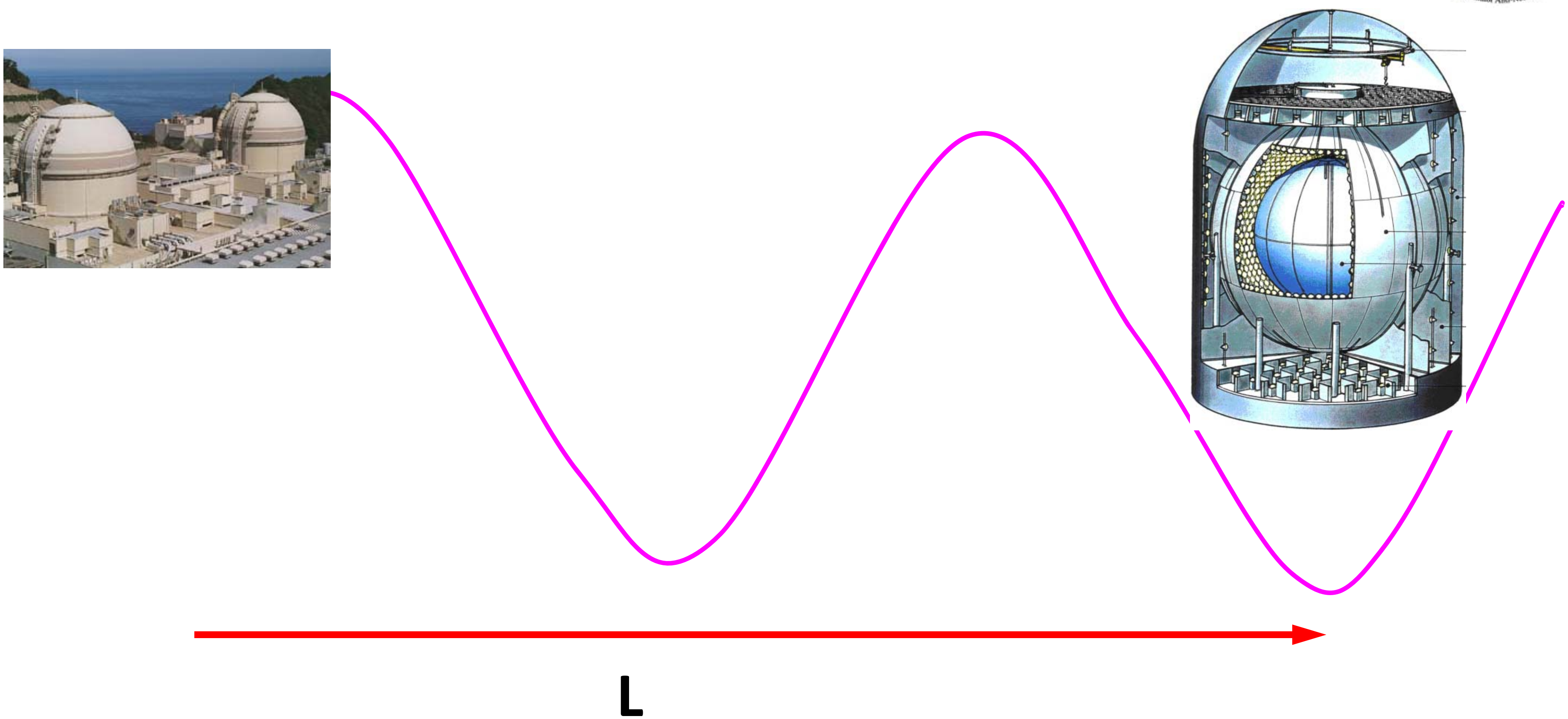
Neutrino Oscillations on Earth



$$P_{\mu} = |A_{\mu}|^2 = \sin^2 2\theta \ \sin^2\left(\frac{\Delta m^2 L}{4 E_{\nu}}\right) \qquad (\Delta m^2 = m_1^2 - m_2^2)$$

$$P_e = |A_e|^2 = 1 - \sin^2 2\theta \ \sin^2\left(\frac{\Delta m^2 L}{4 E_{\nu}}\right)$$

Length & Energy Scales

$E_\nu = 1$ MeV, $\Delta m^2 = 1$ eV^2, → L = 1.24 meters

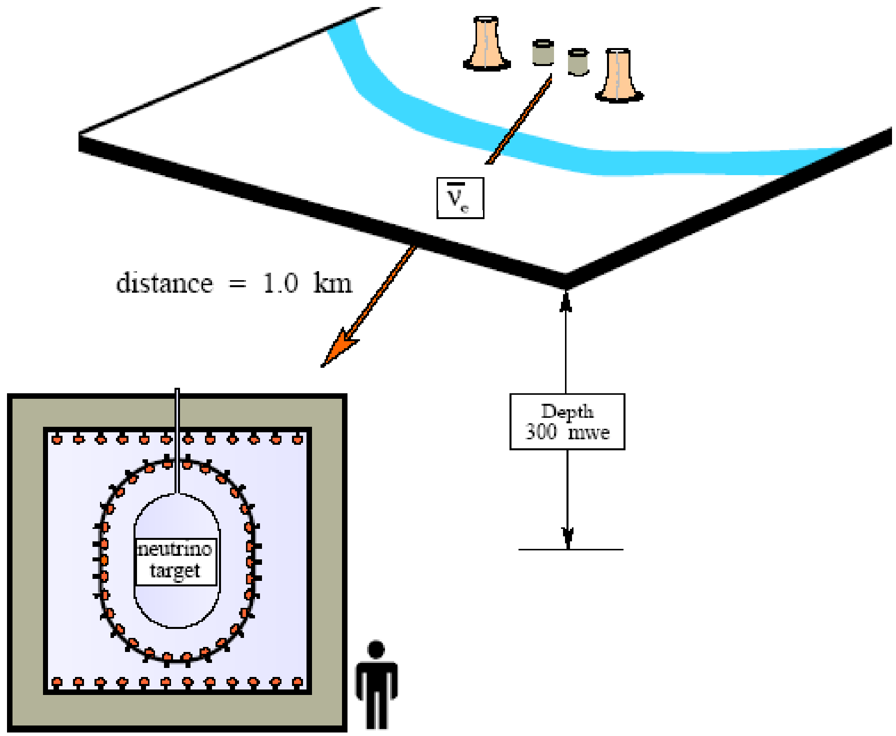
(P_e →minimum)

E_ν= 1 GeV, Δm^2=10^{-3} eV^2 , L = 1240 km Super-K

E_ν= 1 MeV, Δm^2=10^{-3} eV^2 , L = 1.2 km Chooz, Palo Verde

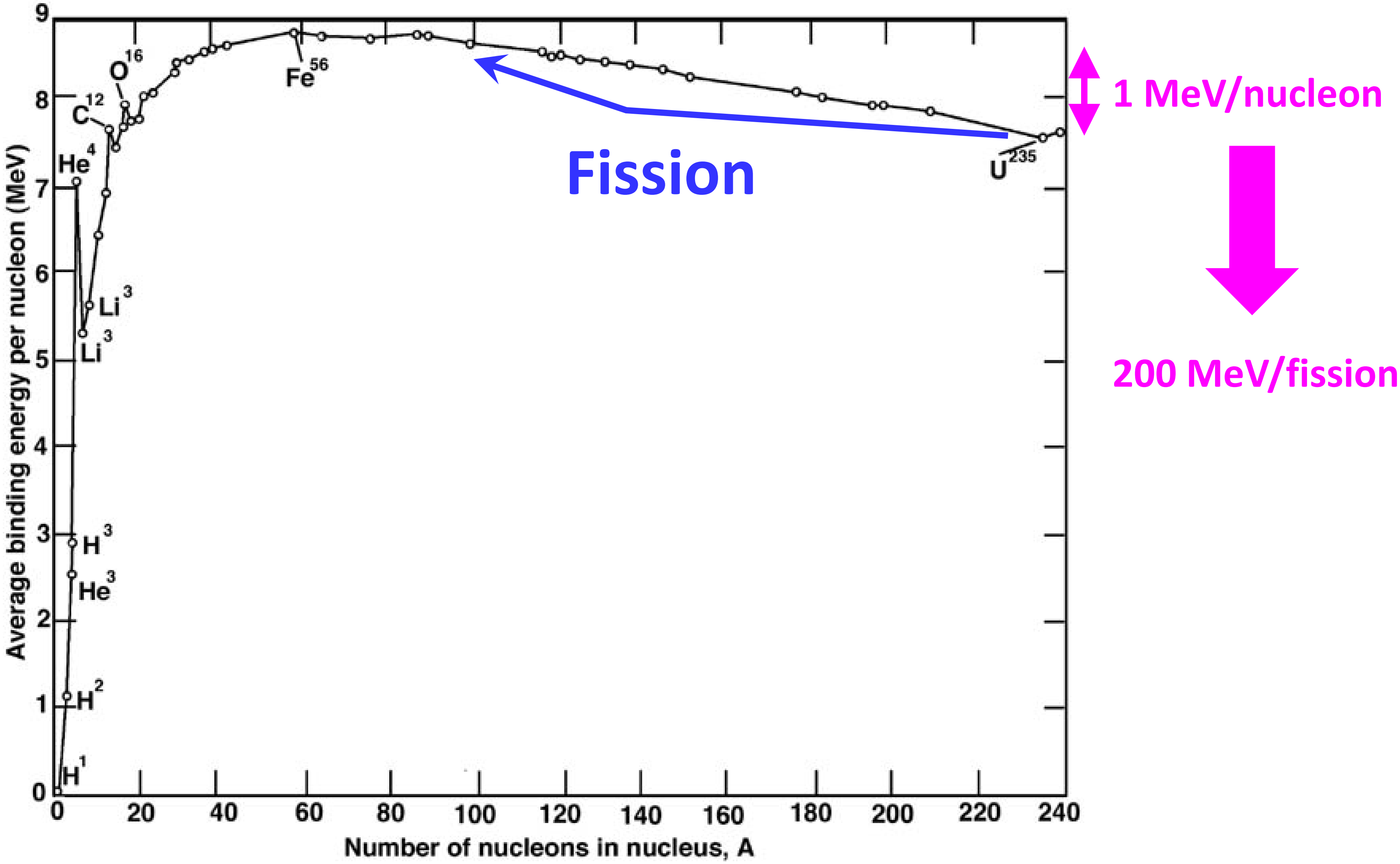
E_ν= 1 MeV, Δm^2=10^{-5} eV^2 , L = 125 km KamLAND

Reactor Neutrino Experiments

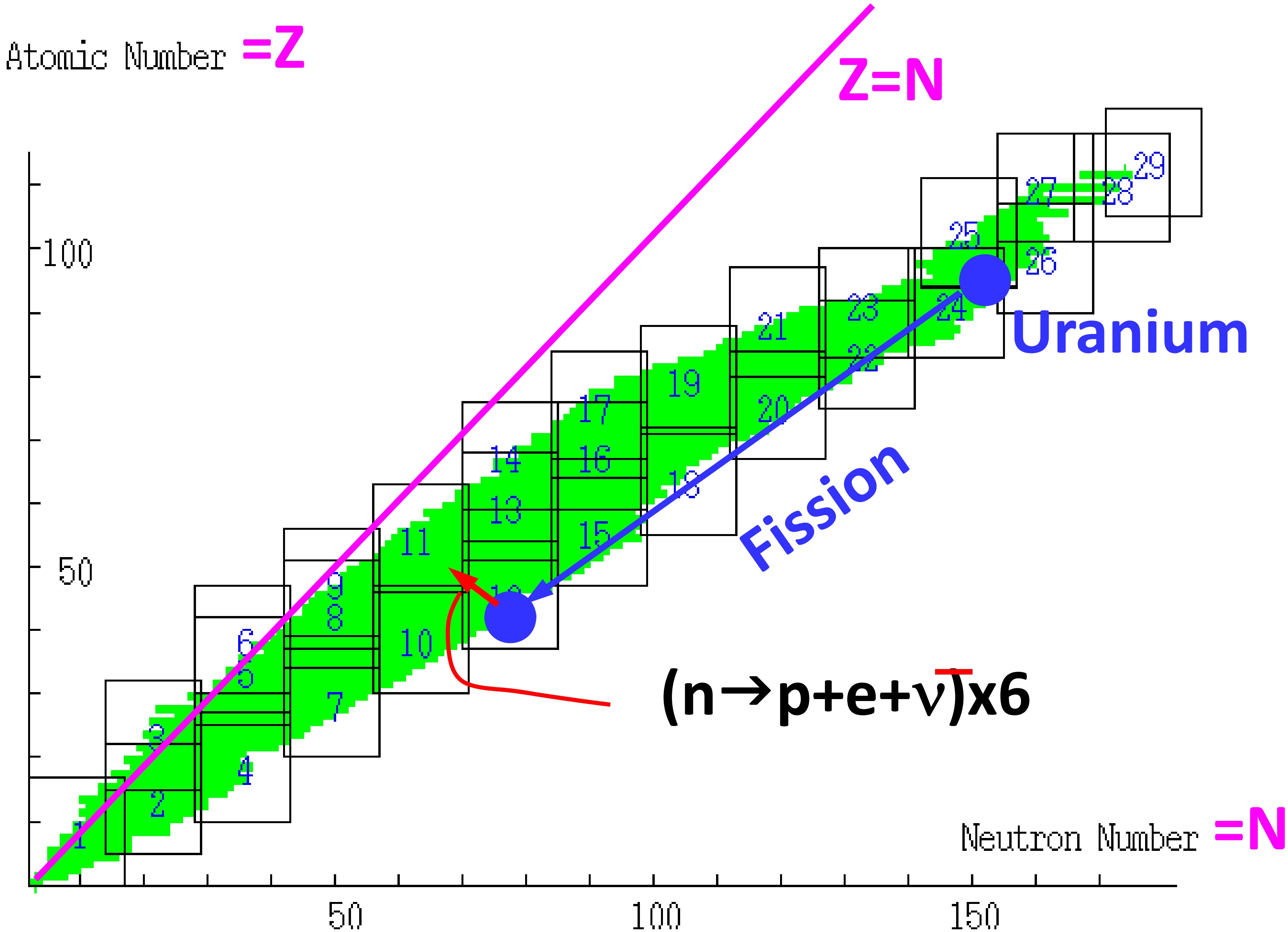


- $\overline{\nu}_e$ from n-rich fission products
- detection via inverse beta decay ($\overline{\nu}_e + p \rightarrow e^+ + n$)
- Measure flux and energy spectrum
- Variety of distances L= 10-1000 m

Binding Energy of Nuclei



Nuclear Reactors make Antineutrinos



$\bar{\nu}_e$ Energy Spectra

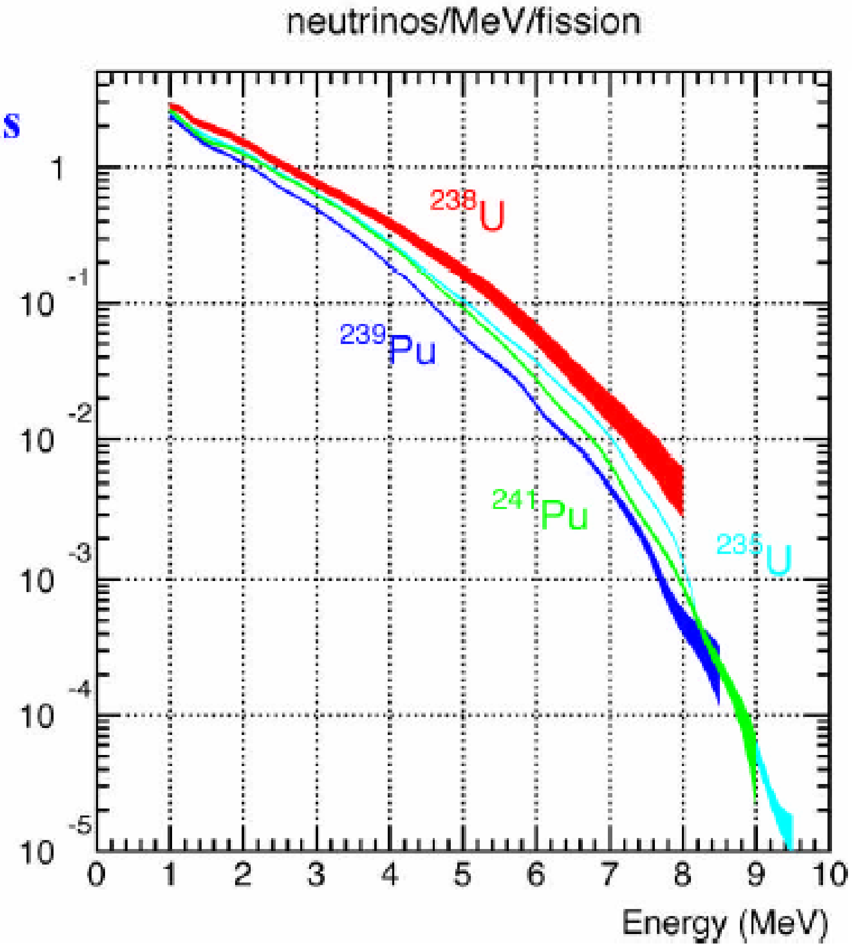
- $\bar{\nu}_e$ associated with ^{235}U, ^{239}Pu and ^{241}Pu

  measured β – spectra
  from thermal neutron fissions

  conversion $E_e \longrightarrow E_\nu$

- $\bar{\nu}_e$ associated with ^{238}U

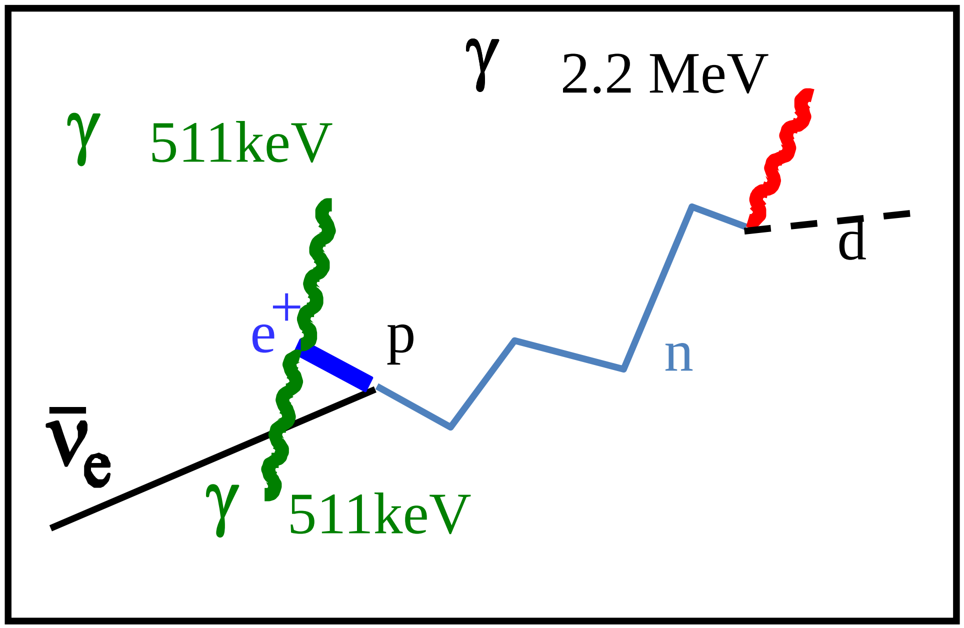
  calculation based on
  744 unstable fission products



K. Schreckenbach et al., PL B160 (1985) 635.
A.A. Hahn et al., PL B218 (1989) 365.
P. Vogel et al., PR C24 (1981) 1543

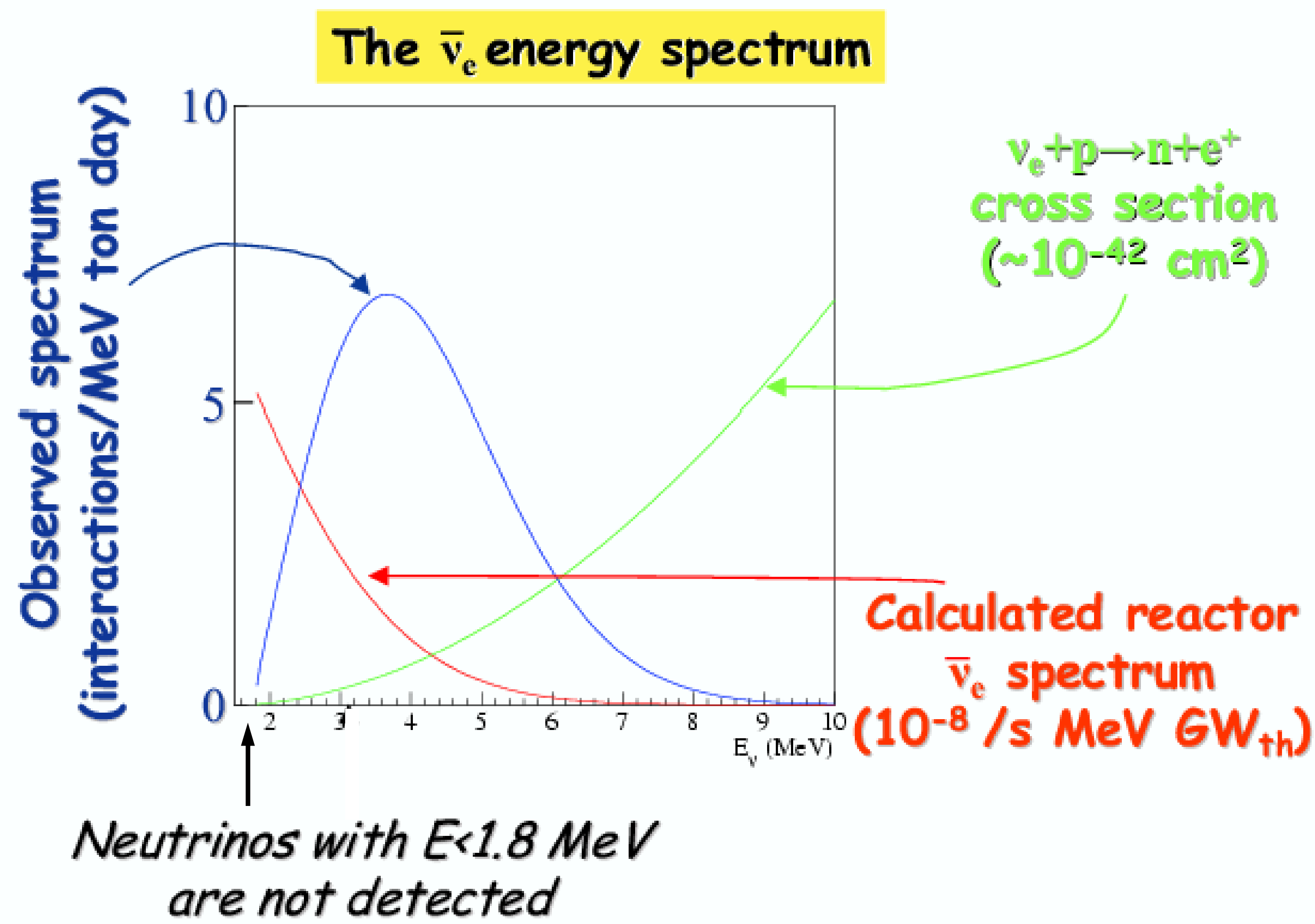
Detection Signal

$$\bar{\nu} + p \rightarrow n + e^+$$



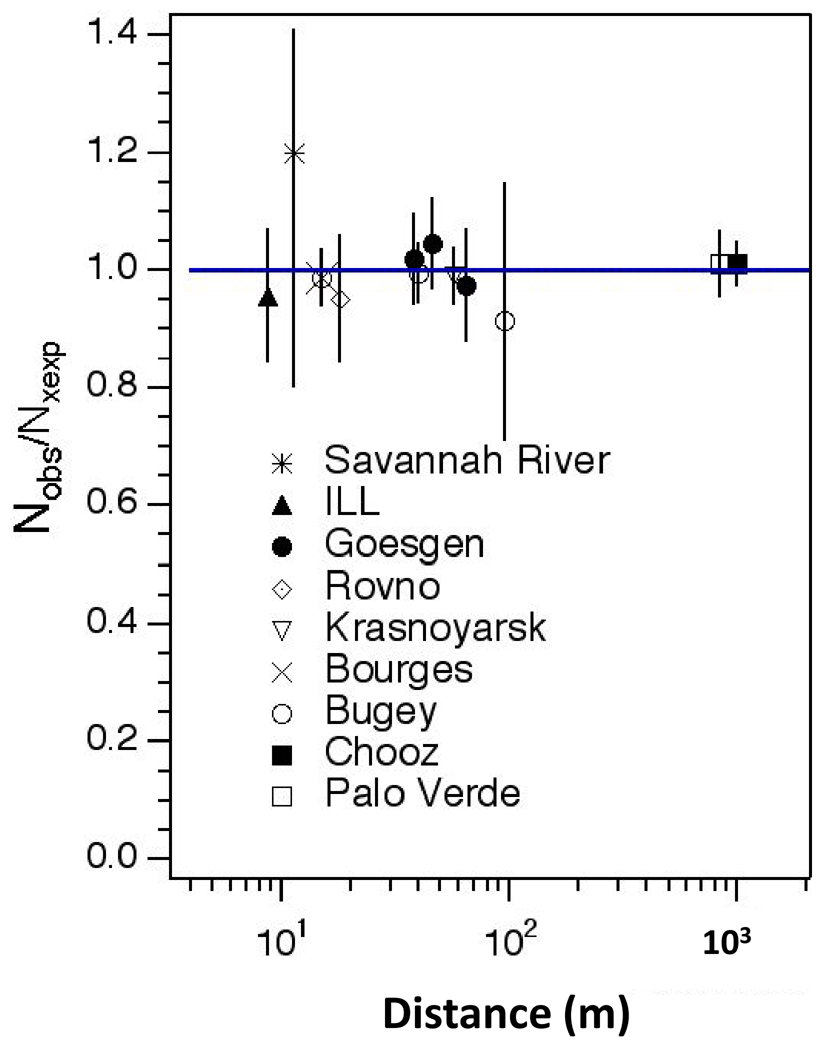
Coincidence signal: detect

- Prompt: e^+ annihilation → $E_\nu = E_{prompt} + \overline{E}_n + 0.8$ MeV
- Delayed: **n** capture 180 μs capture time



The $\bar{\nu}_e$ energy spectrum
Observed spectrum (interactions/MeV ton day)
10
5
0
2 3 4 5 6 7 8 9 10
E_ν (MeV)
$\nu_e + p \rightarrow n + e^+$ cross section ($\sim 10^{-42}$ cm^2)
Calculated reactor $\bar{\nu}_e$ spectrum (10^{-8} /s MeV GW$_{th}$)
Neutrinos with E<1.8 MeV are not detected

Previous Oscillation Searches



Previous Measurements

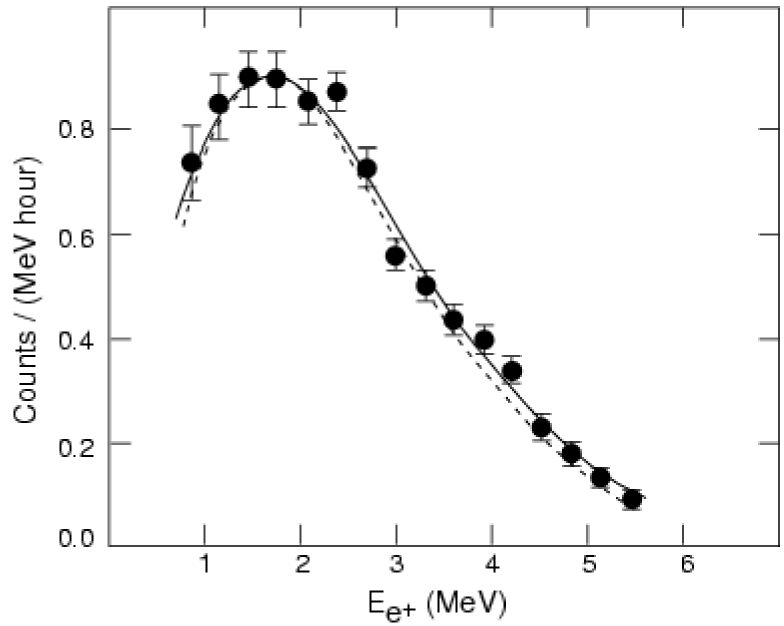
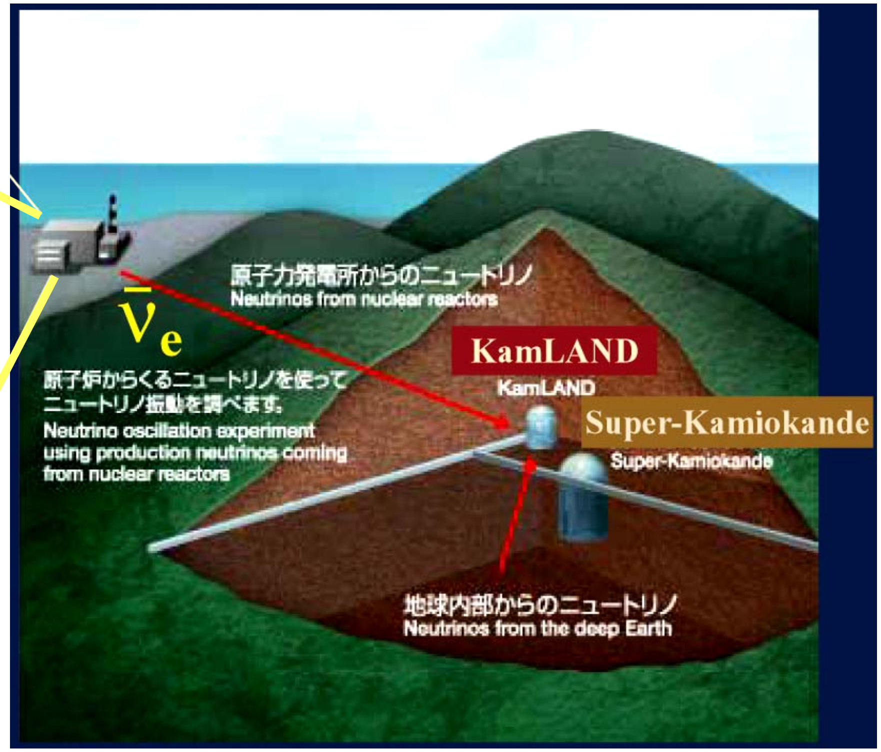


Figure 5: Positron spectrum measured at 45.9 m from the core of the Gösgen reactor [36]. Data points are obtained after background subtraction, errors are statistical only. The solid curve is a fit to the data assuming no oscillations. The dashed curve is derived independently by β-spectroscopy.

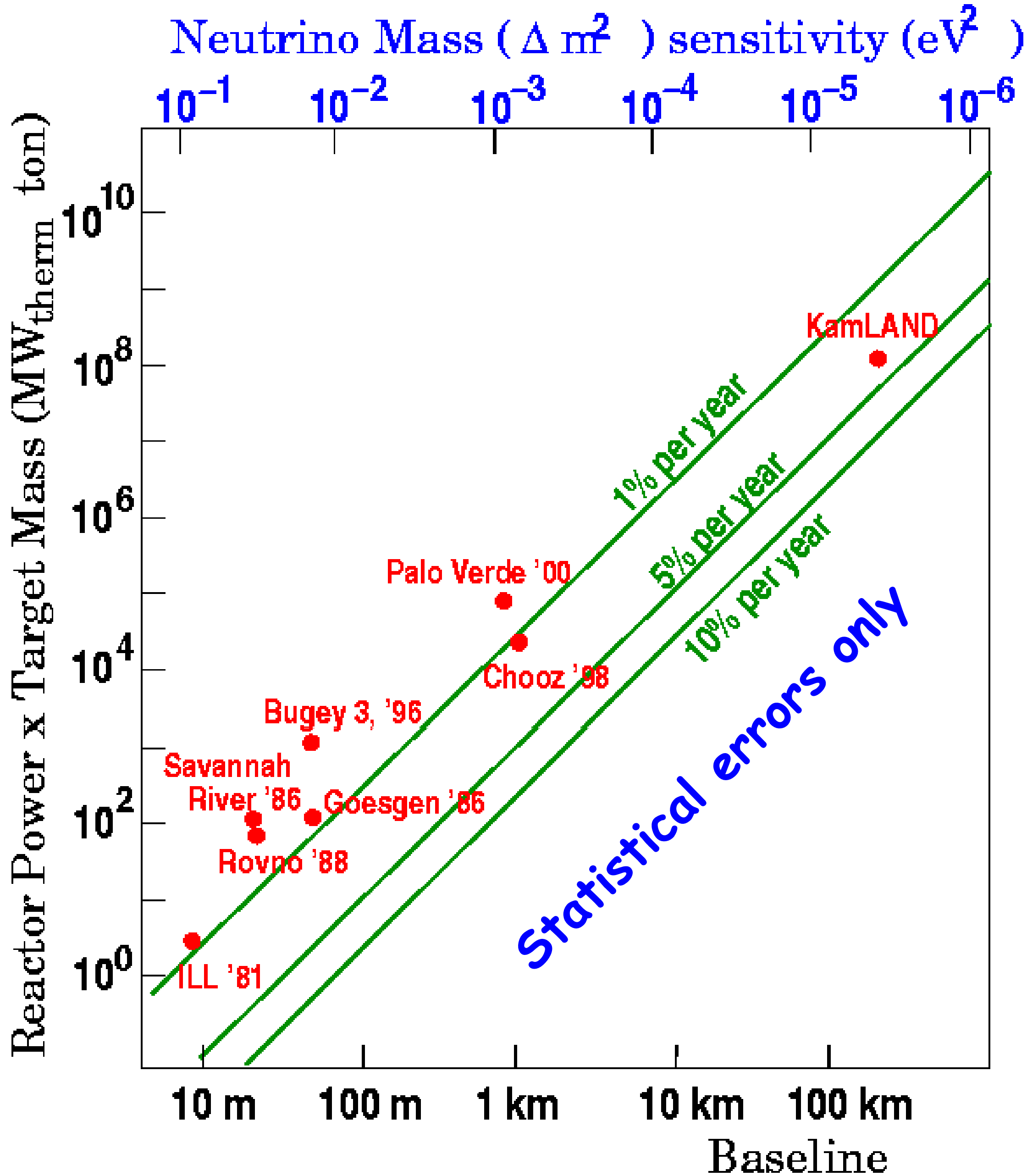
Flux and Energy Spectrum → ~1-2 %

Enter KamLAND

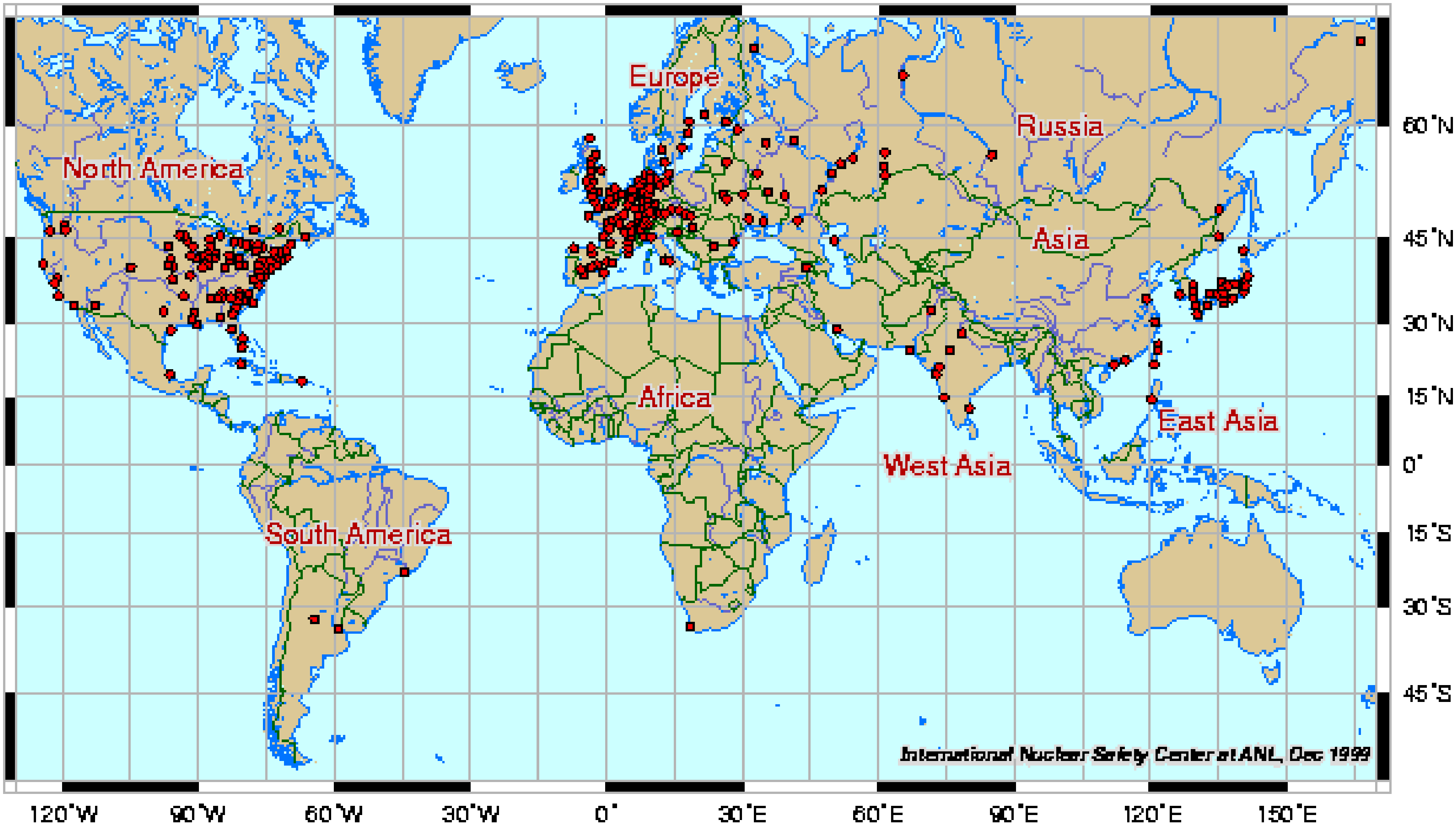


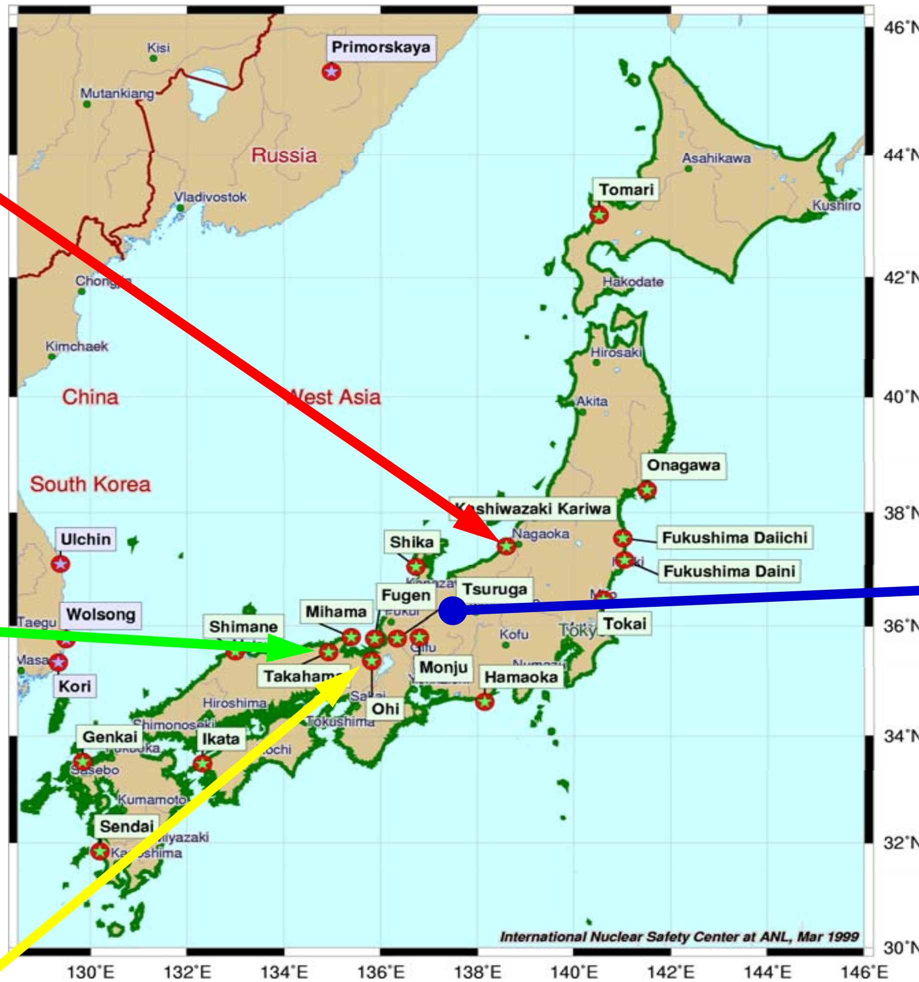
- Long Baseline (180 km)
- Calibrated source(s)
- Large detector (1 kton)
- Deep underground (2700 mwe)

Designed to
test solar neutrino
oscillation parameters
on Earth (!)
KamLAND has a much
longer baseline
than previous
(reactor) experiments



Only a few places in the World could host an experiment like KamLAND...





KamLAND uses the entire Japanese nuclear power industry as a long-baseline source

KamLAND

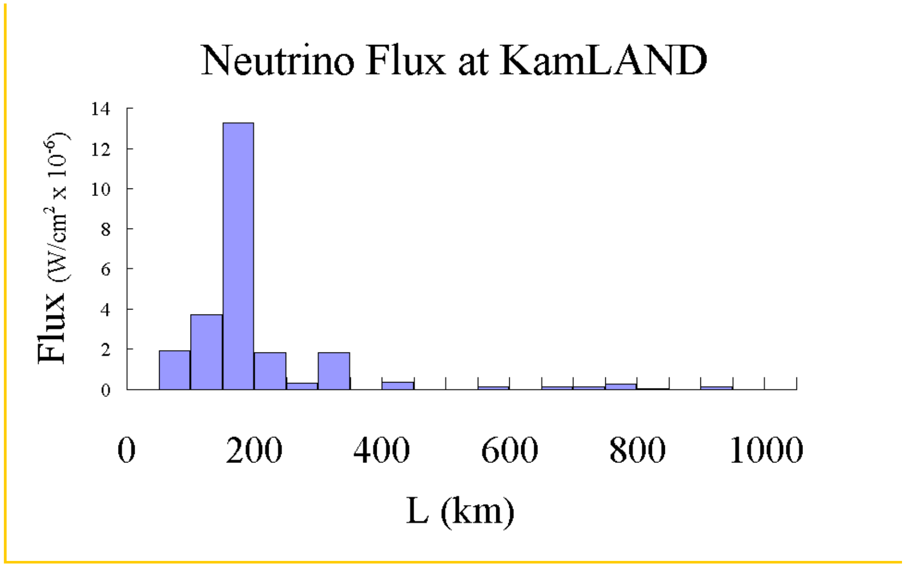
Kamioka Liquid Scintillator Anti-Neutrino Detector

Application: Neutrino signal from nuclear explosion in N. Korea

- 6 ν's/200 MeV $\rightarrow$ $2x10^{23}$ ν/ Terajoule
- 1kton TNT = 4 Terajoule
- $\rightarrow$ $8x10^{23}$ ν/kt
- KamLAND = 10^{32} protons
- R = 1000 km

$$\text{Signal} = \frac{N_\nu N_p \sigma}{4\pi R^2} = 0.003\ \nu/10\text{kt}$$

Narrow baseline range:
85.3% of signal has
140 km < L < 344 km



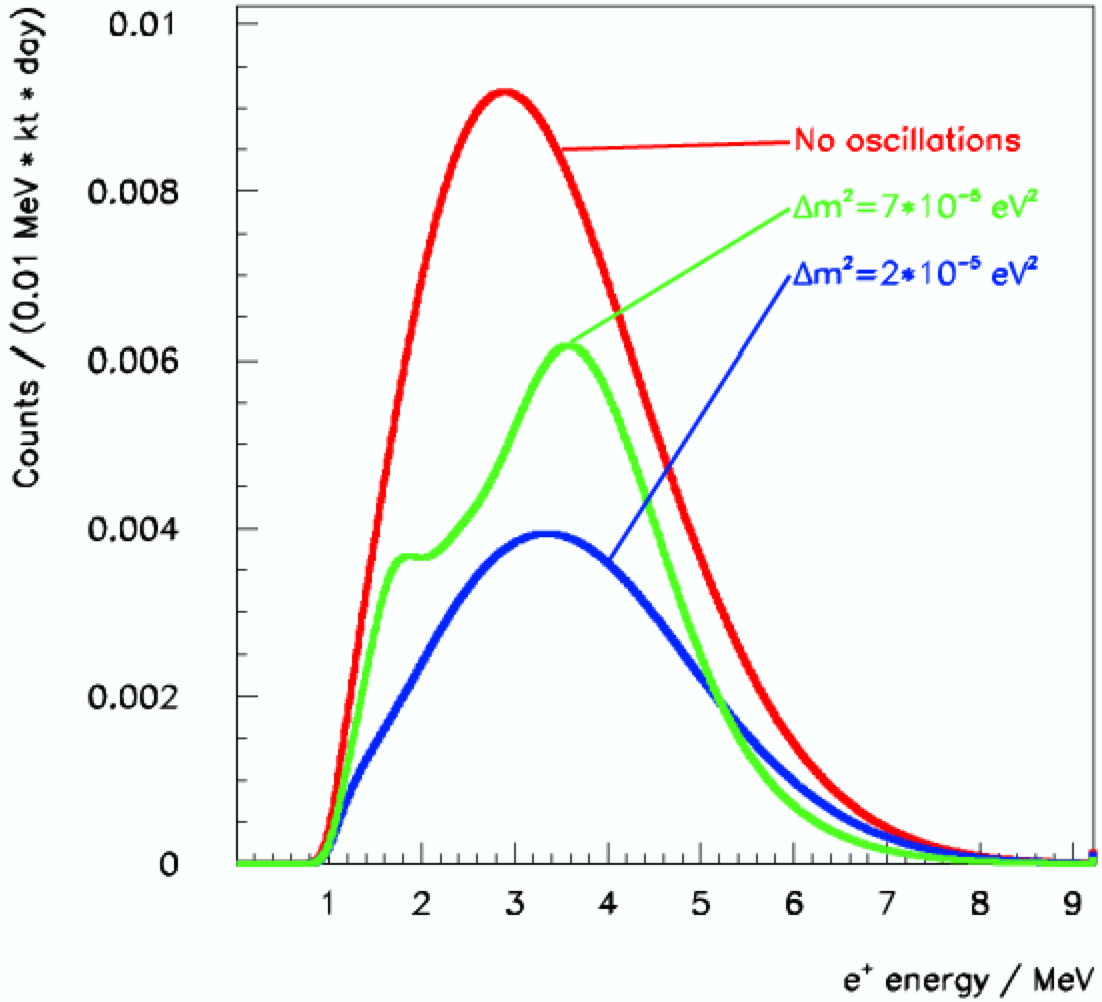
The total electric power produced "as a by-product" of the ν's is:

- ~60 GW or...
- ~4% of the world's manmade power or...
- ~20% of the world's nuclear power

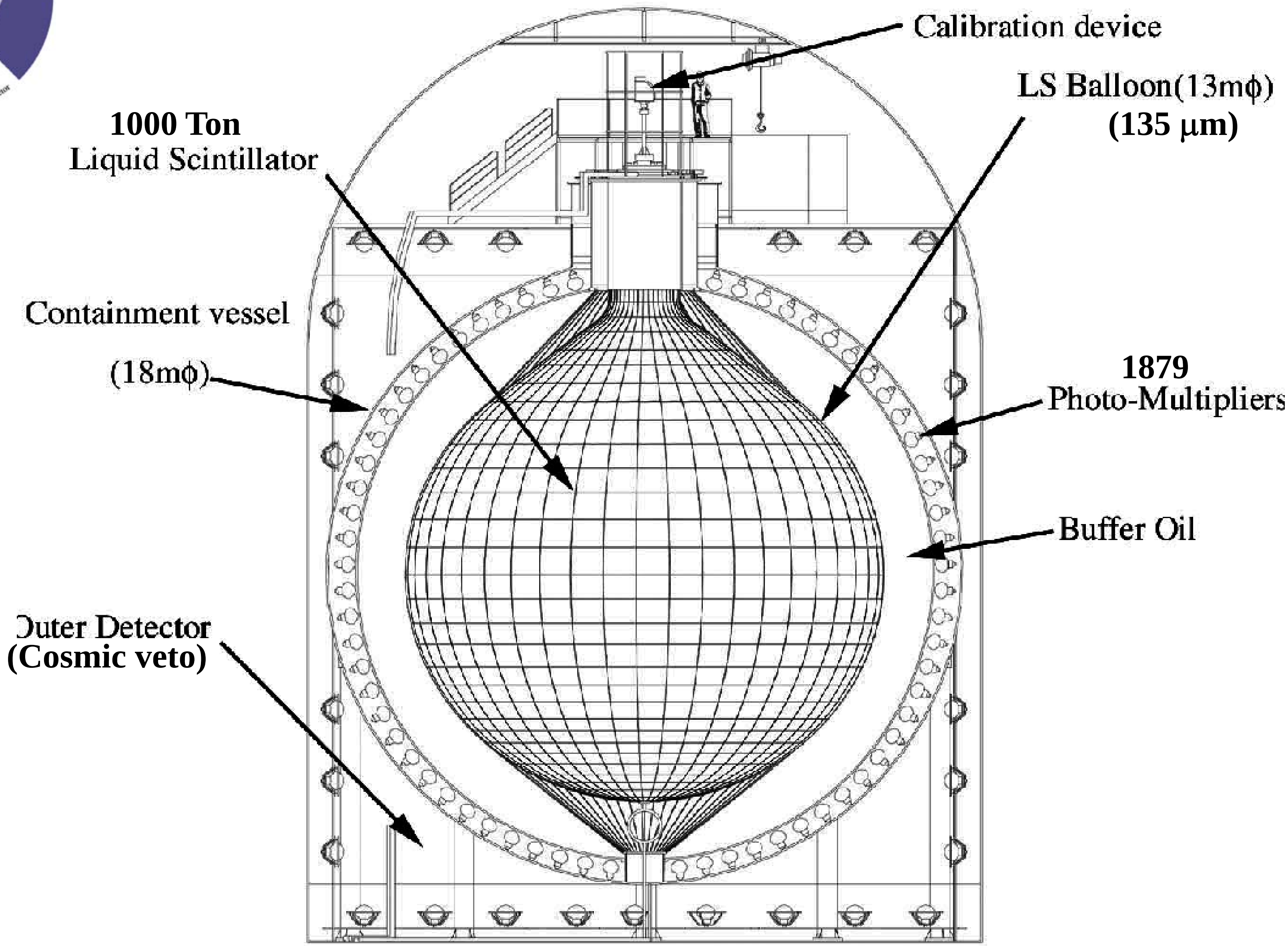
Neutrinos are "free of charge"!

Spectrum Distortion

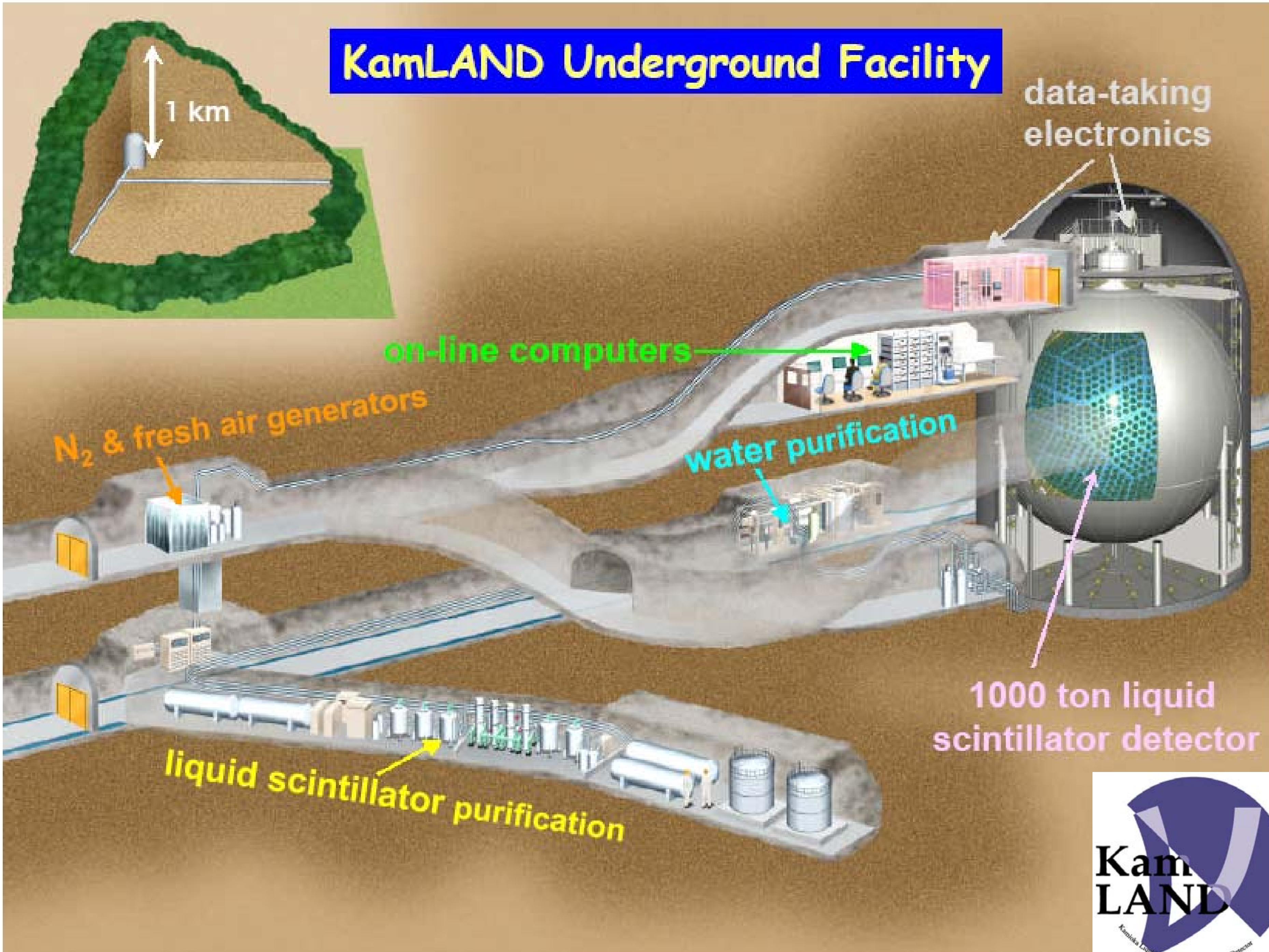
Neutrino oscillations in KamLAND could result in distortion of the energy spectrum along with a deficit of detected events



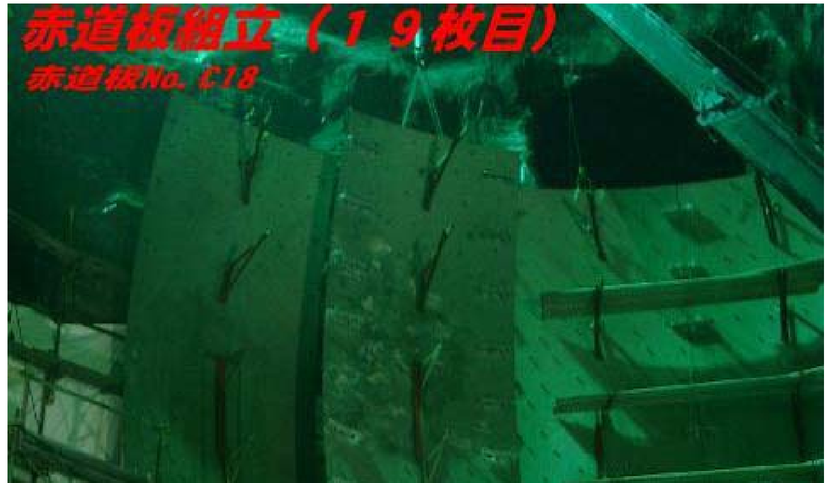
KamLAND Detector

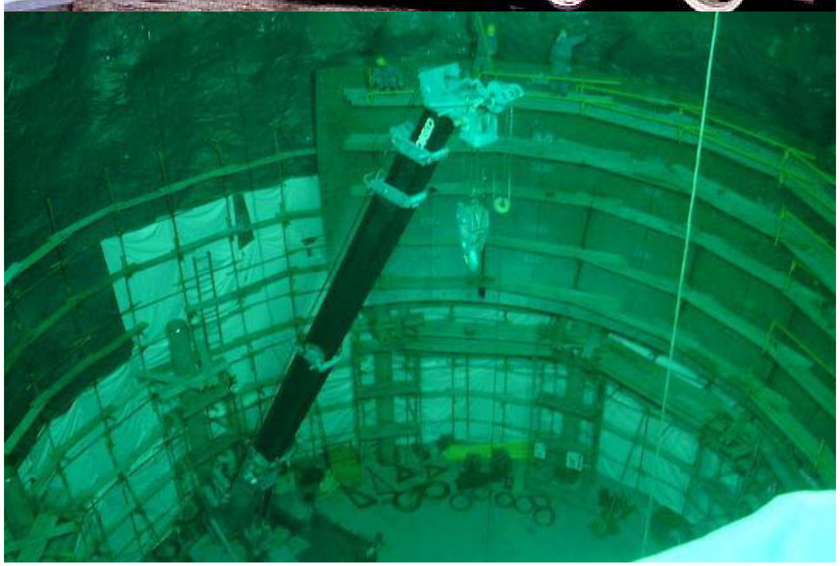


KamLAND Underground Facility

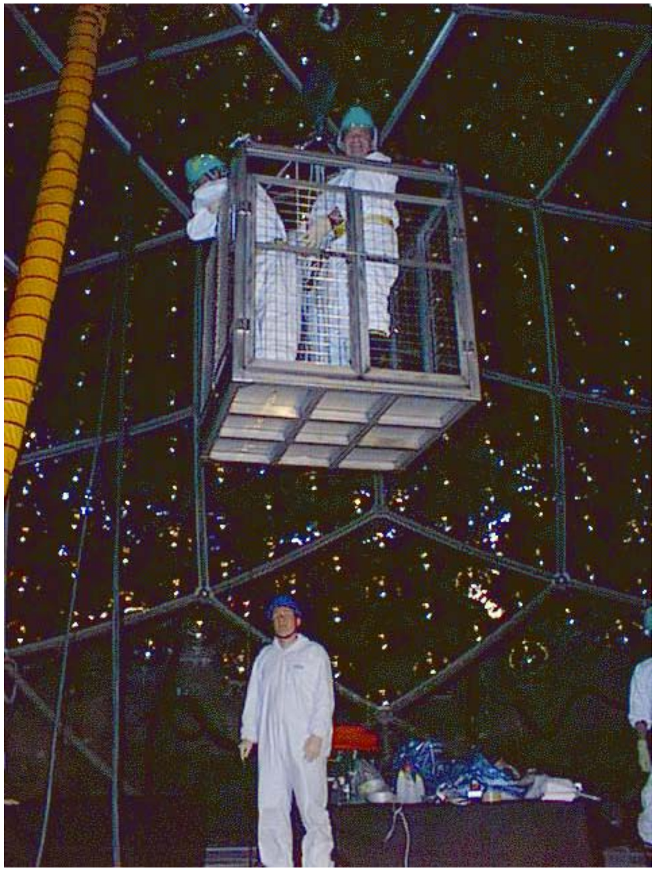


Sphere Construction

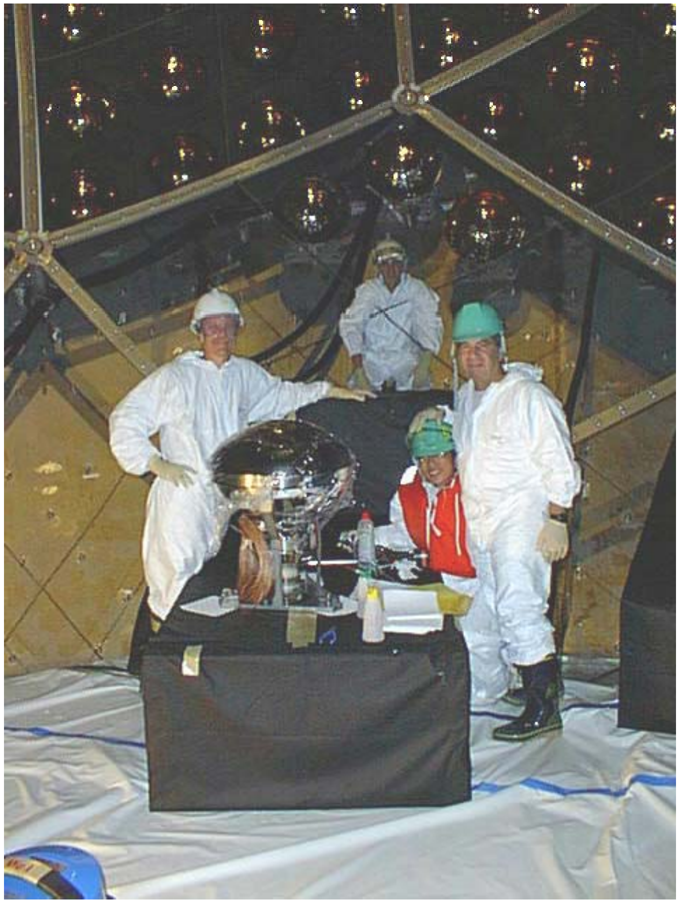




PMT Installation



PMT Installation



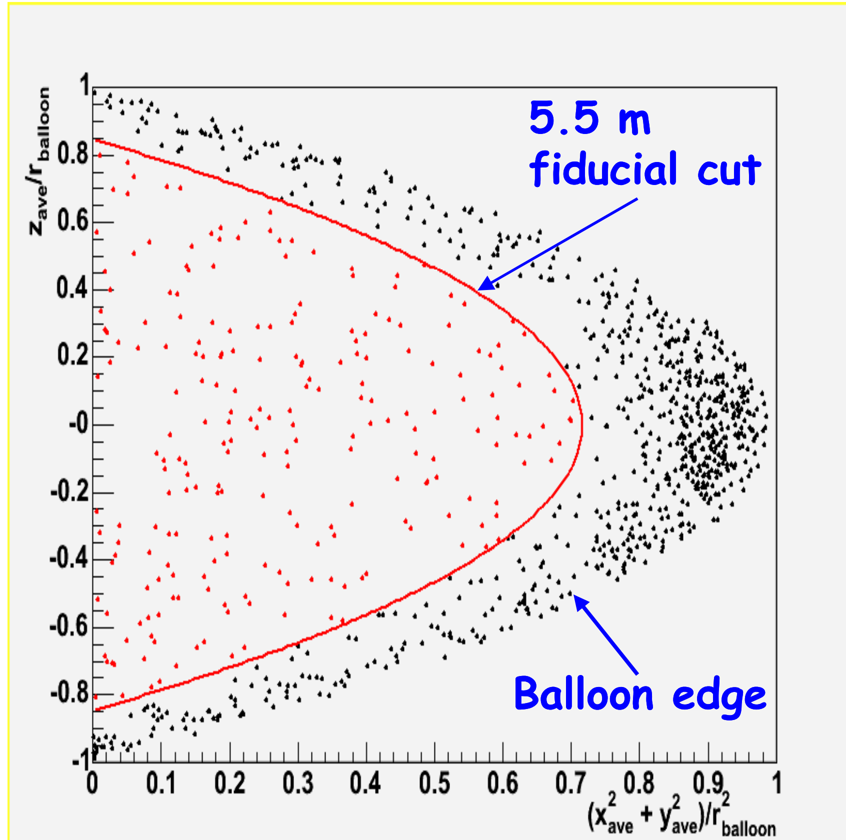
Kam
LAND
Kamioka Liquid scintillator Anti-Neutrino Detector

Filling the Detector

Selecting antineutrinos, E_{prompt}>2.6MeV

- $R_{prompt,\ delayed}$ < 5.5 m (543.7 ton)
- ΔR_{e-n} < 2 m
- 0.5 μs < ΔT_{e-n} < 1 ms
- 1.8 MeV < $E_{delayed}$ < 2.6 MeV
- 2.6 MeV < E_{prompt} < 8.5 MeV

Tagging efficiency 89.8%

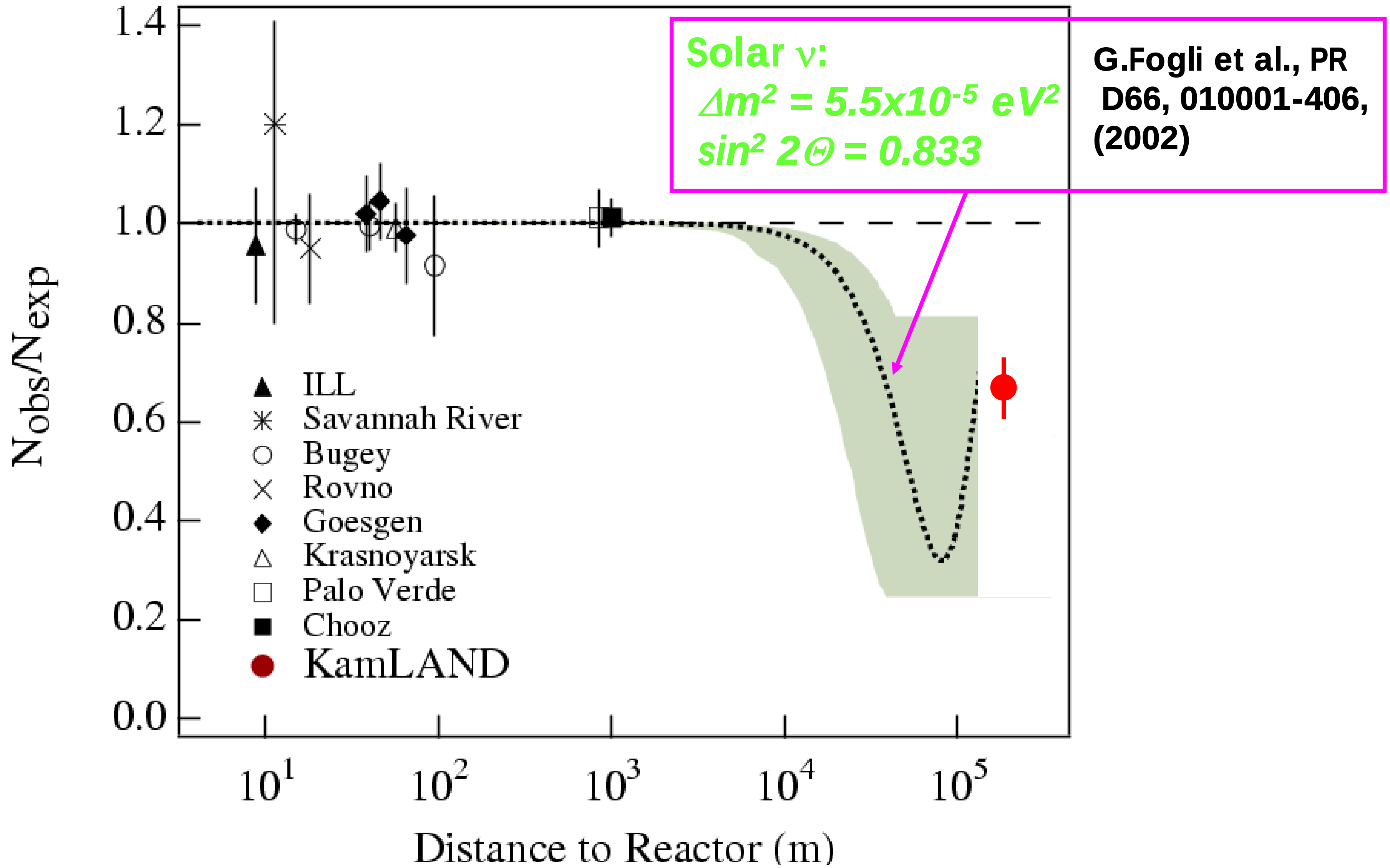


...In addition:

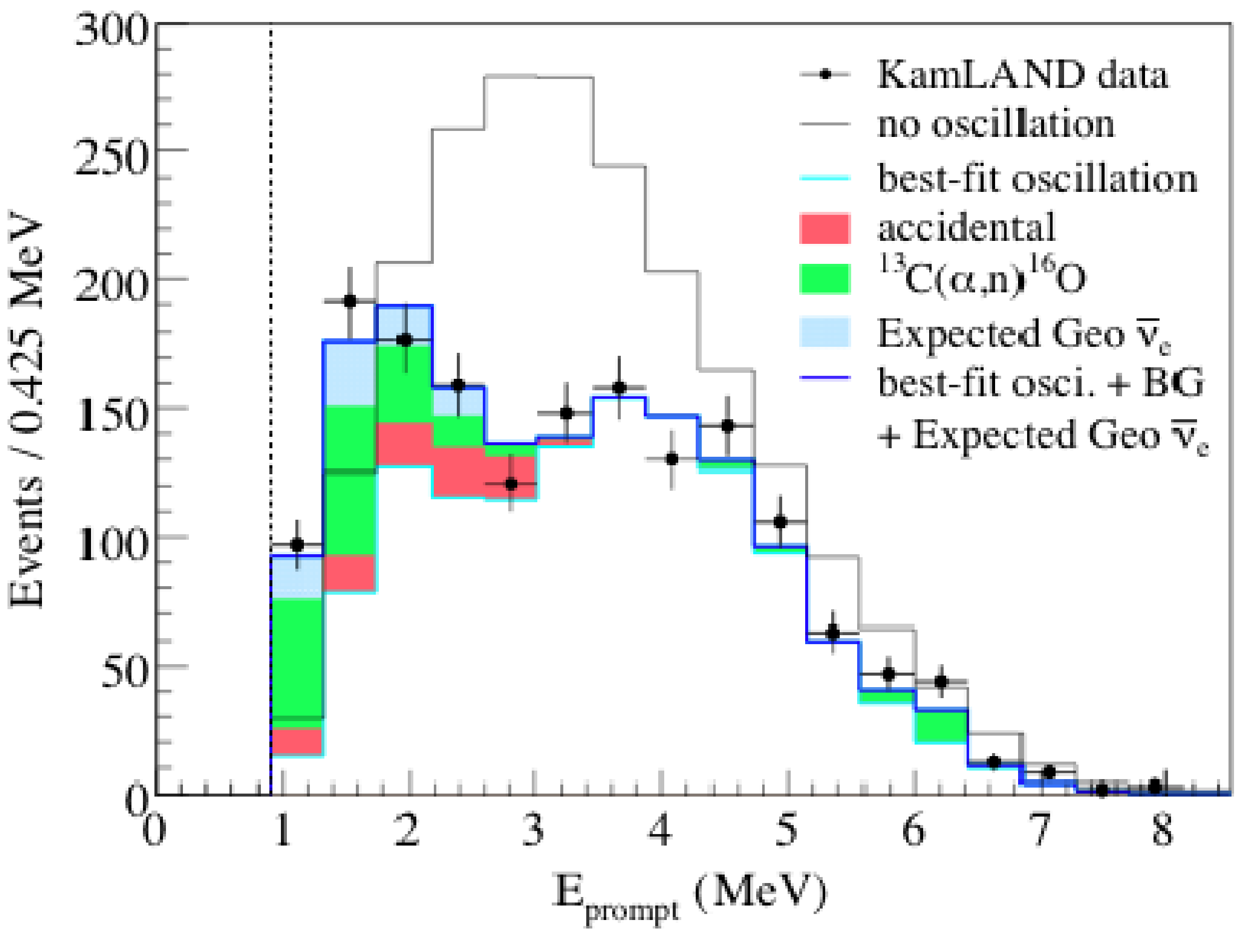
- 2s veto for showering/bad μ
- 2s veto in a R = 3m tube along track

Dead-time 9.7%

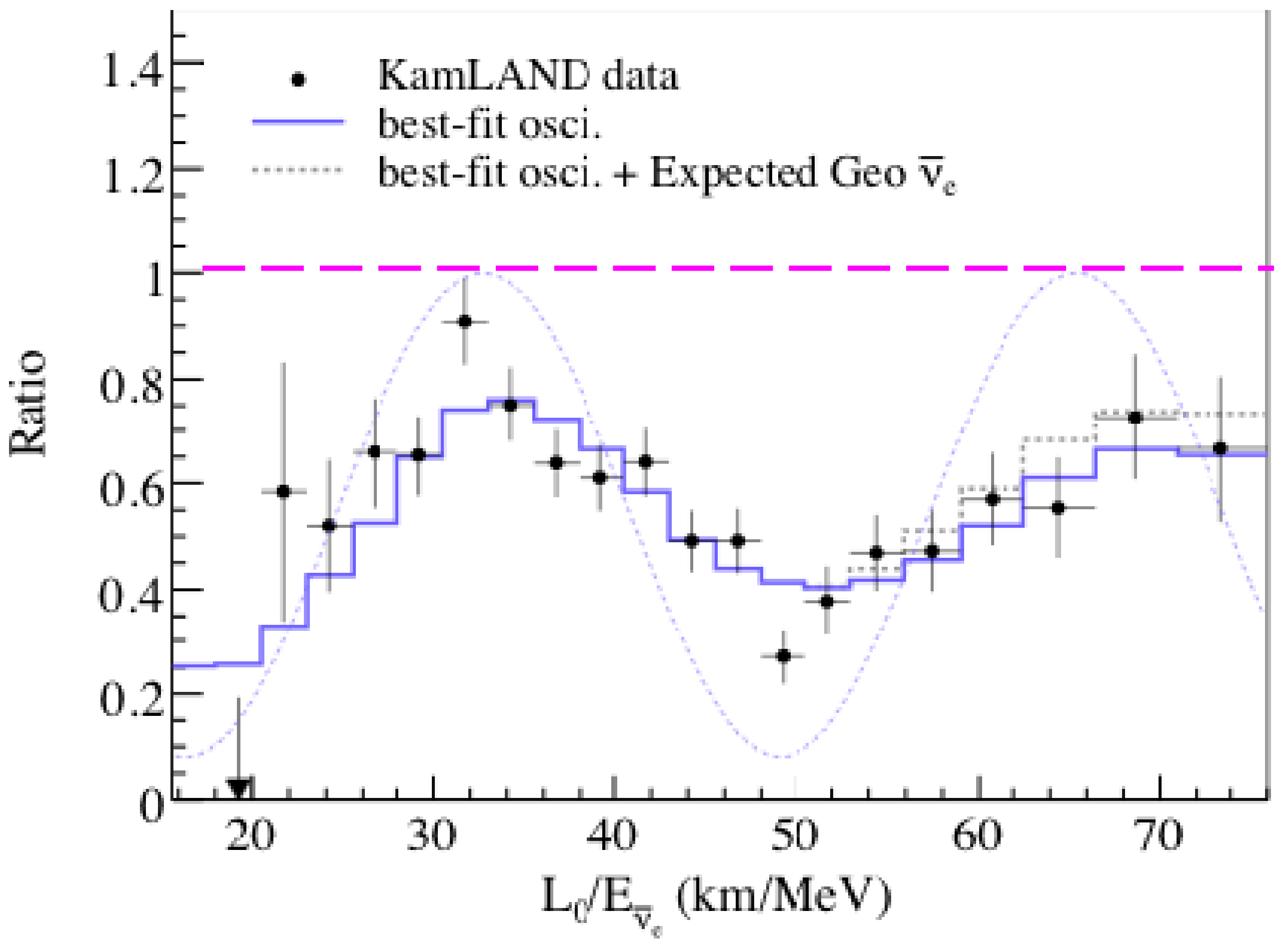
Ratio of Measured and Expected $\bar{\nu}_e$ Flux from Reactor Neutrino Experiments



Measurement of Energy Spectrum



Oscillation Effect



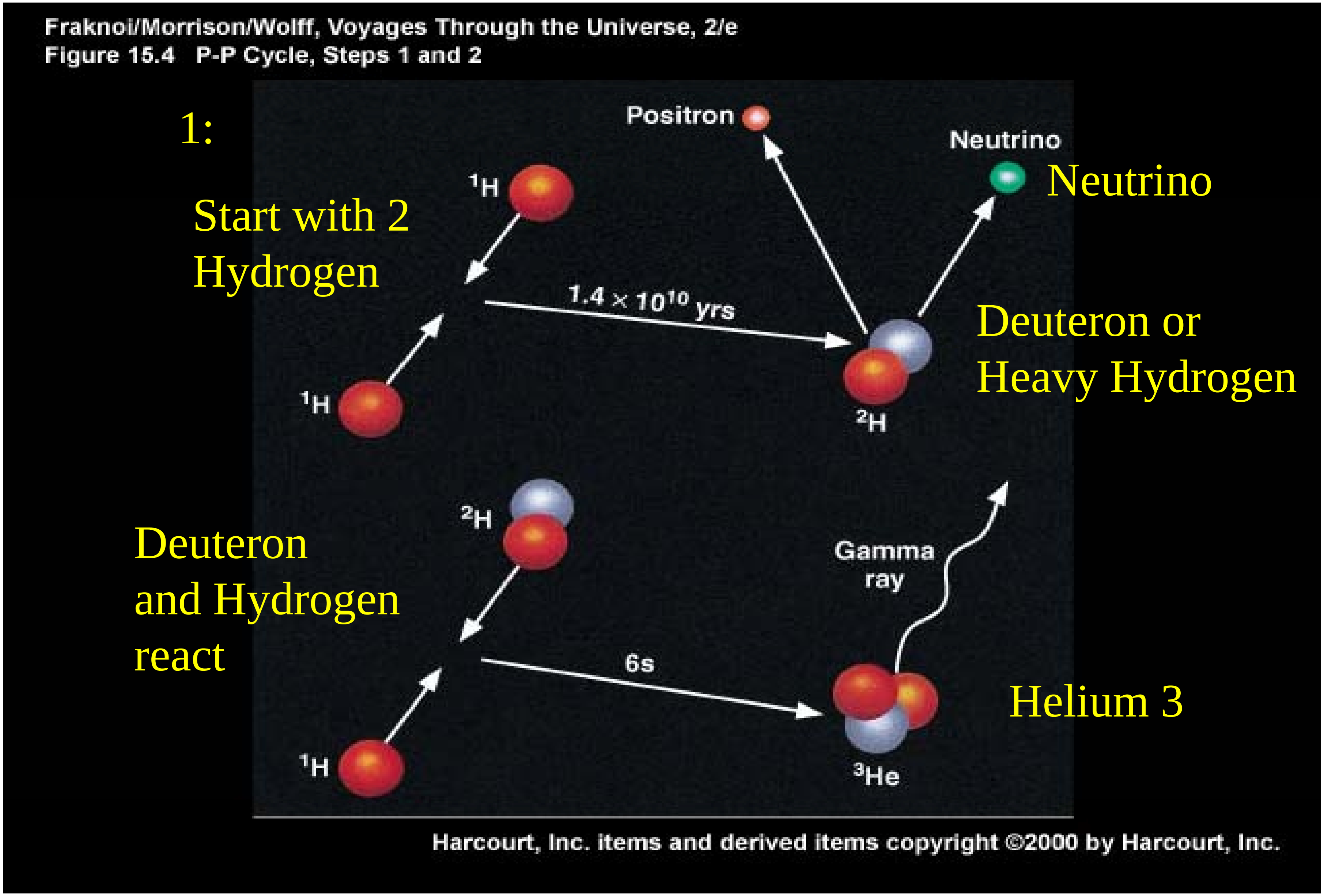
Δm^2 = 7:58x10^{-5} eV^2 (*Note SuperK result:* $\Delta m^2 = 2.5 \times 10^{-3}$ eV^2)

$\sin^2 2\theta = 0.92$

Why Does the Sun Shine?

- Sun ≡ Ball of H gas compressed by gravity → HEAT (not enough)
- HEAT → Nuclear fusion

$$p + p \rightarrow d + e^{+} + \nu + \text{energy}$$


Fraknoi/Morrison/Wolff, Voyages Through the Universe, 2/e
Figure 15.4 P-P Cycle, Steps 1 and 2
1:
Start with 2 Hydrogen
Positron
Neutrino
Neutrino
^{1}H
^{1}H
1.4 × 10^{10} yrs
^{2}H
Deuteron or Heavy Hydrogen
Deuteron and Hydrogen react
^{2}H
^{1}H
6s
Gamma ray
^{3}He
Helium 3
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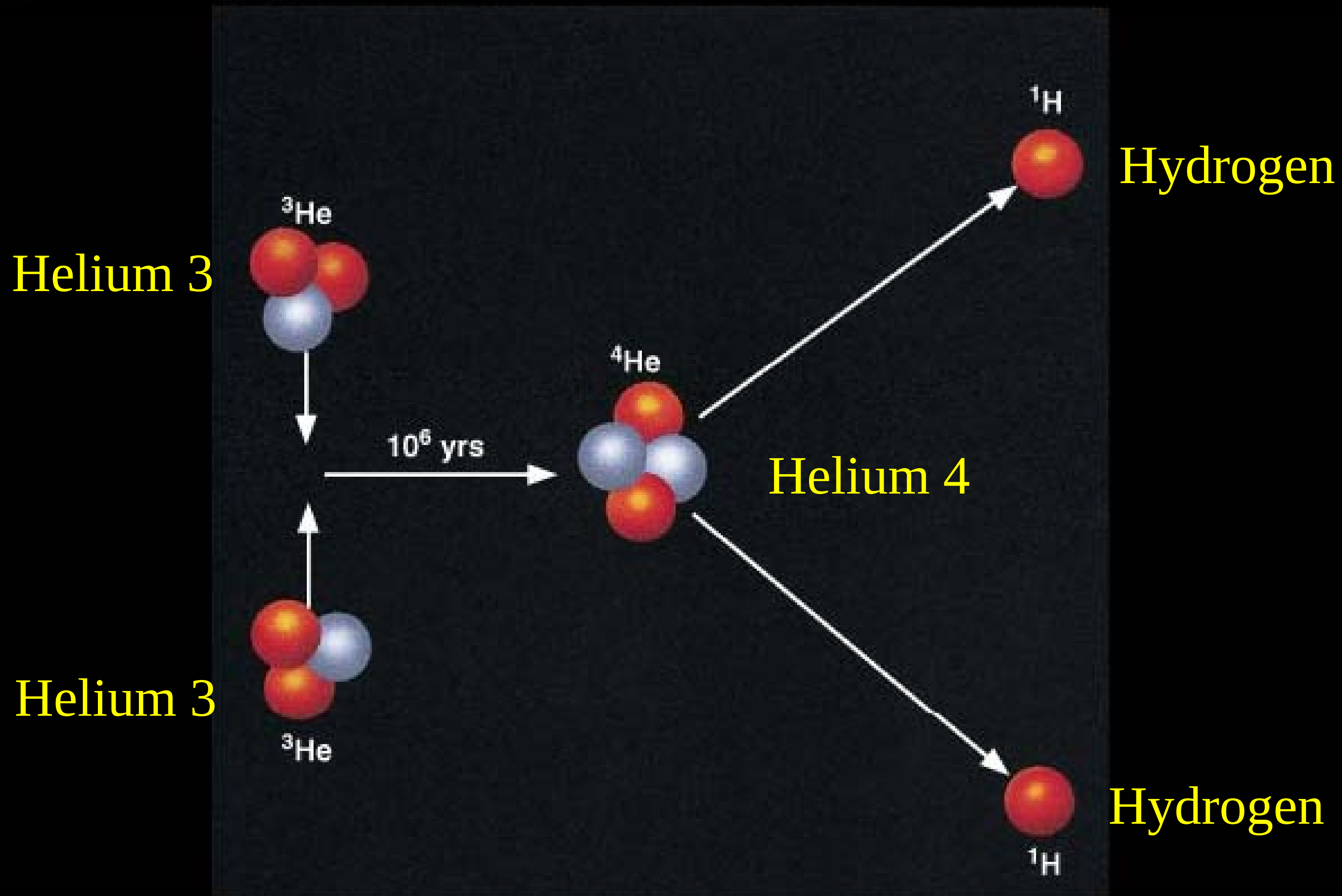
Fraknoi/Morrison/Wolff, Voyages Through the Universe, 2/e

Figure 15.5 P-P Cycle, Step 3



Solar Properties

$L_{\odot} = 3.8 \times 10^{26}$ W $= 2.4 \times 10^{39}$ MeV/s

$M_{\odot} = 2 \times 10^{30}$ kg $= 10^{57}$ M_p

Solar Fusion Cycle:

$2e^- + 4p \rightarrow {}^4He + 2\nu + 26.7$ MeV (thermal)

R_ν = Neutrino rate = $2\,L_{\odot}/26.7$MeV $= 1.8 \times 10^{38}$ ν/sec

Flux at earth = $R_\nu/(4\pi R^2) = 6 \times 10^{10}/\text{cm}^2/\text{sec}$

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SOLAR NEUTRINO FLUX*

The discovery by Holmgren and Johnston (1958, 1959) of an unexpectedly large cross-section for the $He^3(\alpha, \gamma)Be^7$ reaction led to studies by Fowler (1958) and Cameron (1958) which showed that the proton-proton chain in the present sun is frequently completed by a series of reactions involving Be^7. Fowler and Cameron also discussed the possibility that the decay of B^8, formed by $Be^7(p, \gamma)B^8$ reactions in the interior of the sun, produces a terrestrially measurable flux of high-energy neutrinos ($0 < E_\nu < 14$ Mev). The detection of solar neutrinos is the only experiment that we can think of which could provide *direct* evidence of specific nuclear reactions occurring in the interior of a star.

We have made use of recently obtained accurate values for the Be^7 electron-capture cross-section (Bahcall 1962) and the Be^7 formation cross-section (Parker and Kavanagh 1962) to make a detailed calculation of the expected B^8 solar neutrino flux. Other relevant nuclear cross-sections have been taken from the report of Fowler (1960). The cross-section constants, corrected for shielding factors, which we have used are, in units of kev-barns, as follows: $S_{11} = 3.5 \times 10^{-22}$, $S_{33} = 1300$, $S_{34} = 0.5$, $S_{17} = 0.03$. The Be^7 decay rate is

$$\lambda_e(Be^7) = 2.12 \times 10^{-9} \rho (1 + x_H) T_6^{-1/2} \text{ sec}^{-1} .$$

The rate of neutrino emission per gram has been integrated over a new model for the interior of the present sun (Iben and Sears 1962); this model has a central temperature of 16.2×10^6 ° K, a central density of 142 gm/cm^3, and a central composition $x_H = 0.333$, $x_{He} = 0.633$, compared with a surface composition of $x_H = 0.630$, $x_{He} = 0.336$. The opacity and energy-generation rates with B^8 reactions included were taken from the work of Iben and Ehrman (1962); we find that 1.0×10^{35} high-energy neutrinos are generated in the sun per second and that the expected neutrino flux at the earth from B^8 decays in the sun is

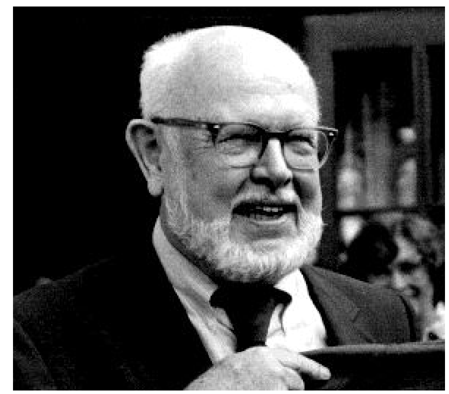
$$\phi_\nu(B^8) = 3.6 \times 10^7 \text{ neutrinos cm}^{-2} \text{ sec}^{-1} .$$

The neutrino generation corresponds approximately to 1 neutrino for every 1500 proton-proton reactions, and the flux should be compared with the value of 6.4×10^{10} low-energy neutrinos cm^{-2} sec^{-1} from the pp-reaction and the Be^7-decays. The flux is a factor of 10 less than could be detected with current experimental techniques using the $Cl^{37}(\nu, e)A^{37}$ reaction and a detector consisting of 10^5 gallons of perchlorethylene (Davis 1962).

However, Davis (1962) has pointed out to us that the more energetic Be^7 neutrinos ($E_\nu = 0.861$ MeV, 88 per cent; 0.383 MeV, 12 per cent) are just above threshold for detection by Cl^{37} absorption ($Q = -0.814$ MeV). The Be^7 solar neutrino flux above the Cl^{37} threshold is

$$\phi_\nu(Be^7; 0.861 \text{ MeV}) = 1.0 \times 10^{+10} \text{ cm}^{-2} \text{ sec}^{-1} .$$

Since the Cl^{37} neutrino-absorption cross-section for the 0.861-MeV Be^7 neutrinos is about a factor of 200 less than the average absorption cross-section for B^8 neutrinos, about one-half of the detectable solar neutrinos are from B^8 decays, according to the model of Iben and Sears (1962).



J. N. BAHCALL
WILLIAM A. FOWLER
I. IBEN, JR.
R. L. SEARS

December 1, 1962

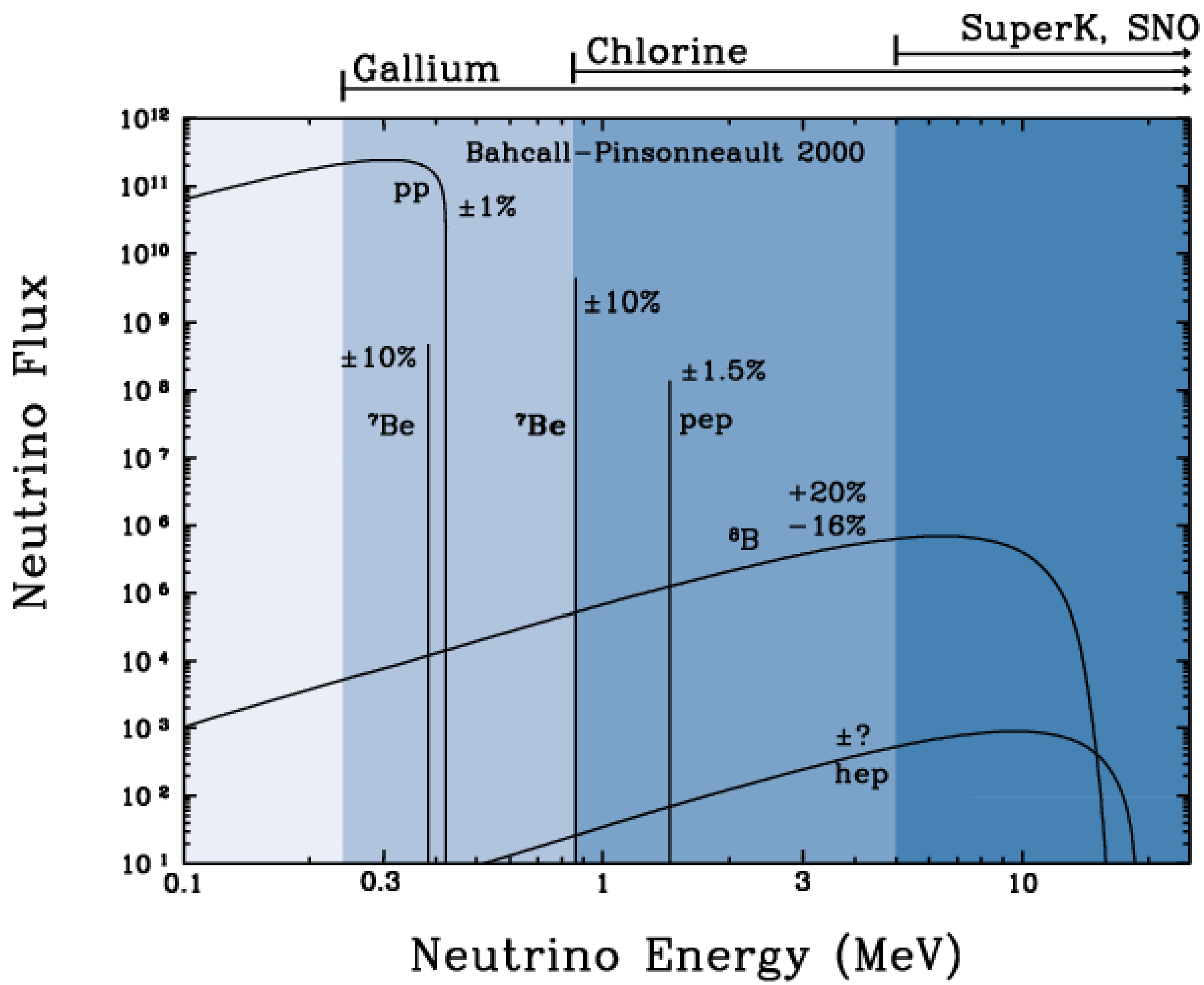
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

Solar Neutrinos

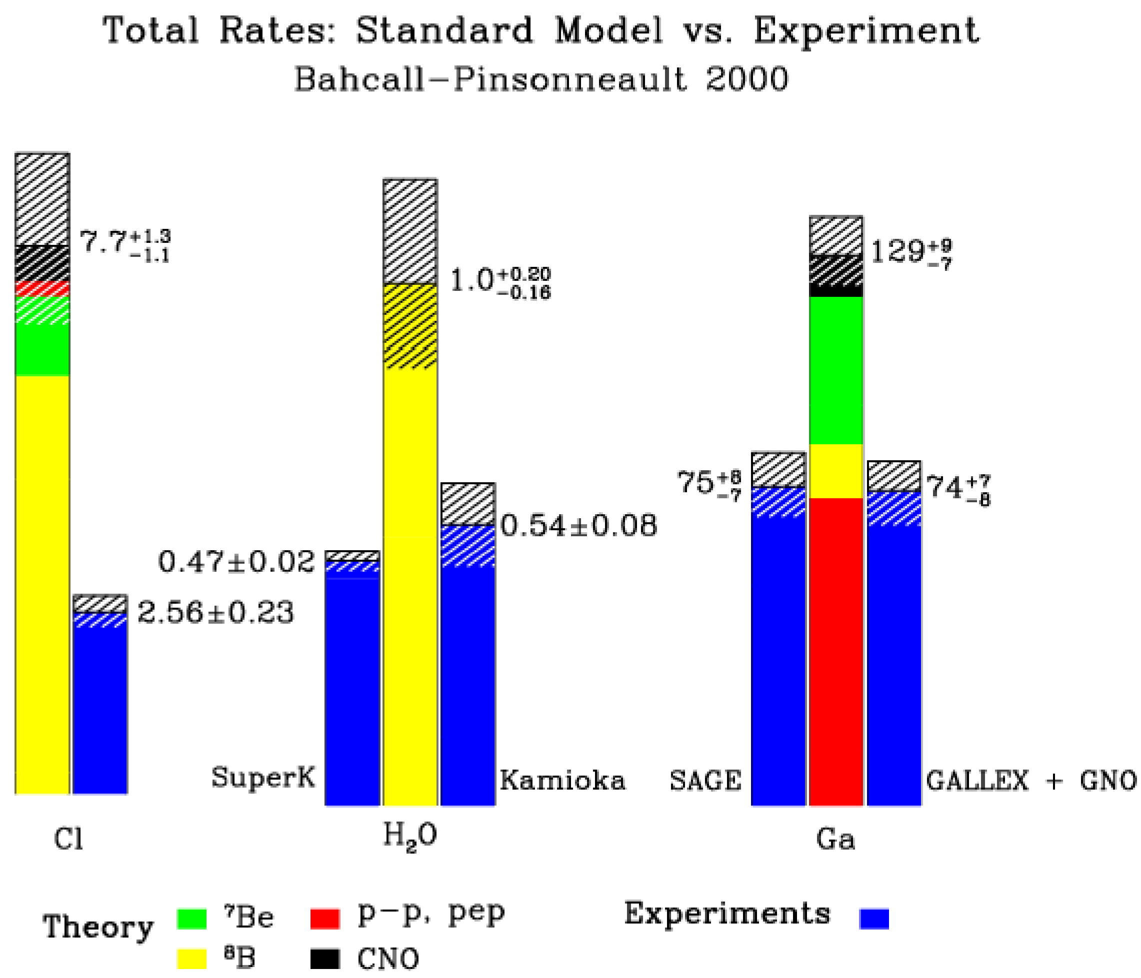
REACTION	TERM (%)	ν ENERGY (MeV)
$p + p \rightarrow {}^{2}H + e^{+} + \nu_e$	(99.96)	≤ 0.423
or		
$p + e^{-} + p \rightarrow {}^{2}H + \nu_e$	(0.44)	1.445
${}^{2}H + p \rightarrow {}^{3}He + \gamma$	(100)	
${}^{3}He + {}^{3}He \rightarrow \alpha + 2p$	(85)	
or		
${}^{3}He + {}^{4}He \rightarrow {}^{7}Be + \gamma$	(15)	
${}^{7}Be + e^{-} \rightarrow {}^{7}Li + \nu_e$	(15)	$\begin{cases} 0.863 & 90\% \\ 0.385 & 10\% \end{cases}$
${}^{7}Li + p \rightarrow 2\alpha$		
or		
${}^{7}Be + p \rightarrow {}^{8}B + \gamma$	(0.02)	
${}^{8}B \rightarrow {}^{8}Be^{*} + e^{+} + \nu_e$		< 15
${}^{8}Be^{*} \rightarrow 2\alpha$		
or		
${}^{3}He + p \rightarrow {}^{4}He + e^{+} + \nu_e$	(0.00003)	< 18.8

Neutrino terminations from BP2000 solar model. Neutrino energies include solar corrections: J. Bahcall, Phys. Rev. C, 56, 3391 (1997).

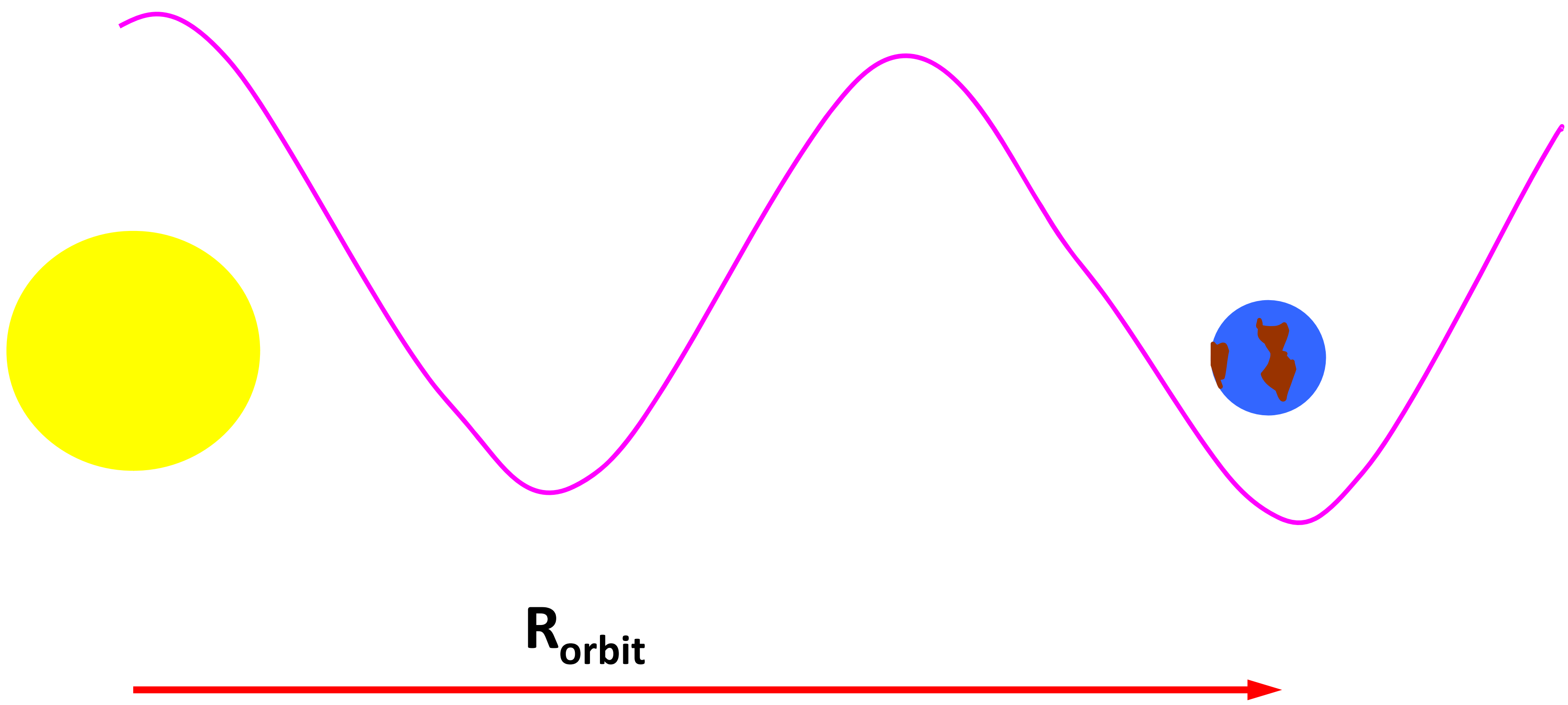
Solar Neutrino Energy Spectrum



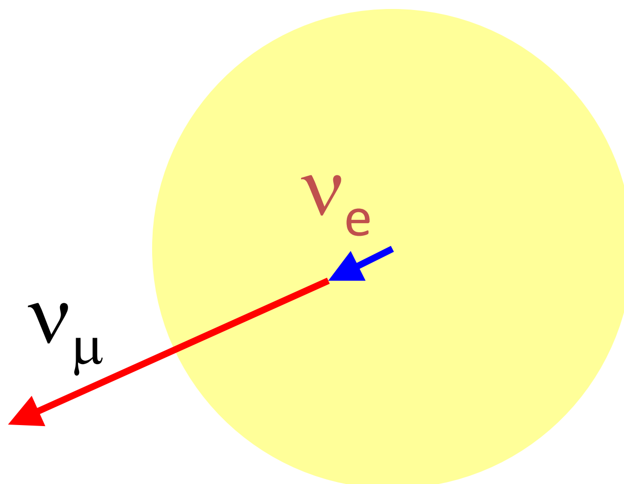
Missing Solar Neutrinos…



Neutrino "Oscillations"?



Solar Neutrino Disappearance



Matter Effect:

$$\nu_e + e^- \neq \nu_\mu + e^-$$

→ resonant matter oscillations

10.4 Matter Enhanced Oscillations

In dense matter (like the solar interior), there is a substantial density of electrons present. The ν_e interact with electrons differently than do the ν_μ. This affects the phase slippage of ν_e relative to ν_μ and can have a dramatic effect on the flavor transformation dynamics.

It is useful to recast the oscillation formulae in a matrix form as follows. (We will simplify this discussion to just 2 flavors.) The transformation between basis states can be written ($C \equiv \cos\theta,\ S \equiv \sin\theta$)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} C & S \\ -S & C \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \tag{10.37}$$

with the inverse transformation

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} C & -S \\ S & C \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}. \tag{10.38}$$

The time evolution of the mass eigenstates can be written

$$|\nu_i(t)\rangle = e^{-i(E_i-p)t}|\nu_i(0)\rangle \tag{10.39}$$

$$= e^{-im_i^2 t/2p}|\nu_i(0)\rangle; \quad i = 1, 2, \tag{10.40}$$

or

$$\frac{d|\nu_i\rangle}{dt} = -i\frac{m_i^2}{2p}|\nu_i\rangle. \tag{10.41}$$

Therefore, we can express the time evolution in the $\{\nu_e,\ \nu_\mu\}$ basis as

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}C & S\\ -S & C\end{pmatrix} i\frac{d}{dt}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} \tag{10.42}$$

$$= \frac{1}{2p}\begin{pmatrix}C & S\\ -S & C\end{pmatrix}\begin{pmatrix}m_1^2 & 0\\ 0 & m_2^2\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} \tag{10.43}$$

$$= \frac{1}{2p}\begin{pmatrix}C^2m_1^2+S^2m_2^2 & CS\Delta m^2\\ CS\Delta m^2 & C^2m_2^2+S^2m_1^2\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}. \tag{10.44}$$

The effect of dense matter is then included by modifying the diagonal matrix element

$$C^2m_1^2+S^2m_2^2 \to C^2m_1^2+S^2m_2^2+2\sqrt{2}G_Fn_ep \tag{10.45}$$

where n_e is the number of e^- per unit volume. If we now define

$$L_0 \equiv \frac{2\pi}{\sqrt{2}G_Fn_e} \tag{10.46}$$

the solutions (for fixed n_e) can be rewritten as

$$|\langle\nu_e|\nu(t)\rangle|^2 = 1-\sin^2 2\theta_m \sin^2\frac{\pi x}{L_m} \tag{10.47}$$

$$|\langle\nu_\mu|\nu(t)\rangle|^2 = \sin^2 2\theta_m \sin^2\frac{\pi x}{L_m} \tag{10.48}$$

with

$$L_m \equiv L\left[1-2\frac{L}{L_0}\cos 2\theta+\left(\frac{L}{L_0}\right)^2\right]^{-\frac{1}{2}} \tag{10.49}$$

The corresponding matter oscillation length [28] is therefore

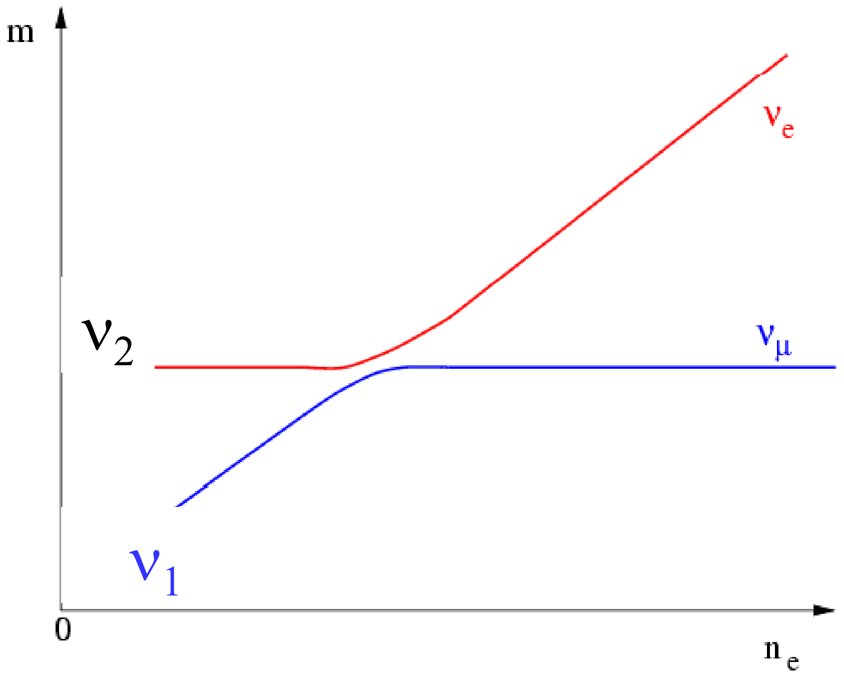
$$L_0 = \frac{2\pi}{\sqrt{2} G_F N_e} \simeq \frac{1.7 \times 10^7 (\mathrm{m})}{\rho(\mathrm{g/cm^3})\ Y_e} \ . \tag{21}$$

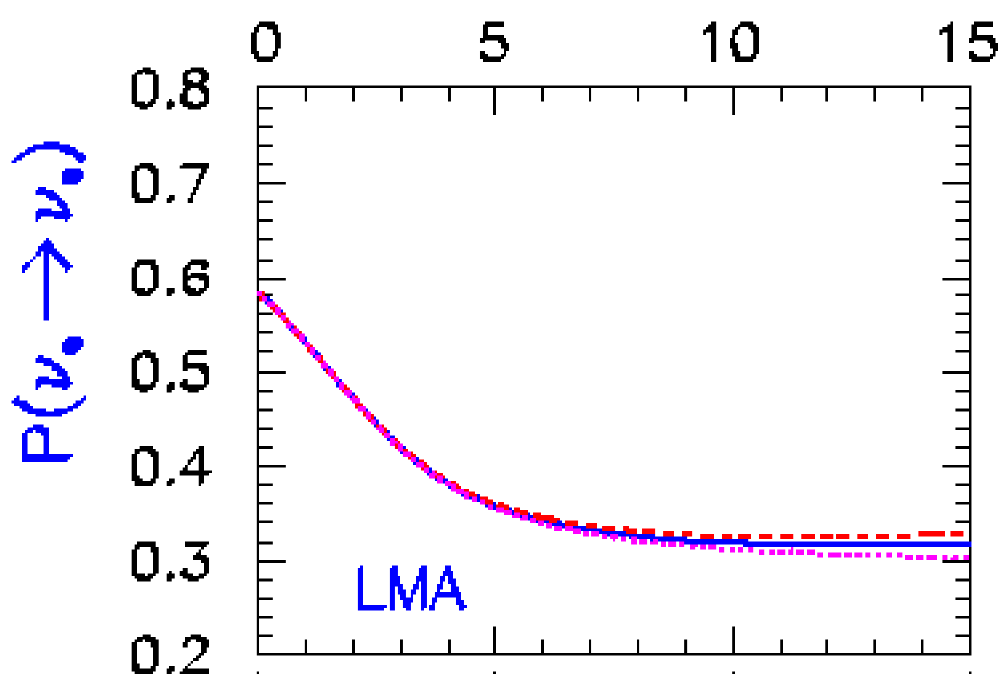
Unlike the vacuum oscillation length, Eq.(8), the matter oscillation length L_0 is independent of the neutrino energy. Note that the matter oscillation length in rock is $L_0 \approx 10^4$ km, and in the center of the Sun $L_0 \approx 200$ km.

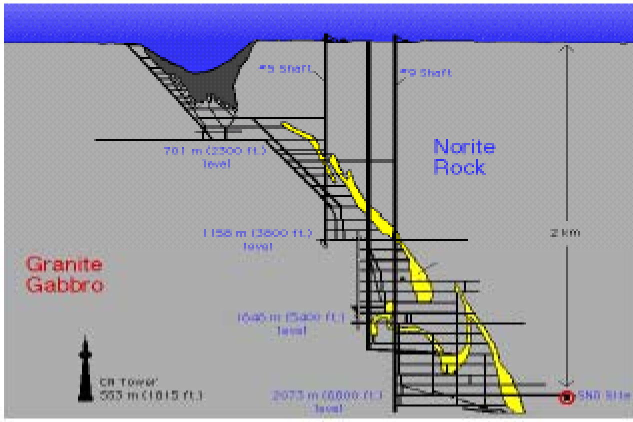
Matter Enhanced Oscillation (MSW)

Mikheyev, Smirnov, Wolfenstein

$$\mathcal{H} = \mathcal{H}_V + \mathcal{H}_M(r)$$
$$= \frac{\Delta m_\odot^2}{4E}\begin{bmatrix} -\cos 2\theta_\odot & \sin 2\theta_\odot \\ \sin 2\theta_\odot & \cos 2\theta_\odot \end{bmatrix} + \begin{bmatrix} V(r) & 0 \\ 0 & 0 \end{bmatrix}$$
$$V = \sqrt{2}\, G_F N_e$$







Sudbury Neutrino Observatory

1000 tonnes D_2O

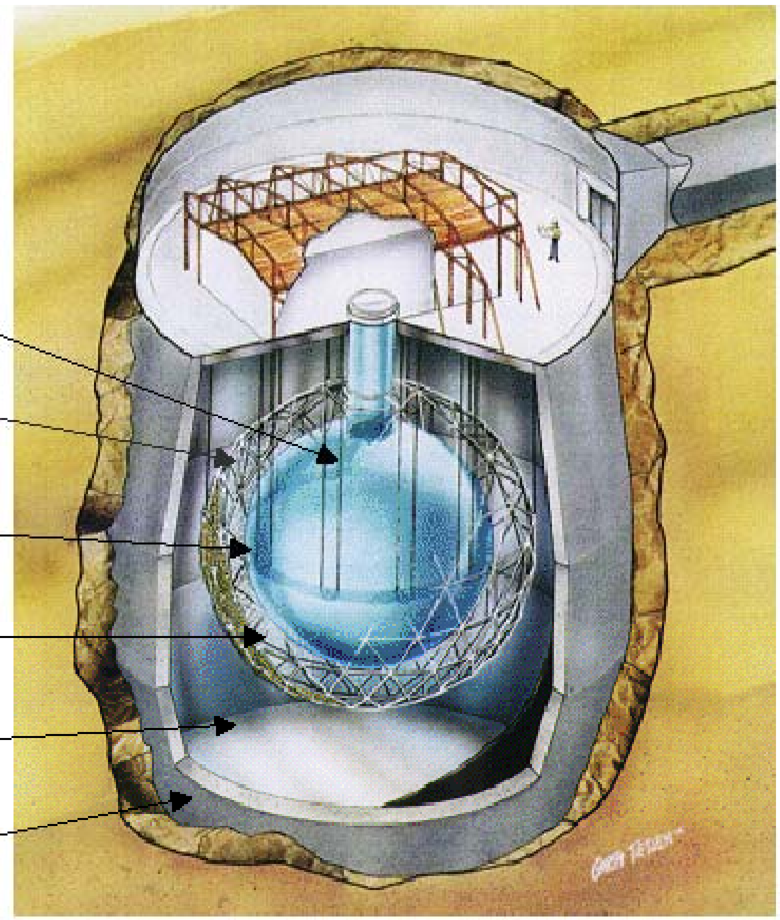
Support Structure for 9500 PMTs, 60% coverage

12 m Diameter Acrylic Vessel

1700 tonnes Inner Shielding H_2O

5300 tonnes Outer Shield H_2O

Urylon Liner and Radon Seal



ν Reactions in SNO

CC $\nu_e + d \Rightarrow p + p + e^-$

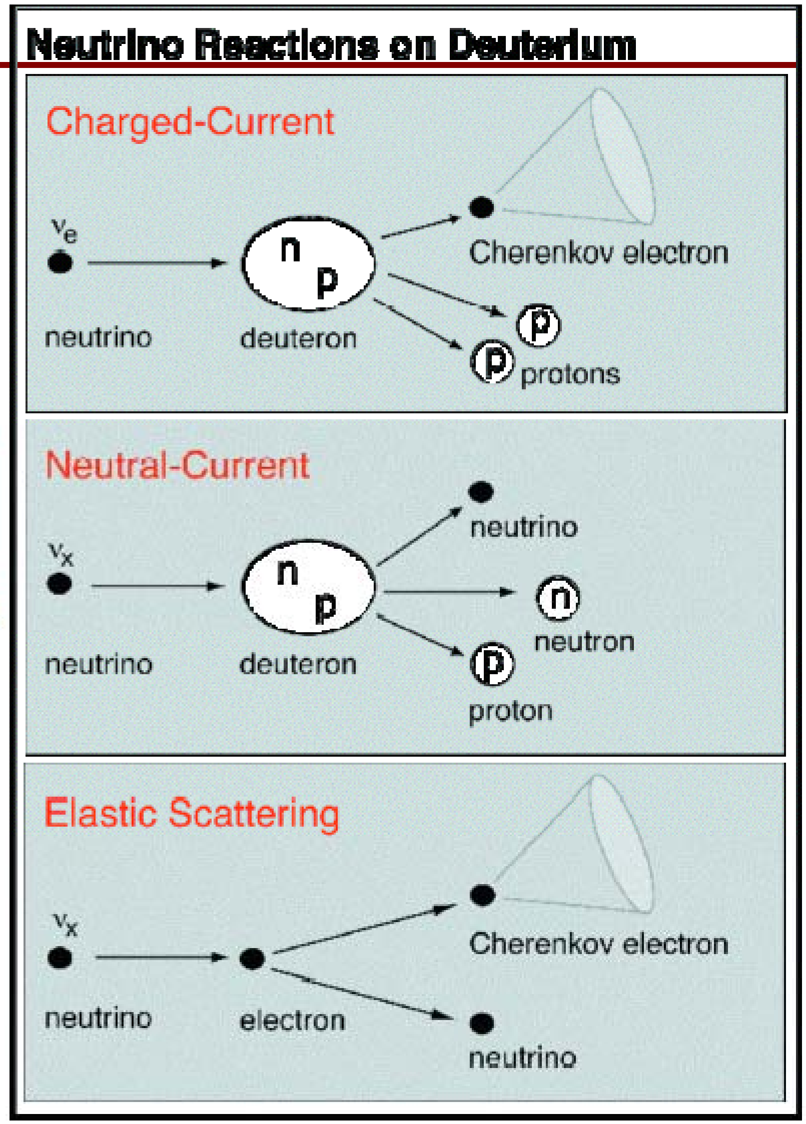
-Gives ν_e energy spectrum well
-Weak direction sensitivity $\propto$ 1-1/3cos(θ)
- ν_e only.

NC $\nu_x + d \Rightarrow p + n + \nu_x$

-Measure total ^8B ν flux from the sun.
- Equal cross section for all ν types

ES $\nu_x + e^- \Rightarrow \nu_x + e^-$

-Low Statistics
-Mainly sensitive to ν_e, some
-sensitivity to ν_μ and ν_τ
-Strong direction sensitivity

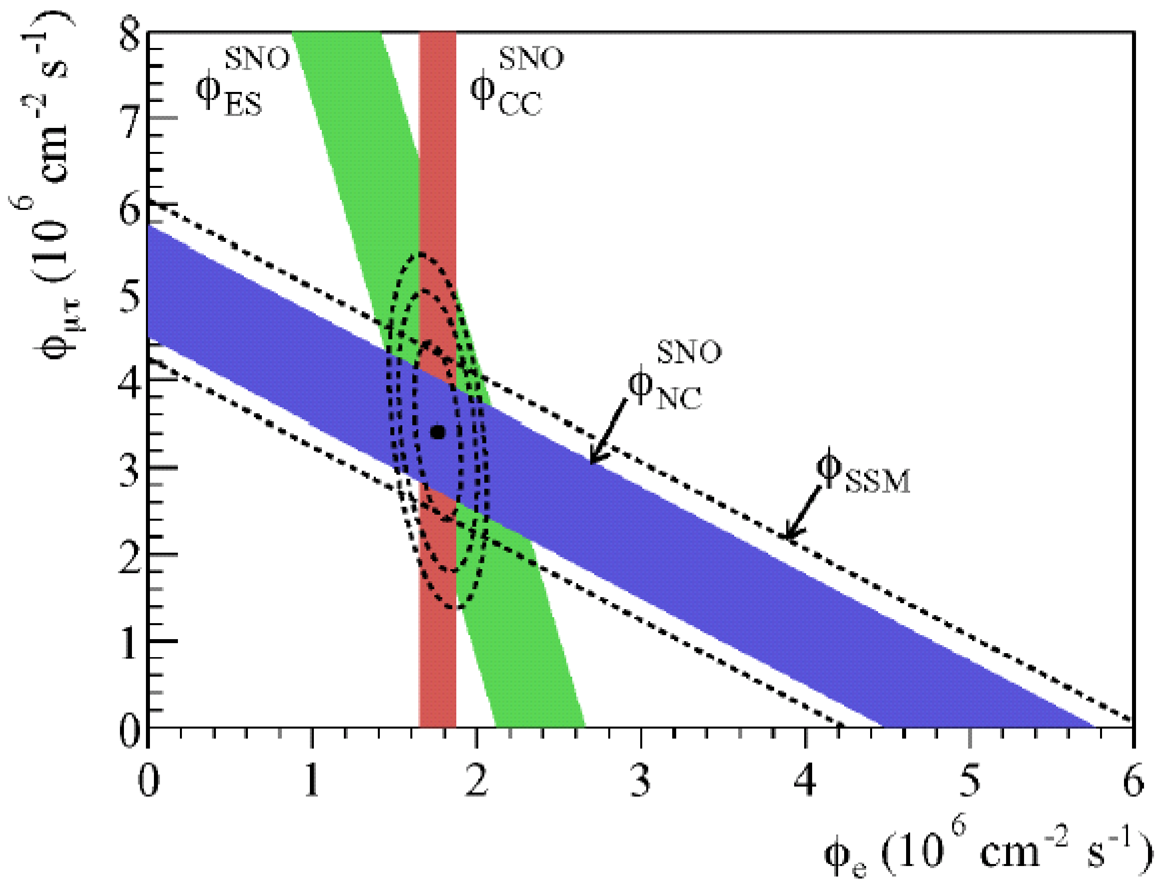


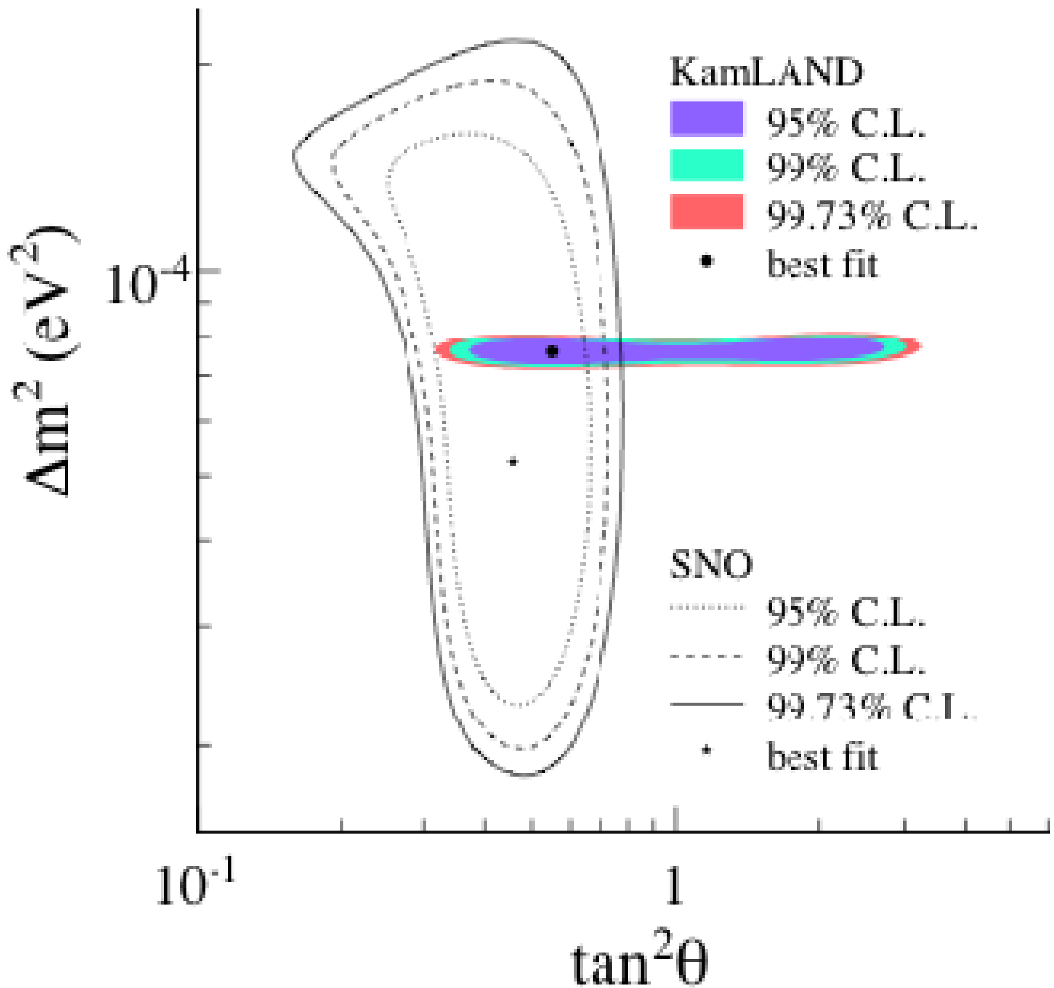
Neutrino Flavor Composition of ^8B Flux

Fluxes

(10^6 cm^{-2} s^{-1})

ν_e:	1.76(11)
$\nu_{\mu\tau}$:	3.41(66)
ν_{total}:	5.09(64)
ν_{SSM}:	5.05





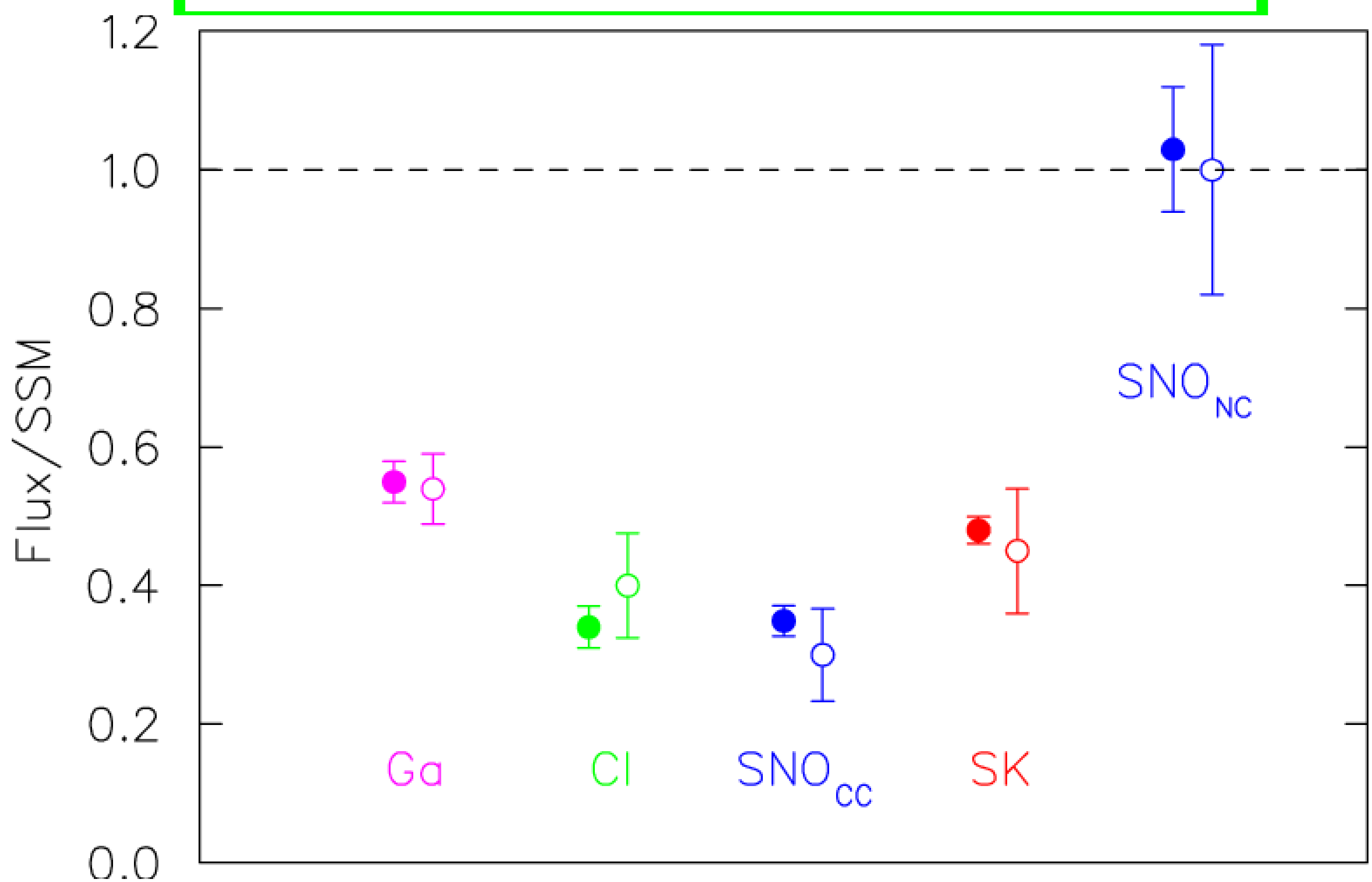
Best fit values:

$\Delta m^2 = 7.59^{+0.2}_{-0.21} \times 10^{-5}\ eV^2$

$\tan^2\theta = 0.49^{+0.07}_{-0.05}$

Summary of Solar Neutrino Results

Open circles: **combined best fit**
Closed circles: **experimental data**



Now that m≠0, we have a choice:

- Dirac neutrinos: ν_L, ν_R, $\bar{\nu}_L$, $\bar{\nu}_R$.
- Majorana Neutrinos: $\nu_L = \bar{\nu}_R$, $\bar{\nu}_L = \nu_R$.

Note that since neutrinos have q=0, the distinction between ν and $\bar{\nu}$ is more subtle.

What is the difference between ν and $\bar{\nu}$?

Massive fermions are usually described by the Dirac equation, where the chirality eigenstates ψ_R and ψ_L are coupled and form a four-component object of mass m,

$$i(\hat{\sigma}^{\mu}\partial_{\mu})\psi_R - m\psi_L = 0 \ , \ i(\sigma^{\mu}\partial_{\mu})\psi_L - m\psi_R = 0 \ , \tag{4}$$

where $\hat{\sigma}^{\mu} = (\sigma^0, \vec{\sigma})$, $\sigma^{\mu} = (\sigma^0, -\vec{\sigma})$ and $(\sigma^0, \vec{\sigma})$ are the Pauli matrices. As written, $\psi_{L(R)}$ are two-component spinors; the usual four-component bispinors are defined as:

$$\Psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix} \ ; \ \Psi_R = \begin{pmatrix} \psi_R \\ 0 \end{pmatrix} \ ; \ \Psi_L = \begin{pmatrix} 0 \\ \psi_L \end{pmatrix} \ , \tag{5}$$

where $\Psi_{L(R)}$ are just the chiral projections of Ψ, i.e. the eigenstates of $P_{L(R)} = (1 \mp \gamma_5)/2$.

However, Majorana's suggestion (16) allows one to use an alternative description of those massive fermions which do not have any additive quantum numbers as either two-component ψ_R (mass m), or ψ_L (mass m'), which obey independent equations

$$i(\hat{\sigma}^{\mu}\partial_{\mu})\psi_R - m\epsilon\psi_R^* = 0 \ ; \ i(\sigma^{\mu}\partial_{\mu})\psi_L + m'\epsilon\psi_L^* = 0 \ , \tag{6}$$

where $\epsilon = i\sigma_y$.

Elliot and Vogel

The Majorana fields can be also expressed in the four-component form

$$\Psi_L(x) = \begin{pmatrix} -\epsilon\psi_L^*(x) \\ \psi_L(x) \end{pmatrix}, \text{ and/or } \Psi_R(x) = \begin{pmatrix} \psi_R(x) \\ \epsilon\psi_R^*(x) \end{pmatrix}. \tag{7}$$

Such a four-component notation is a convention useful to express the charged weak current in a compact form. It is then clear that the Dirac field Ψ, Equation 5, is equivalent to a pair of Majorana fields with $m = m'$ and $\psi_L = \epsilon\psi_R^*$.

The four-component Majorana fields, Equation 7, are selfconjugate, $\Psi^c_{L(R)}(x) = \Psi_{L(R)}(x)$, where charge conjugation is defined as $\Psi^c_{L(R)}(x) = i\gamma^2\gamma^0\bar{\Psi}^T_{L(R)}$. The fields $\Psi_L(x)$ and $\Psi_R(x)$ are eigenstates of CP with opposite eigenvalues.

The Lorentz invariant mass term in the neutrino Langrangian can appear in three forms:

$$M_D[\bar{\nu}_R\nu_L \; + \; (\bar{\nu}_L)^c\nu_R^c] \, , \; M_L[(\bar{\nu}_L)^c\nu_L + \bar{\nu}_L\nu_L^c] \, , \; M_R[(\bar{\nu}_R)^c\nu_R + \bar{\nu}_R\nu_R^c] \, , \tag{8}$$

where we have introduced the notation $\nu_{L(R)}$ for the corresponding neutrino annihilation operators. The first expression in Equation 8 is the Dirac mass term (with the mass parameter M_D) which requires the existence of both chirality eigenstates ν_L and ν_R and conserves the lepton quantum number. The second (and third) mass terms are Majorana mass terms, which violate the lepton number and can be present even without the existence of ν_R (for the term with mass parameter M_L) or ν_L (for the term with mass parameter M_R). In general, all three terms might coexist, and then the mass Langrangian must be diagonalized resulting in two generally nondegenerate mass eigenvalues for each flavor. (That is the situation with the generic see-saw mass (34), where it is assumed that $M_R \gg M_D \gg M_L \sim 0$, and the light neutrino acquires the mass $m_\nu \sim M_D^2/M_R$.)

Elliot and Vogel

Back to Neutrino Mixing…

$$|\nu_\ell\rangle = \sum_i U_{\ell i} |\nu_i\rangle \ . \tag{4}$$

When the standard model is extended to include neutrino mass, the mixing matrix U is unitary.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2}\ \nu_1 \\ e^{i\alpha_2/2}\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{9}$$

In vacuum, the mass eigenstates propagate as plane waves. Leaving out the common phase, a beam of ultrarelativistic neutrinos $|\nu_i\rangle$ with energy E at the distance L acquires a phase

$$|\nu_i(L)\rangle \sim |\nu_i(L=0)\rangle \exp(-i\frac{m_i^2}{2}\frac{L}{E}) \tag{5}$$

Given that, the amplitude of the process $\nu_\ell \to \nu_{\ell'}$ is

$$A(\nu_\ell \to \nu_{\ell'}) = \sum_i U_{\ell i} e^{-i\frac{m_i^2 L}{2E}} U^*_{\ell' i} \ , \tag{6}$$

and the probability of the flavor change for $\ell \neq \ell'$ is the square of this amplitude. It is obvious that due to the unitarity of U there is no flavor change if all masses vanish or are exactly degenerate. The existence of the oscillations is a

The general formula for the probability that the “transition” $\ell \to \ell'$ happens at L is

$$P(\nu_\ell \to \nu_{\ell'}) = |\sum_i U_{\ell i} U^*_{\ell' i} e^{-i(m_i^2/2E)L}|^2$$
$$= \sum_i |U_{\ell i} U^*_{\ell' i}|^2 + \Re \sum_i \sum_{j \neq i} U_{\ell i} U^*_{\ell' i} U^*_{\ell j} U_{\ell' j} e^{i\frac{|m_i^2 - m_j^2|L}{2E}} \quad . \tag{10}$$

Clearly, the probability (10) is independent of the Majorana phases α. The oscillations described by the Eq.(10) violate the individual flavor lepton numbers, but conserve the total lepton number. The oscillation pattern is identical for Dirac or Majorana neutrinos.

Maki - Nakagawa - Sakata Matrix

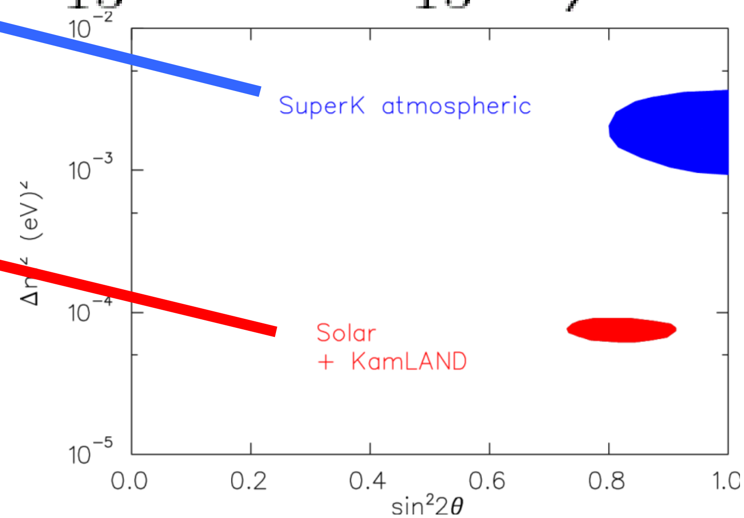
$$U_{MNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Gateway to CP Violation!

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}$$

CP violation

$$\times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$



Neutrino Mixing versus Quark Mixing

Leptons

$$U_\ell = \begin{pmatrix} 0.85 & 0.52 & < 0.053 \\ 0.33 & 0.62 & -0.72 \\ -0.40 & 0.59 & 0.70 \end{pmatrix}$$

Why so different???

Quarks

$$V_q = \begin{pmatrix} 0.976 & 0.22 & 0.003 \\ -0.22 & 0.98 & 0.04 \\ 0.007 & -0.04 & 1 \end{pmatrix}$$

Tri-bimaximal neutrino mixing:

$$U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

(Harrison, Perkins, Scott 1999)

Neutrinos: The Best is Yet to Come?

- Absolute mass scale
- θ_{13} – the last mixing angle
- Mass hierarchy
- CP violation
- Antineutrino=neutrino (Majorana)?