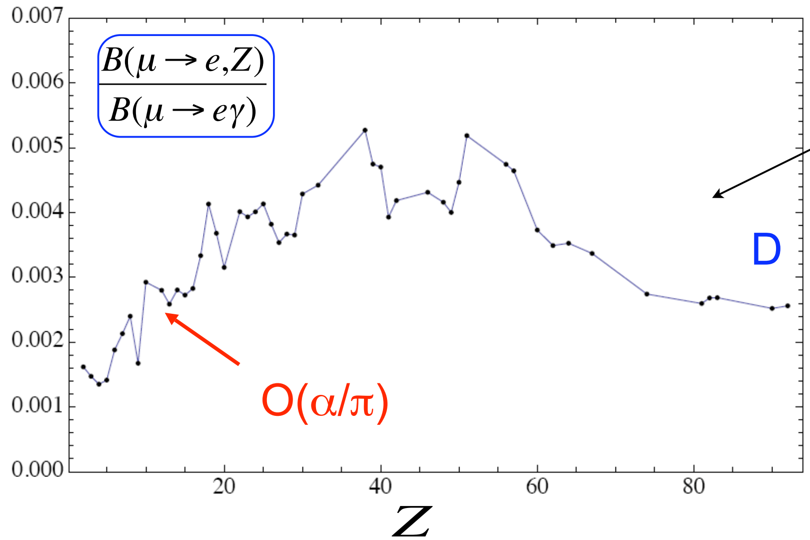
- Conversion amplitude has non-trivial dependence on target nucleus, that distinguishes D,S,V underlying operators

$$\langle e^-; A, Z | \hat{O}_\ell \, \hat{O}_q | \mu^-; A, Z \rangle \sim \int d^3x \; \bar{\psi}_e O_\ell \psi_\mu \; \langle A, Z | \hat{O}_q | A, Z \rangle$$

Relativistic components of muon wave-function give different contributions to D,S,V overlap integrals

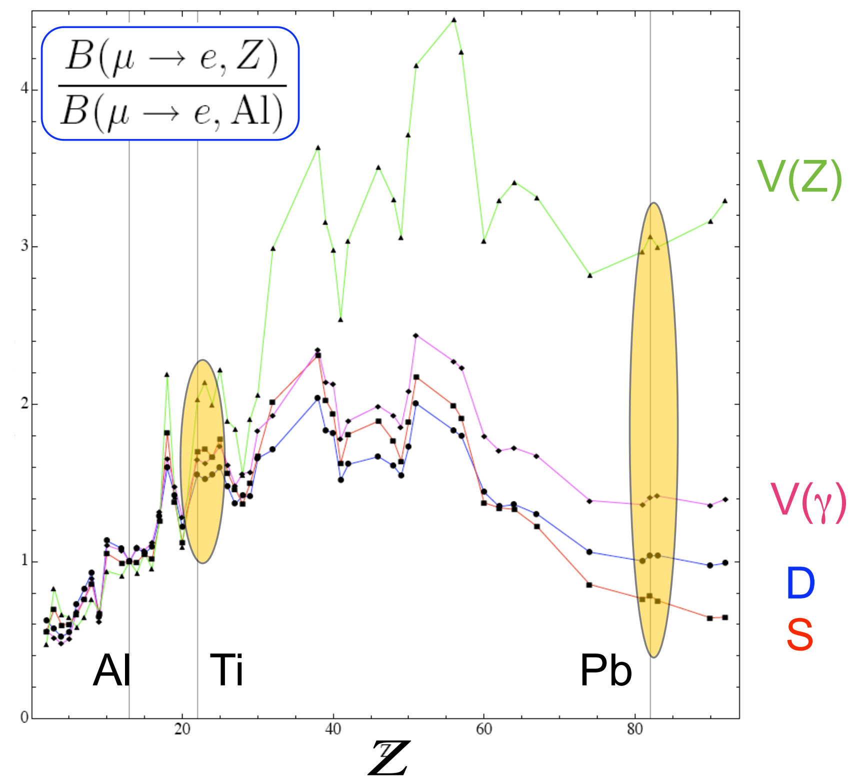
$\rho^{(p,n)}(r)$

- Models in which a single operator dominates can be tested with one double ratio (two LFV measurements):



Deviation from this pattern indicates presence of scalar and/or vector contributions

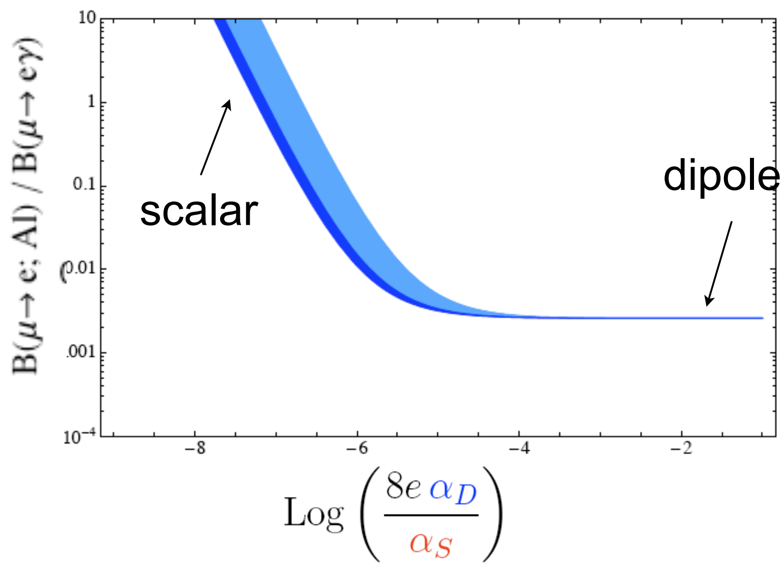
VC-Kitano-Okada-Tuzon '09

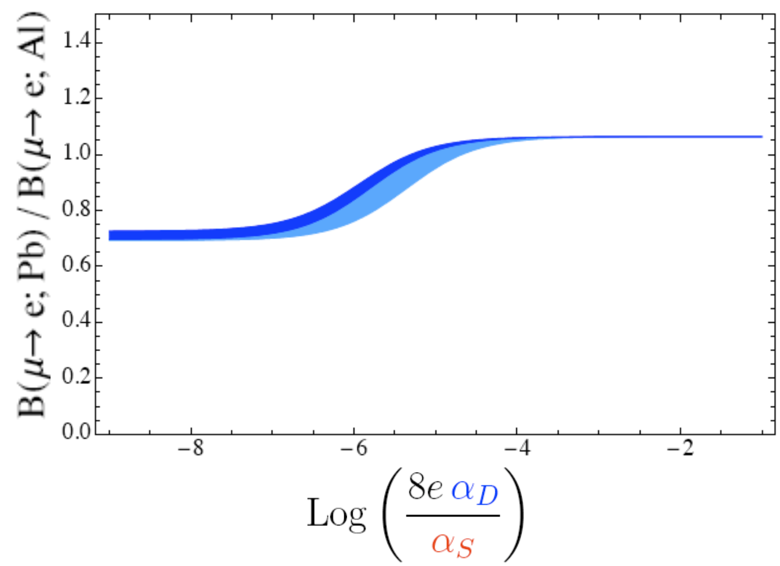


- Essentially free of theory uncertainty (largely cancels in ratios)
- Discrimination: need 5% measure of Ti/Al or 20% measure of Pb/Al
- Ideal world: use Al and a large Z-target (D,V,S have largest separation)

- Models in which two operators dominate can be tested with two double ratios (three LFV measurements!).
- Consider S and D: realized in SUSY via competition between dipole and scalar operator (mediated by Higgs exchange)

Relative sign: +





- Uncertainty from strange form factor largely reduced by lattice QCD

$$y = \frac{2\,\langle p|\bar{s}s|p\rangle}{\langle p|\bar{u}u + \bar{d}d|p\rangle} \in [0, 0.4] \rightarrow [0, 0.05]$$

JLQCD 2008

[0, 0.4] ↓ fat error band

[0, 0.05] ↓ thin error band

- Models in which two operators dominate can be tested with two double ratios (three LFV measurements!).
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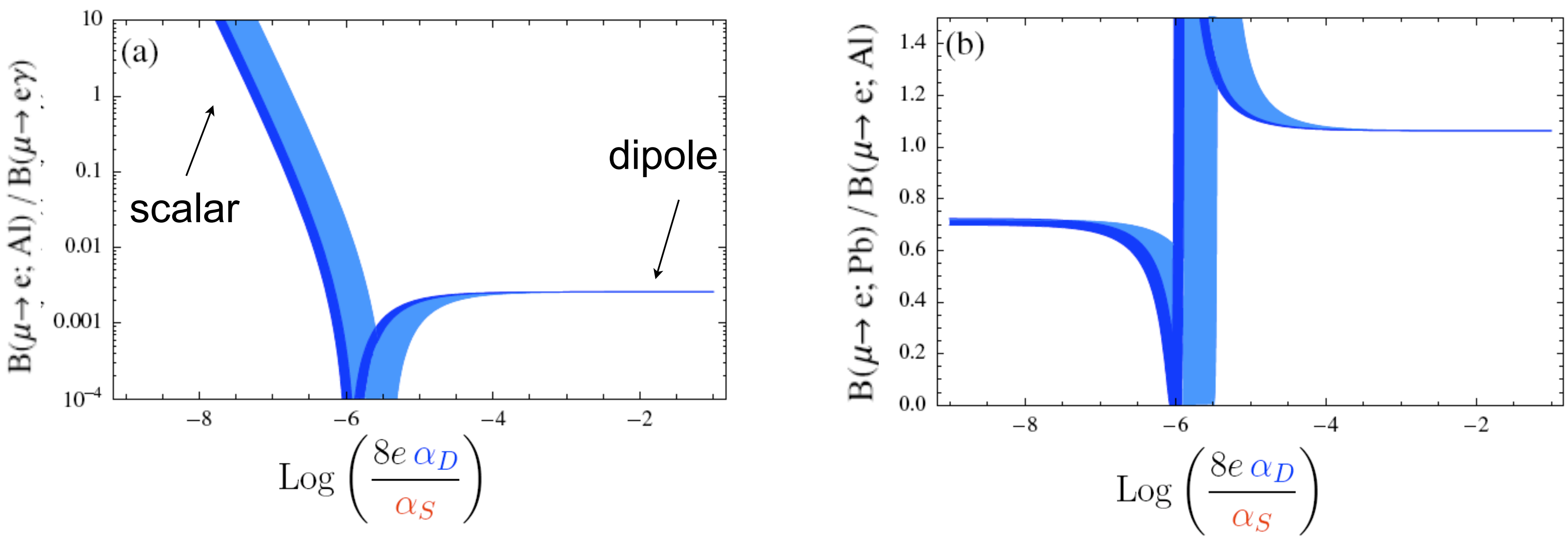
Relative sign: -



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- Models in which two operators dominate can be tested with two double ratios (three LFV measurements!).
- Consider S and D: realized in SUSY via competition between dipole

In summary:

- Theoretical hadronic uncertainties under control (OK for 1-operator dominance, need Lattice QCD for 2-operator models)

- Realistic model discrimination requires measuring Ti/Al at <5% or Pb/Al at <20%: challenge for future experiments

- Uncertainty from strange form factor largely reduced by lattice QCD

$$y = \frac{2\ \langle p|\bar{s}s|p\rangle}{\langle p|\bar{u}u + \bar{d}d|p\rangle} \in \quad [0, 0.4] \quad \rightarrow \quad [0, 0.05]$$

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