Superallowed Fermi β Decay and CKM Unitarity

NNPSS Student Talk
June 26, 2008
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CKM Matrix

For quarks,

mass eigenstates ≠ weak eigenstates

That means there's a mixing matrix that connects the two bases.
This is called the CKM (Cabibbo-Kobayashi-Maskawa) matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

N. Cabibbo (1963), M. Kobayashi and T. Maskawa (1973)

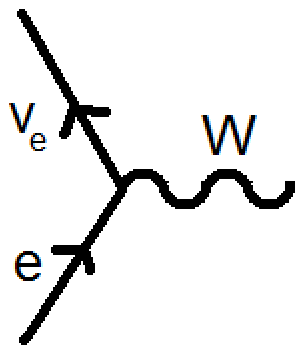
What effect does the CKM matrix have?

Weak charged current is

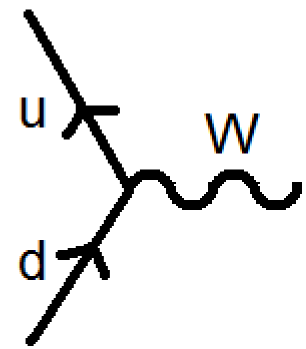
$$J_{cc} = \sum_{m=1}^{3} \left[\bar{\nu}_m \gamma^\mu (1-\gamma_5) l_m + \sum_{n=1}^{3} V_{mn} \bar{u}_m \gamma^\mu (1-\gamma_5) d_n \right]$$

where m and n are generation indices

What effect does the CKM matrix have?



$$\propto e_W$$



$$\propto V_{ud} e_W$$

Compared to muon decays, nuclear/neutron β decay amplitudes are suppressed by V_{ud}

Why should we care about CKM unitarity?

- Self-consistency check for Standard Model

  If there are just 3 generations for quarks, 3x3 CKM matrix should be unitary. Each row and column should satisfy relations like this

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- Constraints on physics beyond SM

  4th generation of quarks

  Exotic muon decay

I'll focus on extracting V_{ud} from nuclear β decay

2 Types of β Decay

In $q \rightarrow 0$ (allowed) limit

- Fermi

  Vector current, g_V

$$M_F = \sum_k \tau_+(k)$$

  $\Delta J = 0$

  $\Delta T = 0$

  $\Delta T_3 = 1$

  same parity

- Gamow-Teller

  Axial current, g_A

$$\vec{M}_{G-T} = \sum_k \tau_+(k)\vec{\sigma}(k)$$

  $\Delta J = \pm 1, 0$ (except $0 \rightarrow 0$)

  $\Delta T = \pm 1, 0$ (except $0 \rightarrow 0$)

  $\Delta T_3 = 1$

  same parity

Can we get rid of one of them?

Yes!

For $J_i = J_f = 0$, we'll have no Gamow-Teller (Sorry GWU)

And if the initial and final nuclei are in the same isospin multiplet, then it's called a “superallowed” Fermi β decay

There are many isotopes you (not I) can measure:

^{14}O, $^{26}Al^m$, ^{34}Cl, $^{38}K^m$, ^{42}Sc, ^{46}V, ^{50}Mn, ^{54}Co...

Now everything's simple, right?

- From vector current conservation (recall the nucleon structure lectures), g_V=1
- So for all J=0, T=1 decay nuclei, the leading-order decay amplitudes are identical
- Just factor out the phase space dependence, and the normalized lifetimes (ft value) will be identical as well

$$
\begin{aligned}
ft &\equiv \left[m_e^{-5}\int_{m_e}^{Q} d\epsilon\sqrt{\epsilon^2 - m_e^2}\,\epsilon(Q-\epsilon)^2\right]\frac{\ln 2}{\Gamma} \\
&= \frac{2\pi^3 \ln 2}{m_e^5 G_F^2 |V_{ud}|^2 |M_F|^2}
\end{aligned}
$$

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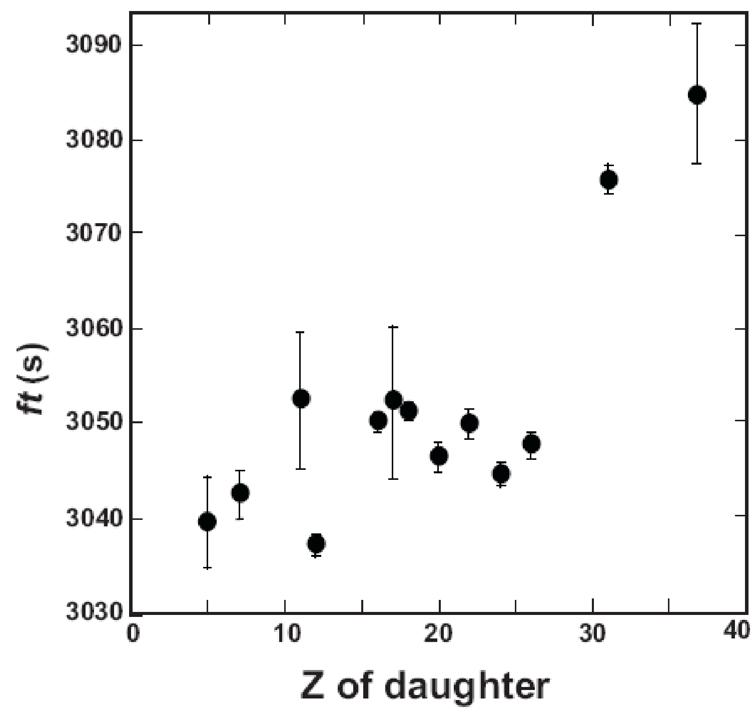
Uh...not quite

Uncorrected ft Values



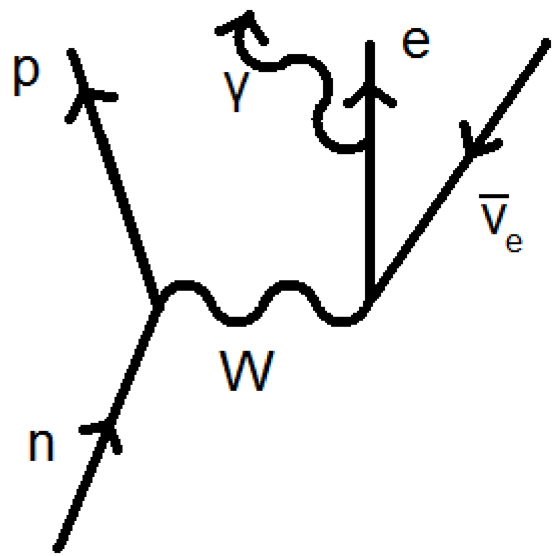
J.C. Towner and I.S. Hardy
arXiv:0710.3181

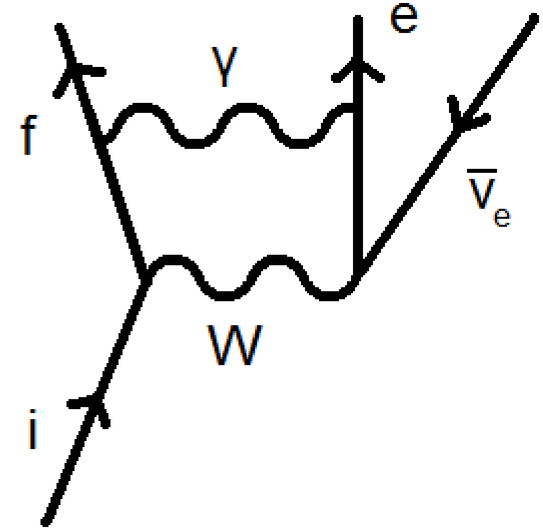
Those ft values don't look so identical

There's more to β decay than just the leading order

Theoretical Corrections

We need to calculate how higher-order processes affect the amplitude, e.g.

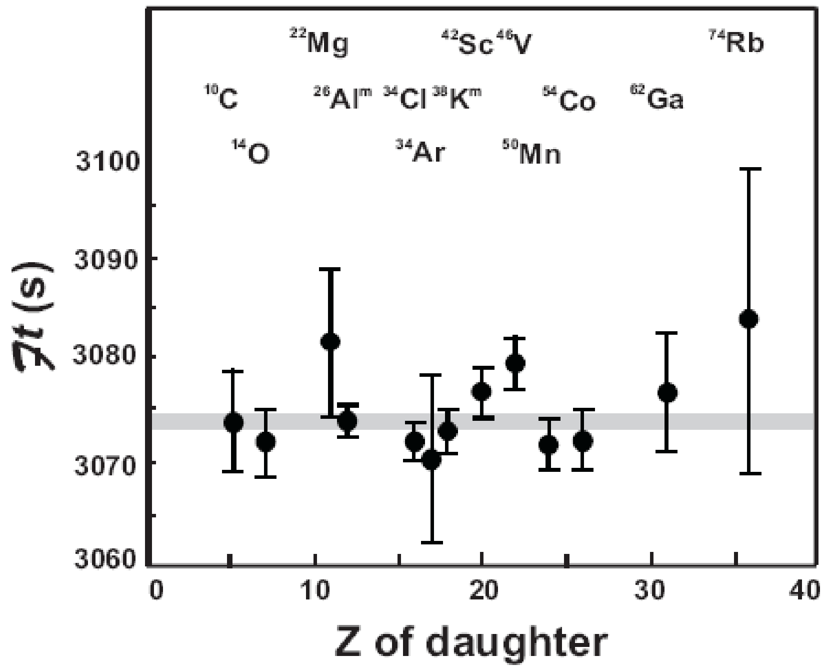




Some diagrams are nucleus-independent (left), but others depend on the nucleus (right)

Theoretical Corrections

Almost all the important terms have been calculated, and...



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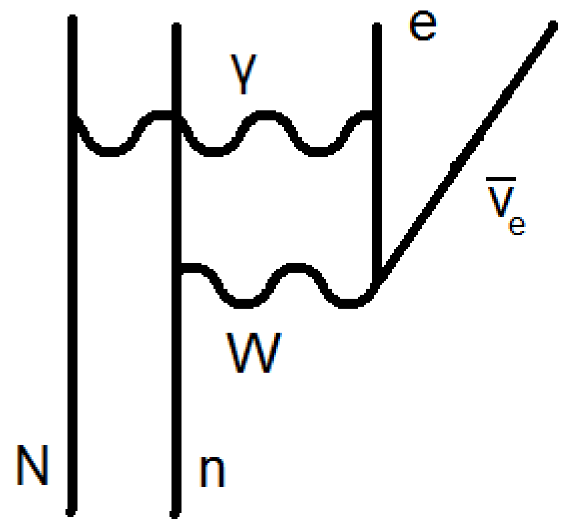
Corrections appear to be working

Nucleus Dependence

But because of the origin of this problem in particle physics, nuclear structure often has been taken lightly



This 2-nucleon diagram has been considered, but the nucleons are treated as being almost free

Nucleus Dependence

- Simple dependence on Z

  Coulomb correction, QED-type corrections, etc.

  Well-established

- Isospin symmetry breaking

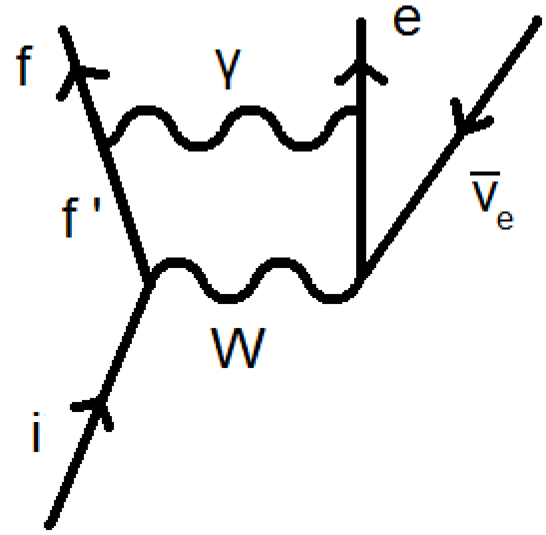
  Isospin mixing and radial overlap

  (I.S. Towner and J.C. Hardy arXiv:0710.3181,

  G.A. Miller and A. Schwenk arXiv:0805.0603)

What I'm trying to do

Calculate the contribution to the amplitude from excited intermediate states



There is some overlap with earlier calculations, but this has never been done

What I'm trying to do

From non-relativistic P.T.

$$T_{fi} = \sum_n \frac{< f|H'_{em}|n >< n|H_W|i >}{E_n - E_i}$$

Subtlety here is that I need to subtract out

$$< f|H_{em}|f >$$

from the EM Hamiltonian.

Otherwise the T matrix above can't be derived

What I'm trying to do

- Multipole expansion of the Hamiltonians

  Formalism available from electron-nucleus scattering

- Looking at limits

  High loop momentum: 2 momentum transfers are nearly back-to-back

  Low loop momentum: expansion in powers of loop momentum

More explicitly

$$
\begin{aligned}
& < f|H'_{em}|n >< n|H_W|i > \\
= \;& -\frac{e^2 G_F}{q^2}\left[4\pi < f|j_{em,0'}M'_{00'}|n >< n|(j_{W,0}M_{00} - j_{W,3}L_{00})|i >\right. \\
+ \;& \sum_{J=1}^{\infty} 2\pi(2J+1)(-1)^J \left\{< f| \left[\sum_{\lambda'=\pm 1}(j_{em,-\lambda'}T^{el}_{J\lambda'}) + \sqrt{2} j_{em,0'}M_{J0'}\right]|n >\right. \\
\times \;& < n| \left[\sum_{\lambda=\pm 1} j_{W,\lambda}(\lambda T^{mag5}_{J,-\lambda} + T^{el}_{J,-\lambda}) + \sqrt{2}(j_{W,0}M_{J0} - j_{W,3}L_{J0})\right]|i > \\
+ \;& < f| \sum_{\lambda'=\pm 1} j_{em,\lambda'}\lambda' T^{mag}_{J,-\lambda'}|n > \\
\times \;& \left.\left.< n| \left[\sum_{\lambda=\pm 1} j_{W,\lambda}(\lambda T^{mag}_{J,-\lambda} + T^{el5}_{J,-\lambda}) + \sqrt{2}(j_{W,0}M^5_{J0} - j_{W,3}L^5_{J0})\right]|i >\right\}\right]
\end{aligned}
$$

Outlook

- Nuclear-structure dependence calculations are coming along (both ISB and our calculation)

  V_{ud} = 0.97378(27) from nuclear decays, and uncertainties are dominated by theory

- Neutron decay rate measurements might yield more precise values in the near future

  No nuclear structure here, so we know the corrections better

- Current top-row unitarity test

  0.9483(5)+0.0509(9)+0.0000(1)=0.9992(11)

  It's satisfied...for now

Good Reads

Reviews:

I.S. Towner and J.C. Hardy, nucl-th/0412056

"The CKM Quark-Mixing Matrix" and "Vud, Vus, The Cabibbo Angle, and CKM Unitarity" from Particle Data Group (pdg.lbl.gov)

Nucleus-dependent Corrections:

I.S. Towner and J.C. Hardy nucl-th/0209014