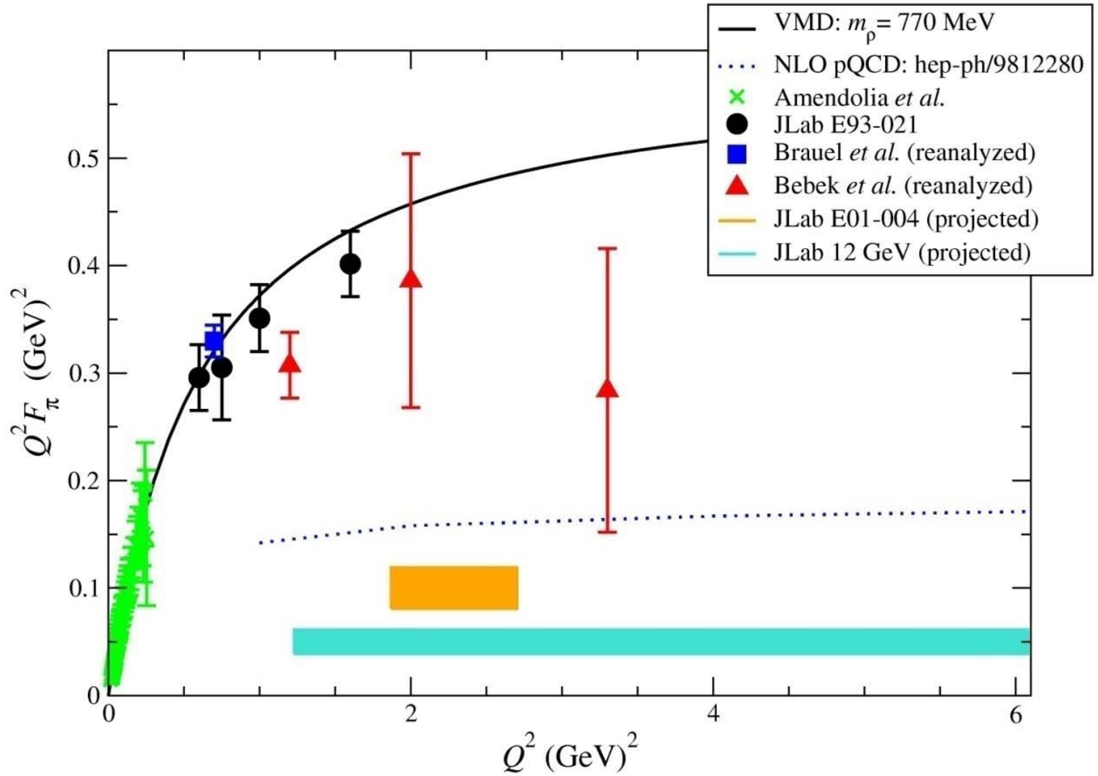
Lecture 3: Hadron Structure

- Hadron Structure Recipe
- Form Factors, DIS and GPDs
- New Insights – orbital angular momentum
- Future

Pion Form Factor

$$F_\pi(Q^2) = \frac{8\pi\alpha_s(Q^2) f_\pi^2}{Q^2} \quad \text{as} \quad Q^2 \to \infty.$$

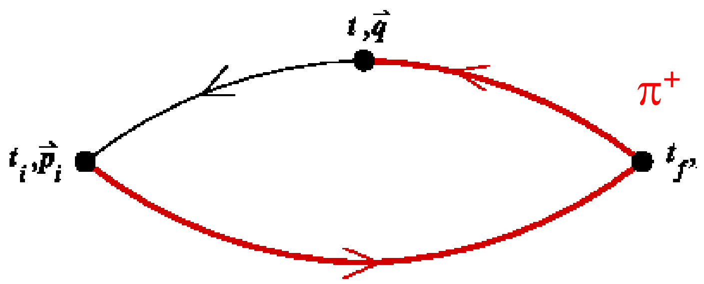
$$F_\pi(Q^2) \approx \frac{1}{1 + Q^2 / m_{\mathrm{VMD}}^2} \quad \text{for} \quad Q^2 \ll m_{\mathrm{VMD}}^2.$$



Anatomy of a Matrix Element Calculation

Pion Interpolating Operator

$$\left\{\begin{array}{rcl} \phi(x) & = & \bar{d}(x)\gamma_5 u(x) \\ \phi^\dagger(x) & = & -\bar{u}(x)\gamma_5 d(x) \\ V_\mu(x) & = & e_u\bar{u}(x)\gamma_\mu u(x) + e_d\bar{d}(x)\gamma_\mu d(x). \end{array}\right.$$



$$\Gamma_{\pi^+\mu\pi^+}(t_f, t; \vec{p}, \vec{q}) = \sum_{\vec{x},\vec{y}} \langle 0|\phi(\vec{x}, t_f)V_\mu(\vec{y}, t)\phi^\dagger(\vec{0}, 0)|0\rangle e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{q}\cdot\vec{y}},$$

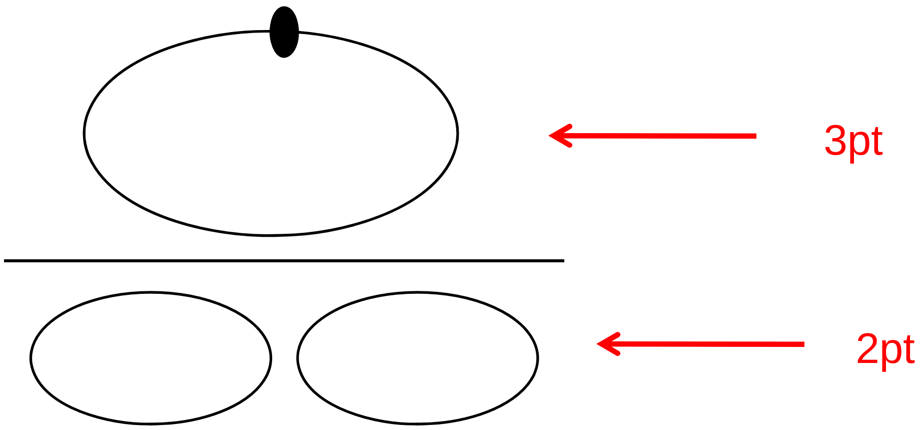
Resolution of unity – insert states

$$\langle 0 \mid \phi(0) \mid \pi, \vec{p}+\vec{q}\rangle\langle \pi, \vec{p}+\vec{q} \mid V_\mu(0) \mid \pi, \vec{p}\rangle\langle \pi, \vec{p} \mid \phi^\dagger \mid 0\rangle e^{-E(\vec{p})(t-t_i)} e^{-E(\vec{p}+\vec{q})(t_f-t)}$$

Anatomy of a Matrix Element Calculation

$$\Gamma_{\pi^+\pi^+}(t,0;\vec{p}) = \sum_{\vec{x}} \langle 0 \mid \phi(\vec{x},t_f)\phi^\dagger(0) \mid 0\rangle e^{-i\vec{p}\cdot\vec{x}}$$

$$\propto |\langle 0|\phi(0)|\pi,\vec{p}\rangle|^2 e^{-E(\vec{p})t}$$

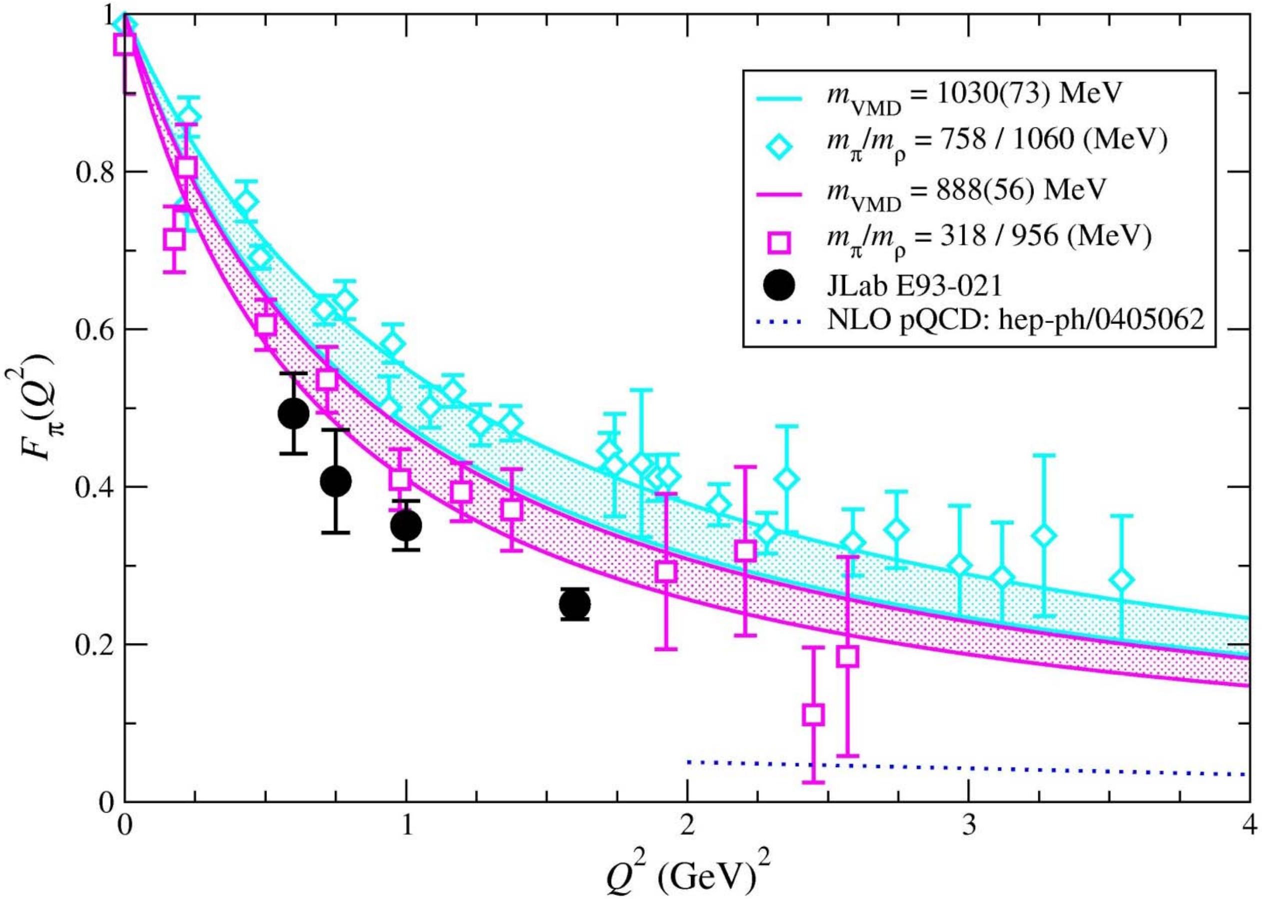


Both 3pt and 2pt built from Quark Propagators through Wick Expansion

$$G^{ij}_{\alpha\beta}(x,y) = \langle 0|\psi^i_\alpha(x)\bar{\psi}^j_\beta(y)|0\rangle$$

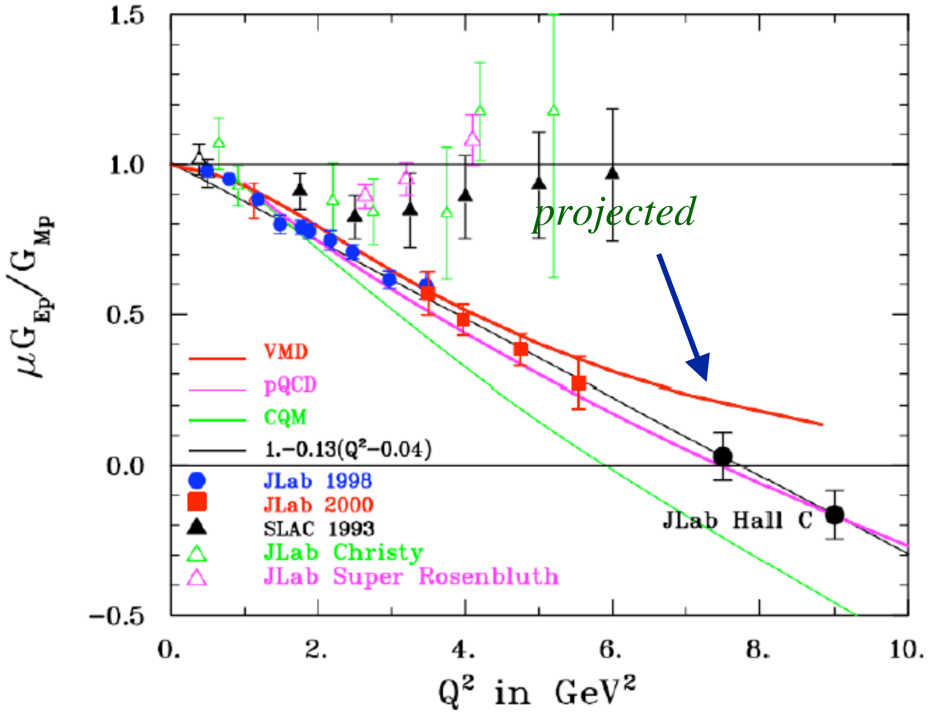
Pion Form Factor

LHPC, Bonnet *et al,*
Phys.Rev. D72 (2005) 054506

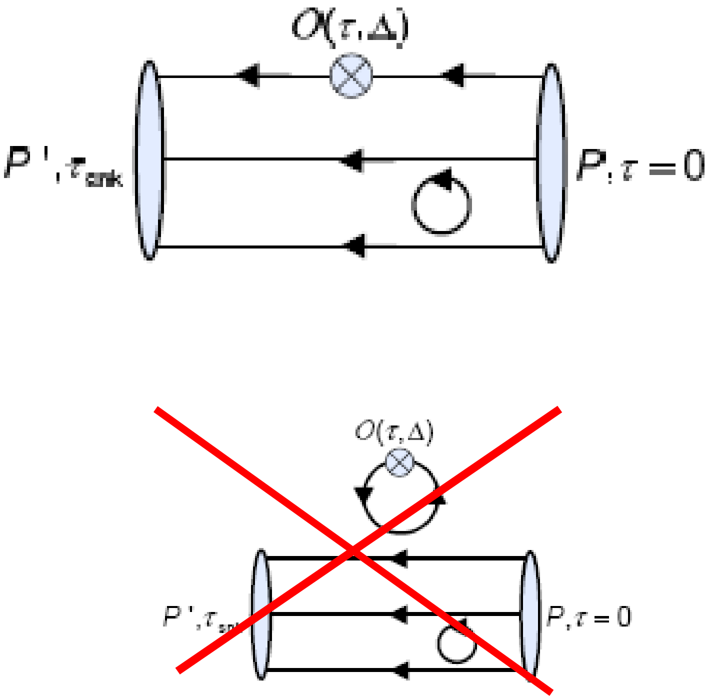


Proton EM Form-Factors - I

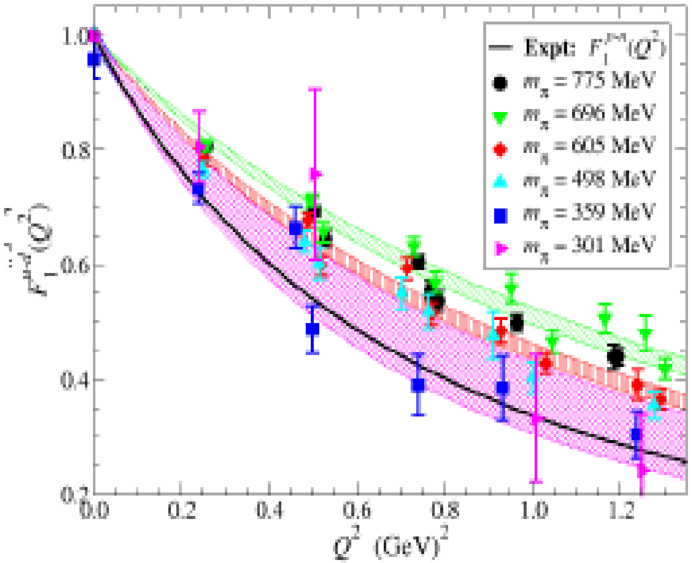
EM Form Factors describe the distribution of *charge* and *current* in the proton: 2 form factors

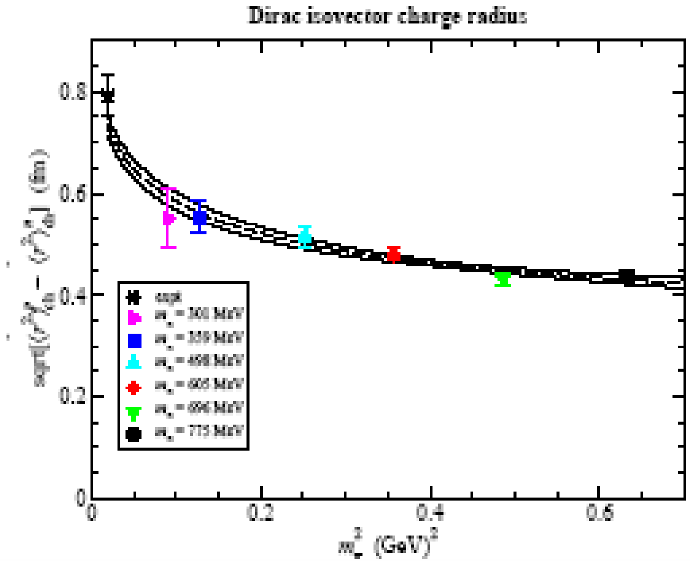


C. Perdrisat (W&M) , JLab Users Group Meeting, June 2005



Proton EM Form Factors - II





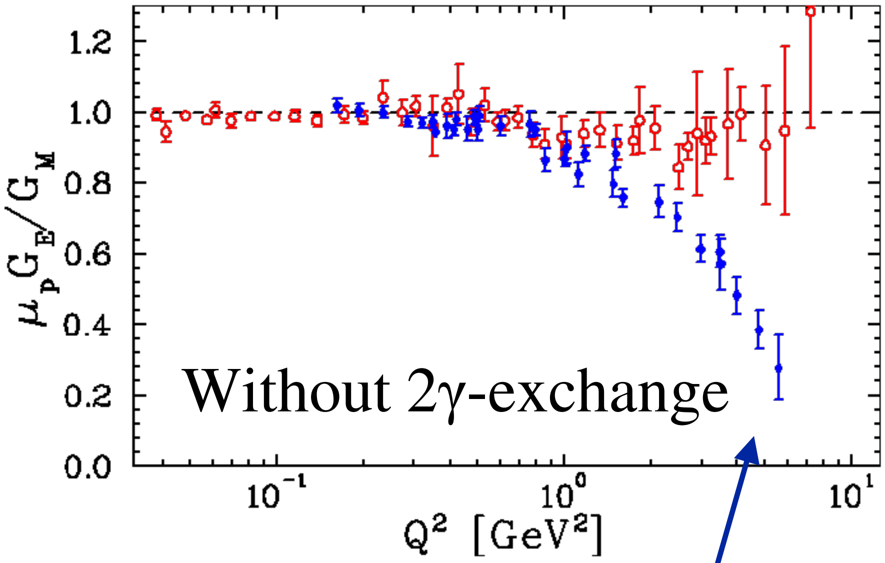
- **Lattice QCD computes the *isovector* form factor**
- **Hence obtain** Dirac charge radius $\langle r^2\rangle^{u-d}_{ch}$ **assuming dipole form**

LHPC, hep-lat/0610007

$$\langle r^2\rangle^{u-d}_{ch} = a_0 - 2\frac{\left(1+5g_A^2\right)}{(4\pi f_\pi)^2}\frac{1}{2}\log\left(\frac{m_\pi^2}{m_\pi^2+\Lambda^2}\right)$$

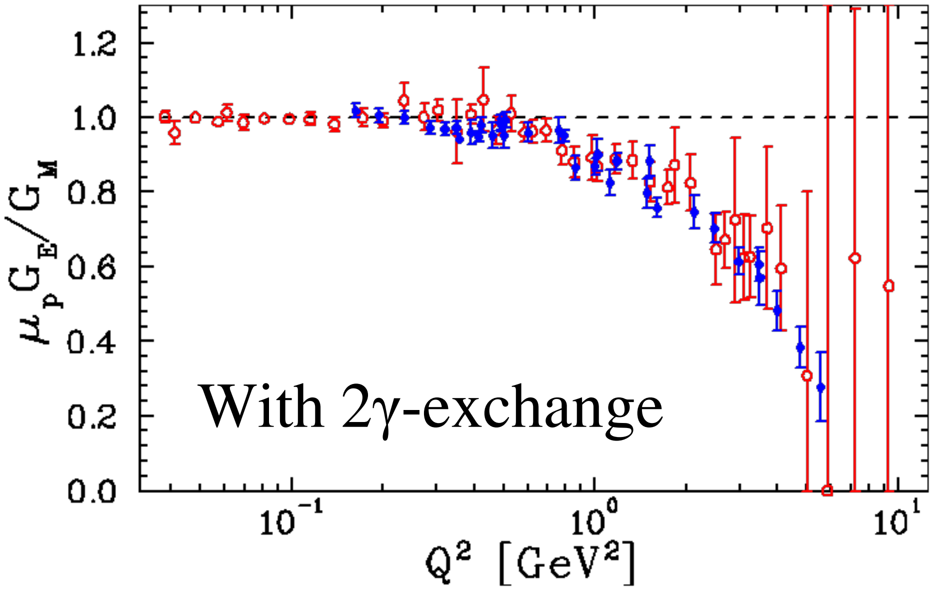
Leinweber, Thomas, Young, PRL86, 5011

Form Factor – III



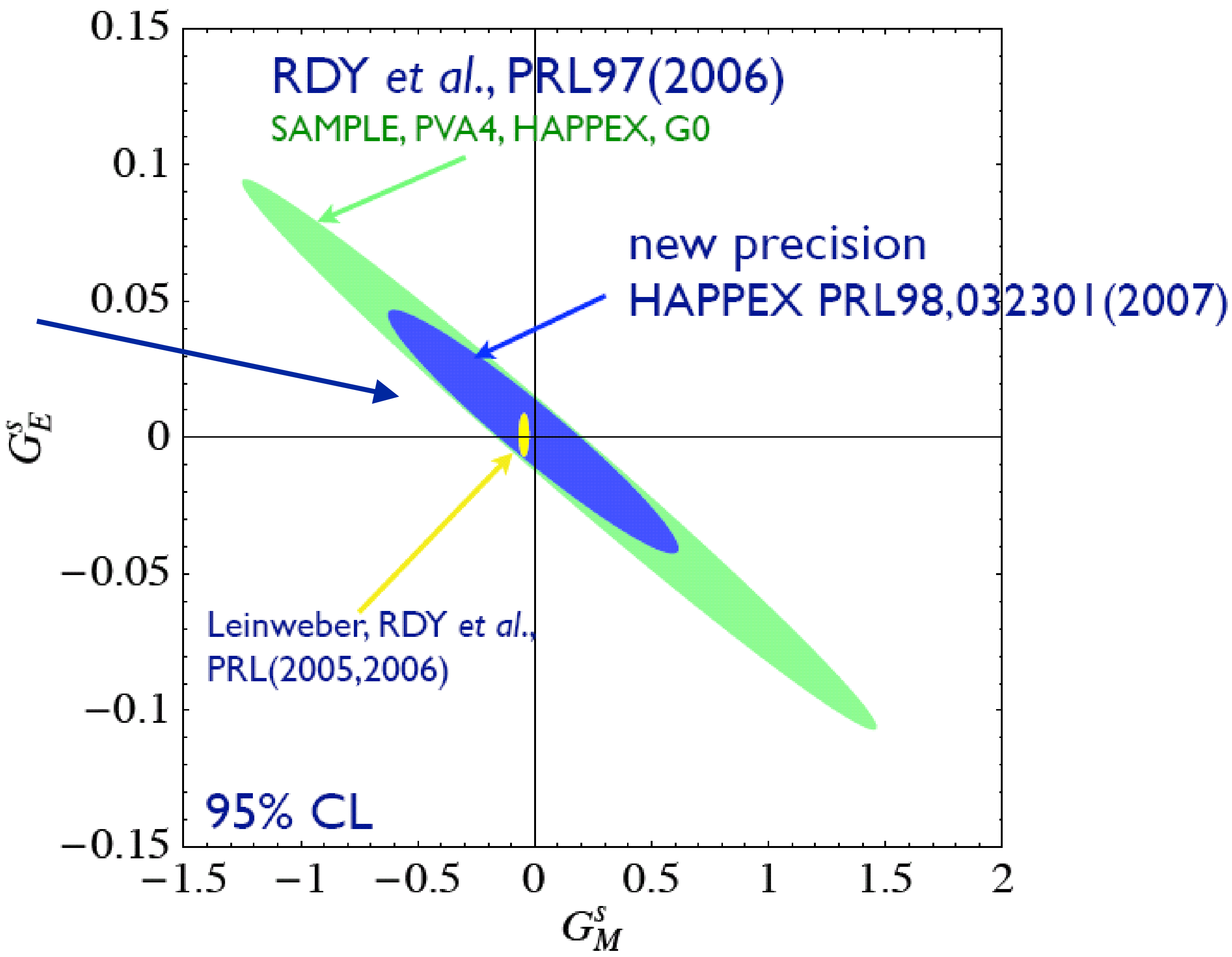
Rosenbluth

Polarization Transfer

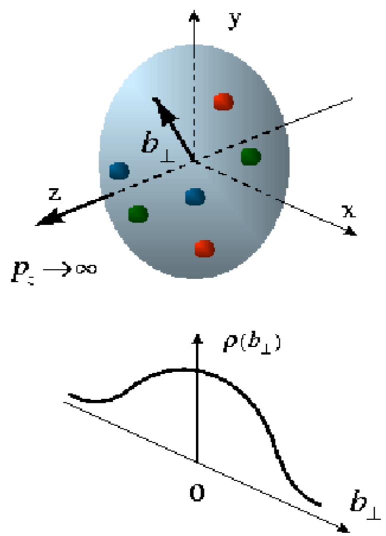


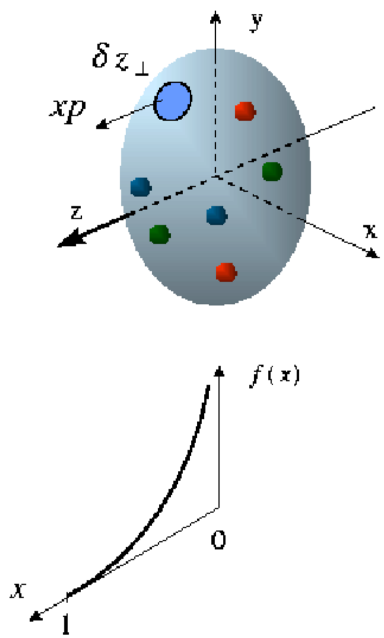
Disconnected Contributions

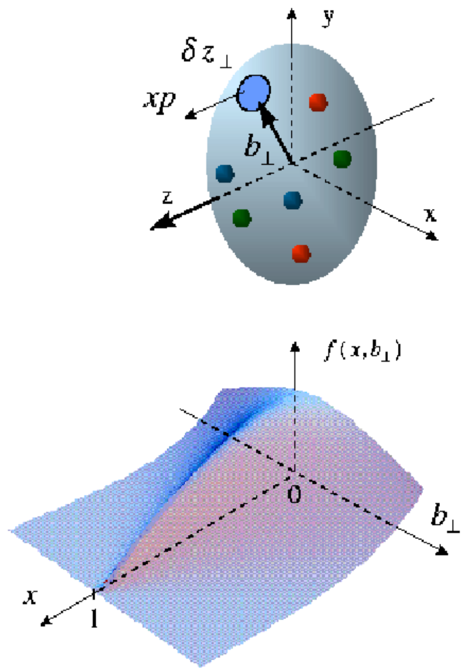
Amalgam of Lattice QCD and Phenomenology by Leinweber *et al.*



Different Regimes in Different Experiments







Form Factors
transverse quark distribution in Coordinate space

Structure Functions
longitudinal quark distribution in momentum space

GPDs
Fully-correlated quark distribution in both coordinate and momentum space

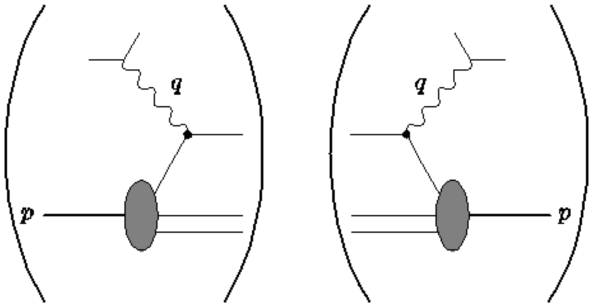
Moments of Structure Functions and GPD's

- **Describe distribution of longitudinal momentum and spin in proton**
- **Matrix elements of** **light-cone correlation functions**

$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}\left(-\frac{\lambda}{2}n\right) n P e^{-ig\int_{\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(\alpha n)} \psi\left(\frac{\lambda}{2}n\right)$$

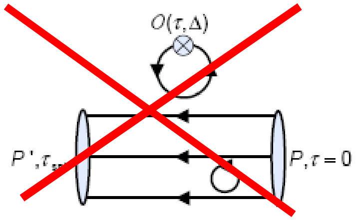
- **Expand *O(x)* around light-cone**

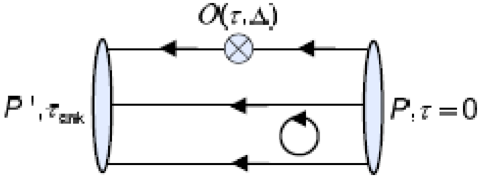
$$O_q^{\{\mu_1\mu_2\dots\mu_n\}} = \bar{\psi}_q \gamma_5 \gamma^{\{\mu_1} iD^{\mu_2}\dots D^{\mu_n\}}\psi_q$$



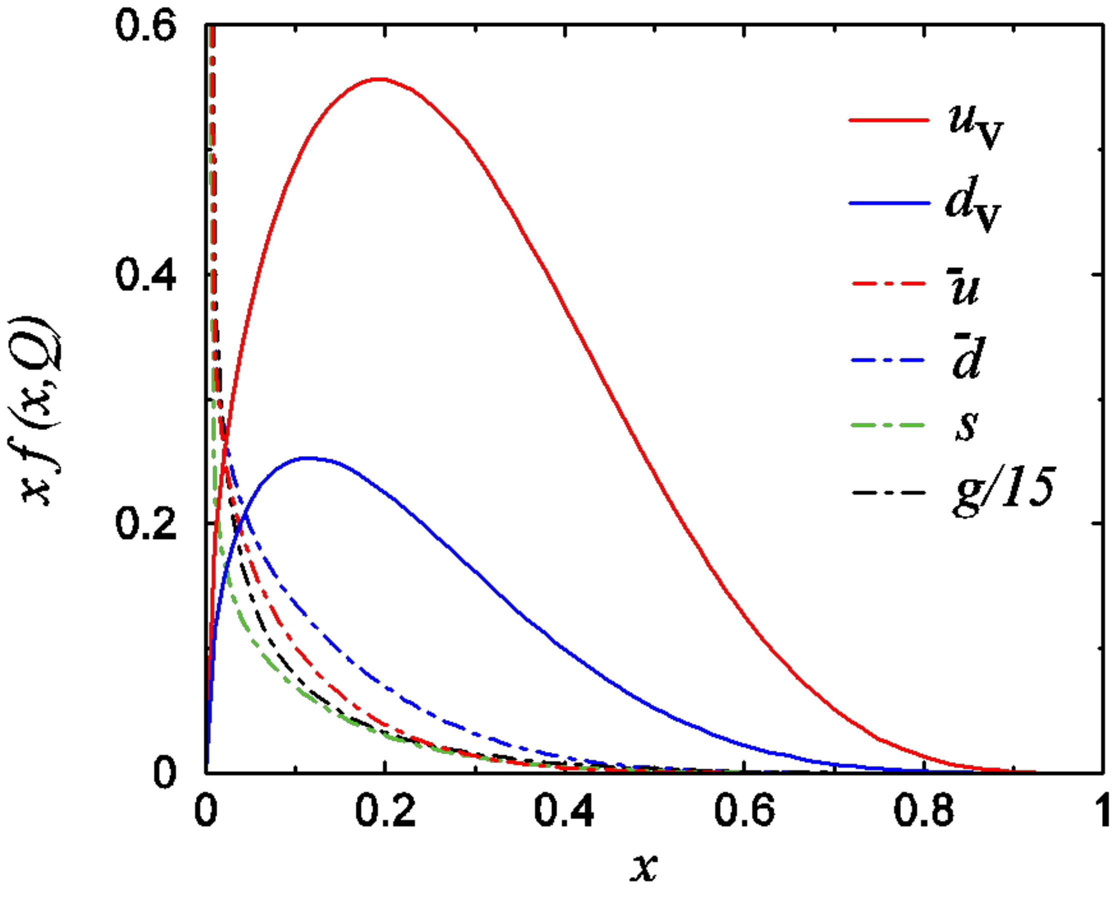
- **Diagonal matrix element**

$$\langle P|O_q^{\{\mu_1\dots\mu_n\}}|P\rangle \simeq \int dx\, x^{n-1} q(x)$$





Moments of Parton Distributions



Distributions at 5 GeV

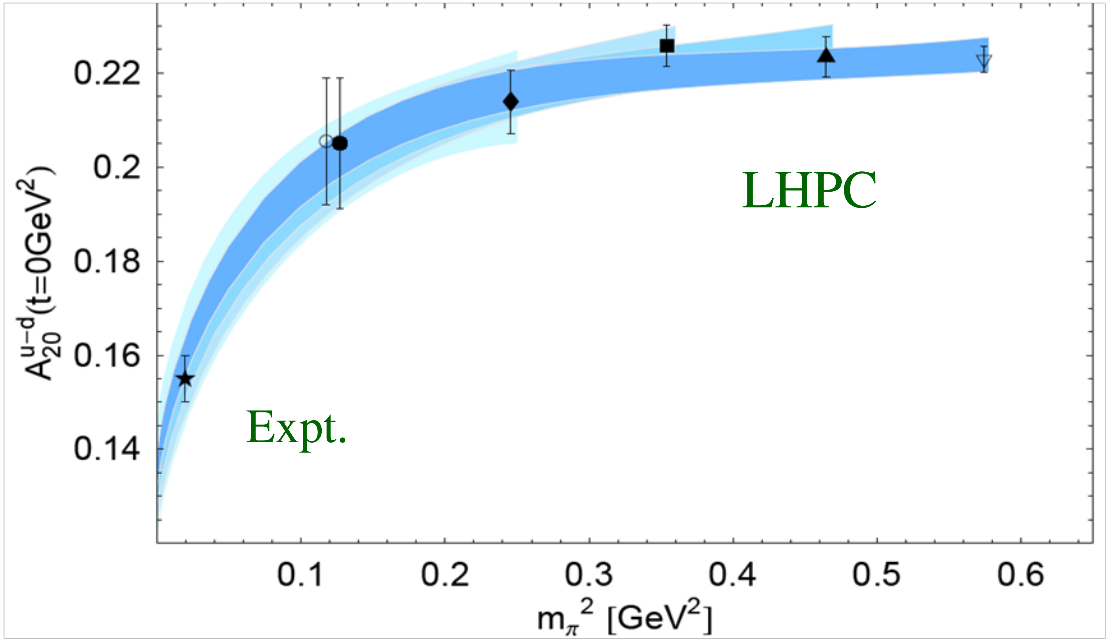
Iso-vector Momentum Fraction

Isovector momentum fraction

$$\langle x \rangle_{u-d} = \langle x \rangle^0_{u-d} \left(1 - \frac{(3g_A+1)}{4\pi f_\pi^2} m_\pi^2 \ln m_\pi^2 \right) + \ldots$$

- Dominates behavior at low mass
- g_A, f_π well-determined on lattice

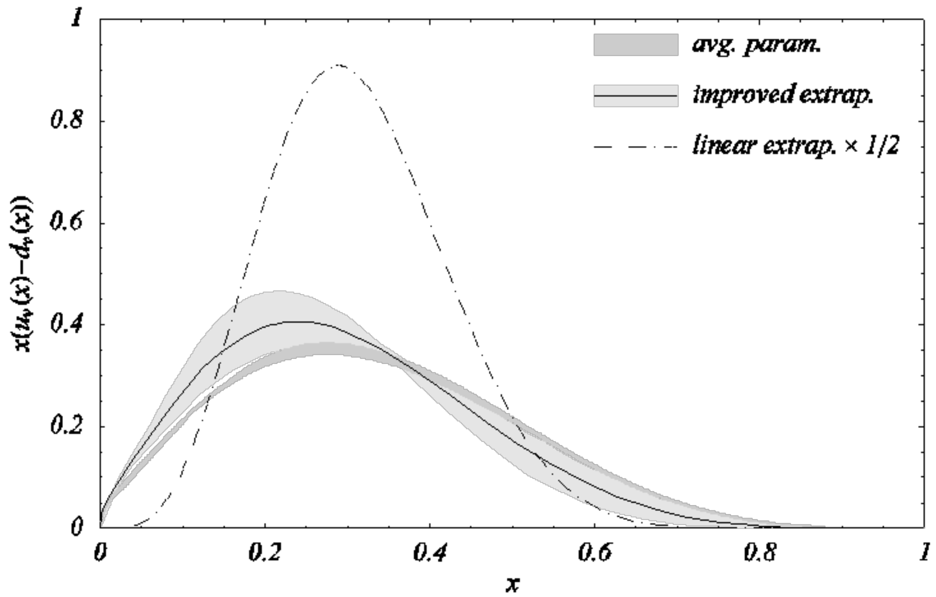
$\langle x \rangle_{u-d}$



Shape…

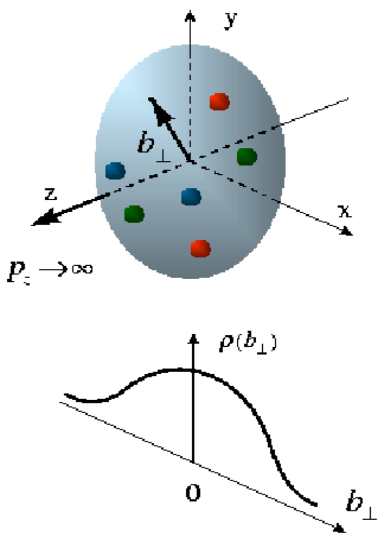
- **Calculations give moments of distributions**
- **Higher moments harder - hypercubic symmetry…**
- **Can we recover shape from knowledge of, say, first three moments?**

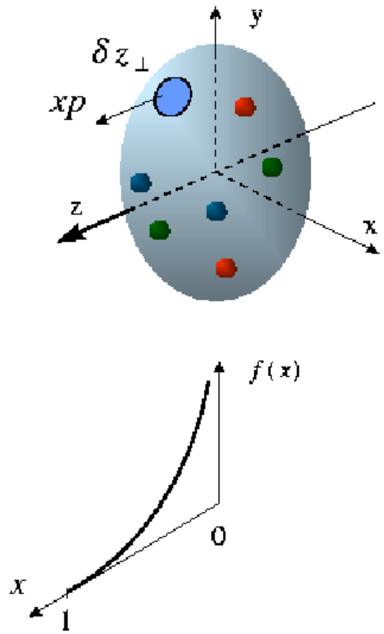
Detmold, Melnitchouk, Thomas

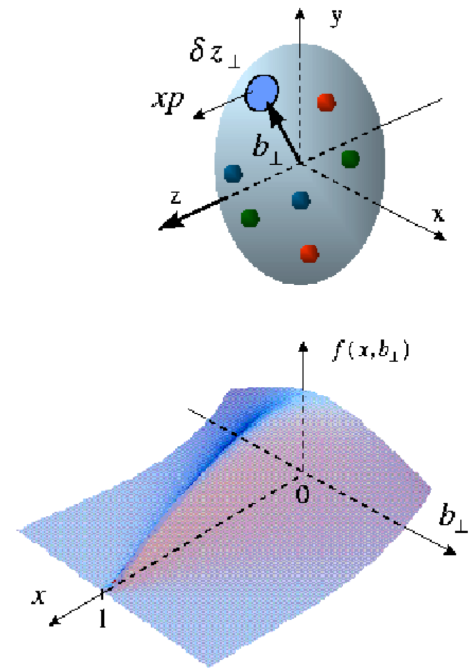


$$x(u_v(x)-d_v(x)) = a\,x^b(1-x)^c(1+\varepsilon\sqrt{x}+\gamma\,x)$$ ← Model dependence

Different Regimes in Different Experiments







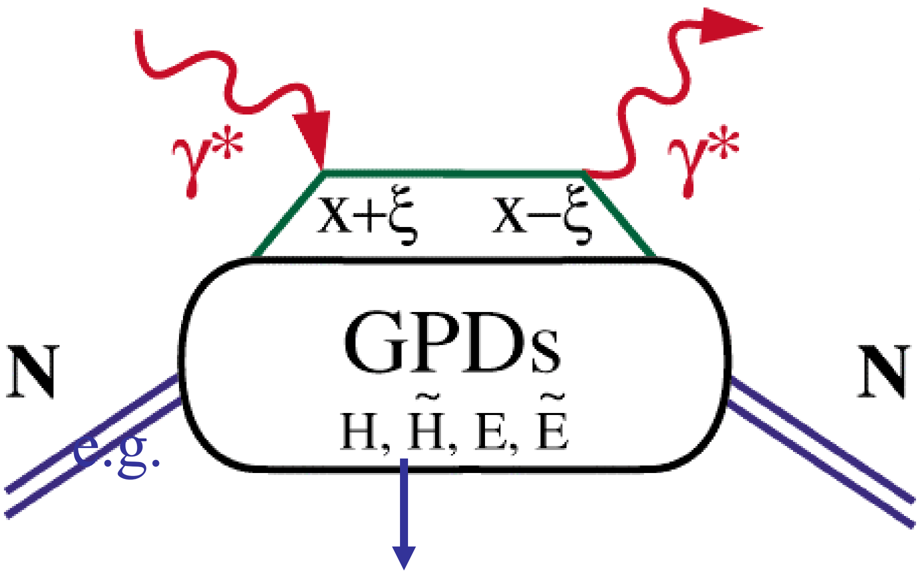
Form Factors
transverse quark distribution in Coordinate space

Structure Functions
longitudinal quark distribution in momentum space

GPDs
Fully-correlated quark distribution in both coordinate and momentum space

Generalized Parton Distributions (GPDs)

Measured in, e.g., Deeply Virtual Compton Scattering



D. Muller *et al* (1994), X. Ji & A. Radyushkin (1996)

X. Ji, PRL 78, 610 (1997)

Quark angular momentum

$$J^q = \frac{1}{2} - J^G = \frac{1}{2}\int_{-1}^{1} x\,dx\,\left[H^q(x,\xi,0) + E^q(x,\xi,0)\right]$$

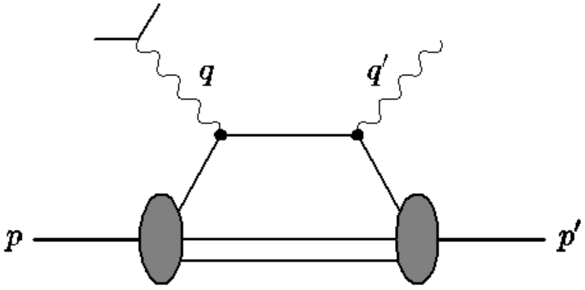
Moments of Structure Functions and GPD's

- **Matrix elements of light-cone correlation functions**

$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}\left(-\frac{\lambda}{2}n\right) nP e^{-ig\int_{\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(\alpha n)} \psi\left(\frac{\lambda}{2}n\right)$$

- **Expand *O(x)* around light-cone**

$$O_q^{\{\mu_1\mu_2\ldots\mu_n\}} = \bar{\psi}_q \gamma^{\{\mu_1} iD^{\mu_2}\ldots D^{\mu_n\}}\psi_q$$



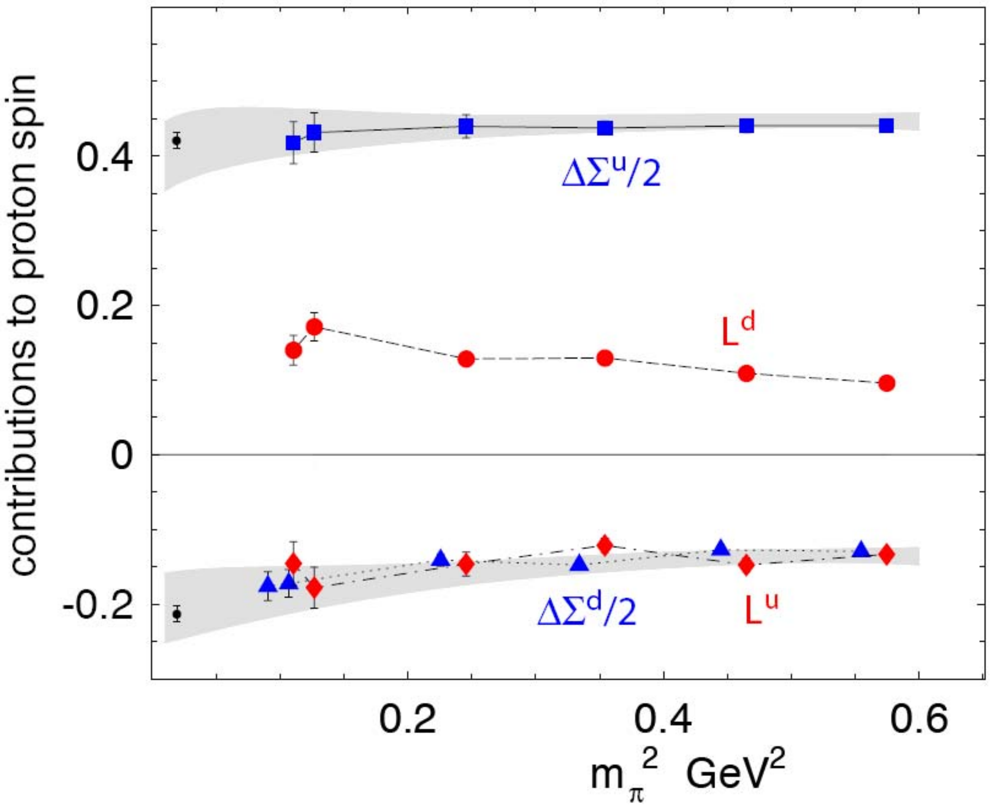
- **Off-diagonal matrix element**

$$\langle P'|O_q^{\{\mu_1\ldots\mu_n\}}|P\rangle \simeq \int dx\, x^{n-1}[H(x,\xi,t), E(x,\xi,t)]$$

$$\longrightarrow A_{ni}(t), B_{ni}(t), C_n(t), \tilde{A}_{ni}(t), \tilde{B}_{ni}(t), \tilde{C}_n(t)$$

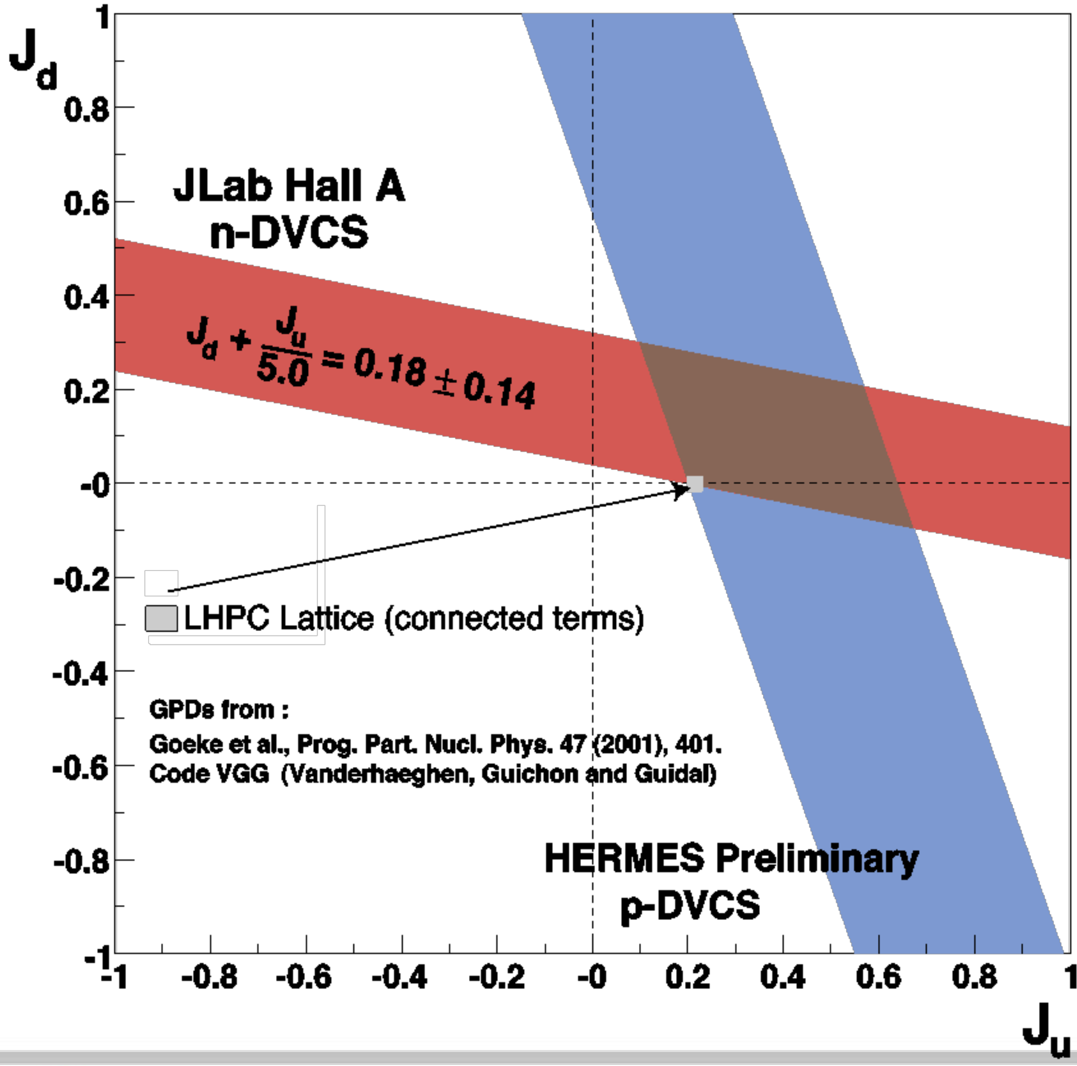
Origin of Nucleon Spin - I

How does the spin of the nucleon arise from quark spin, quark orbital angular momentum, and gluons?



LHPC, Haegler et al., Phys. Rev. D 77, 094502 (2008)

Origin of Nucleon Spin - II

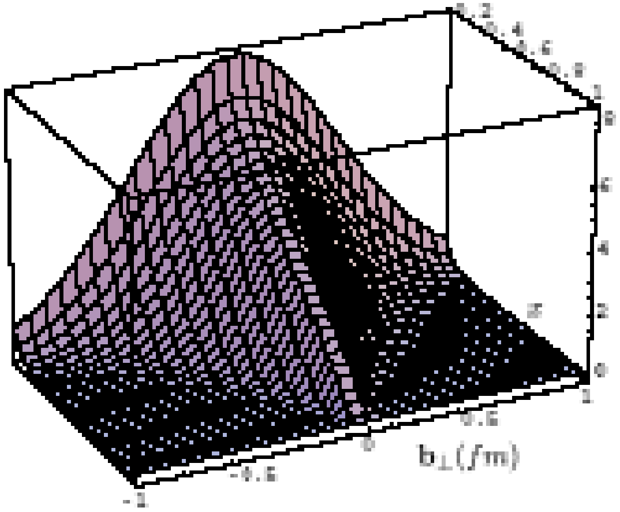


Lattice QCD + expt.

Discovering origin of nucleon spin

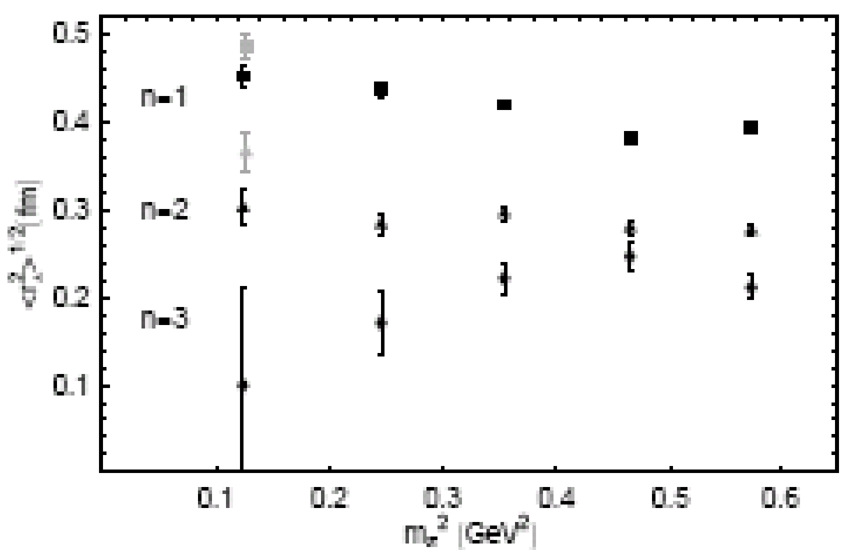
LHPC, Haegler et al., Phys. Rev. D 77, 094502 (2008)

Transverse Distribution - I



HP 2008

• Lattice QCD can compute moments of GPDs and PDFs, and the *t*-dependence



$$A^q_{n0}(-\vec{\Delta}_\perp^2) = \int d^2 b_\perp \, e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp} \int_{-1}^{1} dx \, x^{n-1} q(x, \vec{b}_\perp)$$

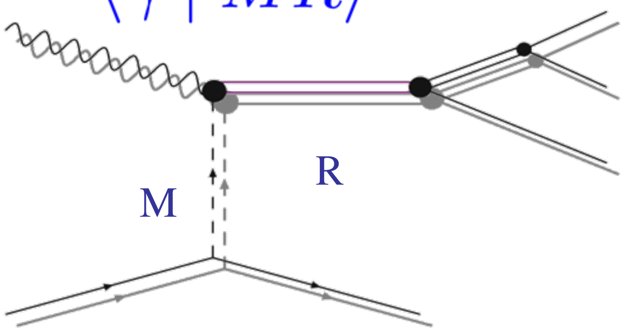
Compare to phenomenological models

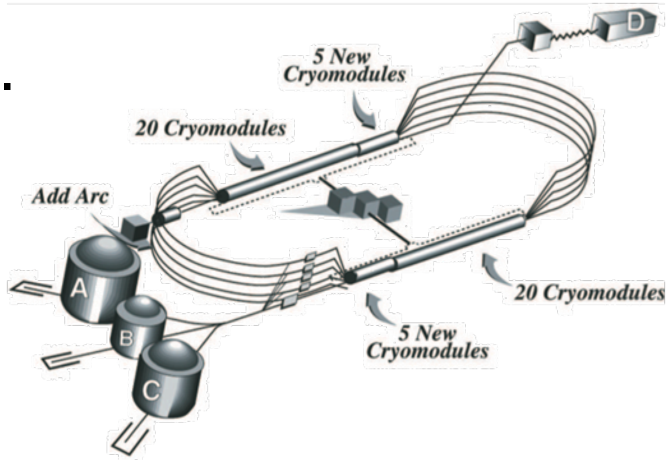
Decrease slope : decreasing transverse size as x → 1
Burkardt

Photo-transitions

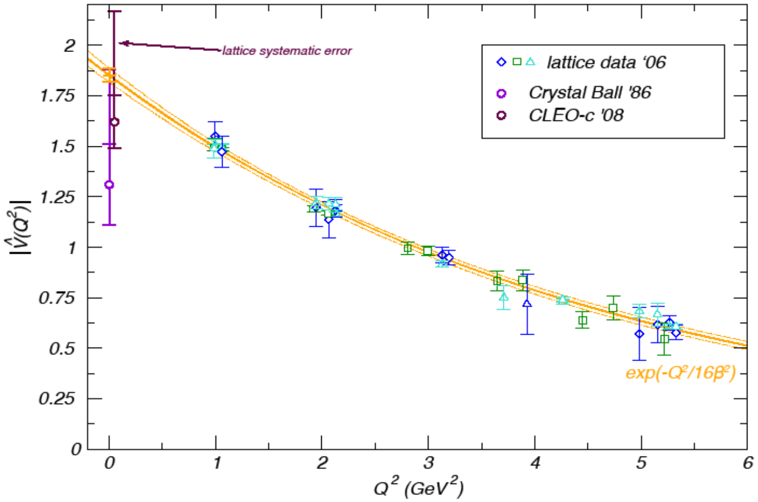
- Lattice can compute photocouplings.
- Initial computations in charmonium

$$\langle \gamma \mid M R \rangle$$





hep-ex/0805.252



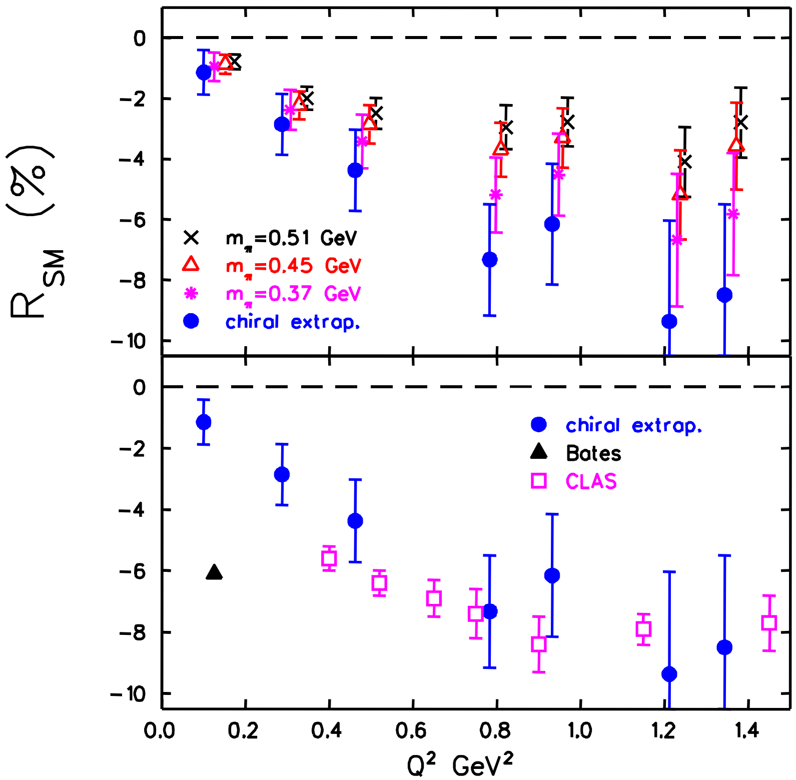
Dudek, Edwards, Richards, PRD73, 074507

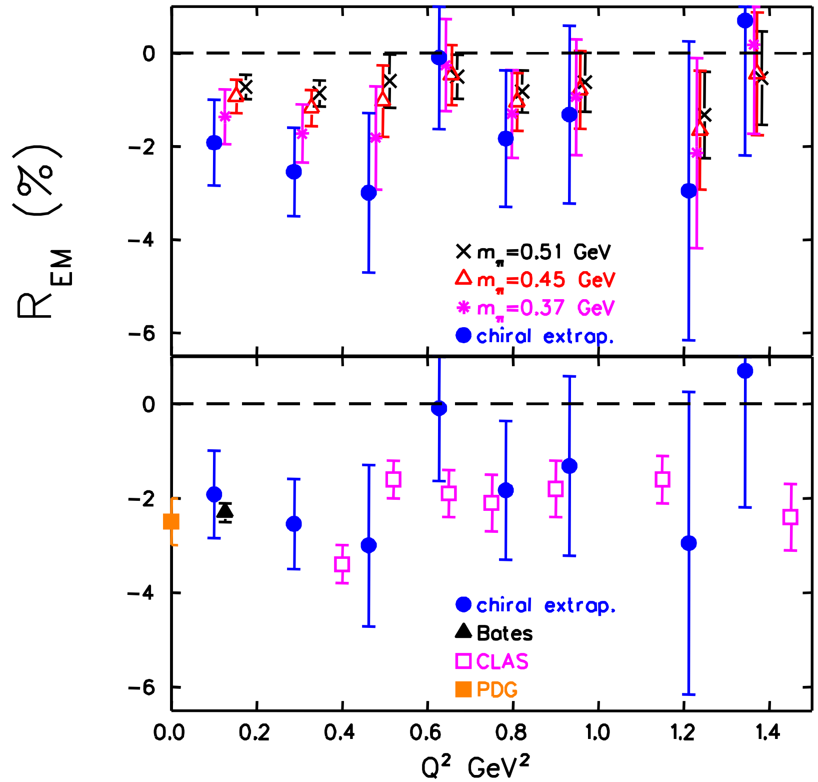
$$J/\psi \longrightarrow \eta_c \gamma$$

Transition FF: γ N - Δ

$R_{SM}=G_C/G_M$

$R_{EM}=-G_E/G_M$





Alexandrou et al. (2005)

Statistics for Hadron Structure

- Signal to noise degrades as pion mass decreases

$$\frac{\text{Signal}}{\text{Noise}} = \frac{\langle J(t)J(0)\rangle}{\frac{1}{\sqrt{N}}\sqrt{\langle |J(t)J(0)|^2\rangle - \langle J(t)J(0)\rangle^2}}$$

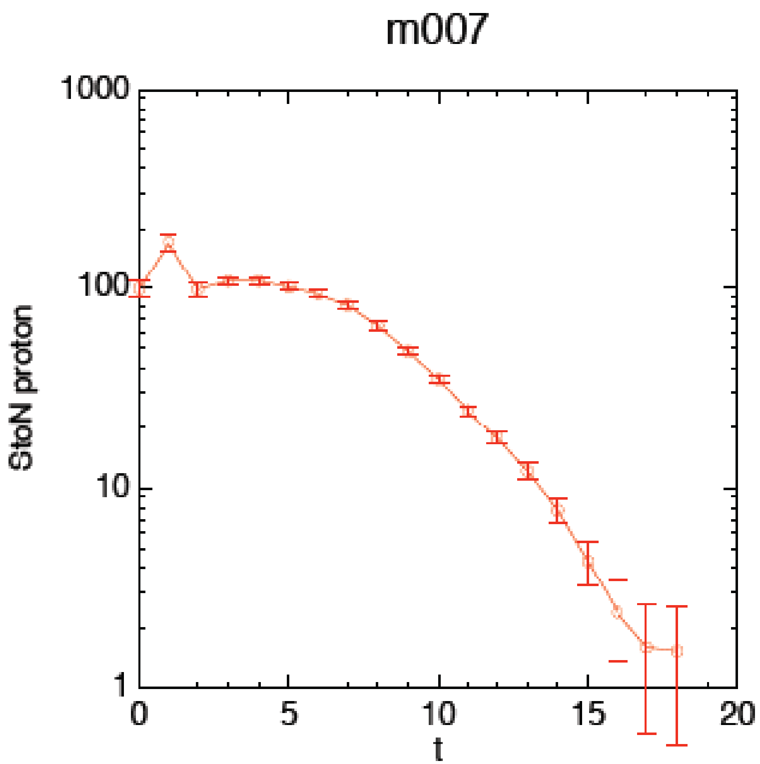
$$\sim \frac{Ae^{-M_n t}}{\frac{1}{\sqrt{N}}\sqrt{Be^{-3m_\pi t} - Ce^{-2M_n t}}}$$

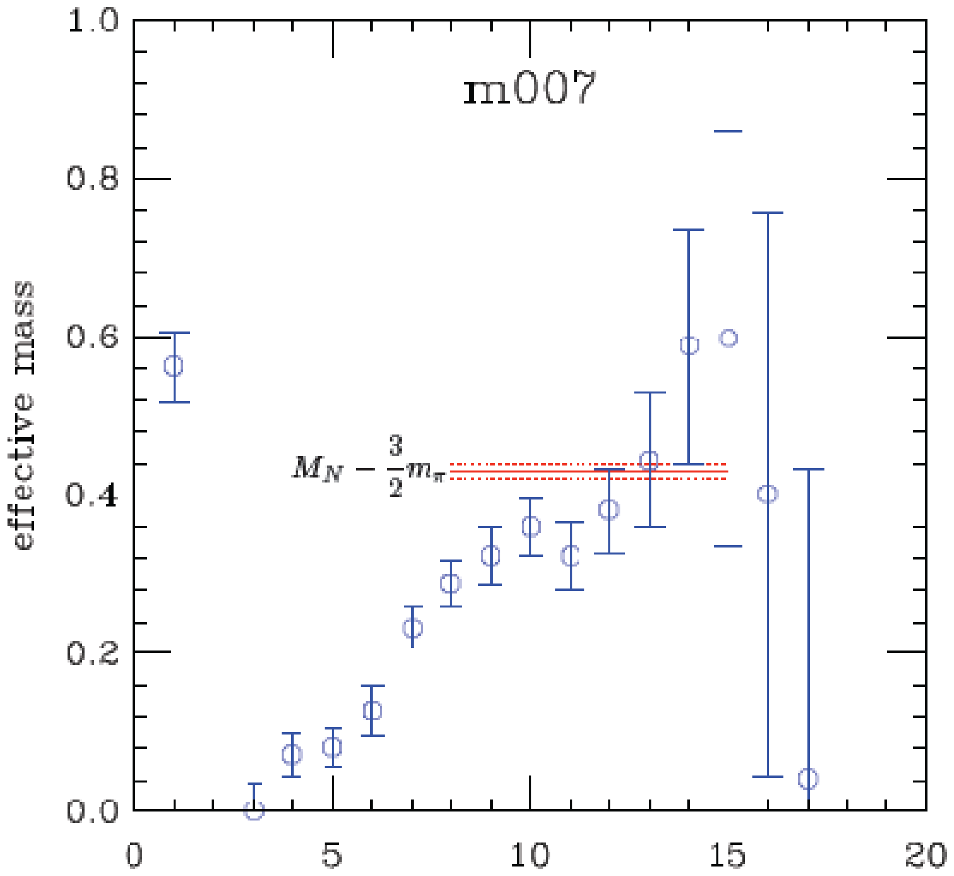
$$\sim \sqrt{N} De^{-(M_n - \frac{3}{2}m_\pi)t}$$

- Due to different overlap of nucleon and 3 pions also have volume dependence: $\sqrt{V}$

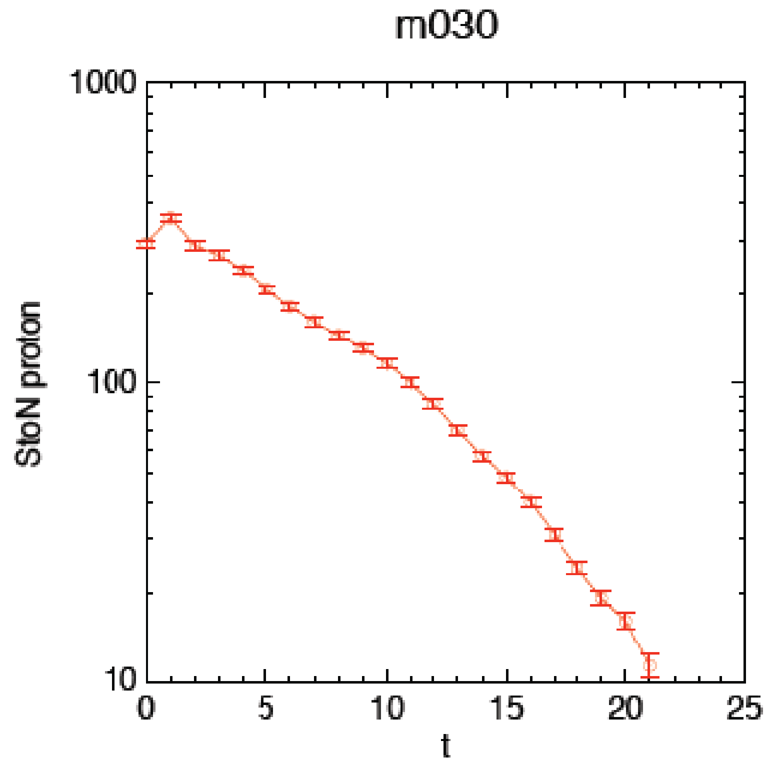
300 MeV pions

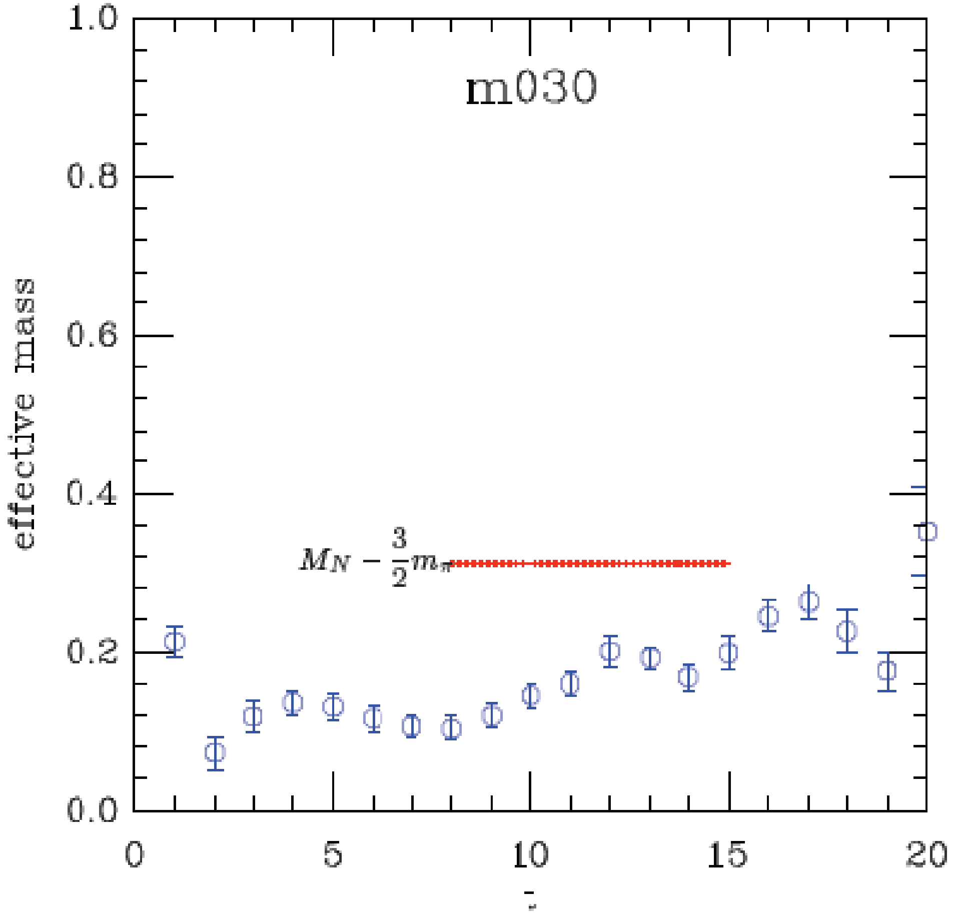
K. Orginos





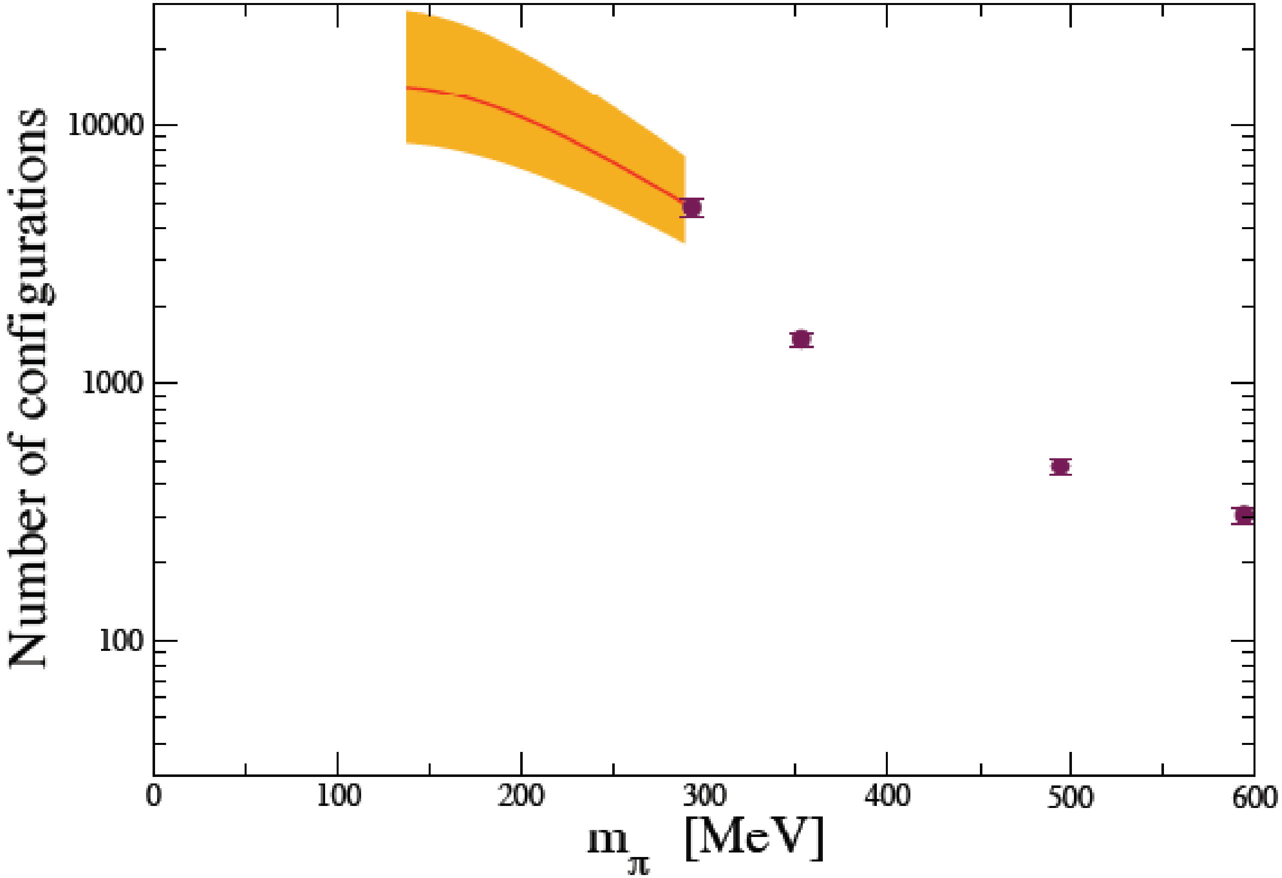
550 MeV pions





Required Measurements

- Measurements required for 3% accuracy at T=10
- May need significantly more



Proceeding to Physical Masses

$$\text{Cost}_{\text{traj}} = \left(\frac{\text{fm}}{a}\right)^6 \cdot \left[\left(\frac{L_s}{\text{fm}}\right)^3 \left(\frac{L_t}{\text{fm}}\right)\right]^{5/4} \cdot [C_1 + C_2/m_l] \, .$$

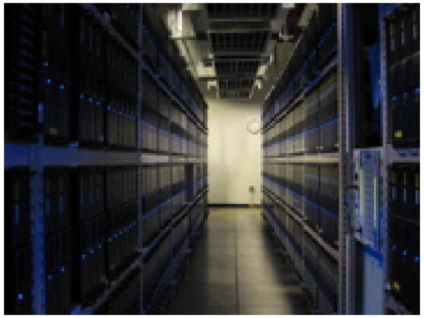
a (fm)	m_ℓ/m_s	Size	m_π(GeV)	L (fm)	Lm_π	MC traj.	TF-Yrs
0.086	0.150	$48^3 \times 64 \times 16$	276	4.1	5.8	6000	7
0.086	0.093	$48^3 \times 64 \times 16$	217	4.1	4.5	2400	3
0.086	TOTAL						10
0.125	0.134	$32^3 \times 64 \times 24$	250	4.0	5.1	4000	2
0.125	0.102	$32^3 \times 64 \times 24$	218	4.0	4.4	4000	2
0.125	0.102	$48^3 \times 64 \times 24$	218	6.0	6.6	4500	9
0.125	0.071	$48^3 \times 96 \times 24$	181	6.0	5.5	4500	18
0.125	0.039	$48^3 \times 96 \times 24$	135	6.0	4.1	6000	34
0.125	TOTAL						65
0.094	0.102	$64^3 \times 96 \times 24$	218	6.0	6.6	6700	65
0.094	0.071	$64^3 \times 96 \times 24$	181	6.0	5.5	7000	78
0.094	0.039	$64^3 \times 96 \times 24$	135	6.0	4.1	7600	115
0.094	TOTAL						258

Possible ensemble of DWF gauge configurations for joint HEP/Hadron Structure investigations

Computing Resources

Leadership-class (ORNL, ANL) – petaflop at ORNL

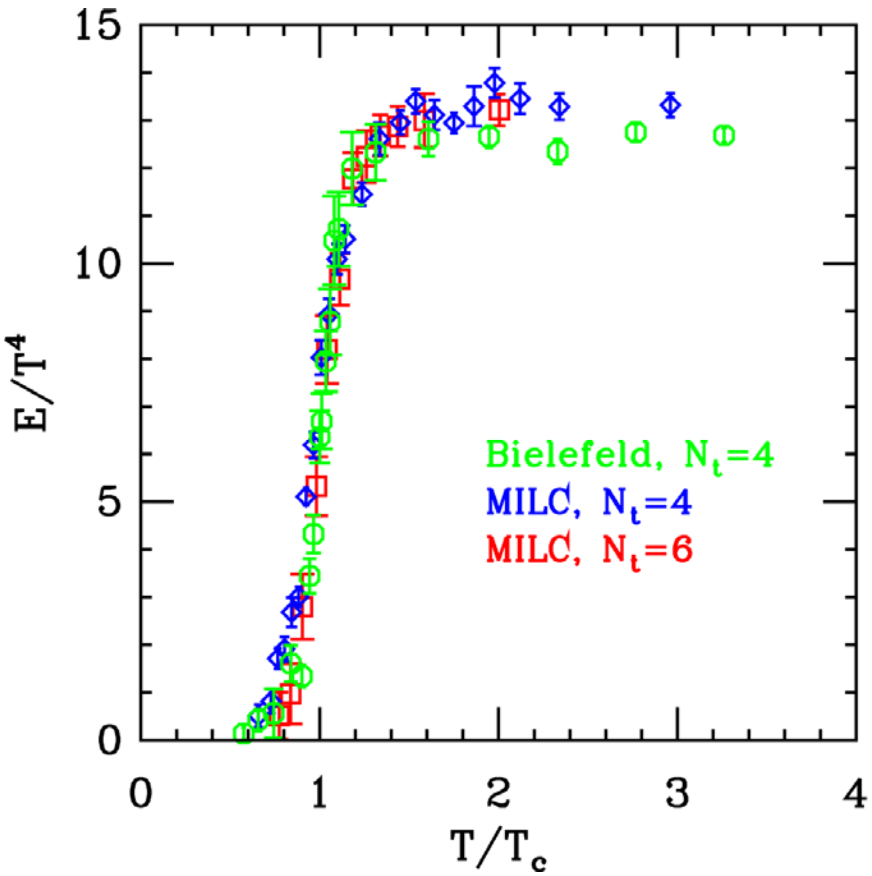
QCDOC



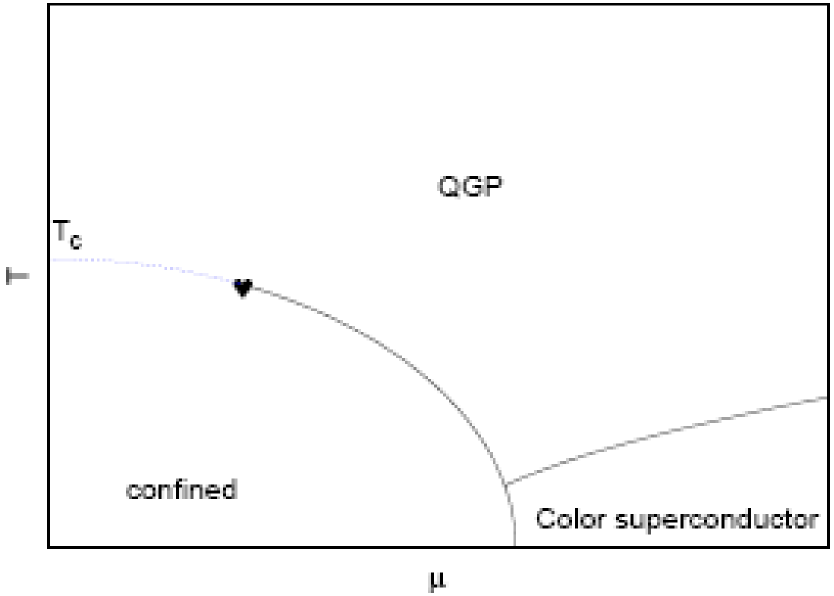
www.usqcd.org

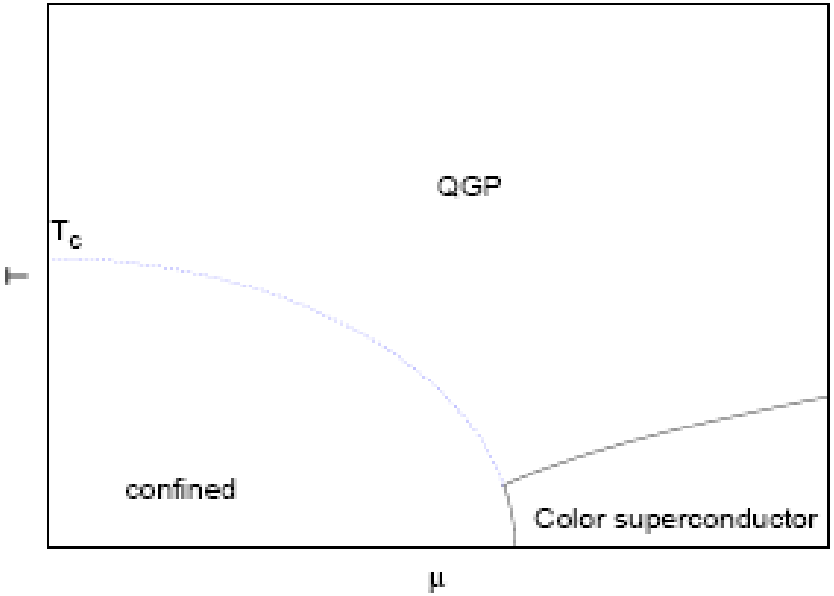
- Chroma, MILC,QDP

QCD at Finite Temperature and Density



Phase Diagram of QCD





O. Philipsen, arXiv:0708.1293