The two-nucleon current operator in chiral effective field theory

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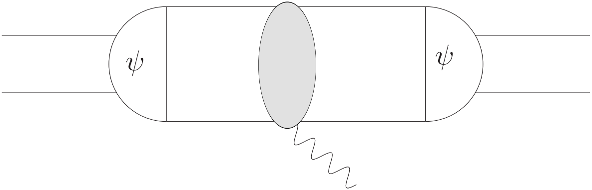
Motivation

- Calculate processes like $n + p \rightarrow d + \gamma$
- Many phenomenological works since the 70's
- Pioneering work of Park, Min and Rho showed how to apply χ-EFT on 2N electromagnetic currents, but still a hybrid approach
- Successful derivation of nuclear forces using the method of unitary transformation
- Consistent derivation of nuclear currents in this framework $\rightarrow$ controlled uncertainty



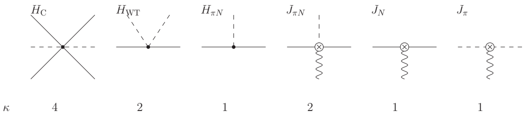
Power counting scheme

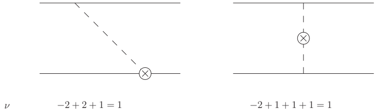
- Classify contributions according to $\left(\frac{Q}{\Lambda}\right)^{\nu}$
- Naive power counting

$$\nu = -2 + \sum_i V_i \kappa_i \quad \text{with} \quad \begin{aligned} \kappa_i &= e_i + d_i + \frac{3}{2} n_i + b_i - 4 \\ \kappa_i &> 0 \end{aligned}$$

counts the power of the hard scale Λ

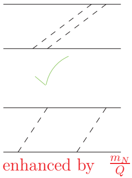
- More convenient for algebraic calculations

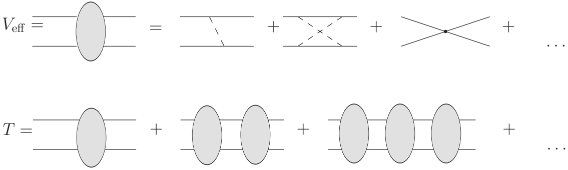




Shallow bound states?

- We follow Weinberg's description to cope with bound states
  $\longrightarrow$ calculate only irreducible graphs and iterate





Disadvantages:

- Effective potential is energy dependent
- Wave functions are not orthogonal

Method of unitary transformation

- Decompose Fock space

$$
\begin{aligned}
|\phi\rangle &= |N, NN, NNN, \dots\rangle \\
|\varphi\rangle &= |N\pi\rangle + |N\pi\pi\rangle + \dots + |NN\pi\rangle + \dots
\end{aligned}
$$

$$
\begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \begin{pmatrix} |\phi\rangle \\ |\varphi\rangle \end{pmatrix} = E \begin{pmatrix} |\phi\rangle \\ |\varphi\rangle \end{pmatrix}
$$

- Decouple the two subspaces by a unitary transformation

$$
H_{\mathrm{eff}} = U^\dagger H U = \begin{pmatrix} \eta H_{\mathrm{eff}} \eta & 0 \\ 0 & \lambda H_{\mathrm{eff}} \lambda \end{pmatrix}
$$

Method of unitary transformation

- Parameterize U by (Okubo '54):

$$U = \begin{pmatrix} \eta\left(1+A^\dagger A\right)^{-\frac{1}{2}} & -A^\dagger\left(1+AA^\dagger\right)^{-\frac{1}{2}} \\ A\left(1+A^\dagger A\right)^{-\frac{1}{2}} & \lambda\left(1+AA^\dagger\right)^{-\frac{1}{2}} \end{pmatrix}$$

$$A = \lambda A \eta$$

- Obtain decoupling equation

$$\lambda\left(H + [H, A] - AHA\right)\eta = 0$$

- Expand Hamiltonian and A in powers of κ

$$H = H_0 + \sum_{\kappa=1}^{\infty} H_\kappa \quad \text{and} \quad A = \sum_{\kappa=1}^{\infty} A_\kappa$$

Method of unitary transformation

$$A_1 = -\frac{\lambda}{\omega} H_1 \eta$$
$$A_2 = -\frac{\lambda}{\omega} \left(H_2 - A_1 \eta H_1 + H_1 \lambda A_1 \right)$$

Advantages:

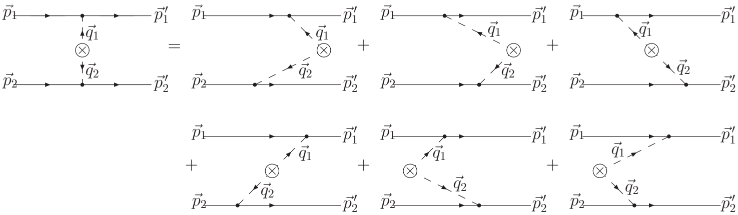
- Wave functions orthogonal
- Effective potential energy independent

$$J^{\mu}_{\text{eff}} = \eta \left(1 + A^{\dagger} A\right)^{-\frac{1}{2}} \left(J^{\mu} + A^{\dagger} J^{\mu} + J^{\mu} A + A^{\dagger} J^{\mu} A\right) \left(1 + A^{\dagger} A\right)^{-\frac{1}{2}} \eta$$

Example of the calculation

Consider the one-pion exchange contribution

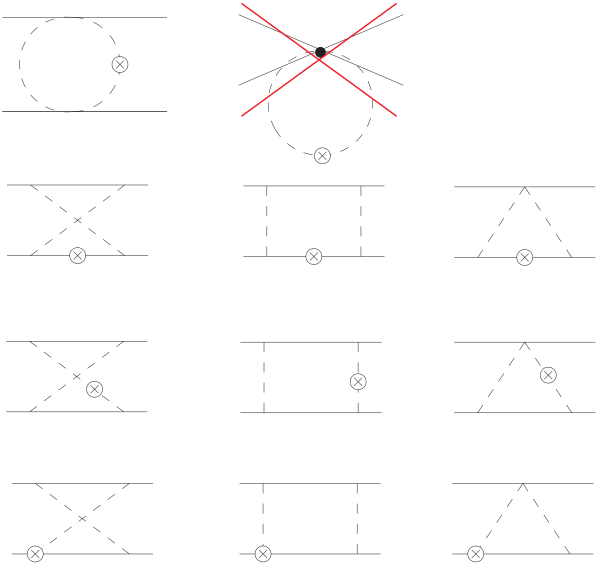
$$J_{\mathrm{eff},\,0} \;=\; \eta\left[J_\pi \frac{\lambda^2}{\omega} H_{\pi N} \frac{\lambda^1}{\omega} H_{\pi N} + H_{\pi N} \frac{\lambda^1}{\omega} J_\pi \frac{\lambda^1}{\omega} H_{\pi N} + H_{\pi N} \frac{\lambda^1}{\omega} H_{\pi N} \frac{\lambda^2}{\omega} J_\pi\right]\eta$$



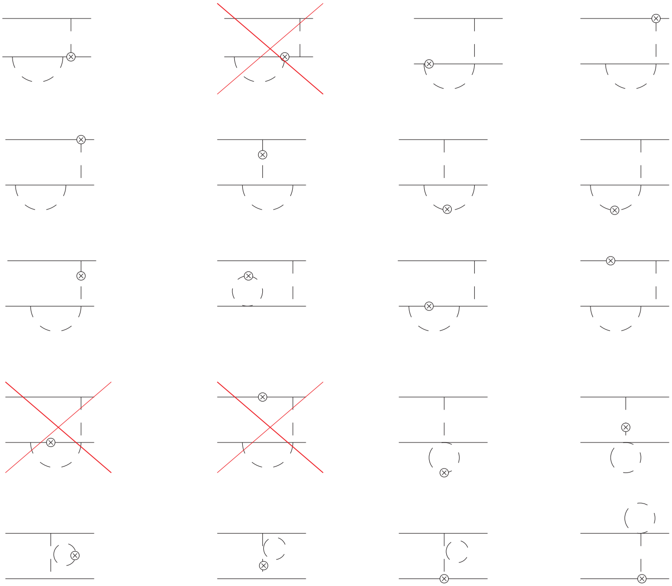
$$\sim \qquad V^\mu F(\omega_1, \omega_2)$$

$$F(\omega_1,\omega_2) = \frac{1}{2}\left(\frac{1}{\omega_2(\omega_1+\omega_2)} + \frac{1}{\omega_1\omega_2} + \frac{1}{\omega_1(\omega_1+\omega_2)}\right) = \frac{1}{\omega_1\omega_2}$$

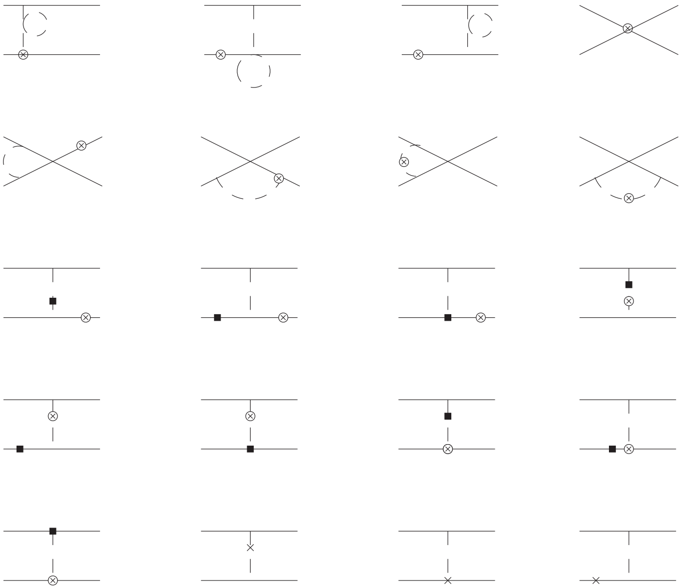
Next-to-leading order



Next-to-leading order



Next-to-leading order



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Next-to-leading order

Example:

$$F_{\mathrm{cbox}} = \frac{\omega_+^2 + \omega_+\omega_- + \omega_-^2}{2\omega_+^2\omega_-^2\,(\omega_+ + \omega_-)} \qquad F_{\mathrm{box}} = -\frac{\omega_+^2 + \omega_+\omega_- + \omega_-^2}{2\omega_+^2\omega_-^2\,(\omega_+ + \omega_-)}$$

$$\omega_\pm = \sqrt{(\vec{q}_1 \pm \vec{l})^2 + 4M_\pi^2}$$

Adding all contributions together:

$$\sim \int \frac{d^3 l}{(2\pi)^3}(q_1^2 - l^2)\frac{\omega_+^2 + \omega_+\omega_- + \omega_-^2}{2\omega_+^3\omega_-^3\,(\omega_+ + \omega_-)}$$

$$\sim \int \frac{d^4 l}{(2\pi)^4}(q_1^2 - l^2)\frac{1}{(l+q_1)^2 - M_\pi^2}\,\frac{1}{(l-q_1)^2 - M_\pi^2}\left(\frac{1}{l^0 + i\epsilon}\right)^2$$

Conclusion

- Systematic treatment of currents in χ-EFT using the method of unitary transformation
- Calculation of the two-pion exchange
- (Most) Loop contributions of one-pion exchange in NLO
- Still some work to be done

Thank you!