Introduction to Relativistic Heavy Ion Physics

Lecture 1: QCD Thermodynamics

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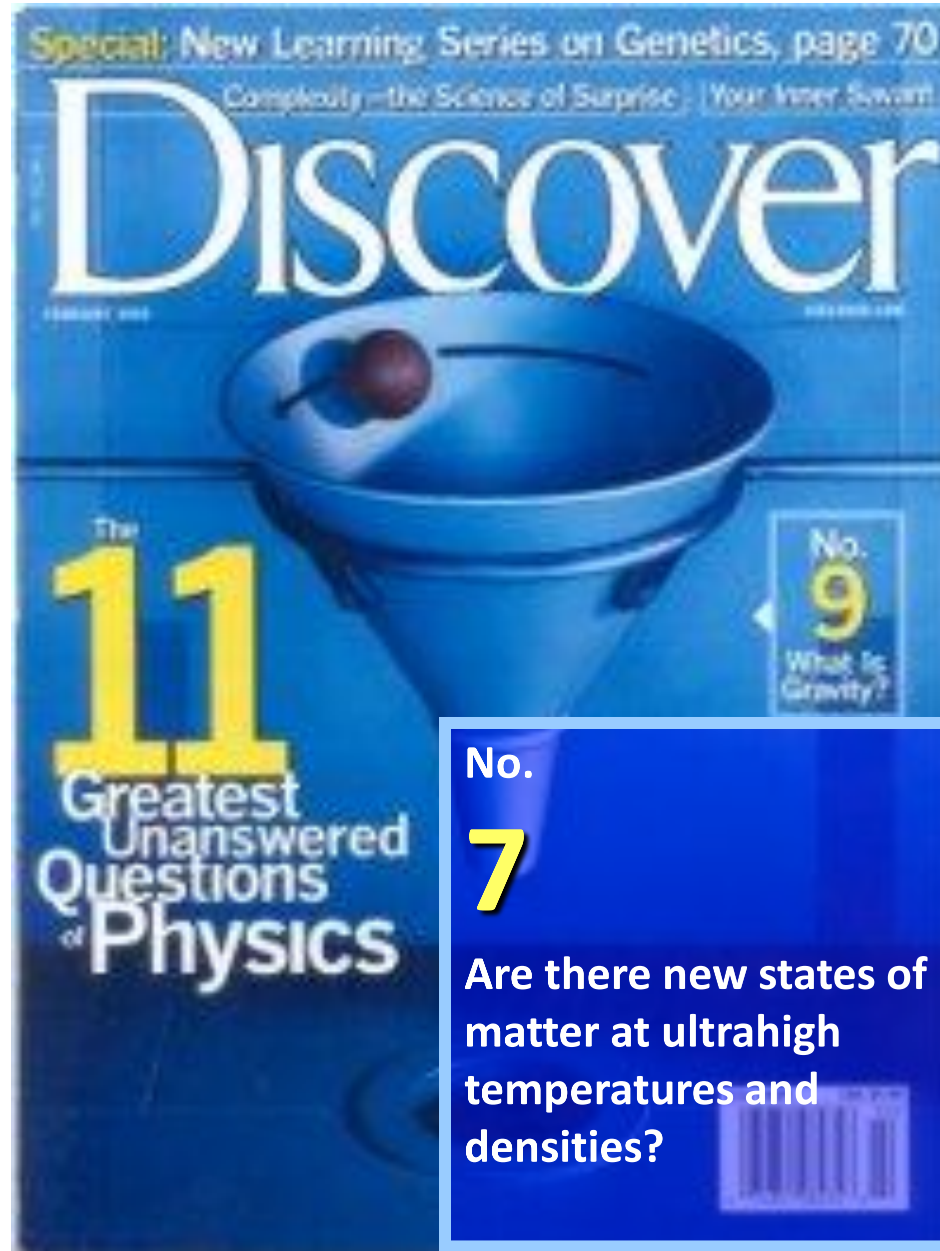
Science Questions

- ✕ Uninteresting question:
  - What happens when I crash two gold nuclei together?
- ✓ Interesting question:
  - ➡ Are there new states of matter at the highest temperatures and densities?

New States of Matter ?

Eleven Science Questions for the New Century
Committee on the Physics of the Universe
NATIONAL RESEARCH COUNCIL *OF THE NATIONAL ACADEMIES…*

What Are the New States of Matter at Exceedingly High Density and Temperature?



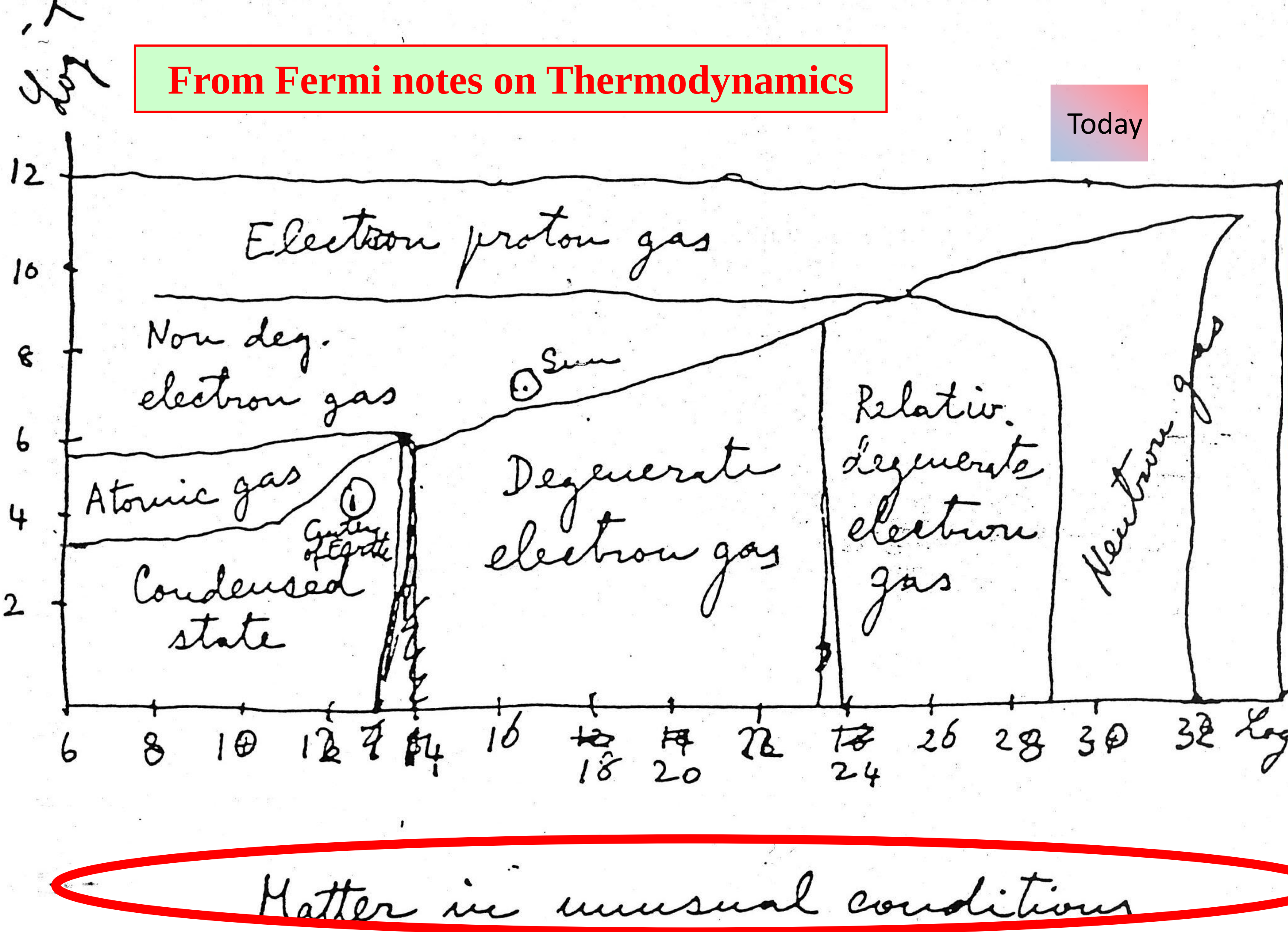
No.
7
Are there new states of matter at ultrahigh temperatures and densities?

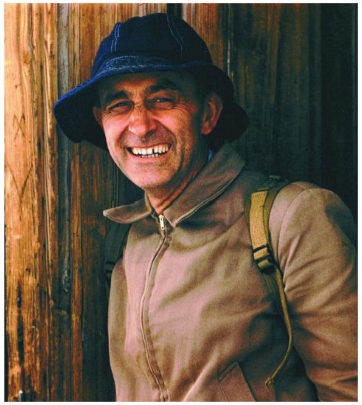
Fermi's Vision

- ~1950: (Almost) included physics of 2009
- See also remarks in his "statistical model" paper

From Fermi notes on Thermodynamics

Today





dyne/cm^2 !

(Thanks to A. Melissinos)

A. Zajc

Science Questions

- ✕ Uninteresting question:
  - What happens when I crash two gold nuclei together?
- ✓ Interesting question:
  - ➡ Are there new states of matter at the highest temperatures and densities?
- $ Compelling question:
  - ☞ What fundamental *thermal* properties of our gauge theories of nature can be investigated experimentally?
    - ☞ Hint: *Gravity* is a gauge theory…

1973

- 1973 = Birth of QCD
- Gross, Politzer, Wilczek

VOLUME 30, NUMBER 26 PHYSICAL REVIEW LETTERS 25 JUNE 1973

[14] Y. Nambu and G. Jona-Lasino, Phys. Rev. 122, 345 (1961); S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).

[15] K. Symanzik (to be published) has recently suggested that one consider a $\lambda\varphi^4$ theory with a negative λ to achieve UV stability at $\lambda=0$. However, one can show, using the renormalization-group equations, that in such theory the ground-state energy is unbounded from below (S. Coleman, private communication).

[16] W. A. Bardeen, H. Fritzsch, and M. Gell-Mann, CERN Report No. CERN-TH-1538, 1972 (to be published).

[17] H. Georgi and S. L. Glashow, Phys. Rev. Lett. 28, 1494 (1972); S. Weinberg, Phys. Rev. D 5, 1962 (1972).

[18] For a review of this program, see S. L. Adler, in Proceedings of the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Illinois, 1972 (to be published).

Reliable Perturbative Results for Strong Interactions?*

H. David Politzer
Jefferson Physical Laboratories, Harvard University, Cambridge, Massachusetts 02138
(Received 3 May 1973)

An explicit calculation shows perturbation theory to be arbitrarily good for the deep Euclidean Green's functions of any Yang-Mills theory and of many Yang-Mills theories with fermions. Under the hypothesis that spontaneous symmetry breakdown is of dynamical origin, these symmetric Green's functions are the asymptotic forms of the physically significant spontaneously broken solution, whose coupling could be strong.

Renormalization-group techniques hold great promise for studying short-distance and strong-coupling problems in field theory.[1,2] Symanzik[2] goes to zero, compensating for the fact that there are more and more of them. But the large-p^2 divergence represents a real breakdown of

PHYSICAL REVIEW D VOLUME 8, NUMBER 10 15 NOVEMBER 1973

Asymptotically Free Gauge Theories. I*

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and Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540

Frank Wilczek
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540
(Received 23 July 1973)

Asymptotically free gauge theories of the strong interactions are constructed and analyzed. The reasons for doing this are recounted, including a review of renormalization-group techniques and their application to scaling phenomena. The renormalization-group equations are derived for Yang-Mills theories. The parameters that enter into the equations are calculated to lowest order and it is shown that these theories are asymptotically free. More specifically the effective coupling constant, which determines the ultraviolet behavior of the theory, vanishes for large spacelike momenta. Fermions are incorporated and the construction of realistic models is discussed. We propose that the strong interactions be mediated by a "color" gauge group which commutes with SU(3) × SU(3). The problem of symmetry breaking is discussed. It appears likely that this would have a dynamical origin. It is suggested that the gauge symmetry might not be broken and that the severe infrared singularities prevent the occurrence of noncolor singlet physical states. The deep-inelastic structure functions, as well as the electron-positron total annihilation cross section are analyzed. Scaling obtains up to calculable logarithmic corrections, and the naive light-cone or parton-model results follow. The problems of incorporating scalar mesons and breaking the symmetry by the Higgs mechanism are explained in detail.

Quantum Chromodynamics (QCD)

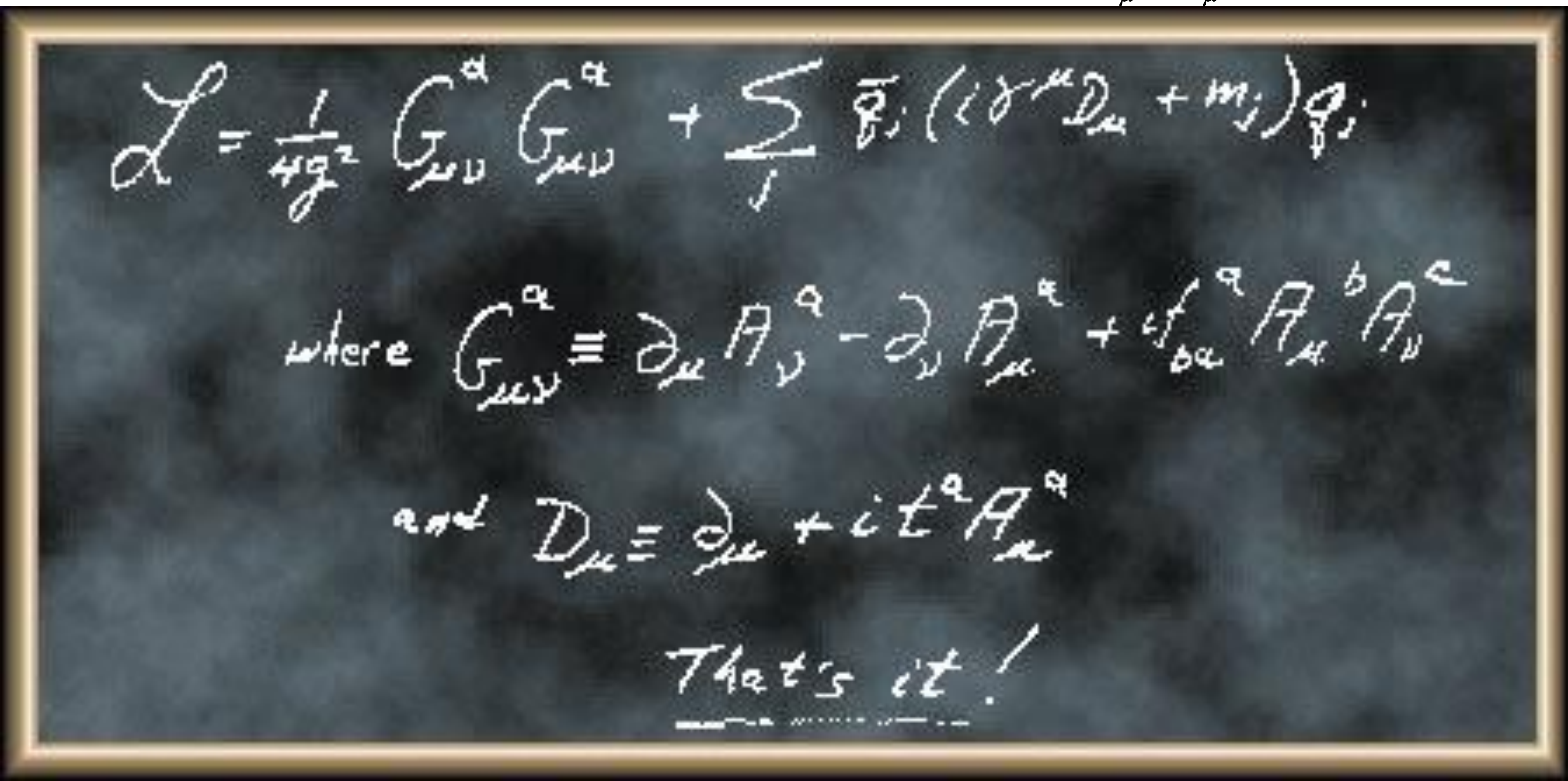
- Sure looks like QED:

$$L = \frac{1}{4e^2} F_{\mu\nu} F_{\mu\nu} + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

$$\textit{where} \quad F_{\mu\nu} \equiv \partial_\mu A^\nu - \partial_\nu A^\mu$$

$$\textit{and} \quad D_\mu = \partial_\mu - iA^\mu$$

Warning: Non-standard definition of A^μ !



A Nobel Cause

The Nobel Prize in Physics 2004

The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics for 2004 "for the discovery of asymptotic freedom in the theory of the strong interaction" jointly to David J. Gross, H. David Politzer and Frank Wilczek

BACK

David J. Gross
Kavli Institute for Theoretical Physics
University of California, Santa Barbara, USA

H. David Politzer
California Institute of Technology (Caltech), Pasadena, USA

Frank Wilczek
Massachusetts Institute of Technology (MIT), Cambridge, USA

A good start ...

Frank Wilczek and David Politzer were barely 20 years old and still PhD students when their discovery of asymptotic freedom was published. These were their very first scientific publications!

A colourful connection

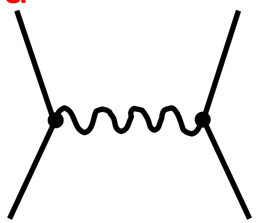
The scientists awarded this year's Nobel Prize in Physics have solved a mystery surrounding the strongest of nature's four fundamental forces. The three quarks within the proton can sometimes appear to be free, although no free quarks have ever been observed. The quarks have a quantum mechanical property called colour and interact with each other through the exchange of gluons - nature's glue.

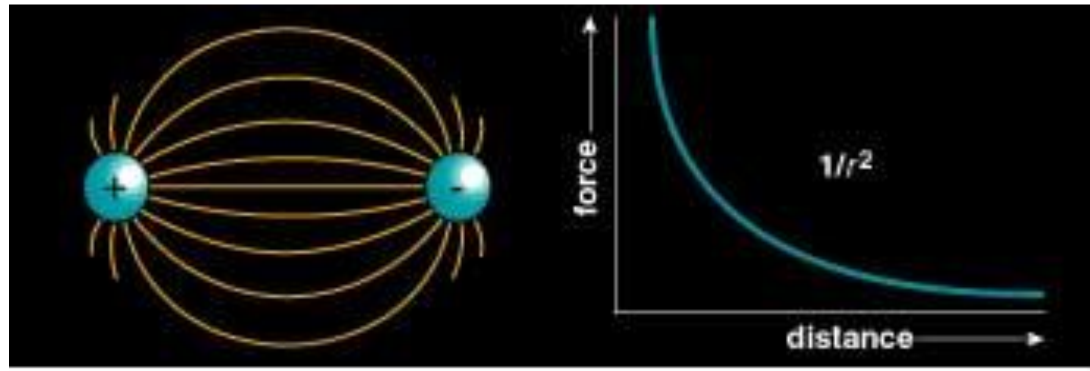
QCD is *not* QED

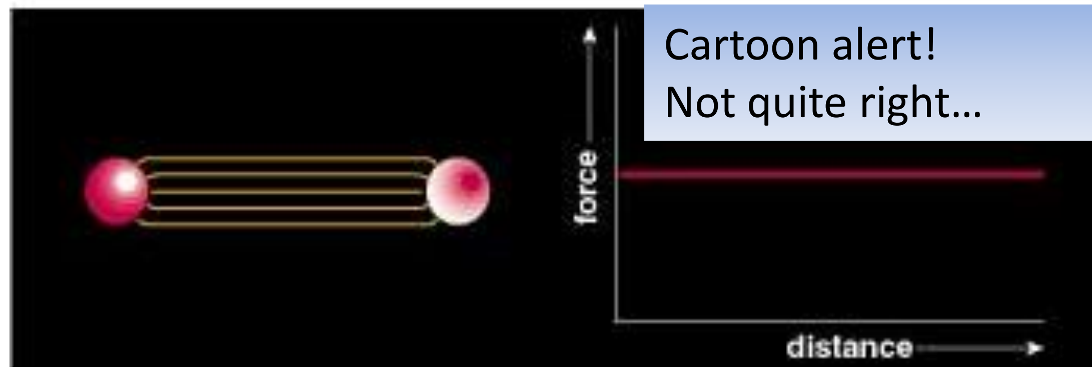
- QED (Abelian):
  - Photons do not carry charge
  - Flux is not confined

    ⇒ 1/ r potential

    ⇒ 1/ r ^{2} force

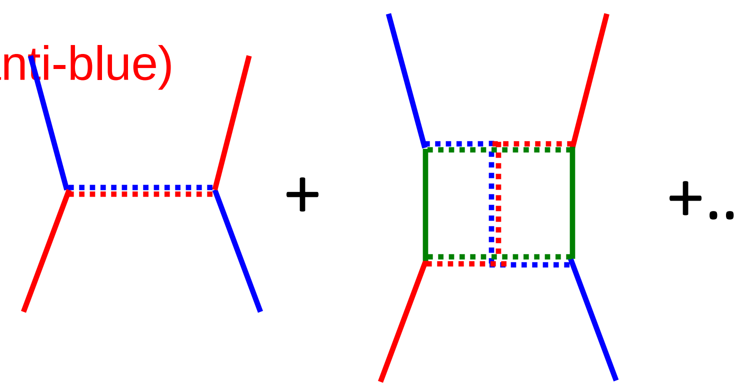






Cartoon alert!
Not quite right…

- QCD (Non-Abelian):
  - Gluons do carry charge
    (red, green, blue) ⊗ (anti-red, anti-green, anti-blue)
  - Flux tubes form



  ⇒ potential ~ r

  ⇒ *constant* force (at ‘large’ distances)

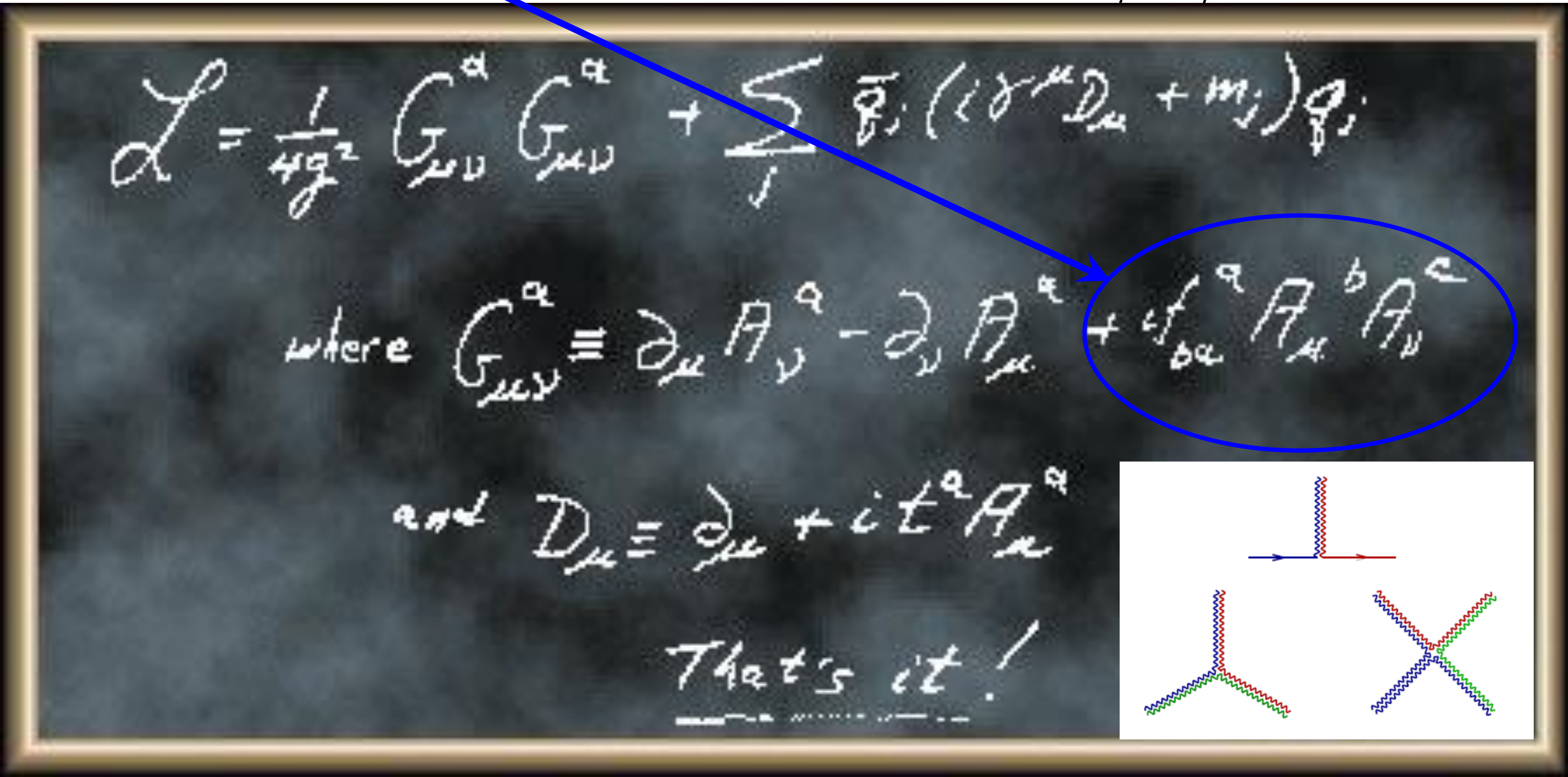
Quantum Chromodynamics (QCD)

- Sure looks like QED:
- Except for this !

$$L = \frac{1}{4e^2} F_{\mu\nu} F_{\mu\nu} + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

$$\text{where} \quad F_{\mu\nu} \equiv \partial_\mu A^\nu - \partial_\nu A^\mu$$

$$\text{and} \quad D_\mu = \partial_\mu - iA^\mu$$



The Consequence

- Linear potential (at large distances) ⇒

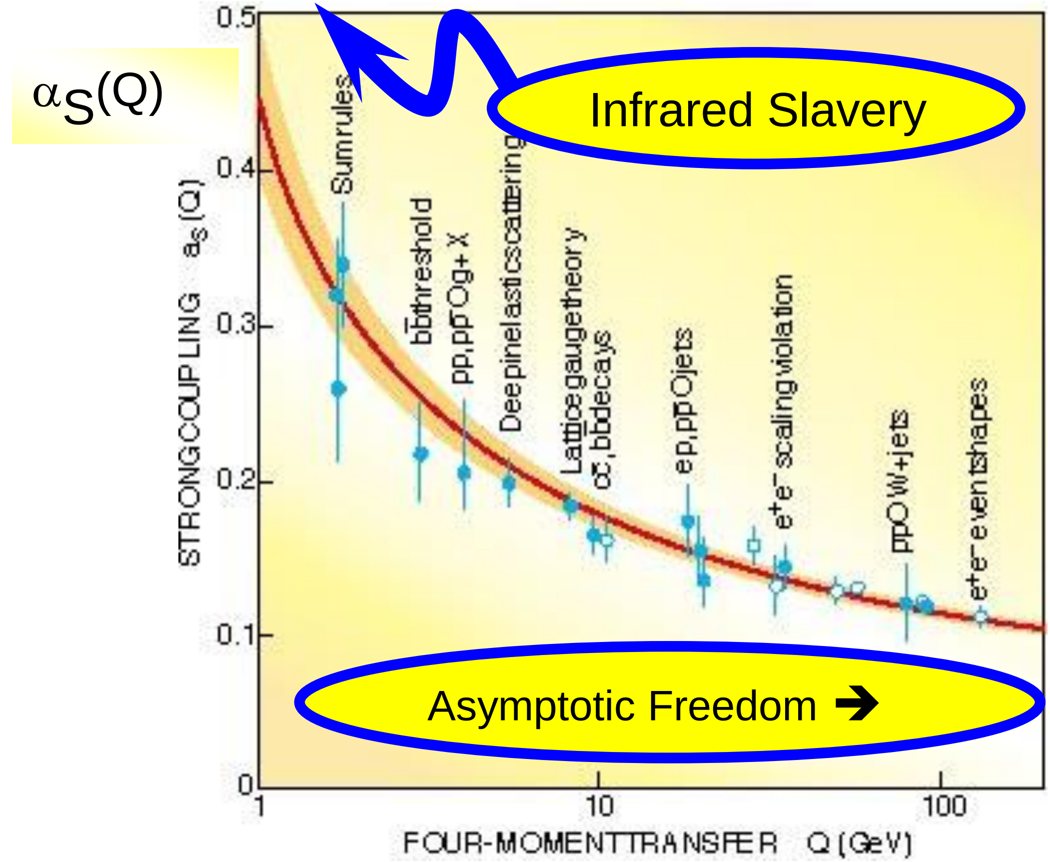
  “Single” (aka “isolated” aka “bare” aka “free”) quarks are *never* observed.

- A “direct” consequence of the non-Abelian terms in the QCD Lagrangian

- Instead
  - Mesons : *Confined* quark-antiquark pairs
  - Baryons: *Confined* 3q combinations

QCD's Essential Feature

- Hadron sizes
  ~ 10^{-15} meters
  aka 1 femtometer
  aka 1 fermi = 1 fm
- Planck's constant
  $\hbar$ c = 0.2 GeV-fm
  ➔ $1\ fm^{-1} \Leftrightarrow 200$ MeV
  ➔ 200 MeV
  ~ characteristic scale of *confinement*



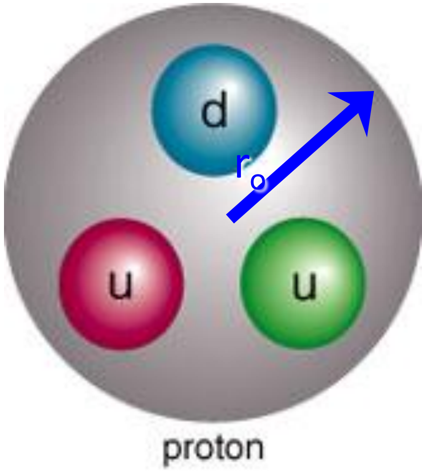
0.2 fm 0.02 fm 0.002 fm

- As reflected in the "running coupling constant" of QCD

$$\alpha_S(Q^2) = \frac{12\pi}{(33-2N_F)\log(\frac{Q^2}{\Lambda^2})} \sim \frac{1}{\log(\frac{Q^2}{\Lambda^2})} \qquad \Lambda \approx \frac{\hbar c}{r_o} \approx 0.2\ GeV$$

Required Hadron Physics

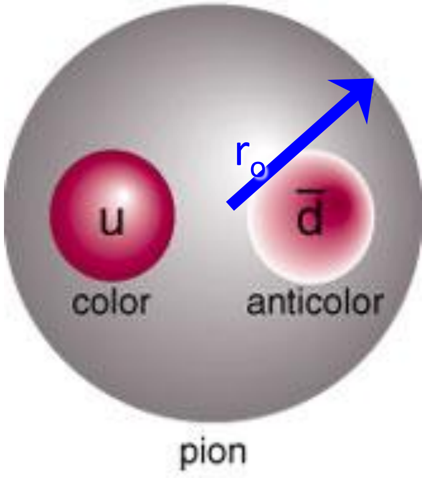
- One magic number: "all" hadrons have the same radius r_o
  - Characteristic length scale r_o ~ 1 fm
  - Characteristic energy scale $\hbar c$ / (1 fm) ~ 200 MeV



- *'Observation':* Quarks (and gluons) are *confined* (color neutral) *bags* of radius ~ r_o

- Parameterize confinement by "bag constant" B

$$m_H c^2 = potential + kinetic = B\left(\frac{4\pi}{3} r_0^3\right) + a\frac{\hbar}{r_0}$$



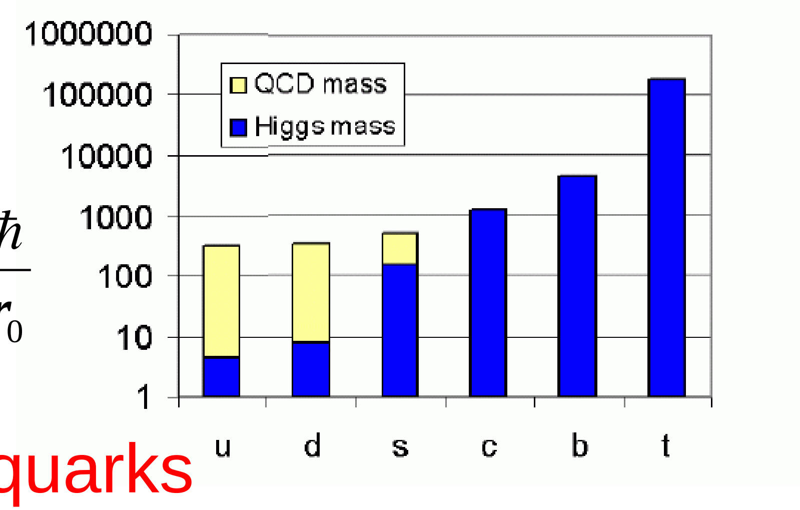
- Hadron masses mc^2 ~ 1 GeV , "a" ~ 1

➡ B ~ 200 MeV / fm^3 = 0.2 GeV / fm^3

A Consequence

- (Well, really an assumption)
  - Built into this expression

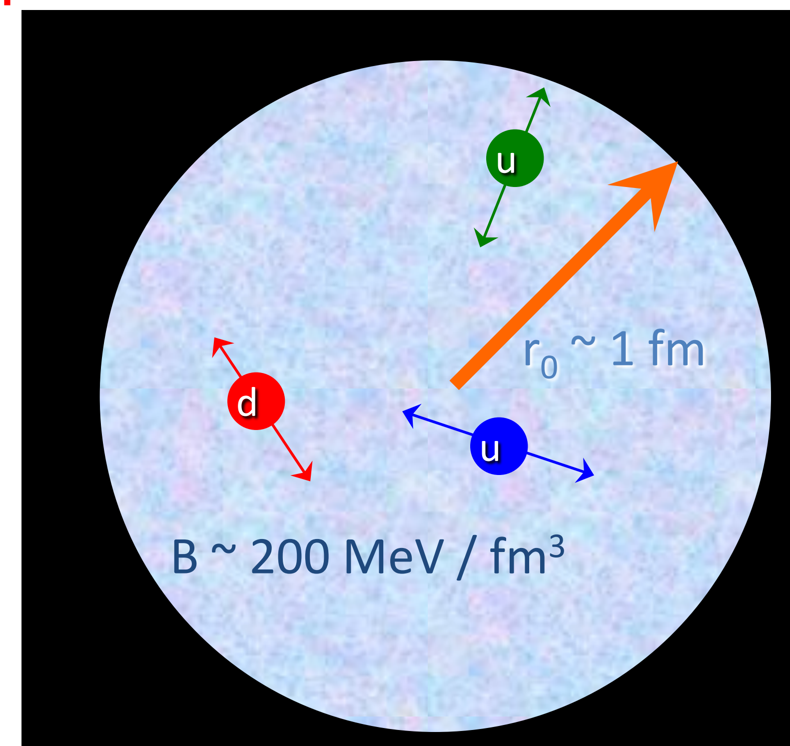
$$m_H c^2 = potential + kinetic = B\left(\frac{4\pi}{3} r_0^{\ 3}\right) + a\frac{\hbar}{r_0}$$



  - is the assumption of *massless* quarks

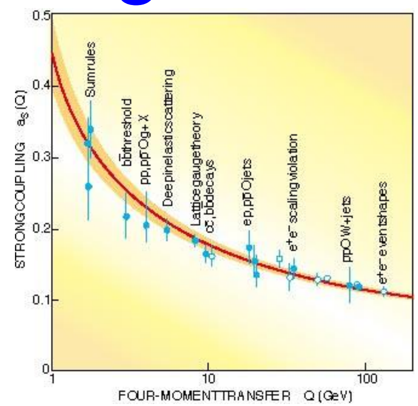
$$kinetic\ energy \sim momentum \sim \frac{h}{\lambda} \sim a\frac{\hbar}{r_0}$$

- This (strange) assumption *consistent* with properties of hadrons:
  - $m_{UP} \sim m_{DOWN} \sim$ few MeV
  - $m_{PROTON} \sim 940$ MeV $\sim m_{BAG}$!!!



Liberation Movement

- Not long after 1973 . . .
- Running of QCD coupling constant suggests possibility of building a new state of "QCD Matter" with "free" quarks and gluons at:
  - Sufficiently high temperature T
  - Sufficiently high baryon density ρ_B



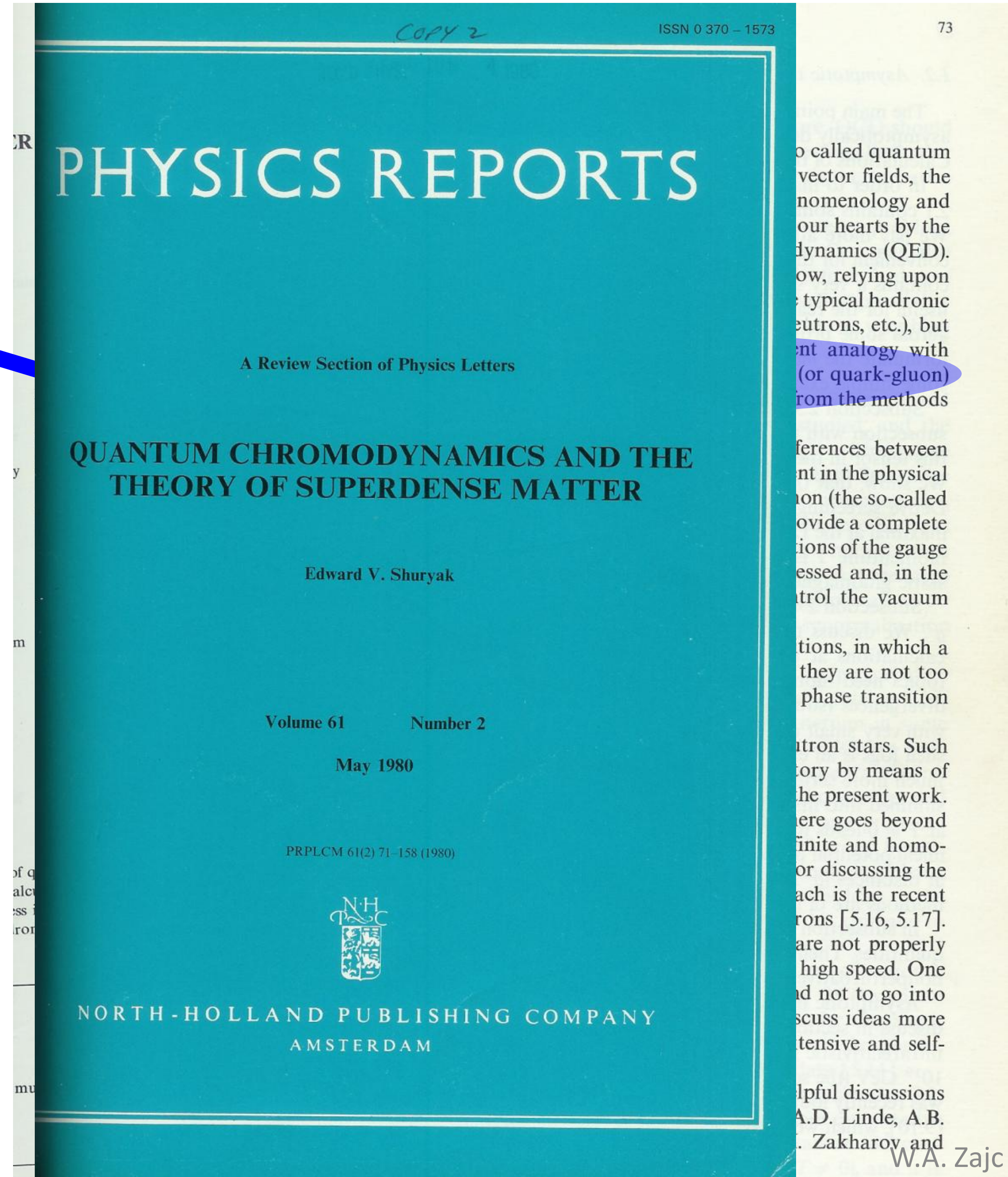
- What T or ρ_B ?
  - For massless quarks and gluons, *only* scale in QCD is confinement scale ~ 1 fm
    - $T \sim \hbar c$ / (1 fm) ~ 200 MeV
    - $\rho_B \sim T^4$ ~ (200 MeV) / fm^3
- Rest of this lecture- 'improving' these estimates

Naming It

- Shuryak publishes first "review" of thermal QCD- and coins a phrase:

  "Because of the apparent analogy with similar phenomena in atomic physics, we may call this phase of matter the QCD (or quark-gluon) plasma."

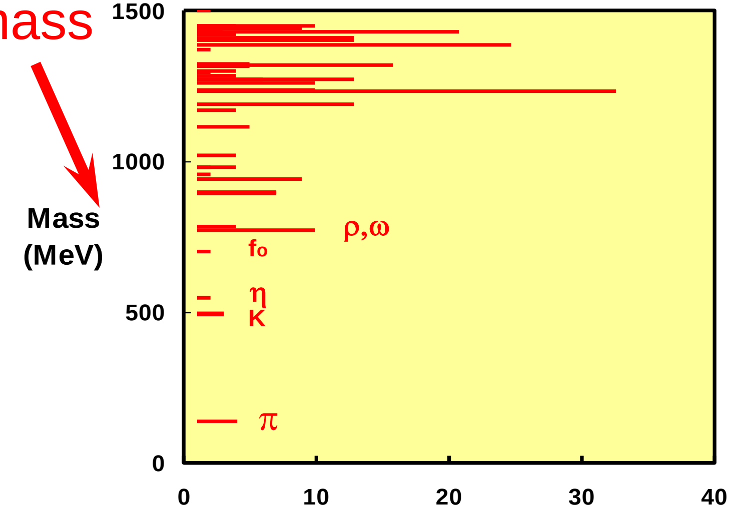
QGP



A "STRONG" Hint (prior to QCD)

- Hagedorn (~1968) : An ultimate temperature?
- The very rapid increase of hadron levels with mass
- ~ equivalent to an exponential level density

Hadron 'level' diagram

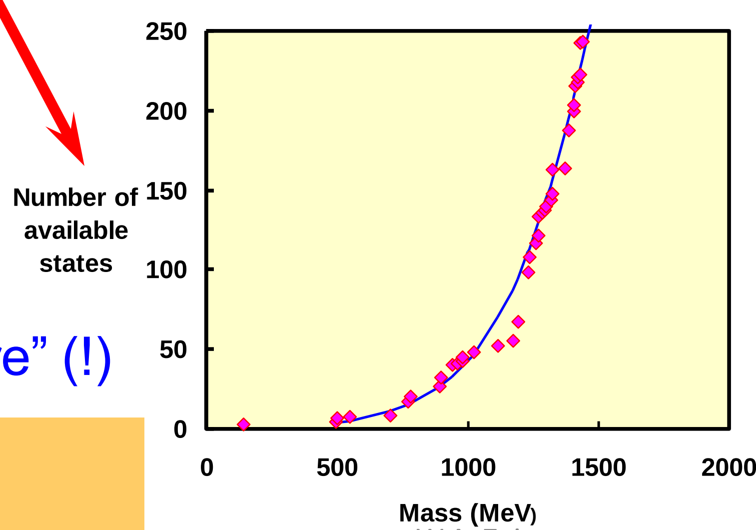


$$\rho(m) \equiv \frac{dn}{dm} \sim m^a e^{m/T_H}$$

$$\Rightarrow Z \sim \int \rho(m)\ e^{-m/T} dm$$

$$\sim \int m^a e^{m\left(\frac{1}{T_H} - \frac{1}{T}\right)} dm$$

Density of States vs Energy



- And thus would imply an "Ultimate Temperature" (!) $T_H \sim 170$ MeV

Hagedorn,

S. Fraustchi, Phys.Rev.D3:2821-2834,1971

Puzzles from pre-History

- Huang and Weinberg (1970):
  - *Ultimate Temperature and the Early Universe*, Phys. Rev. Lett. 25, 896 (1970)
  - Difficulties in constructing a consistent theory of the early universe with a limiting temperature
  - Its own fine-tuning problem(s)
    - "A curious tentative view of cosmic history emerges from these considerations… at earlier times ($T \sim T_0$), ρ was, once again, dominated by non-relativistic baryons!"

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ULTIMATE TEMPERATURE AND THE EARLY UNIVERSE*

Kerson Huang and Steven Weinberg
Laboratory for Nuclear Science and Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 5 August 1970)

The early history of the universe is discussed in the context of an exponentially rising density of particle states.

There are now plausible theoretical models[1] for the thermal history of the universe back to the time of helium synthesis, when the temperature was 0.1 to 1 MeV. Our present theoretical apparatus is really inadequate to deal with much earlier times, say when $T \gtrsim 100$ MeV, and in lieu of any better ideas it is usual to treat the matter of the very early universe as consisting of a number of species of essentially free particles. But how many species?

At one extreme, it might be assumed that the number of particle species stays fixed (perhaps just quarks, antiquarks, leptons, antileptons, photons, and gravitons). In this case, the temperature T will vary with the cosmic scale factor[2] $R(t)$ according to the relation $T \propto 1/R$. The present universe should then contain various relics of the early inferno: There would be a 1°K blackbody gravitational radiation,[3] if TR stayed roughly constant between the times that the gravitons and the photons decoupled from the rest of the universe; also, according to Zeldovich,[4] the leftover quarks should be about as common as gold atoms. The gravitational radiation would not have been seen, but the quarks would have been, unless, of course, quarks do not exist.

At the other extreme, one might assume that the number of species of particles with mass between m and $m+dm$ increases as $m \to \infty$ as fast as possible:

$$N(m)dm \to Am^{-B}e^{\beta_0 m}dm. \quad (1)$$

If $N(m)$ increased any faster, the partition function would not converge. With the increase (1), the partition function converges only if the temperature[5] is less than $1/\beta_0$. The quantity $T_0 = 1/\beta_0$ is thus a maximum temperature for any system in thermal equilibrium.

Support for this latter sort of model comes from two quite different directions:

(1) The transverse momentum distribution of secondaries in very high energy collisions is observed to be roughly $\exp(-|p_\perp|/160\text{ MeV})$. Hagedorn[6] interprets this distribution in terms of a statistical model with $T_0 \approx 160$ MeV and $B = \frac{5}{2}$.

(2) If particles fall on families of parallel linearly rising Regge trajectories, their masses take discrete values $m_1, m_2, \cdots$, where

$$\alpha' m_n^2 + \alpha_0 = n. \quad (2)$$

Here $\alpha' \approx 1$ GeV^{-2} is the universal Regge slope and α_0 is a number, of order unity, characterizing the family. The extension of the Veneziano model[7] to multiparticle reactions requires[8] that the number of particle states with mass m_n equals the degeneracy of the eigenvalue n of the operator

$$N = \sum_{\mu=1}^{D} \sum_{k=1}^{\infty} k a_{\mu k}^\dagger a_{\mu k}, \quad (3)$$

where $a_{\mu k}$ and $a_{\mu k}^\dagger$ are an infinite set of annihilation and creation operators. For $n \to \infty$, this number is[9]

$$P_{nD} \approx 2^{-1/2}(D/24)^{(D+1)/4} n^{-(D+3)/4} \times \exp[2\pi(\tfrac{1}{6}Dn)^{1/2}]. \quad (4)$$

Equations (2) and (4) lead to an asymptotic level density of form (1), with

$$\beta_0 = 2\pi(\tfrac{1}{6}D\alpha')^{1/2}, \quad B = \tfrac{1}{2}(D+1). \quad (5)$$

The value of D is not certain—originally Fubini and Veneziano[9] had $D = 4$, but Lovelace[10] argues that D is larger, possibly $D = 5$.

Table I summarizes the values of T_0 and B for these various models. Lovelace[10] has emphasized the striking agreement between the values of T_0 derived in such different ways. We now see that

Table I. Possible values of the parameters in the level-density formula (1).

Model	$T_0 = 1/\beta_0$	B
(1) Hagedorn[a]	~160 MeV	$\frac{5}{2}$
(2) Veneziano[b] (with $\alpha' = 1$ GeV^{-2})		
$D = 4$	190 MeV	$\frac{5}{2}$
$D = 5$	174 MeV	3
$D = 6$	160 MeV	$\frac{7}{2}$
$D = 7$	147 MeV	4

[a] Ref. 6. [b] Ref. 8.

"…a veil, obscuring our view of the very beginning."
Steven Weinberg, *The First Three Minutes* (1977)

Towards A "Better" Estimate

Q: How to compute location of transition from

A gas of hadrons at temperature T

- to

A gas of deconfined quarks and gluons at T ?

Answer:

- Compute the pressure P in each phase
- The phase with the higher pressure wins

- Next few slides:
  - Review of requisite statistical mechanics and thermodynamics

Statistical Mechanics I

- Density of states: $dN = \frac{d^3r\,d^3p}{h^3}$
  - (Incredibly ubiquitous and useful)

- Boson occupation factor: $f_B(p) = \frac{1}{e^{(E(p)-\mu)/T} - 1}$

- Fermion occupation factor: $f_F(p) = \frac{1}{e^{(E(p)-\mu)/T} + 1}$

- Then

$$N = \int \frac{1}{e^{(E(p)-\mu)/T} \pm 1} \frac{d^3r\,d^3p}{h^3} = \frac{V}{h^3} \int \frac{1}{e^{(E(p)-\mu)/T} \pm 1} \frac{d^3p}{h^3}$$

$$U = \int \frac{E(p)}{e^{(E(p)-\mu)/T} \pm 1} \frac{d^3r\,d^3p}{h^3} = \frac{V}{h^3} \int \frac{E(p)}{e^{(E(p)-\mu)/T} \pm 1} \frac{d^3p}{h^3}$$

Statistical Mechanics II

- Huge simplification for (non-interacting) massless quanta at zero chemical potential μ .
- Mathematics: $\int \frac{s^a\,ds}{e^s-1} = \Gamma(a+1)\zeta(a+1)$

Exercise 1: 'Prove' this.

$$\int \frac{s^a\,ds}{e^s+1} = (1-\frac{1}{2^a})\Gamma(a+1)\zeta(a+1)$$

Exercise 2: 'Prove' this.

- Physics: $n_B(T) \equiv \frac{N_B}{V} = \frac{\xi(3)}{\pi^2}T^3 \quad , \quad n_F(T) = \frac{3}{4}n_B(T)$

Exercise 3: Derive these relations using above

$$\varepsilon_B(T) \equiv \frac{U_B}{V} = \frac{3\xi(4)}{\pi^2}T^4 \quad , \quad \varepsilon_F(T) = \frac{7}{8}\varepsilon_B(T)$$

- Mathematics: $\xi(2) = \frac{\pi^2}{6} \quad , \quad \xi(4) = \frac{\pi^4}{90}$

Exercise 4: 'Prove' this. (Either you know the trick or …)

Statistical Mechanics III

- End result (for massless bosons)
  - Number density $$n(T) = \frac{1.202}{\pi^2} T^3 \approx \left(\frac{T}{2}\right)^3$$
  - Energy density $$\varepsilon(T) = \frac{\pi^2}{30} T^4$$
  - Pressure $$P(T) = \frac{1}{3}\varepsilon(T) = \frac{\pi^2}{90} T^4$$

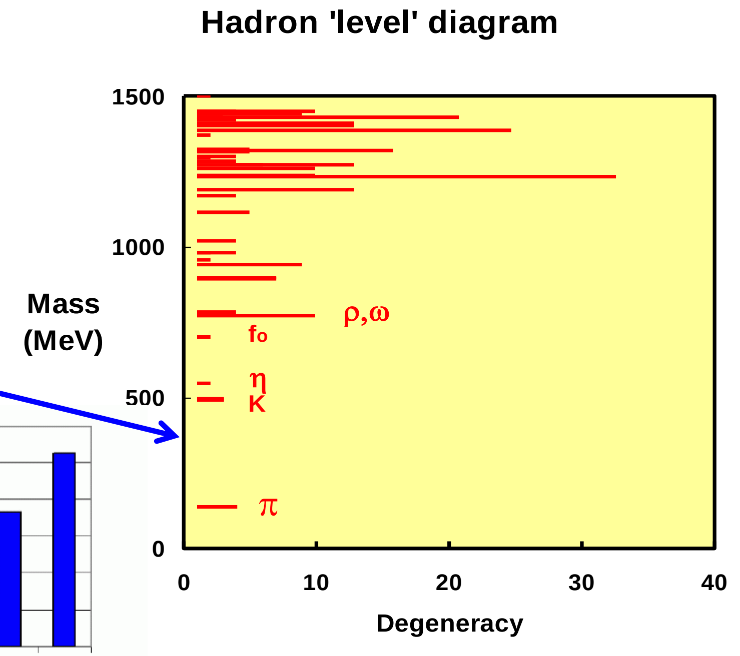
Exercise 5: Show this (handy pocket formula). Use it to compute density of 2.7 K photons.

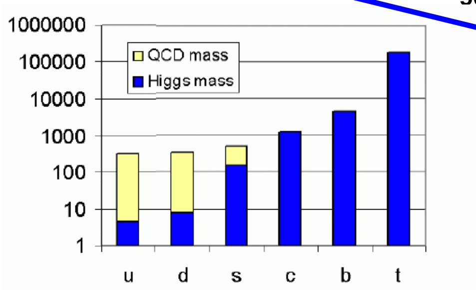
☞ *Per degree of freedom*

⇒Next step: Counting degrees of freedom

Counting Degrees of Freedom

- Hadronic phase
  - Assume relevant T << 500 MeV
  - ndf = 3 (π^-, π^0, π^+)





- Quark-gluon phase
  - Gluons: ndf = $2_s \times 8_c$ = 16
  - Quarks: ndf = $(7/8) \times 2_s \times 2_f \times 2_a \times 3_c$ = <u>21</u>
  - Total ndf = 37

- Bottom line: $ndf_{QGP} \sim 10 \times ndf_{Hadrons}$

Hadron Gas Versus Quark-Gluon Plasma

- *Compare*

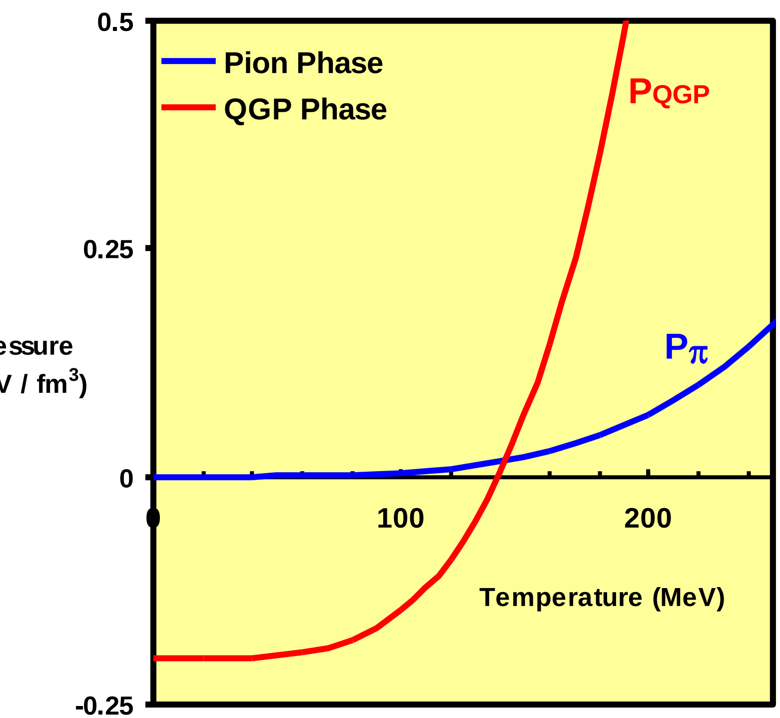
$$P_\pi = 3\frac{\pi^2}{90}T^4$$

Pressure of "pure" pion gas at temperature T

$$P_{QGP} = g\frac{\pi^2}{90}T^4 - B, \ g = 37$$

Pressure in plasma phase with "Bag constant" B ~ 0.2 GeV / fm^3

Select system with higher pressure:



Exercise 6: Show this.

➔ Phase transition at T ~ 145 MeV with latent heat ~0.8 GeV / fm^3

Compare to (c. 2000) best estimates (Karsch, QM01) from lattice calculations:
T ~ 150-170 MeV
latent heat ~ 0.7±0.3 GeV / fm^3

A Question

- Why does the system select the *higher* pressure ?
- After all, systems tend to 'select' the lowest energy…
- Possible answers:
  - To get the right answer
  - Higher pressure pushes harder on lower pressure…
  - It's more chaotic
  - 2^{nd} Law of Thermodynamics

Thermodynamics I

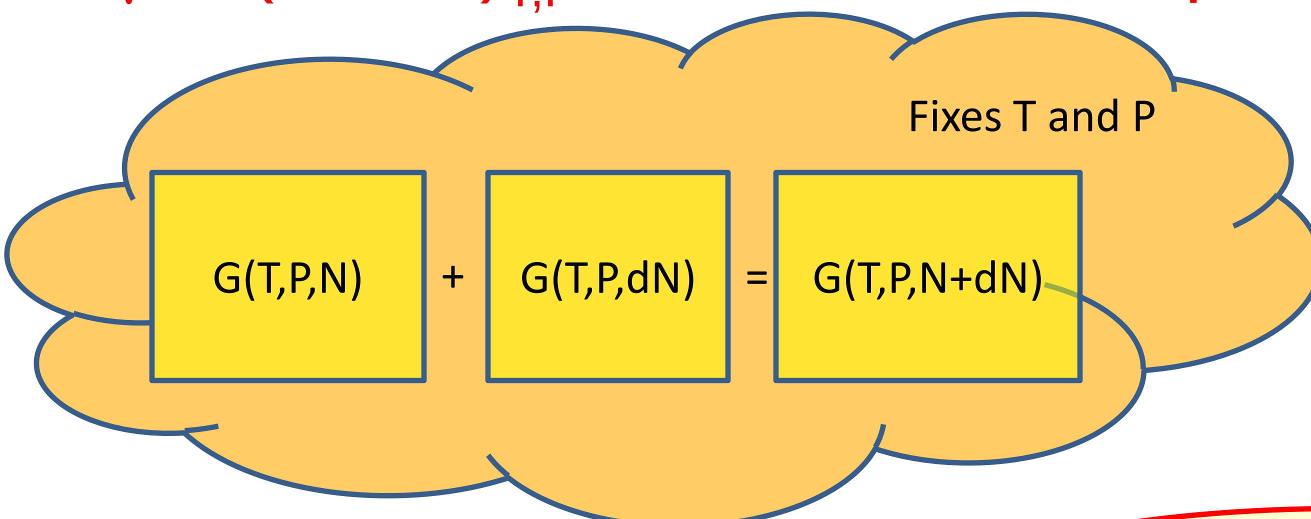
- $U \equiv$ Internal energy of a system
  - $dU = dQ + dW$ (1st Law, energy conservation)
  - $dU = T\,dS - P\,dV + \mu\,dN \Rightarrow U(S,V,N)$

Physicists: Always remember this form, derive the rest.

- Enthalpy $H \equiv U + PV$
  - $dH = T\,dS + V\,dP + \mu\,dN \Rightarrow H(S,P,N)$
- Free Energy $F \equiv U - TS$
  - $dF = -S\,dT - P\,dV + \mu\,dN \Rightarrow F(S,T,N)$
- Gibbs Free Energy $G \equiv F + PV$
  - $dG = -S\,dT + V\,dP + \mu\,dN \Rightarrow G(T,P,N)$
- Grand Potential $\Phi \equiv F - \mu N$
  - $d\Phi = -S\,dT + V\,dP - N\,d\mu \Rightarrow \Phi\,(T,P,\mu)$

Thermodynamics II

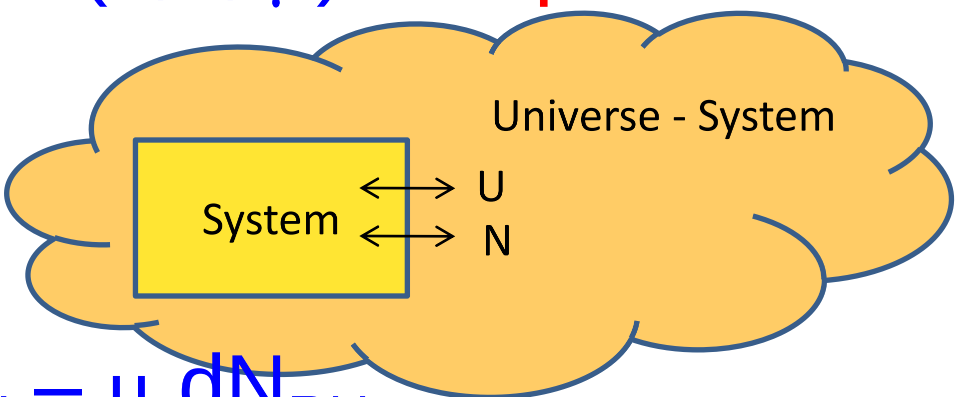
- Hiding in the Legendre Transformation formalism is some very useful physics:
- Gibbs Free Energy $G \equiv F + PV = U - TS + PV$
  - $dG = -S\,dT + V\,dP + \mu\,dN \Rightarrow G(T,P,N)$
  - So $\mu = (\partial G/\partial N)_{T,P}$, which in turn implies…



- So $G = \mu N = U - TS - PV \Rightarrow U = TS - PV + \mu N$

Thermodynamics III

- Now use $U = TS - PV + \mu N$ together with definition of grand potential $\Phi \equiv F - \mu N$:
- $\Phi = \Phi(T,P,\mu) \equiv F - \mu N = U - TS - \mu N = -PV$
- Consider system at fixed (T,P,μ) in equilibrium with rest of universe:



- $dS_{TOT} = dS_S + dS_{RU}$
- $T\, dS_{RU} = dU_{RU} + P\, dV_{RU} - \mu\, dN_{RU}$
  $= dU_{RU} + \qquad - \mu\, dN_{RU}$ (Since V_{RU} fixed)
- Then $dS_{TOT} = \quad dS_S + (dU_{RU} - \mu\, dN_{RU})/T$
  $= (T\, dS_S - dU_S \quad + \mu\, dN_S \quad)/T \; = - d\Phi/T$

Hadron Gas Versus Quark-Gluon Plasma

- *Compare*

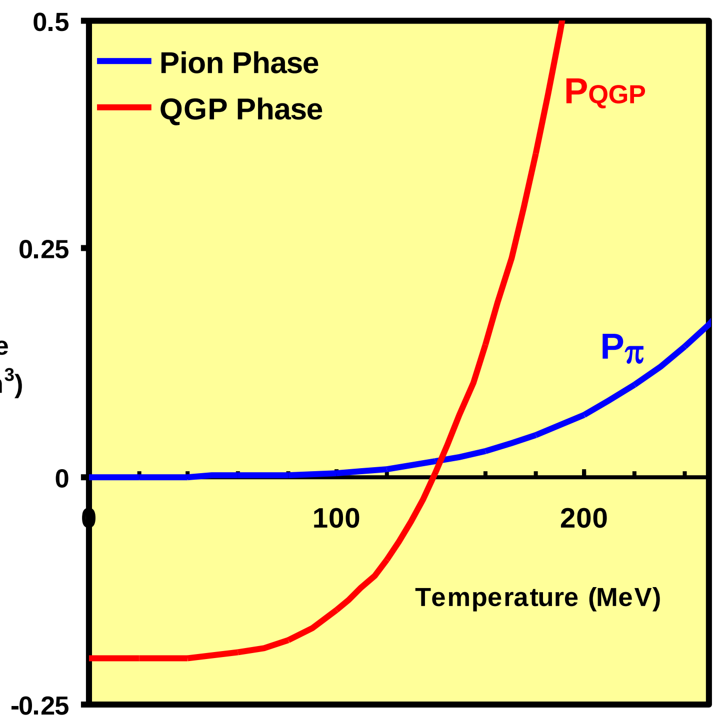
$$P_\pi = 3\frac{\pi^2}{90}T^4$$

Pressure of "pure" pion gas at temperature T

$$P_{QGP} = g\frac{\pi^2}{90}T^4 - B, \; g = 37$$

Pressure in plasma phase with "Bag constant" B ~ 0.2 GeV / fm^3

Select system with higher pressure:



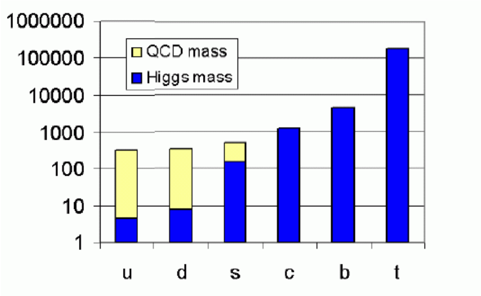
Exercise 6: Show this.

➔ Phase transition at T ~ 140 MeV with latent heat ~0.8 GeV / fm^3

Compare to (c. 2000) best estimates (Karsch, QM01) from lattice calculations:
T ~ 150-170 MeV
latent heat ~ 0.7±0.3 GeV / fm^3

Damage Control

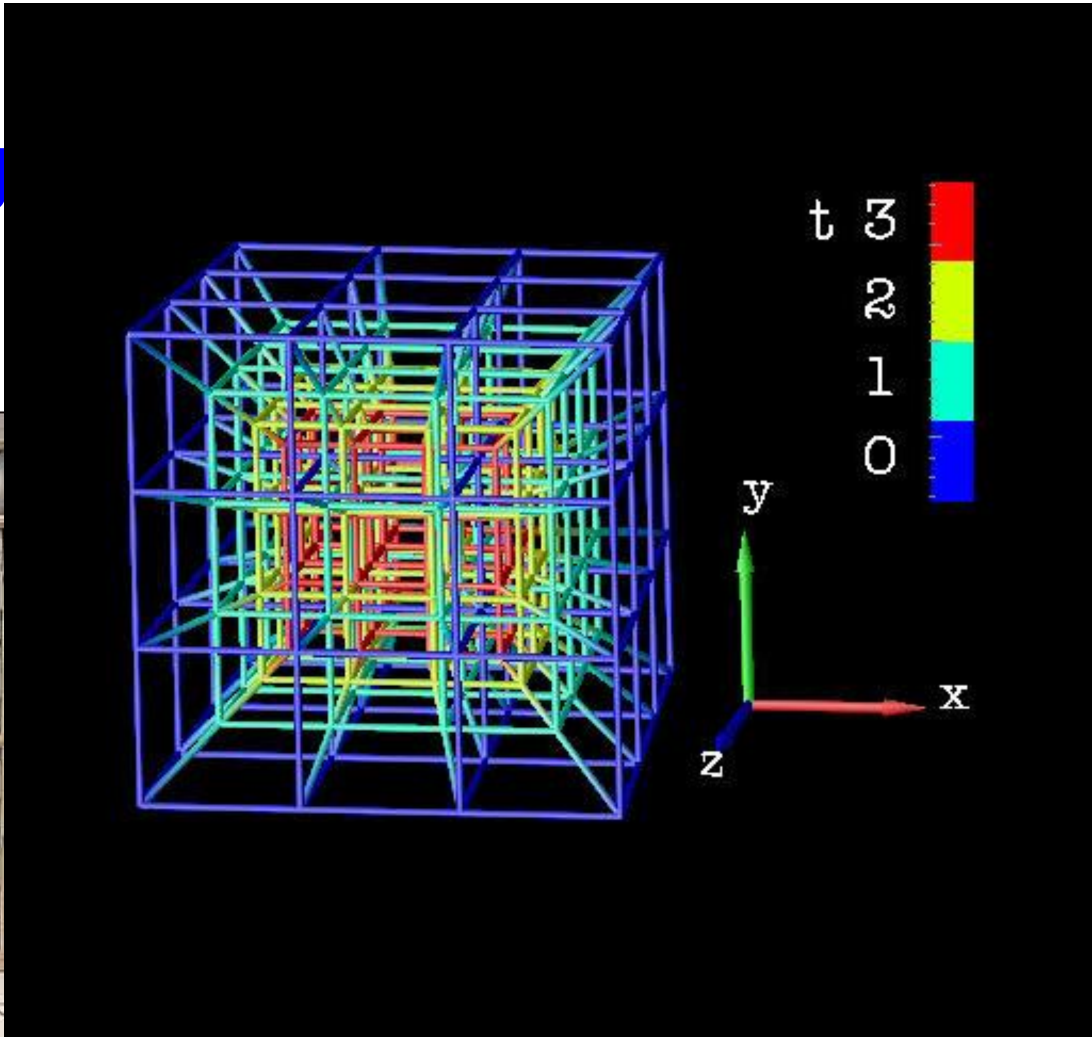
- In reality, this is not such a great estimate:
  - Hadron side
    - Pions hardly massless relative to 145 MeV ☺
    - Ignores exponential growth at higher T
    - Ignores (strong!) interactions
  - QGP side
    - Strange quark neither massless nor massive
    - Bag constant stand-in for QCD vacuum fluctuations
    - Ignores (strong!) interactions



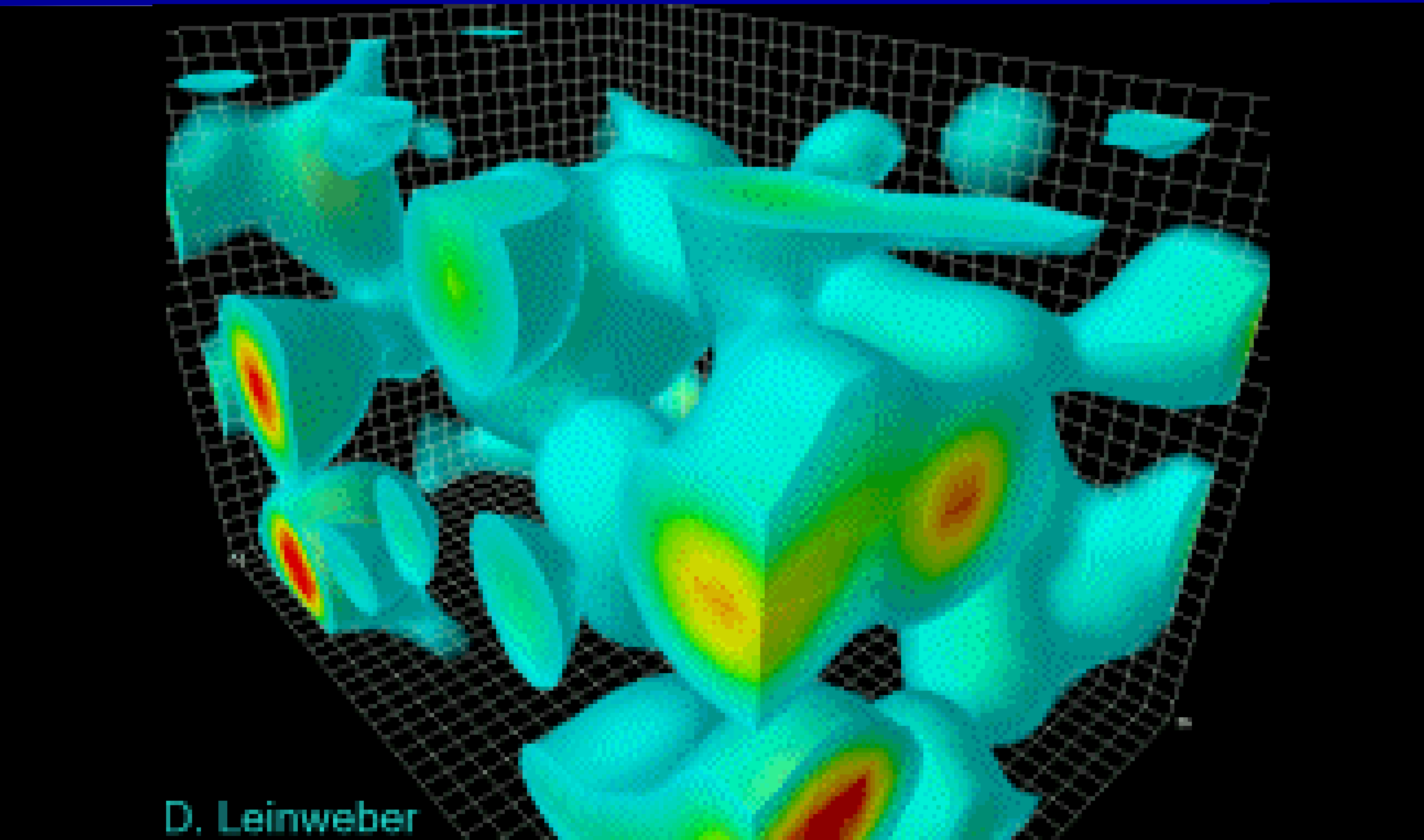
- To do better
  - Program some of this in *Mathematica*
  - Calculate ~ 1 TeraFlops x 100 days ~ 10^{19} Flop

Lattice QCD

- "Solve" the theory on a discrete space-time lattice
- Requires massive (parallel) computing



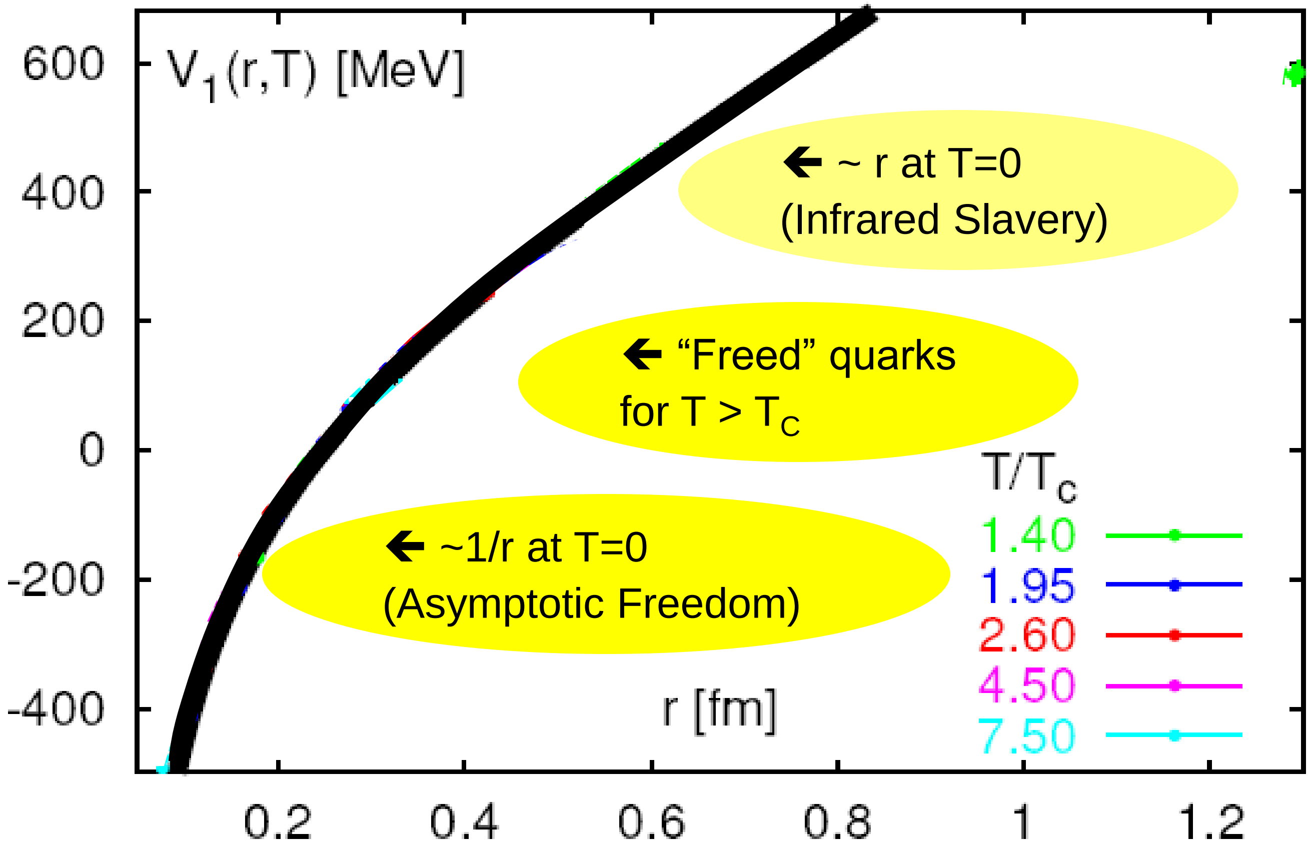
Lattice QCD



☞ See "Visualization of Four Dimensional Quantum Chromodynamics Data " at http://www.ccd.bnl.gov/visualization/gallery/qcd/

Lattice Results for q-qbar Potential

- Lattice QCD results for the quark-antiquark potential:
  - T=0 : a linear "confining" term appears in the potential
  - T> ~100 MeV: "confinement" vanishes



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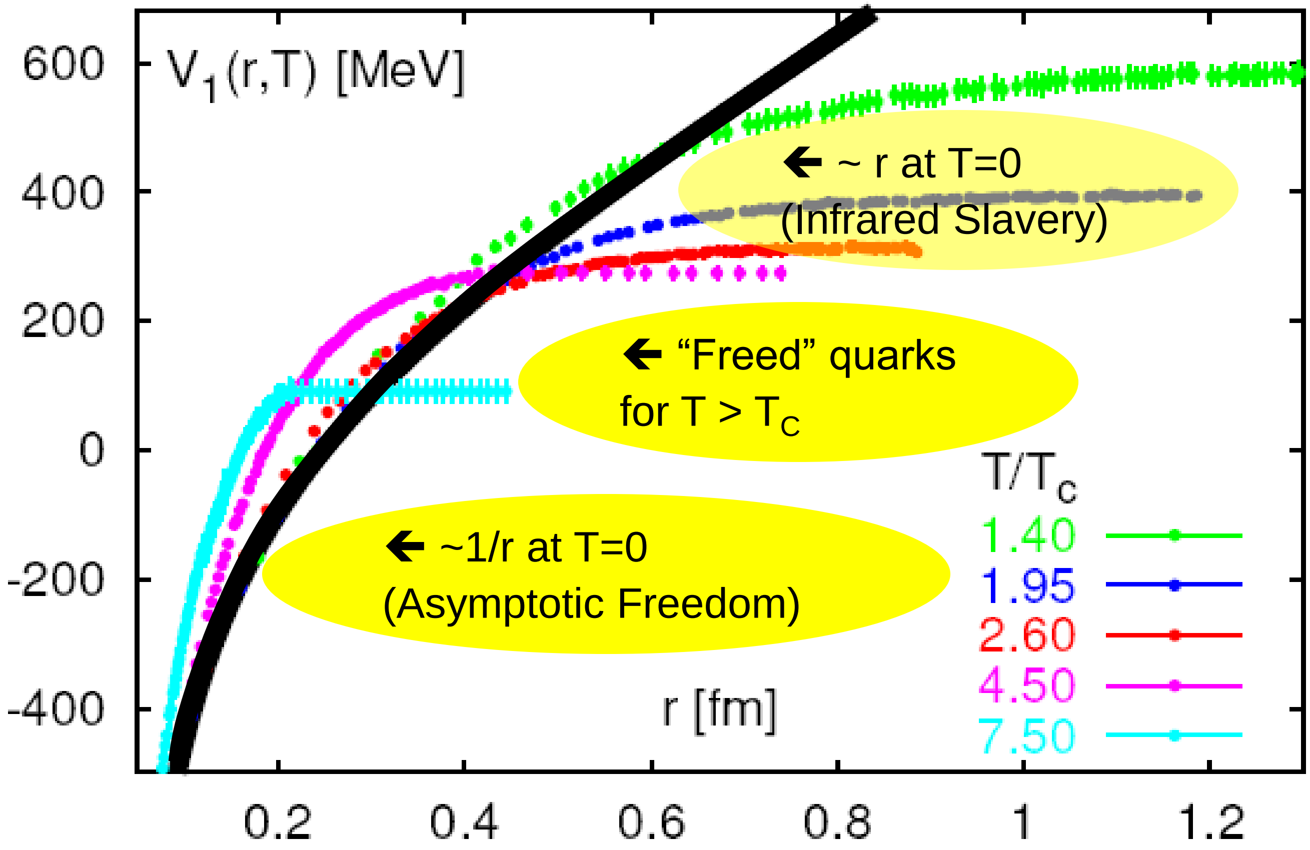
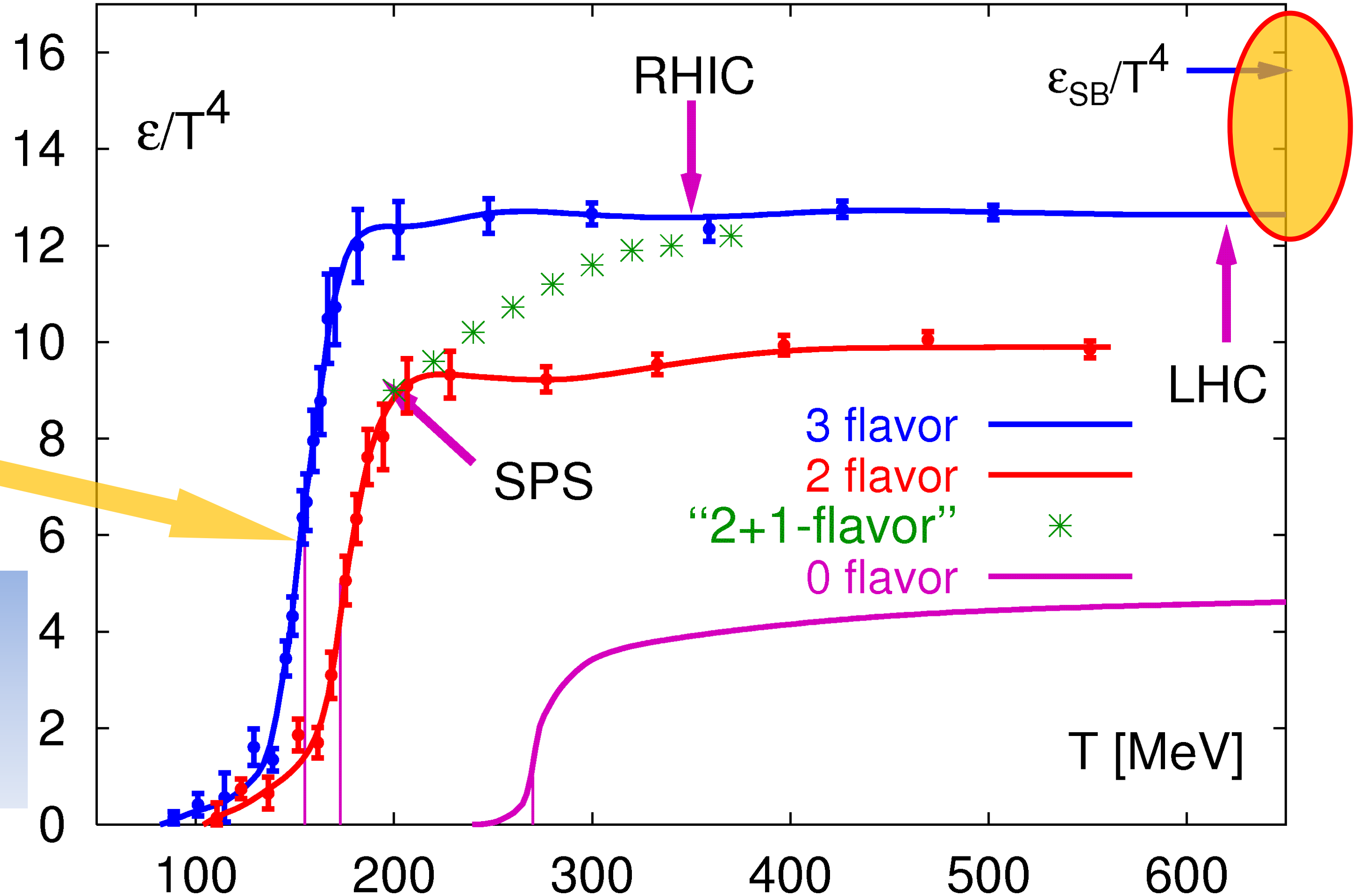


Lattice Results on QCD Transition

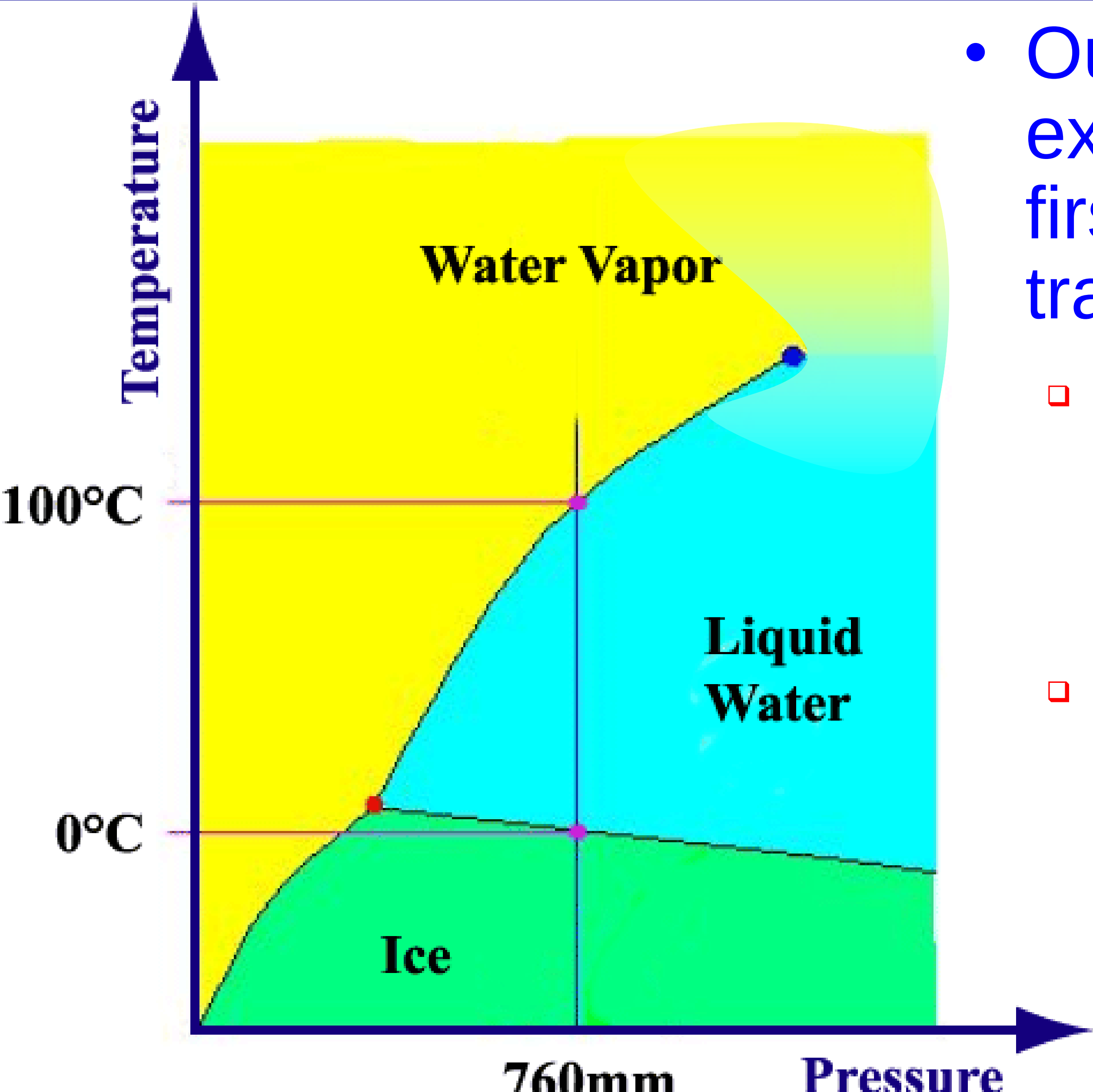
- Rapid rise in d.o.f at T ~ 170 MeV
- Latent heat = 0 (i.e., a smooth "cross-over")

Energy density
ε ~ 1 GeV/fm^3
~ (6-7) x ε_0

Exercise 7: a) Check if I've got this right.
b) Compute ε at the "top".



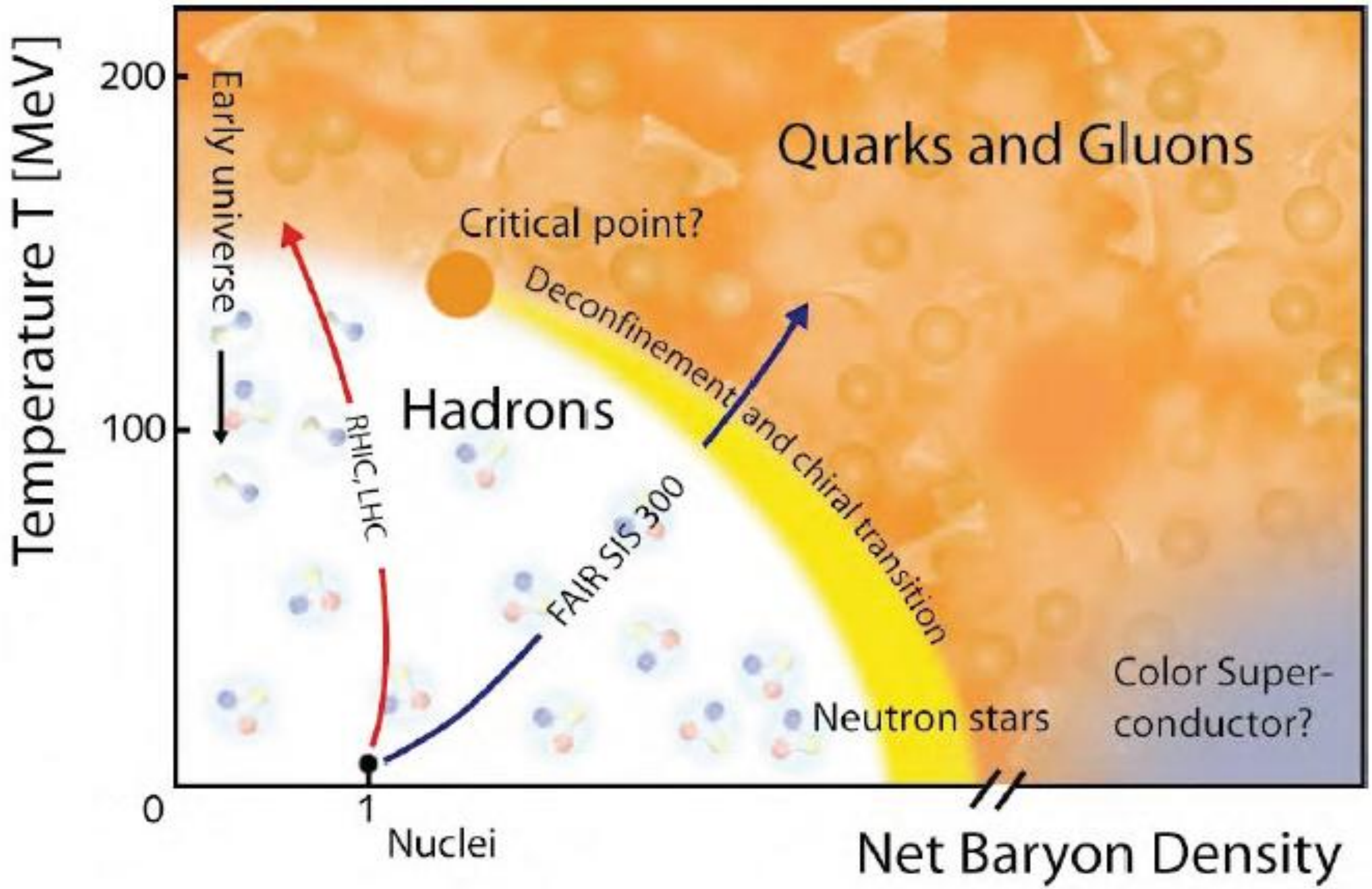
A Familiar Phase Transition



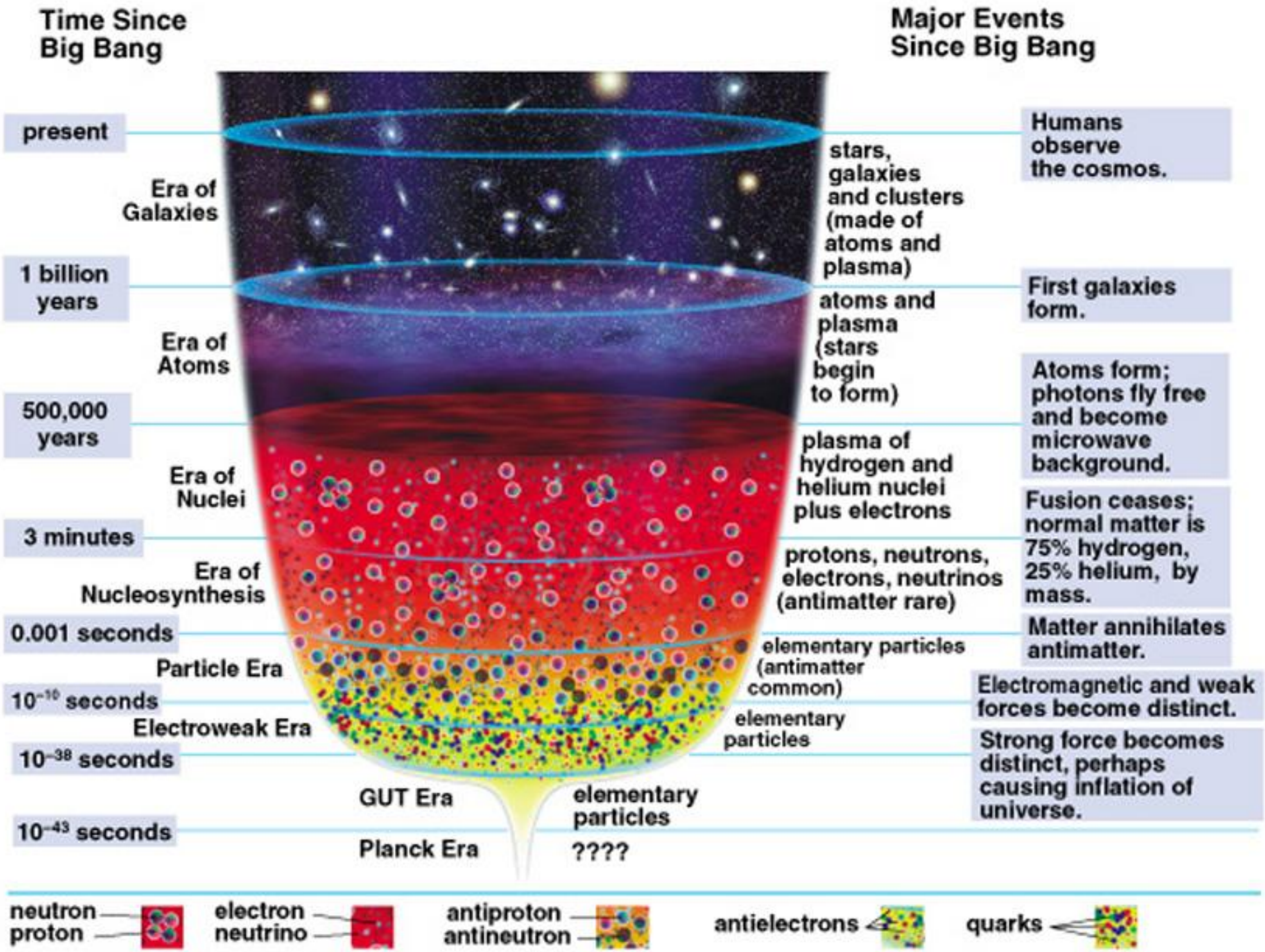
- Our best known examples of first-order phase transitions
  - Note that here we can independently vary T and P (why?)
  - Note also presence of critical point ⇒ vanishing of 1st order transition

Exercise 8: Answer this question.

The QCD Phase Diagram

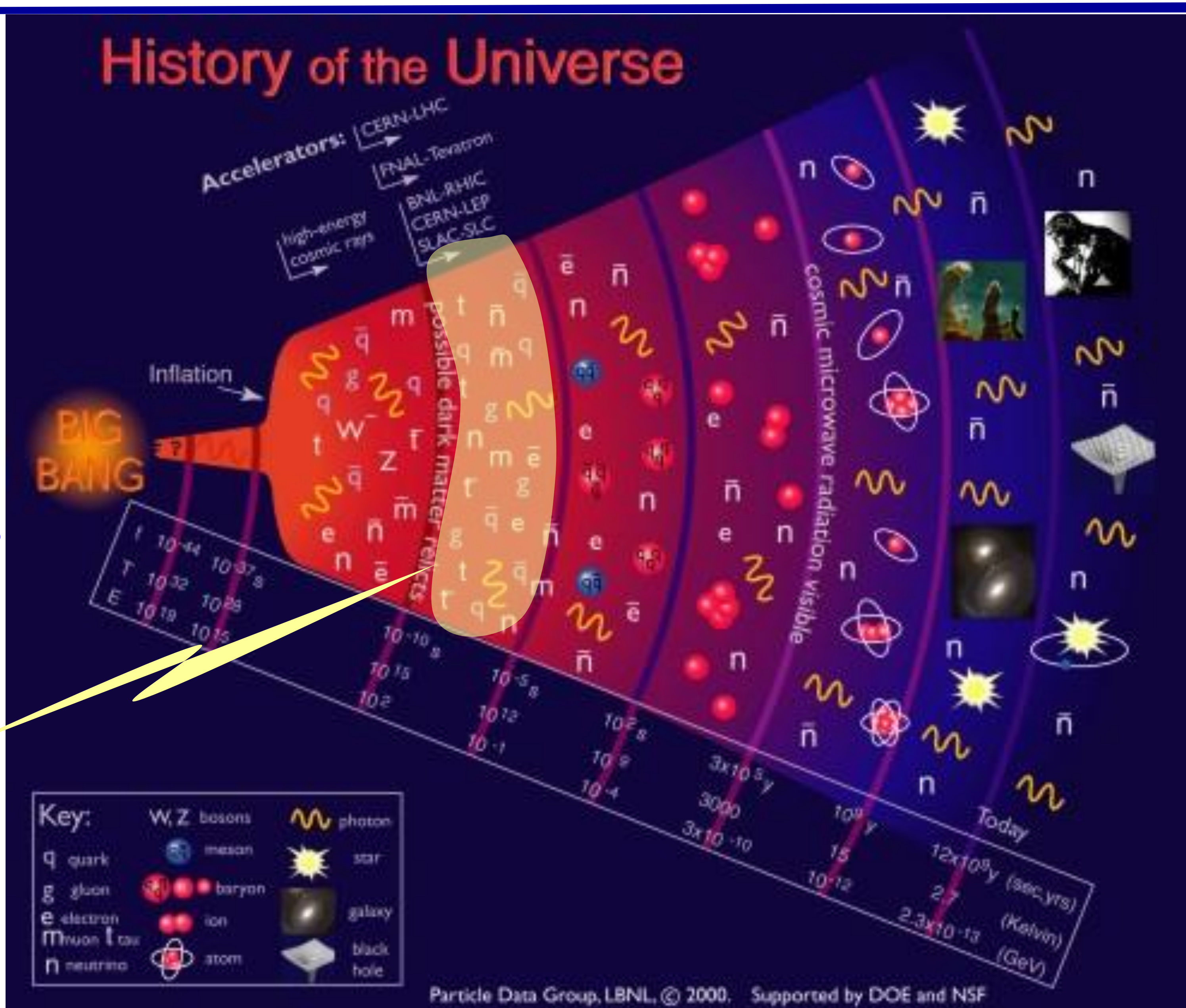


Major Events Since Big Bang

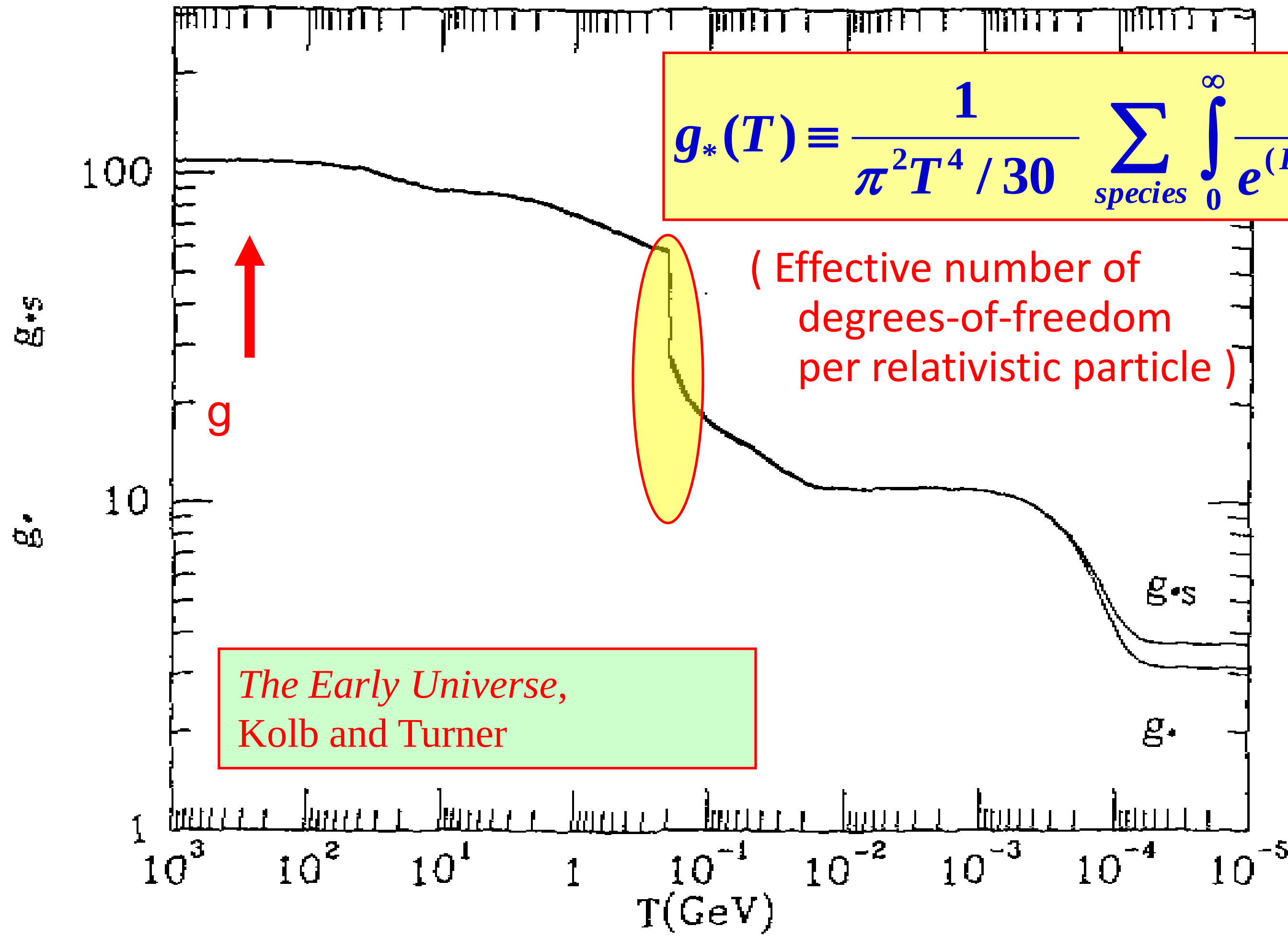


History of the Universe

- Density
  1 GeV / fm^3
  ~ 10^{15} gm/cm^3
- Temperature
  ~ 160 MeV
  ~ 10^{12} K
- Conditions that prevailed ~ 10 μs after the Big Bang



History of the Massless Species



$$g_*(T) \equiv \frac{1}{\pi^2 T^4/30} \sum_{species} \int_0^\infty \frac{E_i(p)}{e^{(E_i-\mu_i)/T_i} \pm 1} \frac{d^3p}{(2\pi)^3}$$

(Effective number of degrees-of-freedom per relativistic particle)

The Early Universe,
Kolb and Turner

Fig. 3.5: The evolution of $g_*(T)$ as a function of temperature in the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ theory.

Summary- Lecture 1

- The intrinsic scale of QCD is that of *confinement* : 1 fm ⇔ 200 MeV .
- General arguments suggest that for temperatures T ~ 200 MeV, nuclear matter will undergo a *deconfining* phase transition.
- Lattice QCD is the only theoretical tool that provides a rigorous procedure for moving from general arguments to quantitative results in this (non-perturbative) regime.