The Physics of Nuclei – I: Building Nuclei

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Synopsis: Three Lectures

I. Building Nuclei

Nucleons, NN forces, Effective Forces
Few-body dynamics, Halo Nuclei

II. Nuclear Structure

Liquid Drop,
Shell model,
Density Functional Descriptions

III. Nuclear Reactions

Types of Reactions,
Scattering, Mechanisms

Who am I?

Ian Thompson

- I am a theorist
- Ph.D. in New Zealand; Postdoc. at Daresbury Lab (UK)
- Teaching at Bristol, then Surrey University, up to 2006
- Solving quantum reaction problems (also halo structure)
- Comparisons of ‘good theories’ with experiments.
- 1 year ago: moved to Lawrence Livermore Lab (CA)
- Writing a Book on Reaction Theory

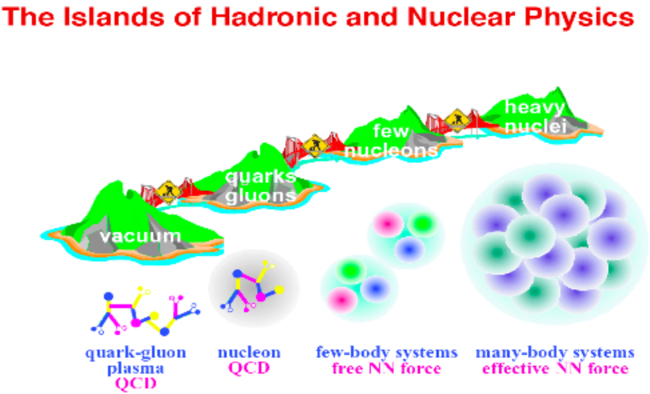
Who are you?

Many students!

- Theory or Experiment?
- Beginning Ph.D. or mid Ph.D., or postdoc? (where in the U.S., or overseas?)
- Which experiments are interesting?
- What do you hope to learn?
-

Building Nuclei

We look for how the quarks makes nucleons, which interact to make nuclei.



Using Nuclei

Beams and Targets (or electrons)
: Know your target!

Test fundamental symmetries
: E.g. by mixing intrinsic nuclear symmetries

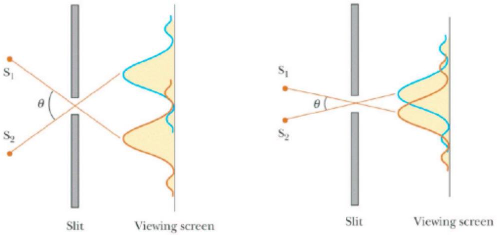
Nuclear astrophysics
: Nucleosynthesis, supernovae, neutron stars

New structures of exotic nuclei
: E.g. near the proton and neutron drip lines

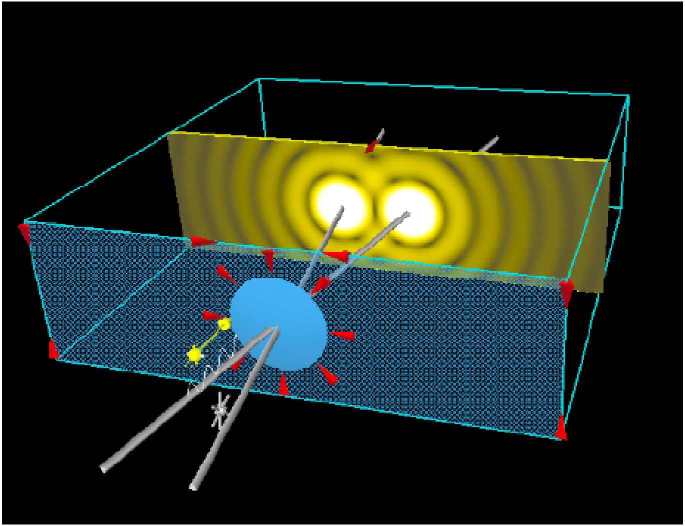
Fermionic Many-Body Systems

Resolution determines level of Dynamical Detail.

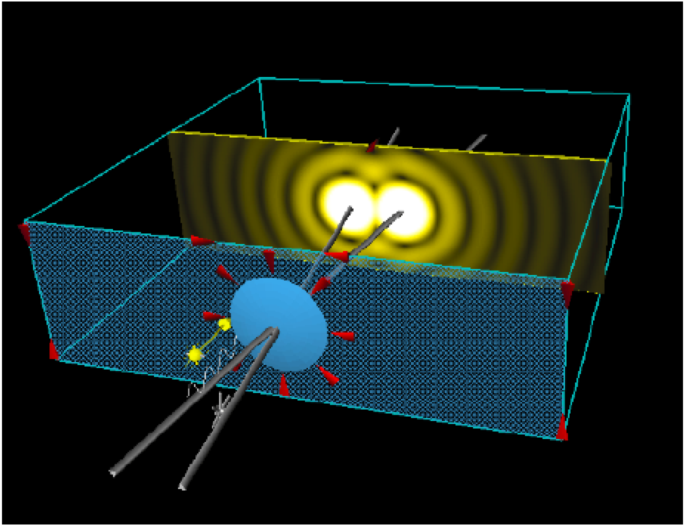
Entities and Effective Interactions also vary with resolution



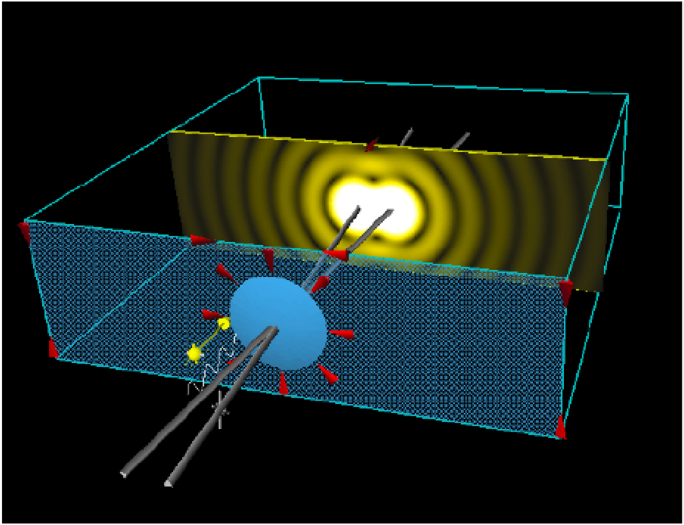
Diffraction and Resolution 1



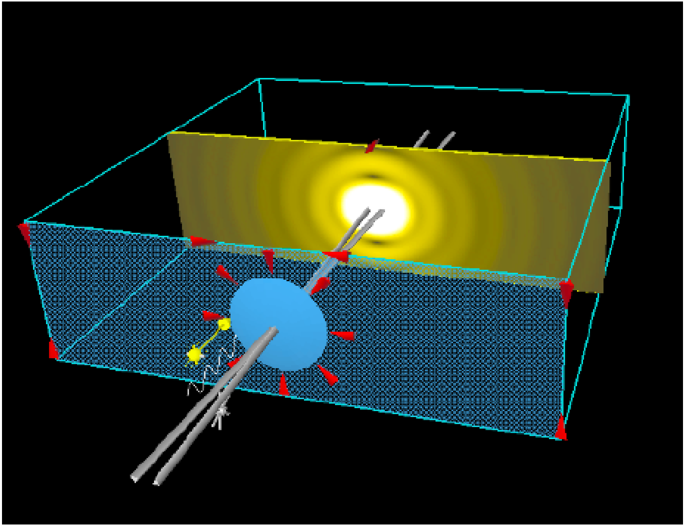
Diffraction and Resolution 2



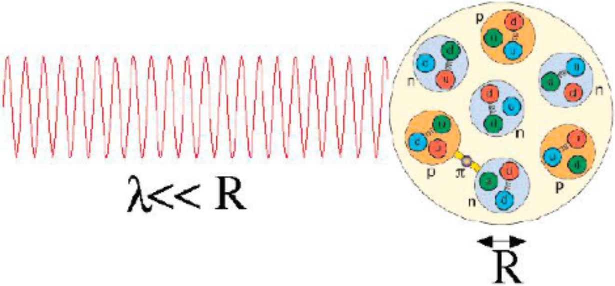
Diffraction and Resolution 3



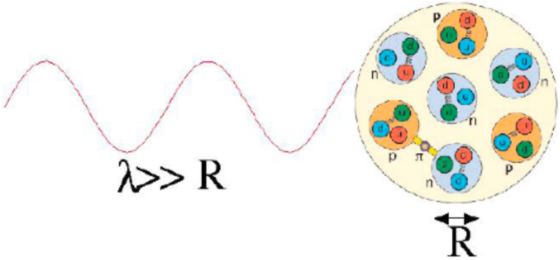
Diffraction and Resolution 4



Principles of Effective Theories 1

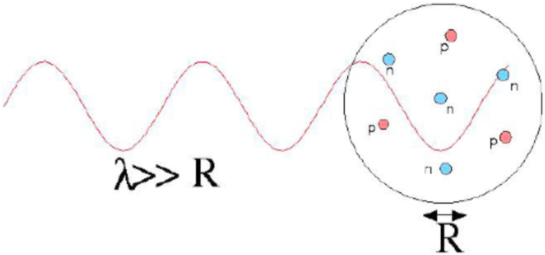


Principles of Effective Theories 2



If system is probed at low energies, fine details not resolved

Principles of Effective Theories 3



If system is probed at low energies, fine details not resolved

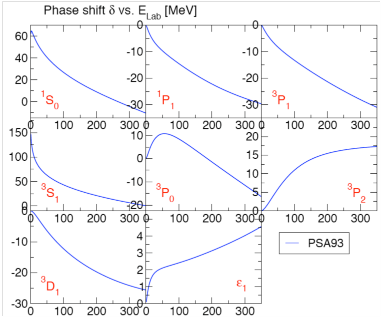
- use low-energy variables for low-energy processes
- short-distance structure can be replaced by something simpler without distorting low-energy observables

Two-nucleon phenomena

Nuclei start when nucleons are resolved

Start from the simplest experiments:

- NN Scattering
  (nn, np, pp)
  Phase shift analysis:
- Deuteron Bound State:
  Binding 2.224 MeV,
  Quadrupole moment 0.282 fm^2.



Phenomenological NN Potentials

Use meson exchange forms

Adjust parameters and cutoffs

Reproduce low-energy scattering lengths etc

$a_{pp} = -17.3 \pm 0.4$ fm; $a_{nn} = -18.8 \pm 0.3$ fm;
$a_{np} = -23.75 \pm 0.1$ fm;
Note that $V_{pp} \neq V_{np} \neq V_{nn}$.

Main features

Strong tensor force,
Strong repulsive core at short distances.

Examples of NN Potentials

Argonne potentials

Wiringa, Stoks, Schiavilla, PRC 51, 38 (1995)
Coulomb + One-pion exchange + intermediate- and short-range

Bonn potential

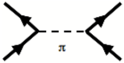
R. Machleidt, PRC63, 024001 (2001)
Based on meson-exchange, Non-local

Effective field theory

Ordóñez, Ray, van Kolck, PRC 53, 2086 (1996); Epelbaoum, Glöckle, Meissner, NPA 637, 107 (1998)
Based on Chiral Lagrangians
Expansion in momentum up to cutoff $\sim$ 1 GeV
Generally has a soft core

Theory of NN Potentials

Look at main part – π-exchange:



Elastic scattering in momentum space

$$V^{\pi NN}_{local}(\mathbf{q}=\mathbf{k}'-\mathbf{k}) = -\frac{g_\pi^2}{4M^2}\frac{(\sigma_1\cdot\mathbf{q})(\sigma_1\cdot\mathbf{q})}{\mathbf{q}^2+m_\pi^2}$$

or through a Fourier transform, in coordinate space:

$$V_\pi = \frac{g_\pi^2}{4M^2}\frac{1}{3}m_\pi\left[\sigma_i\cdot\sigma_j + \left(1+\frac{3}{\mu r}+\frac{3}{(\mu r)^2}\right)(3\sigma_i\cdot\hat{\mathbf{r}}\sigma_i\cdot\hat{\mathbf{r}}-\sigma_i\cdot\sigma_j)\right]\frac{e^{-\mu r}}{\mu r}$$

Off-shell component present in the Bonn potentials

$$V^{\pi NN}(\mathbf{k}',\mathbf{k}) = -\frac{g_\pi^2}{4M^2}\frac{(E'+M)(E+M)}{(\mathbf{k}'-\mathbf{k})^2+m_\pi^2}\left(\frac{\sigma_1\cdot\mathbf{k}'}{E'+M}-\frac{\sigma_2\cdot\mathbf{k}}{E+M}\right)\times\left(\frac{\sigma_2\cdot\mathbf{k}'}{E'+M}-\frac{\sigma_1\cdot\mathbf{k}}{E+M}\right)$$

Non-local (depends on initial and final momenta). Plays a role in many-body applications: more binding

Three-Nucleon Interactions

Look at main part, **pion-exchange:**

Needed to Bind $A = 3$ nuclei
: Two-nucleon interactions under-bind
: Note CD-Bonn has a little more binding due to non-local terms

Further evidence
: from by ab initio calculations for ^{10}B:
: NN-interactions give the wrong ground-state spin!

Example: Tucson-Melbourne Force
: S.A. Coon and M.T. Peña, PRC 48, 2559 (1993)
: Based on two-pion exchange and intermediate Δs
: The exact form of NNN is not known

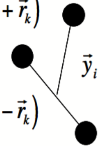
Three-Body Dynamics

For two particles we use Schrdinger equation

For three and four, there are Faddeev and Faddeev-Yakubovsky formulations

Three-body Jacobi Coordinates:

$$\vec{R} = \tfrac{1}{3}\left(\vec{r}_i + \vec{r}_j + \vec{r}_k\right)$$



$$\vec{y}_i = \sqrt{\frac{2}{3}}\left(\vec{r}_i - \tfrac{1}{2}\left(\vec{r}_j + \vec{r}_k\right)\right)$$

$$\vec{r}_i = \tfrac{1}{\sqrt{2}}\left(\vec{r}_j - \vec{r}_k\right)$$

$$\Psi = \psi_1 + \psi_2 + \psi_3$$

$$\psi_i = \frac{1}{E - H_0} T_i \left(\psi_j + \psi_k\right)$$

$$H_0 = \sum_i \frac{\vec{p}_i^{\,2}}{2m_i}$$

$$T_i = V_{jk} + V_{jk} \frac{1}{E - H_0} T_i$$

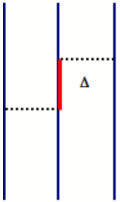
$$\left(E - H_0 - V_{23}\right)\psi_1 = V_{23}\left(P_{12}P_{23} + P_{13}P_{23}\right)\psi_1$$

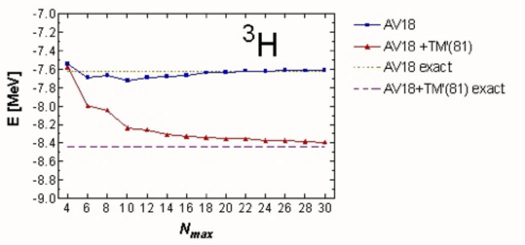
W. Glöckle in Computational Nuclear Physics, Springer-Verlag, Berlin, 1991

Exact methods exist for $A \leq 4$.

Effects of Three-Nucleon Force

Binding of Triton (^{3}H) without and with Tucson-Melbourne Force





More than Four Bodies?

Synopsis of what we can do:

- Cluster Models.
- Liquid-drop Models: see lecture II.
- Greens Function Monte Carlo $\langle \Psi_{exact} | \hat{O} | \Psi_{exact} \rangle = \lim_{\beta \to \infty} \frac{\langle \psi_{trial} | \hat{O} e^{-\beta \hat{H}} | \psi_{trial} \rangle}{\langle \psi_{trial} | e^{-\beta \hat{H}} | \psi_{trial} \rangle}$
- Coupled-cluster $\Psi = e^{\sum_{ij} C_{ij} a_i^+ a_j + \sum_{ijkl} C_{ijkl} a_i^+ a_j^+ a_k a_l + \ldots} \psi_{ref}$
- Shell model (lecture II.)

$$\phi = \frac{1}{\sqrt{A!}} \begin{vmatrix} \phi_i(\mathbf{r}_1) & \phi_i(\mathbf{r}_2) & \cdots & \phi_i(\mathbf{r}_A) \\ \phi_j(\mathbf{r}_1) & \phi_j(\mathbf{r}_2) & & \phi_j(\mathbf{r}_A) \\ & \vdots & \ddots & \vdots \\ \phi_l(\mathbf{r}_1) & \phi_l(\mathbf{r}_2) & \cdots & \phi_l(\mathbf{r}_A) \end{vmatrix}$$
$$= a_l^+ \ldots a_j^+ a_i^+ |0\rangle$$

- Mean-field (energy density functional) methods (lecture II.)

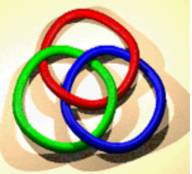
Cluster Models for Halo Nuclei

Definition Weakly-bound nuclei near drip line that are large

Composition One or two neutrons (or protons) outside a core nucleus.

Interesting New physics away from valley of stability

Borromean Borromean three-body systems bound, even though no pairwise (two-body) bound states:



Examples of Halo Nuclei

One neutron:

^{11}Be ($S_n = 0.504$ MeV)

Two-neutron Borromean:

^{6}He ($S_{2n} = 0.97$ MeV),

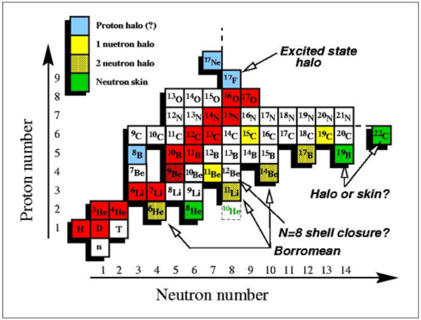
^{11}Li ($S_{2n} = 0.30$ MeV),

One-proton

^{8}B ($S_p = 0.137$ MeV),

Two-proton Borromean:

^{17}Ne ($S_{2p} = 0.96$ MeV),

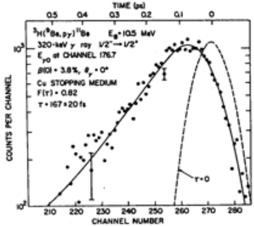


Why Study Haloes?

- Good few-body system:
  Continuum is near to bound states, long tails to bound states, so large cross sections & dynamic distortion in reactions.
- See prominent single-particle states
- See pairing outside nuclear surface:
  in two-neutron halo ground states;
  in two-neutron continuum via breakup; and
  in two-proton decay via tunnelling
- See bound states in classically forbidden regions.

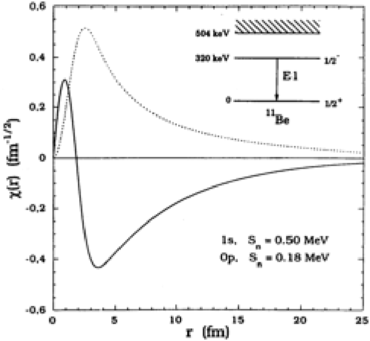
First Halo: ^{11}Be

Strong E1 transition in ^{11}Be:



τ = 168(17) fs: B(E1) = 0.36(3) W.u.

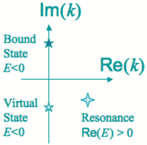
Millener et al., PRC **28** (1983) 497

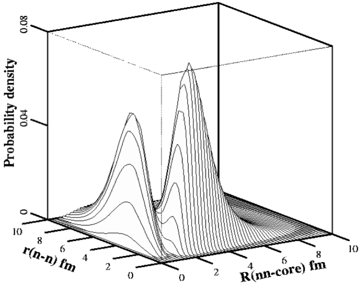


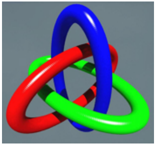
"We note that to obtain the 1$s_{1/2}p_{1/2}$ matrix element for low binding energies it is necessary to integrate out to large radii"

Borromean Halo: ^{6}He

- Two Neutrons and an α particle bound at $S_{2n} = 0.97$ MeV
- n-α unbound, but $p_{3/2}$ resonance at 0.8 MeV
  n-n unbound, but virtual state $a_{nn} = -18.8 \pm 0.3$ fm

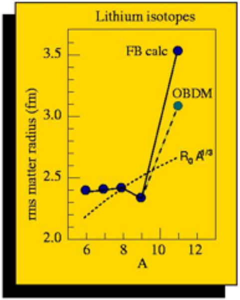


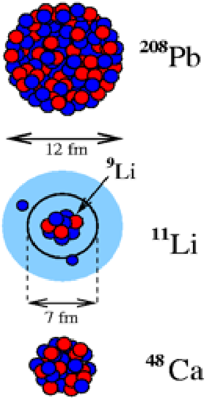




Experimental Evidence

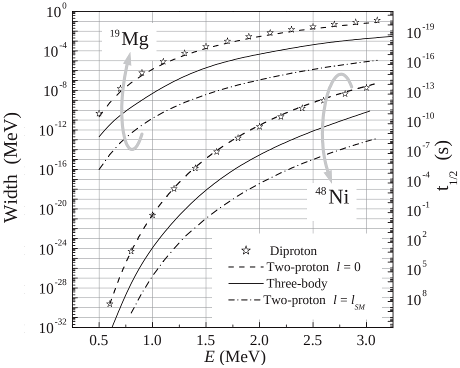
Study of halo nuclei (officially) began with measurement of interaction cross sections in Berkeley in 1985.





Two-proton Decay

- Not via point diproton
- Need three-body models with pairing in exterior
- Prediction: pairing acts to correlate the protons to enhance $L = 0$ cluster-nucleus relative motion.



Using Few-Body Methods for More Bodies

Summary:

- Cluster Models.
- Greens Function Monte Carlo

$$\langle \Psi_{exact} | \hat{O} | \Psi_{exact} \rangle = \lim_{\beta \to \infty} \frac{\langle \psi_{trial} | \hat{O} e^{-\beta \hat{H}} | \psi_{trial} \rangle}{\langle \psi_{trial} | e^{-\beta \hat{H}} | \psi_{trial} \rangle}$$

Monte Carlo Methods

- For smaller systems (finite or in a box with pbc's)
  $\Longrightarrow$ approximate energy and full many-body wave function
  - Variational, diffusion, path integral, Green's function MC
  - All use the Metropolis algorithm: random walkers
- Variational Monte Carlo (VMC): Estimate $\langle E \rangle = \int d\mathbf{R}\, \rho(\mathbf{R})\, E_L(\mathbf{R})$
  - local energy $E_L(\mathbf{R}) = \frac{H\psi_T(\mathbf{R})}{\psi_T(\mathbf{R})}$ with trial wave function $\psi_T(\mathbf{R})$
  - probability distribution $\rho(\mathbf{R}) = \frac{\psi_T^2(\mathbf{R})}{\int d\mathbf{R}\, \psi_T^2(\mathbf{R})}$
  - accept step to $\mathbf{R}'$ if $p = \psi_T^2(\mathbf{R}')/\psi_T^2(\mathbf{R}) \geq 1$,
    else if $p < 1$ accept with probability p
  - minimize $\langle E \rangle$ or variance of $E_L(\mathbf{R})$ with respect to variational parameters in $\psi_T(\mathbf{R})$
  - gives upper bound to ground state E
- Requires very good trial wave function for reliable results

Finding the Ground State

- DMC and GFMC exploit S–equation in imaginary time $\Longrightarrow$ diffusion!

$$-\hbar \frac{\partial}{\partial \tau} \Psi(\mathbf{R}, \tau) = -\frac{\hbar^2}{2M} \nabla^2_{\mathbf{R}} \Psi(\mathbf{R}, \tau) + V(\mathbf{R}) \Psi(\mathbf{R}, \tau)$$

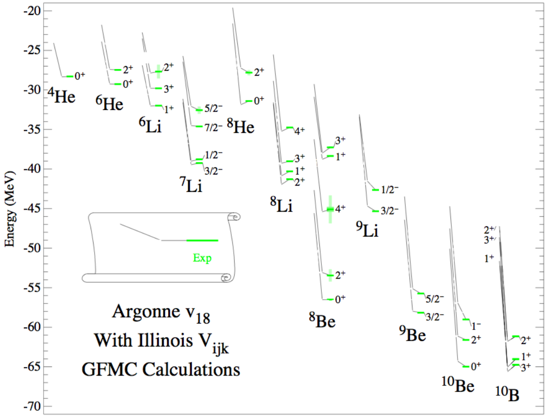
  - Use Metropolis to propagate to large $\tau \Longrightarrow$ projects ground state

$$\Psi(\mathbf{R}, \tau) = \int d\mathbf{R}' \, G(\mathbf{R}, \mathbf{R}', \tau) \, \Psi(\mathbf{R}', \tau)$$

  - Take many steps with small τ approximation to G
  - Generates “walker representation” of wave function (a set of $\mathbf{R}_i$’s) $\Longrightarrow$ can only represent a positive density
- Fermion sign problem for diffusion, path integral, GFMC
  - for fermions, even ground-state wavefunction changes sign (anti-symmetric)
  - if trial function provides good representation of nodes, solve in regions with nodal boundary conditions (“fixed node”)

Results for Light Nuclei

Large GFMC computations by Argonne group of Carlson, Pandharipande, Pieper & Wiringa.



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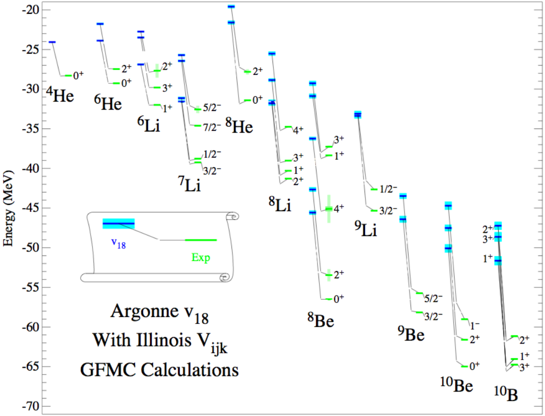

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