20th National Nuclear Physics Summer School

Pion-Pion Scattering and Vector Symmetry

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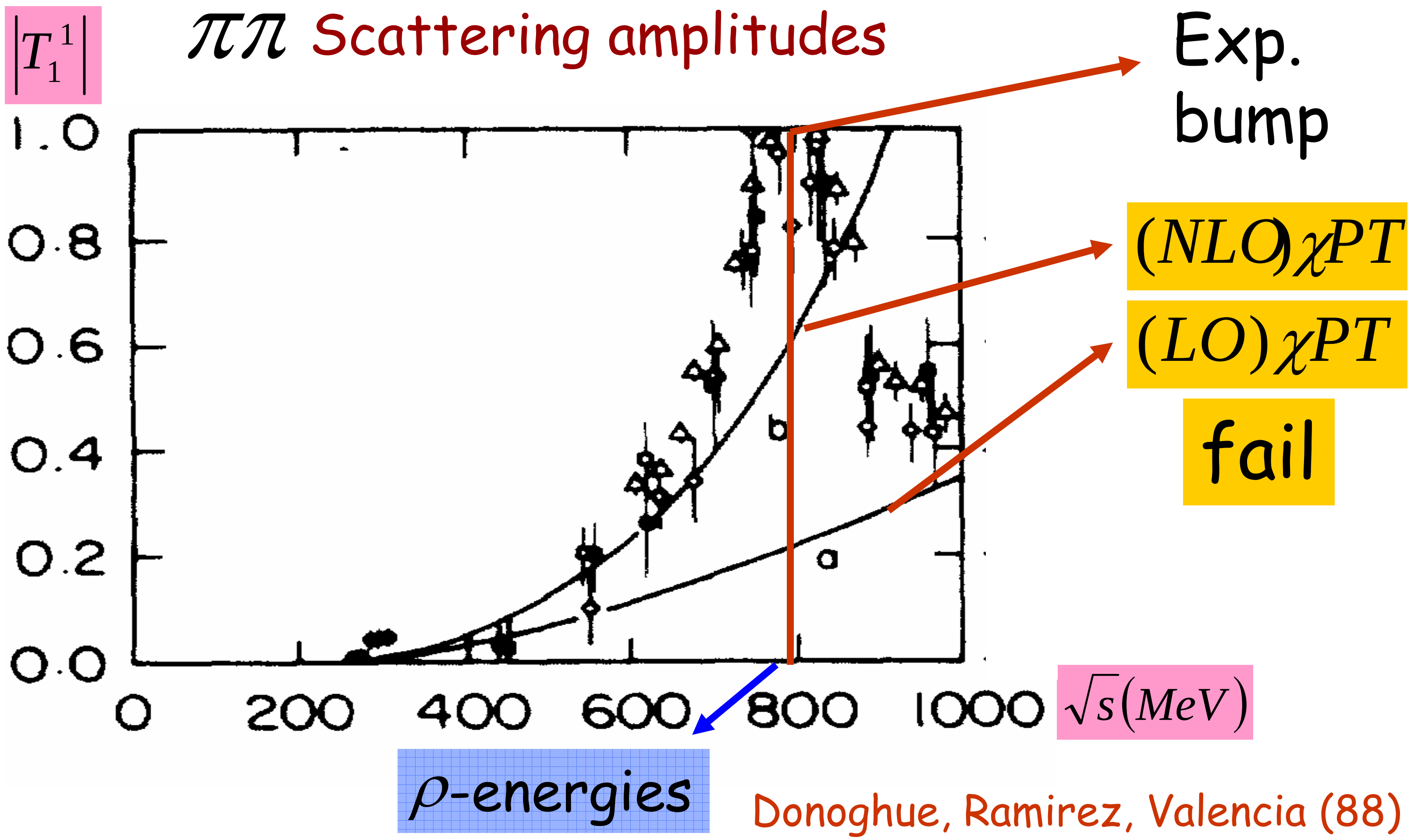
Outline

- Motivation
- Chiral Perturbation Theory
- Vector Model
- Breaking Vector Symmetry
- Pion-Pion Interaction
- Conclusion and Outlook

Experimental data and χPT theory

$\pi\pi$ Scattering amplitudes



Donoghue, Ramirez, Valencia (88)

IDEA: Include ρ-meson in EFT with a rational power counting and low-energy degree of freedom

Might be important for nuclear physics?

Chiral Perturbation Theory

Weinberg (79),
Gasser + Leutwyler (84)
...

Global Groups

$G = SU(2)_L \times SU(2)_R$

$q_L = \frac{1-\gamma_5}{2} q \rightarrow \underbrace{e^{i\vec{\alpha}_L \cdot \vec{\tau}/2}}_{L} q_L$

$L \leftrightarrow R$

$\underbrace{H = SU(2)_V}_{L = R}$

Spontaneously broken isospin

Massless particles

Goldstone bosons: pions

Pauli matrices in isospin space

$\pi = \vec{\pi} \cdot \vec{\sigma}/2$

Field matrix $\Sigma = e^{2i\pi/f} \rightarrow L \Sigma R^{+}$

$f = 93.1\mathrm{MeV}$ decay constant of the pion

Vector Symmetry

H. Georgi (90), P. Cho (90)

Global/Local Groups

$$G = SU(2)_L \times SU(2)_{G_L} \times SU(2)_R \times SU(2)_{G_R}$$

$SU(2)_{G_L}$, $SU(2)_{G_R}$ → dynamic groups; only hadronic level

$$H = SU(2)_{L+G_L} \times SU(2)_{R+G_R}$$

Vector Symmetry break

Massless particles

Goldstone bosons: Pions (pseudoscalars) and Scalars?

$$\Sigma_L \equiv e^{i\pi/f} e^{is/f} \to L \Sigma_L G_L^+$$

$$\Sigma_R = e^{-i\pi/f} e^{is/f} \to R \Sigma_R G_R^+$$

Eaten by Higgs mechanism

longitudinal ρ

Building EFT Lagrangian

- Infinite powers of series operators of increasing dimension with unspecified coefficients
- Each derivative → Q / M_{QCD}
- Each term → $f^2 M_{QCD}^2$
- Each mass → Q / M_{QCD}

$$Q^2 \sim m_\pi^2, m_\rho^2$$

$$L_{eff} = \sum_n L_{VS}^{(n)} \rightarrow L_{VS}^{(n)} = O\left(f^2 M_{QCD}^2 \left[Q^2 / M_{QCD}^2 \right]^n \right)$$

$n = 1$ LO

$n = 2$ NLO

$\vdots$ $\vdots$

$$L_{VS}^{(1)} = \frac{1}{2} f^2 Tr(L_\mu L^\mu + R_\mu R^\mu)$$

$$L_\mu \equiv -i\Sigma_L^+ \partial_\mu \Sigma_L \rightarrow G_L L_\mu G_L$$

$$R_\mu \equiv -i\Sigma_R^+ \partial_\mu \Sigma_R \rightarrow G_R R_\mu G_R^+$$

Breaking Vector Symmetry

$$L^{(1)} = \frac{1}{2} f^2 Tr(L_\mu L^\mu + R_\mu R^\mu)$$

$$L_\mu \equiv -i\Sigma_L^+ D_\mu \Sigma_L \to G_L L_\mu G_L^+$$

$$D_\mu \Sigma_L = \partial_\mu \Sigma_L - ig\Sigma_L \rho_\mu^L$$

$$R_\mu \equiv -i\Sigma_R^+ D_\mu \Sigma_R \to G_R R_\mu G_R^+$$

$$D_\mu \Sigma_R = \partial_\mu \Sigma_R - ig\Sigma_R \rho_\mu^R$$

Give mass to the ρ − meson

$$+ \frac{1}{2} f^2 Tr(\mu \mathcal{M} + \mathcal{M}^+ \mu)$$

$$\mathcal{M} = \Sigma_R^+ M \Sigma_L \to G_L \mathcal{M} G_R^+$$

$$\mu \to G_L \mu G_R^+$$

Give mass to the pions

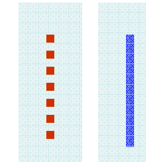
$$-\frac{1}{2} Tr(\rho^L_{\mu\nu} \rho^{L\mu\nu} + \rho^R_{\mu\nu} \rho^{R\mu\nu})$$

$$\rho_{\mu\nu}^{L,R} = \partial_\mu \rho_\nu^{L,R} - \partial_\nu \rho_\mu^{L,R} + ig\left[\rho_\mu^{L,R}, \rho_\nu^{L,R}\right]$$

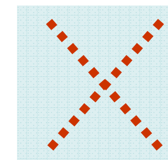
$$g\rho_\mu^{L,R} \to G_{L,R}\, g\rho_\mu^{L,R} G_{L,R}^+ - iG_{L,R} \partial_\mu G_{L,R}^+$$

$$+ \quad L^{(2)} \quad + \quad \ldots$$

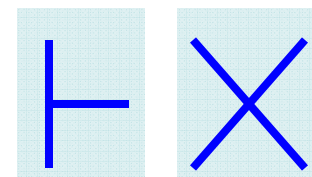
$$L^{(1)}_{eff} = Tr\partial_\mu \pi \partial^\mu \pi - \underbrace{2\mu\hat{m}}_{m_\pi^2} Tr\pi^2 - \frac{1}{2} Tr\left(\partial_\mu \rho_\nu - \partial_\nu \rho_\mu\right)^2 + \underbrace{(gf)^2}_{m_\rho^2} Tr\rho_\mu \rho^\mu$$



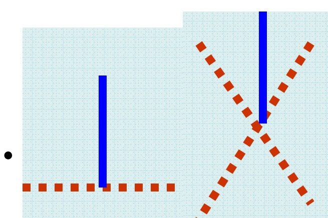
$$+ \frac{1}{6f^2} Tr\pi\partial_\mu \pi\left[\pi, \partial^\mu \pi\right] + \frac{2\mu\hat{m}}{3f^2} Tr\pi^4 + \ldots$$



$$-2igTr\partial^\mu \rho^\nu \left[\rho_\mu, \rho_\nu\right] + \frac{1}{2} g^2 Tr\left[\rho^\mu, \rho^\nu\right]\left[\rho_\mu, \rho_\nu\right]$$



$$+ igTr\rho^\mu \left[\pi, \partial_\mu \pi\right] - \frac{ig}{12 f^2} Tr\rho^\mu \left[\pi, \left[\pi, \left[\pi, \partial_\mu \pi\right]\right]\right] + \cdots$$

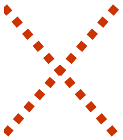


Pion-Pion Scattering

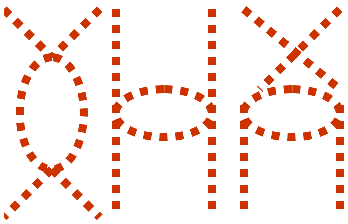
R. Azevedo, U. van Kolck

Power Counting

χPT $\quad Q^2 \sim m_\pi^2$



$$\sim \frac{Q^2}{f_\pi^2}$$ **LO**



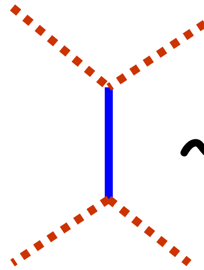
$$\sim \frac{Q^2}{f_\pi^2}\left(\frac{Q}{4\pi f_\pi}\right)^2$$ **NLO**

Vector Realization $\quad Q^2 \sim m_\pi^2 \sim m_\rho^2$

$$Q^2 \sim m_\rho^2\left(1+\frac{\delta Q^2}{m_\rho^2}\right)$$

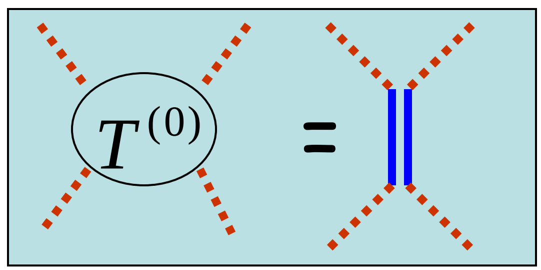
$\frac{\delta Q^2}{m_\rho^2} << 1$

Close to resonance

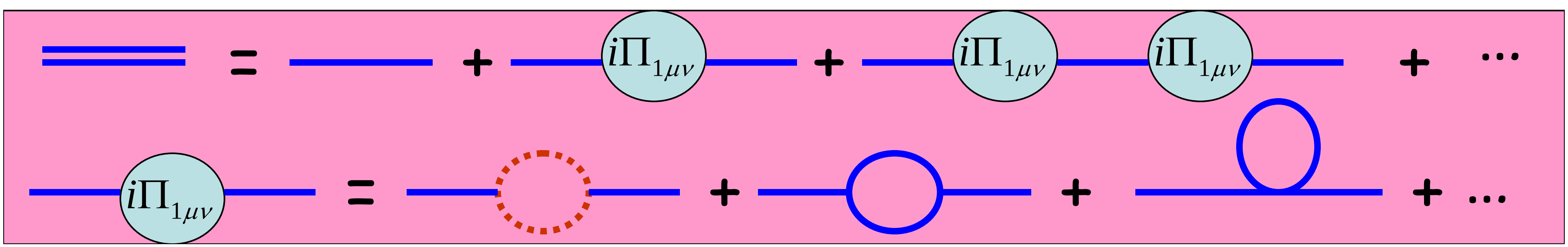


$$\sim \frac{Q^2}{f_\pi^2}\frac{m_\rho^2}{\delta Q^2}$$

$>> 1$ enhancement



I=1, l=1



$$-2\pi i T_1^1 = e^{2i\delta_1^1} - 1 = -\frac{2i m_\rho^R \Gamma\left(m_\rho^{2R}\right)}{s - m_\rho^{2R} + i m_\rho^R \Gamma\left(m_\rho^{2R}\right)}$$

$$m_\rho^{2R} = m_\rho^2 + \mathrm{Re}(\Pi_{1\mu\nu})$$

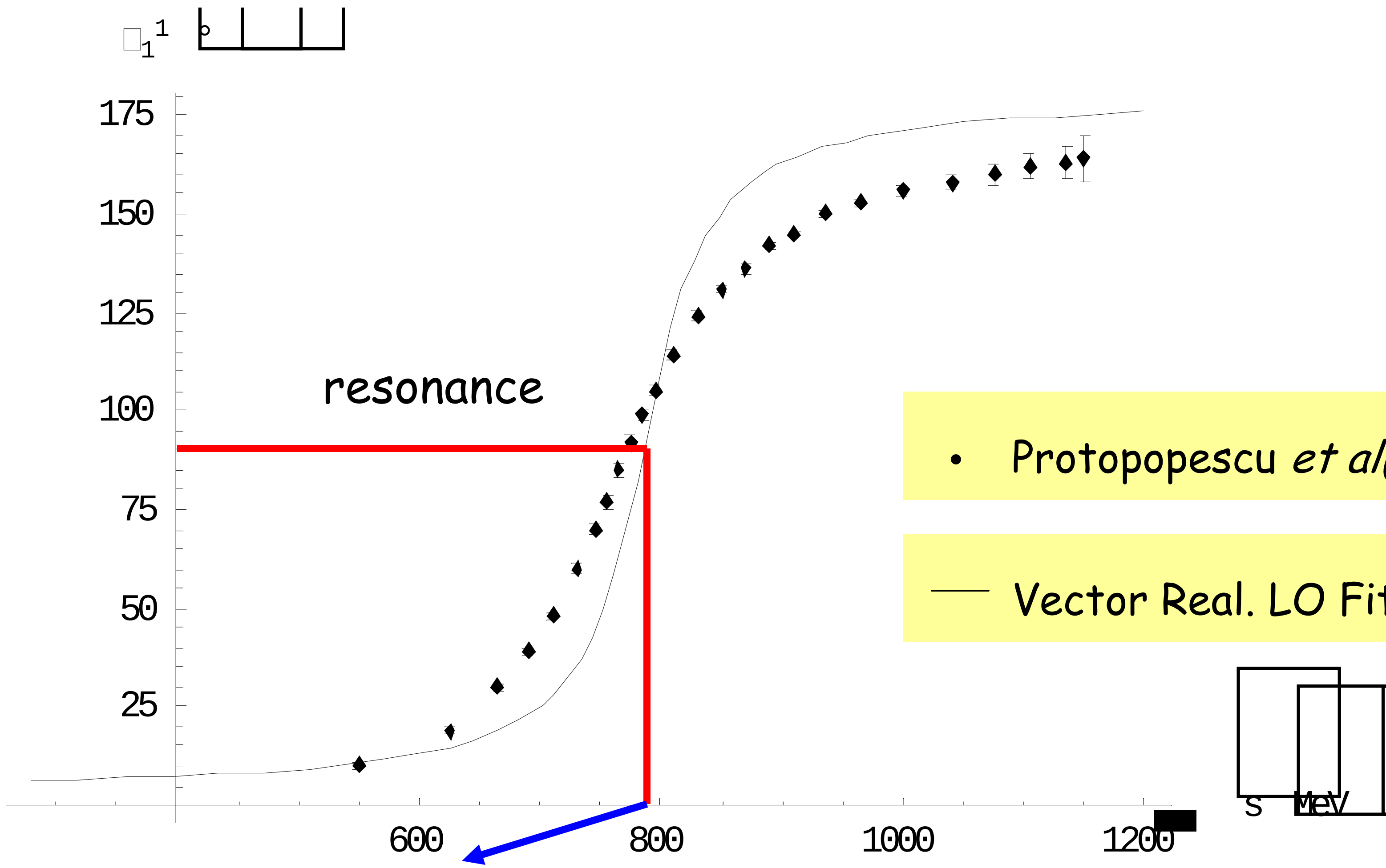
Bare mass – a parameter of the Lagrangian

Renormalized mass – observable

$$m_\rho^R \Gamma\left(m_\rho^{2R}\right) = \mathrm{Im}\left(\Pi_{1\mu\nu}\right)$$

$$\Gamma\left(m_\rho^{2R}\right) = \frac{\pi}{12} m_\rho^R \left(\frac{m_\rho^R}{4\pi f_\pi}\right)^2 \left(1 - 4\frac{m_\pi^2}{m_\rho^{2R}}\right)^{3/2}$$

Preliminaries

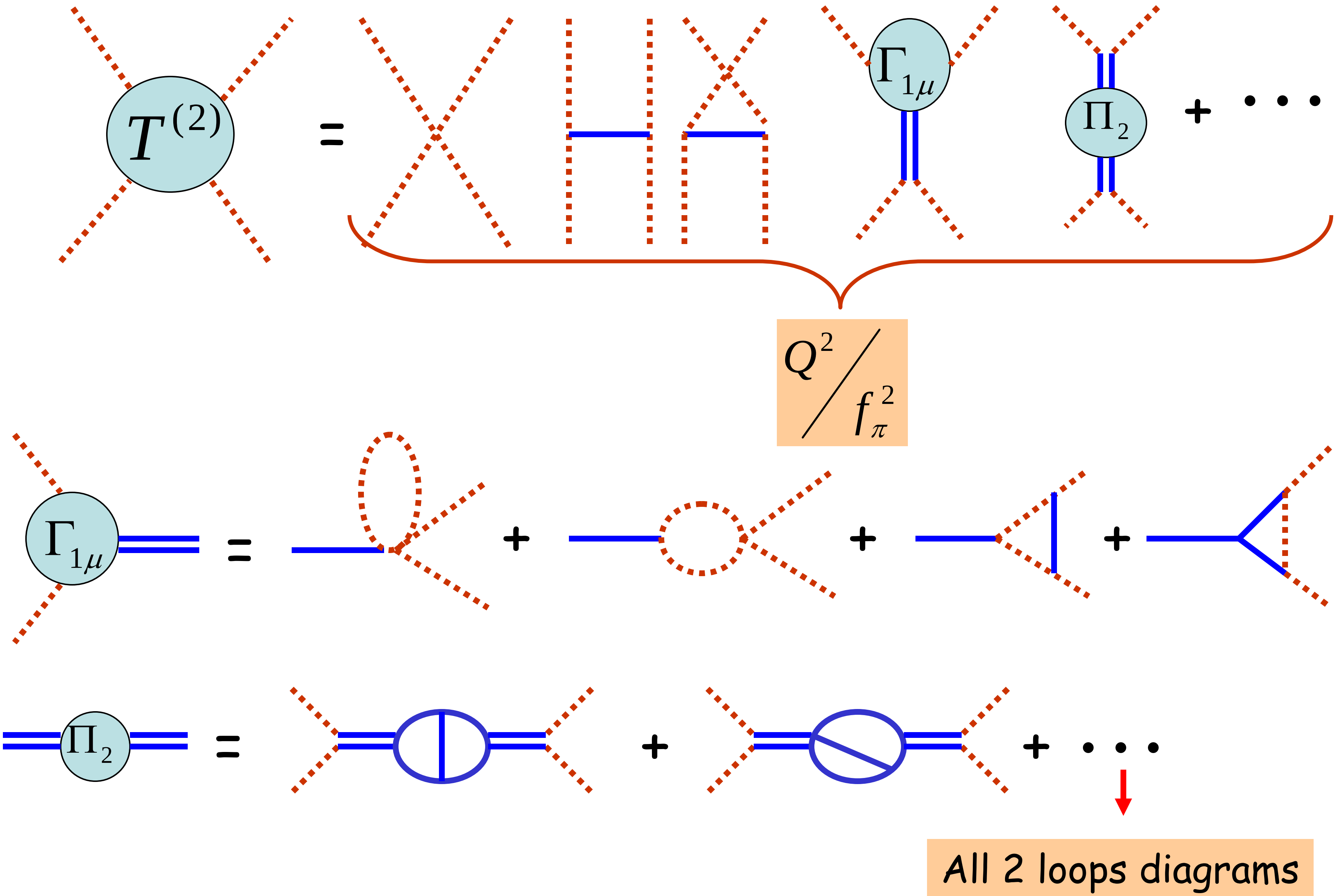


fit : $m_\rho = 785.4\,\mathrm{MeV}$
exp.: $m_\rho = (770 \pm 3)\mathrm{MeV}$

fit : $\Gamma(\rho \to \pi\pi) = 75.8\,\mathrm{MeV}$
exp. : $\Gamma(\rho \to \pi\pi) = (153 \pm 2)\mathrm{MeV}$

OK qualitatively, quantitatively: NLO

Next-to-leading order (NLO)



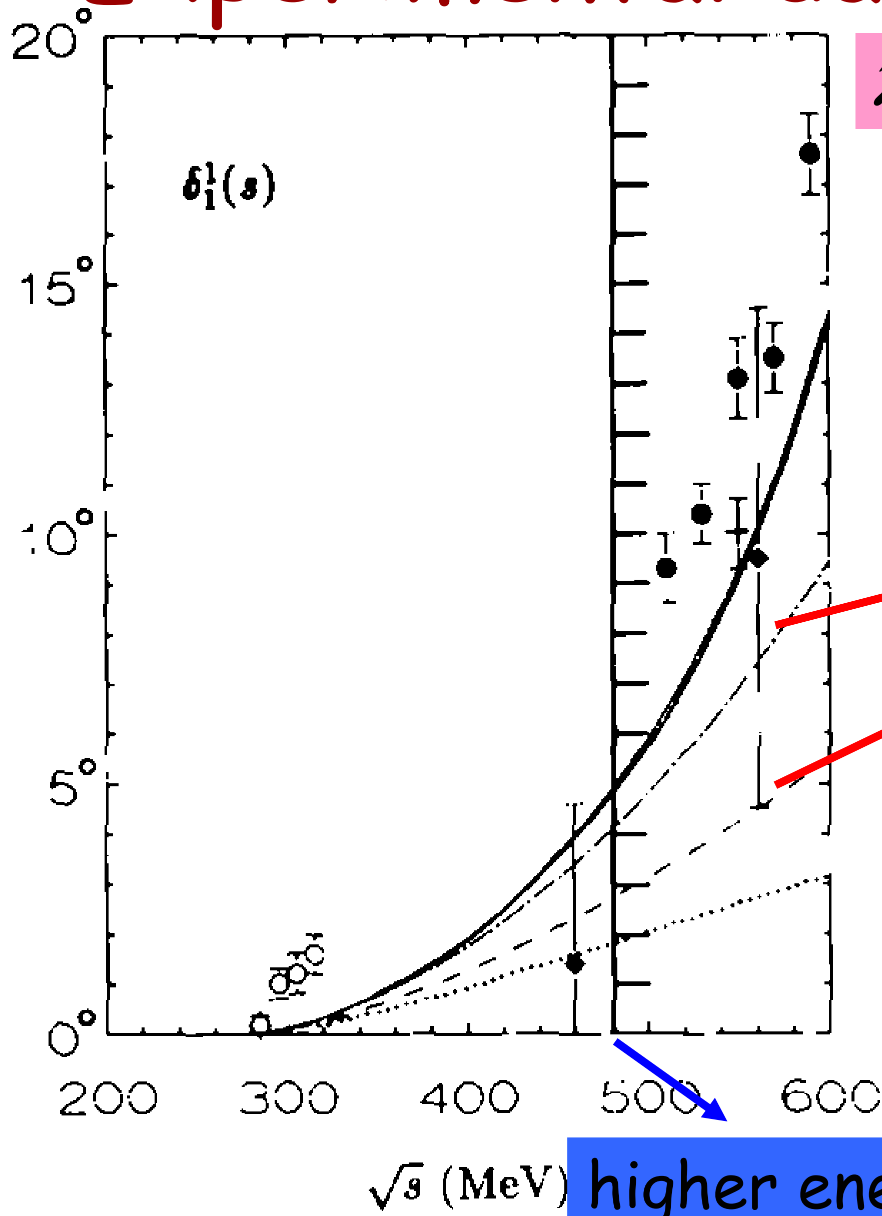
Conclusions and Outlook

- Vector Realization: a possible way to introduce ρ in the EFT with a rational power counting;
- $\pi\pi$ scattering: LO $-m_\rho$ OK

  $-\Gamma\left(m_\rho^2\right)$ Not OK yet

  NLO next ;
- Include ρ -nucleon interactions in the theory----nuclear forces.

Experimental data and χPT theory



$\pi\pi$ Phase Shift (l=1, I=1)

$(NLO)\,\chi PT$

$(LO)\,\chi PT$

fail

higher energies

IDEA: Include ρ in EFT

Might be important for nuclear physics?

Gasser, Meissner (91)

Motivation

- Leading order (LO) for the finite range of the nucleon-nucleon interaction exchanging one pion

$$V_{OPE}(r) = f_\pi^2 m_\pi \tau_1 \bullet \tau_2 \left[\left(\frac{1}{3m_\pi r} + \frac{1}{(m_\pi r)^2} + \frac{1}{(m_\pi r)^3} \right) e^{-m_\pi r} S_{12} + \frac{1}{3} \sigma_1 \bullet \sigma_2 \frac{e^{-m_\pi r}}{m_\pi r} \right]$$

$$S_{12} = 3\sigma_1 \bullet \hat{r} \sigma_2 \bullet \hat{r} - \sigma_1 \bullet \sigma_2$$

The only other meson with isospin one in the nucleon-nucleon interaction is the ρ-meson

Also, ρ-mass is large and the ρ-interaction, therefore, short-ranged.

The tensor terms arise from one-pion-exchange and one-rho-exchange potentials

Inclusion of the rho in EFT

J. Speth, V. Klemt, J. Wambach, G.E.Brown Nucl. Phys. A343, 382 (1980)

$$\Downarrow$$

should reduce the 2N tensor force at short distances

The idea is simplifying the renormalization of the LO nuclear potential by canceling them with the counterterms found in the pionful EFT

We need power expansions of $\dfrac{m_\rho}{M_{QCD}}$

Suggestion: A new realization of Chiral Symmetry – The Vector Phenomenology

H. Georgi Nucl. Phys. B331, 311 (1990); P. Cho Nucl. Phys. B358, 383 (1991)