Nuclear Reactions

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Lecture 1

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Motivation: Nuclear Reactions – a wide field

→ Compound nuclear reactions

System equilibrates the degrees of freedom, and then decays statistically (formation and decay well separated).

→ Direct reactions

reactions involving few or simple degrees of freedom, e.g. single particle or collective.

→ Seminar by W.Nazarewicz)

→ Heavy Ion collisions:

nucleus-nucleus collisions, with all degrees of freedom involved, but equilibrium is not reached. Depending on the incident energy one distinguishes roughly:

discussed in this lecture:

Deep inelastic collisions (DIC): barrier energies. The reaction is essentially binary.

Fermi energy regime (FE): energies of the order of the Fermi energy in nuclei, i.e. about 35 MeV/A.

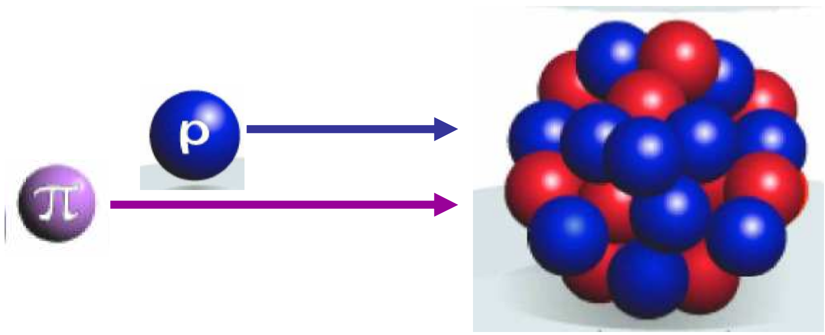
Relativistic regime (RHIC): energies, where only hadronic dof play a role (100 MeV/A to a few Gev/A).

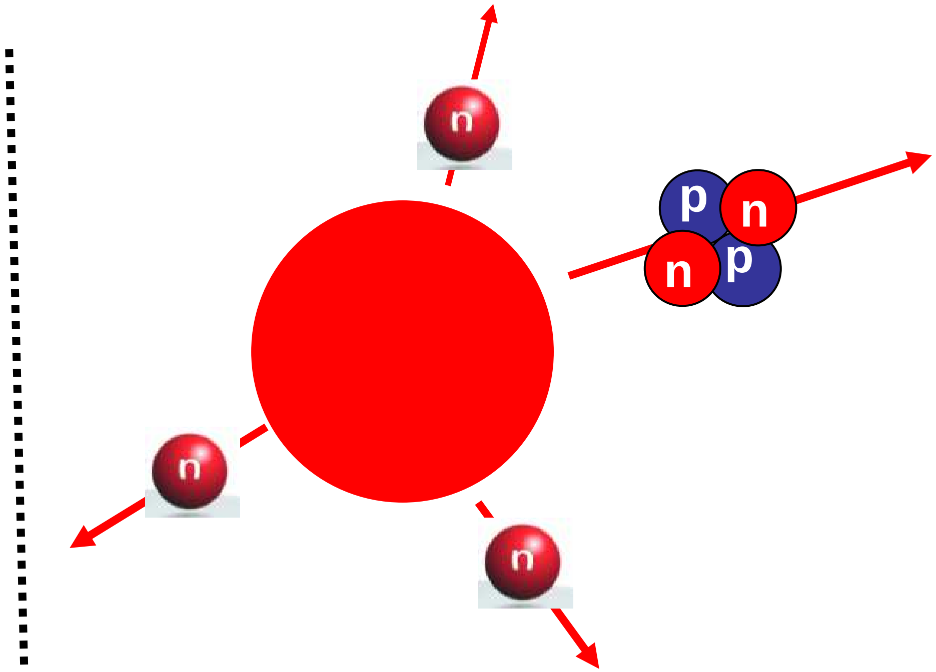
Ultarrel. Collisions (UrHIC): highest energies to study the deconfinement transition and the Quark-Gluon Plasma.

→ Lecture by B. Zajc

O.1 Compound nuclear reactions

System equilibrates the degrees of freedom, and then decays statistically (formation and decay well separated). Produce well defined excited nuclei and observe statistical properties, slow 10^{-16} sec





also: Fusion

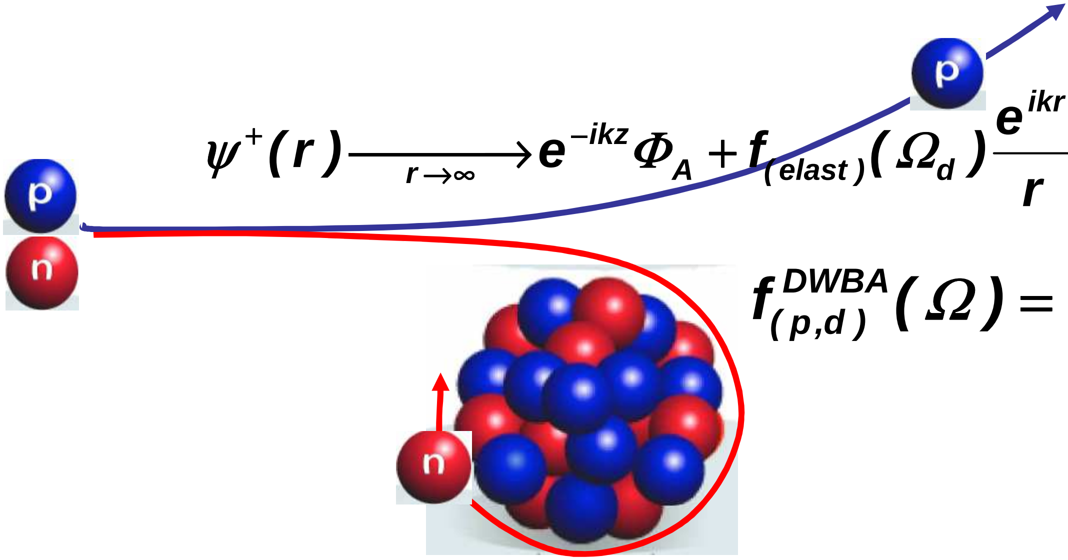
CN: statistical decay of excited nucleus, by sequential emission of light particles

MF multifragmentation (later): simultaneous decay into many fragments

higher excitation energies in HIC

Direct Reactions

Reactions involving few or simple degrees of freedom, e.g. single particle or collective, e.g. $^{16}O(d,p)^{17}O$, usually using light probes. Treated quantum-mechanically with scattering theory, e.g. optical model, transfer reactions with DWBA.



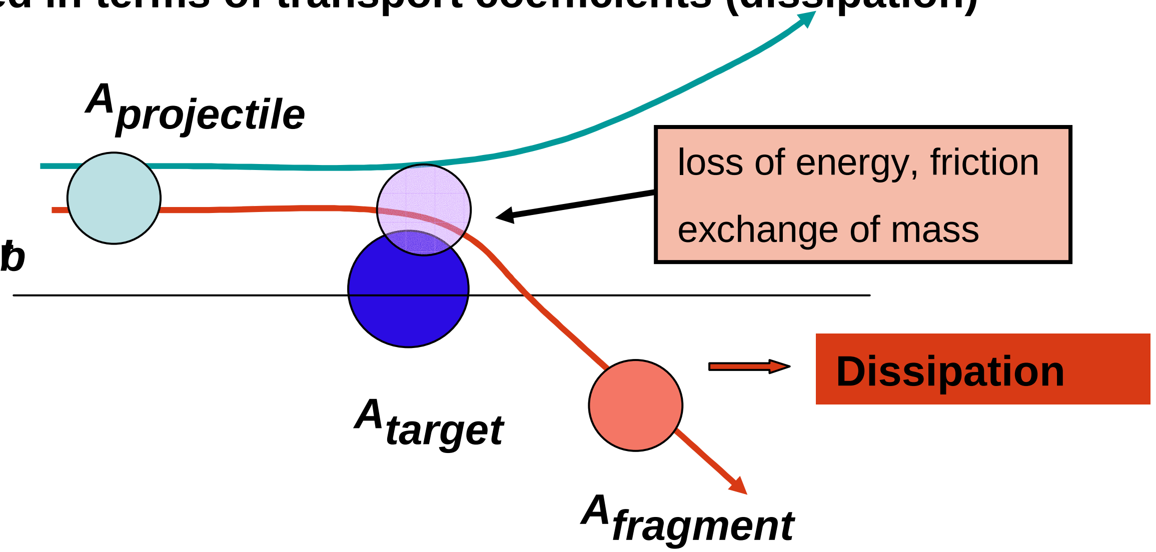
$$\psi^{+}(r)\xrightarrow[r\to\infty]{} e^{-ikz}\Phi_A + f_{(elast)}(\Omega_d)\frac{e^{ikr}}{r}\Phi_A + f_{(d,p)}(\Omega_p)\frac{e^{ikr}}{r}\Phi_{A+n}$$

$$f^{DWBA}_{(p,d)}(\Omega) = \left\langle \psi_p \Phi_{A+n} \middle| V_{pn} \middle| \psi_d \Phi_A \right\rangle$$

Today of interest to study structure of exotic, weakly bound nuclei, where the closeness of the continuum play an important role (→ seminar Nazarewicz).

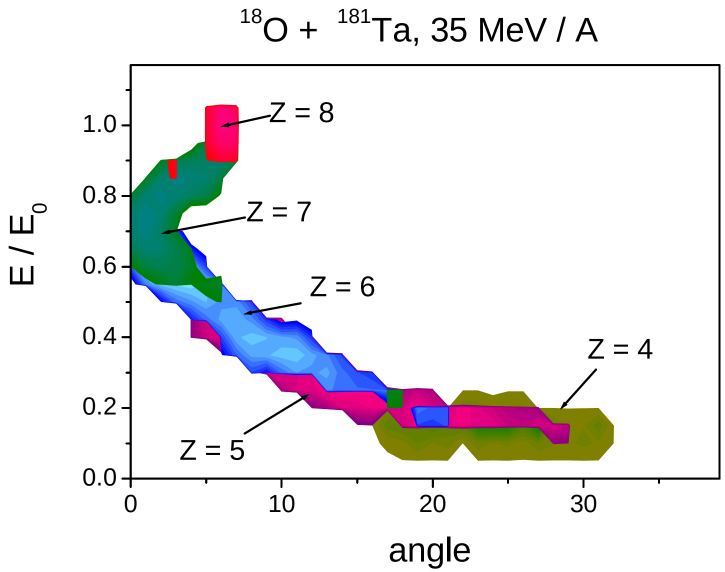
O.3 Heavy Ion Collisions: Deep Inelastic Collisions (DIC)

At barrier energies the reaction is essentially binary with small transfer of energy and mass. Expressed in terms of transport coefficients (dissipation)



Usually treated with classical eom for relative motion and qm calculated dissipation coefficents. Recently also treated with transport approach.

e.g. Energy-loss ← → deflection angle correlation (Wilczinski plot)

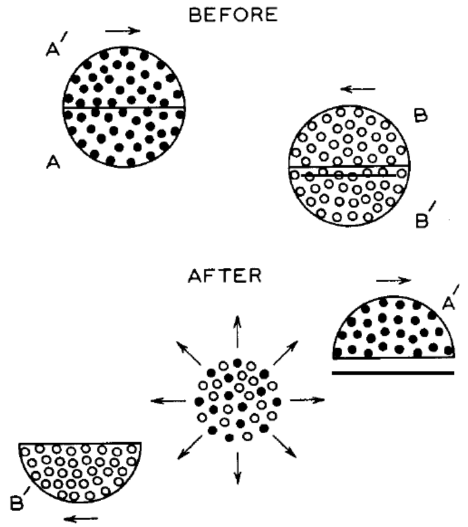


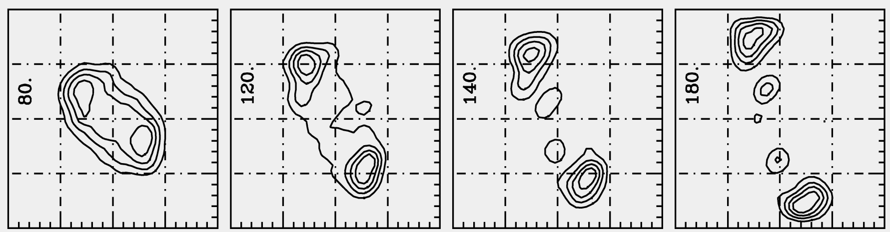
O.4 Heavy Ion Collisions: Fermi Energy Regime

Energies of the order of the Fermi energy in nuclei, i.e. about 35 MeV/A. Moderate compression, special interest in the expansion phase and phase transitions (NSCL, GANIL, Tamu, future FRIB)

multifragmentation in central collisions

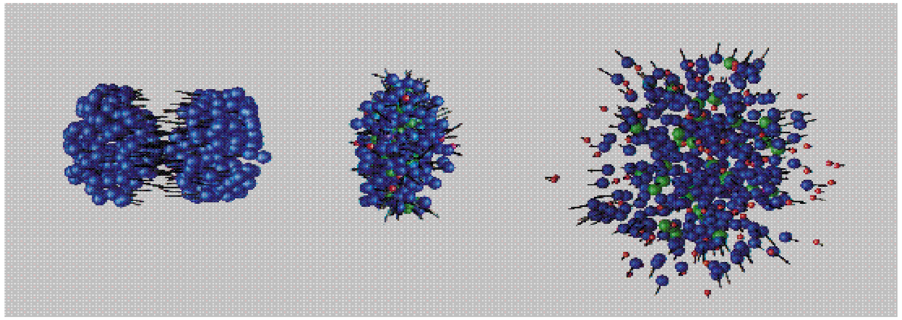
participant-spectator picture in peripheral collsions





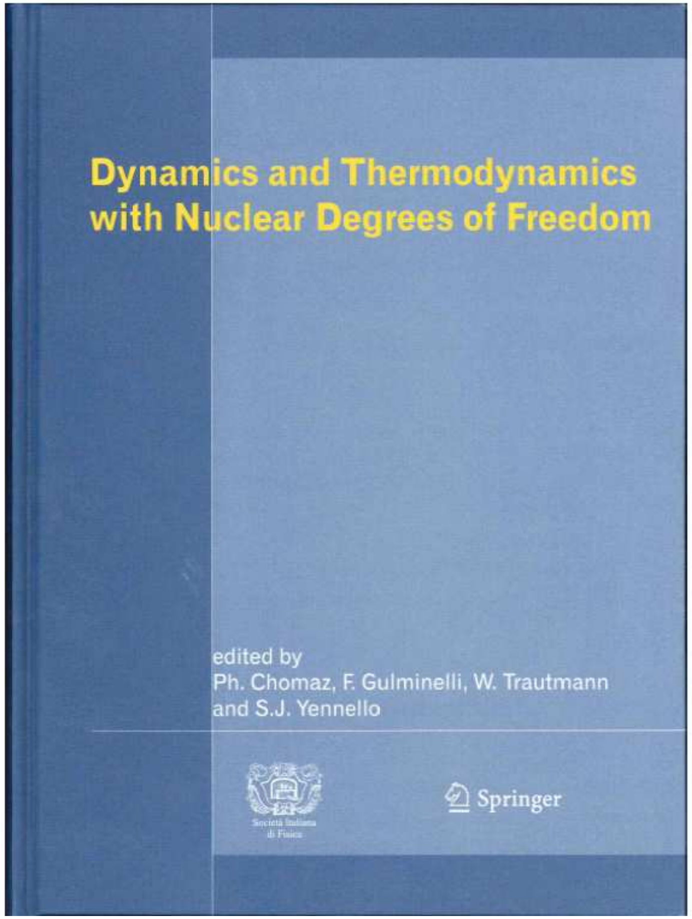
Neck fragmentation

energies, where only hadronoc dof play a role (100 MeV/A to a few Gev/A). Study of dense nuclear matter and hadron properties in dense matter (GSI, Riken)



WCI – „World Consensus Initiative“

A good collection of review articles about low and intermediate energy HIC

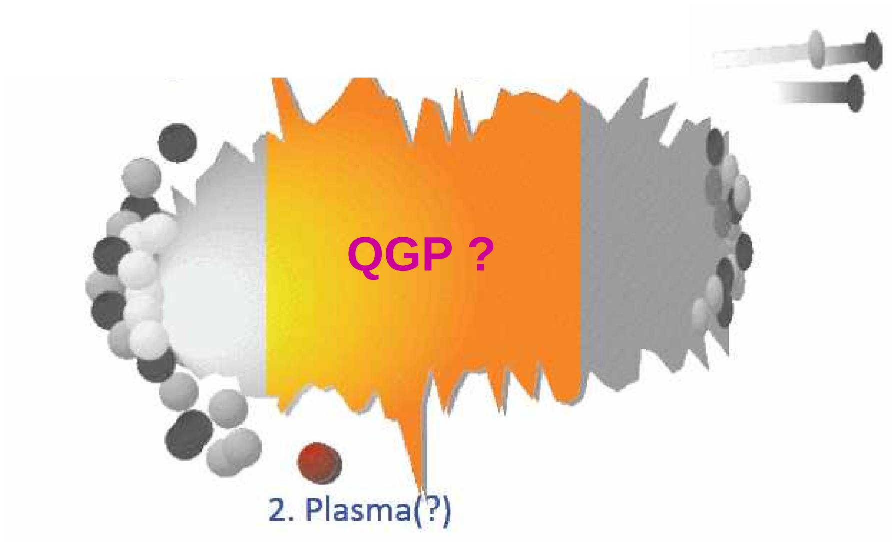


European Physics Journal A - Hadrons and Nuclei, Vol. 30

O.6 Heavy Ion Collisions: GSI/FAIR Facility

Heavy Ion Collisions: Ultra-Relativistic Collisions

highest energies to study the deconfinement transition and the Quark-Gluon Plasma (RHIC, LHC, FAIR)

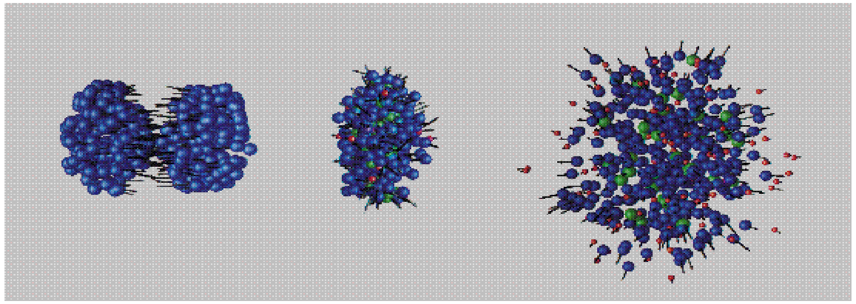


→lecture Zajc

→Hydrodynamics (seminar Teaney)

→Transport codes with subnuclear degrees of freedom (not discussed here)

Heavy Ion Collisions: Why study??



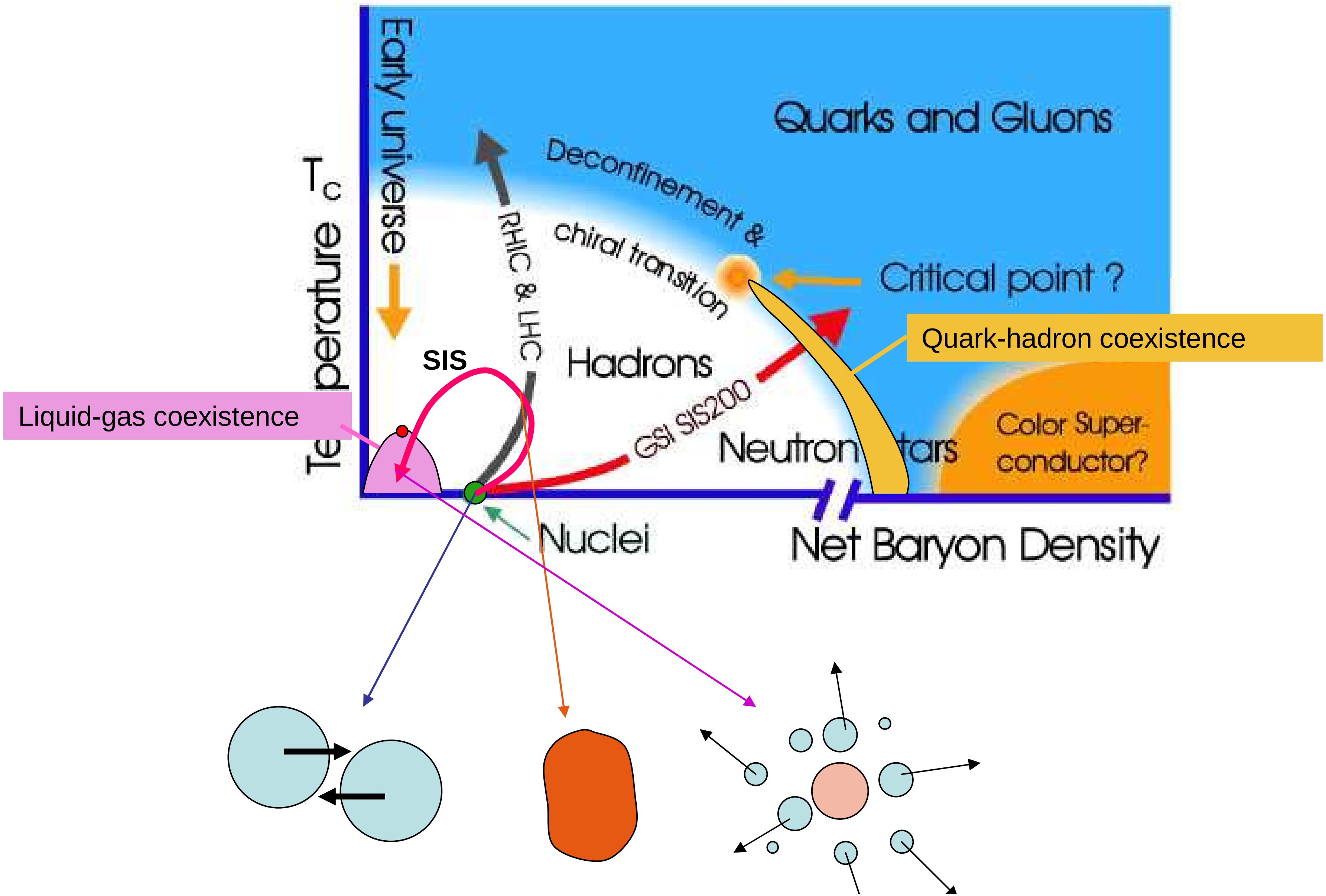
- ensemble reactions → (semi-)classical description possible
- time dependent, i.e. non-equilibrium processes

→ use transport or kinetic theory, dissipation and fluctuation

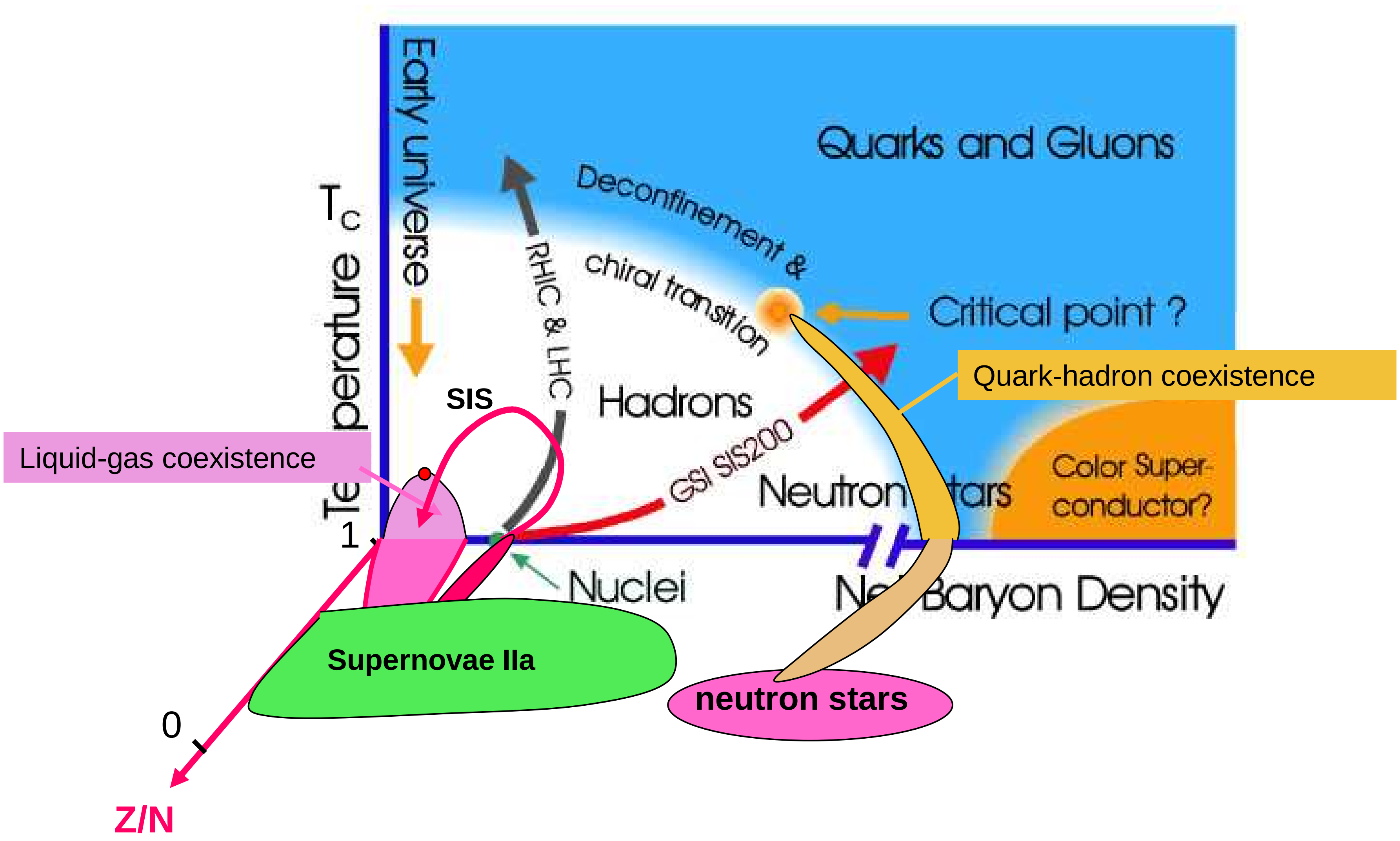
→ complex, so why study?

1. see seminar of Sherry Yennello
2. explore phase diagram of strongly interacting matter in the hadronic world
3. nuclear matter out of saturation point. determine Equation-of-State (EOS) and hadronic properties in dense medium
4. interest in itself, i.e. phase transitions in finite systems
5. importance for astrophysics: supernovae and neutron stars

O.9 Schematic Phase Diagram of Strongly Interacting Matter



O.9 Schematic Phase Diagram of Strongly Interacting Matter



Aim of these lectures

1. understand theoretical treatment of HIC in this energy range
2. get an idea of implementations (difference in codes, ingredients, uncertainties)
3. non-relativistic vs. relativistic treatment
4. information gained and how
5. selection of significant results and open questions (but not complete overview)

apologies:

rather „theoretical“

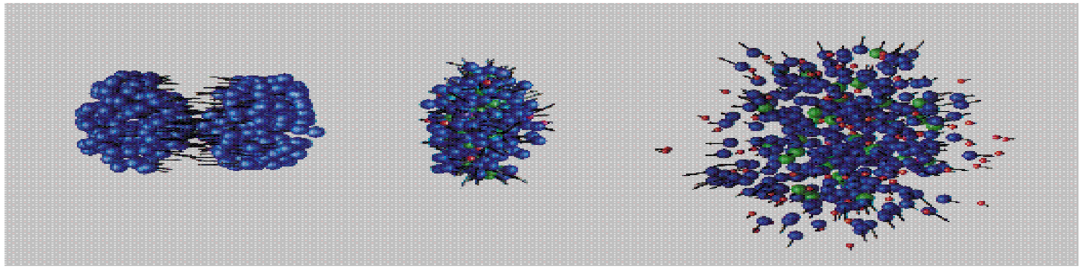
imperfect, short time for preparation

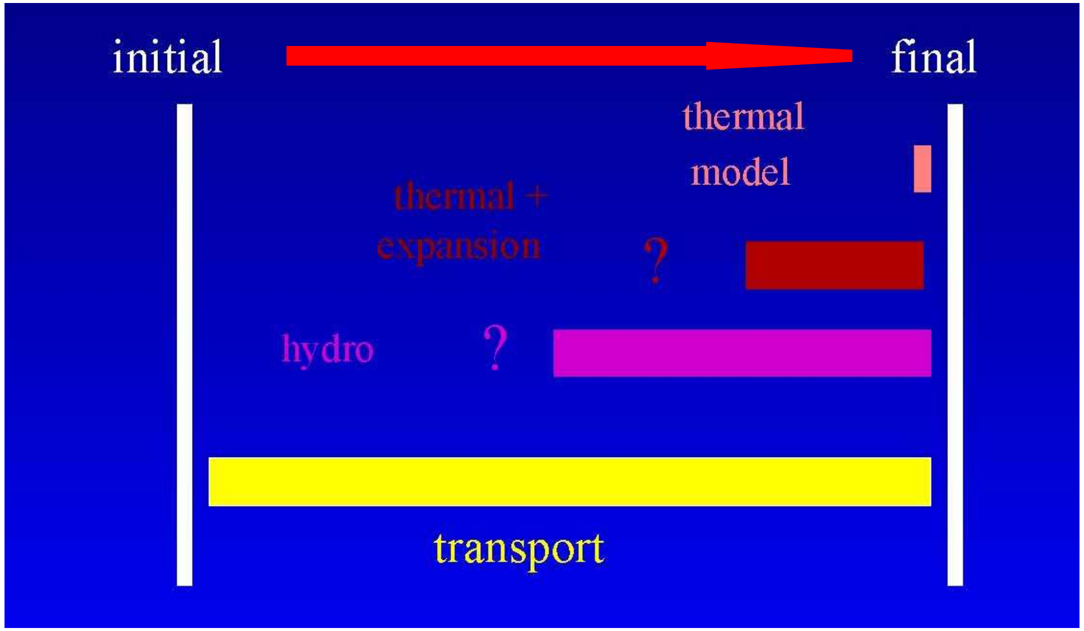
Contents

may be too much!!

I.1 Descriptions of heavy ion collisons

Levels of description of evolution from initial to final state:





Statistical models, e.g. SMM, Botvina, et al.

Statistical emission in expanding system, e.g. EES, Friedmann

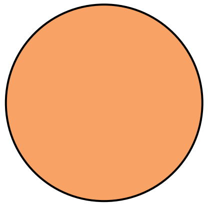
Hydrodynamical model, e..g. Stöcker, Maruhn, et al.

Transport models, e.g. BUU, QMG, AMD, etc

→Discussed in these lectures

Thermodynamical Models

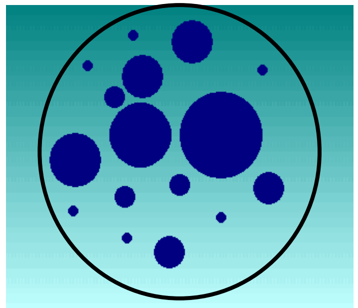
excited, expanded nucleus or residue at end of a collision (not necc. spherical)



assumed all nuclear interactions cease: freeze-out configuration:

A,Z,E^*,ρ_c,(shape)

assume thermo-dynamical equilibrium, with temperature T

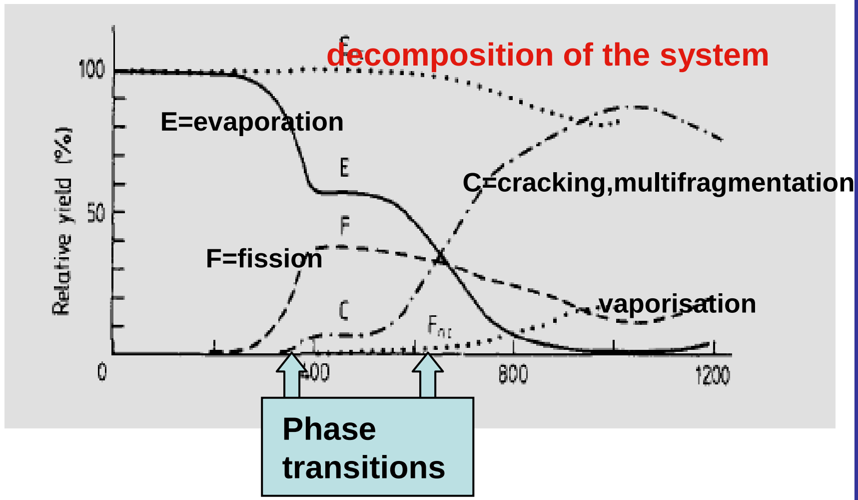


Final state event constructed with statistical sampling according to free energy

Models (ensembles):

microcanonical MMMC (D. Gross)

canonical SMM (Bondorf, Botvina, Mishustin)



Hydrodynamical Models

assume local thermal equilibrium, and uses conservation equations for

particle number

$$\partial f/\partial t + \nabla \cdot (\rho \boldsymbol{u}) = 0 .$$

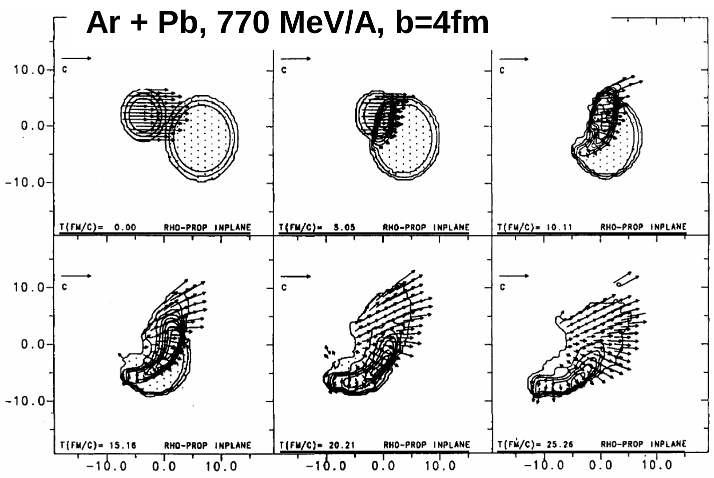
momentum

$$\frac{\partial}{\partial t}(\rho \boldsymbol{u}) + \nabla \cdot (\rho \boldsymbol{u}\boldsymbol{u}) = - \nabla \cdot \mathbf{P} + \frac{\rho}{m} \boldsymbol{F} .$$

energy

$$\frac{\partial}{\partial t}(\rho E) + \nabla \cdot (\rho E \boldsymbol{u}) = - \nabla \cdot (\boldsymbol{u} \cdot \mathbf{P}) + \rho \boldsymbol{u} \cdot \boldsymbol{F} .$$

E.g. early prediction of nuclear shock wave phenomena in heavy ion collisions (Stöcker, Greiner, 1978) → Mach cones



assumption of local thermodynamical equilibrium usually, esp. at high energy too simple → transport decriptions

Transport Models

Transport theory describes the non-equilibrium aspects of the temporal evolution of a collision. The central quantitiy is the phase space density (coordinate and momentum distribution). This will be discussed in greater detail in the following.

Demonstrate two aspects:

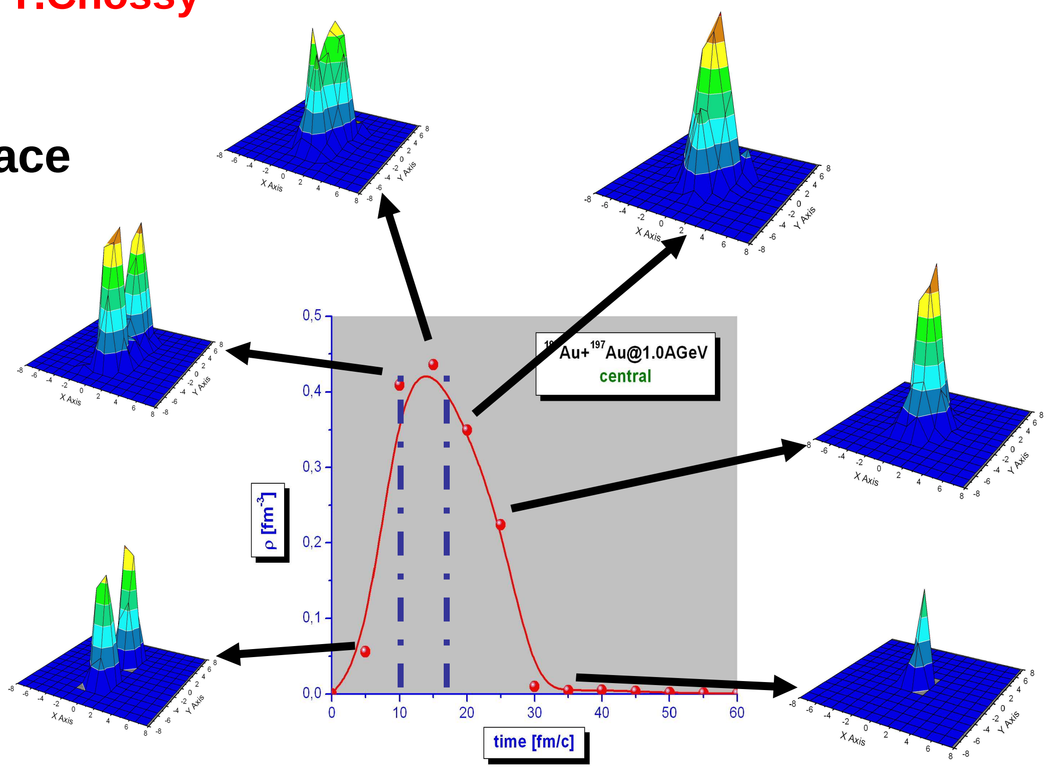
1. Evolution in coordinate space:

→ movies curtesy T. Gaitanos, T.Chossy

2. Evolution in momentum space

non-equilibrium,

non-shericity of local momentum distributions



II.1 Heuristic Derivation of Transport

aim: microscopic discription of nucleus-nucleus collisions

here: make plausible without a derivation

main ingredients:

individual N-N collisions	→	**Cascade model**
nucleons move in mean field	→	**Vlasov equation**
both simultaneously	→	**Boltzmann equation and variants**

reference:

PHYSICS REPORTS (Review Section of Physics Letters) 160, No. 4 (1988) 189–233. North-Holland, Amsterdam

A GUIDE TO MICROSCOPIC MODELS FOR INTERMEDIATE ENERGY HEAVY ION COLLISIONS

G.F. BERTSCH

Cyclotron Laboratory and Physics Department, Michigan State University, East Lansing, MI 48824, U.S.A.

and

S. DAS GUPTA

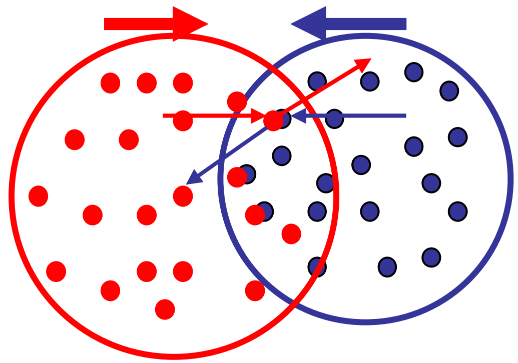
Physics Department, McGill University, Montreal, Quebec, Canada H3A 2T8

Received September 1987

→ materials

Cascade Model

simplest and first model: Cascade model (e.g. Cugnon, et al., NPA 532 (1981))



nucleons of nucleus (A,Z) distriuted randomly in sphere of Radius R_A

nucleons interact in a time interval δt if:

- they pass their distance of closest approach
- and this distance is less than $b < b_{max} = \frac{1}{\pi}\sqrt{\sigma^{tot}(\sqrt{s})}$

- the scattering can be elastic or inelastic

NN	→	NN	
	→	NΔ,	Δ → Nπ
	→	NΛK	
NΔ	→	NΔ	
ΔΔ	→	ΔΔ	
πN	→	ΛK	
etc.			

scattering channel and scattering angle are chosen randomly from experimental (free) cross sections or models

no mean field effects!
model valid only at very high energies !

1-body phase space

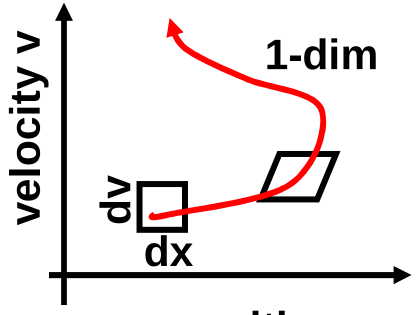
central quantitiy: 1-body phase space distribution: $f_i(\vec{r},\vec{p};t)$

= probability to find at time t a particle if type i at point r with momentum p

motion of phase space cell in phase space

deformation but no change of area (Liouville theorem),

→phase space density is constant in time (prove it!)



then

$$df = \frac{\partial f}{\partial r} dr + \frac{\partial f}{\partial p} dp + \frac{\partial f}{\partial t} dt$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + v \frac{\partial f}{\partial r} + F \frac{\partial f}{\partial v}$$

or generally in a potential U(r):

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla}^{(r)} f - \vec{\nabla}^{(r)} U(r) \, \vec{\nabla}^{(p)} f = 0$$

(Vlasov equ.)

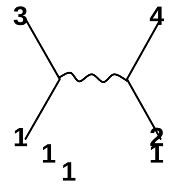
drift term — acceleration by the field

„streaming derivative“

However, collisions will change the phase space density!

Collision term

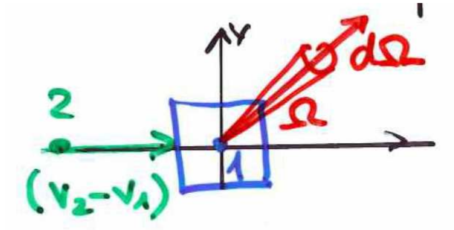
change of phase space probability through collisions: e.g. loss

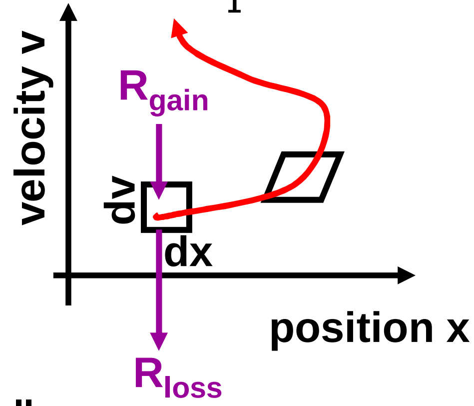


$$j_0 = \overbrace{f(r,v_2;t)}^{f_2}\,\overbrace{|v_2 - v_1|}^{v_{21}}$$

$$j_\Omega = j_0\,\sigma(\Omega; v_{21})$$

$$R_{loss} = \underbrace{f(r,v_1;t)}_{f_1} \int d\vec{v}_2\, d\Omega\, j_\Omega = \int d\vec{v}_2\, d\Omega\, v_{21}\, \sigma(\Omega)\, f_1\, f_2$$





correspondingly: gain term R_{gain} by scattering into phase space cell

Pauli principle: phase space cell into which the scattering occurs must be available: **blocking factors** $(1 - f(r,v_i;t)) \equiv (1 - f_i) := \bar{f}_i$

energy momentum conservation: $p_1 + p_2 = p_{1'} + p_{2'}$

total collision term:

$$I_{coll} = \int d\vec{v}_2\, d\vec{v}_{1'}\, d\vec{v}_{2'}\, |v_2 - v_1|\, \sigma(\Omega)\, (2\pi)^3\, \delta(p_1 + p_2 - p_{1'} - p_{2'}) \left[f_{1'} f_{2'} (1 - f_1)(1 - f_2) - f_1 f_2 (1 - f_{1'})(1 - f_2') \right]$$

Transport equation: Boltzmann-Uehling-Uhlenbeck (BUU)

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla}^{(r)} f - \vec{\nabla} U(r)\, \vec{\nabla}^{(p)} f = I_{coll}$$

II.5 Remarks on BUU-Equation

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\vec{\nabla}^{(r)} f - \vec{\nabla} U(r)\,\vec{\nabla}^{(p)} f(\vec{r},\vec{p};t) = \int d\vec{v}_2\, d\vec{v}_{1'}\, d\vec{v}_{2'}\, v_{21}\, \sigma_{12}(\Omega)\,(2\pi)^3 \delta(p_1 + p_2 - p_{1'} - p_{2'})$$
$$\left[f_{1'} f_{2'} (1-f_1)(1-f_2) - f_1 f_2 (1-f_{1'})(1-f_2')\right]$$

numology: **Boltzmann: collision term (without blocking), but no mean field potential**

Vlasov: mean field but no collision (rhs)

Nordheim, Ü&U: Pauli plocking factors

Landau: dissipation (collisions, in an approx.)

thus many names: Boltzmann-Uehling-Uhlenbeck (BUU), Landau-Vlasov (LV)

Boltzmann-Nordheim-Vlasov (BNV), VUU Vlasov-UUU

→ all the same equation, but also different ways to solve (rather simulate) it (→later).

assumptions:

1. **essentially classical (quantum derivation later)**
2. **quantum aspects only in blocking factors → semiclassical**
3. **only two-body collisions, indep. of previous history: Markov assumption, no memory effects**

   only valid in sufficiently dilute medium (no 3-body collisions)

ingredients:

1. **mean field potential *U*, self consistent mf potential (HF), or parametrized as function of mean density → from here obtain the Equation-of-State (EOS), i.e. E(ρ,T=0)**
2. σ, **cross section in-medium, thus not directly obtainable from experiment, local collisions**

II.6 Solutions of BUU Equation

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\vec{\nabla}^{(r)} f - \vec{\nabla} U(r)\,\vec{\nabla}^{(p)} f(\vec{r},\vec{p};t) = \int d\vec{v}_2\, d\vec{v}_{1'}\, d\vec{v}_{2'}\, v_{21}\, \sigma_{12}(\Omega)\,(2\pi)^3\, \delta(p_1 + p_2 - p_{1'} - p_{2'})$$
$$\left[f_{1'} f_{2'} (1 - f_1)(1 - f_2) - f_1 f_2 (1 - f_{1'})(1 - f_2') \right]$$

non-linear integro-differential equation, no closed solutions

- deterministic ! (for later discussion)

a) solution on a lattice: has been used for low-dimensional model systems, but too expensive for realistic cases

b) test particle method (Wong 82) $f(r,p;t) = \frac{1}{N_{TP}} \sum_{i=1}^{AN_{TP}} \delta(r - r_i(t))\, \delta(p - p_i(t))$

where $\{ r_i(t), p_i(t) \}$ are the positions and momenta of the TP as a funct. of time,

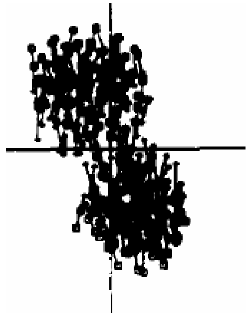
and N_{TP} is the number of TP per nucleon (usually around 50 – 100)

→ approximate a (continuous) phase space distribution by a swarm of δ-functions

→ if one plugs ansatz into Vlasov eq. (lhs of BUU-eq.), one sees (show!)

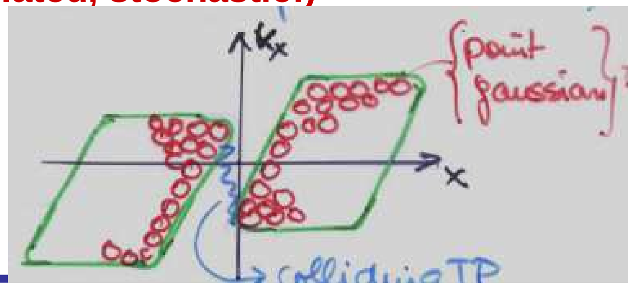
that the TP centers obey

Hamiltonian equations of motion (eom): $\frac{\partial r_i}{\partial t} = \frac{p_i}{m}; \qquad \frac{\partial p_i}{\partial t} = -\nabla U\big|_{r_i}$



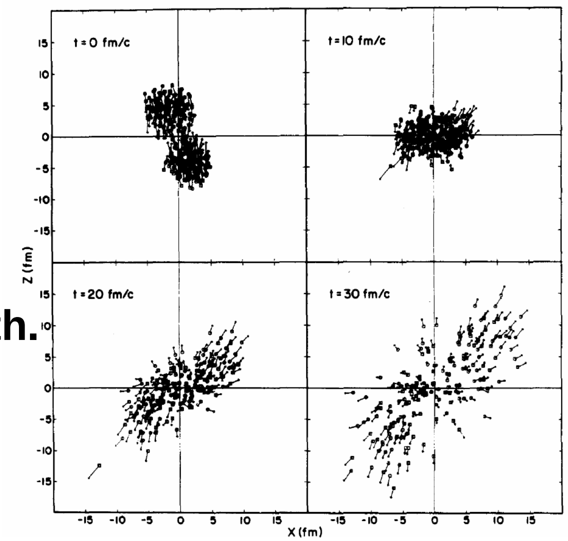
the rhs (collision term is treated like in cascade (i.e.simulated, stochastic!)

ex: 1-dim slab movement in phase space



Discussion of TP Method

example of TP evolution in a collision (Nb+Nb, 400 AMeV, b~5 fm)



Difficulty in test particle method:

- need U(ρ(r)) at the position r of a TP, but ρ(r) is not smooth.
- two methods

a) average of little volume ΔV~1 fm^3

$$\rho(r) = \frac{1}{N_{TP}} \frac{1}{\Delta V} \sum_{r_i \in \Delta V} 1$$

N_{TP} large (~500)

$$f = \frac{1}{N_{TP}} \sum_{i=1}^{AN_{TP}} g(r - r_i(t))\, \tilde{g}(p - p_i);$$

$$g(r) = \frac{1}{(\sigma\sqrt{\pi})^3} e^{-r^2/\sigma^2}; \quad \tilde{g} = \ldots$$

b) TP of finite extension

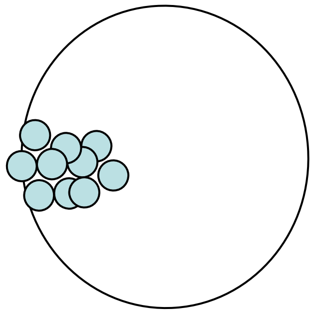
(shape,i.e.Gaussian))

then $\rho(r;t) = \sum_i g(r - r_i(t))$ → a sort of self-averaging!

N_{TP} smaller (~50-100)

also used to obtain smooth momentum distributions locally,

and to have a better representation of the surface



II.8 Simulation of Collision Term

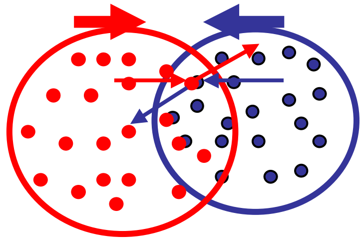
as in cascade model:

nucleons interact in a time interval dt if:

a) they pass their distance of closest approach

b) and this distance is less than $b < b_{max} = \frac{1}{\pi}\sqrt{\sigma^{tot}(\sqrt{s})}$

c) select final scattering state and angle according to cross section and angular distribution



two strategies:

1) collide test particles (so-called full ensemble method) with cross section σ/N_{TP}

→ closer to solution of original BUU eq., in part. small fluctuations

→ expensive numerically, $\sim(AN_{TP})^2$

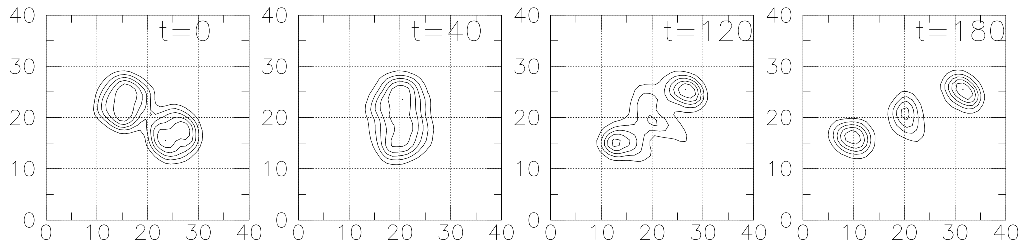
2) divide all TP into N_{TP} ensemples, and collide particles only in their ensemble (parallel ensemble method), but calculate mean field from all TP

→ easier numerically $N_{TP}A^2$

→ introduces more fluctuations into phase space distribution („numerical fluctuations“)

→ each ensemble is a separate „event“, but cross talk

→ discuss fluctuations later



II.9 How to gain information from HIC?

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\vec{\nabla}^{(r)} f - \vec{\nabla} U(r)\vec{\nabla}^{(p)} f(\vec{r},\vec{p};t) = \int d\vec{v}_2\, d\vec{v}_{1'}\, d\vec{v}_{2'}\, v_{21}\, \sigma_{12}(\Omega)\,(2\pi)^3 \delta(p_1 + p_2 - p_{1'} - p_{2'})$$
$$\left[f_{1'} f_{2'} (1-f_1)(1-f_2) - f_1 f_2 (1-f_{1'})(1-f_2') \right]$$

Transport calculation has physical input:

→Mean field, U(r), usually parametrized as a function of local density U(ρ(r)),

from this obtain Equation-of-state, i.e. energy density of uniform nuclear matter

as

$$E/A = \langle T \rangle + \int_0^{\rho} U(\rho)\, d\rho$$

→the cross sections σ_{12}, which are cross sections in the medium of density ρ, which are not obtainable directly from experiment. It is of interest to learn about them.

→Compare results of calculations with experimental data (observables → see later)

→Claim to have learned something, if results agree?!

Perhaps, but there are many other uncertainties about the meaning of the input and the solution of the transport equation. This will be discussed later.

Next

Sect.II: **Heuristic construction of Boltzmann equation**

Next to come:

Sect. III: **Elementary derivation of transport equation, starting from quantum mechanics.**

In a non-relatistic formalism,

and then relativistically from a hadronic field theory

Sect. IV: Derivation of a quantum transport theory

(from Kadanoff-Baym equations)