Introduction to Relativistic Heavy Ion Physics

Lecture 4: New Dimensions

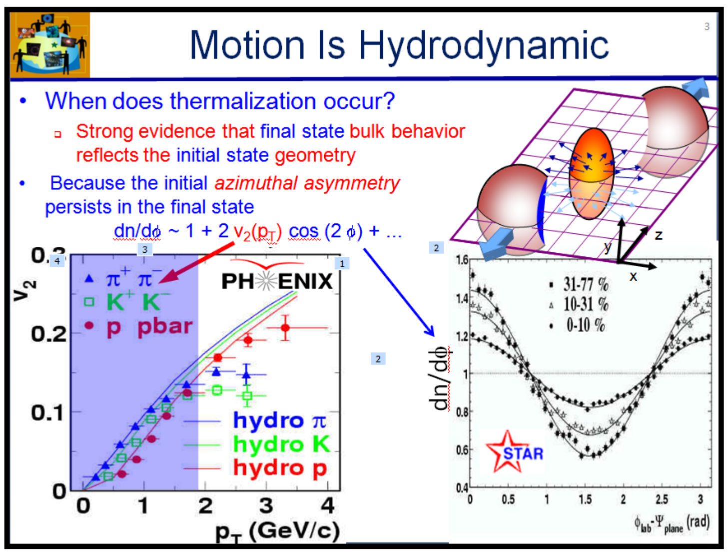
W.A. Zajc
Columbia University

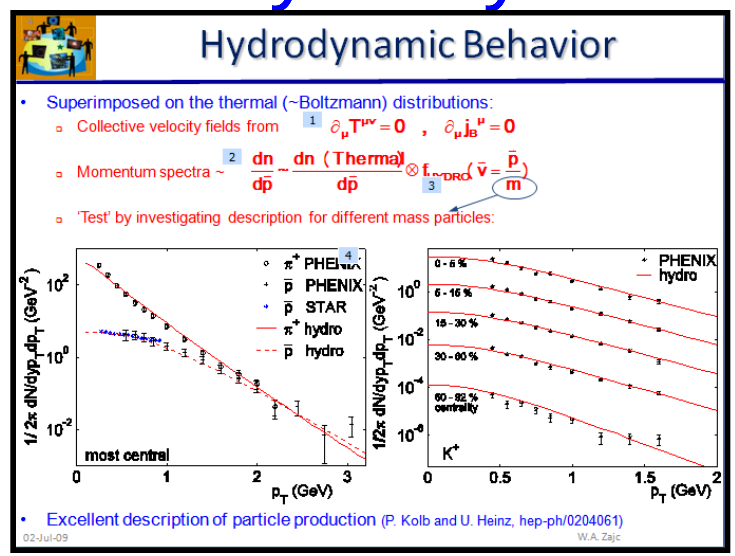
Reminder- From Lecture 3

- Strong evidence that initial-state spatial asymmetry appears as final-state “flow”
- The flow properties of QGP in Au+Au collisions at top RHIC energy is roughly consistent with perfect fluid (η=0) hydrodynamics
  - Particle mass dependence of $v_2(p_T)$
  - Scaling of same with KE_T
- Theoretical argument (Landau) suggests applicability of hydrodynamics to relativistic systems is approximately equivalent to requiring perfect fluid behavior.

What's The Problem ?

- Landau makes plausible argument for neglecting viscous effects in relativistic hydrodynamics
  - Caveat: Subject to condition R / λ >> 1
- Data well-described by ideal hydrodynamics:





- Are we done ?

What Is The Viscosity at RHIC?

- "Perfect fluid" (and/or "ideal hydrodynamics")

  ~ defined as "zero viscosity".

$$\left.\begin{array}{l}\eta_{QGP} \sim 2\times10^{11}\ \mathrm{Pa\cdot s}\\ \eta_{H_2O} \sim 1\times10^{-3}\ \mathrm{Pa\cdot s}\end{array}\right\} \Rightarrow \frac{\eta_{QGP}}{\eta_{H_2O}} \sim 2\times10^{14}$$

$$\eta_{Pitch} \sim 2.3\times10^{8}\ \mathrm{Pa\cdot s} \qquad \eta_{Glass(A.P.)} \sim 10^{12}\ \mathrm{Pa\cdot s}$$

?

A Check Of This Strange Conclusion

- Recall $\eta \sim n\,\bar{p}\,\lambda_{mfp}$
- But $$\lambda_{mfp} = \frac{1}{n\sigma} \Rightarrow \eta \sim \frac{\bar{p}}{\sigma}$$
  - Very Important Point !!
  - To get small viscosity you need LARGE σ
- Using above
- $$\frac{\eta_{QGP}}{\eta_{H_2 0}} \sim \frac{\bar{p}_{QGP}}{\sigma_{QGP}}\frac{\sigma_{H_2 0}}{\bar{p}_{H_2 0}} \sim \frac{T_{QGP}}{\pi\left(\frac{1}{T_{QGP}}\right)^2}\frac{\pi a^2{}_{H_2 0}}{\sqrt{2\pi m k T_{H_2 0}}} \sim 10^{14}$$

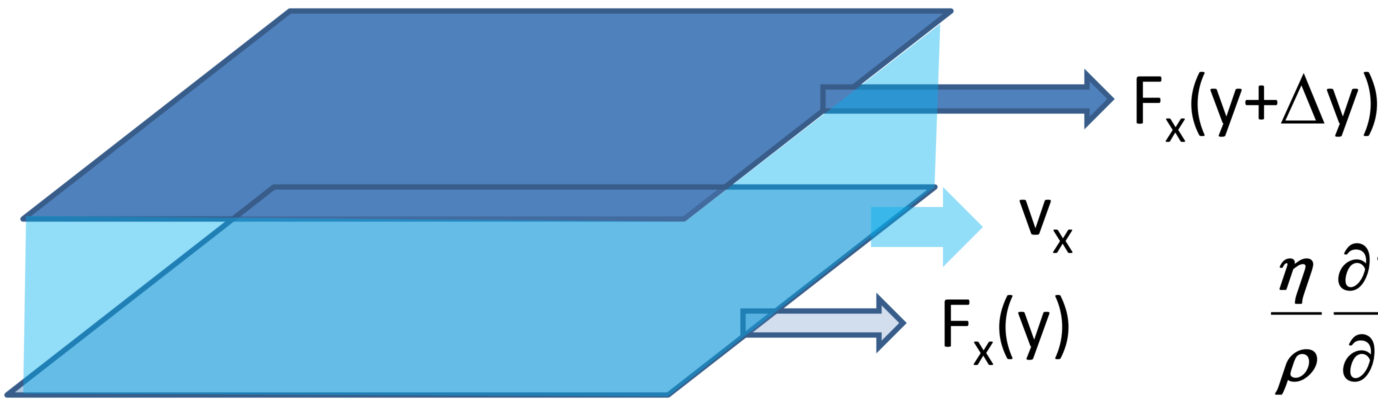
Exercise 1: Check the above value , putting in plausible estimates for the various parameters.

Kinematic Viscosity

- In Reynolds Number: $\mathrm{Re} \equiv \dfrac{\rho V L}{\eta}$

- Determines relaxation rate

Exercise 2: At what velocity does it feel as if you are 'swimming' in water? (Save yourself some work- kinematic velocities are tabulated.)



$$\frac{\eta}{\rho}\frac{\partial^2 v_x}{\partial y^2} = \frac{\partial v_x}{\partial t}$$

Exercise 3: a) Use F=ma and the definition of viscosity to show that the relaxation of the velocity field $v_x(y)$ follows the diffusion equation.

b) verify solution to same

$$\Rightarrow v_x(y,t) \sim \frac{1}{\sqrt{4\frac{\eta}{\rho}t}} e^{-y^2/(4\frac{\eta}{\rho}t)}$$

Why η/s Matters

- Any engineer will tell you
  - *Kinematic* viscosity $\eta / \rho \sim$ [Velocity] x [Length] is what matters (see Landau's remark on Reynolds number)
- Any relativist will tell you
  - $\rho \rightarrow \varepsilon + P$
- Any thermodynamicist will tell you
  - $\varepsilon + P = T s$ (at $\mu_B = 0$)
- So
  - $\eta/\rho \rightarrow \eta/(\varepsilon + p) \rightarrow (\eta/sT) = (\eta/s)(1/T)$

    ~ **(**damping coefficient x thermal time**)**

Exercise 4: a) Use this and previous statistical mechanics results for massless quanta to find an analytic result for entropy density s.
b) Show that s = 3.6 n
c) Instead of statistical expression for n, *define* n via P = (N/V) T = n T. Show that with this definition s = 4 n for massless quanta. Comment.

A Long Time Ago (1985)

- Miklos Gyulassy and Pawel Danielewicz:
  - *Dissipative Phenomena in Quark-Gluon Plasmas*
    P. Danielewicz, M. Gyulassy Phys.Rev. D31, 53,1985.

  noted several restrictions on smallest allowed η :
- Most restrictive:
- **λ > h/<p> $\Rightarrow$ η > ~ n / 3**
- But for the quanta they were considering s = 3.6n
- **$\Rightarrow$ η/s > 1 / (3.6 x 3) ~ 1 / (4 π) !!**

Before estimating λ_i via Eq. (3.2) we note several physical constraints on λ_i. First, the uncertainty principle implies that quanta transporting typical momenta $\langle p \rangle$ cannot be localized to distances smaller than $\langle p \rangle^{-1}$. Hence, it is meaningless to speak about mean free paths smaller than $\langle p \rangle^{-1}$. Requiring $\lambda_i \gtrsim \langle p \rangle_i^{-1}$ leads to the lower bound

$$\eta \gtrsim \tfrac{1}{3} n \ , \qquad (3.3)$$

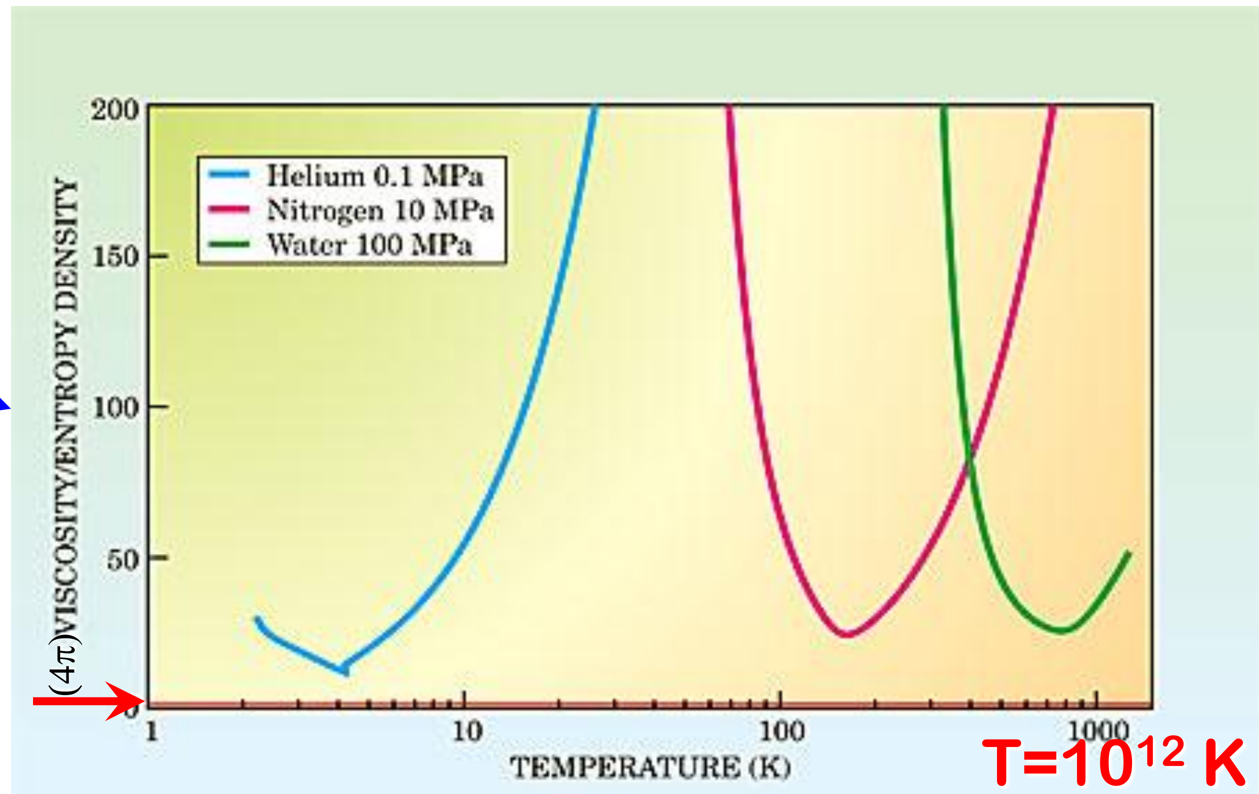
where $n = \sum n_i$ is the total density of quanta. What seems amazing about (3.3) is that it is independent of dynamical details. There is a finite viscosity regardless of how large is the free-space cross section between the quanta. See Refs. 21 and 22 for examples illustrating how the thermalization rate of many-body systems is limited by the uncertainty principle.

KSS Bound

- This bound is (now) *very* well-known in the nuclear physics community:
- "*A Viscosity Bound Conjecture*", P. Kovtun, D.T. Son, A.O. Starinets, hep-th/0405231

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi} \frac{1}{k_B}$$

- Where do "ordinary" fluids sit wrt this limit?
- RHIC "fluid" *might* be at ~2-3 on this scale (!)



Putting Viscosity Into Formalism

- The stress-energy tensor now contains off-diagonal terms:
  - $T^{\mu\nu}$ will contain a piece called the shear stress tensor $\pi^{\mu\nu}$:

$$\pi^{\mu\nu} = T^{<\mu\nu>} \equiv \left[\frac{1}{2}\left(\Delta_\alpha{}^\mu \Delta_\beta{}^\nu + \Delta_\alpha{}^\nu \Delta_\beta{}^\mu \right) - \frac{1}{3} \Delta^{\mu\nu} \Delta_{\alpha\beta} \right] T^{\alpha\beta}$$

$$\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$$

Exercise 5: Show that Δ projects out the 3-volume orthogonal to four-velocity u .

  - The existence of velocity gradients will *produce* a shear stress
  - Parameterize this via a ‘constitutive equation’ :

$$\pi^{\mu\nu} = 2\eta \nabla^{<\mu} u^{\nu>}$$

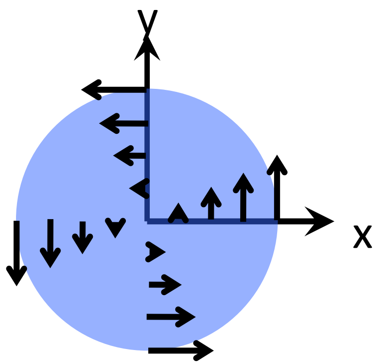
  - Then equation of motion is $\partial_\mu [(T^{\mu\nu}_{\substack{perfect\\fluid}}) + \pi^{\mu\nu}] = 0$

- This is a simplified form of the relativistic Navier-Stokes eq.
  - Ignores heat conduction, bulk viscosity

An Aside on Formalism

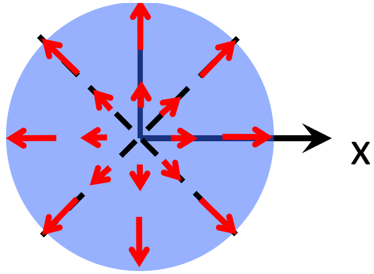
- Q. What are these weird index manipulations ?
- A. They produce a *symmetrized, traceless* 'gradient' :
  - Recall we're interested in velocity gradients: $\frac{F_i}{A} = \mu \frac{\partial v_i}{\partial x_j}$
  - Remove uniform rotation:

$$\frac{\partial v_i}{\partial x_j} = \frac{1}{2}[\partial_j v_i + \partial_i v_j] + \frac{1}{2}[\partial_j v_i - \partial_i v_j]$$



  - Remove uniform (Hubble) expansion:

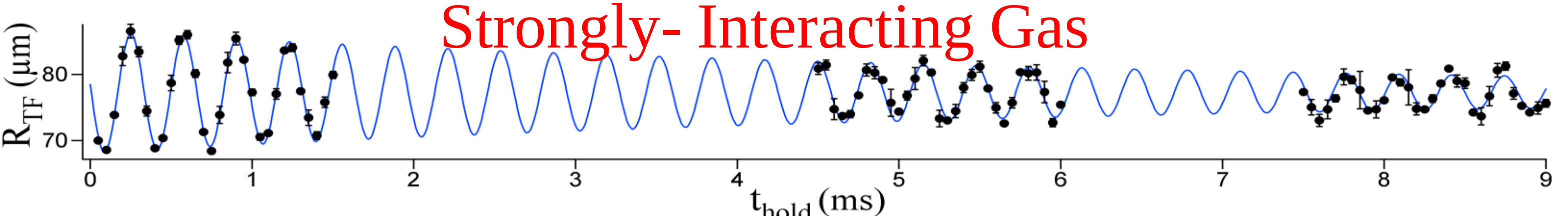
$$\frac{1}{2}[\partial_j v_i + \partial_i v_j] \rightarrow \frac{1}{2}[\partial_j v_i + \partial_i v_j - \frac{2}{3}\delta_{ij}(\partial_k v_k)]$$

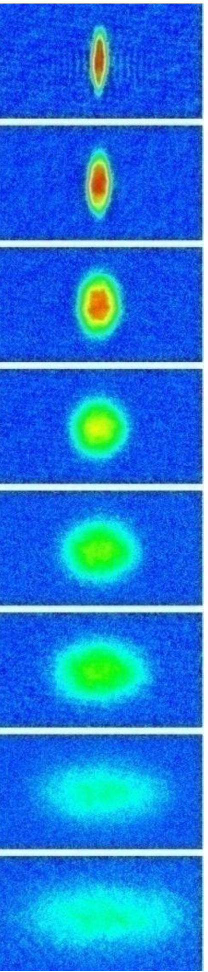


Exercise 6: Check these properties

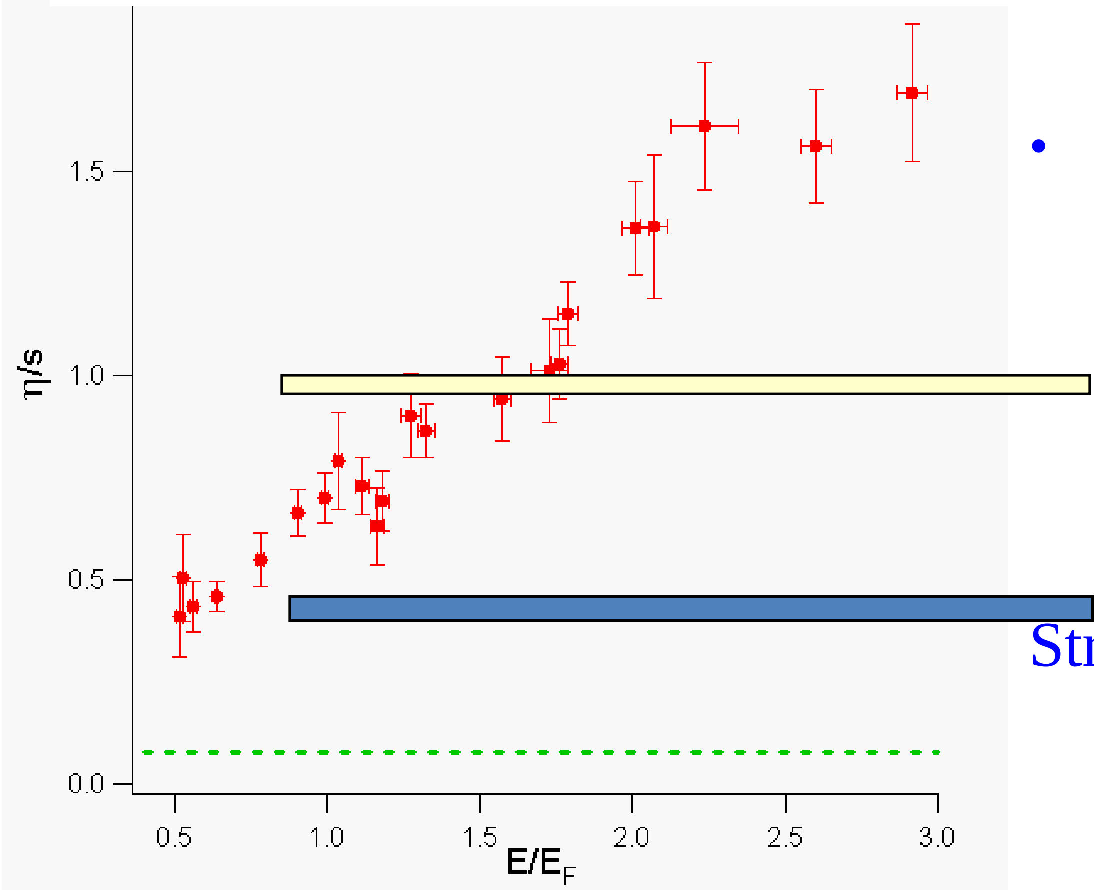
Apply to a ‘Real’ Fluid

Strongly- Interacting Gas





$$\dot{E} = -\frac{1}{2}\int d^3x\,\eta(x)\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2$$



- Damping of breathing mode in cold Fermi gas
- (All figures courtesy of John Thomas, Duke University)

^{3}He,^{4}He near λ–point

QGP simulations

String-theory 1/4π

Estimating η/s For RHIC

- **Damping** (flow, fluctuations, heavy quark motion) ~ η/s

  - FLOW: *Has the QCD Critical Point Been Signaled by Observations at RHIC?*, R. Lacey *et al.*, Phys.Rev.Lett.98:092301,2007 (nucl-ex/0609025)

$$\frac{\eta}{s} = (1.1 \pm 0.2 \pm 1.2)\frac{1}{4\pi}$$

  - *The Centrality dependence of Elliptic flow, the Hydrodynamic Limit, and the Viscosity of Hot QCD*, H.-J. Drescher *et al.*, (arXiv:0704.3553)

$$\frac{\eta}{s} = (1.9 - 2.5)\frac{1}{4\pi}$$

  - FLUCTUATIONS: *Measuring Shear Viscosity Using Transverse Momentum Correlations in Relativistic Nuclear Collisions*, S. Gavin and M. Abdel-Aziz, Phys.Rev.Lett.97:162302,2006 (nucl-th/0606061)

$$\frac{\eta}{s} = (1.0 - 3.8)\frac{1}{4\pi}$$

  - DRAG, FLOW: *Energy Loss and Flow of Heavy Quarks in Au+Au Collisions at* $\sqrt{s_{NN}}$ = 200 *GeV* (PHENIX Collaboration), A. Adare *et al.*, to appear in Phys. Rev. Lett. (nucl-ex/0611018)

$$\frac{\eta}{s} = (1.2 - 2.0)\frac{1}{4\pi}$$

Viscous Relativistic Hydrodynamics

- Why not do a 'real' (that is, Navier-Stokes) hydrodynamic calculation at RHIC?
  - Incorporate non-zero viscosity
  - 'Invert' to determine allowed range for η / s.
- Two little problems:
  - It's wrong
    - Solutions are acausal
    - Needed patch:
  - It's wrong
    - Solutions are *intrinsically* unstable
    - No patch, must take *all terms* to 2^{nd} order in gradients

$$v_x(y,t) \sim \frac{1}{\sqrt{4\frac{\eta}{\rho}t}} e^{-y^2/(4\frac{\eta}{\rho}t)}$$

$$\frac{\eta}{\rho}\frac{\partial^2 v_x}{\partial y^2} = \frac{\partial v_x}{\partial t} \rightarrow \frac{\eta}{\rho}\frac{\partial^2 v_x}{\partial y^2} = \frac{\partial v_x}{\partial t} + \tau_R \frac{\partial^2 v_x}{\partial t^2}$$

A Partial List of “All Terms”

- Relativistic, Causal, second-order expansion:
  - Relativistic Fluid Dynamics: Physics for Many Different Scales
- Neglect various terms at your own risk:
  - Baier et al., Relativistic viscous hydrodynamics, conformal invariance, and holography
  - Natsuume and Okamura, Comment on “Viscous hydrodynamics relaxation time from AdS/CFT correspondence”

Working out the divergence of the entropy current, and making use of the equations of motion, we arrive at

$$\begin{aligned}\nabla_\mu s^\mu = &-\frac{1}{T}\tau\left[\nabla_\mu u^\mu + \beta_0 u^\mu \nabla_\mu \tau - \alpha_0 \nabla_\mu q^\mu - \gamma_0 T q^\mu \nabla_\mu\left(\frac{\alpha_0}{T}\right) + \frac{\tau T}{2}\nabla_\mu\left(\frac{\beta_0 u^\mu}{T}\right)\right] \\ &-\frac{1}{T}q^\mu\left[\frac{1}{T}\nabla_\mu T + u^\nu \nabla_\nu u_\mu + \beta_1 u^\nu \nabla_\nu q_\mu - \alpha_0 \nabla_\mu \tau - \alpha_1 \nabla_\nu \tau^\nu{}_\mu\right. \\ &\qquad \left. + \frac{T}{2} q_\mu \nabla_\nu\left(\frac{\beta_1 u^\nu}{T}\right) - (1-\gamma_0)\tau T \nabla_\mu\left(\frac{\alpha_0}{T}\right) - (1-\gamma_1) T \tau^\nu{}_\mu \nabla_\nu\left(\frac{\alpha_1}{T}\right)\right] \\ &-\frac{1}{T}\tau^{\mu\nu}\left[\nabla_\mu u_\nu + \beta_2 u^\alpha \nabla_\alpha \tau_{\mu\nu} - \alpha_1 \nabla_\mu q_\nu + \frac{T}{2}\tau_{\mu\nu}\nabla_\alpha\left(\frac{\beta_2 u^\alpha}{T}\right) - \gamma_1 T q_\mu \nabla_\nu\left(\frac{\alpha_1}{T}\right)\right]. \end{aligned} \tag{300}$$

In this expression it should be noted that we have introduced (following Lindblom and Hiscock) two further parameters, γ_0 and γ_1. They are needed because without additional assumptions it is not clear how the “mixed” quadratic term should be distributed. A natural way to fix these parameters is to appeal to the Onsager symmetry principle [58], which leads to the mixed terms being distributed “equally” and hence $\gamma_0 = \gamma_1 = 1/2$.

Denoting the comoving derivative by a dot, i.e. using $u^\mu \nabla_\mu \tau = \dot{\tau}$ etc. we see that the second law of thermodynamics is satisfied if we choose

$$\tau = -\zeta\left[\nabla_\mu u^\mu + \beta_0 \dot{\tau} - \alpha_0 \nabla_\mu q^\mu - \gamma_0 T q^\mu \nabla_\mu\left(\frac{\alpha_0}{T}\right) + \frac{\tau T}{2}\nabla_\mu\left(\frac{\beta_0 u^\mu}{T}\right)\right], \tag{301}$$

$$\begin{aligned} q^\mu = -\kappa T \perp^{\mu\nu} &\left[\frac{1}{T}\nabla_\nu T + \dot{u}_\nu + \beta_1 \dot{q}_\nu - \alpha_0 \nabla_\nu \tau - \alpha_1 \nabla_\alpha \tau^\alpha{}_\nu + \frac{T}{2} q_\nu \nabla_\alpha\left(\frac{\beta_1 u^\alpha}{T}\right)\right. \\ &\left. -(1-\gamma_0)\tau T \nabla_\nu\left(\frac{\alpha_0}{T}\right) - (1-\gamma_1) T \tau^\alpha{}_\nu \nabla_\alpha\left(\frac{\alpha_1}{T}\right) + \gamma_2 \nabla_{[\nu} u_{\alpha]} q^\alpha\right], \end{aligned} \tag{302}$$

$$\tau_{\mu\nu} = -2\eta\left[\beta_2 \dot{\tau}_{\mu\nu} + \frac{T}{2}\tau_{\mu\nu}\nabla_\alpha\left(\frac{\beta_2 u^\alpha}{T}\right) + \left\langle \nabla_\mu u_\nu - \alpha_1 \nabla_\mu q_\nu - \gamma_1 T q_\mu \nabla_\nu\left(\frac{\alpha_1}{T}\right) + \gamma_3 \nabla_{[\mu} u_{\alpha]} \tau_\nu{}^\alpha \right\rangle\right], \tag{303}$$

where the angular brackets denote symmetrization as before. In these expression we have added yet another two terms, representing the coupling to vorticity. These bring further “free” parameters γ_2 and γ_3. It is easy to see that we are allowed to add these terms since they do not affect the entropy production. In fact, a large number of similar terms may, in principle, be considered (see note added in proof in [53]). The presence of coupling terms of the particular form that we have introduced is suggested by kinetic

Complete Set of Terms

- Daunting:

$$\tau_\Pi \, \dot{\Pi} \; + \; \Pi \; = \; \Pi_{\rm NS}$$

$$\nu$$

$$\tau_q \, \Delta^{\mu\nu} \dot{q}_\nu \; + \; q^\mu \; = \; q^\mu_{\rm NS}$$

$$+ \; \hat{\delta}_{1,2} \, \Pi$$

$$\tau_\pi \, \dot{\pi}^{<\mu\nu>} \; + \; \pi$$

$$- \, 2 \, \tau_\pi \, \pi_\lambda{}^{<\mu} \sigma^{\nu>\lambda} \; - \; 2 \, \lambda_{\pi q} \, q^{<\mu} \nabla^{\nu>} \alpha \; + \; 2 \, \lambda_{\pi\Pi} \, \Pi \, \sigma^{\mu\nu}$$

$$+ \; \hat{\delta}_{2,2} \, \Pi \, \pi^{\mu\nu} \; - \; \hat{\eta}_2 \, \pi_\lambda{}^{<\mu} \pi^{\nu>\lambda} \; - \; \hat{\epsilon}_2 \, q^{<\mu} q^{\nu>}$$

- And still subject to
  - Poorly constrained initial Conditions
  - Eccentricity fluctuations
  - Poorly constrained equation of state
  - Hadronic rescattering effects
  - Bulk viscosity
  - Numerical viscosity
  - Finite size, core/corona effects

Complete Set of Terms

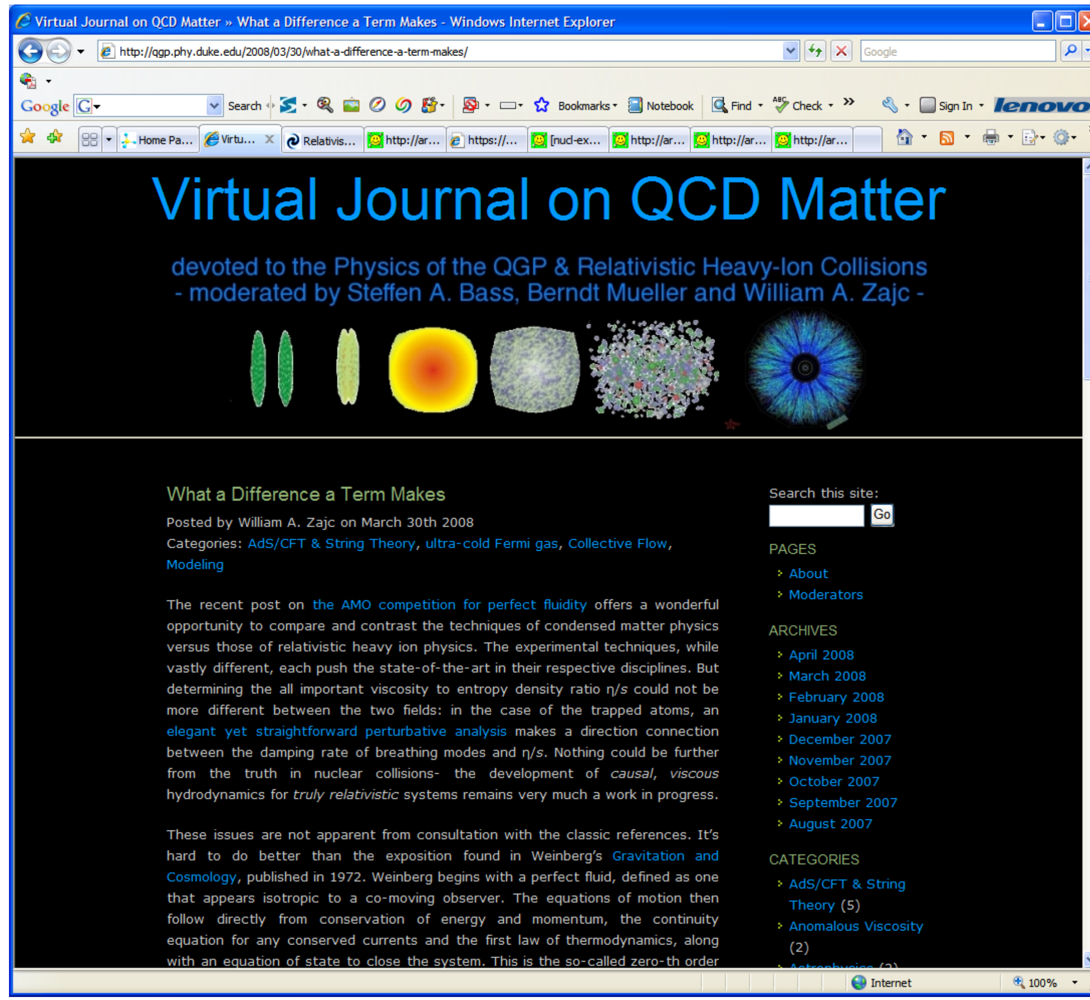
- Daunting:

$$
\begin{aligned}
\tau_\Pi \dot{\Pi} + \Pi &= \Pi_{\mathrm{NS}} + \tau_{\Pi q}\, q\cdot\dot{u} - \ell_{\Pi q}\,\partial\cdot q - \zeta\,\hat{\delta}_{0,1}\,\Pi\,\theta \\
&\quad + \lambda_{\Pi q}\, q\cdot\nabla\alpha + \lambda_{\Pi\pi}\,\pi^{\mu\nu}\sigma_{\mu\nu} + \hat{\delta}_{0,2}\,\Pi^2 + \hat{\epsilon}_0\, q\cdot q + \hat{\eta}_0\,\pi^{\mu\nu}\pi_{\mu\nu} \\
\tau_q\,\Delta^{\mu\nu}\dot{q}_\nu + q^\mu &= q^\mu_{\mathrm{NS}} - \tau_{q\Pi}\,\Pi\,\dot{u}^\mu - \tau_{q\pi}\,\pi^{\mu\nu}\,\dot{u}_\nu \\
&\quad + \ell_{q\Pi}\,\nabla^\mu\Pi - \ell_{q\pi}\,\Delta^{\mu\nu}\,\partial^\lambda\pi_{\nu\lambda} + \tau_q\,\omega^{\mu\nu}\,q_\nu - \frac{\kappa}{\beta}\,\hat{\delta}_{1,1}\,q^\mu\,\theta \\
&\quad - \lambda_{qq}\,\sigma^{\mu\nu}\,q_\nu + \lambda_{q\Pi}\,\Pi\,\nabla^\mu\alpha + \lambda_{q\pi}\,\pi^{\mu\nu}\,\nabla_\nu\alpha \\
&\quad + \hat{\delta}_{1,2}\,\Pi\,q^\mu + \hat{\eta}_1\,\pi^{\mu\nu}\,q_\nu \\
\tau_\pi\,\dot{\pi}^{<\mu\nu>} + \pi^{\mu\nu} &= \pi^{\mu\nu}_{\mathrm{NS}} + 2\,\tau_{\pi q}\,q^{<\mu}\dot{u}^{\nu>} \\
&\quad + 2\,\ell_{\pi q}\,\nabla^{<\mu}q^{\nu>} + 2\,\tau_\pi\,\pi_\lambda{}^{<\mu}\omega^{\nu>\lambda} - 2\,\eta\,\hat{\delta}_{2,1}\,\pi^{\mu\nu}\,\theta \\
&\quad - 2\,\tau_\pi\,\pi_\lambda{}^{<\mu}\sigma^{\nu>\lambda} - 2\,\lambda_{\pi q}\,q^{<\mu}\nabla^{\nu>}\alpha + 2\,\lambda_{\pi\Pi}\,\Pi\,\sigma^{\mu\nu} \\
&\quad + \hat{\delta}_{2,2}\,\Pi\,\pi^{\mu\nu} - \hat{\eta}_2\,\pi_\lambda{}^{<\mu}\pi^{\nu>\lambda} - \hat{\epsilon}_2\,q^{<\mu}q^{\nu>}
\end{aligned}
$$

- And still subject to
  - Poorly constrained initial Conditions
  - Eccentricity fluctuations
  - Poorly constrained equation of state
  - Hadronic rescattering effects
  - Bulk viscosity
  - Numerical viscosity
  - Finite size, core/corona effects

For Further Details

- See "Virtual Journal on QCD Matter"
  - Steffen A. Bass, Berndt Mueller, William A. Zajc
  - qgp.phy.duke.edu



- On the topic of 2^{nd} order hydro:
  - What a Difference a Term Makes

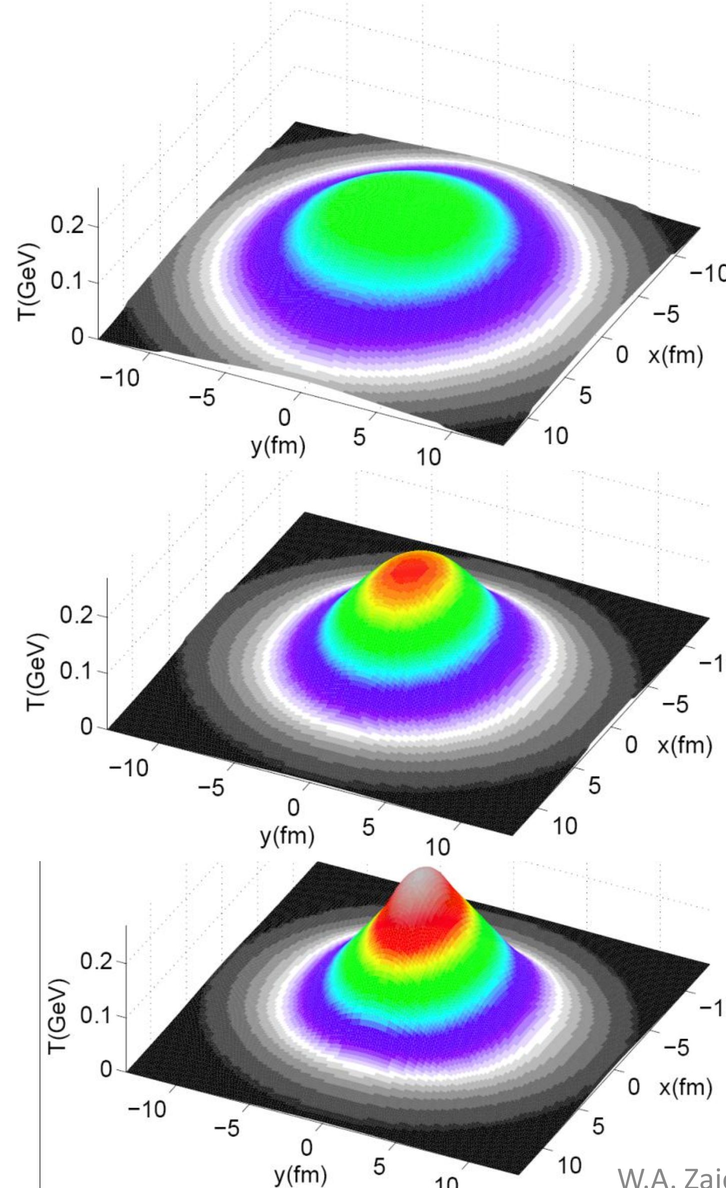
Implementing and Testing

- Examples
  - P. Romatschke and U. Romatschke, Phys. Rev. Lett. 99:172301, 2007
  - H. Song and U. Heinz, Phys. Rev. C78, 024902, 2008
  - M. Luzum and P. Romatschke hys.Rev.C78:034915,2008.

7 fm/c

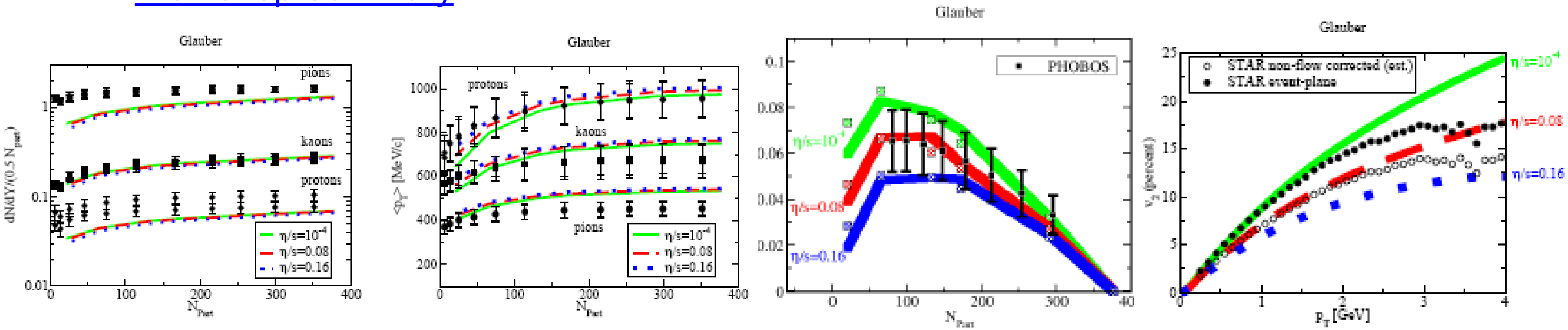
3 fm/c

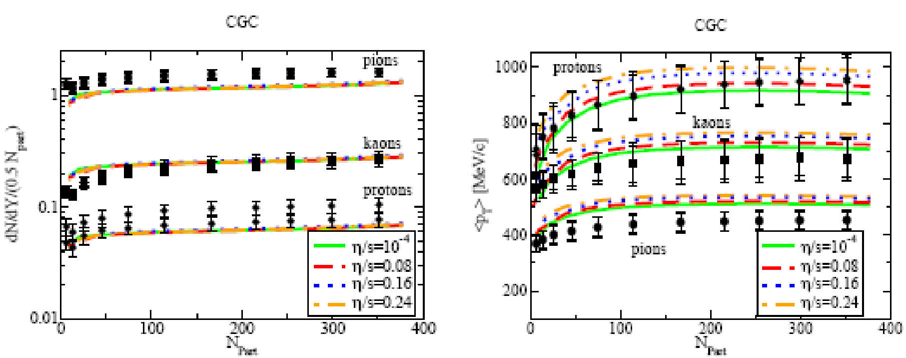
1 fm/c

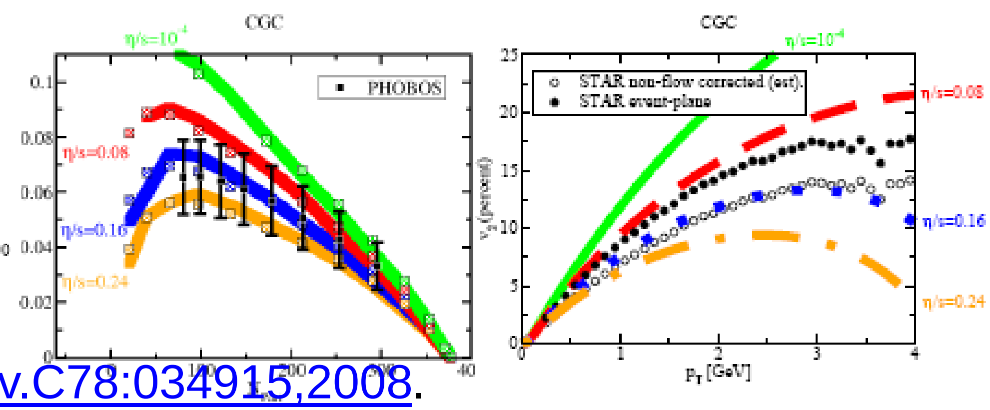


Concordance

- BNL, April 2008:
  - *Workshop on Viscous Hydrodynamics and Transport Models in Heavy Ion Collisions*
  - Workshop Summary







M. Luzum and P. Romtschke Phys.Rev.C78:034915,2008.

Concordance → *Quantifying* η/s

Q : How?

A : By performing detailed and systematic hydrodynamic simulations to understand sensitivity to η/s (and "everything" else)

- *Viscosity Information from Relativistic Nuclear Collisions: How Perfect is the Fluid Observed at RHIC?, P. Romatschke and U. Romatschke, Phys. Rev. Lett. 99:172301, 2007*

$$\frac{\eta}{s} = (0-2)\frac{1}{4\pi}$$

- *Multiplicity Scaling in Ideal and Viscous Hydrodynamics, H. Song and U. Heinz, Phys. Rev. C78, 024902, 2008*

$$\frac{\eta}{s} = (1-2)\frac{1}{4\pi}$$

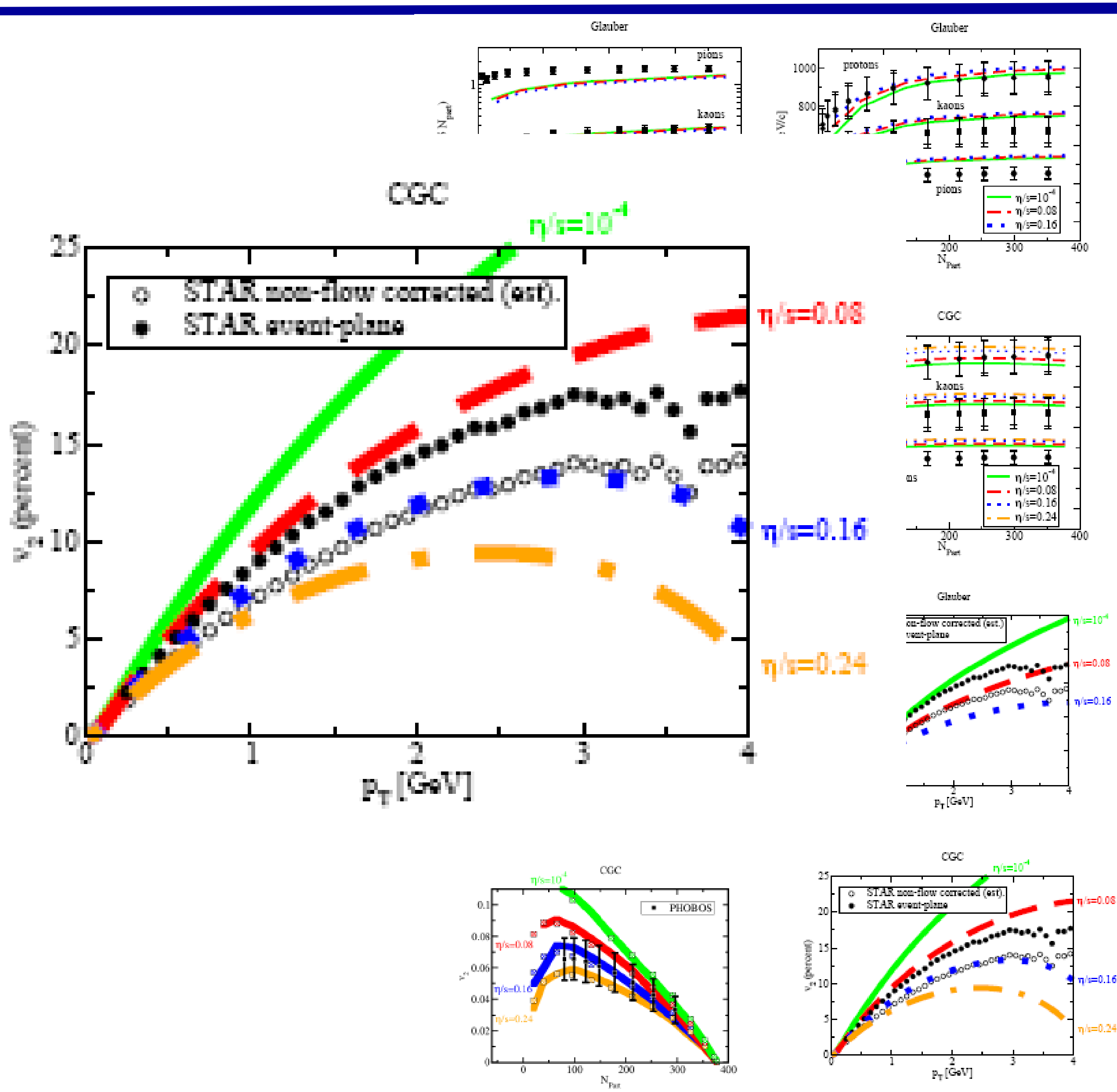
- *Conformal Relativistic Viscous Hydrodynamics: Applications to RHIC results at $\sqrt{s_{NN}}$ = 200 GeV, M. Luzum and P. Romatschke, Phys.Rev.C78:034915,2008.*

$$\frac{\eta}{s} = (1.3 \pm 1.3 \pm 1)\frac{1}{4\pi}$$

Viscosity Information from Relativistic Nuclear Collisions: How Perfect is the Fluid Observed at RHIC?, P. Romatschke and U. Romatschke, [Phys. Rev. Lett. 99:172301, 2007]

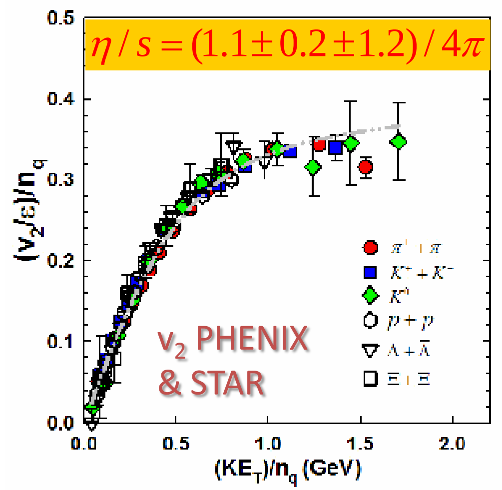
- Signatures: dN/dy, v_2, $<p_T>$
- Calculation: 2nd order causal viscous *conformal* hydro:

  (Glauber and CGC IC's)
- Payoff Plots:

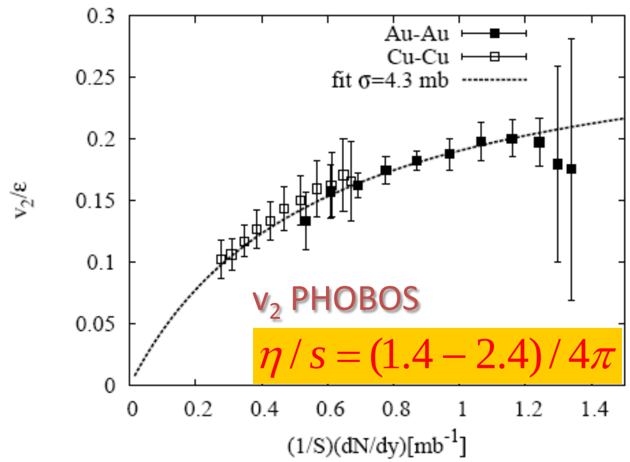


Comparison of Estimates

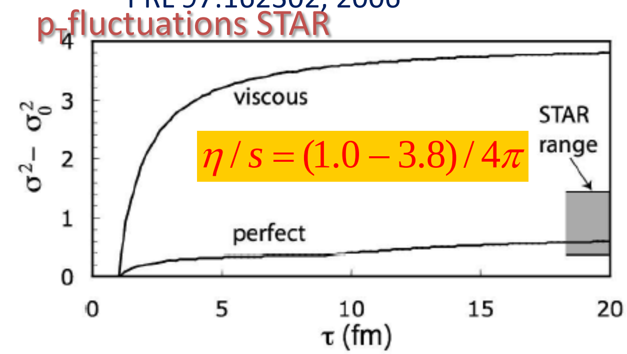
R. Lacey et al.: PRL 98:092301, 2007



H.-J. Drescher et al.: arXiv:0704.3553

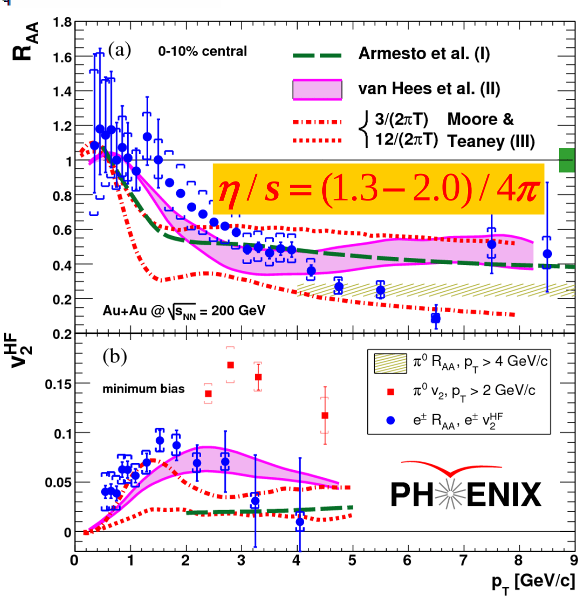


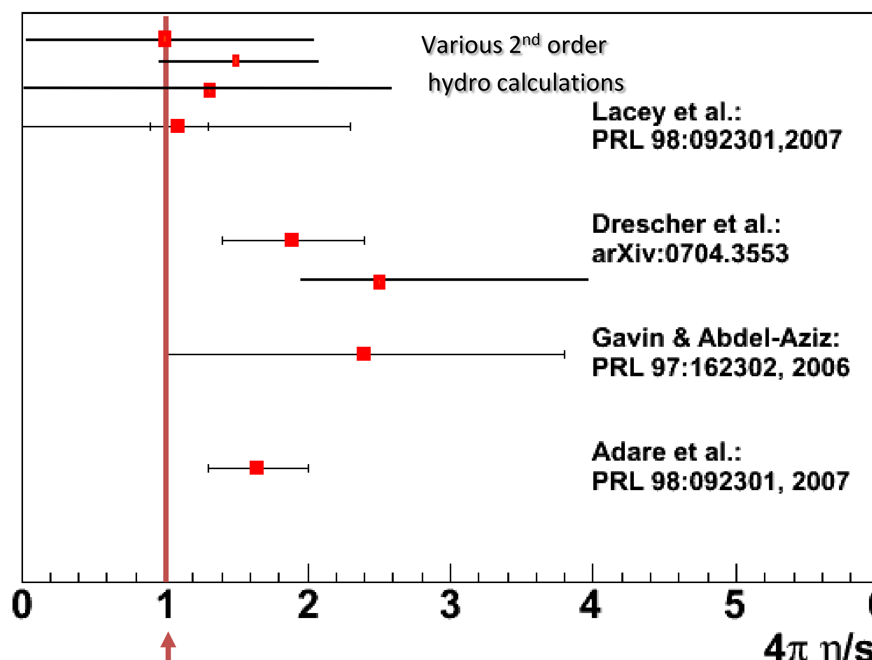
S. Gavin and M. Abdel-Aziz:
PRL 97:162302, 2006



A. Adare *et al*, PRL 98:172301, 2007

Heavy flavor drag, flow; PHENIX





conjectured quantum limit

Numerical Value of RHIC Viscosity

- **Input :** $\eta_{QGP} \sim \frac{\hbar}{4\pi} s_{QGP}$ $\quad T_{QGP} \sim 200\ \mathrm{MeV}$ $\quad \hbar c = 200\ \ \mathrm{Mev \cdot fm}$

- **Estimating s_{QGP} (per degree of freedom):**
  - **Method 1:**

$$s_{QGP} = (\varepsilon + p)/T_{QGP} \sim \frac{4}{3}\frac{\varepsilon}{T_{QGP}} = \frac{4}{3T_{QGP}}\left(\frac{\pi^2}{30}T_{QGP}{}^4\right) = \frac{2\pi^2}{45}T_{QGP}{}^3$$

  - **Method 2 (based on handy rule of thumb):**

$$n \sim \left(\frac{T}{2}\right)^3 \qquad s \sim 3.6n \Rightarrow s \sim \frac{3.6}{2^3}T^3 = 1.03\left(\frac{2\pi^2}{45}T^3\right)$$

- **Number of (assumed massless) degrees of freedom:**

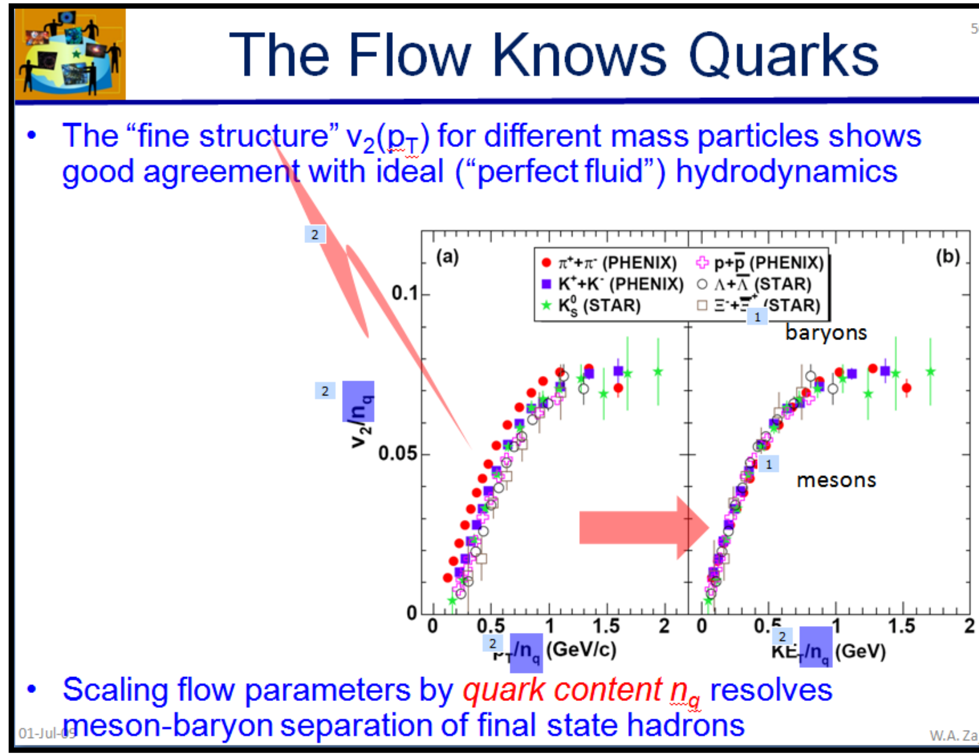
$$n_{d.o.f.} \sim \left\{2_s \cdot 8_g + \frac{7}{8}\cdot 2_s \cdot 2_{a.p.} \cdot 3_c \cdot 3_f\right\} = 47.5 \sim 45$$

$$\Rightarrow \eta_{QGP} \sim \frac{\hbar}{4\pi} s_{QGP} \sim \frac{\hbar}{4\pi} n_{d.o.f.}\left(\frac{2\pi^2}{45}T^3{}_{QGP}\right) \sim \frac{\pi}{2}\frac{\hbar c}{c}T^3{}_{QGP}$$

$$= \frac{\pi \cdot (2\cdot 10^8\ \mathrm{eV} \cdot 10^{-15}\ \mathrm{m}) \times (1.6\cdot 10^{-19}\ \mathrm{J/eV})}{2(3\cdot 10^8\ \mathrm{m/s})}(10^{15}\ \mathrm{m})^3 = 1.6\times 10^{11}\ \frac{\mathrm{J\cdot s}}{\mathrm{m}^3}$$

A Deep Puzzle

- Apparent role of quark degrees of freedom:



- *Incompatible* (! ?) with η/s near the quantum bound :

- 'Good' quasi-particles have widths << mass

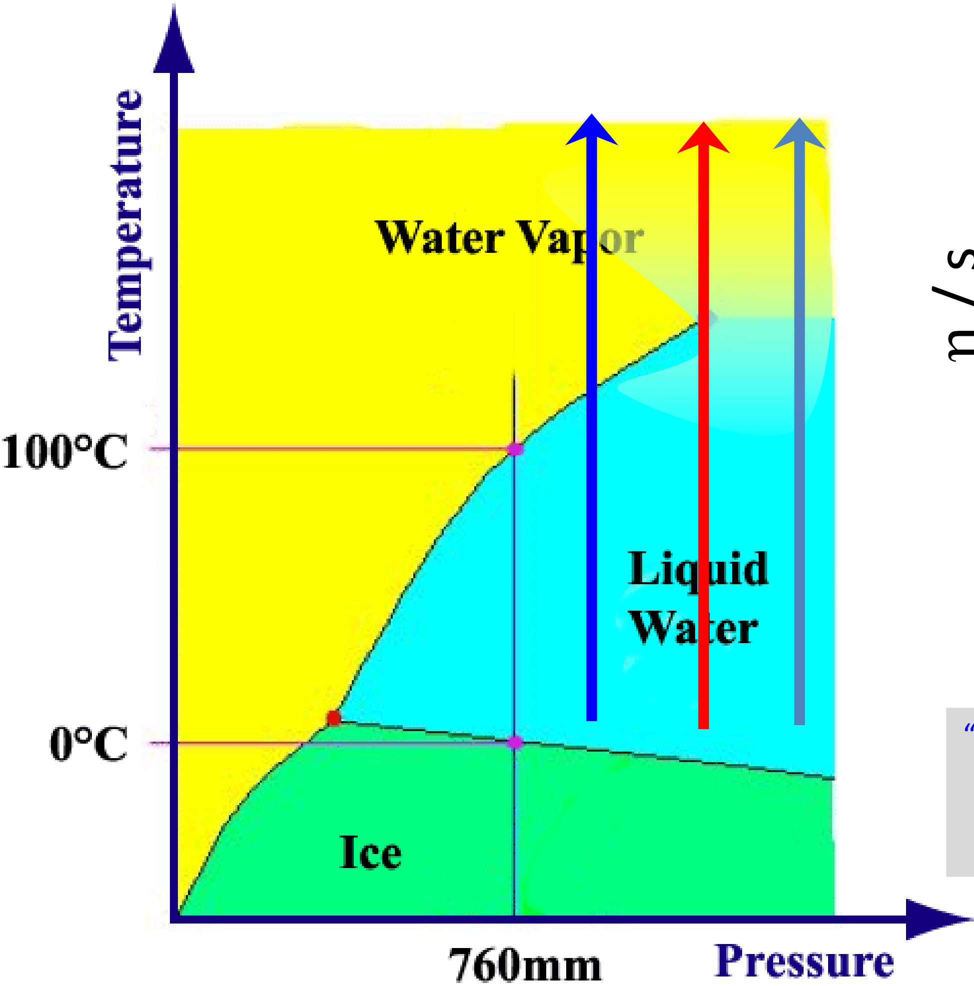
- Minimal mfp's $\Rightarrow$ widths ~ mass

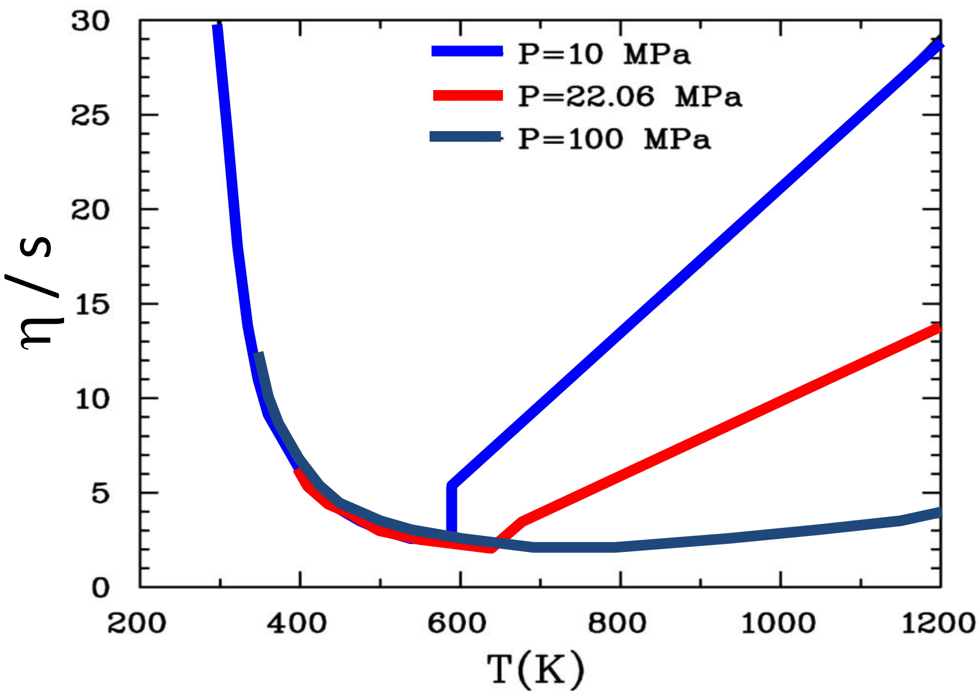
Implications for D.O.F

- While tempting to identify the coalescence patterns with "underlying quark degrees of freedom"…
- Much work still needed to reconcile with 'absence' of quasiparticles when η/s near quantum bound
  - *Quasi-Particle Degrees of Freedom versus the Perfect Fluid as Descriptors of the Quark-Gluon Plasma*, L.A. Levy *et al*., Phys.Rev.C78:044905,2008. 0709.3105
  - *Quantum Criticality and Black Holes*, S. Sachdev and M. Mueller, 0810.3005 :
    - →"The theory of the quantum critical region shows that the transport coefficients, and the relaxation time to local equilibrium, *are not proportional to a mean free scattering time between excitations*, as is the case in the Boltzmann theory of quasiparticles

Water → RHIC → Water → RHIC

- The search for QCD phase transition of course was informed by analogy to ordinary matter
- Results from RHIC are now "flowing" back to ordinary matter





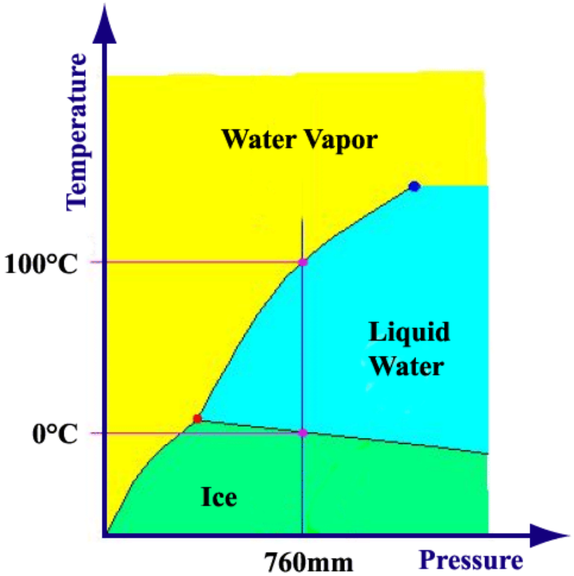
"On the Strongly-Interacting Low-Viscosity Matter Created in Relativistic Nuclear Collisions",
L.P. Csernai, J.I. Kapusta and L.D. McLerran,
Phys.Rev.Lett.97:152303,2006, nucl-th/0604032

Is There a QCD Critical Point?

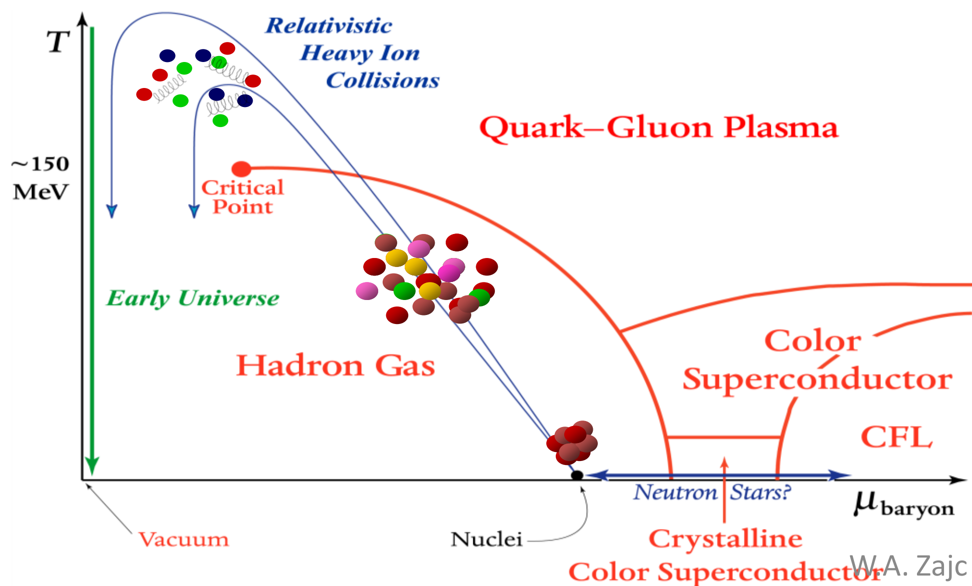
- Here the analogy with phase transitions in ordinary matter breaks down:
  - Recall " Properties of the medium are (at zero baryon number) uniquely determined by T "

  ⇨ Pressure = P(T) can't vary independently (unlike water)
  - But if baryon number is non-zero

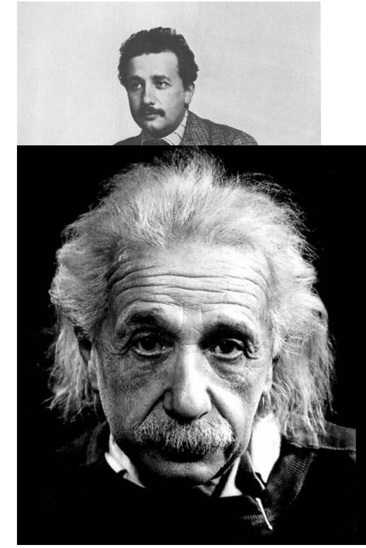
    ⇨ (intensive order parameter) baryon chemical potential μ_B :



- To increase μ_B :
  - *Lower* collision energy
  - *Raise* atomic mass
  - *Both* part of RHIC II and GSI-FAIR



A Spooky Connection



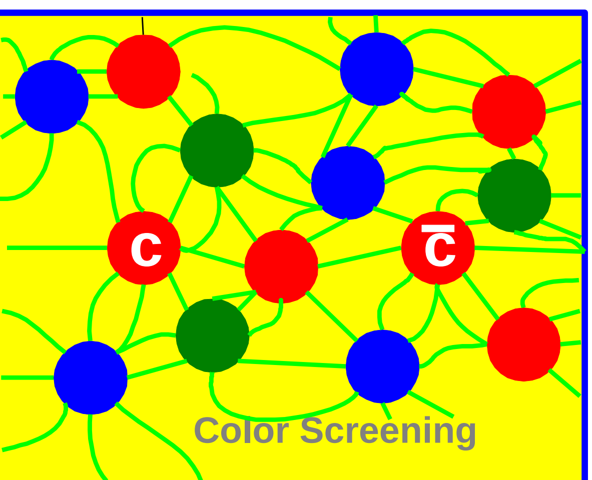
- RHIC physics clearly relies on
  - The quantum nature of matter (Einstein, 1905)
  - The relativistic nature of matter (Einstein, 1905)

but presumably has no connection to
  - General relativity (Einstein, 1912-7)

- Wait ! Both sides of this equation

$$(Viscosity)_{RHIC} \approx \frac{\hbar}{4\pi} (Entropy\ Density)_{RHIC}$$

were calculated using black hole physics (in 10 dimensions)

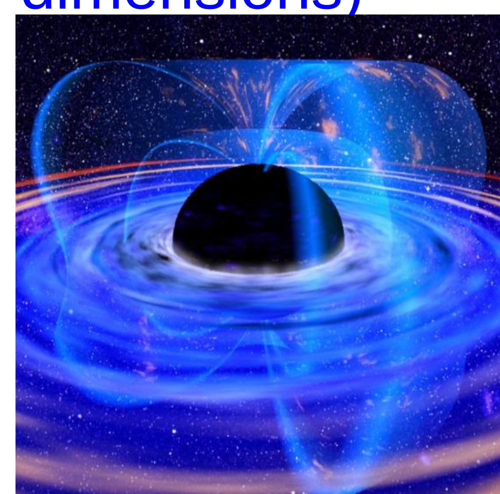


MULTIPLICITY

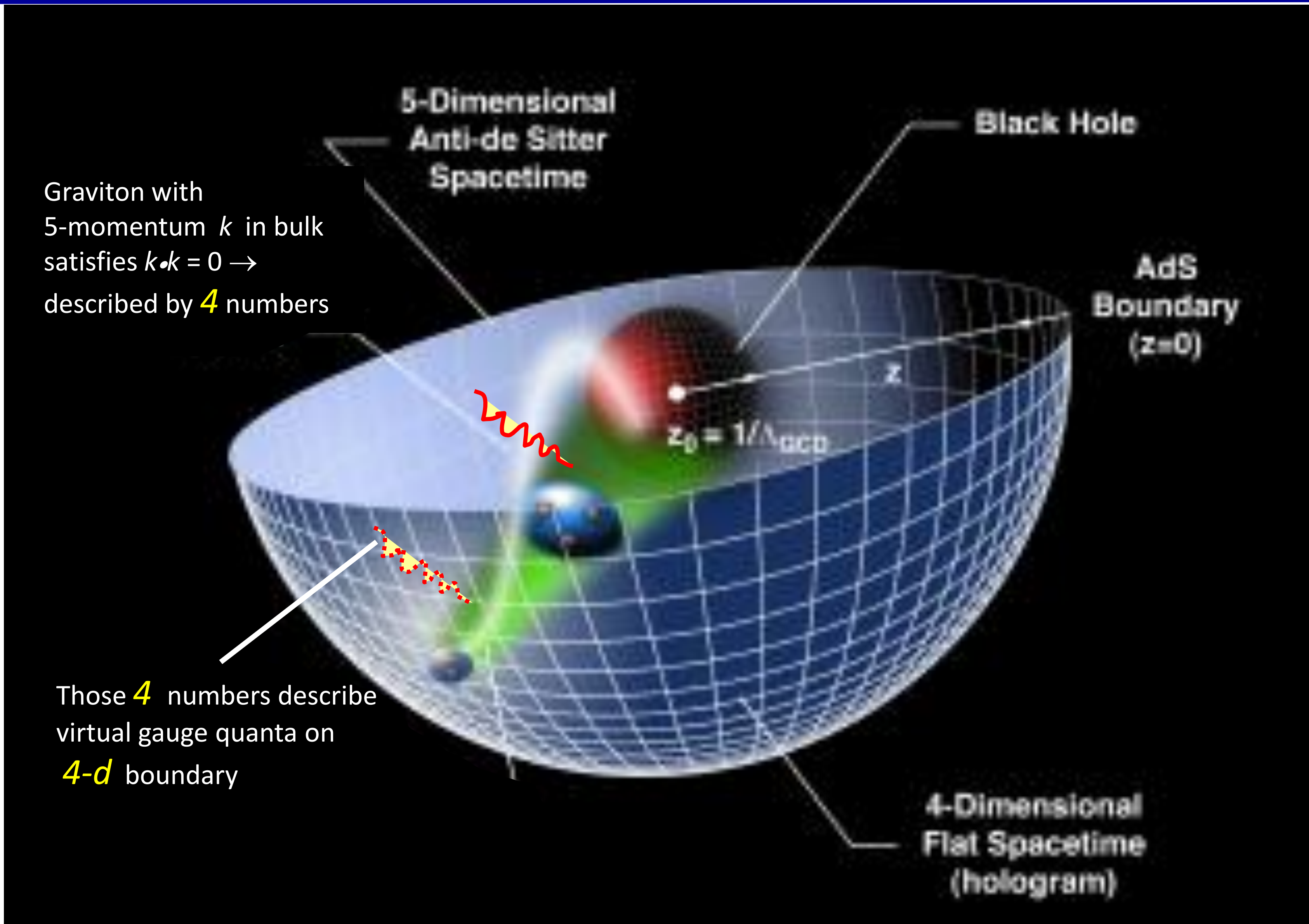
Entropy $\leftrightarrow$ Black Hole Area

DISSIPATION

Viscosity $\leftrightarrow$ Graviton Absorption



(Minimal) AdS / CFT $\Rightarrow \frac{\eta}{s} \geq \frac{1}{4\pi}$



(Adopted from S. Brodsky figure)

In Words

- A stringy theory of gravity in N space-time dimensions (the "bulk" AdS)

  is "dual" (that is, equivalent to)

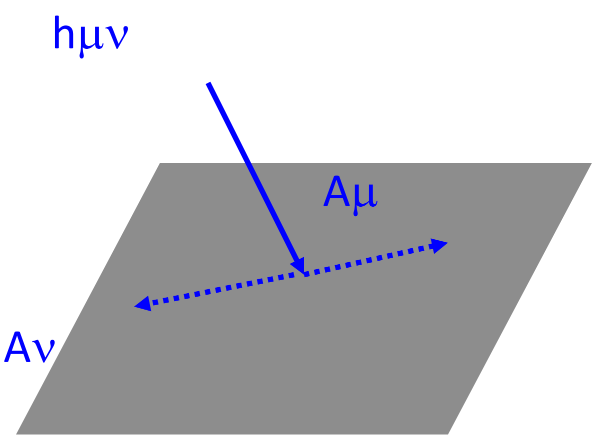
- A gauge theory without gravity in N-1 space-time dimensions (the "boundary" CFT)
- Notes:
  - AdS ≡ Anti de Sitter space ; CFT ≡ Conformal Field Theory
  - "Equivalent" means "equivalent" – all phenomena in one theory have corresponding "dual" descriptions in the other theory.
  - Maldacena's AdS/CFT correspondence is a realization of hypotheses from both 't Hooft and Susskind that the ultimate limit on the number of degrees of freedom in a spacetime region is proportional to the area of its boundary, *not* its volume (!)

Why Does This Work??

- The easy part:
  - Recall $$\frac{F_x}{A} = -\eta \frac{\partial v_x}{\partial y}$$
  - that is, viscosity ~ x-momentum transport in y-direction ~ T^{xy}
  - There are standard methods (Kubo relations) to calculate such dissipative quantities

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d\mathbf{x}\, e^{i\omega t} \langle [T_{xy}(t, \mathbf{x}), T_{xy}(0, 0)] \rangle$$

- The hard part:
  - This calculation is difficult in a strongly-coupled gauge theory



- The weird part:
  - A (supersymmetric) pseudo-QCD theory can be mapped to a 10-dimensional classical gravity theory on the background of black 3-branes
  - The calculation can be performed there as the absorption of gravitons by the brane
  - THE SHEAR VISCOSITY OF STRONGLY COUPLED N=4 SUPERSYMMETRIC YANG-MILLS PLASMA., G. Policastro, D.T. Son , A.O. Starinets, Phys.Rev.Lett.87:081601,2001 hep-th/0104066

The Result

- Viscosity η = “Area”/16πG

Infinite “Area” !

- Normalize by entropy (density) S = “Area”/4G
- Dividing out the infinite “areas” :

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi}\frac{1}{k_B}$$

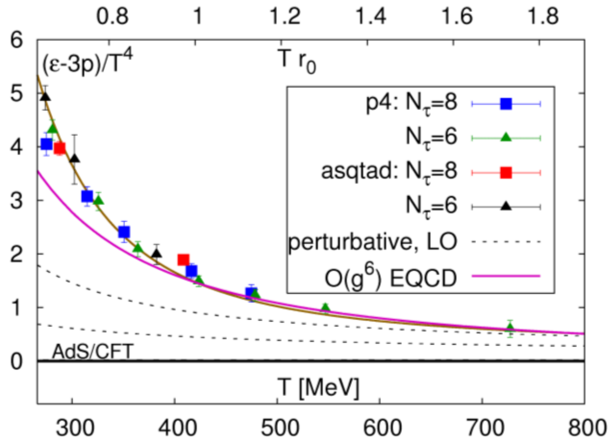
- *Conjectured* to be a lower bound “*for all relativistic quantum field theories at finite temperature and zero chemical potential*”.
- See “*Viscosity in strongly interacting quantum field theories from black hole physics*”, P. Kovtun, D.T. Son, A.O. Starinets, Phys.Rev.Lett.94:111601, 2005, hep-th/0405231

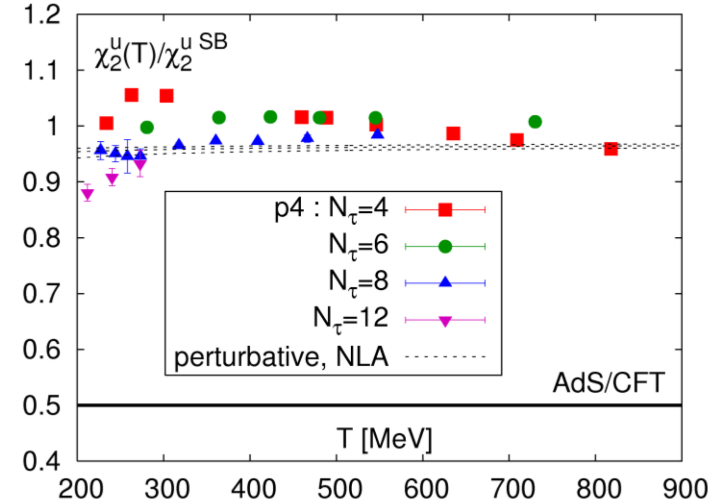
AdS/CFT - Pro

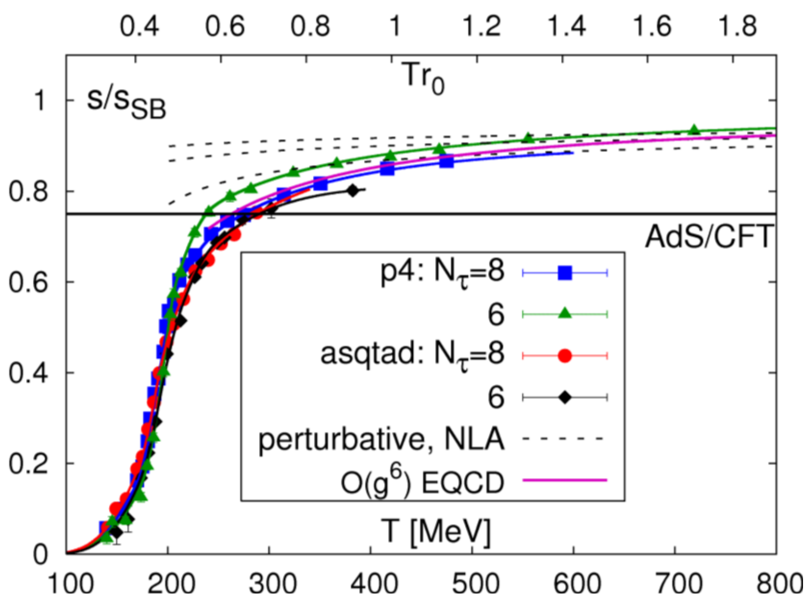
- *Gauge/gravity duality*, G.T. Horowitz and J. Polchinski, gr-qc/0602037)
  - "Hidden within every non-Abelian gauge theory, even within the weak and strong nuclear interactions, is a theory of quantum gravity."
- *Stringscape*, by Matthew Chalmers, in *Particle World*:
  - "Susskind says that by studying heavy-ion collisions you are also studying quantum gravity that is 'blown up and slowed down by a factor of 10^{20}' "
- *The Black Hole War*, L. Susskind, ISBN 978-0-316-01640-7 :
  - "…the Holographic Principle is evolving from radical paradigm shift to everyday working tool of – surprisingly – nuclear physics."

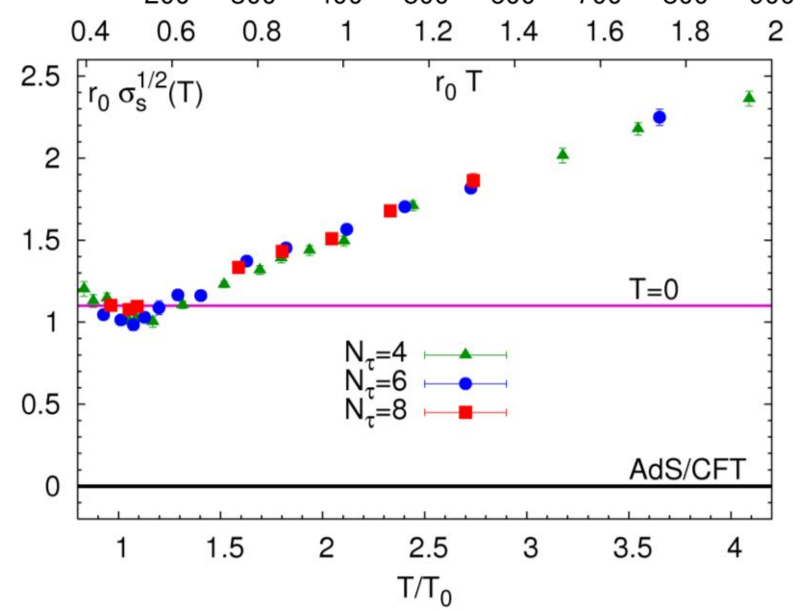
AdS/CFT - Con

- P. Petreczsky, QM09: “AdS/CFT is consistently wrong. ”









Editorial Opinion

- Reality may lie between these two extremes ☺
- At the moment:
  - Hard physics (high p_T energy loss)
    - Fragile predictions
    - Robust extractions
  - Soft physics (hydrodynamics, viscosity)
    - Robust predictions
    - Fragile extractions
- AdS/CFT has led to qualitatively new insights
- Not covered:
  - AdS/QCD (application to confinement, masses)

Why AdS/CFT Matters…

- All the thermal parts are built upon Bekenstein and Hawking's (unproven) assertion that black holes have entropy:

$$S_{BH} = \frac{A}{4G} = \frac{A}{4L_P^{\ 2}}$$

⇨Black holes have a temperature

⇨Black holes can radiate

⇨Black holes don't lose information

- Important to test these very underpinnings

Summary- Lecture 4

- A minimal η/s is predicted for a dense thermal fluid in the quantum regime.
- Major advances in relativistic viscous hydrodynamics are helping to bound the observed η/s value at RHIC.
- There is a striking connection between this physics and similar phenomena predicted via the AdS/CFT correspondence.

Summary

- There is qualitative and quantitative theoretical support for a deconfining phase transition in QCD.
- The matter produced at RHIC is indeed matter, imbued with unusual properties beyond those predicted for ‘QGP’.
- Most compelling of these is the fluid behavior of the densest matter ever studied.
- There are more discoveries to come as we continue to explore thermal QCD matter.