A manuscript of lectures on

Effective Field Theories in Few-Nucleon Systems

by

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Warning: While considerable effort has been invested to ensure the accuracy of the Physics presented, this script bears witness to my limited understanding of the subject. I am most grateful to every reader who can point out typos, errors, omissions or mis-conceptions.

Very few of what is written here is my original idea. A list of references can be found at the end – please consult it.

Some References to H. W. Grießhammer: Effective Field Theories in Few-Nucleon Systems

This is an incomplete, biased list of references I found useful when preparing the lectures, mostly of lecture notes and review articles. Please consult them as exhaustive bibliographic and historic resources for the original publications. I do usually not mention original articles. Ordering by date of publication only. No guarantees as to completeness, usefulness or relevance. Your mileage may vary.

Introductions to EFT and Its Applications to Nuclear Physics

[1] D. B. Kaplan, *Five lectures on effective field theory,* arXiv:nucl-th/0510023. Very pedagogic, updated version of [10] which has more emphasis on Chiral Perturbation Theory, EFT($\not\pi$) and χEFT, including exercises – a template for my presentation.

[2] D. R. Phillips, *Building light nuclei from neutrons, protons, and pions,* Czech. J. Phys. **52**, B49 (2002) [arXiv:nucl-th/0203040]. A very nice and pedagogic review which grew out of the 2001 Praha Summer School, with very instructive exercises which focus on EFT($\not\pi$)– a template for my presentation.

[3] P. F. Bedaque and U. van Kolck, *Effective field theory for few-nucleon systems,* Ann. Rev. Nucl. Part. Sci. **52**, 339 (2002) [arXiv:nucl-th/0203055]. Focus on Nuclear EFT($\not\pi$) and χEFT– a template for my presentation.

[4] U. van Kolck, L. J. Abu-Raddad and D. M. Cardamone, *Introduction to effective field theories in QCD,* arXiv:nucl-th/0205058. Written in close collaboration between students and lecturer; focuses on χEFT and does not use many formulae.

[5] B. R. Holstein, *Effective interactions are effective interactions,* arXiv:hep-ph/0010033. Pedagogic introduction to EFTs from its beginnings to χEFT.

[6] S. R. Beane, P. F. Bedaque, W. C. Haxton, D. R. Phillips and M. J. Savage, *From hadrons to nuclei: Crossing the border,* arXiv:nucl-th/0008064. Exhaustive review of the status of the field in 2000.

[7] U. van Kolck, *Effective field theory of nuclear forces,* Prog. Part. Nucl. Phys. **43**, 337 (1999) [arXiv:nucl-th/9902015].

[8] G. P. Lepage, *How to renormalize the Schroedinger equation,* arXiv:nucl-th/9706029. Very influential and pedagogic account of the key ideas of Renormalisation and EFTs, building on the uses of the Schrödinger equation in QED, QCD and χEFT.

[9] A. V. Manohar, *Effective field theories,* arXiv:hep-ph/9606222. Pedagogic account for high-energy physicists, focusing on EFTs in QCD.

[10] D. B. Kaplan, *Effective field theories,* arXiv:nucl-th/9506035. Renormalisation-group aspects of EFTs, including exercises.

[11] H. Georgi, *Effective field theory,* Ann. Rev. Nucl. Part. Sci. **43**, 209 (1993). A top-cited classic which puts concepts before formulae.

[12] G. P. Lepage, *What is Renormalization?,* arXiv:hep-ph/0506330. Originally published in 1989, this talk on the EFT philosophy proved so influential that the author set it on the arXiv in 2005.

[13] A. Manohar and H. Georgi, *Chiral Quarks And The Nonrelativistic Quark Model,* Nucl. Phys. B **234**, 189 (1984). First mention of "Naïve Dimensional Analysis".

[14] S. Weinberg, *Phenomenological Lagrangians,* Physica A **96**, 327 (1979). The paper which started it all. . .

Some Trends in Nuclear Physics: Renormalisation Group Perspectives of EFT($\not\pi$)

[15] H. W. Hammer, D. R. Phillips and L. Platter, *Pion-mass dependence of three-nucleon observables,* Eur. Phys. J. A **32**, 335 (2007) [arXiv:0704.3726 [nucl-th]]. Strikingly simple account why it's very useful.

[16] H. W. Griesshammer, *Naive Dimensional Analysis for Three-Body Forces Without Pions,* Nucl. Phys. A **760**, 110 (2005) [arXiv:nucl-th/0502039]. Not so interesting by itself, but exhaustively cites the relevant literature.

Some Trends in Nuclear Physics: Renormalisation Group Perspectives of χEFT

[17] M. C. Birse, *Deconstructing triplet nucleon-nucleon scattering,* arXiv:0706.0984 [nucl-th].

[18] M. C. Birse, *Power counting with one-pion exchange,* Phys. Rev. C **74**, 014003 (2006) [arXiv:nucl-th/0507077].

[19] A. Nogga, R. G. E. Timmermans and U. van Kolck, *Renormalization of One-Pion Exchange and Power Counting,* Phys. Rev. C **72**, 054006 (2005) [arXiv:nucl-th/0506005].

[20] S. R. Beane, P. F. Bedaque, M. J. Savage and U. van Kolck, *Towards a perturbative theory of nuclear forces,* Nucl. Phys. A **700**, 377 (2002) [arXiv:nucl-th/0104030].

Some Further, Phenomenologically Oriented Recent Developments in Nuclear Physics

[21] V. Bernard, *Chiral Perturbation Theory and Baryon Properties,* arXiv:0706.0312 [hep-ph]. The most up-to-date introduction into Chiral Perturbation Theory and its extension to the one-baryon sector.

[22] R. Machleidt, *Nuclear forces from chiral effective field theory,* arXiv:0704.0807 [nucl-th]. A sometimes provocative, hands-on account of χEFT.

[23] M. J. Ramsey-Musolf and S. A. Page, *Hadronic parity violation: A new view through the looking glass*, Ann. Rev. Nucl. Part. Sci. **56** (2006) 1 [arXiv:hep-ph/0601127]. Review on χEFT in hadronic, parity-violating processes.

[24] E. Epelbaum, *Few-nucleon forces and systems in chiral effective field theory,* Prog. Part. Nucl. Phys. **57**, 654 (2006) [arXiv:nucl-th/0509032]. Another hands-on interpretation and summary of χEFT.

[25] S. Scherer and M. R. Schindler, *A chiral perturbation theory primer,* arXiv:hep-ph/0505265. Exhaustive and pedagogic lectures on Chiral Perturbation Theory.

Some Recent Developments in Atomic Physics

[26] E. Braaten and H. W. Hammer, *Universality in Few-body Systems with Large Scattering Length,* Phys. Rept. **428**, 259 (2006) [arXiv:cond-mat/0410417]. Much on more conventional models, but EFT is fully embedded.

Exercises to Lecture I: Concept of EFT and First Examples

Number of stars indicates level of difficulty, but also of reward.

1. Rayleigh-Scattering and Dimensional Analysis: [*] Convince yourself that writing the simplest interaction responsible for scattering a photon from a very small, neutral object as in the notes via

$$H_{\text{int}} = R_0^3 \left[c_E\ \vec{E}^2 + c_M\ \vec{B}^2\right]$$

leads in the natural system of units $\hbar = c = 1$ indeed to dimension-less interaction strengthes c_E, c_M. Repeat the exercise when the particle is represented by a very massive, neutral scalar field Φ which couples to the photon via

$$\mathcal{L}_{\text{int}} = R_0^3 \left[c_E\ \vec{E}^2 + c_M\ \vec{B}^2\right] \Phi^2 \quad .$$

Hint: The action $S = \int \mathrm{d}^4x\ \mathcal{L}$ has mass-dimension zero.

2. Threshold Expansion and Dimensional Regularisation: [**] Let's have some fun with the fact that integrals without internal scales disappear in dimensional regularisation, i.e. $\int \mathrm{d}^dq\ q^\alpha = 0$ for all dimensions d and exponents α. The integral corresponding to a one-dimensional loop

$$I(a,b) := \int \mathrm{d}q\ \frac{1}{q^2 - a^2 + \mathrm{i}\epsilon}\ \frac{1}{q^2 - b^2 + \mathrm{i}\epsilon} = \frac{\mathrm{i}\pi}{ab(a+b)}$$

is usually solved by contour integration. Assume now $v^2 := \frac{a^2}{b^2} < 1$. By saddle-point approximation, the dominating contributions come from the regions where $|q|$ is close to a or b:

$$I(a,b) \approx \left[\int\limits_{|q|\sim a} + \int\limits_{|q|\sim b}\right] \mathrm{d}q\ \frac{1}{q^2 - a^2 + \mathrm{i}\epsilon}\ \frac{1}{q^2 - b^2 + \mathrm{i}\epsilon} \quad .$$

In the first integral, $q \sim a$ is small against b, so that we can perform a Taylor expansion *to all orders* in $\frac{q}{b} \sim v < 1$. If $q^2 \gtrsim b^2$, the expansion breaks down, so that the approximated integrals can *not* be solved by contour integration. In general, the (arbitrary) borders of the integration régimes (the "cutoffs") lead to power-divergences. If one treats however

$$\frac{-1}{b^2}\sum_{n=0}^{\infty}\int\limits_{|q|\sim a} \mathrm{d}q\ \frac{1}{q^2 - a^2 + \mathrm{i}\epsilon}\ \frac{q^{2n}}{b^{2n}} \to \frac{-1}{b^2}\sum_{n=0}^{\infty}\int \mathrm{d}^dq\ \frac{1}{q^2 - a^2 + \mathrm{i}\epsilon}\ \frac{q^{2n}}{b^{2n}}$$

as a d-dimensional integral over all q with $d \to 1$ only at the end of the calculation, the contribution one obtains in the contour integration from the pole at $|q| = a$ emerges as a power series in $v = \frac{a}{b}$.

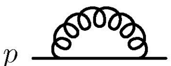
Convince yourself of this: First, extend the integration régime of the approximation around $q \sim a$ to the whole d-dimensional space. Then, calculate the integral order by order in the expansion, still treating $\frac{q^2}{b^2} \sim v^2$ as formally small. Make extensive use of the identities

$$q^{2n} = \sum_{m=0}^{n} \binom{n}{m} a^{2m}(q^2 - a^2)^{n-m} \quad \text{and} \quad \frac{q^2}{q^2 - a^2} = 1 + \frac{a^2}{q^2 - a^2}$$

The same can be done for the integration around b, $\frac{a}{q} \sim \frac{1}{v} > 1$. This is a pedagogical example of a very useful general formalism developed by Beneke and Smirnov: Nucl. Phys. **B522**, (1998) 321-344, taken from Grießhammer: Phys. Rev. **D58**, (1998) 094027 [Sorry about my self-centredness.].

3. SELF-ENERGY FROM "FULL THEORY" OR EFT: [***] We discussed in the lecture one diagram of a simple, relativistic two-scale theory with Lagrangean

$$\mathcal{L} = \bar{\psi}[\mathrm{i}\partial\!\!\!/ - m]\psi + g\,\bar{\psi}\psi\Phi + \frac{1}{2}\left[(\partial^\mu\Phi)^2 - M_\Phi\,\Phi^2\right] \quad .$$

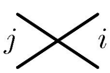
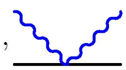
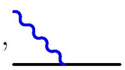
p 

Consider now the lowest-order fermionic self-energy diagram at low energies $p \ll M_\Phi$ both in the "full theory" and in the low-energy EFT (see above). Compare as in the lecture the "full answer" with the one you obtain with a "saddle-point approximation" of the integrand. Is the fermion still massless in the EFT? Like in the lecture, you can limit your efforts to a qualitative discussion and do not have to do all the integrals – although they are not hard. If you do them, you should use dimensional regularisation and your experience from problem 2.

Exercises to Lecture II: EFT($\pi\!\!\!/$)

Lagrangean and some Feynman rules of EFT($\pi\!\!\!/$) for the $^3\mathrm{S}_1$ (deuteron) channel of NN scattering to LO: (E: N kinetic energy, $\vec{k}$: N momentum)

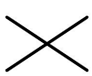
$$\mathcal{L} = N^\dagger\big[\mathrm{i}\partial_0 + \frac{\vec{D}^2}{2M}\big]N - C_0(N^T P^i N)^\dagger(N^T P^i N) \tag{1}$$

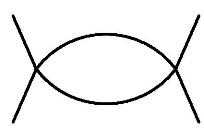
prop. $\dfrac{\mathrm{i}}{E - \frac{\vec{k}^2}{2M} + \mathrm{i}\epsilon}$, j  i : $-\mathrm{i}\,C_0\,P^i\,P^{j\dagger}$,  : $-\dfrac{\mathrm{i}\,e^2}{M}\,Q\,\vec{\epsilon}\cdot\vec{\epsilon}'^{*}$,  : $-\dfrac{\mathrm{i}\,e}{2M}\,Q\,(\vec{k}_{\mathrm{in}} + \vec{k}_{\mathrm{out}})\cdot\vec{\epsilon}$

with $N = \binom{p}{n}$ the nucleon doublet , $P^i = \frac{1}{\sqrt{8}}\,\tau_2\,\sigma_2\sigma^i$ the projector onto the $^3S_1$ channel, τ/σ the Pauli matrices for iso-spin/spin, $\vec{D} = \vec{\partial} + \mathrm{i}eQ\vec{A}$ the minimally substituted derivative, $Q = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ the nucleon charge matrix, $2\,\mathrm{tr}[P^{i\dagger}P^j] = \delta^{ij}$, $4\,\mathrm{tr}[P^{i\dagger}QP^j] = \delta^{ij}$. Photons have energy ω and polarisations $\vec{\epsilon}, \vec{\epsilon}'$. If you are deterred by spin and iso-spin, eliminate P^i from the Lagrangean and treat N as a complex scalar field, representing a particle with charge $Q = e$. You will arrive at the same formulae.

1. NN SCATTERING FOR THE $^3\mathrm{S}_1$ (DEUTERON) CHANNEL AT LO IN EFT($\pi\!\!\!/$):

a) [**] Verify the result in the lecture for the tree and bubble graphs in the centre-of-mass frame:

 : $\mathrm{i}\mathcal{A}_{\mathrm{tree}} = -\mathrm{i}\,C_0$,

$(\frac{E}{2} = \frac{\vec{k}^2}{2M}, \vec{k})$

$(\frac{E}{2} = \frac{\vec{k}^2}{2M}, -\vec{k})$

: $\mathrm{i}\mathcal{A}_{1\,\mathrm{loop}} = -\mathrm{i}\,C_0\,I_0(k)\,C_0$, (2)

where after performing the energy integration one is left with an integration in D space-dimensions,

$$I_0(k) = \left(\frac{\mu}{2}\right)^{3-D}\int\frac{\mathrm{d}^D q}{(2\pi)^D}\,\frac{M}{k^2 - \vec{q}^2 + \mathrm{i}\epsilon} \quad , \; D = 3 \text{ the "real" world.} \tag{3}$$

b) Calculate the divergent integral $I_0(k)$ using a cut-off [*], or using dimensional regularisation [***]. In the latter case, subtract a divergence in $D = 2$ [sic!] dimensions to get $I_0(k) = -M/(4\pi)\,(\mu + \mathrm{i}k)$. Why is it prudent to keep the μ dependent term, which is absent in the usually employed MS scheme?

c) [*] Construct now the LO scattering amplitude by summing all bubbles. Determine (in PDS) the parameter $C_0 = \frac{4\pi}{M}\frac{1}{\gamma - \mu}$ by demanding that the amplitude has a pole at the deuteron binding energy.

d) [*] Convince yourself that the NN scattering amplitude at LO is (in PDS) given in terms of observables only, and all cut-off dependence drops out.

e) [*] Calculate the LO phase shift in PDS and cutoff-regularisation. How accurate do you estimate your result to be, and at which momenta do you expect it to fail? Plot it as function of the cutoff at $k = 0$; 50; 100 MeV. Compare to "real life".

2. ESTIMATING THEORY-UNCERTAINTIES BY VARYING INPUT PARAMETERS: As discussed in the lecture, one way to check the accuracy of an EFT prediction is to determine the parameters from different data sets. These data must of course be taken in the régime of validity of the EFT. We exemplify this method by NN-scattering in the ${}^3\mathrm{S}_1$-wave at NLO.

   a) [***] At NLO, a new interaction appears, as discussed in the lecture:

$$\frac{C_2}{8}\left[(N^T P^i N)^\dagger \left(N^T P^i (\overleftarrow{\partial} - \overrightarrow{\partial})^2 N\right) + \text{H.c.}\right]$$

   Show that the amplitude is to NLO in terms of the LO-parameter C_0 and the NLO-interaction C_2:

$$\mathcal{A}_{NN}(k) = -\frac{C_0}{1+\frac{C_0 M}{4\pi}(\mu + \mathrm{i}k)}\left[1 + \frac{C_2\, k^2}{1+\frac{C_0 M}{4\pi}(\mu+\mathrm{i}k)}\right] \quad .$$

   b) We now discuss 3 scenarios to determine the parameters:

   (i) [*] Fit to the scattering length and effective range, i.e. the coefficient of the k^2-term of $k\cot\delta(k)$, expanded around $k=0$: $a = 5.4$ fm, $r_0 = 1.7$ fm. (H.A. Bethe, Phys. Rev. 76 (1949), 38)

   (ii) [*] Fit to position of the pole of $\mathcal{A}$ (i.e. the deuteron binding energy $B_d = 2.225$MeV) and its effective range, i.e. now the coefficient of the k^2-term of $k\cot\delta(k)$, expanded around $k = \mathrm{i}\gamma$: $\gamma = \sqrt{MB_d}$, $\rho_0 = 1.7$ fm. (H.A. Bethe, Phys. Rev. 76 (1949), 38)

   (iii) [**] Fit to position and residue of the pole of $\mathcal{A}$, i.e. to a Laurent expansion about the pole:

$$\mathcal{A} = -\frac{4\pi}{M}\,\frac{Z}{\gamma + \mathrm{i}k} + \text{ terms which are finite for } k \to \mathrm{i}\gamma$$

   One (you?) can show by explicit Fourier transformation that the residue $Z = 1.6$ is related to the asymptotic normalisation of the S-wave component of the (radial) deuteron wave-function:

$$u(r\to\infty) = \sqrt{2\gamma Z}\ \exp -\gamma r$$

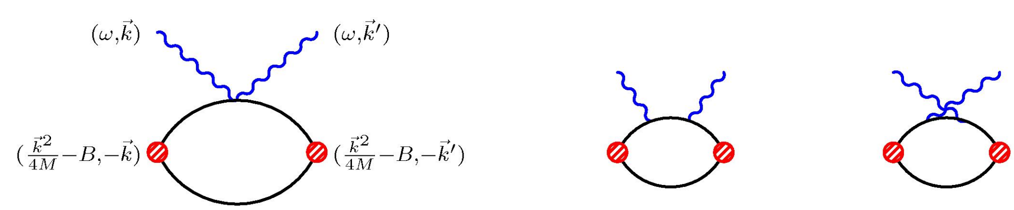
   Obviously, one can only fit γ at LO. Show that the residue is then $Z = 1$. This is called "Z-parameterisation". (D.R. Phillips, G. Rupak and M.J. Savage, Phys. Lett. B 473 (2000), 209 [arXiv: nucl-th/9908054])

   c) [**] Compare now the NN-scattering phase shifts in the three schemes at LO, and to the Nijmegen phase-shift analysis. Then the same for NLO. Note that the three results are now closer to each other. Compare in each case also to your estimate of higher-order corrections from power-counting. Which scenario would you pick for scattering processes? Is your choice a matter of taste?

   d) [*] Compare pole position and residue of each scheme, at LO and NLO. Compare in each case also to your estimate of higher-order corrections from power-counting. Which scenario is a prudent choice for high-accuracy calculations of deuteron properties?

3. DEUTERON COMPTON SCATTERING IN THE CM FRAME:

   a) [*] Power-count the LO ("seagull") diagram and the "ants", and estimate their relative strengths at $\omega = 1$ MeV $\sim \frac{\gamma^2}{M}$ and $\omega = 30$ MeV $\sim \gamma$. It is prudent to explicitly keep ω in the propagators.



   b) [**] With θ the angle between incoming and outgoing photon, show that the seagull amplitude is

$$\mathcal{A}_{\text{seagull}} = \frac{2\sqrt{2}\,e^2\gamma}{M\,\omega\sqrt{1-\cos\theta}}\,\vec{\epsilon}\cdot\vec{\epsilon}'^{*}\ \arctan\frac{\omega\sqrt{1-\cos\theta}}{2\sqrt{2}\gamma} \quad . \tag{4}$$

Don't forget to multiply by the wave function renormalisation at LO, $Z = -\frac{8\pi\gamma}{M^2}$.

c) [*] Confirm from the result of b) your power counting for the seagull in a).

4. Three Nucleons in EFT($\not\pi$): (or, if you fear spin-iso-spin, three bosons)

a) [**] Show that the nd-scattering amplitude (i.e. all the "pinball diagrams") in an S-wave is of order Q^{-2} in the power-counting.

b) [*] The dependence of the three-body force H_0 on the cut-off Λ at LO is derived in P.F. Bedaque, H.-W. Hammer and U. van Kolck, Nucl. Phys. A 646 (1999), 444 [arXiv: nucl-th/9811046]:

$$H_0(\Lambda) = -\frac{\sin[s_0 \ln \Lambda/\Lambda_* - \arctan \frac{1}{s_0}]}{\sin[s_0 \ln \Lambda/\Lambda_* + \arctan \frac{1}{s_0}]} \quad \text{with } s_0 = 1.0062\ldots$$

This particular running is called a renormalisation group limit *cycle* because the strength of the three-body force parameter is periodic in $\ln \Lambda$. Determine the period.

Effective Field Theories in Few-Nucleon Systems

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Lecture I: Effective Field Theories: Concept and First Examples

Lecture II: QCD at Very Low Energies: EFT($\not\pi$)

Lecture III: A Dash into χEFT: Pions and A Few Nucleons

How to root Nuclear Physics in QCD?

How to get reliable predictions & extractions of nucleonic & nuclear properties?

How does that serve our understanding?

Office of Science U.S. DEPARTMENT OF ENERGY

Goals: Explain foundations, spell out "trade secrets".

Allow you to follow seminars on recent developments.

Last lecture: small, biased survey of some recent developments.

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Effective Field Theories in Few-Nucleon Systems

Lecture I: Effective Field Theories: Concept and First Examples

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1. Teaser: What's the Big Deal About Effective Field Theories Anyway?
2. Example: Two Scales, Quantum Effects and Renormalisability
3. The EFT Philosophy

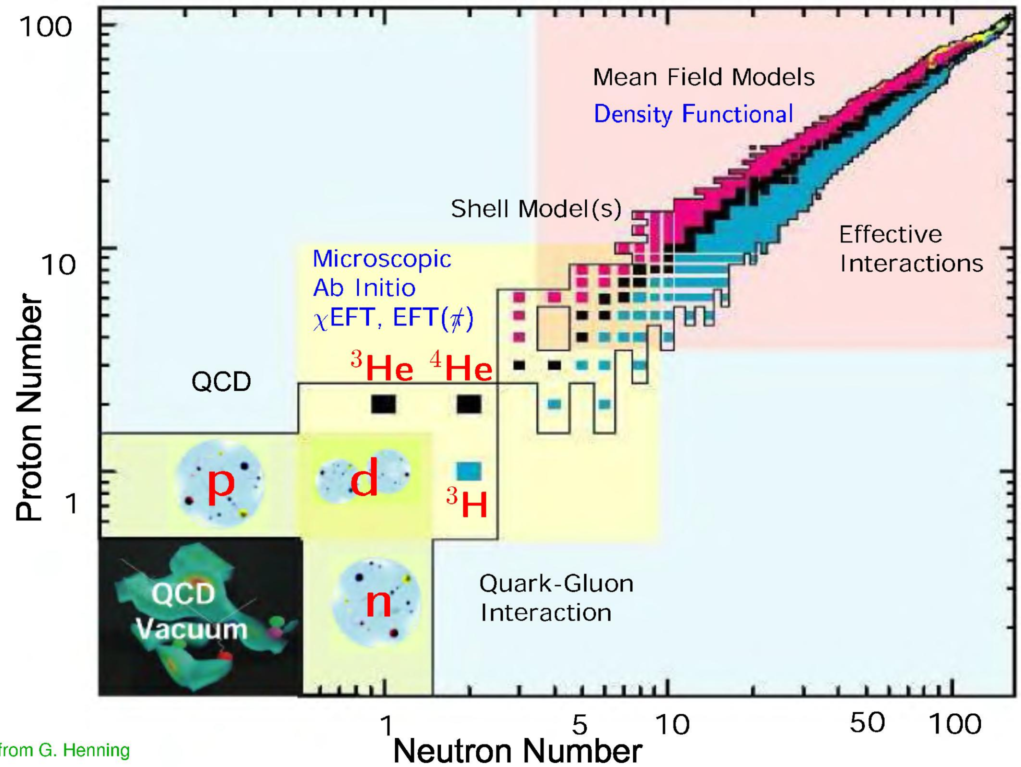
The EFT Philosophy: Separation of scales, symmetries & effective degrees of freedom.

Power-Counting for error-estimates: Naïve dimensional analysis for interactions and loops.

Construction principle: Matching, integrating out, or fitting.

1. Teaser: What's the Big Deal About Effective Field Theories Anyway?

(a) Wanted: Error-Bars for Nuclear Physics!



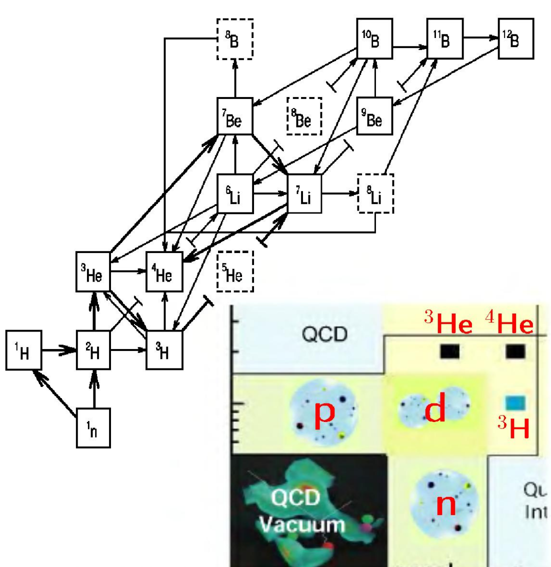
adapted from G. Henning

1. Teaser: What's the Big Deal About Effective Field Theories Anyway?

(a) Wanted: Error-Bars for Nuclear Physics!

Goal: Model-independent, reliable, unified, systematic low-energy description of few-body systems.

Example Nucleo-synthesis 3 minutes after Big Bang: $E_{\text{typ}} = 0.02 - 0.2\ \text{MeV}$ $\Longrightarrow$ difficult experiments.



adapted from G. Henning

Problems:

Bridge from Nuclear Physics to QCD?

System much larger than constituents,

e.g. deuteron $5.5\ \text{fm} \gg 1\ \text{fm}$ nucleon

Rewards:

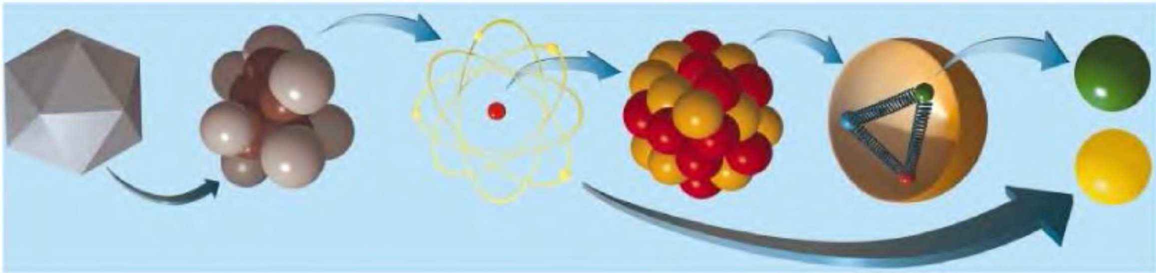
Reliable high-accuracy predictions & extractions of properties of system and its constituents.

New phenomena & concepts, better understanding.

Universal formalism $\Longrightarrow$ Atomic Physics, BEC,...

Effective Field Theories

(b) What You See Is What You Get



To probes with wavelength λ, object of size R appears point-like for $\lambda \gg R$, blurry for $\lambda \gtrsim R$, composed for $\lambda \lesssim R$.

Refined by Wilson, Weinberg 1967:

Effective Field Theories: method for **multiple, separate scales**:

Identify those **degrees of freedom and symmetries** which are **appropriate to resolve the relevant Physics** at the scale of interest.

Turn into **systematic approximation** of real world, allowing for **estimate of theoretical uncertainties involved**.

Lecture I: Concept & First Examples

1. Basic Idea

Fundamental Tenet of Natural Sciences:

Phenomena at low energies / long wavelengths cannot probe details at high energies / short distances:

Slide

Resolution-dependence of answers

→ eg optics, QM: $\Delta x \Delta p \gtrsim \hbar$; reduce problems to "spherical cows"

- Example multipole expansion of charge distribution with size R_0:

$$\Phi(\vec{r}) = \int d^3r' \, \frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}$$

Taylor: $|\vec{r}\,| \gg R_0 \geq |\vec{r}\,'|$

$$= \frac{Q}{r} + \frac{\vec{d}\circ\vec{e}_r}{r^2} + \dots$$

: encode complicated internal charge distribution into simple, intuitively accessible coefficients:

charge Q

dipole moment $\vec{d}$ $\hat{=}$ separation between centres of pos. & neg. charges

etc.

rapidly converging expansion in $\frac{R_0}{r} \ll 1$ ⇒ error-estimate.

- Refined into principle exactly 40 years ago by Wilson, Weinberg 1967:

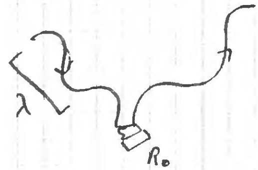
Slide

EFTs are theories for problems with multiple, separate scales: Identify those degrees of freedom and symmetries which are appropriate to resolve the relevant physics at the scale one is interested in. Turn this into a systematic approximation of the real world, allowing for an estimate of the theoret. uncertainties involved.

Concept 2

2. First Example: Rayleigh Scattering, or Why the Sky is Blue

a) Setting



Low-energy scattering of photon
by massive, neutral particle:
electric & magnetic fields induce
separation of pos. & neg. charges over typ. distance set by size R_0
$\Rightarrow$ induced dipole moment.

Manohar hep-ph 2606.222
Kaplan: lectures p. 41
Phillips p. –

b) Building blocks

(α) Relevant degrees of freedom (dofs)

- (point-like) particle (with dipole moment)
- photon A^μ (with wavelength λ)

(β) $\Rightarrow$ Separation of scales

$$Q = \frac{R_0}{\lambda} \simeq \omega R_0 \ll 1 \qquad \text{(natural system of units: } \hbar = c = 1)$$

eg light in atmosphere: $\lambda \simeq 4000 - 7000\,\text{Å} \gg \underbrace{R_0 \simeq 1\,\text{Å}}_{\text{atomic scale}} \gg \underbrace{\frac{1}{30\,\text{GeV}}}_{N_2, O_2 \text{ mass}}$

(γ) Symmetries which constrain the possible interactions

- gauge $\Rightarrow$ $\partial_\mu - i\, Q A_\mu$; $\vec{E}$, $\vec{B}$ and its derivatives
- particle neutral $\Rightarrow$ $Q = 0$: no coupling via A_μ.

c) Constructing the interaction

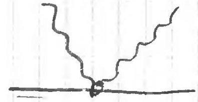
$\Rightarrow$ simplest is $\vec{E}^2$, $\vec{B}^2$, $\cancel{\vec{E}\cdot\vec{B}}$ violates parity conservation of EM.

$$\Rightarrow H_{int} = R_0^3 \left[c_E \vec{E}^2 + c_B \vec{B}^2 \right] + \text{terms with more } \vec{E}, \vec{B}, \partial_\mu$$

$\hookrightarrow$ power chosen such that c_E, c_B are pure numbers:
converts energy densities $\vec{E}^2, \vec{B}^2$ into energies H_{int}

R_0 is the only intrinsic scale available.

$\Rightarrow$ QM scattering amplitude from H_{int}:



Feynman diagram

$$\mathcal{A} \sim (c_E, c_B)\, R_0^3\, \omega^2$$

↳ couplings to $\vec{E}^2, \vec{B}^2$

↳ "scales like":
functional form (& dimensions) correct, pure numbers dropped.

$\Rightarrow$ cross-section $\quad \sigma \propto |\mathcal{A}|^2 \propto c_{E,B}^2\, R_0^6\, \omega^4$

has correct dimension length^2 ✓

: Result from <u>Naïve Dimensional Analysis</u> NDA; prefactors by full calc.

Suffices to explain why the sky is blue:

$$\omega_{blue} \simeq 1.8\, \omega_{red} \implies \sigma_{blue} \approx 10\, \sigma_{red}$$

: blue light much more strongly scattered than red one.

d) How accurate?

<u>Error-estimate</u> again by NDA:

Sources of corrections:

- higher-order interactions, eg $R_0^5\, c_2\, \dot{\vec{E}}^2 \Rightarrow \mathcal{A}_{corr.} \sim c_2 R_0^5 \omega^4$
  suppressed by $(R_0 \omega)^2 \simeq \left(\frac{1}{1000}\right)^2 \ll 1$
- internal structure of particle suppressed by bad resolution
  $\frac{\omega}{\text{typ. excitation energy } \Delta E} \sim \frac{1\,eV}{10\,eV} \ll 1$
- loops / "re-scattering"  $\propto \alpha^2$, while  $\propto \alpha = \frac{1}{137}$
- gravity, nucleus-structure, QCD, strings, ...
  $\frac{\omega,\ \Delta E,\ \frac{1}{R_0}}{\text{a typical, very high scale } \bar{\Lambda}} \ll 1.$

  eg $\bar{\Lambda}_{Nuclear} \sim 1\,GeV$

  $\bar{\Lambda}_{string} \sim 10^{30}\,TeV.$

3. Example: Two Scales, Quantum Effects & Renormalisability

a) Setup

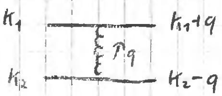
Low-Energy exchange of a heavy boson Φ between massless fermions Ψ

$\hat{=}$ simplified W/Z exchange of GSW model

$$\mathcal{L}_{Full} = g\,\bar{\Psi}\,\Phi\,\Psi - \frac{1}{2} M_\phi^2\,\Phi^2 \qquad , g \ll 1 \Rightarrow \text{perturbation theory ok.}$$

Phillips p. 7
Kaplan: 5 lectures p. 10

b) tree-level $\hat{=}$ classical FT



$$= g^2 \frac{1}{q^2 - M_\phi^2}$$

small momentum-transfer $q \Rightarrow$ Taylor-expansion in

$$\boxed{Q = \frac{\text{(low-E) momentum transfer } q}{\text{(high-E) boson mass } M_\phi} \ll 1}$$

$$\simeq -\frac{g^2}{M_\phi^2}\left[1 + \frac{q^2}{M_\phi^2} + \frac{q^4}{M_\phi^4} + \ldots\right]$$

$$Q^0 \gg Q^2 \gg Q^4 \gg \ldots$$

Φ-propagator "eliminated": fermions cannot resolve Φ-exchange

$\Rightarrow$ mimic at low-E by point-like interactions:

$$\mathcal{L}_{EFT} = -\frac{g^2}{M_\phi^2}(\bar{\Psi}\Psi)^2 - \frac{g^2}{M_\phi^4}(\bar{\Psi}\partial_\mu\Psi)^2 - \ldots$$

: Low Energy Coefficients LEC of interactions derived from $\mathcal{L}_{full}$

$$\Rightarrow \sigma \propto \frac{g^4}{M_\phi^4}\,(k_1+k_2)^2\left[1 + Q^2 + \ldots\right] \text{ at low energies}$$

↳ to get overall dimension fm^2: total cm-energy = Mandelstam-s

If Ψ has a small mass $m_\psi \ll M_\phi$:

$$\sigma(\vec{k}_i \to 0) \propto \frac{g^4}{M_\phi^2}\,\frac{m_\psi^2}{M_\phi^2}$$

$$\overset{!}{=} 4\pi a^2 \text{ with } a = \text{scatt. length}$$

$$\Rightarrow a \sim \frac{1}{M_\phi}\,\frac{m_\psi}{M_\phi} \sim \text{range of interaction mediated by } \Phi.$$

Application: Weak interactions are weak on the nucleon scale

because $M_Z \simeq 90$ GeV $\gg m_e, m_q, M_N$

$$\Rightarrow \sigma_{weak} \simeq 10^{-11} \text{ barn}$$
$$\approx 10^{-(7..8)} \text{ of typ. nuclear xsect (mbarn)}$$

but strong at LEP (CERN): $s \simeq 90\,\text{GeV} = M_Z$

$\Rightarrow$ Q-expansion inapplicable.

: Fermi's theory of weak interactions

ok for nuclear physics, but not for CERN.

: Result by Naïve Dimensional Analysis NDA;

prefactors # (pure numbers) by explicit calculation.

Problem: Couplings of $\mathcal{L}_{EFT}$ have mass-dimension

$$\left[\frac{g^2}{M_\phi^{2+2n}}\right] = -2-2n \,, \; n \in \mathbb{N}_0$$

$\hookrightarrow$ # of derivatives

$\Rightarrow$ tradition has it that such theories are

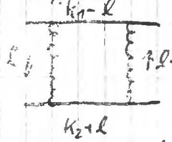
"non-renormalisable"

when quantised.

c) Quantum corrections: loops $\hat{=}$ expansion in $\hbar$

[Here only qualitative arguments – work out details yourself: HW.]

"Typical" (a simple) graph in "full theory" – same principle for all graphs.



$$\propto g^4 \int \frac{d^4 \ell}{(2\pi)^4} \frac{1}{\not{k}_1 - \not{\ell}} \frac{1}{\ell^2 - M_\phi^2} \frac{1}{\not{k}_2 + \not{\ell}} \frac{1}{(\ell+q)^2 - M_\phi^2}$$

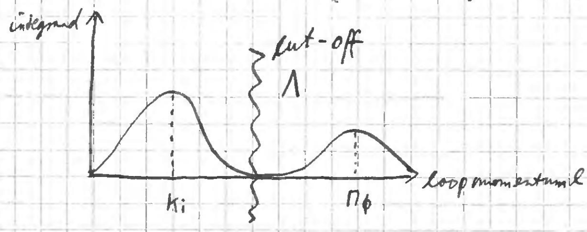
: loop mom. ℓ seems to blur separation between

low scales k_i, q & high scale M_ϕ.

– Cannot do integral exactly, but that is not necessary at low E:

Approximate by expansion about saddle-points of integrand,

i.e. points where propagators diverge $\hat{=}$ particles on-shell



$\Rightarrow$ separate integration region into

Régime I: <u>slow modes</u>/ low-energy : $\ell \sim k_i < \Lambda$: Ψ on-shell

Régime II: <u>fast modes</u>/ high-energy : $\ell \sim M_\phi > \Lambda$: Φ on-shell

Choice of value & form of cutoff Λ (& momentum-routing)

arbitrary (inside window for traditional cutoff; really for dim reg $\to$ HW)

"<u>Λ is regulator / cut-off</u>"

(α) Régime I: slow modes : expand again in powers of Q

$$\to g^4 \int\limits_{\ell<\Lambda} \frac{d^4\ell}{(2\pi)^4}\; \frac{1}{\not k_1 - \not\ell}\; \frac{1}{m_\phi^2}\; \frac{1}{\not k_2 + \not\ell}\; \frac{1}{m_\phi^2}\left[1 + \mathcal{O}(Q^2)\right]$$

Identical to  in EFT : same long-distance / IR physics

leading tree-level vertex $-\frac{g^2}{m_\phi^2}$. but much simpler to calculate.

Estimate cutoff-dependence by superficial degree of divergence (NDA):

$$\Rightarrow \frac{g^4}{m_\phi^4}\Big[\underbrace{\#\,\Lambda^2}_{\text{quad. div.}} + \underbrace{\#(q,k)\ln\Lambda}_{\text{ln. div.}} + \text{finite terms as "}\Lambda\to\infty\text{"}\Big]$$

↳ eg $\ln q$: contain IR physics.

same structure as tree-level EFT vertices, plus additional ones

(β) Régime II: fast modes

$$\to g^4 \int\limits_{\ell>\Lambda} \frac{d^4\ell}{(2\pi)^4}\; \frac{1}{-\not\ell}\left[1 + \frac{\not k_1}{\not\ell} + \dots\right]\frac{1}{\ell^2 - m_\phi^2}\; \frac{1}{\not\ell}\left[1 + \dots\right]\frac{1}{\ell^2 - m_\phi^2}\left[1 + \dots\right]$$

: low-E scales k_i, q only in polynomials in numerator.

$\Rightarrow$ out of integral. Never terms which are non-analytical in k_i^2, q^2.

$$\xrightarrow{\text{NDA}} g^4\left[\#\,\frac{1}{m_\omega^2} + \#\,\frac{k_i^2, q^2}{m_\omega^4} + \dots\right] \quad \text{(and } \Lambda\text{-dependences)}$$

: same structure as tree-level $\mathcal{L}_{EFT}$, no new vertices generated.

$\Rightarrow$ Absorb régime II & divergences of régime I into tree-level LECs:

 $= -\frac{g^2}{m_\phi^2}\left[1 - \#\,\frac{g^2\Lambda^2}{m_\phi^2} + \#\,g^2 + \dots\right]$

 $= -\frac{g^2 q^2}{m_\phi^2}\left[1 - \#\,g^2\ln\Lambda + \#\,g^2 + \dots\right]$

: corrections suppressed by powers of $Q^2 \ll 1$, $g^2 \ll 1$.

$\Rightarrow$ Total amplitude  $\approx$  +  +  + .. Λ-indep. up to $g^4 Q^2$.

↖ exactly Λ-independent.

[This was an exception: Usually, only the total amplitude = sum of all graphs in the "full theory" is Λ-independent.]

– This expansion is of course useful only when corrections indeed small:

- $k_i \ll m_\phi \iff Q \ll 1$: limits regime of validity of EFT to low-E.

$\Rightarrow m_\phi$ represents <u>breakdown-scale $\overline{\Lambda}_{EFT}$</u>

where new, <u>dynamical</u> d.o.f.s enter: Φ propagates!

Not to be confused with <u>arbitrary, unphysical cutoff parameter Λ</u>!

- pure numbers "#" should be "not too large":

Σ of all Q^2-graphs $\ll$ Σ of all Q^0-contributions.

: <u>Naturalness - Assumption</u> : coefficients do not destroy expansion. (cf Taylor expansion)

d) Result: an EFT

Low-Energy EFT which order-by-order in g, Q

– reproduces all aspects of "full theory" in <u>infrared</u>;

– simplifies "full theory" in <u>ultraviolet</u>

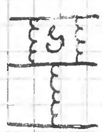
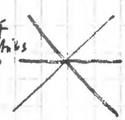
by "<u>integrating out</u>" high-E modes into LECs

: generates <u>any local interaction</u> between the effective d.o.f.'s which is compatible with the <u>symmetries</u> of the "full theory".

<u>Symmetries scale-independent!</u> here: $\Psi \to e^{i\alpha}\Psi$: U(1) global (particle number conservation)

<u>Any interaction</u>: • <u>Reg-Ren scheme</u> dependent LECs (Λ vs dim reg ...)

• <u>Swiss basic law</u>: "All that is not forbidden is compulsory."

eg high-E part of  one of the contributions : $\ell \to m$  $\sim \# g^6 \frac{1}{m_\phi^5} (\bar{\Psi}\Psi)^3$

↳ dimensional analysis!

suppressed against  $\sim \frac{g^2}{m_\phi^2} \frac{1}{q} \frac{g^2}{m_\phi^2}$

↳ fermion prop.

by $g^2 Q \ll 1$.

e) A radical step: EFT from Matching.

$\Rightarrow$ Turn this around: Start with most general low-E $\mathcal{L}$ compatible with symmetries:

$$\mathcal{L}_{EFT} = \bar\Psi[i\partial\!\!\!/ - m]\Psi + \frac{c_1}{\bar\Lambda^2_{EFT}}(\bar\Psi\Psi)^2 + \frac{c_2}{\bar\Lambda^2_{EFT}}\left[\bar\Psi\frac{\partial^\mu}{\bar\Lambda_{EFT}}\Psi\right]^2 + \frac{c_3}{\bar\Lambda_{EFT}}[\bar\Psi\Psi]\left[\bar\Psi\frac{\partial^2}{\bar\Lambda^2_{EFT}}\Psi\right] + \dots$$

$$+ \frac{d_1}{\bar\Lambda^5_{EFT}}(\bar\Psi\Psi)^3 + \frac{d_2}{\bar\Lambda^5_{EFT}}(\bar\Psi\Psi)\left[\bar\Psi\frac{\partial^\mu}{\bar\Lambda_{EFT}}\Psi\right]^2 + \dots + (\bar\Psi\Psi)^n \dots$$

Use NDA ~~of~~ LECs to make high-E scale transparent.

Matching to "full theory" (perturbative in g) gives

- pure numbers (Reg/Ren Scheme dependent): LECs

  $c_1 \approx g^2[1 + \mathcal{O}(g^2, Q^2)](\Lambda)$; c_i, d_i

- breakdown scale $\bar\Lambda_{EFT} = M_\phi$

$\Rightarrow$ Establishes relative importance of terms, and hence

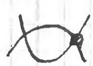
Power-Counting in Q : NDA of LECs (can be surprising, $\to$ later)

$\oplus$ NDA of loops (usually not too difficult)

: Allows us to estimate order at which diagram enters without explicit calculation,

and hence truncation of $\mathcal{L}_{EFT}$ to finite number of terms at each order.

- Example :



$$\sim \frac{g^2}{M_\phi^2}\;\frac{g^2 q^2}{M_\phi^4}\;\frac{q^4}{q\cdot q} \sim \frac{g^4}{M_\phi^2}\,Q^4$$

is N^3LO against

$$\sim \frac{g^2}{M_\phi^2}\,Q^0$$

: Calculations in EFT simpler than in "full theory".

: efficiency as advantage.

: provides systematic error-bar of calculation.

If your calculation does not have an error-bar, you are not a theorist.

f) EFT from fitting

or: "Pragmatically turning your whims into principles"
(Calvin & Hobbes)

– Often, we do not know underlying theory, or cannot match to it.
(QCD: Confinement; QGravity)

$\Rightarrow$ Develop Power-Counting & fit $c_i, d_i, .., \bar{\Lambda}$ to low-E experiments.

– Consistency condition: Renormalisability at each order in Q.

Already seen above: needed at Q^2 to absorb
Λ-dependence of EFT

: LECs serve as counter-terms of BPHZ-regularisation scheme, whose finite pieces are scheme-dependent.

$\Rightarrow$ Wilson's Criterion for order-by-order renormalisability:

Theory needs at each order in $Q \ll 1$ only a finite number of counter-terms = LECs to allow for predictive power.
(finite # of data-points necessary for fitting)

This replaces the narrow definition of "strict renormalisability" to all orders simultaneously,
and allows use of any Reg/Ren scheme, not only cutoff!

$\rightarrow \boxed{HW}$

$\Rightarrow$ Minimum number of parameters at given accuracy, i.e. given order,
with error-estimate from neglect of details
$\hat{=}$ systematic uncertainty of theory

: Space for Improvement

$\oplus$ Serious Physicists quote error-bars.

:

In contradistinction to Mathematics, "exactness" is irrelevant, and even a fake, in Physics:

Even "full theory" only solved perturbatively in g, and inappropriate at high-energies: Theory of Everything!

$\Rightarrow$ Every theory is an EFT which neglects some effects and has to be replaced at high-E.

What we need are

not precise results $\hat{=}$ (numerically) exact solutions of a model of Nature,

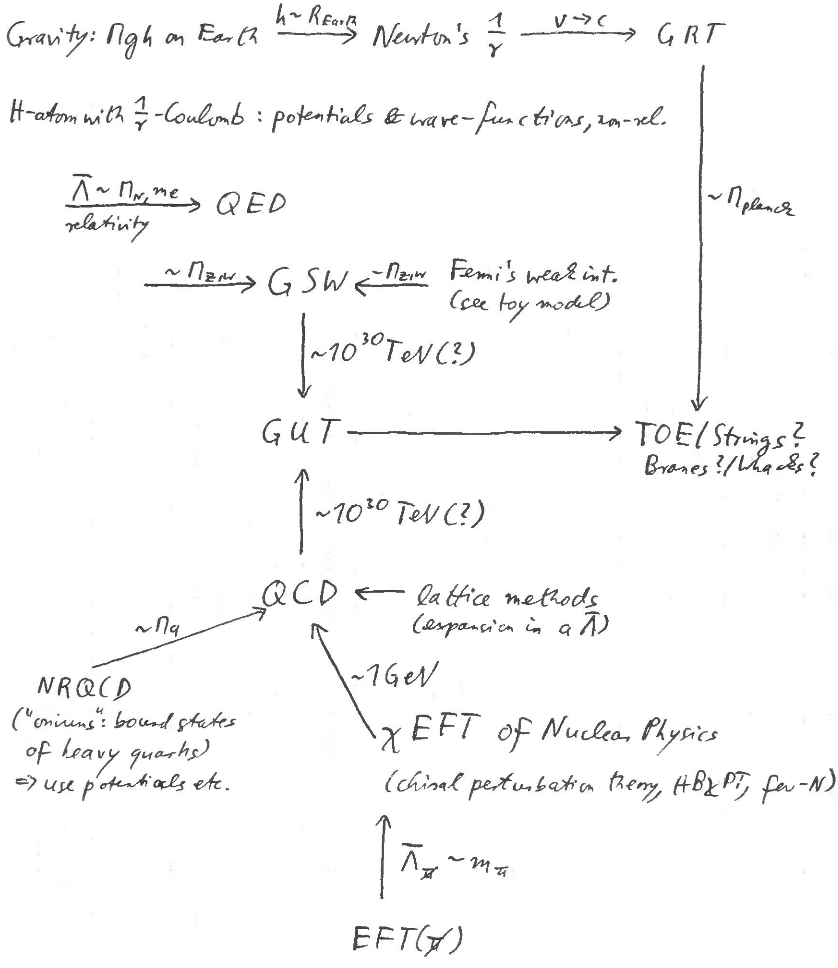
but accurate results $\hat{=}$ solutions with reliable estimates of neglected effects, by small parameter

$$Q = \frac{\text{typ. low-E scale } k}{\text{typ. high-E scale } \bar{\Lambda}_{\text{EFT}}} \ll 1$$

4. The EFT Philosophy

a) Some EFTs & their advantages : The Onion We Call Nature

Efficiency: Convenient, simple methods save time & effort.



: The World is effective. Wherever you look, our understanding is based on theories which are applicable only in a limited energy range.
Only one theory would not be "effective", namely the Theory of Everything.
But its present candidate (Strings/Branes/M/?) is rather in-effective.

b) The EFT Cookbook : $\boxed{\text{Slide}}$

⇒ EFT is more than "just fitting":

Most general interactions compatible with symmetries and minimum set of parameters & error-estimate.

⇒ model-independent results.

: predictable error-bars from boring systematics

⟺ model with intuition built by lifetime experience, not derived from basic principle
But often needed to establish d.o.f.'s, symmetries, P.C. for EFT, or when EFT unknown.

(Field-Theory advantages : straight-forward extension to include gauge fields, finite-Temp., relativistic corrections, ...)

(b) An EFT Cookbook

Wilson, Weinberg 1967, 1979; Georgi, Manohar, ... 1982-

"EFT $=$ Symmetries $+$ Parameterisation of Ignorance"

Ingredients:

Separation of scales by breakdown-scale $\bar{\Lambda}_{\text{EFT}}$:

high momenta $q \gtrsim \bar{\Lambda}_{\text{EFT}}$ $\longrightarrow$ simplify complicated/unknown UV into local LECs.

low momenta $q_{\text{low}} \ll \bar{\Lambda}_{\text{EFT}}$

Effective (relevant) degrees of freedom $\longrightarrow$ correct IR-Physics

Symmetries at low scales constrain interactions. Lorentz, gauge,...

Recipe:

Write down **most general Lagrangean** permitted by ingredients. $\Longrightarrow$ infinitely many terms

Order in small expansion parameter $Q = \dfrac{\text{typ. low momenta } q_{\text{low}}}{\text{breakdown scale } \bar{\Lambda}_{\text{EFT}}} \ll 1$:

Power-counting for loops & LECs (loops: usually simple; LECs: some fun!)

from underlying theory **or** order-by-order renormalisability $+$ minimum of phenomenology.

$\Longrightarrow$ Estimate importance of LECs & graphs without explicit calculation by

Naïve Dimensional Analysis & Naturalness Assumption.

Determine LECs at desired accuracy from underlying theory or (simple) low-mom. observables.

Result:

Model-independent, universal, systematic, unique: Predictions with estimate of uncertainties.

Finite accuracy with minimal number of parameters at each order. "Space from Improvement"

Serve With Caution:

Check assumptions: $Q \ll 1$?, jungle of scales: no separation?, wrong constituents?

Check consistency: results must match predicted convergence patter. $\longrightarrow$ later

Consider only observables. Scattering matrix between on-shell particles.

c) Limitations: When EFT does not work

: Unreasonable answers when underlying assumptions violated:

- $Q \not\ll 1 \implies p_{typ} \simeq \Lambda_{EFT}$

  : carry "seed of their own destruction" (D. Phillips).

- too many / djungle of scales / no scale-separation

  $\Rightarrow \nexists\, Q \ll 1$

  Example: N^*-spectrum at 2 GeV (?)

- Incorrect constituents

  Examples: • For $p \sim M_Z$, new propagating d.o.f.: Z-boson.

  • change of d.o.f.'s over phase-transition.

  nucleons $\Leftrightarrow$ quarks.

Effective Field Theories in Few-Nucleon Systems

Lecture II: QCD at Very Low Energies: EFT($\not\pi$)

H. W. Grießhammer

Center for Nuclear Studies
The George Washington University, DC, USA

It's natural to have unnaturally large scattering lengthes.

Power Counting for non-perturbative EFTs: Don't be too naïve!

Beyond the Effective-Range Expansion: Model-independence for data analysis & predictions.

(a) An EFT Cookbook

Wilson, Weinberg 1967, 1979; Georgi, Manohar, . . . 1982-

"EFT $=$ Symmetries $+$ Parameterisation of Ignorance"

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Consider only observables. Scattering matrix between on-shell particles.

Lecture II. QCD at Very Low Energies I: EFT($\not\pi$) for 2 Nucleons

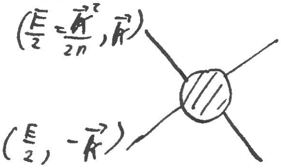
1. Recap

a) Cookbook $\to$ $\boxed{\text{slide}}$

b) Low-Energy scattering theory

$\boxed{\text{slide}}$ 2 identical particles with only short-range-interactions

$\Rightarrow$ usually dominated by s-wave



cm kinematics

$\vec{k}$: cm momentum

$E = \frac{\vec{k}^2}{M}$ total cm kinetic energy.

unitarity of $S = \mathbb{1} + \frac{iMk}{2\pi} \mathcal{A}(k)$

$$\Rightarrow \quad \mathcal{A}(k) = \frac{4\pi}{M} \frac{1}{\underbrace{k\cot\delta(k)}_{\delta:\ \text{phase-shift}} - ik}$$

(see textbooks, eg Sakurai)

low-energy $\Rightarrow$ expand analytical function in k^2:

$$k\cot\delta(k) = -\frac{1}{a} + \frac{r_0 k^2}{2} + \dots$$

a: scattering length. geometrical meaning:

$\boxed{\text{slide}}$ $\sigma(k=0) = 4\pi a^2$: "target size" in hard-sphere collision

r_0: effective range $\hat{=}$ typ. size of short-range interactions.

$\Rightarrow$ expect $a \sim r_0$ "of natural scale", i.e. set by length-scale of interaction.

$\Rightarrow$ expand in $(ak), (r_0 k) \ll 1$:

$$\mathcal{A}_{nat}(k\to 0) = -\frac{4\pi}{M} a \left[1 - i(ak) - (ak)^2\left(1 - \frac{r_0}{2a}\right) + \dots\right]$$

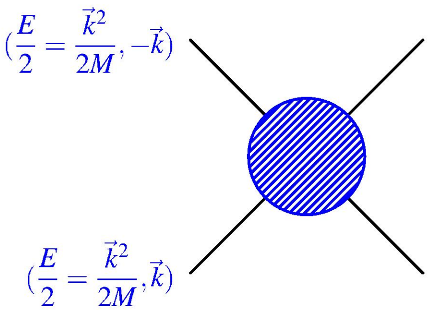
<u>Problem</u>: interesting systems with scatt. length $a \gg$ range of interaction r_0

$\Rightarrow$ (ap)-expansion breaks down very early.

(b) Recap: Low-Energy Scattering Theory

Schwinger 1947, Bethe,. . . : any QM textbook, e.g. Sakurai

Non-relativistic, S-wave, cm-frame: total kinetic energy E, relative momentum $\vec{k}$

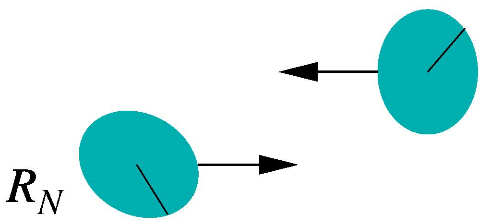


Scattering amplitude $\mathcal{A}(k) = \frac{4\pi}{M} \frac{1}{k\cot\delta(k) - \mathrm{i}k}$

$k\cot\delta(k) = -\frac{1}{a} + \frac{r_0}{2}k^2 + \ldots$ analytic function in k^2.

a: scattering length; r_0: effective range

Example: Scattering between identical particles of size R_N at zero energy:



$\sigma(k=0) = 4\pi\, a^2$

hard-sphere collisions: $a = 2R_N$

geometrical meaning of scattering length a: "target size"

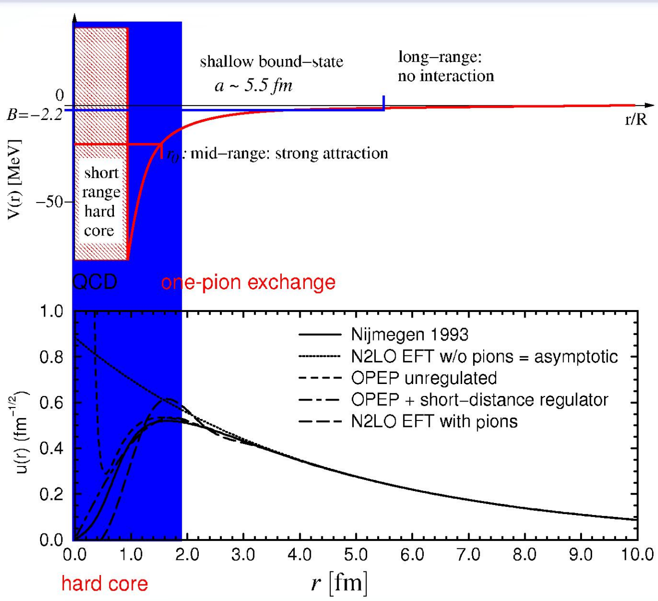
$\Longrightarrow$ expect $a \sim R_N \sim r_0$: "a, r_0 have natural size",

set by interaction range/mass of exchange-particle.

Expand for $ak,\ r_0k \ll 1:\ \mathcal{A}_{\text{natural}}(k \to 0) = -\frac{4\pi}{M}\, a \left[1 - \mathrm{i}\,(ak) - (ak)^2\left(1 - \frac{r_0}{2a}\right) + \ldots\right]$

(c) It's Natural to Have Unnatural Scales

Bethe 1949, Kaplan/Savage/Wise & van Kolck 1997



***NN*-scattering:**

$a(^1\mathrm{S}_0) = -24$ fm

$a(^3\mathrm{S}_1) = 5.4$ fm $\quad\Longleftrightarrow$ range $\sim \frac{1}{m_\pi} = 1.5$ fm

deuteron $B_d = 2.225$ MeV

hard-core + one-pion exchange $\propto -\frac{1}{r^{1\text{ to }3}}$

$^4\mathrm{He}_2$-molecule: $a \approx 104$ Å $\gg r_0 \approx 7$ Å

hard-core + van-der-Waals $\propto -\frac{1}{r^{6\text{ to }12}}$

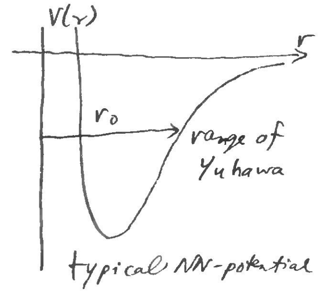
Fine-tuning, Separation of Scales: $Q = \frac{\text{typ. momentum/resolution } p_{\text{typ}} \sim 1/a}{\text{breakdown-scale/constituent size } \Lambda_{\text{break}} \sim 1/r_0} \ll 1$

$\Longrightarrow$ **Effective Field Theory of Point-Like Interactions, EFT($\not\pi$)**

$$\mathcal{L} = N^\dagger\left[\mathrm{i}\partial_0 + \frac{\vec{\partial}^2}{2M}\right]N - C_0(N^TP^iN)^\dagger(N^TP^iN) + \frac{C_2}{8}\left[(N^TP^iN)^\dagger\left[N^TP^i(\overleftarrow{\partial} - \overrightarrow{\partial})^2N\right] + \text{H.c.}\right] + \ldots$$

c) It's Natural to have Unnatural scales $\boxed{\text{slide}}$

(1) Nuclear Physics



Attractive One-Pion-Exchange over range $\sim \frac{1}{m_\pi}$

fine-tuned with repulsive core

– origin not understood

$\Rightarrow$ unnaturally large scattering lengths:

$$a(^1S_0) = -24\,\text{fm} = -\frac{1}{8.3\,\text{MeV}} \gg \frac{1}{m_\pi} \sim 1.5\,\text{fm}$$

$$a(^3S_1) = 5.4\,\text{fm} = \frac{1}{45\,\text{MeV}} > \frac{1}{m_\pi} \sim 1.5\,\text{fm}$$

$\hookrightarrow$ mass of exchange particle.

$\hat{=}$ deuteron as low-lying NN bound-state

with size $5.5\,\text{fm} \simeq a(^3S_1)$

$$B = \frac{1}{M a^2} = 2.225\,\text{MeV} \ll \frac{m_\pi^2}{M_N} \sim 10\,\text{MeV}$$

expected from Yukawa

(2) Neutral Atoms

Repulsive hard-core + van-der-Waals attraction $\propto -\frac{1}{r^{6..12}}$ from QED

$\Rightarrow$ eff. range $r_0(^4He_2) \simeq 7\,\text{Å} \ll$ scatt. length $a(^4He_2) = 104\,\text{Å}$

$\Rightarrow$ $^4He_2$ dimer molecule very loosely/shallowly bound.

also in Li, Na, K, Rb, Cs, ...

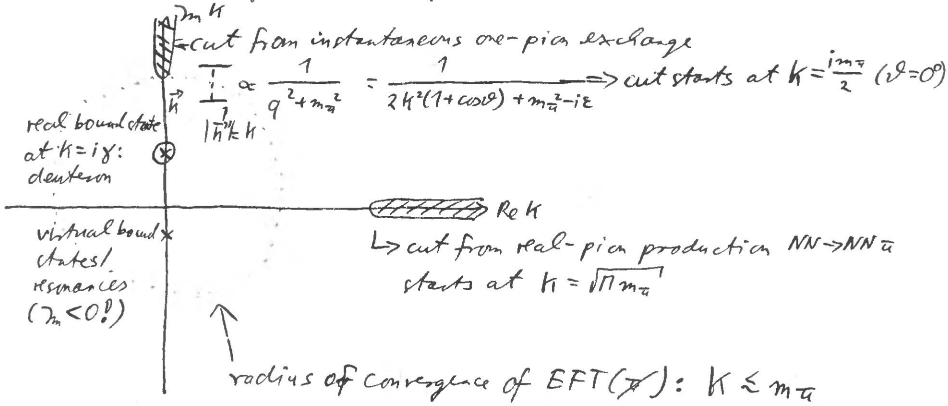
In both cases, there are no new interactions at the scale $k \sim \frac{1}{a}$

$\Rightarrow$ Re-order expansion of scattering amplitude $\mathcal{A}$ such that

single pole $\hat{=}$ bound state accomodated below $r_0 \sim \frac{1}{\Lambda_{EFT}} \sim \frac{1}{m_\pi}$

: non-perturbative in $(ak) \not< 1$!

– Poles & Cuts of $\mathcal{A}$ in complex-k-plane



⇒ <u>E</u>ffective-<u>R</u>ange <u>E</u>xpansion ERE = Laurent-Series of $\mathcal{A}$ in k
(Schwinger, Bethe, .. 1947)

$$\mathcal{A}_{ERE} = -\frac{4\pi}{M}\,\frac{1}{\gamma+ik}\left[1+\frac{\rho_0}{2}\,\frac{\gamma^2+k^2}{\gamma+ik}+\dots\right]$$
$$\sim\frac{1}{Q}\qquad\left[Q^0+\frac{Q^2}{Q}+\dots\right]$$

Clever Matching: Position (& residue) of pole at position of deuteron pole:

$\gamma = -ik = -i\sqrt{ME} = \sqrt{MB_{deut}} \simeq 45$ MeV for $B_d = 2.2248$ MeV
↳ "binding momentum", $\gamma \approx \frac{1}{a(^3S_1)}$

with $r_0 \sim \overline{\Lambda}_{\not\pi}$, $a \sim \gamma$: anticipate LO $= \frac{1}{Q}$, NLO $= Q^0$

How to reproduce this in EFT of short-range interactions only?

2. Fine-Tuning in EFT: Two Particles

a) EFT($\not\pi$)

scale-separation: $p_{typ} \ll m_\pi \simeq \bar{\Lambda}_{\not\pi}$: lightest particle mediating NN force

$\Rightarrow$ pion-interactions "integrated out" into LECs:

 encodes all Nucl. Potential, QCD, ..

degrees of freedom: N with mass M only.

symmetries: – $M \gg p_{typ}$ $\Rightarrow$ nonrelativistic kin. energy $\frac{p^2}{2M}$

no anti-nucleons

– isospin : $M_p = 938$ MeV $\approx M_n = 940$ MeV from QCD.

$\Rightarrow$ small breaking $\frac{M_n - M_p}{m_\pi \sim \bar{\Lambda}_{\not\pi}} \approx \frac{1}{70}$.

$\Rightarrow N = \begin{pmatrix} p \\ n \end{pmatrix}$ is isovector of Weyl-2-spinors

Concentrate on s-waves (dominant, others can be included systematically):

projectors $\mathcal{P}^i(^3S_1) = \frac{1}{\sqrt{8}} \underbrace{(\sigma_2 \sigma^i)}_{\text{spin}} \underbrace{\tau_2}_{\text{isospin}}$

$\mathcal{P}^i(^1S_0) = \frac{1}{\sqrt{8}} (\tau_2 \tau^i) \sigma_2$

$$\Rightarrow \mathcal{L} = N^\dagger \left[i\partial_0 + \frac{\vec{\partial}^2}{2M} + \frac{\vec{\partial}^4}{8M^3} + \dots \right] N$$

$\boxed{\text{Lorentz-symmetry fixes relative strength of kinetic terms!}}$

$$- \sum_{\alpha = {}^3S_1, {}^1S_0} C_0^{(\alpha)} \left(N^T \mathcal{P}^i_\alpha N\right)^\dagger \left(N^T \mathcal{P}^i_\alpha N\right)$$

\+  $p^2 C_2$ with projectors etc. $\frac{C_2}{8} \left(N^T \mathcal{P}^i_\alpha N\right)^\dagger \left[N^T (\overleftarrow{\partial} - \overrightarrow{\partial})^2 \mathcal{P}^i_\alpha N\right]$

\+ ...

Very similar for atomic scattering: $N \to$ scalar field, no isospin.

Concentrate now only on one partial wave, suppress all indices.

b) Accommodating unnatural scales in EFT($\not\pi$)

(Kaplan/Savage/Wise, van Kolck 1997 : 10 years ago!)

(α) failed trial

- low momenta $\Rightarrow$ C_0 and its bubbles should dominate

$i\mathcal{A} =$ 

$$= -iC_0 - iC_0[I_0(k)C_0] - iC_0[I_0(k)C_0]^2 - \dots$$

$$Q^0 \qquad\qquad Q^1 \qquad\qquad Q^2$$

with basic loop integral

$$I_0(p) = i\int\frac{dq_0}{(2\pi)}\frac{d^3\vec{q}}{(2\pi)^3}\,\frac{1}{\frac{\vec{k}^2}{M}+q_0-\frac{\vec{q}^2}{2M}+i\epsilon}\,\frac{1}{-q_0-\frac{\vec{q}^2}{2M}+i\epsilon}$$

P.C.: $\quad \frac{Q^2}{M} \quad Q^3 \quad \frac{1}{Q^2/M} \quad \frac{1}{Q^2/M} \sim MQ$

Contour-integration in complex q_0-plane

$$= \int\frac{d^3q}{(2\pi)^3}\,\frac{M}{\vec{k}^2-\vec{q}^2+i\epsilon} \qquad \text{reveals linear divergence}$$

$$= -\frac{M}{4\pi}[\mu + ik] \qquad \rightarrow \underline{\text{HW for DimReg \& Cutoff}}$$

some arbitrary scale which enters by regularisation, eg cutoff $\Lambda = \frac{\pi\mu}{2}$

$$\left(I_0(k,\Lambda) = -\frac{M}{2\pi^2}\left[\Lambda + ik\arctan\frac{i\Lambda}{k}\right] \right)$$

- μ seems irrelevant and will "somehow" disappear by Renormalisation.

$\Rightarrow$ $C_0 I_0(k) = \mathcal{O}(Q)$ for $k \to 0$,

i.e. loop suppressed against X $= Q^0$

$\Rightarrow$ LO is X $= -iC_0 \underset{\text{match to } \mathcal{A}_{\text{natural}}}{=} -\frac{4\pi}{M}a \Rightarrow C_0 = \frac{4\pi a}{M}$.

<u>can't</u> match to $\mathcal{A}_{\text{ERE}}$: wrong k-dependence!

: Works only if $a \sim \frac{1}{\bar\Lambda}$ <u>natural</u>: $C_0 \sim \frac{1}{M\bar\Lambda}$ UV encoding

$\Rightarrow$ "<u>count only powers of</u> $p \to Q$": $\quad k^{2n}C_{2n} \sim \frac{1}{M\bar\Lambda}\left(\frac{k}{\bar\Lambda}\right)^{2n} \sim \frac{1}{M\bar\Lambda}Q^{2n}$ suppressed:

$\Rightarrow$ perturbative EFT, no fine-tuning: M_Z-EFT, χPT, HBχPT, but not Nuclear Physics!

(B) What went wrong?

– Cannot change NDA for loops, but for <u>LECs</u>!

: Neglected regularisation-scheme (RS) dependence of $I_0(p;\mu)$

Physical amplitude μ-dependent $\Rightarrow$ $C_0(\underset{\S}{\underline{\mu}})$ depends on RS.

Separation between finite, analytical & infinite part in loop arbitrary

$\Rightarrow$ Shuffle strength between $C_0(\mu) \Leftrightarrow I_0(p;\mu)$

and <u>turn this into your advantage</u>!

$\Rightarrow$ Blindly resum all bubbles via geometric series – justify later.

$$i\mathcal{A} = \frac{-iC_0(\mu)}{1-I_0(p;\mu)} = -\frac{4\pi}{M}\frac{i}{\frac{4\pi}{MC_0(\mu)}+\mu+ik} \overset{!}{=} -\frac{4\pi}{M}\frac{i}{\gamma+ik} = \mathcal{A}_{ERE}$$

same structure: good!

$$\Rightarrow C_0(\mu) = \frac{4\pi}{M}\frac{1}{\underbrace{\gamma-\mu}} \sim \frac{1}{MQ}$$

low-E scale for <u>choice</u> $\mu\sim\gamma\sim Q$.

$\to C_0(\mu) \to \infty$ for $Q\to 0$: "<u>relevant operator</u>" in R Group

$\Rightarrow C_0(\mu)\, I_0(k;\mu) \sim \frac{1}{MQ}\, MQ = Q^0$, $\mathcal{A} = -\frac{4\pi}{M}\frac{1}{\gamma+ik} \sim \frac{1}{Q}$: μ-indep. LO

: all C_0-bubbles of same order in P.C.!

$\Rightarrow \mu\sim\gamma\sim Q$ gives <u>convenient justification</u> to sum geometric series.

– If $\mu\sim\bar\Lambda\sim Q^0$: $C_0 \sim \frac{1}{M\bar\Lambda}$ as naively expected, but resummation <u>not</u> justified

[One can show that one has to resum even in this case because the fine-tuning of a transcends into one for the <u>bare</u> C_0-bubbles. But our P.C. makes the resummation manifest already in the bare couplings $C_0(\mu)$.]

<u>NB</u> : Used <u>one piece of phenomenology</u> as input :

<u>Qualitatively large</u> scattering length / shallow bound state.

: P.C. from minimum of phenomenology, because could not (yet) derive from QCD.

- Extension to the other operators again via comparison to ERE:

 $k^2 C_2(\mu) = \frac{4\pi}{M}\frac{r_0}{2}\frac{k^2}{(\gamma-\mu)^2} \sim \frac{1}{M\bar\Lambda_{\not\pi}} \sim Q^0$; $\sim Q^0 (!)$

[Recap ERE: $\mathcal{A}(k\to 0) = -\frac{4\pi}{M}\frac{1}{\gamma+ik}\left[1 + \underbrace{\frac{r_0}{2}\frac{k^2+\gamma^2}{\gamma+ik}}_{} + \dots\right]$

 $\sim Q^0$; $\sim C_2 \Rightarrow$ should enter at Q^0.

and indeed:  $\sim \underset{C_0}{\frac{1}{Q}}\ \underset{C_2}{\frac{1}{M\Lambda}}\ \frac{Q^2 Q^3}{Q^2 Q^2} \sim Q^0$: indeed NLO ✓

note divergences introduced:  $\sim \frac{\Lambda^3\Lambda^2\Lambda^2}{\Lambda^2\Lambda^2} \sim \cancel{\#}\Lambda^3 + \cancel{\#}\Lambda k^2 + \text{finite}$

absorb into C_0 ; absorb into C_2

 $\sim \frac{Q^5 Q^5 Q^2}{(Q^2)^2(Q^2)^2}\ \underset{C_0}{\frac{1}{Q}}\ \underset{C_0}{\frac{1}{Q}}\ \underset{C_2}{Q^0} \sim Q^0$

& no new divergence structures

- dito $k^{2n} C_{2n} \sim \frac{1}{M\bar\Lambda_{\not\pi}} Q^{n-1}$ even more suppressed.

$\Rightarrow$ It's <u>fallacious</u> to just count powers of momenta in nonperturbative EFTs: <u>fine-tuning</u>.

$\Rightarrow \mathcal{A}_{LO}(k) = -\frac{4\pi}{M}\frac{1}{\gamma+ik} = \mathcal{A}_{ERE}$ $\rightarrow$ <u>HW</u>.

represent bubbles by <u>"deuteron propagator"</u>: double-line  at LO.

: μ-independent ✓

: error-estimate : $1 \pm \frac{k_{typ}\sim\gamma}{\bar\Lambda_{\not\pi}\sim m_\pi} \simeq 1 \pm \frac{1}{3..5}$

$\rightarrow$ $\boxed{\text{slides}}$: NN phase-shifts for S-waves & convergence checks.

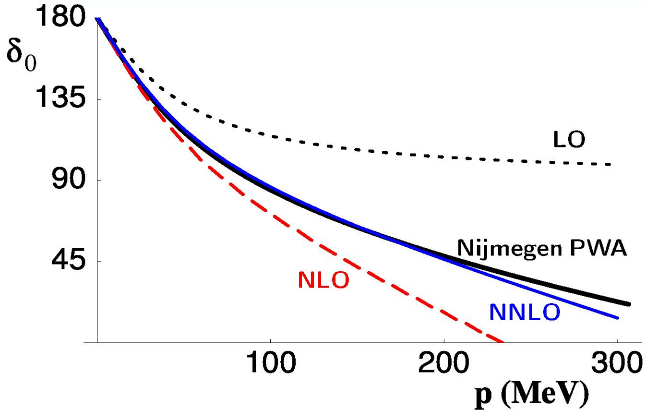
- So we have reproduced ERE - so what??

$\rightarrow$ life is more than NN : electroweak probes, more nucleons!

$\rightarrow$ $\boxed{\text{slides}}$ Beyond ERE.

(c) NN Phase Shifts in EFT($\not{\pi}$): Example $^3S_1$

figure: Phillips et al, 1999



Cross-checks:

- Convergence to Nature. o.k.
- Order by order smaller μ-dependence. Exact.
- Order by order smaller corrections to confirm error-estimate:

$$Q = \frac{\text{typ. momentum } \gamma}{\text{breakdown scale } \Lambda_{\not{\pi}} \sim m_\pi} \approx \frac{1}{3}$$ o.k.

EFT($\not{\pi}$) exactly reproduces Effective-Range Expansion.

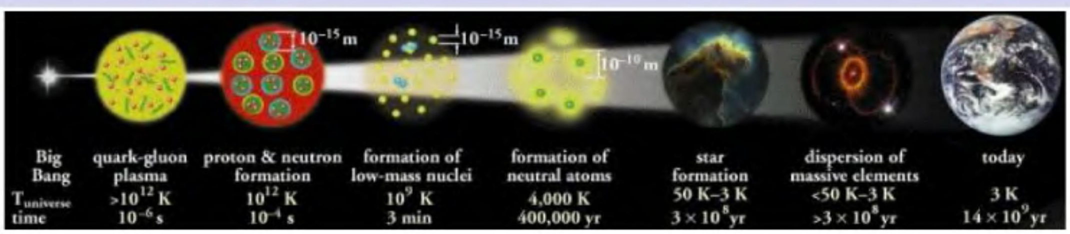
Matches data even for $p \to 300\ \text{MeV} \gg \Lambda_{\not{\pi}}$.

$\Longrightarrow$ **Use Advantages of Field Theories: unambiguous inclusion of**

- gauge-currents, relativistic corrections, finite-temperature,...
- more nucleons

(d) Why Bother?: Big-Bang Nucleo-Synthesis and $np \to d\gamma$

Chen/Savage 1999, Rupak 2000

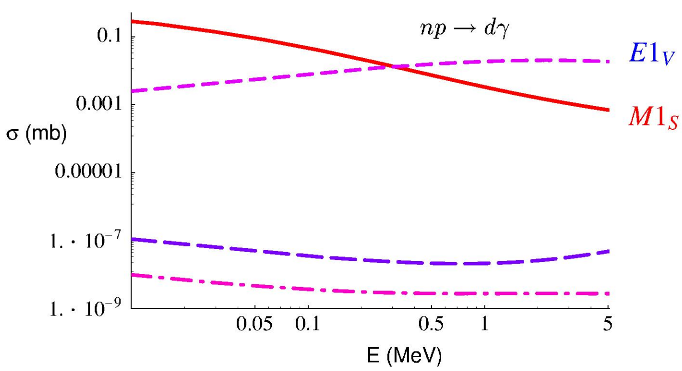


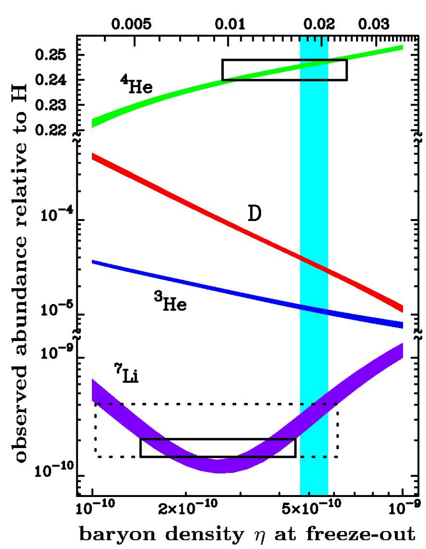
$E_{\text{typ}} \approx 0.02 - 0.2\ \text{MeV}$, light-element abundances sensitive to baryon density.

Accurate theoretical determination necessary: error-estimate!

$np \to d\gamma$ biggest uncertainty, but "impossible" to measure.

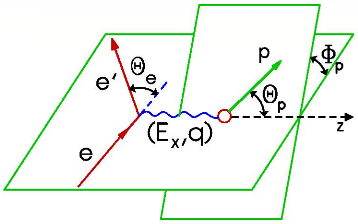
EFT($\not{\pi}$) to N^{4}LO in closed form: accuracy $\lesssim 1\%$. Rupak 1999





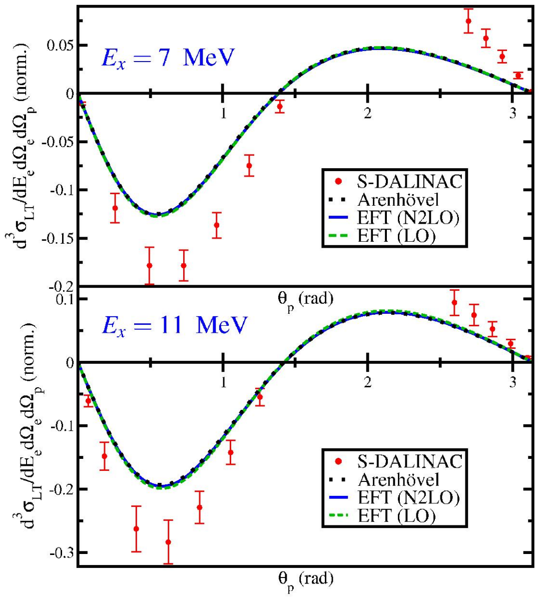
(e) Problem-Spotting: Deuteron Electro-Disintegration $d(e,n)pe'$

S-DALINAC 2002; EFT($\not\pi$): hg/Christlmeier 2004/05



$$\left.\begin{array}{l} E_p \leq 16 \text{ MeV} \\ E_x = E_{\gamma^*} = [8\dots16] \text{ MeV} \end{array}\right\} \Longrightarrow \text{low-energy: } \Delta x \gtrsim 4 \text{ fm}$$

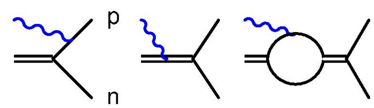
$$\frac{d^3\sigma}{dE_e^{\text{lab}}\, d\Omega_e^{\text{lab}}\, d\Omega_p^{\text{np}}} = (\sigma_L + \sigma_T) + \sigma_{LT}\, \cos\phi_p^{\text{np}} + \sigma_{TT}\, \cos 2\phi_p^{\text{np}}$$



Experiment vs. Bonn-potential Arenhövel et al. 1995

– same total cross-section; 30% **discrepancy in** σ_{LT}.

EFT($\not\pi$) at LO+NLO ($\approx 10\%$): minimal coupling only



– automatically gauge-invariant etc.;

– error-estimate: parameter $Q^{\not\pi} = \frac{p_{\text{typ.}}}{\Lambda_{\not\pi}} \approx \frac{1}{3\dots5}$.

EFT is universal: Within uncertainty, same result for **any model** with same B_{deut}, Z_{deut}.

Found data normalisation problem.

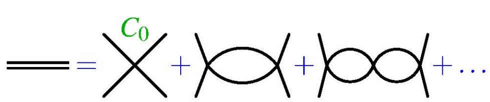
3. The Problem with Three-Nucleon Forces

(a) EFT($\not\pi$): QCD at Very Low Energies

Schwinger 1947, Bethe 1949, Kaplan/Savage/Wise & van Kolck 1997

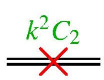
2-Body Sector: Extension of Effective Range Expansion to include external currents, well-understood.

LO (Q^{-1}, $\lesssim 30\%$) :



$\propto \frac{1}{\frac{1}{a} + ik}$ scattering lengthes a

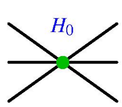
NLO (Q^0, $\lesssim 10\%$) :



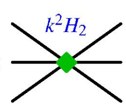
$\propto \frac{1}{\frac{1}{a}+ik}\ \frac{k^2\rho_0}{2}\ \frac{1}{\frac{1}{a}+ik}$ effective ranges ρ_0

3-Body Sector: All interactions permitted by symmetries $\Longrightarrow$ 3-body forces

$H_0\,(N^\dagger N)^3$:



, $k^2\, H_2\,(N^\dagger N)^3$:

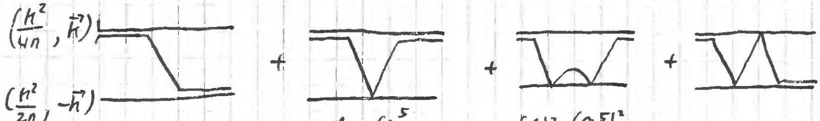


, etc.

How important? $\Longleftrightarrow$ **Which observables most sensitive?**

b) Three Nucleons at LO in EFT($\not\pi$)

given by "pinball diagrams": "exchange" of a nucleon



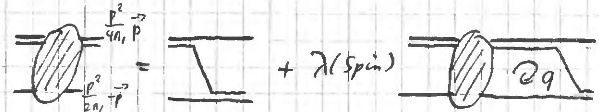
$\frac{1}{Q^2}$ + $\frac{1}{Q}\frac{Q^5}{(Q^2)^3} \sim Q^{-2}$ (C_0) + $\left(\frac{1}{Q}\right)^2 \frac{(Q^5)^2}{(Q^2)^5} \sim Q^{-2}$ (C_0) + $\sim Q^{-2}$

⇒ sum 2-body bubbles again into deuteron

⊕ use iteration ⇒ Faddeev integral equation for half off-shell amplitude

(Skorniakov / Ter-Martirosian 1957)

$\mathcal{A}(k;p) =$



$\sim Q^{-2}$

≙ Schrödinger equation with "potential"

but 3body force appears $H_0 (N^\dagger N)^3$: momentum-independent ?

$\Rightarrow \quad H_0 \sim Q^0$?? i.e. N^2LO ??

but that was already fallacious in NN

⇒ check cutoff-dependence without 3body-force.

Consider S-wave scattering between N and d:

∃ 2 channels: (1) $^4S_{3/2}$: spins of N and d parallel ⇒ $S_{total} = \frac{3}{2}$

expect no 3NF at LO because

d spin ≈ n-spin + p-spin ⇒ Pauli-blocking

⇒ ∄ H_0

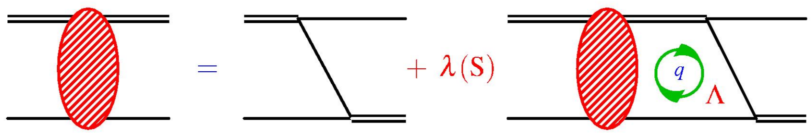
(2) $^2S_{1/2}$: spins of N and d anti-parallel ⇒ $S_{total} = \frac{1}{2}$

expect no 3NF at LO because

$H_0 \sim Q^0$, while $\sim Q^{-2}$.

(c) Cut-off Dependence: nd scattering length, Quartet-S Wave

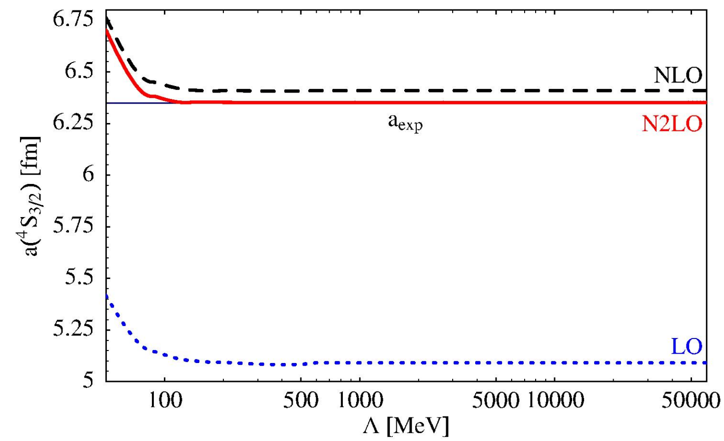
LO:



nd scattering length,
Quartet-S wave

Observable cut-off independent,
good convergence.

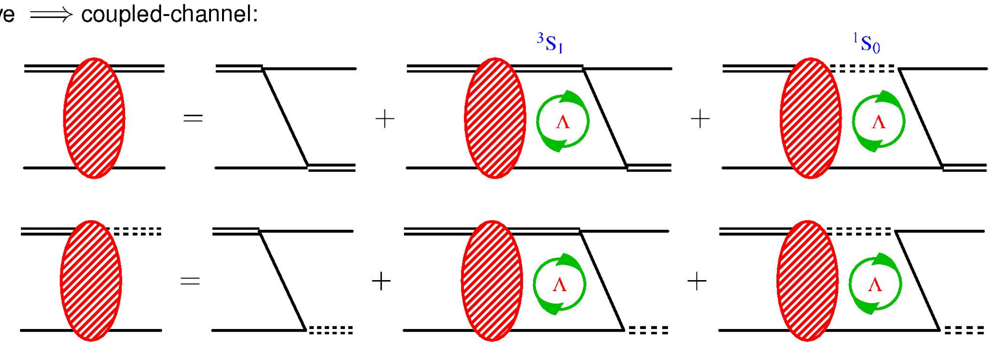
parameter-free

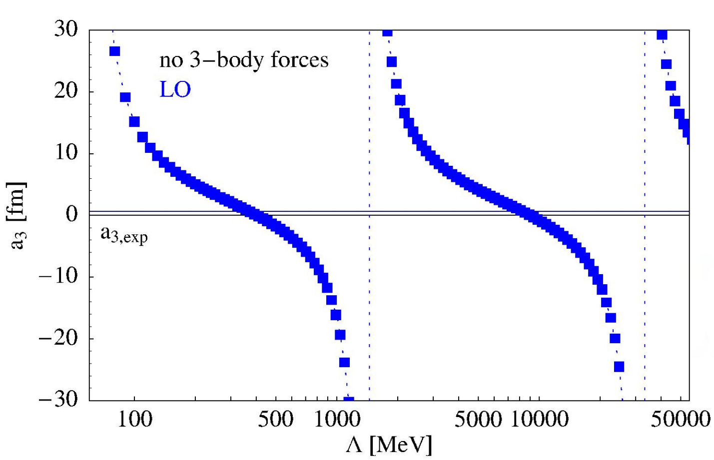


(LO: Skorniakov/Ter-Martirosian 1957, NLO: Efimov 1991, N2LO: Bedaque/van Kolck 1998, hg 2005)

(d) The Problem: nd-Scattering, ${}^2\mathrm{S}_{\frac{1}{2}}$ Wave ("triton channel")

${}^2\mathrm{S}_{\frac{1}{2}}$-wave $\Longrightarrow$ coupled-channel:





Slight cut-off variation has dramatic effect on scattering length $a({}^2\mathrm{S}_{\frac{1}{2}})$.

Danilov, Minlos/Faddeev 1961

$\Longrightarrow$ No self-consistent Effective Range Expansion in 3-body system!

e) Alternative Picture: Coordinate Space

In hyper-spherical coordinates, the Schrödinger Equation for the distance between N and d takes in the limit $R \to 0$ (very short distance $\hat{=}$ unphysical) the form:

$$\left[-\frac{1}{R}\frac{\partial}{\partial R} R \frac{\partial}{\partial R} + \frac{s_\ell^2}{R^2} \right] F(R) = ME\, F(R)$$

This looks like a radial Schrödinger equation with a centrifugal barrier, if $s_\ell^2 = (\ell+1)^2$

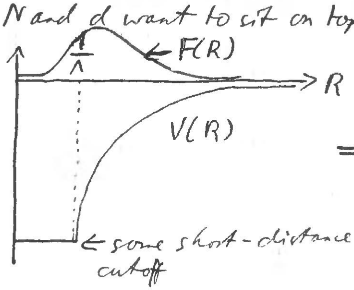
(not $\ell(\ell+1)$ because of hyperspherical coordinates)

– But one finds (→ p. Three N4)

$s(^2S_{1/2}) = 1.0062\ldots i$ ⇒ attractive $\frac{1}{R^2}$ - potential

⇒ wavefunction $F(R)$ wants to collaps to $R \to 0$ as $\Lambda \to \infty$.

$\hat{=}$ N and d want to sit on top of each other.



⇒ need repulsive 3BI at LO to "push" N and d away from each other!

– Converse situation: $s_\ell \gg \ell + 1$ for eg $^4S_2$-wave

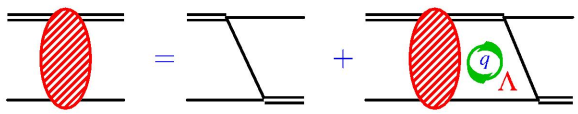
⇒ potential more repulsive than naive centrifugal barrier

⇒ $F(R)$ less sensitive to short-distance physics

⇒ 3BI less important than expected

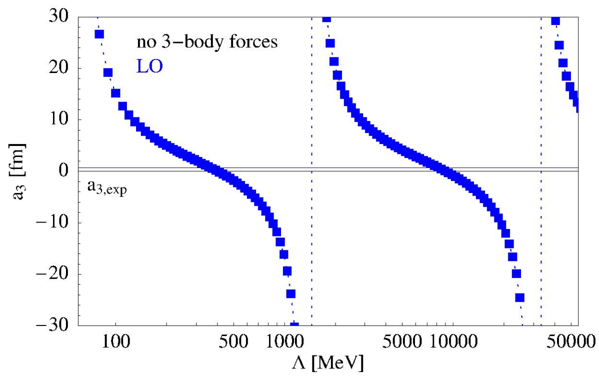
⇒ "demoted" to higher orders.

(e) Alternative Picture: Co-ordinate Space



Hyper-spherical co-ordinates, n–d-distance $R \to 0$:

$$\left[-\frac{1}{R}\frac{\partial}{\partial R}R\frac{\partial}{\partial R}+\frac{s_l^2}{R^2}\right]F(R)=ME\,F(R)$$



Danilov, Minlos/Faddeev 1961

Naïve: centrifugal barrier $s_l^2 = (l+1)^2$

$s_l(\lambda) = 1.0062\,\mathrm{i}$ **imaginary,** $|s_l(\lambda)| > \frac{1}{2}$**: attractive**

$\Longrightarrow$ collaps to $R \to 0$

$\Longrightarrow$ infinitely many deeply bound states:

Thomas & Efimov Effects (1935, 1971)

$\Longrightarrow$ dramatic low-energy effect varying high scale Λ.

Promote $H_0(\Lambda)$ to stabilise/absorb Λ-dep.

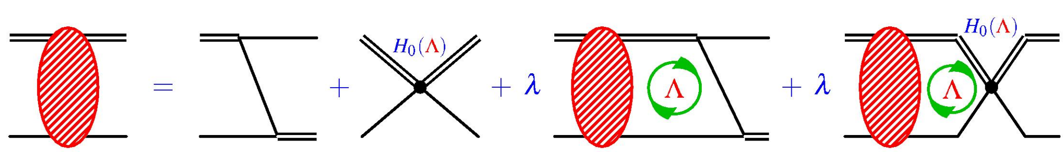
Converse situation: $s_l(\lambda) \gg l+1$ (e.g. $s_0(^4\mathrm{S}_{\frac{3}{2}}) = 2.16\dots \gg 1$)

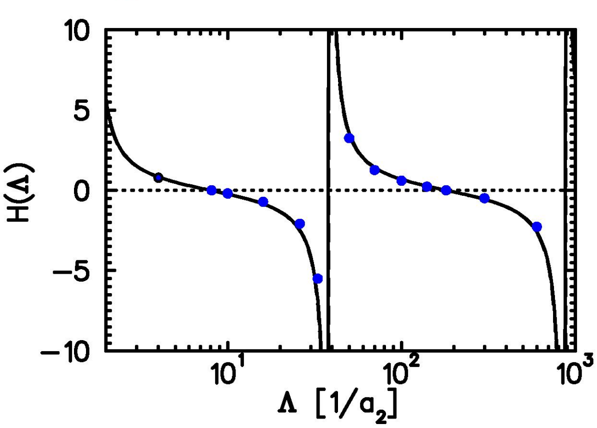
$\Longrightarrow$ more repulsive than estimated, less sensitivity to short-distance $\Longrightarrow$ 3body forces demoted.

(f) The Solution: EFT Wilson 1971+93, Efimov 1971-95, Bedaque/Hammer/van Kolck 1998, Bedaque/hg/Hammer/Rupak 2004

Tenet: Include specific 3BF if and only if needed to cancel off-shell dependence of observables.

$H_0(\Lambda) \sim Q^{-2}$: Momentum-independent 3BF absorbs cut-off dependence at LO.





$\mathcal{A}$ analytical in UV $\Longrightarrow$ UV Λ-independent for

$$H_0(\Lambda) = -\frac{\sin[s_0 \ln \Lambda/\Lambda_* - \arctan\frac{1}{s_0}]}{\sin[s_0 \ln \Lambda/\Lambda_* + \arctan\frac{1}{s_0}]}$$

New RG phenomenon: Limit Cycle

Not 3NF "large", but effect on observables!

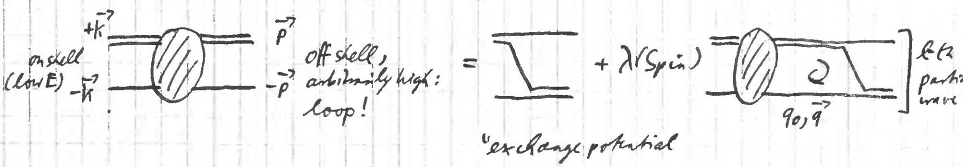
Naïve dimensional analysis too naïve!

Low-energy datum determines physical scale Λ_*.

LO and NLO ($\lesssim 10\%$ accuracy): One free parameter H_0: triton binding energy.

N2LO and N3LO ($\lesssim 1\%$ accuracy): One more free parameter H_2: scattering length.

– Translate diagrams into integral equation



– Interested only in leading UV-asymptotics:

$$(\Lambda \gg)\; p, q \gg \overline{\Lambda}_{\not\pi} \gg k, \gamma, \frac{1}{a({}^1S_0)} \to 0$$

$\Rightarrow$ NN-amplitude in loop $\quad \mathcal{A}_{NN} \propto \dfrac{1}{\gamma \text{ or } \frac{1}{a} + ip} \to -\dfrac{2}{\sqrt{3}\; p}$

independent of scatt. length: same for ${}^1S_0$ & ${}^3S_1$

"Wigner's SU(4) spin-isospin symmetry"

Close contour of q_0-integration $\;: q_0 = \dfrac{\vec{q}^{\,2}}{2M}$: eliminates N-propagator

$$\Rightarrow \mathcal{A}^{(\ell)}_\lambda(p) = \underbrace{\divideontimes}_{\text{some factor}} - \frac{4}{\pi}\,\lambda(\text{spin}) \int\limits_{\min \to 0}^{\max \to \infty} dq\, q^2 \left(-\frac{2}{\sqrt{3}\,q}\right) \mathcal{A}^{(\ell)}_\lambda(q)$$

$$\times \underbrace{\frac{1}{2}\int\limits_{-1}^{1} d\cos\vartheta\; \frac{P_\ell(\cos\vartheta)}{p^2+q^2+pq\cos\vartheta}}_{= \frac{(-1)^\ell}{pq}\, Q_\ell\left(\frac{p}{q}+\frac{q}{p}\right)}$$: Legendre polynomial

: Legendre 2nd kind.

$$= \divideontimes + \frac{8\lambda(\text{spin})}{\sqrt{3}\,\pi}\,(-)^\ell \int\limits_0^\infty \frac{dq}{p}\; Q_\ell\left(\frac{p}{q}+\frac{q}{p}\right) \mathcal{A}^{(\ell)}_\lambda(q)$$

– Ansatz: $\mathcal{A}(p) = p^{s-1}$ $\qquad$ "Mellin transform"

$$\Rightarrow 1 = \frac{\divideontimes}{p^{s-1}} + \frac{8\lambda(\text{spin})}{\sqrt{3}\,\pi}\,(-)^\ell \underbrace{\int\limits_0^\infty \frac{dq}{p} \left(\frac{q}{p}\right)^{s-1} Q_\ell\left(\frac{p}{q}+\frac{q}{p}\right)}_{\text{a number which depends on } s(\ell,\lambda)}$$

$\;\;\;\;\;\;\;\;\;\;\llcorner\!\to 0$ intuitively for $p \to \infty$ (can be shown)

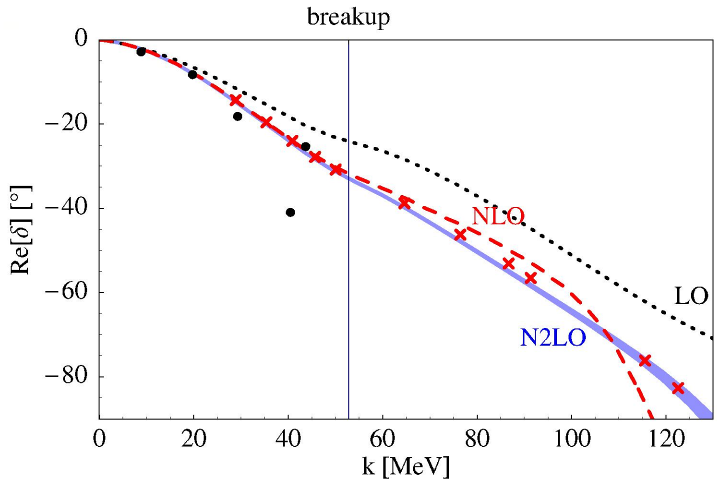
$\Rightarrow$ integral equation in UV $\longrightarrow$ algebraic problem with transcendental equation

$$\lambda(\text{spin}) = \begin{cases} 1 & \text{for 3 bosons \& one diagonalised component of (Nd) in doublet} \\ -\frac{1}{2} & \text{for (Nd) in quartet \& the other diagonalised component in doublet} \end{cases}$$

(g) Doublet-S Wave nd Phase Shift

Bedaque/hg/Hammer/Rupak 2002, hg 2004

$\times$: AV18+U IX (Kievsky 2002) $\quad\bullet$: PWA 1967 (Seagrave/van Oers) $\quad$ ▬: N2LO, $\Lambda \in [200;\infty]$ MeV



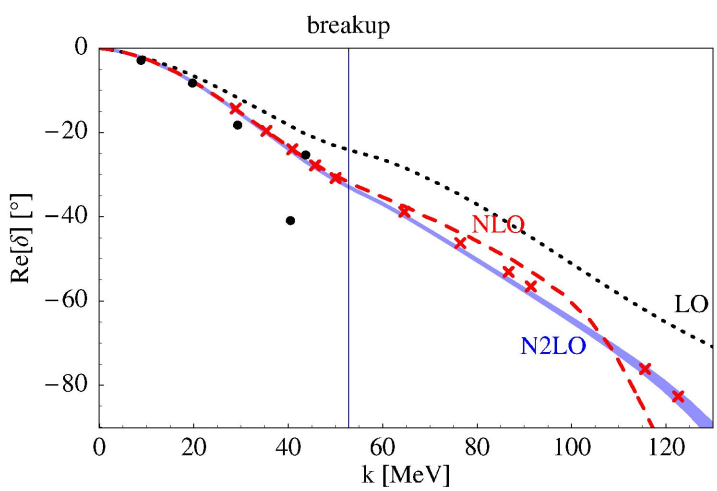
Convergence Criteria: EFT Self-Consistent on Quantitative Level.

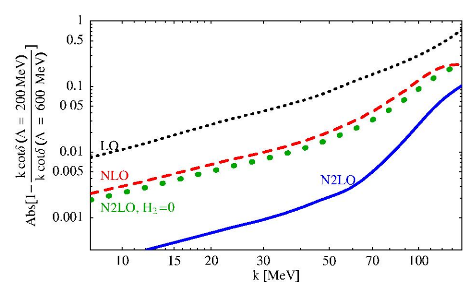
- From outside EFT: Convergence to Nature — No data here.
- Order by order smaller corrections (also with selected higher-order terms). — O.k.
- Order by order less dependence on particular low-E data taken for LECs. — O.k.
- Order by order less cut-off/RG-scheme dependence: **Wilson's Renormalisability** — quantify in "Lepage plot".

(g) Doublet-S Wave nd Phase Shift

Bedaque/hg/Hammer/Rupak 2002, hg 2004

$\times$: AV18+U IX (Kievsky 2002) $\quad\bullet$: PWA 1967 (Seagrave/van Oers) $\quad$ ▬: N2LO, $\Lambda \in [200;\infty]$ MeV





Wilson's Renormalisability Criterion quantified by "Lepage plot"

$$\left|1-\frac{k\cot\delta(\Lambda=200\ \mathrm{MeV})}{k\cot\delta(\Lambda=\infty)}\right| \sim \underbrace{\left(\frac{p_{\mathrm{typ.}}}{\Lambda_{\not\pi}}\right)^{n}}_{Q^n}$$

$\Longrightarrow$ Fit to $k \in [70;100\ldots130]$ MeV

	LO	NLO	N^{2}LO	N^{2}LO without H_2
n fitted	~ 1.9	2.9	4.8	3.1
n expected	2	3	4	4!!

Some cross-checks for serious EFT-results

If EFT from integrating out / "matching" to "full theory", very trustworthy.

If EFT from general principles, have to convince the skeptics:

Show that EFT keeps <u>promise of self-consistency</u>:

Theoretical accuracy $\simeq Q = \frac{k_{typ}}{\bar{\Lambda}_{EFT}}$ of difference between N^nLO & $N^{n+1}LO$-contributions.

$\Rightarrow$ <u>Checks</u>: (1) Include "selected" (Λ-independent) higher-order contributions & check that indeed small.

(2) Order-by-order reduced dependence on particular low-E data taken to fix LECs.

$\rightarrow$ <u>HW</u>: fit to (a, r_0) vs (γ, ρ_0)

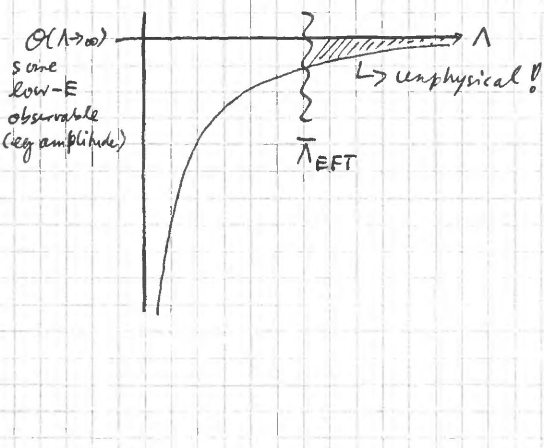
(3) Order-by-order smaller dependence on <u>value</u> & <u>scheme</u> used for cutoff / regularisation. $\hat{=}$ <u>Wilson's renormalisability</u>

If used <u>cutoff</u>, vary $\Lambda \in [\bar{\Lambda}_{EFT}; \infty[$

& cutoff-function used (sharp vs e^{-p^2/Λ^2} ..)

: Ensure that <u>physical amplitude</u> dominated by integration over momenta in regime when EFT applicable : $q \lesssim \bar{\Lambda}_{EFT}$.

$\Leftrightarrow$ reduced sensitivity to short-distance $\hat{=}$ <u>unreasonable Physics</u>



: $\mathcal{O}(\Lambda)$ <u>saturated</u> above $\bar{\Lambda}_{EFT}$.

(3a) Carries <u>quantitative</u> predictions in range of validity:

Observables expanded in powers of Q: $\mathcal{A} = \sum\limits_{n=n_{min}}^{\infty} \mathcal{A}_n Q^n$

short-distance sensitivity $\hat{=}$ cutoff-dependence of Q^{n_0}-calculation must enter at Q^{n_0+1}

$\Rightarrow$ for variation of cutoff, eg in scattering:

$$\left| 1 - \frac{k\cot\delta(\Lambda)}{k\cot\delta(\Lambda')} \right| \sim \left(\frac{p_{typ.} \sim \gamma, k}{\overline{\Lambda}_{EFT}} \right)^{n_0+1} = Q^{n_0+1}$$

$\Rightarrow$ double-logarithmic "Lepage-plot" of observable as fu. of k.

: for $\overline{\Lambda} \gg k \gg \gamma$: slope $\hat{=}$ order n_0

$\overline{\Lambda}_{EFT} \approx$ point where 1 reached.

$$\text{eg } k\cot\delta(\Lambda) = \underbrace{\sum_{n=n_{min}}^{n_0} \mathcal{A}_n(k_{low})\, Q^n}_{\text{cutoff-independent by construction.}} + \underbrace{\mathcal{A}_{n_0+1}(\Lambda)}_{\text{cutoff-dependent}}\, Q^{n_0+1} + \dots$$

$$\Rightarrow \frac{k\cot\delta(\Lambda) - k\cot\delta(\Lambda')}{k\cot\delta(\Lambda)} = \#\, Q^{n_0+1}$$

$\hookrightarrow$ dimension-less number

(4) Order-by-order closer to <u>Nature</u>:

should lie within the bands assigned by Q^n-corrections.

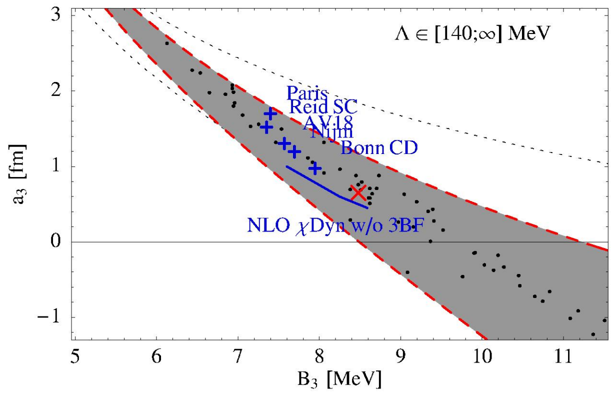
But this is a constraint imposed from <u>outside</u>, not from <u>self</u>-consistency.

$\rightarrow$ (may) determine power-counting.

(h) EFT Uncovers Correlations

Efimov 1988, Bedaque et al 1999-, Platter/Hammer/Meißner 2005,...

EFT: Interactions from Consistency instead of Phenomenology.



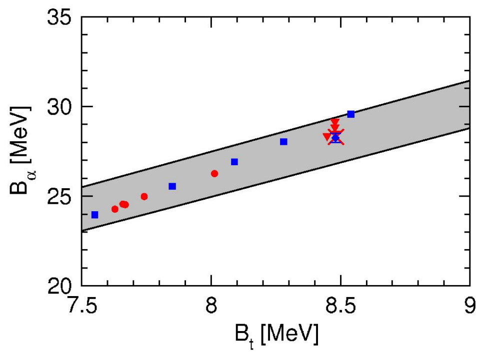
$H_0(\Lambda_*)$ explains **Phillips line** (1969):

2N-data insufficient to predict 3N observables, but

correlation in $^2S_{\frac{1}{2}}$: scatt. length $\Longleftrightarrow$ binding energy

Traditional potentials: ad-hoc 3BF for correct value.

Sets number of new parameters at given accuracy.



^{4}He at LO cutoff-independent without 4BF **Platter et al. 2004**

$\Longrightarrow$ no 4NF at LO, correlations ^{3}H $\Longleftrightarrow$ ^{4}He

explains e.g. **Tjon line** (1975) of binding energies

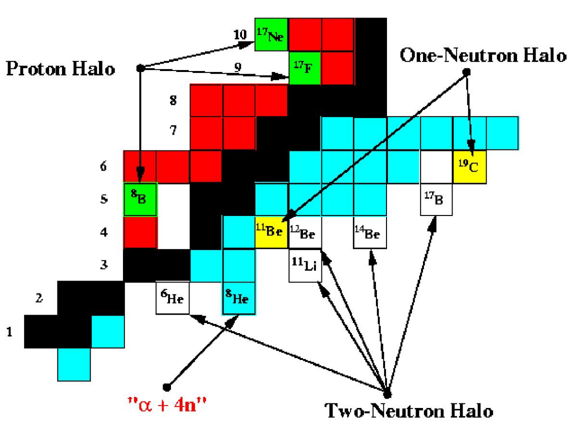
Separate between "trivial" and "interesting" observables.

(i) EFT of Point-Like Interactions: The Lessons of Universality

Universal aspects of systems with anomalously large 2-body scattering length constrain models & data.

Hypertriton Λnp: $B_3 = [0.13 \pm 0.05]$ MeV (exp.) $\Longrightarrow a_{\Lambda d} = 16.8^{+4.4}_{-2.4}$ fm, $r_{\Lambda d} = [2.3 \pm 0.3]$ fm **Hammer 2001**

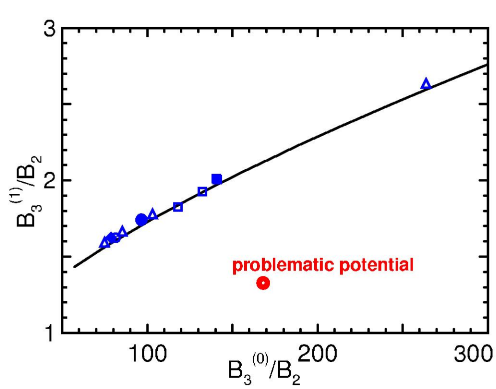
Halo Nuclei: Borromean, Samba, Tango,... **Bertulani/..., Frederico/..., Jansen/..., Bedaque/...,...2002-**



Molecular Dimer ^{4}He$_2$ **Bedaque/Hammer/van Kolck 1999**

scatt. length $a = 104$ Å $\gg$ eff. range $r_0 = 7$ Å

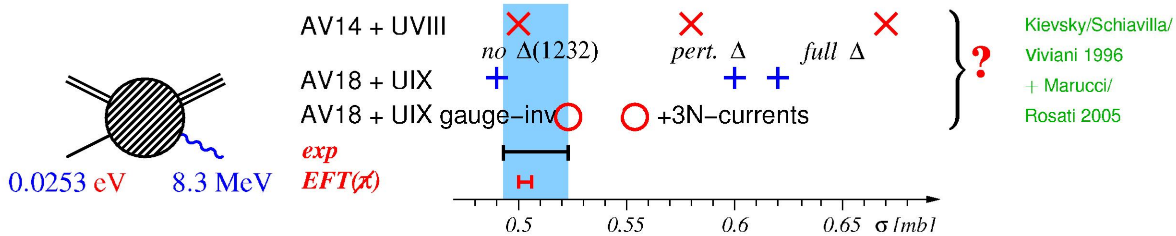
Correlations sort experiments and models.



(j) The Problem With Triton Radiative Capture

Sadeghi/hg/Bayegan: *Phys. Lett.* **B643** (2006), 263

A Problem Solved: $nd \to t\gamma$ at thermal energies: $E_n = 0.0253$ eV



experiment Jurney 1982 $[0.508 \pm 0.015]$ mb

Merritt 1968: $[0.521 \pm 0.009]$ mb
Jurney 1963: $[0.60 \pm 0.05]$ mb

N^{2}LO EFT($\not\pi$) $[0.503 \pm 0.003]$ mb $= [\underbrace{0.485}_{\text{LO}} + \underbrace{0.011}_{\text{NLO}} + \underbrace{0.007}_{\text{N}^2\text{LO}}]$ mb

– Prediction: **No new 3BFs up to N^{3}LO**. H_0, H_2 fixed by B_3, a_3.

– Manifestly gauge-invariant.

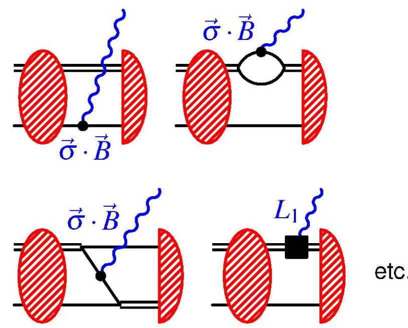
– $M1$-transition dominates, S-wave only.

– Cut-off independence: $\pm 10^{-5}$ for $\Lambda \in [150; 500]$ MeV.

– Short-distance term $L_1\, \vec{d} \cdot \vec{B}\, s_3$ from thermal $np \to d\gamma$.

– γ polarisation $R_c = -[0.412 \pm 0.003]$ Sadeghi 2007

exp: $-[0.42 \ \pm 0.03\ ]$ Konijnenberg/…1998



New problem: Models with same input should give same result.

4. Today's Summary: "Pion-less Theory"

EFT-Cookbook applied to few-body systems with **large scattering lengths/shallow bound-states**.

EFT($\not\pi$) is ***the*** EFT of QCD at very low energies: local interactions between nucleons only.

Principle: effective degrees of freedom + symmetries + power-counting.

Power-counting in non-perturbative EFT at LO is more than counting derivatives:

Constructed from one, qualitative input: **shallow bound-state**. Could not (yet) derive from QCD.

Criterion: LECs/3BFs only as counter-term for cutoff-independence of observables. Renormalisability.

Model-independence for data analysis, predictions & model-constraining.

Convergence-checks for error-estimates: Order by order smaller cut-off dep., corrections, "Lepage plots",…

Simplicity: Few parameters, determined by simple observables, often analytical results.

Correlations in observables signal few-N forces; EFT($\not\pi$) to separate between "trivial" and "interesting" Physics.

Limit Cycle is new RG phenomenon beyond relevant/marginal/irrelevant operators.

Universal to systems with large scattering lengths:

Exotic and hyper-nuclei/radioactive beams, molecular systems, Bose-Einstein Condensates,…

Plethora of pivotal physical processes for prediction & extraction of fundamental nucleon properties, e.g.:

d-Compton, $\nu/\bar{\nu}$-d scattering, $pp \to de^+\nu_e$, $ed \to ed \& enp$, $nd \to t\gamma$, big-bang nucleo-synthesis, properties of ^{3}H and ^{3}He; neutrinos and light nuclei (calibrating SNO), A_y-problem,…

$\Longrightarrow$ **Next lecture: Pions and Nucleons.**

Effective Field Theories in Few-Nucleon Systems

Lecture III: A Dash into χEFT: Pions and A Few Nucleons

H. W. Grießhammer

Center for Nuclear Studies
The George Washington University, DC, USA

Learning from other people's mistakes: Non-perturbative power-counting with pions.

Biased review & what's cooking right now.

Alternative worlds: Controlling the pion-mass in χEFT and on the lattice.

What holds the nucleus together?

In the past quarter century physicists have devoted a huge amount of experimentation and mental labor to this problem – probably more man-hours than have been given to any other scientific question in the history of mankind. [. . .]

The glue that holds the nucleus together must be a kind of force utterly different from any we yet know.

HANS A. BETHE: "What holds the nucleus together?", *Scientific American* **189** (1953), no. 2, p. 58

1. Pion-ful Physics: Chiral EFT

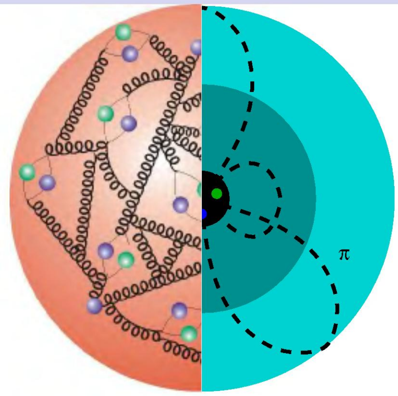
Wilson, Weinberg, ...

(a) Effective Field Theory from QCD

– Theory of strong interactions: Quantum Chromo Dynamics QCD

$$\mathcal{L}_{\text{QCD}} = \bar{\Psi}_q [\mathrm{i}\, \not{\partial} + g \not{A} - m_q] \Psi_q - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}$$

$\Longrightarrow$ Effective low-energy degrees of freedom: Nucleons, Pions, $\Delta(1232)$

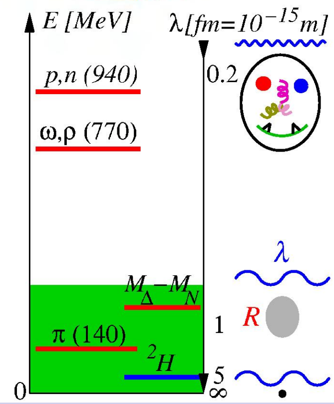


Systematic ordering in $Q = \dfrac{\text{typ. momentum } m_\pi}{\text{breakdown scale 1 GeV}} \approx \dfrac{1}{5 \ldots 7}$

$$\mathcal{L}_{\chi\text{EFT}} = (D_\mu \pi^a)(D^\mu \pi^a) - m_\pi^2\, \pi^a \pi^a + \ldots$$

$$+ N^\dagger [\mathrm{i}\, D_0 + \frac{\vec{D}^2}{2M} + \frac{g_A}{2 f_\pi}\, \vec{\sigma} \cdot \vec{D}\pi + \ldots] N$$

$$+ C_0 \left(N^\dagger N\right)^2 + \cdots + H_0 \left(N^\dagger N\right)^3 + \ldots$$



Symmetries constrain:

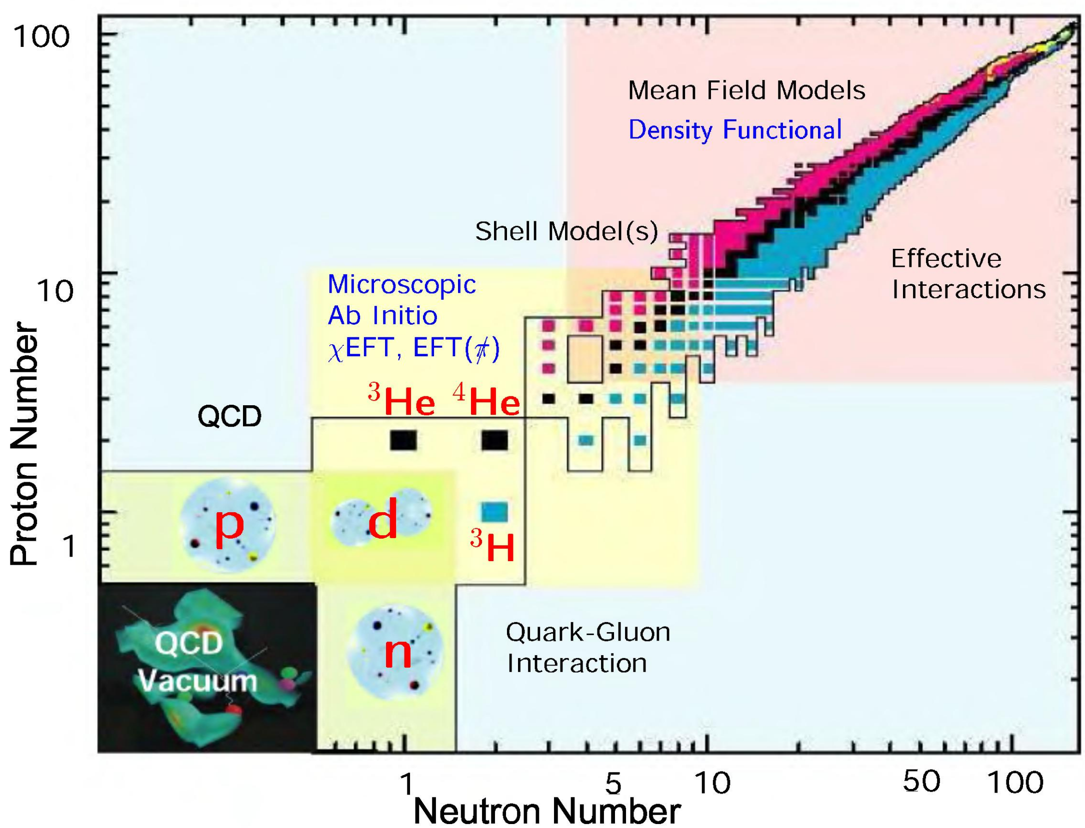
– gauge, Lorentz, iso-spin, ...

– chiral: pions light & weakly coupled: Goldstone bosons of Chiral SSB

$\Longrightarrow$ Chiral Effective Field Theory χEFT $\equiv$ low-energy QCD

Parameterisation of ignorance: determine short-distance coefficients.

(b) The Nuclear Chart from QCD



b) Chiral Symmetry in 30 Seconds (see Scherer in Bibliography for more)

$$\mathcal{L}_{QCD} = \bar{\Psi}_q [i\not{\partial} - m_q] \Psi_q + \dots, \quad \Psi_q = \begin{pmatrix} u \\ d \end{pmatrix} \text{ for 2 quarks}$$

for massless quarks ($m_u \sim m_d \sim 5$ MeV $\ll$ 1 GeV: typ. QCD scale)
invariant under the following global flavour rotations:

$$\Psi_q \to e^{i\sigma^a \xi^a} \Psi_q \quad \text{and} \quad \Psi_q \to e^{i\gamma_5 \sigma^a \xi^a} \Psi_q$$

σ^a ↳ Pauli matrix; ↳ changes parity!

or, equivalently, under separate SU(2) trafos of the right- & left-handed quarks: symmetry group $SU(2)_L \times SU(2)_R$

Chiral Symmetry in flavour space.

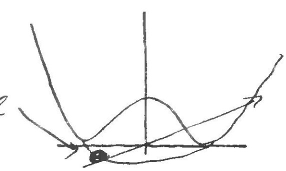
Noether's theorem: symmetry ⇒ conservation law

QCD spectrum should show doublets of mesons with same spin & mass, but different parity.

Nature: pion = lowest pseudo-scalar meson $m_\pi = 140$ MeV
f^0 = lowest scalar meson $m_{f_0} = 600$ MeV ↯

⇒ Chiral symmetry is realised in the Goldstone-mode: spontaneous breaking of global symmetry.

Example Champagne-bottle potential



ground state ≙ a point on bottom of well
motion along bottom of well without effort
≙ massless excitation: Goldstone boson

Pion is ~massless on typ. QCD scale ~1 GeV
⇒ interpret pion as the field excitation along the well (nonlinear because well is bent): $e^{i\sigma^a \pi^a(t,\vec{r})}$

This leads to derivative-couplings between pions, and between pions and nucleons.

⊕ nonlinear field ⇒ constraints on multi-pion couplings like 

All from <u>chiral symmetry of QCD</u>.

⇒ chiral perturbation theory (EFT for pions) (Weinberg 1980–)

baryon chiral perturbation theory (–″–)
(EFT for π's & N's)

Nonzero pion mass from small <u>explicit</u> breaking of chiral symmetry by $m_u \sim m_d \sim 5\,\mathrm{MeV} \neq 0$.

$\hat{=}$ slight "tilt" in potential well
: even oscillations along well cost some effort

$$\Rightarrow \quad m_\pi^2 = -\frac{2 m_q}{f_\pi^2} \underbrace{\langle \bar{q} q \rangle}_{\text{quark condensate}}$$ <u>(Gell-Mann-Oakes-Renner)</u>

<That's all I want to say here – if you want more, consult the references!!>

2. The χEFT Concept

a) Power-Counting in χEFT

- LO LEC in EFT($\not\pi$) : $\times = C_0^{\not\pi} = \frac{4\pi}{M} \frac{1}{\gamma - \mu}$

Increase resolution beyond $p_{typ} \sim \Lambda_{\not\pi} \sim m_\pi$

$\Rightarrow$ pion-exchange resolved as nonlocal.

- Question: How much of $\times|_{\not\pi}$ is coming from  ?

- Consistency argument: Whatever the interaction is, it has to scale as Q^{-1}:

only phenomenological input: non-relativistic system with shallow (real/virtual) bound-state (as in EFT($\not\pi$)).

$\Rightarrow$ all terms must be of same order in PC

if not, we could expand $\Rightarrow$ finite # of diagrams $\Rightarrow$ no boundstate.

$\Rightarrow$ Ansatz  $\sim Q^n$

$$Q^n = Q^n + Q^n \frac{Q^5}{Q^2 Q^2} Q^n \quad \Rightarrow n = -1$$

:  $\sim Q^{-1}$ irrespective of potential used.

EFT($\not\pi$) : $\times \; C_0 \sim Q^{-1}$ ✓

χEFT : additionally one-pion exchange

 $= -\frac{g_A^2}{4 f_\pi^2} \; \vec{\tau}_1 \cdot \vec{\tau}_2 \; \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{\vec{q}^{\,2} + m_\pi^2}$

with $g_A = 1.26$, $f_\pi = 93$ MeV : pion-decay constant.

strengths fixed from πN scattering.

New low-energy parameter m_π which is related to χSB & quark-masses:

Gell-Mann-Oakes-Renner $\quad m_\pi^2 = -\frac{2 m_q}{f_\pi^2} \langle \bar{q} q \rangle$

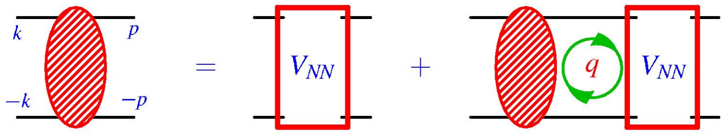
$\hookrightarrow$ quark-condensate

(a) Messages for the Chiral Power-Counting

Weinberg 1991, van Kolck 1992-; cf. hg forthcoming

Only phenomenological input: $\boxed{\text{Non-relativistic system with shallow (real/virtual) bound-state.}}$

$$\boxed{T_{NN}(E \sim \frac{p^2,k^2}{M}) \sim Q^{-1}}$$

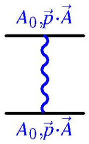


$\sim Q^{-1}$

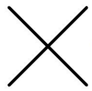
Power-Counting: Q^m $\quad Q^m \quad Q^{2m+3-2} \stackrel{!}{=} Q^m \quad \Longrightarrow m = -1$

Examples: NRQCD/NRQED **EFT($\not\pi$)** **χEFT**

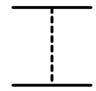
Coulomb



$\sim v^{-1}$



$C_0 \sim Q^{-1}$ $+$



$$-\frac{g_A^2}{4f_\pi^2}\,\vec{\tau}_1\cdot\vec{\tau}_2\,\frac{\vec{\sigma}_1\cdot\vec{q}\;\vec{\sigma}_2\cdot\vec{q}}{\vec{q}^{\,2}+m_\pi^2} \sim Q^{-1}$$

$$\boxed{T_{NN} \text{ non-perturbative only in bound-state dynamics: } E \sim \frac{Q^2}{M}}$$

– 3 Scenarios were considered.

(1) Kaplan-Savage-Wise 1997

only count powers of q in $\Rightarrow \sim Q^0$:

: NLO against X ~ LO, i.e. most of C_0^{π} remains in C_0^{π}

Can be shown to be consistently renormalisable.

Consider $^1S_0$: $\Big|_{^1S_0} = -\frac{g_A^2}{4f_\pi^2}\,\frac{\vec{q}^{\,2}}{\vec{q}^{\,2}+m_\pi^2} = -\frac{g_A^2}{4f_\pi^2}\left[1-\frac{m_\pi^2}{\vec{q}^{\,2}+m_\pi^2}\right]$

independent of m_π $\Rightarrow$ chiral-symmetry conserving

depends on m_π $\Rightarrow \chi$SB-sensitive

Now consider functional dependence on m_π only: "switch $m_\pi \to 0$ off":

(m_π is additional LEC!, choosable at will by tuning m_q)

$\Rightarrow$ only contact piece X $= -\frac{g_A^2}{4f_\pi^2}$ remains,

which has same structure as X $(C_0^{\pi}(\mu)) \Rightarrow$ absorb into it, μ-dependent.

$\Rightarrow$ LO is X $C_0^{\pi}(\mu)$, NLO is $\propto \frac{m_\pi^2}{\vec{q}^{\,2}+m_\pi^2}$, etc.

$\hookrightarrow \frac{1}{r}$ - potential ✓

Correct PC in singlet-channel, with correct m_π-dependence

dictated by χSB.

Fails to match phenomenology in triplet-waves: Nehen/Stewart 2000

(2) Weinberg 1991: "Resum now, ask questions later." (Mike Birse: The Texas Approach)

Phenomenological proposal: long-range dominated by one-pion exchange.

χPT for mesons & one nucleon has no unnatural scales $\Rightarrow$ NDA = counting derivatives.

Take the V_{NN} constructed by these rules to a certain order and

insert it into the integral equation above.

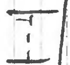
$\rightarrow$ pursued by van Kolck 1991–, Epelbaum/.. 1999–, Machleidt/..2000–

to "$Q^4 = N^3LO$". Excellent phenomenological results.

Problem: if $\propto \frac{(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})}{\vec{q}^{\,2}+m_\pi^2} \sim Q^0$, then no reason to resum besides Weinberg's word.

(3) Construction from self-consistency: Beane/Bedaque/Savage/van Kolck 2002, Nogga/Timmermans/van Kolck 2005, Birse 2005–;

Triplet-channel: one-pion exchange in chiral limit $m_\pi \to 0$:



$$\Big|_{\text{triplet}} \propto -\frac{q^i q^j}{\vec{q}^{\,2}} \quad \text{independent of } m_\pi \text{ (strongest contribution)}$$

$$\xrightarrow[\text{into coordinate space}]{\text{Fourier}} \left(\pm\frac{1}{r^3}, \pm\frac{1}{r^2}, \pm\frac{1}{r}\right) - \text{potential}$$

↳ like Coulomb: regulation unnecessary

↳ attractive $\frac{1}{r^2}$ gives limit-cycle.

$\Rightarrow$ need C_0^π ($\hat{=} H_0$!)

↳ if more attractive, limit-cycle-like behaviour

$\Rightarrow$ need C_0^π, but also more:

Renormalisability: Attraction in low partial waves $^3P_{0,2}$ & $^3D_{2,3}$

so strong that it needs additional LECs to avoid

Λ-dependence of phase-shifts (= observables).

$\Rightarrow$  $\sim p^2 C_2(^3P_{0,2}; {}^3D_{2,3})$ with correct partial-wave structure

must appear at LO, although $\propto p^2 \sim Q^2$ naively:

Again: PC in nonperturbative EFT is more than just counting p's.

: (4) more LECs at LO than Weinberg expected.

Higher orders, repulsive waves → ongoing research.

Understanding m_π-dependence is important for lattice-extractions of few-nucleon-properties! → Bedaque/Beane/Savage/... (NPLQCD collaboration) 2006–

(b) Even Weinberg Can Be Wrong

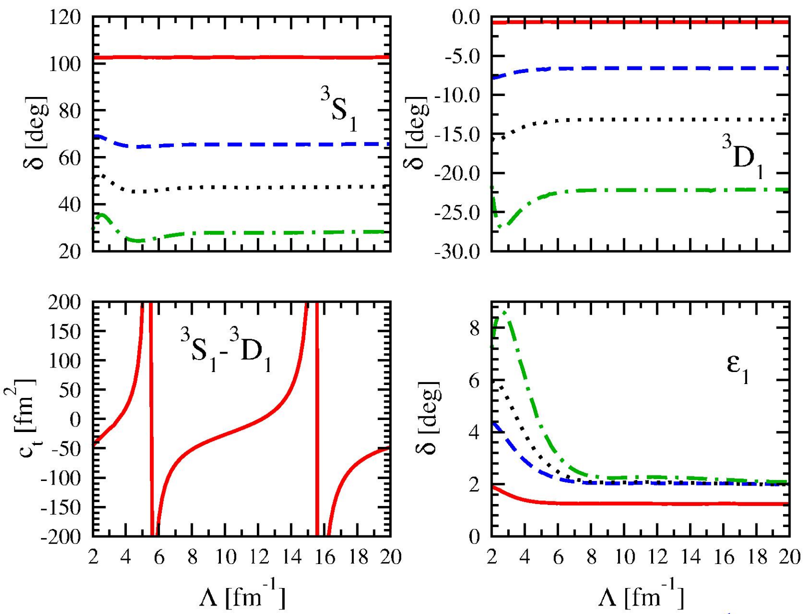
Beane/. . . 2002, Nogga/Timmermans/van Kolck 2005, Birse 2005-07

Check consistency of Weinberg's proposal: Observables cut-off dependent?

$^3S_1$-$^3D_1$**-channel: Potential** $\propto -\dfrac{q^i q^j}{q^2} \to -\dfrac{1}{r^3} \implies$ **Weinberg predicts one LEC**

phase-shift
δ(cut-off Λ) at

- (solid) $E_{\text{lab}} = 10\,\text{MeV}$
- (dashed) $50\,\text{MeV}$
- (dotted) $100\,\text{MeV}$
- (dash-dot) $190\,\text{MeV}$



Similar for all repulsive waves & attractive singlets $(\pm\frac{1}{r})$.

(b) Even Weinberg Can Be Wrong

Beane/. . . 2002, Nogga/Timmermans/van Kolck 2005, Birse 2005-07

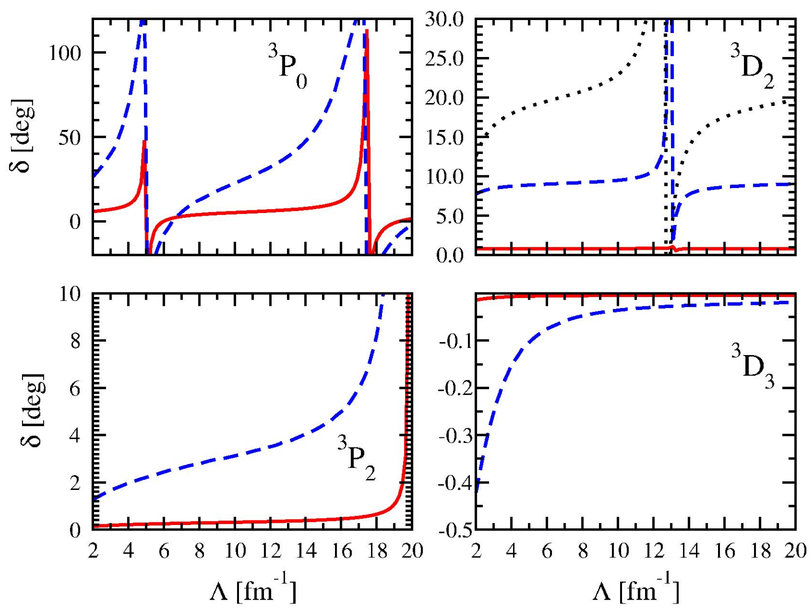
Check consistency of Weinberg's proposal: Observables cut-off dependent?

Low attractive triplets: Weinberg predicts zero LEC at LO (momentum-independence)

phase-shift
δ(cut-off Λ) at

- (solid) $E_{\text{lab}} = 10\,\text{MeV}$
- (dashed) $50\,\text{MeV}$
- (dotted) $100\,\text{MeV}$
- (dash-dot) $190\,\text{MeV}$

$$V(r) \propto -\frac{1}{r^3} + \frac{j(j+1)}{r^2}$$



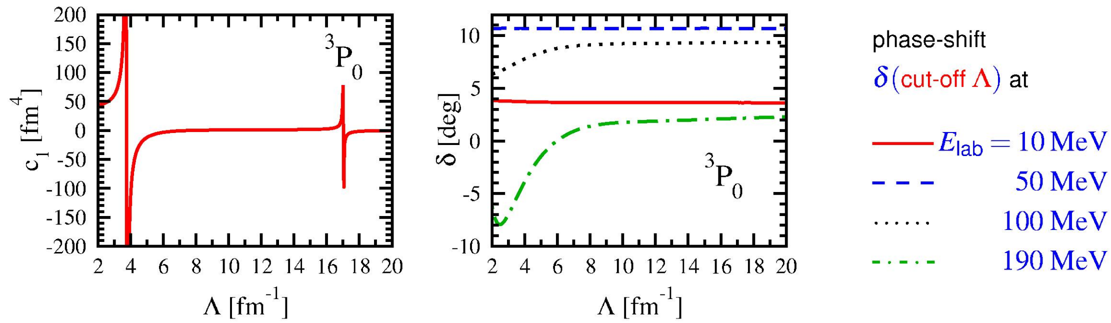
Cutoff-dependent $\implies$ Short-distance missing!

(b) Even Weinberg Can Be Wrong

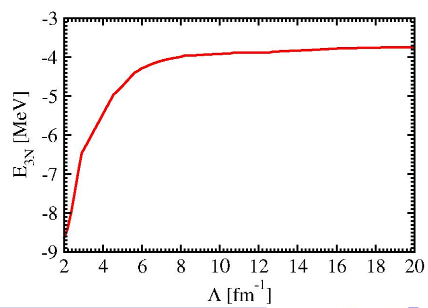
Beane/. . . 2002, Nogga/Timmermans/van Kolck 2005, Birse 2005-07

Check consistency of Weinberg's proposal: Observables cut-off dependent?

Need 4 new, momentum-dependent LECs for low attractive triplets: $^3\mathrm{P}_{0,2}$, $^3\mathrm{D}_{2,3}$



Extension to higher orders in progress.



Triton binding-energy cutoff-independent.

$\Longrightarrow$ Pointlike 3NF in EFT($\not\pi$) resolved at LO as one-pion exchange in χEFT.

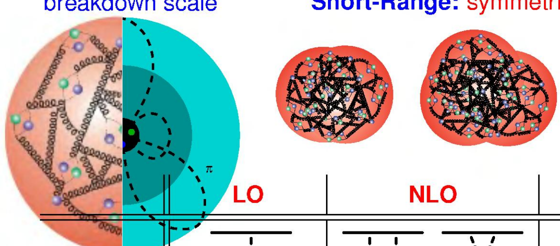
(c) Few-Nucleon Interactions in χEFT

Weinberg, Ordóñez/Ray/van Kolck, Friar/Coon, Kaiser/Brockmann/Weise, Epelbaum/Glöckle/Meißner, Entem/Machleidt, Kaiser, Higa/Robilotta, Epelbaum, . . .

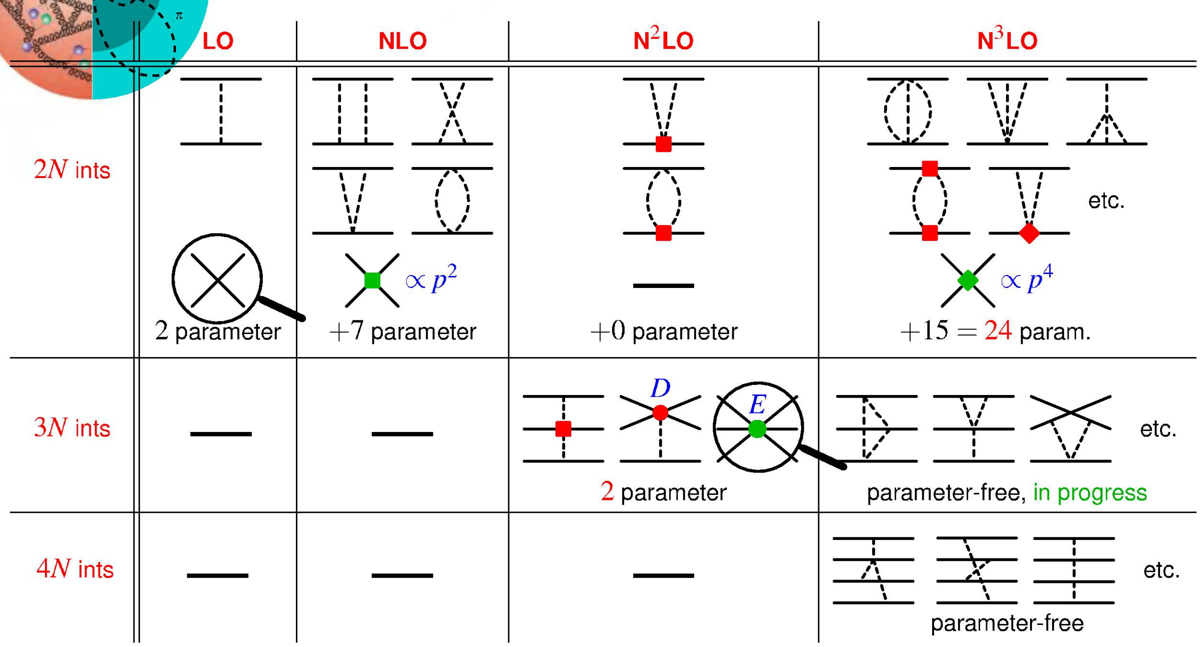
$\dfrac{\text{typ. momentum}}{\text{breakdown scale}} \ll 1$

Long-Range: correct symmetries and IR degrees of freedom: **Chiral Dynamics**

Short-Range: symmetries constrain contact-ints to simplify UV: **Minimal parameter-set**

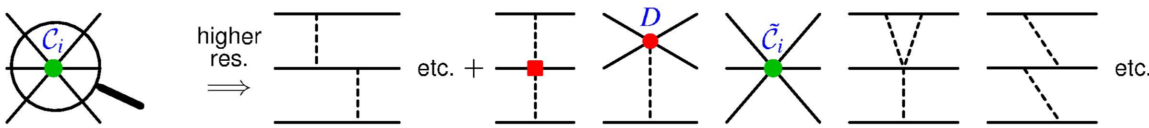


Hierarchy: 2NF-effects $\gg$ 3NF-effects $\gg$ 4NF-effects



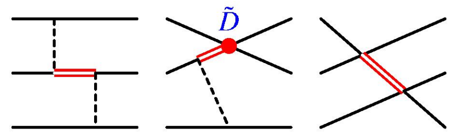
(d) A Question of Resolution

Importance of $\mathcal{C}_i$'s changes with resolution $\Longleftrightarrow$ active degrees of freedom.



"pion-less version" 2-N reducible "$\Delta(1232)$-less version"

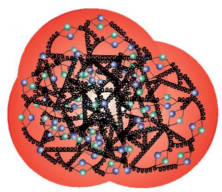
higher res. $\Longrightarrow$ the above (**new numbers!**) + more 2-N reducible graphs +



etc.

"$\Delta(1232)$-ful version"

higher res. $\Longrightarrow$



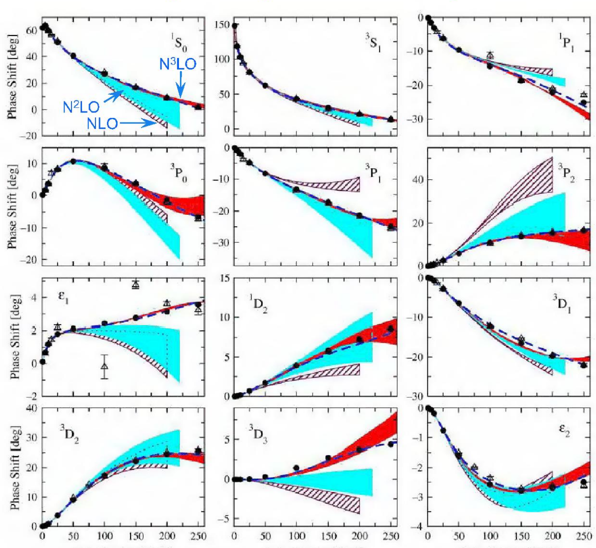
QCD: 9 valence quarks in volume $\lesssim$ resolution^3

3. χEFT: "Selected" (Biased) Applications

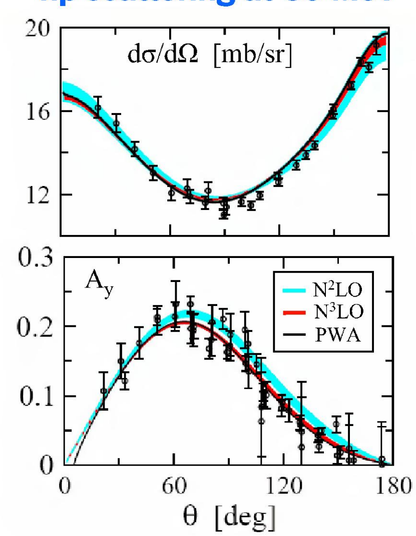
(a) Two Nucleons in χEFT

Entem/Machleidt 2003, Epelbaum/Glöckle/Meißner 2005

Neutron-proton phase shifts up to N^{3}LO



np scattering at 50 MeV



Bands estimate higher-order effects.

	LO	NLO	N^{2}LO	N^{3}LO	AV 18
# of parameters	2	$+7$	$+0$	$+15=24$	~ 40
χ^2/d.o.f in np		36.2	10.1	1.10	1.04

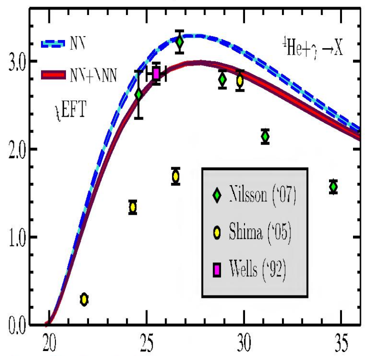
Future: High-accuracy deuteron physics, Dynamical $\Delta(1232)$, P, weak, iso-spin breaking, $T=\frac{3}{2}$ 3NFs,...

(b) Few-Nucleon χEFT: A Merger of Opportunities

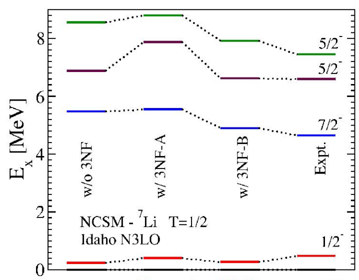
$A \geq 4$: Merge χEFT with well-developed but sophisticated numerical techniques.

Explored by several new collaborations, for example:

Total Photo-Absorption on ^{4}He



Quaglioni/…2007 ω [MeV]



Nogga/…2006

AGS-equations Platter/Hammer 2005- ^{4}He: universal correlations,…

NCSM Navrátil, Quaglioni, Nogga, Stetcu, Barrett, Vary, van Kolck, …2005-

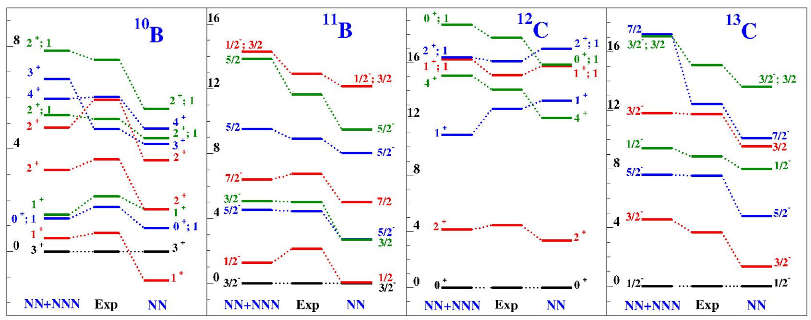
Spectra, radii, halo-nuclei,…also in EFT($\not\pi$)

EIHH-Lorentz-Integral-Transform Leidemann/Orlandini/…, +hg/…

Coupled-Cluster Method D. Dean/…2006-

Resonating Group Model Hofmann/hg/…

χEFT on the Lattice D. Lee/Th. Schäfer/Borasoy/Epelbaum/Krebs/Meißner 2006-

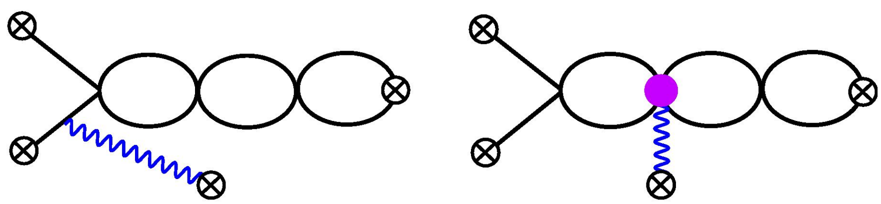


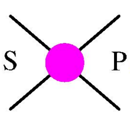
Navrátil/…2007

(c) Bottom-Up: Flavour-Conserving Parity-Violations

Kaplan/Savage 1993; Zhu/Maekawa/Holstein/Ramsey-Musolf/van Kolck 2005; …; Review Ramsey-Musolf/Page 2006; Giusti 2008

EFT($\not\pi$): lowest partial waves, simplest interactions



S  P $\propto p$ 5 parameters $\sim \dfrac{G_F f_\pi^2}{2\sqrt{2}} \sim 10^{-7}$:

$\rho_t^{\not\pi}$: $^3\mathrm{S}_1(I=0)$–$^3\mathrm{P}_1(I=0)$

$\lambda_t^{\not\pi}$: $^3\mathrm{S}_1(I=0)$–$^1\mathrm{P}_1(I=0)$

$\lambda_s^{\not\pi(0,1,2)}$: $^1\mathrm{S}_0(I=1)$–$^3\mathrm{P}_1(I=1)$ with $\Delta I = 0,1,2$

$\Longrightarrow$ Fix by >5 experiments sensitive to different combinations, e.g.:

experiment	$h_\pi^{(1)}$, ρ_t	λ_t	$\lambda_s^{(0)}$	$\lambda_s^{(1)}$	$\lambda_s^{(2)}$
	coefficient in front of				
$\vec{p}p$	$\otimes$	$\otimes$	$4k/M$	$4k/M$	$4k/M$
$\vec{p}^4$He	-1.1	-0.5	-0.7	-0.5	$\otimes$
$np \leftrightarrow d\ \overset{\circlearrowleft}{\gamma}$	$\otimes$	0.6	-0.2	$\otimes$	0.3
$\vec{n}p \to d\gamma$	-0.1	$\otimes$	$\otimes$	$\otimes$	$\otimes$
$\vec{n}^4$He spin rot.	-2.7	1.2	1.8	-1.2	$\otimes$
$\vec{n}d \to {}^3$Hγ	-3.6	-1.4	-1	-0.2	1.2

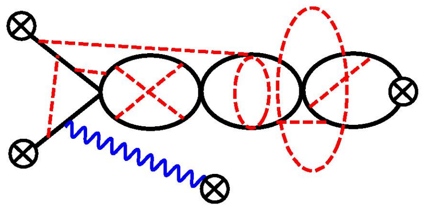
To dis-entangle & check data consistency, need ***accurate*** description of **strong force: EFT($\not\pi$), χEFT**

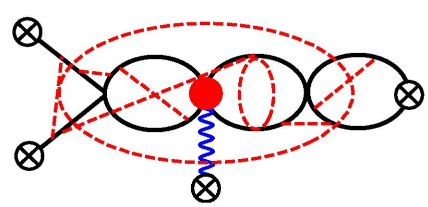
Starting-point for theory of many-body P-violation.

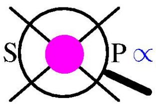
(c) Bottom-Up: Flavour-Conserving Parity-Violations

Kaplan/Savage 1993; Zhu/Maekawa/Holstein/Ramsey-Musolf/van Kolck 2005; ...; Review Ramsey-Musolf/Page 2006; Giusti 2008

χEFT: more resolution for more insight







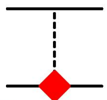
5 parameters $\sim \dfrac{G_F f_\pi^2}{2\sqrt{2}} \sim 10^{-7}$:

$\rho_t^{\not\pi}$: ${}^3\mathrm{S}_1(I=0)$–${}^3\mathrm{P}_1(I=0)$

$\lambda_t^{\not\pi}$: ${}^3\mathrm{S}_1(I=0)$–${}^1\mathrm{P}_1(I=0)$

$\lambda_s^{\not\pi(0,1,2)}$: ${}^1\mathrm{S}_0(I=1)$–${}^3\mathrm{P}_1(I=1)$ with $\Delta I = 0,1,2$

$\Longrightarrow$ **LO** ($\mathcal{O}(Q^{-1})$):

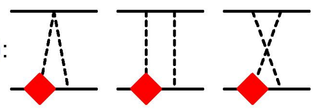


$$-\mathrm{i}\,h_\pi^{(1)}\,\frac{g_A}{\sqrt{2}f_\pi}\left(\frac{\vec{\tau}_1\times\vec{\tau}_2}{2}\right)_z\frac{(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\vec{q}}{\vec{q}^2+m_\pi^2}$$

long-range: P-violating 1π exchange

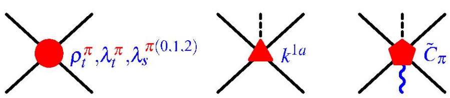
$-\mathrm{i}h_\pi^{(1)}[\bar{p}\pi^+ n + \mathrm{H.c.}]$

N^{2}LO ($\mathcal{O}(Q^1)$, $\lesssim 10\%$):



medium-range: P-violating 2π exchange (Chiral symmetry) parameter-free

$+$



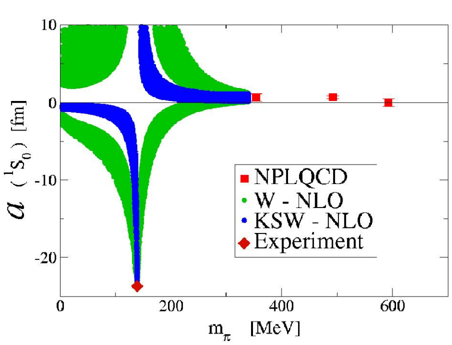
short-range: same structure as EFT($\not\pi$), different values!
resonance saturation (Desplanques/... "DDH")?

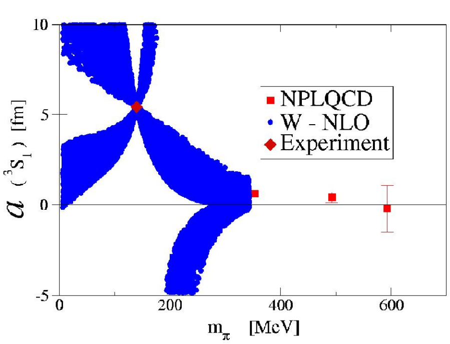
$\Longrightarrow$ **1 parameter $h_\pi^{(1)}$ to $\lesssim 10\%$ accuracy, $+7$ to $\lesssim 1\%$; no P-violating 3NI.**

(d) χEFT, Few-N Interactions and QCD Lattice Simulations: Alternative Worlds

χEFT: long-range correlations well understood, short-distance QCD encoded in minimal parameter-set.

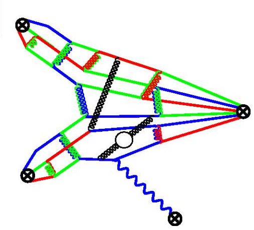
$\Longrightarrow$ χEFT: Chiral symmetry dictates extrapolation in $m_q \propto m_\pi^2, \dots$, volume, lattice-spacing.





m_π-dependence of NN-scatt. lengths from MILC-lattices (unquenched)

NPLQCD 2006



Future: Fully dynamical simulations utilising χEFT for long-range part.

- Explain fine-tuning of NN-scattering lengths, origin of few-N int.'s
- Verify NN-potential from first principles — cf. Ishii/Aoki/Hatsuda 2006 (quenched)
- Fix parameters hard to determine experimentally: **weak int.'s** test SM; πNN- & YN-couplings...
- Implications of QCD-parameter changes on Nuclear Physics, BBN etc. — e.g. Kneller/McLaughlin 2003

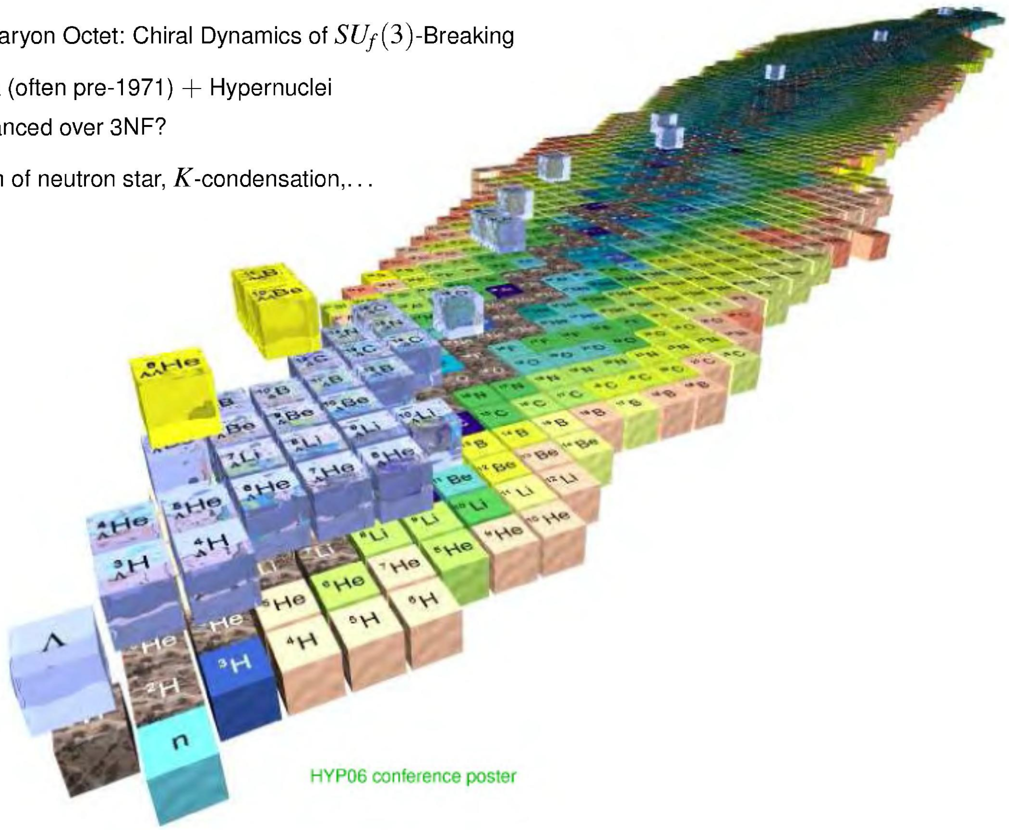
(e) Hypernuclei: “Third Axis” of the Nuclear Chart

Strangeness & Baryon Octet: Chiral Dynamics of $SU_f(3)$-Breaking

Only 35 YN data (often pre-1971) $+$ Hypernuclei

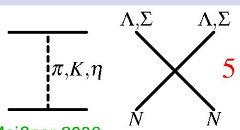
YNN, YYN enhanced over 3NF?

$\Longrightarrow$ Composition of neutron star, K-condensation,. . .



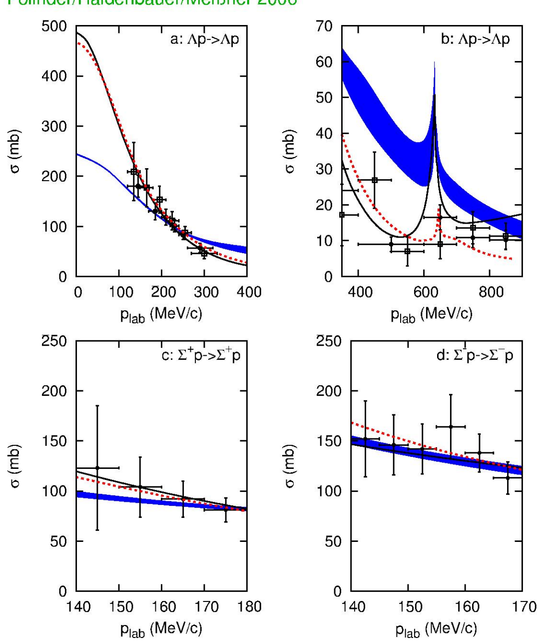
(e) Hypernuclei: “Third Axis” of the Nuclear Chart

LO χEFT for YN:

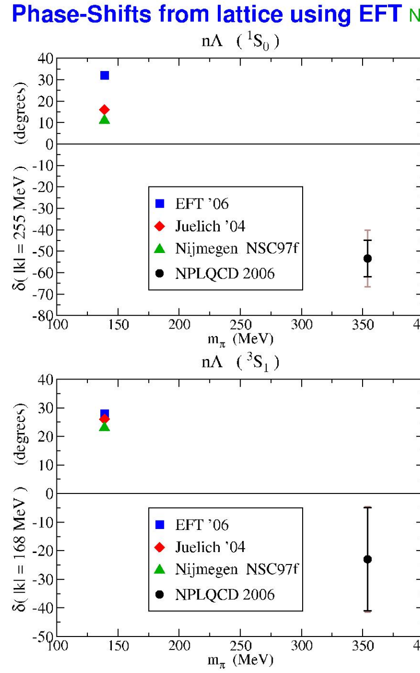


5 parameter

Polinder/Haidenbauer/Meißner 2006



Phase-Shifts from lattice using EFT NPLQCD 2006



$B({}^3_\Lambda\mathrm{H}) = 2.35 \pm 0.01$ MeV χEFT

$B({}^3_\Lambda\mathrm{H}) = 2.35 \pm 0.05$ MeV exp

Determine unknowns from lattice, feed into χEFT, predict.

4. The Big Picture: Error-Bars for Nuclear Physics!

χEFT and EFT($\not\pi$) as low-energy QCD

Unified, systematic description, rooted in QCD.

Universally parameterise short-range int's.

Bridge from (lattice) QCD to Nuclear Structure.

Quantitative studies of few-N systems

Reliable **predictions & extractions** for light nuclei:

neutron properties, iso-spin & P-violation, ...

Unique signals of chirality, 3NF, 4NF,... $\Longleftrightarrow$ QCD

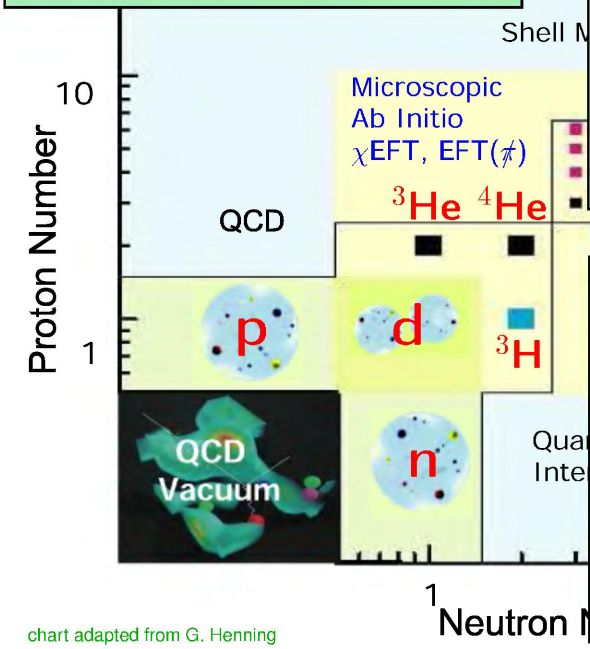
Frontiers of Stability: Fine-Tuning? FAIR@GSI, RIA,...

Halo-nuclei $^{5\text{-}10}$He,... ; Hyper-nuclei $^{3,4}_{\Lambda}$H, $^{4}_{\Lambda}$He,... ; ...

χEFT vs. EFT($\not\pi$): **Chiral Dynamics** or **Universality**?

Need for accurate low-energy data:

Determine parameters, test theory!



Reliable predictions for processes hard-to-access:

Astrophysics: Uncertainties Big-Bang Nucleosynthesis:

$d(p,\gamma)^3$He, ^{3}He$(d,p)^4$He, ^{3}He$(^4$He$,\gamma)^7$Be, ...

Protostar-collapse, supernovae, neutron stars,...

Neutrinos: e.g. hep-process $^3\text{He} + p \to {}^4\text{He} + e^+ + \nu_e$

Physics beyond the Standard Model

Alternative Worlds: spectra, rates, BBN,... sensitive to

(time-variation of) QCD-parameters $m_q,\ \alpha_s,\ N_c, \ldots$

chart adapted from G. Henning

4. The Big Picture: Error-Bars for Nuclear Physics!

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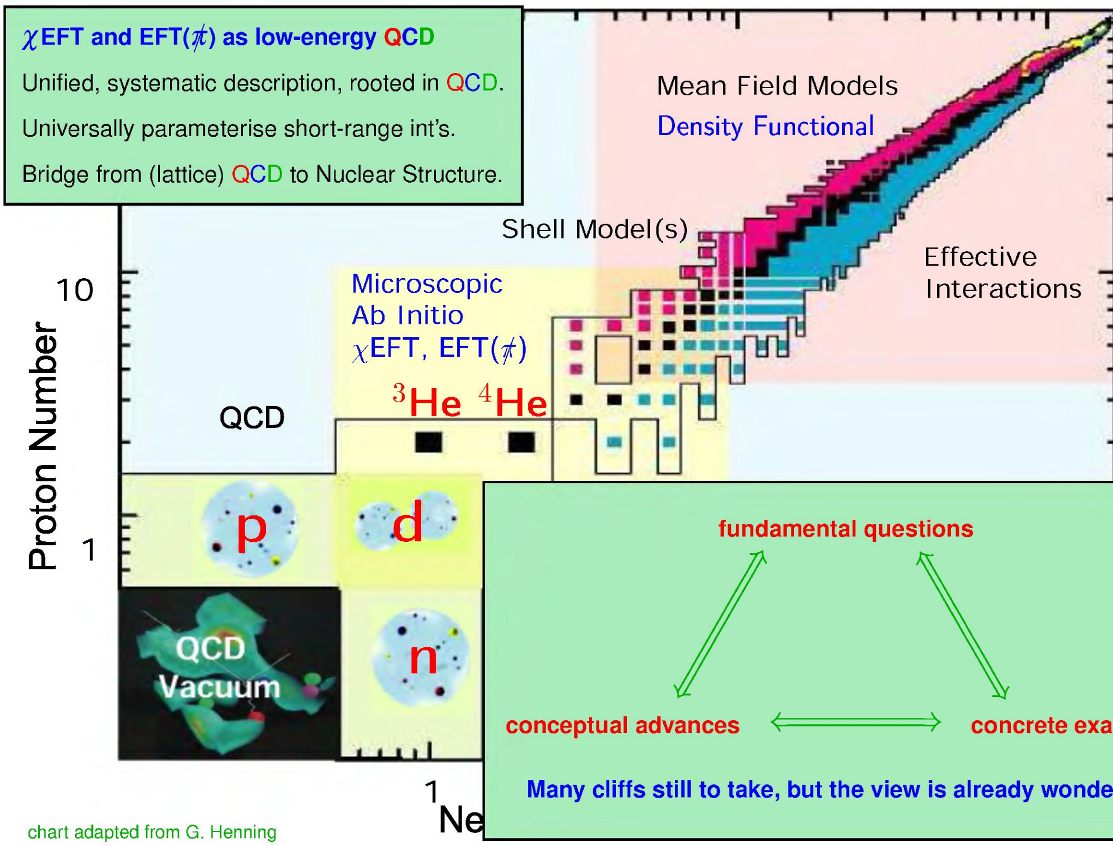
Universally parameterise short-range int's.

Bridge from (lattice) QCD to Nuclear Structure.



fundamental questions

conceptual advances $\Longleftrightarrow$ **concrete examples**

Many cliffs still to take, but the view is already wonderful!

chart adapted from G. Henning