

Relativistic Heavy Ions II - Soft physics

RHI Physics

*The US National Nuclear
Physics Summer School &
TRIUMF Summer Institute*

Vancouver, Canada

Helen Caines - Yale University

June 2010

Outline:

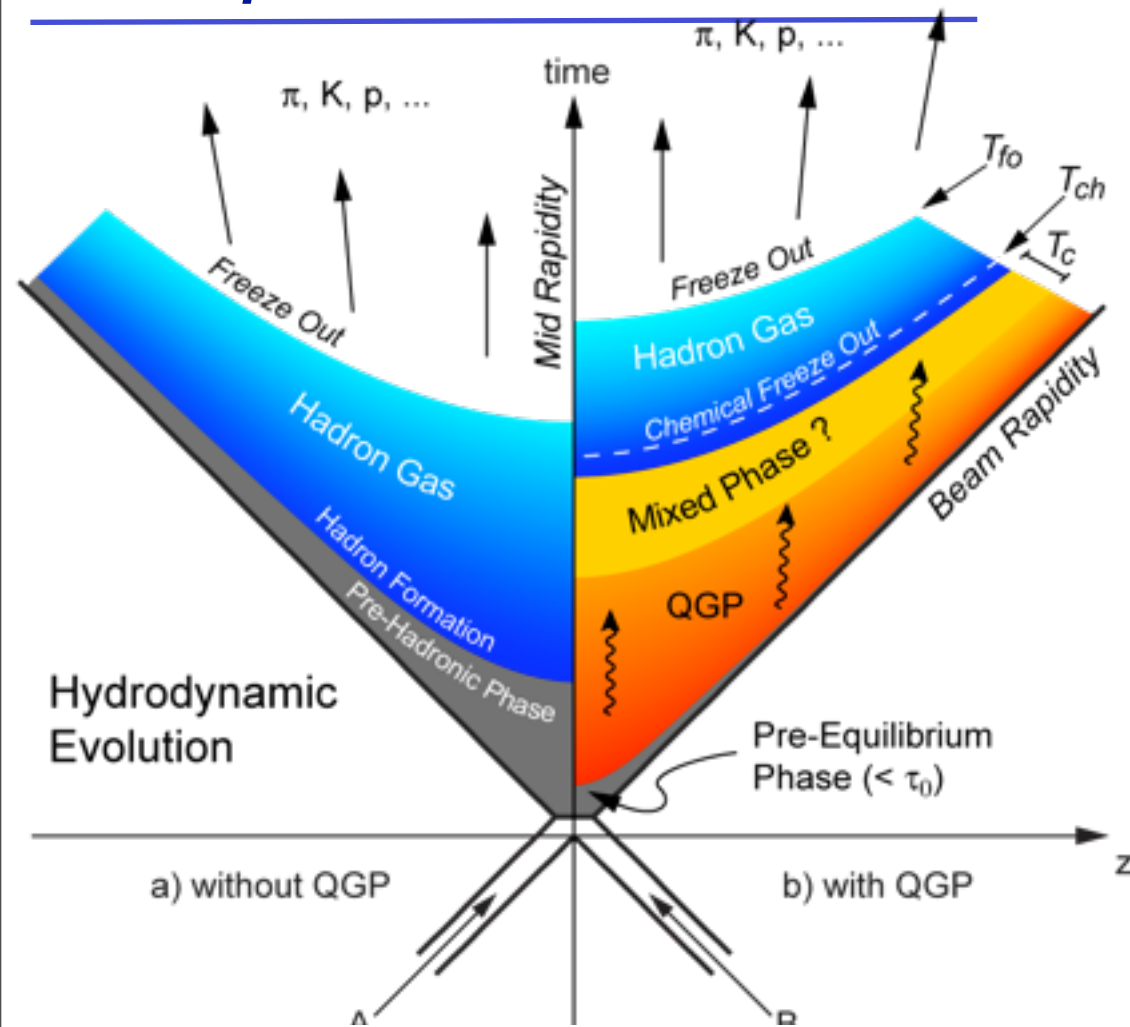
*The Energy Density
The Temperature
Fluid and Flow*



Recap of first lecture

- Looking for evidence of a new state of matter → QGP
- Predicted by QCD to occur, due to screening of colour charge, at high T and/or density
 - $T_c \sim 160$ MeV
- Create in laboratory by colliding ultra-relativistic heavy-ions
- Large multi-purpose experiments necessary to sift through all the data produced

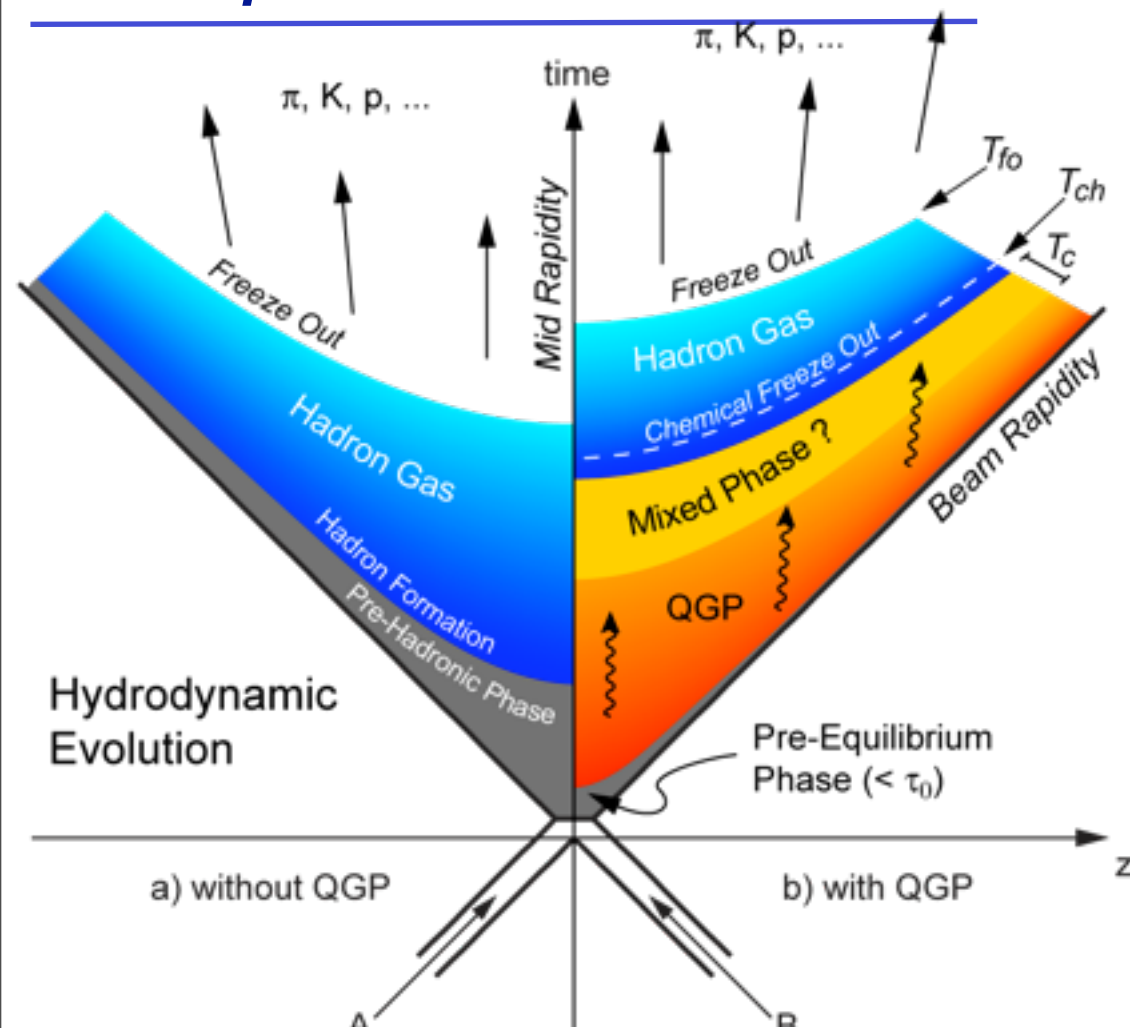
The phase transition in the laboratory



Chemical freeze-out ($T_{ch} \leq T_c$): inelastic scattering ceases

Kinetic freeze-out ($T_{fo} \leq T_{ch}$): elastic scattering ceases

The phase transition in the laboratory



Lattice (2-flavor):

$$T_C \approx 173 \pm 8 \text{ MeV}$$

$$\epsilon_C \approx (6 \pm 2) T^4 \approx 0.70 \text{ GeV/fm}^3$$

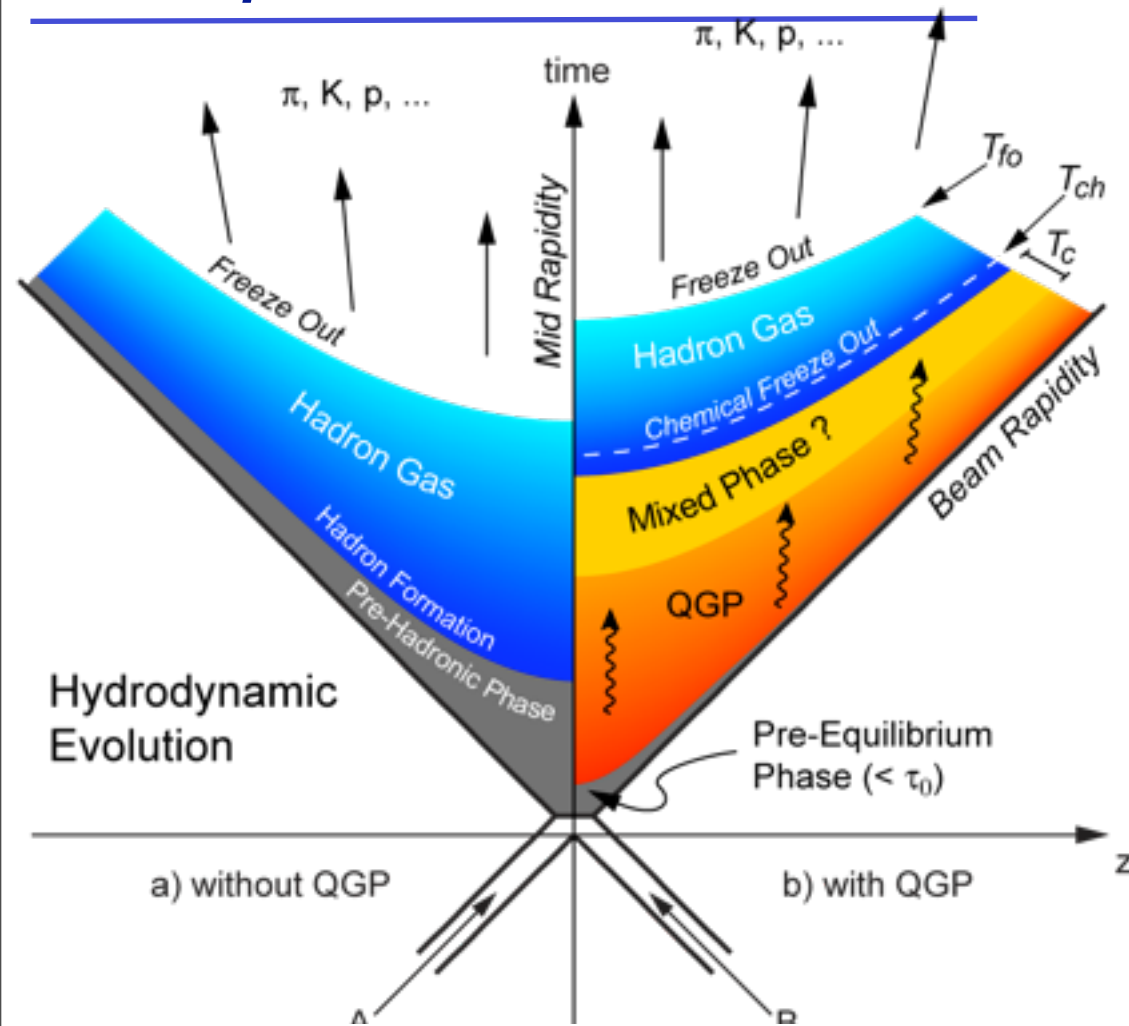
Remember: cold nuclear matter

$$\epsilon_{\text{cold}} \approx u / \frac{4}{3} \pi r_0^3 \approx 0.13 \text{ GeV/fm}^3$$

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Necessary but not sufficient condition

Tevatron (Fermilab)

$$\epsilon(\sqrt{s} = 1.8 \text{ TeV } pp) \gg$$

$$\epsilon(\sqrt{s} = 200 \text{ GeV Au+Au RHIC})$$

Thermal Equilibrium \Rightarrow

many constituents

Size matters !!!

Chemical freeze-out ($T_{\text{ch}} \leq T_C$): inelastic scattering ceases

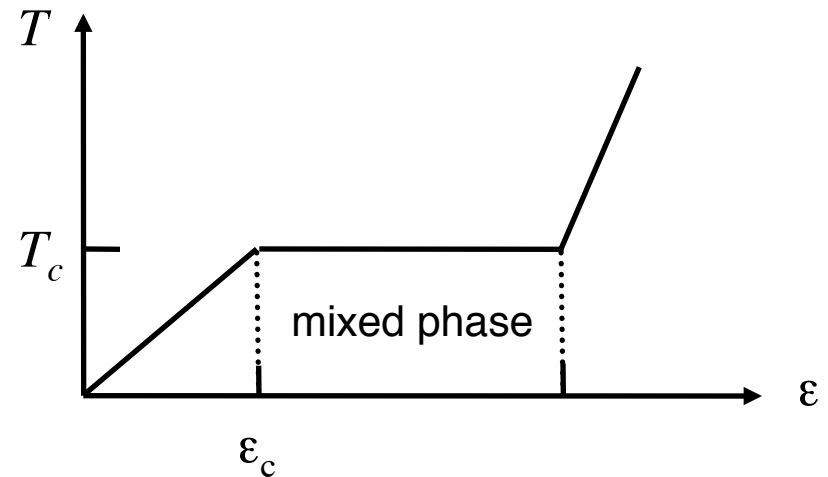
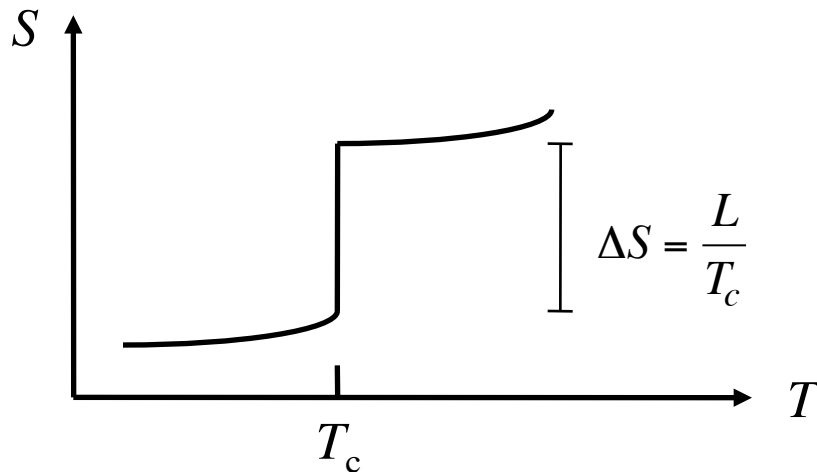
Kinetic freeze-out ($T_{\text{fo}} \leq T_{\text{ch}}$): elastic scattering ceases

Thermodynamics - phase transitions

Phase transition or a crossover?

Signs of a phase transition:

1st order: discontinuous in entropy at T_c → Latent heat, a mixed phase



Higher order: discontinuous in higher derivatives of $\delta^n S / \delta T^n$ → no mixed phase - system passed smoothly and uniformly into new state (ferromagnet)

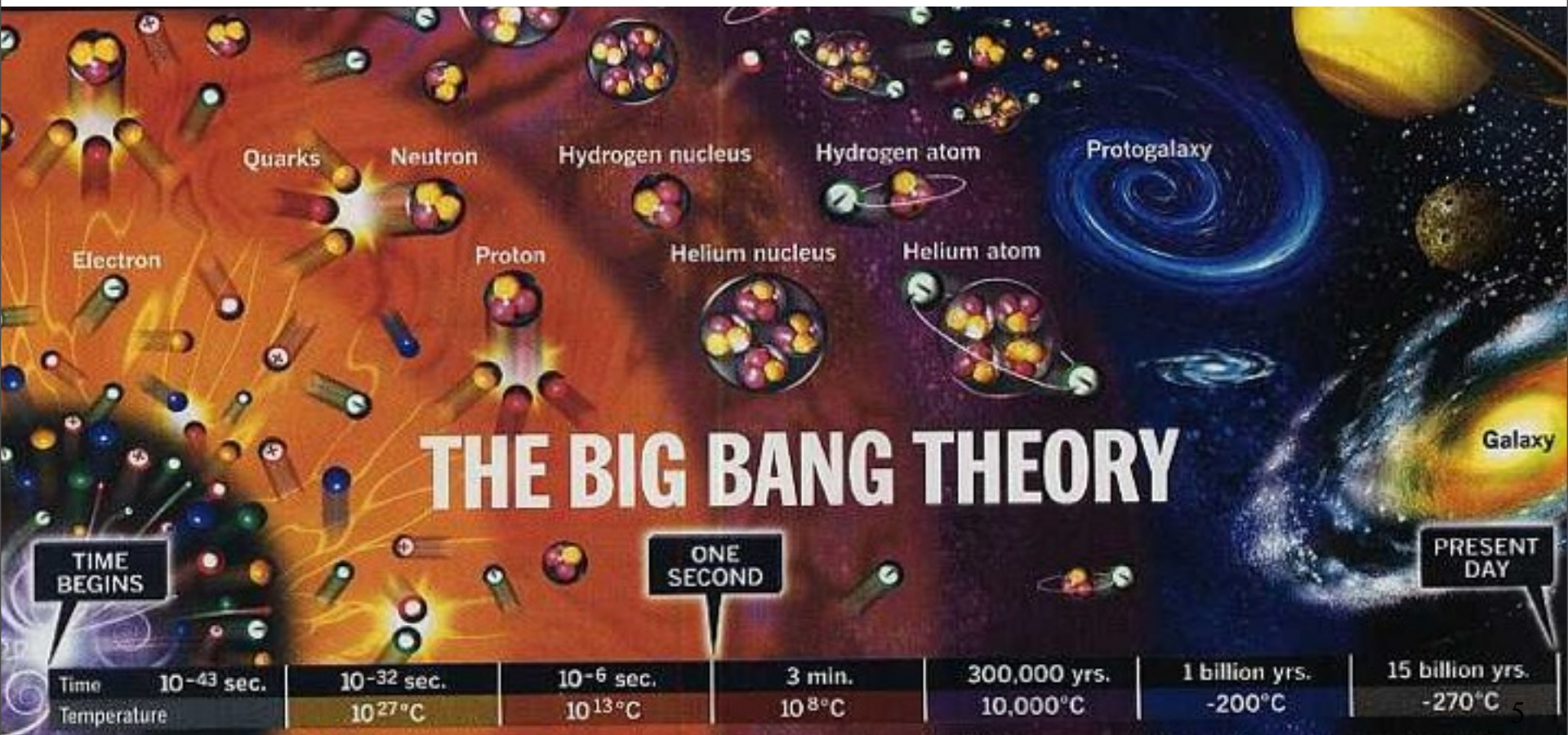
Temperature \Leftrightarrow transverse momentum $T \propto \langle p_T \rangle$

Energy density \Leftrightarrow transverse energy $\epsilon \propto dE_T / dy \cong \langle m_T \rangle dN / dy$

Entropy \Leftrightarrow multiplicity $S \propto dN / dy$

The order of the phase transition

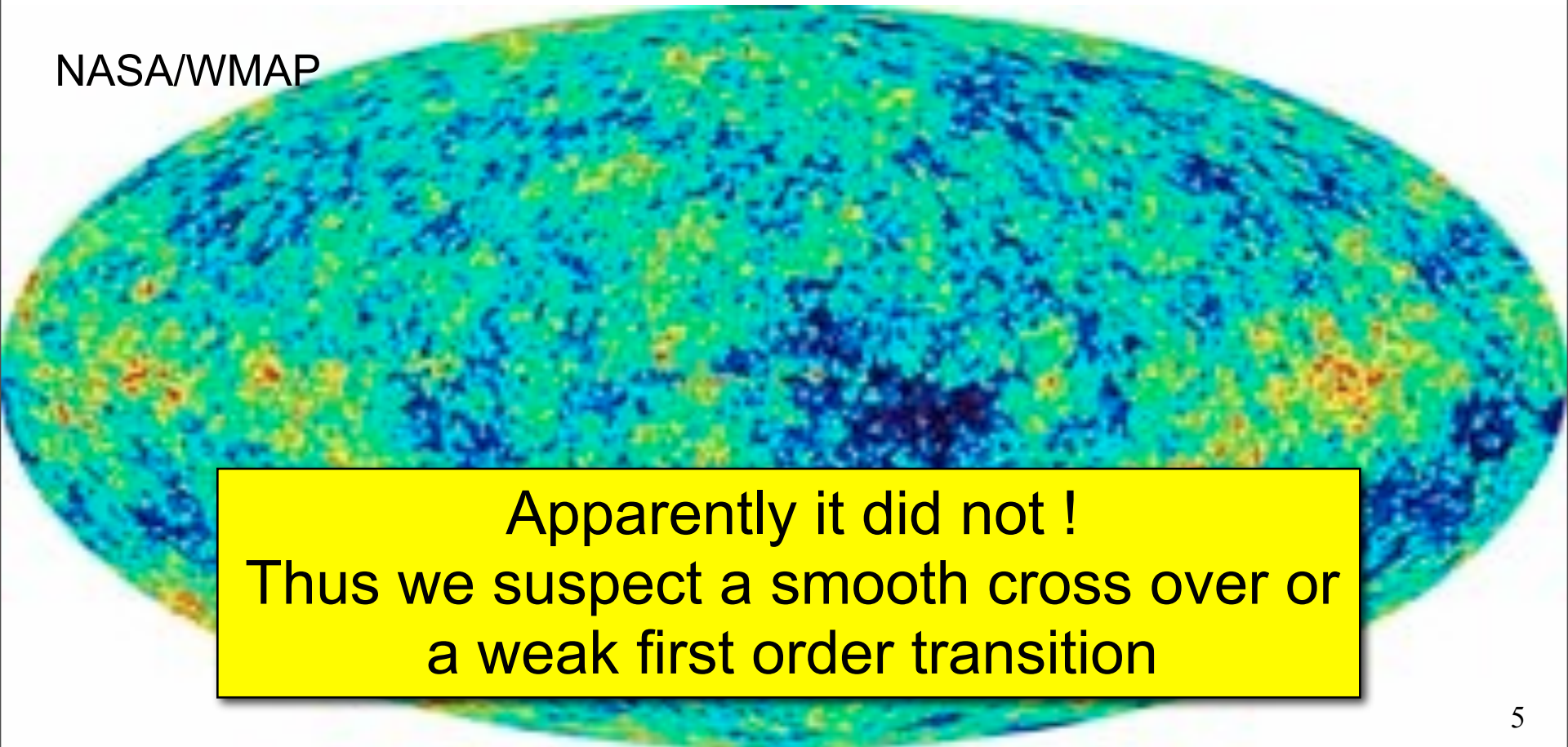
“A **first-order QCD phase transition** that occurred in the early universe would lead to a **surprisingly rich cosmological scenario.**” Ed Witten, Phys. Rev. D (1984)



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NASA/WMAP



The language of RHI collisions

- Before starting, we need to know some specific terminology used in RHI collisions.

- Relativity: Energy: $E^2 = p^2 + m^2$ or $E = T + m$ or $E = \gamma m$

where: $\gamma = \frac{1}{\sqrt{(1 - \beta^2)}}$ and $\beta = \frac{v}{c} = \frac{p}{E}$

- Lorentz Transformations: $E' = \gamma(E + \beta p_z)$

$$p'_z = \gamma(p_z + \beta E)$$

- Kinematics:

$$p_L = p_z$$

$$p_T = \sqrt{(p_x^2 + p_y^2)}$$

$$m_T = \sqrt{(p_T^2 + m^2)}$$

Transverse mass

$$y = \frac{1}{2} \ln \frac{E + p_L}{E - p_L}$$

Rapidity

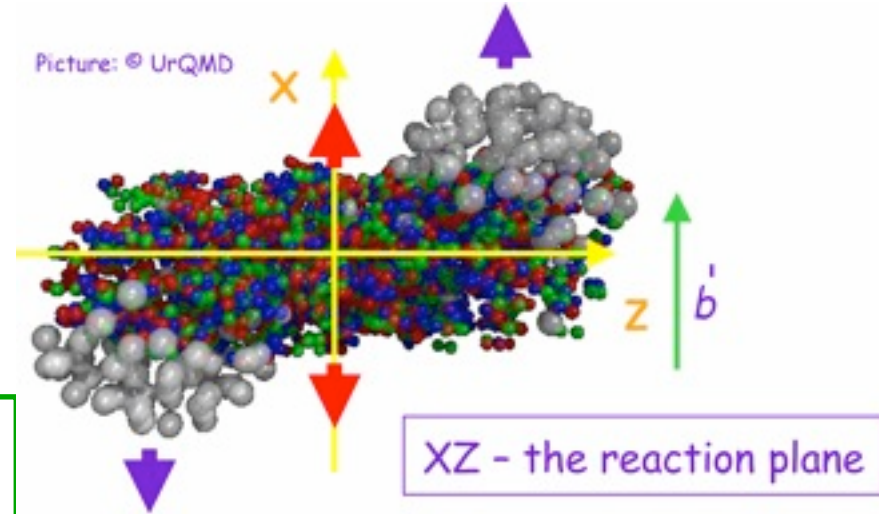
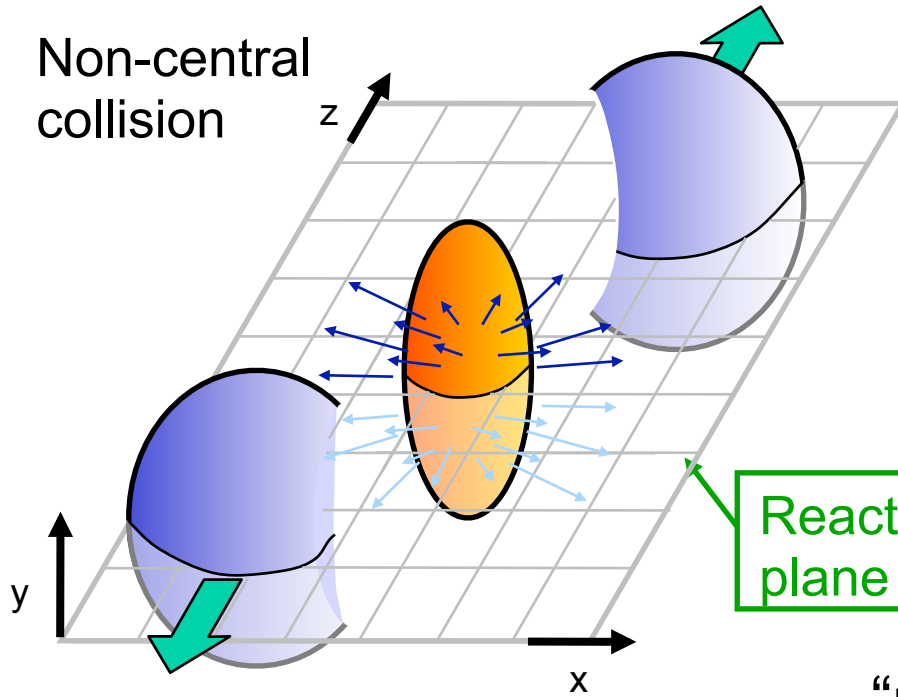
$$y' = y + \tanh^{-1} \beta$$

$$\eta = \frac{1}{2} \ln \frac{p + p_L}{p - p_L}$$

Pseudo-Rapidity
(no particle id required)

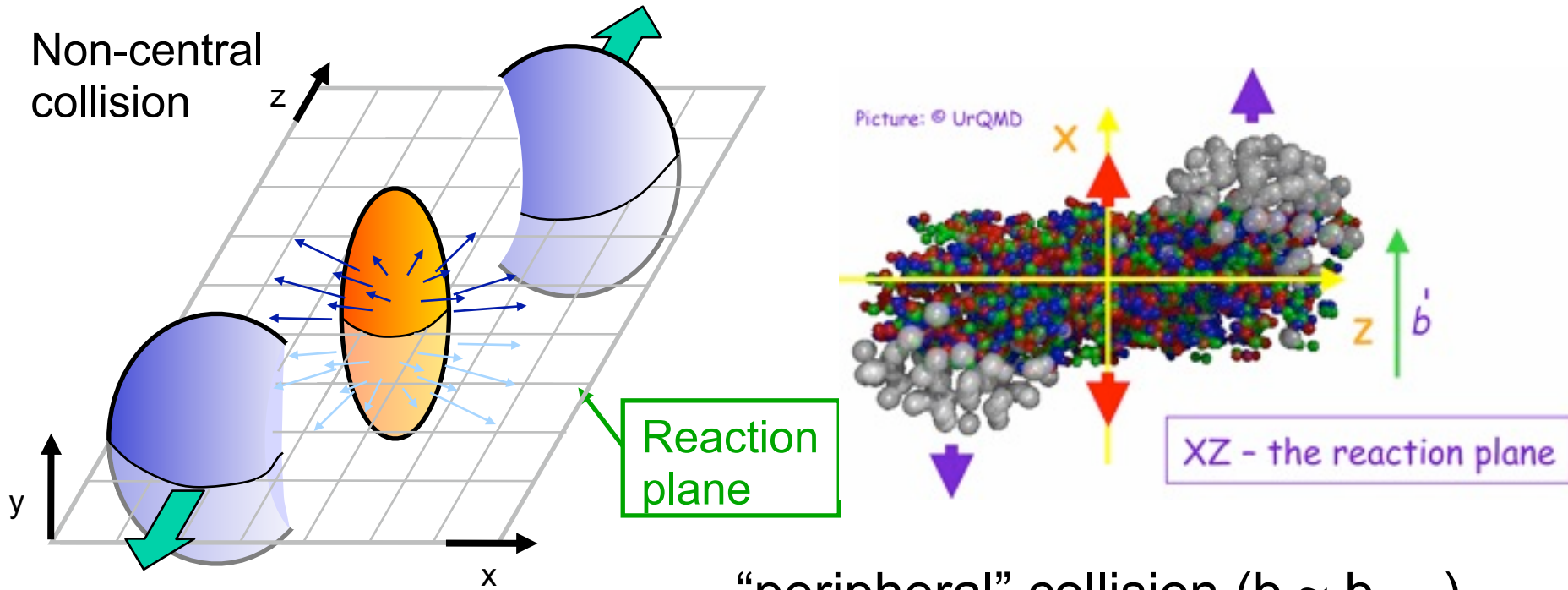
Geometry of a heavy-ion collision

Non-central collision



“peripheral” collision ($b \sim b_{\max}$)
“central” collision ($b \sim 0$)

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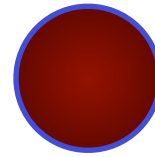
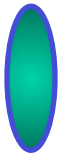
Number of participants (N_{part}): number of incoming nucleons (participants) in the overlap region

Number of binary collisions (N_{bin}): number of equivalent inelastic nucleon-nucleon collisions

$$N_{\text{bin}} \geq N_{\text{part}}$$

Quantifying the geometry

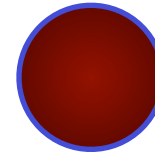
p+p: 2 Participants, 1 Binary Collision



Participants: those nucleons that have interacted at least once
Binary collisions: the number of 1+1 collisions

Quantifying the geometry

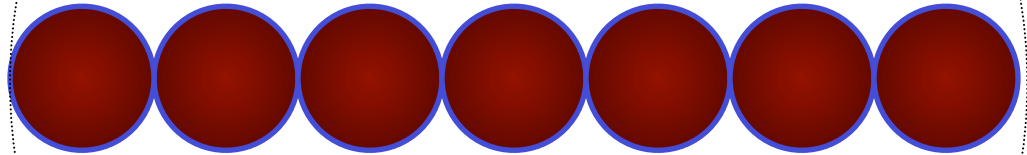
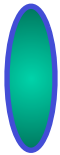
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Quantifying the geometry

p+A: 8 Participants, 7 Binary Collisions



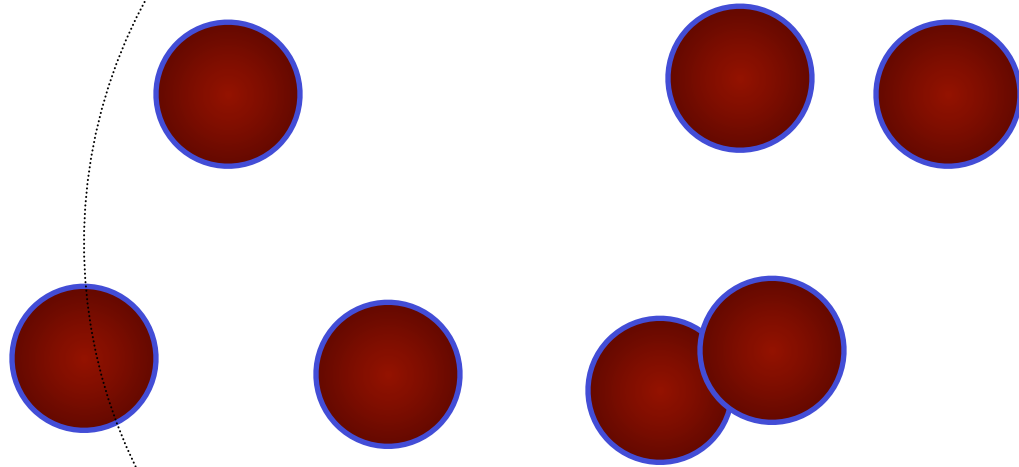
Generically: $N_{\text{part}} = N_{\text{bin}} + 1$

Participants: those nucleons that have interacted at least once

Binary collisions: the number of 1+1 collisions

Quantifying the geometry

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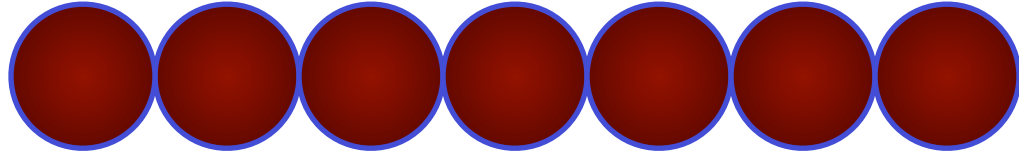
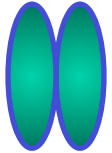
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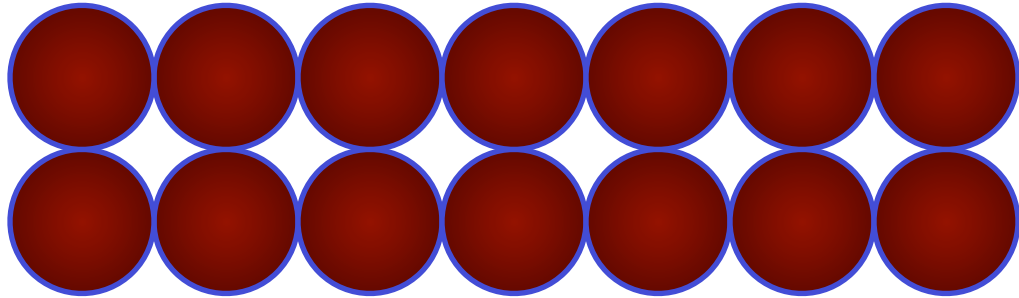
Binary collisions: the number of 1+1 collisions

Quantifying the geometry

A+A: 9 Participants, 14 Binary Collisions



A+A: 16 Participants, 14 Binary Collisions

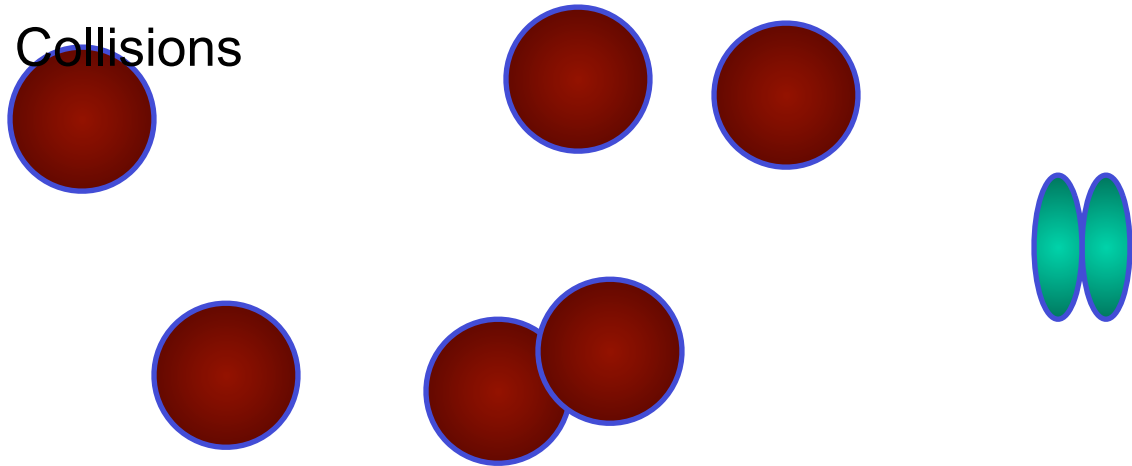


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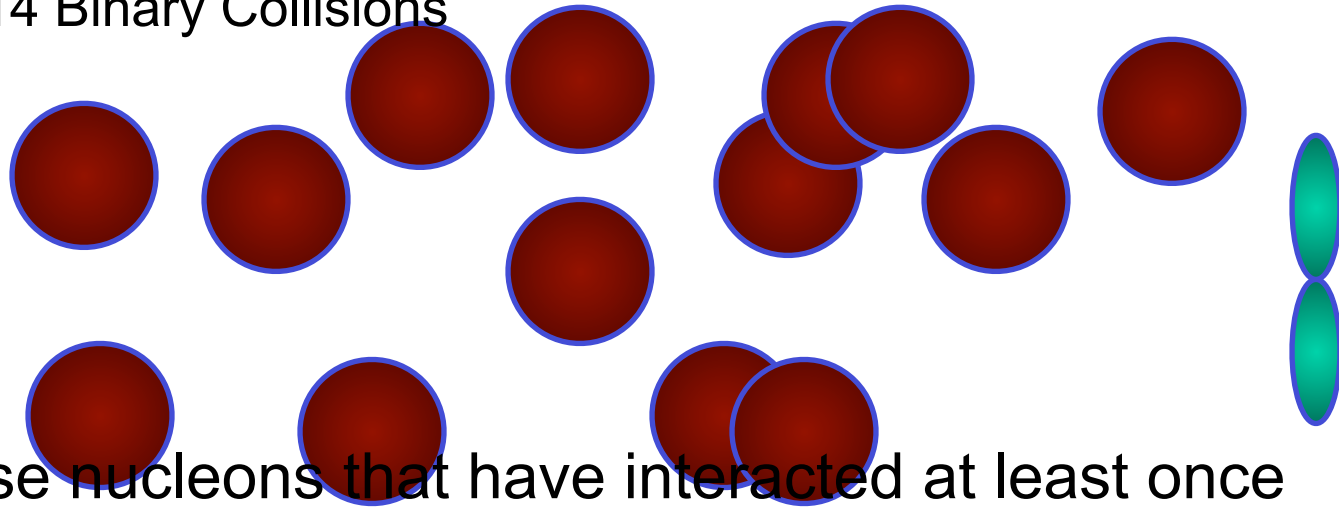
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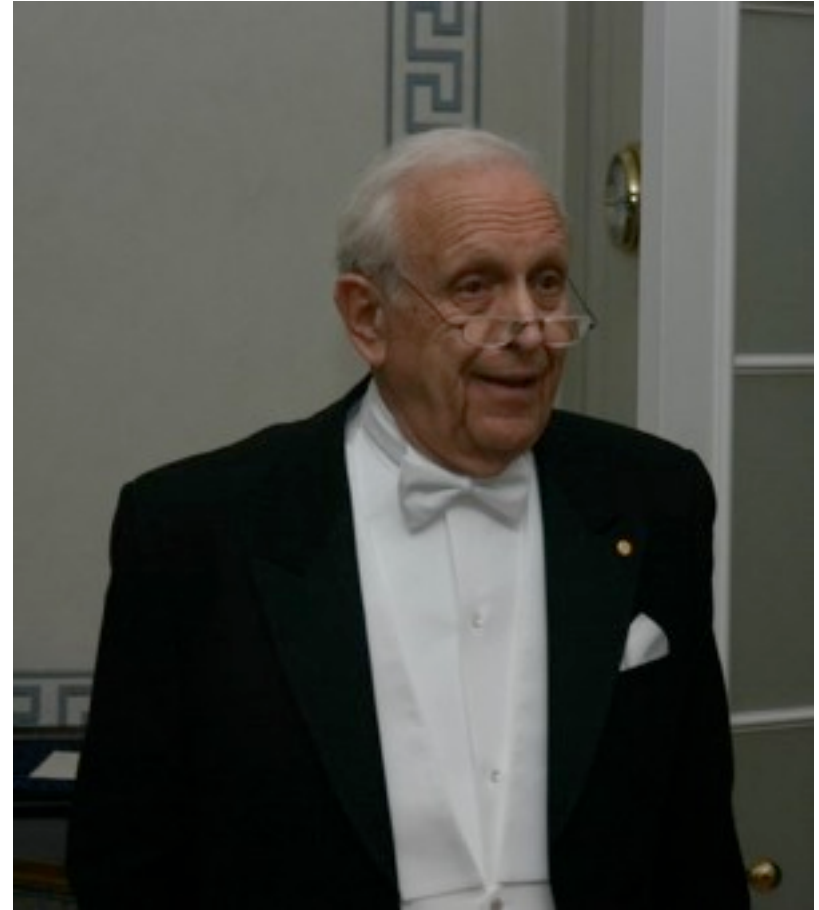


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Binary collisions: the number of 1+1 collisions

Glauber calculations

Use a Glauber calculation to estimate N_{bin} and N_{part}

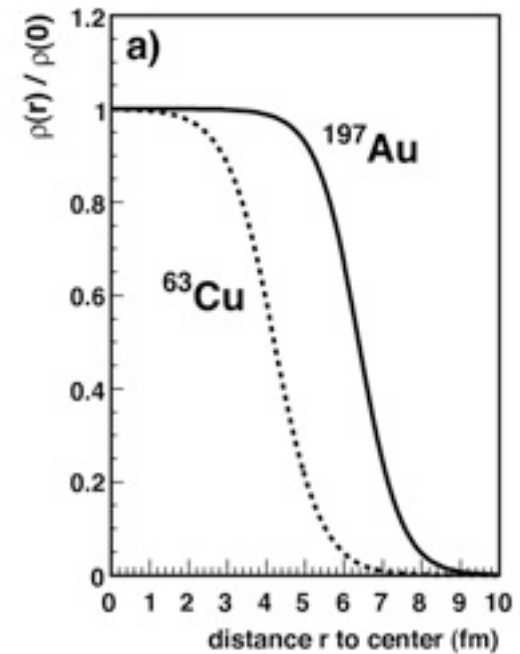
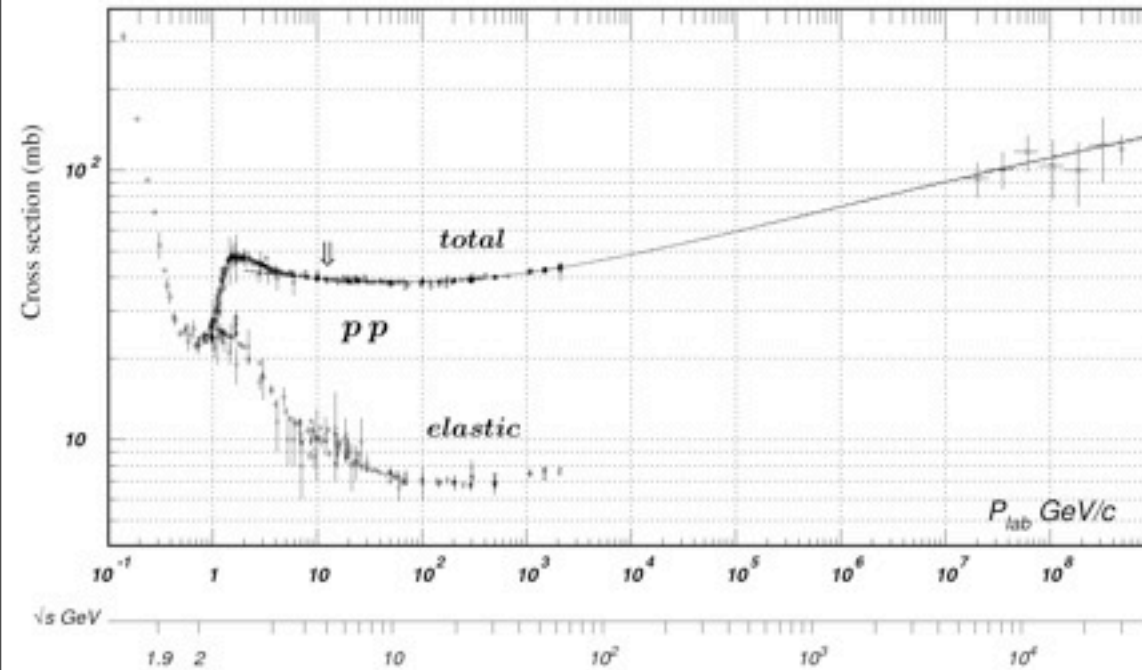
- Roy Glauber: Nobel prize in physics 2005 for “his contribution to the quantum theory of optical coherence”
- Application of Glauber theory to heavy ion collisions does not use the full sophistication of these methods. Two simple assumptions:
 - Eikonal: constituents of nuclei proceed in straight-line trajectories
 - Interactions determined by initial-state shape of overlapping nuclei



Ingredients for Glauber calculations

Particle Data Book: W.-M. Yao et al., J. Phys. G 33,1 (2006) Fig 40.11

M. Miller et al, nucl-ex/0701025



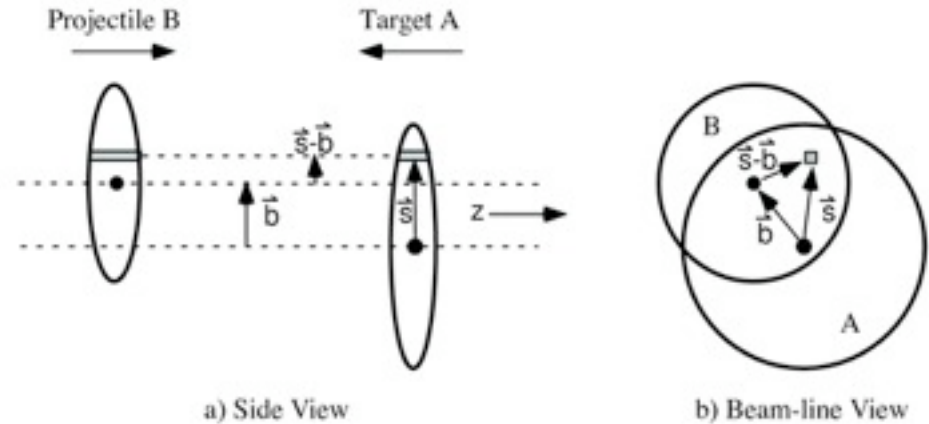
- Assumptions: superposition of straight-line interactions of colliding nucleons
- Need nucleon-nucleon interaction cross section
 - Most use inelastic: 42 mb at $\sqrt{s}=200$ GeV
 - Other choices: Non-singly-diffractive, 30 mb at $\sqrt{s} = 200$ GeV
- Need probability density for nucleons:
 - 'Wood-Saxon' from electron scattering experiments

Implementations of Glauber

M. Miller et al, nucl-ex/0701025

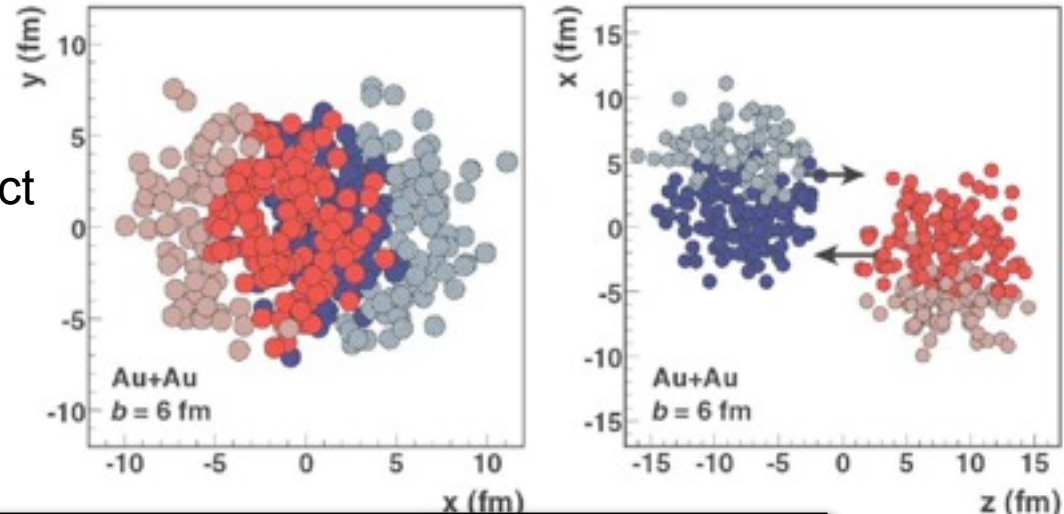
- **Optical Glauber**

- Smooth distribution assumed
- Analytic overlap calculation from integration over nuclear shape functions, weighted with appropriate N-N cross-section



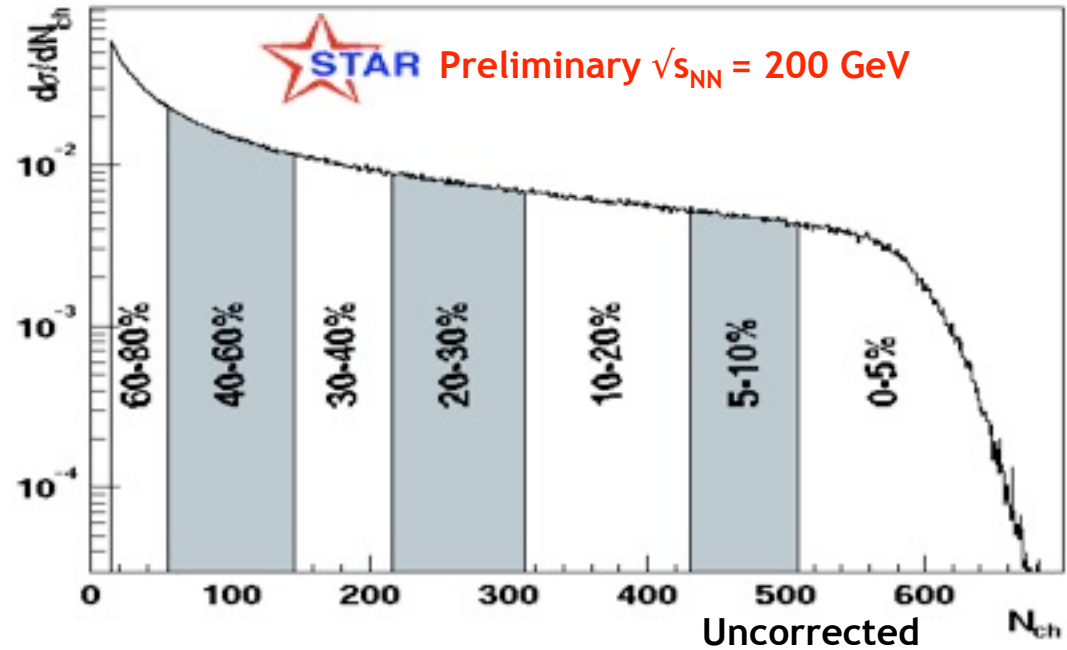
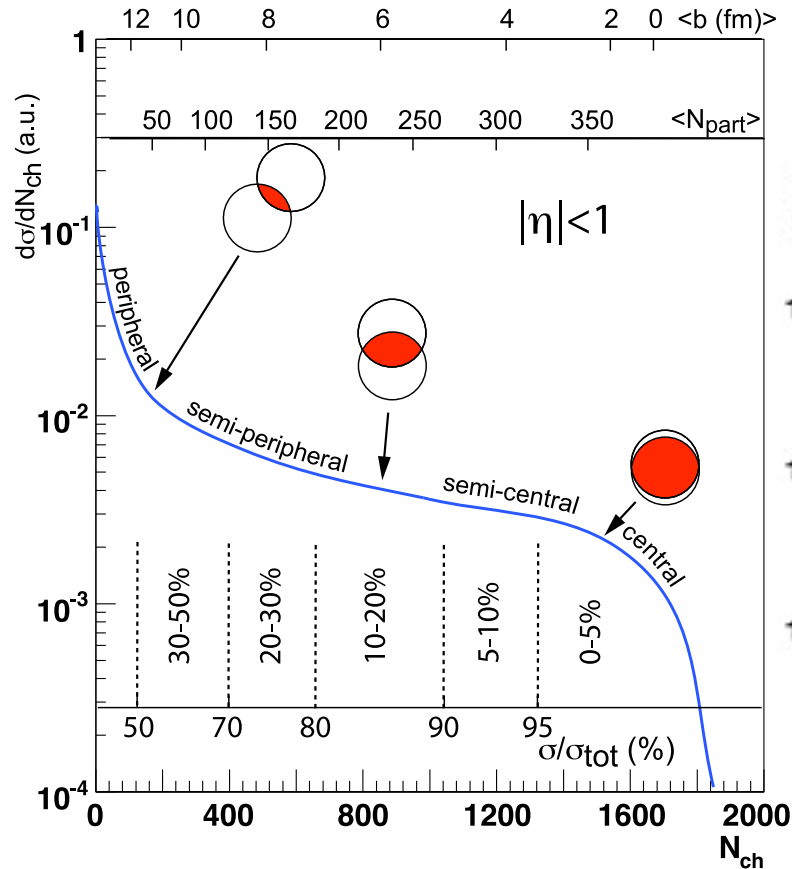
- **Monte Carlo Glauber**

- Randomly initialize nucleons sampling nuclear shape
- At randomly selected impact parameter, allow nuclei to interact
- Randomly sample probability of nucleons to interact from interaction cross-section
 - e.g. if distance d between nucleons is $< \sqrt{\sigma_{int}/\pi}$



Calculate probability that N_{part} or N_{bin} occurs per event

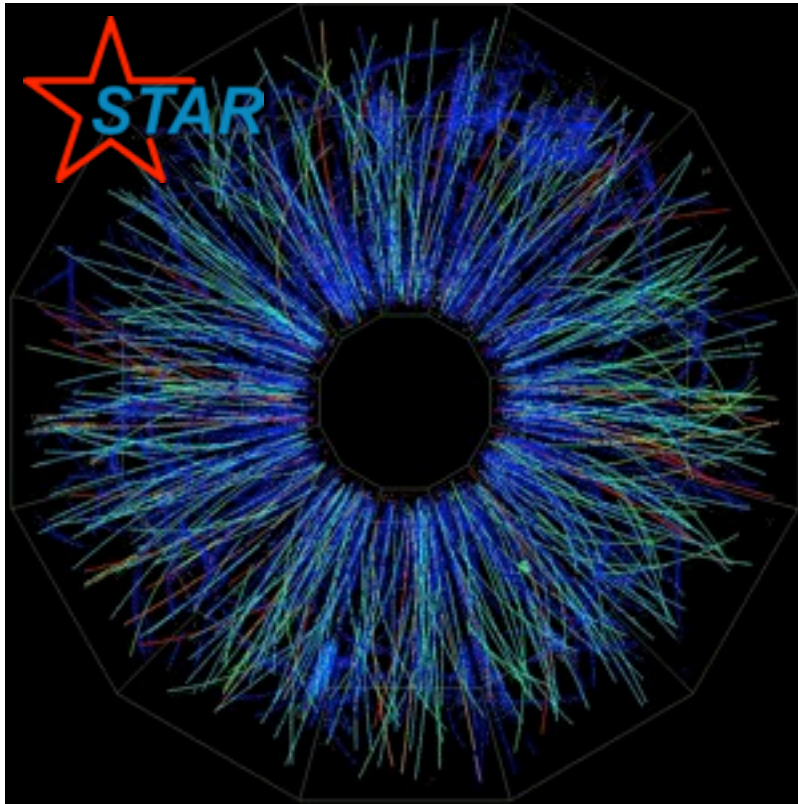
Comparing to data heavy-ion collision



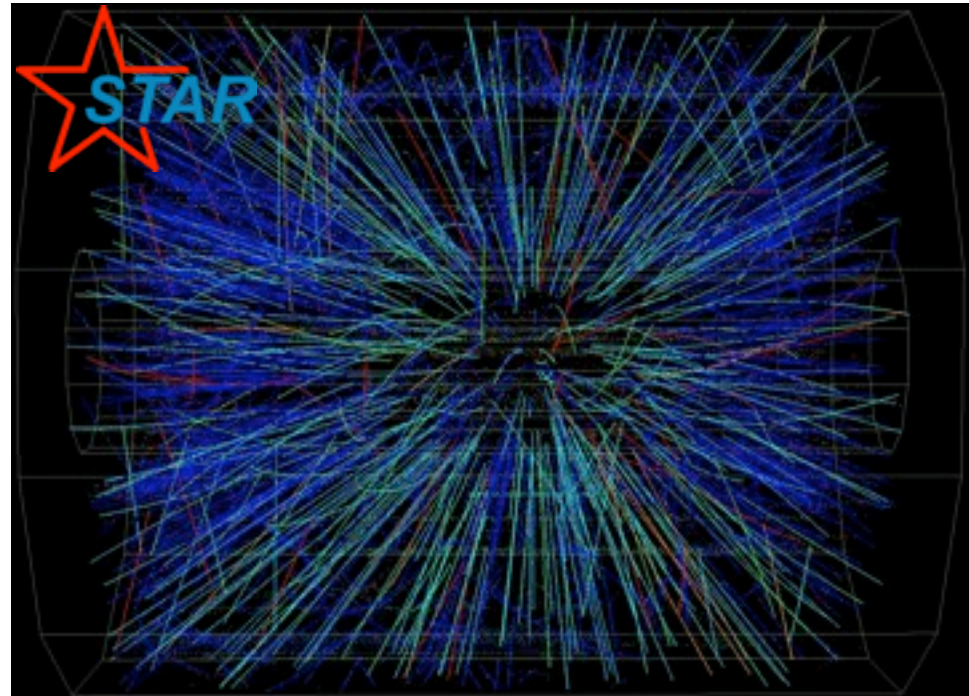
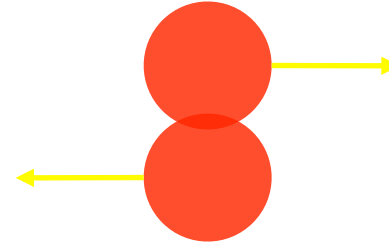
Good agreement between data and calculation

Measured mid-rapidity particle yield can be related to size of overlap region

A peripheral Au-Au collision

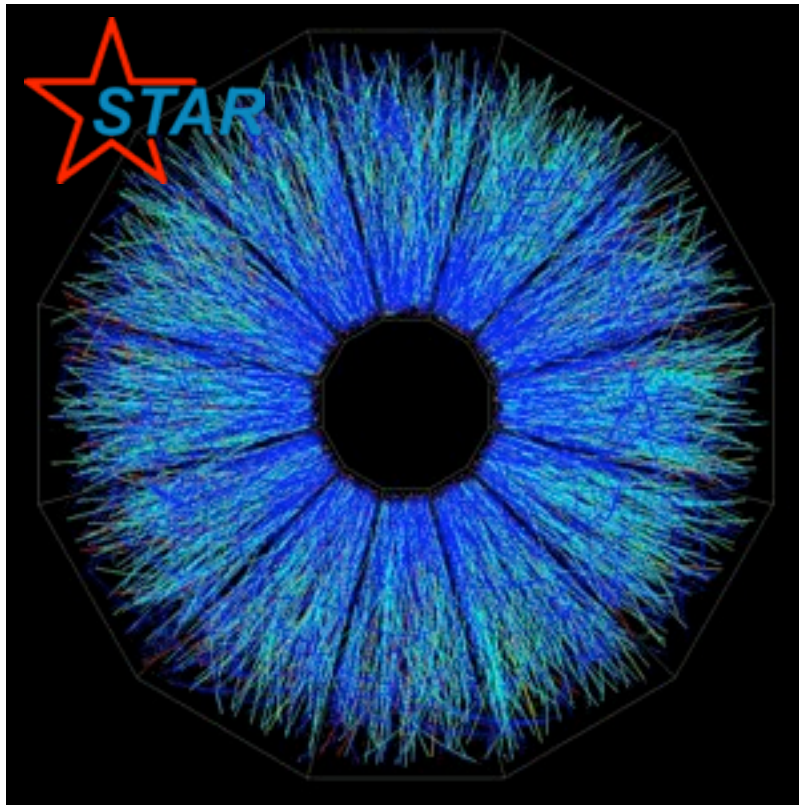


Peripheral Collision



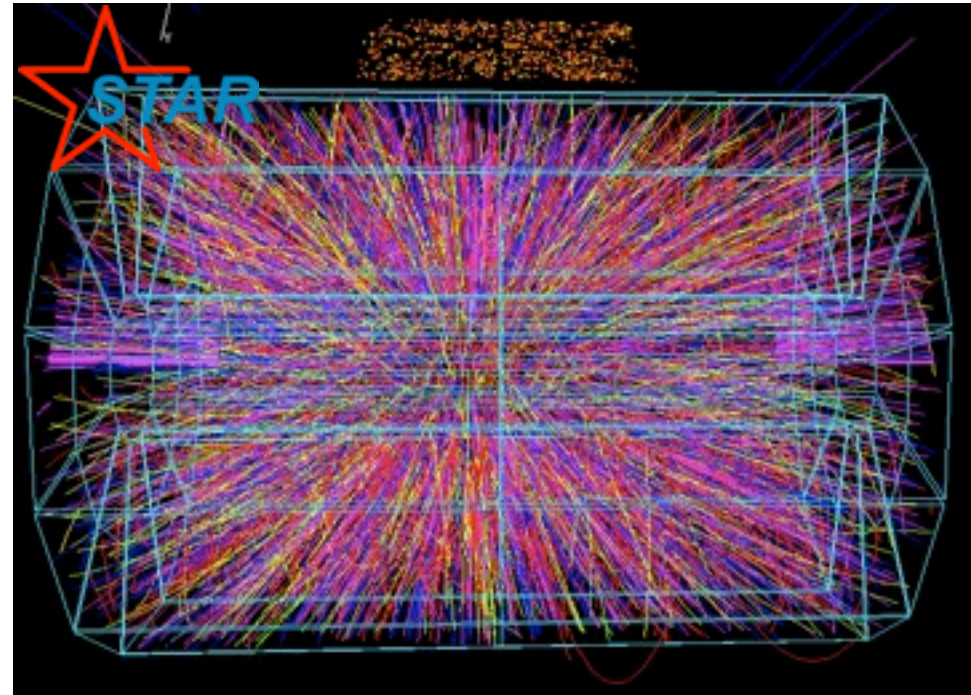
Color \Rightarrow Energy loss in TPC gas

39.4 TeV in central Au-Au collision

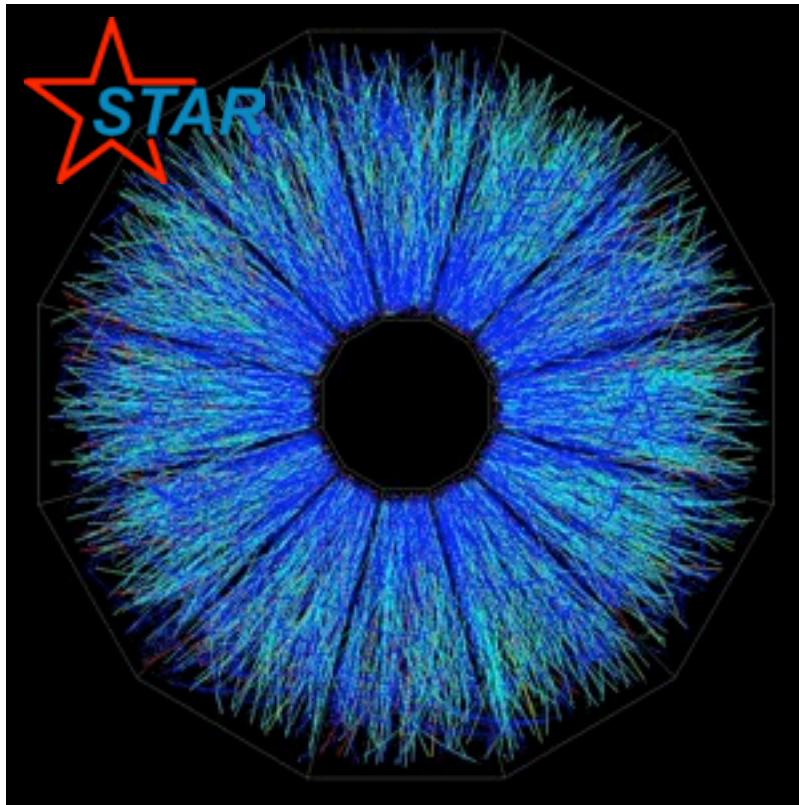


>5000 hadrons and leptons

- Only **charged** particles shown
- Neutrals don't ionise the TPC's gas so are not "seen" by this detector.



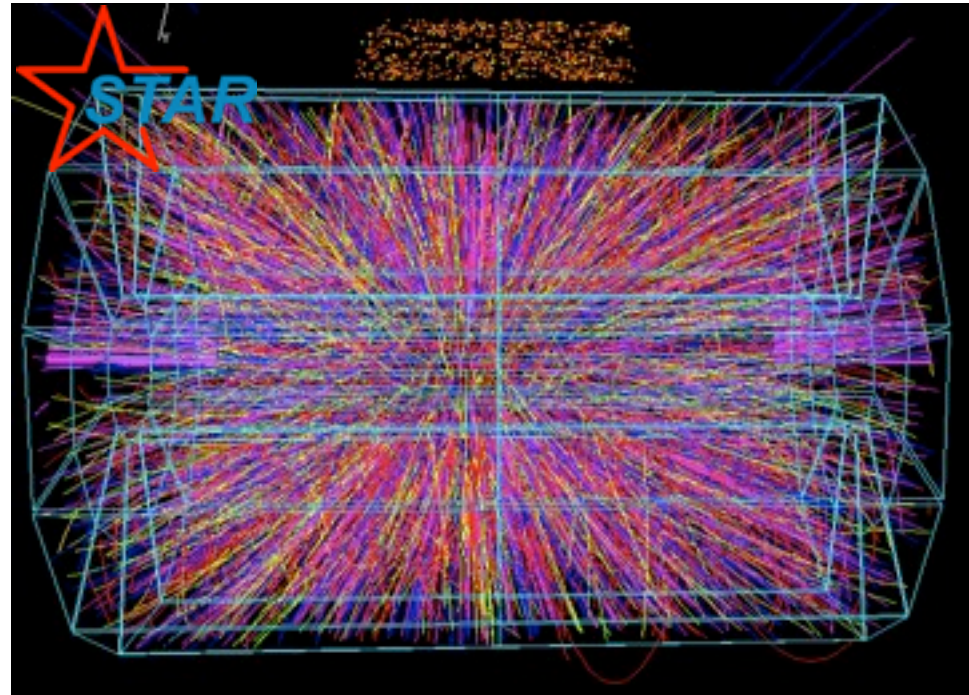
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>5000 hadrons and leptons

26 TeV is removed
from colliding beams.

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The energy is contained in one collision

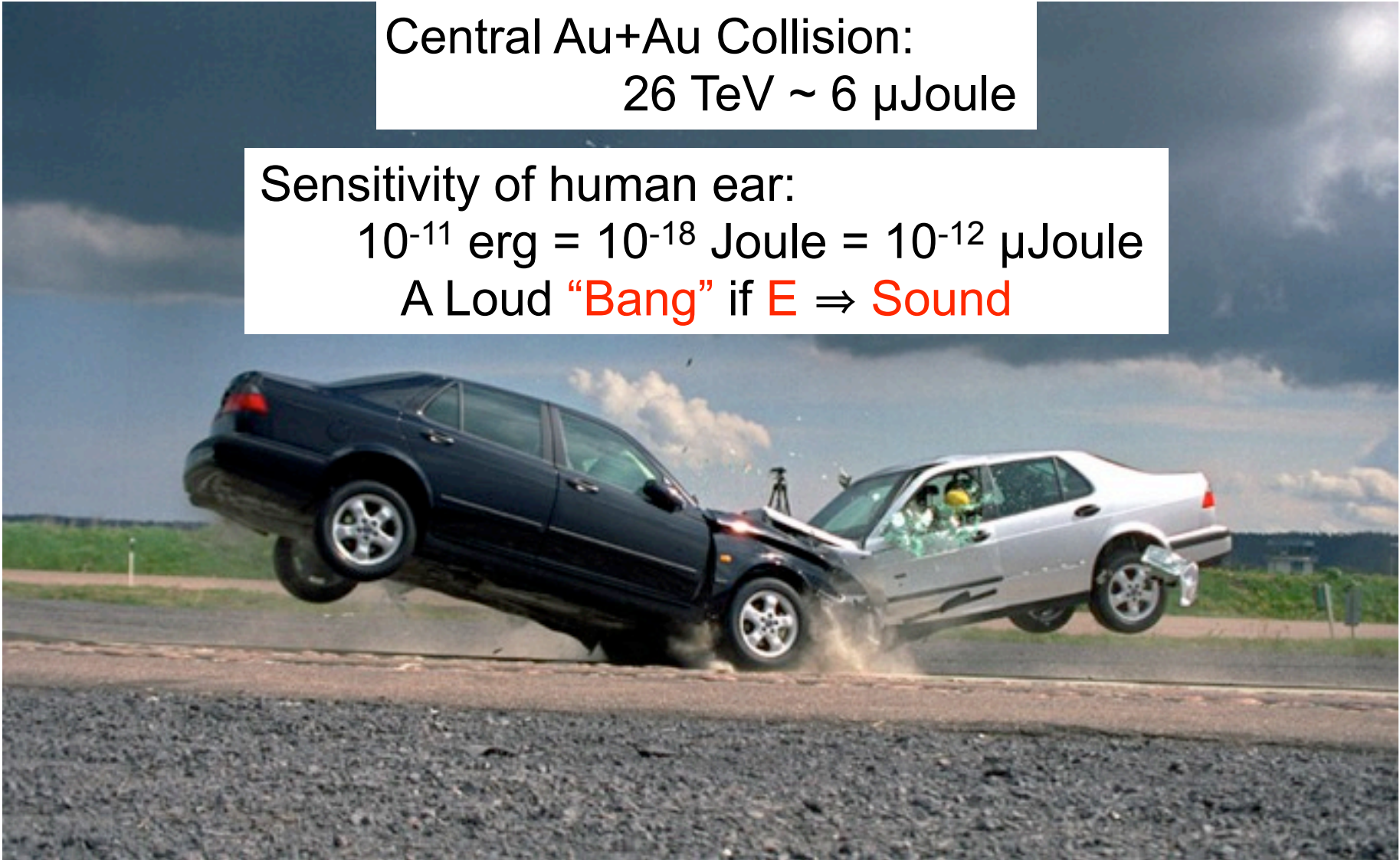
Central Au+Au Collision:
26 TeV \sim 6 μ Joule



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Sensitivity of human ear:
 10^{-11} erg = 10^{-18} Joule = 10^{-12} μ Joule
A Loud “Bang” if $E \Rightarrow$ Sound



The energy is contained in one collision

Central Au+Au Collision:
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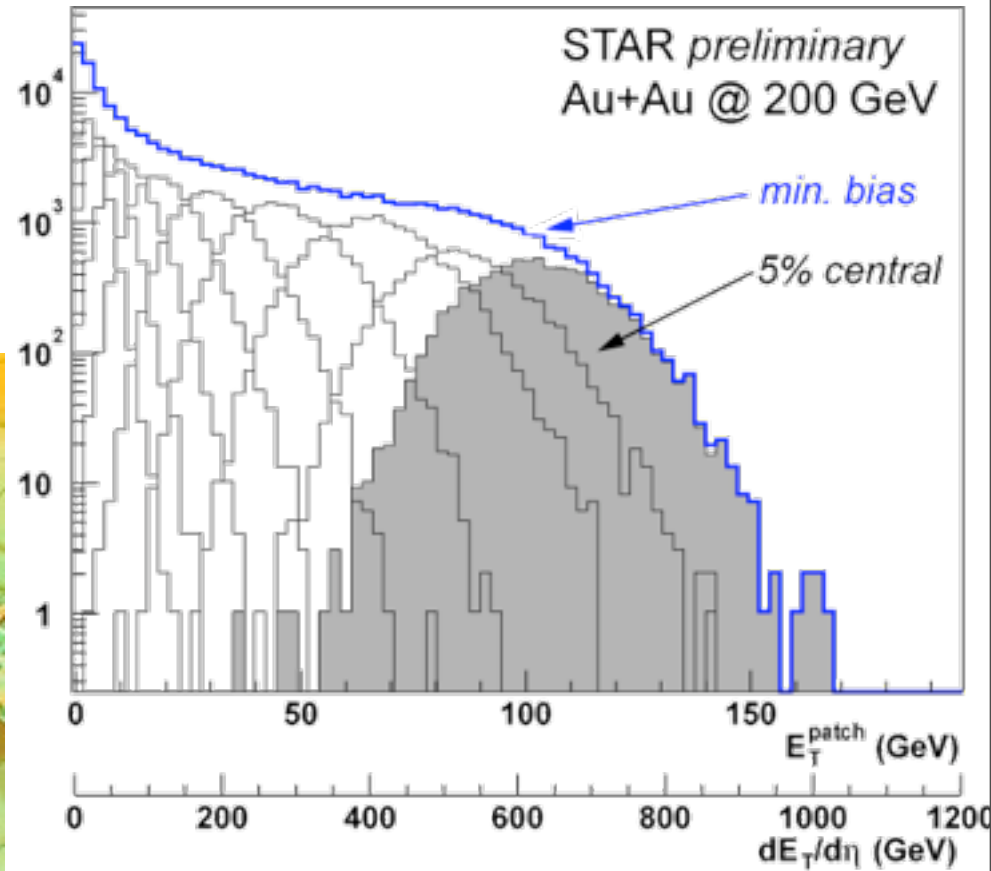
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Most goes into particle creation

Energy density in central Au-Au collisions

- use calorimeters to measure total energy



Energy density in central Au-Au collisions

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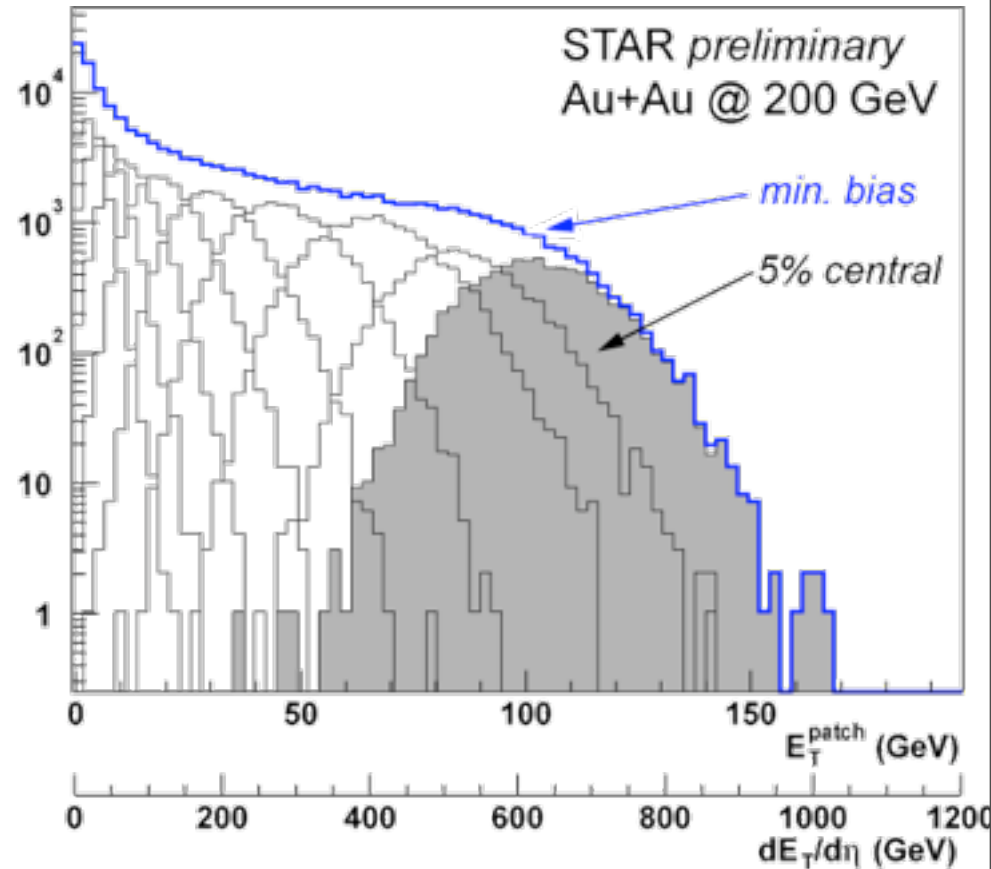
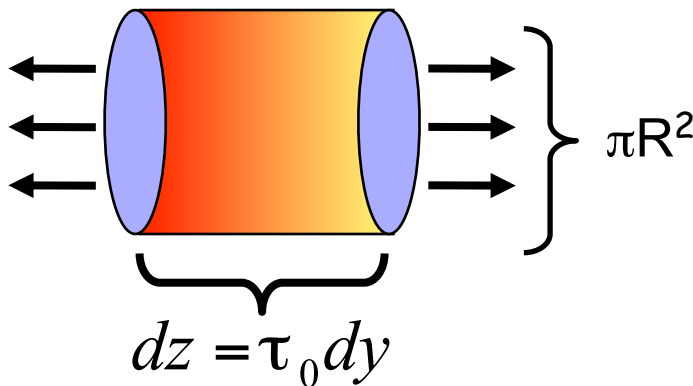
- estimate volume of collision

Bjorken-Formula for Energy Density:

$$\varepsilon_{Bj} = \frac{\Delta E_T}{\Delta V} = \frac{1}{\pi R^2} \frac{1}{\tau_0} \frac{dE_T}{dy}$$

$R \sim 6.5$ fm

Time it takes to thermalize system ($t_0 \sim 1$ fm/c)



Energy density in central Au-Au collisions

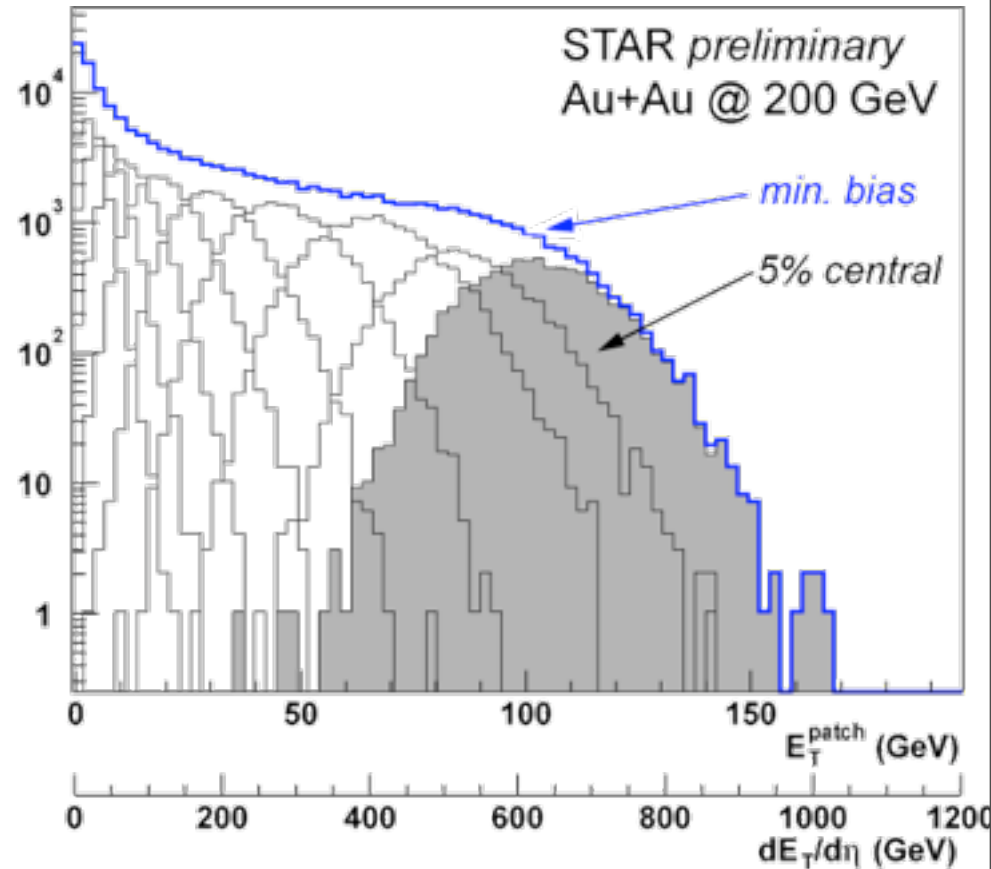
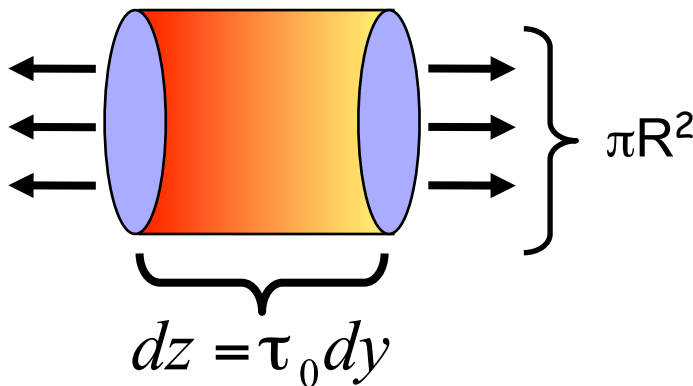
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$\epsilon_{BJ} \approx 5.0 \text{ GeV/fm}^3$
 ~ 30 times normal nuclear density
 ~ 5 times $> \epsilon_{\text{critical}}$ (lattice QCD)

5 GeV/fm³. Is that a lot?

In a year, the U.S. uses ~100 quadrillion BTUs of energy
(1 BTU = 1 burnt match):

$$100 \times 10^{15} \text{ BTU} \times \frac{1060 \text{ J}}{\text{BTU}} \times \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} = 6.6 \times 10^{38} \text{ eV}$$

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At 5 GeV/fm³, this would fit in a volume of:

$$6.6 \times 10^{38} \text{ eV} \div \frac{5 \times 10^9 \text{ eV}}{\text{fm}^3} = 1.3 \times 10^{29} \text{ fm}^3$$

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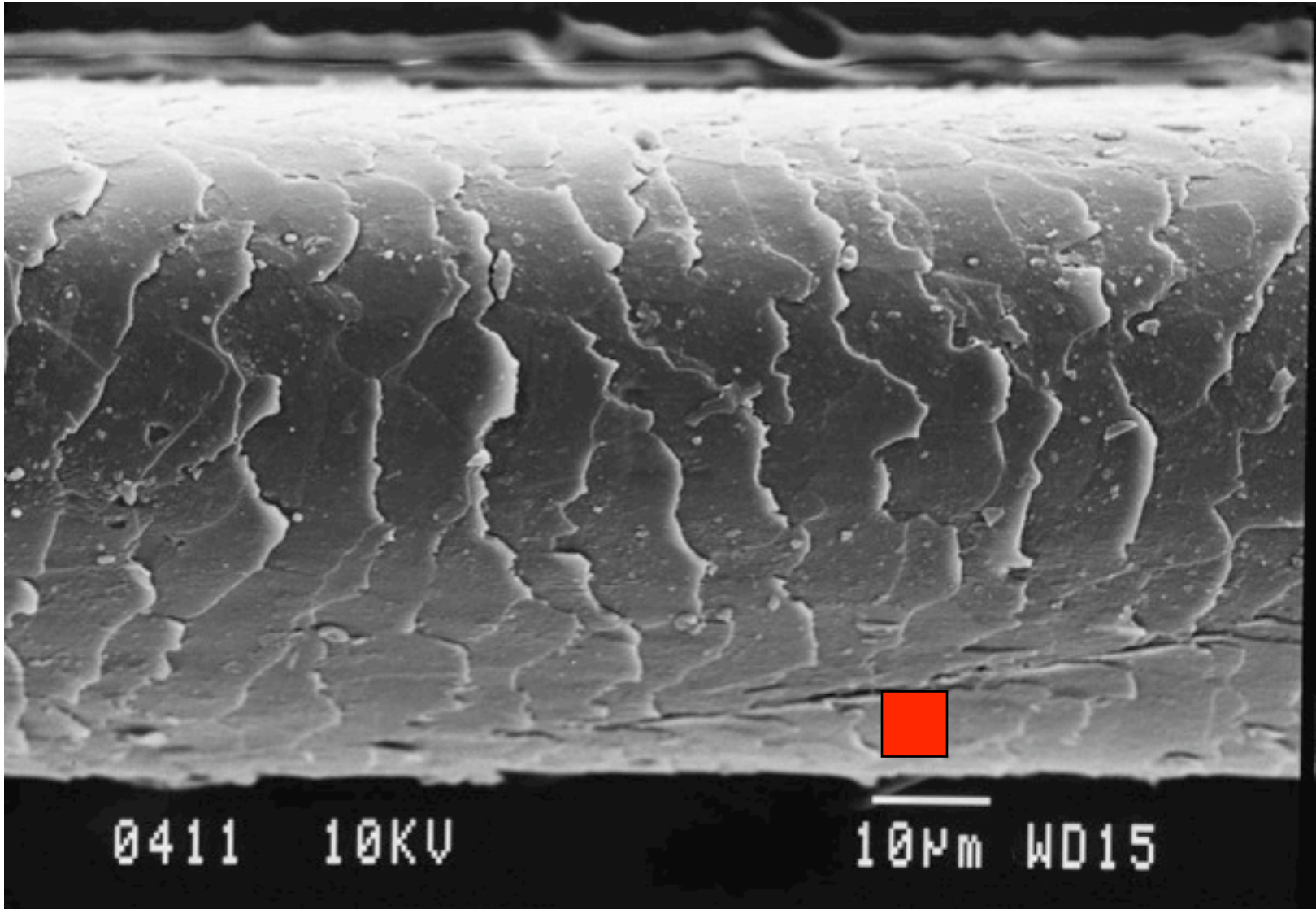
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Or, in other words, in a box of the following dimensions:

$$\sqrt[3]{1.3 \times 10^{29} \text{ fm}^3} = 5 \times 10^9 \text{ fm} = 5 \mu\text{m}$$

A human hair



What is the temperature of the medium?

- **Statistical Thermal Models:**
 - Assume a system that is **thermally** (constant T_{ch}) and **chemically** (constant n_i) **equilibrated**
 - System composed of non-interacting hadrons and resonances
 - Obey conservation laws: Baryon Number, Strangeness, Isospin
- Given T_{ch} and μ 's (+ system size), n_i 's can be calculated in a grand canonical ensemble

$$n_i = \frac{g}{2\pi^2} \int_0^{\infty} \frac{p^2 dp}{e^{(E_i(p) - \mu_i)/T} \pm 1}, \quad E_i = \sqrt{p^2 + m_i^2}$$

Fitting the particle ratios

Number of particles of a given species related to temperature

$$dn_i \sim e^{-(E-\mu_B)/T} d^3p$$

- Assume all particles described by same temperature T and μ_B
- one ratio (e.g., \bar{p} / p) determines μ / T :

$$\frac{\bar{p}}{p} = \frac{e^{-(E+\mu_B)/T}}{e^{-(E-\mu_B)/T}} = e^{-2\mu_B/T}$$

- A second ratio (e.g., K / π) provides $T \rightarrow \mu$

$$\frac{K}{\pi} = \frac{e^{-E_K/T}}{e^{-E_\pi/T}} = e^{-(E_K - E_\pi)/T}$$

- Then all other hadronic ratios (and yields) defined

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A. Adronic *et al.*, NPA772:167

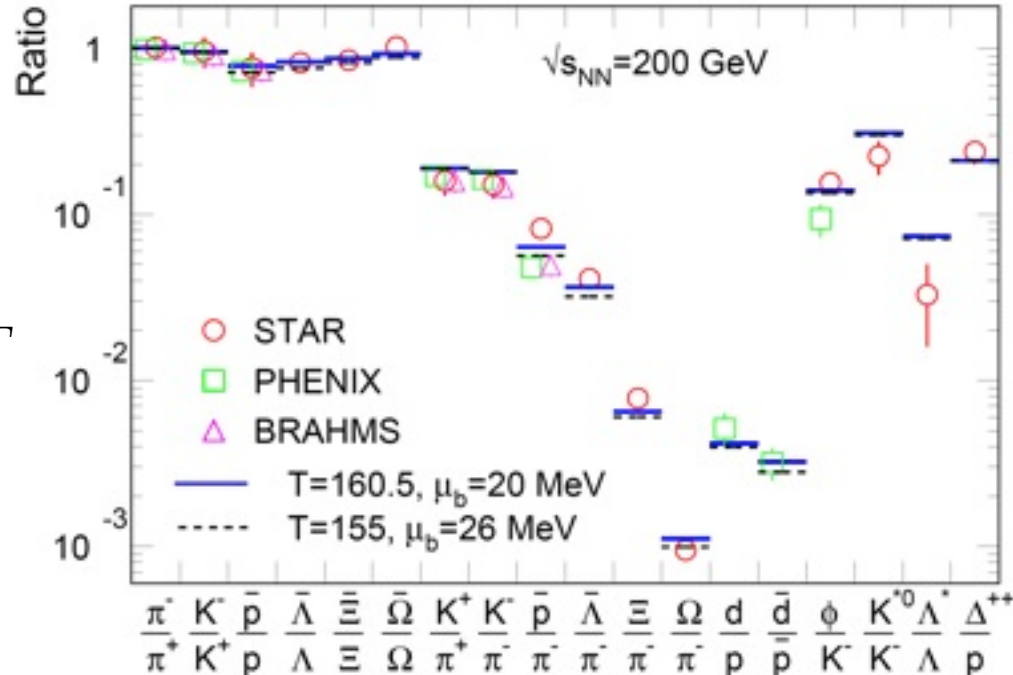
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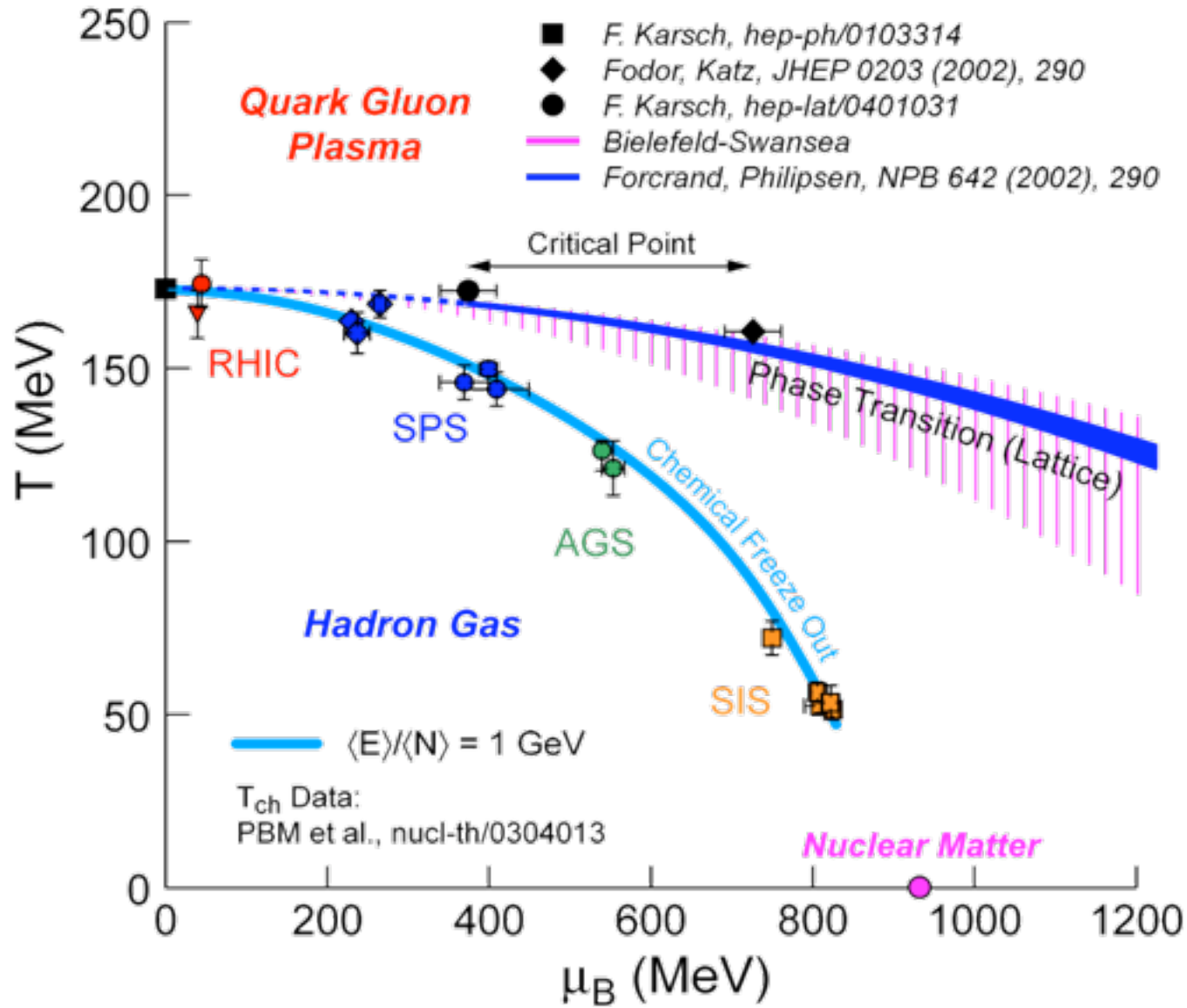
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$T \sim 160 \text{ MeV}, \mu_b \sim 20 \text{ MeV}$

Initial Temperature probably much higher

Where RHIC sits on the phase diagram



Off on a tangent

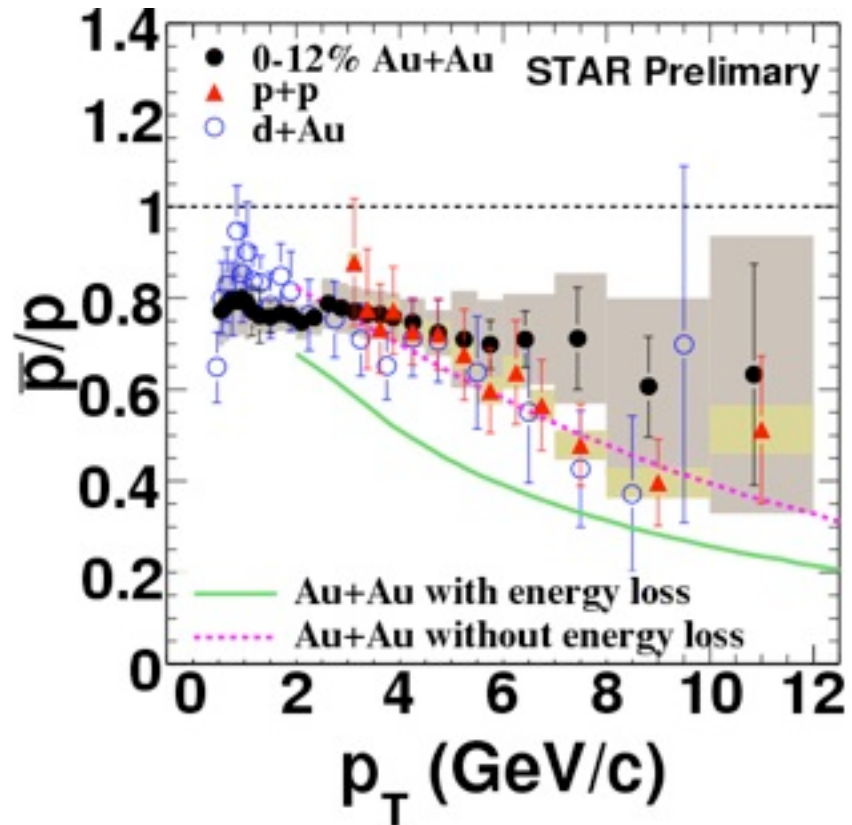
Take a second look at the anti-proton/proton ratio

$$\bar{p}/p \sim 0.8$$

There is a net baryon number at mid-rapidity!!

Baryons number is being transported over 6 units of rapidity from the incoming beams to the collision zone!

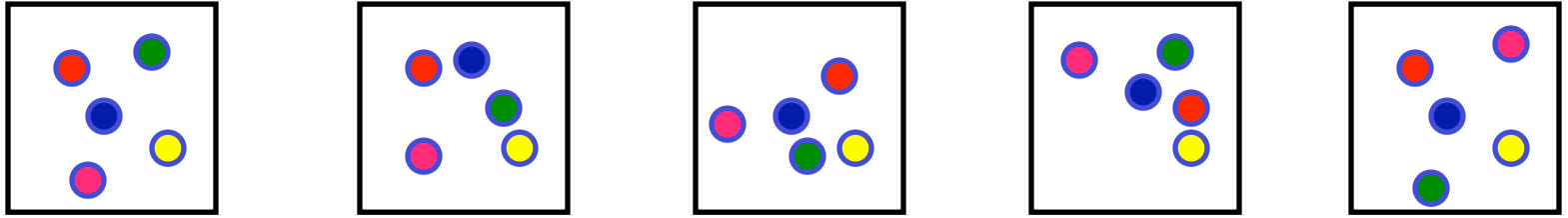
Consider what impulse that must be



Baryon number not carried by quarks
- baryon junctions postulated

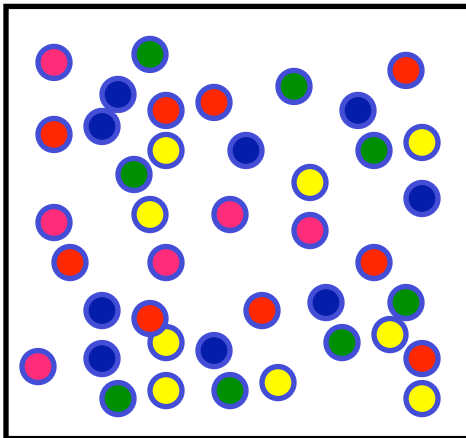
Statistics \neq thermodynamics

$p+p$



Ensemble of events constitutes a statistical ensemble
T and μ are simply Lagrange multipliers
“Phase Space Dominance”

$A+A$



One (1) system is already statistical !

- We can talk about pressure
- T and μ are more than Lagrange multipliers

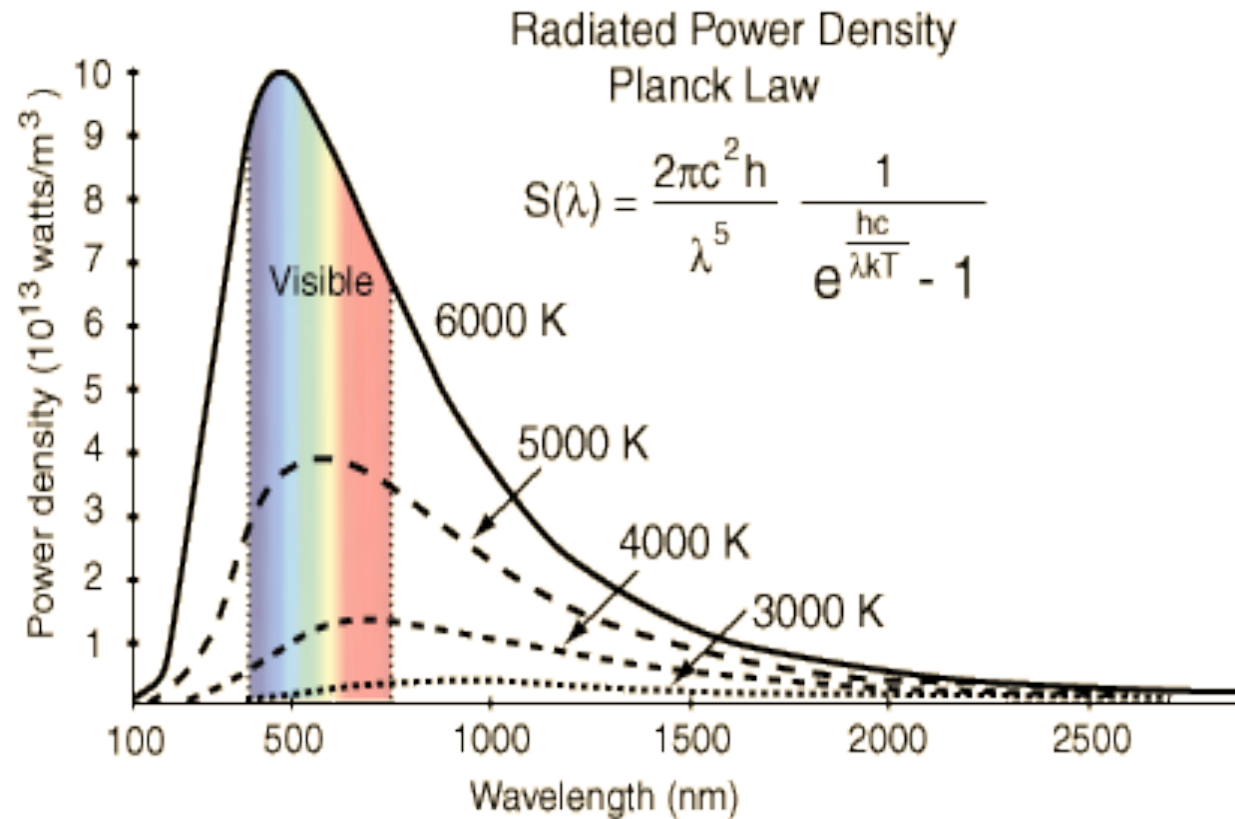
Evidence for thermalization

- Not all processes which lead to multi-particle production are thermal - elementary collisions
- *Any mechanism for producing hadrons which evenly populates the free particle phase space will mimic a microcanonical ensemble.*
- Relative probability to find n particles is the ratio of the phase-space volumes $P_n/P_{n'} = \varphi_n(E)/\varphi_{n'}(E) \Rightarrow$ given by statistics only.
- *Difference between MCE and CE vanishes as the size of the system N increases.*
- Such a system is NOT in thermal equilibrium - to thermalize need interactions/re-scattering

Need to look for evidence of collective motion

Blackbody radiation

Planck distribution describes **intensity** as a **function of the wavelength** of the emitted radiation

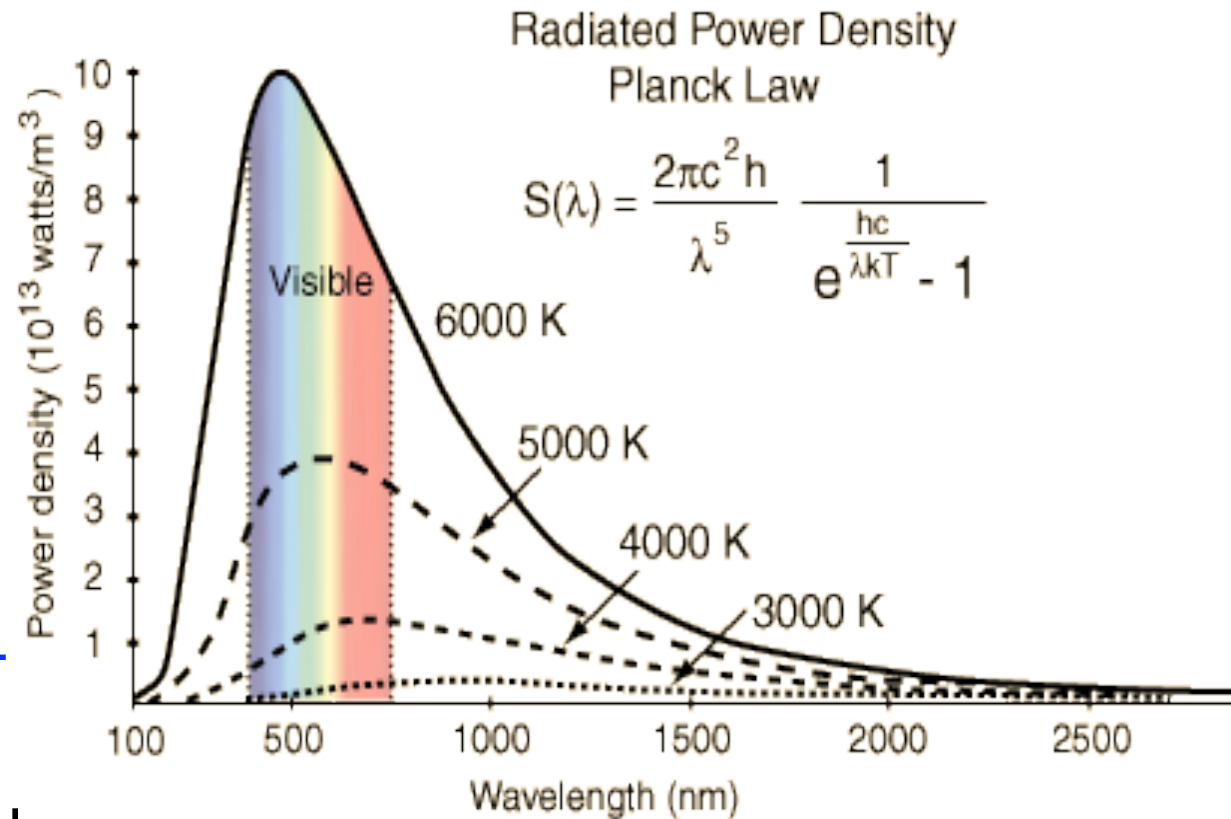


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“Blackbody” radiation is the spectrum of radiation emitted by an object at temperature T

As T increases curve changes

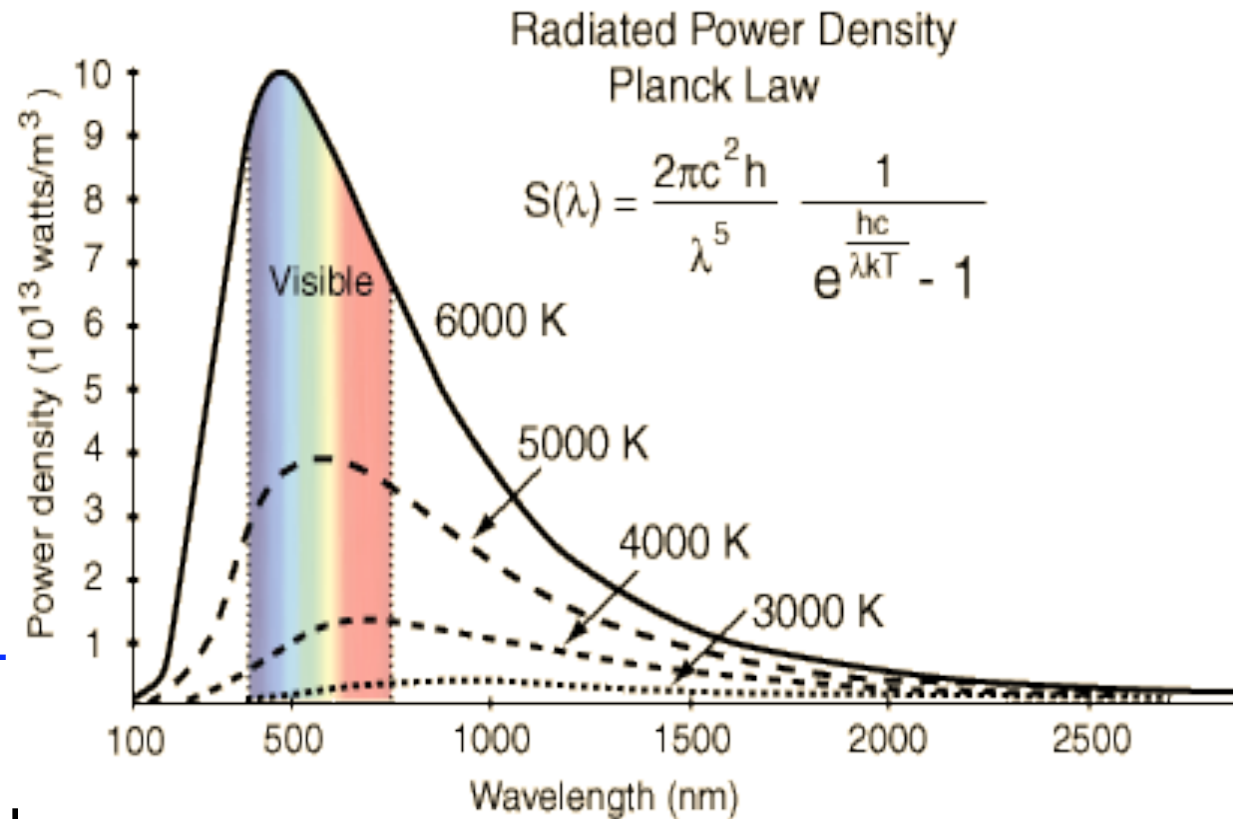


Blackbody radiation

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$$1/\text{Wavelength} \propto \text{Frequency} \propto E \propto p$$

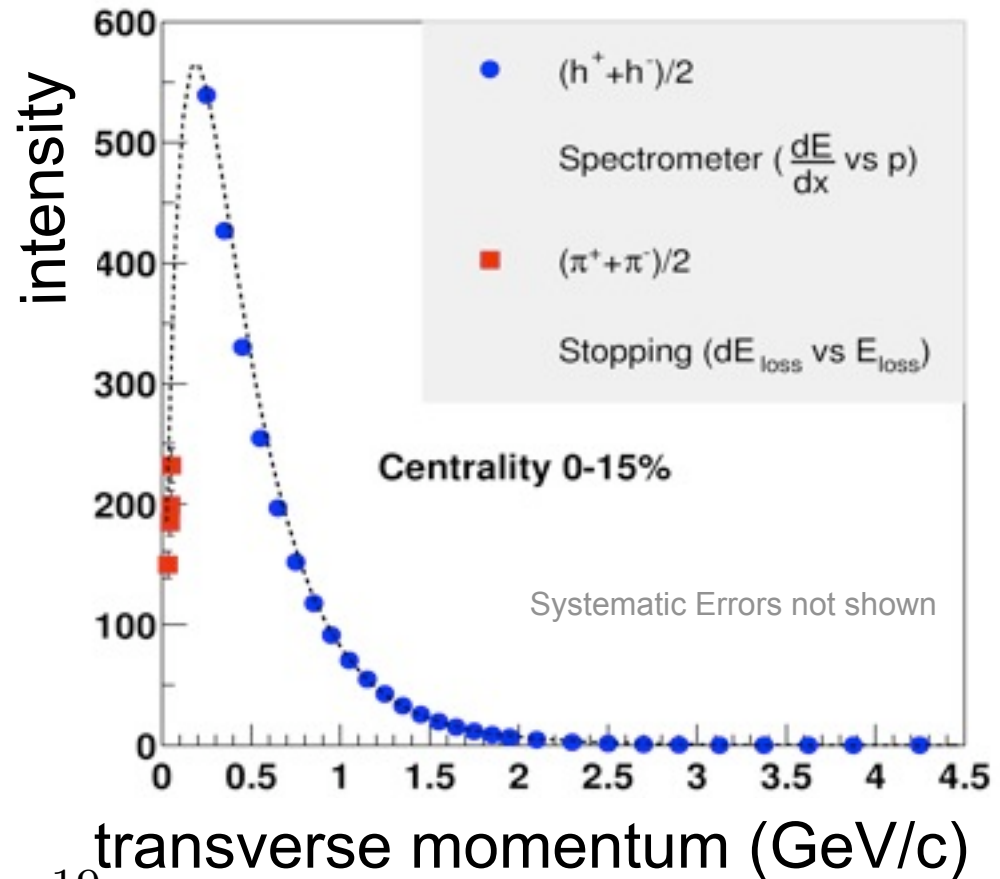
Determining the temperature

From transverse momentum distribution of pions deduce temperature ~ 120 MeV

$$E = \frac{3}{2}kT$$

$$T = \frac{2E}{3k}$$

$$= \frac{2 \times 120 \times 10^6}{3 \times 1.4 \times 10^{-23}} \times 1.6 \times 10^{-19}$$
$$\sim 9 \times 10^{11} K$$



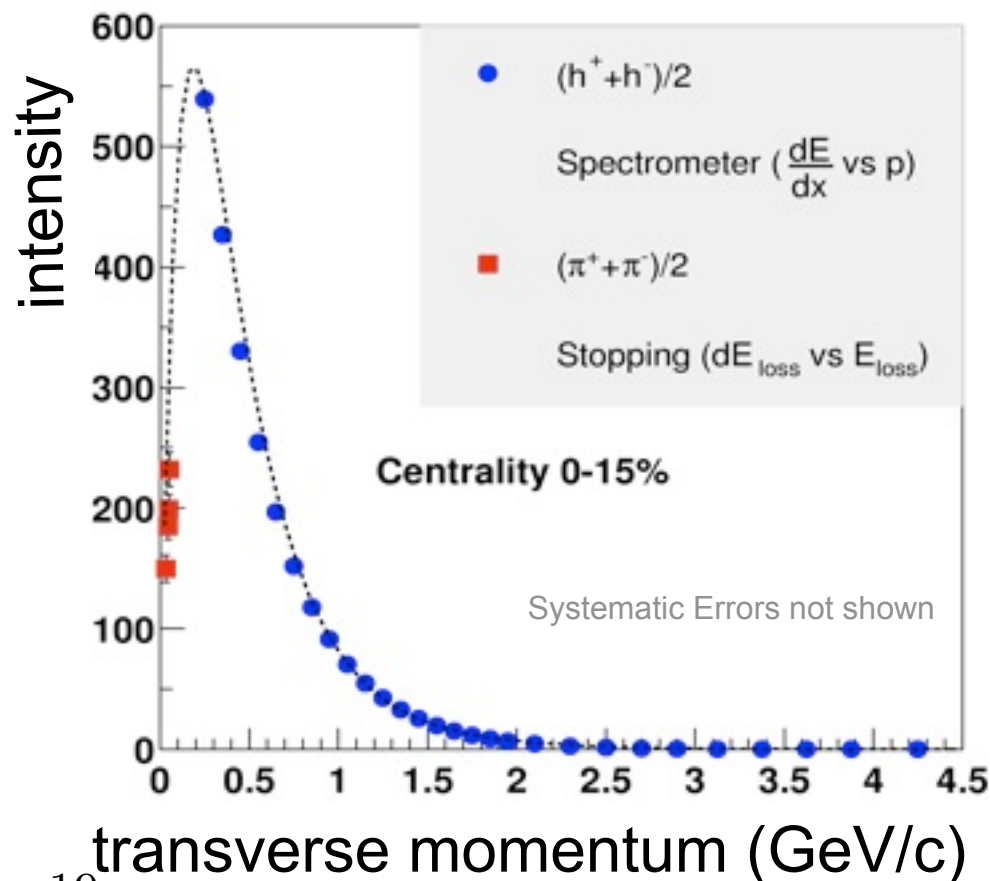
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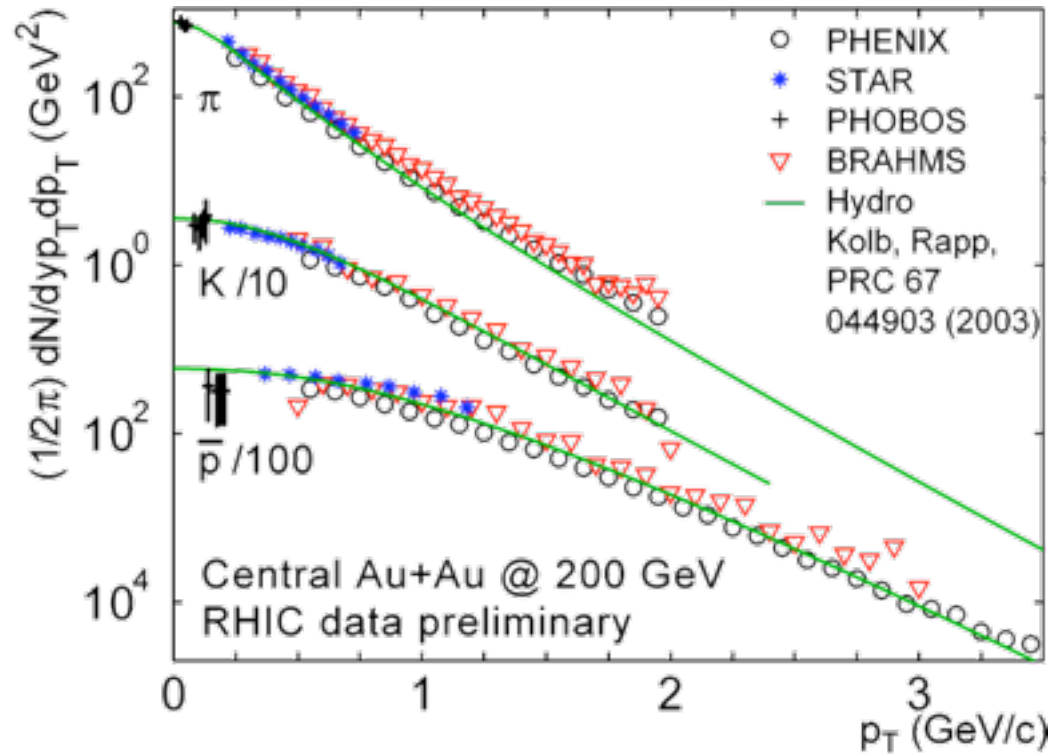
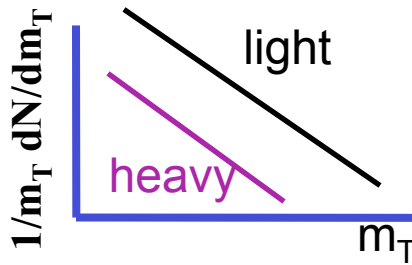


$$T_{\text{ch}} > T_{\text{fo}}$$

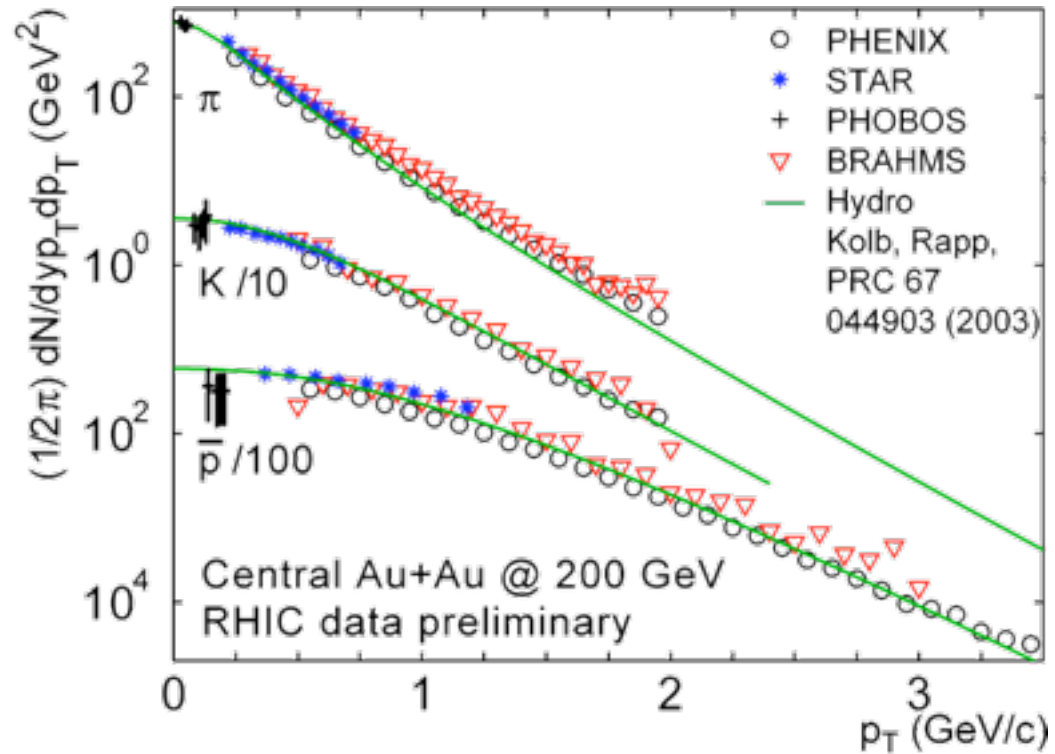
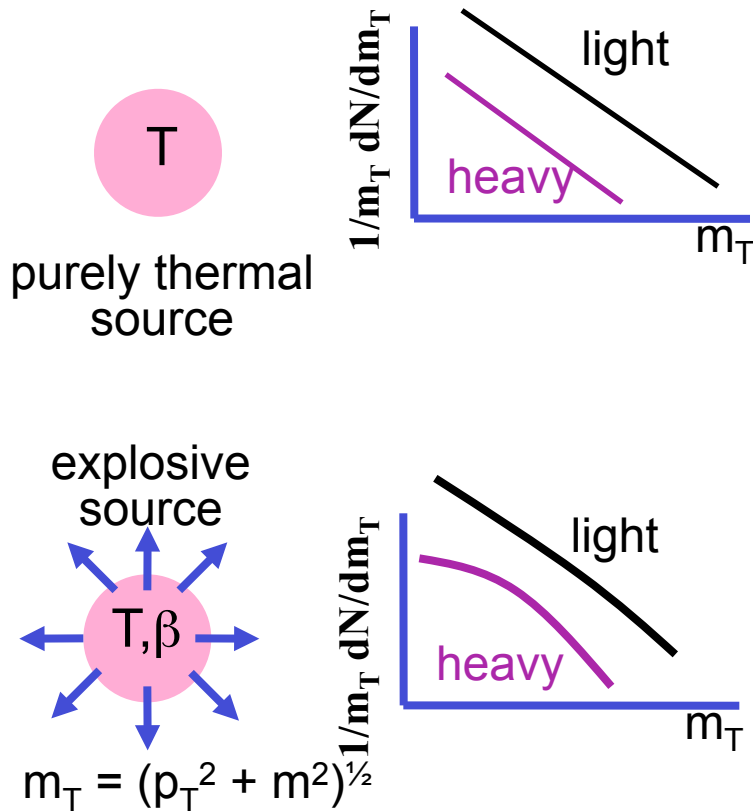
System exist for time in hadronic phase

Strong collective radial expansion


purely thermal
source

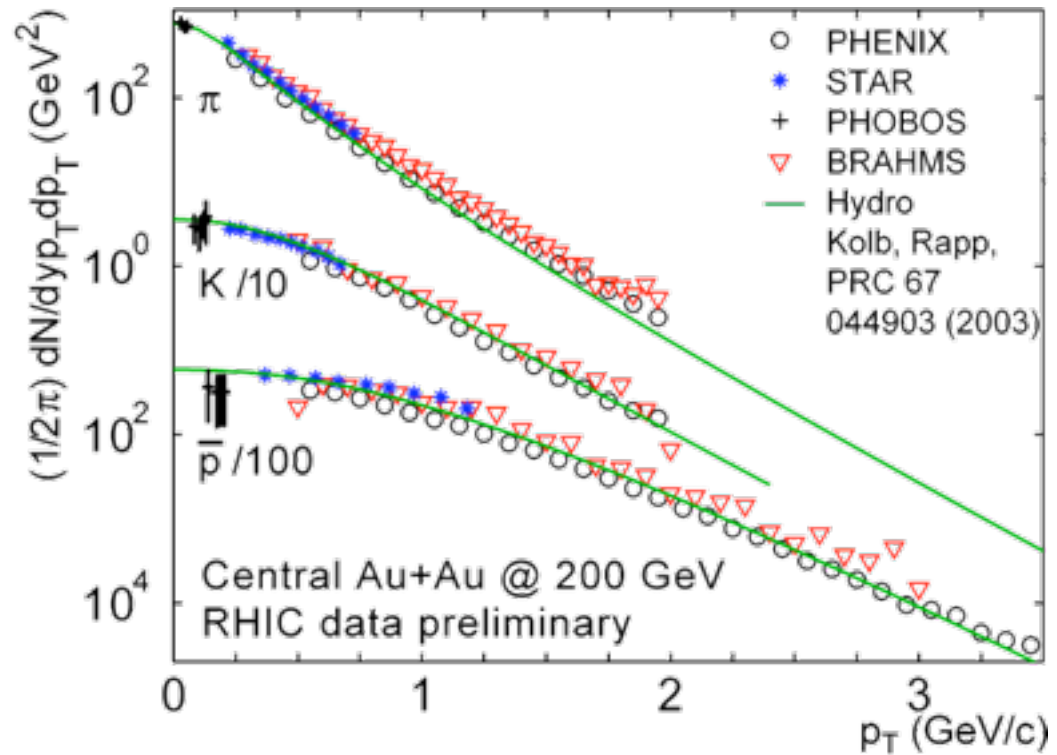
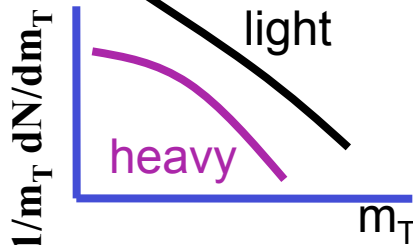
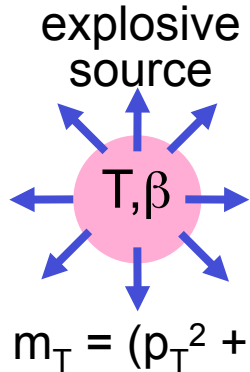
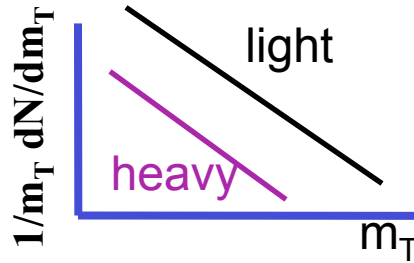
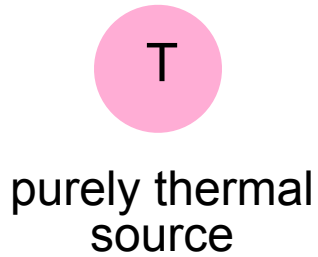


Strong collective radial expansion



- Different spectral shapes for particles of differing mass
 → strong **collective radial flow**

Strong collective radial expansion

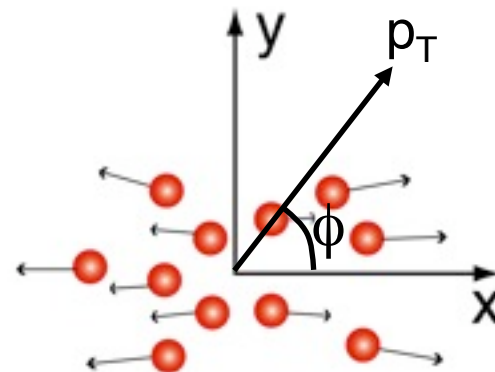
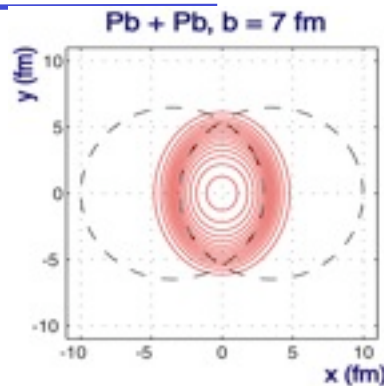
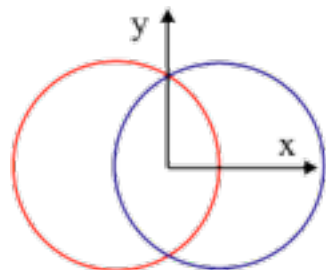


$T_{fo} \sim 100 \text{ MeV}$
 $\langle \beta_T \rangle \sim 0.55 c$

- Different spectral shapes for particles of differing mass
 → strong collective radial flow

Good agreement with hydrodynamic prediction for soft EOS (QGP+HG)

Anisotropic/Elliptic flow



Almond shape overlap
region in **coordinate space**



**Interactions/
Rescattering**



**Anisotropy in
momentum
space**

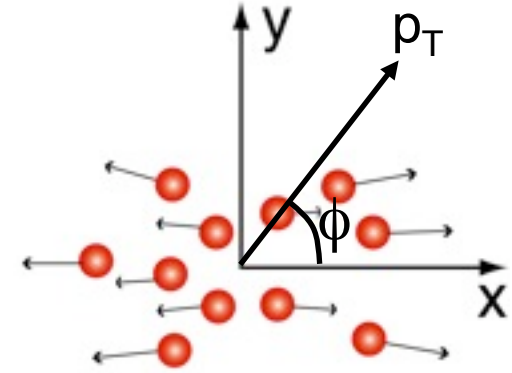
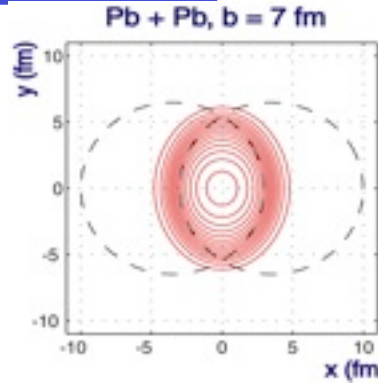
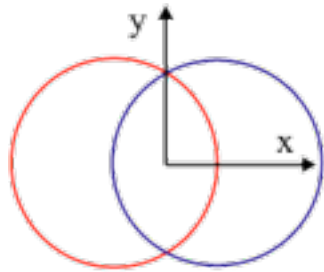
$$dN/d\phi \sim 1 + 2 v_2(p_T) \cos(2\phi) + \dots$$

$$\phi = \text{atan}(p_y/p_x)$$

$$v_2 = \langle \cos 2\phi \rangle$$

v_2 : 2nd harmonic Fourier coefficient in $dN/d\phi$ with respect to the reaction plane

Anisotropic/Elliptic flow



Almond shape overlap region in coordinate space



Interactions/Rescattering



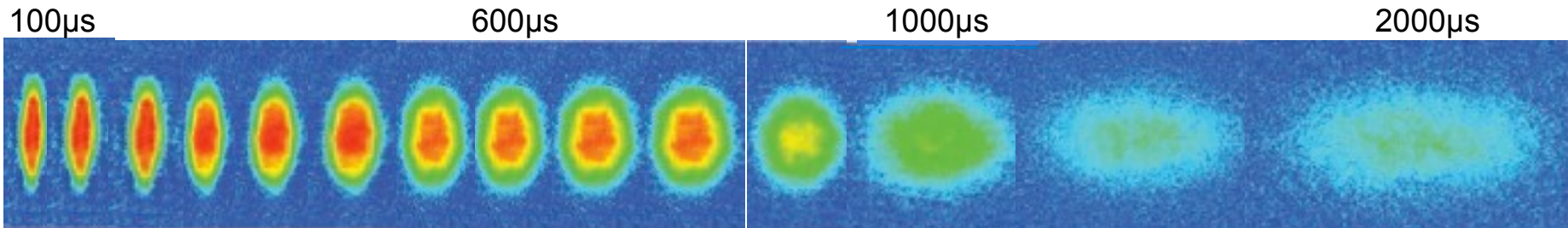
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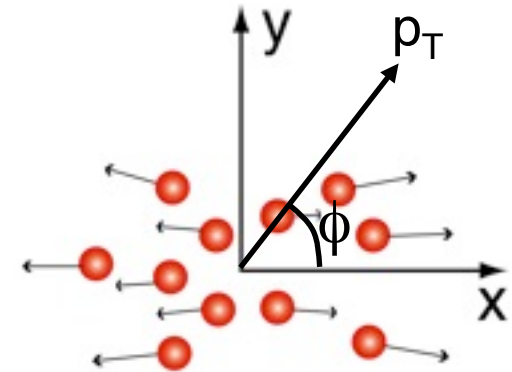
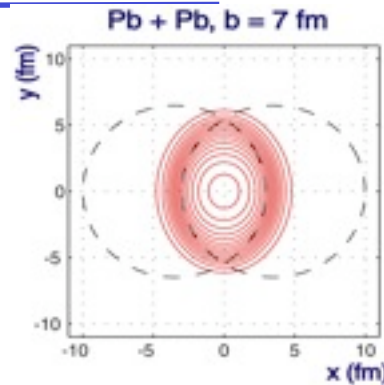
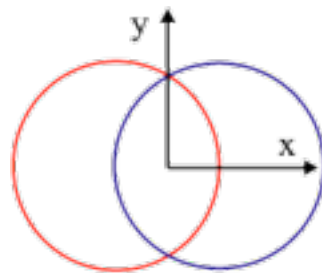
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Time

–M. Gehm, S. Granade, S. Hemmer, K. O’Hara, J. Thomas - **Science** 298 2179 (2002)

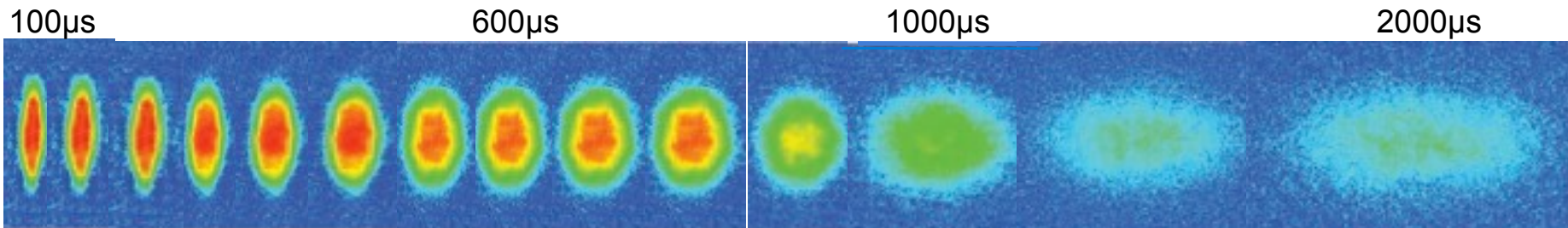
Anisotropic/Elliptic flow



Elliptic flow observable sensitive to early evolution of system

Mechanism is self-quenching

Large v_2 is an indication of **early** thermalization



→ Time

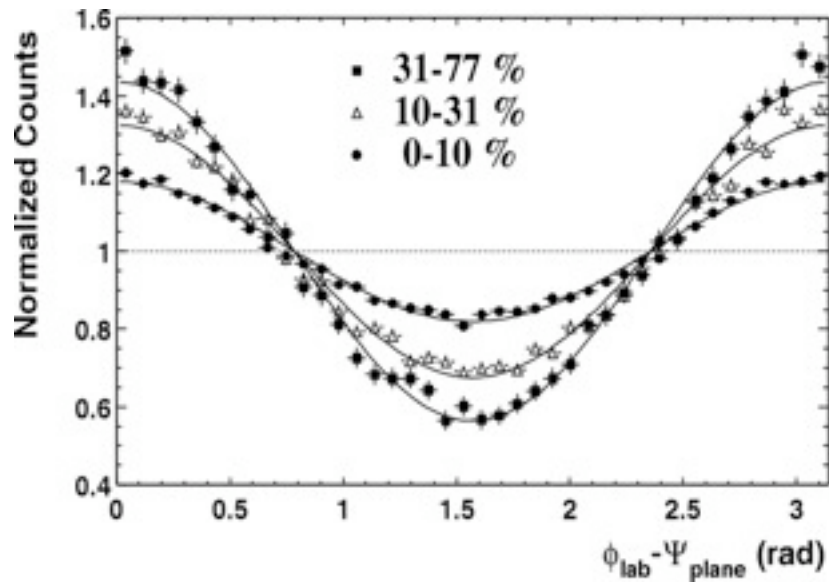
–M. Gehm, S. Granade, S. Hemmer, K. O’Hara, J. Thomas - **Science** 298 **2179** (2002)

Helen Caines - NNPS-SSI - June 2010

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Elliptic flow

Distribution of particles with respect to event plane, $\phi-\psi$, $p_t > 2$ GeV; STAR PRL 90 (2003) 032301

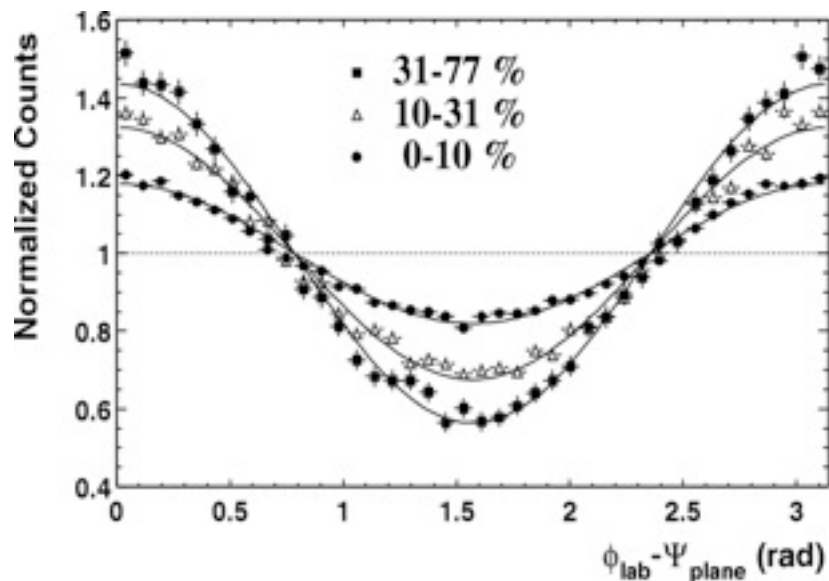


- Very strong elliptic flow → early equilibration

Factor 3:1 peak to valley

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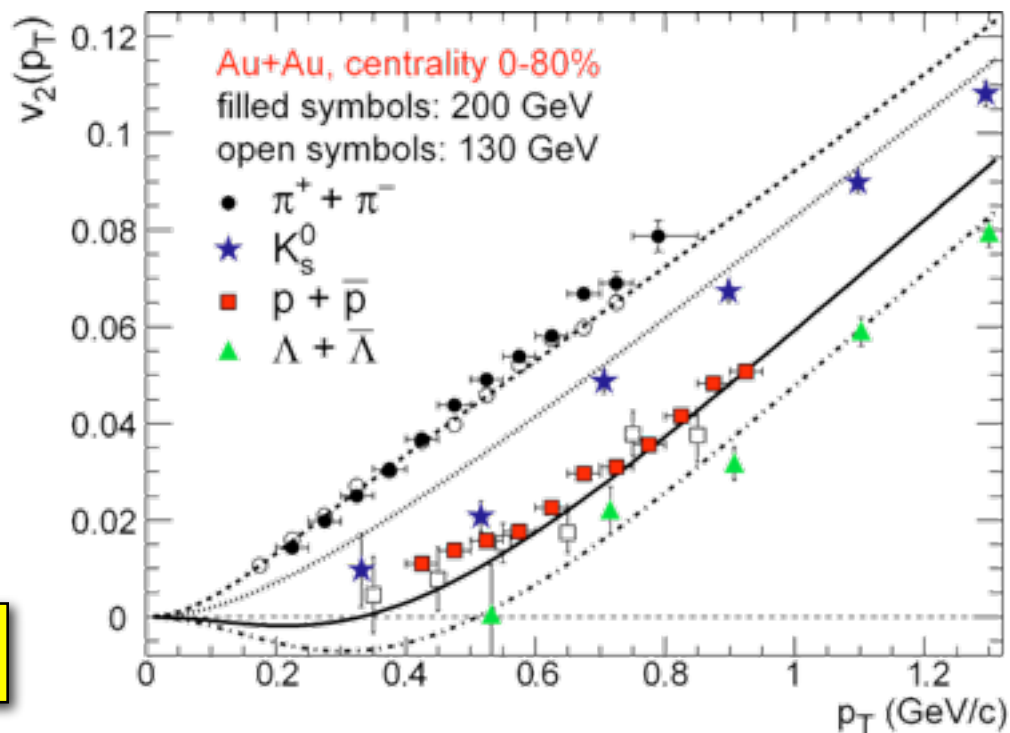


- Pure hydrodynamical models including QGP phase describe elliptic and radial flow for many species

QGP \rightarrow almost perfect fluid

- Very strong elliptic flow \rightarrow early equilibration

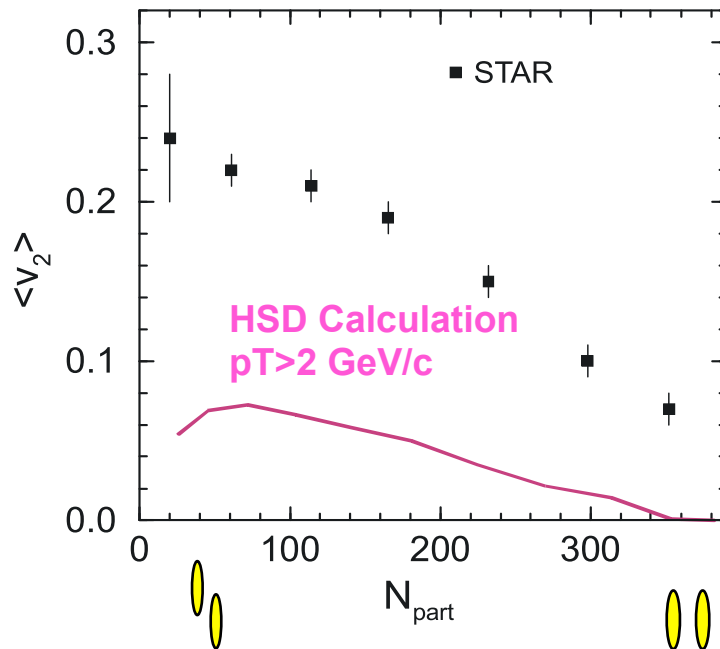
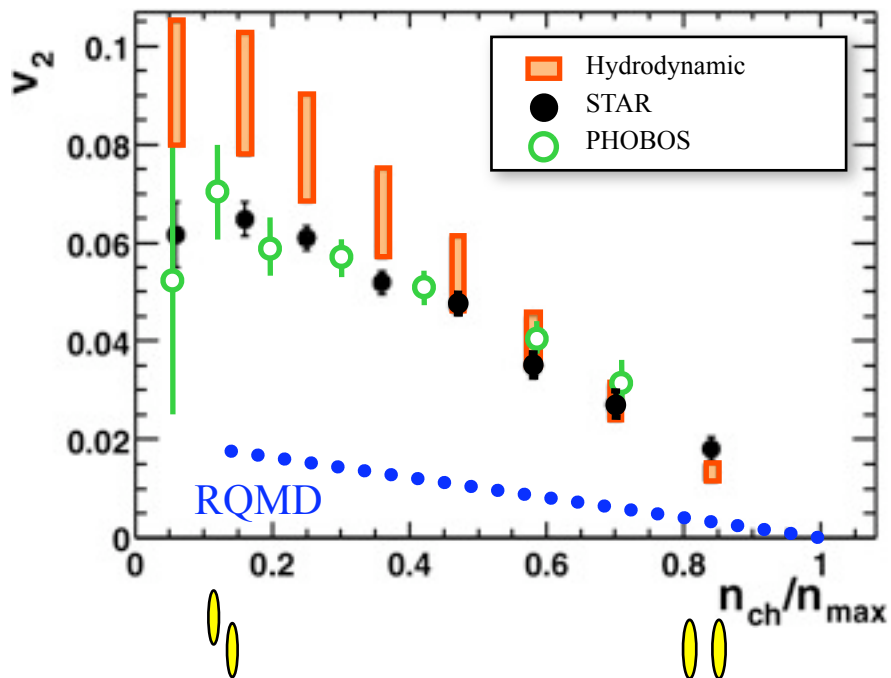
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Just a gas of hadrons?

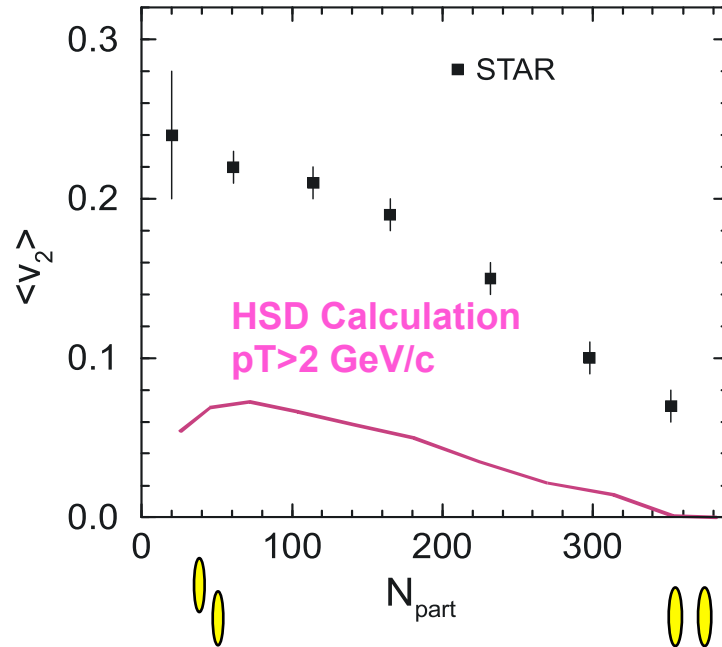
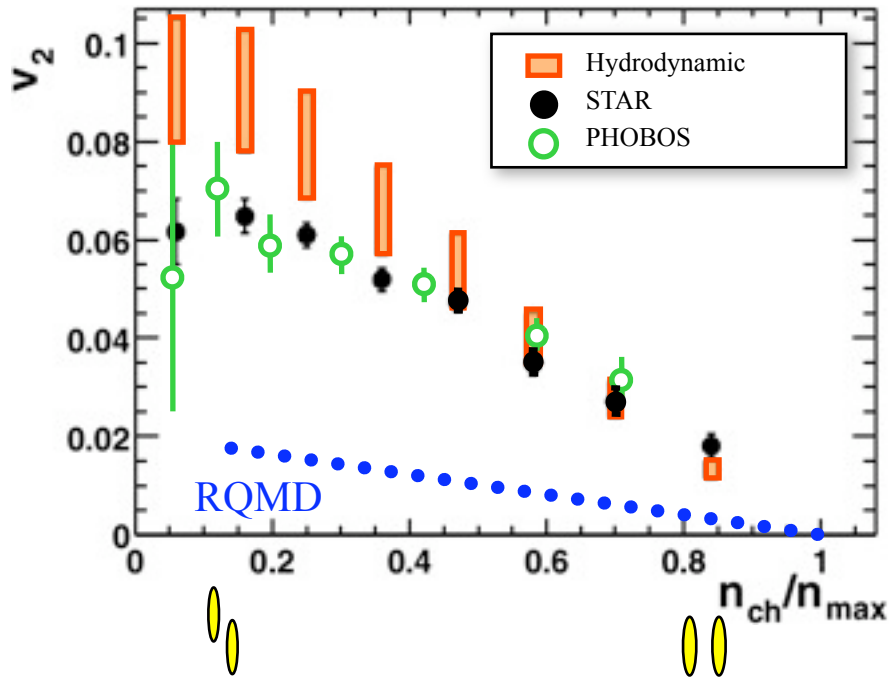
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Hadronic transport models (e.g. RQMD, HSD, ...) with hadron formation times ~ 1 fm/c, fail to describe data.



Just a gas of hadrons?

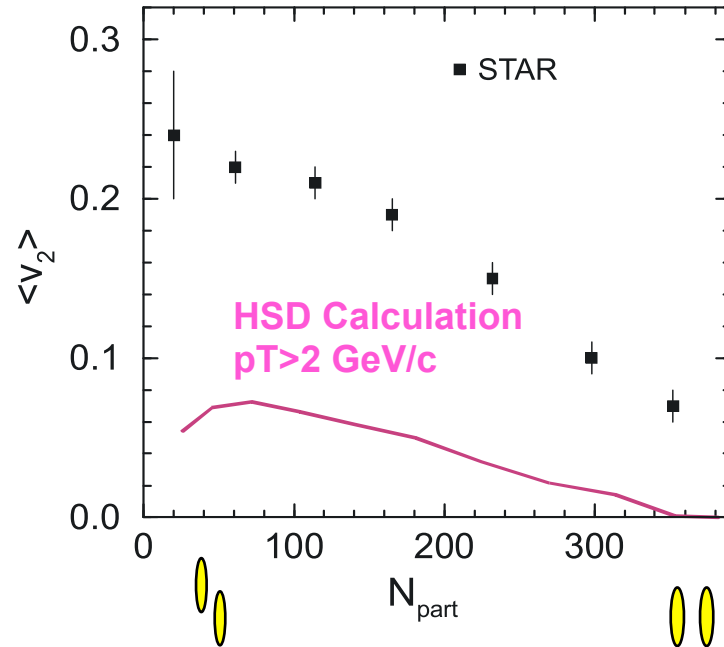
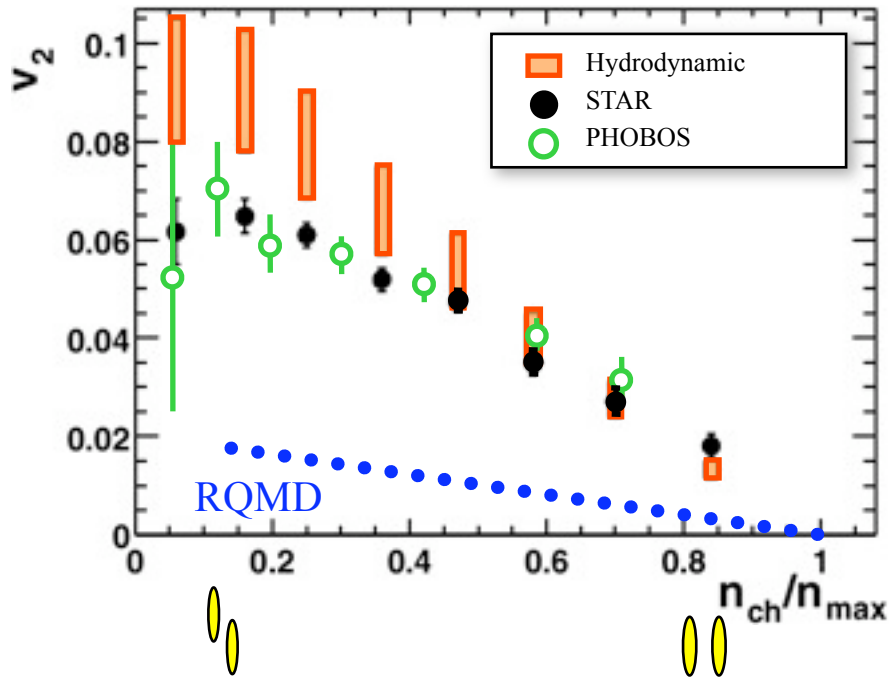
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Clearly the system is not a hadron gas. Not surprising.

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Clearly the system is not a hadron gas. Not surprising.

Hydrodynamical calculations: thermalization time $t=0.6$ fm/c

What interactions can lead to equilibration in < 1 fm/c?

The constituents “flow”

- Elliptic flow is additive.
- If partons are flowing the *complicated* observed flow pattern in $v_2(p_T)$ for hadrons

$$\frac{d^2N}{dp_T d\phi} \propto 1 + 2 v_2(p_T) \cos(2\phi)$$

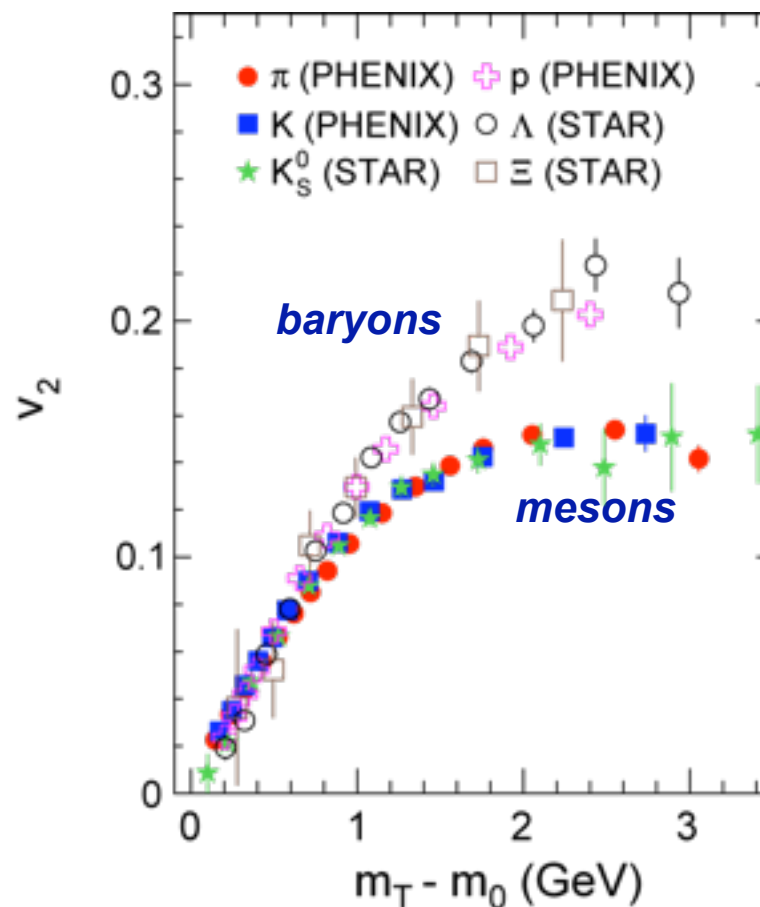
should become *simple* at the quark level

$$p_T \rightarrow p_T / n$$

$$v_2 \rightarrow v_2 / n ,$$

$n = (2, 3)$ for (meson, baryon)

$$m_T = \sqrt{p_T^2 + m_0^2}$$



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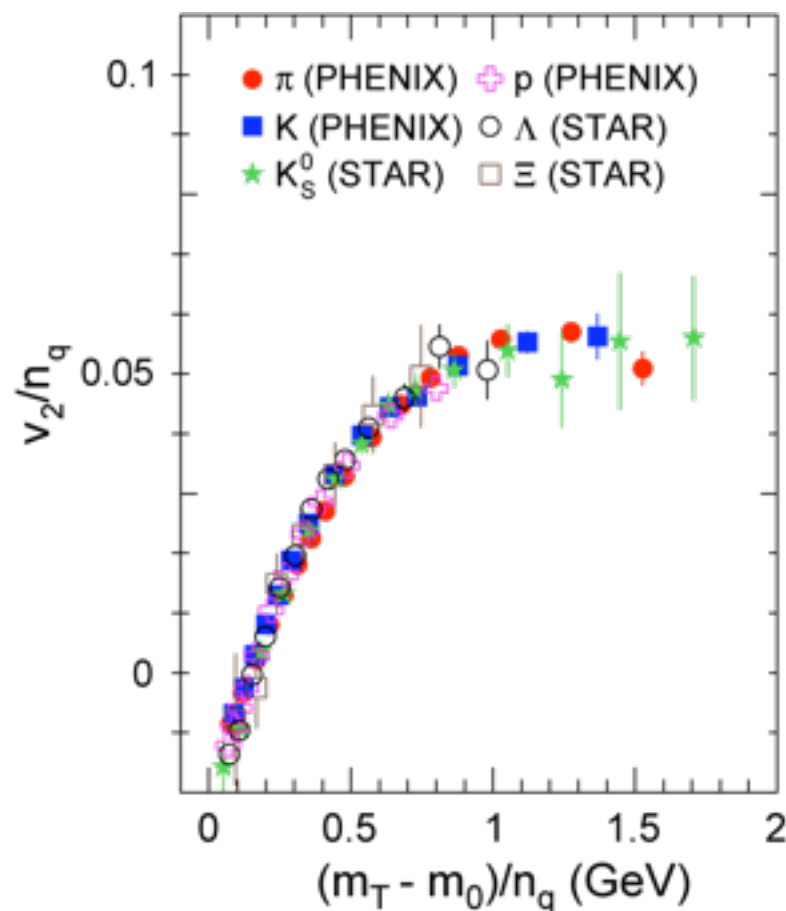
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Works for $p, \pi, K^0_s, \Lambda, \Xi$..

$$v_2^s \sim v_2^{u,d} \sim 7\%$$

$$m_T = \sqrt{p_T^2 + m_0^2}$$



Constituents of QGP are partons

Summary of what we learned so far

- Energy density in the collision region is way above that where hadrons can exist
- The initial temperature of collision region is way above that where hadrons can exist
- The medium has quark and gluon degrees of freedom in initial stages

We have created a new state of matter at RHIC
- the QGP

- The QGP is flowing like an almost “perfect” liquid