Viscosity in Heavy Ion Collisions

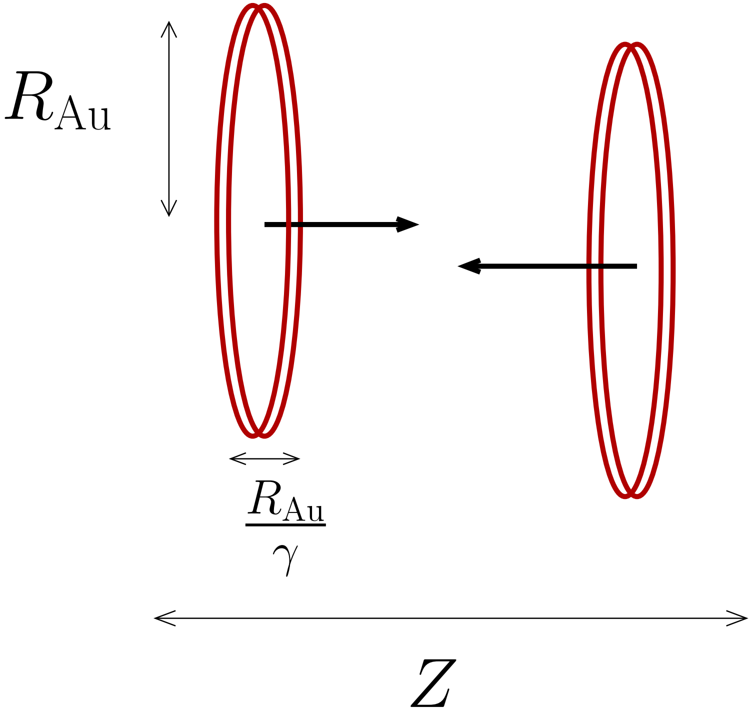
Derek Teaney

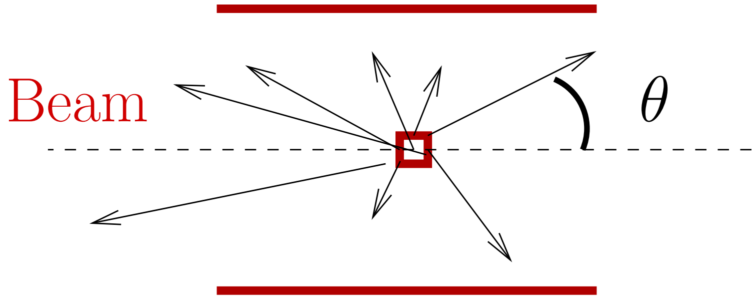
SUNY at Stonybrook and RIKEN Research Fellow

STONY BROOK

STATE UNIVERSITY OF NEW YORK

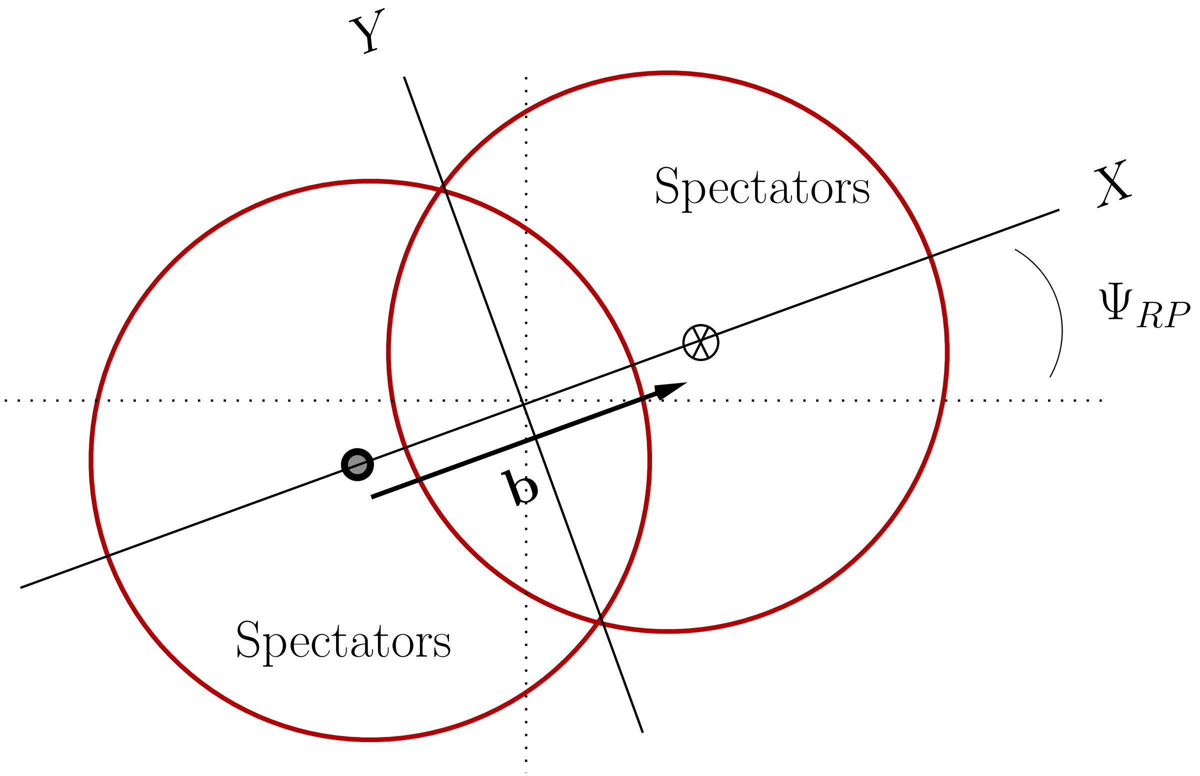
Geometry of Nuclear Collisions – AuAu at RHIC





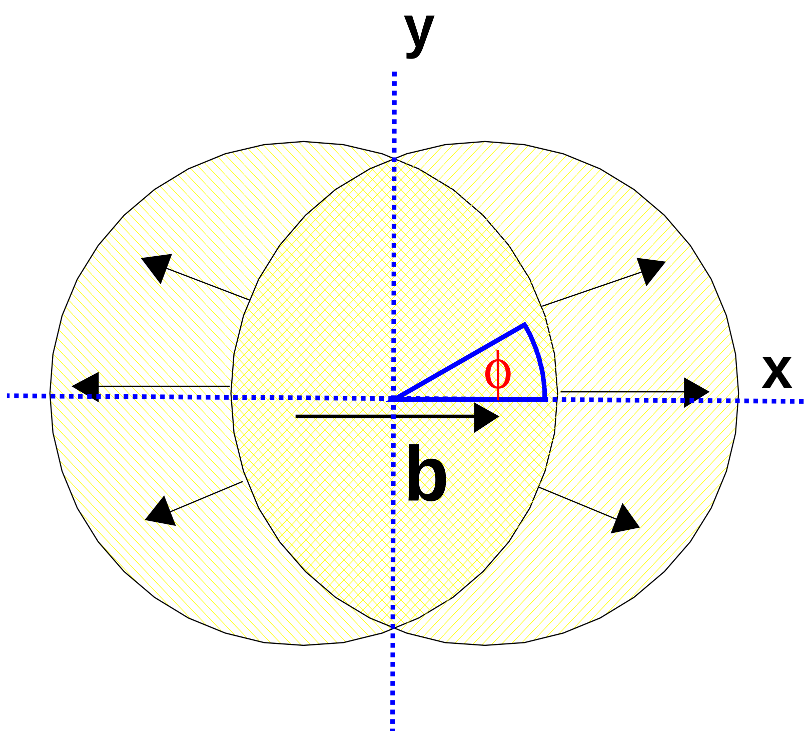
$\gamma \simeq 100 \qquad R_{\mathrm{Au}} \simeq 5\,\mathrm{fm} \qquad N_\pi \approx 10,000$

Transverse Plane



The magnitude and direction of b can be determined on an event by event basis

Observation:



There is a large momentum anisotropy:

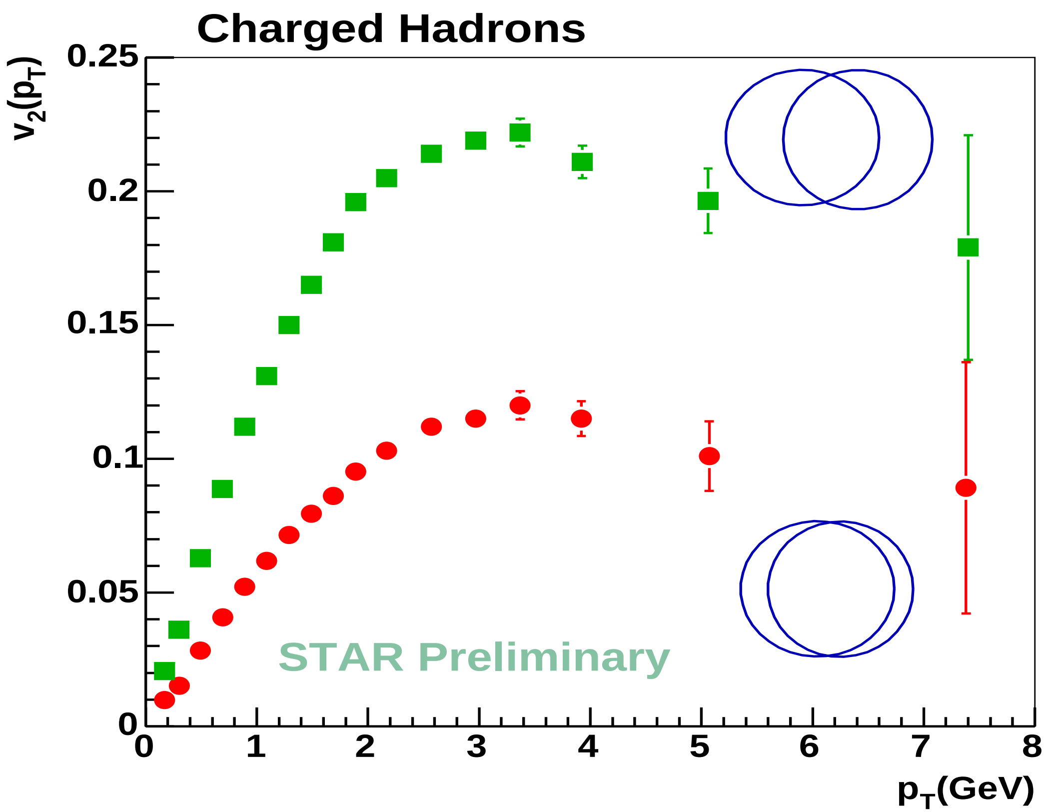
$$v_2 \equiv \frac{\left\langle p_x^2 - p_y^2 \right\rangle}{\left\langle p_x^2 + p_y^2 \right\rangle} \approx 6\%$$

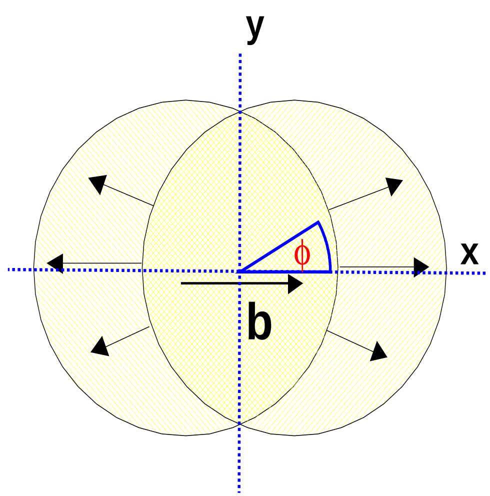
Interpretation

- The medium responds as a fluid to differences in X and Y pressure gradients

Data on Elliptic Flow:

$$\frac{1}{p_T}\frac{dN}{dp_T\,d\phi} = \frac{1}{p_T}\frac{dN}{dp_T}\;(1 + 2\,v_2(p_T)\cos(2\,\phi) + ...)$$

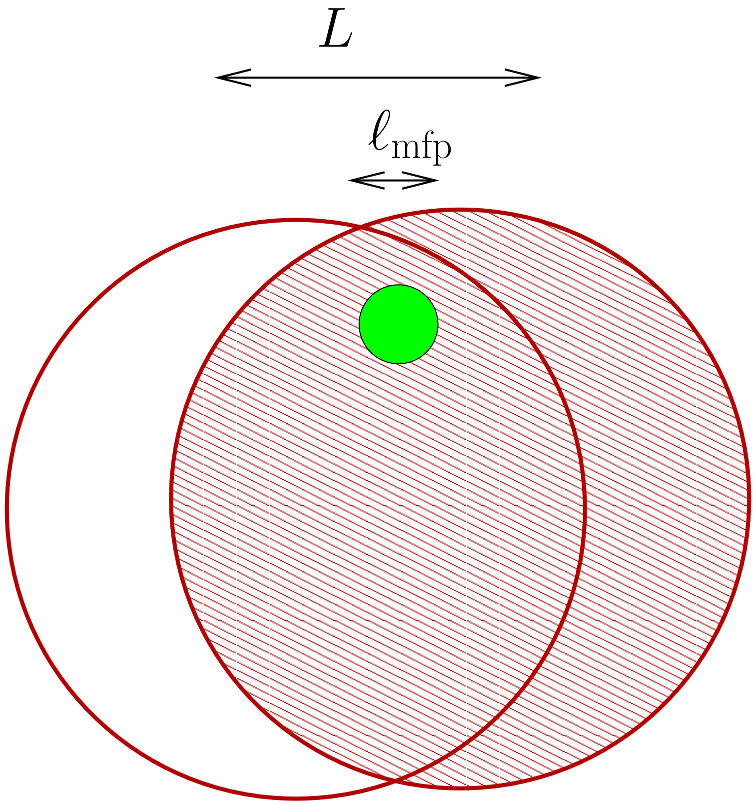




$$\text{X:Y} = (1 + \underbrace{2v_2}_{\sim 0.4} : 1 - \underbrace{2v_2}_{\sim 0.4})$$

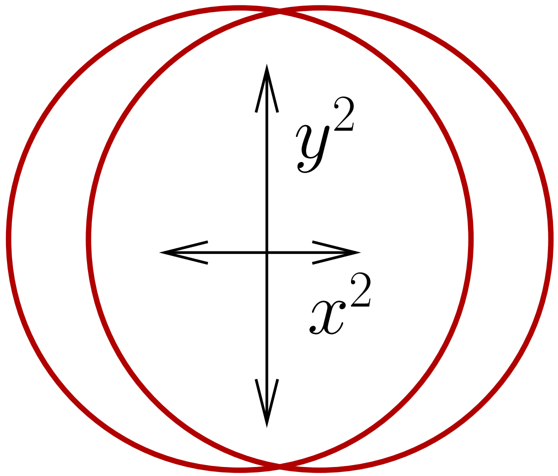
Elliptic flow is large X:Y $\sim 2.0 : 1$

What do we need for hydro?



Need $\frac{\ell_{\rm mfp}}{L} \ll 1$

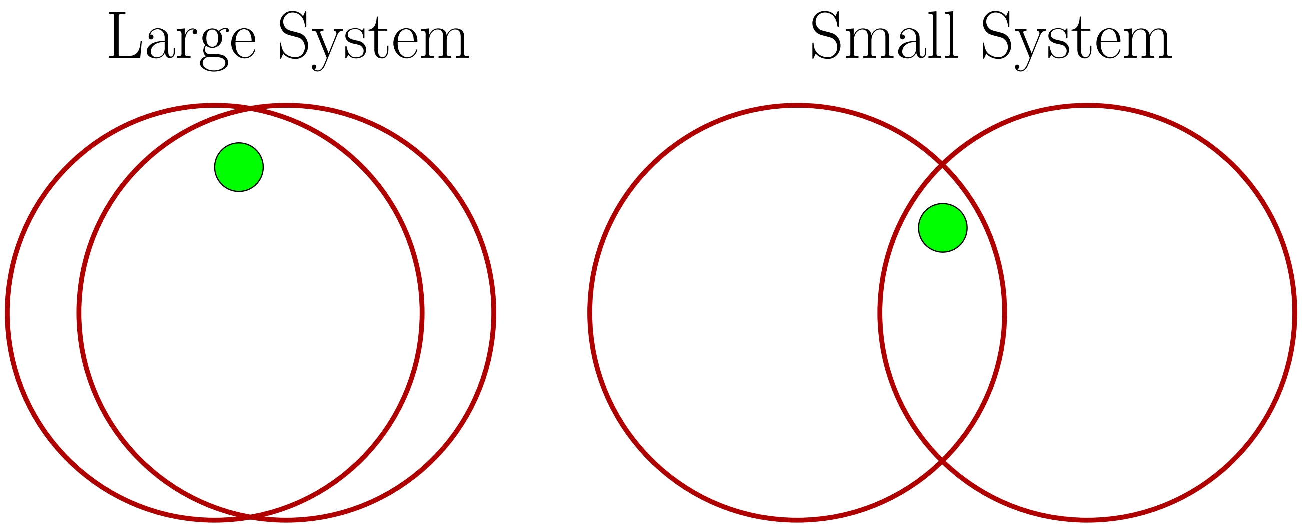
Eccentricity



$$\epsilon \equiv \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

1. $\frac{v_2}{\epsilon}$ is the response of the medium to the spatial anisotropy
2. Expect a strong response for: $\ell_{\rm mfp}/L \ll 1$

Comparing different system sizes – Centrality Dependence

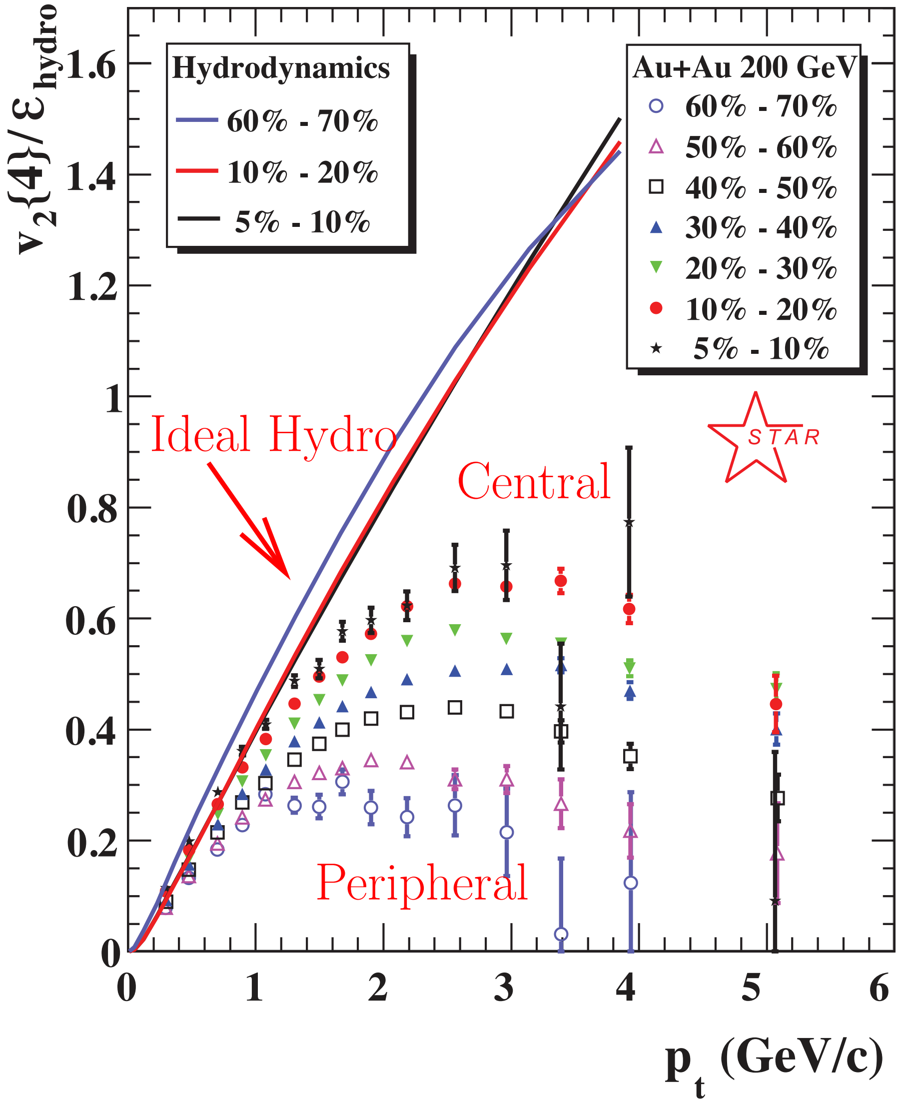


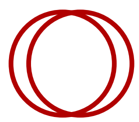
- If the response is the same in the two systems

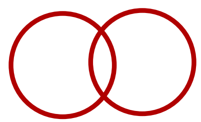
$$\frac{v_2}{\epsilon} \simeq \text{Const}$$

Expect larger system to show a stronger hydrodynamic response

See the hydrodynamic response turn on.







Trends

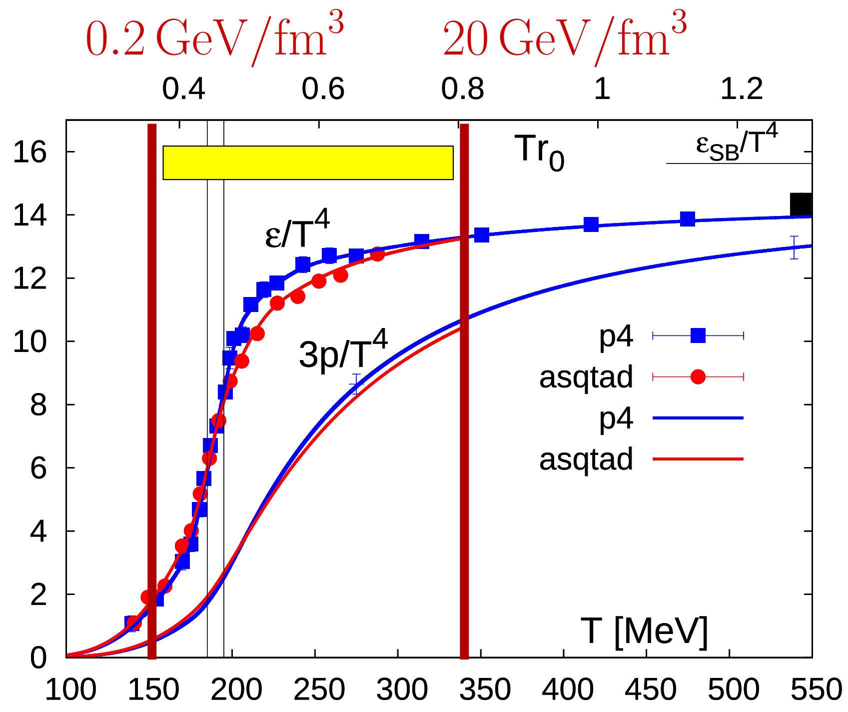
1. Strong response in central.
2. Approaching ideal hydro.
3. Flow out to higher momentum.

Most trends can be understood with a finite mean free path /viscosity

Hydrodynamic Simulations

Equation of State from Lattice

Bazavov et al.

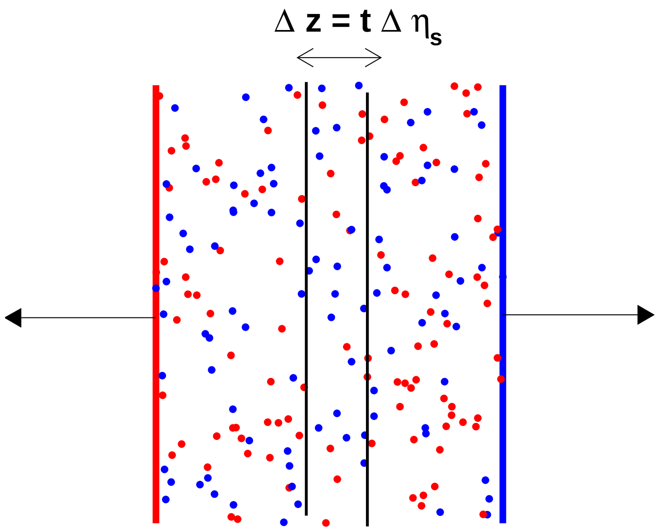


- Low temperature Hadron Resonance Gas
- High temperature Quark Gluon Plasma

$$e_{SB} \propto \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \frac{E_{\mathbf{p}}}{e^{E_{\mathbf{p}}/T} - 1}$$

The transition is very smooth

Bjorken Coordinates



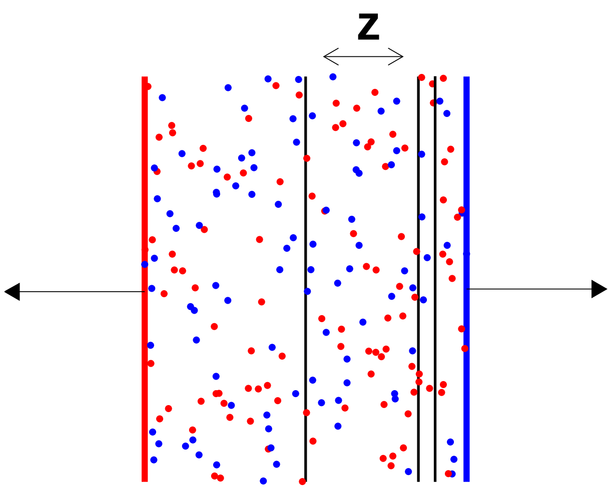
- Instead of t, z we use proper time and space time rapidity

$$\tau = \sqrt{t^2 - z^2} \qquad \text{and} \qquad \eta_s = \frac{1}{2}\log\left(\frac{1 + z/t}{1 - z/t}\right)$$

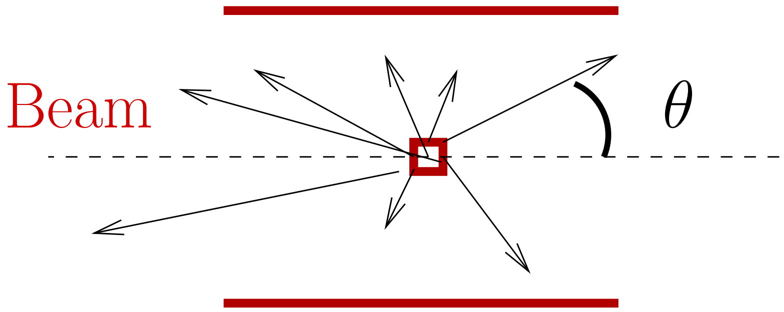
- Near the center (mid-rapditiy)

$$\tau \simeq t \qquad \Delta z = \tau \, \Delta\eta_s$$

The Bjorken expansion



$$z \approx v_z\, t \approx (\frac{p_z}{E})\, t$$

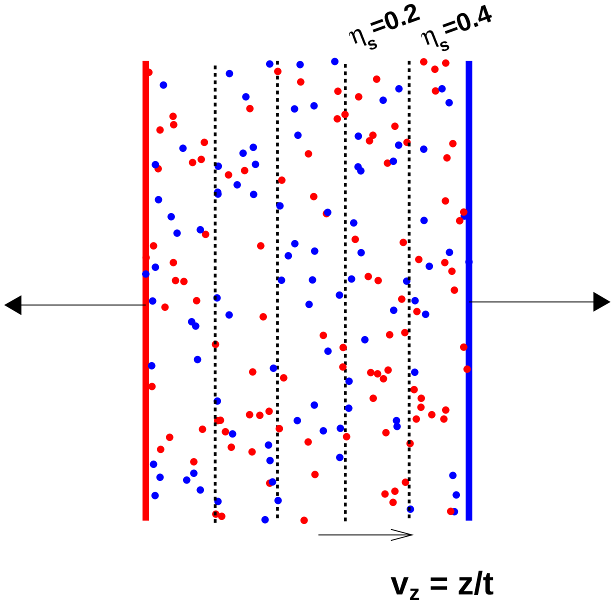


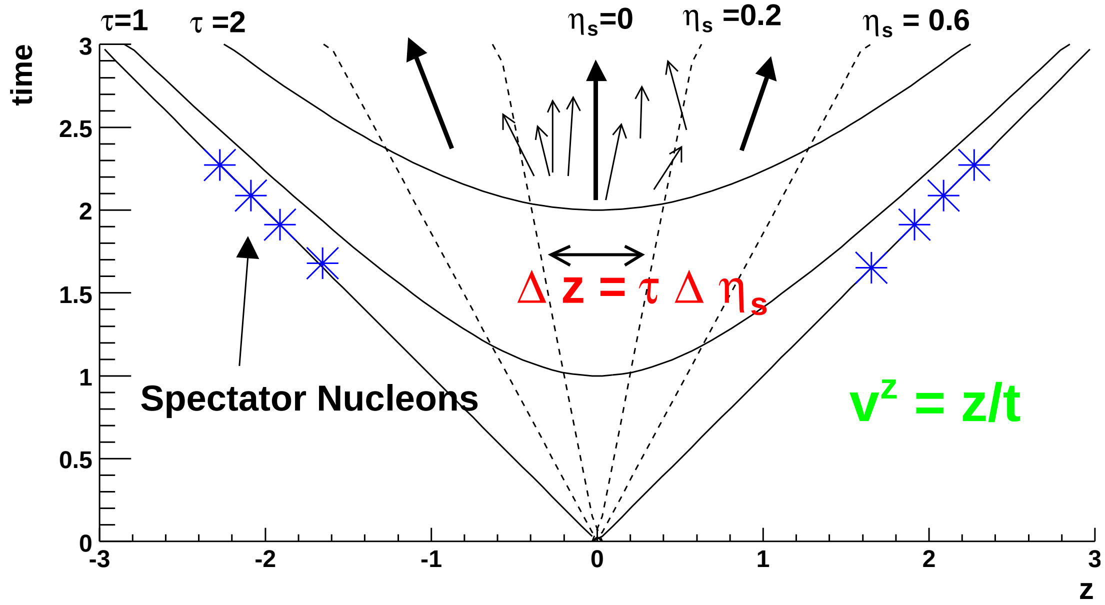
- Each part of the detector is associate with a region of space-time

$$\underbrace{\frac{1}{2}\log\frac{1+z/t}{1-z/t}}_{\eta_s} \approx \underbrace{\frac{1}{2}\log\frac{1+p_z/E}{1-p_z/E}}_{\text{particle rapidity}} \approx \underbrace{\frac{1}{2}\log\frac{1+\cos(\theta)}{1-\cos(\theta)}}_{\eta_{\rm pseudo}}$$

All rapidities are (almost) the same in high energy collision

Bjorken Estimate





Estimate the energy density in a slice (a lower bound)

$$e|_{\tau_o} = \frac{\Delta E}{A \Delta z} = \frac{1}{A\tau_o}\frac{\Delta E}{\Delta \eta_{\rm pseudo}}$$

$$= 5.5\,\frac{\rm GeV}{\rm fm^3} \quad \text{at time} \quad \tau_o = 1\,{\rm fm}$$

Can convert to temperature

$$T_o \simeq 300\,\text{MeV} \quad \text{at time} \quad \tau_o = 1\,{\rm fm}$$

Ideal Hydrodynamics

- The medium has an energy density e, pressure $p(e)$ and four velocity u^μ

$$T^{\mu\nu} = e u^\mu u^\nu + p\,(g^{\mu\nu} + u^\mu u^\nu) \qquad \text{and} \qquad \partial_\mu T^{\mu\nu} = 0$$

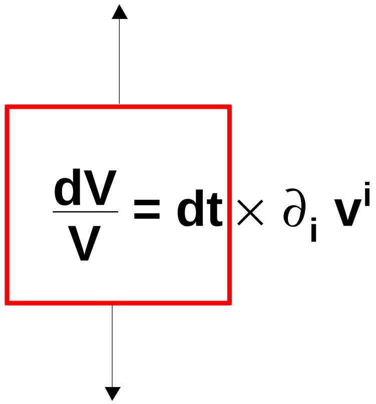
- Can convince yourself that in the rest frame $u^\mu = (1, 0, 0, 0)$

$$T^{\mu\nu} = \begin{pmatrix} e & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$$

- Near the local rest frame $T^{00} = e$ and $T^{0i} = (e+p)v^i$:

$$\partial_t e \quad = \quad -(e+p)\,\partial_i v^i \qquad \Longleftarrow \qquad \text{The Work Equation}$$

The Work Equation:



$$\partial_t e = -(e+p)\,\partial_i v^i$$

$$de = -(e+p)\,\frac{dV}{V}$$

- The EOM reads

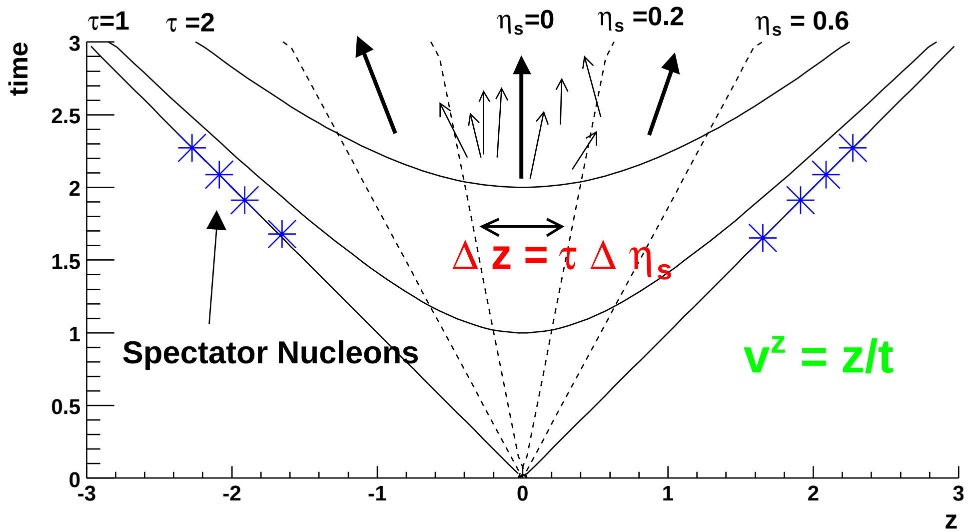
$$d(eV) = -pdV$$

- Compare: $d(eV) = Td(sV) - pdV$ and find

$$d(sV) = 0$$

pdV Work means Entropy is Conserved

1D Bjorken Expansion: (Bjorken)



BJ Ansatz

$$
\begin{aligned}
v^z &= \frac{z}{t} \\
\partial_z v^z &= \frac{1}{t}
\end{aligned}
$$

- The Equation of motion

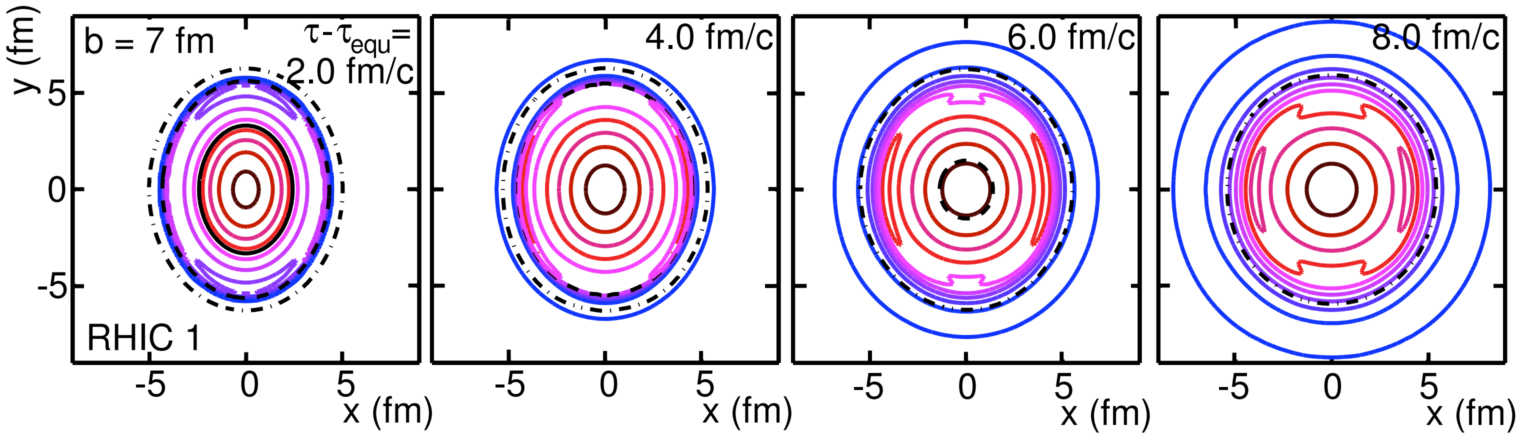
$$
\begin{aligned}
\partial_t e &= -(e+p)\partial_z v^z \\
\frac{de}{d\tau} &= -(e+p)\,\frac{1}{\tau} \\
\frac{d(\tau e)}{d\tau} &= -p
\end{aligned}
$$

Energy per rapidity decreases due to $p\,dV$ work

A full ideal hydro simulation

(Kolb and Heinz)

- Run hydro assuming Bjorken boost invariance in z direction

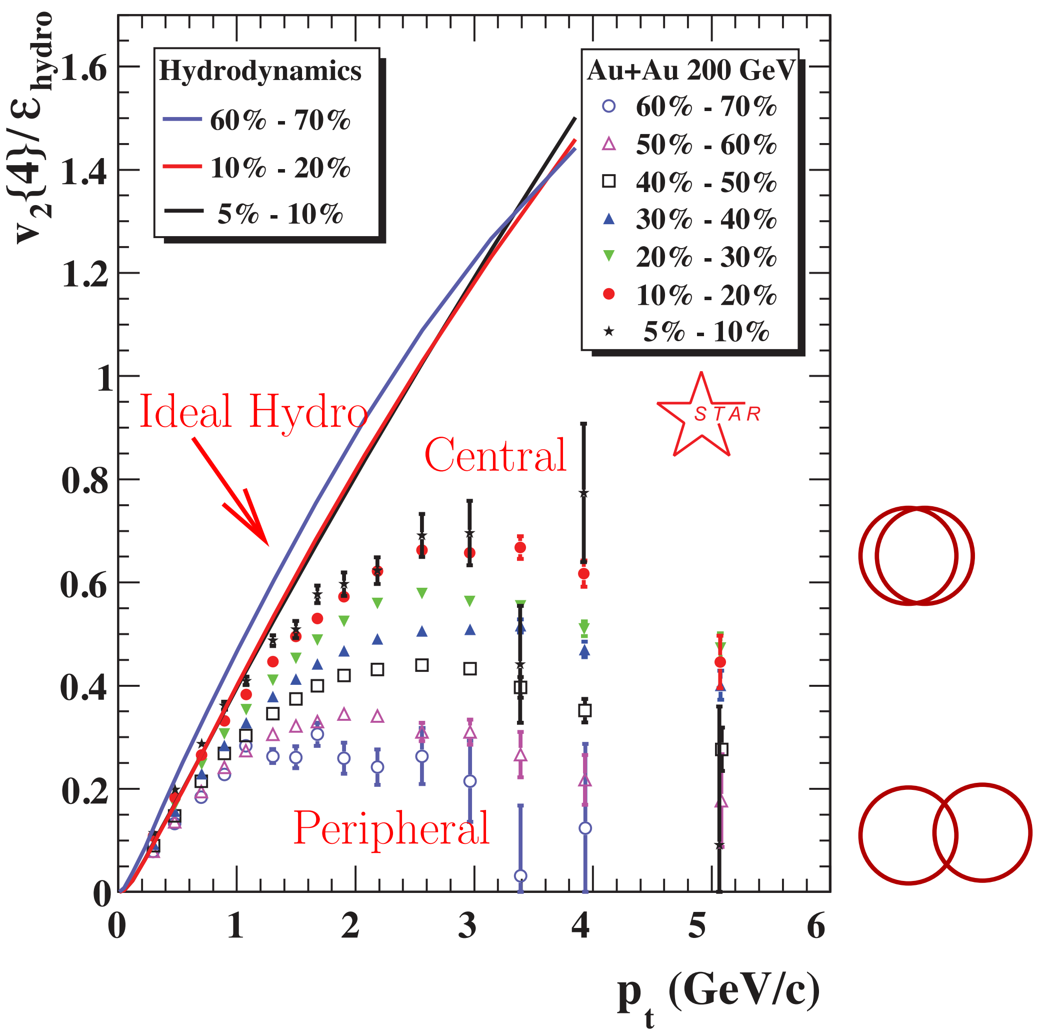


- When reach a "freezeout" temperature in the hadron phase compute spectra:

$$\frac{dN_\pi}{d^3p} = \mathrm{d}V \, \frac{g_\pi}{e^{E_\pi/T} - 1} \qquad \text{covariantly} \qquad E\frac{dN_\pi}{d^3p} = \int_\Sigma P^\mu d\Sigma_\mu \frac{1}{e^{-P\cdot U/T} - 1}$$

- Compare to data!

Ideal hydro simulations by P. Huovinnen



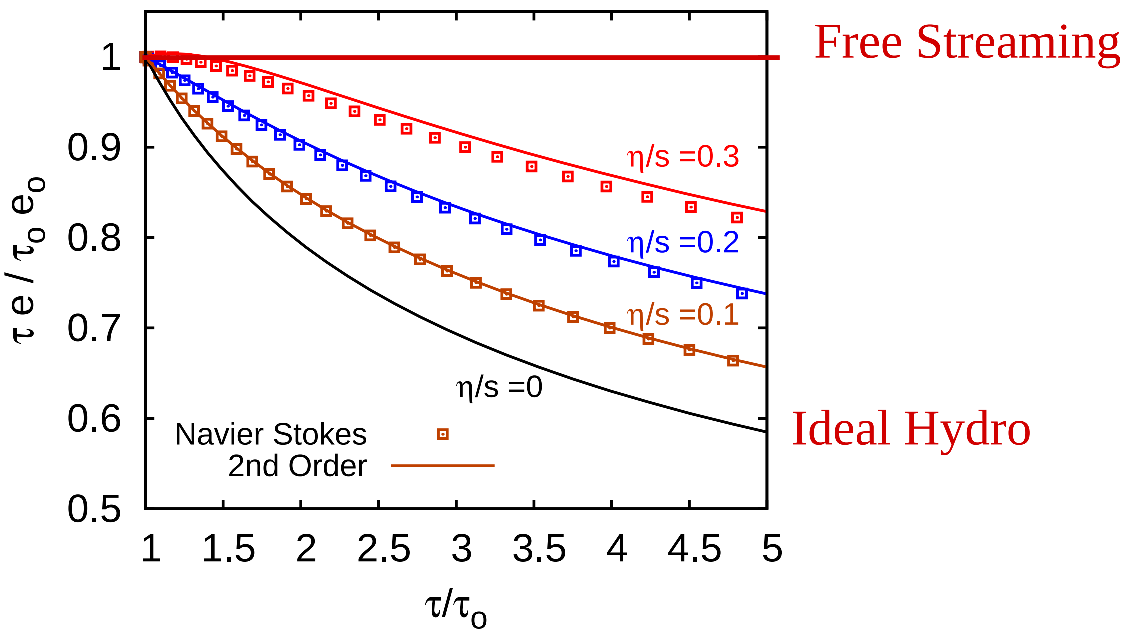
1D Expansion: Hydro vs. Free Streaming vs. Viscous hydro

- Ideal hydrodynamics with ideal gas EOS: $p = e/3$

$$\frac{de}{d\tau} = -\frac{(e+p)}{\tau} \qquad \text{find} \qquad e = e_o \left(\frac{\tau_o}{\tau}\right)^{4/3}$$

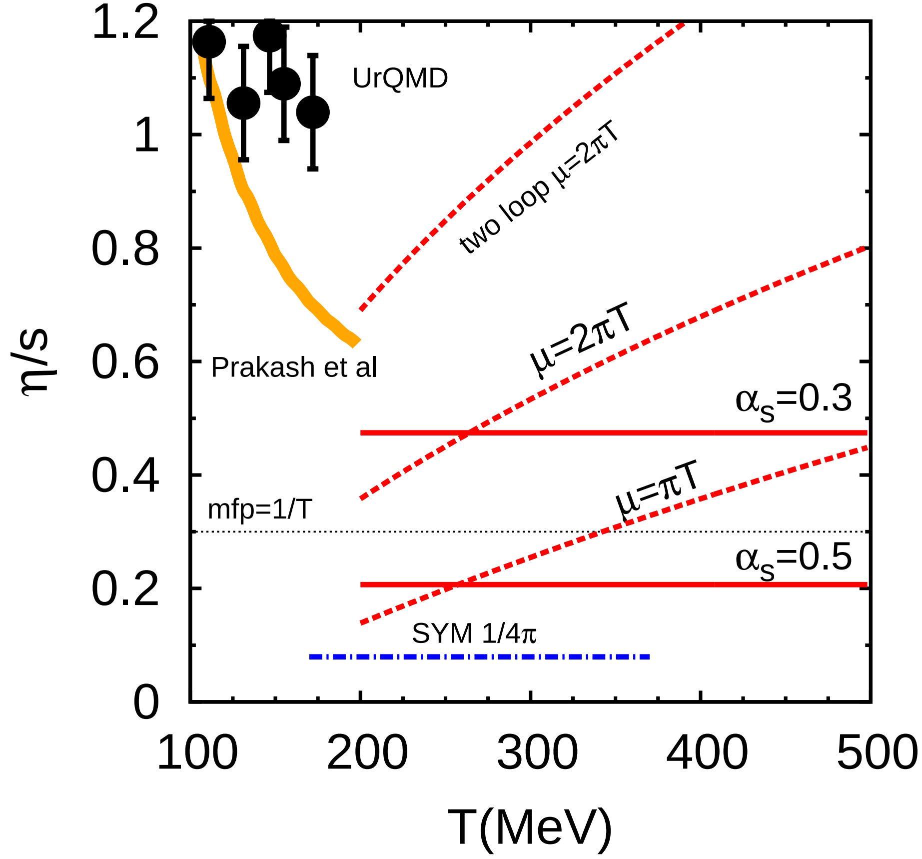
- Compare to non-equilibrium free streaming: $p \simeq 0$

$$\frac{de}{d\tau} \simeq -\frac{e}{\tau} \qquad \text{find} \qquad e = e_o \frac{\tau_o}{\tau}$$



Intro to viscosity (hand written notes)

Viscosity Estimates in QCD



Hydrodynamics with Viscosity

(Gyulassy and Danielewicz)

$$T^{ij} = p\delta^{ij} - \eta\left(\partial^i v^j + \partial^j v^i - \frac{4}{3}\delta^{ij}\partial\cdot v\right) + \text{bulk viscosity}$$

- The Bjorken expansion becomes

$$\frac{de}{d\tau} = -\frac{e+p}{\tau} \qquad \text{becomes} \qquad \frac{de}{d\tau} = -\frac{e+T_{zz}}{\tau}$$

- The pressure get reduced by the expansion

$$T_{zz} = p - \frac{4}{3}\eta \underbrace{\frac{1}{\tau}}_{\partial_z v^z}$$

- The equation of motion is

$$\underbrace{\frac{de}{dt}}_{de} = -\underbrace{(e+p)\frac{1}{\tau}}_{-\text{ideal}} + \underbrace{\frac{4}{3}\frac{\eta}{\tau^2}}_{+\text{viscous}}$$

How valid is Hydrodynamics?

$$\frac{de}{dt} = -(e+p)\frac{1}{\tau} + \frac{4}{3}\frac{\eta}{\tau^2}$$

- Comparing the size of the viscous term to the ideal term need .

$$\frac{\eta}{e+p}\frac{1}{\tau} \ll 1$$

- Function of time, temperature, etc, $(e+p) = sT$

$$\underbrace{\frac{\eta}{s}}_{\text{fluid parameter}} \times \underbrace{\frac{1}{\tau T}}_{\text{experimental parameter} \sim 1/2} \ll 1$$

- Estimate

$$0.2\left(\frac{\eta/s}{0.3}\right)\left(\frac{1\,\mathrm{fm}}{\tau_o}\right)\left(\frac{300\,\text{MeV}}{T_o}\right) \ll 1$$

Need η/s smallish to have hydro at RHIC

A complete viscous hydro simulation

1. Run the evolution the viscous terms

2. Freezeout when viscous terms become large

3. Compute spectra:

   - Viscous corrections modify the distribution function

$$f_o = \frac{1}{e^{E_{\mathbf{p}}/T} - 1} \qquad f_o \to f_o + \delta f$$

   - Maximum momentum also signaled by the equations.

4. Compare with data!

Viscous corrections to the distribution function $f_o \to f_o + \delta f$

- Must be proportional to strains
- Must be a scalar
- General form in rest frame and ansatz

$$\delta f = \chi(p) p^i p^j \sigma_{ij} \Longrightarrow \delta f \propto f_0 \, p^i p^j \sigma_{ij}$$

- Can fix the constant

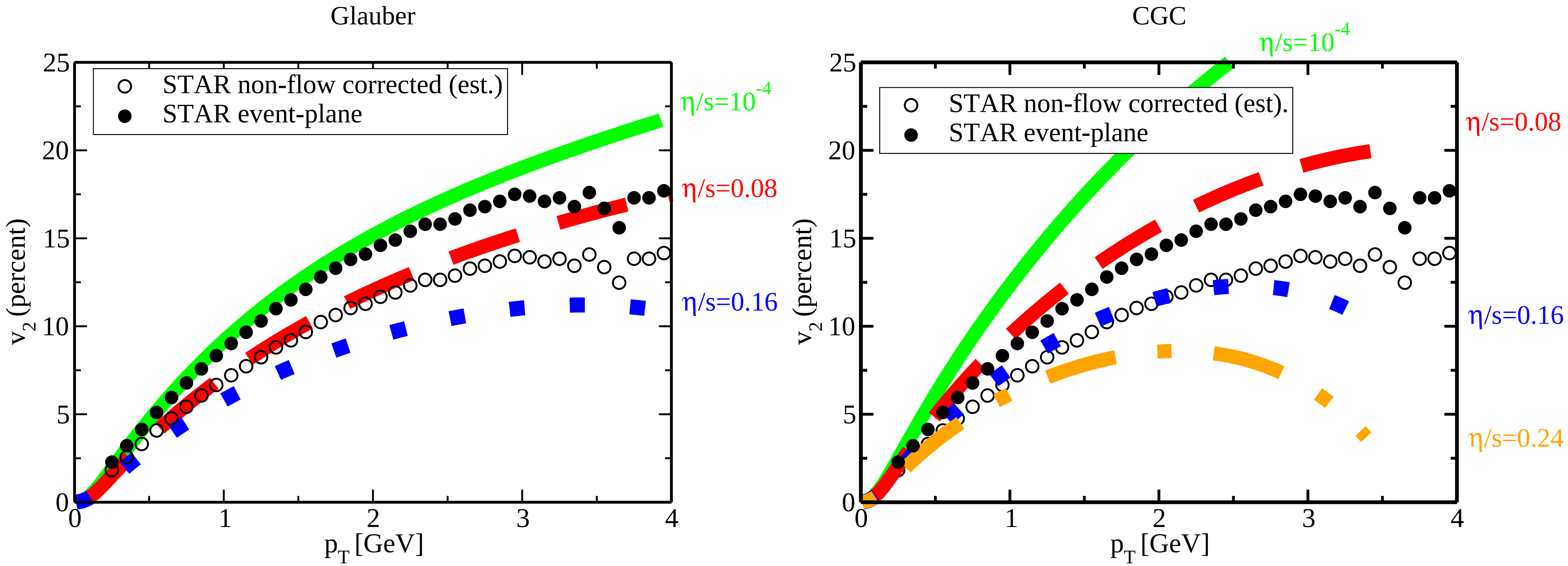
$$T^{ij} = \mathcal{P}\delta^{ij} - \eta\delta^{ij} = \int \frac{d^3p}{(2\pi)^3} \frac{p^i p^j}{E_{\mathbf{p}}} \left(f_0 + \delta f\right)$$

find

$$\delta f = -\frac{\eta}{2(e+p)T^2} \, f_o \, p^i p^j \sigma_{ij}$$

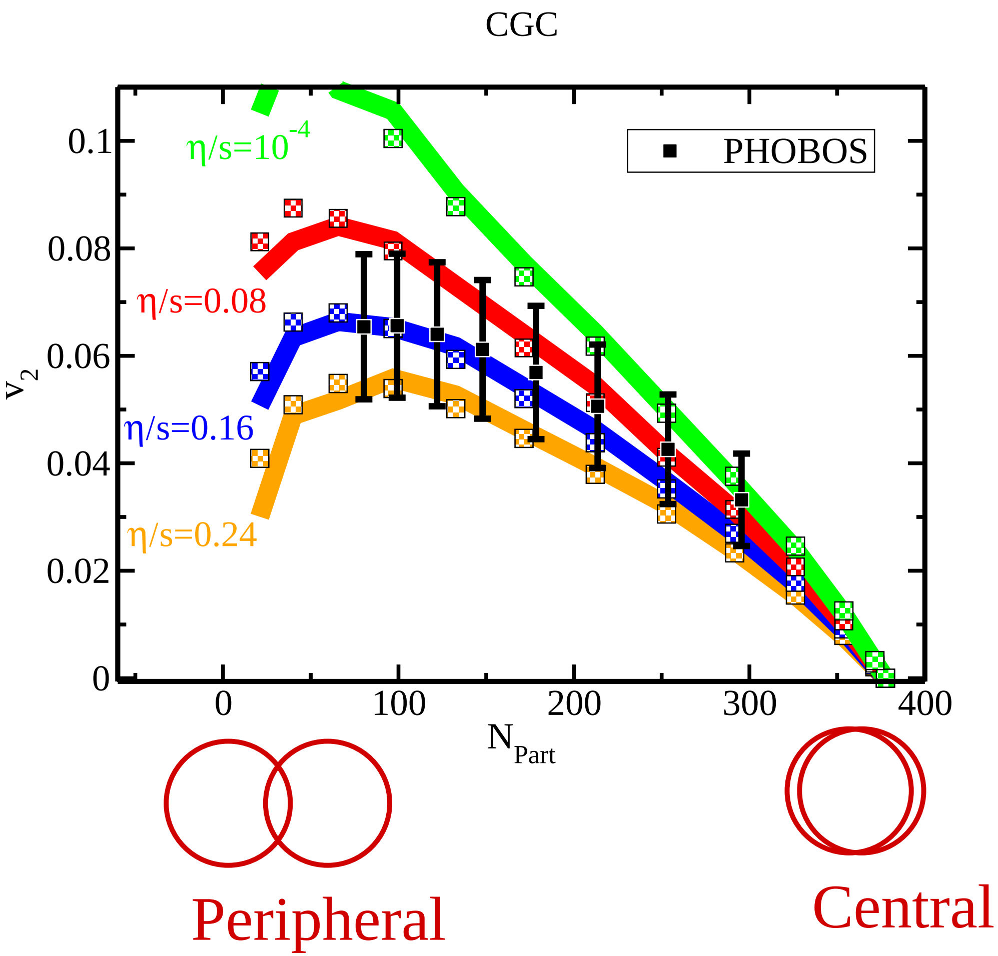
Viscous corrections grow quadratically with momentum

A complete viscous simulation by Romatschke, Romatschke, Luzum



1. Compare open symbols to hydro curves
2. Different simulations represent different initial eccentricity
3. Viscous correction grows with p_T

Romatschke, Romatschke, Luzum – Centrality Dependence



Impossible to accommodate $\eta/s > 0.3$

Computing δf with kinetic theory

$$f_o \rightarrow f_o + \delta f$$

- Work with a relaxation time approximation $v_{\mathbf{p}} = \frac{\mathbf{p}}{E_{\mathbf{p}}}$

$$\partial_t f + v_{\mathbf{p}} \partial_x f = -\frac{f - f_o}{\tau_R(E_{\mathbf{p}})}$$

- Substitute $f_o + \delta f$ and work in a linear approx (see supplement)

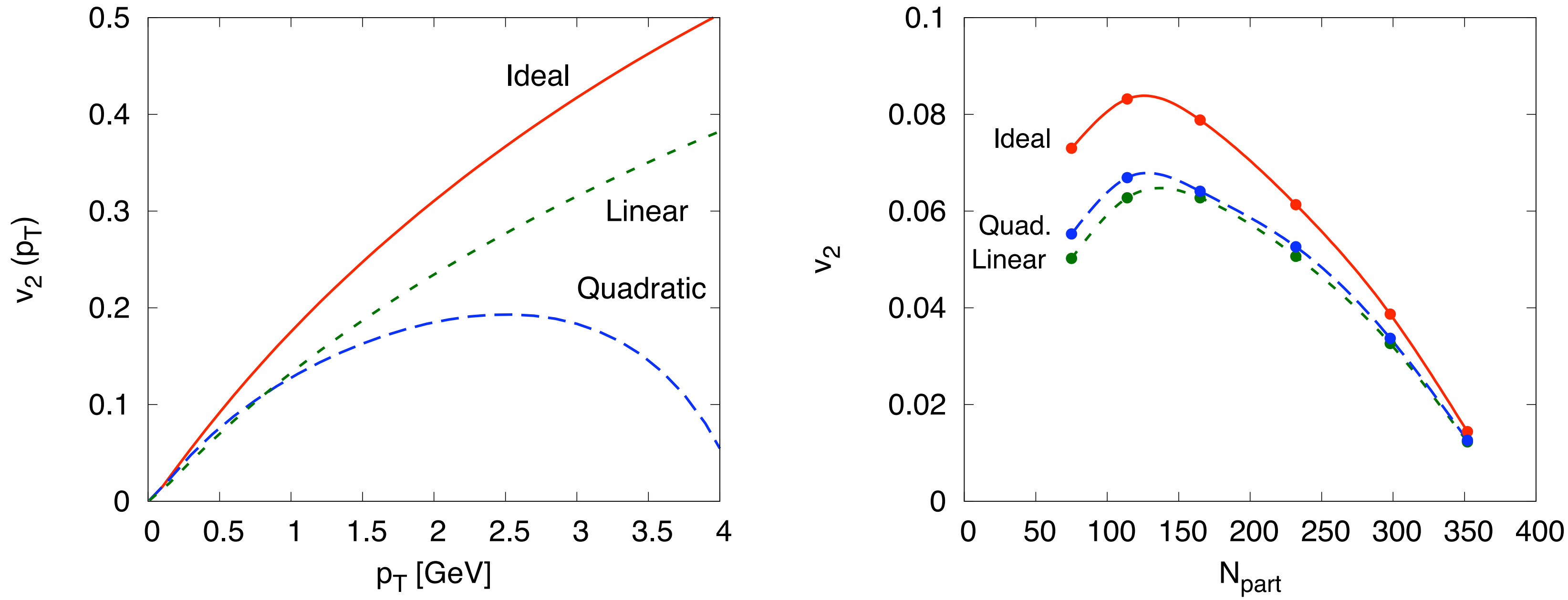
$$\begin{aligned} \partial_t f_o + v_{\mathbf{p}} \partial_x f_o &= -\frac{\delta f}{\tau_R} \\ -f_o \frac{\tau_R(E_{\mathbf{p}})}{2T E_{\mathbf{p}}} p^i p^j \sigma_{ij} &= \delta f \end{aligned}$$

- Two Limiting cases

1. $\tau_R \propto E_{\mathbf{p}}$ – reproduces the quadratic ansatz
2. $\tau_R =$ Const – relaxation time independent of momentum linear ansatz

Sensitivity to δf

K.Dusling, Guy Moore, DT

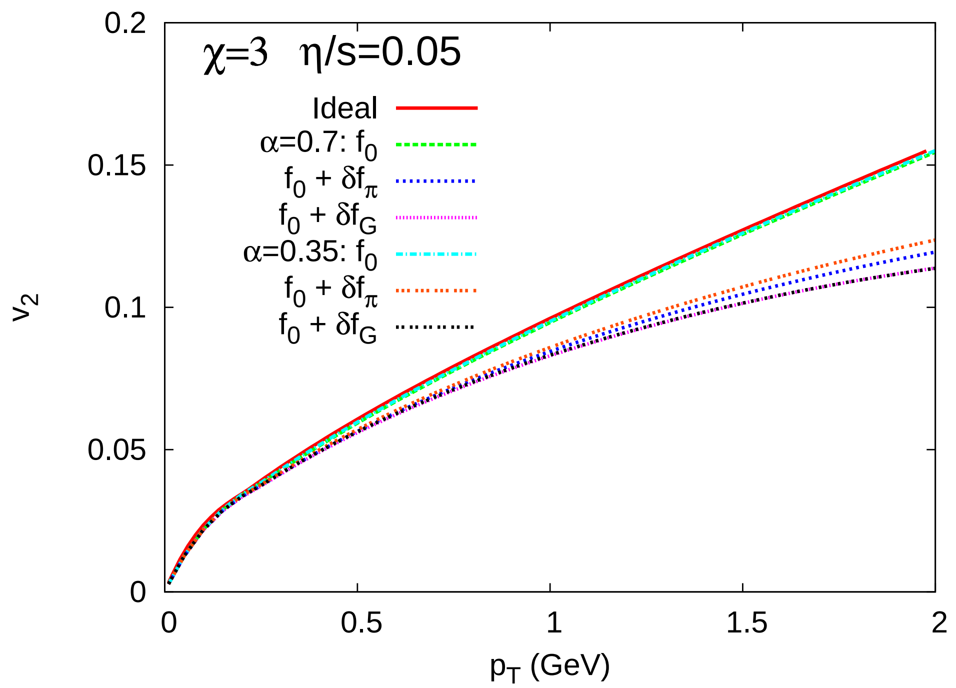


Integrated quantities are insensitive to the precise form of δf

Independent of second derivative terms

(K. Dusling, DT)

$$T^{ij} = \eta\left(\partial^i v^j + \partial^j v^i - \frac{2}{3}\delta^{ij}\partial\cdot v\right) + \text{more derivs}$$
$$= O(\epsilon) \;+\; O(\epsilon^2)$$



Gradient expansion is working. Temperature is a good concept.

Worse at larger viscosities and larger p_T

Hydro Conclusions:

- Many aspects of the heavy ion data support a hydrodynamic interpretation
- Viscous hydro works better than ideal hydro
- Difficult to explain the RHIC data with $\eta/s > 0.3$
- A very interesting regime of quantum transport