Semi Classical Simulation of the Pauli Potential

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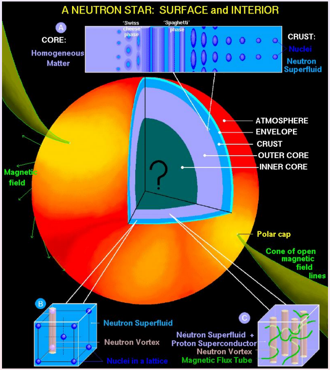
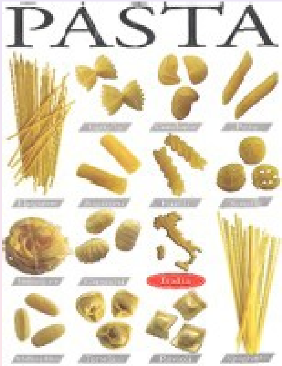
Outline

Pasta Phase in Neutron Stars





Equation of State for Pasta

- In earlier work (by Horowitz, Perez Garcia, and Piekarewicz) the structure and spin-independent neutrino response of the pasta was studied via purely classical simulation.

$$H = T + \sum_{i<j} V_{\rm nuc}(i,j)$$

- V_{nuc}(i,j) is the two body interaction term given as

$$V_{\rm nuc}(i,j) = ae^{-r_{ij}^2/\Lambda} + \left[b + c\tau_z(i)\tau_z(j)\right]e^{-r_{ij}^2/2\Lambda} + V_{\rm coulomb}(i,j)$$

- I am interested in generalizing this work to the study of the spin-dependent response.
- To do so, one must (at least) understand fermionic correlations induced by the Pauli exclusion principle.

- We incorporate Pauli exclusion principle by adding a fictitious momentum-dependent *Pauli potential*

$$H_{\mathrm{new}} = T + \sum_{i<j} V_{\mathrm{nuc}}(i,j) + V_{\mathrm{Pauli}}(i,j)$$

- We will focus our attention to this added term (V_{Pauli}).
  - We will discuss the existing Pauli potential. We will show that the existing model does not reproduce most of the properties of a quantum free Fermi gas.
  - Proposed a modified Pauli potential to overcome this shortcomings.

(This work has been submitted to Phys. Rev. C (archieve nucl.-th/0702086))

Properties of a Free Fermi Gas

- A zero-temperature quantum free Fermi gas can be described by a Slater determinant

$$\Psi_{FG}(\mathbf{p}_1,\ldots,\mathbf{p}_N;\mathbf{r}_1,\ldots,\mathbf{r}_N)=\frac{1}{\sqrt{N!}}\begin{vmatrix}\varphi_{\mathbf{p}_1}(\mathbf{r}_1) & \cdots & \varphi_{\mathbf{p}_1}(\mathbf{r}_N)\\ \vdots & \cdots & \vdots\\ \varphi_{\mathbf{p}_N}(\mathbf{r}_1) & \cdots & \varphi_{\mathbf{p}_N}(\mathbf{r}_N)\end{vmatrix}$$

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  - No two fermions can have the same momentum.
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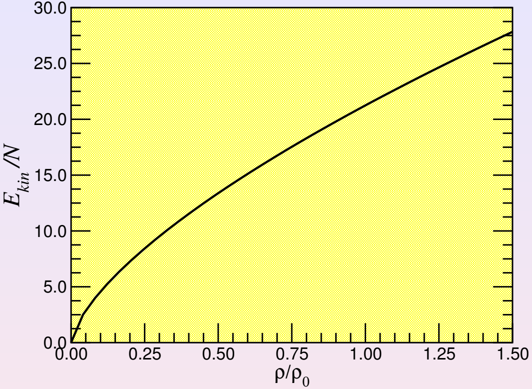

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2. $g2(r) = 1 - \left(\frac{3j_1(p_F r)}{p_F r}\right)^2; \quad j_1(p_F r) = \frac{\sin(p_F r)}{(p_F r)^2} - \frac{\cos(p_F r)}{(p_F r)}$
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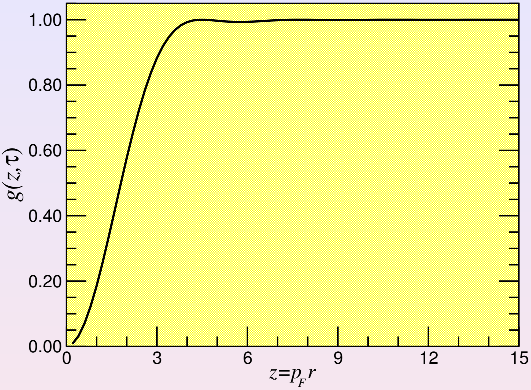

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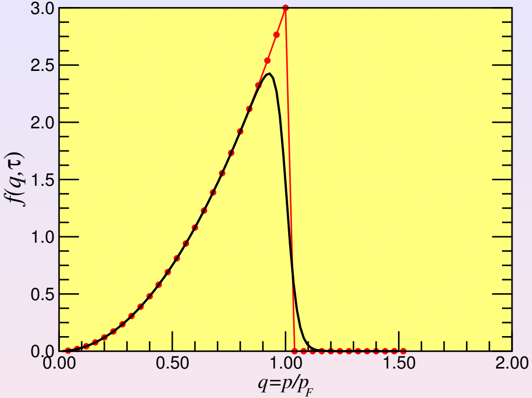

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Pauli Potential by Wilets and collaborators

$$V_{\text{Pauli}}(\mathbf{p}_1,\ldots,\mathbf{p}_N;\mathbf{r}_1,\ldots,\mathbf{r}_N)=\sum_{i<j=1}^{N} V_0 \exp(-s_{ij}^2/2)\delta_{\tau_i\tau_j}\delta_{\sigma_i\sigma_j}$$

s_{ij} is the dimensionless phase-space distance between two particles

$$s_{ij}^2 \equiv \frac{|\mathbf{p}_i-\mathbf{p}_j|^2}{p_0^2}+\frac{|\mathbf{r}_i-\mathbf{r}_j|^2}{r_0^2}$$

- It vanishes for different fermionic species
- When s_{ij} is small, the Pauli potential is big so the particles are penalized for getting too close in phase space.
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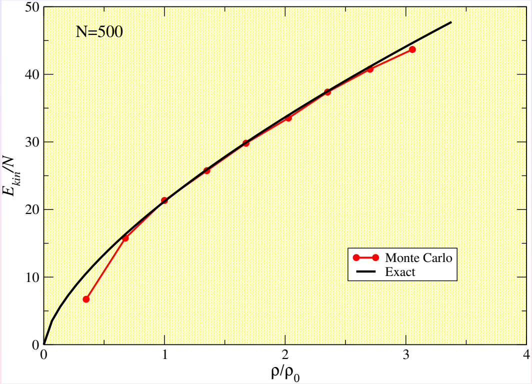

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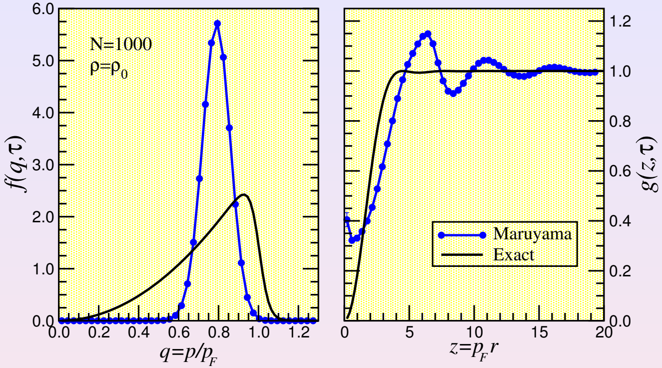

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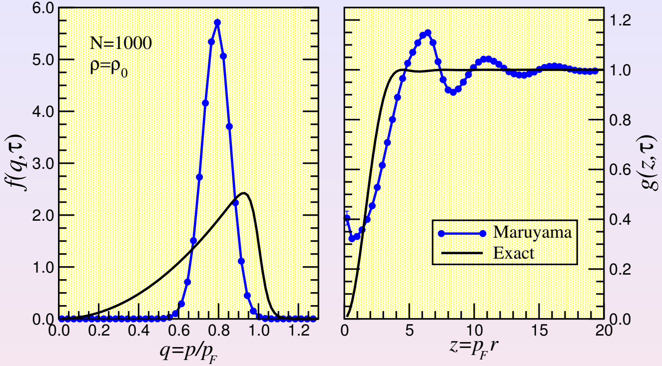
- The Pauli potential does not provide enough suppression between particles at short distances.
- The system crystallizes!



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  - Pauli potential penalizes particles when they are close in phase space and no penalty when they are far apart.
  - However, if particles are close in space ($r_{ij} \ll 1$), as long as momentum between the particles is large ($p_{ij} \gg 1$) (or visa versa) the Pauli potential vanishes.

- In disagreement with Slater determinant properties. Determinant vanishes when two fermions occupy the same location or have the same momentum.

Proposed a new potential to solve this problem.

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New Model for Pauli Potential

$$V_{\text{Pauli}}(\mathbf{p}_1,\ldots,\mathbf{p}_N;\mathbf{r}_1,\ldots,\mathbf{r}_N)=\sum_{i<j}^{N}\mathrm{V_A}\;\; e^{-r_{ij}/r_o}\delta_{\sigma_i\sigma_j}\delta_{\tau_i\tau_j}$$

$$+\sum_{i<j}^{N}\mathrm{V_B}\;\; \mathrm{e}^{-\mathrm{p_{ij}}/\mathrm{p_o}}\delta_{\sigma_i\sigma_j}\delta_{\tau_i\tau_j}+\sum_{i=1}^{N}\frac{\mathrm{V_C}}{1+e^{-\eta(q_i^2-1)}}$$

- The first term penalizes particles if they get too close in space.
- The second term penalizes particles if they get too close in momentum space.
- The last term provides a cut-off for the $p > p_F$ region.

Model parameters are determined by fitting to the properties of a quantum free Fermi gas.

Model parameters (V_A, V_B,V_C,r_o, p_o,η) are determined by fitting to the kinetic energy of a quantum free Fermi gas at various densities.

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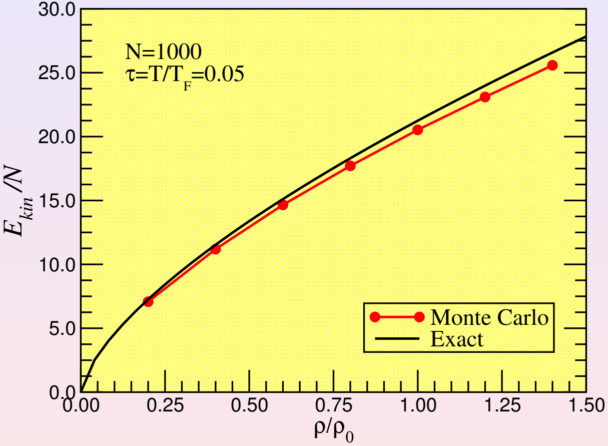

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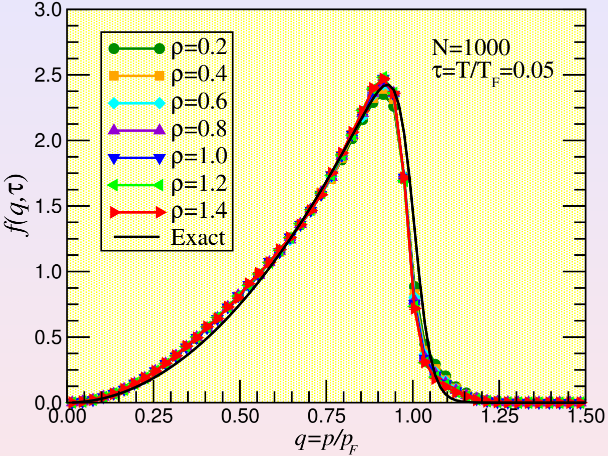
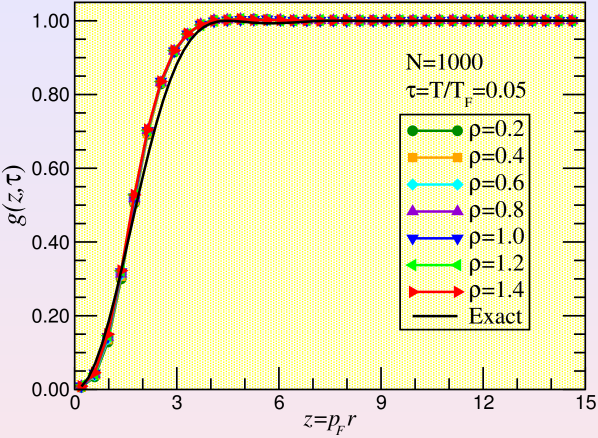


The new Pauli potential reproduces accurately the momentum distribution of a quantum free Fermi gas.



- The new Pauli potential reproduces accurately the two-fermion. correlation function of a quantum free Fermi gas.
- The potential gives enough suppression at r=0.
- There is no crystallization at low (or any) densities.

Conclusions & Future Study

- To include spin into our calculation we need to add a Pauli term into our classical Hamiltonian.
- The “old” Pauli potential fails to accurately describe all properties of a quantum free Fermi gas.
- We proposed a new Pauli potential that has successfully mimicked the Pauli exclusion principle
- We add this new spin dependent potential to our pasta Hamiltonian and proceed (after re-fitting) to calculate the spin dependence response of the nuclear pasta.

  Work is still in progress ...