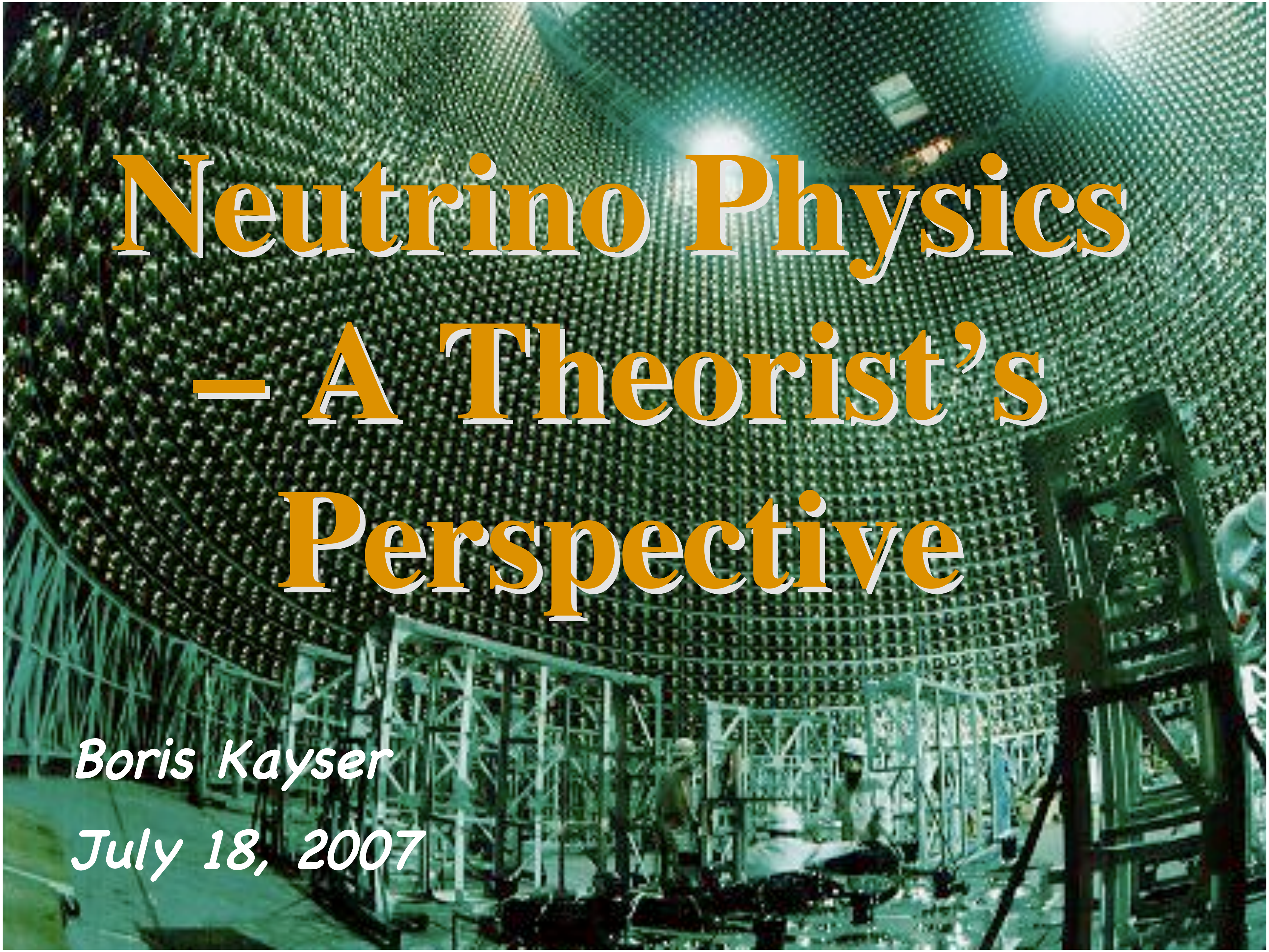


Neutrino Physics – A Theorist's Perspective

Boris Kayser

July 18, 2007

Selected Interesting Questions

- What physics is behind neutrino mass?
- Are neutrinos their own antiparticles?
- Do neutrino-matter interactions violate the symmetry CP? *(Do neutrinos interact differently with antimatter than they do with matter?)*
- Is CP violation involving neutrinos the key to understanding the baryon – antibaryon asymmetry of the universe?
- What totally unexpected discovery will be made on June 3, 2014?

Neutrino Masses and the $\bar{\nu} \overset{?}{=} \nu$ Question

Does $\bar{\nu} = \nu$?

That is, for each ***mass eigenstate*** ν_i , does —

- $\overline{\nu_i} = \nu_i$ (Majorana neutrinos)

or

- $\overline{\nu_i} \neq \nu_i$ (Dirac neutrinos) ?

Equivalently, is the Lepton Number L defined by—
$L(\nu) = L(\ell^-) = -L(\bar{\nu}) = -L(\ell^+) = 1$ conserved?

L is a leptonic analogue of the Baryon Number B that distinguishes the $\bar{n}$ from the n.

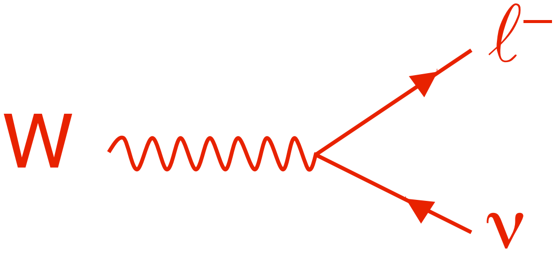
If L is not conserved, then nothing distinguishes $\overline{\nu_i}$ from ν_i . We then have Majorana neutrinos.

Do We Expect That $\bar{\nu}_i = \nu_i$?

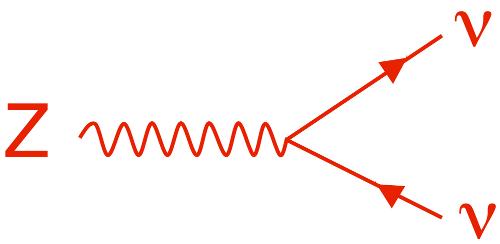
How can the S(tandard) M(odel) be extended to include neutrino masses?

How does the SM become the νSM?

The S(tandard) M(odel)



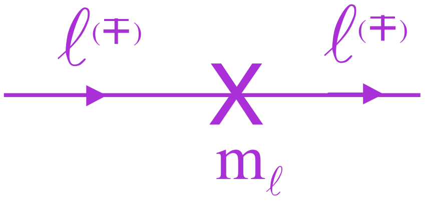
and



couplings conserve the Lepton Number L.

So do the Dirac charged-lepton mass terms

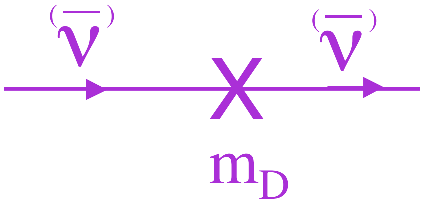
$$m_\ell \bar{\ell}_R \ell_L$$



R(L) ≡ Right(Left) Handed

- Original SM: $m_\nu = 0$.
- Why not add a Dirac mass term,

$$m_D \bar{\nu}_R \nu_L$$



Then everything conserves L, so for each mass eigenstate ν_i,

$$\bar{\nu}_i \neq \nu_i$$

(Dirac neutrinos)

$$[L(\bar{\nu}_i) = -L(\nu_i)]$$

- The SM contains no ν_R field, only ν_L. (Only Left-Handed fermions couple to the W boson.)

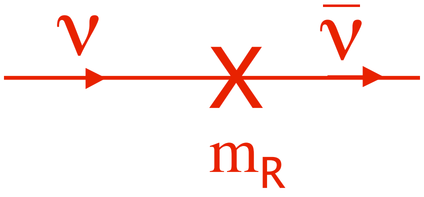
But to add the Dirac mass term, we had to add ν_R to the SM.

Unlike ν_L, ν_R carries no Electroweak Isospin.

Thus, no SM principle prevents the occurrence of the Majorana mass term

$$m_R\overline{\nu_R}^{\,c}\nu_R$$

Charge conjugate



Charge-conjugate fields:

ψc = ψ(Particle ↔ Antiparticle)

The Majorana mass does not conserve L, so now

$\overline{\nu}_i = \nu_i$ (Majorana neutrinos)

[No conserved L to distinguish $\overline{\nu}_i$ from ν_i]

This leads many theorists to expect
Majorana masses, hence $\not{L}$ *and* $\bar{\nu}_i = \nu_i$.

The Standard Model (SM) is defined by the fields it contains, its symmetries (notably Electroweak Isospin Invariance), and its renormalizability.

Leaving neutrino masses aside, anything allowed by the SM symmetries occurs in nature.

If this is also true for neutrino masses,
then neutrinos have *Majorana masses*.

- The presence of Majorana masses
- $\overline{\nu_i} = \nu_i$ (Majorana neutrinos)
- L not conserved

— are all equivalent

Any one implies the other two.

(Recent work: Hirsch, Kovalenko, Schmidt)

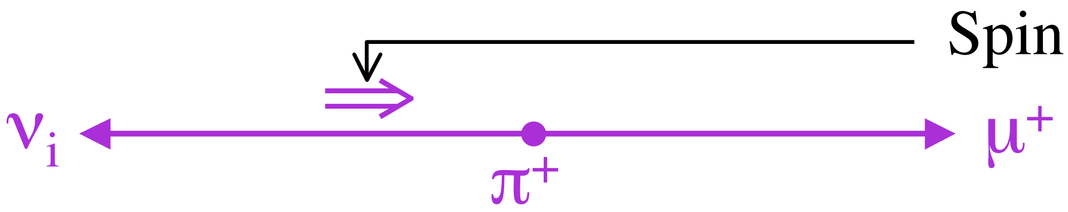
To Determine If Neutrinos Are Their Own Antiparticles

How Can We Demonstrate That $\overline{\nu}_i = \nu_i$?

We assume neutrino interactions are correctly described by the SM. Then the interactions conserve L ($\nu \rightarrow \ell^-$; $\overline{\nu} \rightarrow \ell^+$).

An Idea that Does Not Work
[and illustrates why most ideas do not work]

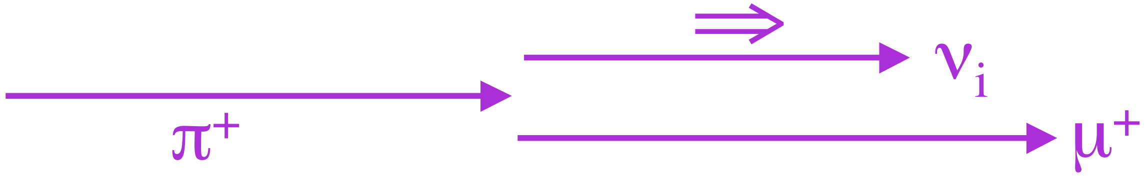
Produce a ν_i via—



Pion Rest Frame

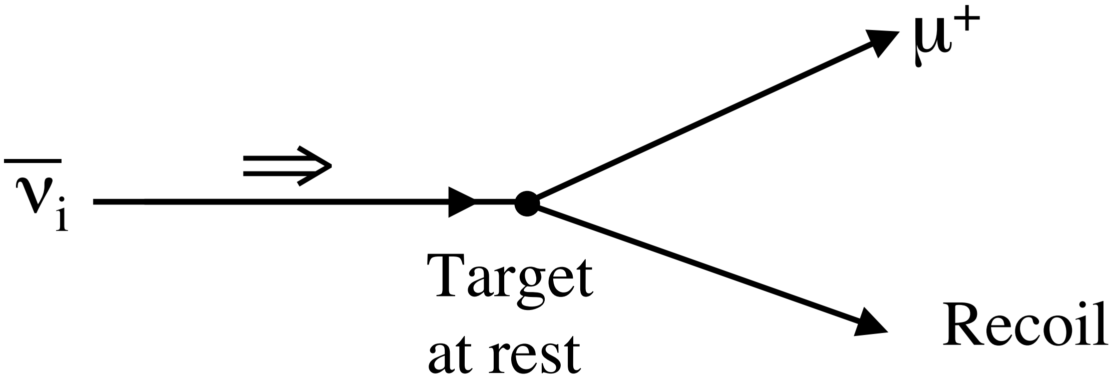
Give the neutrino a Boost:

$\beta_\pi(\text{Lab}) > \beta_\nu(\pi \text{ Rest Frame})$



Lab. Frame

The SM weak interaction causes—



$\nu_i = \bar{\nu}_i$ means that $\nu_i(h) = \bar{\nu}_i(h)$.

helicity

If $\nu_i \xrightarrow{\Rightarrow} = \bar{\nu}_i \xrightarrow{\Rightarrow}$,

our $\nu_i \xrightarrow{\Rightarrow}$ will make μ^+ too.

Minor Technical Difficulties

$$\beta_\pi(\text{Lab}) > \beta_\nu(\pi \text{ Rest Frame})$$

$$\Rightarrow \frac{E_\pi(\text{Lab})}{m_\pi} > \frac{E_\nu(\pi \text{ Rest Frame})}{m_{\nu_i}}$$

$$\Rightarrow E_\pi(\text{Lab}) \gtrsim 10^5 \text{ TeV if } m_{\nu_i} \sim 0.05 \text{ eV}$$

Fraction of all π – decay ν_i that get helicity flipped

$$\approx \left(\frac{m_{\nu_i}}{E_\nu(\pi \text{ Rest Frame})}\right)^2 \sim 10^{-18} \text{ if } m_{\nu_i} \sim 0.05 \text{ eV}$$

Since L-violation comes only from Majorana neutrino *masses*, any attempt to observe it will be at the mercy of the neutrino masses.

(BK & Stodolsky)

The Promising Approach —

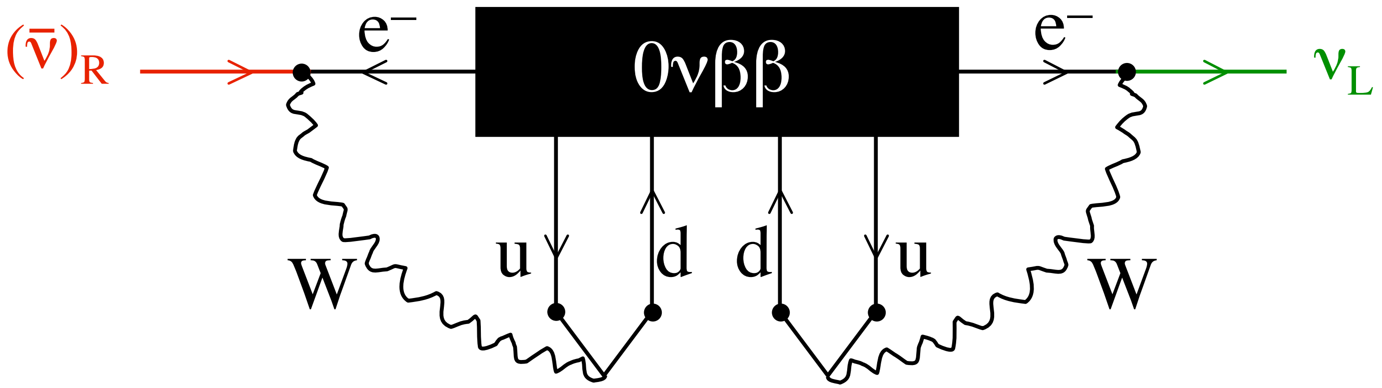
Neutrinoless Double Beta Decay [$0\nu\beta\beta$]

If we start with ***a lot*** of parent nuclei (say, one ton of them), we can cope with the smallness of $\cancel{L}$.

Observation would imply $\cancel{L}$ and therefore $\overline{\nu}_i = \nu_i$.

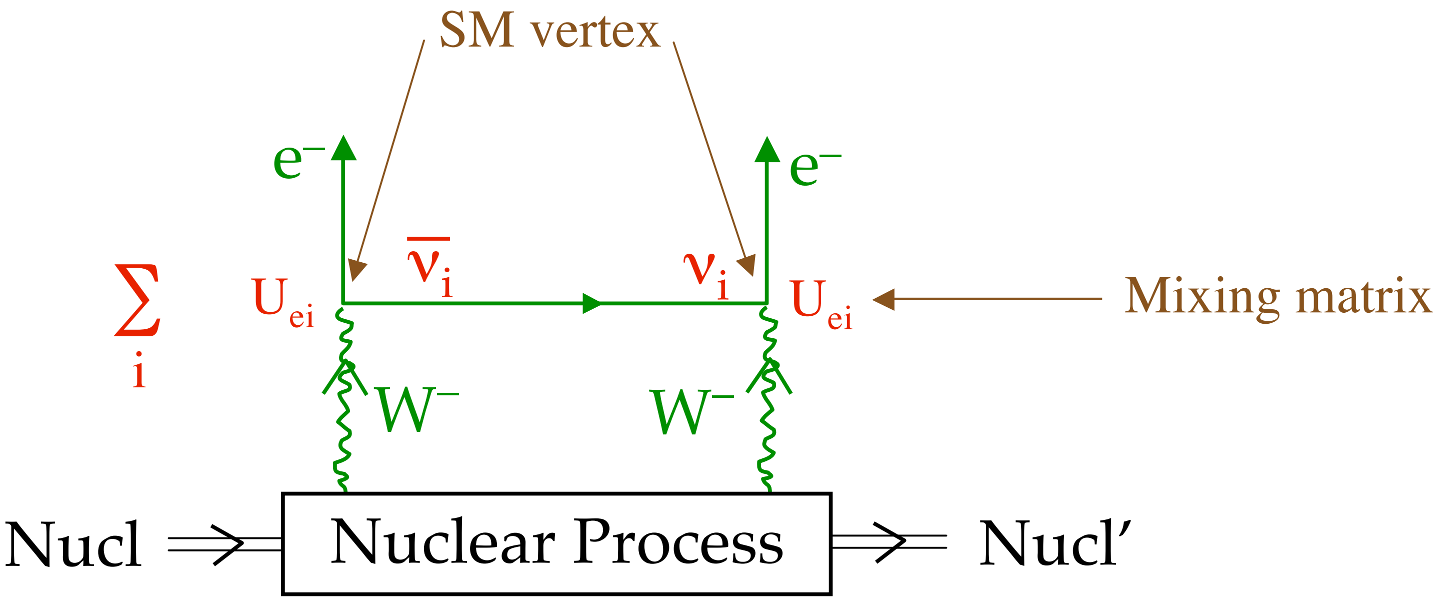
Whatever diagrams cause 0νββ, its observation would imply the existence of a Majorana mass term:

Schechter and Valle

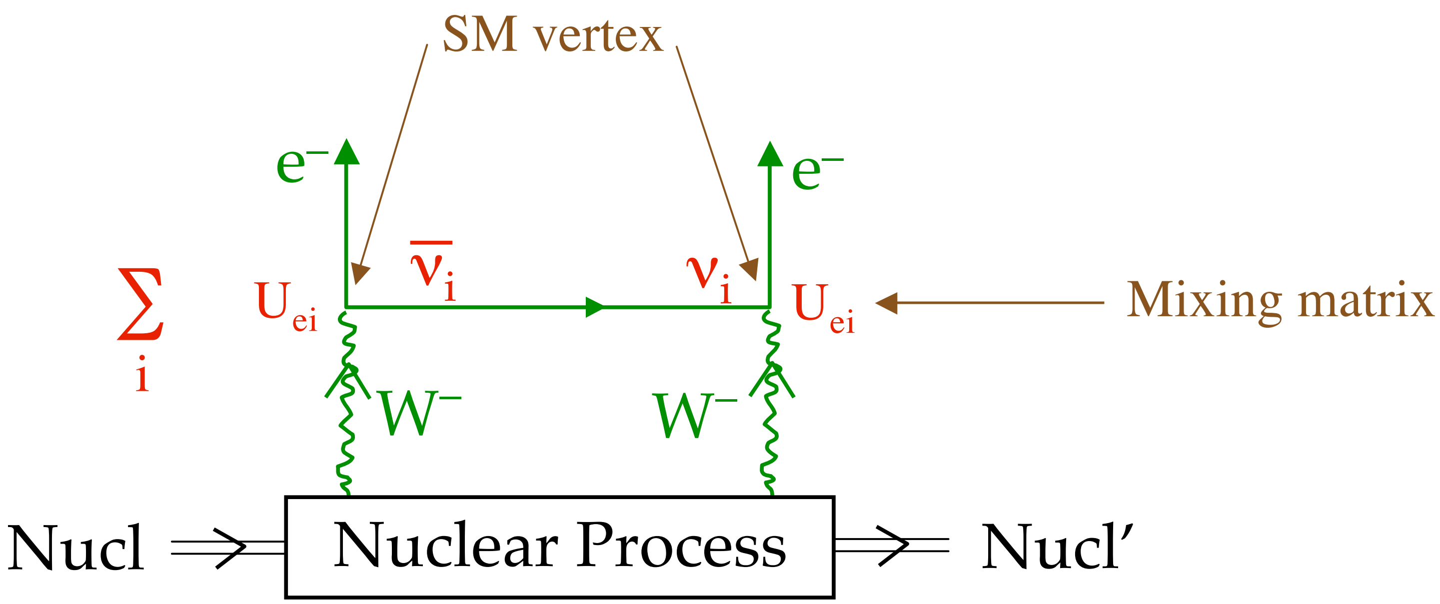


$(\bar{\nu})_R \rightarrow \nu_L$: A Majorana mass term

We anticipate that $0\nu\beta\beta$ is dominated by a diagram with Standard Model vertices:



In —



Mass (ν_i)

the $\bar{\nu}_i$ is emitted [RH + O{m_i/E}LH].

Thus, Amp [ν_i contribution] ∝ m_i

$$\text{Amp}[0\nu\beta\beta] \propto \left|\sum_i m_i U_{ei}^2\right| \equiv m_{\beta\beta}$$

The proportionality of 0νββ to ν mass is no surprise.

0νββ violates L. But the SM interactions conserve L.

The L – violation in 0νββ comes from underlying **Majorana** neutrino mass terms.

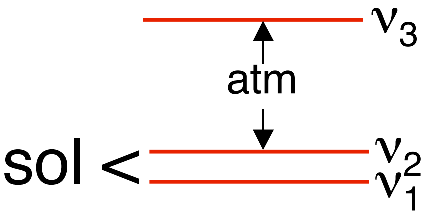
The 0νββ amplitude would be proportional to neutrino mass even if there were no helicity mismatch.

How Large is $m_{\beta\beta}$?

How sensitive need an experiment be?

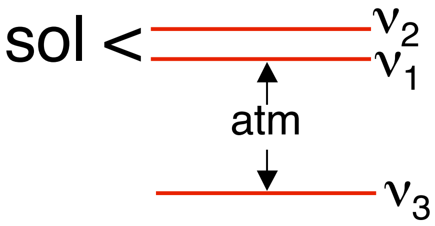
Suppose there are only 3 neutrino mass eigenstates. (More might help.)

Then the spectrum looks like —

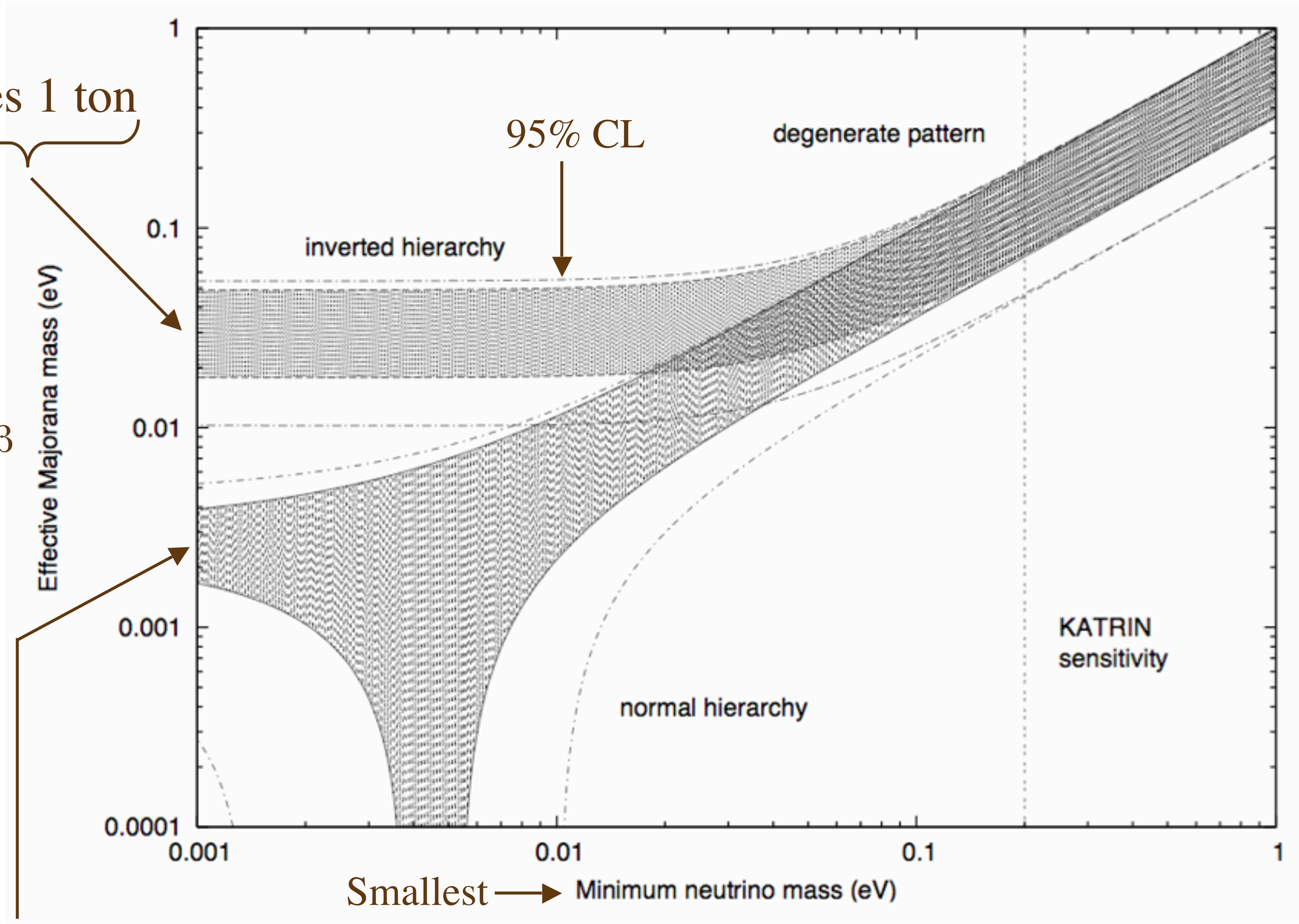


Normal hierarchy

or



Inverted hierarchy



Takes 1 ton

95% CL

$m_{\beta\beta}$

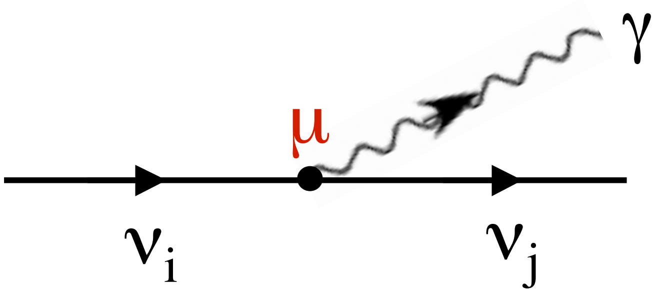
Smallest →

Takes 100 tons

$m_{\beta\beta}$ *For Each Hierarchy*

Possible Information From Neutrino Magnetic Moments

Both Majorana and Dirac neutrinos can have *transition* magnetic dipole moments μ:



For *Dirac* neutrinos, $\mu < 10^{-15}\,\mu_{\mathrm{Bohr}}$

For *Majorana* neutrinos, $\mu <$ Present bound

$$\text{Present bound} = \begin{cases} 7 \times 10^{-11}\,\mu_{\mathrm{Bohr}} \text{ ; Wong et al. (Reactor)} \\ 3 \times 10^{-12}\,\mu_{\mathrm{Bohr}} \text{ ; Raffelt (Stellar E loss)} \end{cases}$$

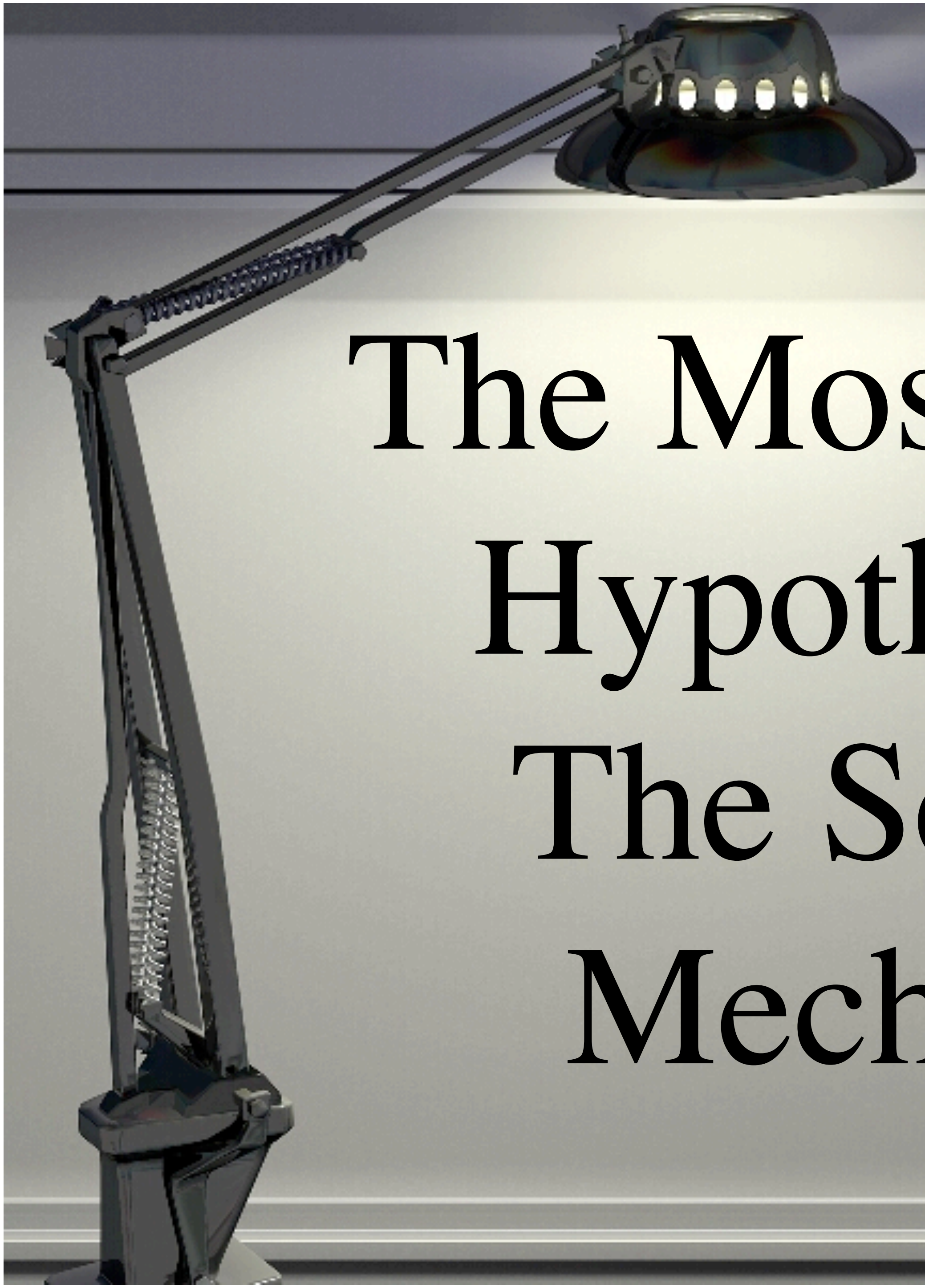
An observed μ below the present bound but well above 10^{-15} μ_{Bohr} would imply that neutrinos are Majorana particles.

However, a dipole moment that large requires L-violating new physics below 100 TeV.

(Bell, Cirigliano, Davidson, Gorbahn, Gorchtein, Ramsey-Musolf, Santamaria, Vogel, Wise, Wang)

Neutrinoless double beta decay at the planned level of sensitivity only requires this new physics at ~ 10^{15} GeV, near the Grand Unification scale.

What Physics Is Behind Neutrino Mass?



The Most Popular Hypothesis — The See-Saw Mechanism

The See-Saw Mechanism
— A Summary —

This assumes that a neutrino has *both*
a Majorana mass term $m_R \overline{\nu_R{}^c}\, \nu_R$
and a Dirac mass term $m_D \overline{\nu}_L \nu_R$.

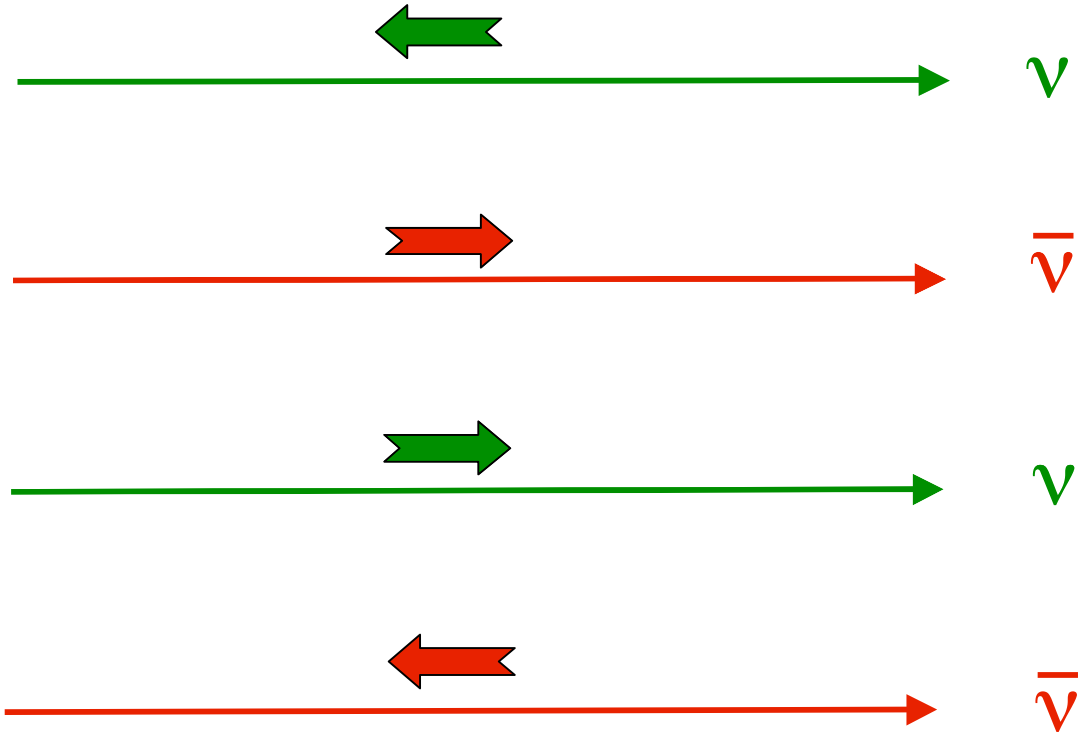
No SM principle prevents m_R from being extremely large.

But we expect m_D to be of the same order as the masses of the quarks and charged leptons.

Thus, we assume that $m_R >> m_D$.

When $\bar{\nu} \neq \nu$

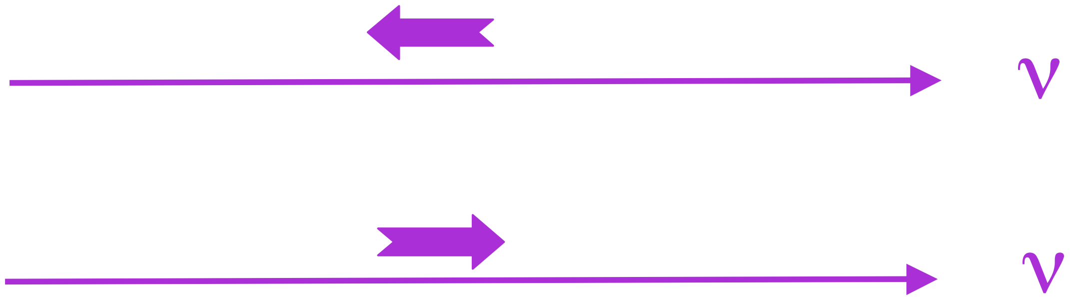
We have 4 mass-degenerate states:



This collection of 4 states is a Dirac neutrino plus its antineutrino.

When $\bar{\nu} = \nu$

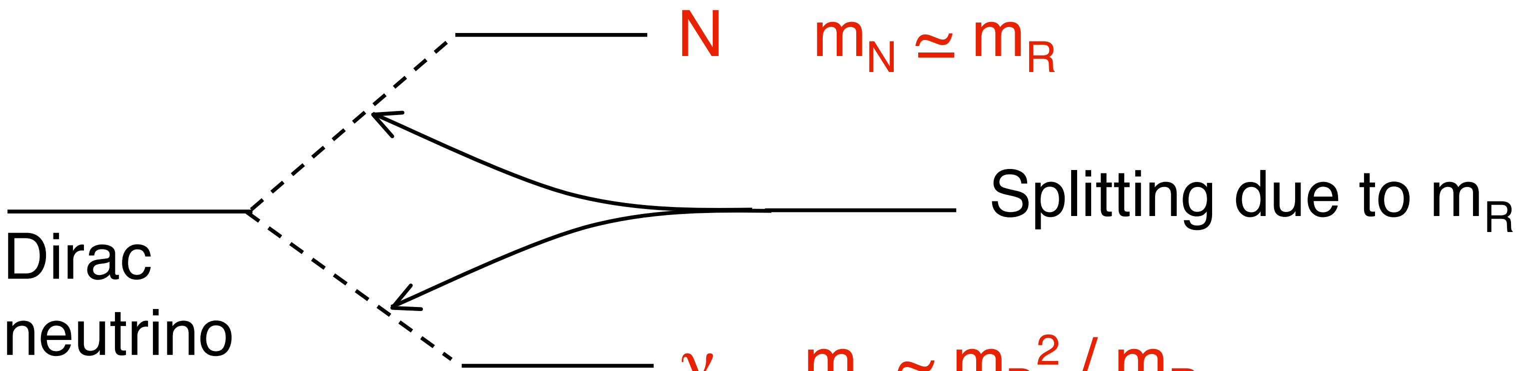
We have only 2 mass-degenerate states:



This collection of 2 states is a Majorana neutrino.

What Happens In the See-Saw?

The Majorana mass term splits a *Dirac* neutrino into two *Majorana* neutrinos.



N $m_N \simeq m_R$

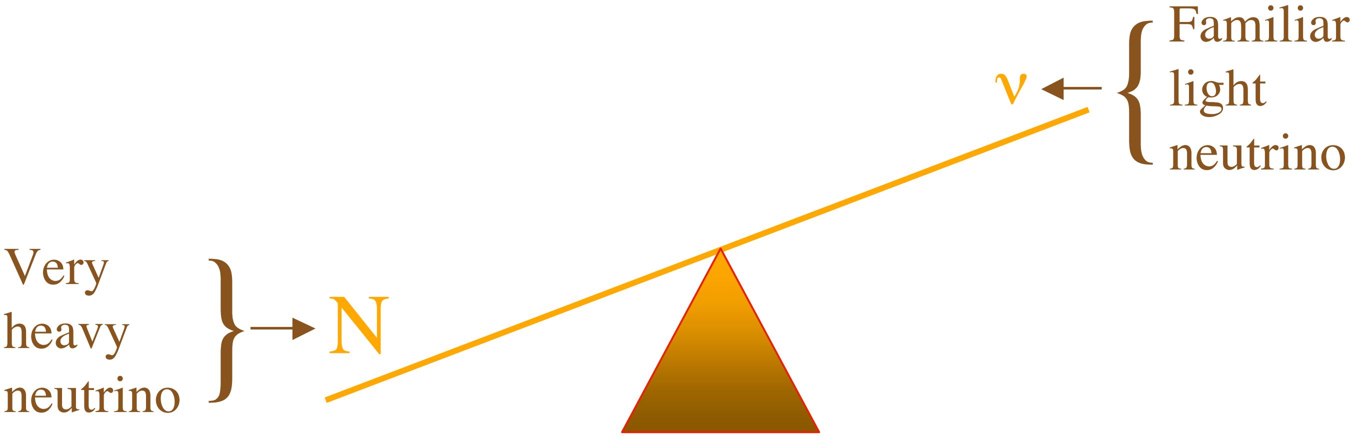
Splitting due to m_R

Dirac neutrino

ν $m_\nu \simeq m_D^2 / m_R$

Note that $m_\nu m_N \sim m_D^2 \sim m_{q\text{ or }l}^2$. ***See-Saw Relation***

The See-Saw Relation



Predictions of the See-Saw

- Each $\bar{\nu}_i = \nu_i$ (Majorana neutrinos)
- The light neutrinos have heavy partners N

How heavy??

$$m_N \sim \frac{m^2_{top}}{m_\nu} \sim \frac{m^2_{top}}{0.05\text{ eV}} \sim 10^{15}\text{ GeV}$$

Near the *Grand Unification* scale, where the strong, electromagnetic, and weak forces appear to unify.

Coincidence??

Do Neutrino Interactions Violate CP?

Suppose that $\bar{\nu} = \nu$. The question is: Do neutrinos interact differently with antimatter than with matter?

The Standard Model (SM) tells us that the neutrinos couple to charged leptons and the W boson according to —

$\ell_e \equiv e$, $\ell_\mu \equiv \mu$, $\ell_\tau \equiv \tau$

$$L_{SM} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left(\bar{\ell}_{L\alpha} \gamma^{\lambda} \nu_{L\alpha} W_{\lambda}^{-} + \bar{\nu}_{L\alpha} \gamma^{\lambda} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

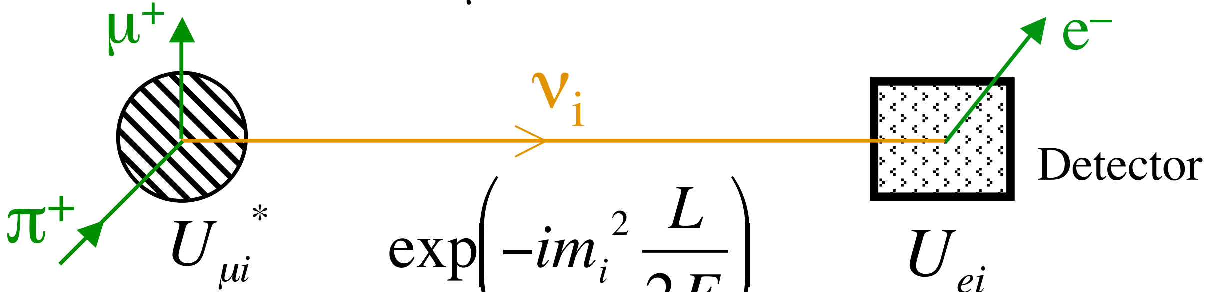
$$= -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(\bar{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} + \bar{\nu}_{Li} \gamma^{\lambda} U_{\alpha i}^{*} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

Mixing matrix

Taking mixing into account

Compare

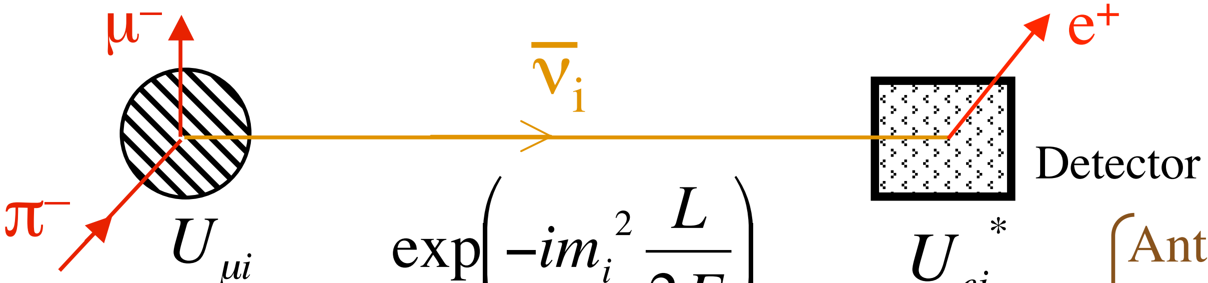
$\nu_\mu \rightarrow \nu_e$



$$\sum_i \; U_{\mu i}^* \; \exp\left(-im_i^2 \frac{L}{2E}\right) \; U_{ei}$$

with

“ $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ ”



$$\sum_i \; U_{\mu i} \; \exp\left(-im_i^2 \frac{L}{2E}\right) \; U_{ei}^*$$

(Anti-Detector is hard to build.)

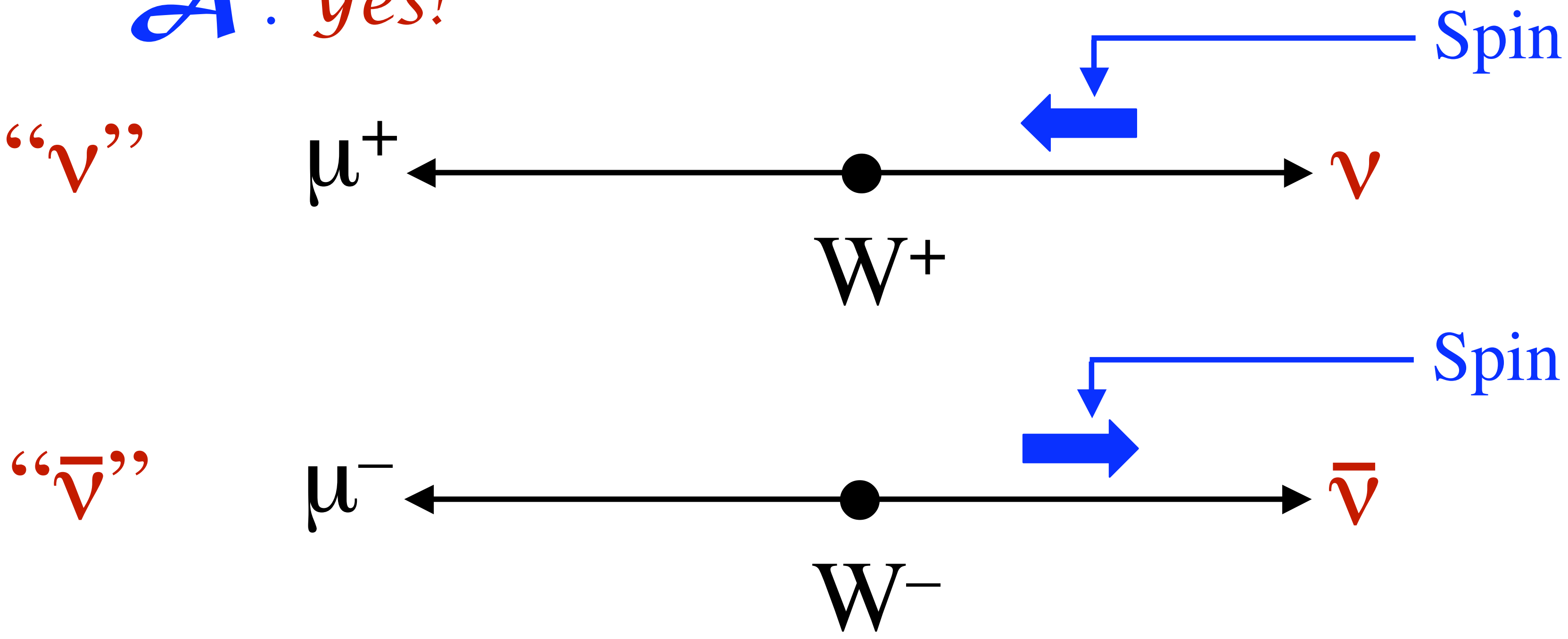
Caution!

Matter will affect the neutrino beams in these two experiments differently, even if there is no genuine CP violation.

This leads to a fake CP violation.

Q : *Does matter still affect the two beams differently when $\bar{\nu} = \nu$?*

A : *Yes!*



The weak interactions violate ***parity***. Neutrino – matter interactions depend on the neutrino ***polarization***.

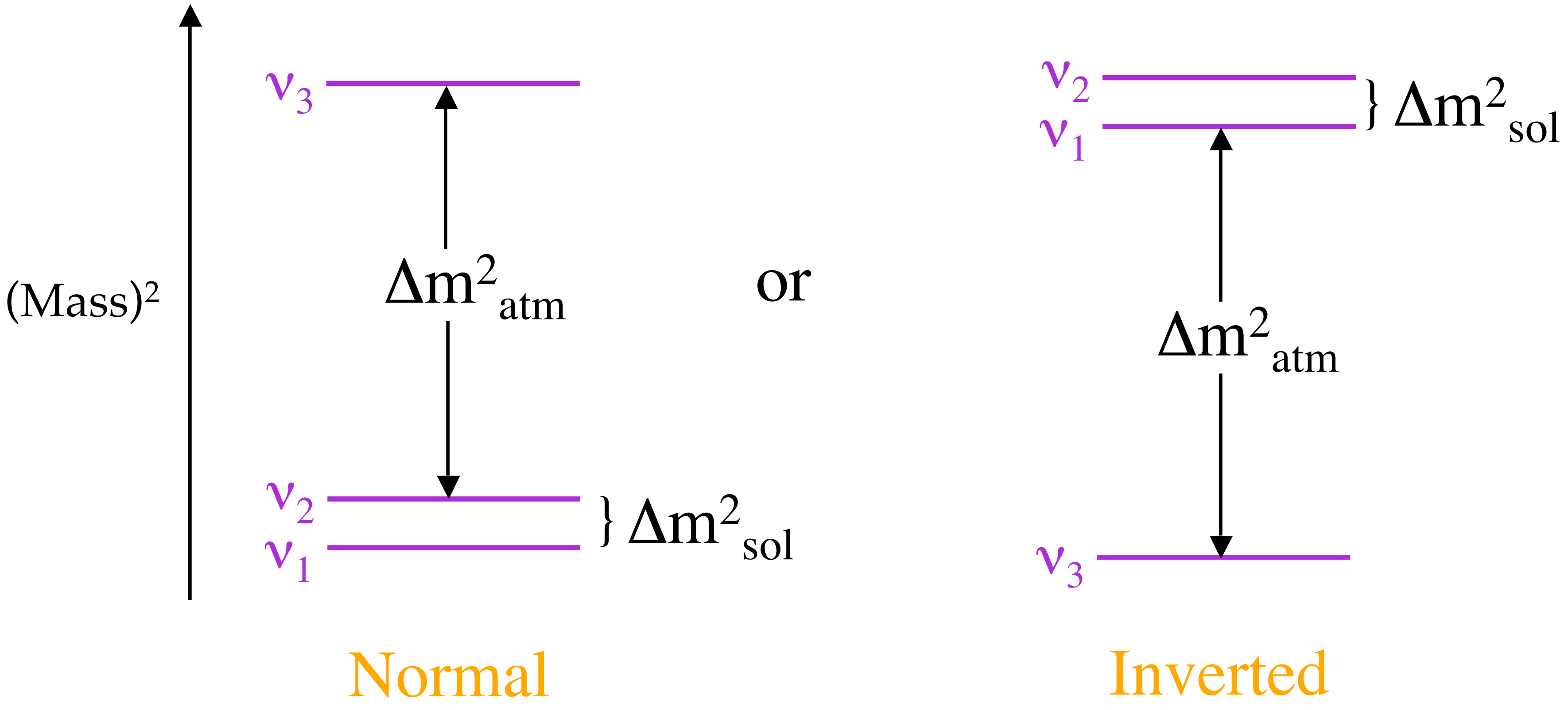
Separating $\cancel{CP}$ From the Matter Effect

Genuine $\cancel{CP}$ and the matter effect both lead to a difference between “ν” and “$\bar{\nu}$” oscillation.

But genuine $\cancel{CP}$ and the matter effect depend quite differently from each other on L and E.

To disentangle them, one must make oscillation measurements at different L and/or E.

The $(\mathrm{Mass})^2$ Spectrum



$(\mathrm{Mass})^2$

ν_3

Δm^2_{atm}

ν_2

ν_1

} Δm^2_{sol}

Normal

or

ν_2

ν_1

} Δm^2_{sol}

Δm^2_{atm}

ν_3

Inverted

$\Delta m^2_{sol} \cong 8 \text{ x } 10^{-5}\ \mathrm{eV}^2, \quad \Delta m^2_{atm} \cong 2.7 \text{ x } 10^{-3}\ \mathrm{eV}^2$

The Mixing Matrix

$$U = \overset{\text{Atmospheric}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}} \times \overset{\text{Cross-Mixing}}{\begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix}} \times \overset{\text{Solar}}{\begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$c_{ij} \equiv \cos\theta_{ij}$$
$$s_{ij} \equiv \sin\theta_{ij}$$

$$\times \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Majorana ~~CP~~
phases

$\theta_{12} \approx \theta_{sol} \approx 34°$, $\theta_{23} \approx \theta_{atm} \approx 37\text{-}53°$, $\theta_{13} \lesssim 10°$

δ would lead to $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$. ~~CP~~

But note the crucial role of $s_{13} \equiv \sin\theta_{13}$.

The Majorana ~~CP~~ Phases

The phase α_i is associated with
neutrino mass eigenstate ν_i:

$U_{\alpha i} = U^0{}_{\alpha i} \exp(i\alpha_i/2)$ for all flavors α.

$$\mathrm{Amp}(\nu_\alpha \rightarrow \nu_\beta) = \sum_i U_{\alpha i}{}^* \exp(-\, i m_i{}^2 L/2E)\, U_{\beta i}$$

is insensitive to the Majorana phases α_i.

Only the phase δ can cause CP violation in
neutrino oscillation.

Accelerator $\overset{(-)}{\nu}$ Oscillation Probabilities

With $\alpha \equiv \Delta m^2_{21} / \Delta m^2_{31}$, $\Delta \equiv \dfrac{\Delta m^2_{31} L}{4E}$, and $x \equiv \dfrac{2\sqrt{2} G_F N_e E}{\Delta m^2_{31}}$ —

$$P\left[\nu_\mu \to \nu_e\right] \cong \sin^2 2\theta_{13}\, T_1 - \alpha \sin 2\theta_{13}\, T_2 + \alpha \sin 2\theta_{13}\, T_3 + \alpha^2\, T_4 \;;$$

$$T_1 = \sin^2\theta_{23} \frac{\sin^2\left[(1-x)\Delta\right]}{(1-x)^2}\,, \quad T_2 = \sin\delta \sin 2\theta_{12} \sin 2\theta_{23} \sin\Delta \frac{\sin(x\Delta)}{x} \frac{\sin\left[(1-x)\Delta\right]}{(1-x)}\,,$$

$$T_3 = \cos\delta \sin 2\theta_{12} \sin 2\theta_{23} \cos\Delta \frac{\sin(x\Delta)}{x} \frac{\sin\left[(1-x)\Delta\right]}{(1-x)}\,, \quad T_4 = \cos^2\theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(x\Delta)}{x^2}$$

$$P\left[\bar{\nu}_\mu \to \bar{\nu}_e\right] \;=\; P\left[\nu_\mu \to \nu_e\right] \text{ with } \delta \to -\,\delta \text{ and } x \to -\,x.$$

(Cervera *et al*., Freund, Akhmedov *et al*.)

Strategies

The matter-effect parameter x has $|x| \approx E/12\ \text{GeV}$.

At L/E of the 1st "atmospheric" oscillation peak, and $E \sim 1$ GeV, the effect of matter on the ***neutrino*** atmospheric oscillation term ($\sin^2 2\theta_{13}\, T_1$) is —

$$1/(1-x)^2 \cong 1 \overset{\text{Normal}}{\underset{\text{Inverted}}{\pm}} (E/6\,\text{GeV})$$

At fixed L/E, genuine ~~CP~~ effects do not change with E, but the matter effect grows, enhancing (suppressing) the oscillation if the hierarchy is Normal (Inverted).

If $E \rightarrow E/3$ at fixed L, we go from the 1st atmospheric oscillation peak to the 2nd one.

When $E \rightarrow E/3$ at fixed L, *the matter effect is reduced by a factor of 3*, *but ~~CP~~ is tripled*.

Why do we care whether neutrino interactions violate CP?

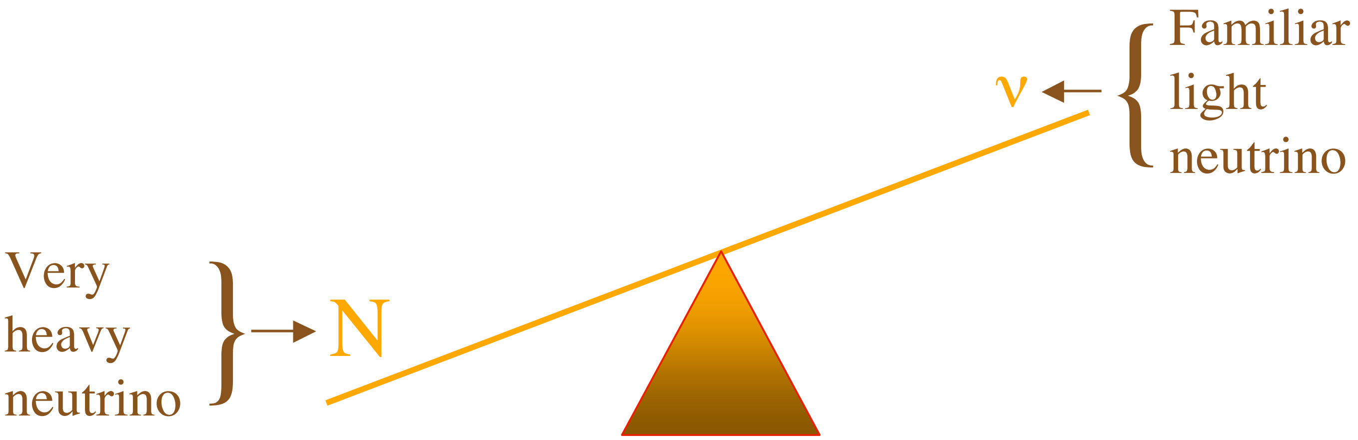
The observed $\cancel{CP}$ in the weak interactions of ***quarks*** cannot explain the *Baryon Asymmetry* of the universe.

Is ***leptonic*** $\cancel{CP}$, through *Leptogenesis*, the origin of the *Baryon Asymmetry* of the universe?

Leptogenesis In 60 Seconds

The most popular theory of why neutrinos are so light is the —

See-Saw Mechanism



The ***very*** heavy neutrinos N would have been made in the hot Big Bang.

The heavy neutrinos N, like the light ones ν, are Majorana particles. Thus, an N can decay into ℓ^- or ℓ^+.

If neutrino oscillation violates CP, then quite likely so does N decay. In the See-Saw, these two CP violations have a common origin.

Then, in the early universe, we would have had different rates for the CP-mirror-image decays –

$$N \rightarrow \ell^- + \ldots \qquad \text{and} \qquad N \rightarrow \ell^+ + \ldots$$

This would have led to unequal numbers of leptons and antileptons (*Leptogenesis*).

Then, Standard-Model ***Sphaleron*** processes would have turned ~ 1/3 of this leptonic asymmetry into a *Baryon Asymmetry*.

We have learned a lot about the neutrinos in the last decade.

What we have learned raises some very interesting questions.

Exciting times lie ahead.