Physics of Nuclei

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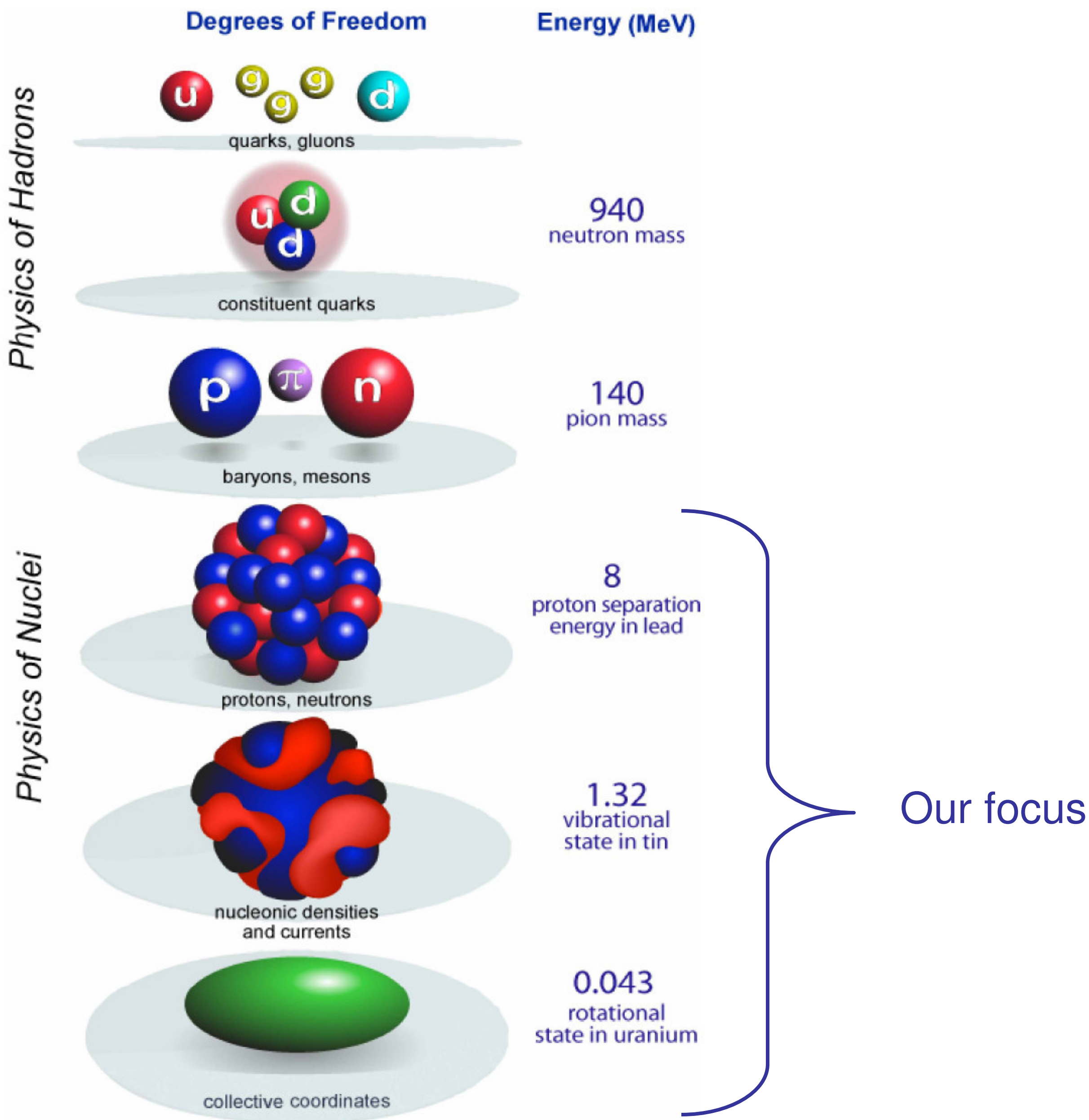
Aim of these lectures:

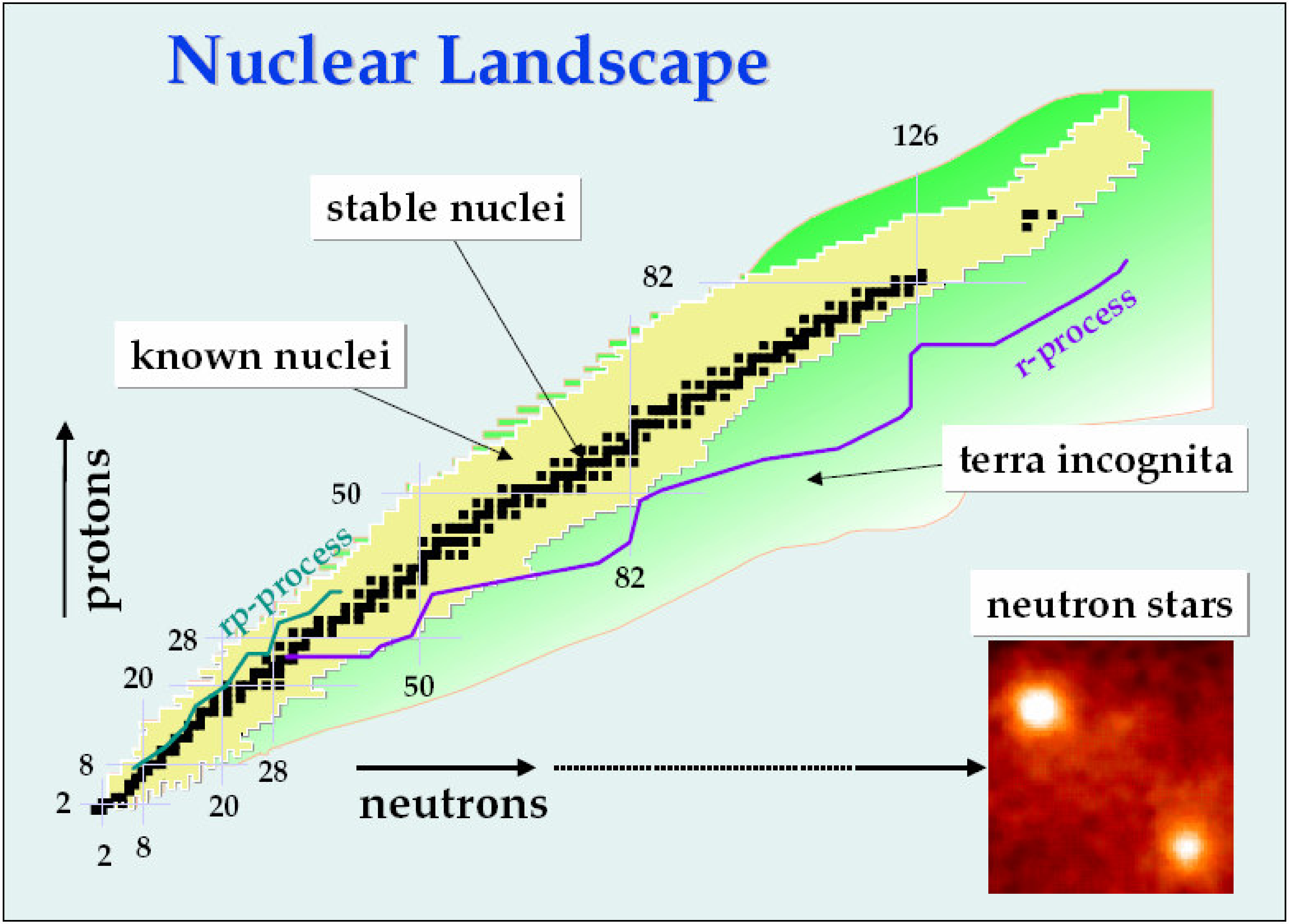
To give an overview of contemporary nuclear structure theory, i. e. effective interactions, methods that solve the nuclear many-body problem, and results of such calculations.

National Nuclear Physics Summer School 2008

George Washington University

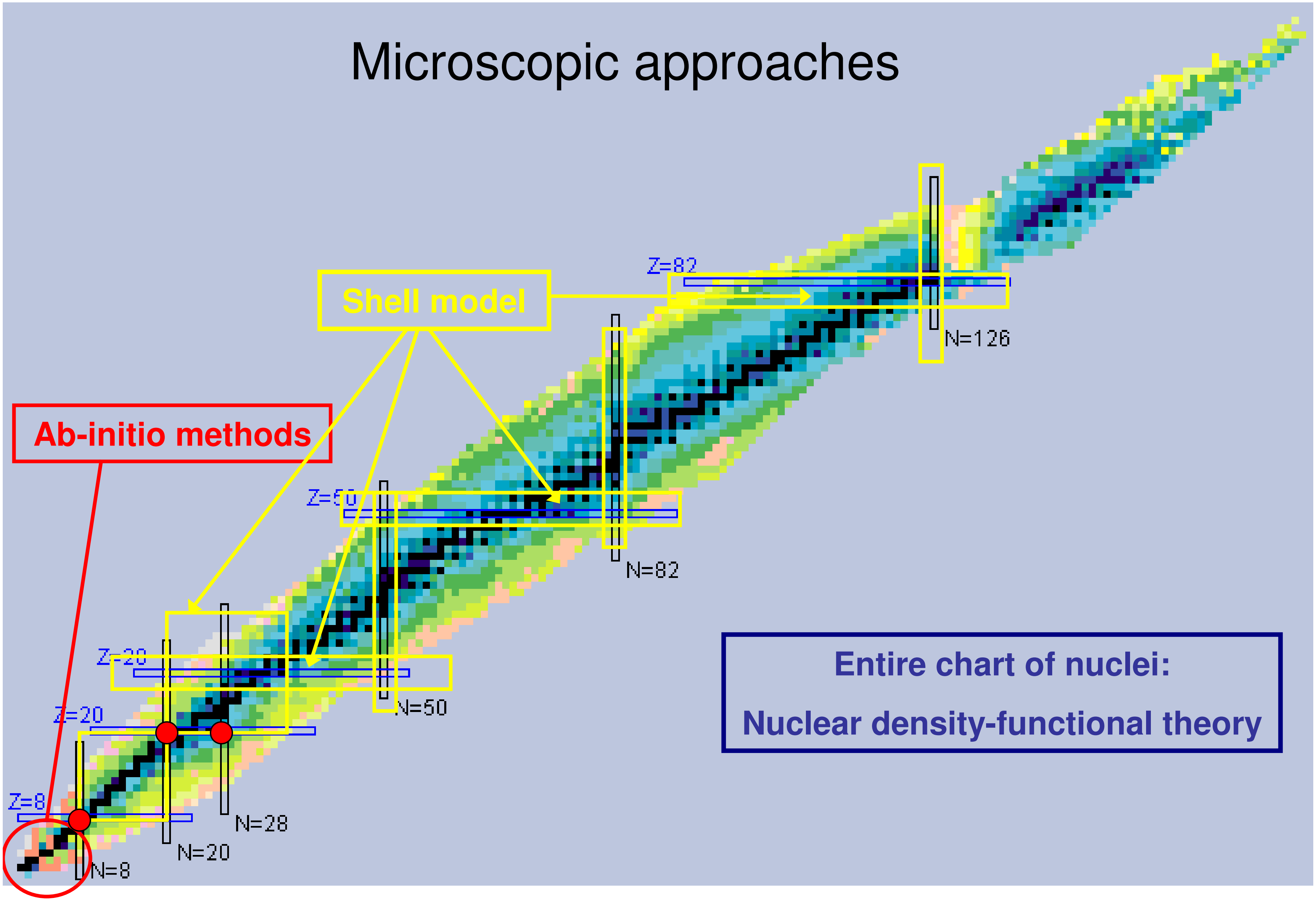
Energy scales and relevant degrees of freedom





Nuclear Landscape
126
stable nuclei
82
known nuclei
r-process
terra incognita
50
protons
rp-process
82
neutron stars
28
20
50
8
28
2
20
neutrons
2
8

Microscopic approaches



Aim: Bottom-up approach to nuclear structure

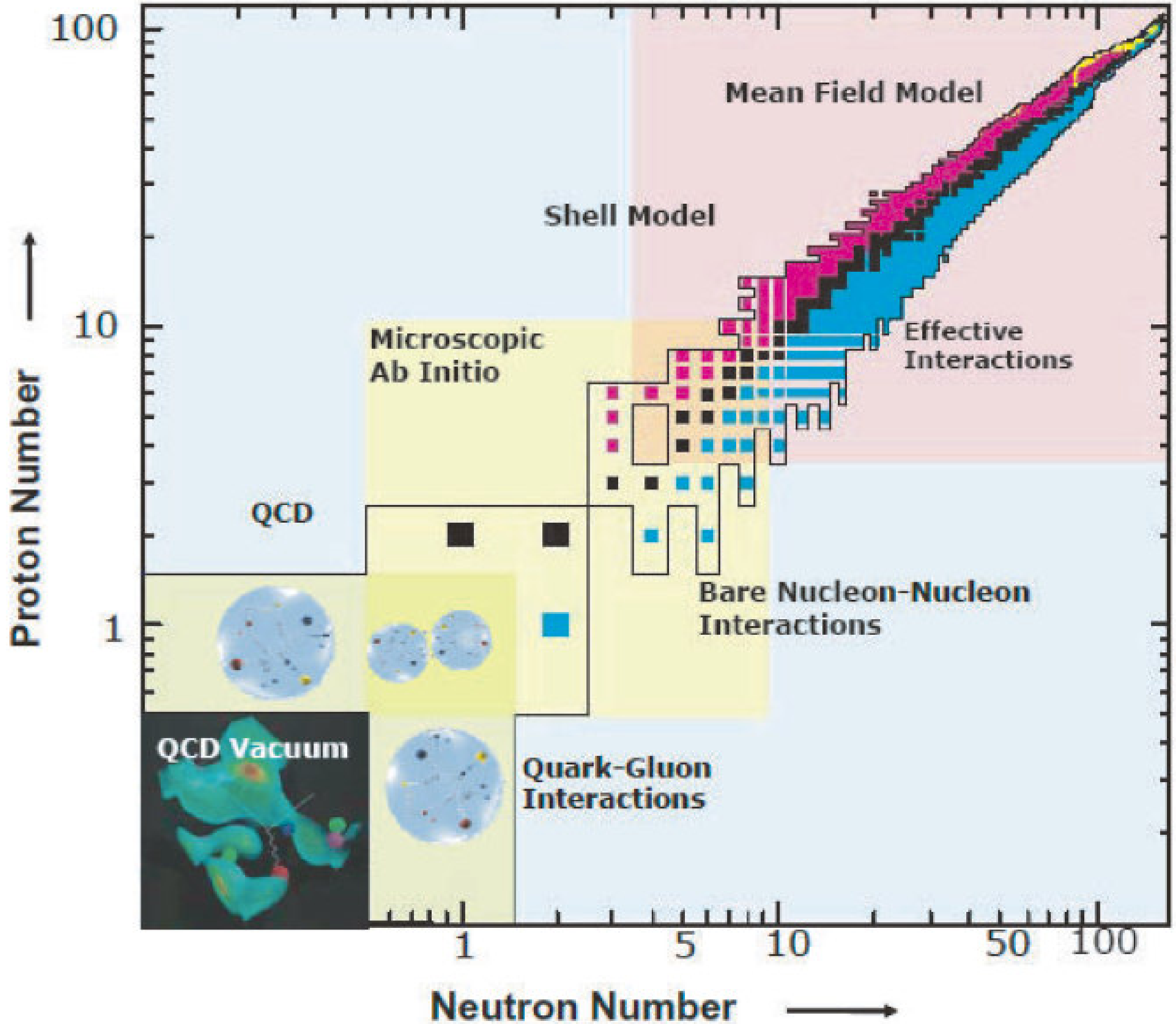


Figure from A. Richter (2004)

DFT

↑

Shell model

↑

Ab-initio

↑

Interactions

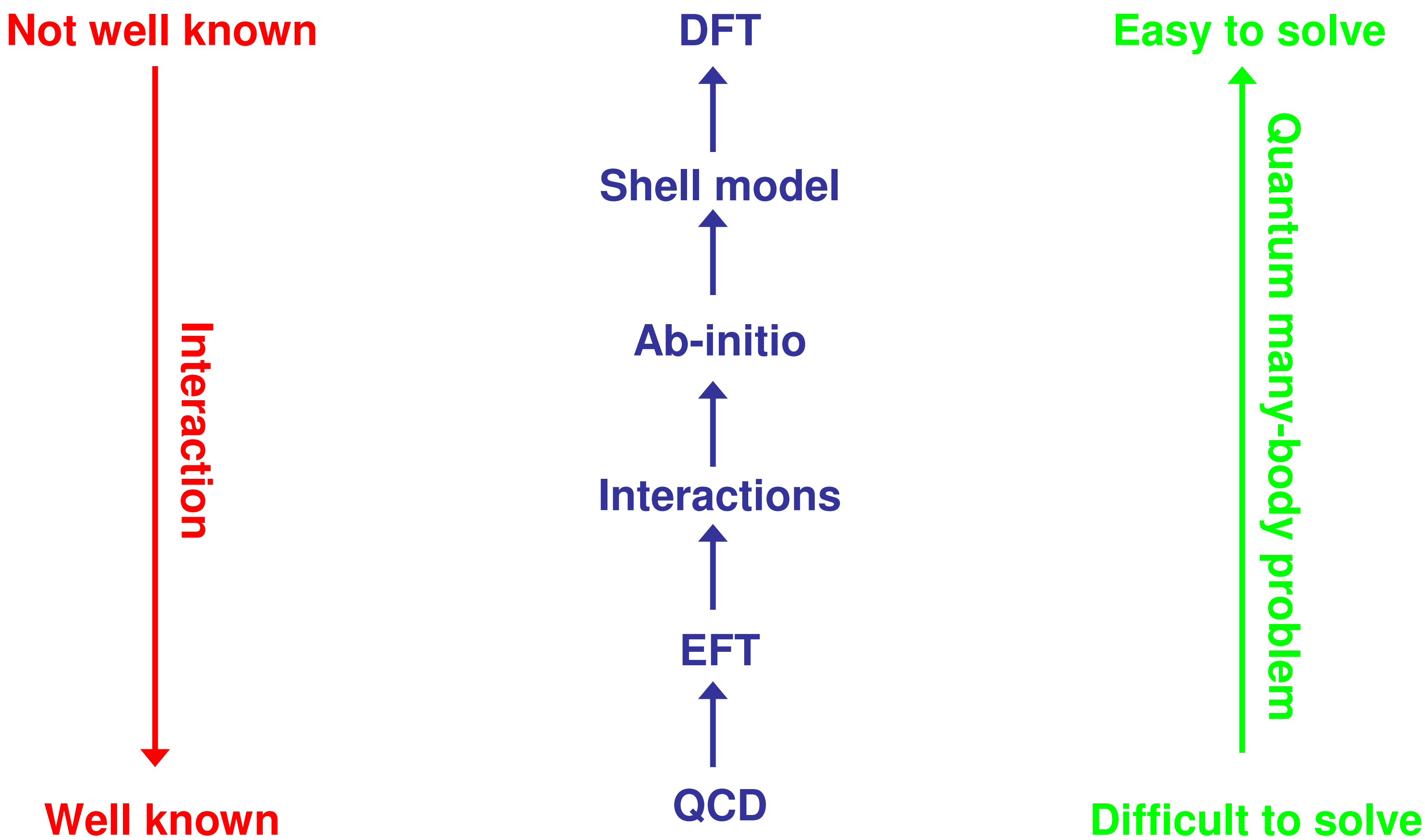
↑

EFT

↑

QCD

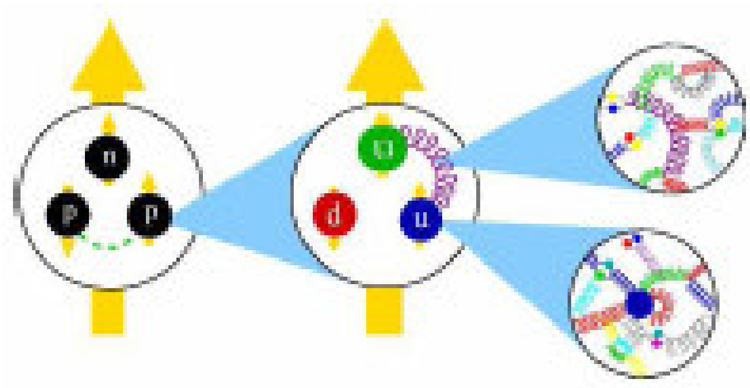
Conservation of misery



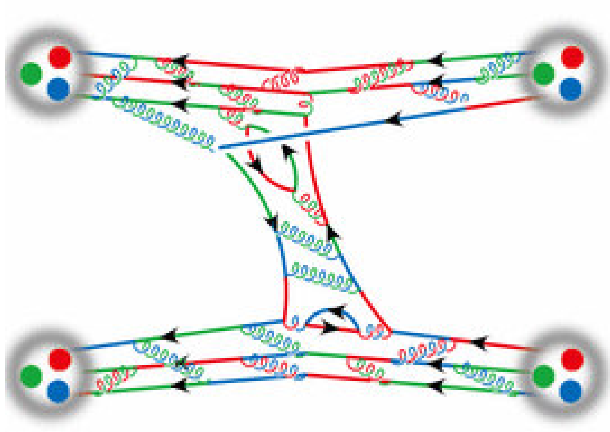
These lectures will present nuclear interactions and methods to solve the nuclear many-body problem!

The effective nucleon-nucleon interaction

- Nuclei are made of protons and neutrons. These are composite particles



Interplay between nucleonic and sub-nucleonic (quarks and gluons) degrees of freedom in few-body nuclear systems

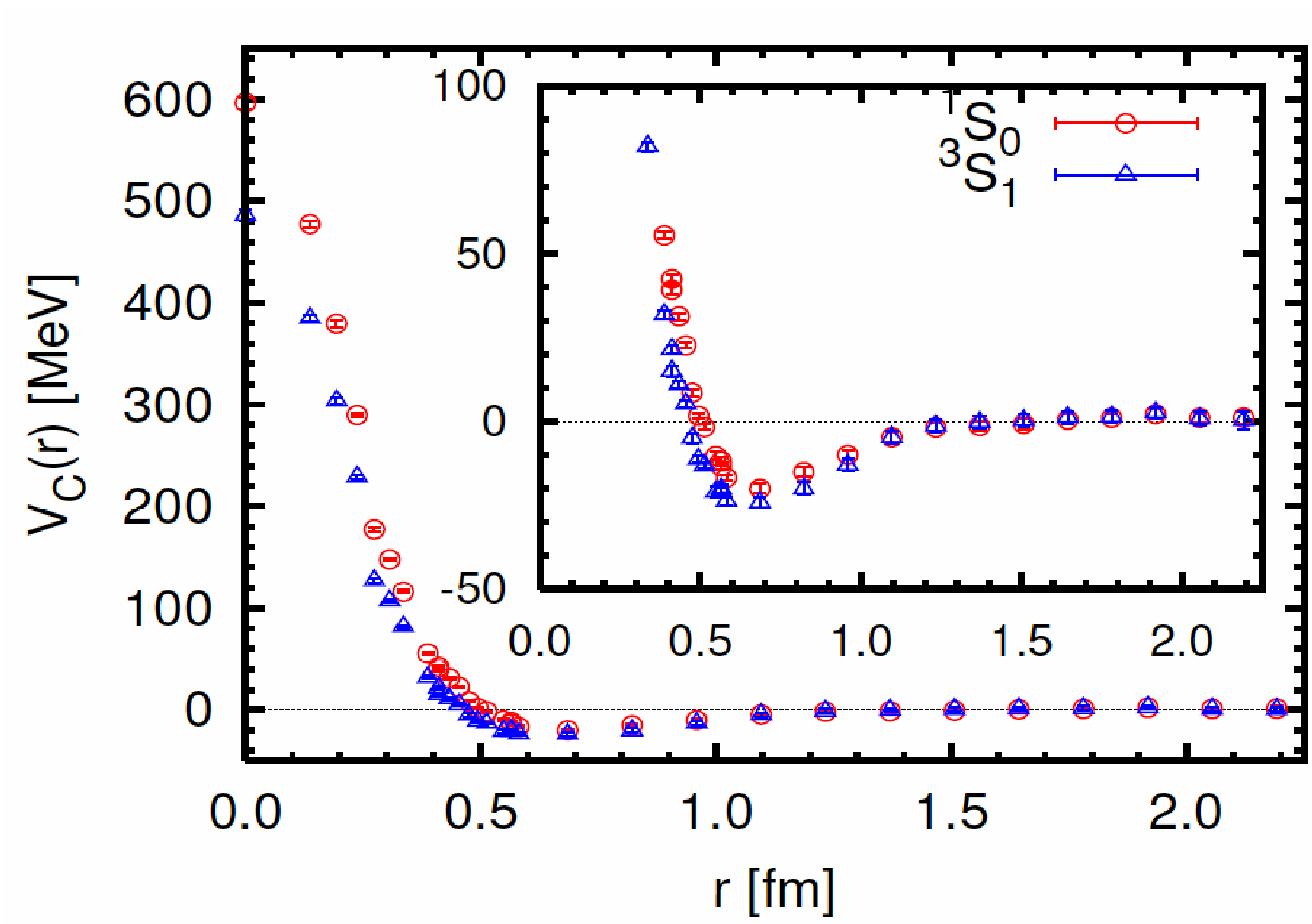


Q: How do we determine the interaction between two nucleons?

A1: Ideally from lattice QCD

A2: Potentials that fit phase shifts

NN central potential $V_c(r)$ for m_π=0.53 GeV *from Lattice QCD*

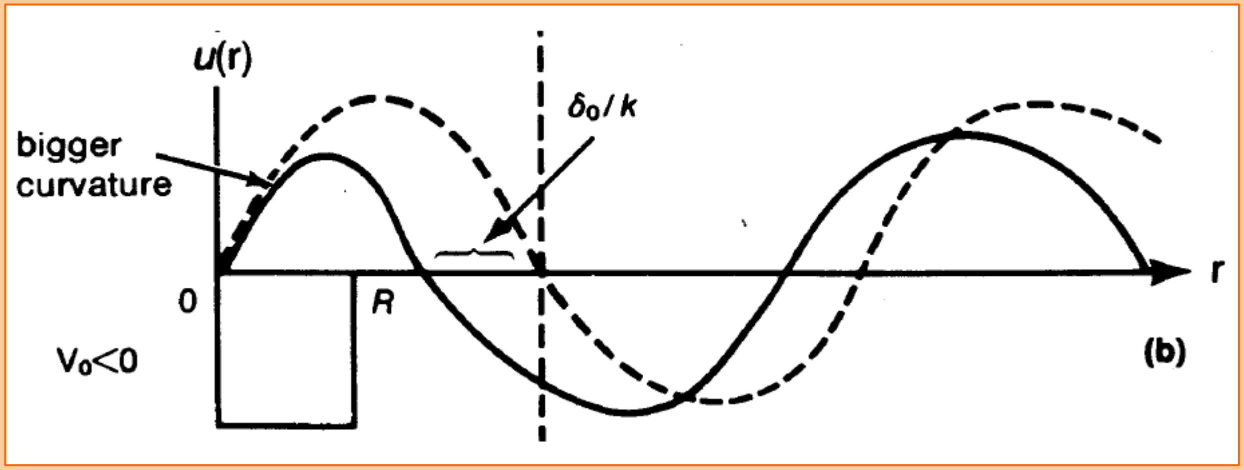


From T. Hatsuda (Oslo 2008)

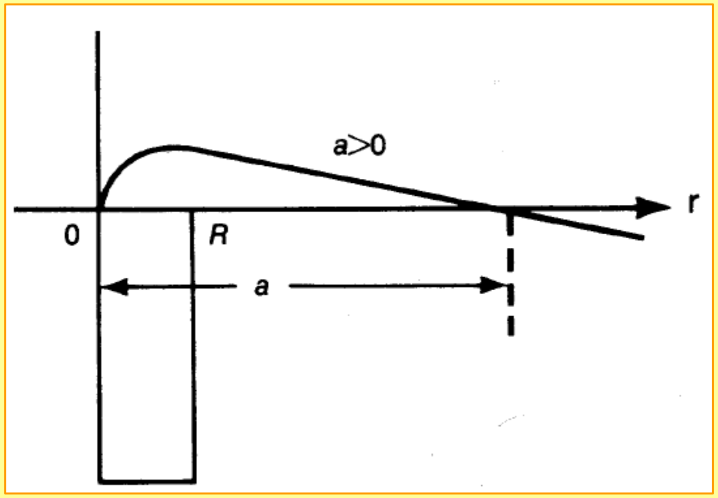
- Ishii, Aoki & T.Hatsuda., Phys. Rev. Lett. 99, 022001 (2007).
- Nemura, Ishii, Aoki & T.Hatsuda, arXiv:0710.3622 [hep-lat]
- Aoki, Ishii & T.Hatsuda, arXiv:0805.2462 [hep-ph]

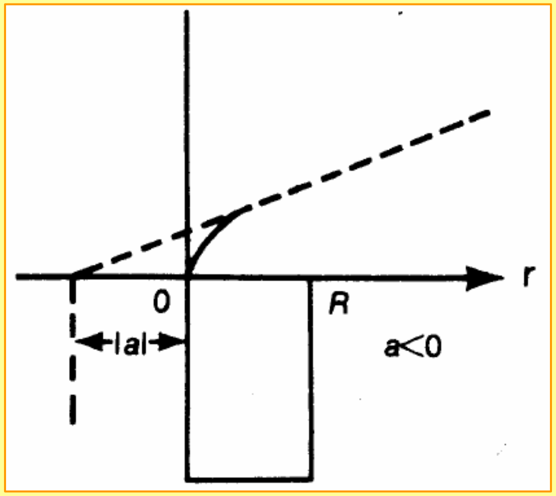
Recapitulation: Scattering theory

Phase shift δ(k) is a function of relative momentum k; Figure shows s-wave.



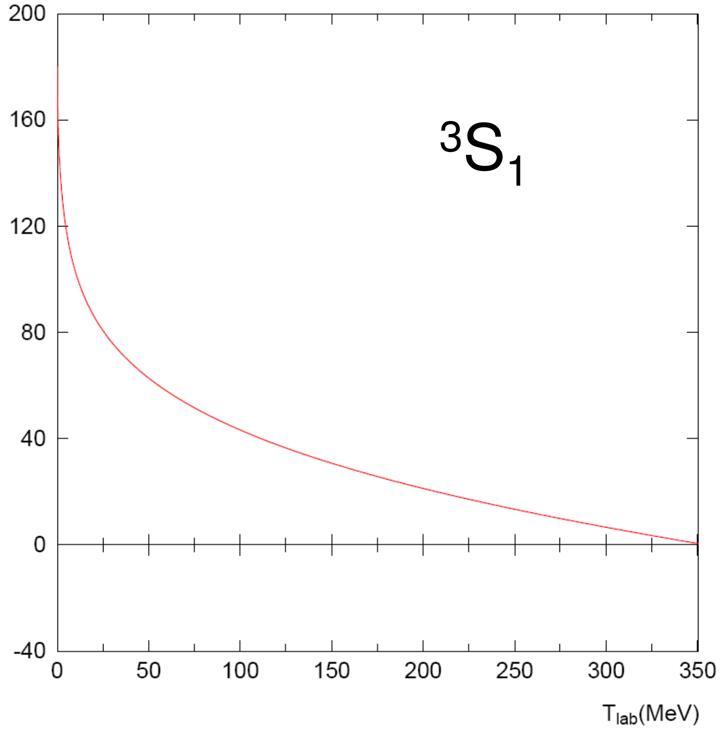
Scattering length: $k \cot \delta(k) \approx -\frac{1}{a}; \quad \sigma_{\text{tot}} \approx 4\pi a^2 \quad \text{for} \quad k \to 0$





Nuclear s-wave phase shifts

http://nn-online.org/

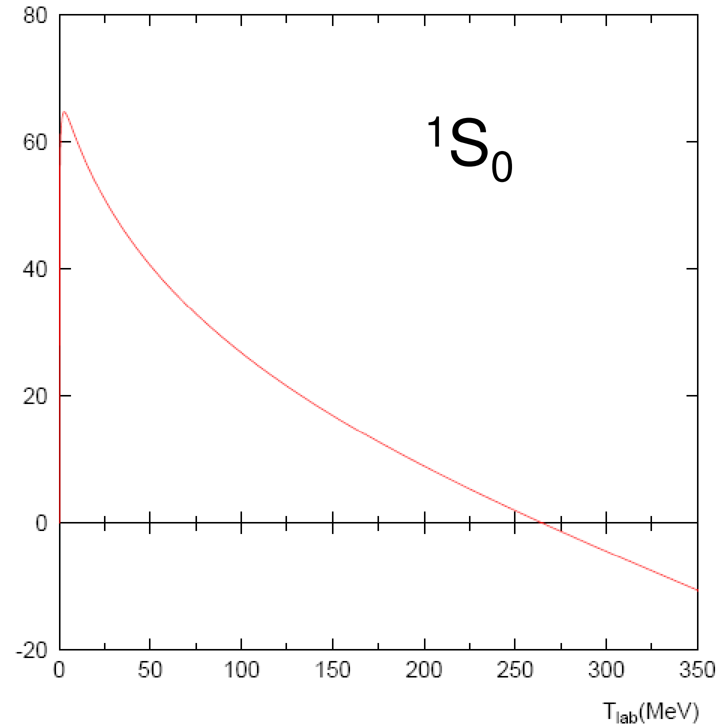


Deuteron is a very weakly bound system!

System has one bound state.

Steep decrease from 180 degrees due to large scattering length a = 5.5 fm.

Acts repulsive due to large (positive) scattering length.

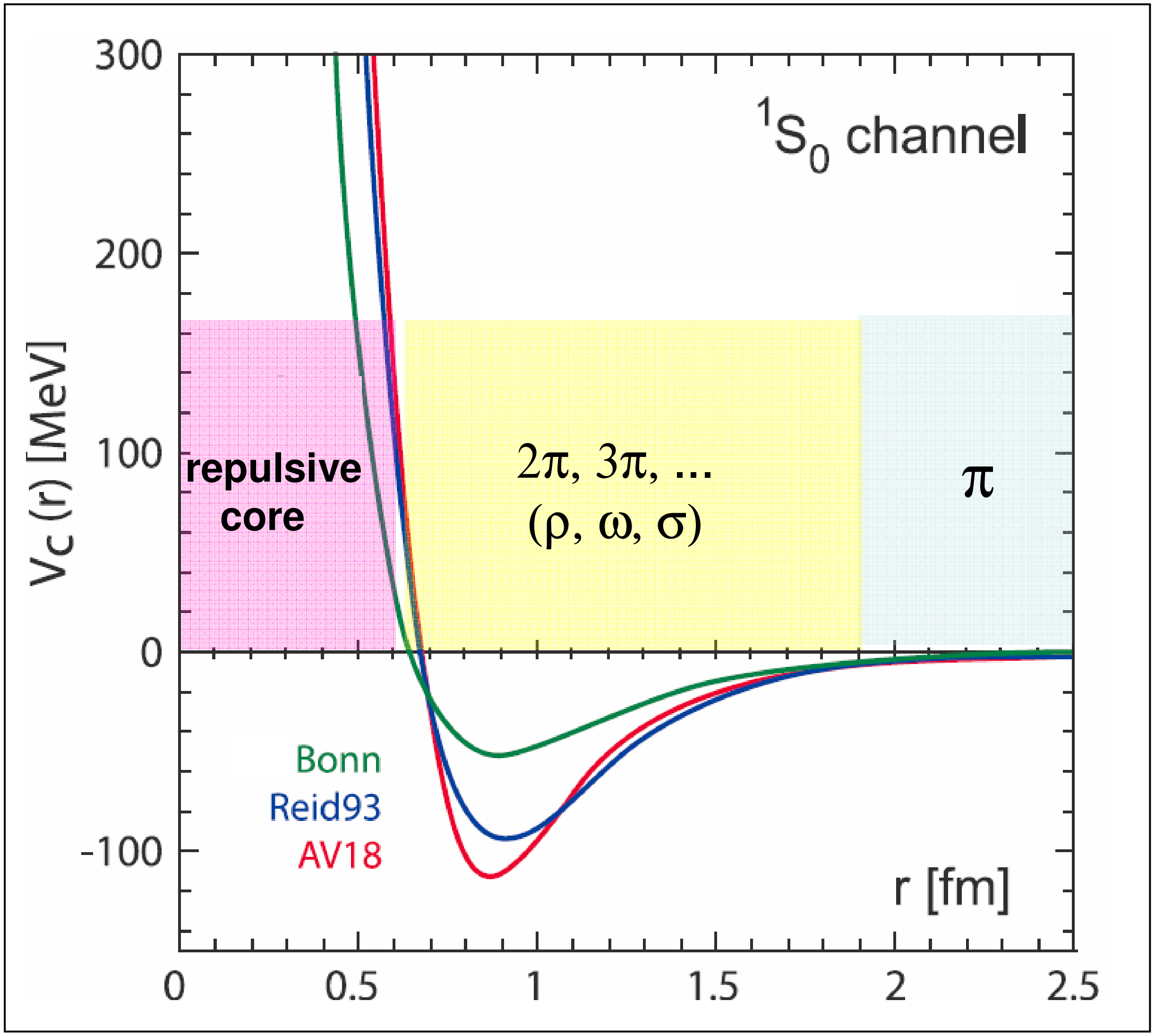


System (barely) fails to exhibit bound state.

Steep rise at 0 due to large scattering length a = -18 fm.

Monotonous decrease due to hard core (if local potential assumed).

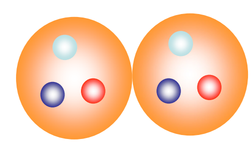
Phenomenological NN potentials



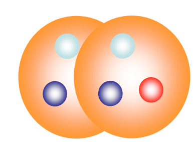
From T. Hatsuda (Oslo 2008)

One-pion exchange by Yukawa (1935)

Multi-pions by Taketani (1951)



Repulsive core by Jastrow (1951)



Phenomenological nucleon-nucleon potentials

Phenomenological potentials (Argonne, Bonn, …)

• contain pion exchange

• model-dependent short-range repulsion

• fit NN phase shifts with high precision

• approximately 20 parameters determined by fit to nucleon-nucleon data

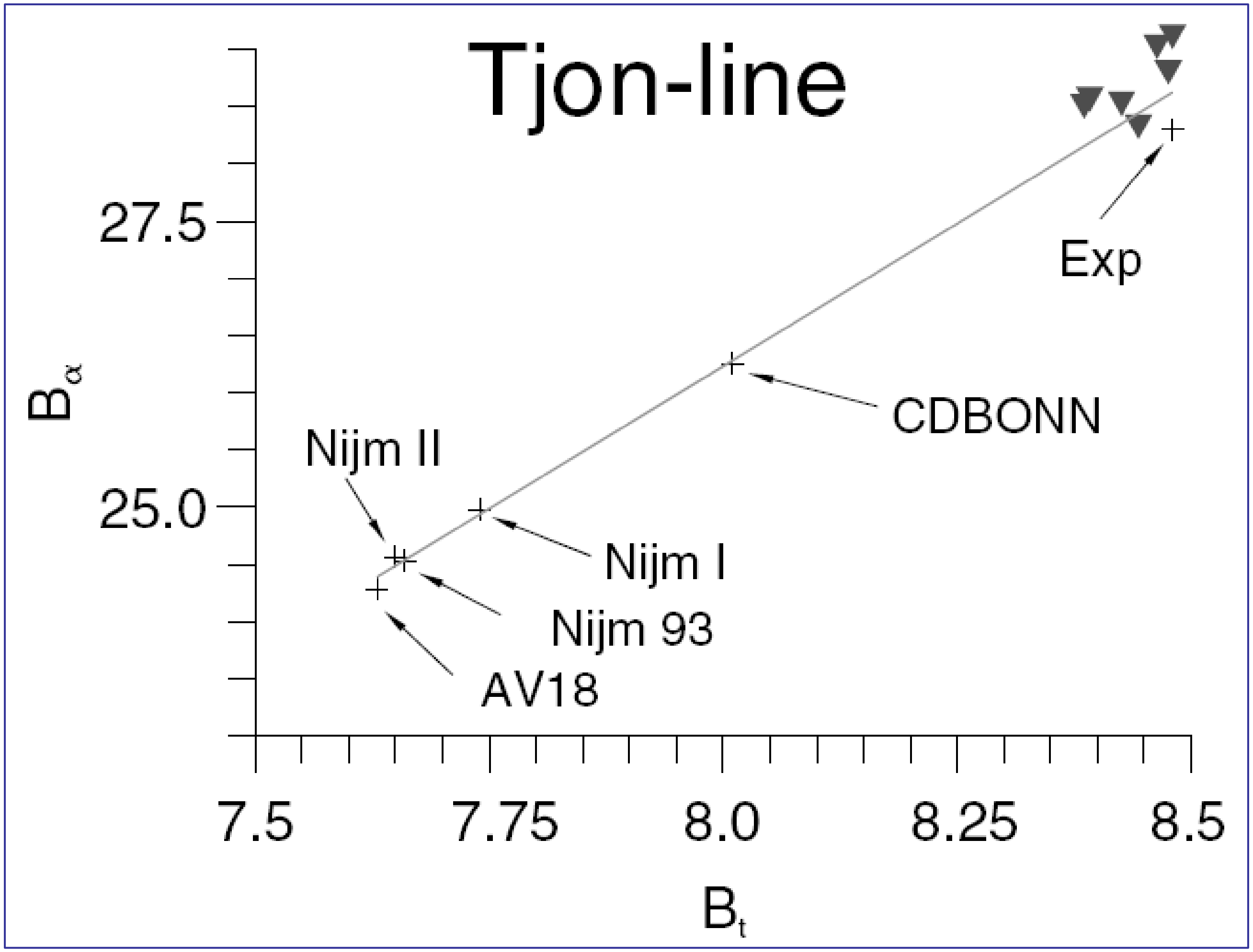
☺ Ab-initio description of light nuclei in 1990s

☺ Argonne potential can be solved by Green's function Monte Carlo method

☹ Difficult to improve systematically → Effective field theory

☹ Difficult to work with, i.e. to solve nuclear many-body problem → Low-momentum potentials and similarity transformations

1990s: High precision nucleon-nucleon potential **models**



A. Nogga et al, PRL 85 (2000) 944

1. Different two-body potential models disagree on structure of triton and alpha particle.

2. With additional **three-nucleon forces**, agreement with experiment is possible.

(Three-nucleon force differs for different two-body potentials.)

3. Four-body forces very small.

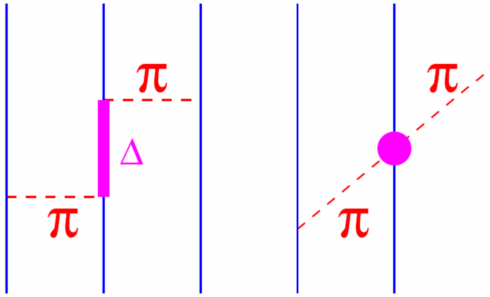
(Tjon line understood as feature of systems with large scattering length and no leading four-body force.)

Illinois three-nucleon force

$$V_{ijk} = V^{2\pi}_{ijk} + V^{3\pi}_{ijk} + V^{R}_{ijk}$$

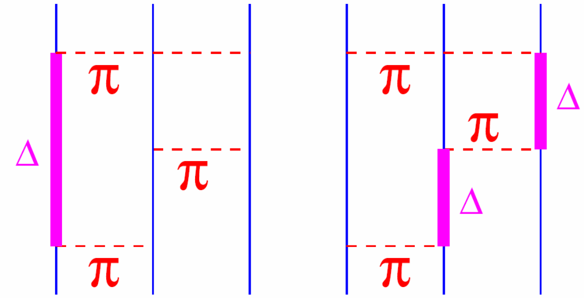
$V^{2\pi}_{ijk}$: Fujita-Miyazawa + s-wave term; in most V_{ijk}

- Longest ranged V_{ijk}
- Attractive in all nuclei studied.



$V^{3\pi}_{ijk}$: 3π rings with Δ's; new in Illinois V_{ijk}

- Extra p-shell, $|N - Z|$ attraction
- One Δ in energy denominator
- $2\Delta, 3\Delta$ denominators not yet considered
- $\langle V^{3\pi}_{ijk} \rangle \lesssim 0.1 \langle V^{2\pi}_{ijk} \rangle$

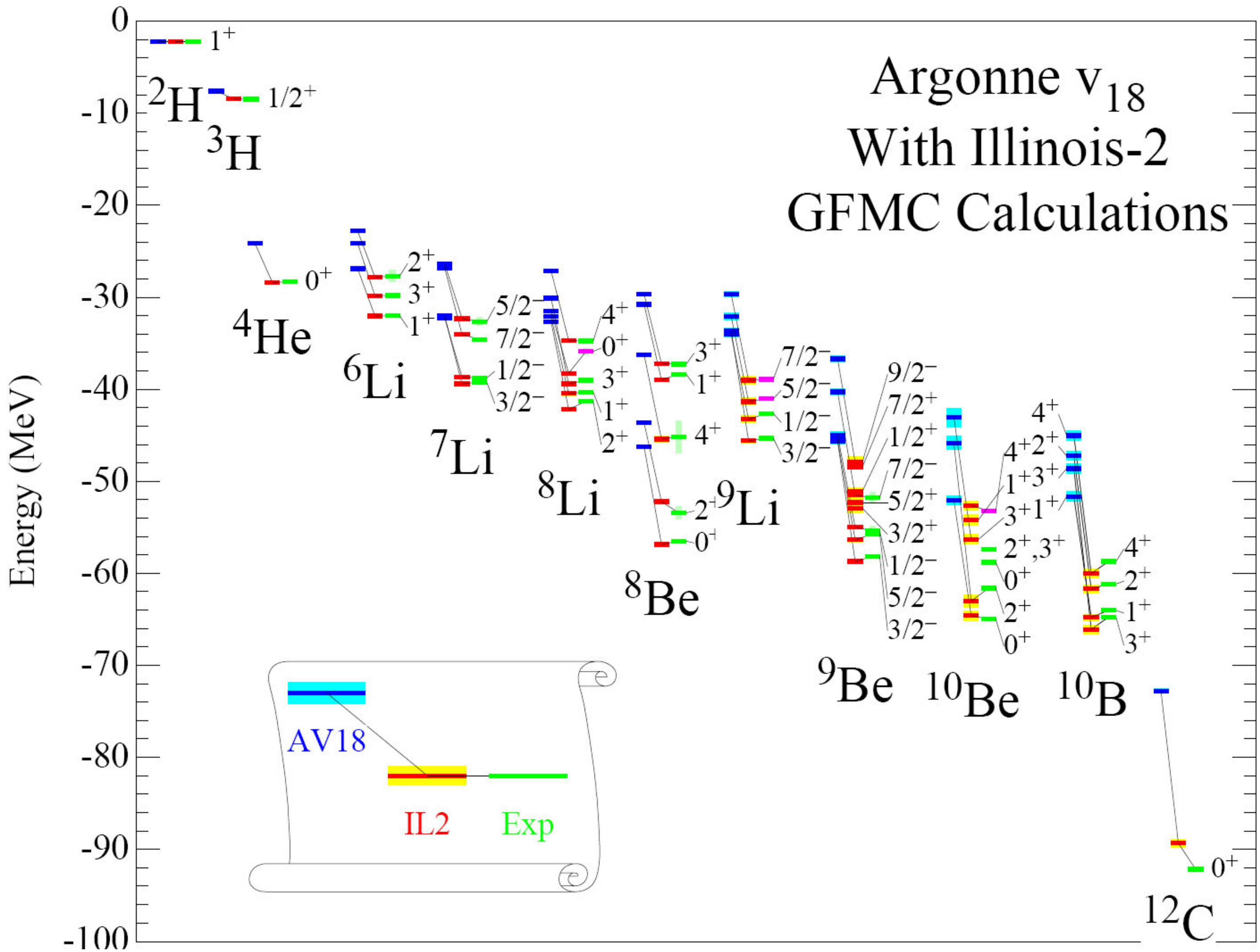


V^{R}_{ijk}: represents all else including relativistic effects – purely central and repulsive

3-4 Couplings adjusted to fit 17 nuclear levels for $A \leq 8$

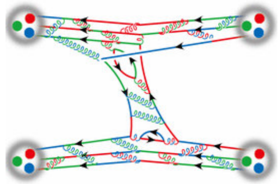
S.C. Pieper, V.R. Pandharipande, R.B. Wiringa, and J. Carlson, Phys. Rev. C **64**, 014001 (2001)

Green's function Monte Carlo results



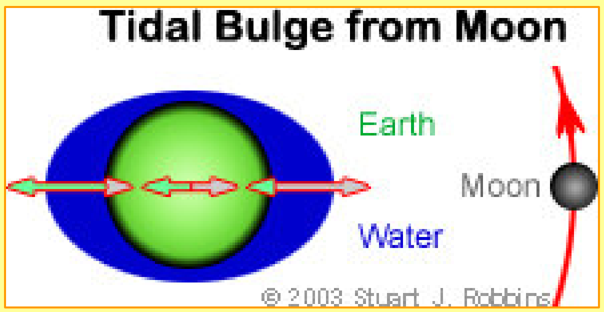
S. C. Pieper and R. B. Wiringa, Ann.Rev.Nucl.Part.Sci. 51 (2001) 53

Three-nucleon forces: Why?

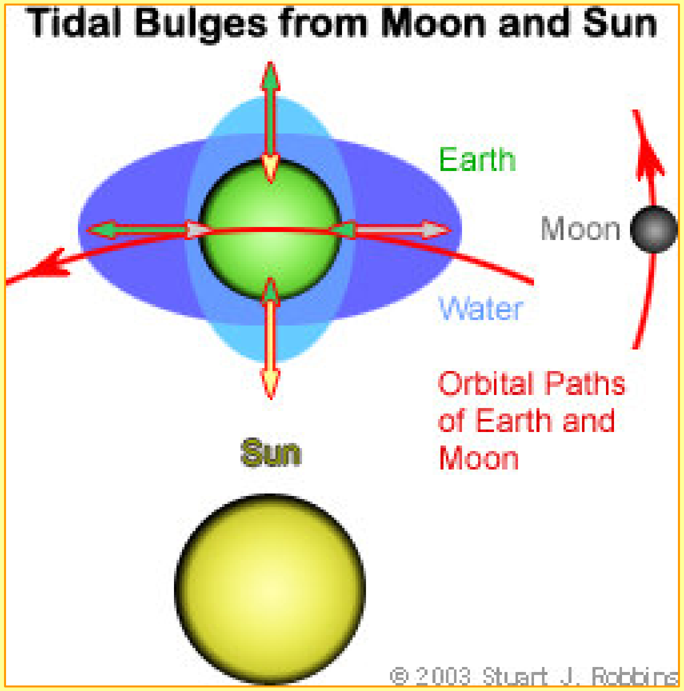


- Nucleons are not point particles (i.e. not elementary).
- We neglected some internal degrees of freedom (e.g. Δ-resonance, "polarization effects", …), and unconstrained high-momentum modes.

Example from celestial mechanics:
Earth-Moon system: point masses and modified two-body interaction



Other tidal effects cannot be included in the two-body interaction! Three-body force unavoidable for point masses.



Renormalization group transformation: Removal of "stiff" degrees of freedom at expense of additional forces.

A theorem for three-body Hamiltonians

Polyzou and Glöckle, Few Body Systems 9, 97 (1990)

Different two-body Hamiltonians can be made to fit two-body and three-body data by including a 3NF into one of the Hamiltonians.

Theorem. *Let*

$$H_{ij} = H_i + H_j + V_{ij} \quad \text{and} \quad \bar{H}_{ij} = H_i + H_j + \bar{V}_{ij} \tag{1.1}$$

be two-body Hamiltonians with the same binding energies and scattering matrices for each pair of particles i and j. Assume that the two-body Hamiltonians are asymptotically complete and that the unitary transformations relating these two-body Hamiltonians, which necessarily exist, have bounded Cayley transforms. Then there exists a three-body interaction, W, such that the two three-body Hamiltonians

$$H = H_1 + H_2 + H_3 + V_{12} + V_{23} + V_{31} \tag{1.2}$$

and

$$\bar{H}' = \bar{H} + W \tag{1.3}$$

with

$$\bar{H} = H_1 + H_2 + H_3 + \bar{V}_{12} + \bar{V}_{23} + \bar{V}_{31} \tag{1.4}$$

have the same binding energies and scattering matrix.

Corollary. *Under the assumptions of the theorem, if $V_{(123)}$ is a three-body interaction then there exists another three-body interaction $\bar{V}_{(123)}$ such that*

$$H = H_1 + H_2 + H_3 + V_{12} + V_{23} + V_{31} + V_{(123)}$$

and

$$\bar{H} = H_1 + H_2 + H_3 + \bar{V}_{12} + \bar{V}_{23} + \bar{V}_{31} + \bar{V}_{(123)}$$

have the same binding energies and scattering matrix.

Implications:

(1) There are no experiments measuring only three-body binding energies and phase shifts that can determine if there are no three-body forces in a three-body system. The question makes no sense. The correct statement is that there may be some systems for which it is possible to find a representation in which three-body forces are not needed.

(2) Different off-shell extensions of two-body forces can be equivalently realized as three-body interactions.

(4) Three-body forces cannot be determined in a manner that is independent of the two-body interaction.

Effective field theory

Q: How can we economically solve a physical problem (by employing appropriate degrees of freedom)?

Examples:

1. Far away from a charge distribution, one employs a multipole expansion for the electromagnetic field
2. Quantum chemistry employs the Coulomb potential and not QED
3. Atomic nuclei are described in terms of protons and neutrons and not via quarks and gluons

A: Employ an effective field theory (EFT). EFTs exploit a separation of scales

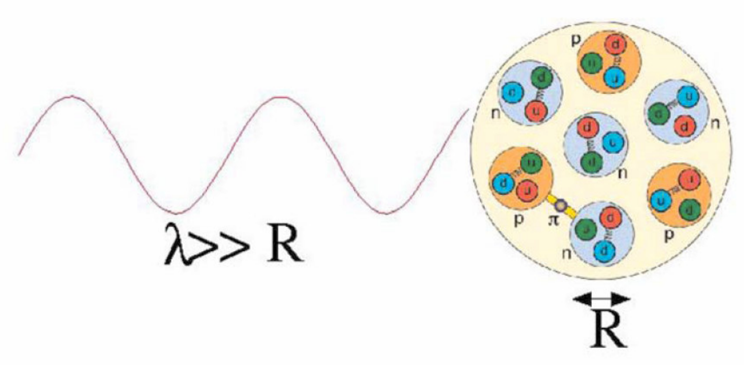
Examples:

1. Distance from charge distribution >> extension of charge distribution
2. $e^+ e^-$ pair production threshold (~1Mev) >> chemical bonds (< eV)
3. Excitation of the nucleon (~300 MeV) >> excitation energies of nuclei (~ 1MeV)

Model-independent approach: chiral NN potentials and effective field theory (EFT)

EFT:

- nuclear structure energies Q well below QCD scale of about Λ ~1GeV



- Further examples of EFTs:
  - Gravitational potential on Earth

$$V(h) = -\frac{mMG}{R+h} \approx V(0) + gmh + O\left(\frac{h^2}{R^2}\right)$$

  - Low-energy potential scattering determined by scattering length alone

$$k \cot\delta(k) \approx -\frac{1}{a}; \quad \sigma_{\text{tot}} \approx 4\pi a^2 \quad \text{for} \quad k \to 0$$

Construction of nuclear potentials via chiral EFT

Weinberg, van Kolck, Epelbaum, Machleidt, …

1. Identify the **relevant degrees of freedom** for the resolution scale of atomic nuclei: **nucleons and pions**.
2. Identify the **relevant symmetries** of low-energy QCD and investigate if and how they are broken: **spontaneously broken chiral symmetry**
3. Construct the most general Lagrangian consistent with those symmetries and the symmetry breaking.
4. Design an **organizational scheme** that can distinguish between more and less important contributions: a low-momentum expansion: **power counting**
5. Guided by the expansion, calculate Feynman diagrams to the desired accuracy for the problem under consideration.

Reviews.:

Bedaque and van Kolck, Ann. Rev. Nucl. Part. Sci. 52 (2002) 339, nucl-th/0205058.

Machleidt, arxiv:0704.0807

See Griesshammer's and Bedaque's lectures!

EFT cont'd

- Low energy degrees of freedom:
  - Nucleons
  - Pions (approximate Goldstone bosons of the spontaneously broken chiral symmetry)

- EFT Lagrangian

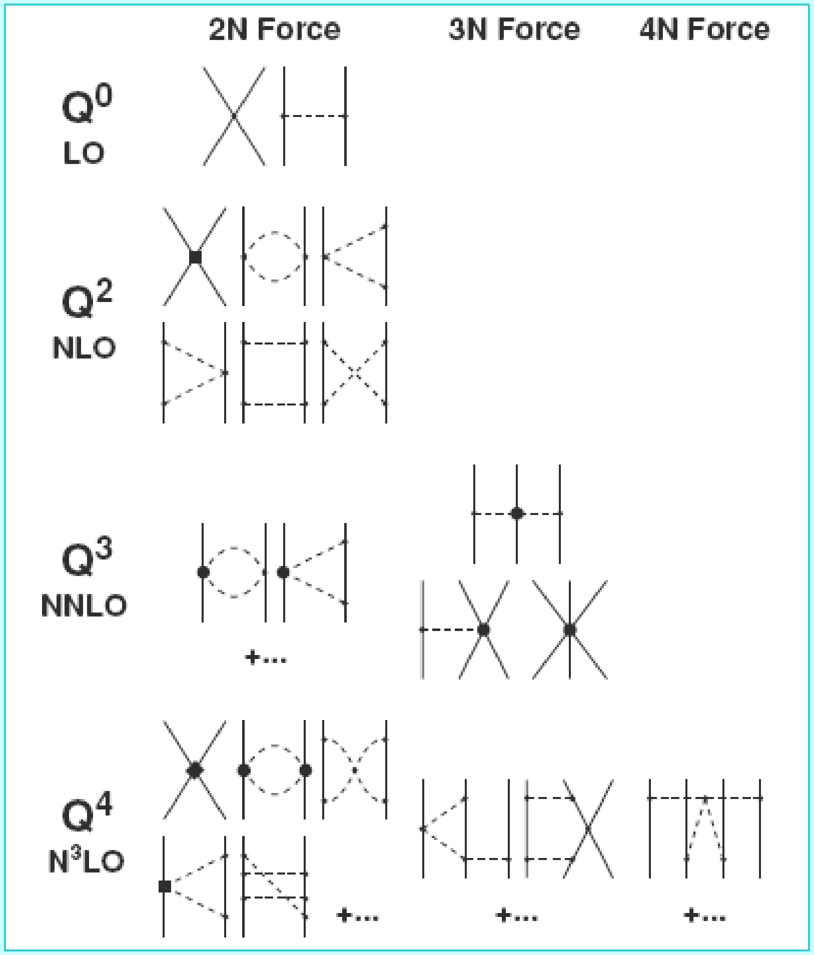
$$\mathcal{L} = \mathcal{L}_{NN} + \mathcal{L}_{N\pi}$$

  - Pion dependence obey rules of broken chiral symmetry
  - Contact terms and derivatives for nucleon fields. (Include effects of short range physics (QCD!) that is not resolved at low energy.)
  - Infinite number of terms
- Order terms of Lagrangian in powers of Q/Λ, and truncate → finite number
- NN-scattering: Feynman diagrams up to given order (finite number)
- Fit low-energy constants to nucleon-nucleon scattering data …

Refs.: S. Weinberg, Nucl. Phys. B 363 (1991) 3.
C.R.L. Ordenez et al, PRC 53 (1996) 2086.
U. van Kolck, Prog. Part. Nucl. Phys. 43 (1999) 337.

Effective field theory: chiral potential at order N^3LO

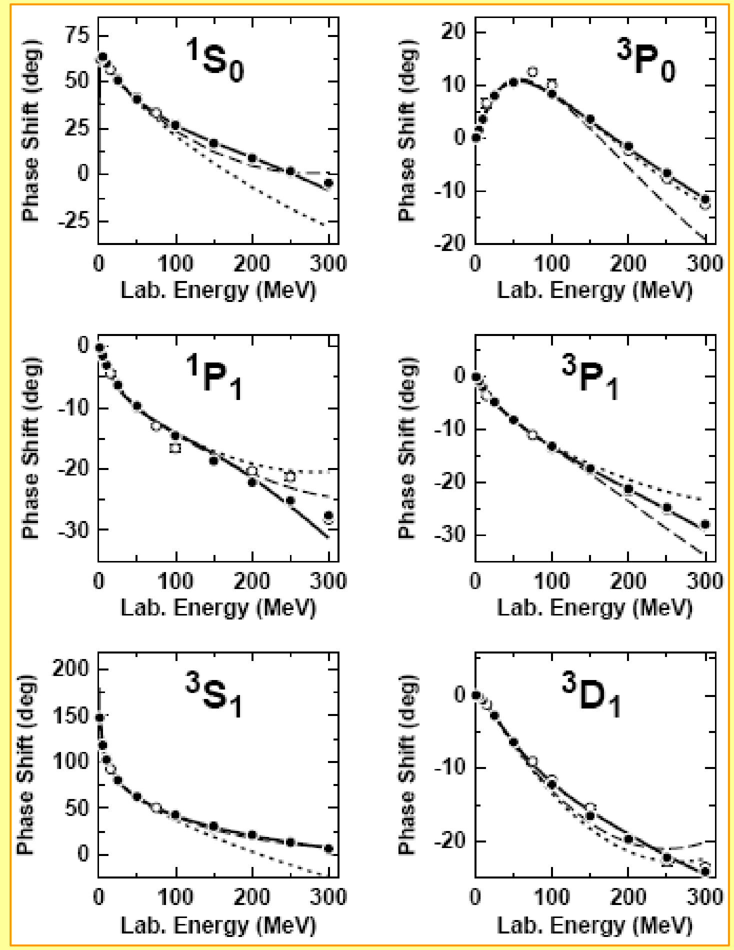
Feynman diagrams



R. Machleidt and D. R. Entem, J. Phys. G 31 (2005) S1235

Phase shifts reproduced to χ^2/datum=1

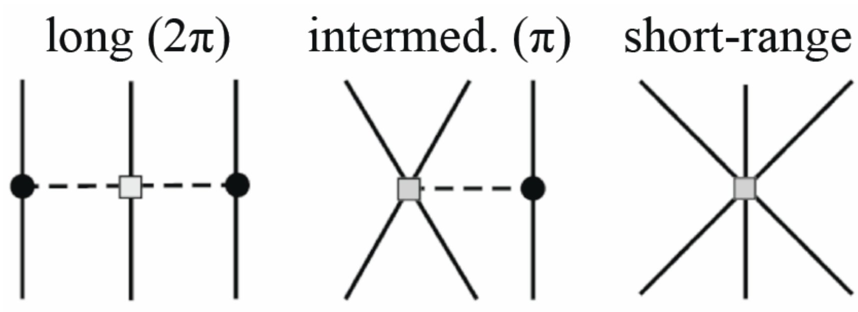
About 40+ parameters



D. R. Entem and R. Machleidt, Phys.Rev. C68 (2003) 041001

Chiral three-nucleon force

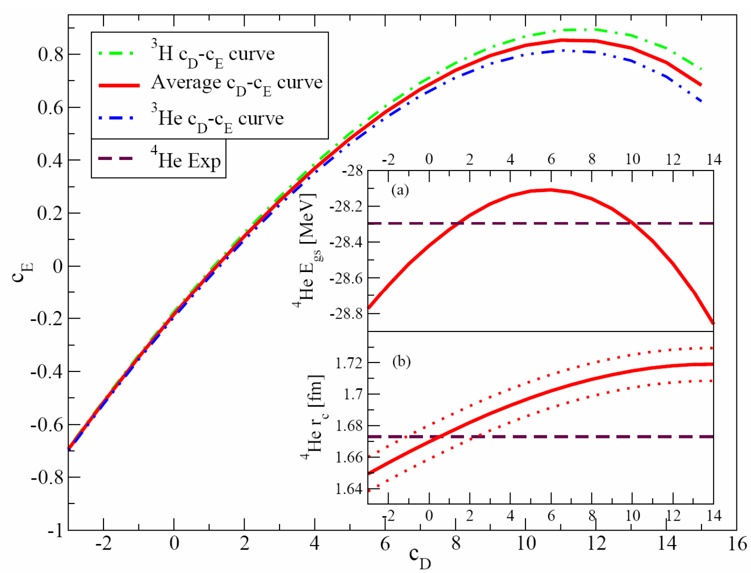
Leading terms at order $N^2LO \sim (Q/\Lambda)^3$ [van Kolck (1994), Epelbaum et al (2002)]

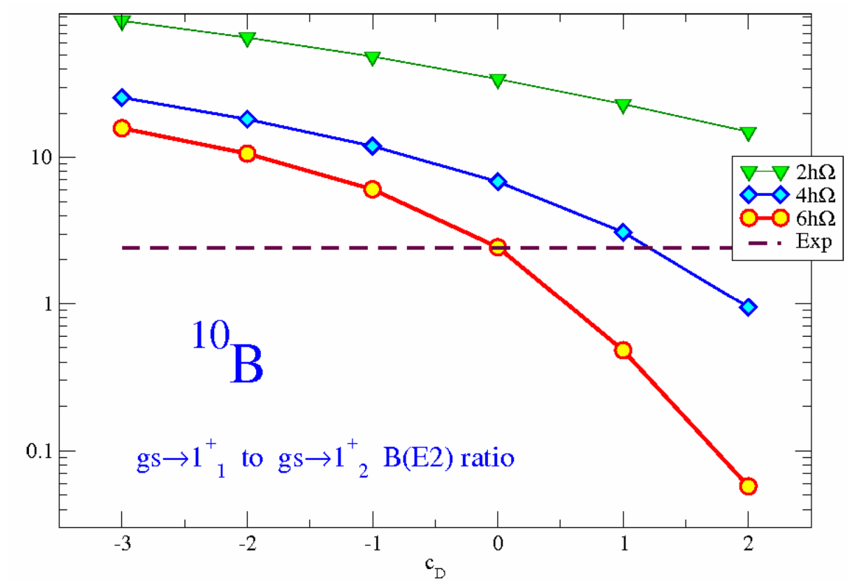


c-terms D E

c-terms (from pion-nucleon scattering) still with considerable uncertainties

Low-energy coefficients D and E of contact terms from A>2 nuclei [Navratil et al 2007]





Light nuclei from chiral interactions with no-core shell model

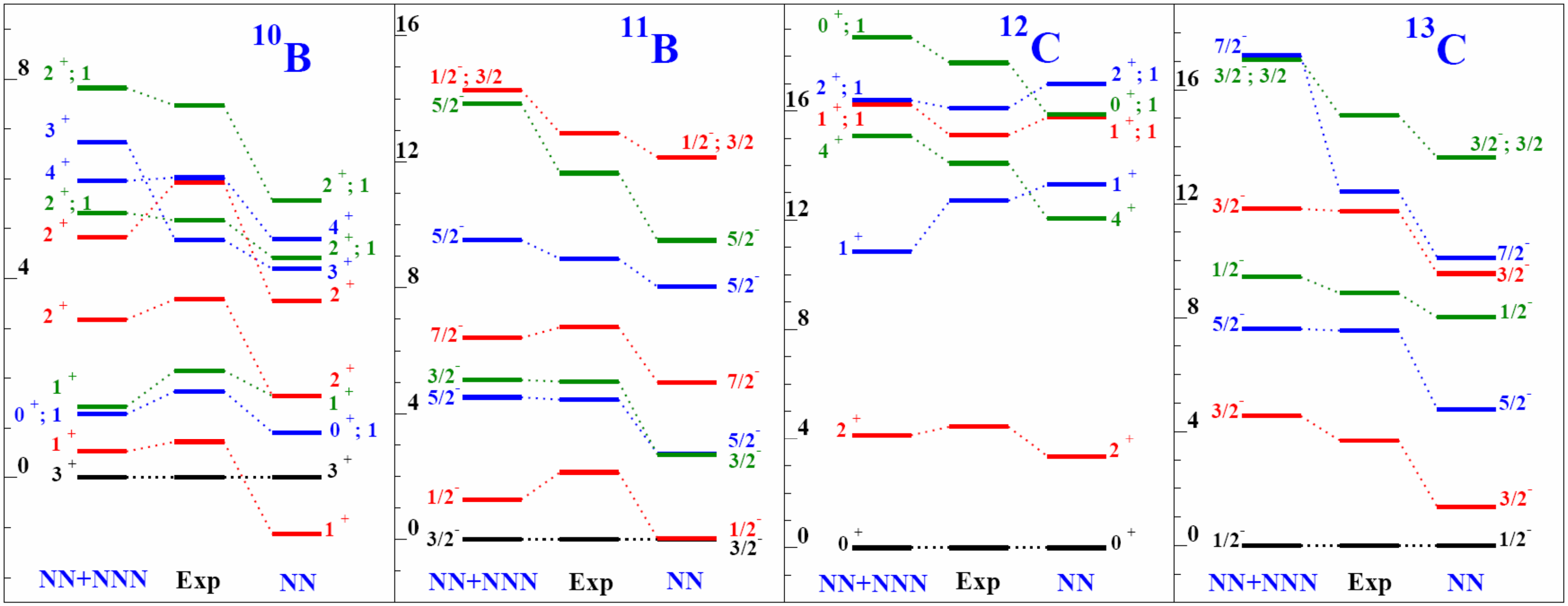
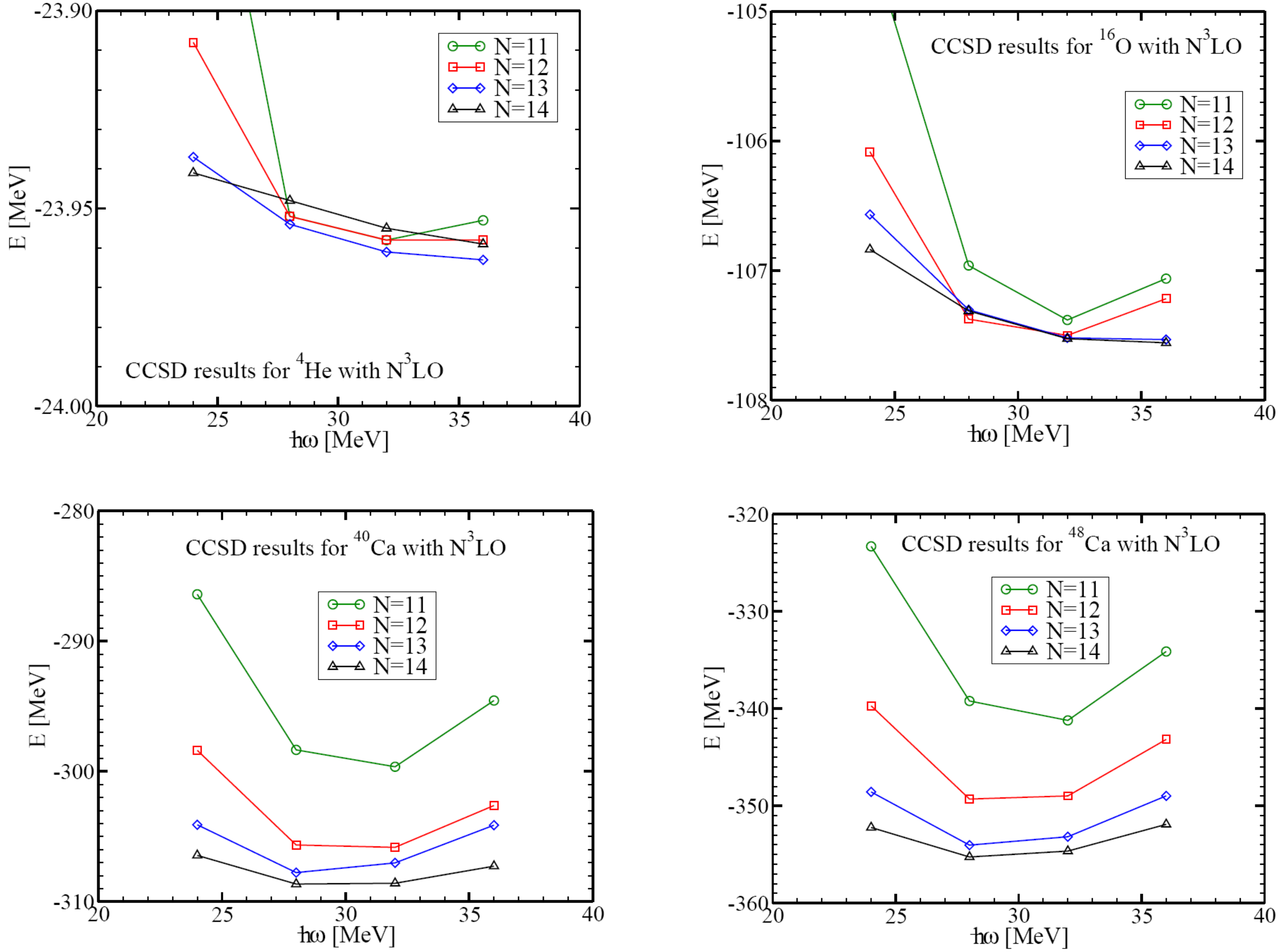


Figure 5. States dominated by p-shell configurations for ^{10}B, ^{11}B, ^{12}C, and ^{13}C calculated at $N_{\max} = 6$ using $\hbar\Omega = 15$ MeV (14 MeV for ^{10}B). Most of the eigenstates are isospin T=0 or 1/2, the isospin label is explicitly shown only for states with T=1 or 3/2. The excitation energy scales are in MeV.

P. Navratil et al., Phys. Rev. Lett. 99, 042501 (2007), nucl-th/0701038.

Medium-mass nuclei from chiral nucleon-nucleon forces with coupled-cluster method



Intermission on nuclear forces

1. Potential models
    1. pion exchange + short-ranged modeling (art and science)
    2. Still useful since solvable via Green's function Monte Carlo
    3. Three-nucleon forces necessary (and practically sufficient for 2 < A <~13)
2. Potentials from effective field theory
    1. Rooted in QCD (→ symmetries)
    2. Systematic expansion
    3. Three-nucleon forces (and higher rank) naturally appear

3. Potentials from QCD? First steps, but not yet there

Issues

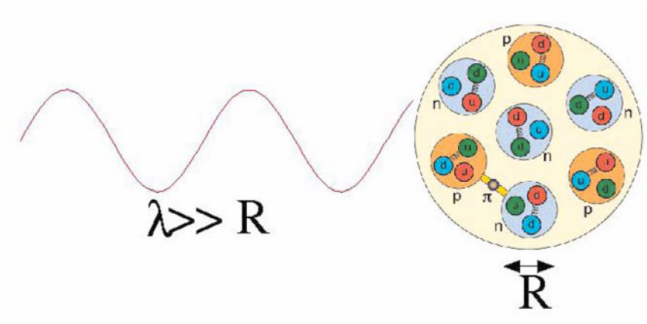
Practical: the nuclear many-body problem (A>12) is still difficult to solve with these potentials

Intellectual: understand cutoff-dependencies and schemes of renormalization

→ Low-momentum potentials and similarity transformations

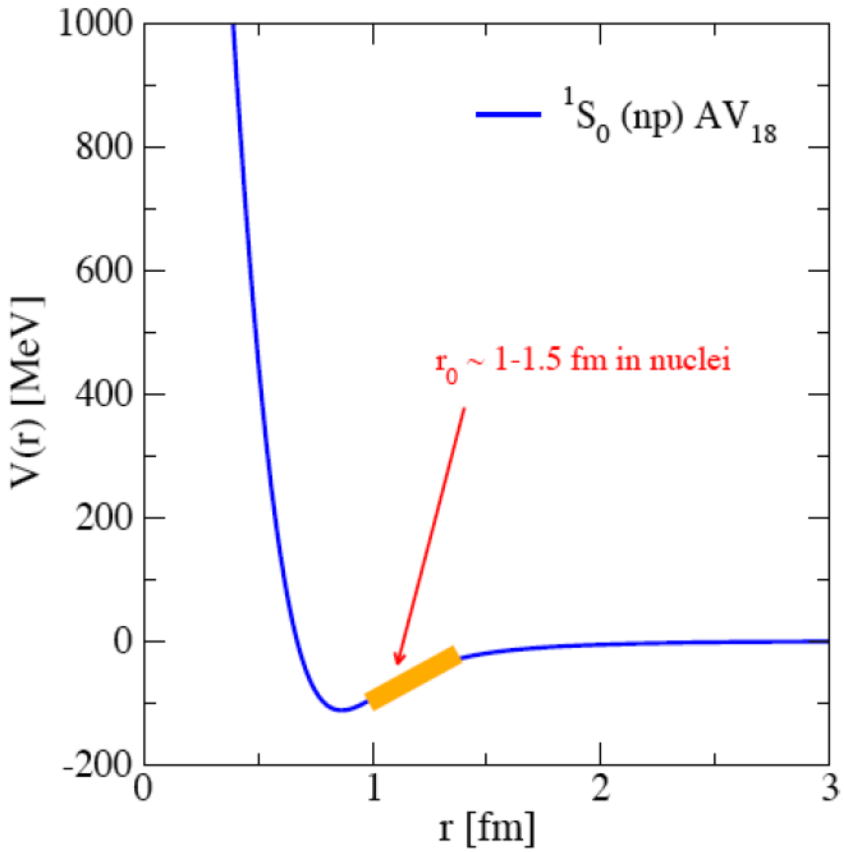
Toward a universal low-momentum potential

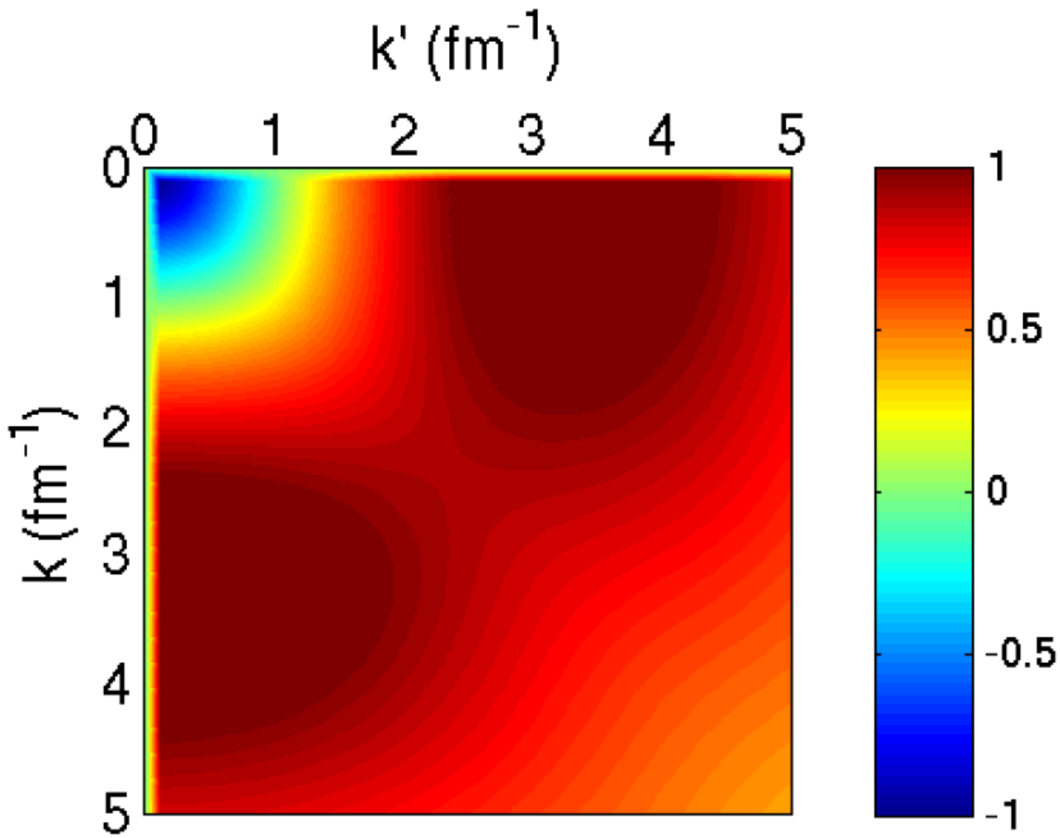
- High-precision potentials contain short-range (high momentum) physics (up to the cutoff-scale of ~ 1 GeV) that is not constrained by phase shifts.
- Is it necessary to know the NN interaction at short distances to understand long wavelength physics?



- Introduce momentum cutoff Λ and integrate out high momentum modes such that low-momentum observables are unchanged (Renormalization group transformation).
- Resulting low-momentum potential $V_{low\text{-}k}$.
- Recall: Fermi momentum at saturation density $k_F = 1.4\ fm^{-1}$.

View on nucleon-nucleon potential





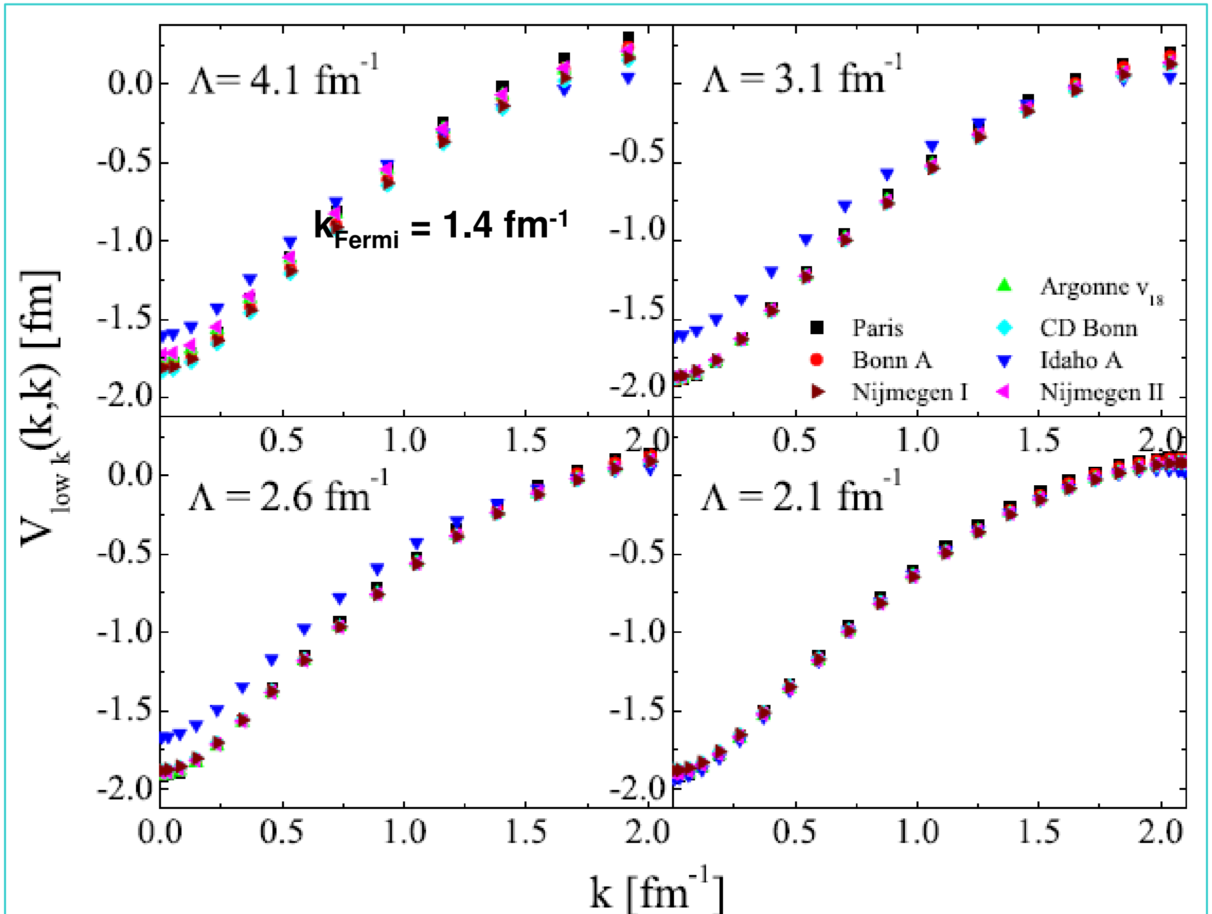
Configuration space ←→ momentum space

Hard core ←→ high-momentum modes

Bogner/Furnstahl (2007)

Low momentum potential V_{low-k}

$$\frac{d}{d\Lambda} V_{\text{low k}}(k', k) = \frac{2}{\pi} \frac{V_{\text{low k}}(k', \Lambda)\, T(\Lambda, k; \Lambda^2)}{1 - (k/\Lambda)^2}$$



Different high-precision potentials

↓

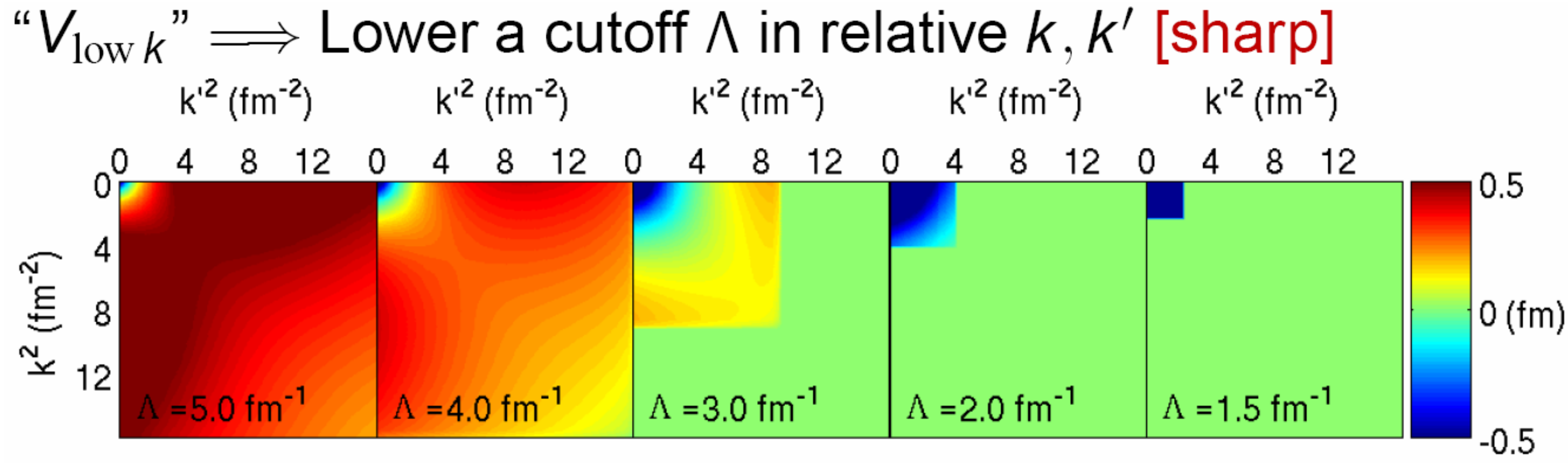
Universal low-momentum potential

Properties of V_{low-k} :

- No hard core
- Nonlocal
- Hartree-Fock already yields bound nuclei.

S. K. Bogner, T. T. S. Kuo, and A. Schwenk, Phys. Rep. 386 (2003) 1

Evolution of $V_{\text{low k}}$ potential with cutoff Λ from Argonne v_{18} potential in $^3S_1$ channel

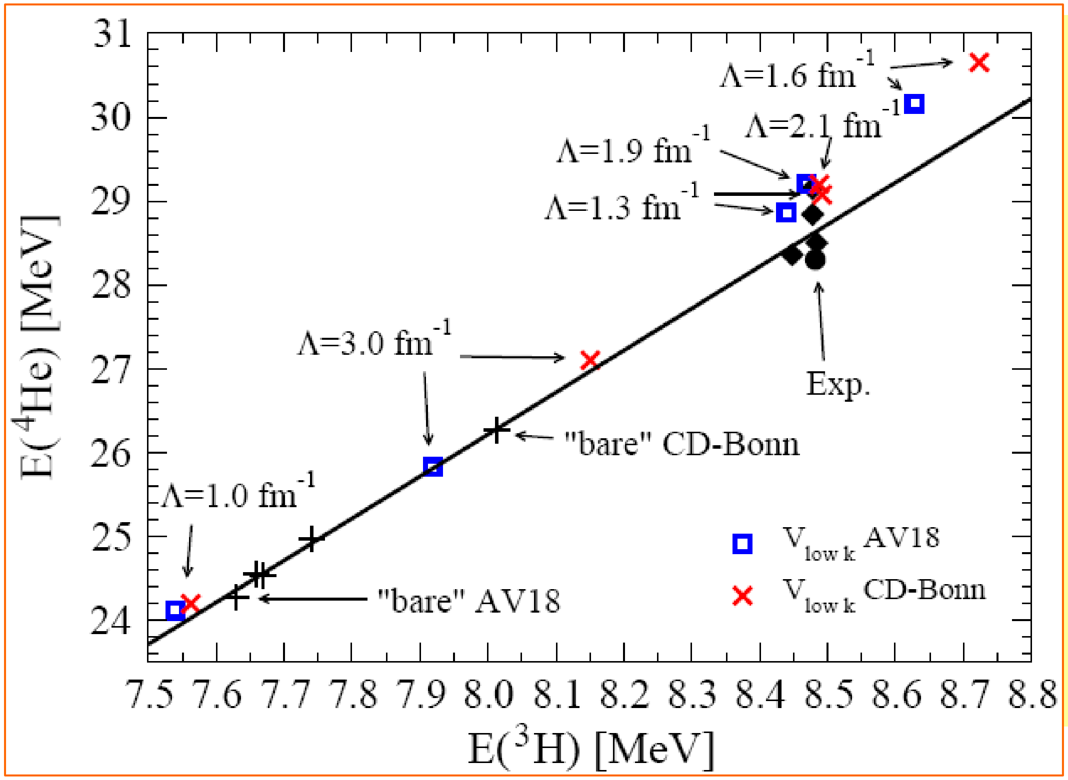


The renormalization group transformation preserves the phase shifts below the cutoff, and sets them to zero above the cutoff.

Momenta above the cutoff are integrated out.

Fig.: Bogner, Furnstahl, …

Light nuclei with V_{low-k} : evolution of binding energy with cutoff



As cutoff Λ is varied, motion along Tjon line.

Addition of Λ-dependent three-nucleon force yields agreement with experiment.

- Three-nucleon force (3NF) necessary.
- There is no “best” potential & 3NF. Choose the most convenient.

A. Nogga, S. K. Bogner, and A. Schwenk, Phys.Rev. C70 (2004) 061002

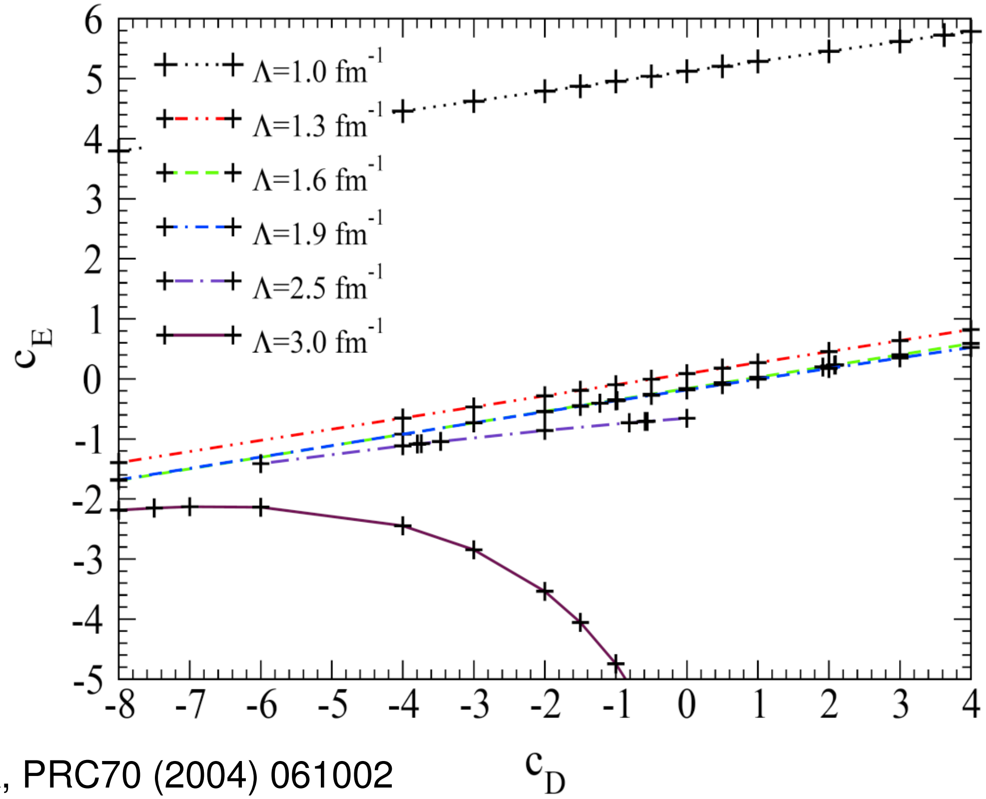
Three-nucleon force within $V_{\text{low k}}$

Purist's point of view

- Must also evolve three-nucleon force to lower cutoffs
- Corresponding RG transformation still in its infancy

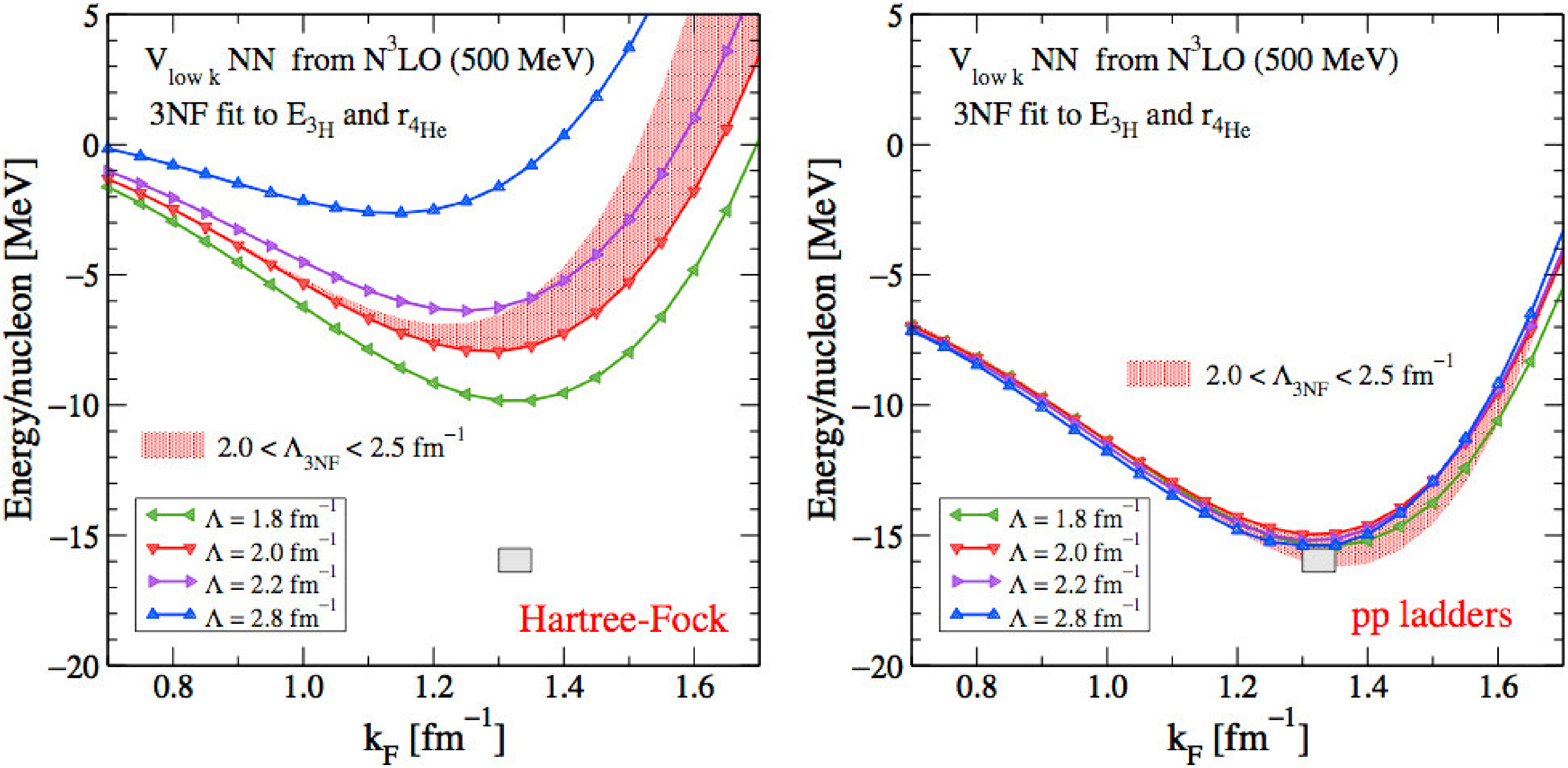
Pragmatic approach

- Low-momentum three-nucleon force must have same structure as predicted by EFT
- Take EFT form for 3NF and determine low energy constants from fit to A=3,4 nuclei



Nogga, Bogner, Schwenk, PRC70 (2004) 061002

Nuclear matter with low-momentum interactions



Bogner, Schwenk, Furnstahl, Nogga (2005 ++)

Similarity renormalization group (SRG) transformation

S. Glazek, K. Wilson, PRD **48** (1993) 5863; **49** (1994) 4214;
F. Wegner, Ann. Phys. **3** (1994) 77

Main idea: decouple low from high momenta via a (unitary) similarity transformation

Unitary transformation

$$\widehat{H}(s) = U(s)\widehat{H}U^\dagger(s) = U(s)\left(\widehat{T} + \widehat{V}\right)U^\dagger(s)$$

Evolution equation

$$\frac{d\widehat{H}(s)}{ds} = \left[\eta(s), \widehat{H}(s)\right] \quad \text{with} \quad \eta(s) \equiv \frac{dU(s)}{ds}U^\dagger(s) = -\eta^\dagger(s)$$

Choice of unitary transformation through

$$\eta(s) = \left[\widehat{T}, \widehat{H}(s)\right]$$

yields scale-dependent potential that becomes more and more diagonal

$$\widehat{H}(s) = \widehat{T} + \widehat{V}(s)$$

Note: Baker-Hausdorff-Campbell implies that SRG of 2-body V generates many-body forces

$$e^{-\eta}\widehat{H}e^{\eta} = \widehat{H} + \left[\widehat{H}, \eta\right] + \frac{1}{2!}\left[\left[\widehat{H}, \eta\right], \eta\right] + \ldots$$

SRG evolution of a chiral potential

(use cutoff $\lambda \equiv s^{-1/4}$ as evolution variable)

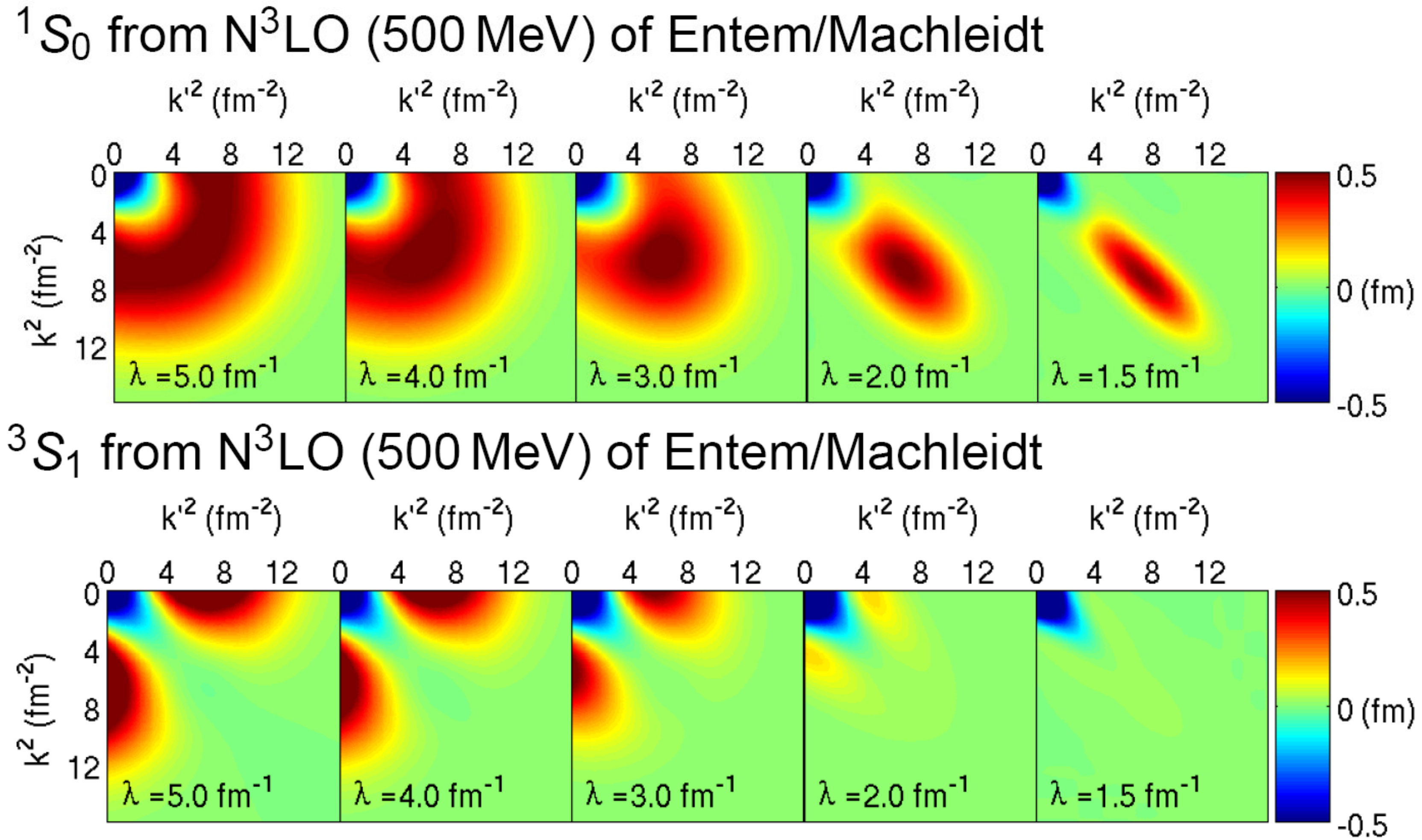
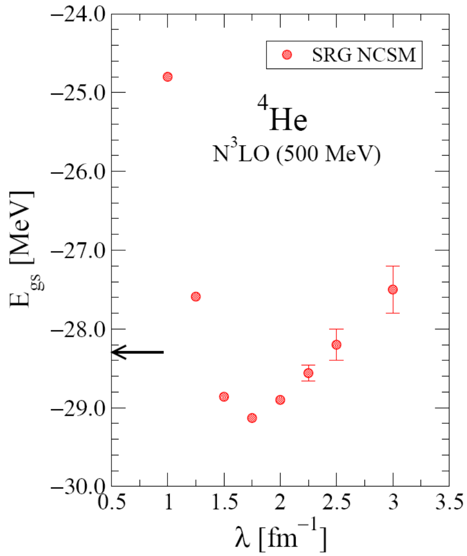
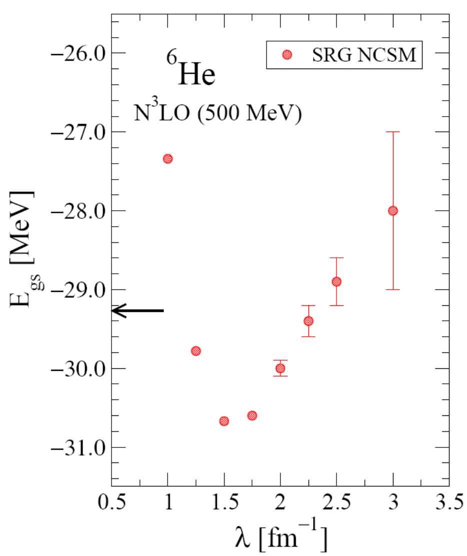


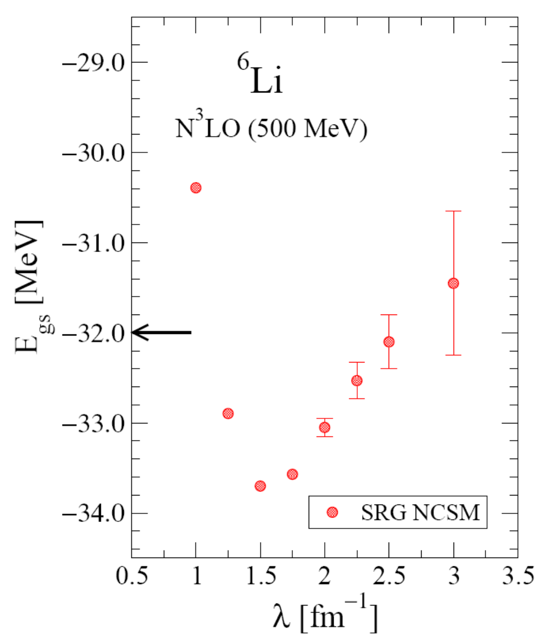
Fig.: Bogner & Furnstahl. See http://www.physics.ohio-state.edu/~ntg/srg

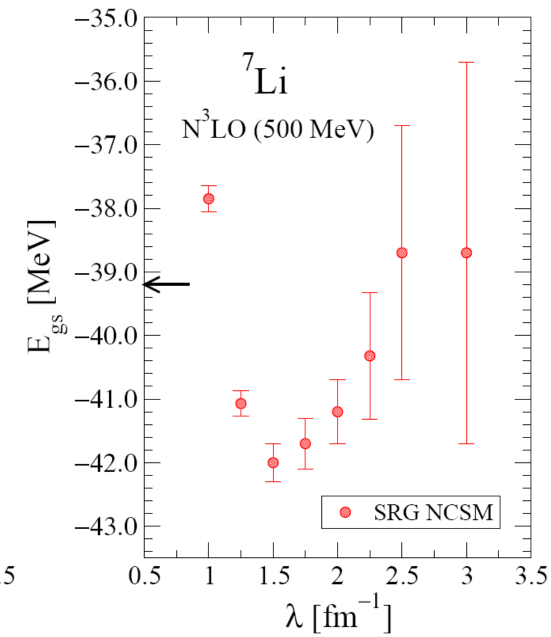
Light nuclei with SRG-transformed chiral nucleon-nucleon interaction





- At small cutoffs the nuclear many-body problem can be solved more easily (evident by reduced extrapolation errors)
- cutoff-dependence demonstrates missing physics (three-nucleon force)
- arrows indicate experimental data





Bogner et al, arxiv:0708.3754

Solving the ab-initio quantum many-body problem

Exact or virtually exact solutions available for:

- A=3: solution of Faddeev equation.
- A=4: solvable via Faddeev-Yakubowski approach.
- Light nuclei (up to A=12 at present): Green's function Monte Carlo (GFMC); virtually exact; limited to certain forms of interactions.

Highly accurate approximate solutions available for:

- Light nuclei (up to A=16 at present): No-core Shell model (NCSM); truncation in model space.
- Light and medium mass region (A=4, 16, 40, 48, 56): Coupled cluster theory; truncation in model space and correlations.

☺ Theorists agree with each other

PHYSICAL REVIEW C, VOLUME 64, 044001

Benchmark test calculation of a four-nucleon bound state

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(Received 20 April 2001; published 27 August 2001)

In the past, several efficient methods have been developed to solve the Schrödinger equation for four-nucleon bound states accurately. These are the Faddeev-Yakubovsky, the coupled-rearrangement-channel Gaussian-basis variational, the stochastic variational, the hyperspherical variational, the Green's function Monte Carlo, the no-core shell model, and the effective interaction hyperspherical harmonic methods. In this article we compare the energy eigenvalue results and some wave function properties using the realistic AV8′ NN interaction. The results of all schemes agree very well showing the high accuracy of our present ability to calculate the four-nucleon bound state.

TABLE I. The expectation values $\langle T \rangle$ and $\langle V \rangle$ of kinetic and potential energies, the binding energies E_b in MeV, and the radius in fm.

Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	−128.33(10)	−25.94(5)	1.485(3)
CRCGV	102.30	−128.20	−25.90	1.482
SVM	102.35	−128.27	−25.92	1.486
HH	102.44	−128.34	−25.90(1)	1.483
GFMC	102.3(1.0)	−128.25(1.0)	−25.93(2)	1.490(5)
NCSM	103.35	−129.45	−25.80(20)	1.485
EIHH	100.8(9)	−126.7(9)	−25.944(10)	1.486

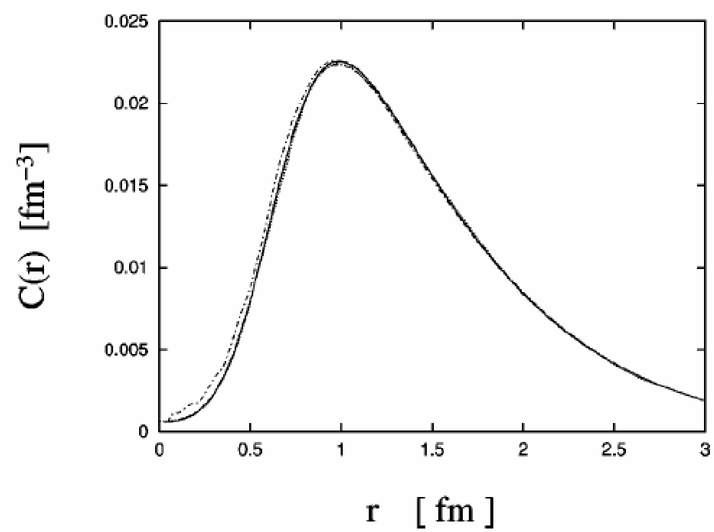
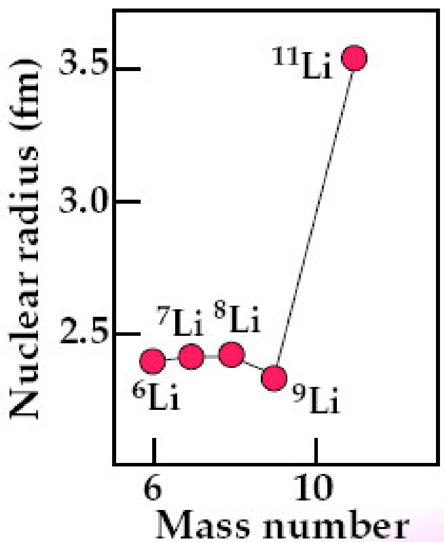


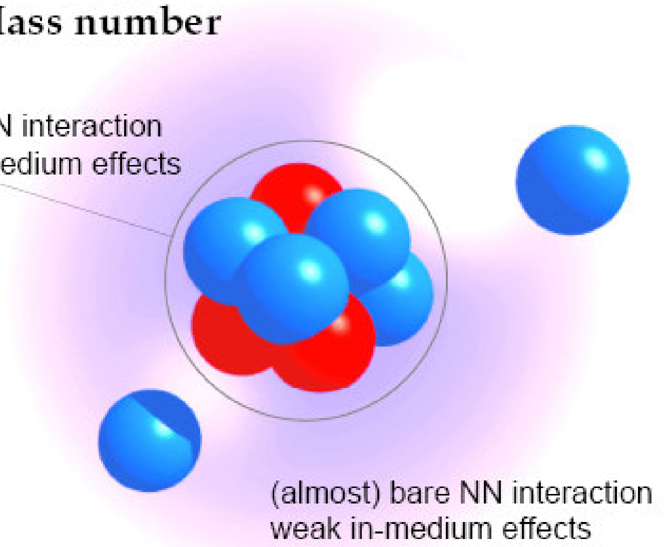
FIG. 1. Correlation functions in the different calculational schemes: EIHH (dashed-dotted curves), FY, CRCGV, SVM, HH, and NCSM (overlapping curves).

Ab-initio calculations of charge radii of Li isotopes



I. Tanihata et al.
Phys. Rev. Lett. 55, 2676 (1985)

Interaction cross section measurements at Bevalac (790 MeV/u)



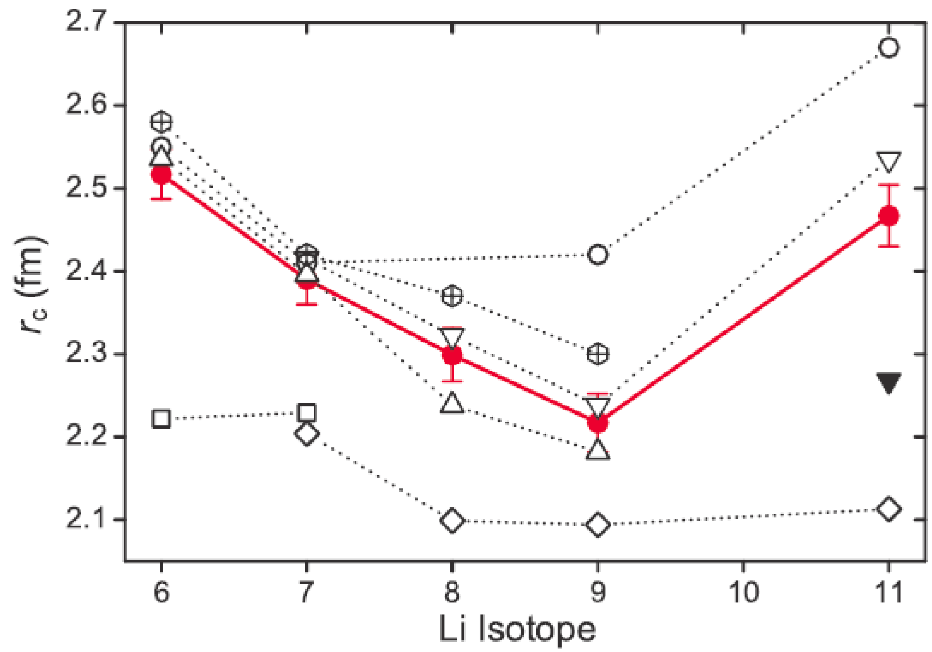
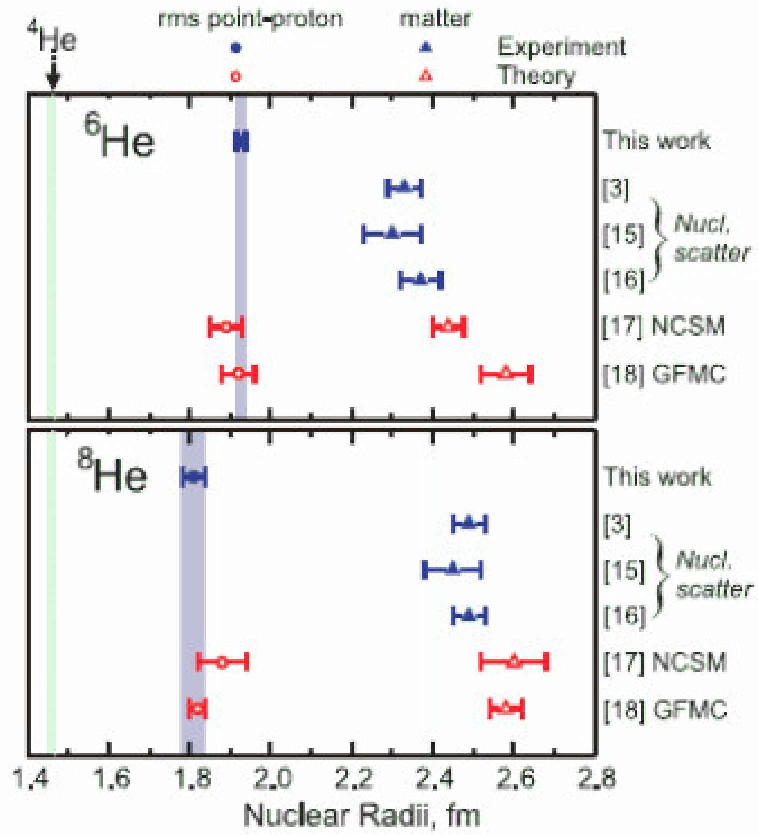


FIG. 2 (color online). Experimental charge radii of lithium isotopes (red, ●) compared with theoretical predictions: △: GFMC calculations [4,22], ▽: SVMC model [27,28] (▼: assuming a frozen ^{9}Li core), ⊕: FMD [26], ○: DCM [19], □ and ◇: *ab initio* NCSM [23,24].

R. Sanchez et al, PRL 96 (2006) 33002.

8He charge radii: theory vs. experiment

8He charge radii measurements using the measured isotope shift with the help of precision atomic theory calculations.



P. Mueller et al., PRL 99, 252501 (2007)

Green's Function Monte Carlo

Idea:

1. Determine accurate approximate wave function via variation of the energy (The high-dimensional integrals are done via Monte Carlo integration).

$$E = \frac{\langle \Psi_{\text{trial}} | \hat{H} | \Psi_{\text{trial}} \rangle}{\langle \Psi_{\text{trial}} | \Psi_{\text{trial}} \rangle}$$

2. Refine wave function and energy via projection with Green's function

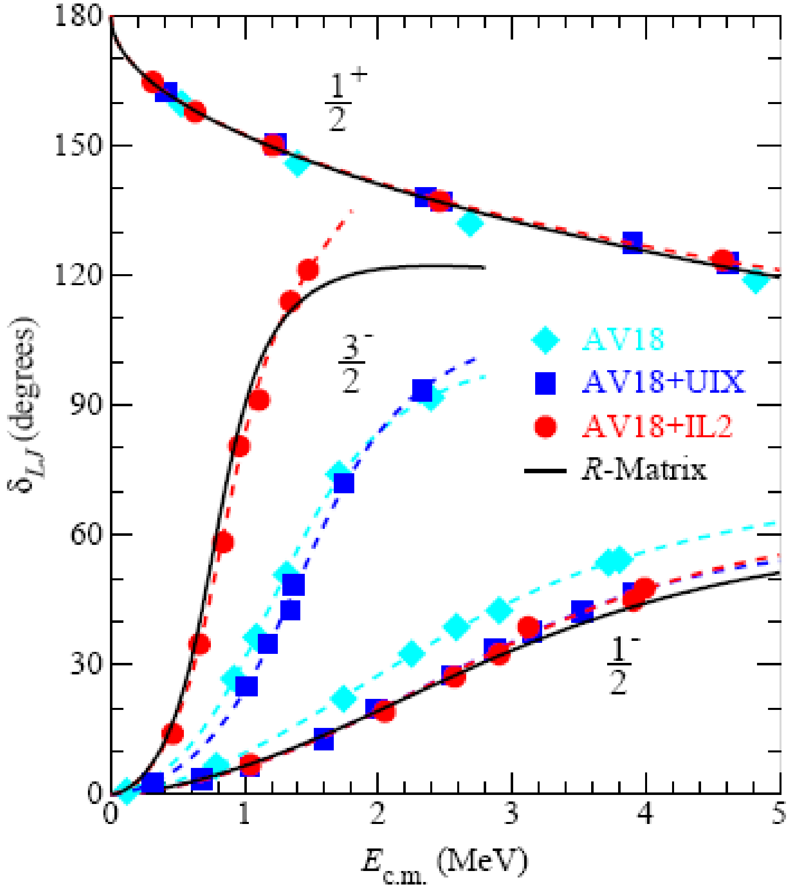
$$|\Psi\rangle = \lim_{\tau \to \infty} e^{-\tau(\hat{H}-E)} |\Psi_{\text{trial}}\rangle$$

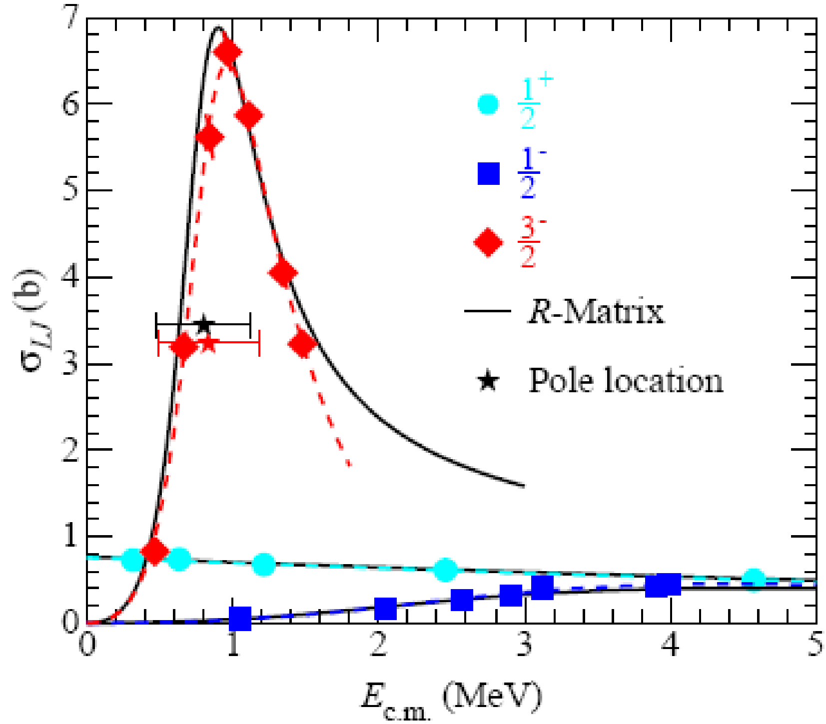
☺ Virtually exact method.

☹ Limited to certain forms of Hamiltonians; computationally expensive method.

GFMC for ^{5}He as n+^{4}He Scattering States

- Black curves: Hale phase shifts from R-matrix analysis up to $J = \frac{9}{2}$ of data
- AV18 with no V_{ijk} underbinds ^{5}He($\frac{3}{2}^-$); overbinds ^{5}He($\frac{1}{2}^-$)
- AV18+IL2 was not fit to ^{5}He, reproduces locations and widths of both P-wave resonances
  - Spin-orbit splitting well reproduced by AV18+IL2





K.M. Nollett, S.C. Pieper, R.B. Wiringa, J. Carlson, G. M. Hale, Phys. Rev. Lett. **99**, 022502 (2007)

Working in a finite model space

NCSM and Coupled-cluster theory solve the Schroedinger equation in a model space with a *finite* (albeit large) number of configurations or basis states.

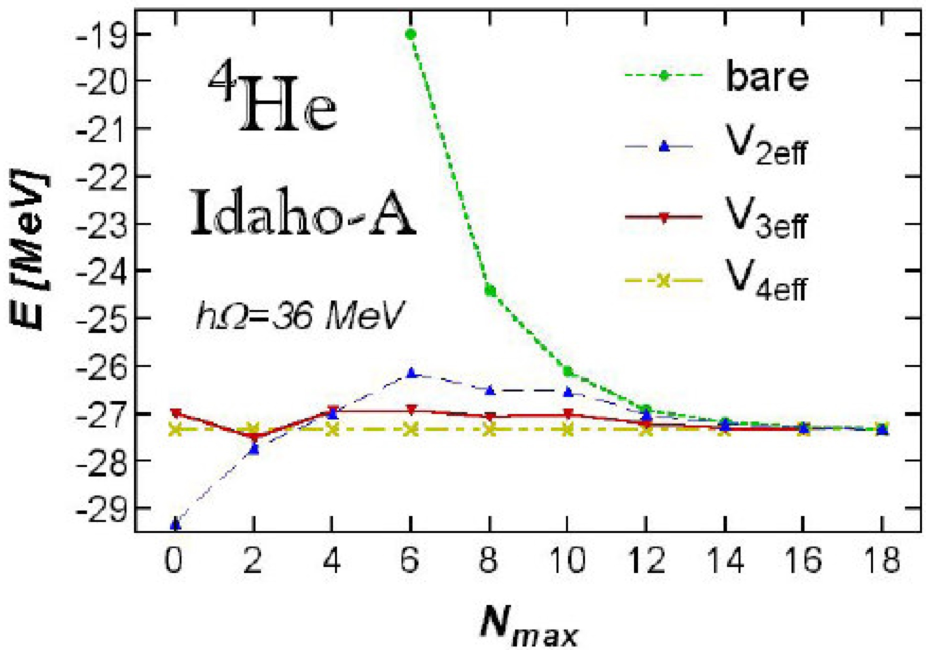
Problem: High-momentum components of high-precision NN interactions require enormously large spaces.

Solution: Get rid of the high-momentum modes via a renormalization procedure. (Vlow-k is an example)

Price tag:

Generation of 3, 4, …, A-body forces unavoidable.

Observables other than the energy also need to be transformed.



E. Ormand
http://www.phy.ornl.gov/npss03/ormand2.ppt

No core shell model

Idea: Solve the A-body problem in a harmonic oscillator basis.

1. Take K single particle orbitals
2. Construct a basis of Slater determinants
3. Express Hamiltonian in this basis
4. Find low-lying states via diagonalization

☺ Get eigenstates and energies

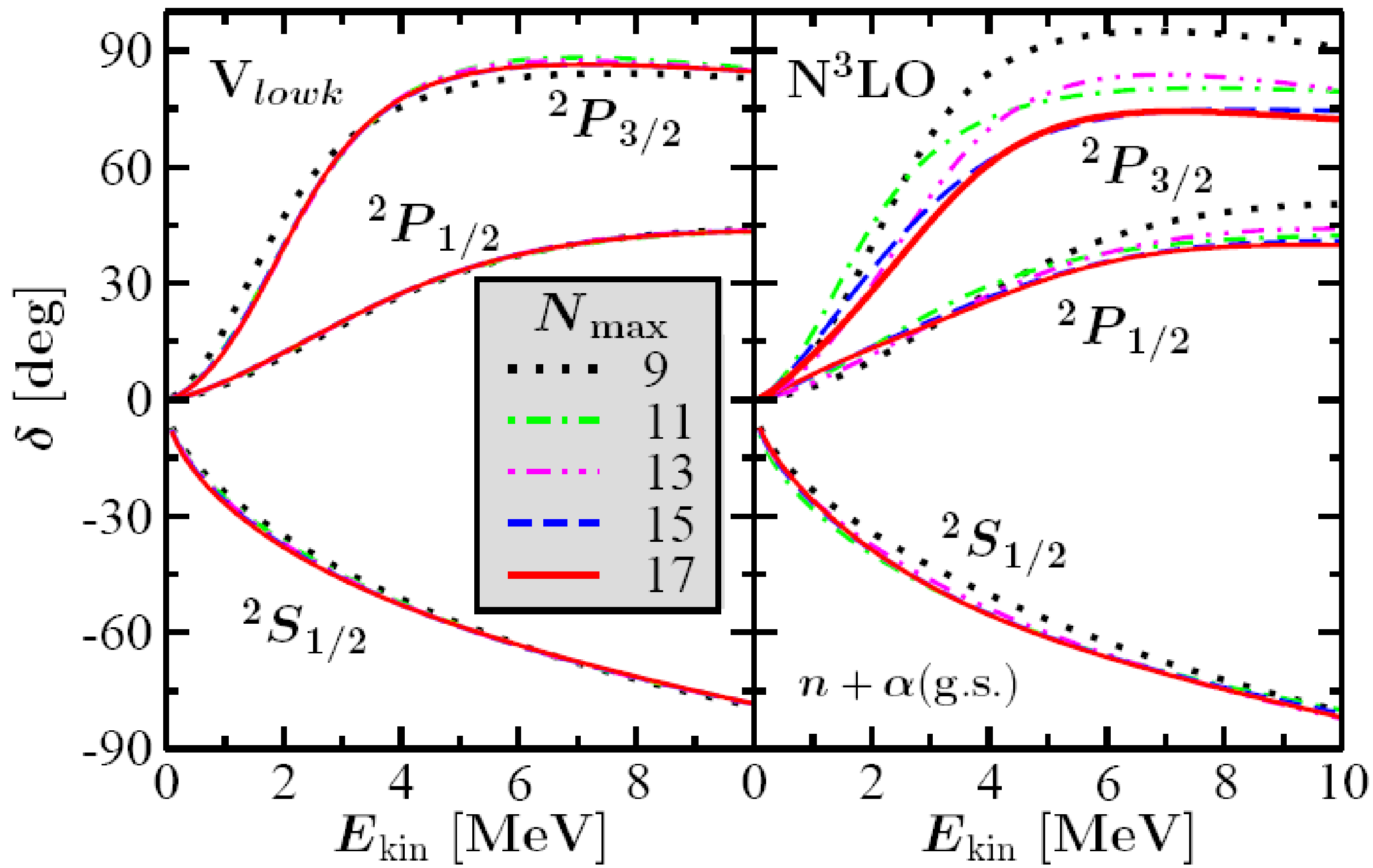
☺ No restrictions regarding Hamiltonian

☹ Number of configurations and resulting matrix very large: There are

$$\binom{K}{A} = \frac{K!}{(K-A)!A!}$$

ways to distribute A nucleons over K single-particle orbitals.

No-core shell model approach to ^{4}He-neutron scattering



S. Quaglione and P. Navratil, arxiv:0804.1560

Motivation

"Exact" ab-initio methods like GFMC and NCSM (in their present forms) are limited to p-shell nuclei.

- GFMC: Spin-isospin configurations ~ 4^A
- NCSM: Configuration space ~ M! / [(M-A)! A!], and M increases with A

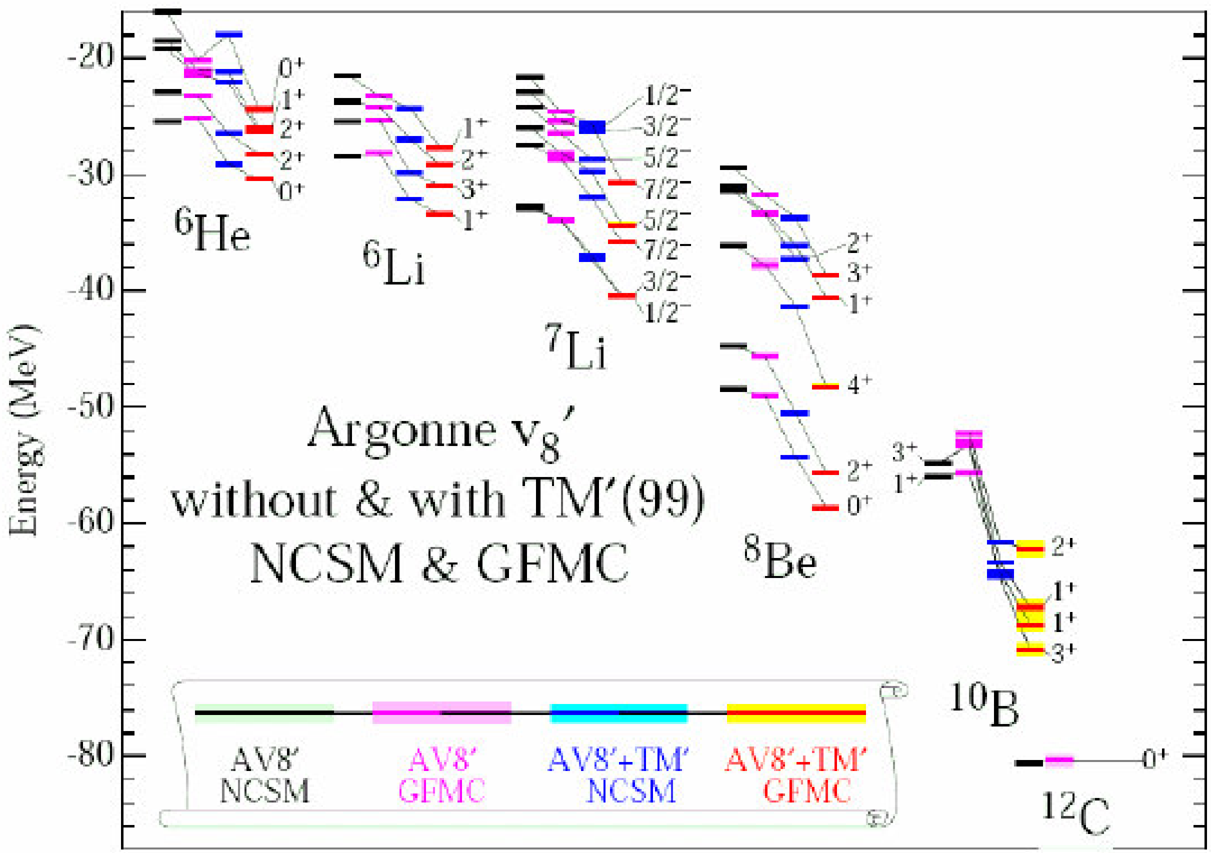
Need theoretical approach that scales more favorable!

(Moore's law is not compatible with above scaling relations)

↓

Coupled-cluster theory (CCSD) scales like $(M-A)^4A^2$

Comparison between NCSM and GFMC



S. Pieper Nucl. Phys. A 751 (2005) 516-532

Figure 6. Comparison of NCSM and GFMC energies for the AV8′ and AV8′+TM′ Hamiltonians.

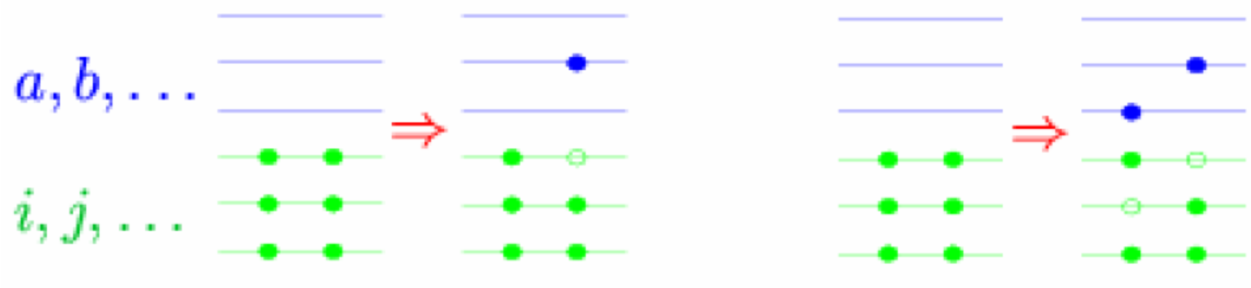
Coupled-cluster theory (CCSD)

Ansatz:

$$|\Psi\rangle = e^{T}|\Phi\rangle$$
$$T = T_1 + T_2 + \ldots$$
$$T_1 = \sum_{ia} t_i^a a_a^\dagger a_i$$
$$T_2 = \sum_{ijab} t_{ij}^{ab} a_a^\dagger a_b^\dagger a_j a_i$$

- ☺ Scales gently (polynomial) with increasing problem size o^2u^4 .
- ☺ Truncation is the only approximation.
- ☺ Size extensive

Correlations are *exponentiated* 1p-1h and 2p-2h excitations. Part of np-nh excitations included!



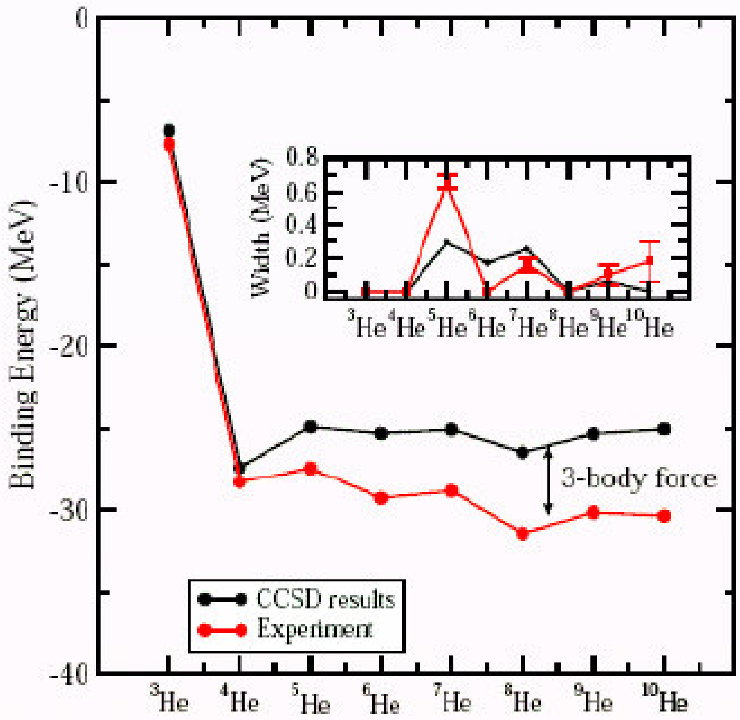
Coupled cluster equations

$$E = \langle\Phi|\overline{H}|\Phi\rangle$$
$$0 = \langle\Phi_i^a|\overline{H}|\Phi\rangle$$
$$0 = \langle\Phi_{ij}^{ab}|\overline{H}|\Phi\rangle$$

Alternative view: CCSD generates similarity transformed Hamiltonian with no 1p-1h and no 2p-2h excitations.

$$\overline{H} \equiv e^{-T}He^{T} = \left(He^{T}\right)_c = \left(H + HT_1 + HT_2 + \frac{1}{2}HT_1^2 + \ldots\right)_c$$

Coupled-Cluster Approach to Weakly bound and unbound nuclear states



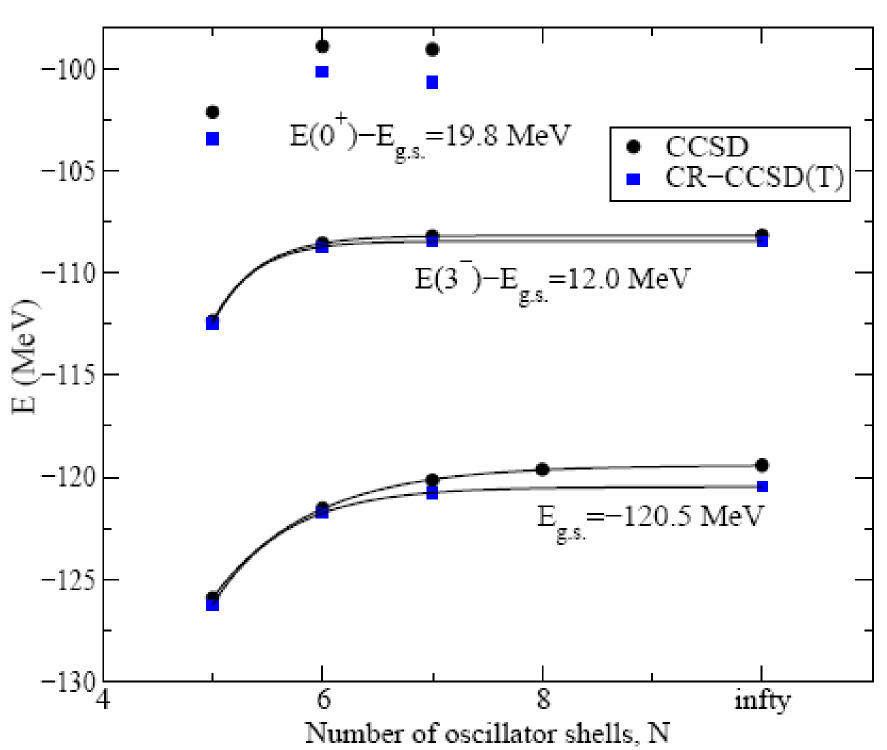
- $V_{\mathrm{low}-k}$ from N3LO with $\Lambda = 1.9\mathrm{fm}^{-1}$.
- G. Hagen et al., Phys. Lett. B 656, 169 (2007). arXiv:nucl-th/0610072.

- First *ab-initio* calculation of decay widths !
- CCM unique method for dripline nuclei.
- ~ 1000 active orbitals
- Underbinding hints at missing 3NF

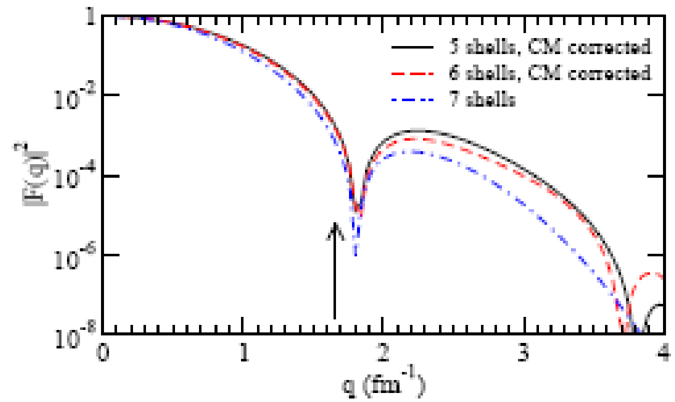
Coupled-cluster calculation for ^{16}O

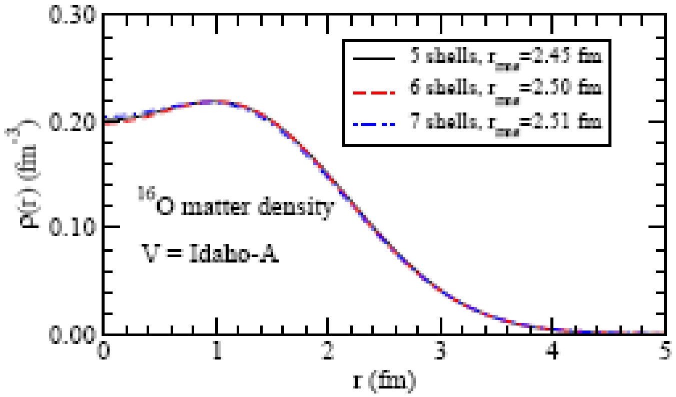
Interaction: Idaho-A based G-matrix

Model space: Up to 8 oscillator shells



M. Wloch et al, Phys. Rev. Lett. 94, 212501 (2005).





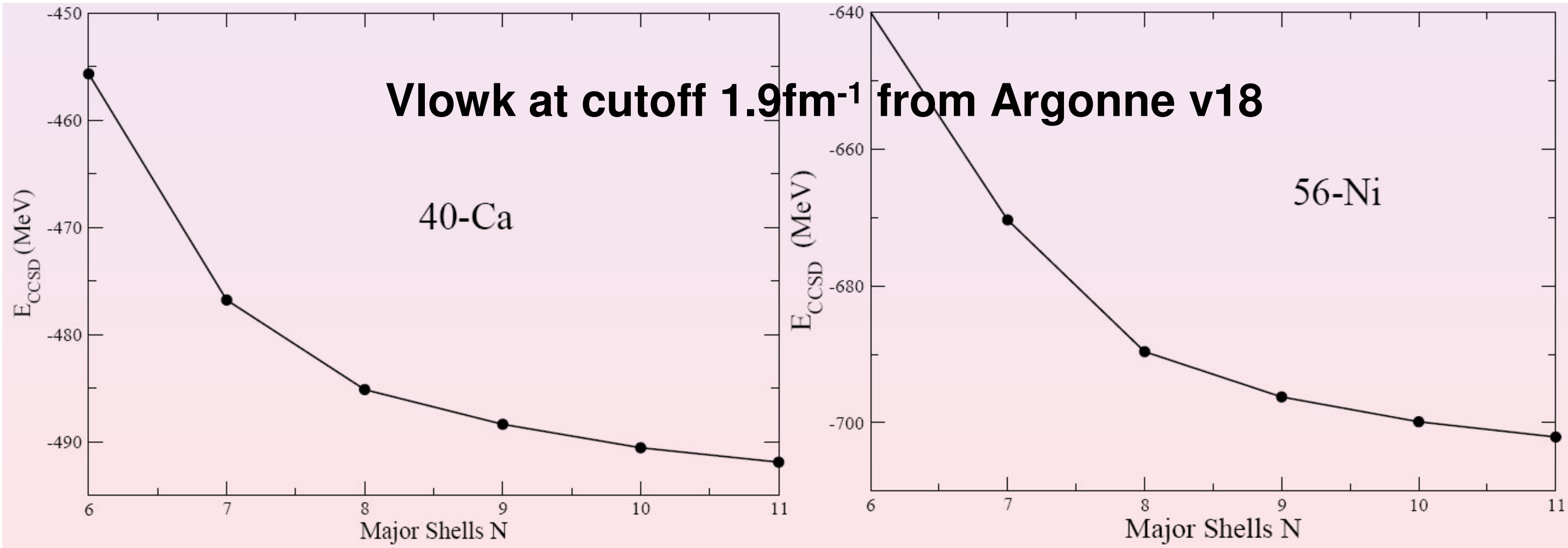
Results converged w.r.t size of model space

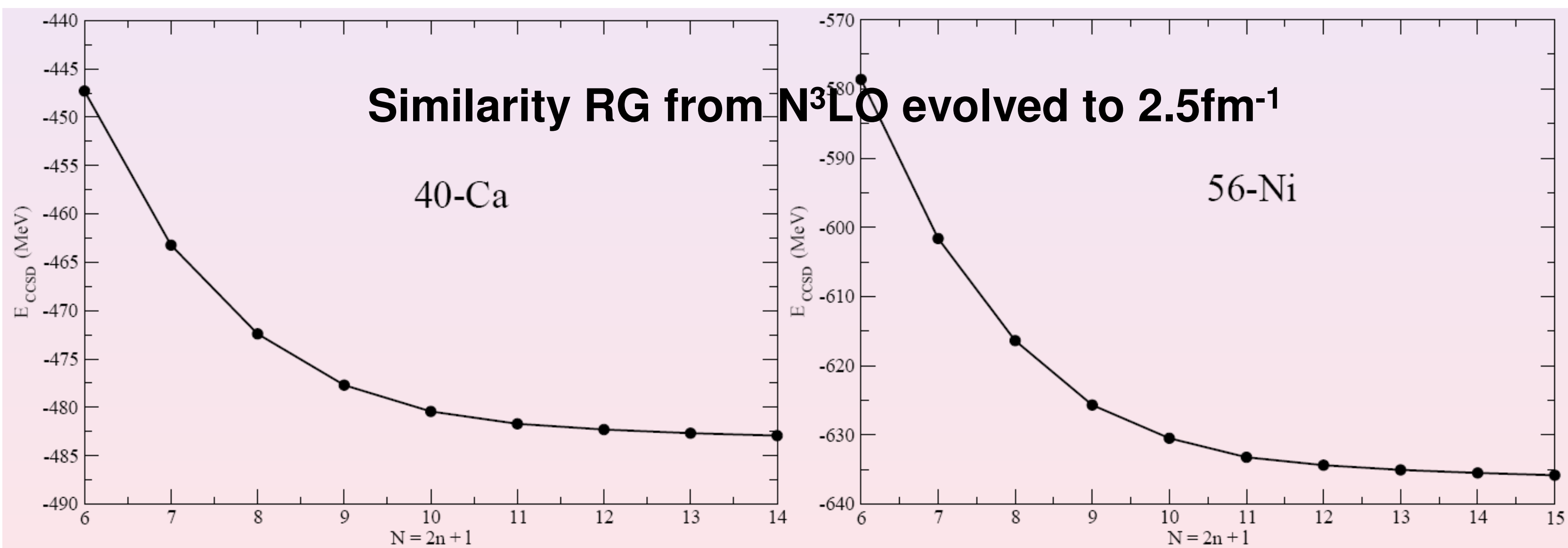
Excited 3⁻ state: 1p-1h, about 6MeV to high

Some deficiencies in form factor.

Three-nucleon force missing.

^{40}Ca and ^{56}Ni with soft nucleon-nucleon interactions





Summary

- High precision NN interactions now available and understood
  - No “best” potential. Choose one that is most convenient
  - Three nucleon forces natural consequence
  - Interplay between three-body forces and high-momentum modes
- Systematic construction of effective interaction via EFT possible
- Several methods that solve the quantum many-body problem
  - Methods agree with each other in results on light systems.
  - Methods differ in accuracy and expense.
  - Agreement with experiment impressive.
  - Reliable predictions can be made.
- Future:
  - Explore three-nucleon forces
  - Heavier nuclei