Nuclear Structure I
experimental

MICHIGAN STATE UNIVERSITY
Advancing Knowledge.
Transforming Lives.

Sunday

Preliminaries

Nuclear binding and masses

→ Indicator of shell structure

How to measure a mass

Monday

Thursday

Friday

Preliminaries (1)

Goal: Establish physical properties of rare isotopes and their interactions to gain predictive power

Experiments: Measure observables

Observables: May or may not need interpretation to relate to physical properties

- e.g., half-life and mass connect directly to physical properties
- e.g., cross sections for reaction processes usually need interpretation to connect to physical properties (model dependencies are introduced)

Preliminaries (2)

Theories and models can relate observables to physical properties – often, experiments are motivated by theoretical predictions that need validation

But: Theories and models have their own realm of applicability that everybody involved in the experiment/data analysis/interpretation should be aware of!

Predictions or systematics come with a warning: Might lead to expectations that can influence the implementation of an experiment and ultimately limit the scope of discovery

Preliminaries (3)

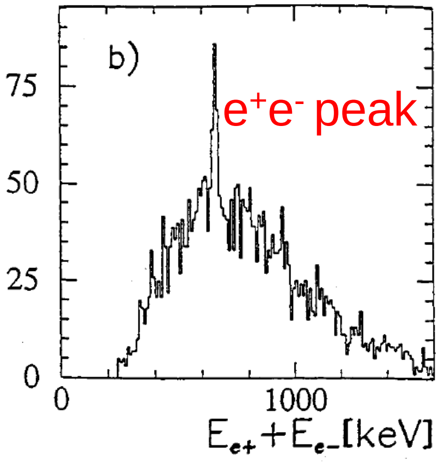
Implementation of experiments can limit the scope of discovery

Examples:

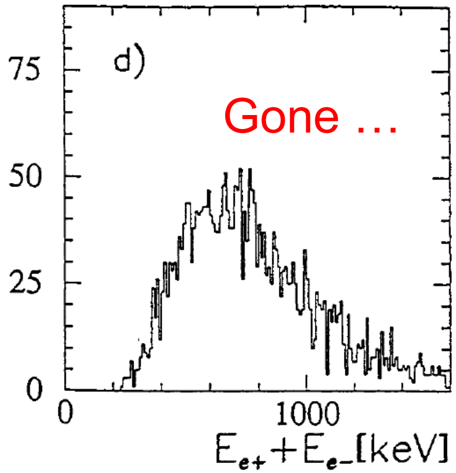
- Lifetime of a nucleus
- Excitation energy

Analysis and “selection” of data (cuts, gates, using subsets, …) can influence the result

Example: e^+e^- resonances in heavy-ion collisions at GSI



First half of the data analyzed



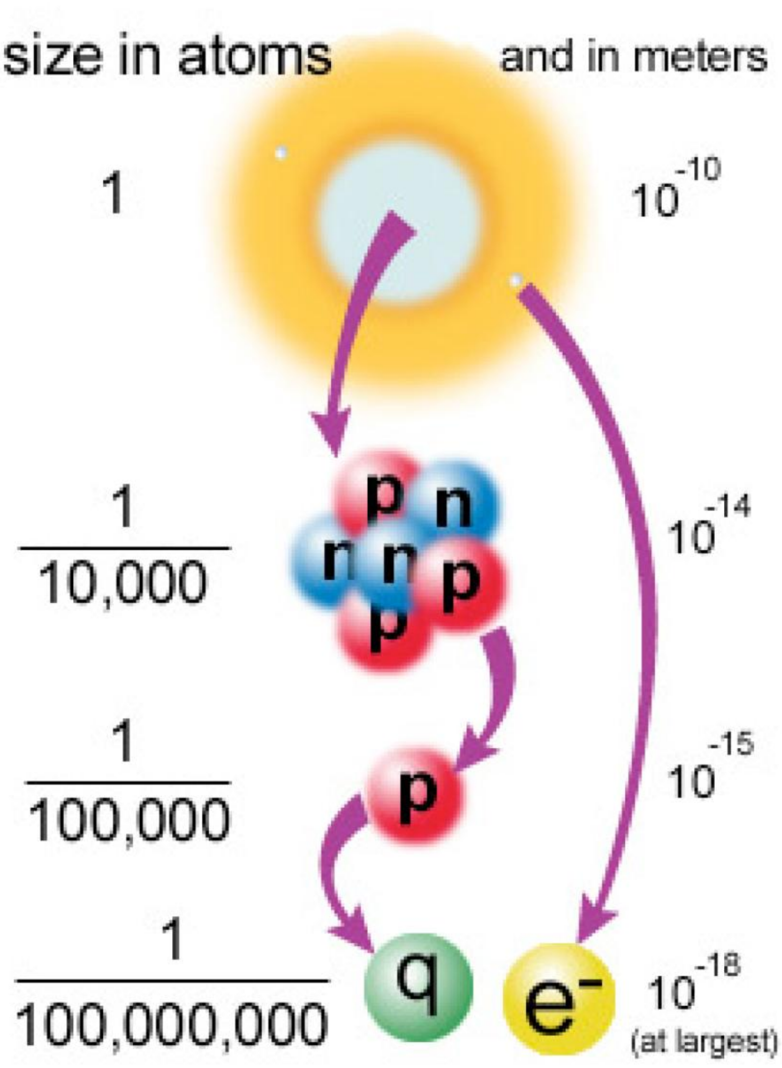
Identical analysis applied to other half of data set

A. FRANKLIN

“Selectivity and the Production of Experimental Results”

Arch. Hist. Exact Sci. 53 (1998) 399

Scales – sizes and energies



Excitation energies in molecules and nuclei

- molecular excitations:

  $E_{rot} << E_{vib} << E_{el}$
  (μeV << meV << eV)

  As a consequence, these different motions can be treated separately and the wavefunction ends up as a product of terms

- In nuclei, the energy scales are much closer

  $E_{rot} \sim E_{vib} \sim E_{sp}$ (MeV)

  Collective and single-particle excitation can be separated but interact strongly

Scales – sizes and energies

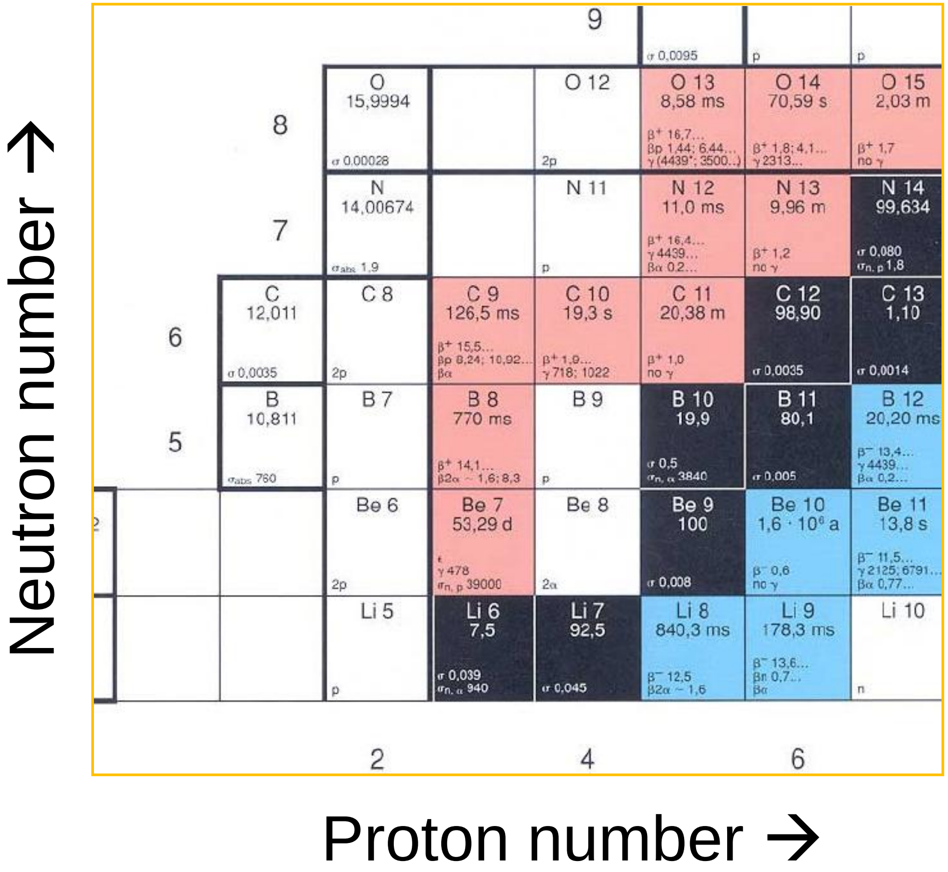
The nucleus is a bound collection of N neutrons and Z protons mass number A=Z+N

^{A}Z

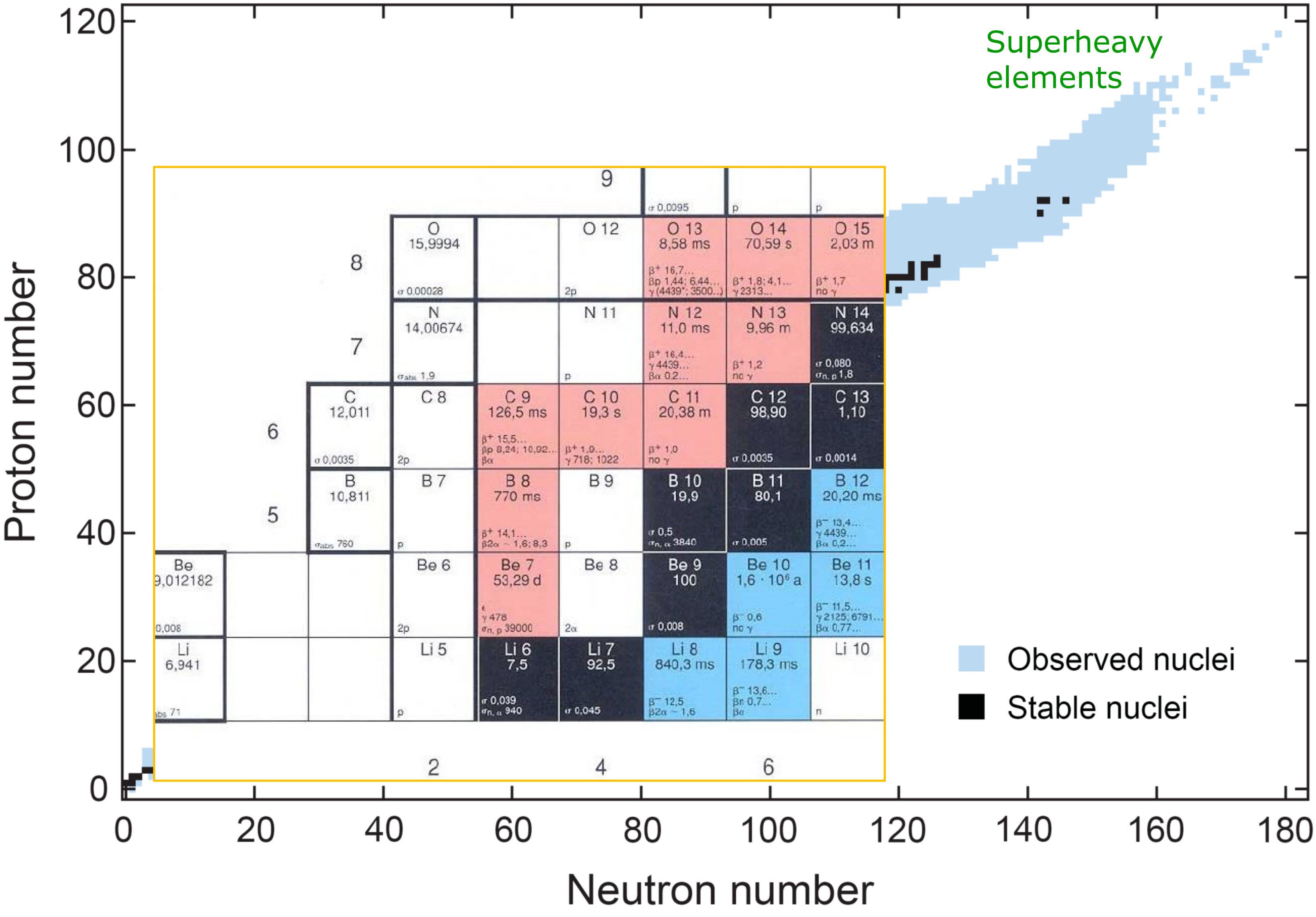
- Isotopes: Nuclei with the same Z but different N – e.g. ^{9}C, ^{10}C, ^{11}C, ^{12}C

- Isotones: Nuclei with the same N but different Z – e.g. ^{9}C, ^{8}B, ^{7}Be, ^{6}Li

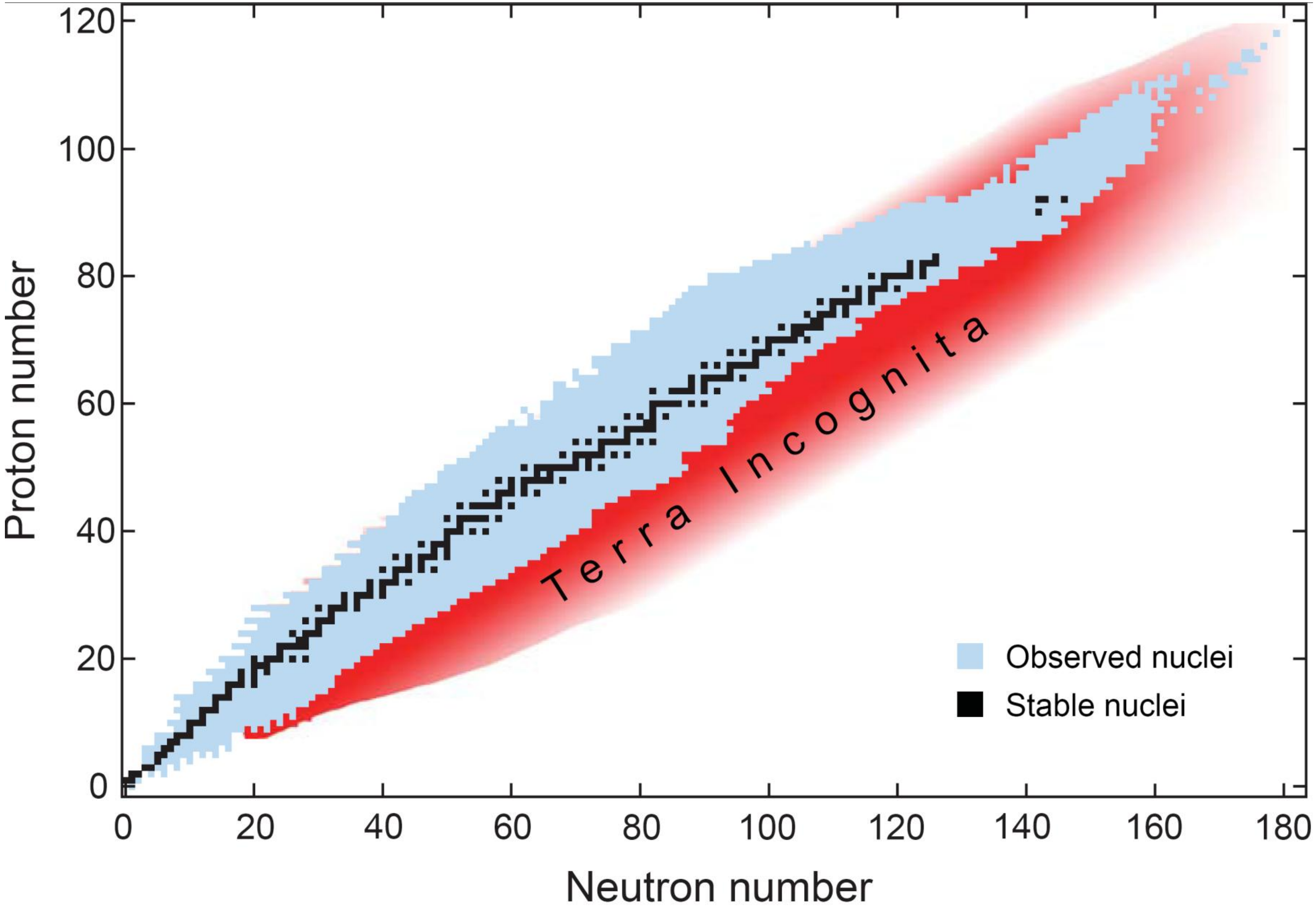
- Isobars: Nuclei with the same mass number – e.g. ^{9}C, ^{9}B, ^{9}Be, ^{9}Li



About 3000 isotopes have been made in laboratories

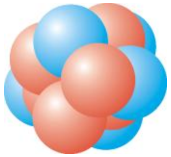


6000-8000 isotopes might be out there



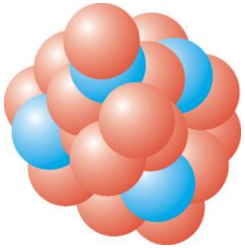
Exotic nuclei

Normal Nucleus:



6 neutrons
6 protons (carbon)
^{12}C
Stable, found in nature

Exotic Nucleus:



16 neutrons
6 protons (carbon)
^{22}C
Radioactive, at the limit of nuclear binding

Characteristics of exotic nuclei: Excess of neutrons or protons, short half-life, neutron or proton dominated surface, low binding

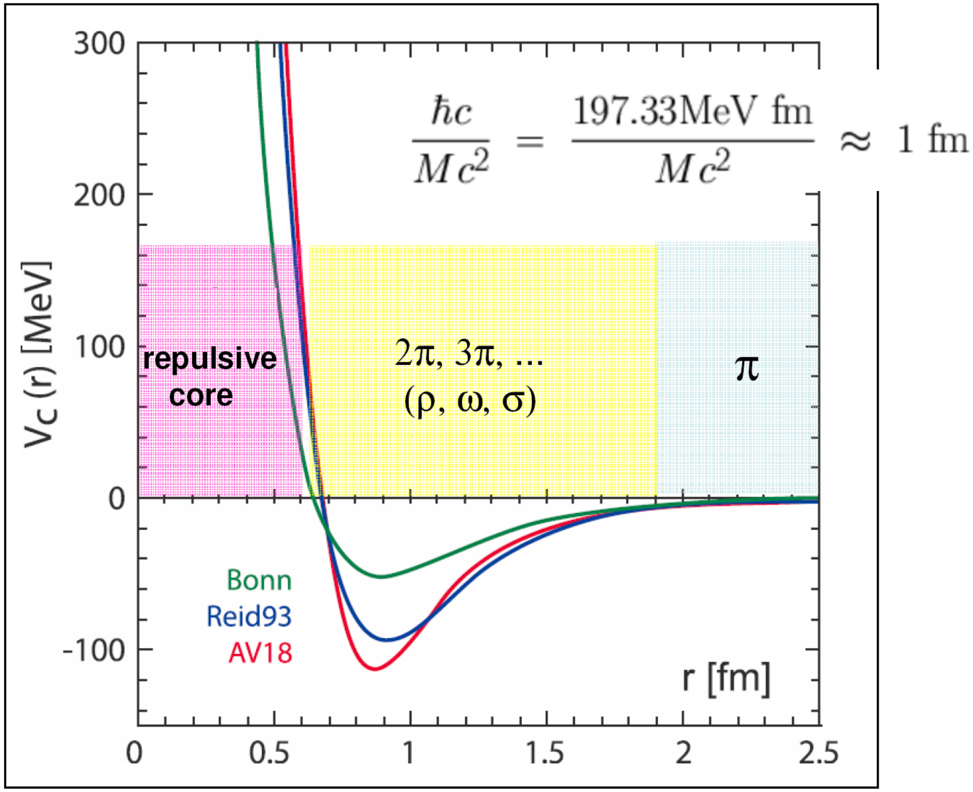
What binds nucleons into nuclei?

The neutrons and protons are bound together by

the *strong or color force*

The strong force between the quarks in one proton and the quarks in another proton is strong enough to overcome the electromagnetic repulsion

Two ways of thinking about the strong force:
As a residual color interaction or as the exchange of mesons



$$\frac{\hbar c}{Mc^2} = \frac{197.33\mathrm{MeV\ fm}}{Mc^2} \approx 1\ \mathrm{fm}$$

From T. Hatsuda (Oslo 2008)

http://particleadventure.org

Binding energy, mass and mass excess

mass $M(N,Z)$ of the neutral atom

Mass excess: $\Delta(N,Z) \equiv M(N,Z) - uA$

Atomic mass unit u:

$$u = M(^{12}\mathrm{C})/12 = 931.49386\,\mathrm{MeV}/c^2.$$

Equivalent to $\Delta(^{12}\mathrm{C})=0$

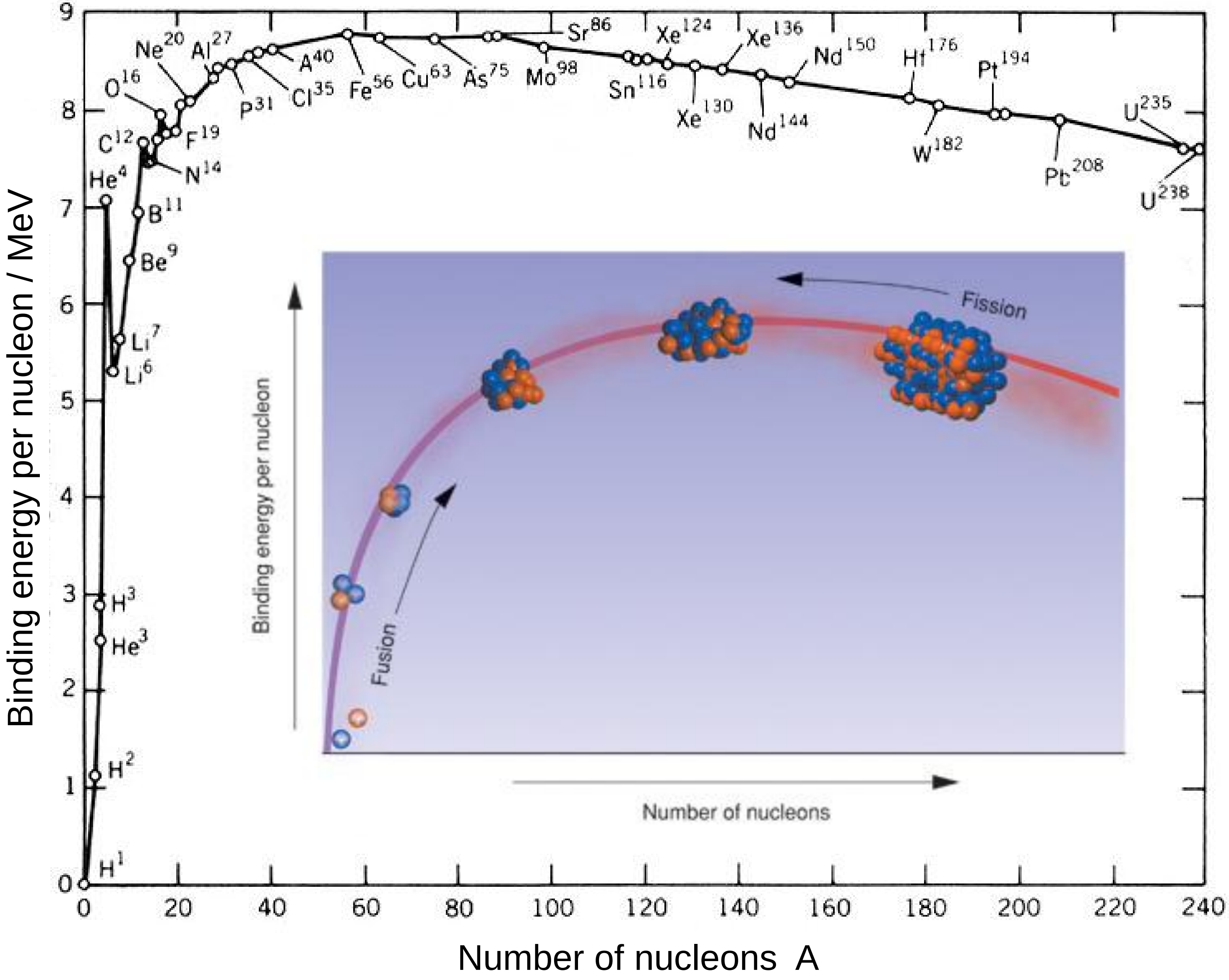
Binding energy:

$$\boxed{B(N,Z) = ZM_Hc^2 + NM_nc^2 - M(N,Z)c^2}$$

$$\Delta_Hc^2 = 7.2890\ \mathrm{MeV} \qquad \Delta_nc^2 = 8.0713\ \mathrm{MeV}$$

$$\boxed{B(N,Z) = Z\Delta_Hc^2 + N\Delta_nc^2 - \Delta(N,Z)c^2}$$

Average binding energy of nuclei



A semi-empirical description of nuclear binding

- $B(Z,A) =$	$+ a_V A$	Volume term
	$+ a_S A^{2/3}$	Surface energy term
	$+ a_C Z^2/A^{1/3}$	Coulomb term
	$+ a_A (N-Z)^2/A$	Asymmetry term
	$- a_P/A^{3/4}$	Pairing term

$\boxed{R \sim A^{1/3}}$

$a_V = -15.68$ MeV

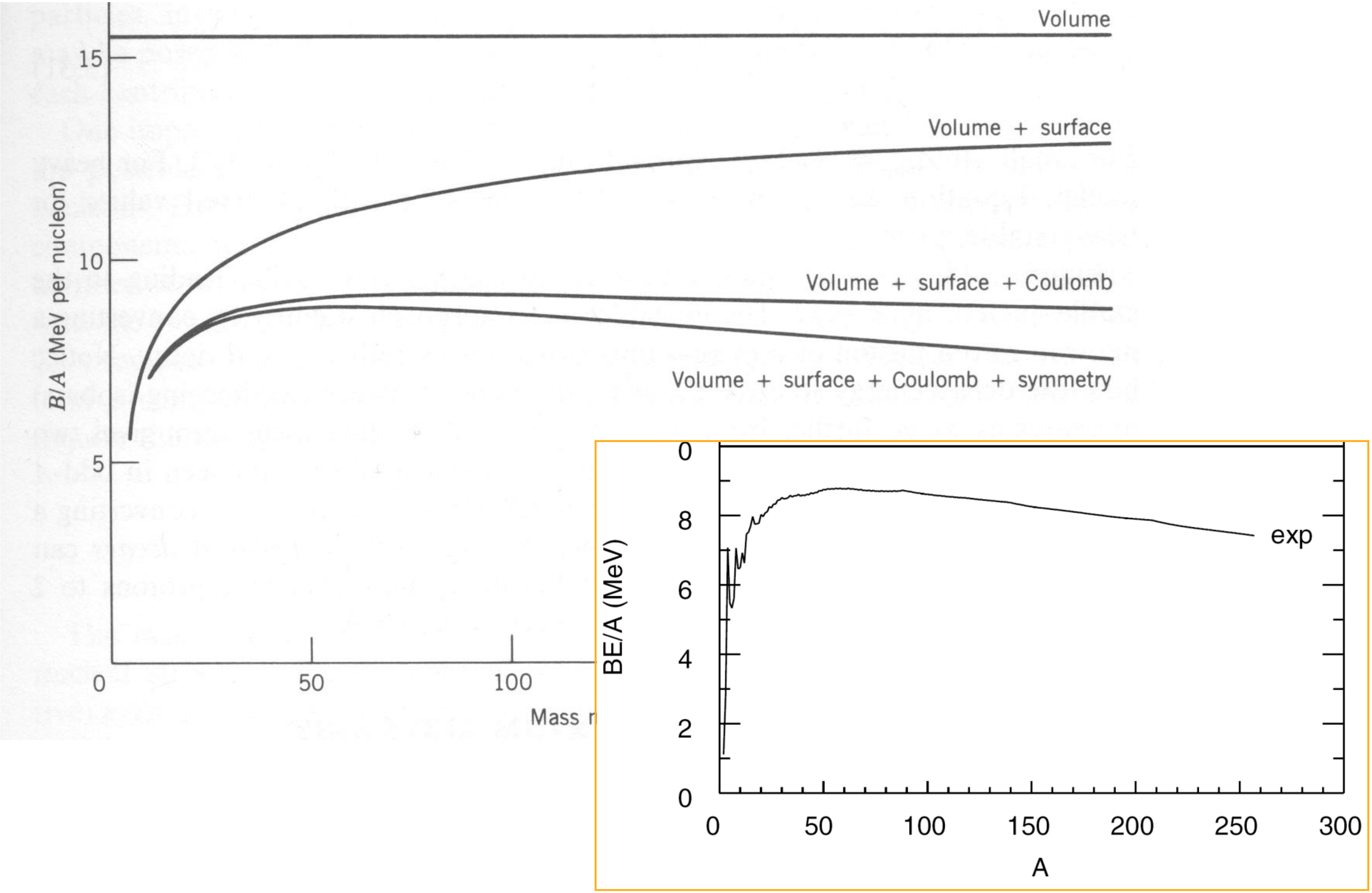
$a_S = 18.56$ MeV

$a_C = 0.717$ MeV

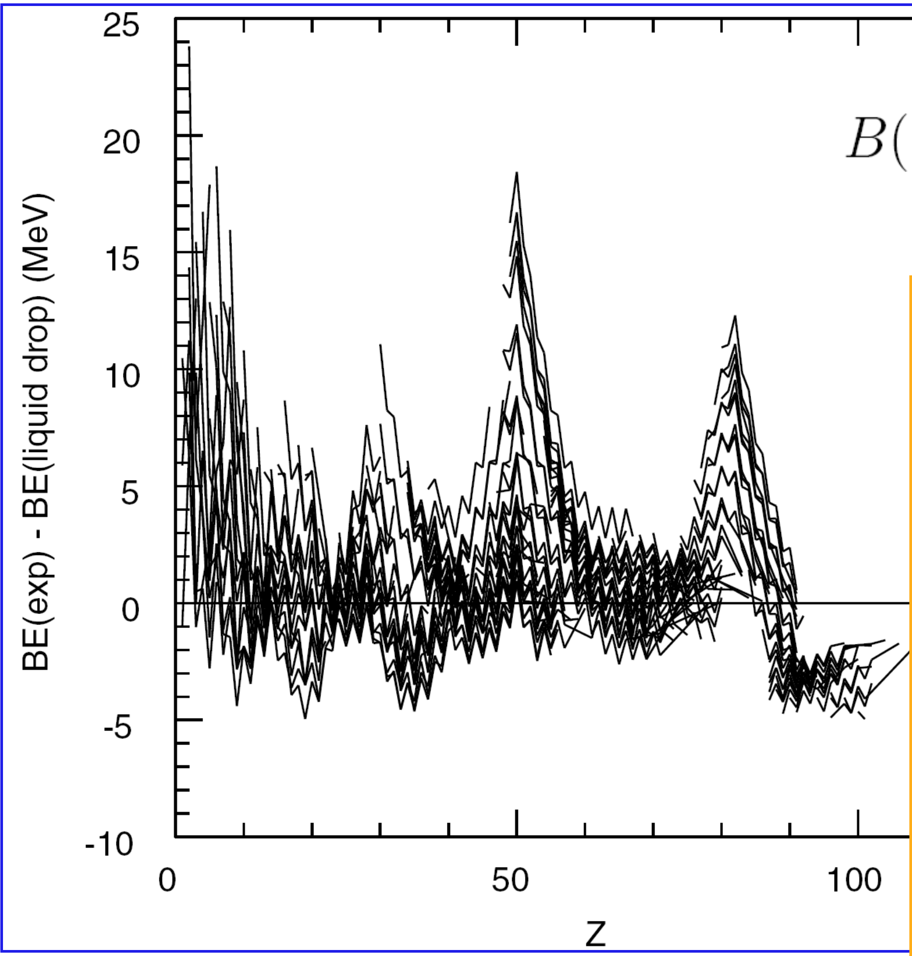
$a_A = 28.1$ MeV

$a_P = 34.0$ MeV for even-even, -34.0 MeV for odd-odd, 0 for even-odd

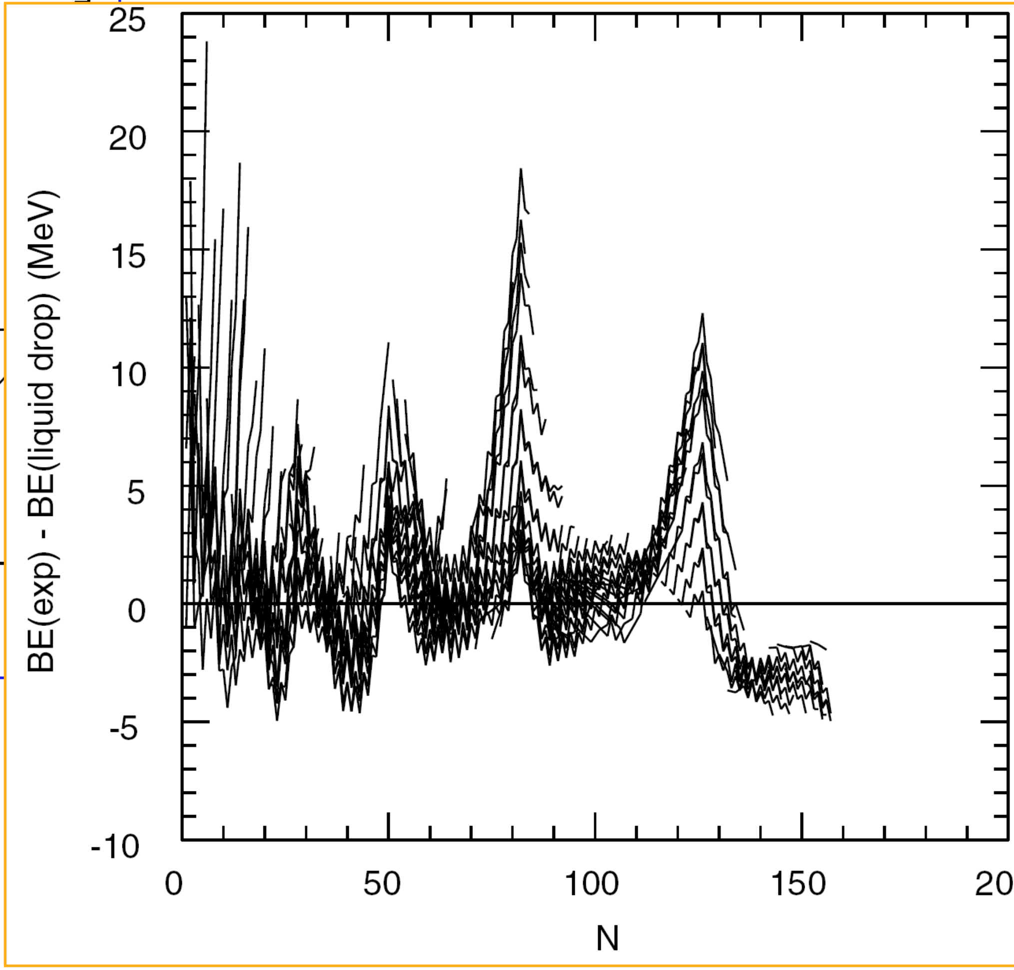
The different contributions to nuclear binding



Semi-empirical mass formula versus experimental reality



$$B(N,Z) = ZM_Hc^2 + NM_nc^2 - M(N,Z)c^2$$



Q values and nucleon separation energies

Q value of a process ($^{A}Z_{i}$ → $^{A}Z_{f}$):

$$Q = \sum_i M(N_i, Z_i)c^2 - \sum_f M(N_f, Z_f)c^2 = \sum_f B(N_f, Z_f) - \sum_i B(N_i, Z_i)$$

Nucleon separation energies:

$$S_n = B(N, Z) - B(N-1, Z),$$

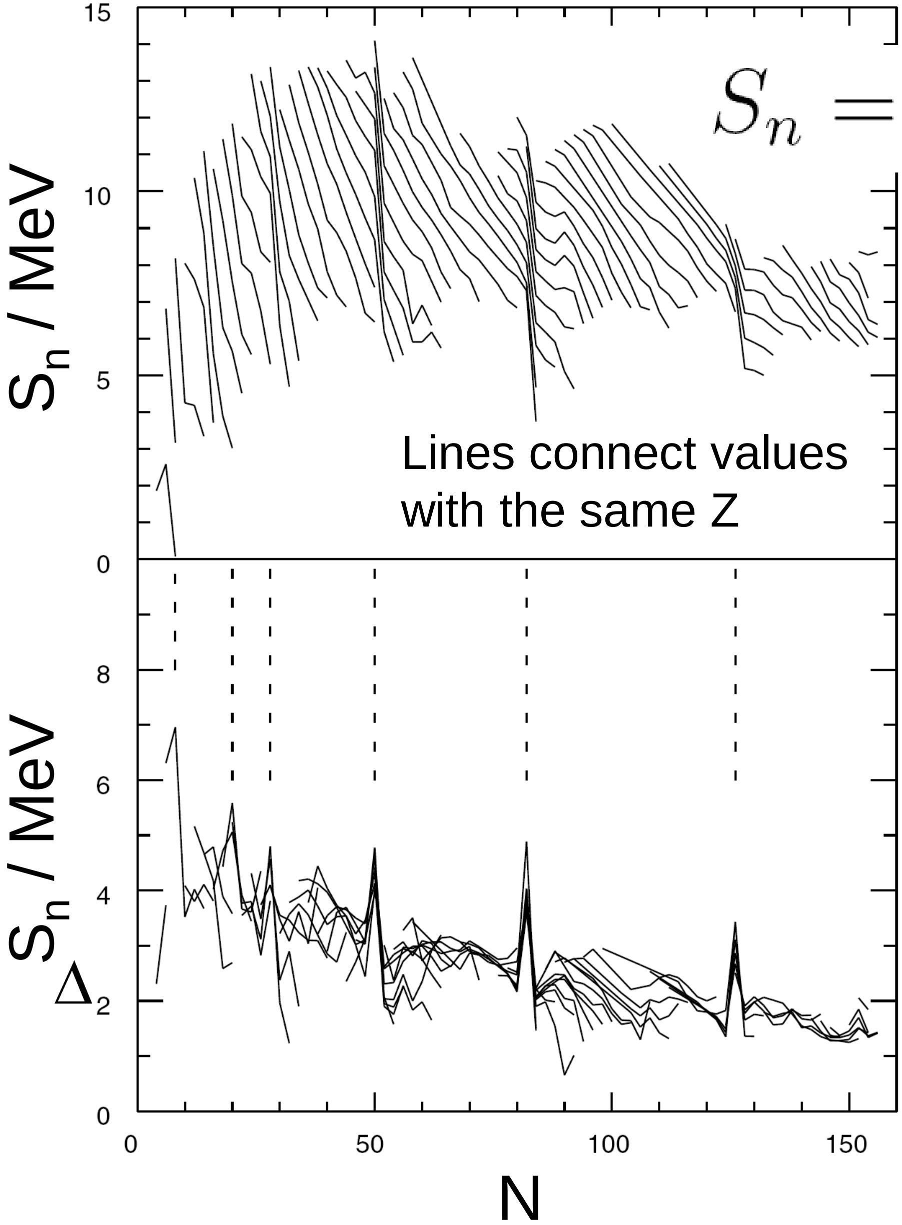
$$S_p = B(N, Z) - B(N, Z-1),$$

$$S_{2n} = B(N, Z) - B(N-2, Z),$$

$$S_{2p} = B(N, Z) - B(N, Z-2).$$

Nucleon separation energies

$$S_n = B(N,Z) - B(N-1,Z)$$

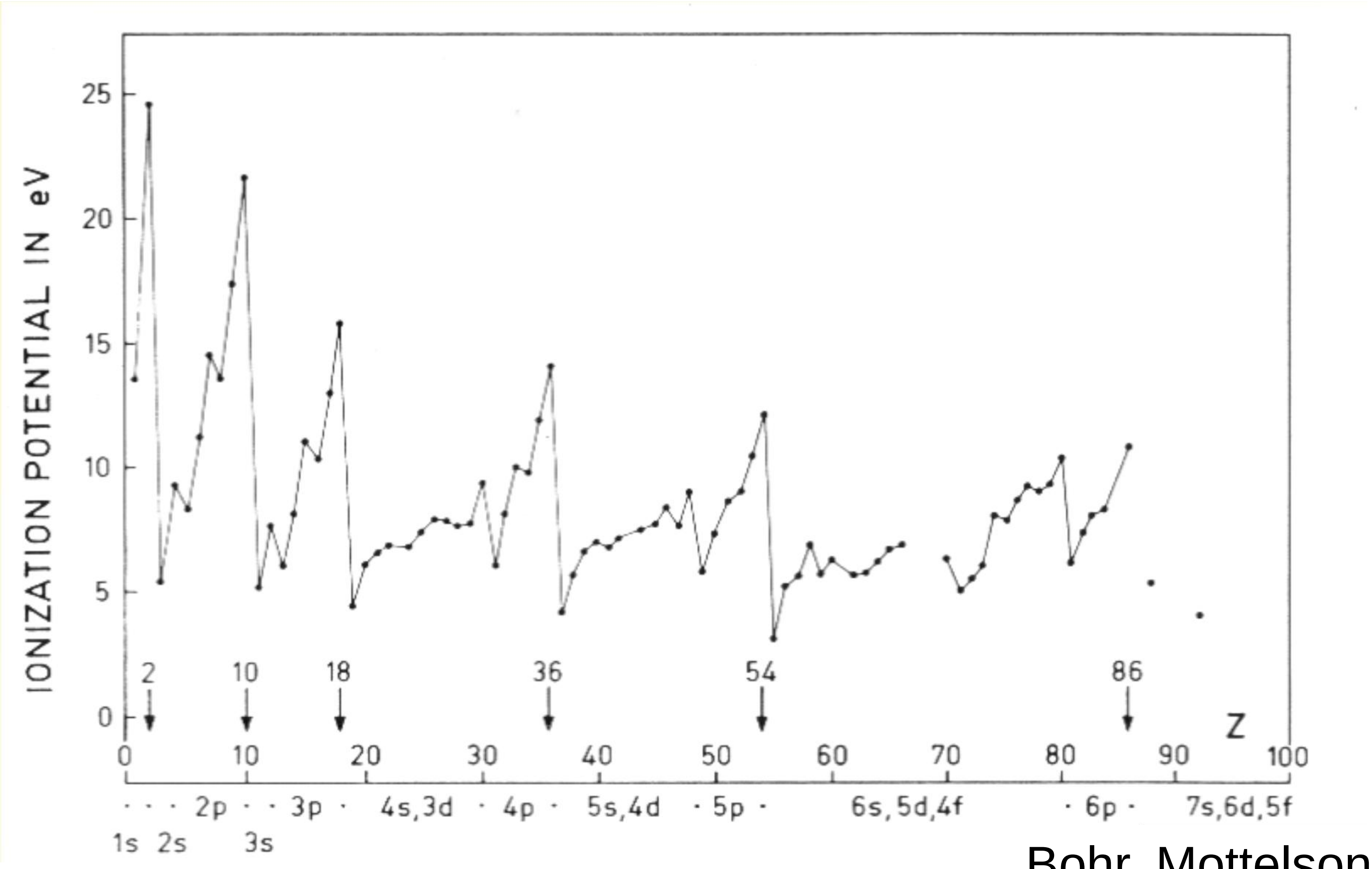


Lines connect values with the same Z

Huge changes in the separation energy at neutron numbers 8, 20, 28, 50, 82, 126

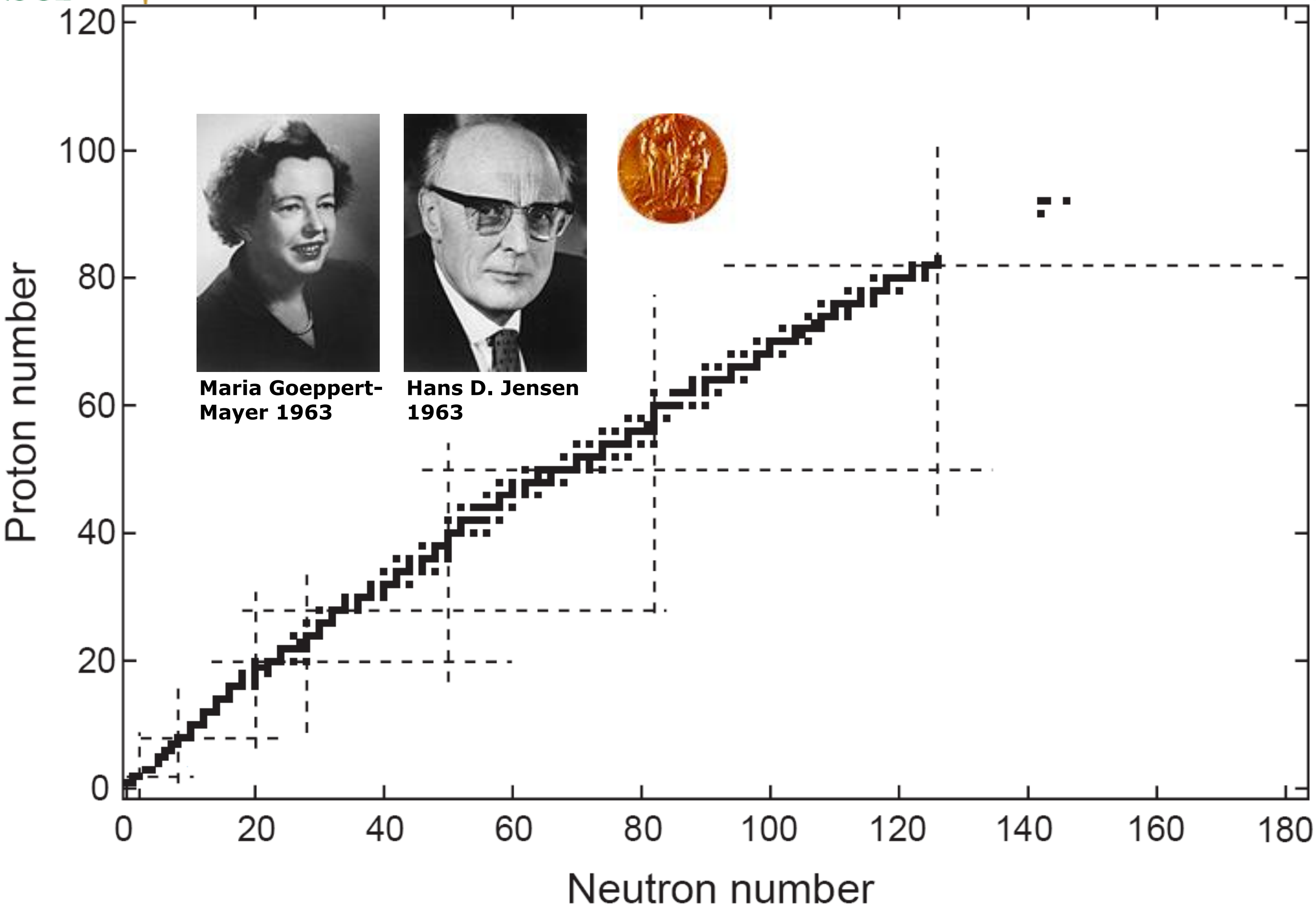
→ Those nuclei are particularly stable

Seen that before … compare to atomic shell structure!



Bohr, Mottelson

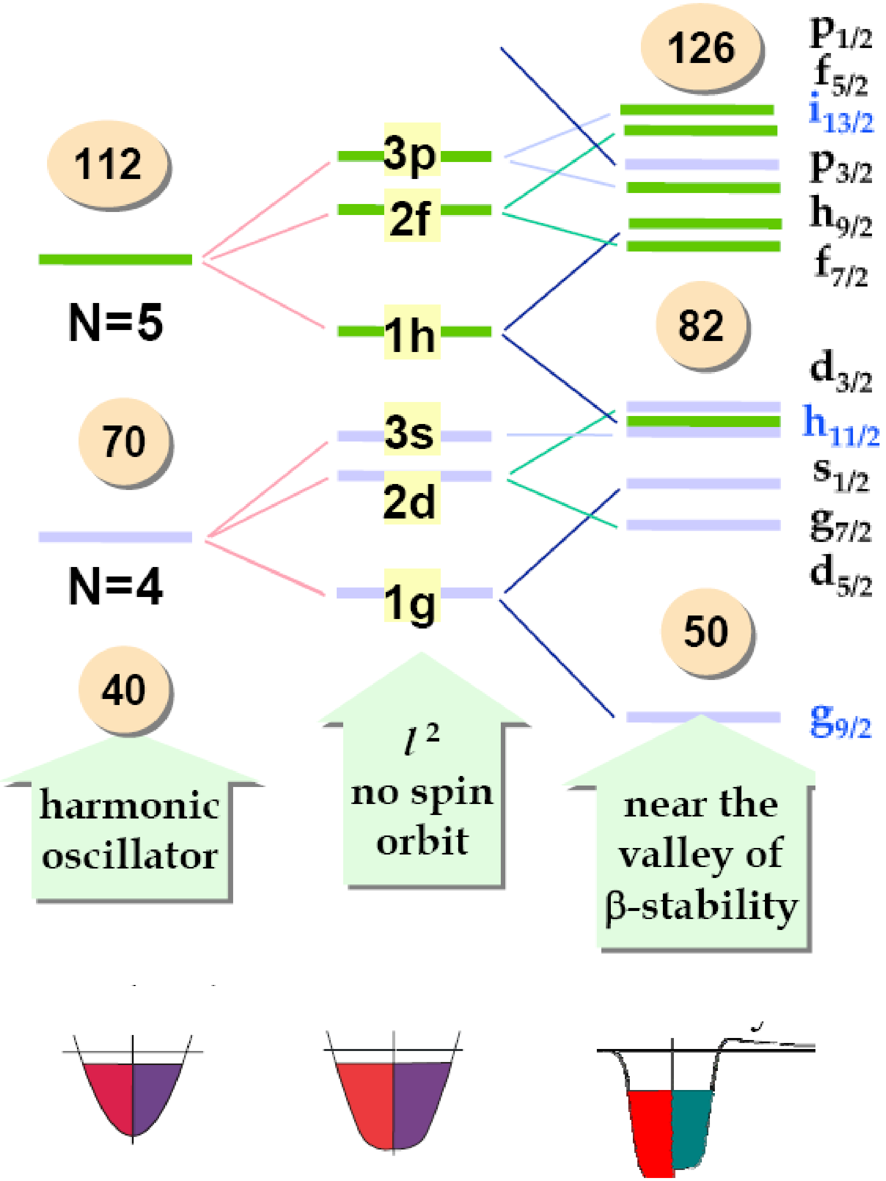
Shell structure – magic numbers



Maria Goeppert-Mayer 1963

Hans D. Jensen 1963

Shell structure – magic numbers



Adapted from D.J. Morrissey

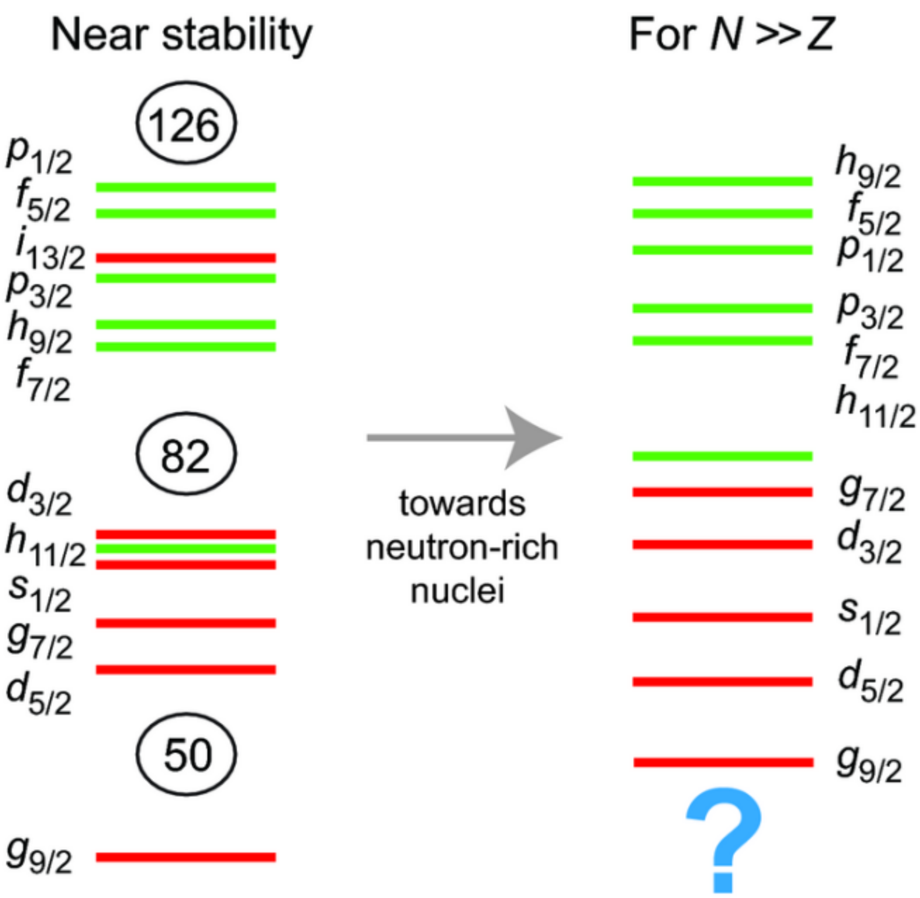
Maria Goeppert-Mayer, Phys. Rev. **75**, 1969 (1949)
O. Haxel, Phys. Rev. **75**, 1766 (1949)

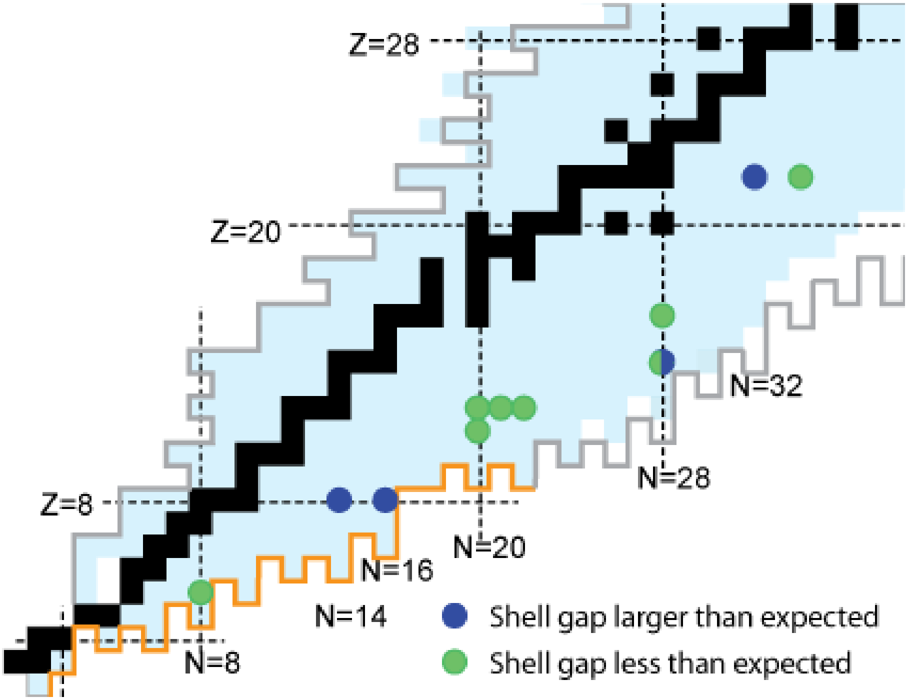
- Single-particle levels in nuclei
  The single-particle levels of this fermionic system are grouped. Large, stabilizing gaps between groups of single-particle states occur at certain occupation numbers of the orbits with a “magic number” of protons and neutrons
- Magic numbers
  Numbers of neutrons and protons in nuclei which correspond to particularly stable structures (2, 8, 20, 28, 50, 82, 126)
- ℓ = 0, 1, 2, 3, ... j= ℓ ± 1/2
  s, p, d, f, ... Max. occupancy: 2j+1
- Experimental signatures of nuclear shells
  - low capture cross sections
  - little collectivity
  - more tightly bound than neighboring nuclei

$$H = H_0 + H_{res} = \sum_{i=1}^{A}\left[\frac{\mathbf{p}_i^2}{2m_i} + U_i(\mathbf{r})\right] + H_{res}$$

Shell structure – magic numbers

Nuclear Shell Structure

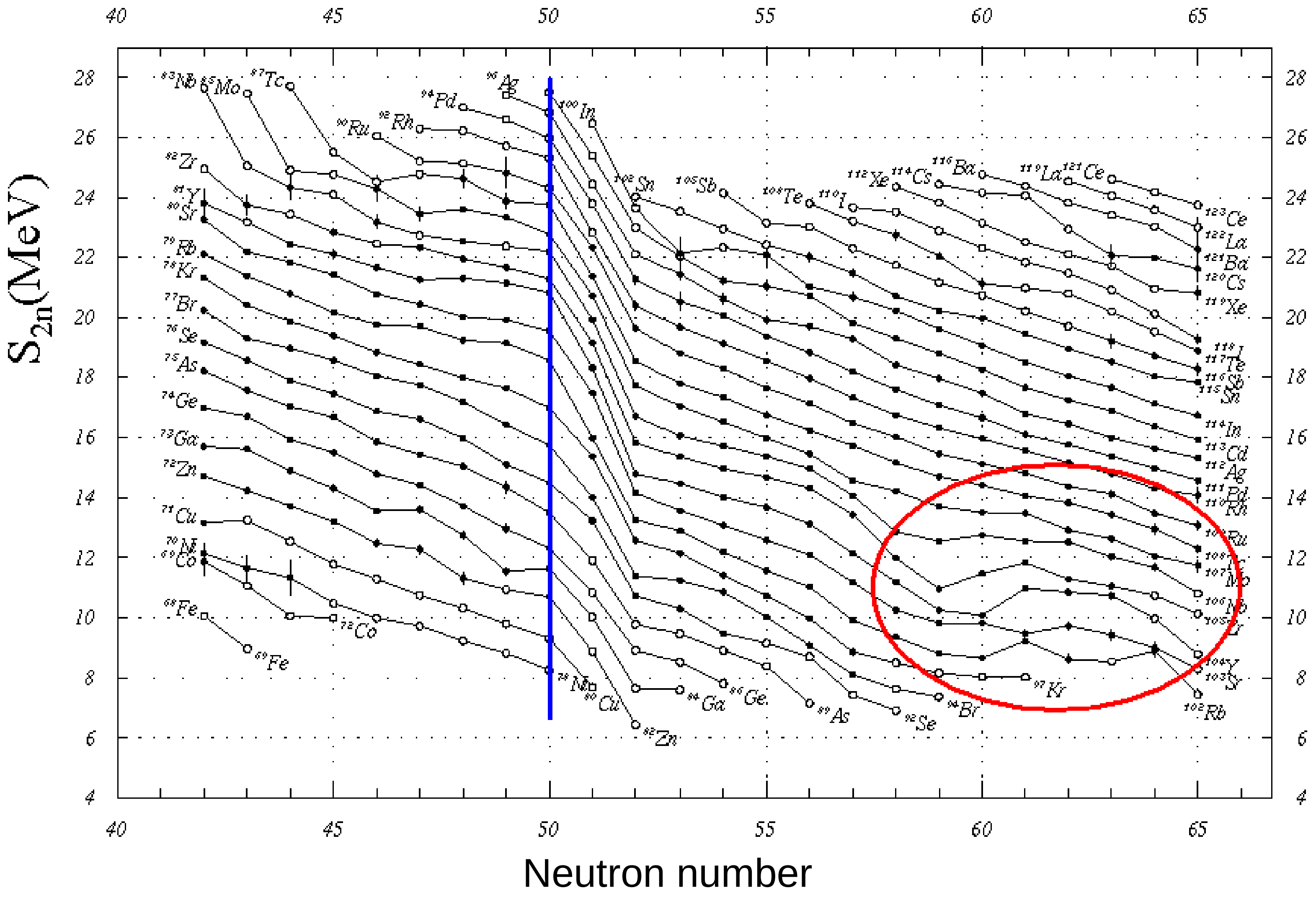




- Mean field near stability
- Strong spin-orbit term

- Mean field for *N* >> *Z*?
- Reduced spin-orbit
- Diffuse density
- Tensor force

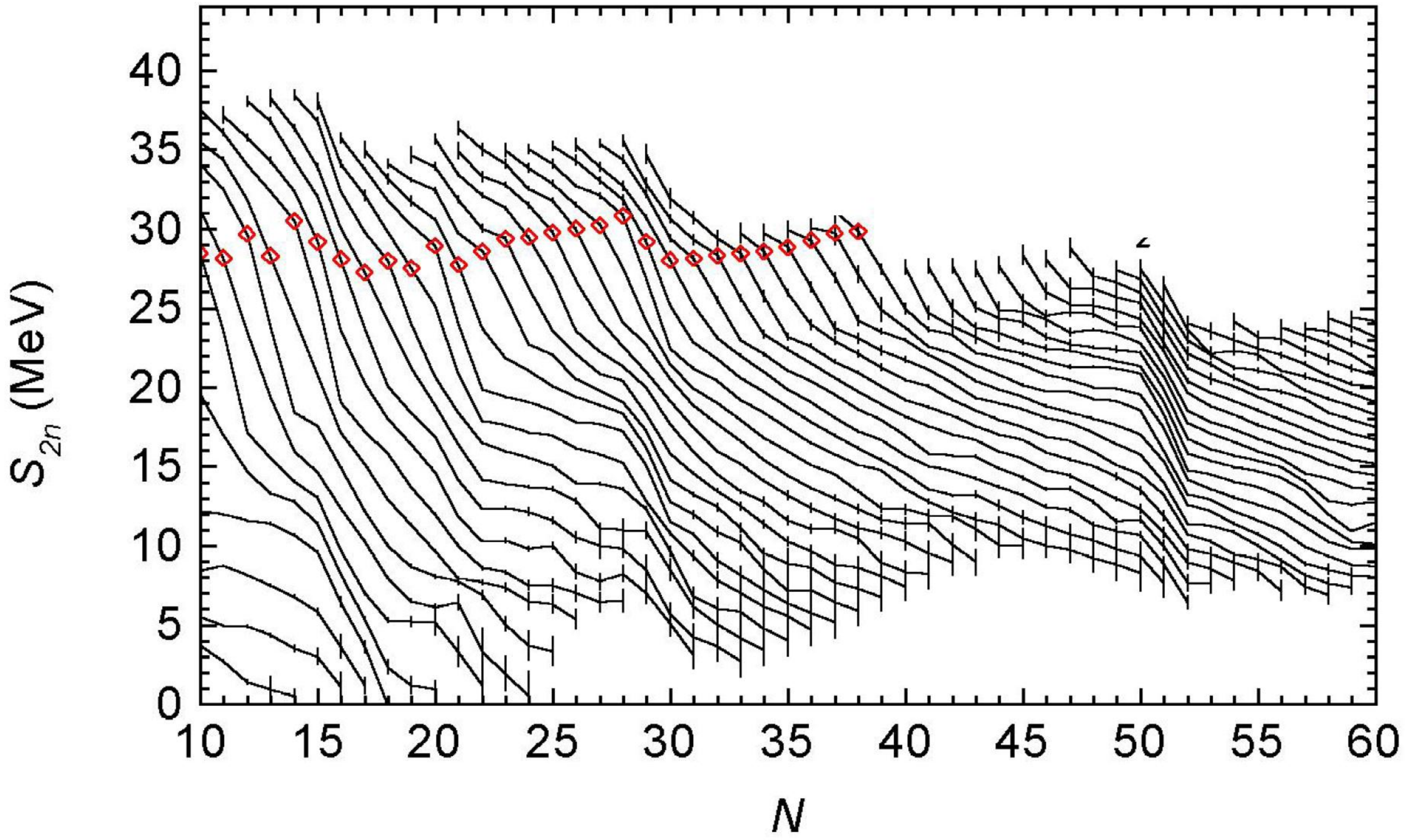
An indicator for changes n nuclear structure



magic number

deformation

Masses – what are they good for?
Nuclear structure



Observables

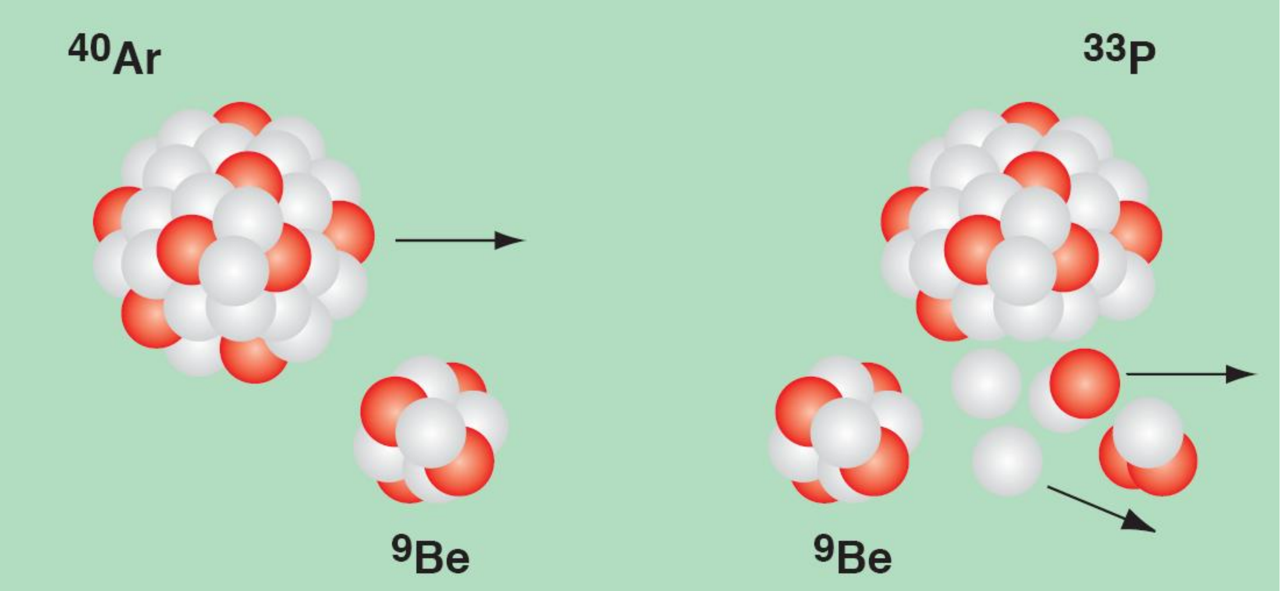
$$
\begin{aligned}
S_{2n} &= B(N,Z) - B(N-2,Z) \\
&= M(N-2,Z) + 2M_N - M(N,Z)
\end{aligned}
$$

Measure nuclear masses of exotic nuclei!

… but first we have to produce them!

Production of exotic nuclei

- Transfer reactions
- Fusion-evaporation
- Fission
- Fragmentation



  - Target fragmentation (HRIBF, TRIUMF, SPIRAL, ISOLDE)
  - Projectile fragmentation (NSCL, GSI, RIKEN, GANIL)

Masses

Indirect

- Decay measurements and kinematics in two-body reactions

reactions:

$A(a,b)B$

$Q = M_A + M_a - M_b - M_B$

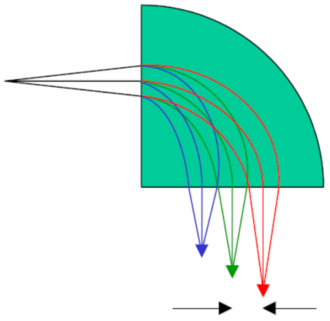
decays:

$B \rightarrow A + b$

$Q_\alpha = M_B - M_A$

Direct

- Conventional mass spectrometry
  - Cern PS, Chalk River
- Time-of-flight
  - spectrometer (SPEG, TOFI, S800)
  - Multi-turn (cyclotrons, storage rings)
- Frequency measurements
  - Penning traps
  - Storage rings



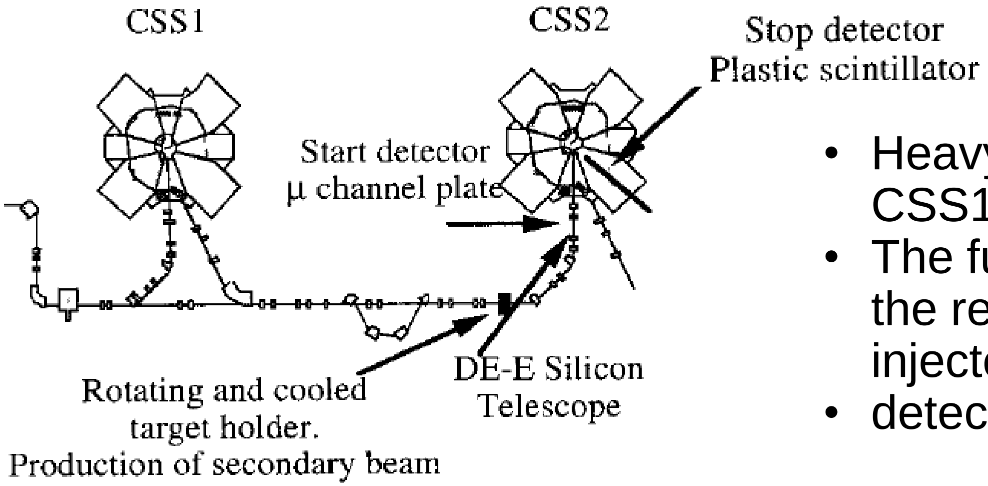
Mass separator
(spectrograph,
spectrometer)

Dispersion

$D = \Delta x\, m / \Delta m$

Adapted from D. Lunney

TOF mass measurement
– *Cyclotrons at GANIL*



A~100 nuclei: ^{50}Cr+^{58}Ni at 250 MeV

- Heavy-ion primary beam delivered by CSS1 with a few MeV/nucleon
- The fusion-evaporation products formed in the reaction with the production target are injected into the CSS2 and accelerated
- detected in a silicon-detector telescope

$$\delta m/m=\delta t/t$$

VOLUME 77, NUMBER 12 PHYSICAL REVIEW LETTERS 16 SEPTEMBER 1996

Mass Measurement of ^{100}Sn

M. Chartier,[1] G. Auger,[1] W. Mittig,[1] A. Lépine-Szily,[2] L. K. Fifield,[3] J. M. Casandjian,[1] M. Chabert,[1] J. Fermé,[1]
A. Gillibert,[4] M. Lewitowicz,[1] M. Mac Cormick,[1] M. H. Moscatello,[1] O. H. Odland,[5] N. A. Orr,[6] G. Politi,[7]
C. Spitaels,[1] and A. C. C. Villari[1]

$$\begin{aligned}
\text{M.E.}(^{100}\text{Cd}) &= -\,74.180 \pm 0.200(\text{syst})\ \text{MeV}\,,\\
\text{M.E.}(^{100}\text{In}) &= -\,64.650 \pm 0.300(\text{syst})\\
&\quad \pm 0.100(\text{stat})\ \text{MeV}\,,\\
\text{M.E.}(^{100}\text{Sn}) &= -57.770 \pm 0.300(\text{syst})\\
&\quad \pm 0.900(\text{stat})\ \text{MeV}\,.
\end{aligned}$$

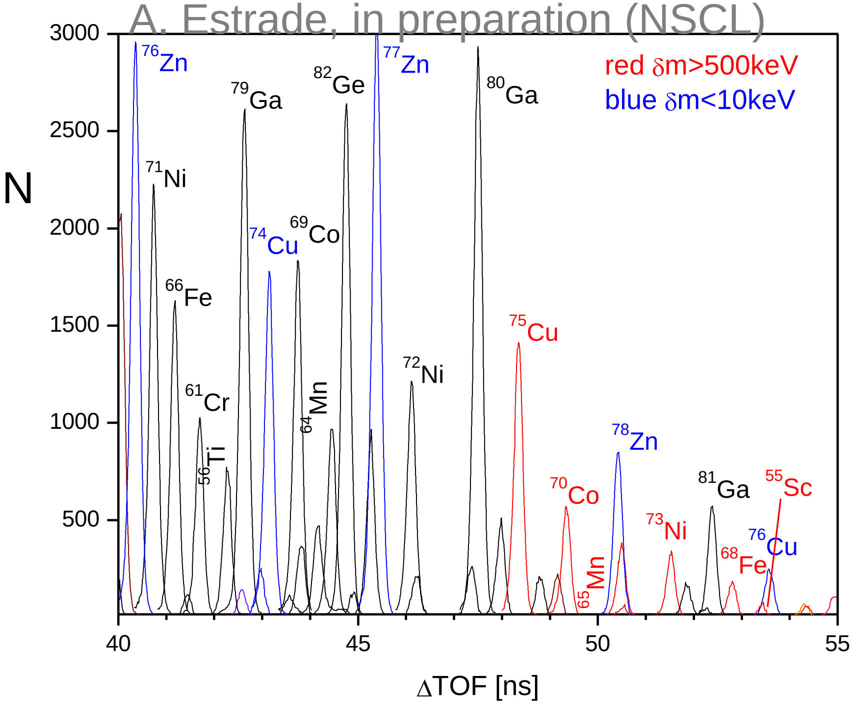
$$\frac{B}{\omega/h} = \gamma\frac{m}{q} = \frac{B\rho}{v}, \qquad \omega_c = \omega/h$$

$$\frac{\delta T_{turn}}{T_{turn}} = \frac{\delta m/q}{m/q}$$

h=#rf periods/turn

TOF mass measurements
– *Spectrographs at NSCL*

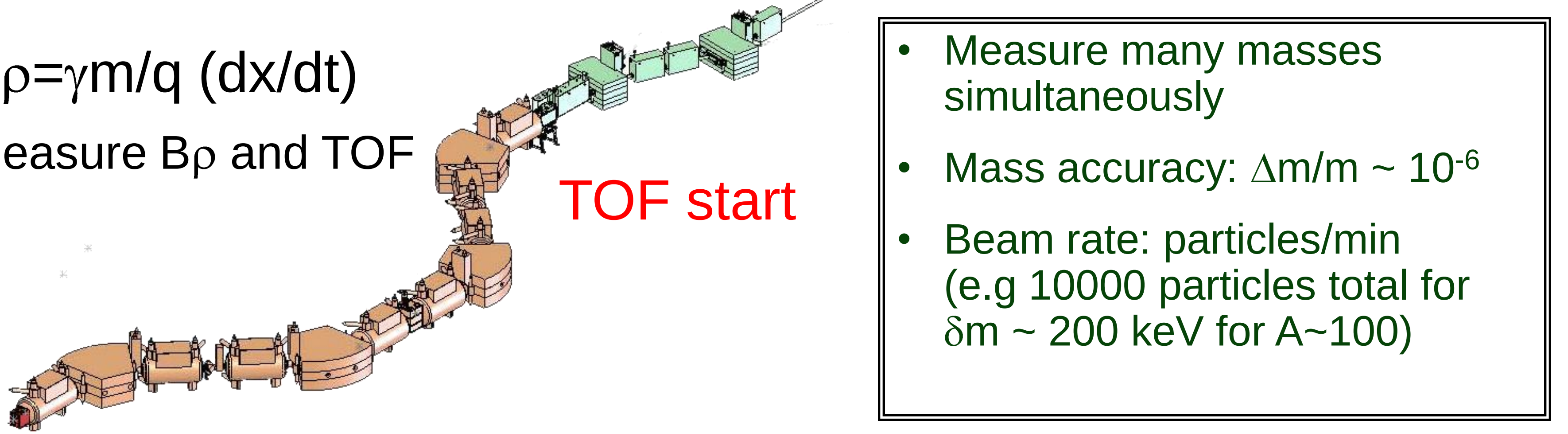
A. Estrade, in preparation (NSCL)



TOF mass measurements on neutron-rich isotopes
goal: δm = 0.2 MeV for A~70
→ δm/m=2 x10^{-6}

TOF stop
58m flight path

A1900+S800 at NSCL



$B\rho=\gamma m/q\,(dx/dt)$

Measure Bρ and TOF

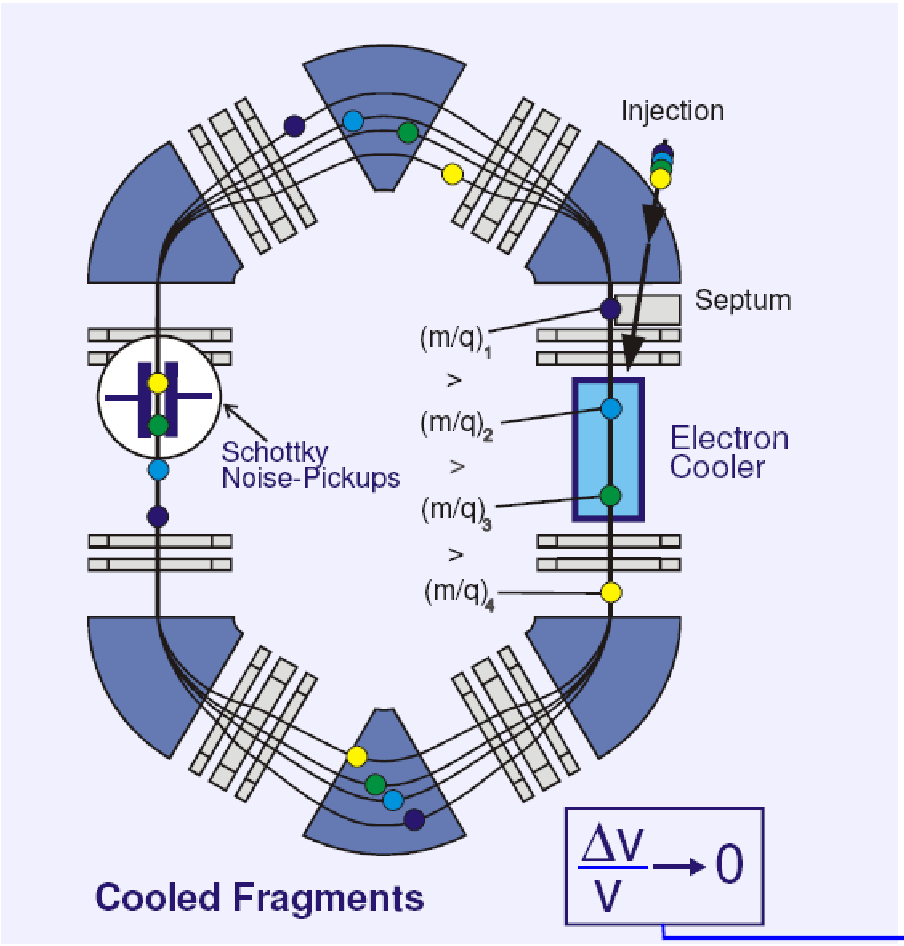
TOF start

- Measure many masses simultaneously
- Mass accuracy: Δm/m ~ 10^{-6}
- Beam rate: particles/min (e.g 10000 particles total for δm ~ 200 keV for A~100)

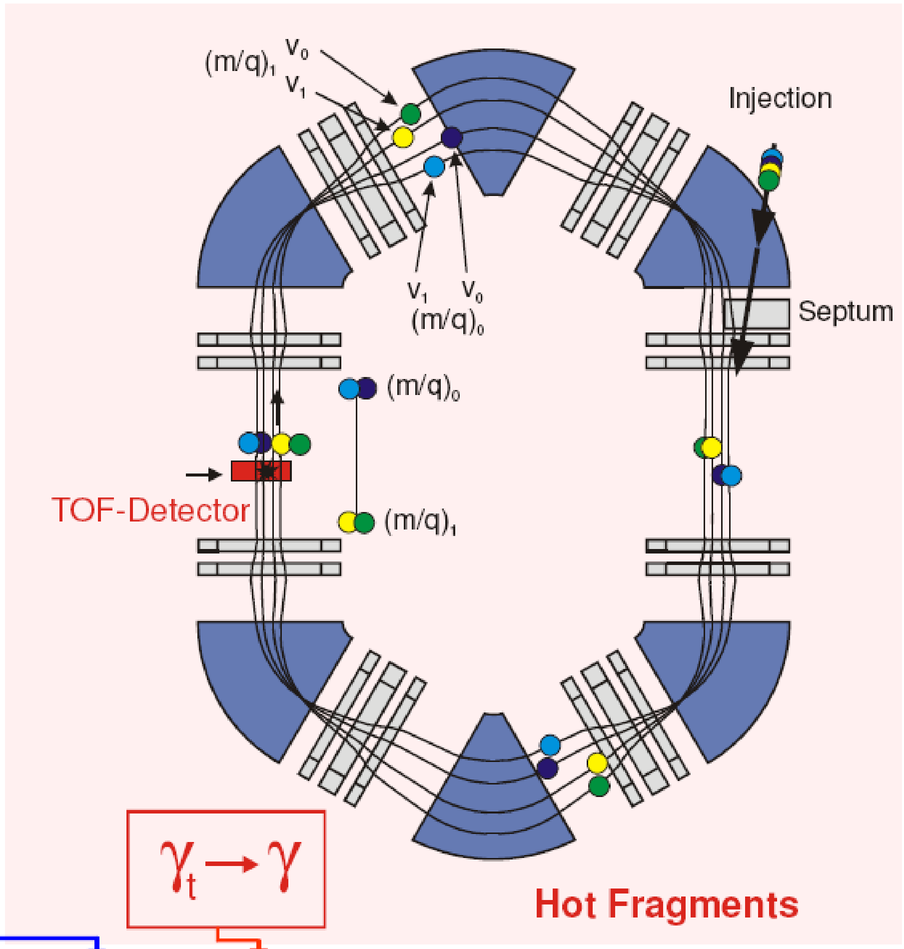
Mass measurements in the ESR storage ring at GSI

F. Bosch, Lect. Notes Phys. 651, 137 (2004)

SCHOTTKY MASS SPECTROMETRY



ISOCHRONOUS MASS SPECTROMETRY



$T_{1/2}>1$ s

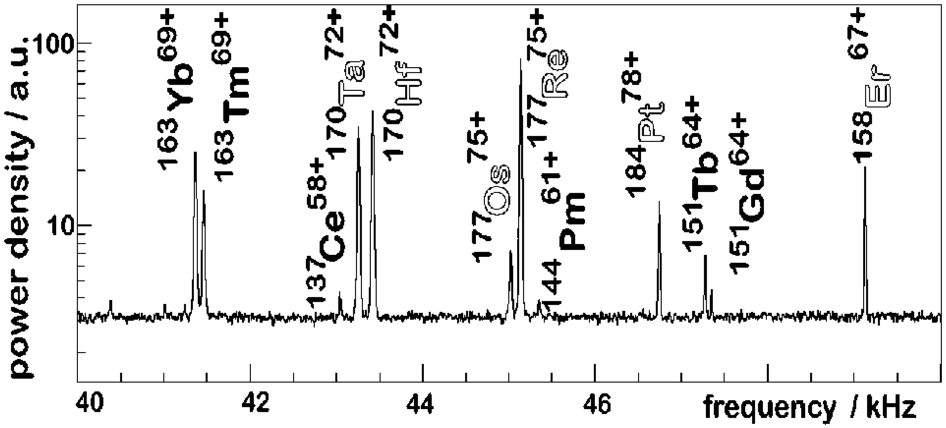
$T_{1/2}>10\ \mu$s

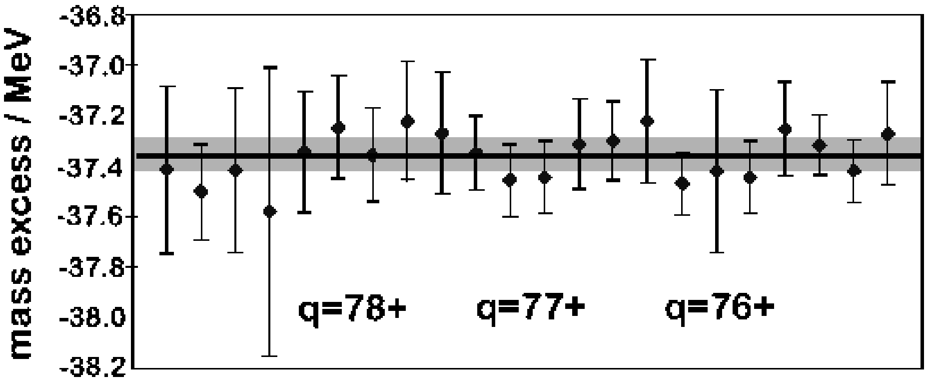
$$\frac{\Delta f}{f} = -\frac{1}{\gamma_t^2}\frac{\Delta(m/q)}{m/q} + \frac{\Delta v}{v}\left(1-\frac{\gamma^2}{\gamma_t^2}\right)$$

γ_t: relative change in path length by turn relative to change in $B\rho$

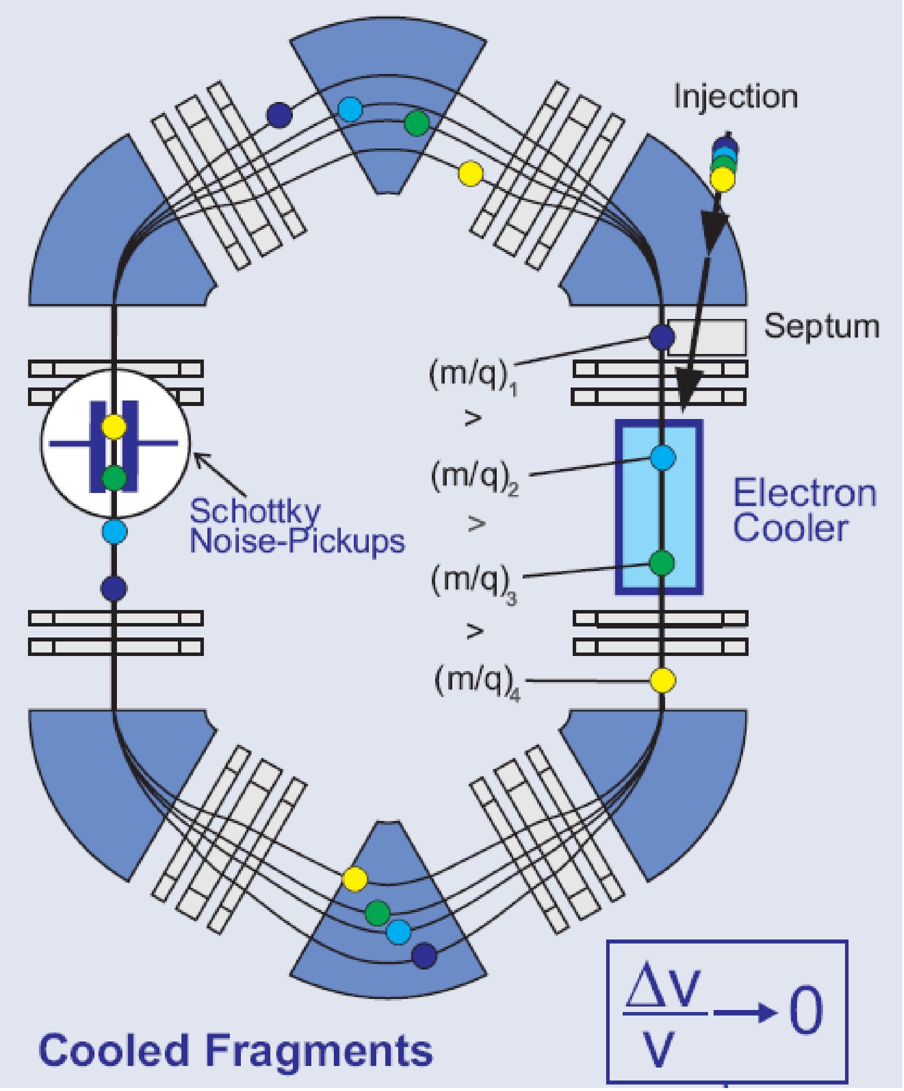
Mass measurements in the storage ring at GSI
I. Schottky mass spectrometry

- Schottky spectrometry in storage ring (GSI), e.g. ^{184}Pt





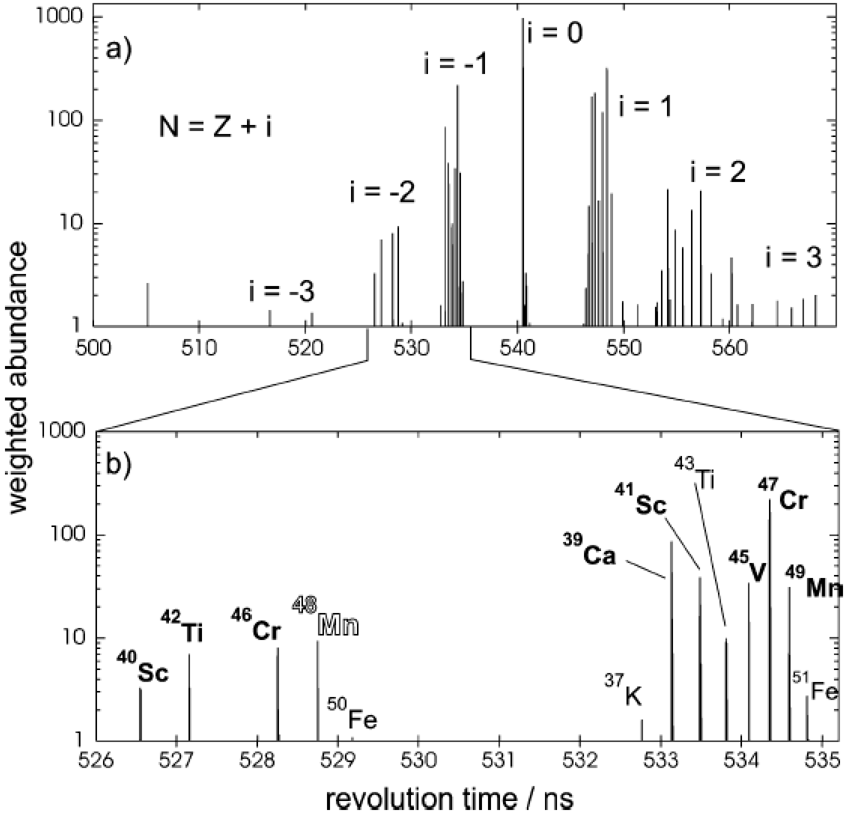
Mass excess for ^{184}Pt as determined in several runs using different reference isotopes and in different ionic charge states *q. (dm/m=5 10^{-7})*

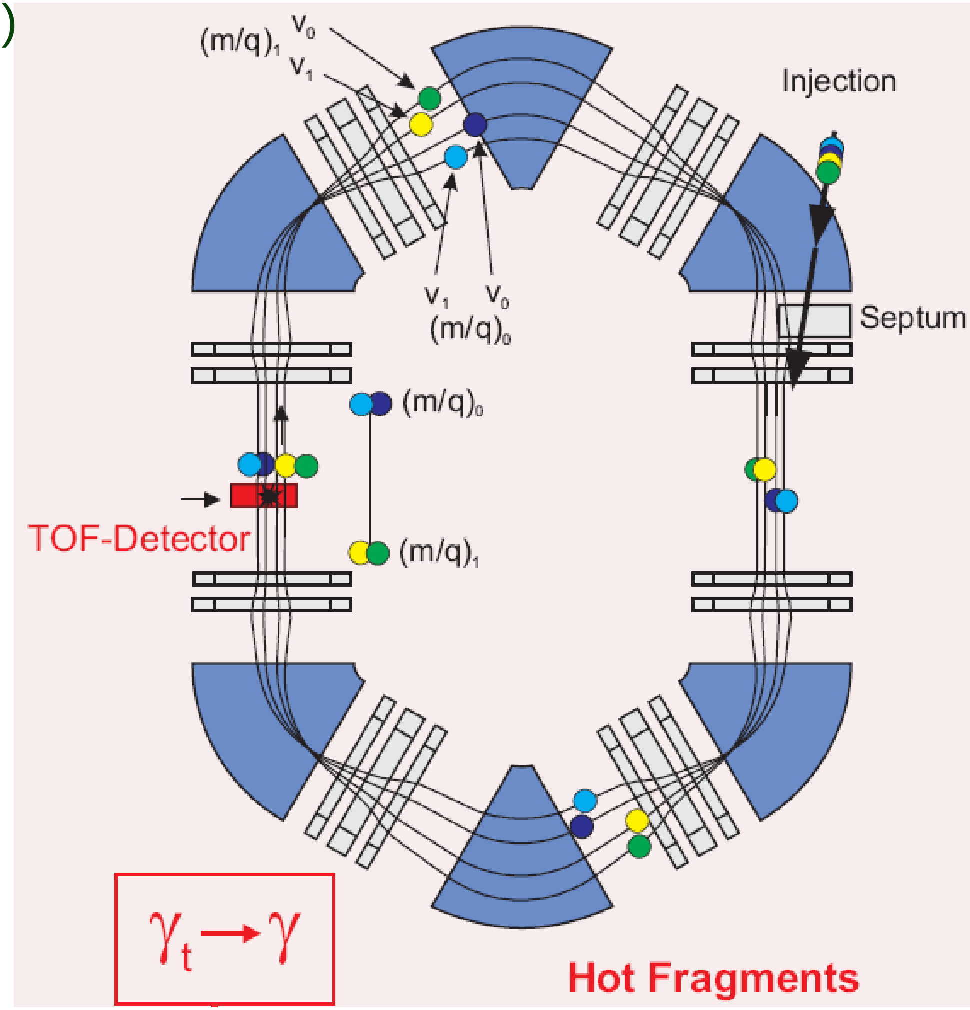


T. Radon et al., PRL 78, 4701 (1997)

Mass measurements in the storage ring at GSI
II. Isochronous mass spectrometry

- Mass measurement of short-lived ^{44}V, ^{48}Mn, ^{41}Ti and ^{45}Cr (X-ray burst models)

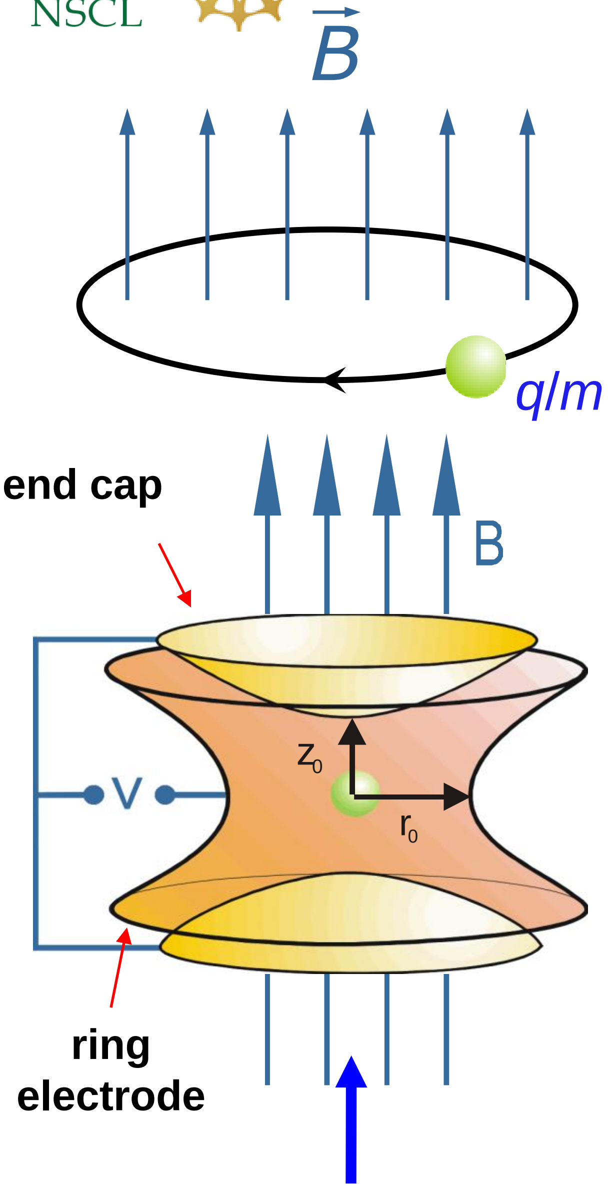




Accuracy of δm= 100-500 keV was achieved (lifetimes ~ 100 ms)

J. Stadlmann et al., PLB 586, 27 (2004)

Mass measurements with Penning traps



end cap

ring electrode

Ion source

Mass measurement via determination of <u>cyclotron frequency</u>

$$f_c = \frac{1}{2\pi} \cdot \frac{q}{m} \cdot B$$

from characteristic motion of stored ions

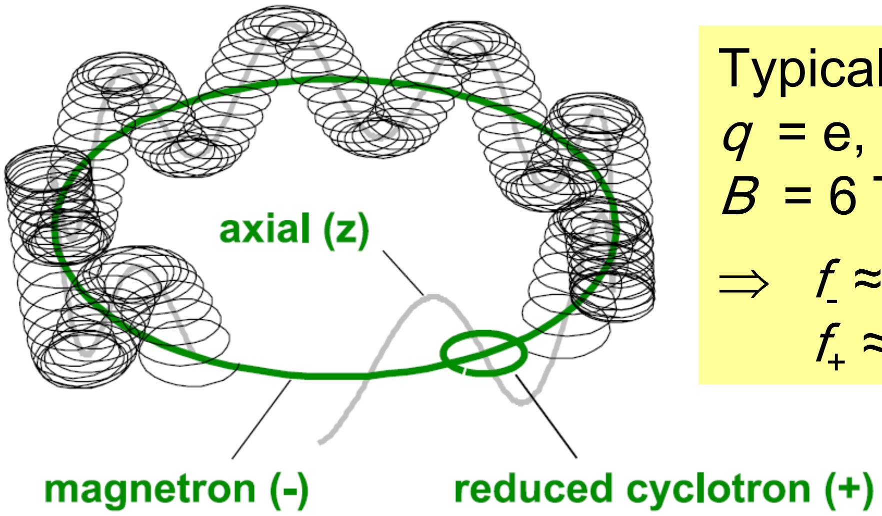
PENNING trap

- Strong homogeneous magnetic field of known strength B provides radial confinement
- Weak electric 3D quadrupole field provides axial confinement

Mass measurements with Penning traps

Motion of an ion is the superposition of three characteristic harmonic motions:

- axial motion (frequency f_z)
- magnetron motion (frequency f_-)
- modified cyclotron motion (frequency f_+)



Typical frequencies
q = e, m = 100 u,
B = 6 T

$\Rightarrow$ $f_- \approx 1$ kHz
$f_+ \approx 1$ MHz

Excite the cyclotron motion with multipolar RF (Goal: excite the cyclotron motion to resonance)

Transform radial to axial energy (gradient dB/dz) and eject ions

Measure time of flight (TOF) - the shorter TOF, the closer is the excitation frequency to the resonance

The frequencies of the radial motions obey the relation

$$f_+ + f_- = f_c = \frac{1}{2\pi} \cdot \frac{q}{\boxed{m}} \cdot B$$

Adapted from K. Blaum

Mass measurements with Penning traps

Eject thermalized ions from buncher

↓

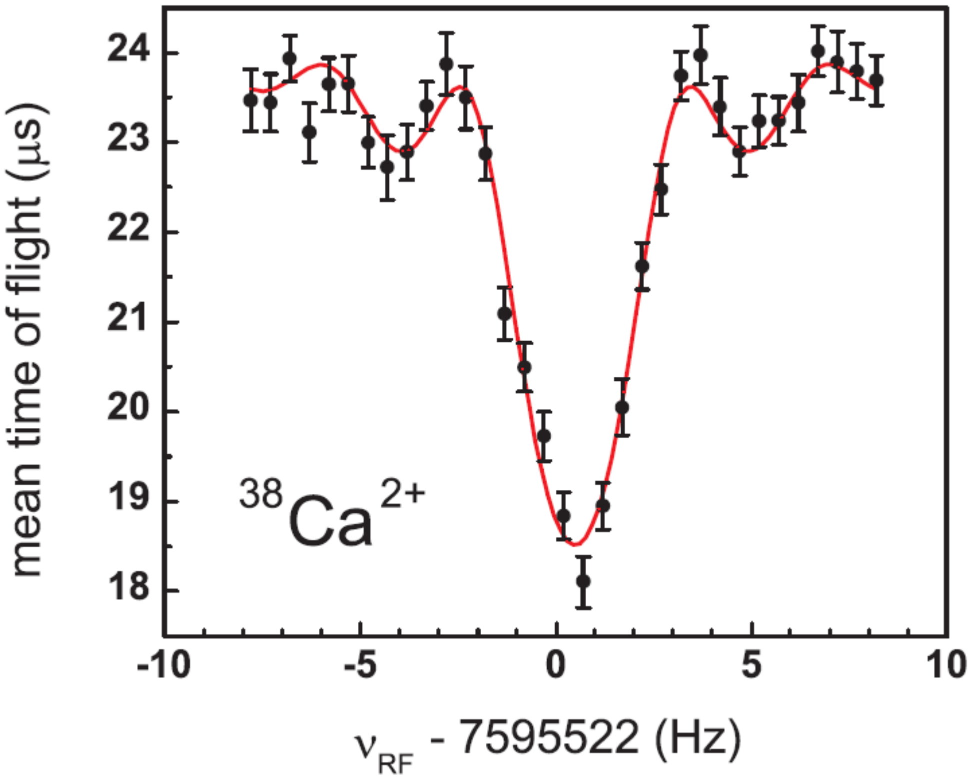
Capture ions in Penning trap

↓

Perform RF excitation

↓

Eject ions and measure TOF



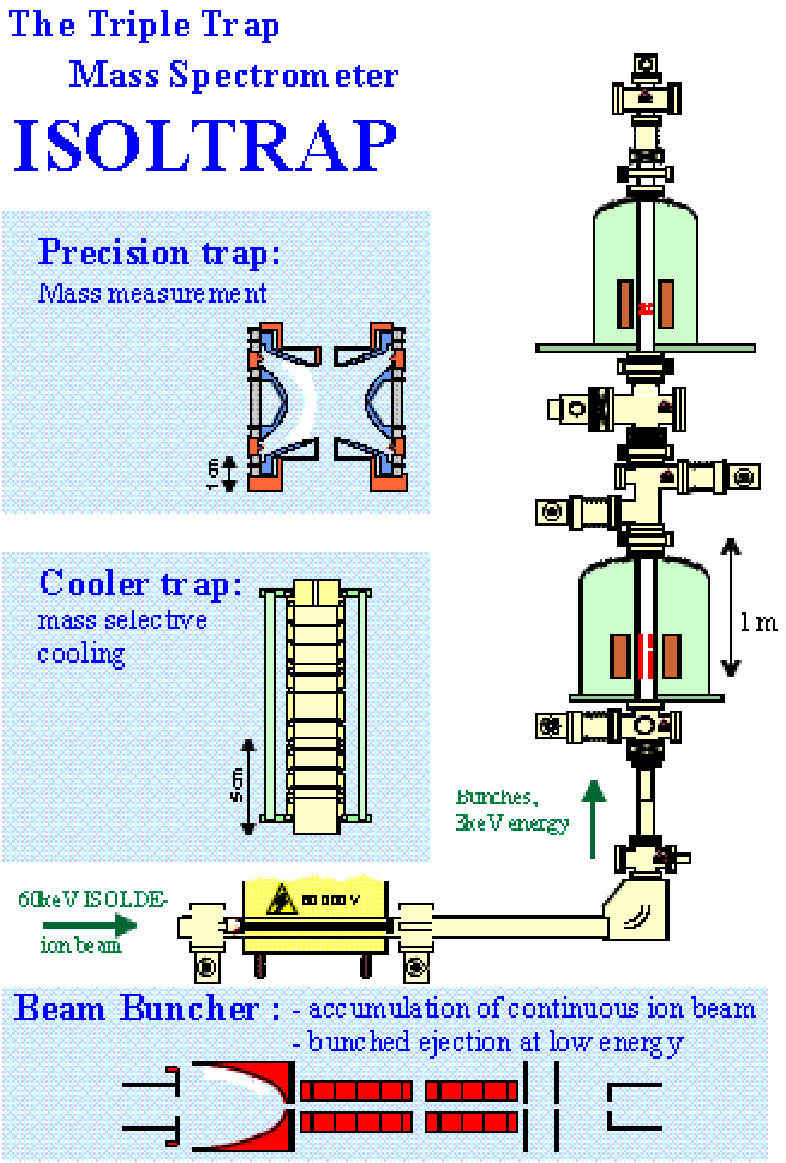
ME=-22058.53(28) keV

δm=280 eV

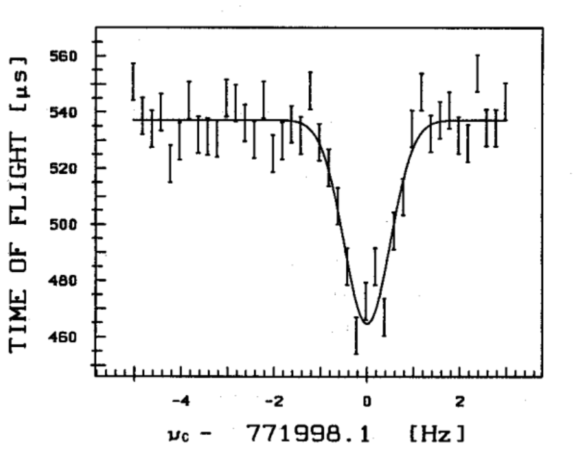
G. Bollen et al., PRL 96, 152501 (2006)

ISOLTRAP at CERN

GSI/Munich/NSCL/Mainz/Greifswald/CSNSM/CERN/McGill/Jyvaskyla



- e.g. ^{123,124,126}Ba

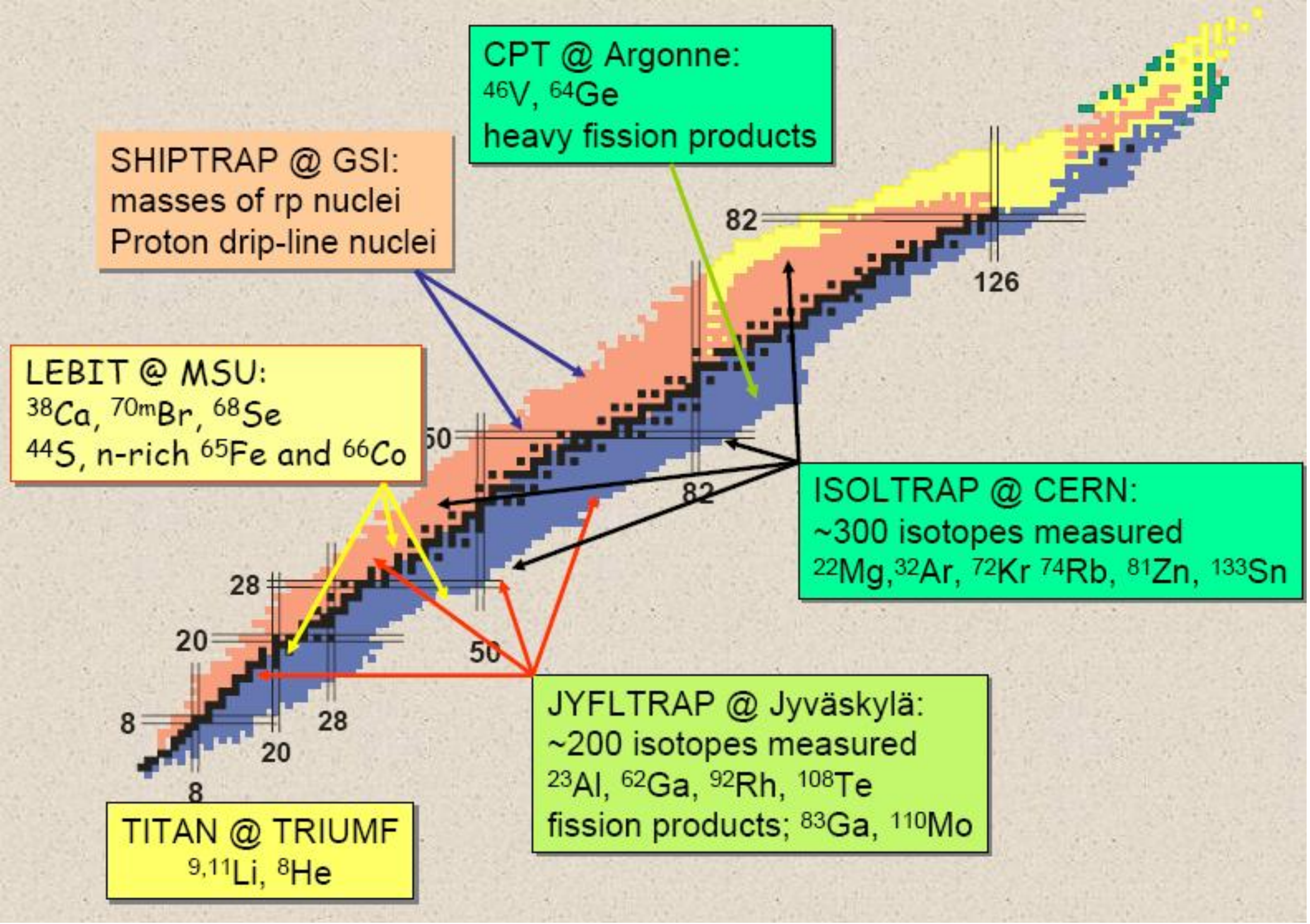


F. Ames et al., NPA 651, 3 (1999)

isoltrap.web.cern.ch/isoltrap/

Many other traps around the world, see e.g. EMIS 14
www.triumf.ca/emis14abs/abstracts.html#Topic06

Trap measurements – Overview



CPT @ Argonne:
^{46}V, ^{64}Ge
heavy fission products

SHIPTRAP @ GSI:
masses of rp nuclei
Proton drip-line nuclei

LEBIT @ MSU:
^{38}Ca, ^{70m}Br, ^{68}Se
^{44}S, n-rich ^{65}Fe and ^{66}Co

ISOLTRAP @ CERN:
~300 isotopes measured
^{22}Mg,^{32}Ar, ^{72}Kr ^{74}Rb, ^{81}Zn, ^{133}Sn

JYFLTRAP @ Jyväskylä:
~200 isotopes measured
^{23}Al, ^{62}Ga, ^{92}Rh, ^{108}Te
fission products; ^{83}Ga, ^{110}Mo

TITAN @ TRIUMF
^{9,11}Li, ^{8}He

J. Aysto (Trento, spring 2008)

Masses – what are they good for?

- Structure information
  - Shell closures and deformation from separation energies ($\delta m/m < 10^{-5}$)
- Astrophysics (Nucleosynthesis)
  - r process ($\delta m/m < 10^{-5}$, $\delta m < 10$ keV)
  - rp process ($\delta m/m \sim 10^{-7}$)
- Fundamental interactions and symmetries ($\delta m/m<10^{-8}$)
  - CVC
  - CKM

Theoretical description of masses

Algebraic: Garvey Kelson (GK), sum and difference relations between masses:

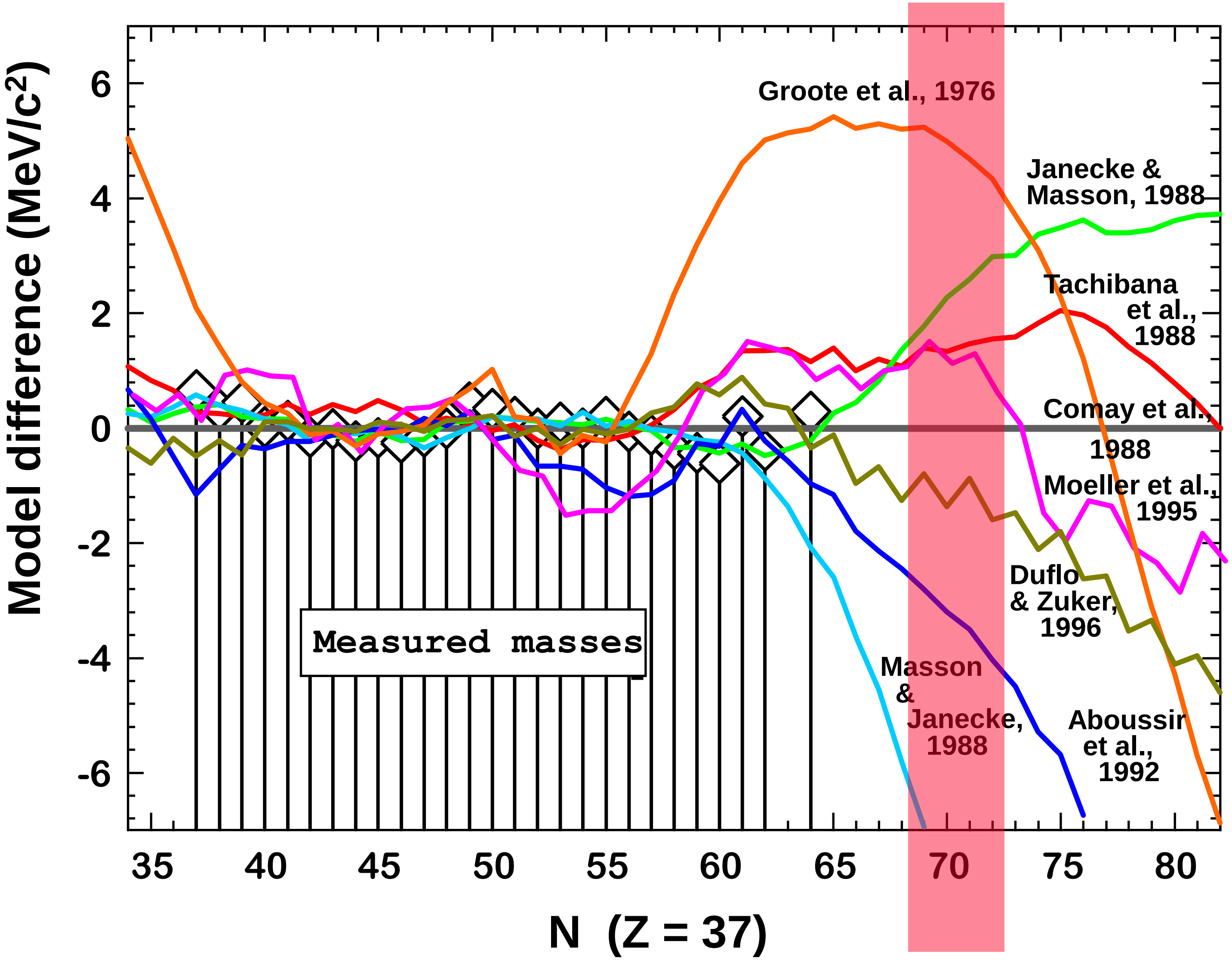
$$M(N+2,Z-2) - M(N,Z) + M(N,Z-1) - M(N+1,Z-2) + M(N+1,Z) - M(N+2,Z-1) = 0$$

Microscopic-macroscopic: For example the finite-range droplet model FRDM (31 parameters, largely fit to known masses), bulk part from liquid drop model (macroscopic) + shell and pairing corrections (microscopic) (extrapolation is dangerous)

Microscopic: Relativistic mean-field (RMF) and Hartree-Fock Bogoliubov (HFB), RMF is based on meson/photon exchange Lagrangian, HFB uses Skyrme or Gogny effective nucleon-nucleon interactions (computationally demanding)

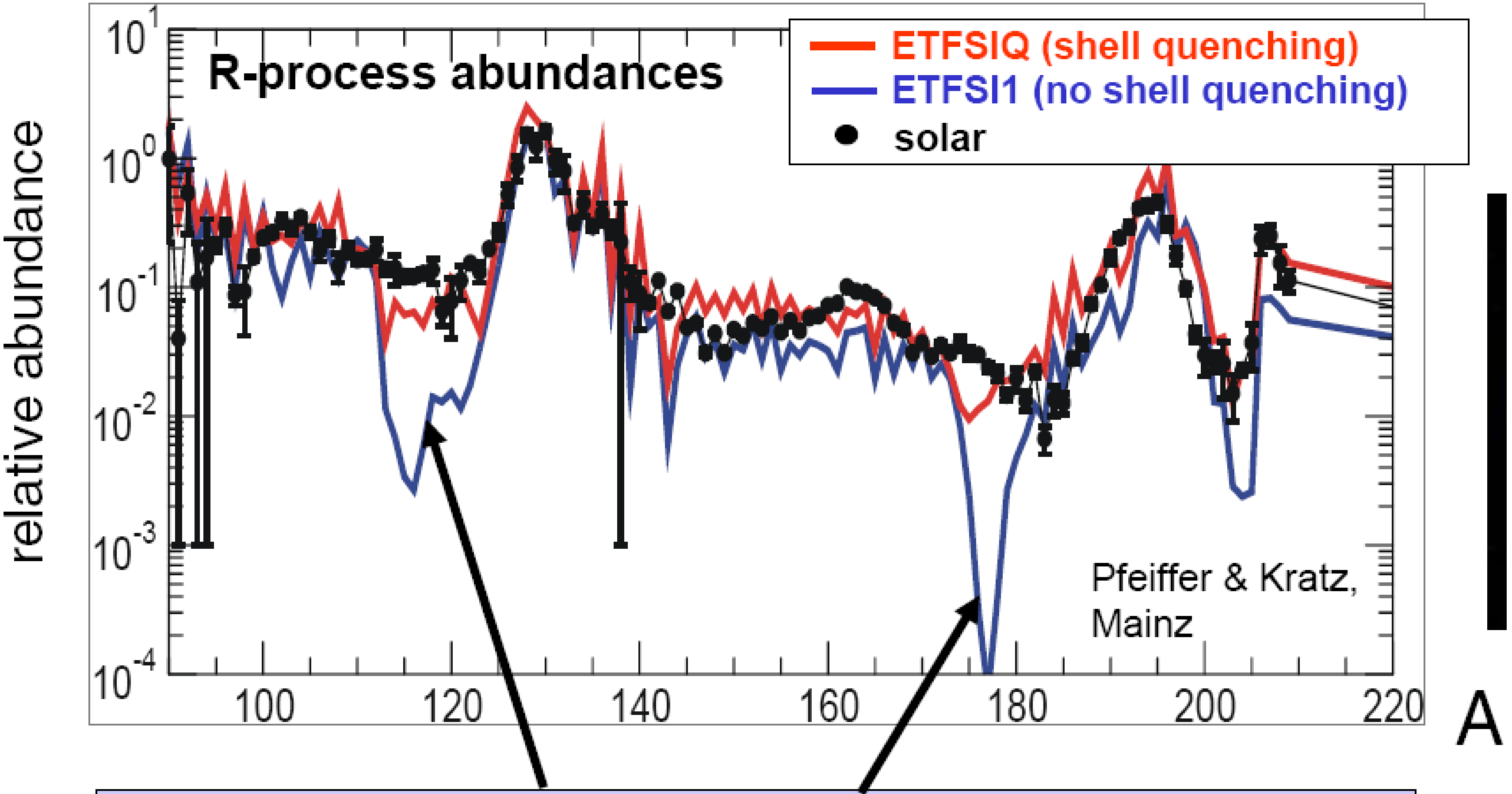
http://www.nuclearmasses.org/resources.html

Masses – what are they good for?
Constrain theory



Needed for r-process

Masses – what are they good for?
Nuclear astrophysics



Difference due to shell quenching for neutron-rich nuclei, or a problem with astrophysical model?

Masses – what are they good for?
Fundamental interactions/symmetries

Physics beyond the Standard Model

(required precision: as good as possible, at least: $\delta m/m<10^{-8}$)

- Conserved vector current (CVC) hypothesis
- Unitarity of the Cabbibo-Kobayashi-Maskawa (CKM) matrix

Details: lecture by Tim Chupp

Takeaway

- Nuclear masses and resulting nucleon separation energies are an indicator of nuclear structure and indicate changes in the shell structure in the exotic regime
- Exotic nuclei can be produced with different methods and reactions
- Masses of short-lived nuclei can be measured in different ways
    - Time of flight mass measurements
    - Storage rings
    - Penning traps
- Masses are important input for nuclear astrophysics and the study of fundamental symmetries

Related review articles

Masses

- Traps for rare isotopes, G. Bollen, Lect. Notes Phys. 651, 169 (2004)
- Measurements of mass and beta lifetime of stored exotic nuclei, F. Bosch, Lect. Notes Phys. 651, 137 (2004)
- Recent trends in the determination of nuclear masses , D. Lunney, J.M. Pearson, C. Thibault, Rev. Mod. Phys. 75, 1021 (2003)
- Mass measurements of short-lived nuclides with ion traps, G. Bollen, NPA 693, 3 (2001)
- Precision nuclear measurements with ion traps, G. Savard and G. Werth, Annu. Rev. Nucl. Sci. 50, 119 (2000)
- Mass measurement far from stability, W. Mittig and A. Lepine-Szily, Annu. Rev. Nucl. Sci. 47, 27 (1997)