

# The Physics of Nuclei – III: Reactions of Nuclei

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# Overview of Nuclear Reactions

Classifications

Compound and Direct

Examples

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## Spherical Potentials

Phase Shifts from Potentials

Integral expressions

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## Physics of Nuclear Reactions

Transfer Reactions

Stripping Reactions

Elastic Scattering

Fusion Reactions

# Classification by projectile

1. **Light-ion Reactions:** projectile mass  $A \leq 4$ :  
smaller momentum and energy transfer
2. **Heavy-ion Reactions:** projectile mass  $A \geq 4$ :  
large momentum and energy transfer

## Classification by Outcome

1. **Elastic scattering:**  
projectile and target stay in their g.s.
2. **Inelastic scattering:**  
projectile or target left in excited state
3. **Transfer reaction:**  
1 or more nucleons moved to the other nucleus
4. **Fragmentation/Breakup/Knockout:**  
3 or more nuclei/nucleons in the final state
5. **Charge Exchange:**  
A is constant but Z (charge) varies, e.g. by pion exchange
6. **Multistep Processes:**  
*intermediate* steps can be any of the above  
(‘virtual’ rather than ‘real’)

7. **Deep inelastic collisions:**  
Highly excited states produced
8. **Fusion:**  
Nuclei stick together
9. **Fusion-evaporation:**  
fusion followed by particle-evaporation and/or gamma emission
10. **Fusion-fission:**  
fusion followed by fission

The first 6 processes are *Direct Reactions* (DI)

The last 3 processes give a *Compound Nucleus* (CN).

# Compound and Direct Reactions

So when two nuclei collide there are 2 types of reactions:

1. Nuclei can coalesce to form highly excited **Compound nucleus (CN)** that lives for relatively long time.  
Long lifetime sufficient for excitation energy to be shared by all nucleons. If sufficient energy localised on one or more nucleons (usually neutrons) they can escape and CN decays.  
Independence hypothesis: CN lives long enough that it loses its memory of how it was formed. So probability of various decay modes independent of entrance channel.
2. Nuclei make 'glancing' contact and separate immediately, said to undergo **Direct reactions(DI)**.  
Projectile may lose some energy, or have one or more nucleons transferred to or from it.



## Location of reactions:

CN reactions at small impact parameter,

DI reactions at surface & large impact parameter.

CN reaction involves the whole nucleus.

DI reaction usually occurs on the surface of the nucleus. This leads to diffraction effects.

### Duration of reactions:

A typical nucleon orbits within a nucleus with a period of  $\sim 10^{-22}$  sec. [as K.E.  $\sim 20$  MeV].

If reaction complete within this time scale or less then no time for distribution of projectile energy around target  $\Rightarrow$  DI reaction occurred. CN reactions require  $\gg 10^{-22}$  sec.

## Angular distributions:

In DI reactions differential cross section strongly forward peaked as projectile continues to move in general forward direction.

Differential cross sections for CN reactions do not vary much with angle (not complete isotropy as still slight dependence on direction of incident beam).

## Examples of direct reactions:

Can identify various types of DI processes that can occur in reactions of interest:

1. **Elastic scattering:**  $A(a, a)A$  – zero Q-value — internal states unchanged.
2. **Inelastic scattering:**  $A(a, a')A^*$  or  $A(a, a^*)A^*$ . Projectile  $a$  gives up some of its energy to excite target nucleus  $A$ . If nucleus  $a$  also complex nucleus, it can also be excited.

[If energy resolution in detection of  $a$  not small enough to resolve g.s. of target from low-lying excited states then cross section will be sum of elastic and inelastic components. This is called **quasi-elastic scattering**].

- Breakup reactions:** Usually referring to breakup of projectile  $a$  into two or more fragments. This may be **elastic** breakup or **inelastic** breakup depending on whether target remains in ground state.
- Transfer reactions:**  
Stripping:  
Pickup:
- Charge exchange reactions:** mass numbers remain the same. Can be elastic or inelastic.

## Some terminology

### Reaction channels:

In nuclear reaction, each possible combination of nuclei is called a **partition**.

Each partition further distinguished by state of excitation of each nucleus and each such pair of states is known as a **reaction channel**.

The initial partition,  $a + A$  (both in their ground states) is known as the incident, or entrance channel. The various possible outcomes are the possible exit channels.

In a particular reaction, if not enough energy for a particular exit channel then it is said to be closed.

# Spherical Potentials

Non-relativistic, 2-body formalism of Schrödinger equation (SE).  
 Look at 2-body system in potential  $V(r)$

$$\mathbf{r} = (\mathbf{r}_1 - \mathbf{r}_2)$$

$$\mathbf{R} = (m_1\mathbf{r}_1 + m_2\mathbf{r}_2)/(m_1 + m_2)$$

The time-independent Schrödinger equation is

$$\hat{H}\Psi = E\Psi \quad (1)$$

The Hamiltonian for the system is

$$\begin{aligned} \hat{H} &= -\frac{\hbar^2}{2m_1}\nabla_{\mathbf{r}_1} - \frac{\hbar^2}{2m_2}\nabla_{\mathbf{r}_2} + V(r) \\ &= -\frac{\hbar^2}{2M}\nabla_{\mathbf{R}} - \frac{\hbar^2}{2m}\nabla_{\mathbf{r}} + V(r) \end{aligned} \quad (2)$$

$$[ m = m_1m_2/(m_1 + m_2) \text{ and } M = m_1 + m_2 ]$$

Thus can look for separable solutions of the form

$$\Psi(\mathbf{R}, \mathbf{r}) = \phi(\mathbf{R})\psi(\mathbf{r}) \quad (3)$$

Substituting for  $\Psi$  back in SE (1) gives LHS function of  $\mathbf{R}$  and RHS function of  $\mathbf{r}$ . Thus both equal to common constant,  $E_{cm}$ . Hence

$$-\frac{\hbar^2}{2M} \nabla_{\mathbf{R}}^2 \phi(\mathbf{R}) = E_{cm} \phi(\mathbf{R}) \quad (4)$$

and

$$\left( -\frac{\hbar^2}{2m} \nabla_{\mathbf{r}}^2 + V(r) \right) \psi(\mathbf{r}) = E_{rel} \psi(\mathbf{r}) \quad (5)$$

where  $E_{rel} = E - E_{cm}$ .

In scattering, if  $m_1$  is projectile incident on stationary target  $m_2$  then

$$E_{cm} = \frac{m_1}{m_1 + m_2} E$$

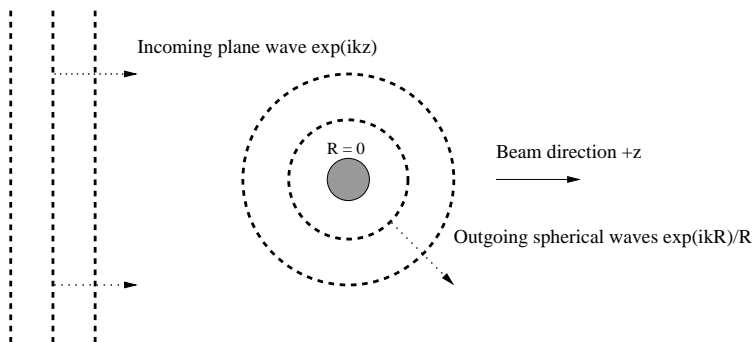
$$E_{rel} = \frac{m_2}{m_1 + m_2} E$$

Solution to (4) is simple:  $\phi(\mathbf{R}) = A e^{i\mathbf{K}\cdot\mathbf{R}}$  which is plane wave. Thus c.o.m. moves with constant momentum  $\hbar\mathbf{K}$  and does not change after scattering. (Note,  $E_{cm} = \hbar^2 K^2 / 2M$ ).

The real physics is in Eq.(5).



## Boundary conditions on $\psi(\mathbf{r})$



If incident beam  $\sim 1$  cm wide, this is  $10^{13}$  fm  $= 10^{12} \times$  nuclear size).

Thus beam can be represented by **plane wave**  $e^{ik \cdot \mathbf{r}}$

As  $|\mathbf{r}| \rightarrow \infty$  (i.e. moving away radially from scattering centre),

$$\psi(\mathbf{r}) \rightarrow N \left( e^{i\mathbf{k}\cdot\mathbf{r}} + f(\theta, \varphi) \frac{e^{ikr}}{r} \right) \quad (6)$$

where  $k$  is defined as  $E_{rel} = \hbar^2 K / 2m$ . (Take  $N = 1$ )

In QM, flux (probability current density) is given by

$$\mathbf{J} = \text{Re} \left[ \psi^* \left( -\frac{i\hbar}{m} \nabla_{\mathbf{r}} \right) \psi \right]$$

For incident flux,  $\psi_{inc} = e^{i\mathbf{k}\cdot\mathbf{r}}$  and

$$\begin{aligned} \mathbf{J}_{inc} &= \text{Re} \left[ e^{-i\mathbf{k}\cdot\mathbf{r}} \left( -\frac{i\hbar}{m} \nabla_{\mathbf{r}} \right) e^{i\mathbf{k}\cdot\mathbf{r}} \right] \\ &= \frac{\hbar\mathbf{k}}{m} . \end{aligned} \quad (7)$$

For scattered flux  $\psi_{scat} = f(\theta, \varphi) \frac{e^{ikr}}{r}$  and hence we obtain

$$\mathbf{J}_{scat} = J_{inc} \frac{|f(\theta, \varphi)|^2}{r^2} \hat{\mathbf{r}} \quad (8)$$

i.e.  $J_{scat} \propto J_{inc}$

and  $\propto 1/r^2$  (traditional fall-off).

**All scattering information contained in  $f(\theta, \varphi)$ ,**  
 known as the SCATTERING AMPLITUDE

# Cross Section

Number of particles collected by detector in unit time

$$= J_{scat} dA \quad (dA \text{ is cross-sectional area of detector})$$

$$= J_{inc} |f(\theta, \varphi)|^2 \frac{dA}{r^2}$$

$$= J_{inc} |f(\theta, \varphi)|^2 d\Omega$$

( $d\Omega$  is solid angle subtended by detector)

Define the **differential cross-section** (in units of area) as

The number of particles scattered into unit solid angle per unit time, per unit incident flux ( $J_{inc} = 1$ ).

$$\text{so} \quad \frac{d\sigma}{d\Omega} = |f(\theta, \varphi)|^2.$$



## What does $f(\theta, \varphi)$ look like?

Go back to SE in Eq.(5)

We know what the solution must look like asymptotically:

$$\psi(\mathbf{r}) \rightarrow N \left( e^{i\mathbf{k}\cdot\mathbf{r}} + f(\theta, \varphi) \frac{e^{ikr}}{r} \right) \quad (10)$$

Choose separable solution (as  $V(r)$  is central)

$$\psi(\mathbf{r}) = \sum_{\ell} \frac{u_{\ell}(r)}{kr} Y_{\ell 0}(\theta) \quad (11)$$

and choose z-axis parallel to incident beam, so  $e^{i\mathbf{k}\cdot\mathbf{r}} = e^{ikz}$ .

Then can write radial SE as

$$\frac{d^2 u_{\ell}}{dr^2} + \left[ k^2 - \frac{2m}{\hbar^2} V(r) - \frac{\ell(\ell+1)}{r^2} \right] u_{\ell} = 0 \quad (12)$$

# Asymptotic Solutions

Choose  $V(r) = 0$  for  $r > r_0$ . Beyond  $r_0$  get **free solution**

$$u_\ell'' + \left[ k^2 - \frac{\ell(\ell+1)}{r^2} \right] u_\ell = 0 \quad (13)$$

Free solution is related to Coulomb functions

$$r > r_0 : \quad u_\ell = A_\ell F_\ell(kr) + B_\ell G_\ell(kr) \quad (14)$$

$$\begin{array}{ccc} & \uparrow & \uparrow \\ & kr j_\ell(kr) & -kr n_\ell(kr) \\ & \text{(regular)} & \text{(irregular)} \end{array}$$

## Phase Shifts

As  $r \rightarrow \infty$

$$\begin{aligned}
 u_\ell &\rightarrow A_\ell \sin(kr - \ell\pi/2) + B_\ell \cos(kr - \ell\pi/2) \\
 &= C_\ell \sin(kr - \ell\pi/2 + \delta_\ell)
 \end{aligned}
 \tag{15}$$

where  $\delta_\ell$  is known as the **phase shift**.

If  $V = 0$  then solution must be valid everywhere, even at origin where it has to be regular. Thus  $B_\ell = 0$ .

So, asymptotically (long way from scattering centre):

$$\begin{aligned}
 \text{For } V = 0 & \quad u_\ell = A_\ell \sin(kr - \ell\pi/2) \\
 \text{and for } V \neq 0 & \quad u_\ell = C_\ell \sin(kr - \ell\pi/2 + \delta_\ell)
 \end{aligned}
 \tag{16}$$

Thus, switching scattering potential 'on' produces a shift in the phase of the wave function at large distances from the scattering centre.

## Scattering amplitudes from Phase Shifts

Now substituting for  $u_\ell$  from Eq.(16) back into Eq.(11) for  $\psi(\mathbf{r})$ , and after some angular momentum algebra, we obtain a scattering wave function which, when equated with the required asymptotic form of Eq.(10) gives

$$f(\theta) = \frac{1}{k} \sum_{\ell} (2\ell + 1) T_{\ell} P_{\ell}(\cos \theta) \quad (17)$$

$$\text{where } T_{\ell} = e^{i\delta_{\ell}} \sin \delta_{\ell} = \frac{1}{2i}(S_{\ell} - 1) . \quad (18)$$

$T_{\ell}$  is the partial wave  $T$ -matrix.

$S_{\ell}$  is the partial-wave  $S$ -matrix.

They are connected to the **T-matrix** (see later).

There is no dependence on  $\varphi$  because of *central* potentials.



## Properties of the S-matrix $S_\ell$ :

The S-matrix element is a complex number  $S_\ell = e^{2i\delta_\ell}$

1. For purely diffractive (real) potentials  $|S_\ell| = 1$ .  
 This is called *unitarity*, and is the conservation of flux.  
 $\delta_\ell$  is usually positive for attractive potentials.
2. For absorptive (complex) potentials,  $|S_\ell| \leq 1$ .  
 The total absorption cross section is

$$\sigma_A = \frac{\pi}{k} \sum_{\ell} (2\ell + 1)(1 - |S_\ell|^2) \quad (19)$$

3. The  $\ell$  value where  $Re(S_\ell) \sim 0.5$  is the **grazing**  $\ell$  value.
4. Partial wave  $\ell$  related to impact parameter  $b$   
 in semiclassical limit:  $\ell = k b$

## Free Green's function $G_0(E)$

Can write SE as

$$(E - H) \psi = 0 \quad \text{or} \quad (E - H_0) \psi = V \psi \quad (20)$$

where  $H = H_0 + V$ . Thus

$$\psi = (E - H_0)^{-1} V \psi = G_0(E) V \psi \quad (21)$$

$G_0(E)$  is the **Green's operator**.

Eq.(21) is not general solution for  $\psi$  as can add on solution of homogeneous equation

$$(E - H_0) \chi = 0 \quad (22)$$

# Lipmann-Schwinger equation

General solution of Eq.(20) is

$$\psi = \chi + G_0(E) V \psi \quad (23)$$

This is iterative

$$\psi = \chi + G_0 V \chi + G_0 V G_0 V \chi + \dots \quad (24)$$

Eq.(23) can be written in integral form as the  
**Lipmann-Schwinger** equation

$$\psi(\mathbf{r}) = \chi(\mathbf{r}) + \int d\mathbf{r}' G(\mathbf{r}, \mathbf{r}') V(\mathbf{r}') \psi(\mathbf{r}') \quad (25)$$

where  $G(\mathbf{r}, \mathbf{r}')$  is the **Green's function**.

## Integral expression for the Scattering Amplitude

The  $\chi(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}$  = incident plane wave,

and we use  $\psi_{\mathbf{k}}^{(+)}(\mathbf{r})$  for the scattering wave function.

(i.e. incident momentum  $\mathbf{k}$  and (+) for outgoing waves solution).

Comparing Eq.(25) with required asymptotic form for  $\psi$  we see that integral term must tend to

$$f(\theta) \frac{e^{ikr}}{r} \quad \text{as } |\mathbf{r}| \rightarrow \infty . \quad (26)$$

Thus, using properties of Green's function we can obtain

$$f(\theta) = -\frac{m}{2\pi\hbar^2} \int d\mathbf{r} e^{-i\mathbf{k}'\cdot\mathbf{r}} V(r) \psi_{\mathbf{k}}^{(+)}(\mathbf{r}) . \quad (27)$$

## Transition matrix element

In Dirac (bra-ket) notation we write this

$$f(\theta) = -\frac{m}{2\pi\hbar^2} \langle \mathbf{k}' | V | \psi_{\mathbf{k}}^{(+)} \rangle \quad (28)$$

$$= -\frac{m}{2\pi\hbar^2} T(\mathbf{k}', \mathbf{k}) . \quad (29)$$

$T(\mathbf{k}', \mathbf{k})$  is known as the **Transition matrix element**.

It can be shown, using some algebra (eg. plane wave expansion formula for  $e^{-i\mathbf{k}'\cdot\mathbf{r}}$ ), that Eq.(27) leads to Eq.(17):

$$f(\theta) = \frac{1}{k} \sum_{\ell} (2\ell + 1) T_{\ell} P_{\ell}(\theta) \quad (30)$$

# Multi-channel Scattering

Use for inelastic, transfer, breakup (etc) in addition to elastic.

Two channel (1=elastic, 2=reaction):

$$\begin{aligned} [T_1 + U_1 - E_1]\psi_1(\mathbf{r}) + V_{12}\psi_2(\mathbf{r}) &= 0 \\ [T_2 + U_2 - E_2]\psi_2(\mathbf{r}) + V_{21}\psi_1(\mathbf{r}) &= 0. \end{aligned} \quad (31)$$

Solve for reaction channel:

$$\begin{aligned} T_{21}(\mathbf{k}'_2, \mathbf{k}_1) &= \langle \mathbf{k}'_2 | V_{21} | \psi_{\mathbf{k}}^{(+)} \rangle \quad :: \text{Exact} \\ &\approx \langle \mathbf{k}'_2 | V_{21} | \chi_{\mathbf{k}_1}^{(+)} \rangle \quad :: \text{DWBA.} \end{aligned}$$

where  $\chi$  is solution of  $[T_1 + U_1 - E_1]\chi_{\mathbf{k}_1}(\mathbf{r}) = 0$ .

This is to neglect 'back couplings'  $V_{12}$ .

## Multi-particle Scattering at High Energies

Some approximations useful at high scattering energies:

**Adiabatic (sudden) approximation:** Neglect excitation energy of projectile compared with beam energy

$$[H_{\mathbf{R}} + h_{\mathbf{r}} + V(\mathbf{R}, \mathbf{r}) - E]\psi \approx [H_{\mathbf{R}} + \varepsilon_0 + V(\mathbf{R}, \mathbf{r}) - E]\psi$$

So can solve  $[H_{\mathbf{R}} + \varepsilon_0 + V(\mathbf{R}, \mathbf{r}) - E]\psi = 0$  for each  $\mathbf{r}$  separately, and average later.

**Eikonal (Glauber) scattering:** Integrate scattering phase along a straight line trajectory with impact parameter  $b$ :

$$S(b) = \exp i \left[ -\frac{1}{\hbar v} \int_{-\infty}^{\infty} V(b, z') dz' \right] \quad (32)$$

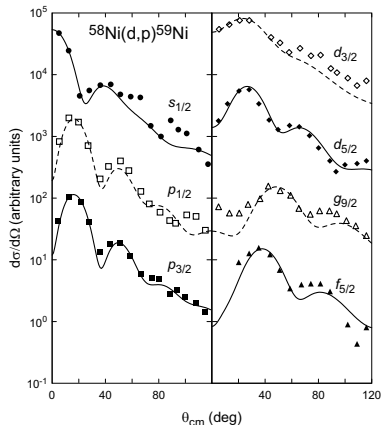
# Physics of Nuclear Reactions

- ▶ Transfer Reactions
- ▶ Breakup Reactions
- ▶ Halo Scattering: Elastic
- ▶ Halo Total Reaction Cross Section
- ▶ Book



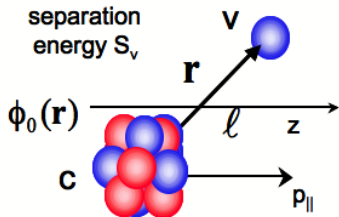
# Transfer Reactions to Probe Single-Particle Structure

- ▶ Weak, so use DWBA
- ▶ One-nucleon transfers, (p,d)  
 shape shows L-value of orbital  
 magnitude gives spectroscopic factor
- ▶ Two-neutron transfers, (p,t)  
 Magnitude depends on s-wave  
 pairing in halo  
 Only relative magnitudes  
 reliably modeled.
- ▶ But: full analysis requires  
 multi-step calculations



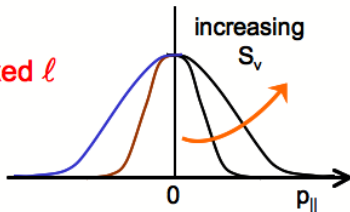
# Stripping Reactions: Measuring Momentum

Probing the momentum content of bound states

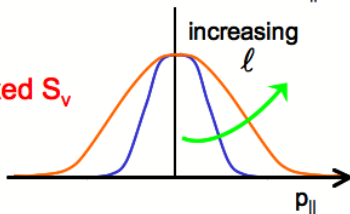


consider momentum components  $p_{\parallel}$  of the heavy residue parallel to the beam direction. In the projectile rest frame .....

Fixed  $l$

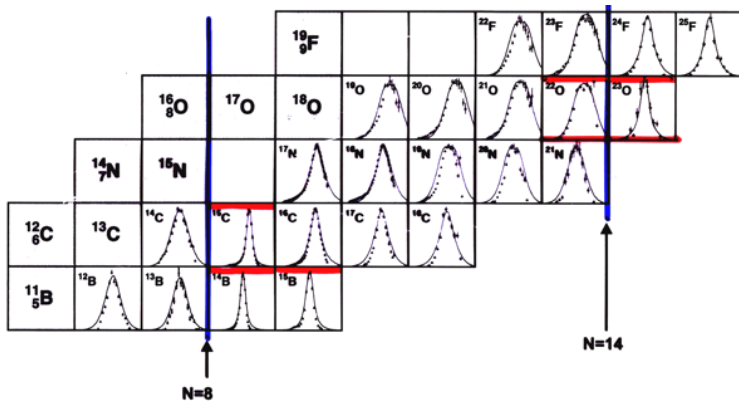


Fixed  $S_v$



# Stripping Reactions: Nuclear Structure

Glauber (eikonal) theory of breakup:

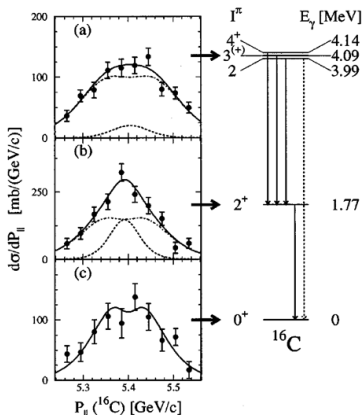


E.Sauvan et al., Phys Lett B 491 (2000) 1

# Stripping Reactions: Removing a Neutron

Reaction  ${}^9\text{Be}({}^{17}\text{C}, {}^{16}\text{C}\gamma)X$

Measured  $\gamma$  from core decays helps to fix the final state



$\ell=0,2$   
 admixture

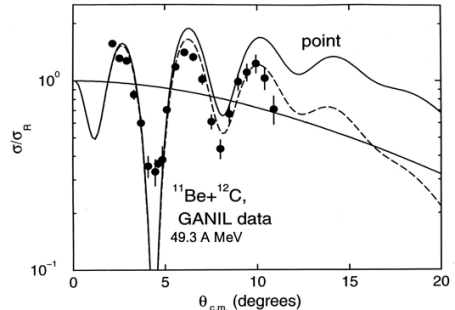
$\ell=0,2$   
 admixture

pure  $\ell=2$   
 but large!!

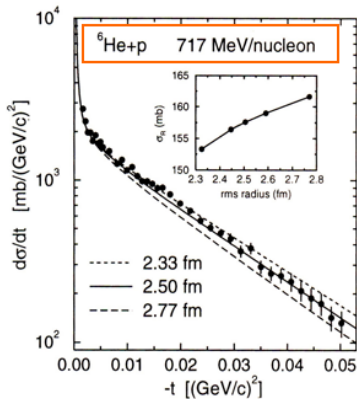
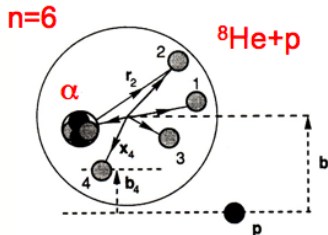
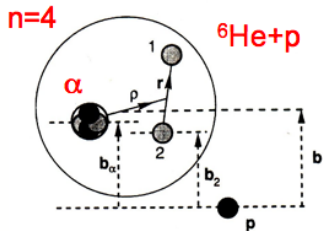
# Halo Scattering: Elastic

Depends on

- ▶ Folded potential from densities
- ▶ Halo breakup effects, i.e.
- ▶ Polarisation potential from breakup channel



# Four- and Six-body Scattering

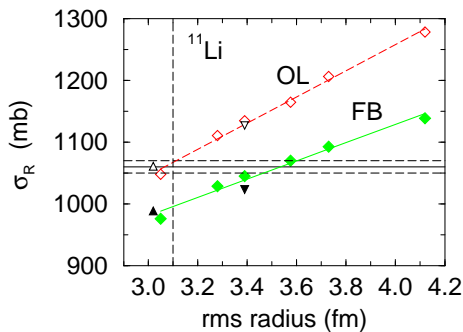


Al-Khalili and Tostevin, PRC **57** (1998) 1846  
 Tostevin et al., PRC **56** (1997) R2929

# Halo Total Reaction Cross Section

Depends on

- ▶ Densities and NN scattering, as usual
- ▶ But: Effects of Halo Breakup (virtual and real) are big!
- ▶ Use few-body Glauber, not Optical Limit Glauber
- ▶ New radii are larger.



# Halo Fusion: an Unsolved Problem

In low-energy Halo Fusion (near the Coulomb barrier):  
Halo neutrons should affect fusion:

- ▶ **Increase fusion**, from neutron attractions & neutron flow
- ▶ **Decrease complete fusion**, from breakup
- ▶ **Increase fusion**, from molecular states & resonances

So: need experiments + good theories!

Some experiments already performed with  ${}^6\text{He}$  and  ${}^9\text{Be}$ , but theoretical interpretations are still unclear.



# Coupled Channels Calculations

## Fresco

*Coupled Reaction Channels Calculations*

[www.fresco.org.uk](http://www.fresco.org.uk)

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### About Fresco

**Fresco** is a program developed by Ian Thompson over the period 1983 - 2006, to perform coupled-reaction channels calculations in nuclear physics. It uses Fortran 90 or Fortran 95 on Unix, Linux, Vax and Windows machines.

**Sfresco** is an additional version of Fresco, to provide Chi-squared searches of potential and coupling parameters, and to fit additional R-matrix terms in hybrid models.

Free!

Book: to be published 2008

# Nuclear Reactions for Astrophysics

Principles, Calculation and Applications of Low-Energy Reactions

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