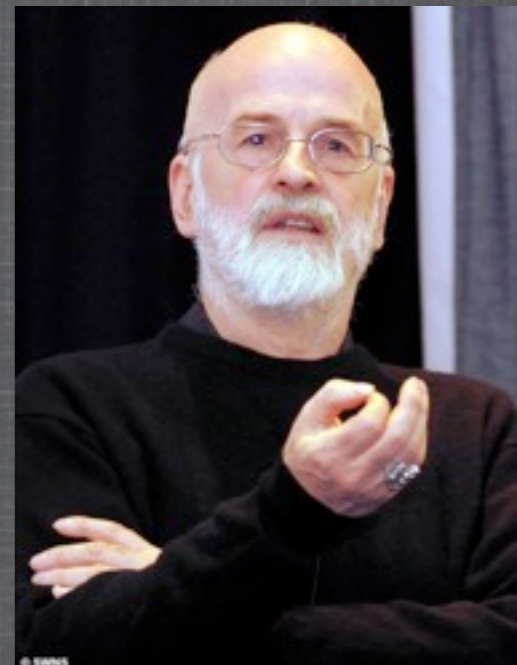


Granddad was superstitious about books.  
He thought that if you had enough of them around,  
education leaked out, like radioactivity.

Terry Pratchett




# Nuclear Astrophysics: Reaction Networks

Frank Timmes

## Outline for 29Jun2009


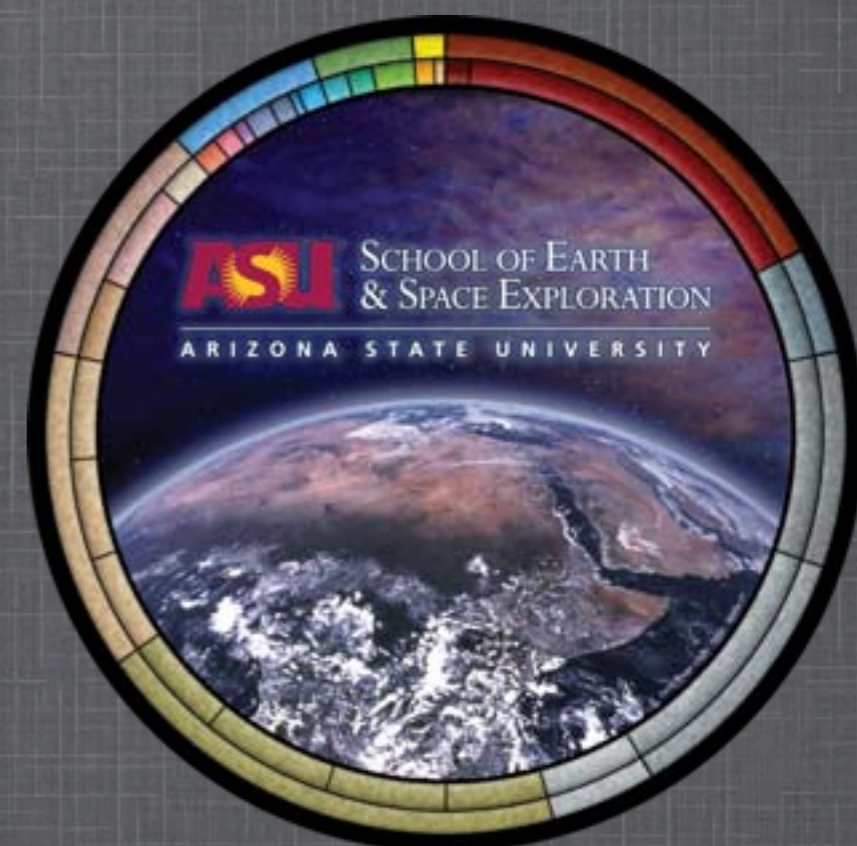
1. Overall theme: putting research tools in your hands
2. Some nomenclature
3. Forming a nuclear reaction network
4. Proton-proton chains



**2009 NATIONAL NUCLEAR PHYSICS SUMMER SCHOOL**  
Lectures, activities, and networking opportunities for nuclear physicists 1-2 years on either side of the PhD

**June 28-July 10, 2009**  
National Superconducting Cyclotron Laboratory (NSCL)  
Michigan State University | East Lansing, Michigan

<http://meetings.nsl.msui.edu/NNPSS09>  
organizers: [nnpss@nsl.msui.edu](mailto:nnpss@nsl.msui.edu)  
Hendrik Schatz (Chair), Wolfgang Bauer, Shari Conroy, Michael Thoennessen  
topics: Hadron Physics, Nuclear Reactions, Nuclear Structure, QCD,  
Neutrinos, Nuclear Astrophysics, Fundamental Symmetries  
invited lecturers: Betsy Beise, University of Maryland; Philippe Chomaz, GANIL; Alexandra Gade,  
Michigan State University; Bob McKeown, California Institute of Technology;  
Frank Timmes, University of Arizona; Bill Zang, Columbia University; John Hardy, Texas A&M University;  
Dave Morrissey, Michigan State University; Wittek Nazarewicz, University of Tennessee; Derek Teaney,  
Stony Brook University; Michael Wiescher, University of Notre Dame; Sherry Yennello, Texas A&M University  
sponsors: National Science Foundation; US Department of Energy's Institute for Nuclear Theory;  
Michigan State University; National Superconducting Cyclotron Laboratory (NSCL)

My overall purpose is to put a research level,  
nuclear reaction network toolkit in your hands.

By the end of my 4 lectures you will (hopefully) have these  
networks under your control and in your knowledge base:

Hydrogen burners: PP chains, CNO cycles

Alpha chains: 13 isotopes, 19 isotopes

Big Bang nucleosynthesis

General reaction network

These reaction networks are written in Fortran 90, so you will need  
a suitable compiler: gfortran, g95, ifort, xlf, absoft, portland, etc.

Reaction networks are an key tool in nuclear astrophysics and other areas of physics, astronomy, chemistry, biology, and geology.

Networks are relevant for modeling nucleosynthesis processes and their associated energy generation in stars.



These talks will provide an overview of the nuclear astrophysics, mathematics, and computational techniques of reaction networks. In only 4 talks, however, they will not be complete.

## Stuff of the day

[cococubed.asu.edu](http://cococubed.asu.edu)

click on “some astronomy codes” and/or “some astronomy talks”

How the Sun Shines

[nobelprize.org/physics/articles/fusion/index.html](http://nobelprize.org/physics/articles/fusion/index.html)

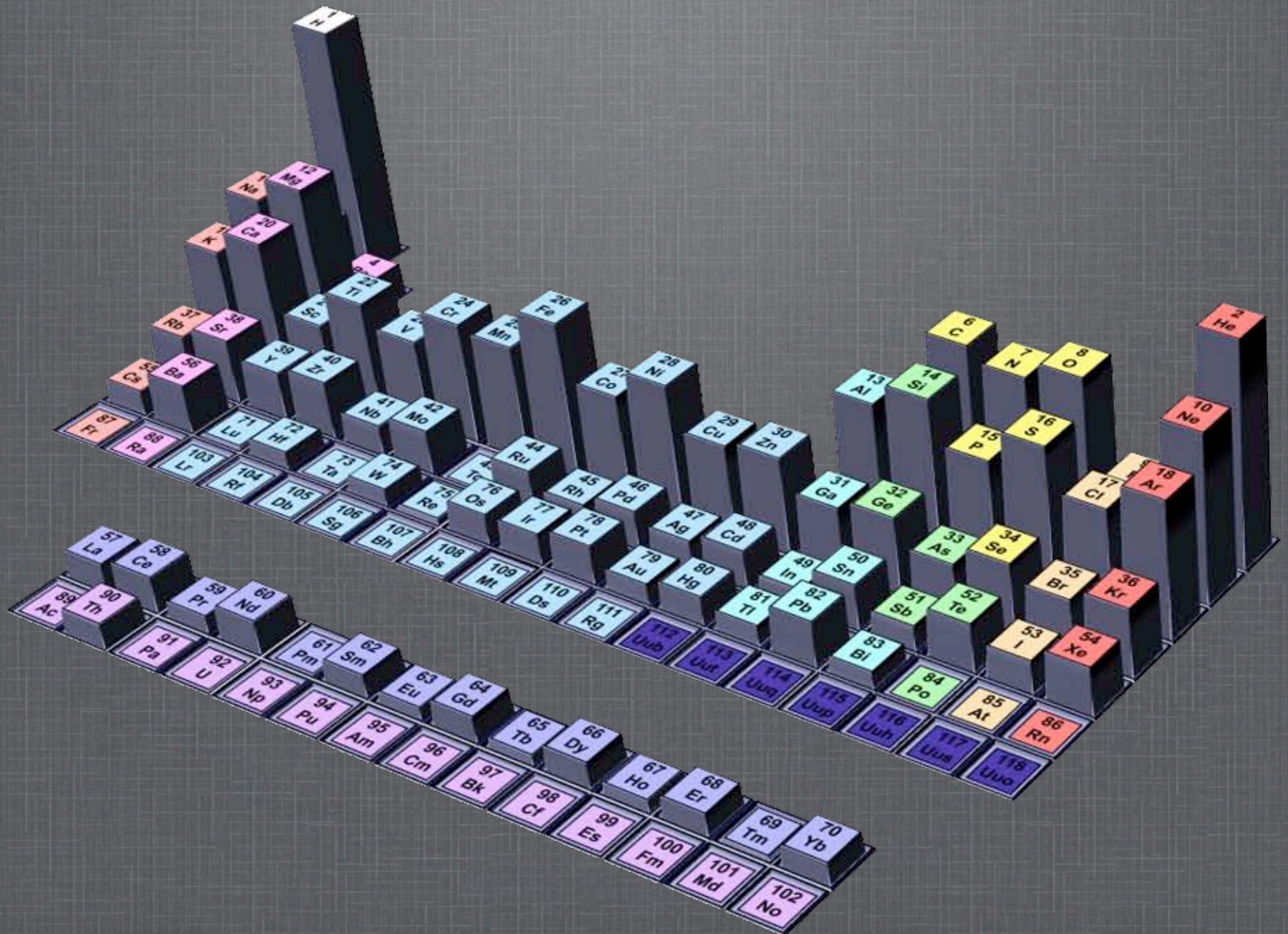
Thermonuclear Kinetics in Astrophysics

[cococubed.asu.edu/papers/hix\\_meyer.pdf](http://cococubed.asu.edu/papers/hix_meyer.pdf)

Integration of Nuclear Reaction Networks...

[www.iop.org/EJ/abstract/0067-0049/124/1/241/](http://www.iop.org/EJ/abstract/0067-0049/124/1/241/)

# Interlude



## Some nomenclature

An isotope can be characterized by the dimensionless integers

$Z$  = number of protons = atomic number

$N$  = number of neutrons

$A = Z + N$  = number of nucleons

The Avogadro number, from the 2006 CODATA values,

$$N_A = 6.02214179 \pm 0.00000030 \times 10^{23} \quad 1/\text{mole}$$

is the number of "entities" in one mole. When an individual entity has a mass  $m$  in grams, the atomic weight or molar mass is

$$W = mN_A \quad \text{g/mol.}$$

The mass of all entities is the number of moles times the molar mass.

The atomic mass unit (amu) is defined as 1/12 mass of an isolated  $^{12}\text{C}$  atom at rest and in its ground state. For  $^{12}\text{C}$ , we define the molar mass to be  $W=12.0$  g/mol. An amu then has  $W=1$  g/mol. Hence,

$$1\text{amu} = 1/N_A = 1.660538782 \pm 0.0000000083 \times 10^{-24} \quad \text{g.}$$

Thus, one can say  $N_A$  has units of grams but care must be taken to apply the implicit mol/g conversion to other quantities of interest.

In this system of units, the molar mass  $W$  is dimensionless. Mixing the [1/mol] and [1/g] systems of units will cause confusion.



The rest mass of a single isotope  $k$  is

$$\begin{aligned} m_k &= Nm_n + Zm_p + Z(1 - f)m_e - \Delta m \\ &= Nm_n + Zm_p + Z(1 - f)m_e - \frac{B}{c^2} \quad \text{g,} \end{aligned}$$

$m_n$  is the neutron mass

$m_p$  is the proton mass

$m_e$  is the electron mass

$f$  is the ionization fraction (0 for a neutral atom, 1 for full ionization),

$\Delta m$  is the mass deficit

$B$  is the nuclear binding energy in erg.

Sometimes terms like  $[15.7 Z^{5/3} - 13.6 Z \text{ eV}]$  are added to estimate the electronic binding energy. Such terms are usually negligible.

The molar mass of the isotope is  $W_k = m_k N_A$ .

For a mixture of isotopes, define

$$\rho = \frac{\sum n_i A_i}{N_A} \text{ g cm}^{-3}, \text{ baryon mass density}$$

where  $n_i$  is the number density of species  $i$ .

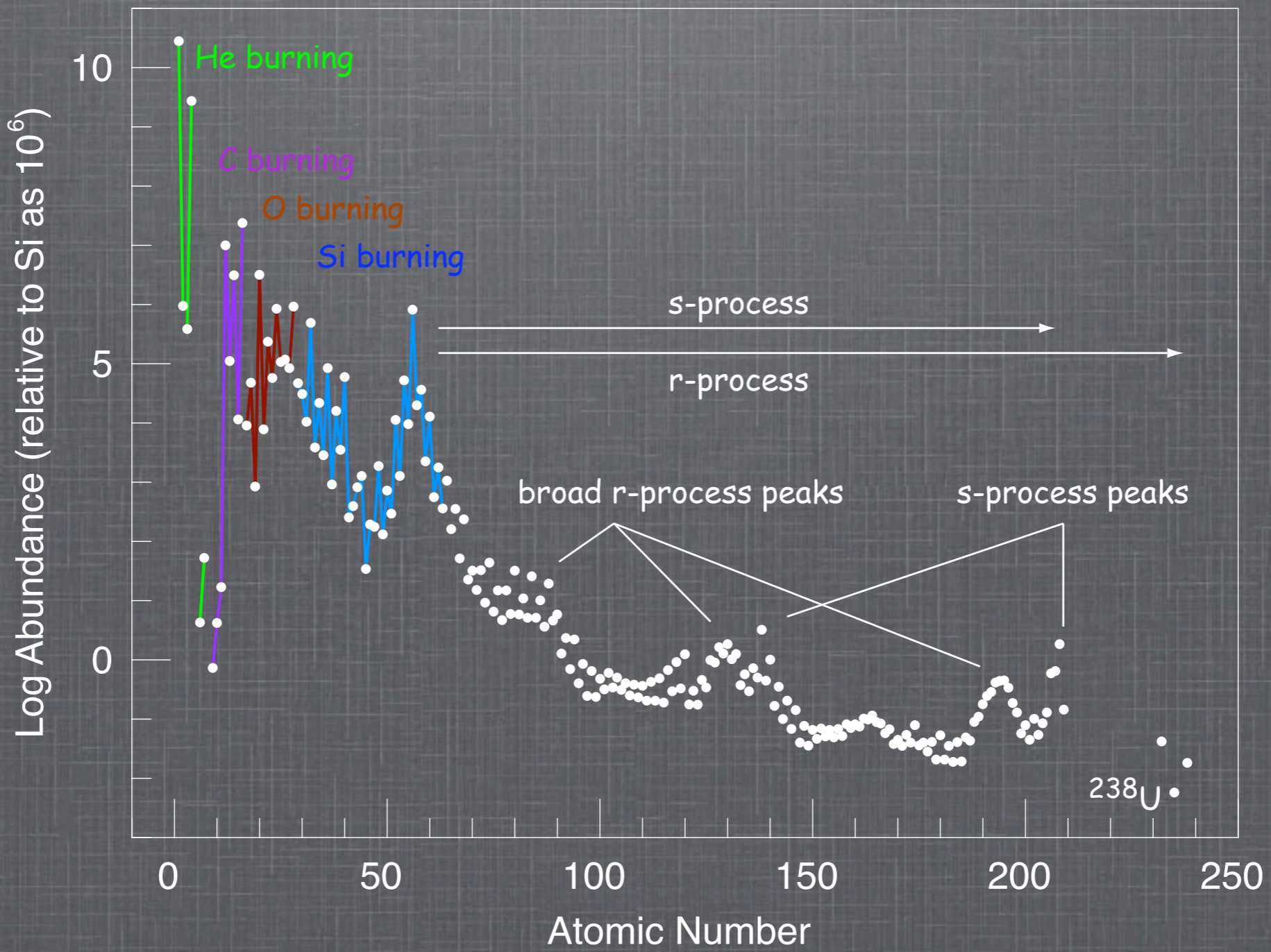
$$X_i = \frac{A_i n_i}{\rho N_A} = \frac{\rho_i}{\rho} \text{ mass fraction, dimensionless}$$

$$Y_i = \frac{X_i}{A_i} = \frac{n_i}{\rho N_A} \text{ molar fraction, dimensionless}$$

$$\sum_{i=1}^k X_i = 1 \text{ mass conservation}$$

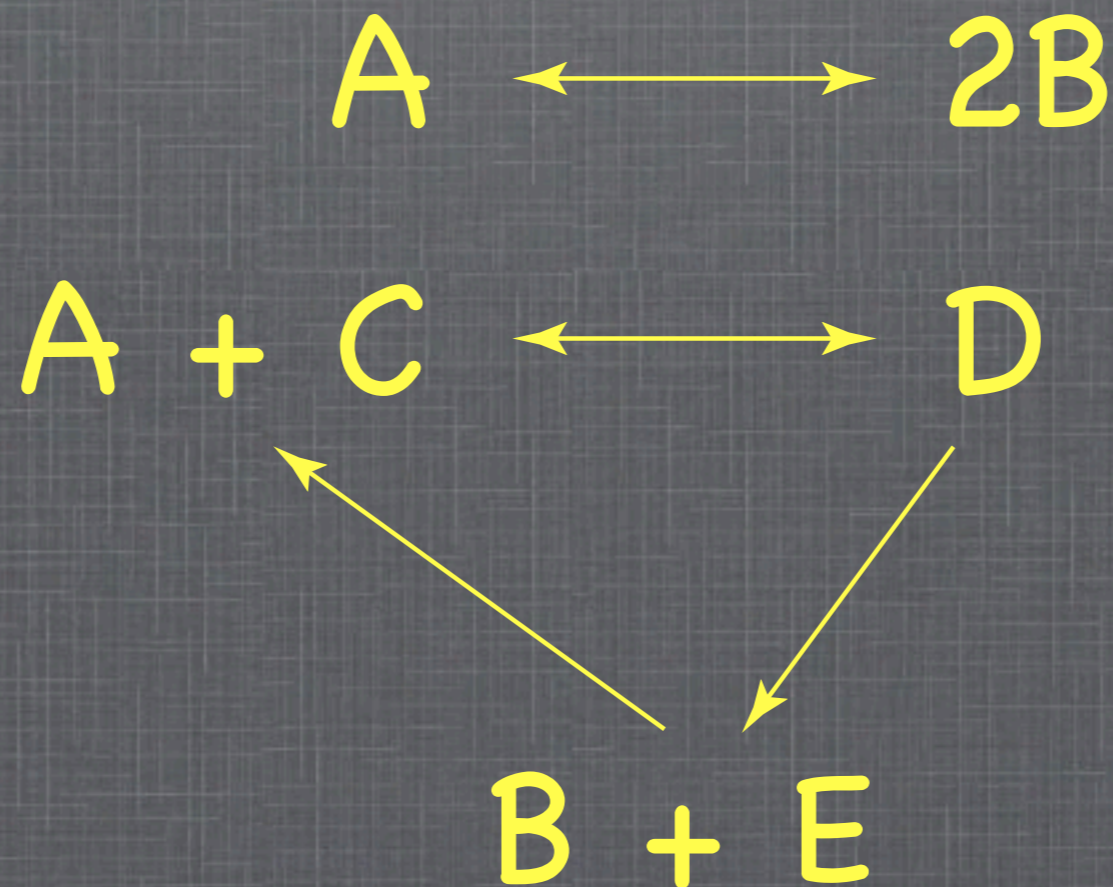
And finally (for today)

$$\bar{A} = \frac{\sum n_i A_i}{\sum n_i} = \frac{1}{\sum Y_i} \quad \bar{Z} = \frac{\sum n_i Z_i}{\sum n_i} = \bar{A} \sum Y_i Z_i$$



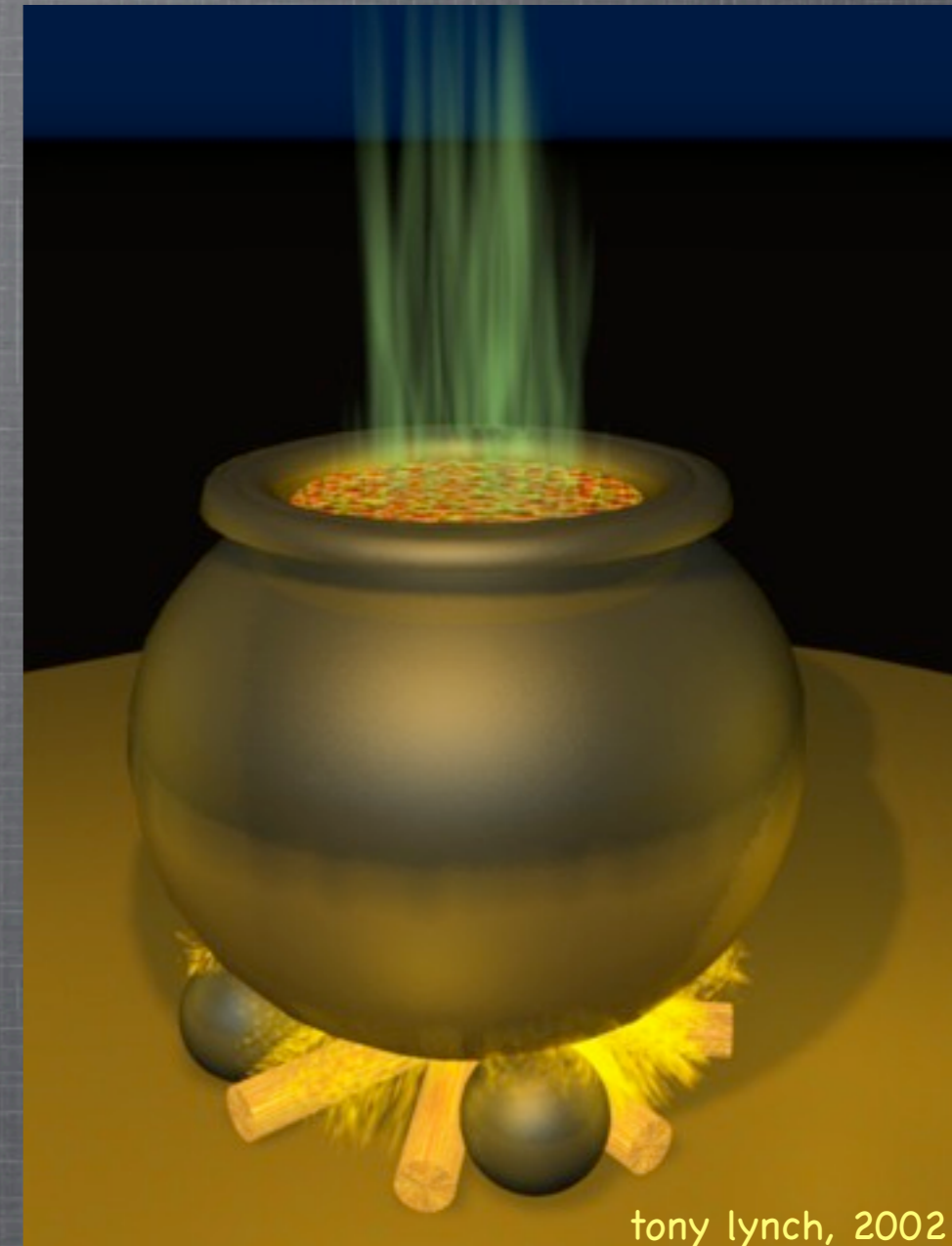
Let's start talking about a special, but rather large class of ordinary differential equations (ODEs) - those that derive from nuclear / chemical / biological reaction networks.

Let's walk through an example of a reaction network and indicate informally how it induces a system of ODEs.



Suppose we throw the various species in a pot that is constantly stirred so its contents remain spatially homogeneous for all time.

We'll also assume that the contents are kept at constant temperature and volume (constant density); hydrostatic burning.



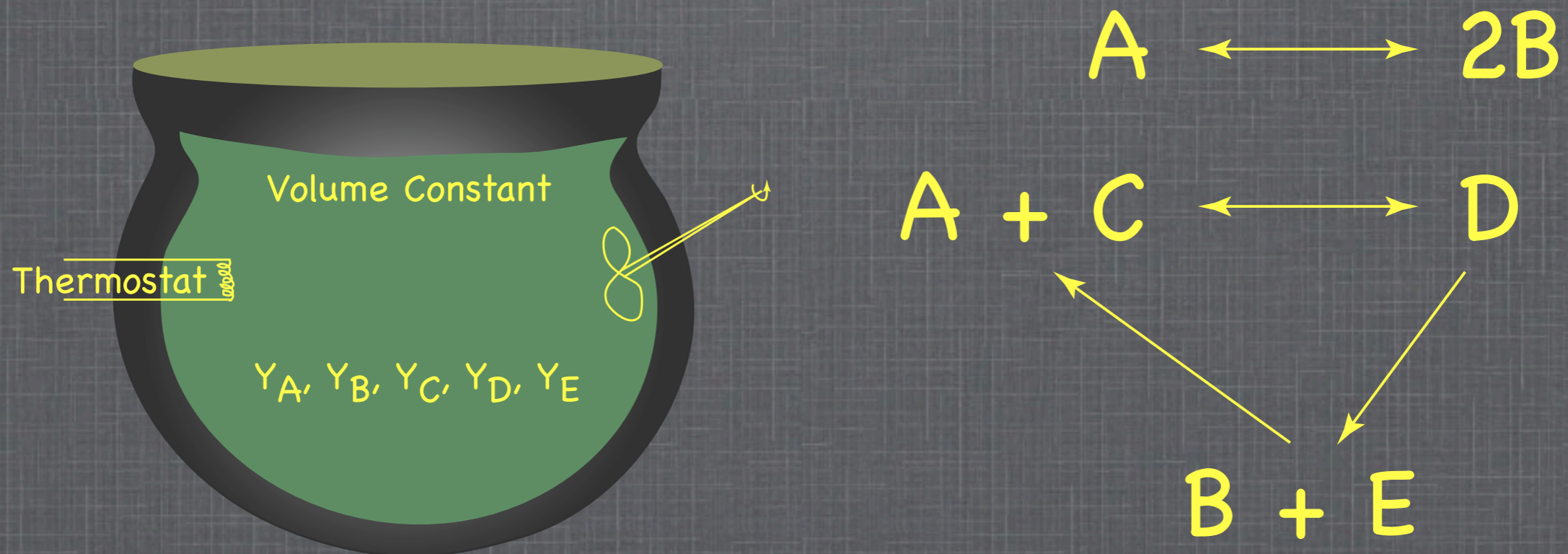
tony lynch, 2002

This is not to say the composition remains constant in time. Reactions will consume some species and generate others. In fact, it is the time evolution of the composition that we wish to investigate.

Denote the instantaneous values of the molar abundances by  $Y_A$ ,  $Y_B$ ,  $Y_C$ ,  $Y_D$ , and  $Y_E$ . We want to write down five ODEs that describe the evolution of the five mole fractions.

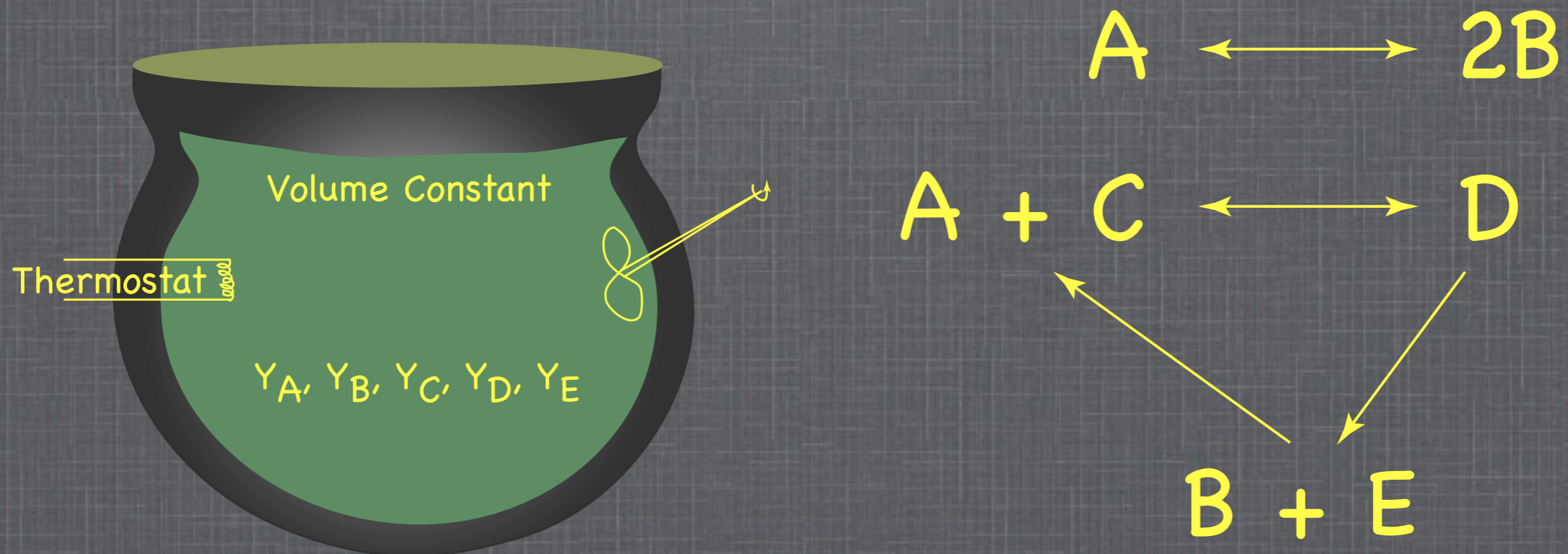
Let's begin by considering the instantaneous rate of change of  $Y_A$ .

Every time  $A \rightarrow 2B$  we lose one unit of A and this reaction occurs with an instantaneous, non-negative, real valued rate of  $K_{A \rightarrow 2B}$ .



Similarly the reaction  $A + C \rightarrow D$  loses a unit of species A, while  $2B \rightarrow A$ ,  $B + E \rightarrow A + C$ ,  $D \rightarrow A + C$  produces a unit of species A. So we write

$$\dot{Y}_A = -K_{A \rightarrow 2B} + K_{2B \rightarrow A} - K_{A+C \rightarrow D} + K_{D \rightarrow A+C} + K_{B+E \rightarrow A+C}$$



Continuing in this way, we can write down a system of ODEs that govern our reactor:

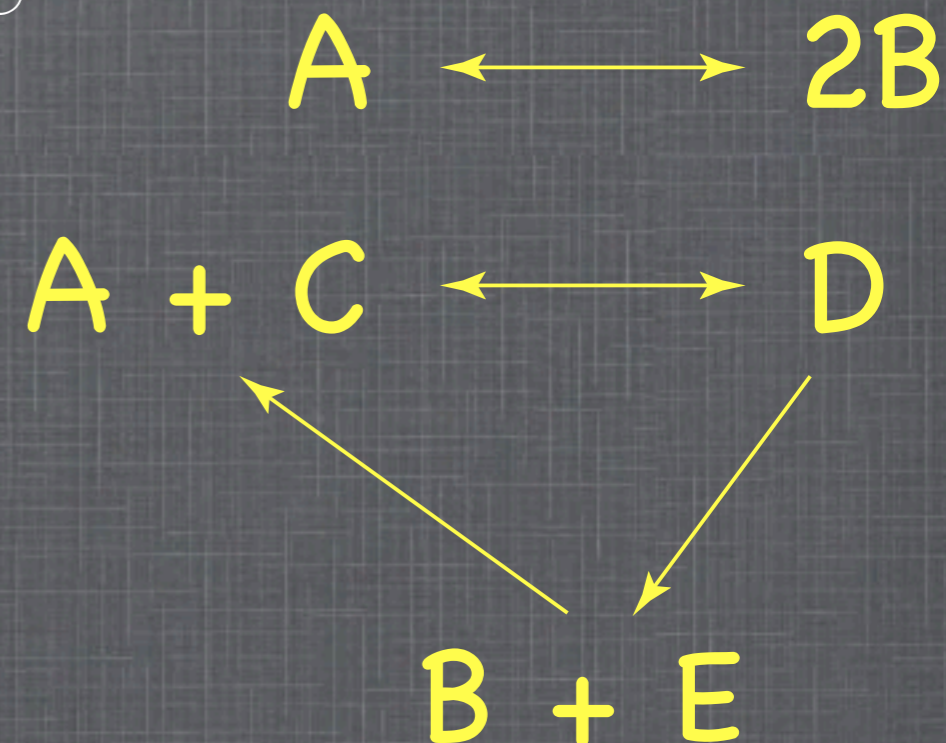
$$\dot{Y}_A = -K_{A \rightarrow 2B} + K_{2B \rightarrow A} - K_{A+C \rightarrow D} + K_{D \rightarrow A+C} + K_{B+E \rightarrow A+C}$$

$$\dot{Y}_B = 2K_{A \rightarrow 2B} - 2K_{2B \rightarrow A} + K_{D \rightarrow B+E} - K_{B+E \rightarrow A+C}$$

$$\dot{Y}_C = -K_{A+C \rightarrow D} + K_{D \rightarrow A+C} + K_{B+E \rightarrow A+C}$$

$$\dot{Y}_D = K_{A+C \rightarrow D} - K_{D \rightarrow A+C} - K_{D \rightarrow B+E}$$

$$\dot{Y}_E = K_{D \rightarrow B+E} - K_{B+E \rightarrow A+C}$$





We haven't said anything yet about the nature of the reaction rates.  
For  $A \rightarrow 2B$ , the more  $A$  there is, the more reaction there will be.  
We take the rate of  $A \rightarrow 2B$  to be proportional to  $Y_A$  :  $K_{A \rightarrow 2B} = \alpha Y_A$ .

For  $A + C \rightarrow D$ , a unit of species  $A$  must meet a unit of species  $C$ .  
We take the probability of such an encounter to be proportional to  
the product  $Y_A Y_C$  :  $K_{A+C \rightarrow D} = \gamma Y_A Y_C$ .



Svante August Arrhenius  
Nobel Prize 1903



With mass action kinetics, our rate functions take the form

$$\mathcal{K}_{A \rightarrow 2B} = \alpha Y_A$$

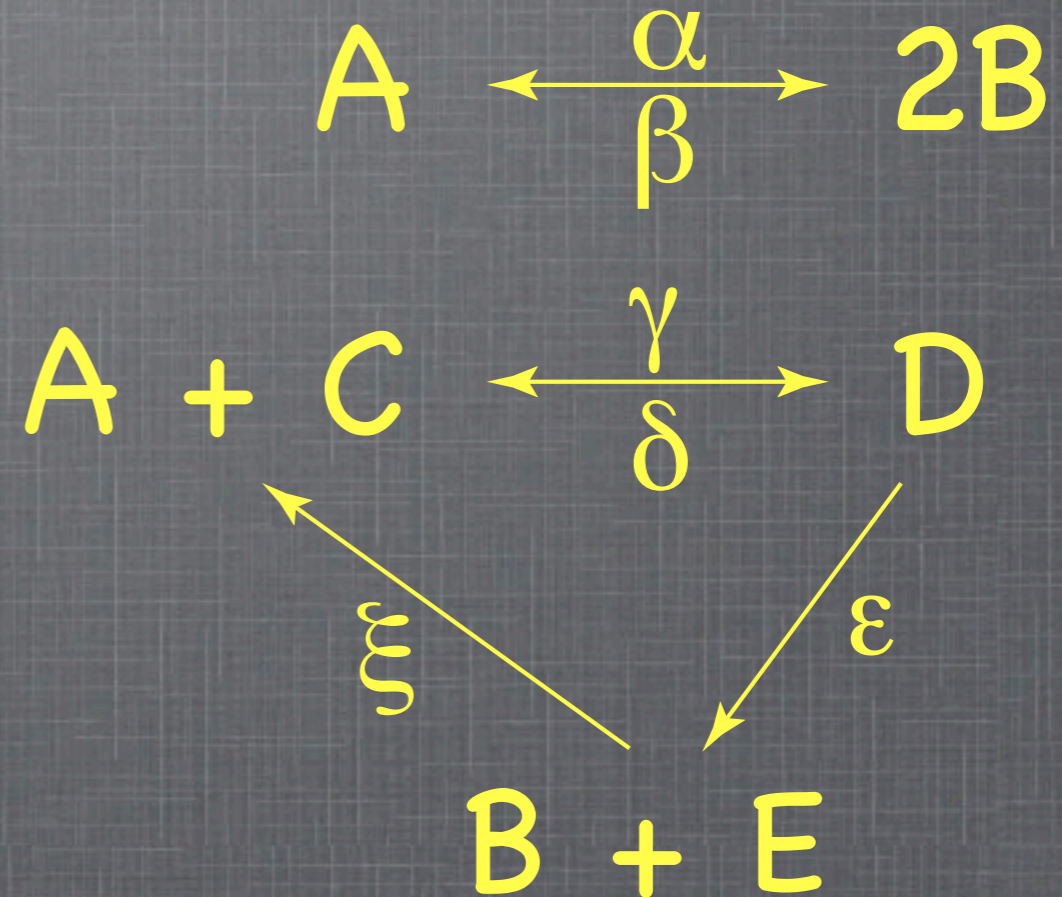
$$\mathcal{K}_{2B \rightarrow A} = \beta Y_B^2$$

$$\mathcal{K}_{A+C \rightarrow D} = \gamma Y_A Y_C$$

$$\mathcal{K}_{D \rightarrow B+E} = \varepsilon Y_D$$

$$\mathcal{K}_{D \rightarrow A+C} = \delta Y_D$$

$$\mathcal{K}_{B+E \rightarrow A+C} = \xi Y_B Y_E$$



The rate “constants”  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ ,  $\varepsilon$ , and  $\xi$  may depend on temperature and density.

And our reaction network takes the form

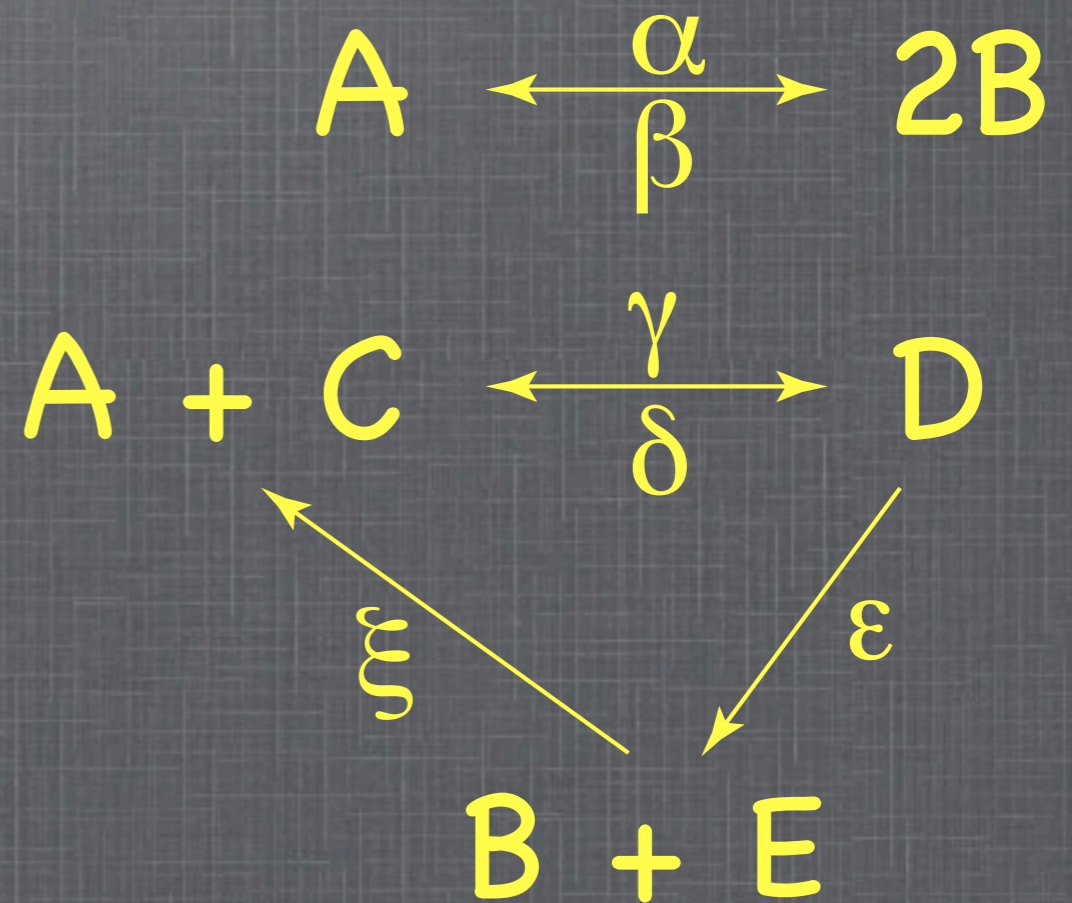
$$\dot{Y}_A = -\alpha Y_A + \beta Y_B^2 - \gamma Y_A Y_C + \delta Y_D + \xi Y_B Y_E$$

$$\dot{Y}_B = 2\alpha Y_A - 2\beta Y_B^2 + \epsilon Y_D - \xi Y_B Y_E$$

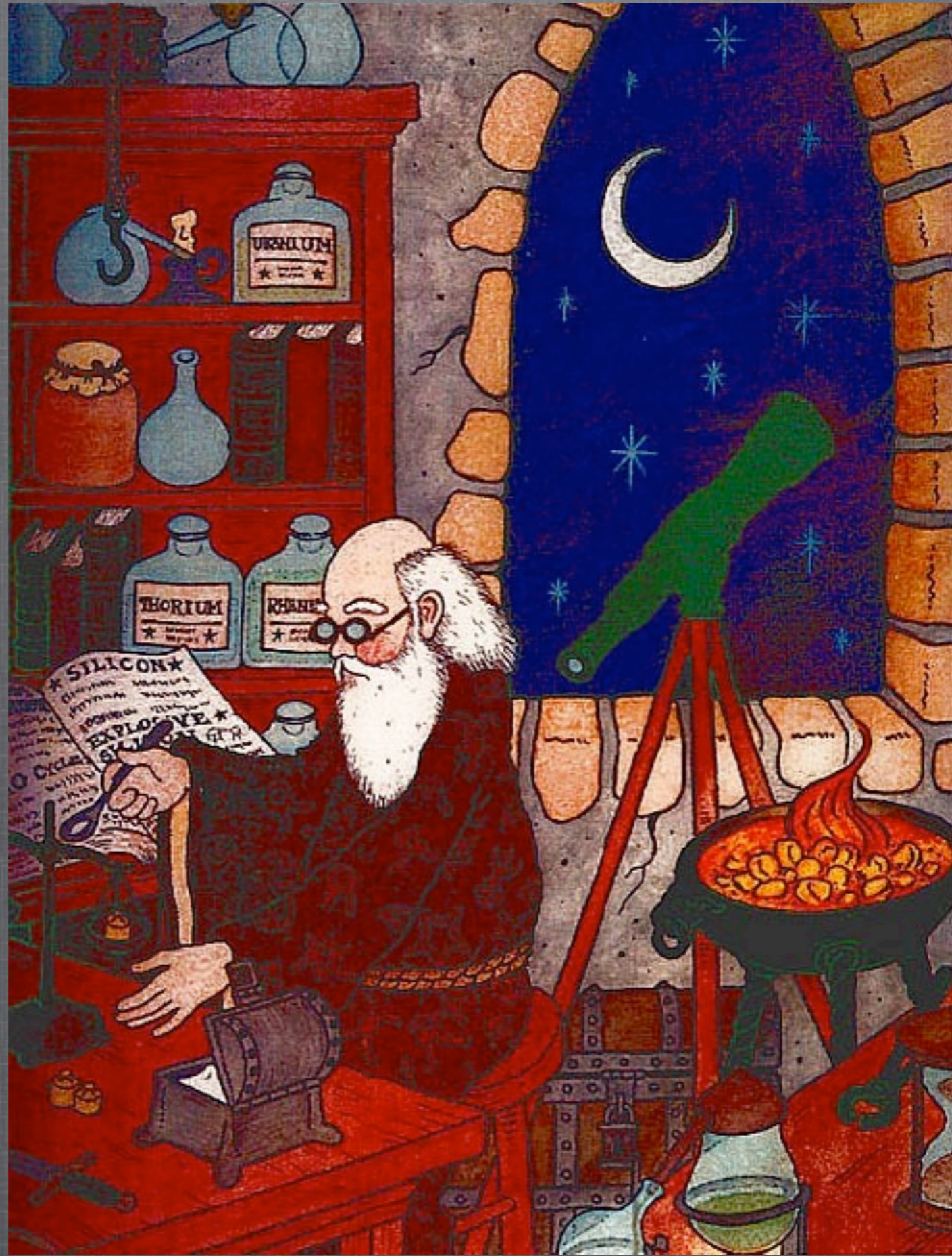
$$\dot{Y}_C = -\gamma Y_A Y_C + \delta Y_D + \xi Y_B Y_E$$

$$\dot{Y}_D = \gamma Y_A Y_C - \delta Y_D - \epsilon Y_D$$

$$\dot{Y}_E = \epsilon Y_D - \xi Y_B Y_E$$



# Interlude



Gerry Wasserburg  
1993

There are different of types of nuclear reactions: emission or absorption of nuclei and nucleons, photons ( $\gamma$ -rays) and leptons (electrons, neutrinos, and their anti-particles).

Nuclear reactions involve three of the four fundamental forces, the nuclear strong, electromagnetic and nuclear weak forces.

Weak interactions (those involving leptons) generally proceed more slowly than those involving nucleons and photons, but these are the only reactions that can change the global proton to neutron ratio.

**1973: Neutral Currents**  
**1983:  $W^\pm$  &  $Z^0$  Bosons**

The anniversary of CERN's discoveries  
and a look into the future

**Tuesday, September 16, 2003 at 9 am**  
Main Auditorium

- **Welcome:** Luciano Maiani
- **The making of the Standard Model:** Steven Weinberg
- **CERN's contribution to accelerators and beams:** Giorgio Brianti
- **The discovery of neutral currents:** Dieter Haidt
- **The discovery of the W & Z, a personal recollection:** Pierre Damiat
- **W & Z Physics at LEP:** Peter Zerwas
- **Physics at the LHC:** John Ellis
- **Challenges of the LHC:**
  - The accelerator challenge of the LHC: Lyn Evans
  - The detector challenge of the LHC: Jos Engelen
  - The computing challenge of the LHC: Paul Messina
- **Particle detectors and society:** Georges Charpak
- **Closing of the Symposium: The future for CERN:** Luciano Maiani

**Panel discussion: Future of Particle Physics**  
With the participation of: Robert Aymar, Georges Charpak, Pierre Damiat, Luciano Maiani, Simon van der Meer, Lev B. Okun, Donald Perkins, Carlo Rubbia, Martinus Veltman, Steven Weinberg

Organizers: Roger Cashmore and Jean-Pierre Reyrol

[www.cern.ch/verndiscoveries](http://www.cern.ch/verndiscoveries)

A key quantity is the cross section  $\sigma$  for a nuclear reaction.

The cross section  $\sigma_{ij}$  for the reaction  $i(j,k)l$  is the number of reactions per target nucleus  $i$  per second divided by the flux of nuclei of type  $j$  (number / cm<sup>2</sup> / s).

$$\sigma(v) = \frac{\text{number of reactions per sec}}{\text{flux of incoming projectiles}} = \frac{r_{ij}/n_i}{n_j v_{ij}}$$

Cross sections are usually reported in “barns”, 10<sup>-24</sup> cm<sup>2</sup>.



The reaction rate per unit volume  $r_{ij}$ , in the simplest case, is then

$$r_{ij} = [\text{flux of } j]n_i\sigma_{ij}(v) = v_{ij}n_jn_i\sigma_{ij}(v) \quad \text{cm}^{-3}\text{s}^{-1}$$

More generally, the targets and projectiles have distributions of velocities, in which case  $r_{ij}$  is given by

$$r_{i,j} = \int \sigma(|\vec{v}_i - \vec{v}_j|)|\vec{v}_i - \vec{v}_j|d^3n_id^3n_j \quad \text{cm}^{-3}\text{s}^{-1}$$

Evaluation of the integrals depends on the particle statistics. For nuclei  $i$  and  $j$  that obey Maxwell–Boltzmann statistics

$$d^3n = n \left( \frac{m}{2\pi k_B T} \right)^{3/2} \exp \left( -\frac{mv^2}{2k_B T} \right) d^3v$$

allowing  $n_i$  and  $n_j$  to be moved outside of the integral.

Then

$$r_{ij} \langle \sigma v \rangle_{ij} n_i n_j = (N_A \rho)^2 \langle \sigma v \rangle_{ij} Y_i Y_j \quad \text{cm}^{-3} \text{s}^{-1}$$

where  $\langle \sigma v \rangle_{ij}$  is the velocity integrated cross section.

The rate of change in the number density of species  $i$  with time is

$$\dot{n}_i = \sum_{j,k} r_{jk} \quad \text{cm}^{-3} \text{s}^{-1}$$

or

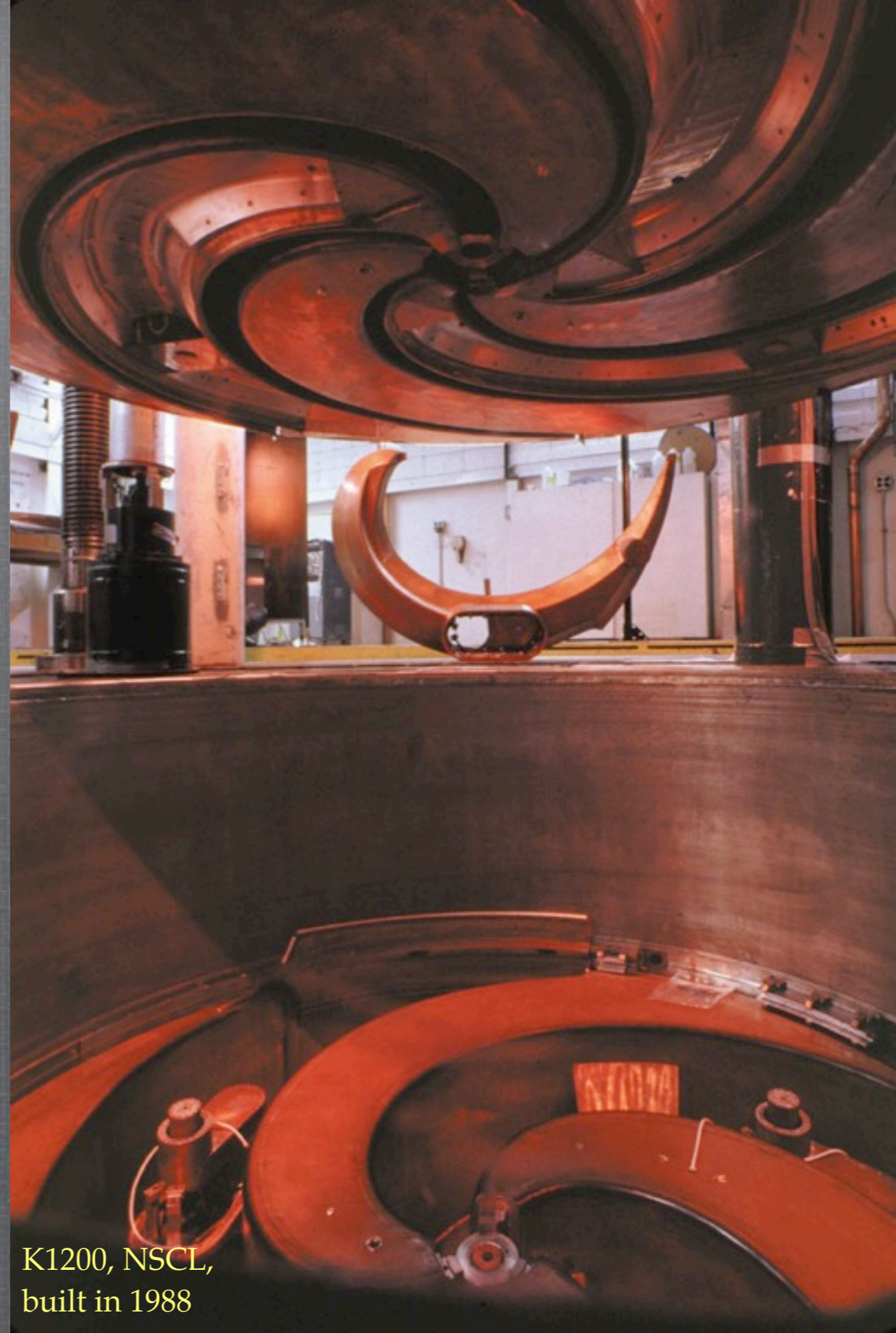
$$\dot{Y}_i = \sum_{j,k} N_A \rho \langle \sigma v \rangle_{ij} Y_j Y_k = \sum_{j,k} \lambda_{ij} \rho Y_j Y_k = \sum_{j,k} R_{ij} Y_j Y_k \quad \text{s}^{-1}$$

where  $\lambda_{ij}$  is what common reaction rate compilations list, and  $R_{ij}$  is “the reaction rate” used in our codes.



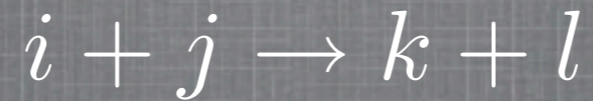
Michael Wiescher will say more about measuring cross sections and Q-values later in this school.

For now, we've established what we need to continue forming a nuclear reaction network.



K1200, NSCL,  
built in 1988

Consider a unidirectional binary reaction with unity coefficients.



$$\dot{Y}_i = -Y_i Y_j R_{ij}$$

$$\dot{Y}_j = -Y_i Y_j R_{ij}$$

$$\dot{Y}_k = Y_i Y_j R_{ij}$$

$$\dot{Y}_l = Y_i Y_j R_{ij}$$

Where the reaction rate  $R_{ij}$  absorbs the density, Avogadro number, and  $\langle \sigma v \rangle_{ij}$  terms.

Now consider the case when the coefficients are not unity.



$$\dot{Y}_i = -\frac{c_i}{c_i! c_j!} Y_i^{c_i} Y_j^{c_j} R_{ij}$$

$$\dot{Y}_j = -\frac{c_j}{c_i! c_j!} Y_i^{c_i} Y_j^{c_j} R_{ij}$$

$$\dot{Y}_k = \frac{c_k}{c_i! c_j!} Y_i^{c_i} Y_j^{c_j} R_{ij}$$

$$\dot{Y}_l = \frac{c_l}{c_i! c_j!} Y_i^{c_i} Y_j^{c_j} R_{ij}$$

If there are identical reactants,  $i=j$ , set  $c_i = 2c_i$  and  $c_j = 0$ .

For a general bidirectional binary reaction



$$\dot{Y}_p = \sum_{r,s} \frac{c_p}{c_r! c_s!} Y_r^{c_r} Y_s^{c_s} R_{rs} - \sum_q \frac{c_p}{c_p! c_q!} Y_p^{c_p} Y_q^{c_q} R_{pq}$$

If there are identical reactants,  $i=j$ , set  $c_i = 2c_i$  and  $c_q = c_s = 0$ .

Reactions can be divided into three categories based on the number of reactants which are nuclei.

Reactions involving a single nucleus - decays, electron and positron captures, photodisintegrations, and neutrino induced reactions - depend on the number density of only the target species.

$$\dot{Y}_i = \sum_j C_i R_j Y_j$$

For a binary reaction,

$$\dot{Y}_i = \sum_{jk} \frac{C_i}{C_j! C_k!} R_{jk} Y_j Y_k$$

The  $C_i$ 's can be positive or negative numbers that specify how many particles of species  $i$  are created or destroyed.

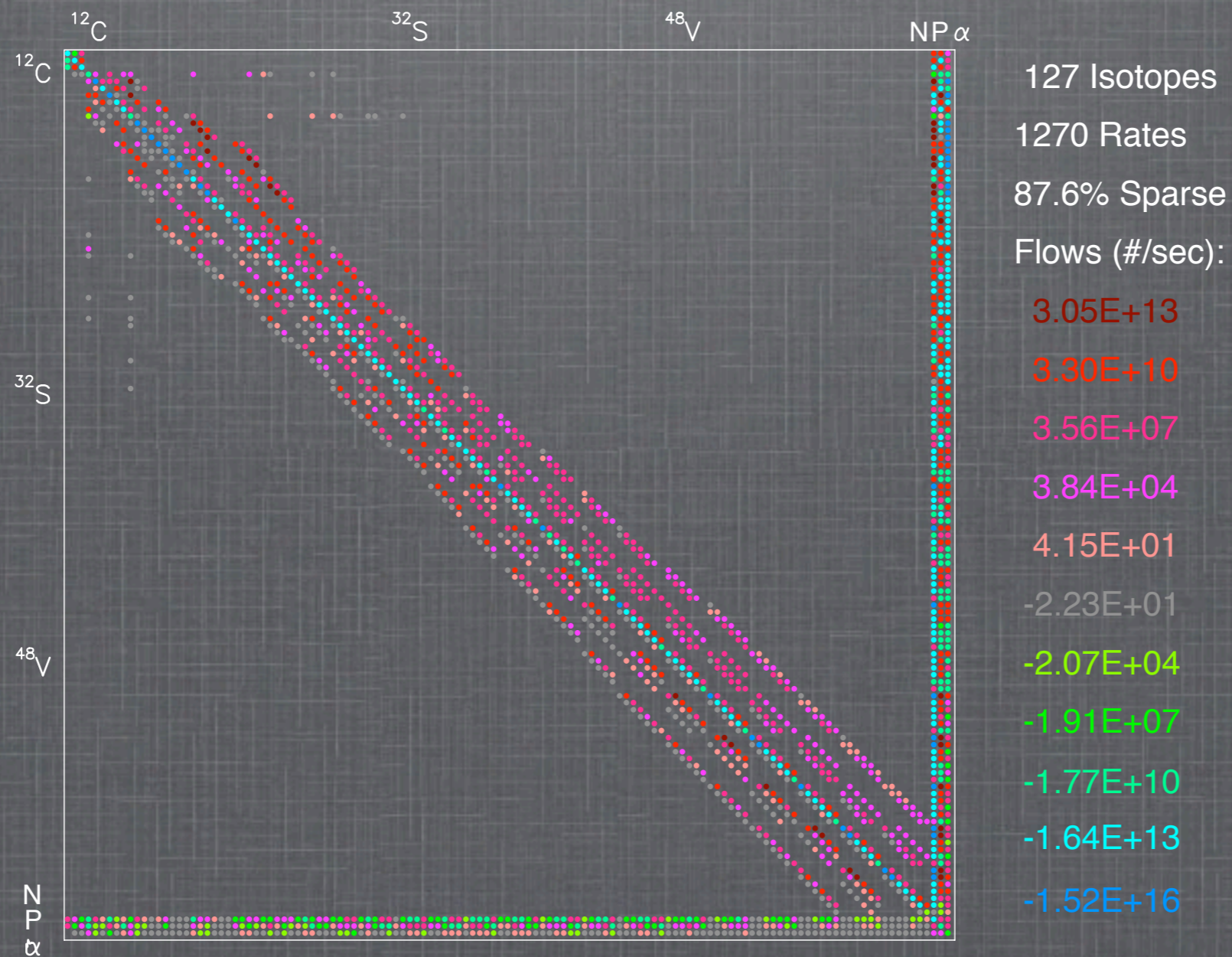
There are also a few important three-particle processes (like the triple- $\alpha$  process) which are commonly successive captures with an intermediate unstable target.

Using an equilibrium abundance for the unstable intermediate, the contributions of these reactions are commonly written in the form of a three-particle processes, depending on a trio of number densities.

$$\dot{Y}_i = \sum_{jk} \frac{C_i}{C_j!C_k!C_l!} R_{jkl} Y_j Y_k Y_l$$

A reaction network may be described by the following set of ODEs

$$\dot{Y}_i = \sum_j C_i R_j Y_j + \sum_{jk} \frac{C_i}{C_j! C_k!} R_{jk} Y_j Y_k + \sum_{jkl} \frac{C_i}{C_j! C_k! C_l!} R_{jkl} Y_j Y_k Y_l$$



# Nuclear Science

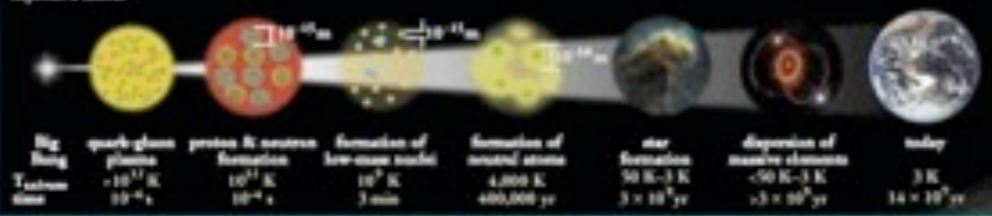
**Nuclear Science** is the study of the structure, properties, and interactions of the atomic nuclei. Nuclear scientists calculate and measure the masses, shapes, sizes, and decays of nuclei at rest and in collisions. They ask questions, such as "Why do nucleons stay in the nucleus?" "What combinations of protons and neutrons are possible?" "What happens when nuclei are compressed or rapidly rotated?" "What is the origin of the nuclei found on Earth?"

**Legend**

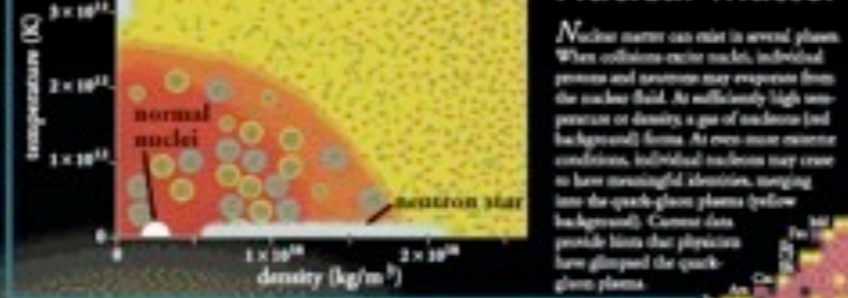
- alpha ( $\alpha$ )
- quark
- gluon field
- proton ( $p$ )
- positron ( $e^+$ )
- gluon
- neutron ( $n$ )
- antineutrino ( $\bar{\nu}$ )
- photon ( $\gamma$ )
- Neutron:  $A - Z$
- Proton:  $Z$
- Atomic Number:  $Z$
- Mass Number:  $A$

## Expansion of the Universe

After the Big Bang, the universe expanded and cooled. At about  $10^{-35}$  second, the universe consisted of a soup of quarks, gluons, electrons, and neutrinos. When the temperature of the Universe,  $T_{univ}$ , cooled to about  $10^9$  K, this soup condensed into protons, neutrons, and electrons. As time progressed, some of the protons and neutrons formed deuterium, helium, and lithium nuclei. Still later, electrons combined with protons and these low-mass nuclei to form neutral atoms. Due to gravity, clouds of atoms contracted into stars, where hydrogen and helium fused into more massive chemical elements. Expanding from supernovae from the most massive elements and dispersing them into space. Our earth was formed from supernova debris.

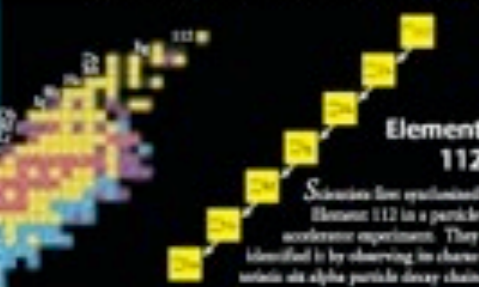


## Phases of Nuclear Matter

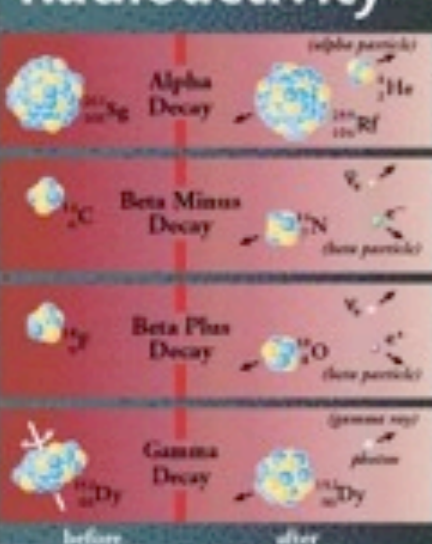


## Unstable Nuclei

Stable nuclei form a narrow white band on the Chart of the Nuclides. Scientists produce unstable nuclei far from this band and study their decays, thereby learning about the extremes of nuclear conditions. In its present form, this chart contains about 2500 different nuclides. Nuclear theory predicts that there are at least 4000 more to be discovered with  $Z \leq 112$ .

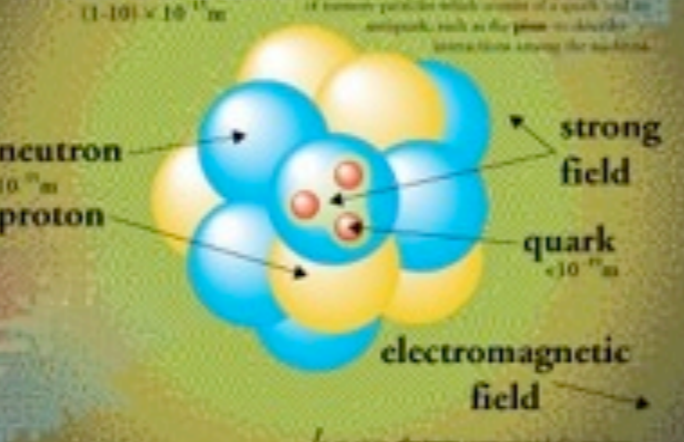


## Radioactivity

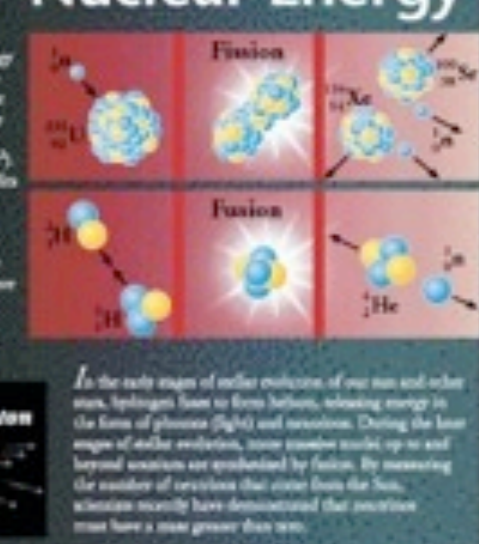


Radioactive decay transforms a nucleus by emitting different particles. In alpha decay, the nucleus releases a  $^4\text{He}$  nucleus—an alpha particle. In beta decay, the nucleus either emits an electron and antineutrino (in beta minus) or captures an electron and emits a positron and neutrino (in beta plus). Antineutrino is composed of anti-particles. Both alpha and beta decays change the original nucleus into a nucleus of a different chemical element. In gamma decay, the nucleus lowers its internal energy by emitting a photon—a gamma ray. This decay does not modify the chemical properties of the atom.

## The Nucleus

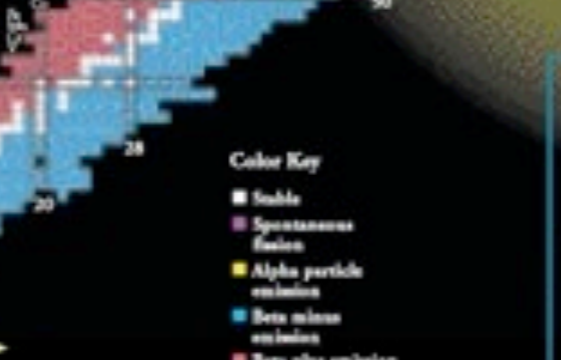


## Nuclear Energy



## Chart of the Nuclides

The Chart of the Nuclides presents in graphic form all known nuclei with atomic number, Z, and neutron number, N. Each nuclide is represented by a box colored according to its predominant decay mode. Magic numbers (2, 8, 20, 28, 50, 82 and 126) are indicated by a rectangle on the chart. They correspond to major shell gaps and show regions of greater nuclear binding energy.



## Applications

- Radioactive Dating:** Naturally occurring radioactive isotopes such as  $^{14}\text{C}$  are used to date objects that were once living, such as wood. For example, from a study of willows found at the site, scientists determined that the bridge was built nearly 4,000 years ago.
- Smoke Detectors:** Many smoke detectors use a small amount of the alpha emitter  $^{241}\text{Am}$  to ionize the air. Smoke entering the detector reduces the current and sets off the alarm.
- Nuclear Medicine:** Radioactive isotopes, such as  $^{99\text{m}}\text{Tc}$ ,  $^{131}\text{I}$ , and  $^{18}\text{F}$ , are commonly used in the diagnosis and treatment of disease. Positron-emitting isotopes such as  $^{18}\text{F}$  are used in Positron Emission Tomography (PET) to generate images of brain activity.
- Space Exploration:** Spacecraft use alpha particles to identify chemical elements present in interplanetary dust. On Earth, nuclear reactors are used in many areas from medical investigations to an automobile.
- Nuclear Reactors:** Nuclear reactors use the fission of  $^{235}\text{U}$  or  $^{239}\text{Pu}$  nuclei to produce electric power. Reactors and other nuclear applications generate radioactive waste, disposal of the waste is subject of intense research.
- Magnetic Resonance Imaging:** Magnetic Resonance Imaging (MRI) makes use of nuclei transitions involving the magnetic field of a nucleus to study the local chemical environment. This technique accurately maps the density of hydrogen in tissues to produce three-dimensional images of the human body.



# How does the Sun shine?

Wood - Ancient Greeks  
lasts 2000 years

Coal - Middle Ages  
lasts 4000 years

Gravitational - 1800's  
lasts 4 million years

Nuclear reactions - 1940's  
lasts 10 billion years

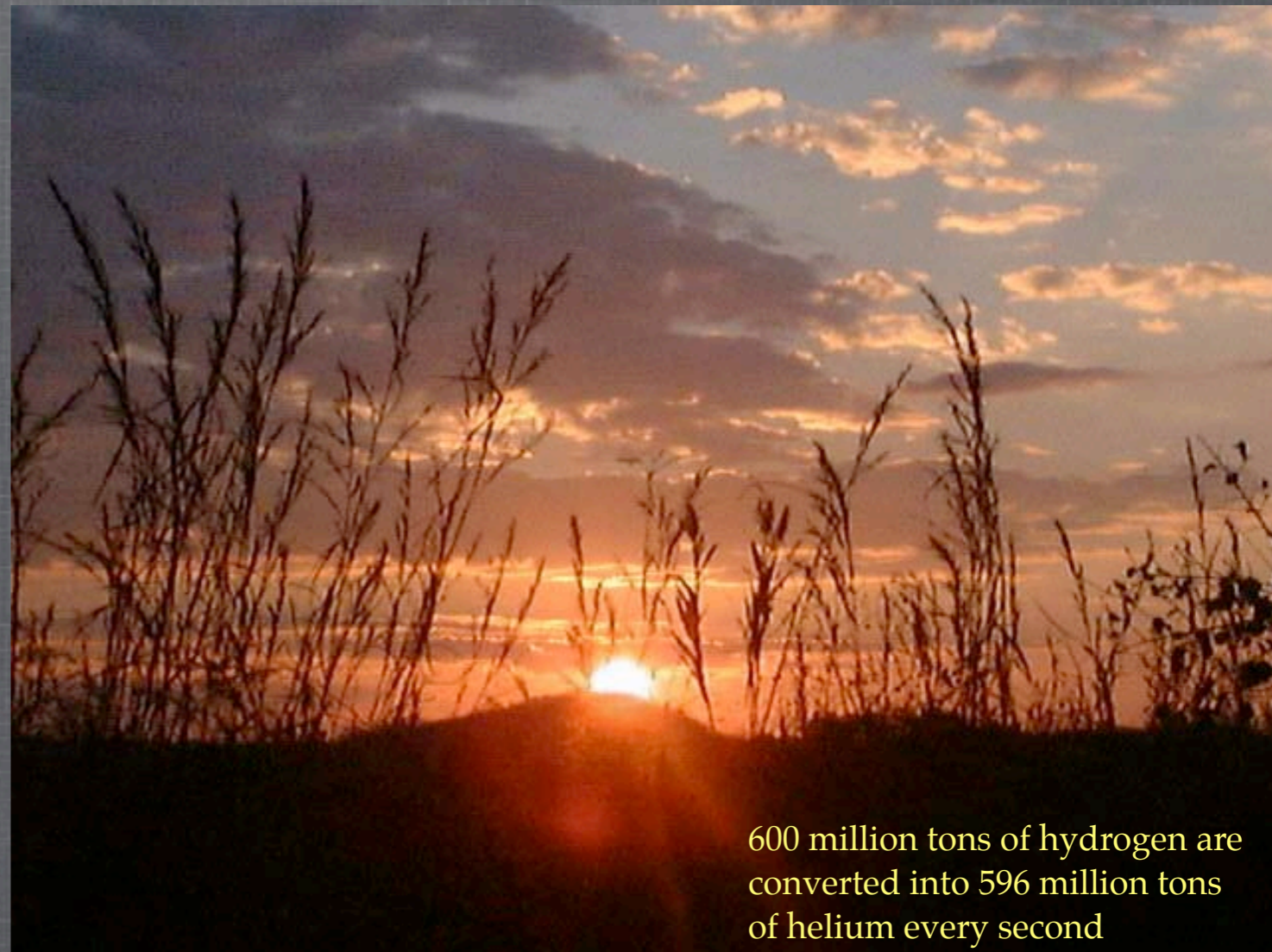


Four hydrogen nuclei get transformed into one helium nucleus.  
The limiting step is a rare reaction; hence a long lived Sun.

But the mass of 4 hydrogen nuclei is larger than the mass of  
1 helium nucleus. Where did the missing mass go?

$$E = mc^2$$

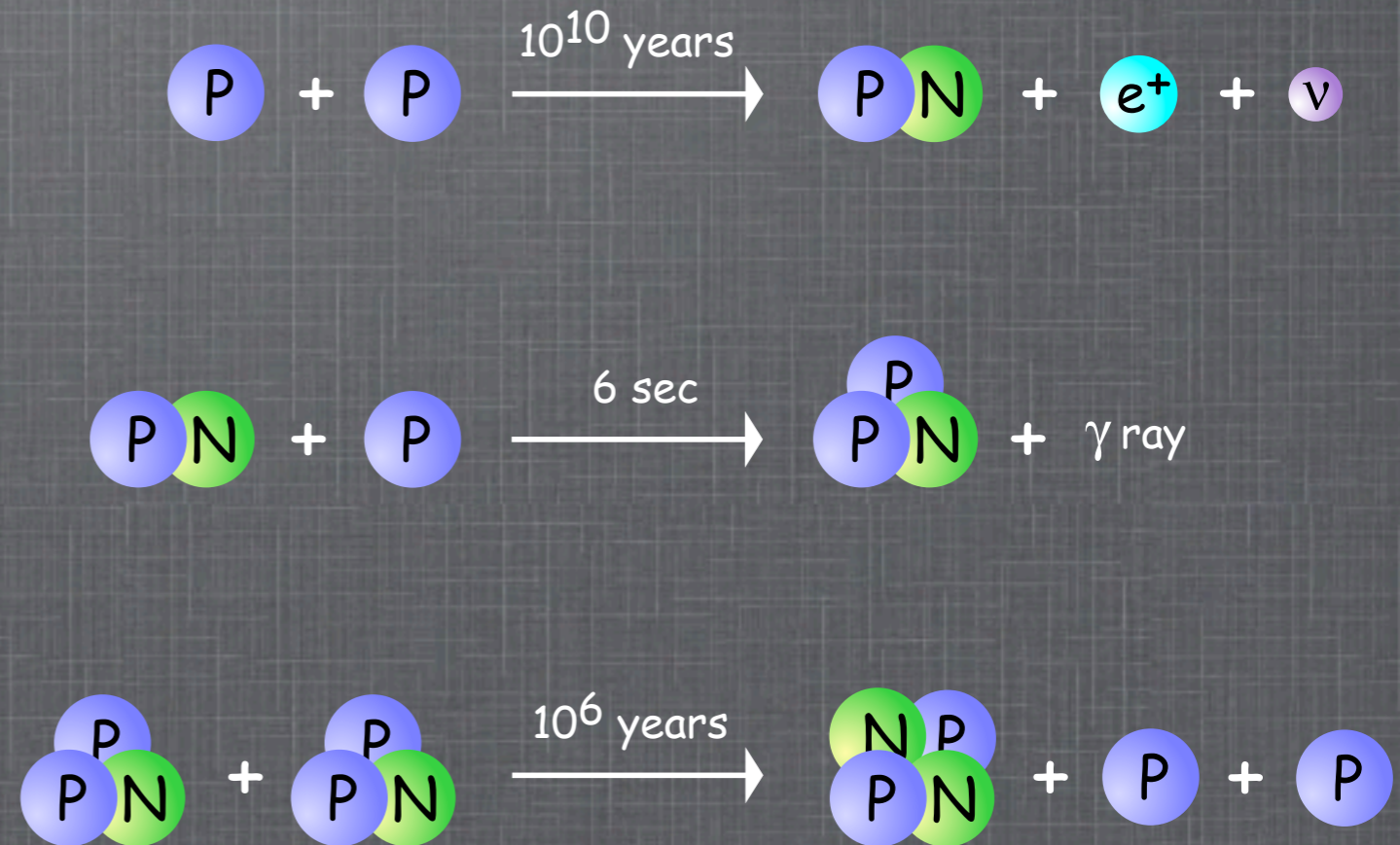
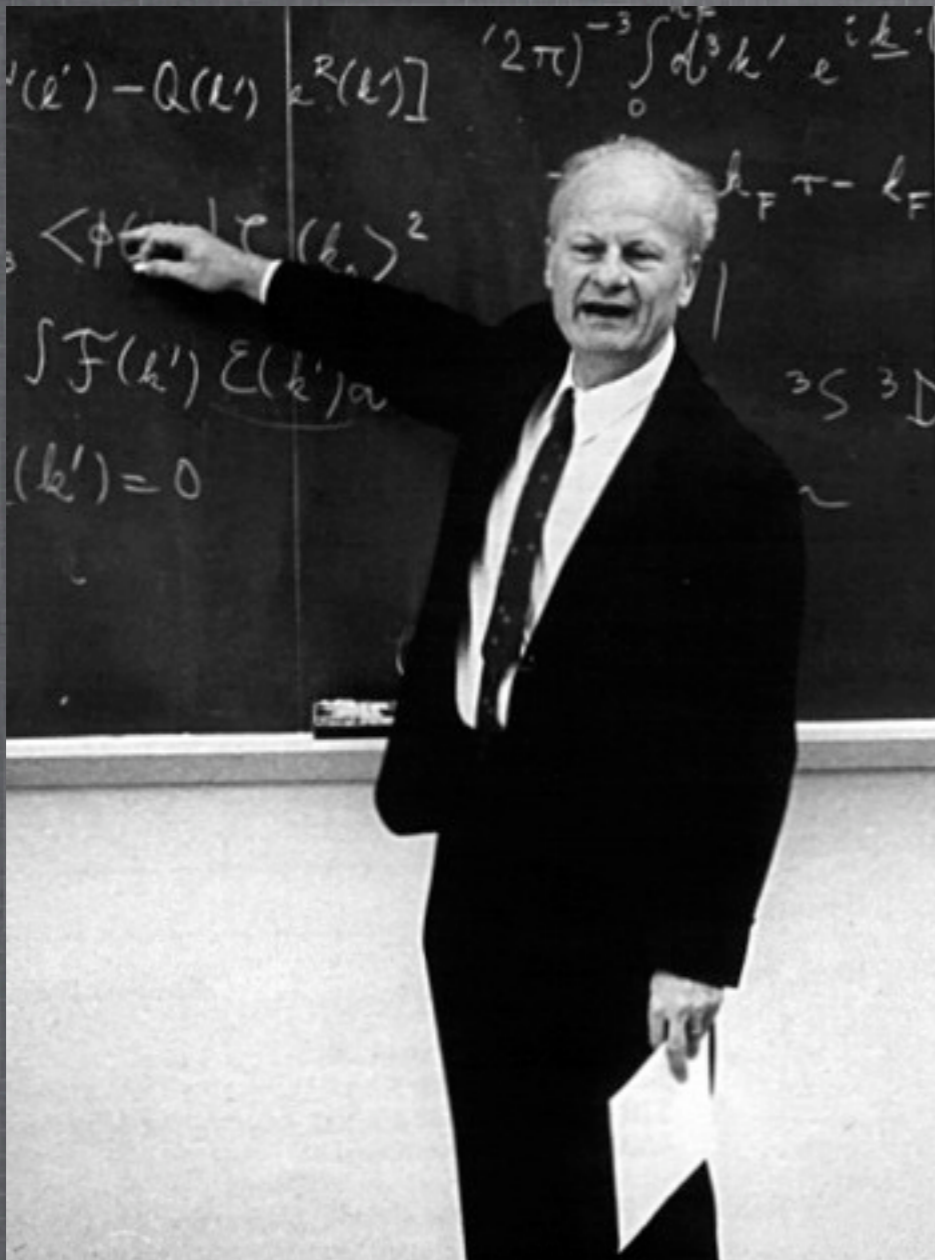
The sun presently shines by  
burning hydrogen (fuel) into  
helium (ash) in its core.



600 million tons of hydrogen are  
converted into 596 million tons  
of helium every second

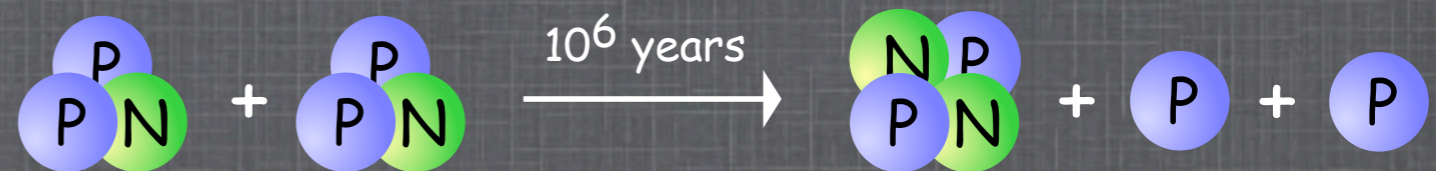
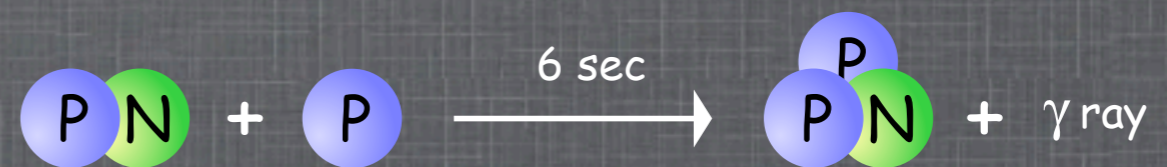
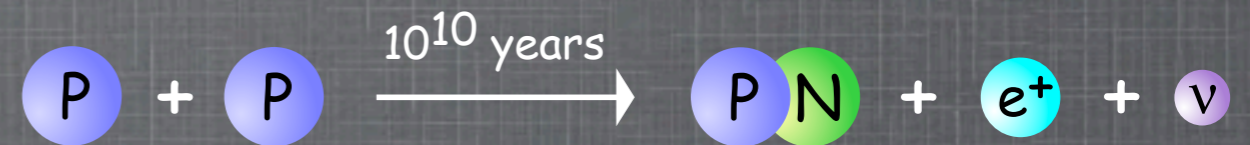
# proton-proton chains

Hans Bethe realized in 1939 that a weak interaction could convert a proton into neutron during the brief encounter of a scattering event.

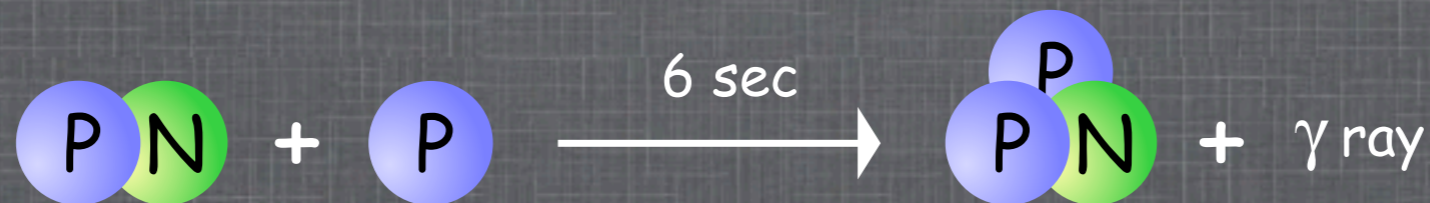


Since the neutron is more massive than a proton, such a decay would require energy (endothermic) except that the neutron can appear in a bound state with the proton in the form of deuterium.

The binding energy is sufficient (2.2245 MeV) to make the reaction exothermic.



We have four species to track ( $^1\text{H}$ ,  $^2\text{H}$ ,  $^3\text{He}$ ,  $^4\text{He}$ ), and three binary reactions that couple these species;  $p(p, e^+ \nu)^2\text{H}$ ,  $^2\text{H}(p, \gamma)^3\text{He}$ , and  $^3\text{He}(^3\text{He}, 2p)^4\text{He}$ .



$$\dot{Y}_p = -Y_p Y_p R_{p,p} - Y_p Y_d R_{p,d} + Y_{3\text{he}} Y_{3\text{he}} R_{3\text{he},3\text{he}}$$

$$\dot{Y}_d = 0.5 Y_p Y_p R_{p,p} - Y_p Y_d R_{p,d}$$

$$\dot{Y}_{3\text{he}} = Y_p Y_d R_{p,d} - Y_{\text{he}3} Y_{\text{he}3} R_{\text{he}3,\text{he}3}$$

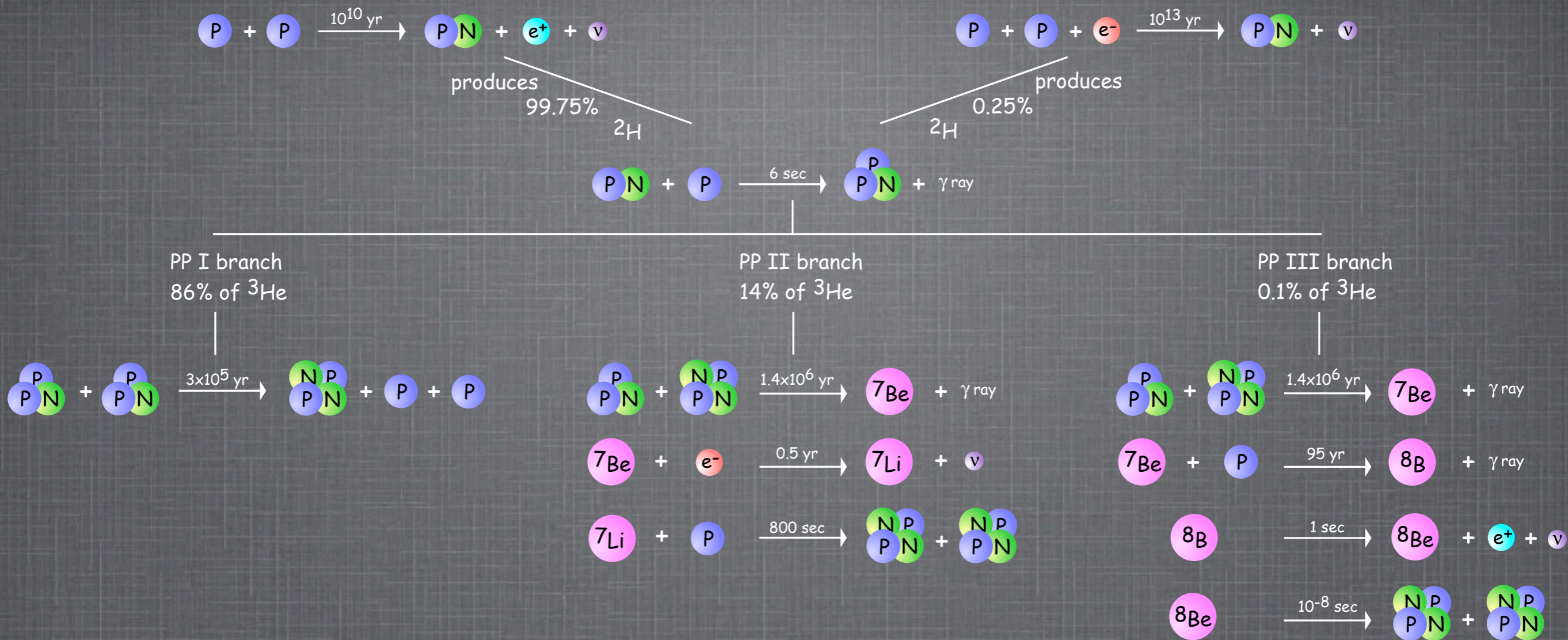
$$\dot{Y}_{4\text{he}} = 0.5 Y_{\text{he}3} Y_{\text{he}3} R_{\text{he}3,\text{he}3}$$



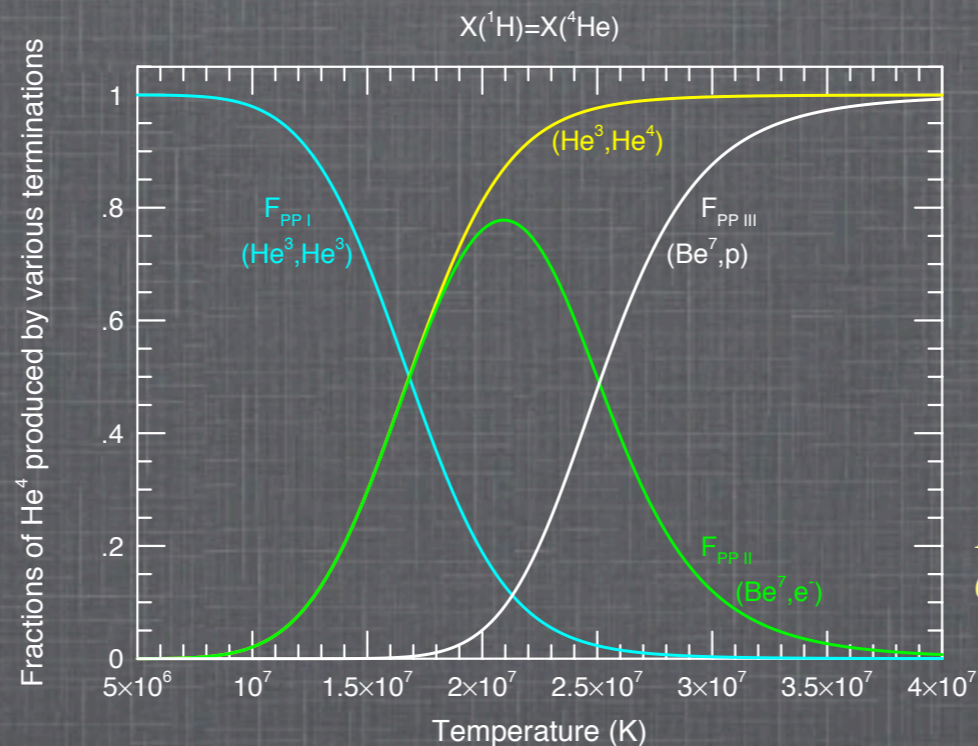
Prior to 1958 it was believed the PPI chain would proceed under most conditions, even if lots of  $^4\text{He}$  were present.

Holmgren & Johnston measured the  $^3\text{He}(\alpha,\gamma)^7\text{Be}$  cross section to be 2500 times larger than the previously accepted value, making this reaction compete with  $^3\text{He}(^3\text{He},2p)^4\text{He}$  for  $^3\text{He}$  nuclei, particularly at higher temperatures.

This leads to two new chains for converting H to He, PPII and PPIII, corresponding to the two possible fates of the  $^7\text{Be}$  nucleus.



The weights of the reactions are given for conditions in the Sun.  
 The PP chains are the most important energy source in stars with masses less than 1.5  $M_{\text{sun}}$ .



After Parker, Bahcall & Fowler ApJ 139, 602, 1964. Also see Clayton figure 5-10.



## Tasks for the day

Derive the ODE equations for the PPI chain.

Download, compile, and run the pp-chain code from [www.cococubed.com/code\\_pages/burn.shtml](http://www.cococubed.com/code_pages/burn.shtml)

Run the code in hydrostatic mode for  $T = 1.5 \times 10^7$  K,  $\rho = 150$  g/cm<sup>3</sup>, and an initial composition of 75% H and 25% He by mass.

Plot the abundance evolution.

How much hydrogen is currently left in the center of the Sun?

How long will the Sun live?

# Questions and Discussion



## 2009 NATIONAL NUCLEAR PHYSICS SUMMER SCHOOL

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