Nuclear Reactions

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Lecture 3

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National Nuclear Physics Summer School,
MSU, June 28 - July 10, 2009

Status of lecture

I. Motivation

  types and characterization of Nuclear reactions

  argument for transport descriptions

II. Heuristic derivation of a transport equation

  test particle solution

III. Elementary derivation of Vlasov eq.

  Relativistic field theory and relativistic transport eq.

IV. Quantum transport theory

V. Characterization and comparison of codes

  Molecular dynamics

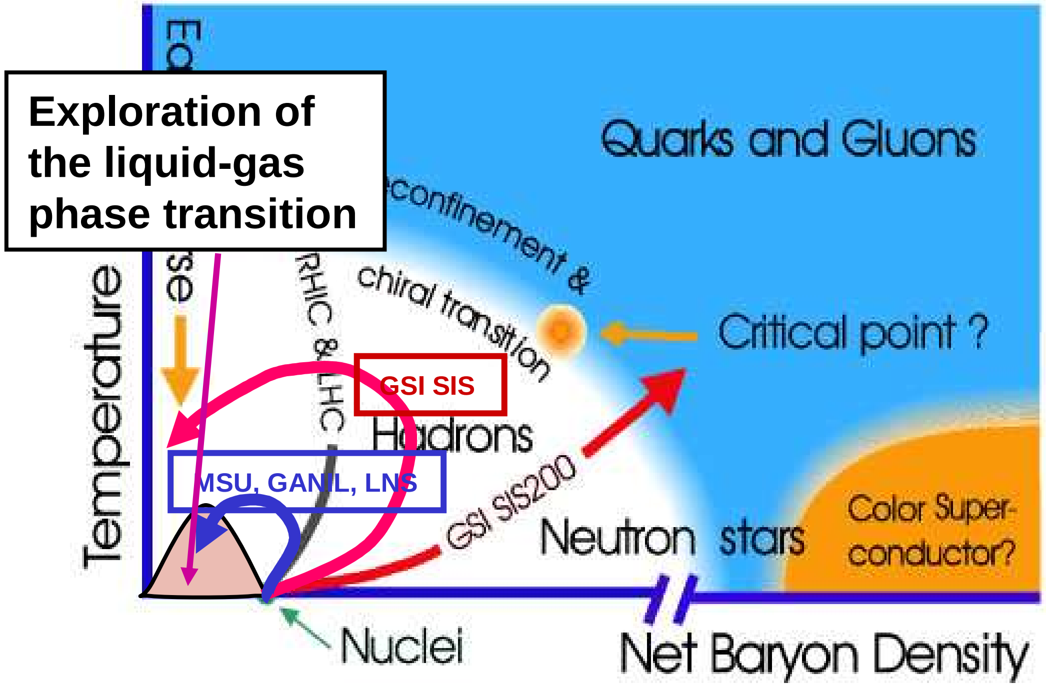
VI. Fluctuations and transport theories

  Instabilities and phase transitions

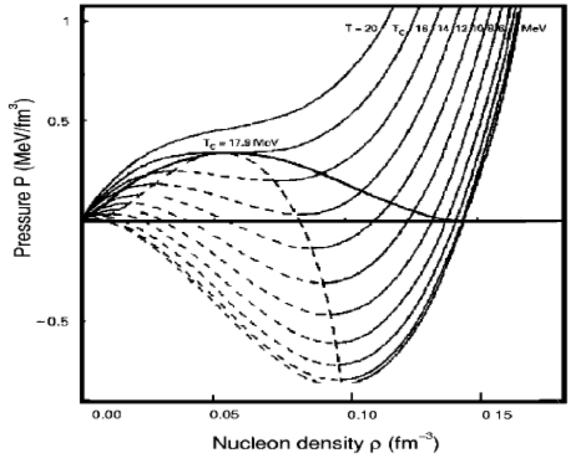
  Boltzmann-Langevin eq. and approx. treatments

VI. Overview over important results in heavy ion collisions

VI.1 Instabilities, Fluctuation and Fragmentation



Van-der-Waals-like EOS

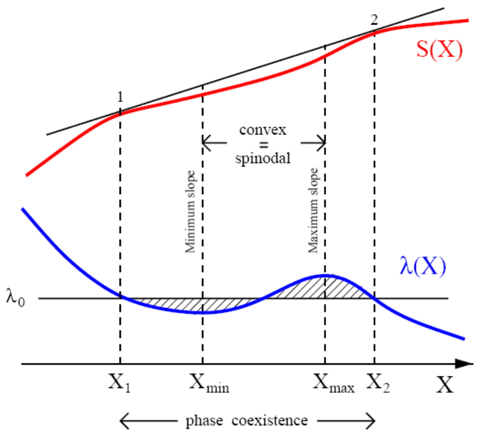


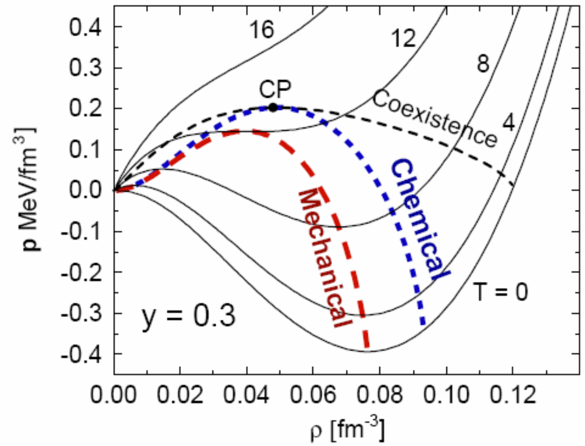
But actually two-component system

$$\text{“Mechanical”}: \left(\frac{\partial P}{\partial \rho}\right)_{T,y} < 0 \,,$$

$$\text{“Chemical”}: \left(\frac{\partial \mu_p}{\partial y}\right)_{T,P} < 0 \,.$$

also discussed in terms of a „convex intruder“ e.g. in the free energy

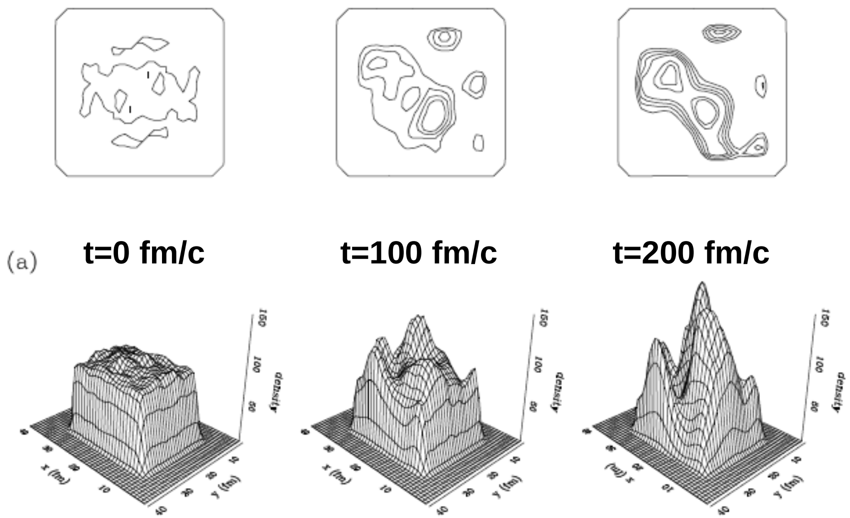




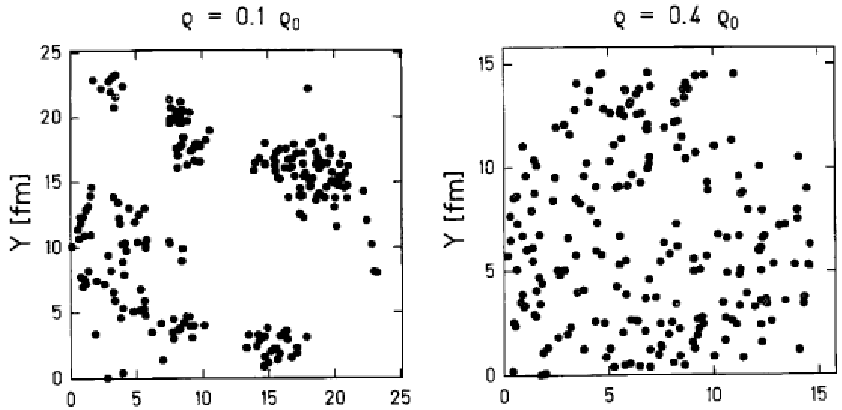
Evidence of Phase Transitions in Calculations

BNV calculation in a box (periodic bounday conditions) with initial conditions in side the instability region: $\rho=\rho_0/3$, T=5 MeV, $\beta=0$

→ Formation of „clusters (fragments)", starting from small (numerical" fluctuations in the density. Time scale shows the growth time of the instable modes



other example as fct. Of initial density

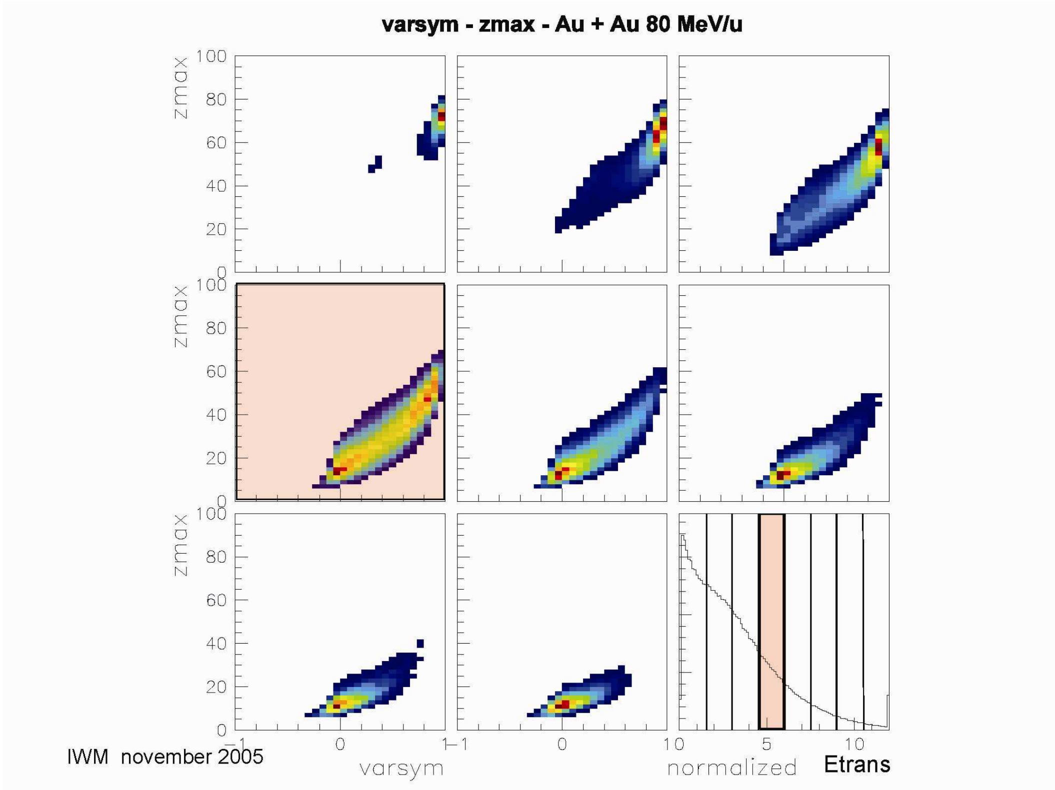


Signatures of phase transition in experiment

→ **Many examples in Sherry's lecture**

Bimodality of the Fragment distribution as a signature of phase transition;

B. Tamain, F. Rivet, GANIL

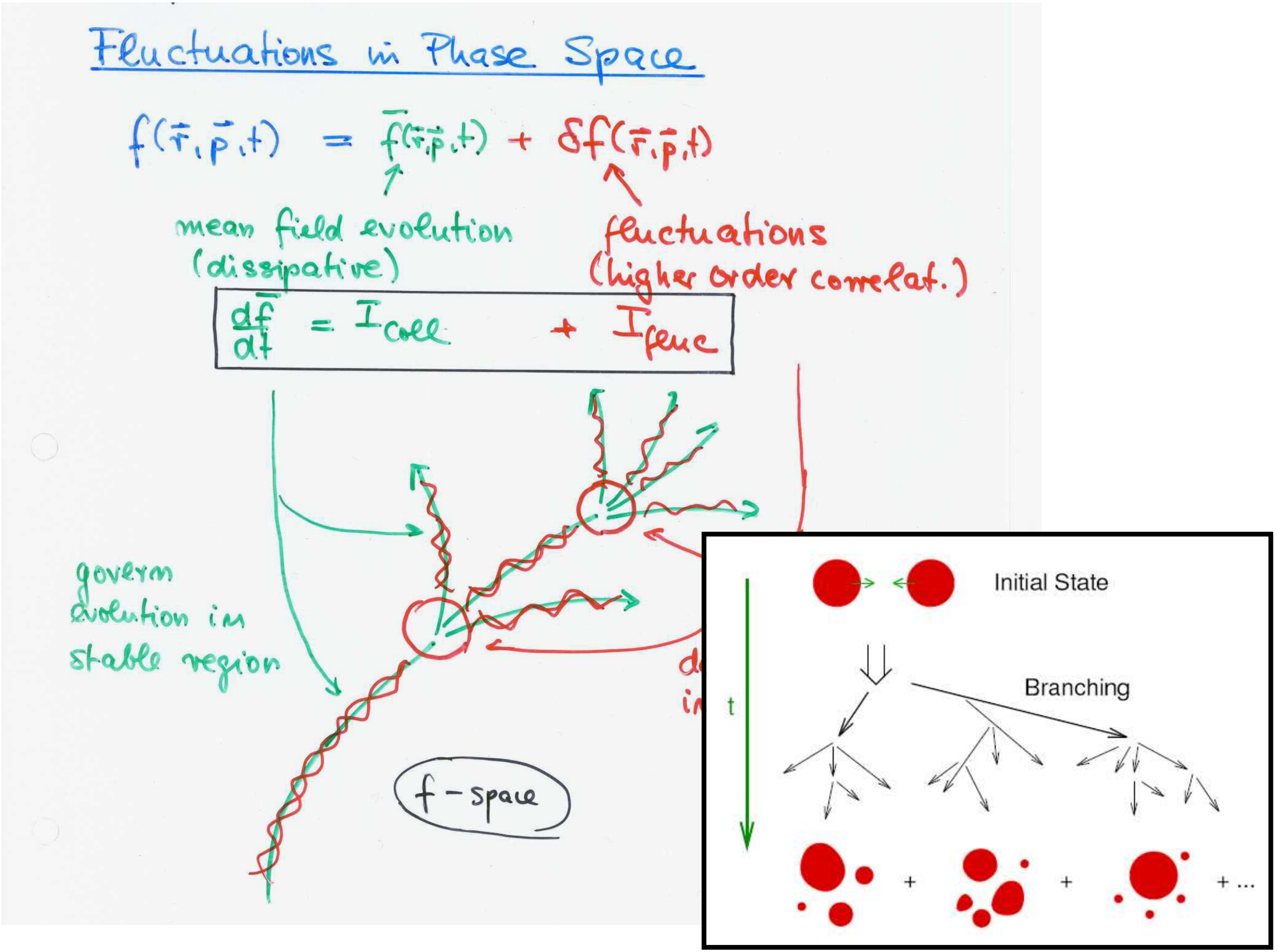


$$\text{varsym} = (Z_{max} - Z_{max-1}) / (Z_{max} + Z_{max-1})$$

varsym ≈ 1 ↔ A big residue + light particles

varsym ≈ 0 ↔ only small fragments

Fluctuations in Phase Space



Fluctuation-Dissipation Theorem

elementary consideration:

Brownian motion with friction and random force *R(t)*

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = -\gamma v + R(t) ,$$

having the properties

$$\langle R(t)\rangle = 0 ,$$

$$\langle R(t)R(t')\rangle = I_R\delta(t-t') ,$$

Solution for average kinetik energy is

$$\tfrac{1}{2}m\langle v^2\rangle = I_R/4\beta + \mathrm{e}^{-\beta t}\langle v(0)^2\rangle . \quad \xrightarrow[t=\infty]{} \tfrac{1}{2}T$$

Then

$$\boxed{\langle R(t)R(t')\rangle = 2\gamma T\,\delta(t-t') .}$$

Fluctuation-Dissipation theorem (Einstein relation)

→ **Dissipation (collisions) and Fluctuations necessarily connected!**

Boltzmann-Langevin Equation

Application of considereations to the Boltzmann transport equation.

Collision term split into average term $\bar{K}$ and a fluctuation term δK

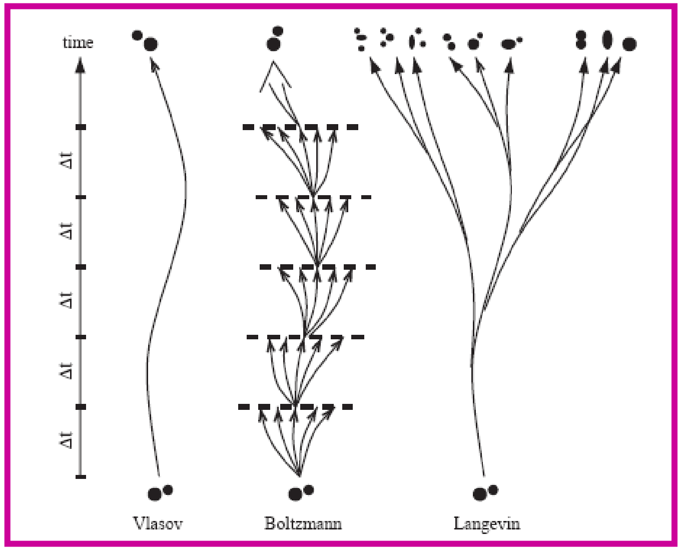
$$\dot{f} \equiv \frac{\partial}{\partial t} f - \{h[f], f\} = K[f] = \bar{K}[f] + \delta K[f] \ ,$$

Boltzmann-Langevin equation

The average term is as before

$$\bar{K}(\boldsymbol{r}, \boldsymbol{p}_1) = g \sum_{234} W(12;34)[\bar{f}_1 \bar{f}_2 f_3 f_4 - f_1 f_2 \bar{f}_3 \bar{f}_4] \ ,$$

$$W(12;34) = v_{12} \left(\frac{d\sigma}{d\Omega}\right)_{12 \to 34} \delta(\boldsymbol{p}_1 + \boldsymbol{p}_2 - \boldsymbol{p}_3 - \boldsymbol{p}_4) \ ,$$



The fluctuating term has the properties

$$\prec \delta K(\boldsymbol{r}, \boldsymbol{p}, t) \succ = 0$$

$$\prec \delta K(\boldsymbol{r}, \boldsymbol{p}, t) \delta K(\boldsymbol{r}', \boldsymbol{p}', t') \succ = C(\boldsymbol{p}, \boldsymbol{p}', \boldsymbol{r}, t) \delta(\boldsymbol{r} - \boldsymbol{r}') \delta(t - t') \ ,$$

$$C(\boldsymbol{p}_a, \boldsymbol{p}_b, \boldsymbol{r}, t) = \delta_{ab} \sum_{234} W(a2;34) F(a2;34) + \sum_{34} [W(ab;34) F(ab;34) - 2W(a3;b4) F(a3;b4)] \ ,$$

VI.7 Implementations of BL Equation

Ref.:

Abe, Ayik, Reinhard, Suraud, Phys.Rep. 275, 49 (96)

P. Chomaz, M. Colonna, J. Randrup, Phys. Rep. 389, 263 (2004)

A. Ono, J. Randrup, Eur. Phys. J. A 30, 109 (2006) (WCI-Book)

Exact studies (on a lattice) only in 2D: Randrup, Burgio, NPA 529 (1991)

Approximate studies:

1. **BOB: replace fluctuation term by fluctuating force, gaged to most unstable mode**

   Colonna, Guarnera

2. **Stochastic MF dynamics: introduce locally statistical fluctuations ito the phase space distribution at certain times $\sigma^2 = f(1-f)$: Colonna, DiToro,…Wolter**

3. **Numerical fluctuations: gauge numerical fluctuations to instability of most unstable mode: Colonna, Di Toro,..**

4. **Molecular dynamics a priori has many body corrlations.**

   QMD: Aichelin, Hartnack: study of formation time of clusters

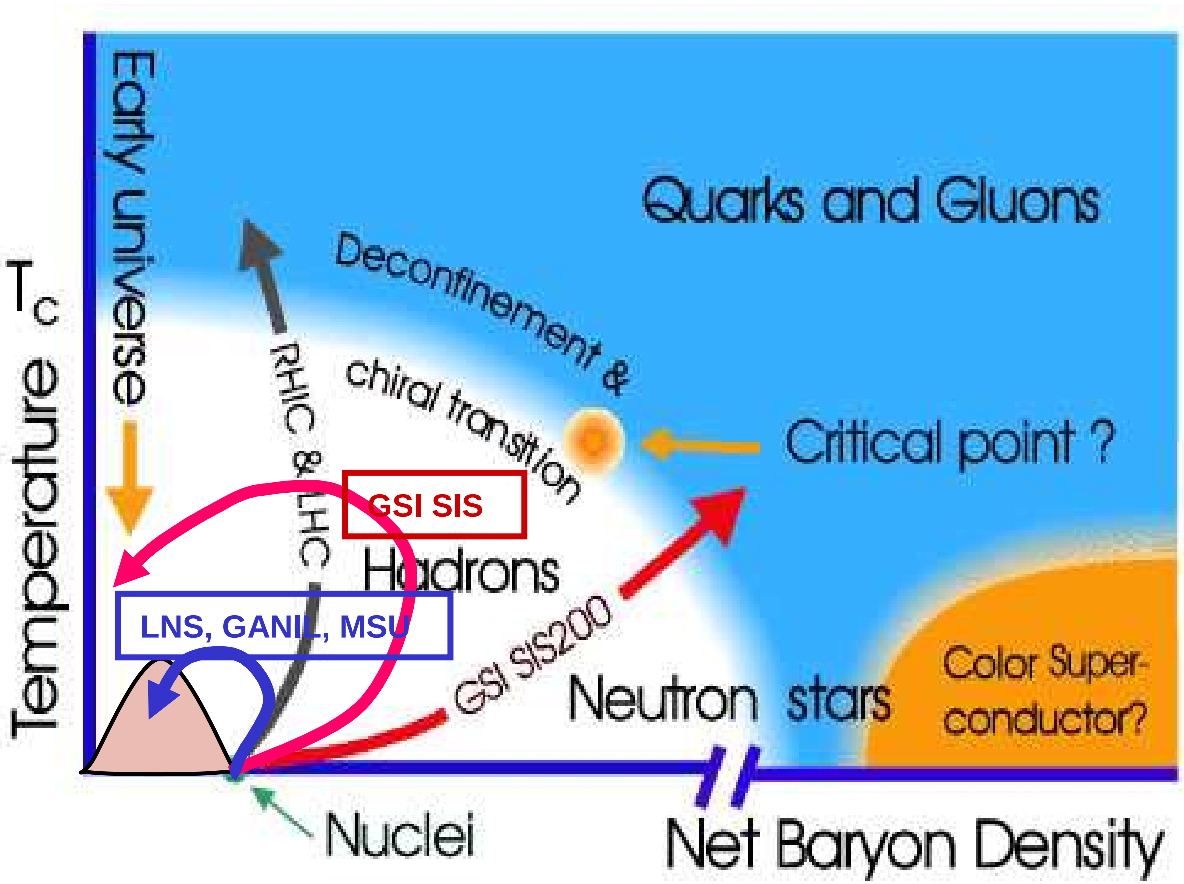
   CoMD: Bonasera, Papa, improve treatment of Pauli principle

   AMD: Ono, fluctuation due to splitting of wave packet.

5. **Studies within BUU an percolation model: W. Bauer, S. Pratt**

A wide field!!

VII.1 What have we learned about the Phase diagram?



Low energy (Fermi regime):

Fragmentation, liquid-gas phase transition,

Deep inelastic

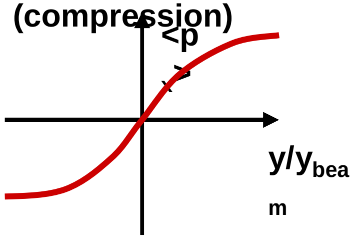
High energy (relativistic):

Compression, particle production, temperature.

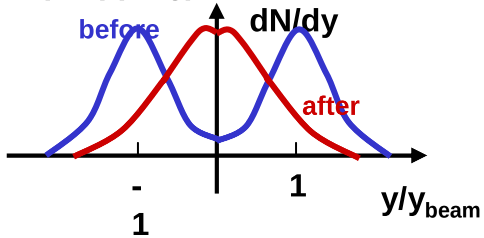
modification of hadron properties (i.e.hadron spectral functions)!

Flow Observables

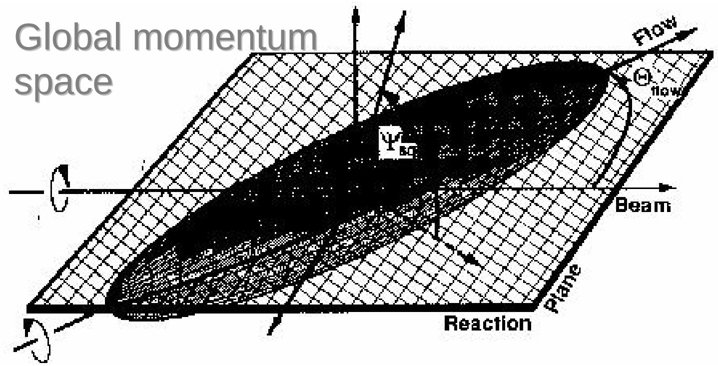
Transverse flow (compression)



Rapidity distribution (stopping)



modern approach:



$$Flow\ : N(\Theta; y, p_t, b) = N_0(1 + v_1(y, p_t)\cos\Theta + v_2(y, p_t)\cos 2\Theta + \ldots$$

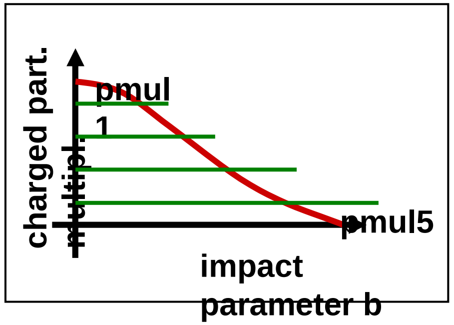
v_1: Sideward flow

v_2: Elliptic flow

$$rapidity \quad y = \frac{1}{2}\ln\frac{1+\beta_z}{1-\beta_z}$$

impact parameter selection

charged particle multiplicity

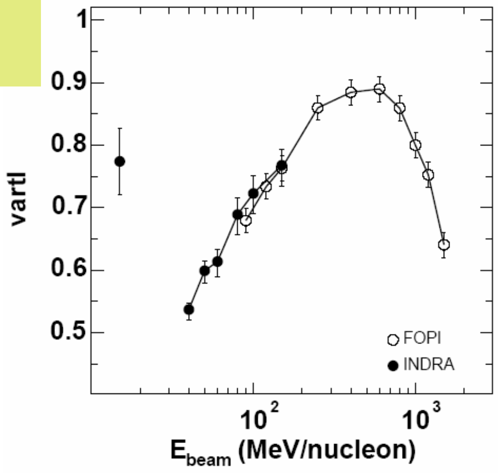


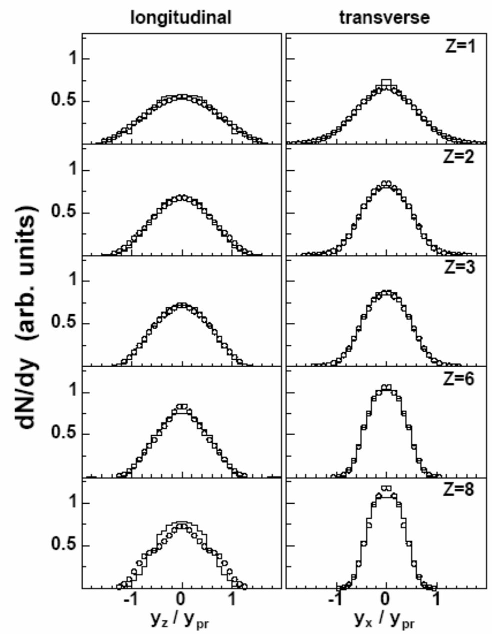
Stopping in HIC

Longitundinal and transverse rapidity distributions:

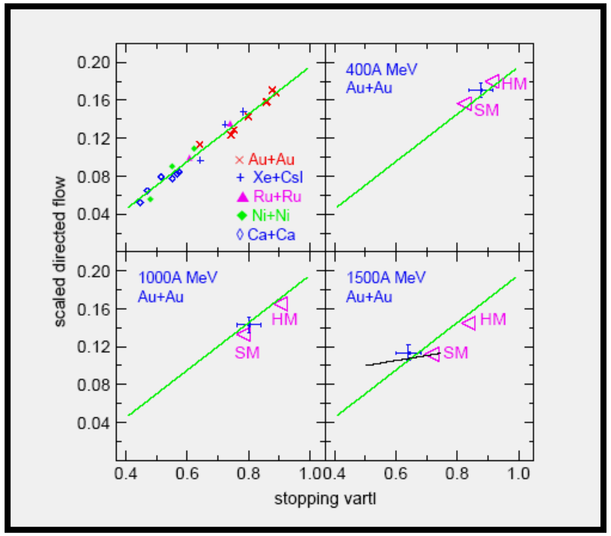
A probe for equilibration

$$\mathrm{var}_{tl}(E_{beam}) = \frac{\sigma_{\perp}^{2}}{\sigma_{z}^{2}}$$

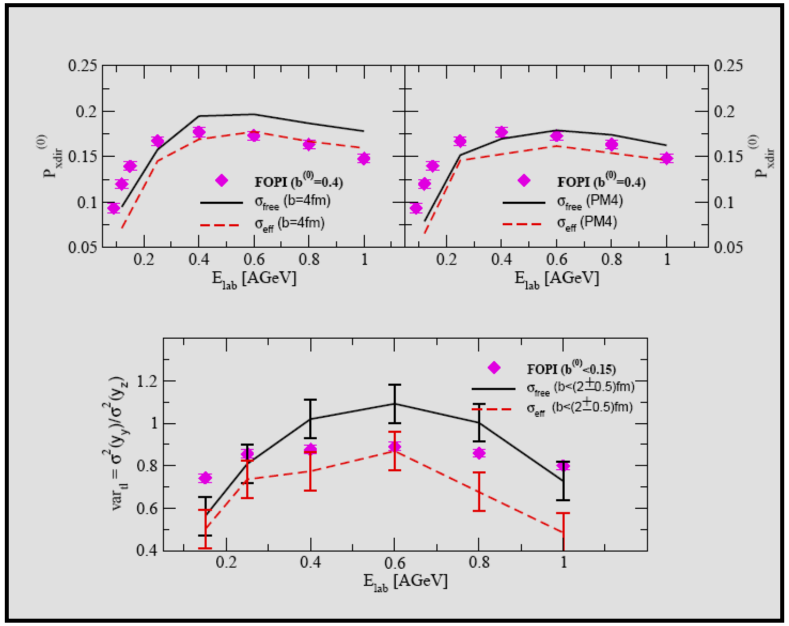




Correlation between stopping and tranverse flow



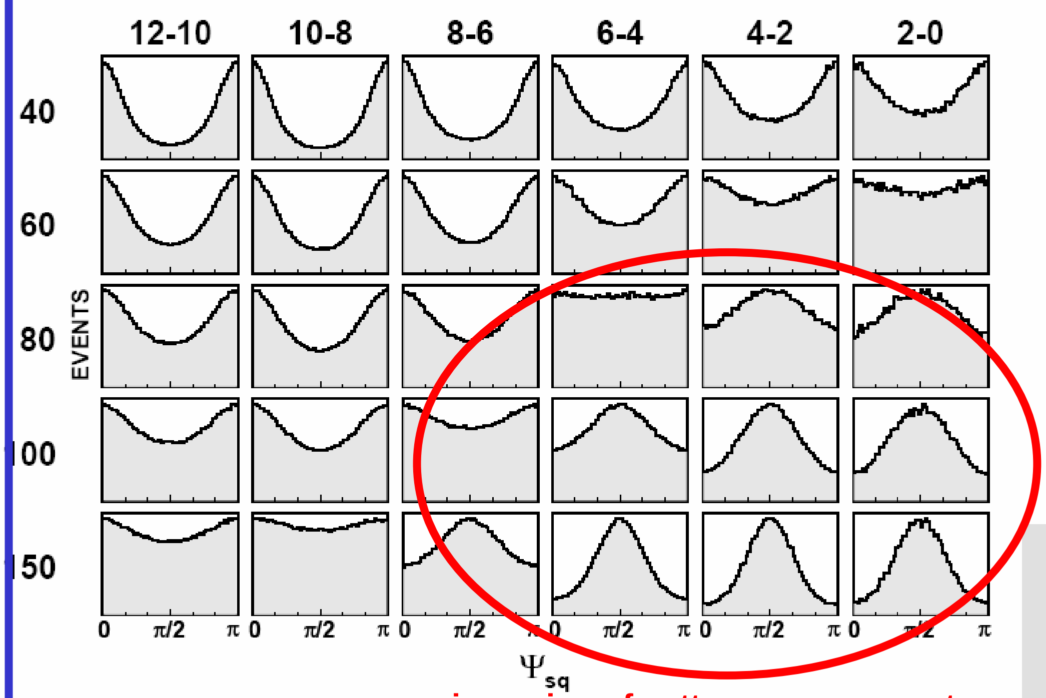
Shown also in calculations, depending on in-medium cross section



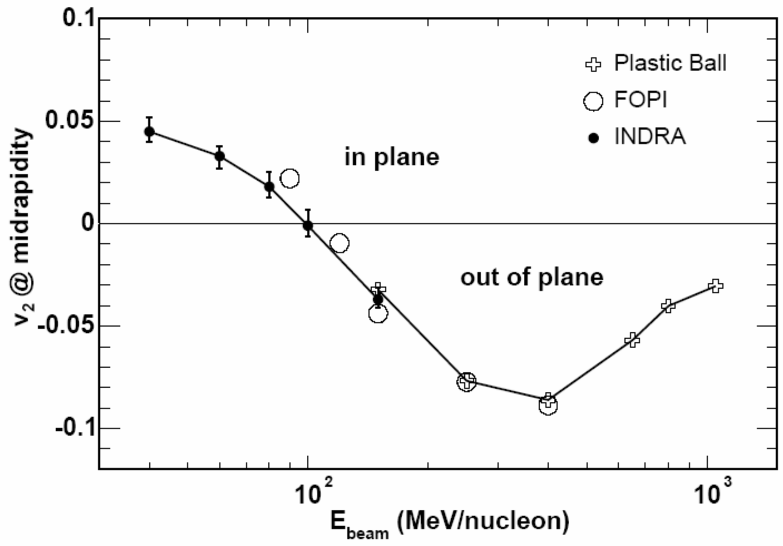
Ref. W. Reisdorf, et al., Nucl.Phys.A781,459,2007

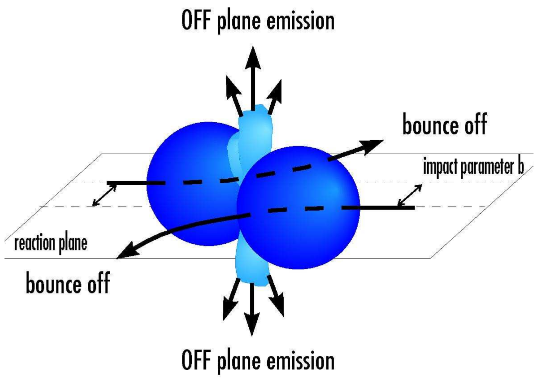
Elliptic Flow

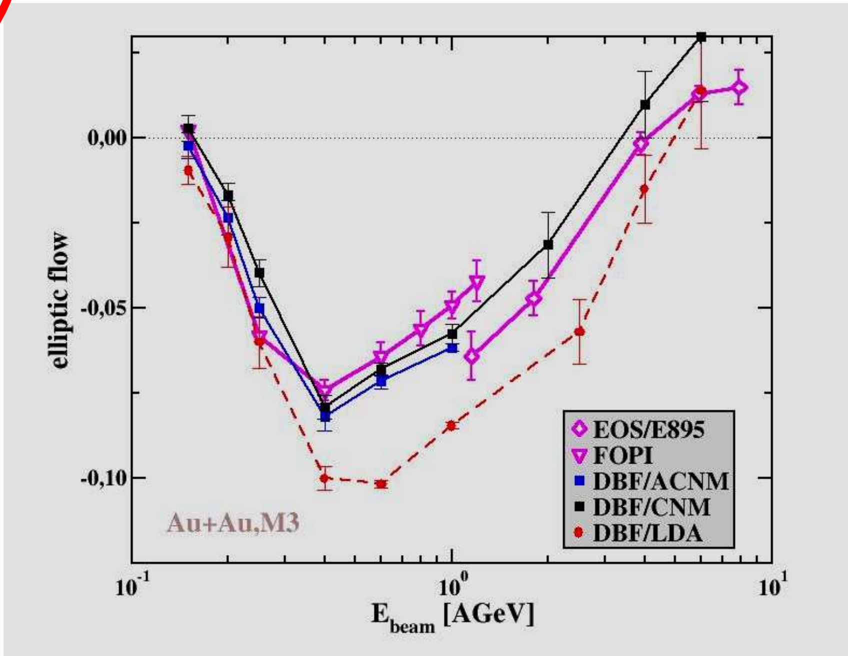
Evolution with impact parameter and energy



inversion of pattern: squeeze out







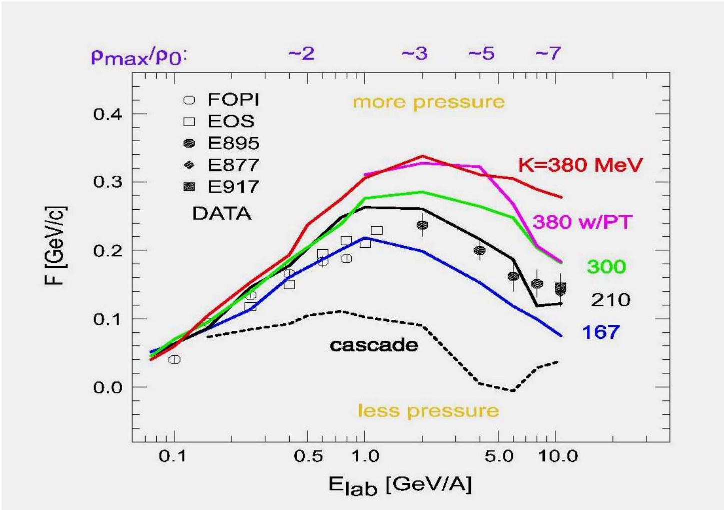
VII.5 Limits for the EOS

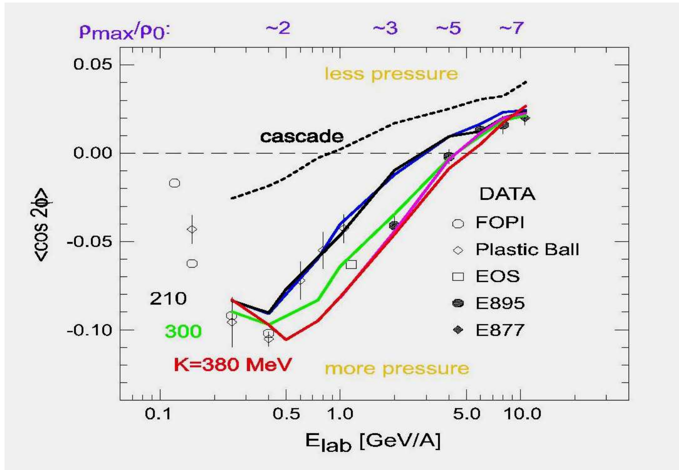
$$Flow\ : N(\Theta; y, p_t, b) = N_0(1 + v_1(y, p_t)\cos\Theta + v_2(y, p_t)\cos 2\Theta + \ldots$$

v_1: Sideward flow

v_2: Elliptic flow

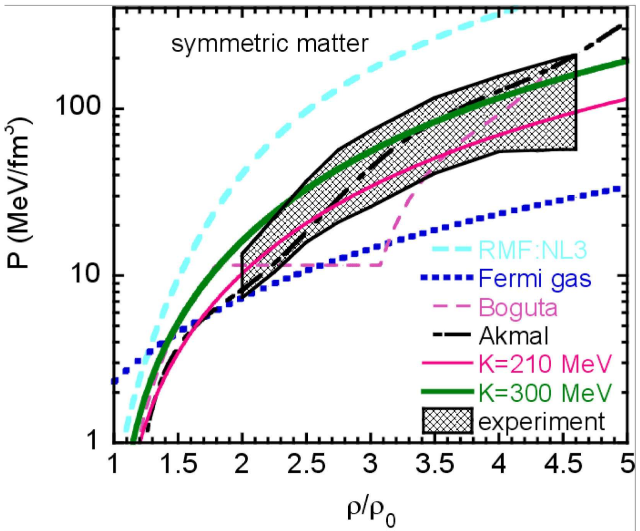
Flow and elliptic flow described in a model which allows to vary the stiffness (incompressibility K), and has a momentum dependence

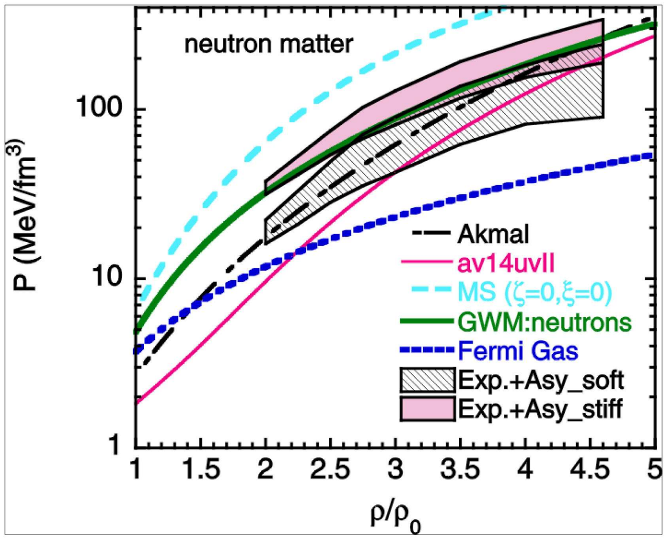




Deduced limits for the EOS (pressure vs. density) for symmetric nm (left).

The neutron EOS (i.e. the symmetry energy) is still uncertain, thus two areas are given for two different assumptions.

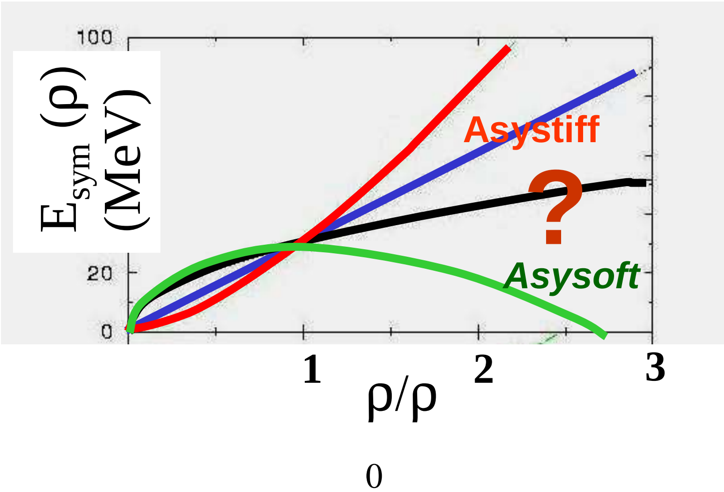




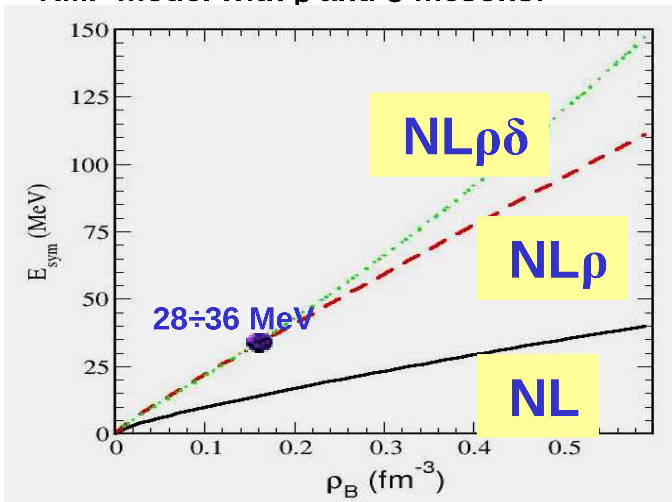
Isospin Flow

Probing the symmetry energy with flow in HIC: iso-flows

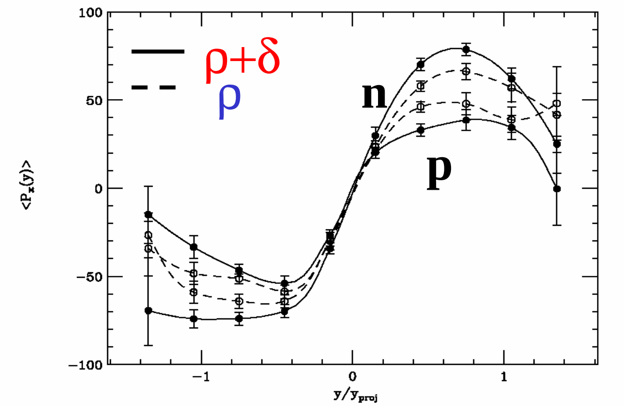
The symmetry energy:

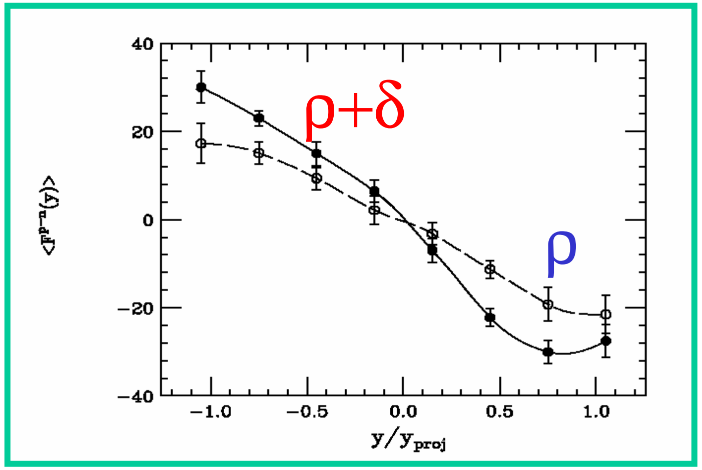


RMF model with ρ and σ mesons:



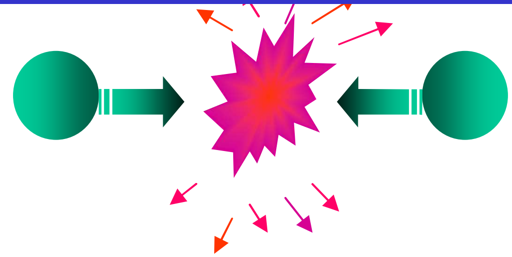
$$E_{sym} = \frac{1}{6}\frac{k_F^2}{E_F^{*2}} + \frac{1}{2}\left[f_\rho - f_\delta \left(\frac{M^*}{E^*}\right)^2 \right]\rho_B$$





Greater E_{sym} ⇒ stiffer F_{np}

Particle Production



What can one learn from different species?

- **photons: high energy: first chance pn-collisions**
- **pions: production at all stages of the evolution via the Δ-resonace**
- **kaons (strange mesons with high mass): subthreshold production, probe of high density phase**
- **ratios of π^+/π^- and K^0/K^+: probe for symmetry energy**

$p, n, d, t, {}^{3,4}He, \ldots,$

$\pi^+, \pi^-, \pi^0, \ldots, K^+, K^0, K^-, \ldots$

Inelastic collisions: Production of particles and resonances: Coupled transport equations

e.g. kaon production;

coupling of Δ and strangeness channels.

$$\frac{d}{dt} f_N(x_\mu) = I_{coll}(\sigma_{NN\to NN}, f_N; \sigma_{NN\to N\Delta} f_\Delta; \ldots.)$$

$$\frac{d}{dt} f_\Delta(x_\mu) = I_{coll}(\sigma_{\Delta N\to NYK} f_Y f_K; \ldots.)$$

etc.

Photons

High energy (hard) photon production in HIC:

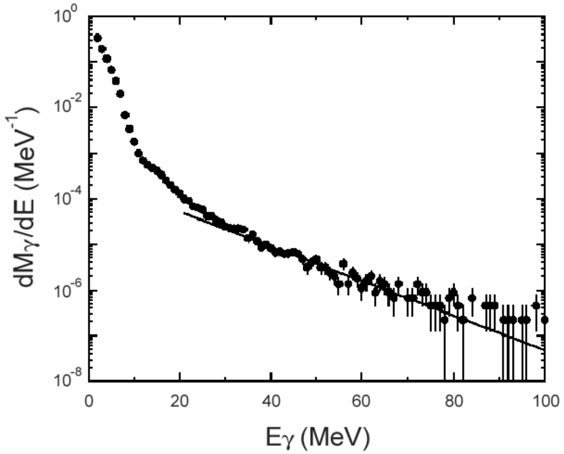


Fig. 7 – Gamma multiplicity spectrum for incomplete fusion reactions in the ^{36}Ar + ^{98}Mo system at 37A·MeV. The continuous line indicates the exponential fit of the hard component (E_γ > 35 MeV). [39Piatelli96]

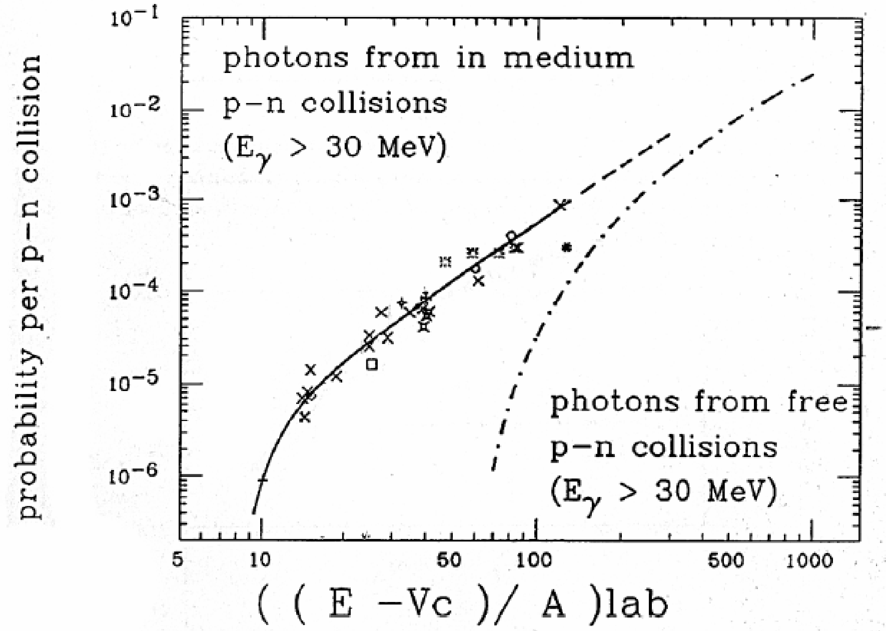


Fig. 8 – Emission probability for photons with E_γ>30 MeV per in-medium nucleon-nucleon collisions as a function of the incident energy per nucleon. Vc is the Coulomb barrier [27Metag88]

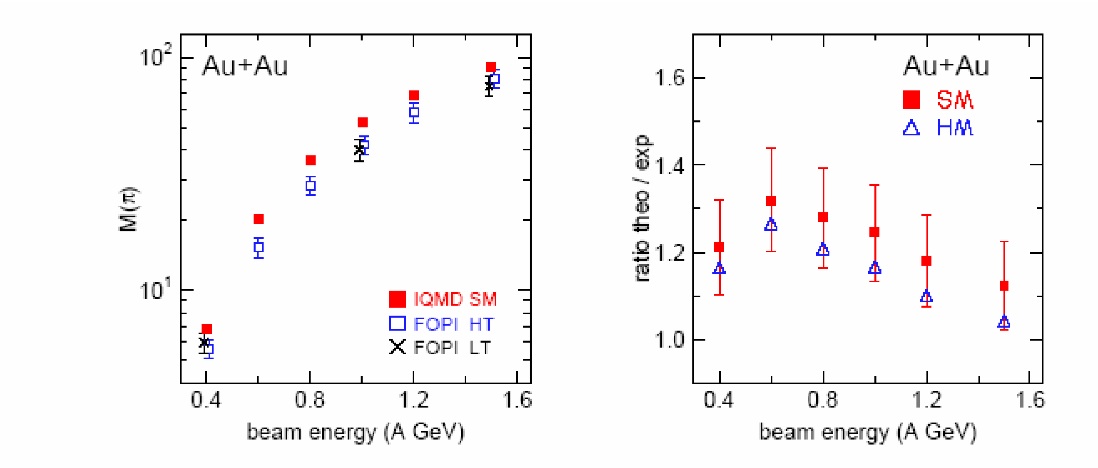
Universal curve, when scaled relative to Coulomb barrier:

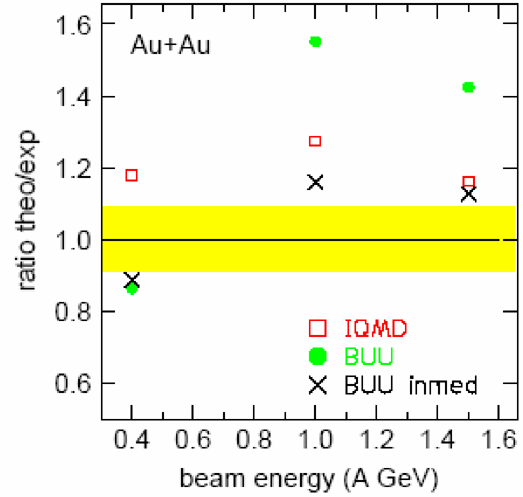
→ First chance pn collisions

→ medium modification of pnγ cross section

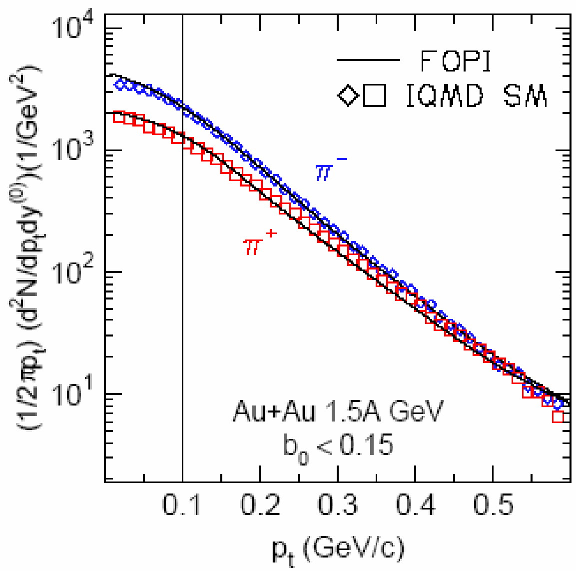
Pions

Pion multiplicity excitation function





energy spectra



Transport calculations reproduce the main features of pion production.

But pion productin can also be used as a probe for the symmetry energy….

VII.10 Pion Ratios and Symmetry Energy

π^{+}/π^{-} **-ratios as a probe for the iso-EOS,**

… and comparison with calculations

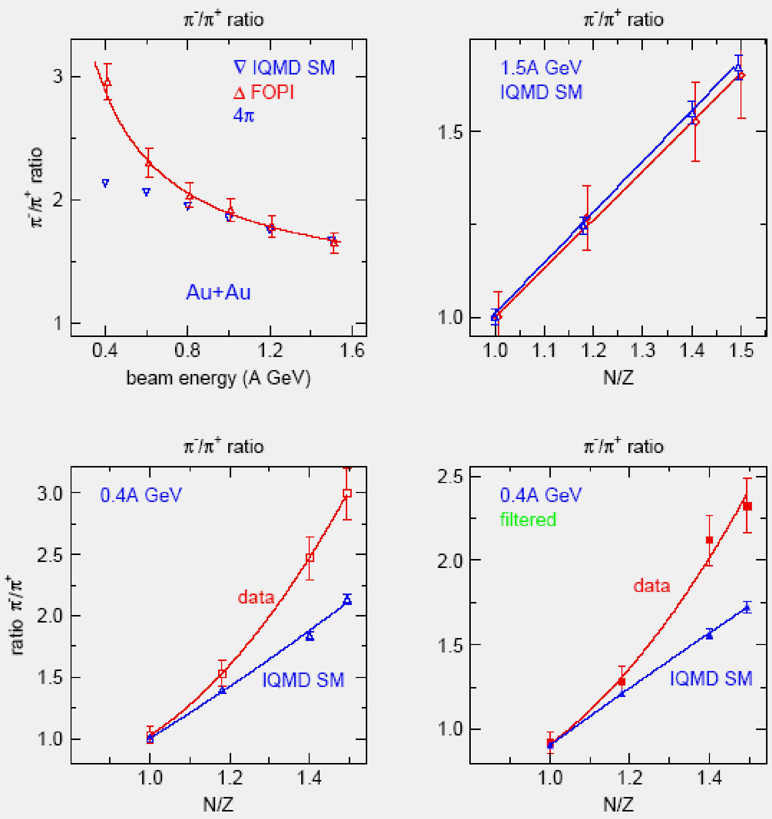
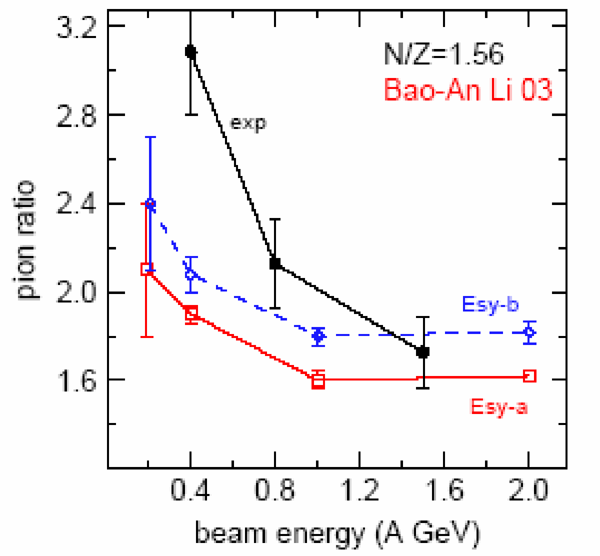
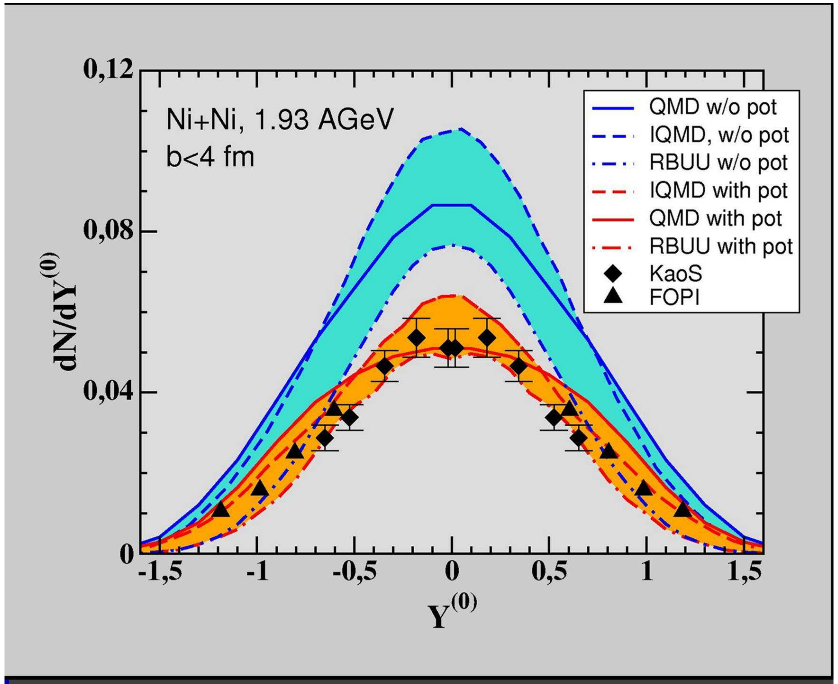


Fig. 25. Upper left panel: Excitation function of the 4π-integrated ratio of π^-/π^+ yields in central Au+Au collisions. The experimental data are joined by a least squares fit of the function $c_0 + c_{-1}(E/A)^{-1}$ excluding the lowest energy point. The IQMD SM prediction (triangles) is also given. Upper right and lower left panels: the N/Z dependence at $1.5A$, respectively $0.4A$ GeV of the π^-/π^+ ratio. The solid lines are least squares fits of linear or quadratic (N/Z) dependence. Lower right panel: same as lower left panel,but for filtered data.



Kaon Production:

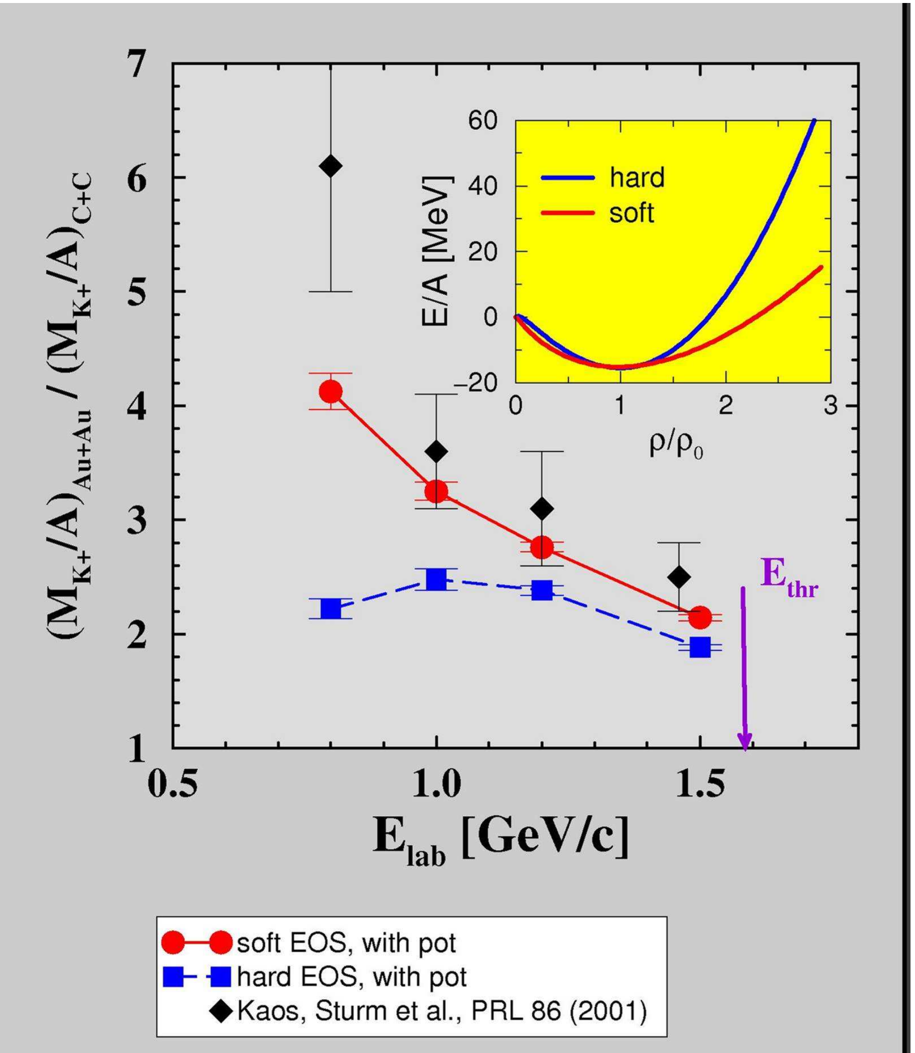
A good way to determine the symmetric EOS (C. Fuchs et al., PRL 86(01)1974)



QMD: Zheng, C.F. et al., PLB 434 (1998) 358

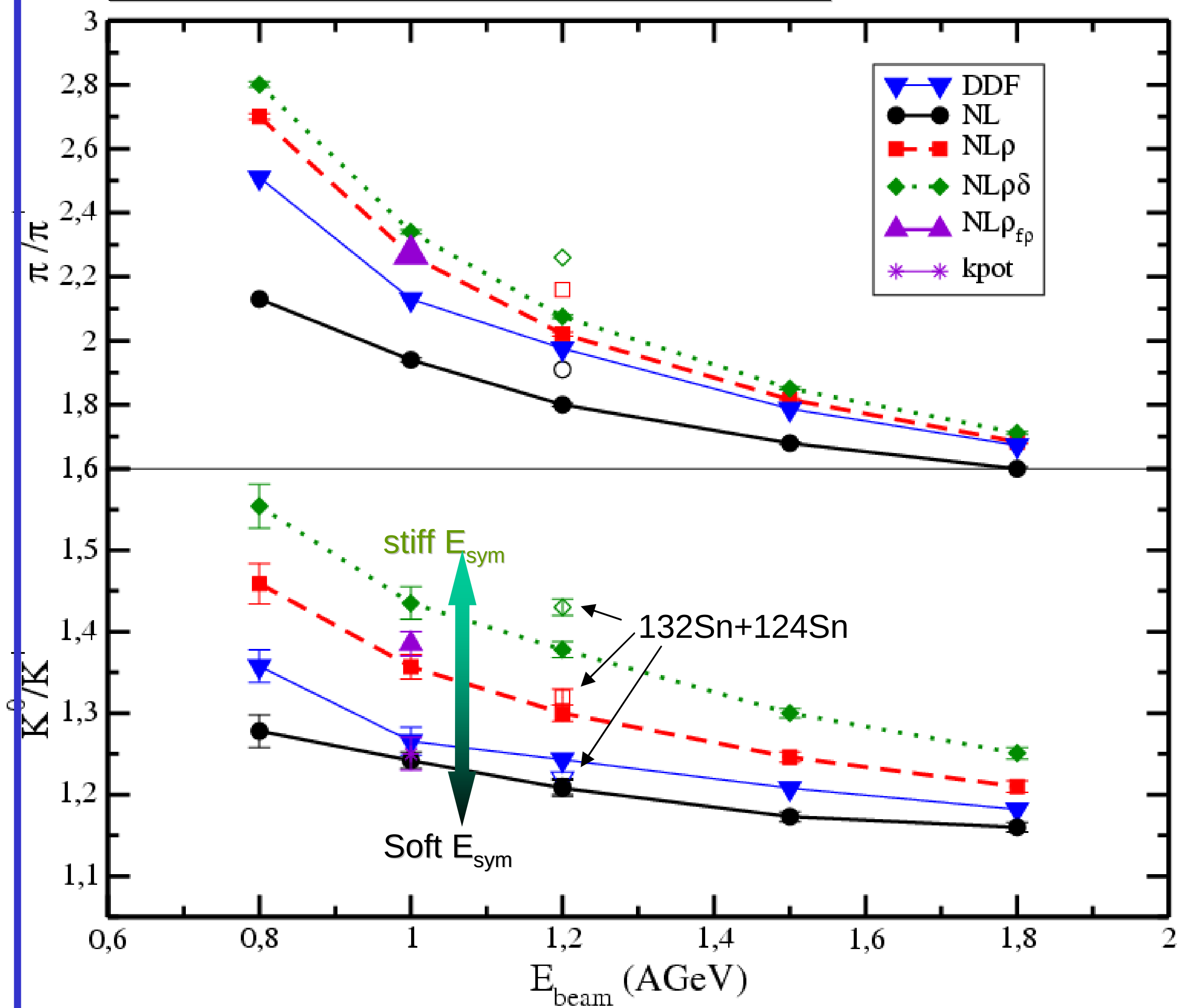
IQMD: Hartnack, Aichelin, JPG 28 (2002) 1649

RBUU: Mishra, Bratkovskaya et al, PRC 70 (2004) 044904



VII.12 Kaon Ratios as Probe of Symmetry energy

Pion and kaon ratios in central Au+Au…



G. Ferini, et al., Phys.Rev.Lett.97:202301,2006, nucl-th/0607005

Kaons:
~15% difference between DDF and NLρδ

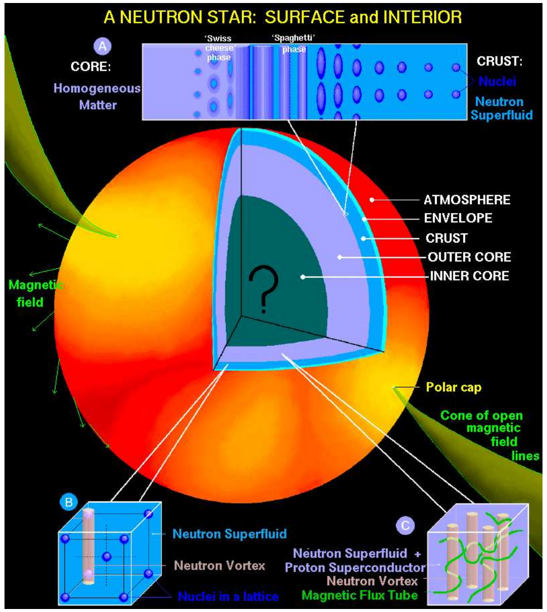
Not sensitive to the K-potential (iso-dep.?)

Pions: less sensitivity ~10%, but larger yields

VII.13 Neutron Star properties and the Symmetry Energy

Astrophysical Connections, esp. for Iso-Vector **EOS**

Neutron Star Structure



Neutron star models

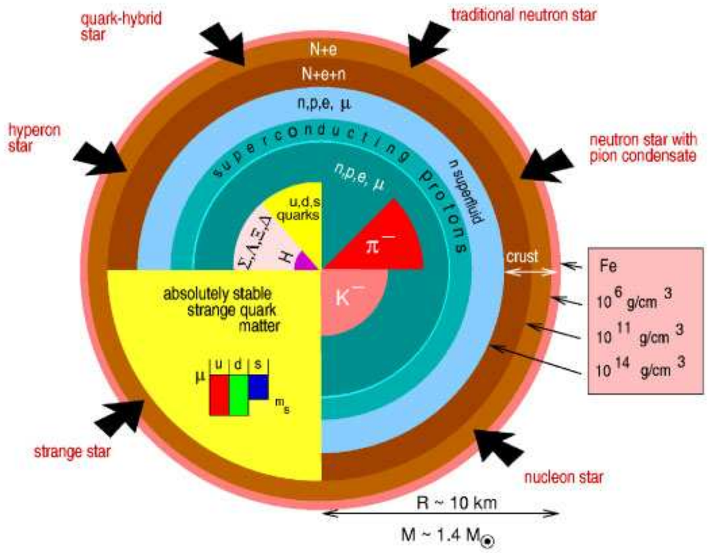


Figure 3.3: Possible novel phases and structures of subatomic matter: (i) a large population of hyperons (Λ,Σ,Ξ), (ii) condensates of negatively charged mesons with and without strange quarks (kaons or pions), (iii) a plasma of up, down, strange quarks and gluons (strange quark matter). Compilation by F. Weber [1].

Constraints on the Equation-of-state

- **from neutron stars:** maximum mass
  - gravitational mass vs baryonic mass
  - direct URCA process
  - mass-radius relation
- **from heavy ion collisions:** flow constraint
  - kaon producton

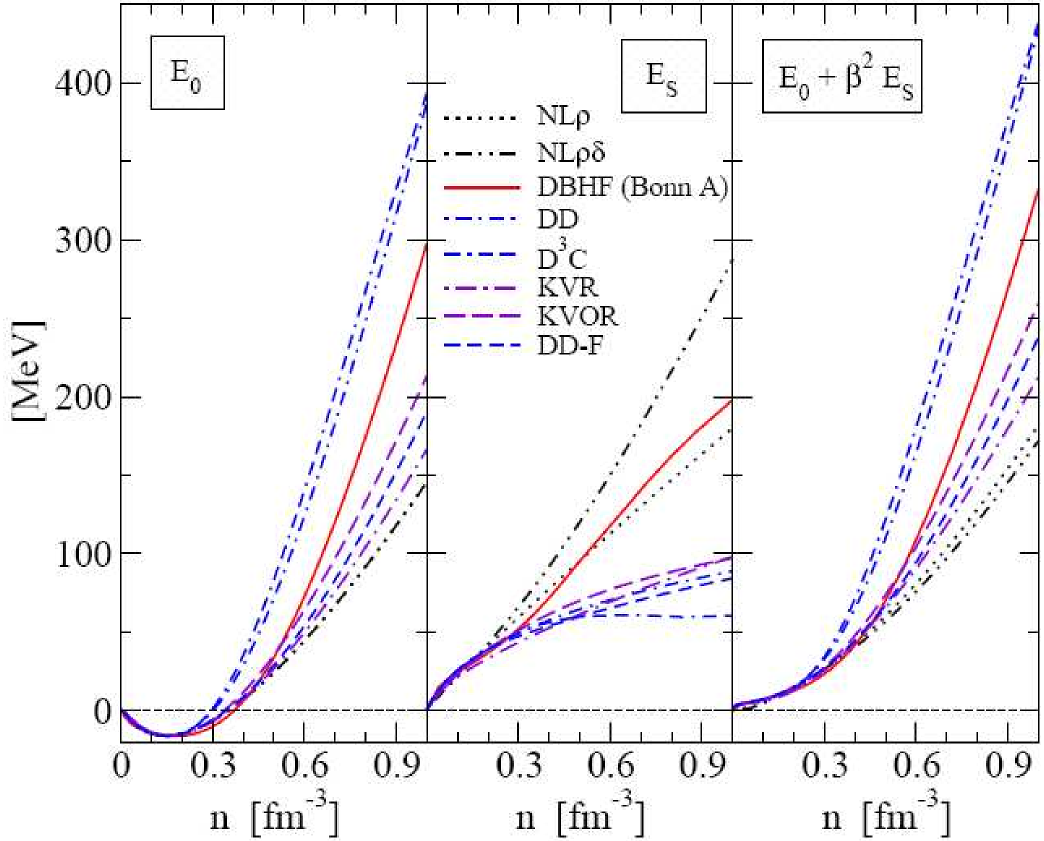
Constraints on the high-density nuclear equation of state from the phenomenology of compact stars and heavy-ion collisions

T. Klähn,[1,2,*] D. Blaschke,[3,4,†] S. Typel,[3] E.N.E. van Dalen,[2] A. Faessler,[2] C. Fuchs,[2] T. Gaitanos,[5] H. Grigorian,[1,6] A. Ho,[7] E.E. Kolomeitsev,[8] M.C. Miller,[9] G. Röpke,[1] J. Trümper,[10] D.N. Voskresensky,[3,11] F. Weber,[7] and H.H. Wolter[5]

Phys.Rev. C74 (2006) 035802

Models for Symmetry Energy for NS

Equations of State tested:



$$\frac{B}{A} = E_0(n) + \beta^2 E_S(n)$$

$$\approx a_V + \frac{K}{18}\epsilon^2 - \frac{K'}{162}\epsilon^3 + \ldots$$

$$\ldots + \beta^2 \left(J + \frac{L}{3}\epsilon + \ldots\right) + \ldots$$

$$\epsilon = (n - n_{sat})/n$$

$$\beta = (n_n - n_p)/(n_n + n_p)$$

Model	$n_{\rm sat}$	a_V	K	K'	J	L	m_D/m
	[fm^{-3}]	[MeV]	[MeV]	[MeV]	[MeV]	[MeV]	
NLρ	0.1459	−16.062	203.3	576.5	30.8	83.1	0.603
NL$\rho\delta$	0.1459	−16.062	203.3	576.5	31.0	92.3	0.603
DBHF	0.1779	−16.160	201.6	507.9	33.7	69.4	0.684
DD	0.1487	−16.021	240.0	−134.6	32.0	56.0	0.565
D^{3}C	0.1510	−15.981	232.5	−716.8	31.9	59.3	0.541
KVR	0.1600	−15.800	250.0	528.8	28.8	55.8	0.800
KVOR	0.1600	−16.000	275.0	422.8	32.9	73.6	0.800
DD-F	0.1469	−16.024	223.1	757.8	31.6	56.0	0.556

NS Masses and Cooling

NS masses and cooling behaviour depends on iso-vector EOS

Tolman-Oppenheimer-Volkov equation to determine mass of neutron star

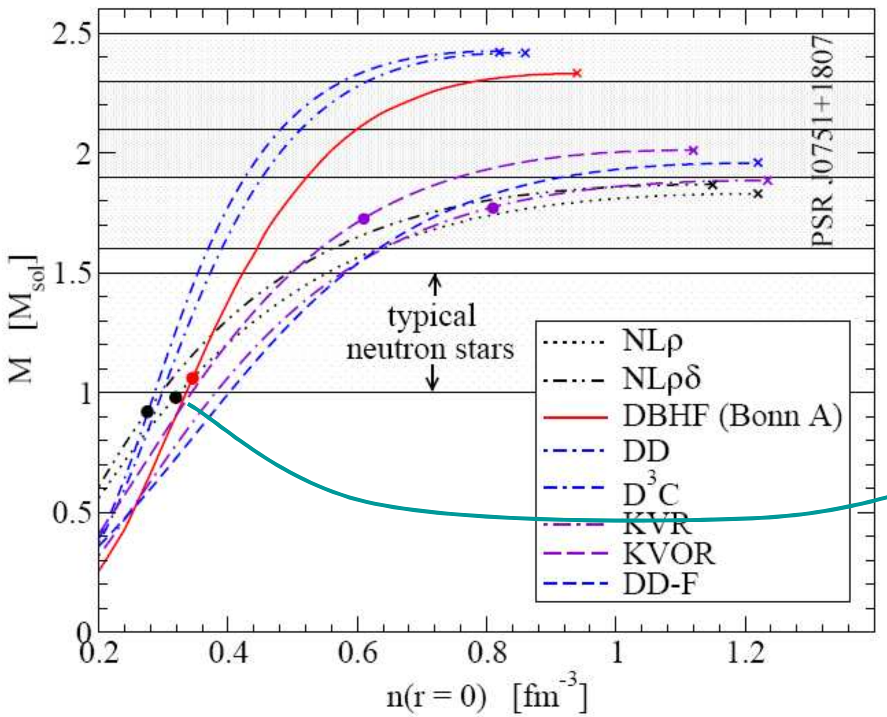
$$\frac{dP(r)}{dr} = -\frac{Gm(r)\varepsilon(r)}{r^2}\left(1+\frac{P(r)}{\varepsilon(r)}\right)\left(1+\frac{4\pi r^3 P(r)}{m(r)}\right)\left(1-\frac{2Gm(r)}{r}\right)^{-1}$$

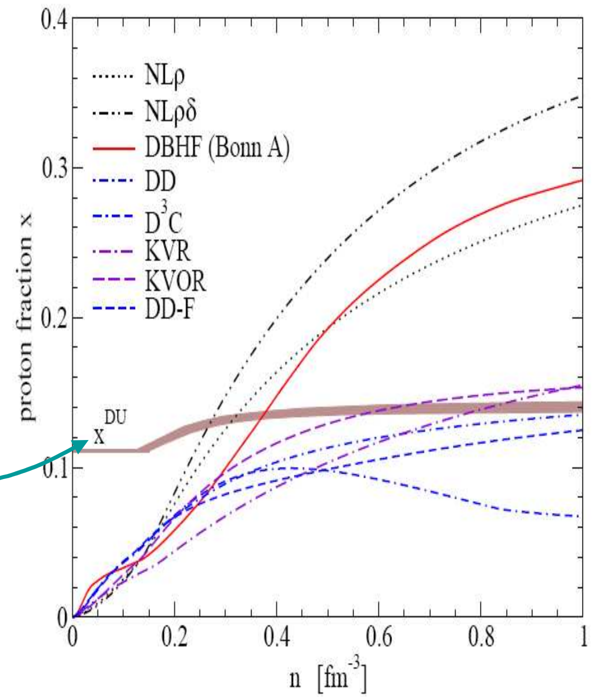
$$m(r) = 4\pi\int_0^r dr'\, r'^2\varepsilon(r').$$

Proton fraction and direct URCA

- β – equilibrium and charge neutrality: $y = \frac{N}{Z} = y(\varepsilon_{sym})$
- direct URCA process: $p \rightarrow n + e^+ + \nu_e$

  threshold: $y \approx 11\%$, fast neutrino cooling





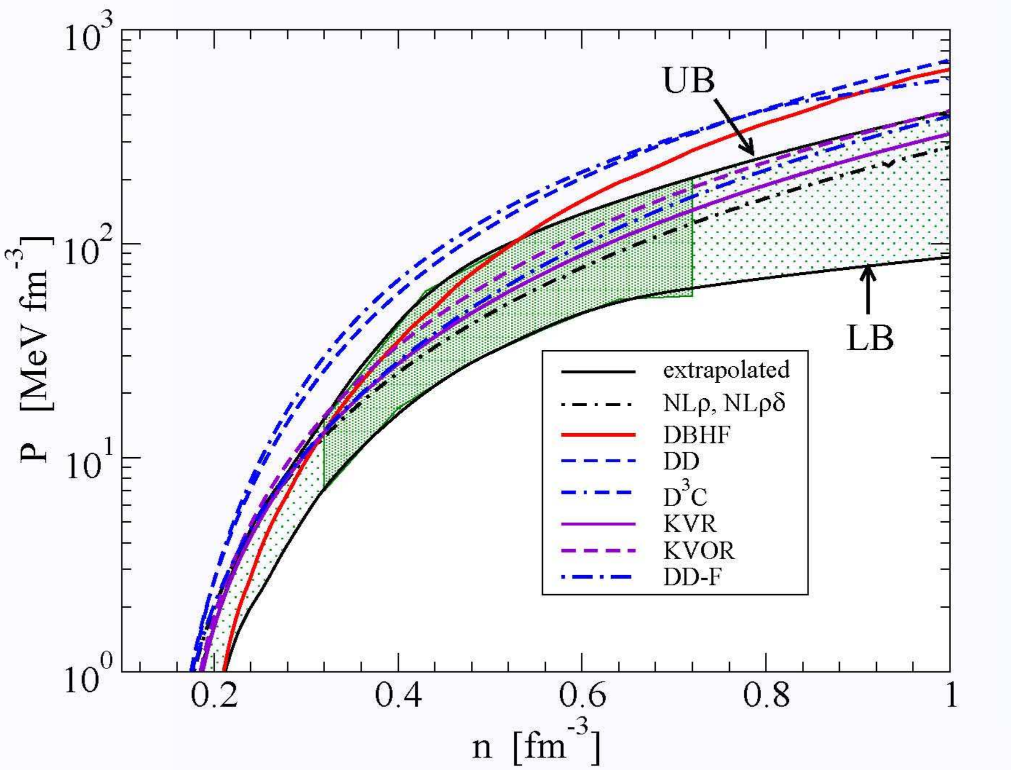
Forbidden by Direct URCA constraint

Klähn, Blaschke, Typel, v.Dalen, Faessler, Fuchs, Gaitanos, Gregorian, Wolter, . Phys.Rev.C74:035802,2006

Consistency between HI and NS Data

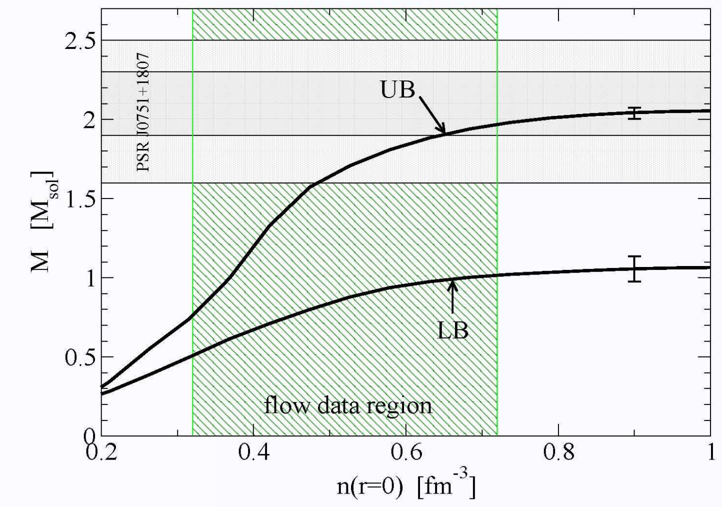
Flow-Constraint from HIC:

(P.Danielewicz, R. Lacey, W.G. Lynch, Science 298, 1592 (2002))



Maximum Mass by Flow Constraint

How strong is the flow constraint?



LB not reliable ↔ Maximum mass constraint demands stiff EoS

(applied "universal" $\beta^2 E_S$ (error bars!))

The End

Thank you very much for the interest

-and I hope the lecture was instructive