A Short Tale of an Effective Field Theory for Nucleons

Johannes Kirscher

The George Washington University, USA

Effective Field Theory (EFT):

- most general Lagrangian with only relevant degrees of freedom, obeying certain symmetries;
  here: low energy $\Rightarrow$ Nucleons only, Galilei invariance;

Effective Field Theory (EFT):

- most general Lagrangian with only relevant degrees of freedom, obeying certain symmetries;
  here: low energy $\Rightarrow$ Nucleons only, Galilei invariance;

- high-momentum/short-distance modes embedded in infinite number of local interaction coefficients;
  here: C_i

Effective Field Theory (EFT):

- most general Lagrangian with only relevant degrees of freedom, obeying certain symmetries;
  here: low energy $\Rightarrow$ Nucleons only, Galilei invariance;

- high-momentum/short-distance modes embedded in infinite number of local interaction coefficients;
  here: C_i

- method only useful with organization schema, i.e. *power counting*;
  here: effective range expansion of the T-matrix;

Wilsonian Effective Theory:

generating functional of a theory valid up to λ ($\to \infty$):

$$Z = \int [D\phi]_\lambda \, e^{-\int d^4x \mathcal{L}(\phi, \partial_\mu \phi)}$$

Wilsonian Effective Theory:

generating functional of a theory valid up to λ ($\rightarrow \infty$):

$$Z = \int [D\phi]_\lambda \, e^{-\int d^4x \mathcal{L}(\phi, \partial_\mu \phi)}$$

$\Downarrow$ integration over momentum shell $b\lambda < |k| < \lambda \;\; ; \;\; b < 1$

Wilsonian Effective Theory:

generating functional of a theory valid up to λ ($\to \infty$):

$$Z = \int [D\phi]_\lambda \, e^{-\int d^4x \mathcal{L}(\phi, \partial_\mu \phi)}$$

$\Downarrow$ integration over momentum shell $b\lambda < |k| < \lambda \;\; ; \;\; b < 1$

$$Z = \int [D\phi]_{b\lambda} \, e^{-\int d^4x \mathcal{L}_{eff}(\phi, \partial_\mu \phi)}$$

with $\mathcal{L}_{eff} = \mathcal{L} + \big($higher order terms in $[\phi, \partial_\mu \phi]\big)$;

Wilsonian Effective Theory:

generating functional of a theory valid up to λ ($\to \infty$):

$$Z = \int [D\phi]_\lambda \, e^{-\int d^4x \mathcal{L}(\phi, \partial_\mu \phi)}$$

$\Downarrow$ integration over momentum shell $b\lambda < |k| < \lambda \;\; ; \;\; b < 1$

$$Z = \int [D\phi]_{b\lambda} \, e^{-\int d^4x \mathcal{L}_{eff}(\phi, \partial_\mu \phi)}$$

with $\mathcal{L}_{eff} = \mathcal{L} + \big($higher order terms in $[\phi, \partial_\mu \phi]\big)$;

important: loop integrals cut off at $b\lambda$;

EFT($\not\pi$) for the Nucleon-Nucleon System:

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \overline{N}\left(i\partial_0 + \frac{1}{2m_N}\vec{\nabla}^2 + (\text{relativistic corrections})\right) N \\
& + C_s \overline{N} N \overline{N} N + C_t \left(\overline{N}\vec{\sigma}N\right) \cdot \left(\overline{N}\vec{\sigma}N\right) \\
& + C_2 \left(\overline{N}\vec{\nabla}N\right) \cdot \left(\overline{\nabla N}N\right) + C_2' \left(\overline{N}\vec{\nabla}N\right) \cdot \left(\overline{N}\vec{\nabla}N\right) \\
& + \ldots
\end{aligned}
$$

EFT($\not\pi$) for the Nucleon-Nucleon System:

$$\begin{aligned}\mathcal{L} &= \overline{N}\left(i\partial_0 + \frac{1}{2m_N}\vec{\nabla}^2 + (\text{relativistic corrections})\right)N \\ &+C_s\overline{N}N\overline{N}N + C_t\left(\overline{N}\vec{\sigma}N\right)\cdot\left(\overline{N}\vec{\sigma}N\right) \\ &+C_2\left(\overline{N}\vec{\nabla}N\right)\cdot\left(\overline{\nabla N}N\right) + C_2'\left(\overline{N}\vec{\nabla}N\right)\cdot\left(\overline{N}\vec{\nabla}N\right) \\ &+\ldots\end{aligned}$$

size estimate of a graph:

$$\text{vertex}: Q^n \qquad \text{loop}: \frac{mQ}{4\pi}$$

$n =$ number of derivatives at the vertex;

$Q =$ low momentum scale;

EFT($\not\pi$) for the Nucleon-Nucleon System:

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \overline{N}\left(i\partial_0 + \frac{1}{2m_N}\vec{\nabla}^2 + (\text{relativistic corrections})\right)N \\
& + C_s\overline{N}N\overline{N}N + C_t\left(\overline{N}\vec{\sigma}N\right)\cdot\left(\overline{N}\vec{\sigma}N\right) \\
& + C_2\left(\overline{N}\vec{\nabla}N\right)\cdot\left(\overline{\nabla N}N\right) + C_2'\left(\overline{N}\vec{\nabla}N\right)\cdot\left(\overline{N}\vec{\nabla}N\right) \\
& + \ldots
\end{aligned}
$$

size estimate of a graph:

$$
\text{vertex} : Q^n \qquad\qquad \text{loop} : \frac{mQ}{4\pi}
$$

$n =$ number of derivatives at the vertex;

$Q =$ low momentum scale;

important: loop integrals and C_i coefficients depend on λ;

T-matrix as an Example:

$$T_{ERE}(k) = \frac{4\pi}{M}\left(-a + ika^2 + \left(\frac{a^2 r_0}{2} + a^3\right)k^3 + \ldots \right)$$

k : c.m. momentum $\quad$ a : scattering length $\quad$ r_0 : effective range

T-matrix as an Example:

$$T_{ERE}(k) = \frac{4\pi}{M}\left(- a + ika^2 + \left(\frac{a^2 r_0}{2} + a^3\right)k^3 + \ldots \right)$$

k : c.m. momentum a : scattering length r_0 : effective range

$$\begin{aligned} T_{EFT}(k) \quad = \quad & -C_0^{(R)}\Bigg\{ 1 - \left(\frac{mC_0^{(R)}}{4\pi}ik\right) + \left(\frac{mC_0^{(R)}}{4\pi}ik\right)^2 + \ldots \\ & +2\frac{C_2^{\prime(R)}}{C_0^{(R)}}k^2\left[1 - 2\left(\frac{mC_0^{(R)}}{4\pi}ik\right)\right] + 2\frac{C_2^{\prime(R)}}{C_0^{(R)}}k^2\vec{p}' \cdot \vec{p} + \ldots \Bigg\} \end{aligned}$$

important: if the two expansions (loop $\rightarrow$, derivative $\downarrow$) can be treated perturbatively depends on the size of the **l**ow **e**nergy **c**oefficients $C_{2n}^{(R)}$;

LEC Scaling Behavior:

loop expansion parameter: $c \propto m C_0^{(R)} k$

mass dimension: $[C_0] = -2$

assumption: one mass scale $M \sim m$

$$C_0^{(R)} \sim \frac{4\pi}{mM} \quad \Rightarrow \quad c \sim \frac{k}{M} \qquad k \ll M$$

LEC Scaling Behavior:

loop expansion parameter: $c \propto m C_0^{(R)} k$
mass dimension: $[C_0] = -2$
assumption: one mass scale $M \sim m$

$$C_0^{(R)} \sim \frac{4\pi}{mM} \quad \Rightarrow \quad c \sim \frac{k}{M} \qquad k \ll M$$

$\Rightarrow$ perturbative treatment possible!

LEC Scaling Behavior:

loop expansion parameter: $c \propto m C_0^{(R)} k$

mass dimension: $[C_0] = -2$

assumption: one mass scale $M \sim m$

$$C_0^{(R)} \sim \frac{4\pi}{mM} \quad \Rightarrow \quad c \sim \frac{k}{M} \qquad k \ll M$$

$\Rightarrow$ perturbative treatment possible!

problem in the NN case: $\exists$ second scale m_π; perturbative treatment only possible if $k \ll m_\pi$, otherwise resummation of diagrams necessary;

LEC Scaling Behavior:

loop expansion parameter: $c \propto mC_0^{(R)}k$
mass dimension: $[C_0] = -2$
assumption: one mass scale $M \sim m$

$$C_0^{(R)} \sim \frac{4\pi}{mM} \quad \Rightarrow \quad c \sim \frac{k}{M} \qquad k \ll M$$

$\Rightarrow$ perturbative treatment possible!

problem in the NN case: $\exists$ second scale m_π; perturbative treatment only possible if $k \ll m_\pi$, otherwise resummation of diagrams necessary;

essence: to establish a correct power counting (observables independent of cutoff up to a certain order) determine size of the various interactions.

Fit/Calculation of the Strength Parameters:

potential: $V \equiv (\sum \text{irreducible diagrams})$

Fit/Calculation of the Strength Parameters:

potential: $V \equiv (\sum$ irreducible diagrams$)$

$$\begin{aligned}\mathcal{L} &= C_s \overline{N} N \overline{N} N + C_t \left(\overline{N}\vec{\sigma}N\right) \cdot \left(\overline{N}\vec{\sigma}N\right) \\ &+ C_2 \left(\overline{N}\vec{\nabla}N\right) \cdot \left(\overline{\nabla N}N\right) + C_2' \left(\overline{N}\vec{\nabla}N\right) \cdot \left(\overline{N}\vec{\nabla}N\right)\end{aligned}$$

$$\Rightarrow \quad V(\vec{q},\vec{k}) = C_s + C_t \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \tilde{C}_2 \vec{q}^2 + \hat{C}_2 \vec{k}^2$$

$$\vec{q} = \vec{p} - \vec{p}' \qquad \vec{k} = \frac{1}{2}\left(\vec{p} + \vec{p}'\right)$$

Fit/Calculation of the Strength Parameters:

potential: $V \equiv \left(\sum \text{irreducible diagrams}\right)$

$$\begin{aligned}\mathcal{L} &= C_s \overline{N} N \overline{N} N + C_t \left(\overline{N} \vec{\sigma} N\right) \cdot \left(\overline{N} \vec{\sigma} N\right) \\ &\quad + C_2 \left(\overline{N} \vec{\nabla} N\right) \cdot \left(\overline{\nabla N} N\right) + C_2' \left(\overline{N} \vec{\nabla} N\right) \cdot \left(\overline{N} \vec{\nabla} N\right)\end{aligned}$$

$$\Rightarrow \quad V(\vec{q}, \vec{k}) = C_s + C_t \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \tilde{C}_2 \vec{q}^2 + \hat{C}_2 \vec{k}^2$$

$$\vec{q} = \vec{p} - \vec{p}' \qquad \vec{k} = \frac{1}{2} \left(\vec{p} + \vec{p}'\right)$$

but: for the variational method used to solve the Schrödinger equation the potential has to be transformed into coordinate space!

Potential Operators:

- momentum space:

$$
\begin{aligned}
V^{(2)}(\vec{q},\vec{k}) &= C_1\vec{q}^2 + C_2\vec{k}^2 + \vec{\sigma}_1\cdot\vec{\sigma}_2\left(C_3\vec{q}^2 + C_4\vec{k}^2\right) \\
&\quad + iC_5\frac{\vec{\sigma}_1+\vec{\sigma}_2}{2}\cdot\vec{q}\times\vec{k} + C_6\vec{q}\cdot\vec{\sigma}_1\vec{q}\cdot\vec{\sigma}_2 + C_7\vec{k}\cdot\vec{\sigma}_1\vec{k}\cdot\vec{\sigma}_2
\end{aligned}
$$

Potential Operators:

- momentum space:

$$V^{(2)}(\vec{q},\vec{k}) = C_1\vec{q}^2 + C_2\vec{k}^2 + \vec{\sigma}_1\cdot\vec{\sigma}_2\left(C_3\vec{q}^2 + C_4\vec{k}^2\right) + iC_5\frac{\vec{\sigma}_1+\vec{\sigma}_2}{2}\cdot\vec{q}\times\vec{k} + C_6\vec{q}\cdot\vec{\sigma}_1\vec{q}\cdot\vec{\sigma}_2 + C_7\vec{k}\cdot\vec{\sigma}_1\vec{k}\cdot\vec{\sigma}_2$$

- coordinate space:

$$V^{(2)}(\vec{r}) = -C_1 I_0 e_1 - C_2 I_0 \vec{\nabla}^2 - \tfrac{1}{4}C_2 I_0 e_1 + C_2 I_0 \tfrac{\Lambda^2}{2}\vec{r}\cdot\vec{\nabla} - C_3 I_0 e_1 \vec{\sigma}_1\cdot\vec{\sigma}_2 + \vec{\sigma}_1\cdot\vec{\sigma}_2\left(-C_4 I_0\vec{\nabla}^2 - \tfrac{1}{4}C_4 I_0 e_1 + C_4 I_0\tfrac{\Lambda^2}{2}\vec{r}\cdot\vec{\nabla}\right) + C_5 I_0\tfrac{\Lambda^2}{2}\vec{L}\cdot\vec{S} + C_6 I_0\tfrac{\Lambda^2}{2}\vec{\sigma}_1\cdot\vec{\sigma}_2 - C_6 I_0\tfrac{\Lambda^4}{4}\vec{\sigma}_1\cdot\vec{r}\vec{\sigma}_2\cdot\vec{r} + C_7 I_0\tfrac{\Lambda^2}{4}\left(\vec{\sigma}_2\cdot\vec{r}\vec{\sigma}_1\cdot\vec{\nabla} + \vec{\sigma}_1\cdot\vec{r}\vec{\sigma}_2\cdot\vec{\nabla} + \tfrac{1}{2}\vec{\sigma}_1\cdot\vec{\sigma}_2 - \tfrac{\Lambda^2}{4}\vec{\sigma}_1\cdot\vec{r}\vec{\sigma}_2\cdot\vec{r}\right) - C_7 I_0\vec{\sigma}_1\cdot\vec{\nabla}\vec{\sigma}_2\cdot\vec{\nabla}$$

Refined Resonating Group Modell:

$$\hat{H}_N = \sum_i^N \hat{T}_i + \hat{V}(\vec{r}_1, \ldots, \vec{r}_N)$$

$\Downarrow$ center of mass motion 'removed'

$$\tilde{H}_N = \hat{H}_{F1} + \hat{H}_{F2} + \hat{T}_r + \frac{Z_1 Z_2 e^2}{R} + \left(\hat{V}_{F1 \leftrightarrow F2} - \frac{Z_1 Z_2 e^2}{R} \right)$$

Refined Resonating Group Modell:

$$\hat{H}_N = \sum_i^N \hat{T}_i + \hat{V}(\vec{r}_1, \ldots, \vec{r}_N)$$

$\Downarrow$ center of mass motion 'removed'

$$\tilde{H}_N = \hat{H}_{F1} + \hat{H}_{F2} + \hat{T}_r + \frac{Z_1 Z_2 e^2}{R} + \left(\hat{V}_{F1 \leftrightarrow F2} - \frac{Z_1 Z_2 e^2}{R} \right)$$

wave function Ansatz:

$$\psi_l = \mathcal{A} \sum_k^{n_k} \phi_{r,k}^{(l)} \Phi_{ch,k}$$

$\mathcal{A}$: anti symmetrizer;

l : determines boundary conditions in the scattering problem;

Refined Resonating Group Modell:

relative motion wave function used for a scattering calculation:

$$\phi_{r,k}^{(l)} = \delta_{lk} F_k(R) + a_{lk} \tilde{G}(R) + \sum_m b_{lkm} \chi_{km}(R)$$

Refined Resonating Group Modell:

relative motion wave function used for a scattering calculation:

$$\phi_{r,k}^{(l)} = \delta_{lk} F_k(R) + a_{lk} \tilde{G}(R) + \sum_m b_{lkm} \chi_{km}(R)$$

channel wave function:

$$\Phi_{ch,k} = \left[\frac{Y_l(R)}{R} \otimes \left[\theta_1^{j_1} \otimes \theta_2^{j_2} \right]^{S_c} \right]^J$$

channel defined by j_1, j_2, S_c, l, J and the fragmentation; fragment wave function:

$$\theta^j = \sum_{\substack{\{l_I\},S \\ \{m\}}} \left[C_m^{l_I l S} \Omega_m^{l_I}(\vec{\rho}) \Xi^{S,T} \right]^J$$