Nuclei as Mesoscopic Systems

Alexander Volya

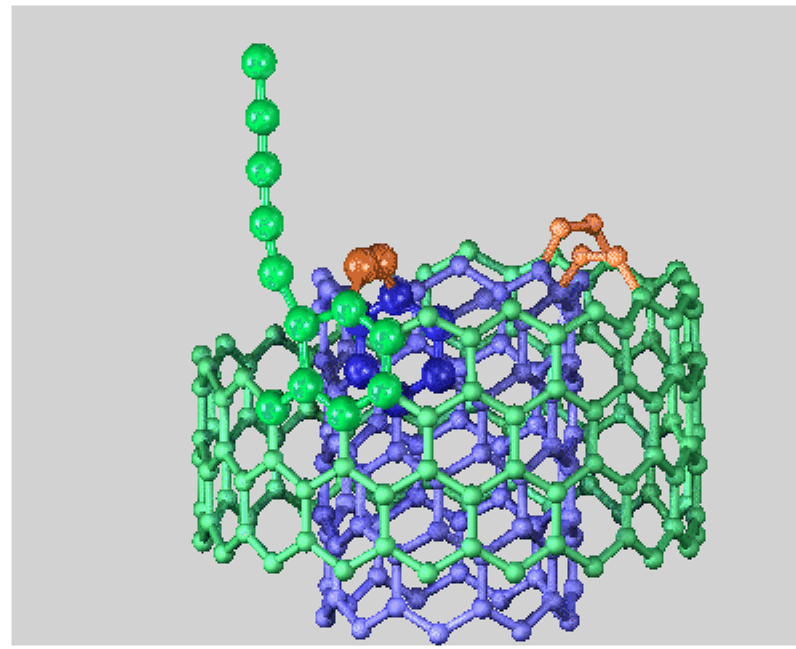
Florida State University

Mesoscopic system

- Quantum many-body system between microscopic (few-body) and macroscopic (thermodynamic limit)
- Quantum many-body system with identifiable individual quantum states, while sufficiently large to reveal regularities of statistical nature.
- Emergence of complexity.

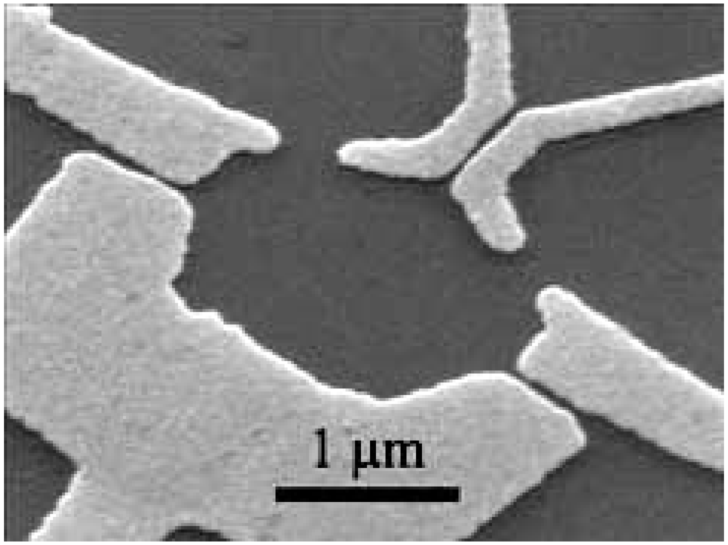
A rich variety of mesoscopic systems

- Nano-wires
- Quantum dots
- Helium drops
- Atomic clusters
- Quantum computers

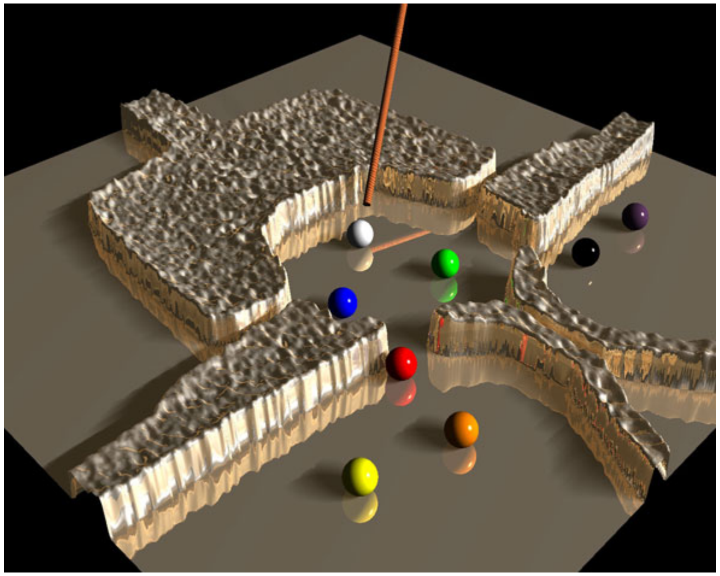


http://www.pa.msu.edu/~tomanek/

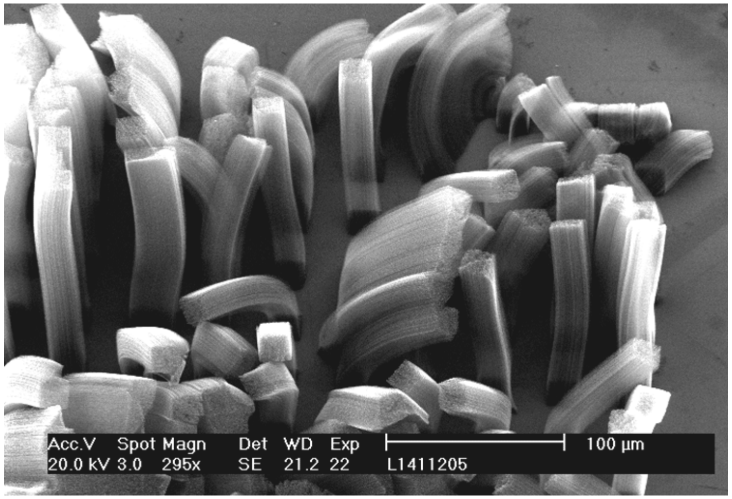
- **Quantum Dot** : 5 metallic gates fabricated on the surface of a GaAs; two dimensional electron gas inside.

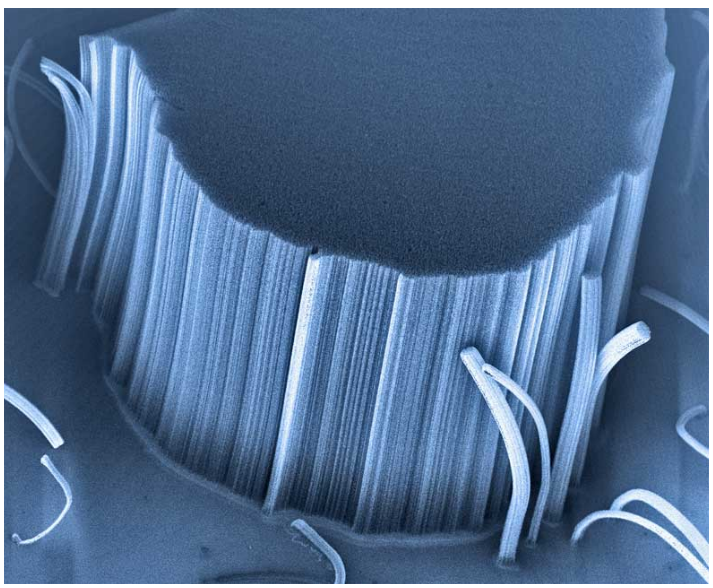


- quantum dot can be seen as a cavity in which electrons bounce at the boundaries similar to a billiard table.



Nanotubes





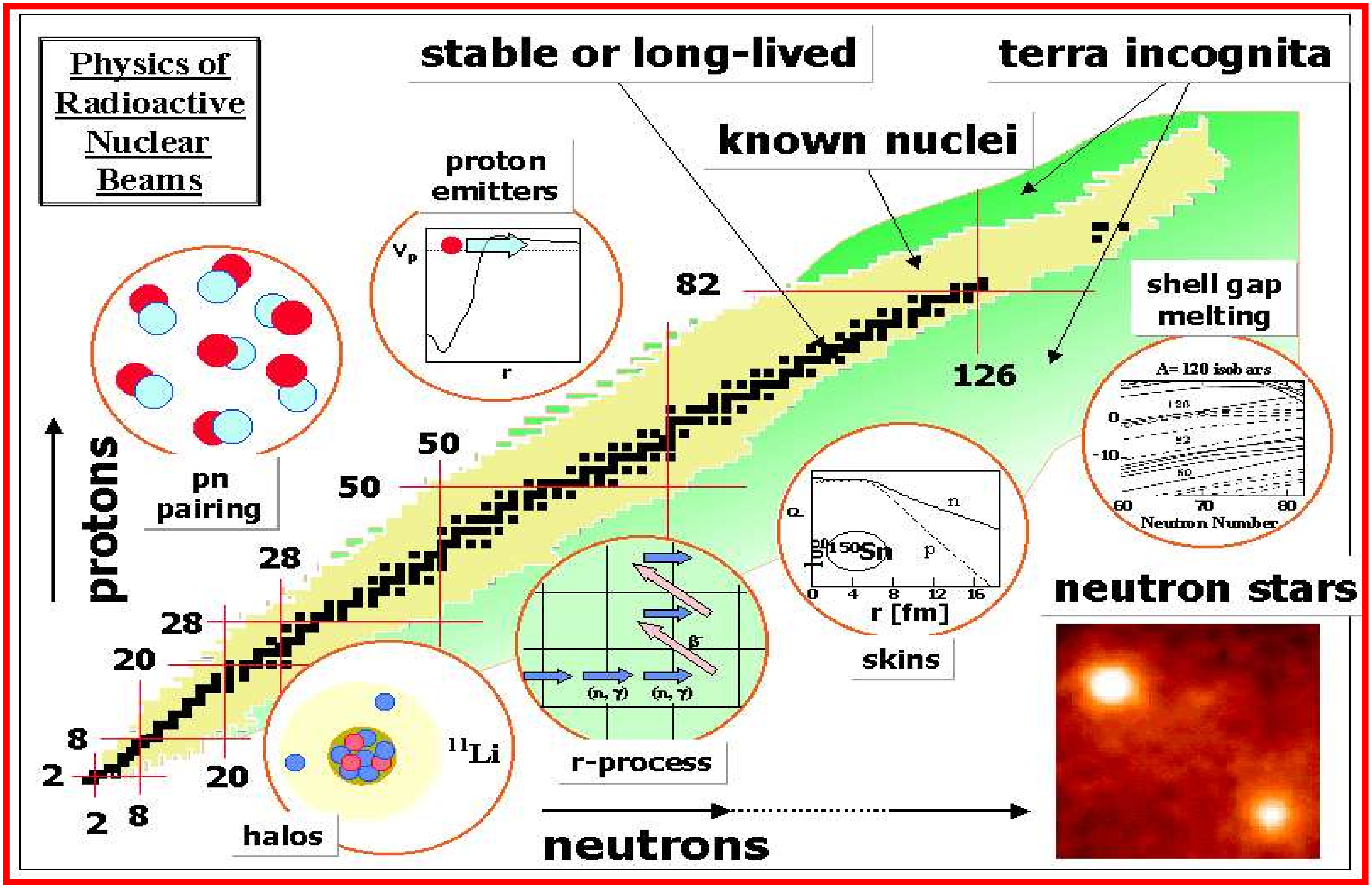
http://pages.unibas.ch/phys-meso/

The nuclear world: the rich variety of natural mesoscopic phenomena

- Predicted: 6000 - 7000 particle-stable nuclides
- Observed: 2932
- even-even 737; odd-A 1469; odd-odd 726.
- Lightest $^{2}_{1}H_{1}$ (deuteron), Heaviest $^{294}_{118}(?)_{176}$
- No gamma-rays known 785.
- Largest number of levels known (578) $^{40}_{20}Ca_{20}$
- Largest number of transitions known 1319 $^{53}_{25}Mn_{28}$
- Highest multipolarity of electromagnetic transition E6 in

  $^{53}_{26}Fe_{27},\ 19/2^{-}\ (3040\,\mathrm{keV}) \rightarrow 7/2^{-}(\mathrm{g.s.}); 2.58\ \mathrm{min}$
- Resut of 100 years of reasearch 182000 citations in Brookhaven database, 4500 new entries per year.

Nuclear Chart

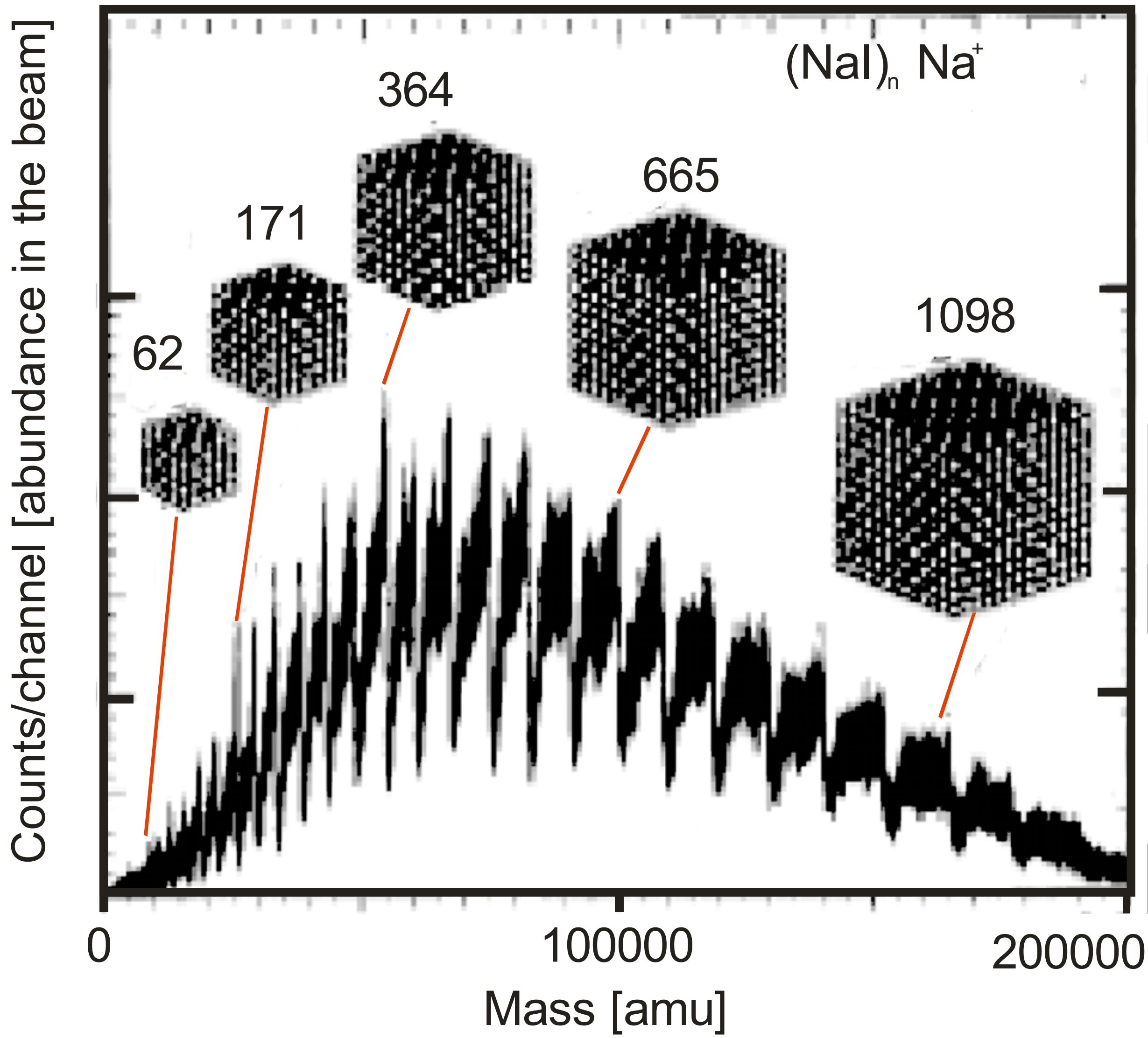


Single-Particle Motion

- Symmetry, surface and shells
- Shells and supershells
- Single-particle modes and magic numbers
- Symmetry and chaos
- Classical periodic orbits

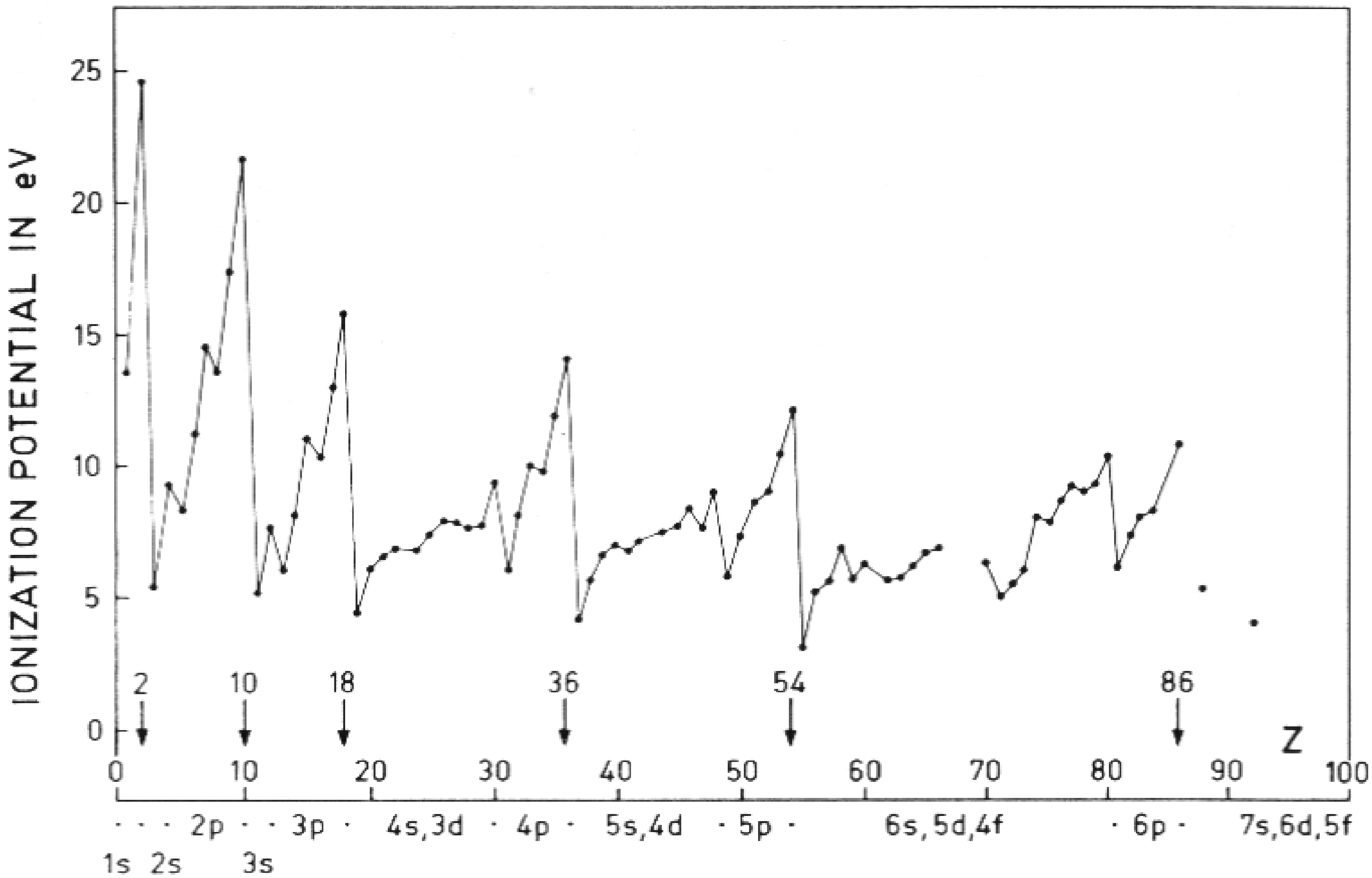
Salt Clusters, transition from small to bulk

- Symmetry
- Surface
- “Shells”



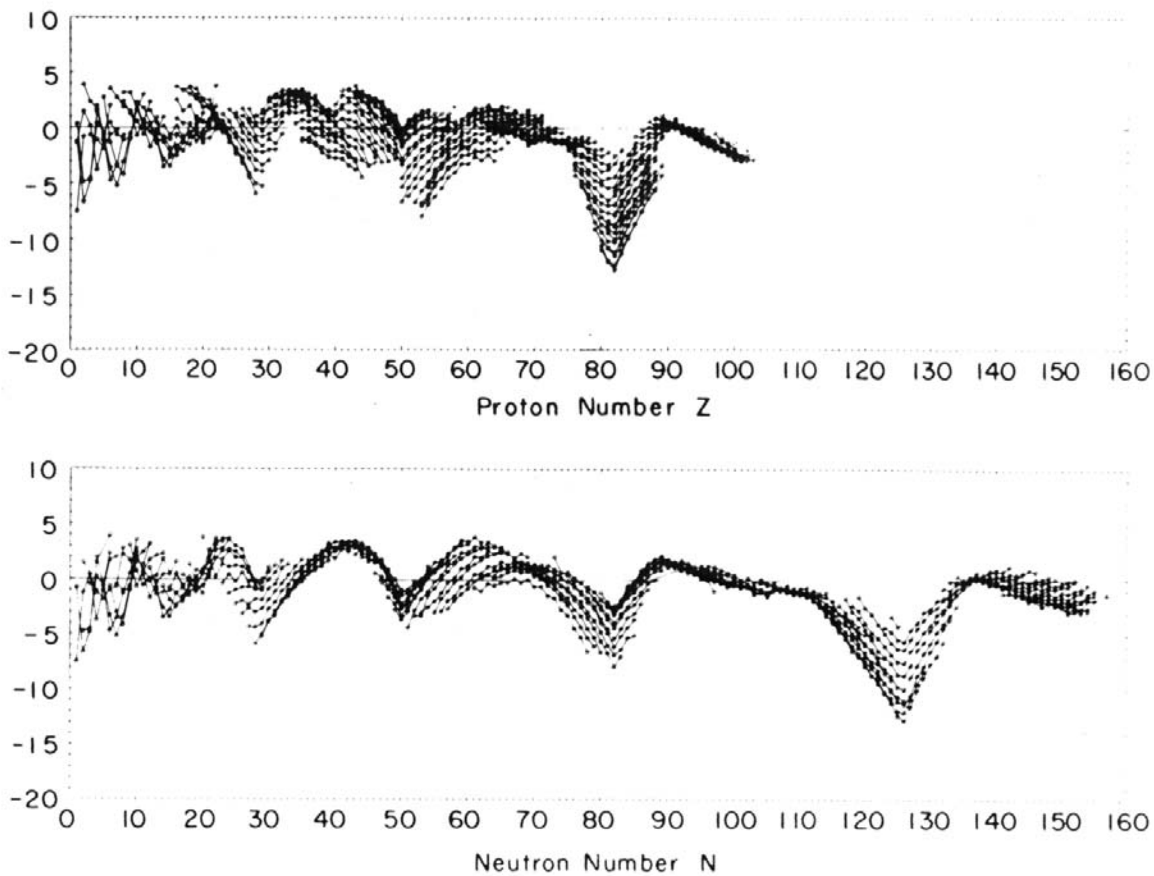
T. P.Martin Physics Reports **273** (1966) 199-241

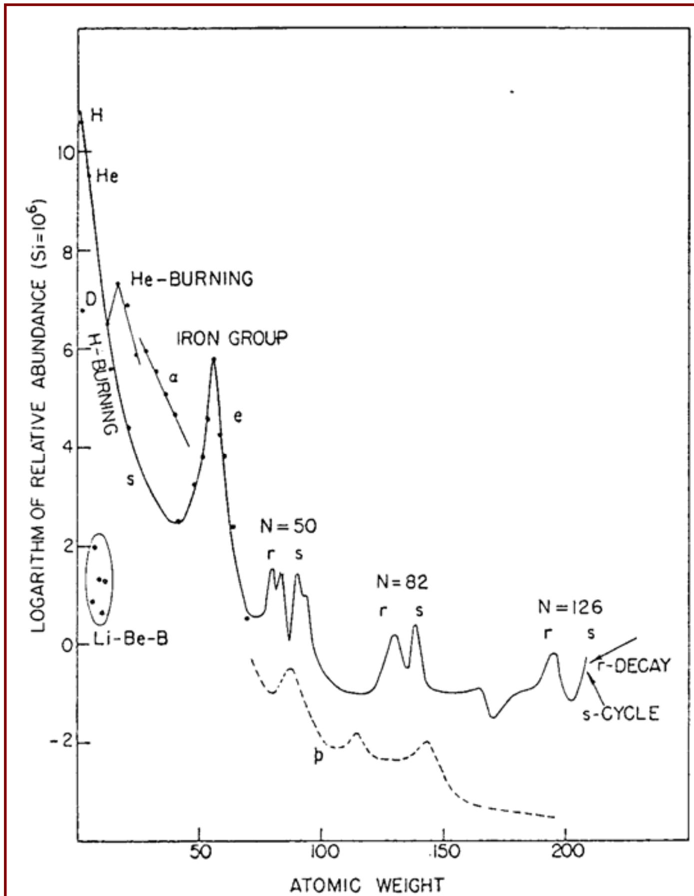
Shell Structure in atoms



From A. Bohr and B.R.Mottleson, *Nuclear Structure*, vol. 1, p. 191 Benjamin, 1969, New York

Nuclear Magic Numbers, nucleon packaging, stability, abundance of elements

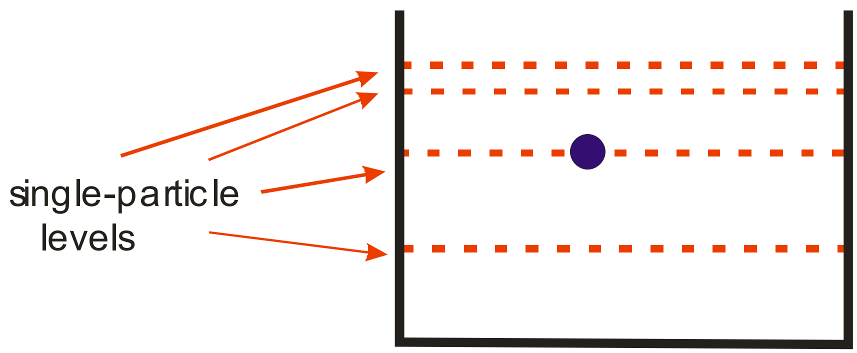


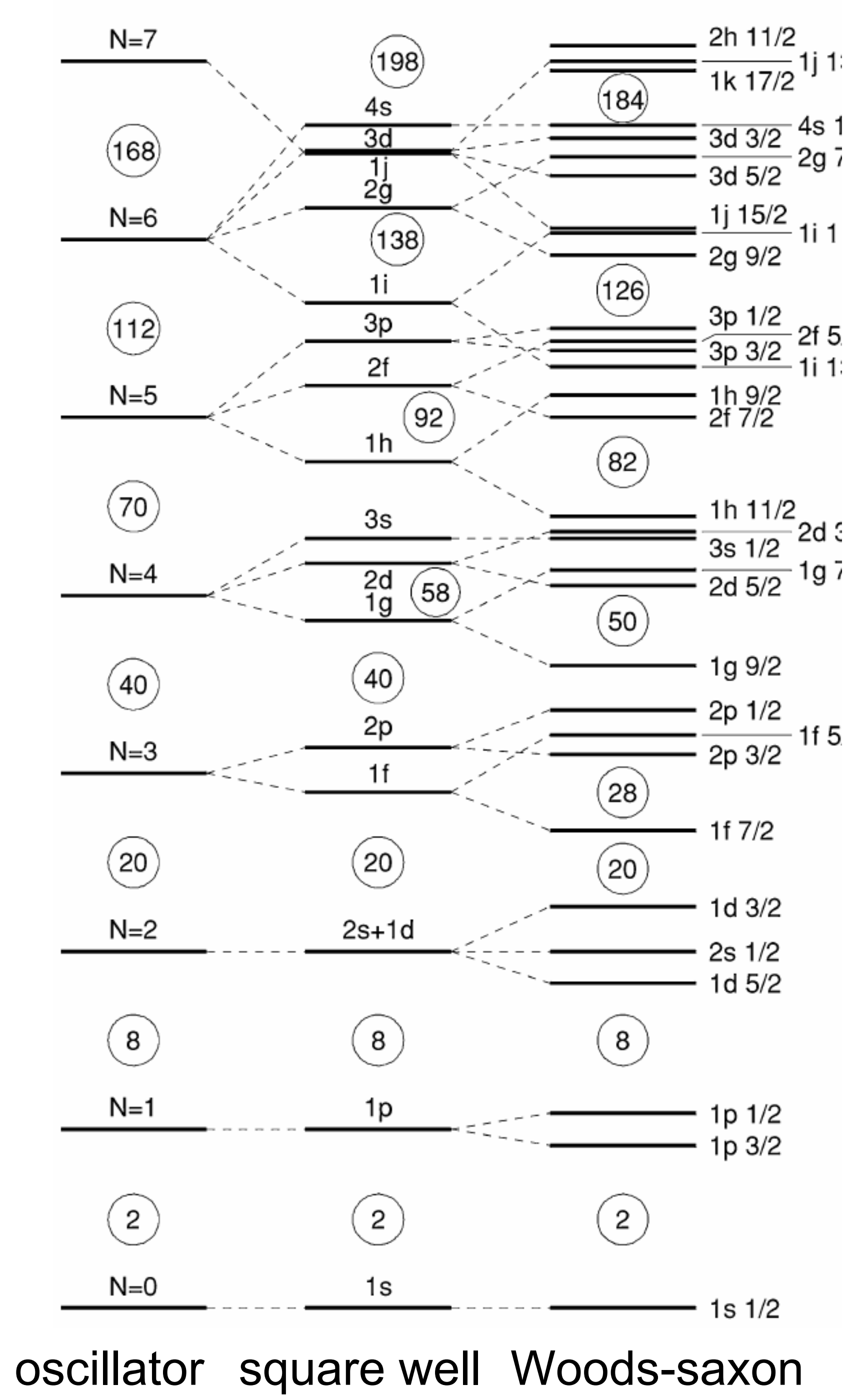


From W.D. Myers and W.J. Swiatecki, Nucl. Phys. **81**, 1 (1966)

Mean field

Nucleon in a box

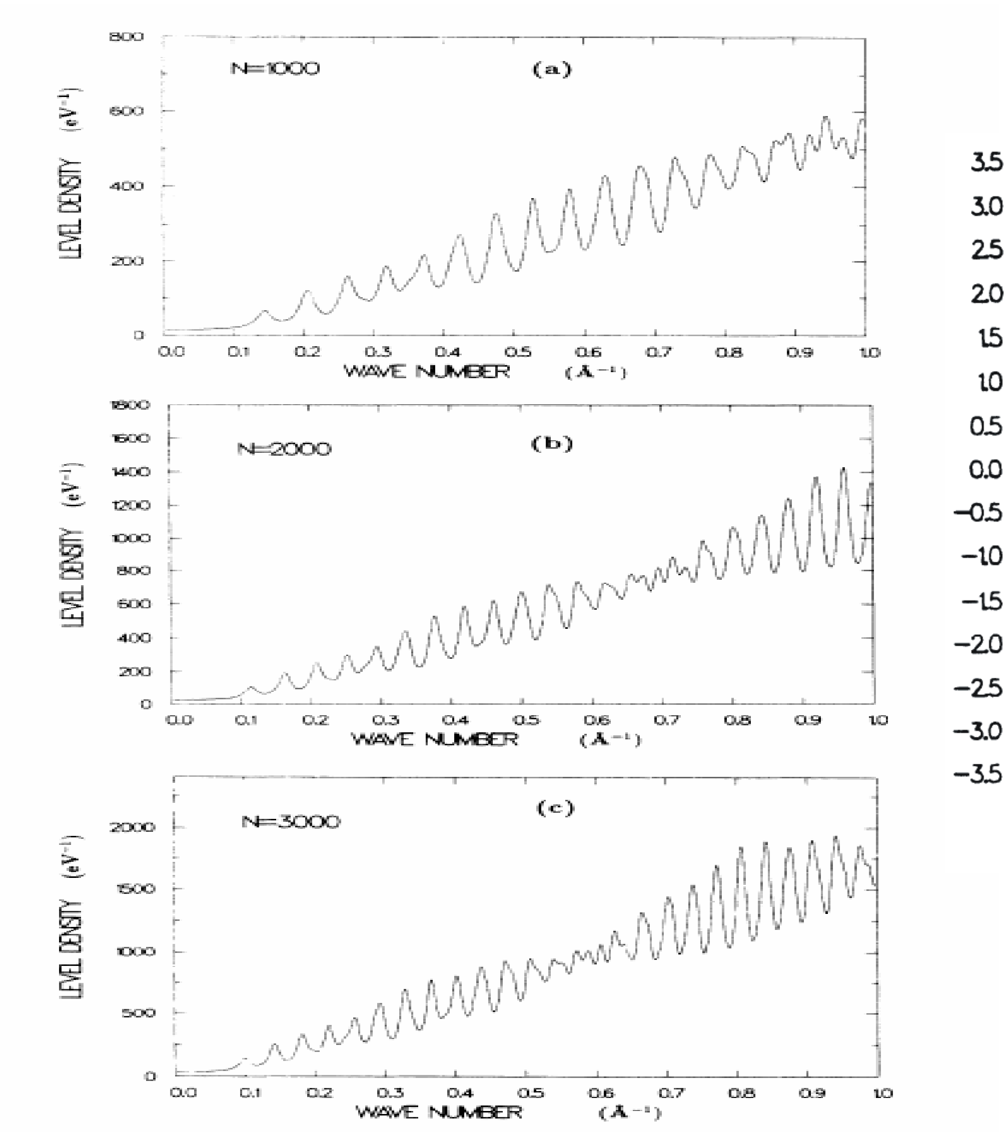




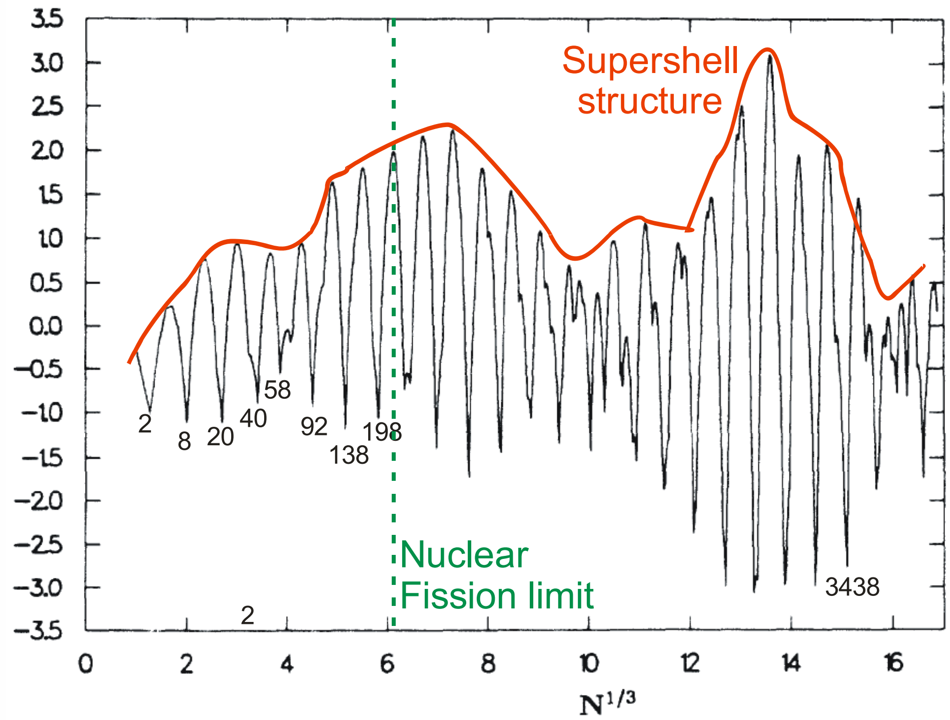
Nuclear Woods-Saxon solver
http://www.volya.net/ws/

Shell gaps N=2,8,20,

Supershells



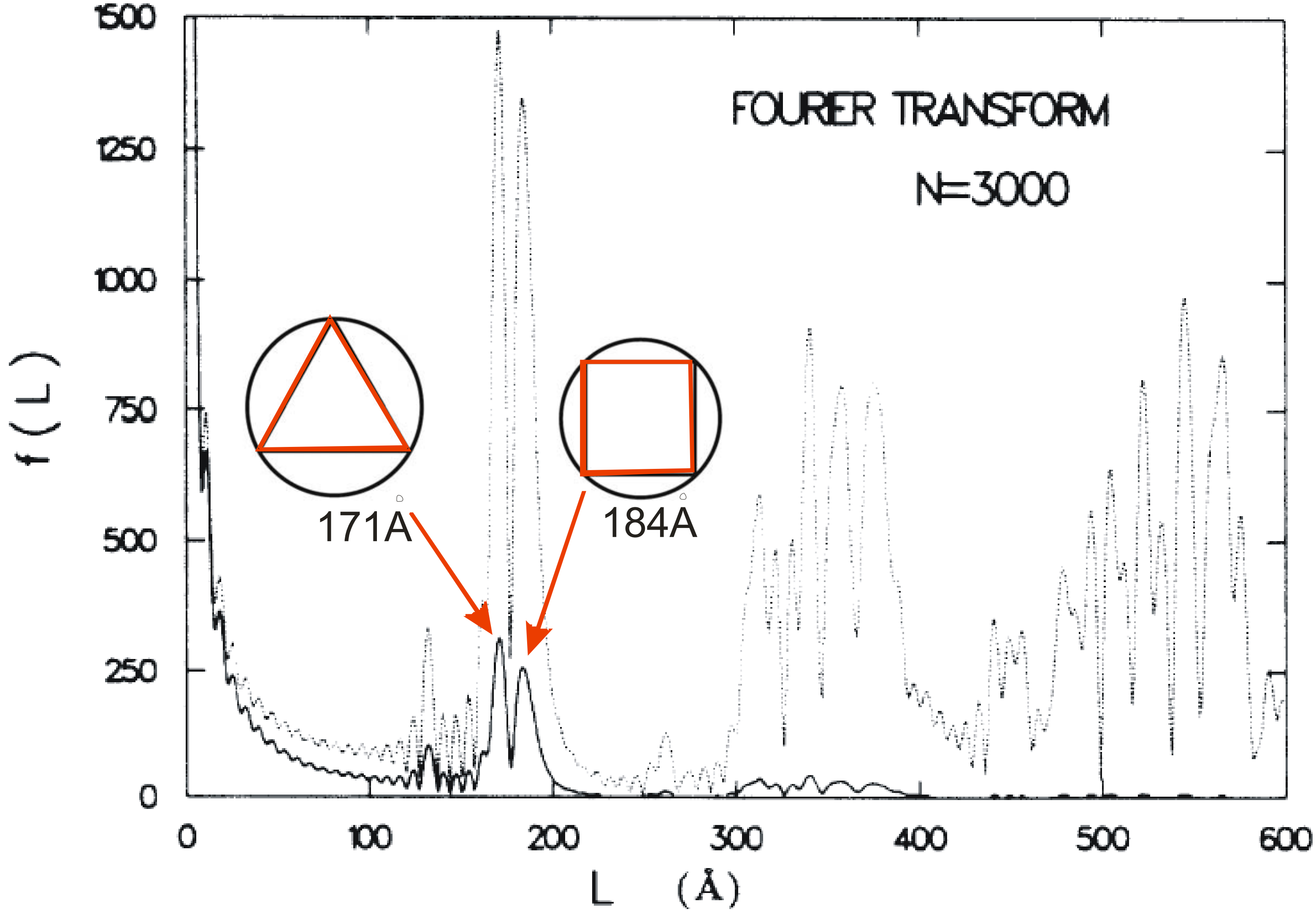
Level density in the Woods-Saxon Potential: N=1000, 2000, and 3000



Binding energy, deviation from average

Nishioka et. al. Phys. Rev. B **42**, (1990) 9377
R.B. Balian, C. Block Ann. Phys. **69** (1971) 76

Supershells and classical periodic orbits



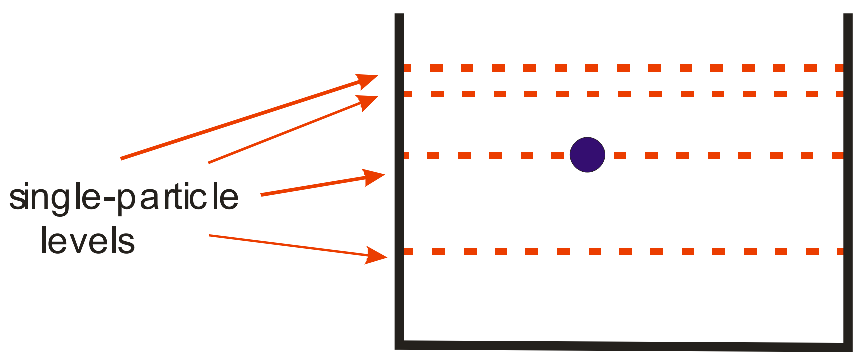
Nucleon in the potential well

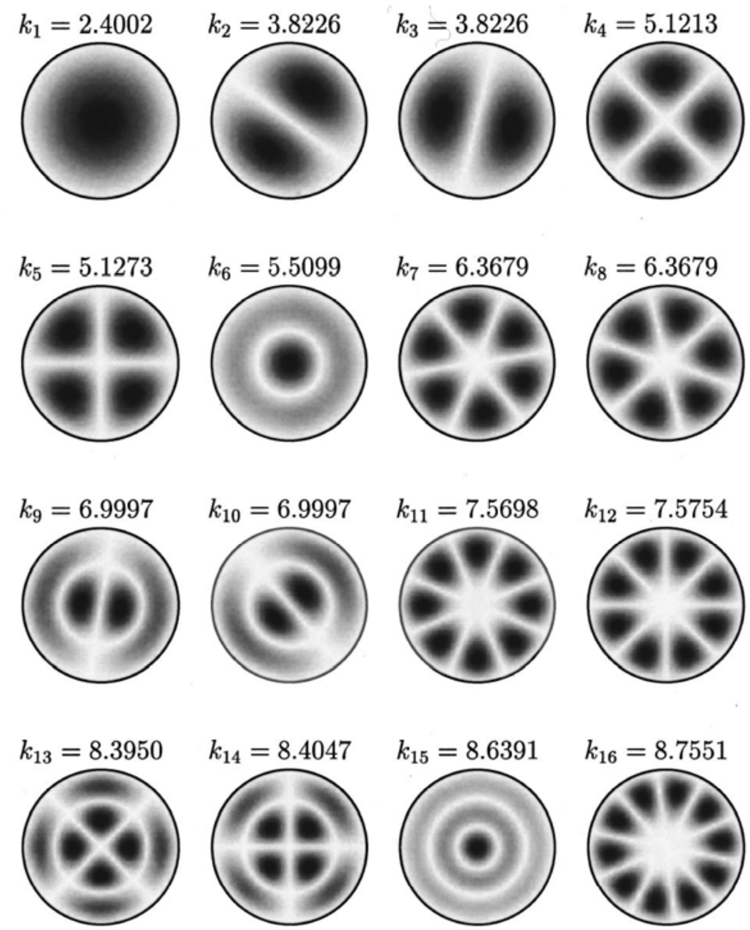
Quantum Billiard

- Shell Model

  Levels in nuclei

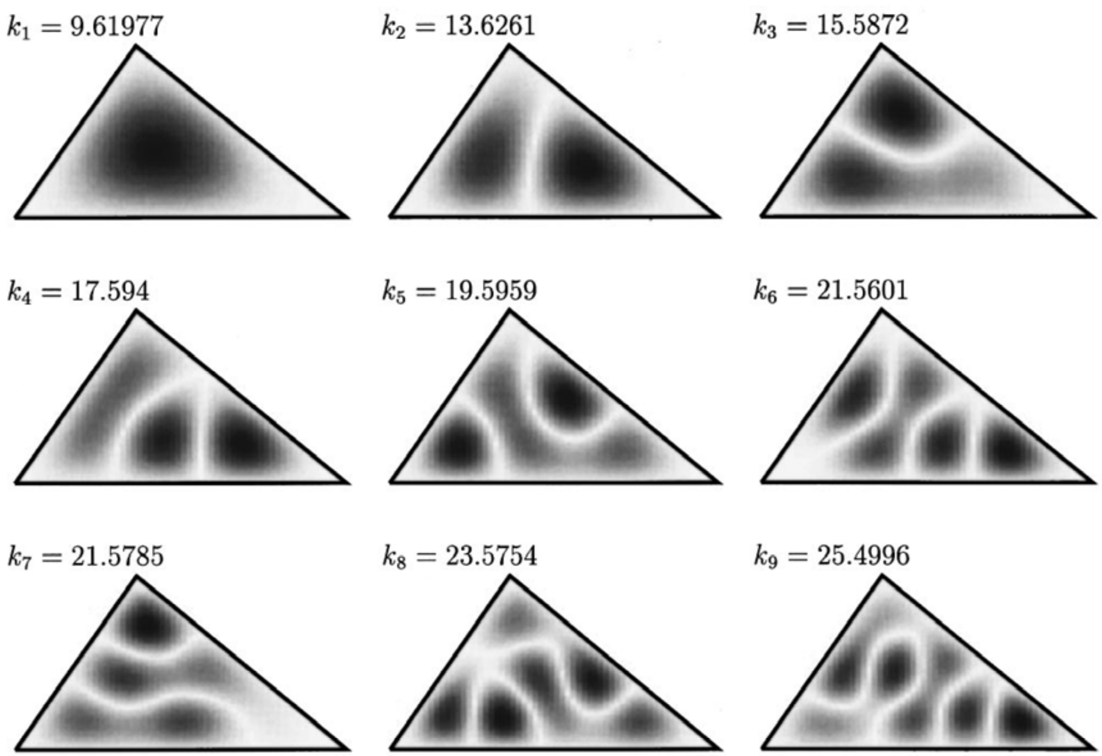
Nucleon in a box





Chaotic motion

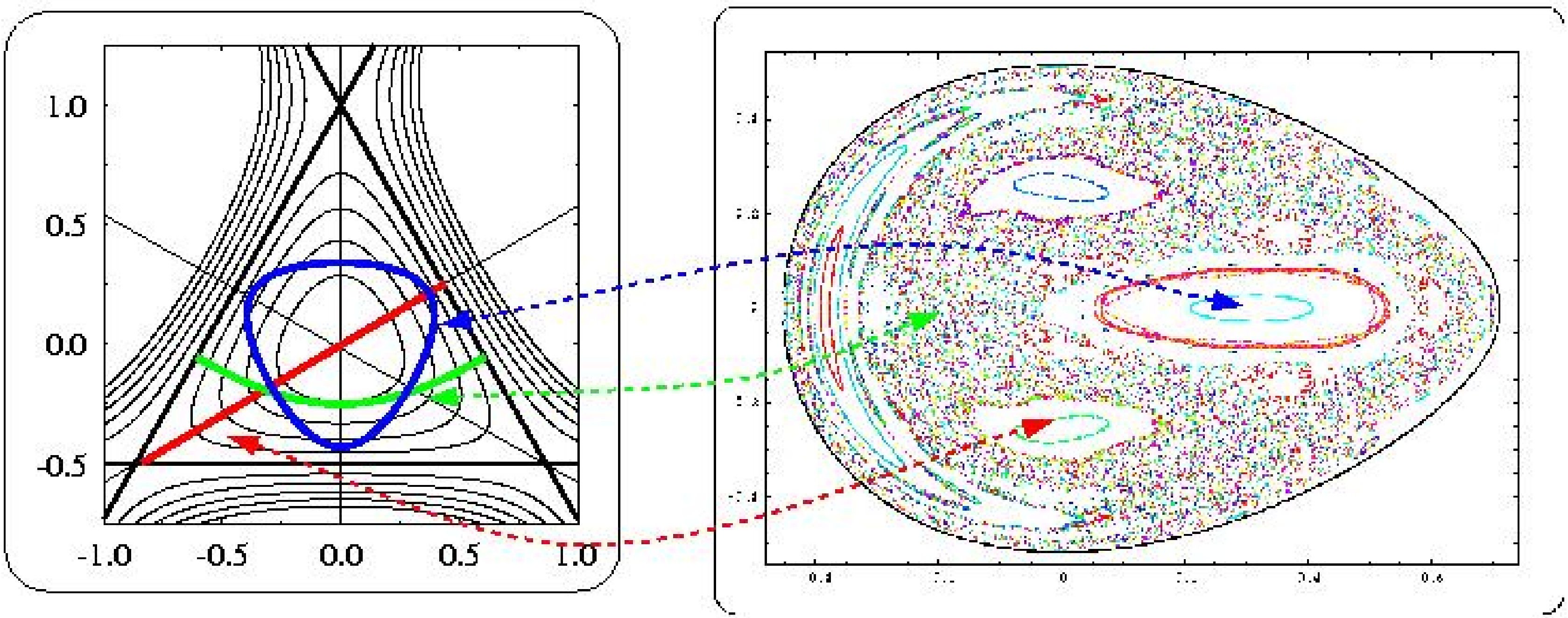
- Non-symmetric shape
  - Shape changes



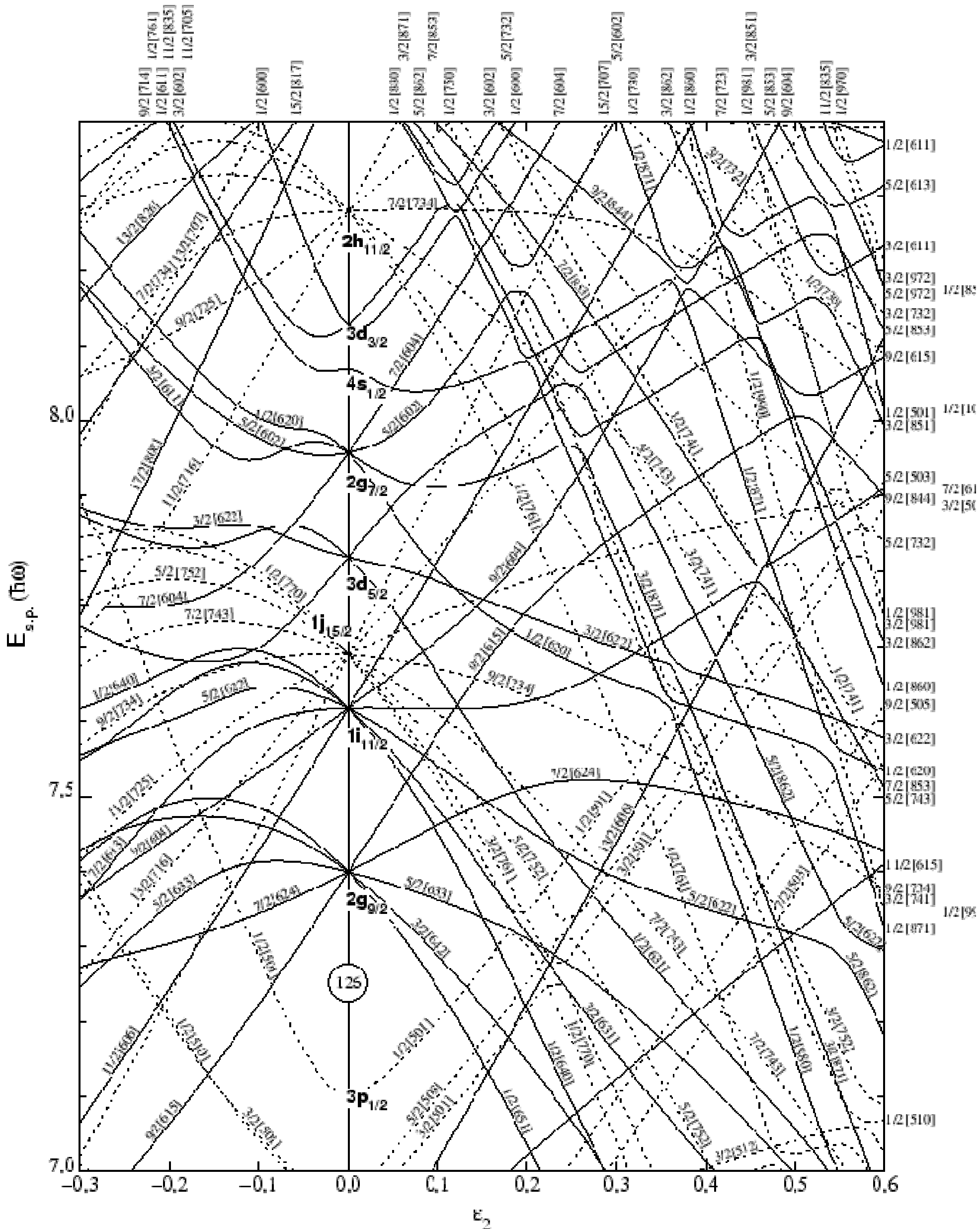
Periodic orbitals and shell structure

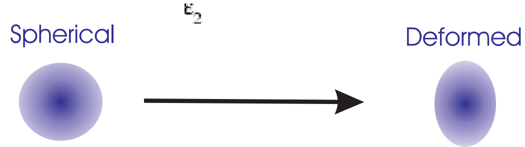
In the realm of chaos

- Why some nuclei are more stable than others?
- Why are there shell effects?



Single-nucleon motion in deformed potential

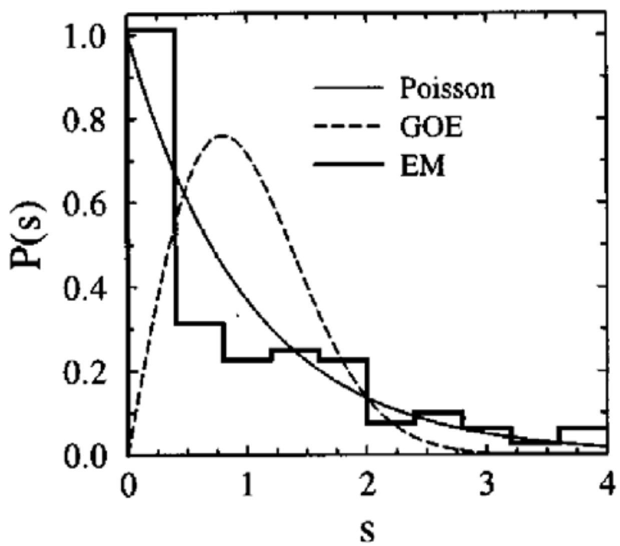




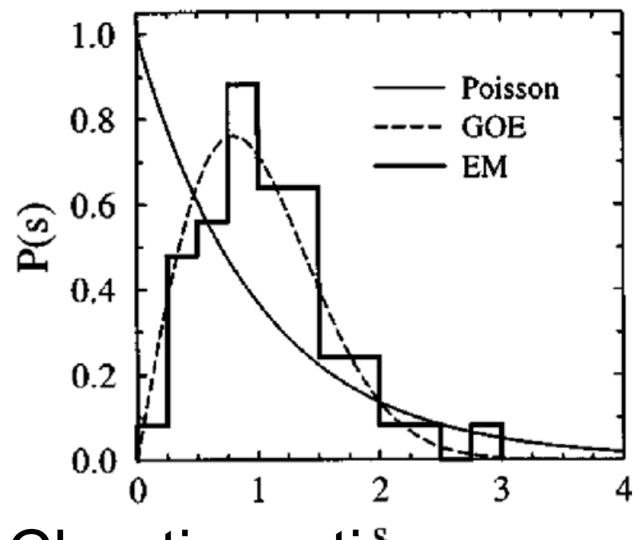
Quantum chaos

Distribution of energy spacing between neighboring states

Circular billiard



Irregular triangle billiard



- Regular motion
  - Analog to integrable systems
  - No level repulsion
  - Poisson distribution P(s)=exp(-s)

- Chaotic motion
  - Classically chaotic
  - Level repulsion
  - GOE (Random Matrix)

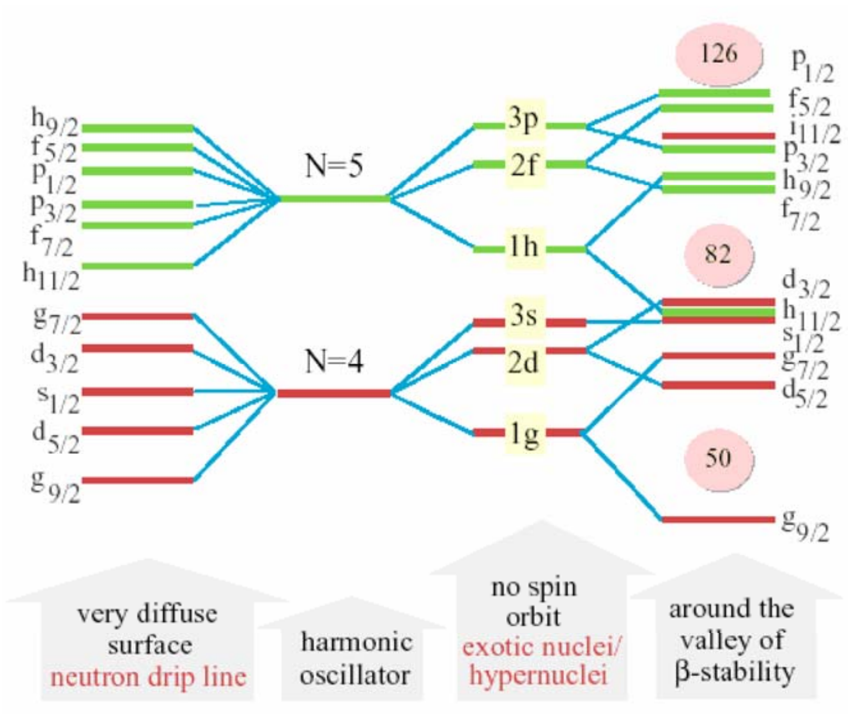
    $P(s)=s\exp(-\pi s^2/4)$

Evolution of shells

- Melting of shell structure
- Shells in deformed nuclei
- Shells in weakly bound nuclei
- Is the mean field concept valid

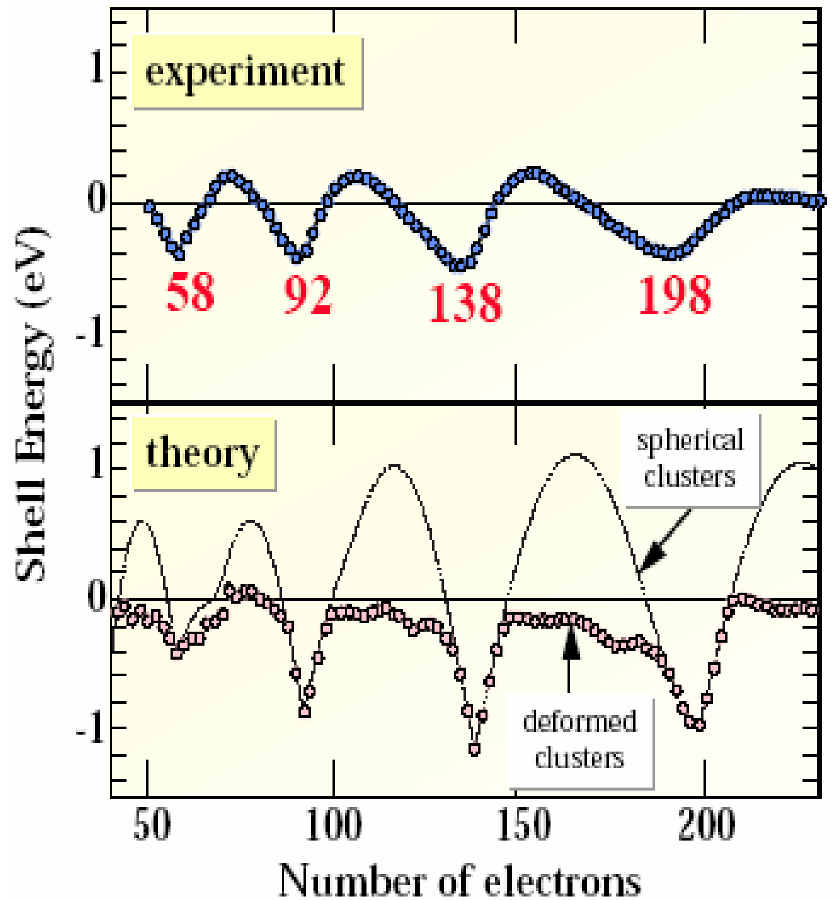
Shell structure in extreme limits

Shells in nuclei far from stability



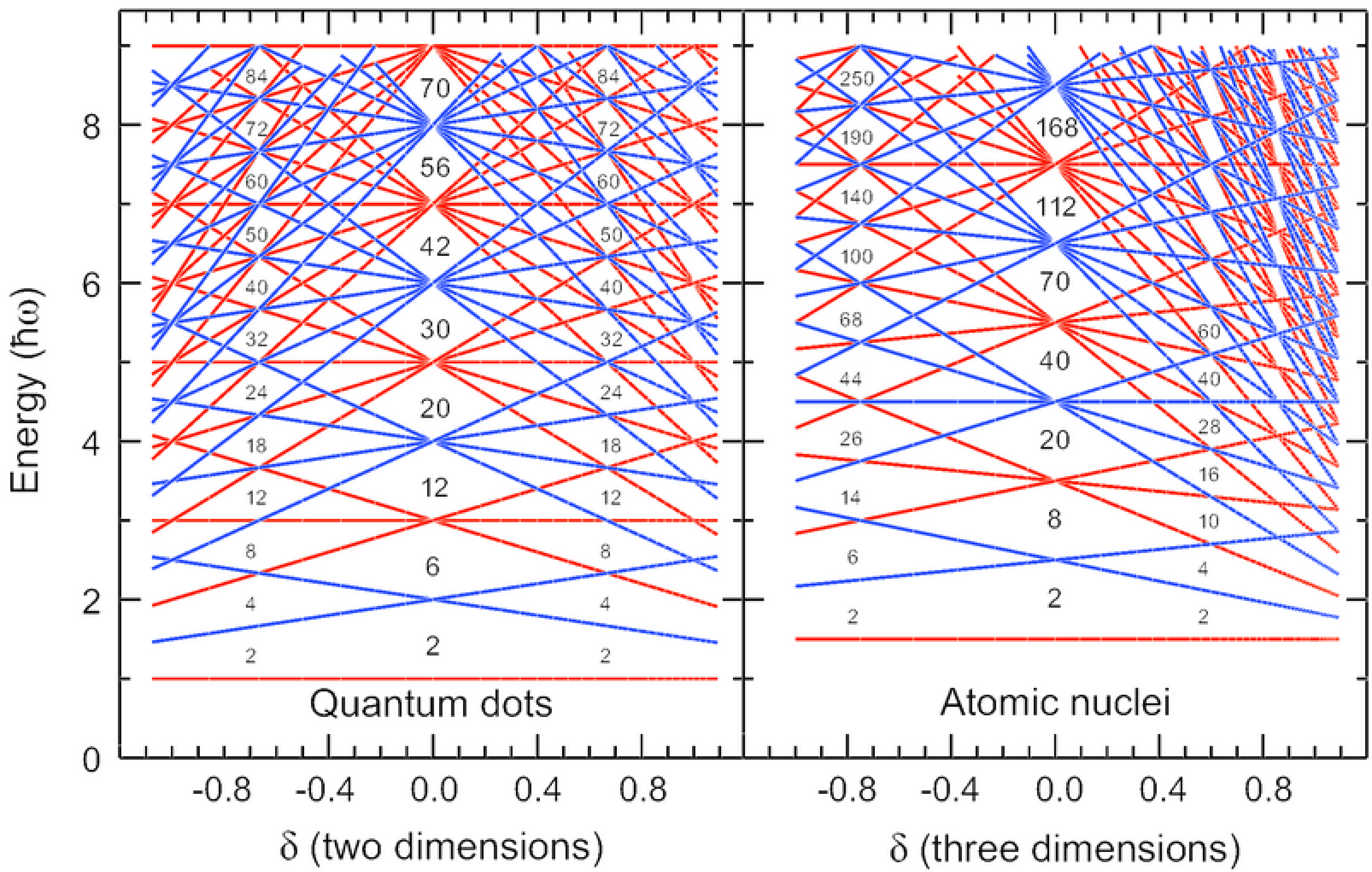
J. Dobaczewski *et al.*, PRC53, 2809 (1996)

Melting of shell structure



T=0 and T=0.4 ev,
Frauendorf S, Pahskevich VV. *NATO ASI Ser. E: Appl. Sci*., ed. TP Martin, 313:201. Kluwer (1996)

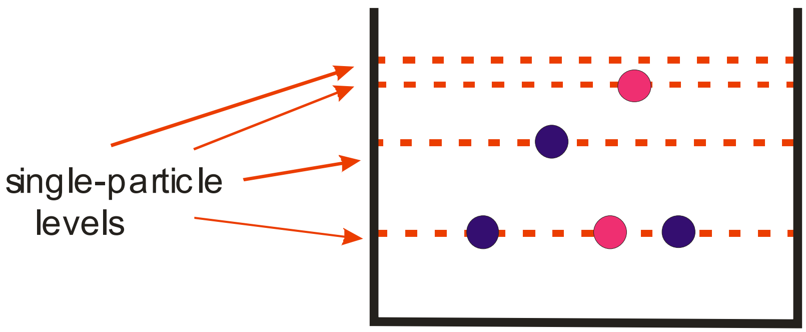
Deformation and shell gaps



Mesoscopic many-body complexity

- Complexity and Chaos
  - Typical level density
  - Chaotization process, geometric chaoticity
  - Random matrix theory
  - Enhancement of weak perturbations
- Collective Motion
  - Pairing and superconductivity
  - Phase transitions
  - Giant resonances
  - Fission
- Shapes
  - Shape change transitions
  - Rotations
- Thermodynamics and phase transitions
  - Features of small systems
  - Thermalization and level density
  - Yang-Lee theory, roots of partition functions

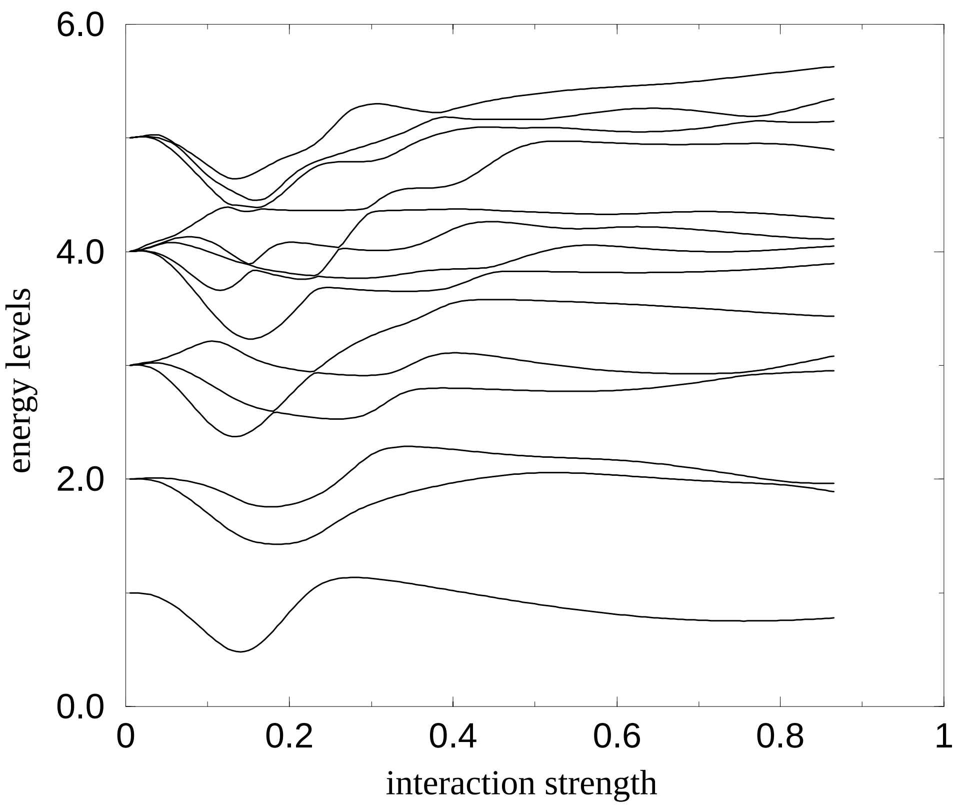
Many-nucleons, two-body scatterings



Nucleons in the box collide (interact)
•Jump from level to level
•Many-body dynamics

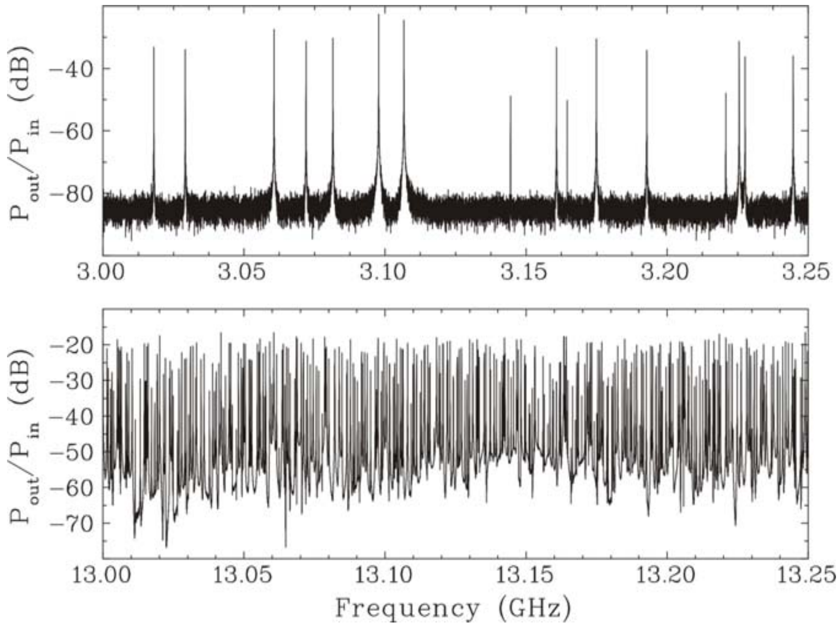
⇓

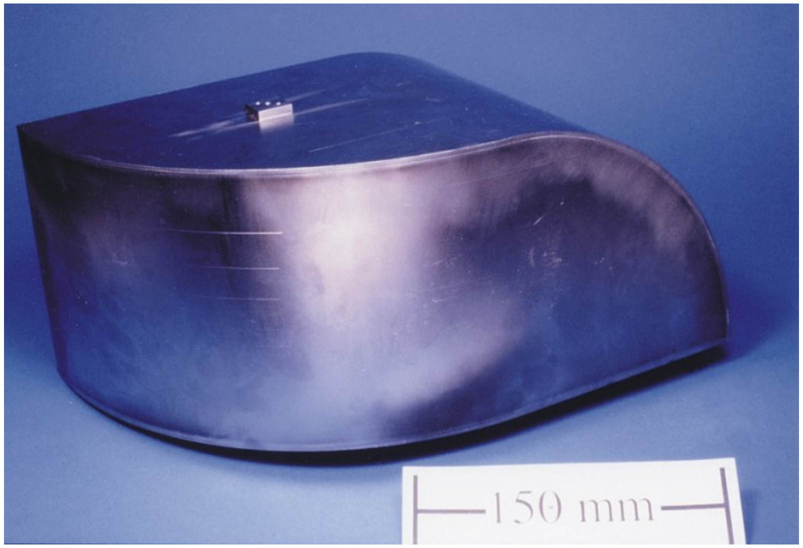
Even more complicated motion



Quantum billiards and neutron resonances n + ^{232}Th

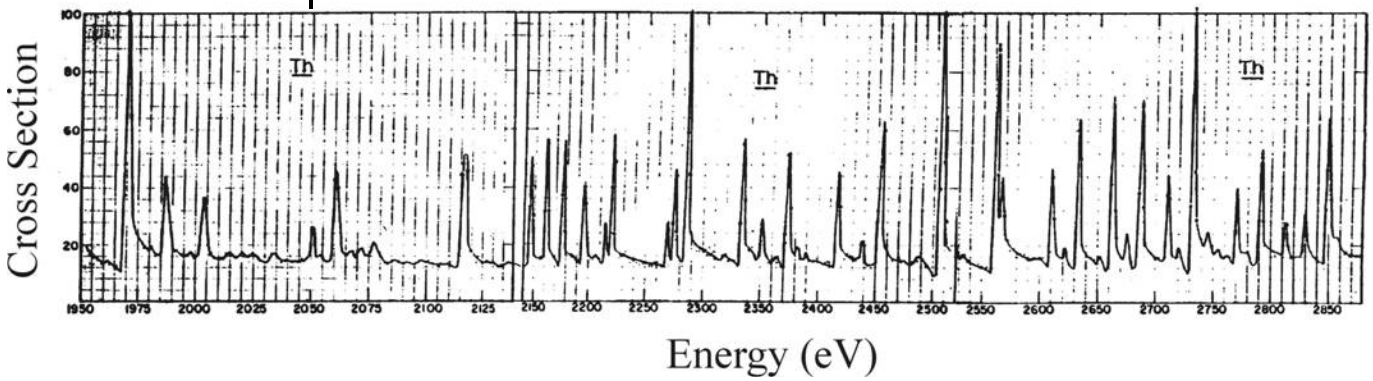
Transmission spectrum of a 3D-stadium billiard





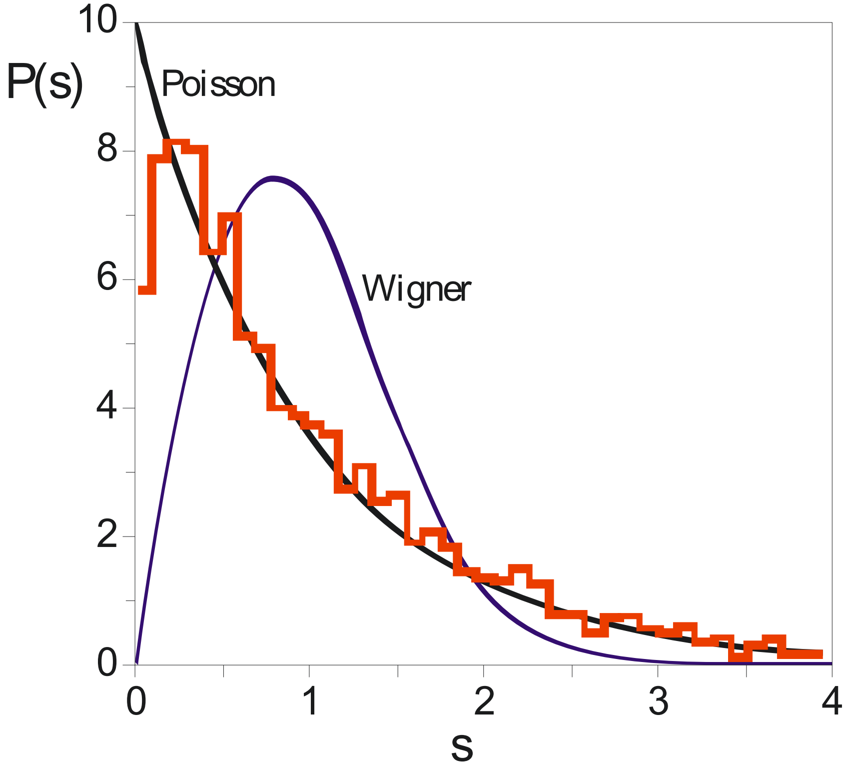
T = 4.2 K

Spectrum of neutron resonances in ^{232}Th + n

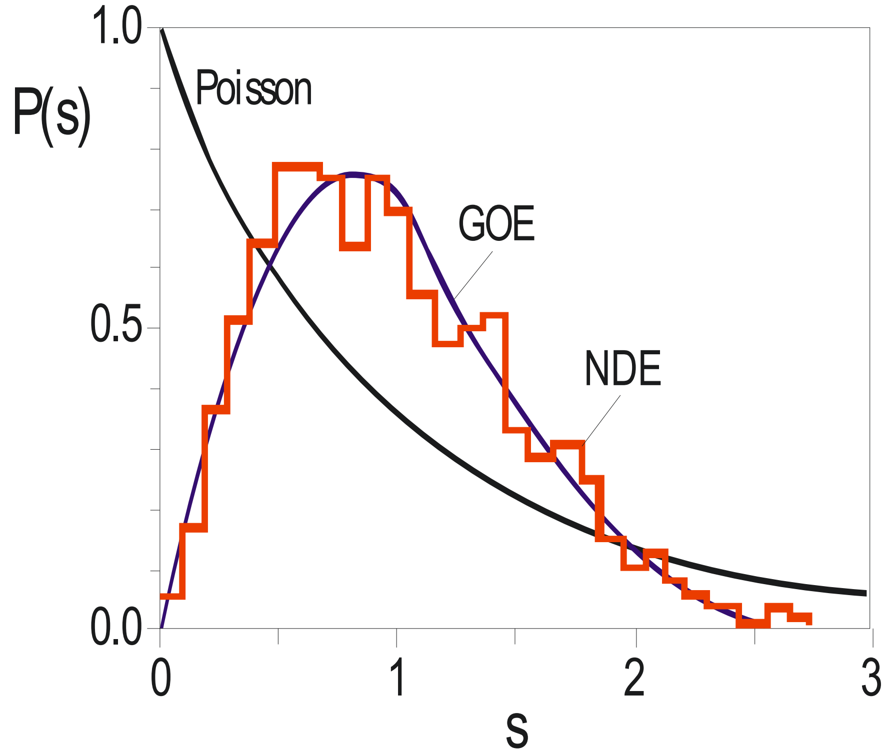


- Great similarities between the two spectra: universal behaviour

Chaotic motion in nuclei



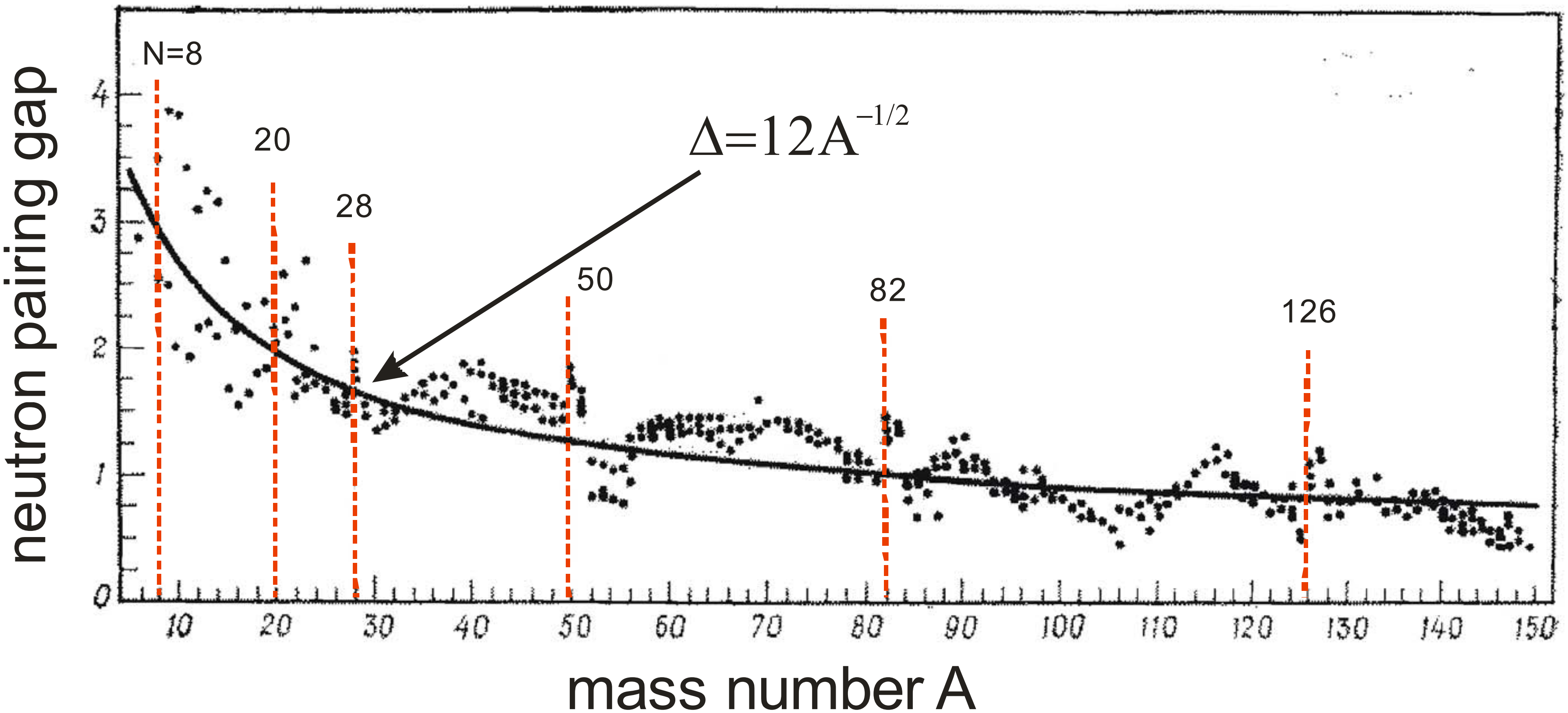
"Cold" (low excitation)
rare-earth nuclei



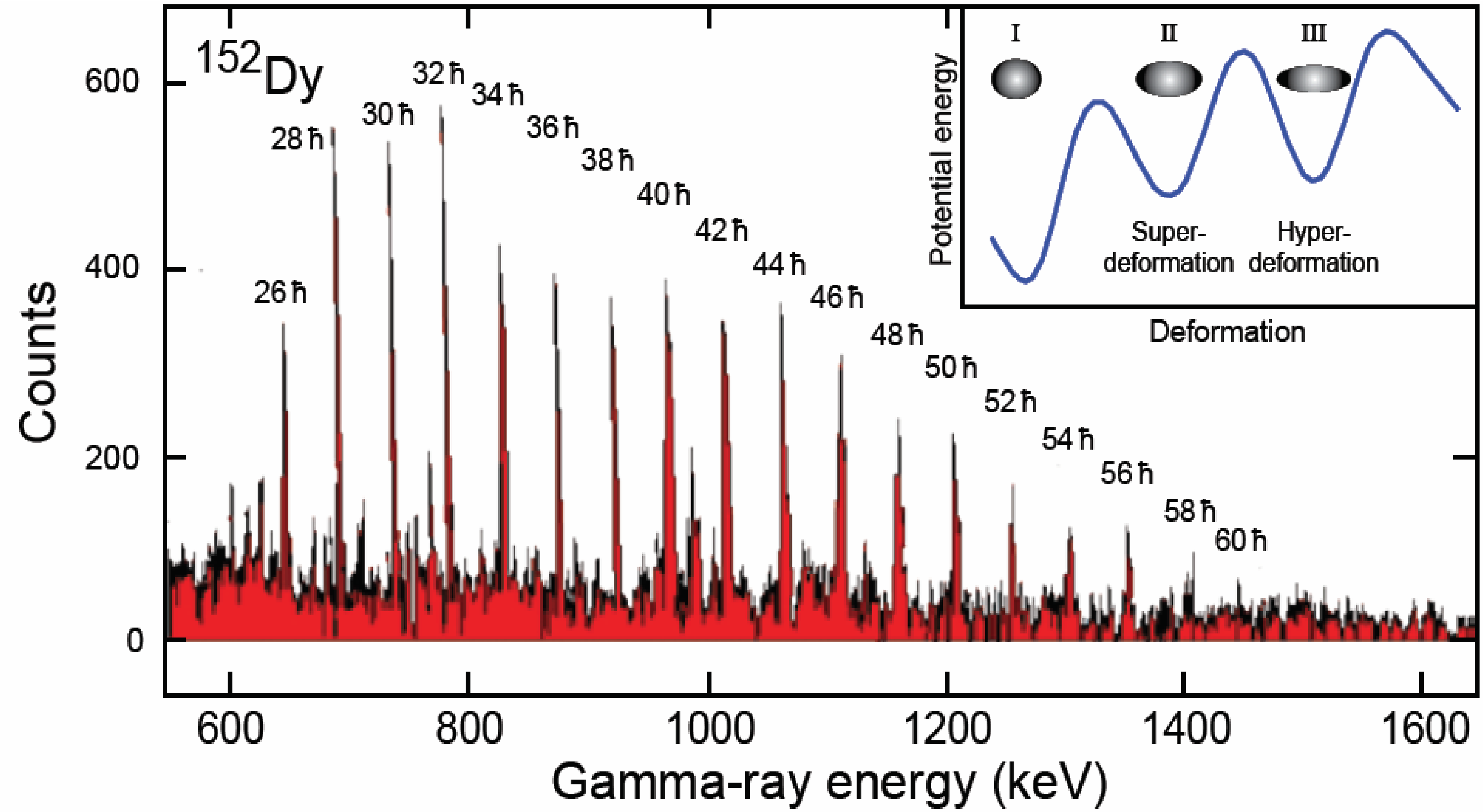
High-Energy region,
Nuclear Data Ensemble
Slow neutron resonant date

Haq. et.al. PRL 48, 1086 (1982)

Pairing interaction in nuclei

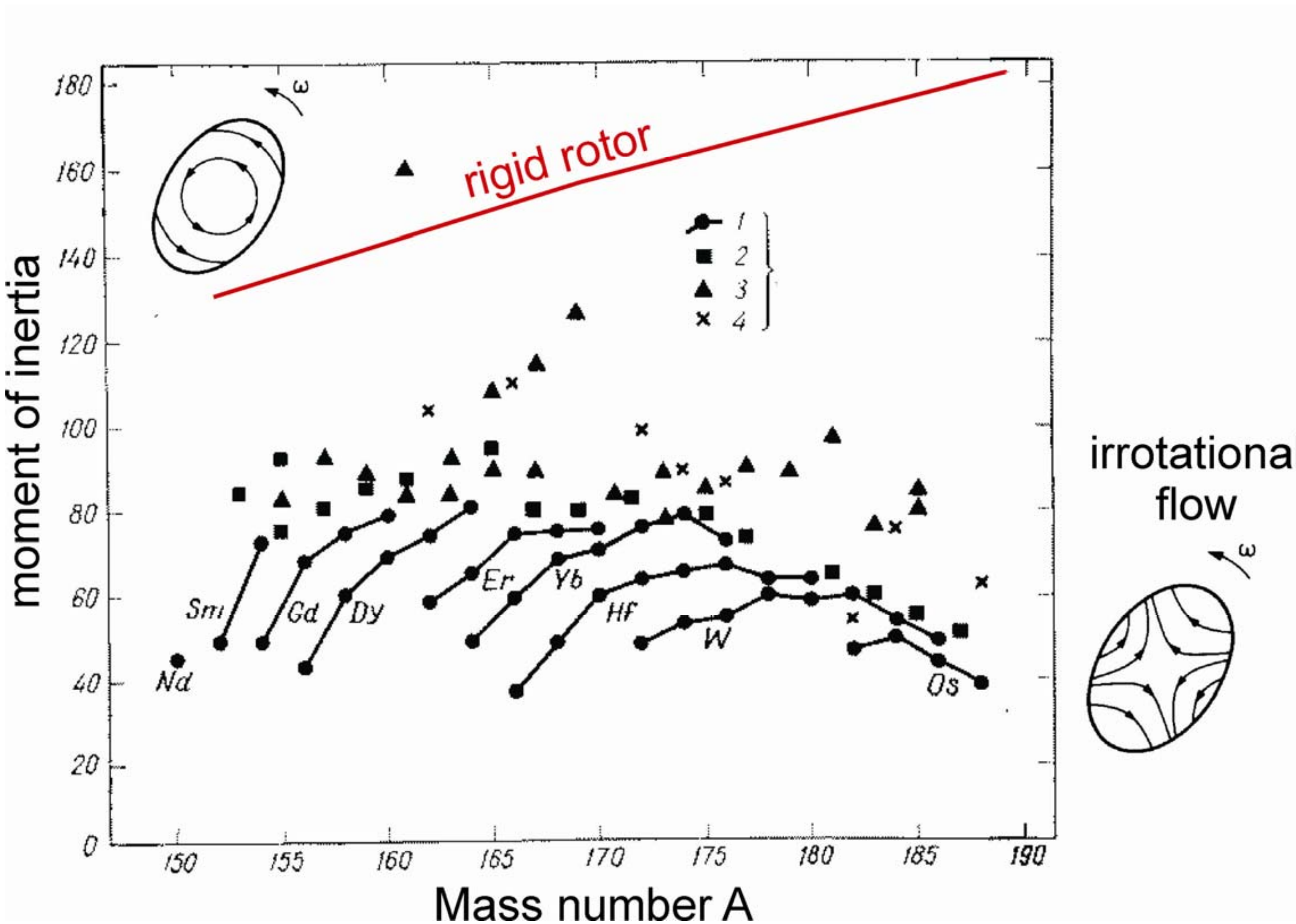


Rotation



^{152}Dy
26ħ
28ħ
30ħ
32ħ
34ħ
36ħ
38ħ
40ħ
42ħ
44ħ
46ħ
48ħ
50ħ
52ħ
54ħ
56ħ
58ħ
60ħ
Counts
0
200
400
600
600
800
1000
1200
1400
1600
Gamma-ray energy (keV)
I
II
III
Potential energy
Super-deformation
Hyper-deformation
Deformation

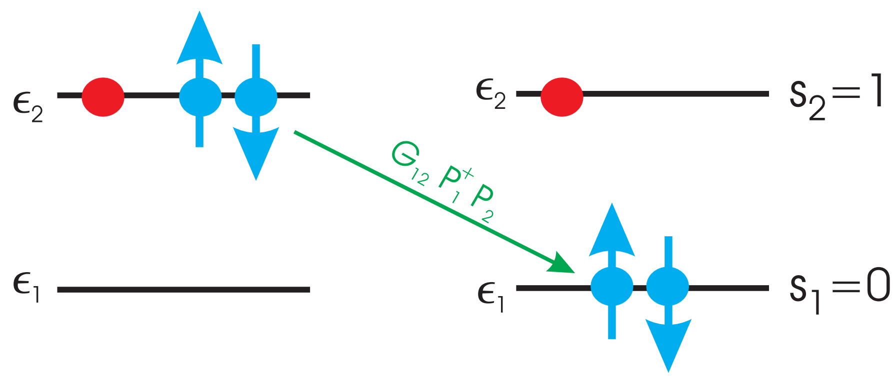
Evidence of nuclear superfluidity



Pairing Hamiltonian

- Pairing on degenerate time-conjugate orbitals
  $|1\rangle \leftrightarrow |\tilde{1}\rangle \quad |\tilde{jm}\rangle = (-1)^{j-m}|j-m\rangle$
- Pair operators $P = (a_1 a_1)_{J=0}$ (J=0, T=1)
- Number of unpaired fermions is seniority s
- Unpaired fermions are untouched by H

$$H = \sum_1 \epsilon_1 N_1 - \sum_{12} G_{12} P_1^\dagger P_2$$



Approaching the solution of pairing problem

- Approximate
  - BCS theory
    - HFB+correlations+RPA
  - Iterative techniques
- Exact solution
  - Richardson solution
  - Algebraic methods
  - Direct diagonalization + quasispin symmetry[1]

[1]A. Volya, B. A. Brown, and V. Zelevinsky, Phys. Lett. B 509, 37 (2001).

BCS theory

Trial wave-function

$$|0) = \prod_{\nu} \left(u_\nu - v_\nu a_\nu^\dagger a_{\tilde{\nu}}^\dagger \right) |0\rangle \,, \quad \text{where} \quad \underbrace{|u_\nu|^2}_{\text{empty}} + \underbrace{|v_\nu|^2}_{\text{occupied}} = 1$$

Minimization of energy determines

$$|v_\nu|^2 = \frac{1}{2}\left(1 - \frac{\epsilon_\nu - \mu}{e_\nu}\right) , \quad |u_\nu|^2 = \frac{1}{2}\left(1 + \frac{\epsilon_\nu - \mu}{e_\nu}\right)$$

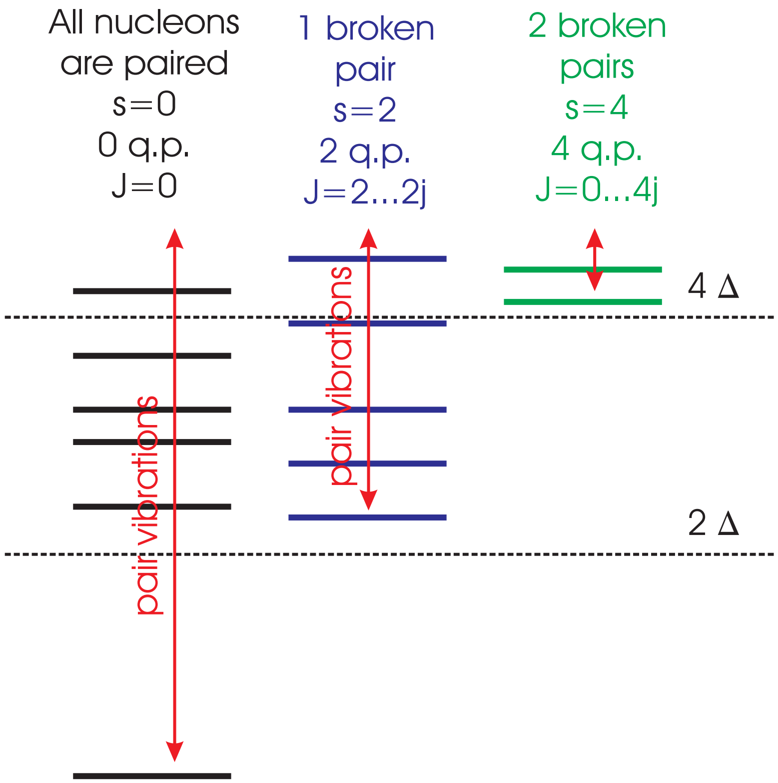
Gap equation

$$\Delta_\nu = \frac{1}{4}\sum_{\nu'} G_{\nu\,\nu'} \frac{\Delta_{\nu'}}{e_{\nu'}} , \text{ where } \quad e_\nu = \sqrt{(\epsilon_\nu - \mu)^2 + \Delta_\nu^2}$$

Low-lying states in paired systems

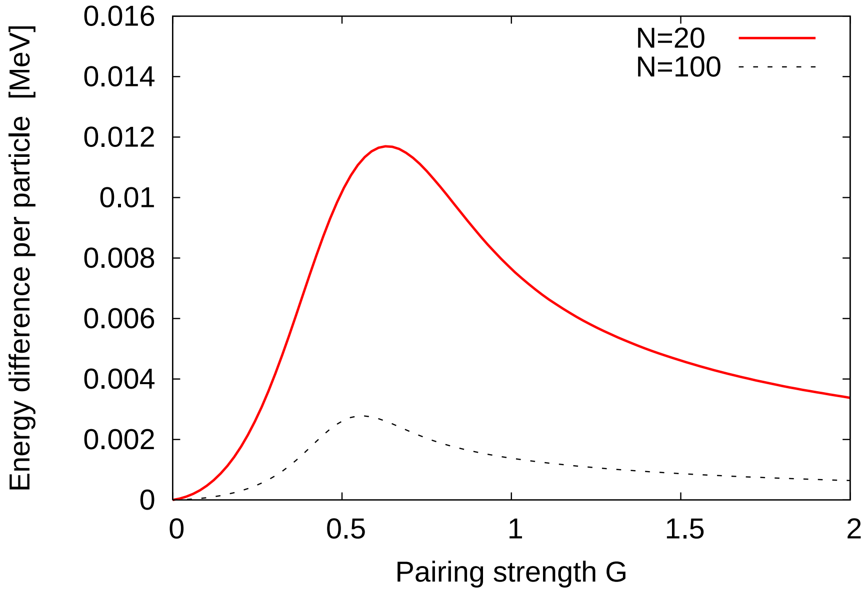
- Exact treatment
  - No phase transition and G_{critical}
  - Different seniorities do not mix
  - Diagonalize for pair vibrations
- BCS treatment

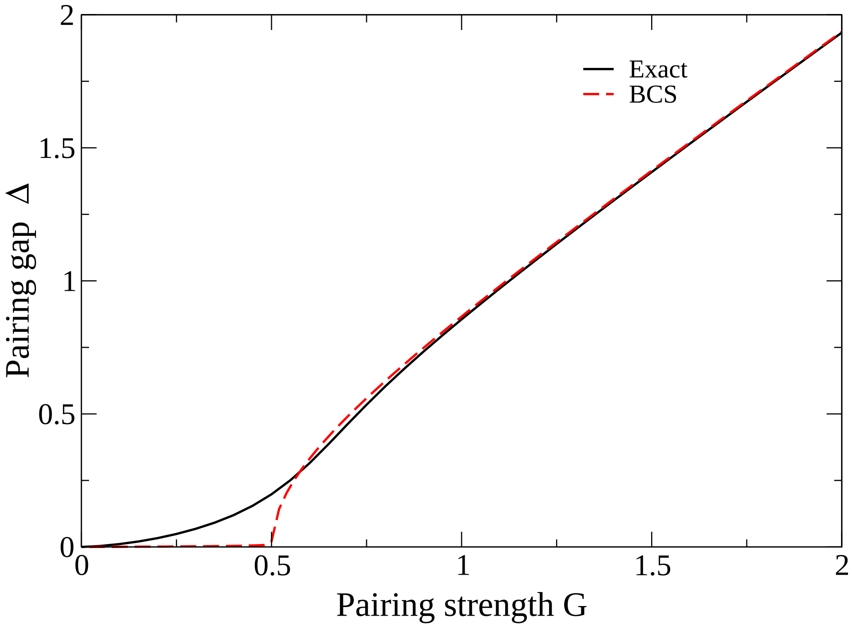
	$G<G_{\text{critical}}$	$G>G_{\text{critical}}$
Ground state	Hartree-Fock	BCS
Elementary excitations	single-particle excitations $E_{s=2}=2\,\varepsilon$	quasiparticle excitation $E_{s=2}=2\,e$
Collective excitations	HF+RPA	HFB+RPA



Cooper Instability in mesoscopic system

BCS versus Exact solution





Statistical treatment of pairing

- Microcanonical $\widehat{\rho}(E, N) = \delta(E - \widehat{H})\delta(N - \widehat{N})$
- Canonical $\widehat{\rho}(\beta, N) = \text{exp}(-\beta\widehat{H})\,\delta(N - \widehat{N})$
- Grand canonical $\widehat{\rho}(\beta, \mu) = \text{exp}\left(-\beta(\widehat{H} - \mu\widehat{N})\right)$

Partition functions

$$Z = \text{Tr}(\widehat{\rho}) \quad \text{and} \quad \widehat{w} = \frac{\widehat{\rho}}{Z}.$$

Statistical averages

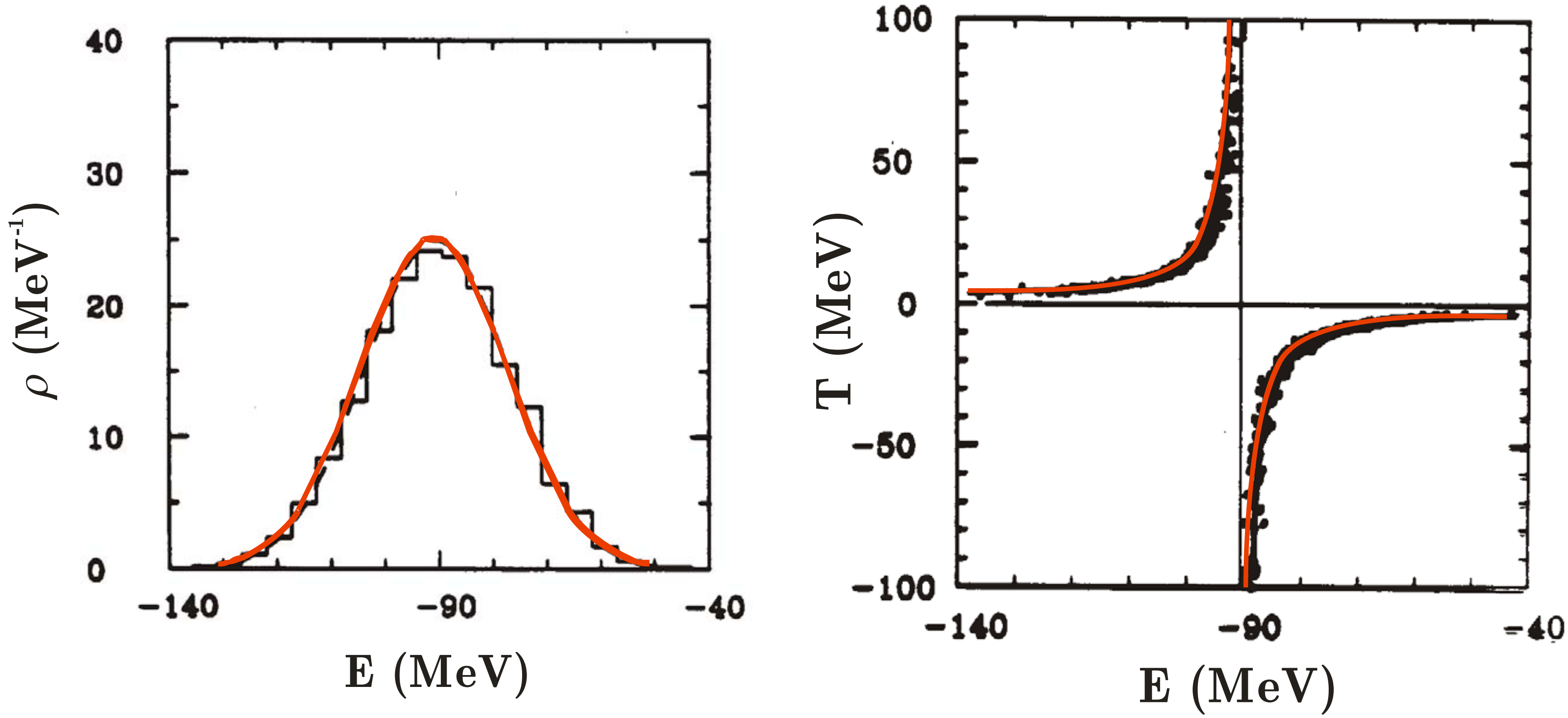
$$\langle\widehat{O}\rangle = \frac{\text{Tr}(\widehat{O}\widehat{\rho})}{\text{Tr}(\widehat{\rho})} = \text{Tr}(\widehat{O}\widehat{w})$$

Entropy

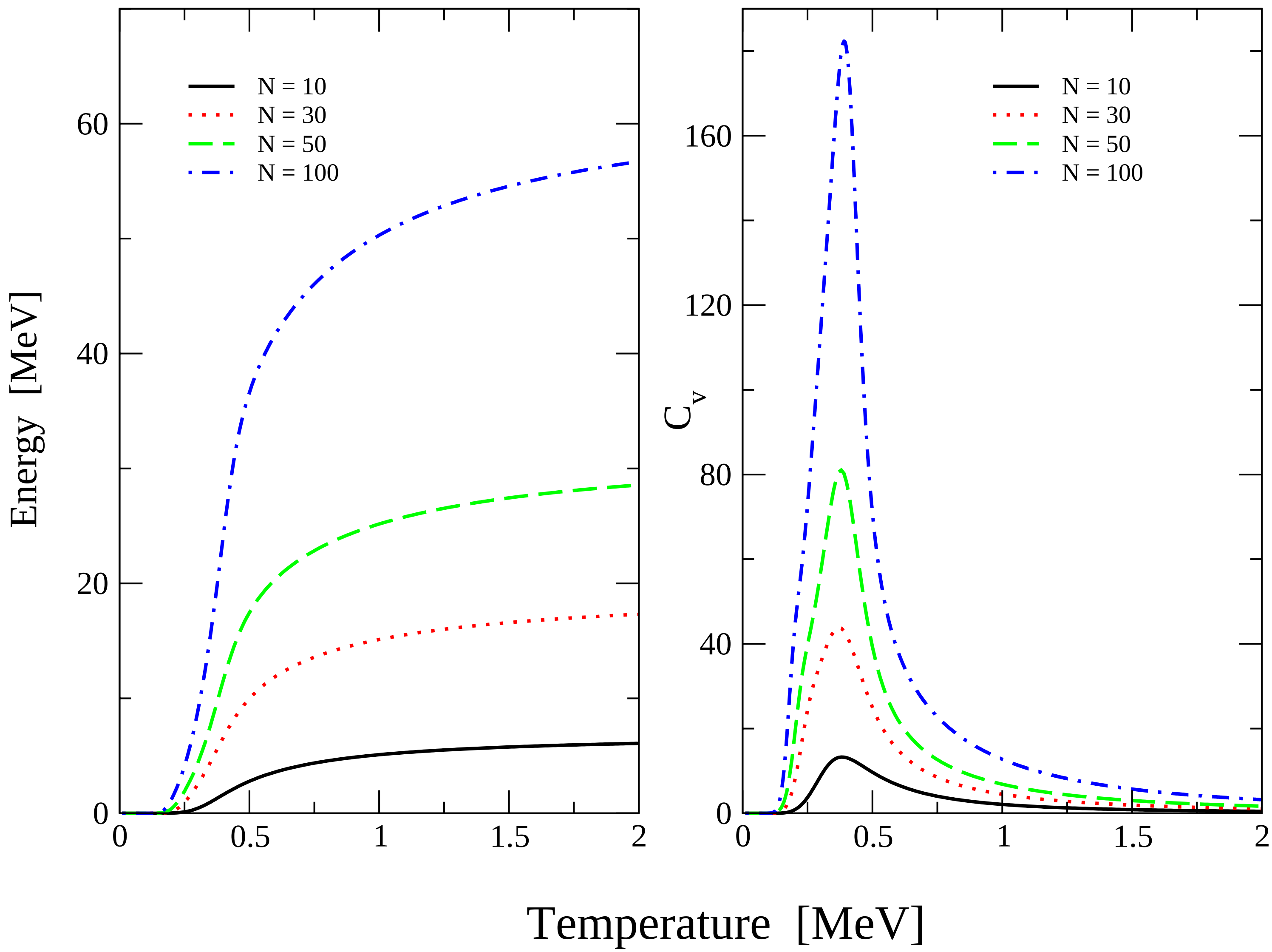
$$S = -\langle\ln(\widehat{w})\rangle = -\text{Tr}(\widehat{w}\,\ln\widehat{w})$$

Is there thermalization?

$$\rho(E) = \rho_0 \, e^{-\frac{(E-E_0)^2}{2\sigma^2}}$$

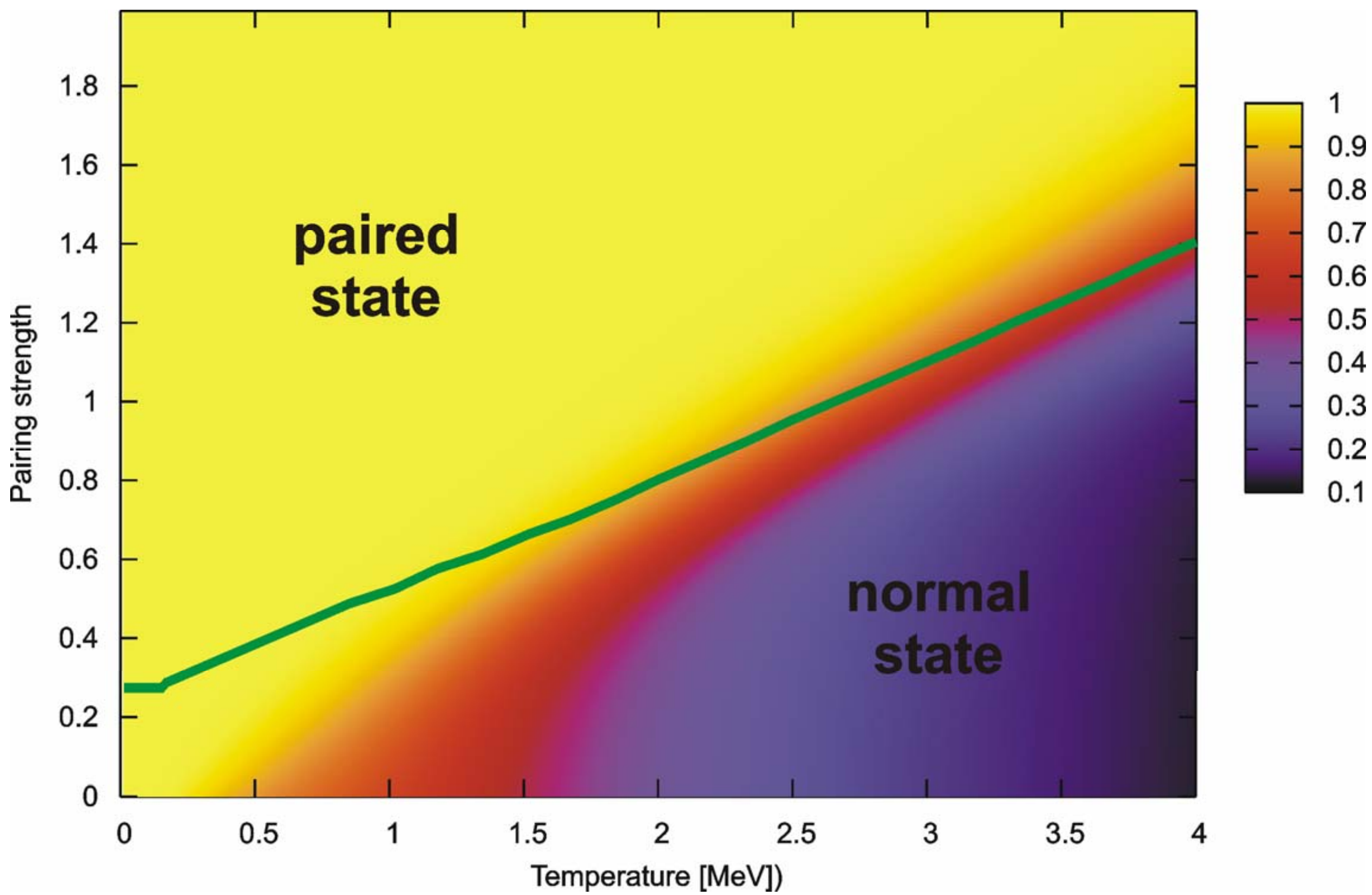


C

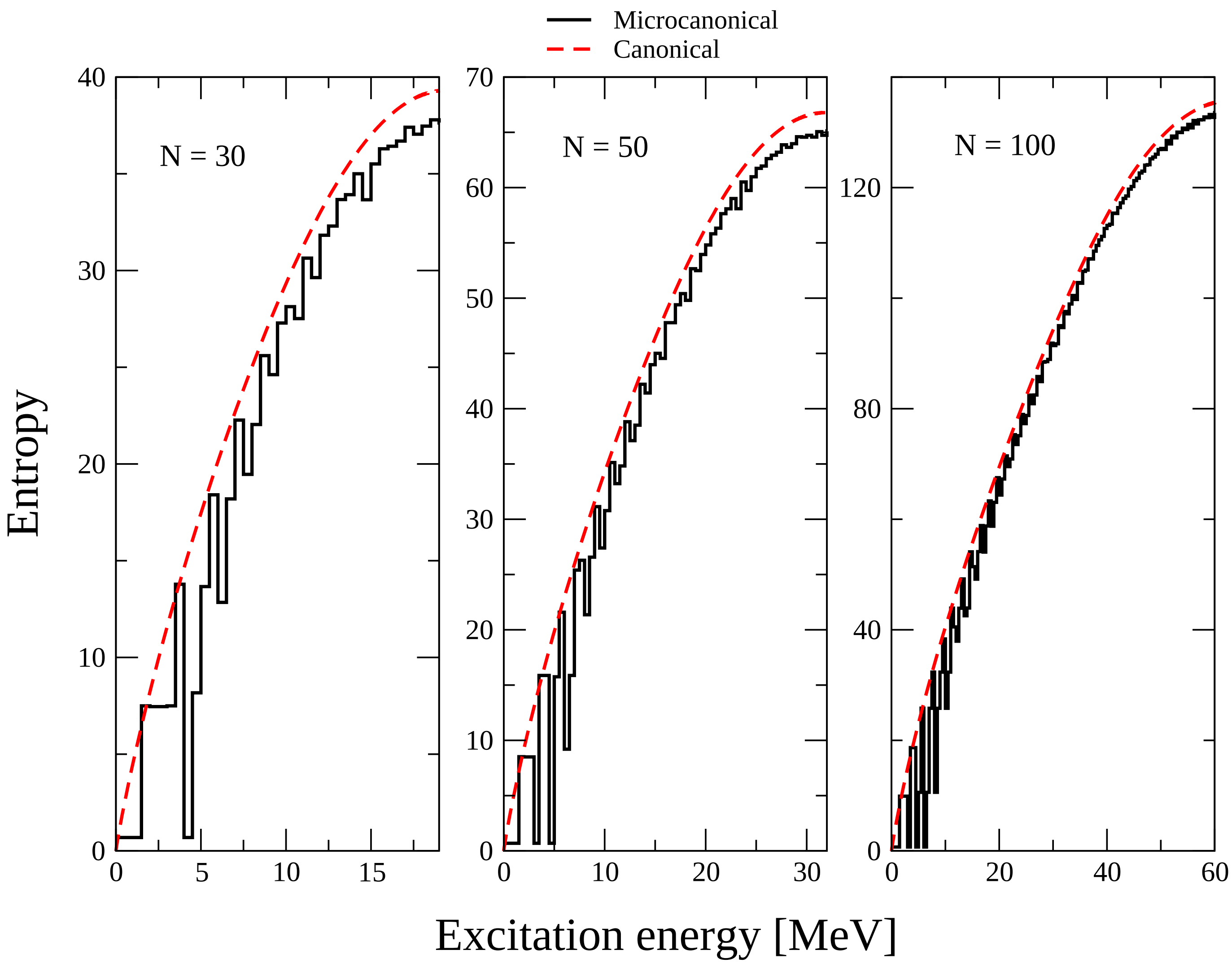


N = 10
N = 30
N = 50
N = 100
Energy [MeV]
60
40
20
0
0
0.5
1
1.5
2
N = 10
N = 30
N = 50
N = 100
C_v
160
120
80
40
0
0
0.5
1
1.5
2
Temperature [MeV]

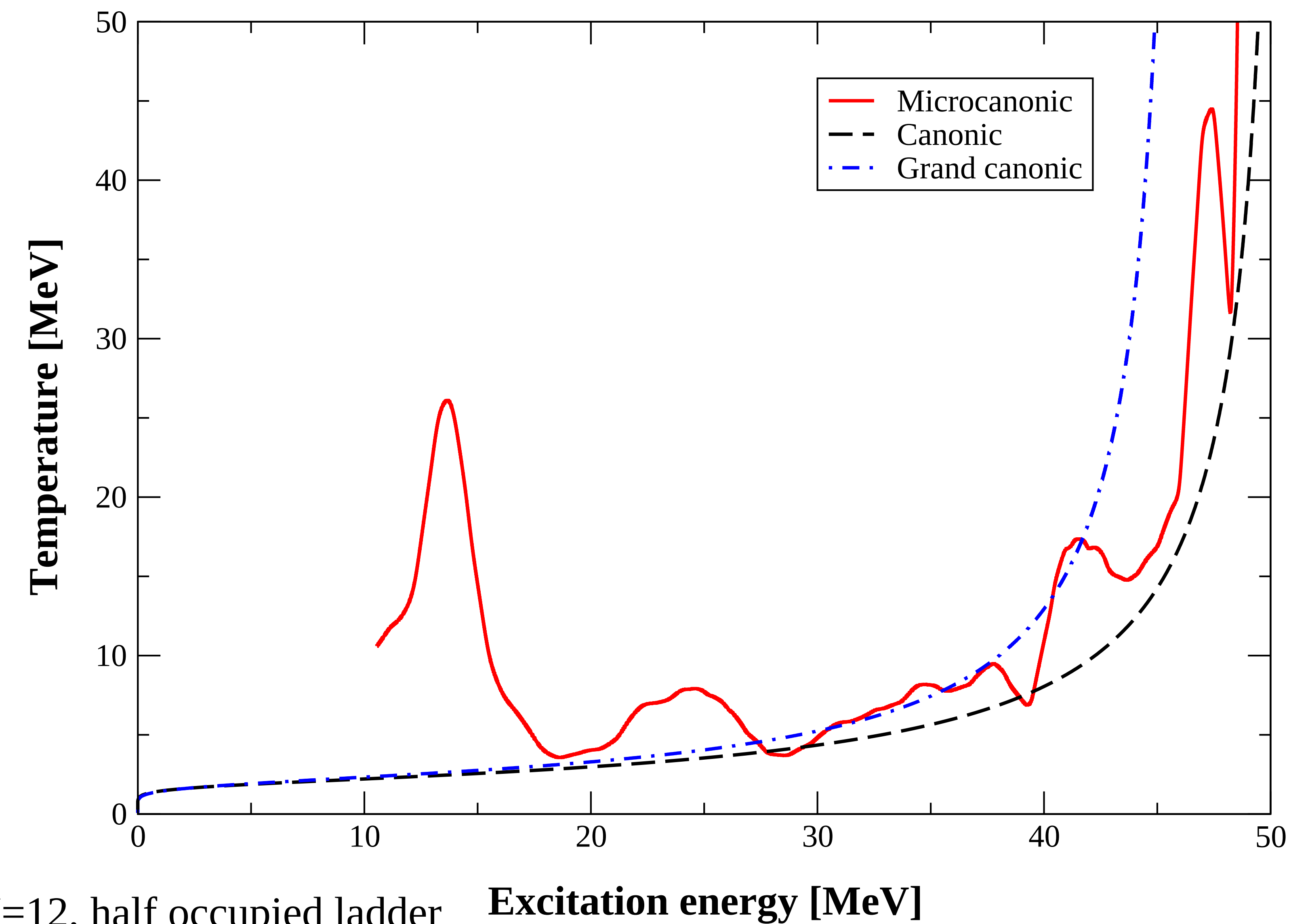
Pairing phase diagram



Microcanonical ensemble and



G=1



N=12, half occupied ladder

Phase Transition in Mesoscopic System

Complex roots- similar to charges
Appear symmetrically,
never exactly on real axis

$$Z(\mathcal{B}_j) = 0\,, \;\; \mathcal{B}_j = \beta_j + i\tau_j$$

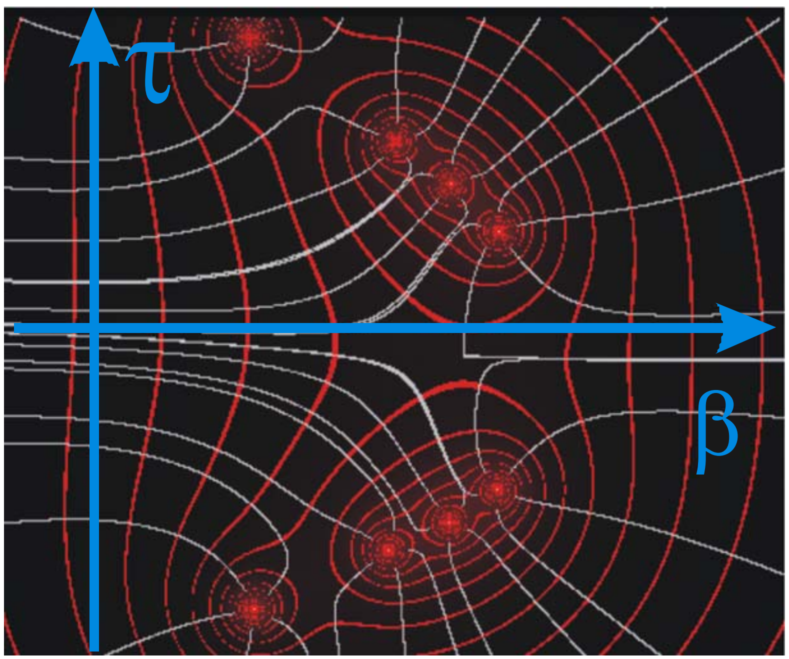
$$\mathcal{Z}(\beta) = \Omega \prod_j \left(1 - \frac{\beta}{\mathcal{B}_j}\right)\left(1 - \frac{\beta}{\mathcal{B}_j^*}\right).$$

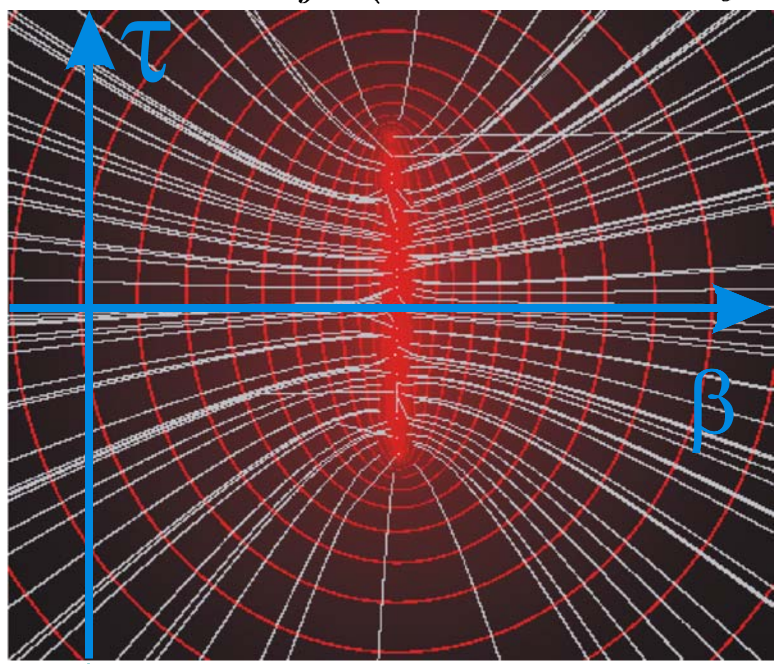
Energy- similar to potential
Roots become poles
Macroscopic accumulation of poles
creates **charged surface**

$$E(\beta) = -\frac{\partial}{\partial\beta}\ln(\mathcal{Z}) = \sum_j \left(\frac{1}{\mathcal{B}_j - \beta} + \frac{1}{\mathcal{B}_j^* - \beta}\right).$$

Heat Capacity – E-filed

$$C_V = \beta^2 \frac{\partial^2 \ln \mathcal{Z}}{\partial \beta^2} = \beta^2 \sum_j \left(\frac{1}{(\mathcal{B}_j - \beta)^2} + \frac{1}{(\mathcal{B}_j^* - \beta)^2}\right).$$





http://www.cco.caltech.edu/~phys1/java/phys1/EField/EField.html

Classification of phase transitions zeros in the complex temperature plane

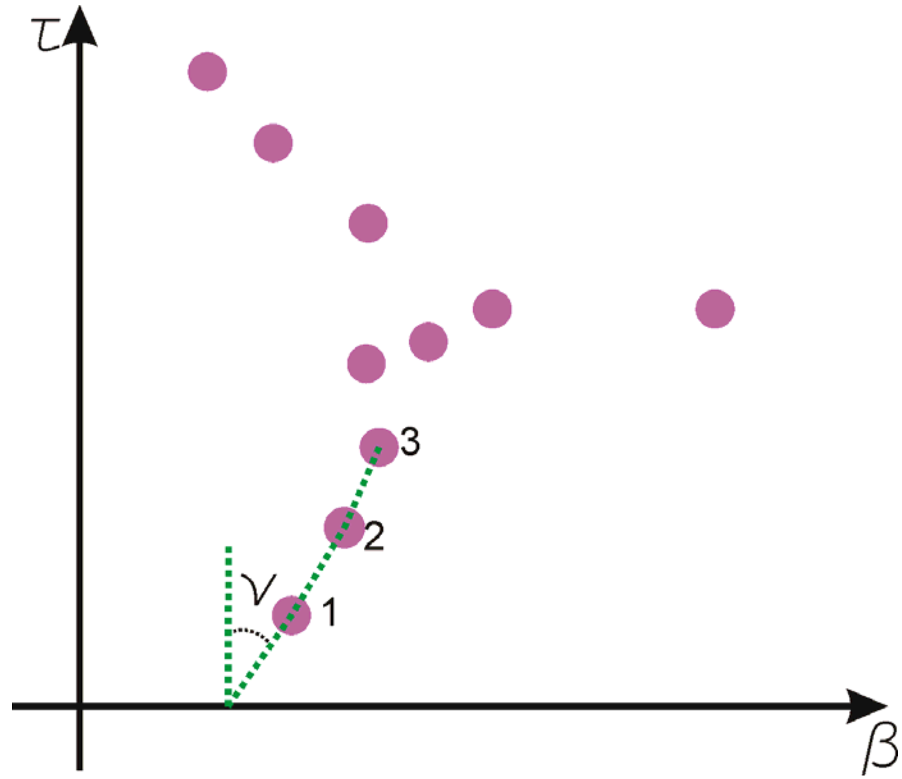
Main Characteristics

- Angle of approach

$$\nu = \text{arctan}\,\frac{\beta_2 - \beta_1}{\tau_2 - \tau_1}$$

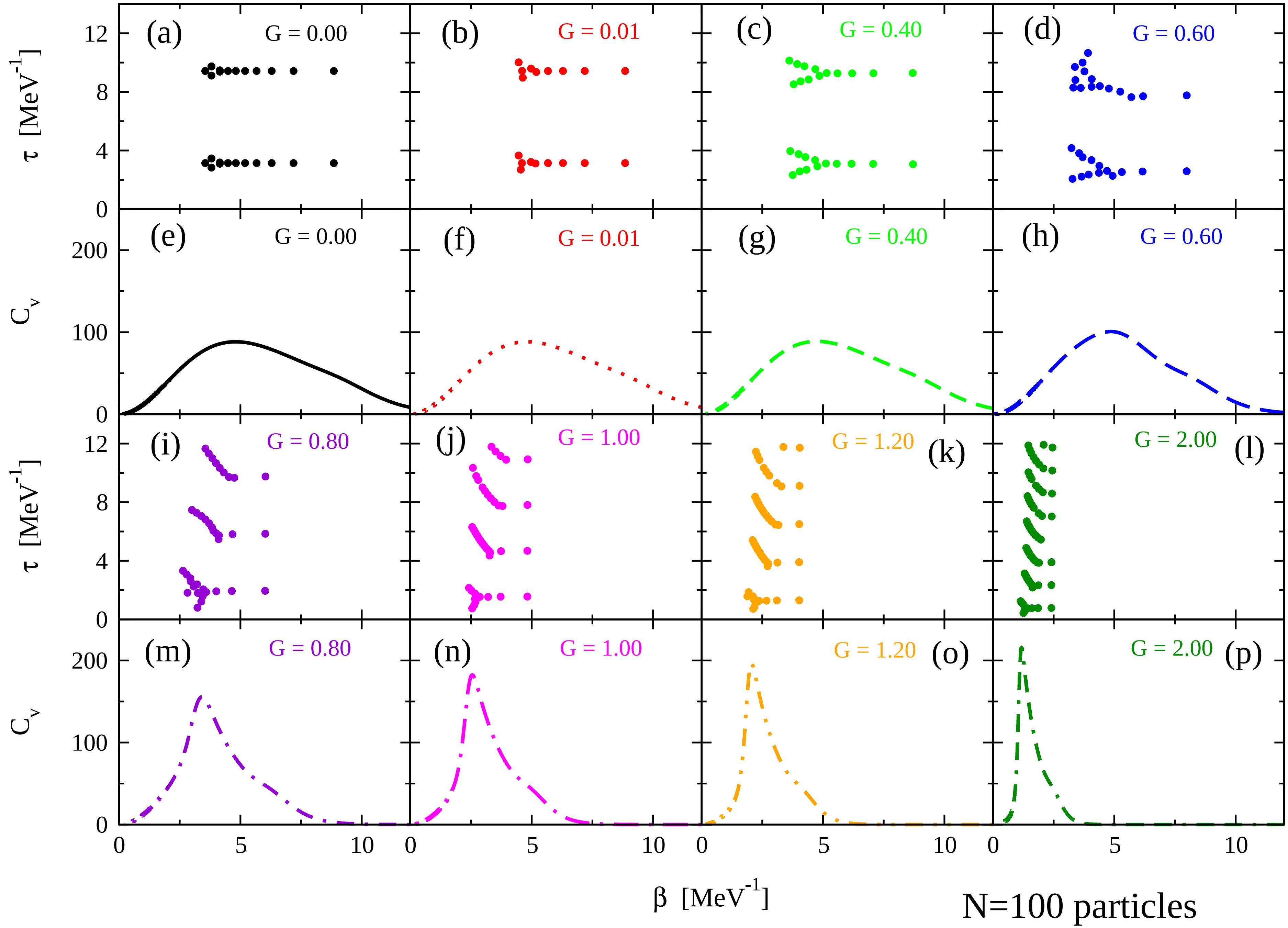
- Congestion of roots

$$|\mathcal{B}_{i+1} - \mathcal{B}_i| \sim \tau_i^{-\alpha}$$



Classification

First order	$\nu = 0 \quad \alpha = 0$
Second order	$0 < \alpha \leq 1$
Higher order	$1 < \alpha$



(a) G = 0.00
(b) G = 0.01
(c) G = 0.40
(d) G = 0.60
(e) G = 0.00
(f) G = 0.01
(g) G = 0.40
(h) G = 0.60
(i) G = 0.80
(j) G = 1.00
(k) G = 1.20
(l) G = 2.00
(m) G = 0.80
(n) G = 1.00
(o) G = 1.20
(p) G = 2.00
τ [MeV^{-1}]
C_v
β [MeV^{-1}]
N=100 particles

End of lecture
continue reading to learn more…

- Invariant correlational entropy
- Phase diagrams
- Open mesoscopic quantum systems
- Superradiance, quasi-stationary states in continuum

Invariant Correlational Entropy

- Parameter-driven equilibration (pairing strength)
- Averaged density matrix

$$|\alpha\rangle = \sum_k C_k^\alpha(G)|k\rangle$$

$$\widehat{\bar{\rho}} = \frac{1}{\delta G}\int_G^{G+\delta G} \hat{\rho}(G) \qquad \rho_{k\,k'}^\alpha = \langle k|\alpha\rangle\langle\alpha|k'\rangle$$

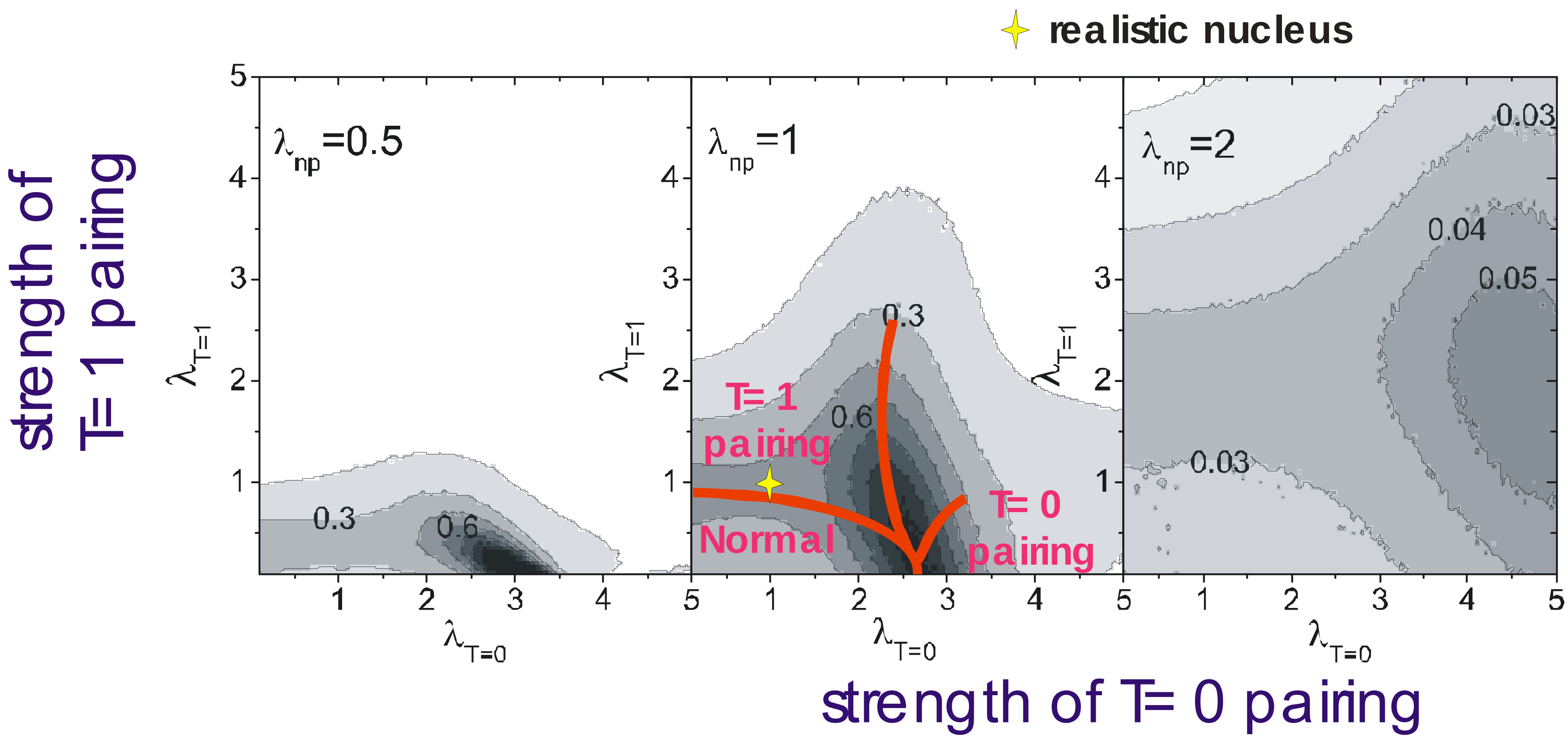
- ICE

$$I^\alpha = -\mathrm{Tr}(\overline{\rho^\alpha}\ln\overline{\rho^\alpha})$$

Advantages

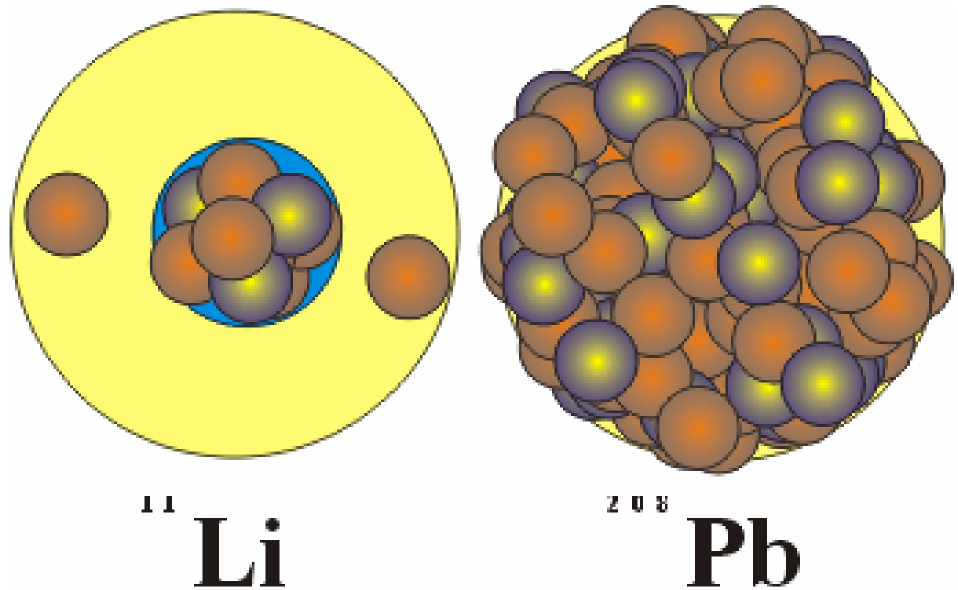
- Basis independent
- Explore individual quantum states
- Needs no heat bath
- No equilibration, thermalization and particle number conservation issues.
- Probe sensitivity of states to noise in external parameter(s)
- Phase transitions -> peaks in ICE

Mg Phase diagram



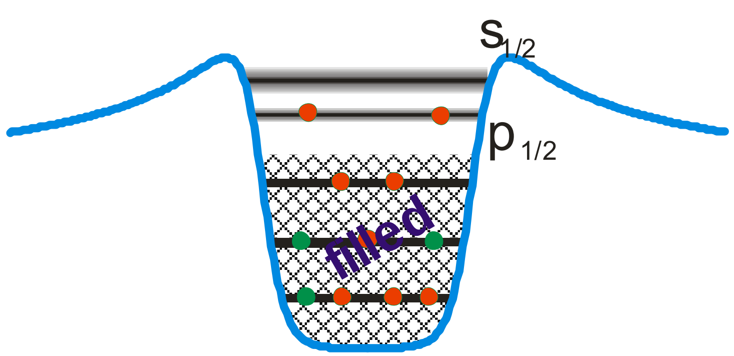
Exotic nuclei: Halo Nucleus ^{11}Li

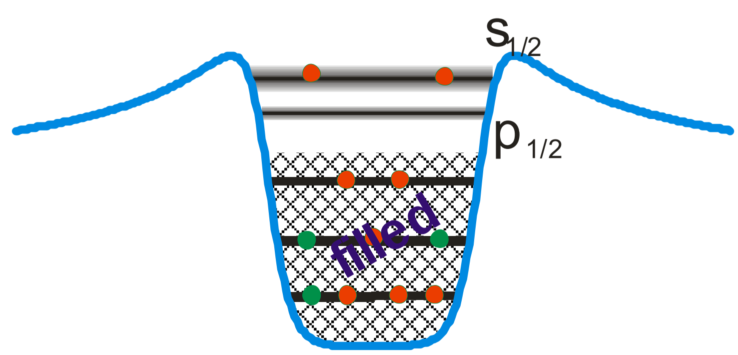
^{11}Li is halo, it is as big a lead



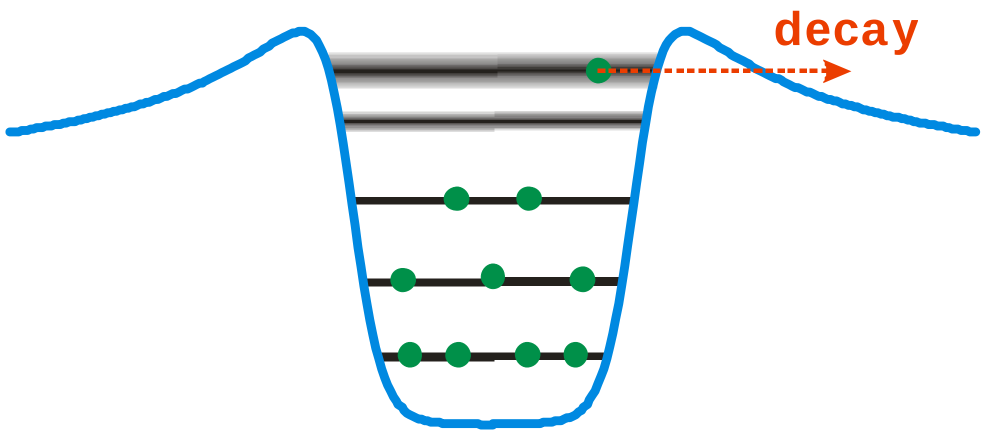
Two neutrons in ^{11}Li are moving on decaying orbitals!

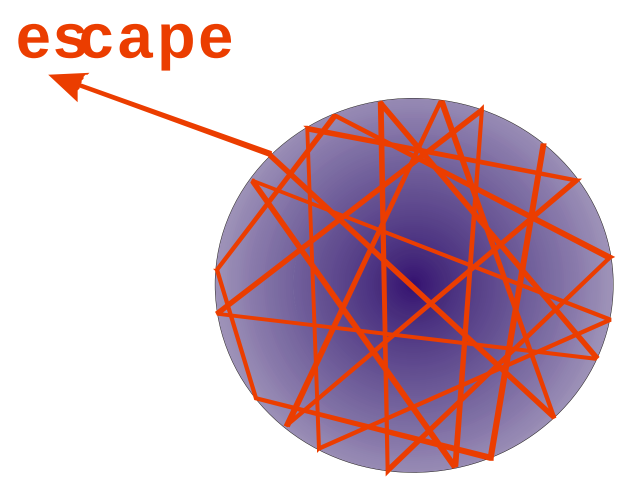
Two "valence" states are possible





Nuclear reaction theory Quantum billiards with particle-leaks



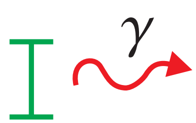


- Due to finite lifetime states acquire width (uncertainty in energy Γ=h/τ)
- Internal complex motion $\Leftrightarrow$ Radiation and decay ?

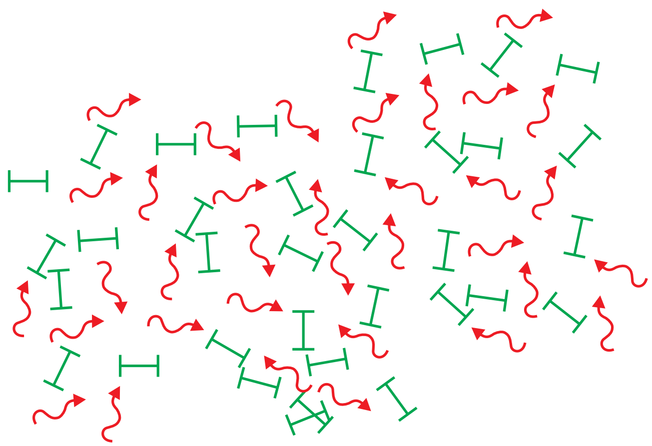
Superradiance, collectivization by decay

Dicke coherent state

N identical two-level atoms
coupled via common radiation

Single atom γ 

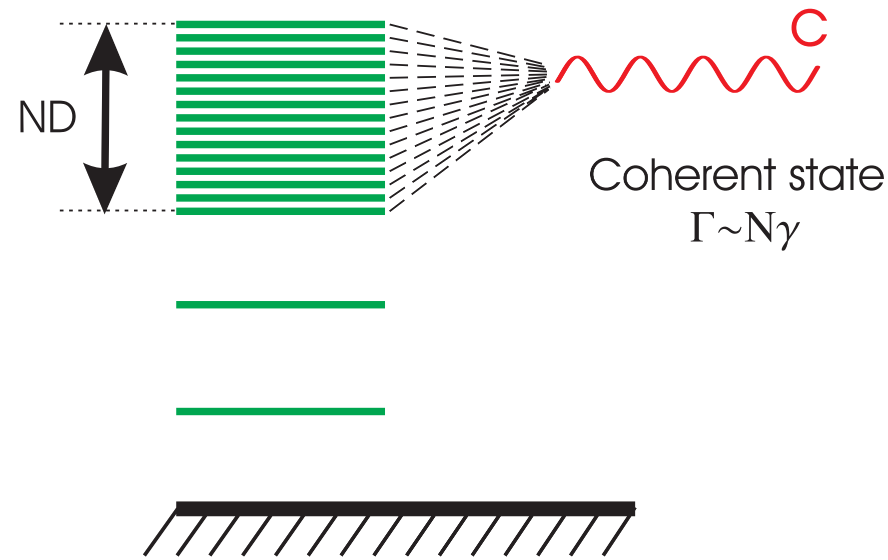
Coherent state $\Gamma \sim N\gamma$



Volume $\ll \lambda^3$

Analog in nuclei

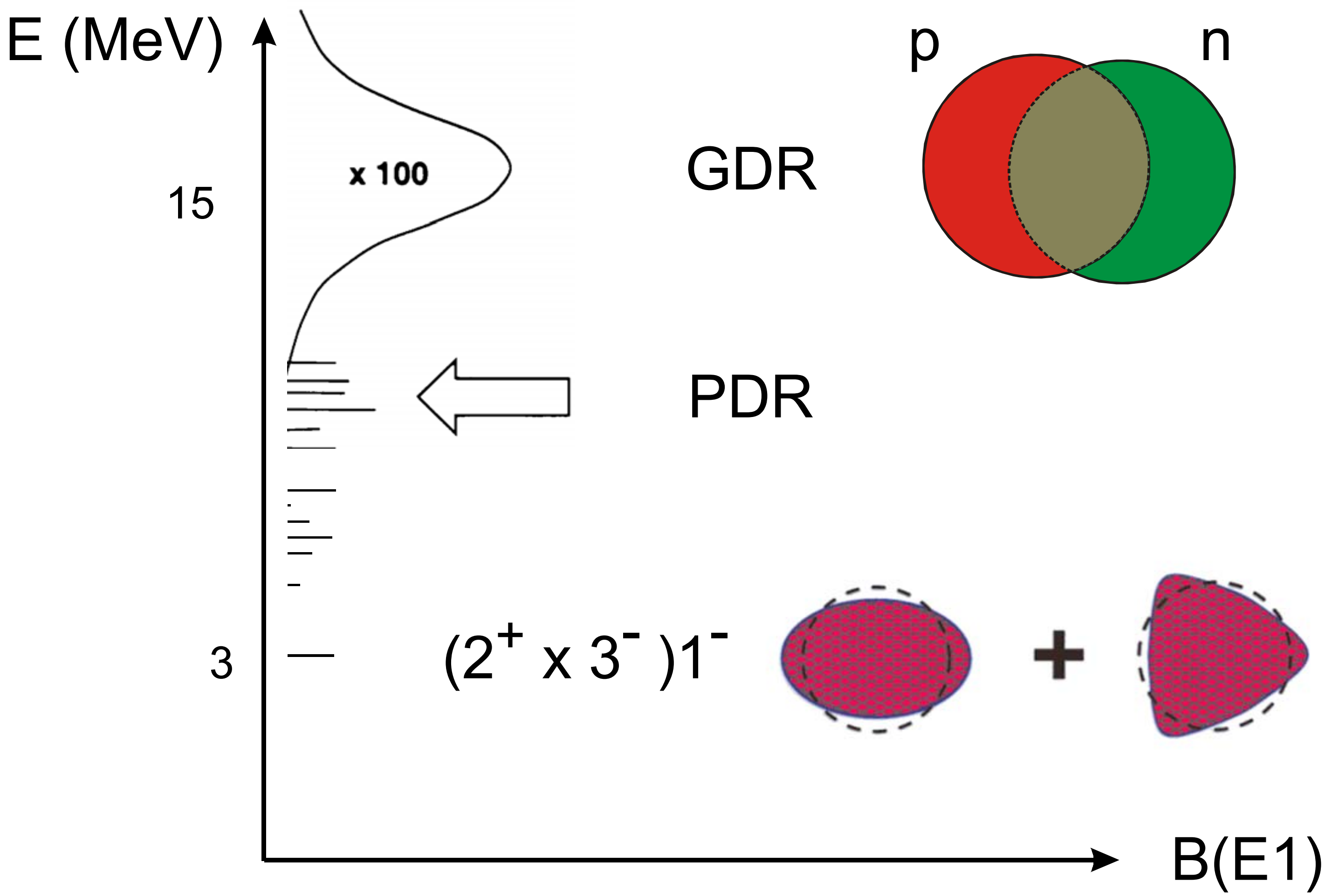
Interaction via continuum
Trapped states $\Rightarrow$ self-organization



γ ~ D and few channels

- Nuclei far from stability
- High level density (states of same symmetry)
- Far from thresholds

Shape vibration and GDR



Simple example: Two-spin system

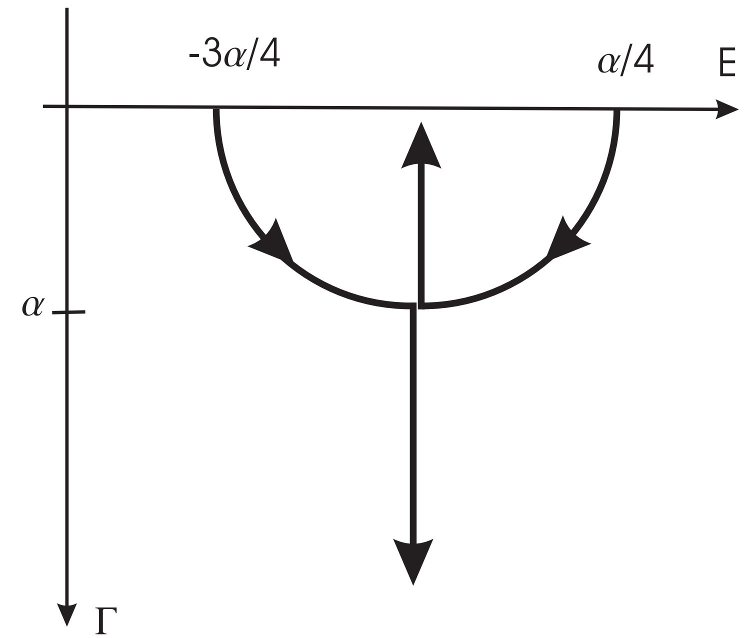
- Two interacting spins $H^{\circ} = \alpha \vec{s}_1 \cdot \vec{s}_2$
  - Spherical symmetry (triplet and singlet states)
- Magnetic field $H^{\mathrm{B}} = \epsilon s_1^z + \epsilon s_2^z = \epsilon S^z$
  - Preserves spherical symmetry $W = -i\frac{\gamma}{4}(s_1^z - s)$
- First spin in s^z=1/2 state decays
  - Reduces symmetry (S^z is preserved but not S^2)

- Hamiltonian for S^z=0

$$\mathcal{H} = -\frac{\alpha}{4} + \frac{1}{2}\begin{pmatrix} -i\gamma & \alpha \\ \alpha & 0 \end{pmatrix}$$

- Complex energies

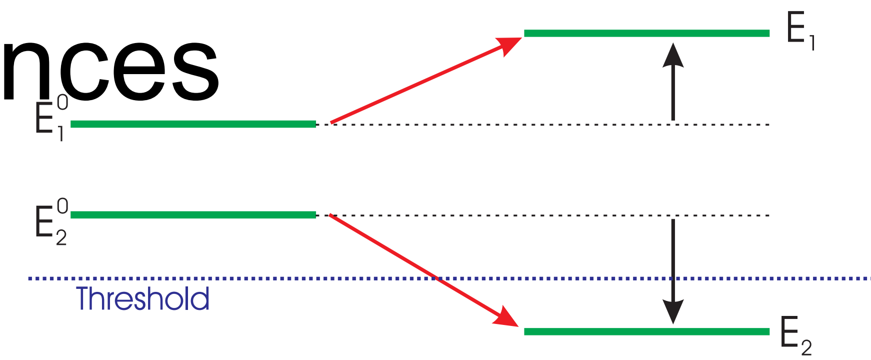
$$\mathcal{E}_{\pm} = -\frac{\alpha}{4} \pm \frac{1}{2}\sqrt{\alpha^2 - \left(\frac{\gamma}{2}\right)^2} - i\frac{\gamma}{4}$$



Features of open system

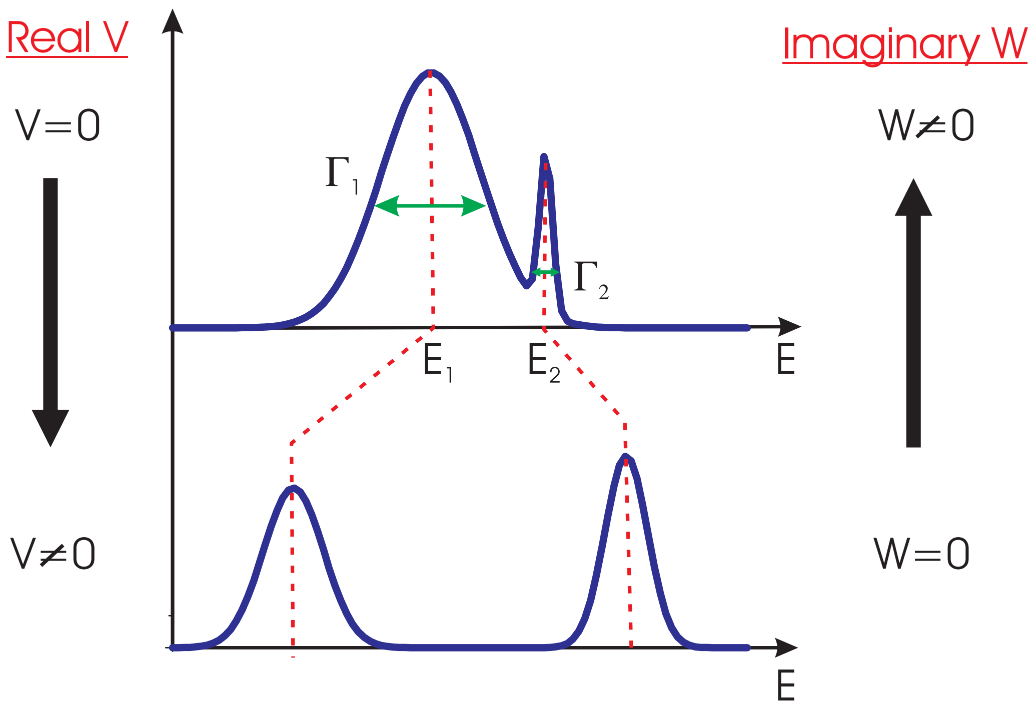
- Incompatible symmetries
- Many-body versus single-spin properties
- Interaction of two resonances
- Superradiance and separation of states
- "Phase transition"

^{11}Li an example of interacting resonances

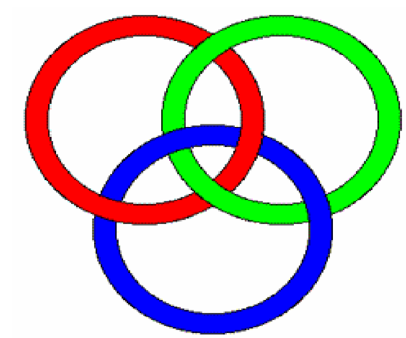


^{11}Li is stable it is held by interaction of resonances

$$\mathcal{H}=H^0+V - iW/2$$



^{11}Li is borromean, if one nuclon is removed it becomes unstable



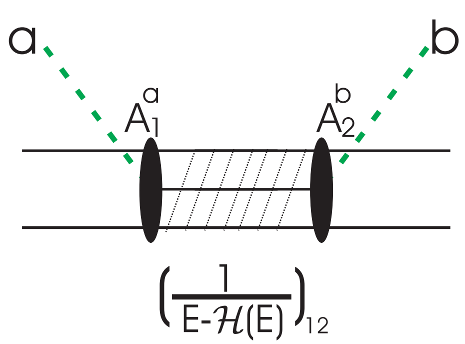
Scattering and cross section near threshold

Scattering Matrix

$$S^{ab} = (s^a)^{1/2}(\delta^{ab} - T^{ab})(s^b)^{1/2}$$

where $s^a = \exp(i\delta_a)$
is smooth scattering phase

$$T^{ab} = \sum_{12} A_1^{a*} \left(\frac{1}{E - \mathcal{H}}\right)_{12} A_2^b$$

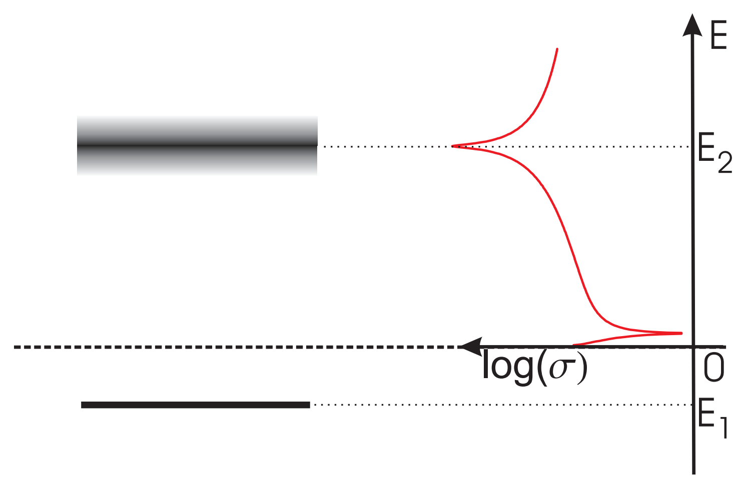


Solution in two-level model

$$T(E) = \frac{E(\gamma_1 + \gamma_2) - \gamma_1\epsilon_2 - \gamma_2\epsilon_1 - 2vA_1A_2}{(E - \mathcal{E}_+)(E - \mathcal{E}_-)}$$

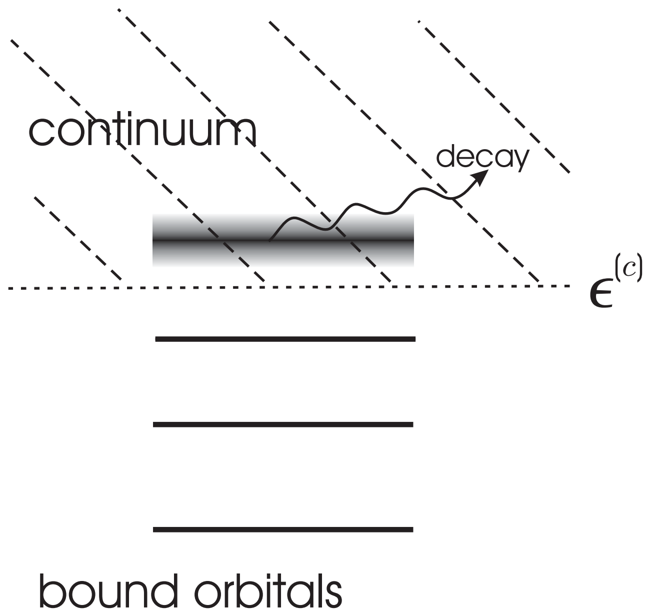
Cross section

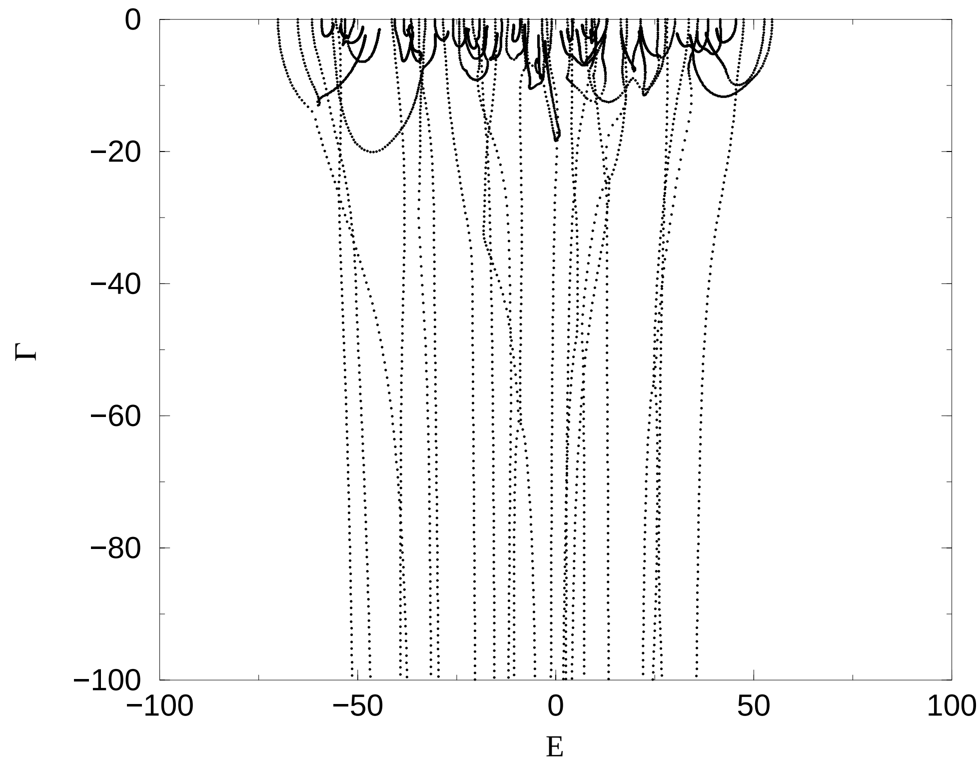
$$\sigma(E) = \frac{\pi}{k^2}|S(E) - 1|^2$$



Single-particle decay in many-body system

Evolution of complex energies $\mathcal{E}$=E-i Γ/2 as a function of γ





- Assume energy independent W
- Assume one channel γ=A^2
- System 8 s.p. levels, 3 particles
- One s.p. level in continuum e=ε –iγ/2

Total states 8!/(3! 5!)=56; **states that decay fast 7!/(2! 5!)=21**